Mark Peletier (Eindhoven)

Large deviations and gradient flows: heat and diffusion

Abstract

Many evolutionary systems described by parabolic partial differential equations can be written as a gradient flow of some functional with respect to some metric. When present, this gradient-flow structure provides both high-level insight into the behaviour of the system, and low-level, practical tools for the analysis of the system and its solutions.

The simplest of parabolic equations is called both the heat equation and the diffusion equation, since it describes both the conduction of heat and the spreading of particles. This equation is known to be a gradient flow of an entropy with respect to the Wasserstein-2 metric, and in earlier work we explained how this gradient-flow structure arises from the fluctuation properties of a system of diffusing particles. This argument shows how the entropy-Wasserstein gradient-flow structure is natural for the diffusion equation.

Heat is not the same as diffusion, however, and in this talk I want to address the question 'what is the **right** gradient-flow structure for the heat equation?' I will explain the arguments and insights for the diffusion case, and then concentrate on the case of heat conduction. Again we will find a natural gradient-flow structure by considering an appropriate stochastic process.