Polynomial Real Root Isolation Using Vincent's Theorem of 1836

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Abstract

In this talk we continue the narrative in a book by the late Jackie Stedall[1] and we first discuss in some detail F.D. Budan's[2] rudimentary attempt of 1807 at isolating the real roots of polynomials. See also `https://en.wikipedia.org/wiki/Budan%27s_theorem`.

Subsequently, we state A.J.H. Vincent's[3] theorem of 1836 for the isolation of the real roots of polynomials using continued fractions and explain the reasons why it was consigned to oblivion for about a century.

In 1948, J.V. Uspensky[4] brought to light and extended Vincent's theorem. However, because he was unaware of the statement of Budan's theorem, Uspensky unwittingly claimed to have developed a "new" method for the isolation of the real roots of polynomials.

A.G. Akritas'[5] Ph.D. thesis of 1978 initiated a long term investigation for a thorough understanding of Vincent's theorem. We discuss the contributions by various people, which led to the development of the method VAS — coined by François Boulier[6] in 2010 — and its acceptance as the fastest method for isolating the real roots of polynomials using exact integer arithmetic.

VAS has been implemented in major computer algebra systems, such as *Mathematica*, maple, sympy, Xcas, to name a few. See also `https://en.wikipedia.org/wiki/Vincent%27s_theorem`.

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[1]From Cardano's great art to Lagrange's reflections: filling a gap in the history of algebra, European Mathematical Society, 2011.

[2]Nouvelle Méthode pour la Résolution des Équations Numériques d' un Degré Quelconque, Courcier, Paris 1807.

[3]Note sur la Résolution des Équations Numériques, Journal de Mathématiques Pures at Appliquées, Vol. 1, 341 – 372, 1836.

[4]Theory of Equations, New York, 1948.

[5]Vincent's Theorem in Algebraic Manipulation, Ph.D. Thesis, NCSU, USA, 1978

[6]Systèmes polynomiaux : que signifie "résoudre"?, Université Lille 1, 2010.