Workshop on Asymptotics of Stochastic Dynamical Systems

Titles and Abstracts

The Workshop is Supporedt by London Mathematical Society and College of Science of Swansea University

- Jianhai Bao (Swansea University): **Asymptotic Log-Harnack Inequality and Ergodicity for Path-Dependent SDEs with Infinite Memory**

  Abstract: The log-Harnack inequality and the gradient estimate for path-dependent SDEs have been extensively investigated, where the length of memory is finite and the diffusion terms depend only on the present state. In this paper, we shall establish the asymptotic log-Harnack inequality for a range of path-dependent SDEs, which allow the length of memory to be infinite and the diffusion terms to be dependent fully on the past history. As a byproduct of the asymptotic log-Harnack inequality, we also derive the asymptotic gradient estimate, which further implies that the semigroup generated by the segment process enjoys the asymptotic strong Feller property. Moreover, via weak Harris' theorem, we discuss the exponential ergodicity under the Wasserstein distance for a wide class of path-dependent SDEs with infinite memory. This is a joint work with Feng-Yu Wang and Chenggui Yuan

- Siyang Cai (University of Strathclyde): **The Application of Stochastic Differential Equation in Epidemic Models**

  Abstract: In this talk I will firstly give an introduction on SDE epidemic models. Based on the previous study on SDE SIS model by A.Gray et al. (2011), we introduce another perturbation to estimate different parameters, with the first perturbation existing. We will then give some results of our new SDE model. We prove that the SDE model has a bounded unique global solution and establish conditions for extinction and persistence of the solution. We also prove that there is a unique stationary distribution if the solution persists. The mean and variance are derived.

- Yongmei Cai (University of Strathclyde): **Stochastic Modelling of Nutrient Dynamics in the Sea Loch**

  Abstract: The deterministic models have been widely applied in the food web dynamics. A natural response to the inevitable variability in the real world would prefer to a stochastic model. It incorporates some representation of randomness. This talk is aimed to set up a stochastic model to describe the dynamics of a nutrient resource in Loch Linnhe, a sea loch on the west coast of Scotland.

- Tomas Caraballo (Universidad de Sevilla): **Random Versus Stochastic Modelling in Epidemiology**

  Abstract: Stochastic and random models are being used to model many realistic phenomena from the real world. In fact, every happening in our world is affected by some randomness or stochasticity. Therefore, it is very important to decide which kind of stochastic or random model is the most appropriate to describe the behavior of the real one in the best way. We will provide some features about this problem in this lecture. Instead of providing a general or abstract theory on this topic, we will consider a random and another stochastic version of an epidemic model previously introduced and analyzed by Kloeden and Kozyakin (2011). In particular, the existence of a random attractor is proved for the random model and the persistence of the disease is analyzed as well. In the stochastic case, we consider some environmental effect on the model, in fact, we assume that one of the coefficients of the system is affected by some stochastic perturbation, and analyze the asymptotic behavior of the solutions. We will emphasize on the comparison between the two different modeling strategies and the usefulness of the theory of random attractors to analyze this and other models from the applied sciences.

- Thomas Cass (Imperial College): **A Stratonovich-to-Skorohod Conversion Formula for Integrals with respect to Gaussian Rough Paths**

  Abstract: Lyons' theory of rough paths allows us to solve stochastic differential equations driven by a Gaussian processes X of finite p-variation. The rough integral of the solutions against X again exists. We show that the solution also belong to the domain of the divergence operator of the Malliavin derivative, so that the 'Skorohod integral' of the solution with respect to X can also be defined. The latter operation has some properties in common with the Ito integral, and a natural question is to find a closed-form conversion formula between this rough integral and its Malliavin divergence. This is particularly useful in applications, where often one wants to compute the (conditional) expectation of the rough integral. In the case of Brownian motion our formula reduces to the classical Stratonovich-to-Ito conversion formula. There is an interesting difference between the formulae obtained in the cases $2 \leq p < 3$ and $3 \leq p < 4$, and we consider the reasons for this difference. We elaborate on the connection with previous work in which the integrand is generally assumed to be the gradient of a smooth function of X_t; we show that our formula can recover these results as special cases.

- Dan Crisan (Imperial College): **Large Time Asymptotics for Diffusion Semigroups Gradient Bounds**

  Abstract: I will present some results related to large time asymptotics for the gradients of the diffusion semigroup. The result complement the work of Kusuoka and Stroock on (possibly) degenerate diffusion semigroups. More precisely, Kusuoka and Stroock showed in the eighties that diffusion semigroups smooth out initial conditions under the so-called UFG condition, which is weaker than the uniform Hormander condition, the smoothing effect taking place only in certain directions.

Moreover, they deduced sharp small time asymptotic bounds for the derivatives of the semigroup in the directions where smoothing occurs. In my talk I will discuss the large time asymptotics for the gradients of the diffusion semigroup in the same set of directions and under the same UFG condition. In particular, I present conditions under which the derivatives of the diffusion semigroup in the smoothing directions decay exponentially in time.

This is a joint work with Michela Ottobre (Heriot Watt) and is based on the paper D Crisan, M Ottobre, Pointwise gradient bounds for degenerate semigroups (of UFG type), Proc. R. Soc. A 472 (2195), 2016.

- David Elworthy (University of Warwick): **Formulae and Asymptotics for Higher Derivatives of Heat Semi-groups: Some Examples and Theory**

  Abstract: I will aim to show that the question of the behaviour in time of spatial derivatives of heat semi-groups can be both more interesting than expected and more complicated.

- Chunrong Feng (University of Loughborough): **Random Periodic Solutions of SDEs**

  Abstract: In my talk, I will talk the existence of random periodic solutions (r.p.s) for SDEs with additive and linear noises, and discuss a few of examples to illustrate the theories. This is a joint work with Huaizhong Zhao.

- Dmitri Finkelshtein (Swansea University): **n-fold Convolutions of Densities with Regular Heavy Tails**

  Abstract: We study the tail asymptotic of sub-exponential probability densities on the whole real line. Namely, we show that the n-fold convolution of a sub-exponential probability density on the real line is asymptotically equivalent to this density times n. We prove Kesten's bound, which gives a uniform in n estimate of the n-fold convolution by the tail of the density. We also introduce a class of regular sub-exponential functions and use it to find an analogue of Kesten's bound for functions on $\mathbb{R}^d$. The results are applied for the study of the fundamental solution to a nonlocal heat-equation. The talk is based on joint preprints with Pasha Tkachov (Bielefeld).

- Martin Grothaus (Technische Universitat Kaiserslautern): **Weak Poincarè Inequalities for Convergence Rate of Degenerate Diffusion Processes**

  Abstract: For a contraction C_0-semigroup on a separable Hilbert space, the decay rate is estimated by using the weak Poincarè inequalities for the symmetric and anti-symmetric part of the generator. As applications, non-exponential convergence rate is characterized for a class of degenerate diffusion processes, so that the study of hypocoercivity is extended. Concrete examples are presented. This is a joint with Feng-Yu Wang.

- Xianping Guo (Sun Yat-Sen Uinversity): **A Probability Criterion for Zero-Sum Stochastic Games**

Abstract: In this talk, we introduce a probability criterion for two-person zero-sum stochastic games, and focus on the probability that the payoff before the first passage time to some target state set exceeds a level formulated by both players, which shows a so called security probability for player 1, and the risk probability for player 2. For the game model based on discrete-time Markov chains, under a suitable condition on the game's primitive data, we establish the Shapley equation, from which the existence of the value of the game and a pair of optimal policies with the maximum security for player 1 and the minimum risk for player 2, is ensured. We also provide a recursive way of computing (or at least approximating) the value of the game. At last, we applied our mail results to an inventory system.

- Des Higham (University of Strathclyde): **Computational Efficiency of Continuous-time Markov Chain Simulations**

  Abstarct: I will analyze and compare the computational complexity of different simulation strategies for continuous time Markov chains. I consider the task of approximating the expected value of some functional of the state of the system over a compact time interval. This task is a bottleneck in many large-scale computations arising in biochemical kinetics and cell biology. In this context, the terms 'Gillespie's method', 'The Stochastic Simulation Algorithm' and 'The Next Reaction Method' are widely used to describe exact simulation methods. For example, Google Scholar records more than 7,500 citations to Gillespie's seminal 1977 paper. I will look at the use of standard Monte Carlo when samples are produced by exact simulation and by approximation with tau-leaping or an Euler-Maruyama discretization of a stochastic differential equation (diffusion approximation). In particular, I will point out some possible pitfalls when computational complexity is analysed. Appropriate modifications of recently proposed multilevel Monte Carlo algorithms will then be studied for the tau-leaping and Euler-Maruyama approaches. I will focus on a parameterization of the problem that, in the mass action chemical kinetics setting, corresponds to the classical system size scaling.

  Co-authors: David F. Anderson (University of Wisconsin-Madison), Yu Sun (University of Wisconsin-Madison)

- Xue-Mei Li (University of Warwick): **Generalised, Semi-Classical and Classical Brownian Bridges**

  Abstract: We study stochastic processes with fixed initial and terminal value on $[0,1]$. These were motivated with heat kernel estimates, the probability distributions of the bridges are candidate for the analysis on the space of pinned paths, by which we mean L_2 analysis in the frame work of Malliavin calculus. One question we ask is whether an integration by parts formula holds (until the terminal value) and whether a Poincare inequality hold? How these bridge measures compare? Are they equivalent? Do they have the same Cameron-Martin space in case the manifold is an Euclidean space and the drift is linear. For example, are the probability distribution of the Brownian motion with drift $-cx/(1-t)$, where c is a constant, equivalent to that of the classical Brownian bridge measure. If so, do their Cameron-Martin spaces coincide?

- Kai Liu (University of Liverpool): **Sensitivity to Small Delays of Pathwise Stability for Stochastic Retarded Evolution Equations**

  Abstract: In this talk, we shall study the pathwise exponential stability property for a class of stochastic functional evolution equations with delays, possibly, in the highest-order derivatives driven by multiplicative noise. Different from the usual methods that ones first consider the moment exponential stability and then proceed to investigate, based on the established moment stability, the pathwise exponential stability of the systems, we develop a direct approach to attack this problem. As a consequence, we can show that some systems, which are not exponential momently stable, have the exponential stability not sensitive to small delays in the almost sure sense.

- Eugene Lytvynov (Swansea University): **Determinantal Point Processes as Particle Density of Quasi-free Representations of the CAR**

  Abstract: We will first discuss the particle density for the gauge-invariant quasi-free states on the algebra of the canonical anticommutation relations (CAR algebra). This particle density is a family of commuting self-adjoint operators and its spectral measure is shown to be a determinantal point process with a Hermitian correlation kernel. We will then perform a particle-hole duality at the level of the creation and annihilation operators. This will imply a new quasi-free state on the CAR algebra. We will discuss how the corresponding particle density should lead to a determinantal point process with a J-Hermitian correlation kernel.

- Yonghua Mao (Beijing Normal University): **Dirichlet Principle for Asymmetric Markov Chains**

  Abstract: We give a new kind of variational formulas for the hitting time of non-reversible Markov chain on countable state space. Some comparison theorems are obtained for the non-reversible Markov chain and its corresponding reversible one. As an application, we prove a stronger version of a conjecture in Aldous-Fill's book.

- Hiroyuki Matsumoto (Aoyama Gakuin University): **Quasiconformal Mappings and Two-dimensional Diffusion Processes**

  Abstract: We show that a class of two-dimensional symmetric diffusion processes, including those generated by uniformly elliptic Laplace-Beltrami operators, can be locally constructed from time-changed processes of two-dimensional standard Brownian motion. This is the property which is well known for one-dimensional diffusion processes. We use quasiconformal mappings in place of the scale functions.

- Bernt Oksendal (University of Oslo): **Stochastic Control of Memory Mean-Field Processes**

  Abstract: We study stochastic control problems of memory mean-field processes. By a memory mean-field process we mean the solution $X(\cdot)$ of a stochastic mean-field equation involving not just the current state $X(t)$ and its law $\mathcal{L}(X(t))$ at time t, but also their paths $X(s)_{t\delta<s<t}$ and $\mathcal{L}(X(s)_{t\delta<s<t}$ respectively, for some constant $\delta > 0$.

(i) We consider the space $\mathcal{M}$ of measures on $\mathbb{R}$ with the norm $\|\cdot\|_{\mathcal{M}}$ introduced by Agram and Oksendal, and prove the existence and uniqueness of solutions of memory mean-field stochastic functional differential equations.

(ii) We prove two stochastic maximum principles, one sufficient (a verification theorem) and one necessary, both under partial information. The corresponding equations for the adjoint variables are a pair of (time-) advanced backward stochastic differential equations involving Fréchet derivatives on path space.

(iii) As an application of our methods, we solve a memory mean-variance problem as well as a linear-quadratic problem of a memory mean-field process.

This is a joint work with Nacira Agram.

- Zhongmin Qian (University of Oxford): **BSDE Method and Semi-linear Parabolic Equations on Fractals**

  Abstract: This talk is based on a joint work with Xuan Liu. In this talk, I will talk about the universal aspects of backward stochastic differential equations in the study of non-linear parabolic equations. A study of some non-linear parabolic equations subject to the Dirichlet boundary condition on the Sierpinski gasket by means of backward stochastic differential equations will be presented. This is kind of completely different parabolic equations and in general two mutually singular measures have to be involved. The existence and uniqueness of solutions of two classes of backward stochastic differential equations driven by Brownian motion on the Sierpinski gasket are established, which give the meaning of solutions to non-linear equations on fractals.

- Panpan Ren (Swansea University): **Matrix-valued SDEs for Foreign Exchange Markets**

  Abstract: We discuss the mathematical settings of the foreign exchange markets. We use matrix-valued time series to model foreign exchange market. We aim to establish an $\mathbb{R}^n \otimes \mathbb{R}^n$-valued SDE motivated by the foregin exchange market. Finally, we establish on-line (universal) portfolio selection and to analyse the possible optimal algorithm. In our forthcoming work, we plan to exam our algorithm with the help of deep machine learning skills. This is a joint work with Jiang-Lun Wu.

- Michael Röckner (Bielefeld University): **Quasi-Linear (Stochastic) Partial Differential Equations with Time-Fractional Derivatives**

  Abstract: In this paper we develop a method to solve evolution equations on Gelfand triples with time-fractional derivative based on monotonicity techniques. Applications include deterministic and stochastic quasi-linear partial differential equations with time-fractional derivatives, including time-fractional (stochastic) porous media equations (including the case where the Laplace operator is also fractional) and p-Laplace equations as special cases (arXiv:1708.05649).

  This is a joint work with Wei Liu and José Luis da Silva. AMS Subject Classification: 60H15, 35K55, 35Q30, 34G20

Keywords: fractional derivative; monotone; pseudo-monotone; porous media equation; p-Laplace equation.

- Sotirios Sabanis (University of Edinburgh): **Numerics for Nonlinear SDEs, Recursive Estimators and MCMC Algorithms**

  Abstract: Some recent advances on numerics for SDEs with superlinear coefficients (joint work with Ying Zhang) and on recursive estimators with discontinuity in the parameters will be discussed (joint work with Huy N. Chau, Chaman Kumar, Miklos Rasonyi). Links with MCMC algorithms will be highlighted.

- Jinghai Shao (Tianjin University): **Stability and Recurrence of Regime-switching Diffusion Processes**

  Abstract: In this talk we shall introduce some results on the stability and recurrence of regime-switching diffusion processes with Markovian swithing or state-dependent switching. Via constructing a coupling process, we also present a result on the existence of invariant probability measure for the state-dependent regime-switching processes.

- Agnes Sulem (Center de Recherche Inria de Paris): **Optimal Stopping Games with Nonlinear Expectations**

  Abstract: We present some recent results for optimal stopping and Dynkin stochastic games under nonlinear expectation and study pricing and hedging issues for game options in an imperfect financial market with default. Imperfections in the market are taken into account via the nonlinearity of the wealth dynamics. In this framework, the pricing system is expressed as a nonlinear $\mathcal{E}^g$-expectation/evaluation induced by a nonlinear Backward Stochastic Differential Equation (BSDE) with jump with driver g. We prove that the superhedging price of a game option coincides with the value function of a corresponding *generalized* Dynkin game expressed in terms of the $\mathcal{E}^g$-evaluation. Th proofs of these results are based on links between generalized Dunkin games and doubly reflected BSDEs. We also address the case of ambiguity on the model, - for example an ambiguity on the default probability - and characterize the superhedging price of a game option as the value function of a *mixed generalized* Dynkin game.

- James Thompson (Luxembourg University): **Derivative and Divergence Formulae for Diffusion Semigroups**

  Abstract: Suppose V is a vector field on a smooth manifold and P_t a semigroup generated by an elliptic operator. Then Bismut's formula provides a probabilistic formula for the derivative $V(P_t f)$, not involving the derivatives of f. Bismut proved it using techniques from Malliavin calculus, before a more elementary approach based on martingales was developed by Elworthy and Li and later by Thalmaier. In this talk, we prove analogous formulae for the derivative $P_t(V(f))$. For non-symmetric generators, such formulae correspond to the derivative of the heat kernel in the forward variable. As an application, our formulae can be used to derive various shift-Harnack inequalities. This is joint work with Anton Thalmaier.

- Bo Wu (Fudan University): **Stochastic Heat Equations Taking Values in a Riemannian Manifold**

  Abstract: In this paper, we prove the existence of martingale solutions to the stochastic heat equation in a Riemannian manifold by using suitable Dirichlet form on the Riemannian path and loop space. Moreover, we also obtain the Log-Sobolev inequality for the associated process. In addition, we present some characterizations for the upper and low bounds of the Ricci curvature by functional inequalities of various associated Dirichlet forms. This is a joint work with M. Rockner, R.-C. Zhu and X.-C. Zhu.

- Guangyu Xi (University of Oxford): **Aronson Type Estimate for Diffusion Processes with Singular Divergence-free Drift**

  Abstract: Diffusion processes are closely related to PDEs in the sense that the transition probability of Diffusion processes is the fundamental solution to the corresponding parabolic equation. In this talk, I will present the Aronson type estimate for a family of Parabolic equations with singular divergence-free drift in BMO^{-1} or Lebesgue spaces. Further, regularity of week solutions can be derived from such Aronson type estimates. This estimate also guarantees the existence and uniqueness of the Diffusion processes. All these results relies critically on the divergence-free condition, which was motivated for applications in fluid dynamics. This talk is based on joint works with Prof Zhongmin Qian.

- Huaizhong Zhao (University of Loughborough): **Ergodicity of Periodic Measures**

  Abstract: In this talk, I will discuss random periodic processes, periodic measures and Poincare sections of Markovian stochastic dynamical systems. I will prove if the periodic measure is PS-mixing and has minimum period, then infinitesimal generator of the Markov semigroup has infinite number of equally placed pure imaginary eigenvalues. Conversely, if the infinitesimal generator of the Markov semigroup has infinite number of equally placed pure imaginary eigenvalues, then the periodic measure has positive minimum period and is PS-ergodic. Thus it is ergodic and the Poincare section and dynamics are determined by the level sets of the eigenfunction corresponding to the imaginary eigenvalue which is most close to 0. This talk is based on a joint work with Chunrong Feng.