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Semi-classical states for the Choquard equation

Abstract

Joint work with Vitaly Moroz (Swansea University, United Kingdom).

The Choquard equations reads as

$$-\varepsilon^2 \Delta u_\varepsilon + V u_\varepsilon = \varepsilon^{-\alpha}\big(I_\alpha * |u_\varepsilon|^p\big)|u_\varepsilon|^{p-2}u_\varepsilon \quad \text{in } \mathbb{R}^N,$$

where $N \geq 1$, $\alpha \in (0, N)$, $I_\alpha(x) = A_\alpha/|x|^{N-\alpha}$ is the Riesz potential and $\varepsilon > 0$ is a small parameter. I will present results on the existence of solutions in the semi-classical régime $\varepsilon \to 0$. If the external potential $V \in C(\mathbb{R}^N; [0, \infty))$ has a local minimum and $p \in [2, (N+\alpha)/(N-2)_+)$ then for all small $\varepsilon > 0$ the problem has a family of solutions concentrating to the local minimum of V provided that: either $p > 1 + \max(\alpha, \frac{\alpha+2}{2})/(N-2)_+$, or $p > 2$ and $\liminf_{|x|\to\infty} V(x)|x|^2 > 0$, or $p = 2$ and $\inf_{x\in\mathbb{R}^N} V(x)(1+|x|^{N-\alpha}) > 0$. I will explain why the assumptions on the decay of V and admissible range of $p \geq 2$ are optimal. I will give the main ideas on the proof which is based on variational methods and required the development of an adequate penalization technique for nonlocal problems.