VIS2024

Localized Evaluation for Constructing Discrete Vector Fields

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Algorithm Properties

- Input: Piecewise Linear Vector Field
- Output: Discrete Vector Field
- Similar results to FastCVT [Reininghaus et al. 2011] Limitation: Slow

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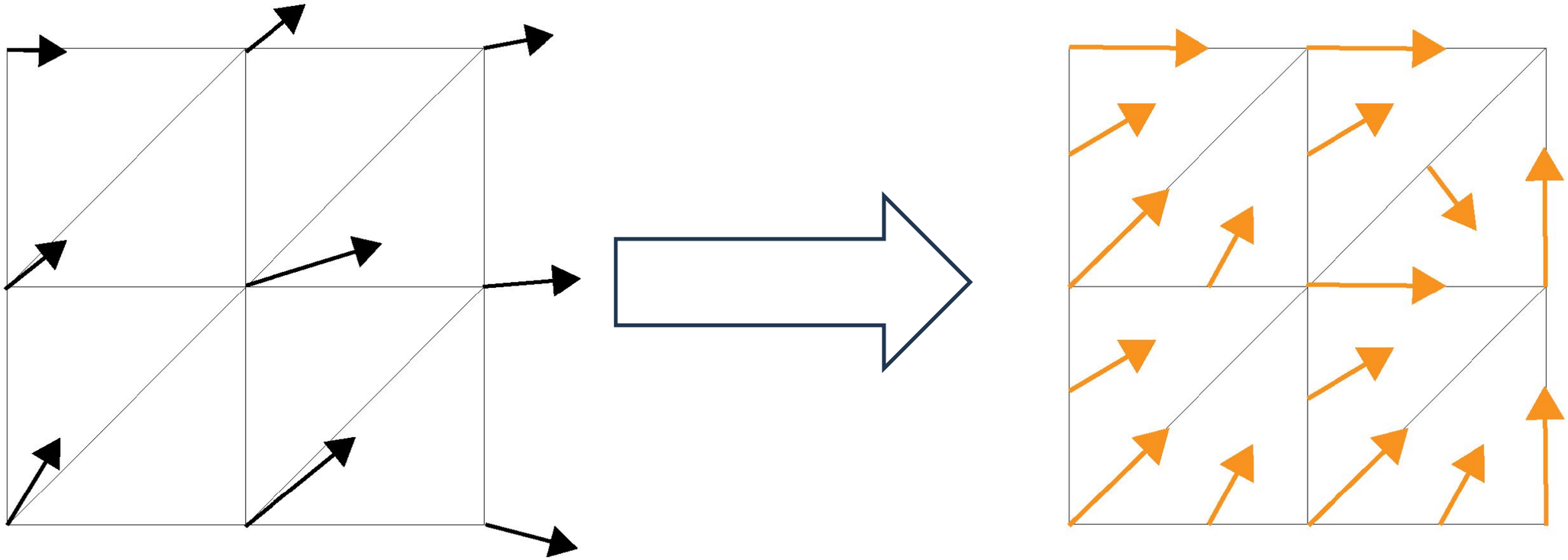

Algorithm Properties

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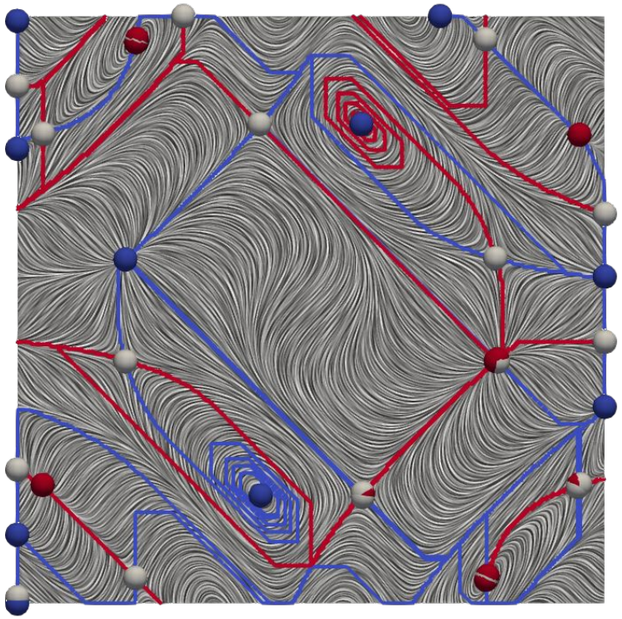
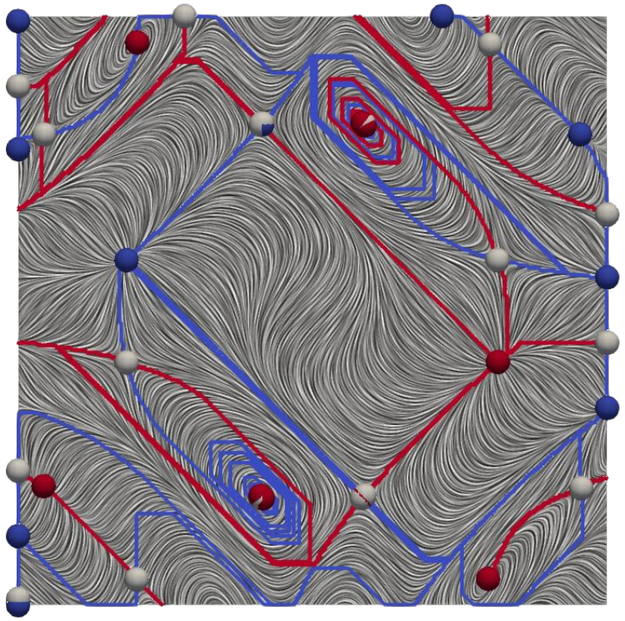
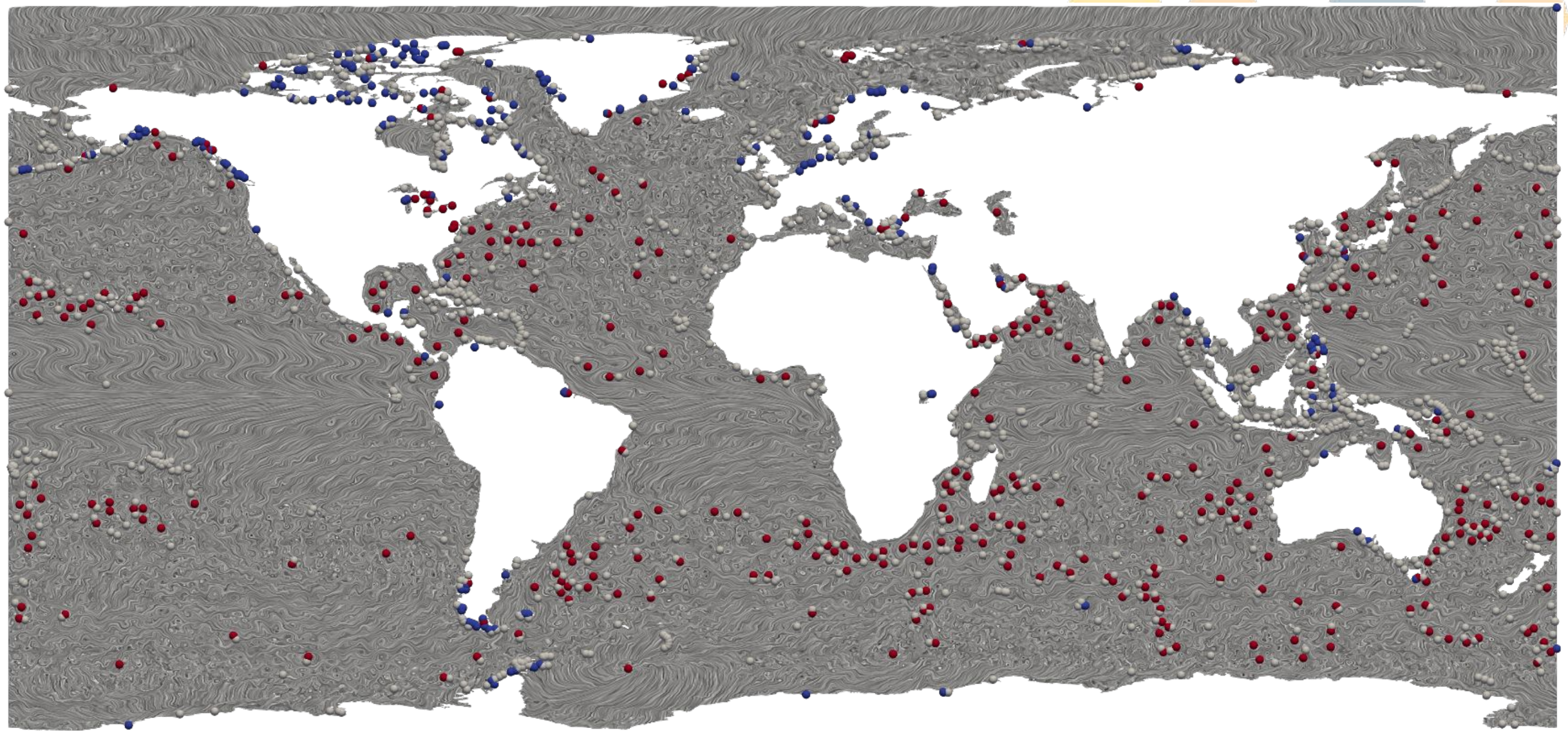


FastCVT(25 s)



Our Algorithm(OSA) (1.6 s)

Fast Computation (48 Million Cells =4 min.)



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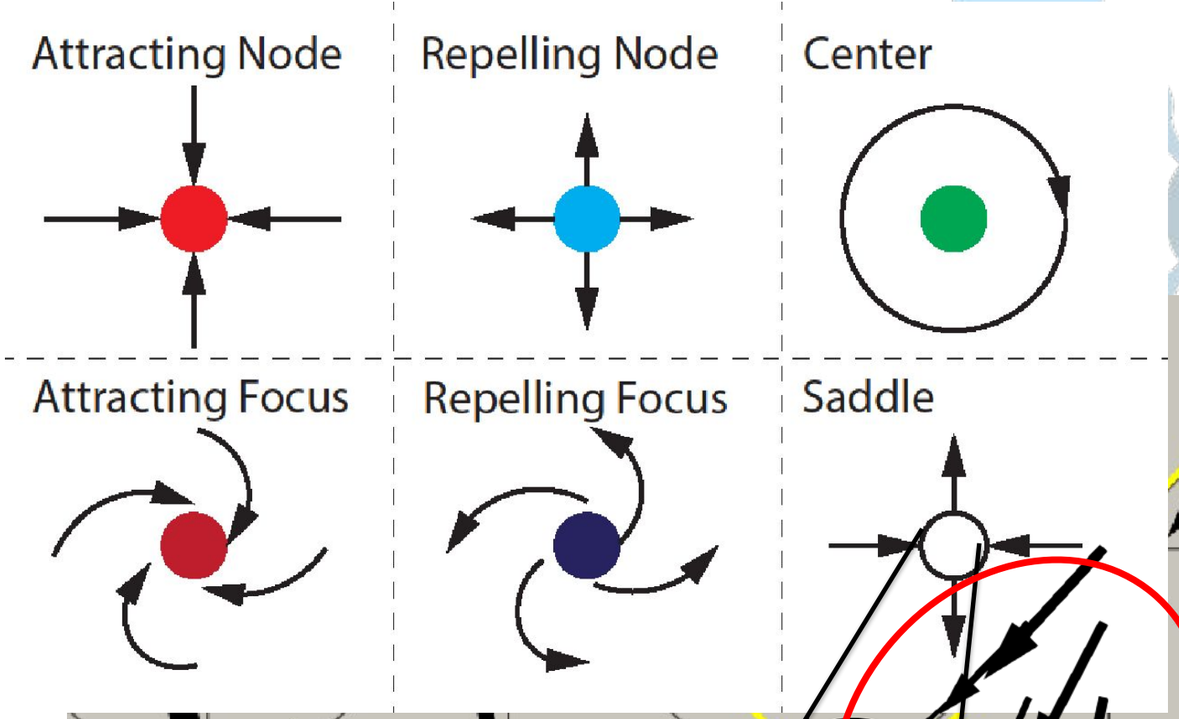
CMIP6 Earth Systems Grid Federation, monthly average of the ocean velocity: Jan. 2014

Input: 2D Piecewise Linear Vector Fields

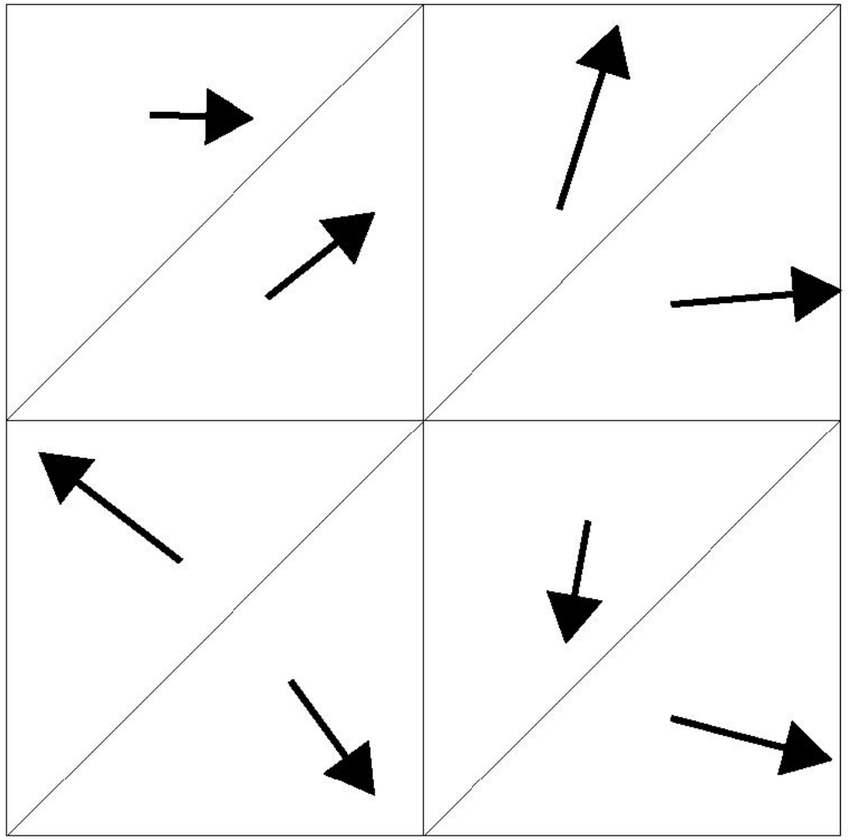
- Triangulated Mesh
- Vectors typically represent flow
- Piecewise Linear Interpolation
- Feature Analysis
  - Critical Points
  - Streamlines(Separatrices)
- Issue when integrating over a continuous range
- Alternative: Discrete



Discrete Representations of Vector Fields

- Piecewise constant

[Tsubone et al. 2000]
Discrete Flow

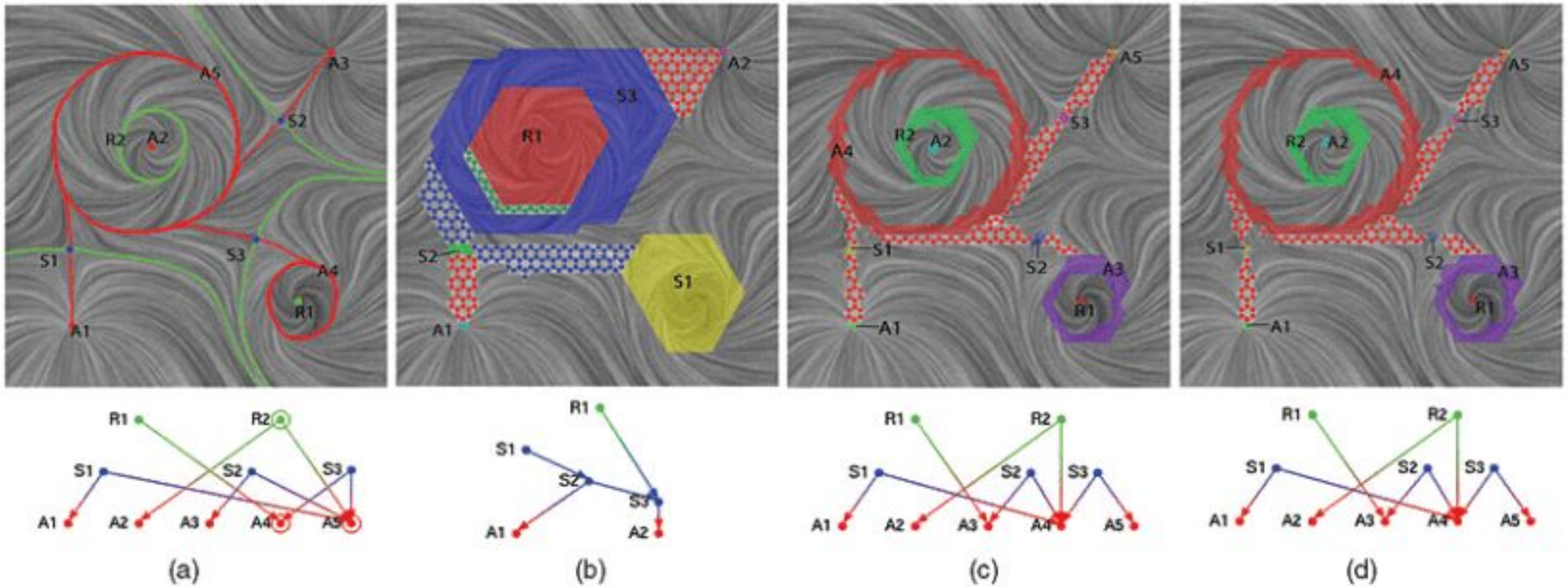


Discrete Representations of Vector Fields

- Morse Decomposition

[Chen et al.]

Discrete Regions



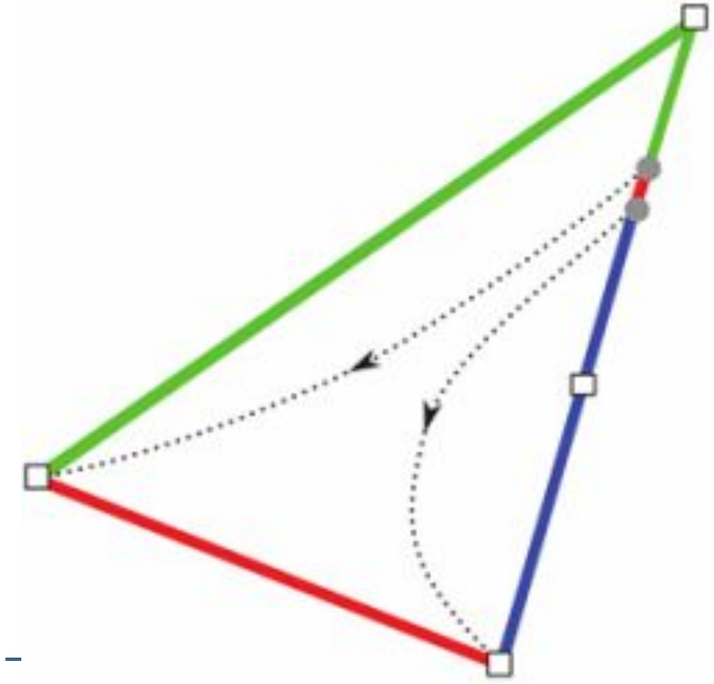
[Chen et al. 2008]

Discrete Representations of Vector Fields

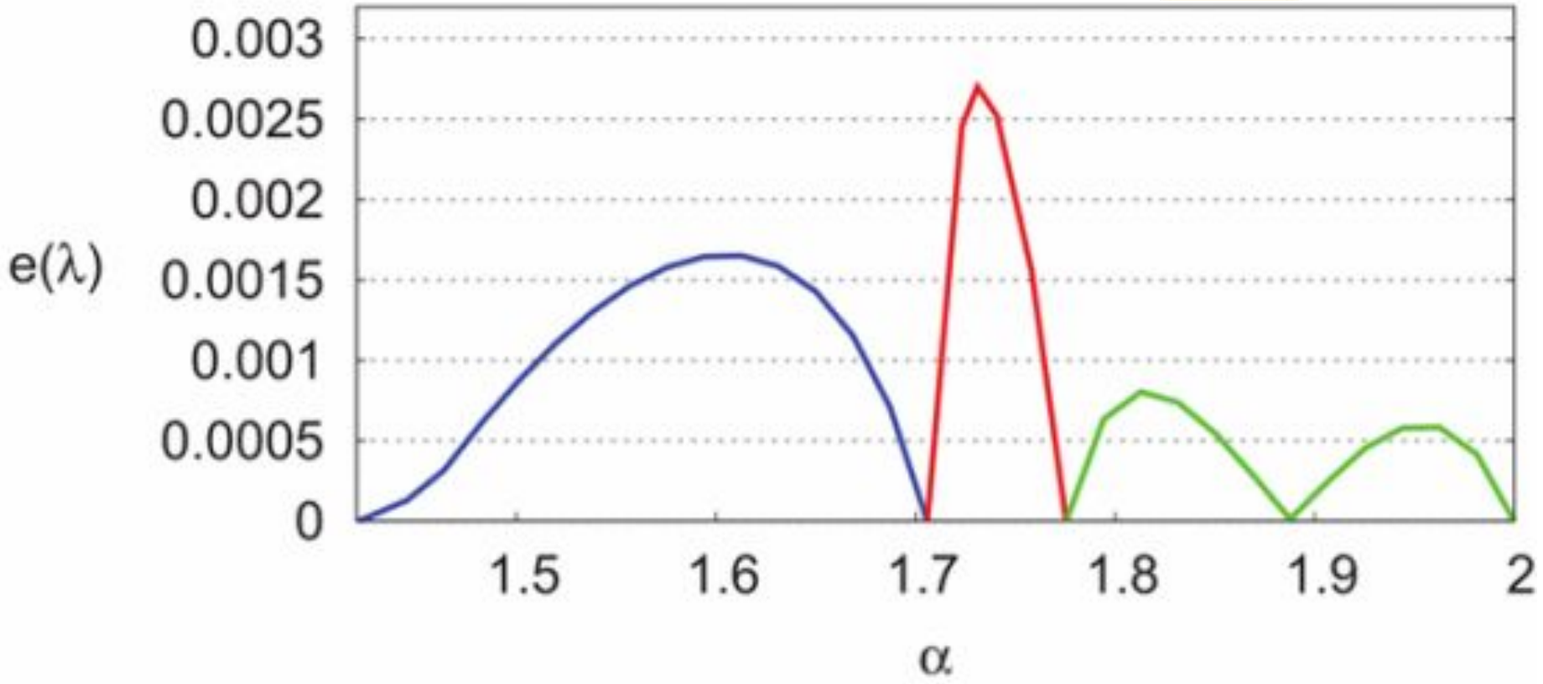
- Edge Maps

[Bhatia et al. 2011]

Discrete Segments Along Triangle Edges



(a)

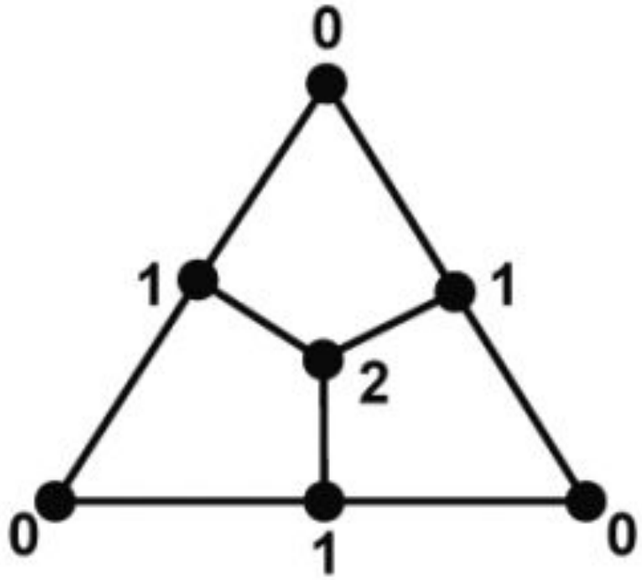


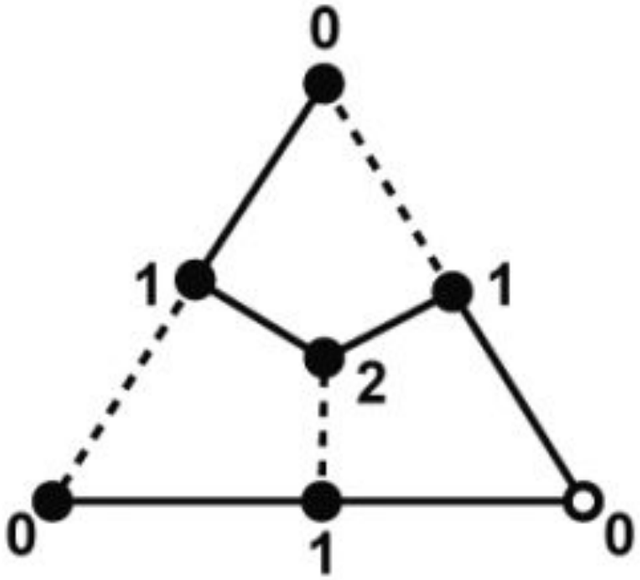
[Bhatia et al. 2011]

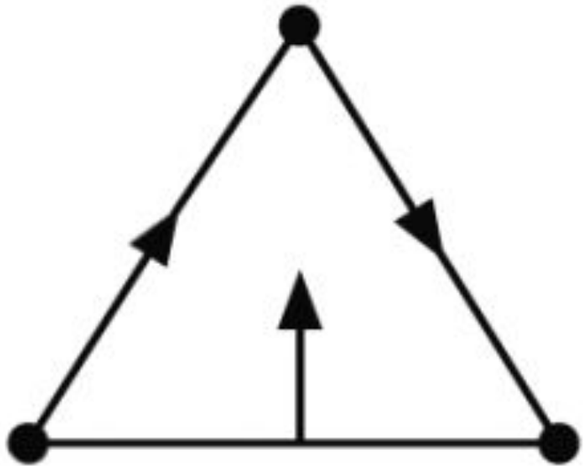
(b)

Discrete Representations of Vector Fields

Fast Combinatorial Vector Field (Discrete Field) [Reininghaus et al. 2011]
Discrete Vector Field



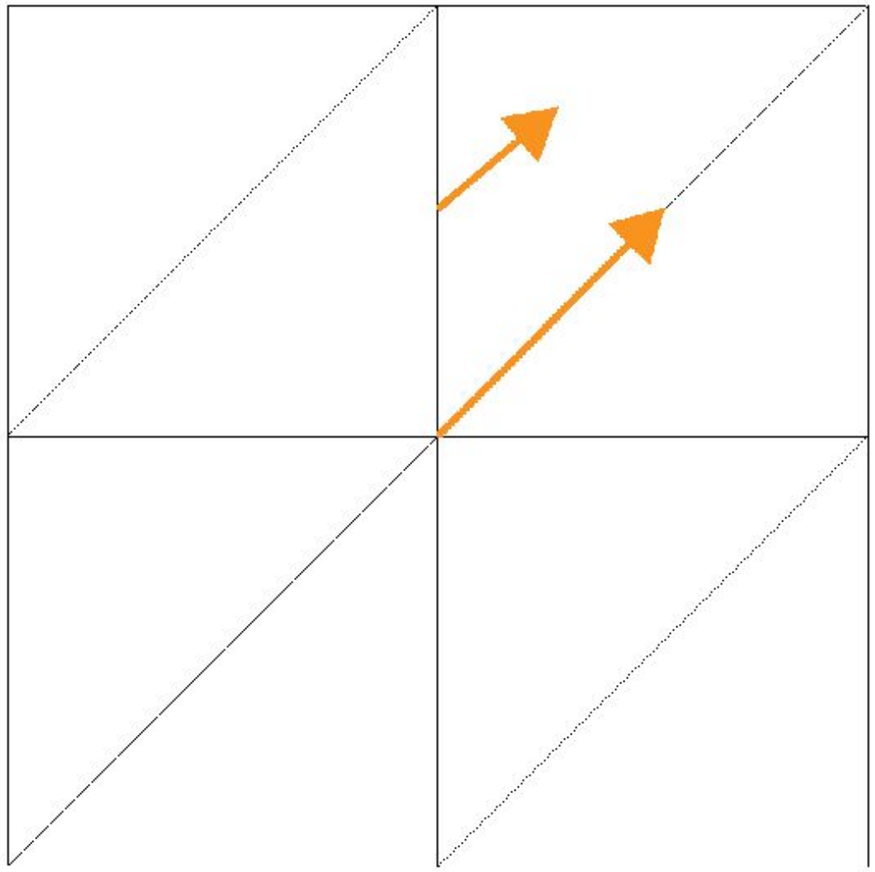




Explaining The Representation: Discrete Vector Field

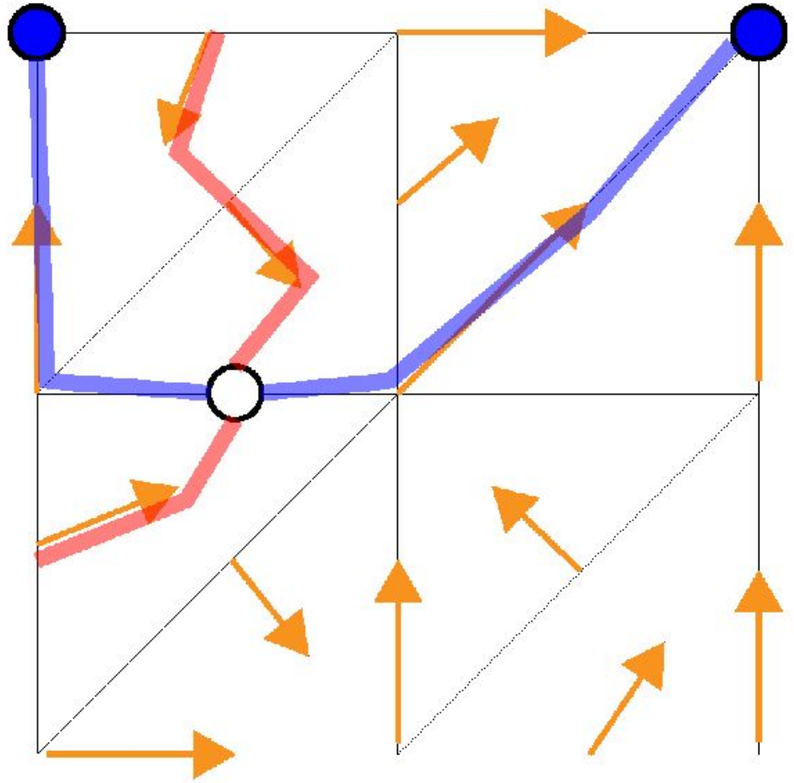
Discrete Morse Theory [Forman 2002]

- Directly Adjacent Simplex Pairs
  - Vertex-Edge
  - Edge-Triangle
- Directionality
  - Lower to Higher Dimension



Discrete Analogues

- Critical Point == Critical Simplex (unpaired)
- Streamline == *V*-path (follow alternating sequence of same type pairs)



How to Construct these Discrete Vectors?

Inspiration from Scalar Field Topology

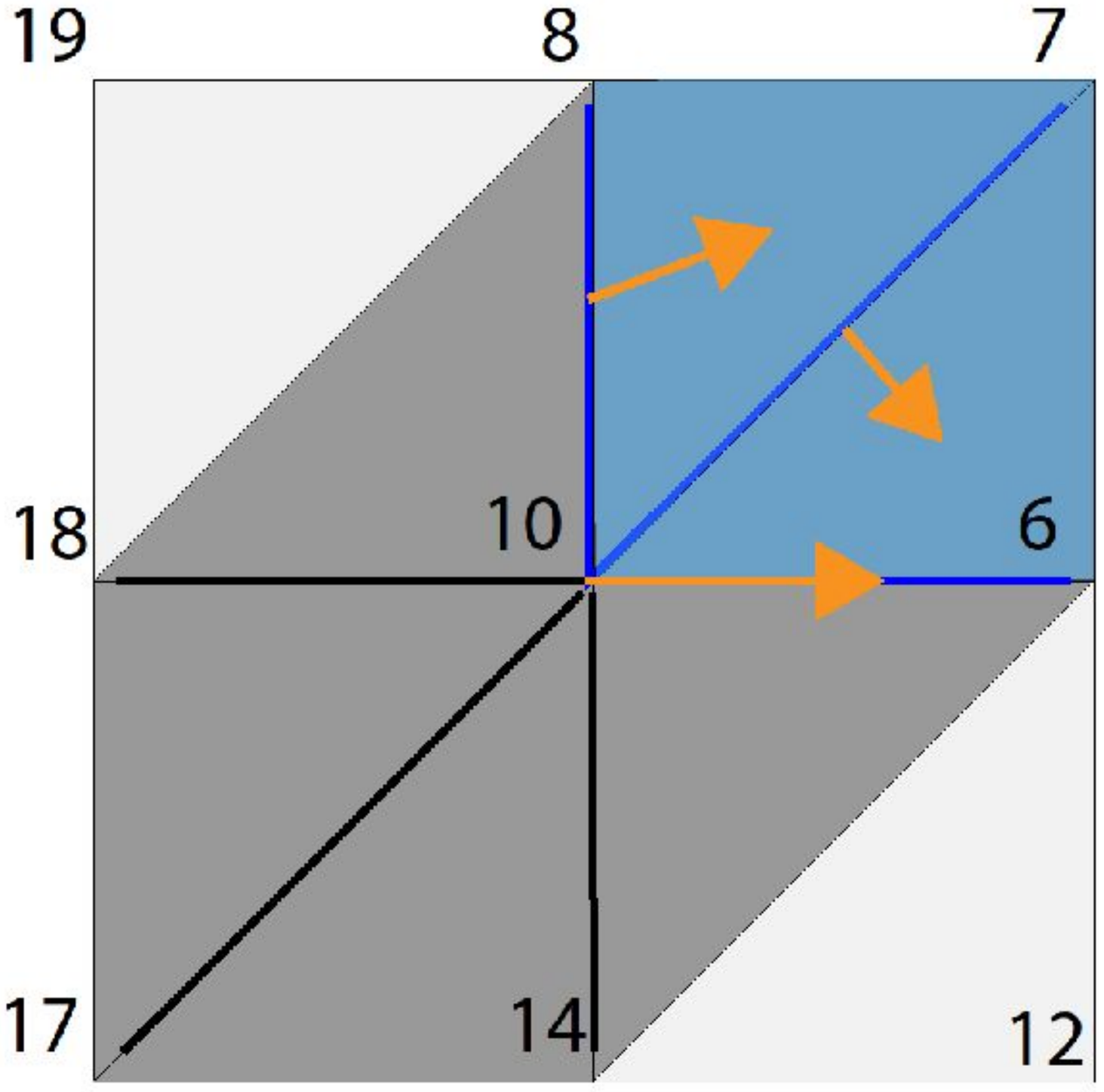
- Lower Star Processing

[Robins et al. *IEEE TPAMI,* 2011]

Star = Neighborhood around a vertex

Lower Star = Only the lower part of the neighborhood

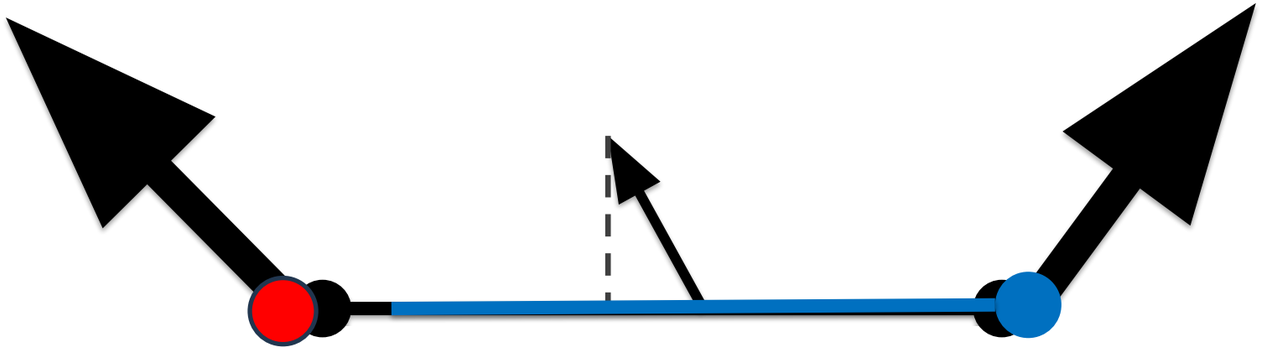
Discrete Vectors chosen in Lower Star because of uniqueness



Defining Outward Star

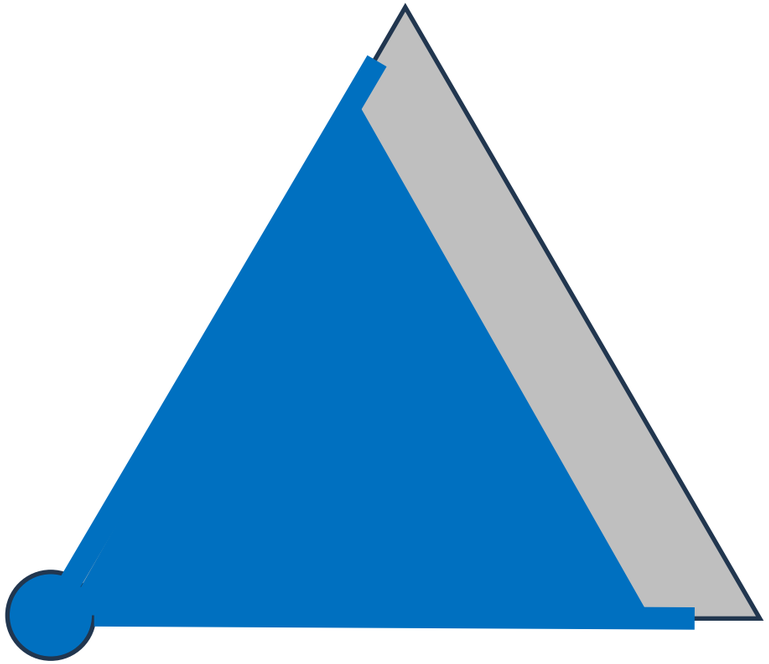
Outward Star Defined for edges

- Outward Flow along an edge
  - Averaged Vector Dotted with Edge Direction (Projects the flow to the edge)

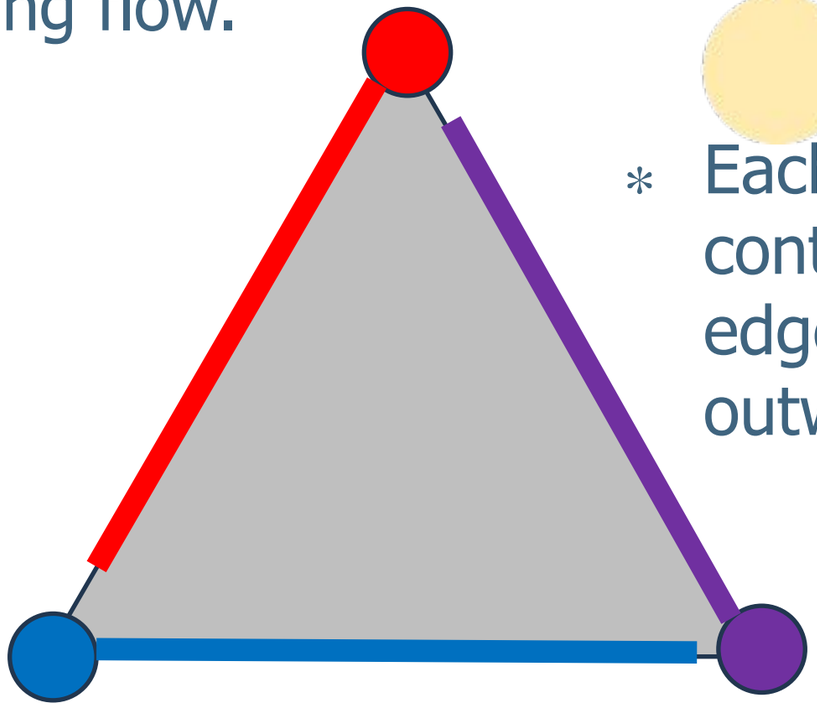


Propagate Outward Flow to Triangles

- Only add triangle to outward star if both edges agree



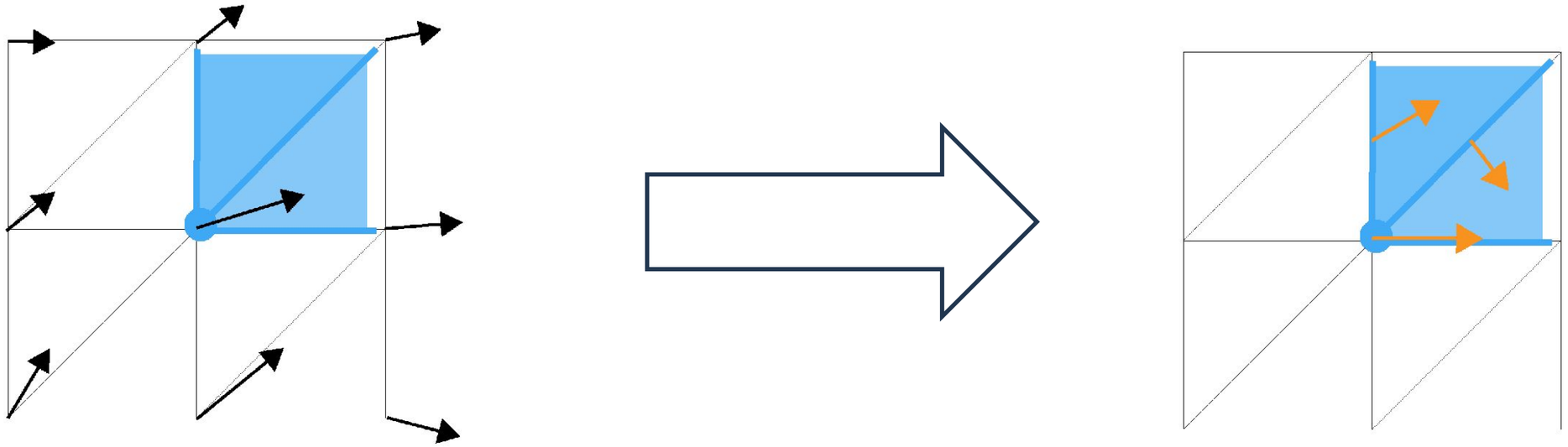
- It’s possible a triangle is not added to an outward star with rotating flow.



- Each vertex contains one edge in their outward star

Outward Star Processing

- Use process on each outward star
  - Minor Modifications[1] to Robins' algorithm to generate discrete vectors

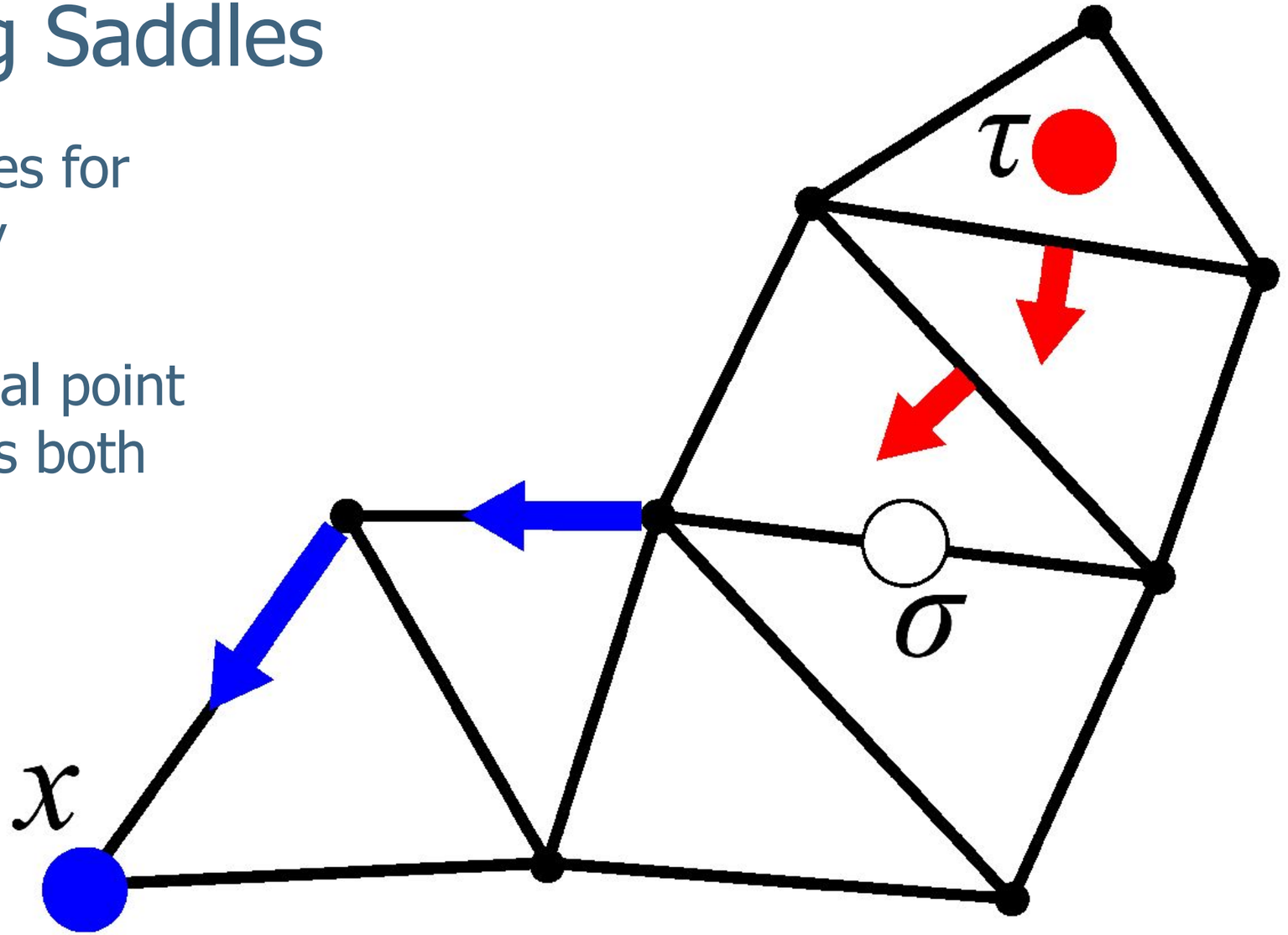


[1]Using outward flow strength to determine 'maximal' edge and the order of processing the simplices

Simplification

Simplification Using Saddles

- Trace saddles' separatrices for possible paths to simplify (uphill and downhill)
- When connected to critical point flipping the path removes both critical points



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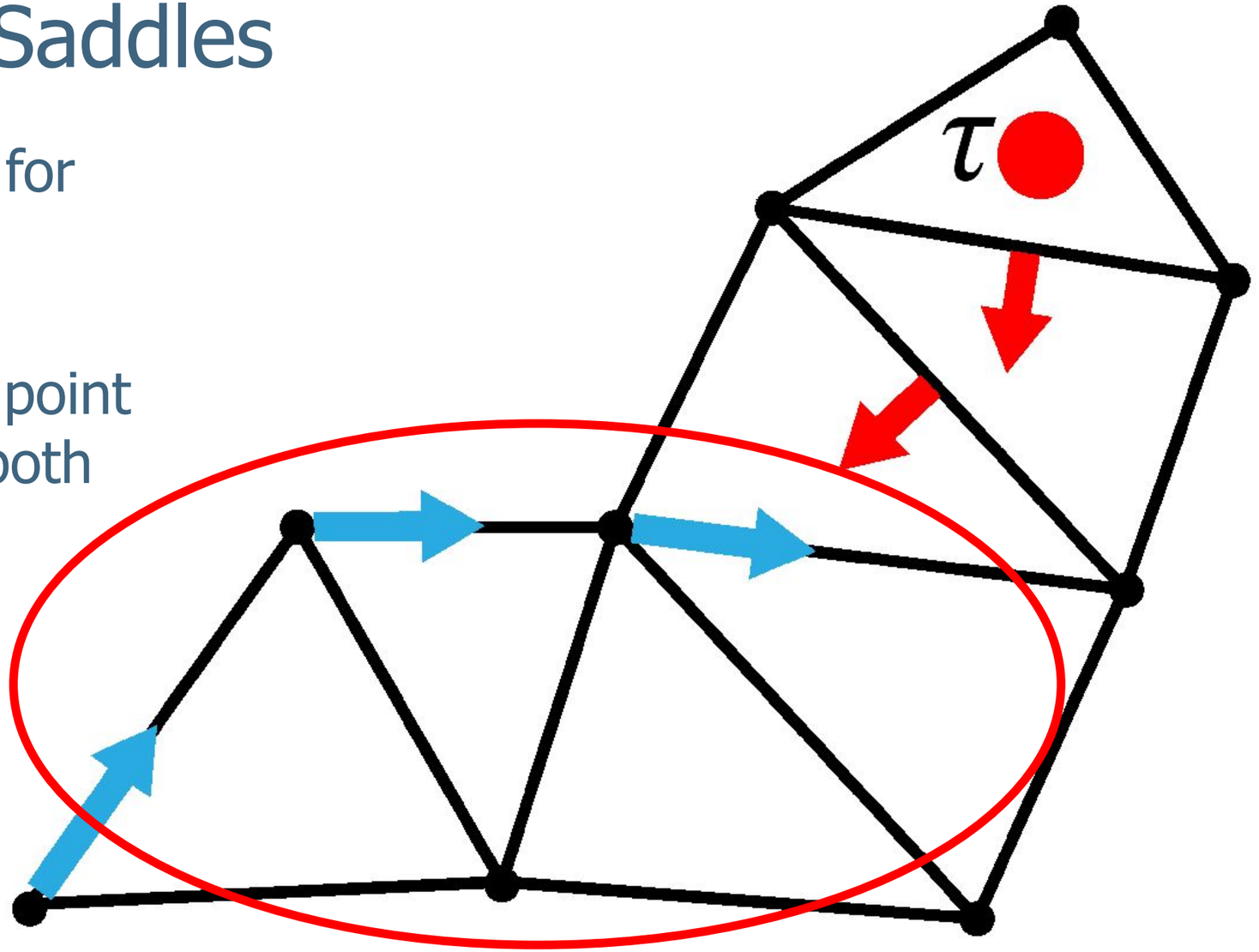
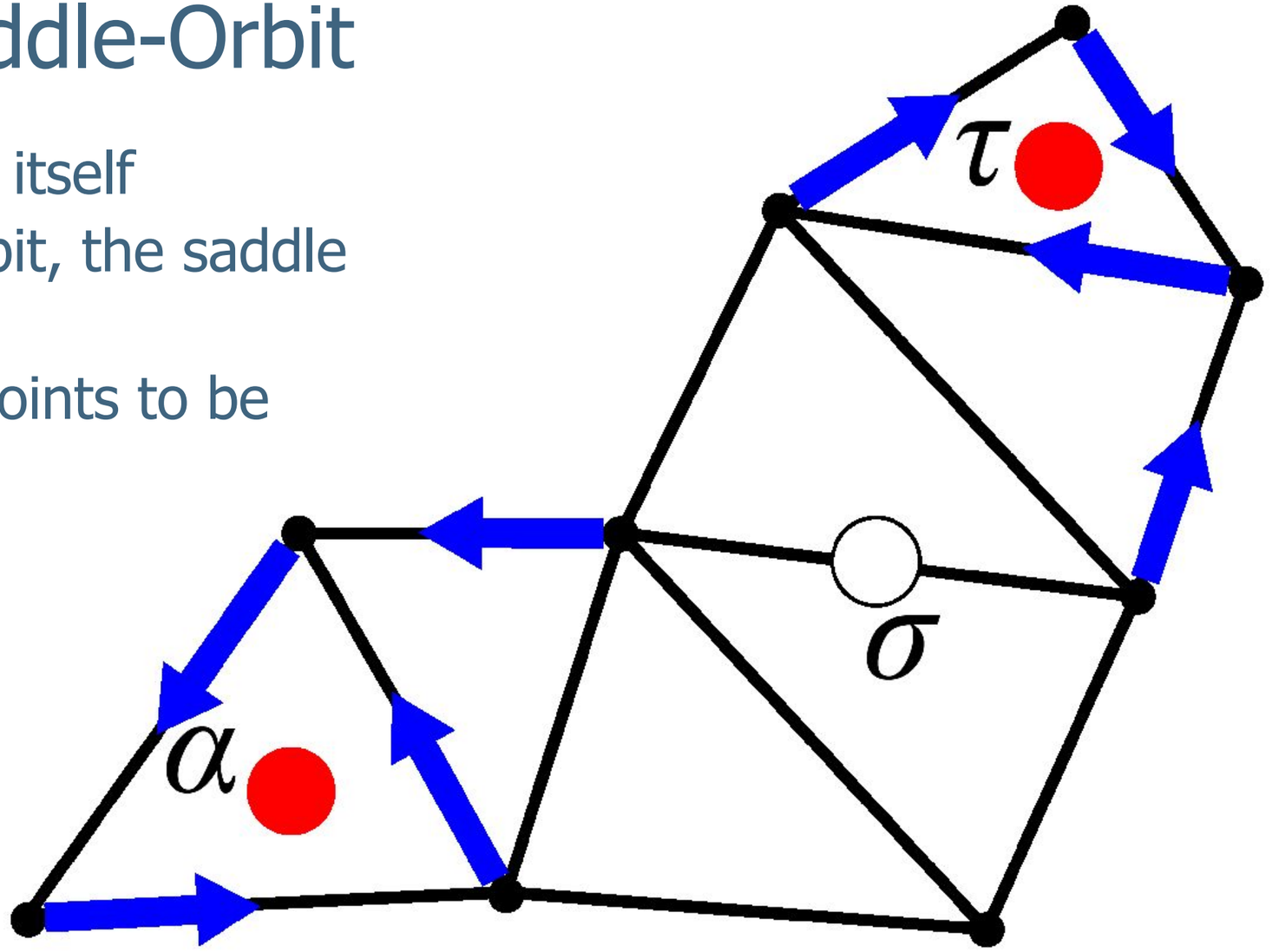


Simplification On Saddle-Orbit

- Saddle's path can return to itself
- When simplifying to the orbit, the saddle will move locations
- Allows all possible critical points to be simplified



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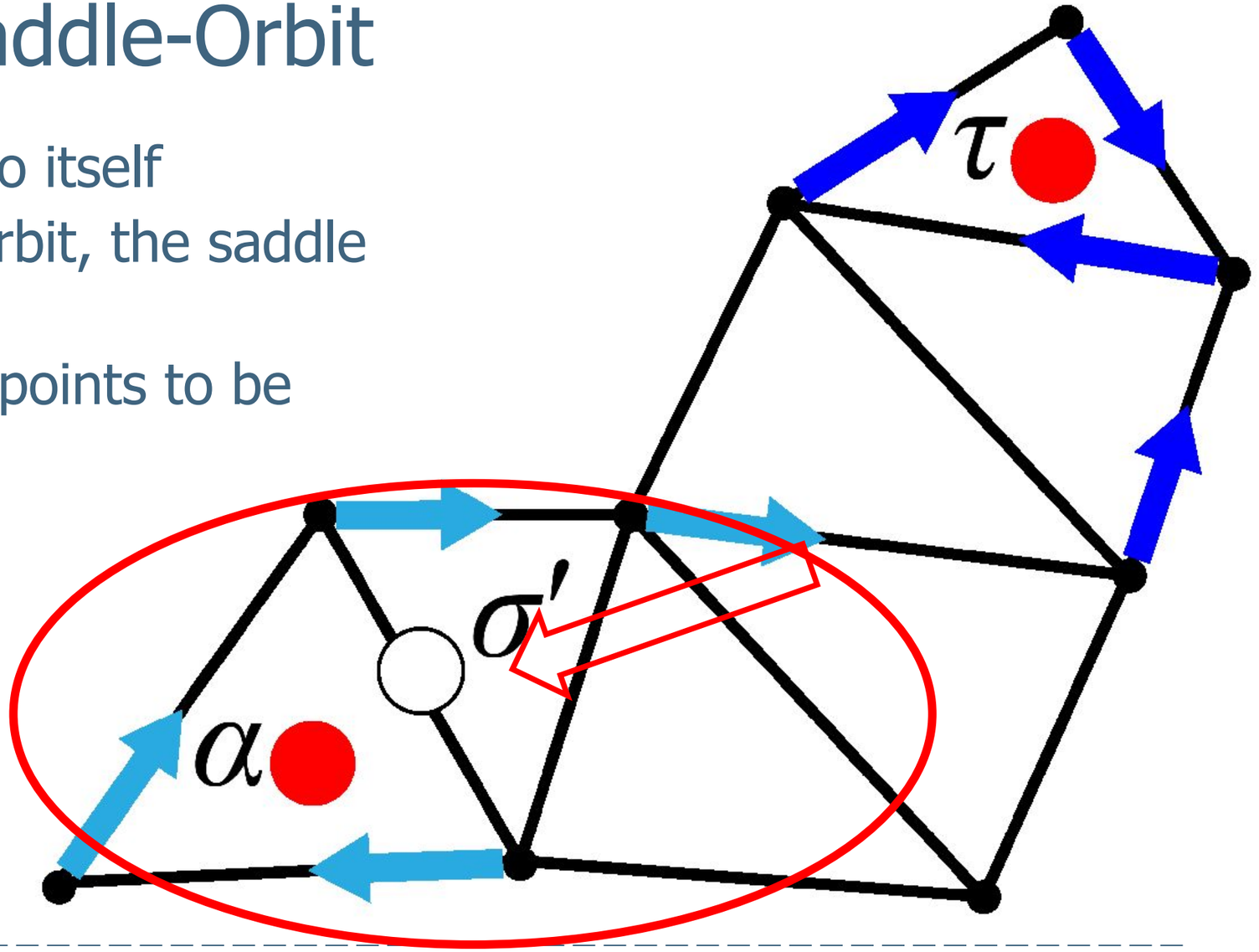
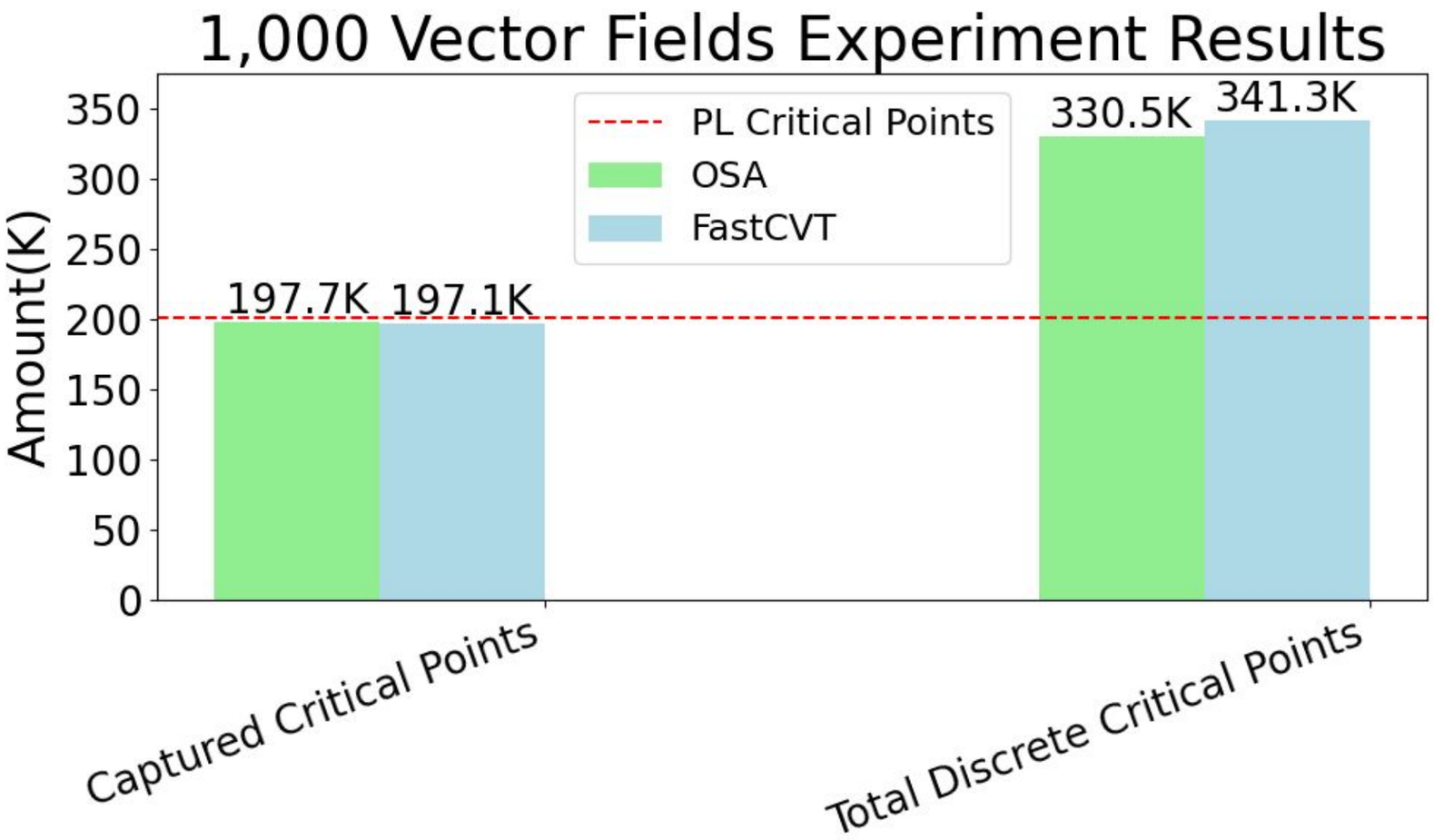


Results

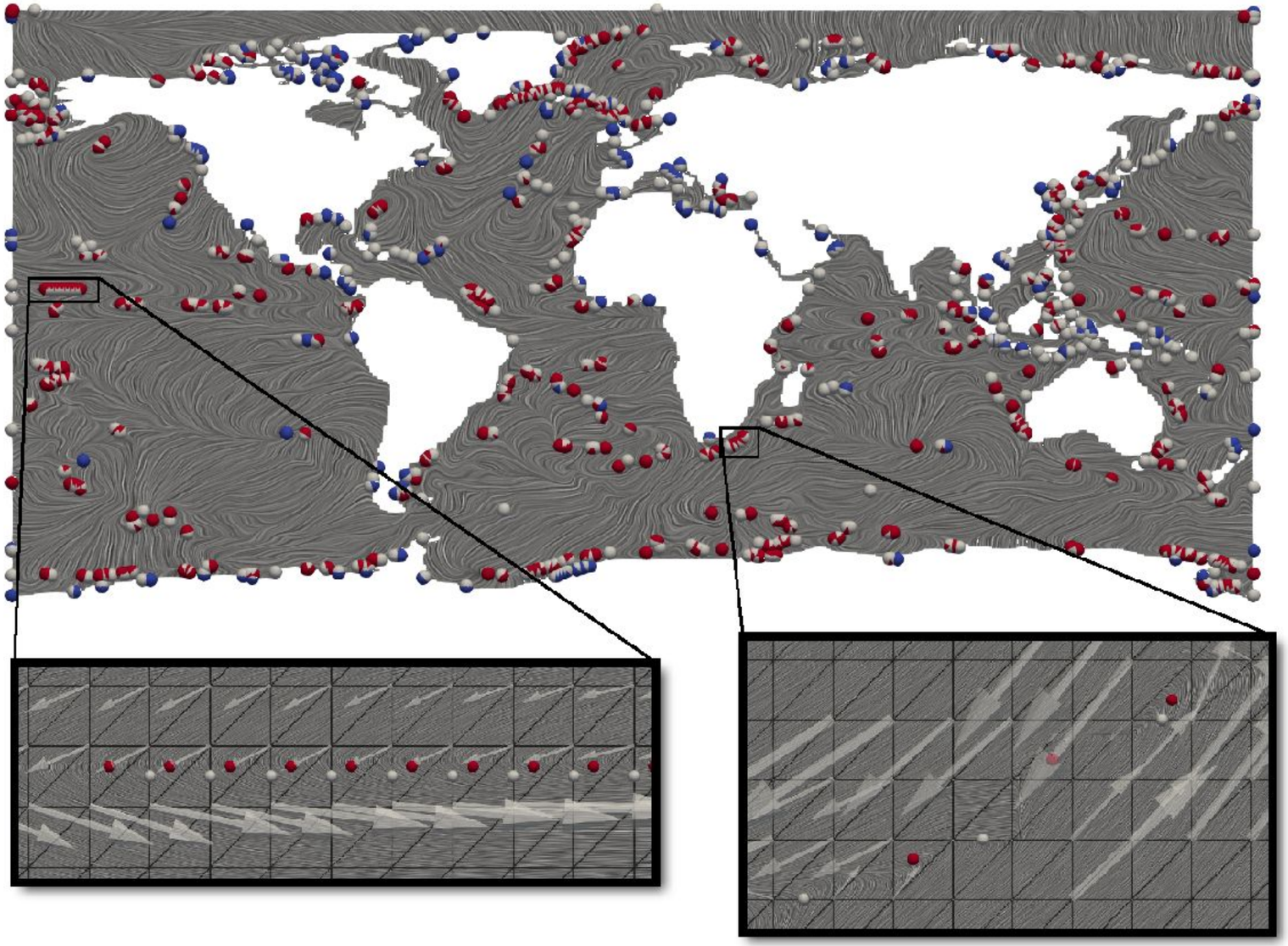
Accuracy Testing (Generated 1,000 Vector Fields with ≈200 Critical Points each)

- >98% of critical points have associated nearby discrete critical simplex
  - Uncaptured=unstable
- Extra Discrete Critical Points
  - Boundary Artifacts
  - Under Represented Curvature



Ocean Flow Dataset

- Real data of Ocean's Flow
- Generated extra discrete critical points around sharply curving flow
- Simplification (to number of PL critical points) removed these extra discrete critical points



Daily mean surface velocity of the ocean from January 8, 1992, by the ECCO Consortium

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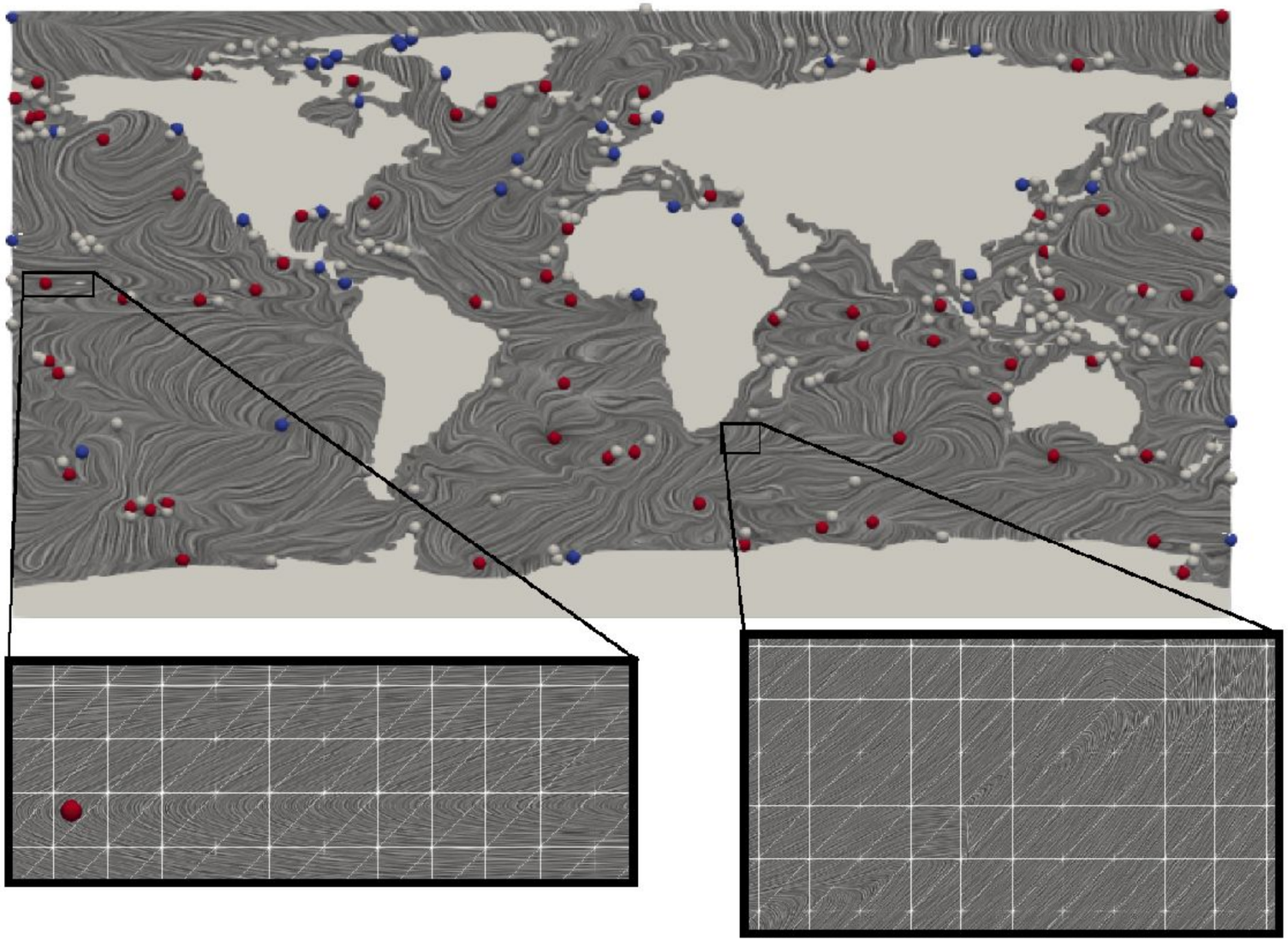

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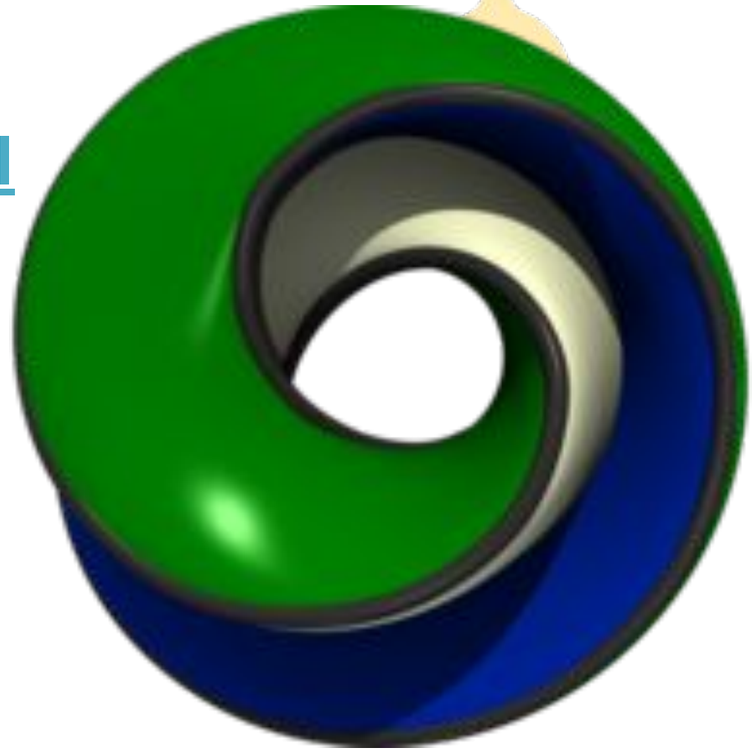
Discussion

- Algorithm is **fast**, easy to parallelize
- Results are similar to FastCVT
  - Implemented in **Topology ToolKit(TTK)**
  - **https://topology-tool-kit.github.io/index.html**
- Limitations (extra discrete critical points)
- Future Work:

Outward Star Formula,

3D, and simplification



Thanks!

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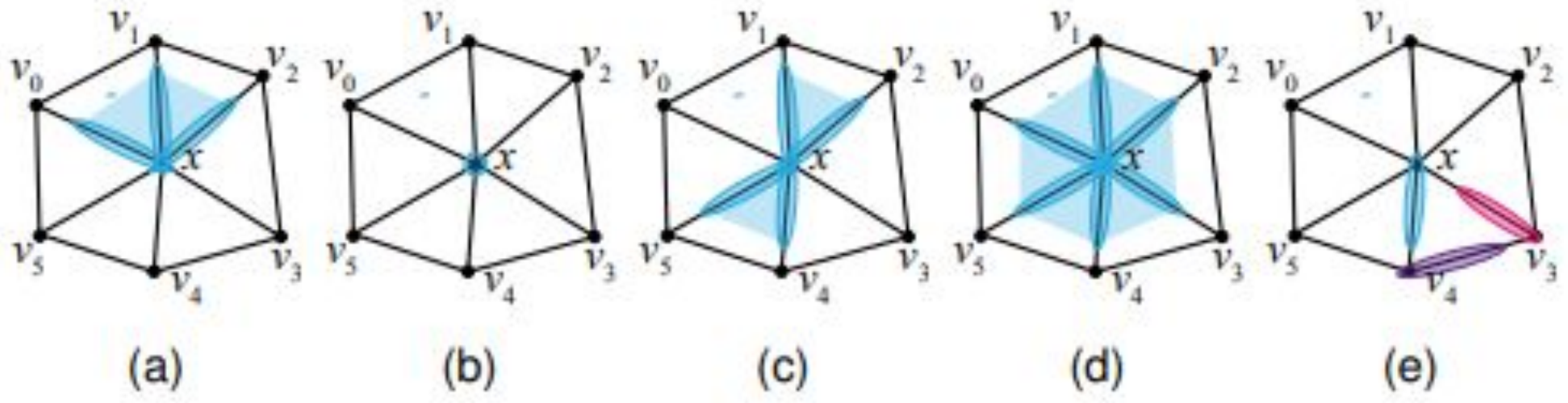
Audience Questions?

Supplemental Slides

- Outward Star Possiblities
- Critical Point Mapping
- Outward Star Processing (Necessary Modifications)
- Overview of Algorithm
- Fast Results
- Accuracy Explanation

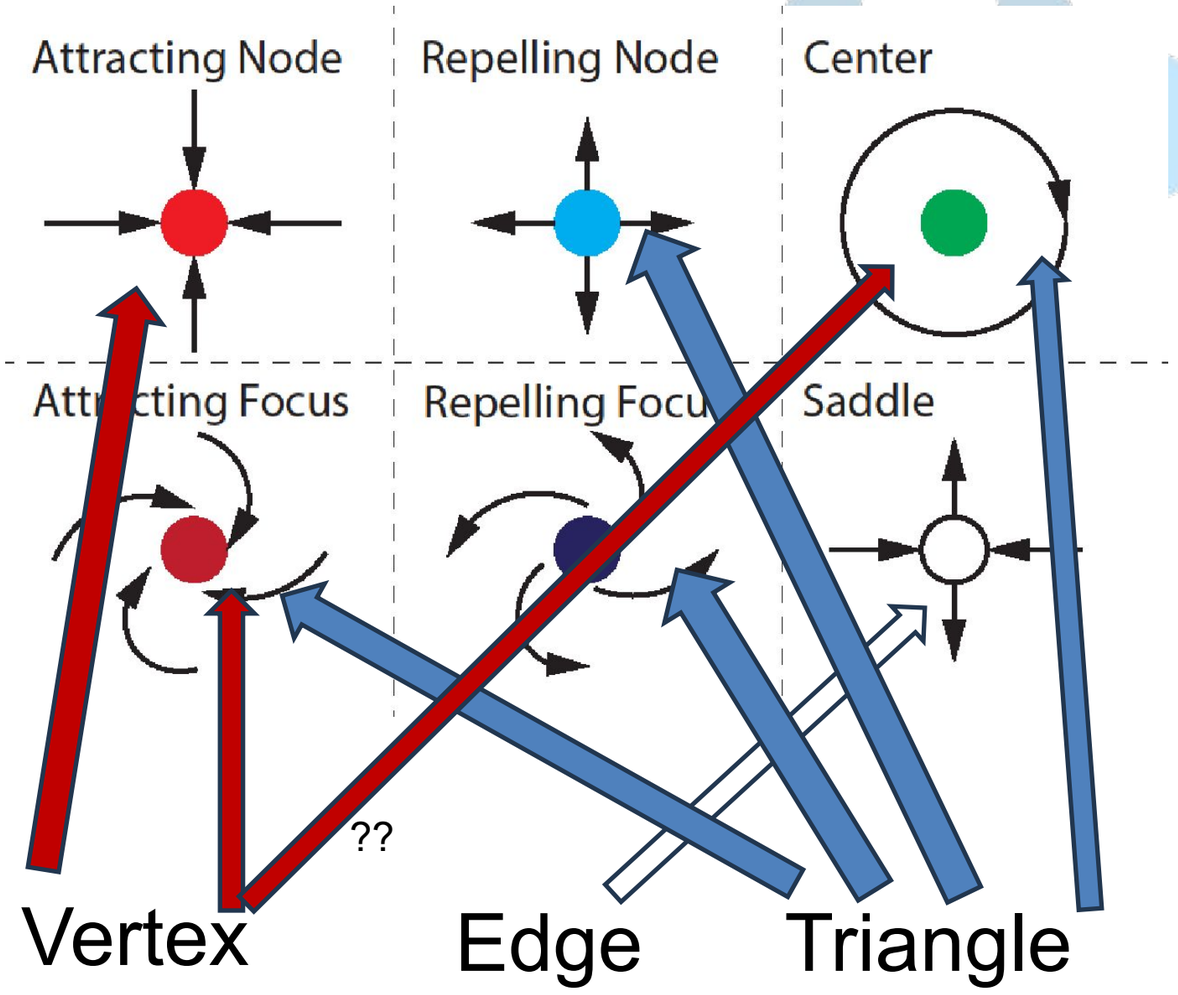
Outward Star Possiblities

- (a) = Standard Flow
- (b) = Critical Vertex
- (c) = Critical Edge(Saddle)
- (d) = Critical Triangle (Source)
- (e) = Critical Triangle (Sink)



Critical Point Mapping

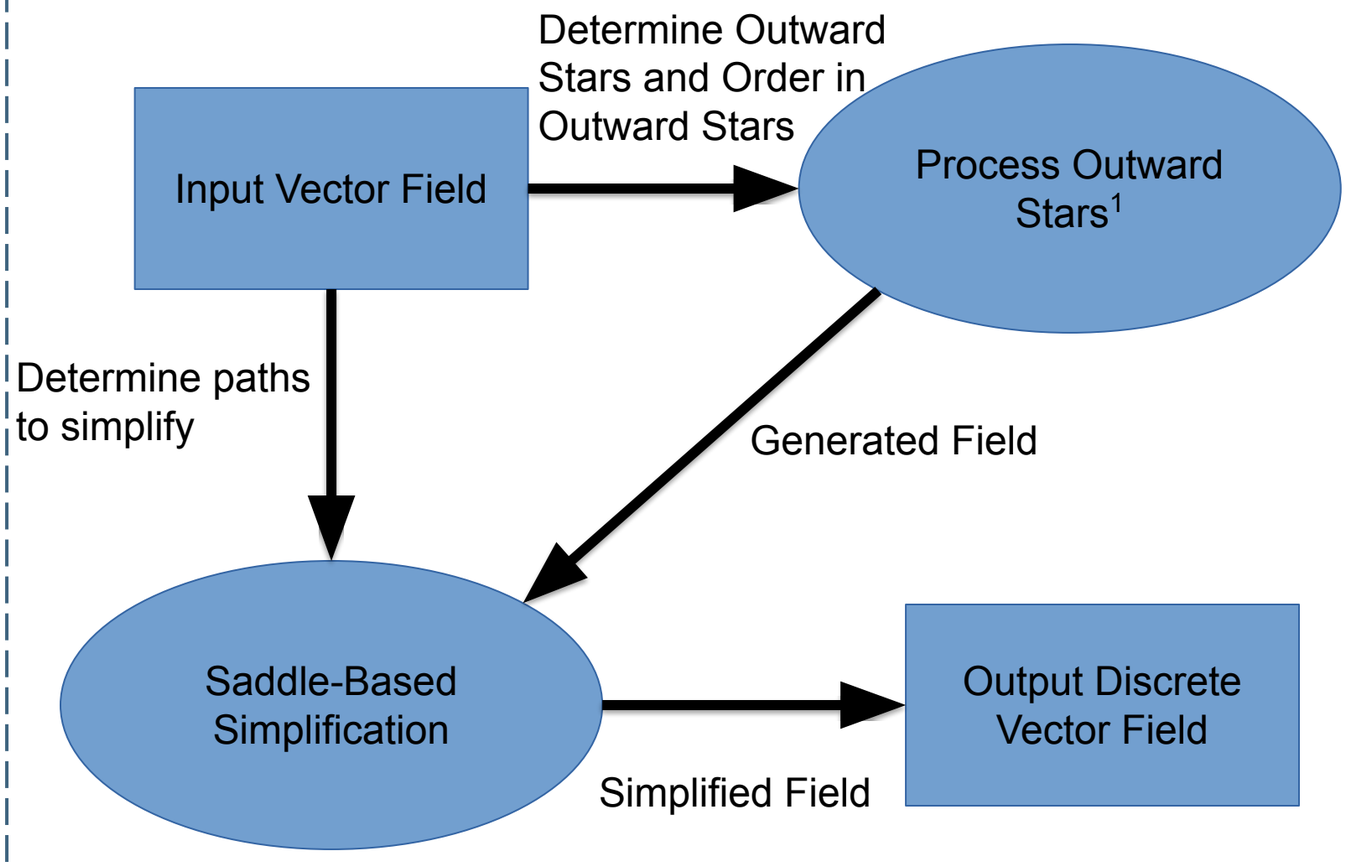
- No direct mapping exists for either algorithm (3=>6)
- Saddles always map to a critical edge
- Sources (Repelling Nodes) map to triangle
- Sinks (Attracting Nodes) map to vertex
- The amount of rotation can change the focus and the centers to triangles.



Outward Star Processing (Modifications)

- 2 Main components(which rely on scalar values) to specify in processing algorithm
  - Which edge to pair the vertex with
    - We chose the edge with the largest weight value
  - How to order the next simplices to pair
    - Order similar to scalar fields with negated weight values (treating them as difference) in lexicographical order

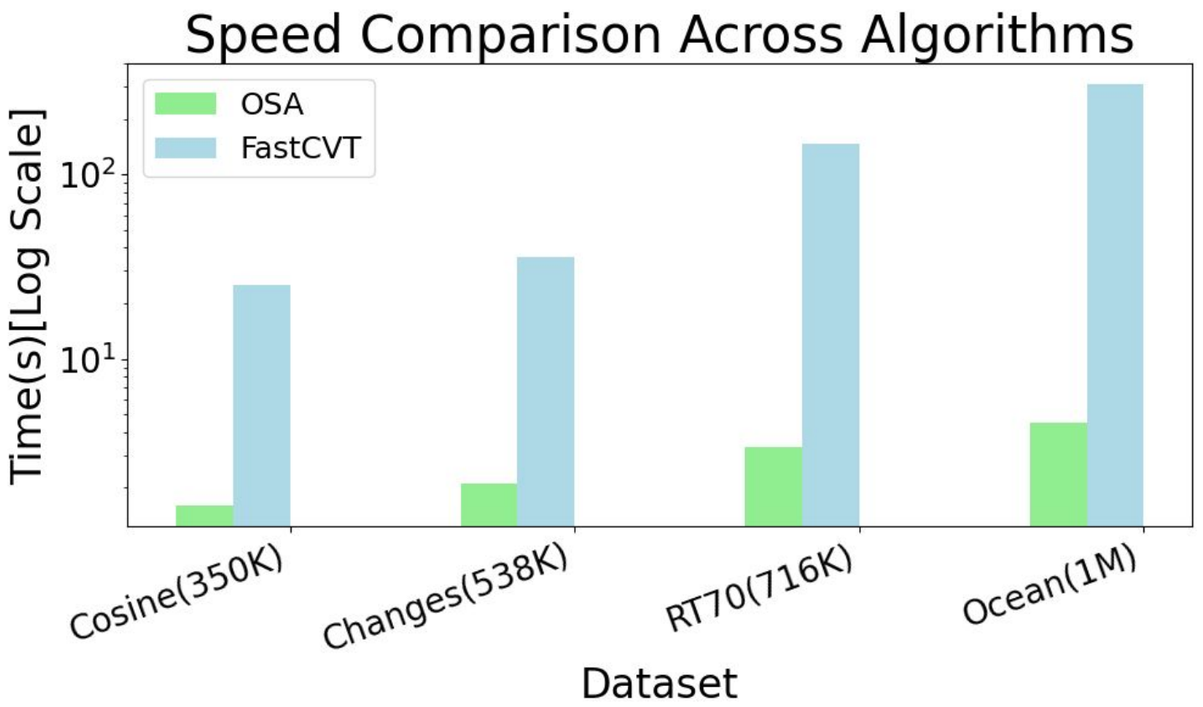
Overview of Algorithm

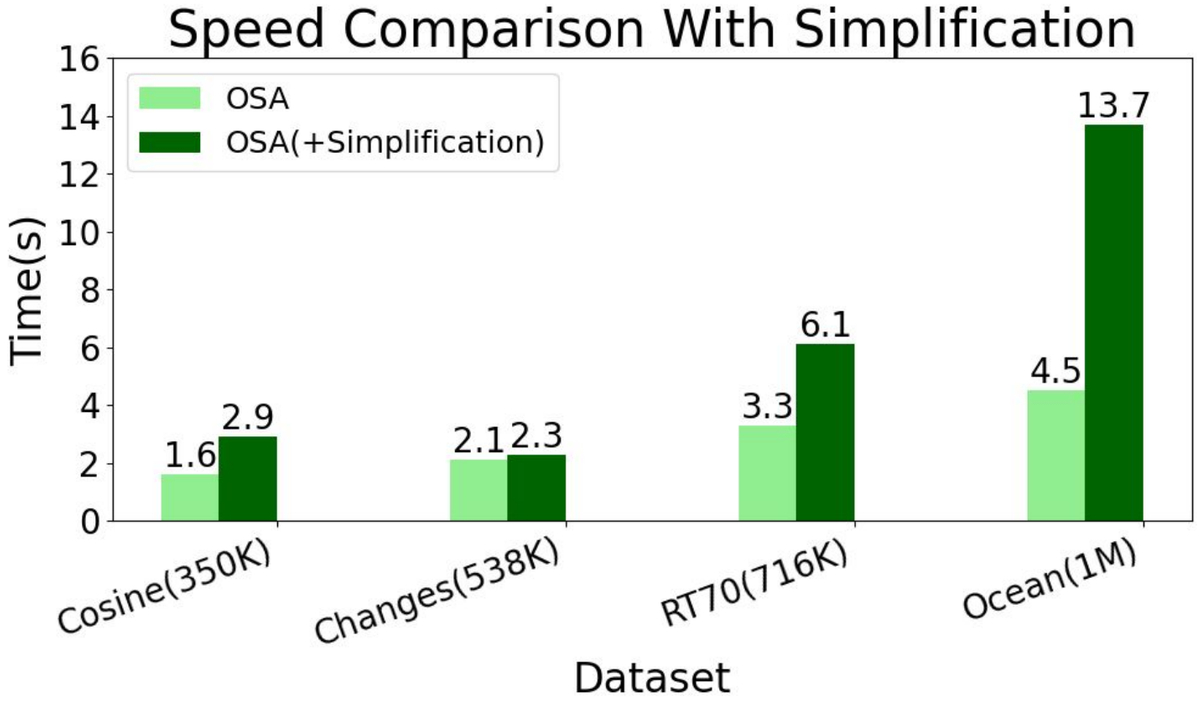


[1] Using a variant of Processing Lower Stars in Scalar Fields

Fast Results

- Implemented in C++ with the same data structures
- Order of Magnitude Improvement
- Linearly Scalable O(n) Generation
- Fast CVT O(n^(3/2) log n)
- Simplification did not incur a drastic slowdown





Accuracy Testing (Additional Explanation)

- What does Captured mean?
- PL Critical Points occur in triangles of the mesh
- Captured Means a discrete critical point is generated in the star of the vertices of the triangle (Simplex contains a point of triangle)