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RESEARCH ARTICLE

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Skewness and index futures return

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Abstract

In this paper, we show that the individual skewness, defined as the average of monthly skewness across firms, performs very well at predicting the return of S&P 500 index futures. This result holds after controlling for the liquidity risk or for the current business cycle conditions. We also find that individual skewness performs very well at predicting index futures returns out-of-sample.

KEYWORDS

average skewness, index futures return, return predictability

JEL CLASSIFICATION

G11; G12; G13; G17

1 | INTRODUCTION

For decades, a substantial literature has explored the dynamics between the U.S. equity index market and the index futures market. The common finding is that the futures market is an important determinant of the equity index market. And there is scant evidence that the equity market performs well at predicting the index futures market. Additionally, predicting the index futures market based on equity market remains a challenging task. However, theoretically the equity market should provide valuable forecasting indicators as the price of an index futures contract is supposed to reflect the expected value of the underlying equity market index.

In this paper, we fill this gap by providing a new finding that the cross-firm average skewness in the equity market plays a prominent role in predicting the S&P 500 index futures market return. Specifically, we find that average skewness significantly predicts the index futures return, that is a one-standard-deviation increase in monthly average skewness predicts a 0.59% decrease in the monthly index futures return. Additionally, we find this significant negative relation between average stock skewness and index futures return holds over subsample periods. It also holds after controlling for the usual variables known to predict market returns such as market illiquidity measures and business conditions. We also find that average skewness outperforms other economic variables proposed by Goyal and Welch (2008) in predicting the index futures returns.

Additionally, we evaluate the out-of-sample performance of the average skewness as a predictor for the index futures return. For this purpose, we perform out-of-sample 1-month ahead forecasts with several predictors, including the equity index return, the average variance, the average skewness, and economic variables proposed by Goyal and Welch (2008). We find evidence that the predictive power of the average skewness dominates that of the other predictors. Additionally, we show that an investor forms her strategy based on timing average skewness does not only make positive investment profit but also enjoys significant economic gains. All these results confirm that the average (individual) skewness is an important predictor of subsequent index futures returns.

In fact, the potential relation between individual skewness and the future asset returns has been investigated in a long-standing literature. Several theories have been proposed to explain why high (individual or idiosyncratic) skewness determines expected returns. First, the role of idiosyncratic skewness has also been put forward to explain why investors actually hold underdiversified portfolios (Conine & Tamarkin, 1981; Kumar, 2009; Mitton & Vorkink, 2007; Simkowitz & Beedles, 1978). That is, investors with preference for skewness may hold underdiversified portfolios to benefit from the upside potential of positively skewed assets. Mitton and Vorkink (2007) argue that investors with preference for skewness underdiversify their portfolio to invest more in assets with positive idiosyncratic skewness. At the equilibrium, stocks with high idiosyncratic skewness will pay a premium. This finding contrasts to the traditional view that individual or idiosyncratic skewness risk should not be priced since it can be diversified away.

Second, investors' beliefs and preferences on skewness influence future asset returns. Brunnermeier, Gollier, and Parker (2007) show that investors choose to have distorted beliefs about the probabilities of future states to maximize their expected utility. They will tend to underdiversify their portfolio by investing in positively skewed assets. In the context of cumulative prospect theory (Kahneman & Tversky, 1979), Barberis and Huang (2008) construct a model in which investors wrongly measure probability weights. This results in an overpricing of positively skewed securities, which earn a negative average excess return at the equilibrium (see also Bali, Cakici, & Whitelaw, 2011; Kumar, 2009). Bordalo, Gennaioli, and Shleifer (2013) develop a theory in which investors overweight the salient payoffs relative to their objective probabilities. In this approach, assets with large upside (positive skewness) are overpriced, whereas assets with large downsides (negative skewness) are underpriced.

Third, the potential growth of firms is also related to skewness which in turn affects firm's stock return. Trigeorgis and Lambertides (2014) and Del Viva, Kasanen, and Trigeorgis (2017) argue that growth options are significant determinants of idiosyncratic skewness. This relation is due to the convexity of the payoff of real options. As investors are willing to pay a premium to benefit from the upside potential of the real option, firms with growth options are generally associated with a negative return premium.[1]

However, to date, the literature on the relation between the average skewness and the subsequent equity index return is very scarce and provides mixed results. Using S&P index options to estimate skewness, Chang, Zhang, and Zhao (2011) find a negative and weakly significant effect of the physical market skewness on the future monthly return for the period 1996–2005. Garcia, Mantilla-Garcia, and Martellini (2014) investigate the ability of cross-sectional variance and a robust measure of skewness, based on the quantiles of the cross-section distribution of monthly returns, to predict the future market return based on CRSP data between 1963 and 2006. The empirical estimate that they find for the skewness parameter is insignificant when predicting the monthly value-weighted market return. In contrast, Stöckl and Kaiser (2016) find that cross-sectional skewness adds to the predictive power of cross-sectional volatility in the short run.

A recent study by Jondeau, Zhang, and Zhu (2019) find a significant negative relation between average skewness and future stock market returns. More negatively skewed returns are associated with subsequent higher returns. So far, no paper has investigated the ability of the average (individual) skewness to predict the subsequent index futures return. We do this by extending the work of Goyal and Santa-Clara (2003), Bali, Cakici, Yan, and Zhang (2005), and Jondeau et al. (2019). We use the same data and methodology and perform a similar robustness analysis. We note that our measure is different from the ones from Jondeau et al. (2019). Their monthly individual firm's skewness is calculated with demeaned individual return, where the mean is the average of individual firm return within each month; however, we use the average of equity market index return within each month as the mean to demean the individual firm return. In this way, we eliminate the effect from the equity market return and focus on the idiosyncratic skewness risk from the current equity market. In the end, we find that average skewness calculated based on this procedure is an effective predictor for index futures returns.

The rest of this paper is organized as follows. Section 2 conducts literature review. Section 3 describes the data and the construction of the variables used in the paper. Section 4 presents empirical evidence that the average skewness negatively predicts subsequent market return. We show that this result is robust to several alterations of the baseline specification. In Section 5, we evaluate the out-of-sample performance of the average skewness as a predictor of the index futures market return. We also find that average skewness generates superior economic performance compared to alternative predictors. Section 6 concludes.

[1]Cao, Simin, and Zhao (2008) and Grullon, Lyandres, and Zhdanov (2012) provide evidence that real options are important drivers of idiosyncratic volatility and may explain the positive relation between stock return and volatility documented by Duffee (1995).

2 | LITERATURE

Given the premise that a typical investor would have a preference for skewness (Scott & Horvath, 1980), earlier studies propose that the systematic component of higher moments should be rewarded and explain the cross-sectional dispersion of expected returns across firms (Harvey & Siddique, 2000; Kraus & Litzenberger, 1976).[2] These papers on the importance of skewness for asset pricing have considered the case of investors with a fully diversified portfolio. In this context, the coskewness of an asset with the market portfolio (systematic risk) should be priced (Barone-Adesi, 1985; Harvey & Siddique, 2000; Kraus & Litzenberger, 1976). In particular, Harvey and Siddique (2000) find that systematic skewness commends a risk premium of 3.6% per year on average. Different from these studies, we focus on individual skewness and explore the predictability of index futures returns.

The other group of literature has investigated the empirical relation between individual or idiosyncratic skewness and future stock return, in a typical cross-section regression, such as Fama-MacBeth regression (Fama & MacBeth, 1973). For instance, Boyer, Mitton, and Vorkink (2010) first measure the expected idiosyncratic skewness based on lagged idiosyncratic volatility and skewness. Then using monthly data (1987–2005), they find a strong negative impact of expected idiosyncratic skewness on portfolios' returns. Amaya, Christoffersen, Jacobs, and Vasquez (2015) report that realized skewness of individual equities have a strong negative impact on next week's stock return. A few papers have used options data to construct a measure of skewness from the risk-neutral density of option prices. Xing, Zhang, and Zhao (2010) find a negative relation between skewness and future individual stock returns. They measure skewness as the difference between the implied volatilities of out-of-the-money puts and at-the-money calls. Using the approach developed by Bakshi, Kapadia, and Madan (2003), Conrad, Dittmar, and Ghysels (2013) construct model-free implied variance and skewness and evaluate the predictive ability of these risk-neutral measures. Again, the paper reports a negative relation between skewness and the subsequent stock return. Instead, we focus on the predictive relation between average skewness and the index futures return.

As mentioned in the previous section, the existing literature documents that futures market is the main source of market wide information for cash market. For example, studies have documented that futures market leads cash market in terms of price (Kawaller, Koch, & Koch, 1987; Wahab & Lashgari, 1993), return (Chan, 1992; Hasbrouck, 2003), and volatility (Koutmos & Tucker, 1996). MacKinlay and Ramaswamy (1988) document that the excess variability in the futures markets induces the autocorrelation in the equity market. Based on intraday data, researchers also find that there is a close relation between equity and futures markets. And the consensus finding is that futures market leads the movement of equity market. Stoll and Whaley (1990) find that intraday futures returns tend to lead stock market returns by 5 min, and the reverse effect also exists but is rather mild. Harris, Sofianos, and Shapiro (1994) investigate the role of program trading in influencing intraday changes in the futures prices and the equity index. Chol and Subrahmanyam (1994) examine the effect of index futures on the returns and volatility of the underlying equity index using intraday data. Researchers also investigate the relation between cash and futures markets while introducing other market such as options (Fleming, Ostdiek, & Whaley, 1996; Ryu, 2015) and Standard & Poor's Depositary Receipts (Chu, Hsieh, & Tse, 1999).

To date, the literature on predicting index futures return based on different indicators is still growing. Stoll and Whaley (1990) find mild evidence on using lagged equity market returns to forecast index futures returns. Another group of literature finds that proxies of investor sentiment can also predict index futures returns. For instance, Simon and Wiggins (2001) construct market-based sentiment indicators such as the volatility index, the put-call ratio, and the trading index to predict subsequent returns on the S&P 500 futures contract. Gao and Yang (2017) consider investor sentiment with mixed frequencies to predict index futures return. Wang, Ye, Zhao, and Kou (2018) use search volume index to predict short-term return reversal in the index futures market.

3 | DATA AND MEASURING RISK

The monthly S&P 500 Futures Index return is calculated as the simple return on the S&P 500 Futures Index level. The monthly futures index return is the cumulative product of all the daily simple returns within each month. S&P 500 Futures Index data are directly downloaded from Datastream. From now on, we denote by $r_{i,t} = R_{i,t} - R_{f,t-1}$ the excess

[2]In usual utility functions, such as CARA or CRRA function, the third-order derivative with respect to wealth is positive, indicating that investors prefer more than less skewness.

return of stock i in month t and by $r_{m,t}$ the index futures return in month t. We also denote by $r_{i,d}$ and $r_{m,d}$, $d = 1, \ldots, D_t$, the daily returns on day d, where D_t is the number of days in month t.

For measuring average variance and skewness, we use daily firm-level returns for all common stocks from the CRSP data set, including those listed on the NYSE, AMEX, and NASDAQ.[3] For a given month, we use all stocks that have at least 10 valid return observations for that month. We exclude the least liquid stocks (firms with an illiquidity measure in the highest 0.1% percentile) and the lowest-priced stocks (stocks with a price less than $1). The sample period ranges from June 1982 to December 2016 due to the availability of S&P 500 index futures contract.[4] The variance of a stock has long been used as the measure of risk. When daily data are available, a common way of calculating the monthly variance of stock i in month t is:

$$V_{i,t} = \sum_{d=1}^{D_t} (r_{i,d} - \bar{r}_{m,t})^2 + 2\sum_{d=2}^{D_t} (r_{i,d} - \bar{r}_{m,t})(r_{i,d} - \bar{r}_{m,t}), \tag{1}$$

where D_t is the number of days in month t, $r_{i,d} = R_{i,d} - R_{f,d-1}$ is the excess return of stock i on day d, and $\bar{r}_{m,t}$ is the mean value of market return within month t, that is $\bar{r}_{m,t} = 1/D_t \sum_{d=1}^{D_t} r_{m,d}$. The second term on the right-hand side corresponds to the adjustment for the first-order autocorrelation in daily returns (see French, Schwert, & Stambaugh, 1987).

The average of monthly variances across firms can be computed in two common ways. The first measure, used by Goyal and Santa-Clara (2003), is based on equal weights: $V_{ew,t} = \frac{1}{N_t}\sum_{i=1}^{N_t} V_{i,t}$, where N_t is the number of firms available in month t. The second measure, adopted by Bali et al. (2005), is based on market capitalization (or value) weights: $V_{vw,t} = \sum_{i=1}^{N_t} w_{i,t}\, V_{i,t}$, where $w_{i,t}$ is the relative market capitalization of stock i in month t.

We define the monthly (standardized) skewness of stock i as:

$$Sk_{i,t} = \frac{1}{V_{i,t}^{3/2}} \sum_{d=1}^{D_t} (r_{i,d} - \bar{r}_{m,t})^3, \tag{2}$$

Using the standardized measure allows the skewness to be compared across firms with different variances. As for the average variance, the average of the monthly skewness is computed as the equal-weighted measure, $Sk_{ew,t} = \frac{1}{N_t}\sum_{i=1}^{N_t} Sk_{i,t}$, or the value-weighted measure, $Sk_{vw,t} = \sum_{i=1}^{N_t} w_{i,t} \quad Sk_{i,t}$.

4 | EMPIRICAL RESULTS

Figure 1 shows that the dynamics of market variance is different from the dynamics of the average stock variance. As most large increases in the market variance coincide with NBER-dated recession periods (with the exception of the 1987 market crash). The subprime crisis has the most influential effect in the market variance. But the average stock variance does not necessarily increase during recessions. The largest jumps of value-weighted average stock variance appear in 1983, 1986, and 2010, which are not years associated with recessions. Additionally, the value-weighted and equal-weighted average variances do not increase significantly during subprime crisis period. However, during the recent period: 2010–2012, there is a large increase in both value-weighted and equal-weighted average stock variances, although the market variance stays at a relatively low level.

As illustrated in Figure 2, the market skewness and the average stock skewness have different dynamic properties. Most of the time the market skewness remains between −0.5 and 0.5. In contrast, the (value-weighted or equal-weighted) average skewness is in general positive (mostly between −0.03 and 0.05). This evidence suggests that there are periods when the average skewness and the market skewness are apart from each other or even have opposite signs. For instance, the most positive market skewness (in 1984) was accompanied by a moderate level of average skewness.

The correlations between various variables are reported in Table 1 (Panel B). It reveals that the current index futures return exhibits a weak and negative correlation with both average variance and skewness. The correlation between average variance and the index futures return is negative and weak (−0.8% and −5.6% for the value-weighted and equal-weighted

[3] CRSP data on individual firms consist of daily returns on common stocks, corrected for corporate actions and dividend payments.

[4] We also investigate the predictability of the E-mini S&P index futures return. We find similar results as that of the S&P 500 index futures contract. Index futures data are downloaded from Datastream.

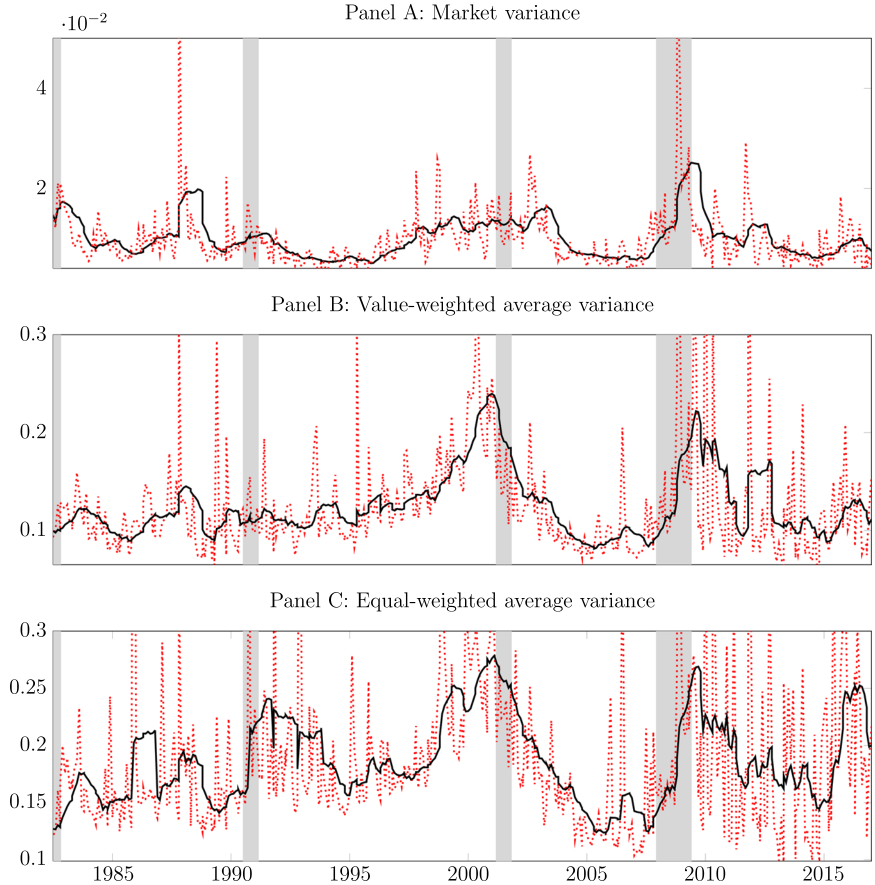

FIGURE 1 Market and average stock variance. This figure presents the 12-month moving-average values (in solid-black line) and raw data series (in dotted-red line) for the squared root of the market variance, the squared root of the value-weighted variance, and the squared root of the equal-weighted variance. Sample: 1982 June–2016 December. NBER recessions are represented by shaded bars [Color figure can be viewed at wileyonlinelibrary.com]

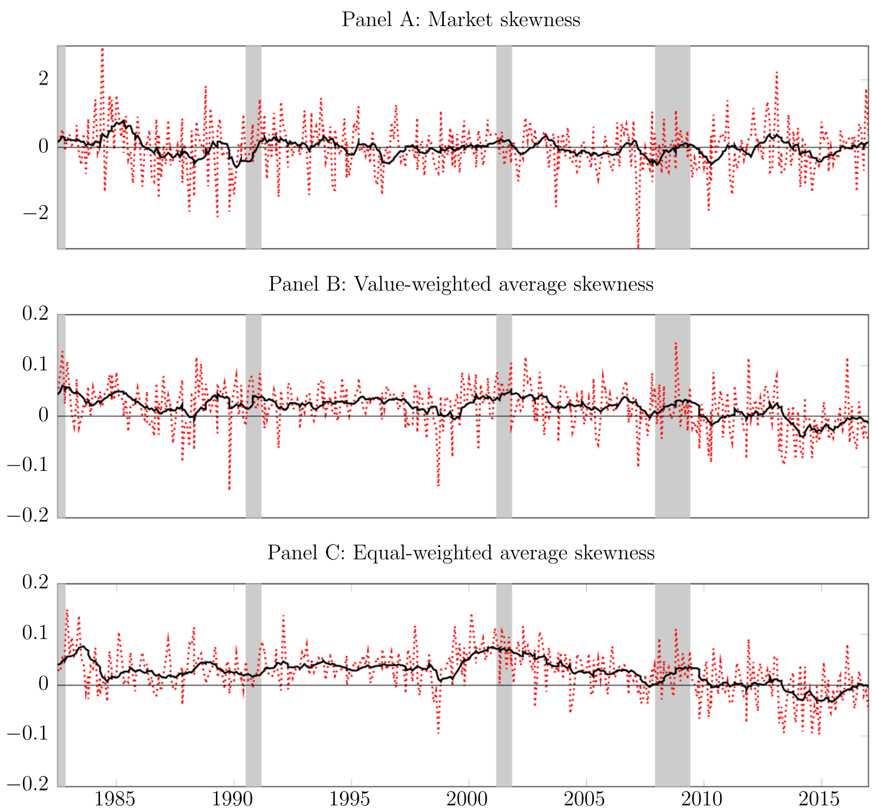

FIGURE 2 Market and average stock skewness. This figure presents the 12-month moving-average values (in solid-black line) and raw data series (in dotted-red line) for market skewness, value-weighted skewness, and equal-weighted skewness. Sample: 1982 June–2016 December. NBER recessions are represented by shaded bars [Color figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Summary statistics and correlation matrix

Panel A: Summary statistics	**Mean**	**Min**	**Med**	**Max**	***SD***	**AR$_1$**
$r_{m,t}$	0.007	−0.223	0.011	0.153	0.045	0.017
$V_{m,t}$	0.014	0.001	0.007	0.709	0.041	0.214
$Sk_{m,t}$	−0.019	−2.997	0.000	3.521	0.693	−0.016
$V_{vw,t}$	0.019	0.003	0.012	0.510	0.032	0.157
$V_{ew,t}$	0.041	0.009	0.028	0.696	0.052	0.090
$Sk_{vw,t}$	0.018	−0.147	0.021	0.145	0.042	0.145
$Sk_{ew,t}$	0.026	−0.097	0.027	0.150	0.040	0.362

Panel B: Correlation	$r_{m,t+1}$	$r_{m,t}$	$V_{m,t}$	$Sk_{m,t}$	$V_{vw,t}$	$V_{ew,t}$	$Sk_{vw,t}$	$Sk_{ew,t}$
$r_{m,t+1}$	1							
$r_{m,t}$	0.018	1						
$V_{m,t}$	−0.136	−0.386	1					
$Sk_{m,t}$	−0.088	0.201	−0.043	1				
$V_{vw,t}$	−0.055	−0.008	0.277	−0.022	1			
$V_{ew,t}$	−0.020	−0.056	0.191	0.092	0.271	1		
$Sk_{vw,t}$	−0.134	−0.035	−0.033	0.461	−0.106	0.060	1	
$Sk_{ew,t}$	−0.033	−0.149	0.013	0.174	−0.015	0.064	0.527	1

Note: This table provides summary statistics and the correlation matrix for the following variables: the subsequent S&P 500 Futures Index return $r_{m,t+1}$, the current S&P 500 Futures Index return $r_{m,t}$, market variance $V_{m,t}$, market skewness $Sk_{m,t}$, value-weighted individual variance $V_{vw,t}$, equal-weighted individual variance $V_{ew,t}$, value-weighted individual skewness $Sk_{vw,t}$, and equal-weighted individual skewness $Sk_{ew,t}$. Sample period: June 1982 to December 2016.

variance, respectively), and the contemporaneous correlation between the index futures return and average skewness is also negative (−3.5% and −14.9% for the value-weighted and equal-weighted skewness, respectively).

The correlation between the index futures return in month $t + 1$ and the average variance or skewness in month t is of a different nature because it involves the time dependence in the return process. The table shows that, as in the contemporaneous case, the correlation of the subsequent index futures return with the current average variance is still negative (−5.5% and −2.0% for the value-weighted and equal-weighted variance, respectively). The correlation with current average skewness is also negative (−13.4% and −3.3% for the value-weighted and equal-weighted skewness, respectively), suggesting that average variance and average skewness may negatively predict market return.

The table also reveals that the correlation between the market skewness and average skewness is relatively low (46.1% and 17.4% for the value-weighted and equal-weighted measures, respectively). These numbers confirm that the market skewness and average skewness convey different types of information, as illustrated in Figure 2.

4.1 | Baseline regressions

We now evaluate the ability of market and average variance (skewness) to predict the subsequent S&P 500 index futures return in a regression. Given the definitions of average variance and skewness based on value and equal weights, the regression can be written, respectively, as below:

$$r_{m,t+1} = a + b \quad V_{vw,t} + c \quad Sk_{vw,t} + d \quad V_{m,t} + e \quad Sk_{m,t} + e_{m,t+1}, \tag{3}$$

$$r_{m,t+1} = a + b \quad V_{ew,t} + c \quad Sk_{ew,t} + d \quad V_{m,t} + e \quad Sk_{m,t} + e_{m,t+1}, \tag{4}$$

where $r_{m,t+1}$ is the subsequent S&P 500 index futures return.

TABLE 2 Predictive regressions of S&P 500 futures index return—baseline case

	I	II	III	IV	V	VI
Constant	0.0067	0.0083	0.0093	0.0092	0.0113	0.0126
	(0.005)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
$r_{m,t}$	0.0177			0.0131	0.0123	−0.0388
	(0.780)			(0.852)	(0.856)	(0.532)
$V_{vw,t}$		−0.0770			−0.0977	−0.0405
		(0.313)			(0.205)	(0.515)
$Sk_{vw,t}$			−0.1425	−0.1420	−0.1500	−0.1389
			(0.008)	(0.013)	(0.008)	(0.018)
$V_{m,t}$						−0.1609
						(0.005)
$Sk_{m,t}$						−0.0018
						(0.564)
Adjusted R^2	−0.211%	0.064%	1.547%	1.325%	1.575%	2.916%
Constant	0.0067	0.0075	0.0077	0.0076	0.0082	0.0092
	(0.005)	(0.002)	(0.001)	(0.005)	(0.006)	(0.002)
$r_{m,t}$	0.0177			0.01320	0.0124	−0.0256
	(0.780)			(0.852)	(0.860)	(0.679)
$V_{ew,t}$		−0.0166			−0.0144	0.0150
		(0.664)			(0.720)	(0.696)
$Sk_{ew,t}$			−0.0365	−0.0343	−0.0332	−0.0226
			(0.483)	(0.513)	(0.525)	(0.652)
$V_{m,t}$						−0.1656
						(0.004)
$Sk_{m,t}$						−0.0057
						(0.057)
Adjusted R^2	−0.211%	−0.205%	−0.136%	−0.362%	−0.579%	1.645%

Note: This table reports results of the 1-month-ahead predictive regressions of the S&P 500 Futures Index return $r_{m,t+1}$. $V_{vw,t}$, and $Sk_{vw,t}$ are the value-weighted average variance and skewness. $V_{ew,t}$ and $Sk_{ew,t}$ are the equal-weighted average variance and skewness. $V_{m,t}$ and $Sk_{m,t}$ are market variance and skewness. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample period: June 1982 to December 2016.

In Table 2, we consider each of the variables in Equations (3) and (4) introduced separately. Panel A reports the results of the regressions for the 1982–2016 sample. The current index futures return fails to predict the subsequent index futures return with a p value equal to 78$ and an adjusted R^2 equal to −0.21%. Consistent with previous literature discussed in Section 2, market skewness does not significantly predict index futures return. Although the market variance has strong predictive power for index futures returns with a low p value, its predictive ability is not robust in subsample periods as shown later in Table 3. The average variance also fails to predict index futures return with a high p value (31.3% for value-weighted variance and 66.4% for equal-weighted variance) and a low adjusted R^2 (0.06% for value-weighted variance and −0.21% for equal-weighted variance). In contrast, the coefficient of the average skewness is highly significant and with a negative value of −0.1425. That is, as the standard deviation of the value-weighted average skewness is equal to 0.042, a one-standard-deviation increase in monthly average skewness results in a 0.59% (=−0.1425 × 0.042) decrease in the monthly index futures return. For the value-weighted average skewness, the p value is low (equal to 0.8%) and the adjusted R^2 is high (equal to 1.547%). However, we notice that the equal-weighted skewness fails to predict the index futures return. The reason may be that implementing equal-weighting scheme to calculate average skewness treats large and small firms equally. Small firm returns are more likely to experience extreme movements and exhibit high volatility. Measuring skewness is admittedly a challenging work as it raises

TABLE 3 Predictive regressions of S&P 500 futures index return—subsamples

	1990–2016 sample						2000–2016 sample					
	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
Panel A: Value-weighted variance and skewness												
Constant	0.0058	0.0069	0.0084	0.0084	0.0097	0.0106	0.0031	0.0044	0.0055	0.0055	0.0071	0.0085
	(0.026)	(0.004)	(0.000)	(0.002)	(0.000)	(0.001)	(0.358)	(0.173)	(0.055)	(0.097)	(0.021)	(0.025)
$r_{m,t}$	0.0235			0.0017	0.0029	−0.0219	0.0446			−0.0026	−0.0016	−0.0364
	(0.745)			(0.983)	(0.971)	(0.755)	(0.616)			(0.979)	(0.987)	(0.678)
$V_{vw,t}$		−0.0475			−0.0619	−0.0320		−0.0499			−0.0658	−0.0360
		(0.473)			(0.365)	(0.577)		(0.475)			(0.341)	(0.546)
$Sk_{vw,t}$			−0.1592	−0.1589	−0.1627	−0.1474			−0.2147	−0.2154	−0.2197	−0.2112
			(0.012)	(0.013)	(0.011)	(0.013)			(0.003)	(0.004)	(0.003)	(0.004)
$V_{m,t}$						−0.1245						−0.1277
						(0.552)						(0.555)
$Sk_{m,t}$						−0.0004						0.0016
						(0.893)						(0.710)
Adjusted R^2	−0.255%	−0.161%	1.976%	1.670%	1.618%	1.395%	−0.298%	−0.285%	3.579%	3.098%	2.982%	2.489%
Panel B: Equal-weighted variance and skewness												
Constant	0.0058	0.0077	0.0075	0.0074	0.0089	0.0092	0.0031	0.0052	0.0061	0.0062	0.0068	0.0064
	(0.026)	(0.005)	(0.002)	(0.014)	(0.006)	(0.015)	(0.358)	(0.254)	(0.038)	(0.081)	(0.140)	(0.236)
$r_{m,t}$	0.0235			0.0084	0.0052	−0.0107	0.0446			−0.0126	−0.0143	−0.0506
	(0.745)			(0.921)	(0.950)	(0.881)	(0.616)			(0.906)	(0.888)	(0.588)
$V_{ew,t}$		−0.0428			−0.0372	−0.0087		−0.0488			−0.0157	0.0691
		(0.287)			(0.404)	(0.864)		(0.592)			(0.879)	(0.600)
$Sk_{ew,t}$			−0.0639	−0.0616	−0.0585	−0.0399			−0.1615	−0.1663	−0.1648	−0.1457
			(0.274)	(0.295)	(0.318)	(0.489)			(0.020)	(0.020)	(0.023)	(0.051)
$V_{m,t}$						−0.1554						−0.2180
						(0.463)						(0.330)
$Sk_{m,t}$						−0.0040						−0.0026
						(0.237)						(0.529)
Adjusted R^2	−0.255%	−0.112%	0.054%	−0.252%	−0.417%	0.027%	−0.298%	−0.368%	1.879%	1.403%	0.920%	1.479%

Note: This table reports results of the one-month-ahead predictive regressions of the S&P 500 Futures Index return $r_{m,t+1}$. $V_{vw,t}$, and $Sk_{vw,t}$ are the value-weighted average variance and skewness. $V_{ew,t}$ and $Sk_{ew,t}$ are the equal-weighted average variance and skewness. $V_{m,t}$ and $Sk_{m,t}$ are market variance and skewness. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p values based on Newey-West adjusted t-statistics. The last row presents the adjusted R^2 values. Sample periods are from January 1990 to December 2016 (Columns I to VI) and from January 2000 to December 2016 (Columns VII to XII).

everything to the third power, which makes skewness sensitive to outliers. Therefore, the failure of the equal-weighted skewness in predicting index futures return may be due to its sensitivity to outliers. Table 3 reveals that, within the two subsample periods: 1990–2016 and 2000–2016, the value-weighted skewness performs even better in predicting index futures returns.[5]

4.2 | Controlling for business cycle and market liquidity risk

Is the significant predictive power of average skewness due to the fact that it is just representing the fundamental business cycle variables? To answer this question, we follow Goyal and Santa-Clara (2003) and investigate the

[5]We also investigate the ability of average skewness in predicting S&P 500 ETF returns. We employ data on Standard & Poor's Depositary Receipts (SPDR) S&P 500 Trust ETF which is designed to track the S&P 500 stock market index, and run the baseline regressions. The predictability of value-weighted average skewness remains powerful, with low p value (equal to 0.3%) and high adjusted R^2 (equal to 2.042%).

relationship between the index futures return and average stock skewness when a set of macroeconomic factors are used as controls for business cycle conditions. The control variables are: the dividend-price ratio, calculated as the difference between the log of the last 12-month dividends and the log of the current level of the market index (*DP*); the default spread, calculated as the difference between a Moody's Baa corporate bond yield and the 10-year Treasury bond yield (*DEF*); the term spread, calculated as the difference between the 10-year Treasury bond yield and 3-month T-bill rate (*TERM*); and the relative 3-month T-bill rate, calculated as the difference between the current T-bill rate and its 12-month backward-moving average (*RREL*). We also introduce the market illiquidity measure proposed by Amihud (2002), which has been found to have some predictive power for future market returns. Bali et al. (2005) show that the predictive power of the equal-weighted variance is partly explained by a liquidity premium, as small stocks dominate the equal-weighted variance.[6]

Table 4 Panel A reports the regression results when all of the business cycle factors are considered with average skewness. First, the results imply that, even if some of these variables are significant as documented in the previous studies (see among others, Bali et al., 2005), their contribution to the predictability of subsequent index futures remains limited when they are introduced together. Second, the significance of the average skewness coefficient is essentially not affected by the introduction of these business cycle variables. Over the 1982–2016 sample, its coefficient estimate is equal to −0.1626 (with a p value of 0.5%) and the adjusted R^2 is equal to 3.691%. When the current index futures return is introduced as an additional control variable, we also obtain similar estimates, with an average skewness coefficient equal to −0.1633 (with a p value of 0.5%) and an adjusted R^2 equal to 3.48%. We also note that, no matter the business cycle factors are introduced in the regression or not, the current index futures return does not help predict subsequent index futures return, with a p value above 15%. Furthermore, Columns II and V report that expected illiquidity has an insignificant estimated coefficient and does not alter the predictive ability of average skewness. When both the expected illiquidity and unexpected illiquidity are introduced, Columns III and VI reveal that both illiquidity variables have significant estimated coefficients and the estimates of the average skewness coefficient remain highly significant. As documented in Panel B, for sample period 2000–2016, although less significant, we find similar predictive performance of average skewness when macrovariables and illiquidity measures are considered.

We also note that illiquidity risk explains well index futures returns. However, it is not an appropriate predictor as unexpected illiquidity is contemporaneous to index futures returns.

4.3 | Comparison with economic variables

Rapach, Strauss, and Zhou (2009) and Rapach and Zhou (2013) argue that model uncertainty and instability may cause the failure of previous papers to find significant out-of-sample gains in forecasting market return. They recommend combining individual predictors and find that a simple equal-weighted combination of 14 standard economic variables succeeds in predicting the monthly market return.[7] We combine these economic variables using the first principal component (denoted by ECON_{PC}) and their equal-weighted average (denoted by ECON_{AVG}) and compare the predictive ability of these variables with that of average skewness.

For the 1982–2016 sample period, Table 5. Panel A reveals that value-weighted average skewness outperforms the first principal component, ECON_{PC}. The adjusted R^2 is equal to 1.55% with average skewness but only 0.76% for ECON_{PC}. Average skewness also outperforms the average of the 14 economic variables ECON_{AVG}, which has an adjusted R^2 equal to 1.04%. Also, the predictability of value-weighted skewness is not essentially affected when the economic variables are considered. That is when the variables are introduced together in the regression, the p value of the average skewness coefficient is equal to 0.3% and 0.2%, when ECON_{PC} and ECON_{AVG} are introduced, respectively. When the current index futures return is added into the regression, average skewness still outperforms the economic factors. For

[6]The illiquidity of a given stock i in month t is defined as $ILLIQ_{i,t} = \frac{1}{D_t}\sum_{d=1}^{D_t}\frac{|r_{i,d}|}{Vol_{i,d}}$, where $Vol_{i,d}$ is the dollar trading volume of firm i on day d. Then, the aggregate illiquidity is the average across all stocks available in month t: $ILLIQ_t = \sum_{i=1}^{N_t} w_{i,t} \quad ILLIQ_{i,t}$. The expected component of the aggregate illiquidity measure is obtained by the following regression (t statistics in parentheses): $\log(ILLIQ_{t+1}) = -0.727(-1.81) + 0.54(2.14)\log(ILLIQ_t) + \text{residual}$, with the adjusted R^2 equal to 23.1%. The expected illiquidity, denoted by $ILLIQ_t^E$, is defined by the first two terms on the right-hand side and $ILLIQ_{t+1}^U$ denotes the residual.

[7]The 14 economic variables are the following: the dividend-price ratio, the dividend yield, the earnings-price ratio, the dividend-payout ratio, the stock variance, the book-to-market ratio, the net equity expansion, the Treasury bill rate, the long-term yield, the long-term return, the term spread, the default yield spread, the default return spread, and the inflation rate.

TABLE 4 Predictive regressions of market return—business cycle and market liquidity

	I	II	III	IV	V	VI
Panel A: 1982–2016						
Constant	0.0782	0.0238	0.0012	0.0798	0.0219	−0.0053
	(0.019)	(0.609)	(0.980)	(0.016)	(0.646)	(0.912)
$R_{m,t}$				−0.0179	−0.0363	−0.0858
				(0.793)	(0.596)	(0.186)
$V_{vw,t}$	−0.0782	−0.0683	−0.0479	−0.0778	−0.0668	−0.0425
	(0.263)	(0.315)	(0.422)	(0.280)	(0.355)	(0.524)
$Sk_{vw,t}$	−0.1626	−0.1538	−0.1342	−0.1633	−0.1546	−0.1345
	(0.005)	(0.006)	(0.011)	(0.005)	(0.007)	(0.013)
DP_t	1.6446	2.7603	3.7163	1.6783	2.9348	4.2063
	(0.017)	(0.010)	(0.001)	(0.016)	(0.007)	(0.000)
DEF_t	0.3258	0.3717	0.4701	0.3215	0.3673	0.4678
	(0.744)	(0.705)	(0.622)	(0.748)	(0.711)	(0.630)
$TERM_t$	−0.3126	−0.3621	−0.4288	−0.3186	−0.3789	−0.4740
	(0.078)	(0.047)	(0.017)	(0.068)	(0.035)	(0.009)
$RREL_t$	−0.7193	−0.6706	−0.6152	−0.7343	−0.6961	−0.6711
	(0.020)	(0.024)	(0.027)	(0.016)	(0.018)	(0.017)
$ILLIQ_t^E$		−0.0063	−0.0100		−0.0069	−0.0117
		(0.136)	(0.018)		(0.110)	(0.007)
$ILLIQ_{t+1}^U$			−0.0258			−0.0279
			(0.000)			(0.000)
Adjusted R^2	3.691%	4.073%	8.433%	3.479%	3.955%	8.866%
Panel B: 2000–2016						
Constant	0.2228	0.4039	0.2560	0.2393	0.4122	0.2480
	(0.014)	(0.046)	(0.245)	(0.016)	(0.048)	(0.263)
$R_{m,t}$				−0.0633	−0.0588	−0.1003
				(0.524)	(0.552)	(0.287)
$Sk_{vw,t}$	−0.1340	−0.1308	−0.1231	−0.1440	−0.1402	−0.1381
	(0.120)	(0.129)	(0.140)	(0.080)	(0.088)	(0.087)
DP_t	4.9216	5.6002	4.9358	5.2833	5.9103	5.3650
	(0.019)	(0.014)	(0.032)	(0.022)	(0.017)	(0.029)
DEF_t	−1.1494	−2.0359	−1.2971	−1.2814	−2.1246	−1.3378
	(0.465)	(0.265)	(0.480)	(0.419)	(0.250)	(0.469)
$TERM_t$	−0.2877	−0.2688	−0.2838	−0.2940	−0.2753	−0.2972
	(0.212)	(0.245)	(0.202)	(0.207)	(0.238)	(0.189)
$RREL_t$	−1.1910	−1.3328	−1.1454	−1.2466	−1.3790	−1.1962
	(0.037)	(0.015)	(0.021)	(0.025)	(0.010)	(0.016)
$ILLIQ_t^E$		0.0090	0.0020		0.0087	0.0004
		(0.278)	(0.827)		(0.301)	(0.968)
$ILLIQ_{t+1}^U$			−0.0184			−0.0211
			(0.027)			(0.005)
Adjusted R^2	7.006%	7.163%	8.891%	6.875%	6.981%	9.278%

Note: This table reports results of the 1-month-ahead predictive regressions of the S&P 500 Futures Index return $r_{m,t+1}$, when additional control variables are considered. $Sk_{vw,t}$ is the value-weighted average skewness. DP_t is the dividend yield of the Standard & Poor's 500 index. DEF_t represents the default spreads, calculated as the difference between Moodys Baa corporate bond yields and 10-year Treasury bond yields. $TERM_t$ is the term spread, calculated as the difference between 10-year Treasury bond yields and 3-month Treasury bill rates. $RREL_t$ is the relative three-month Treasury bill rate, calculated as the difference between current Treasury bill rate and its 12-month backward-moving average. $ILLIQ_t^E$ and $ILLIQ_{t+1}^U$ are the expected market illiquidity and unexpected illiquidity, as described in Section 4.2. Presented are the parameter estimates. The two-sided p values based on Newey-West adjusted t-statistics are in parentheses. Also reported are the adjusted R^2 values. The sample periods are June 1982 to December 2016 (Panel A) and January 2000 to December 2016 (Panel B).

TABLE 5 Comparison with economic variables

	I	II	III	IV	V	VI	VII	VIII	IX	X
Panel A: 1982–2016										
Constant	0.0093	0.012	0.0157	0.0479	0.0574	0.0092	0.0118	0.0156	0.0477	0.0572
	(0.000)	(0.001)	(0.000)	(0.005)	(0.001)	(0.000)	(0.003)	(0.000)	(0.007)	(0.002)
$r_{m,t}$						0.0131	0.0168	0.0115	0.0131	0.0072
						(0.852)	(0.816)	(0.873)	(0.854)	(0.919)
$Sk_{vw,t}$	−0.1425		−0.1580		−0.1595	−0.1420		−0.1576		−0.1591
	(0.008)		(0.003)		(0.002)	(0.013)		(0.005)		(0.005)
$ECON_{PC,t}$		0.0060	0.0071				0.0060	0.0071		
		(0.051)	(0.021)				(0.057)	(0.026)		
$ECON_{AVG,t}$				0.0534	0.0619				0.0532	0.0618
				(0.015)	(0.005)				(0.018)	(0.007)
Adjusted R^2	1.547%	0.765%	2.697%	1.045%	3.019%	1.325%	0.552%	2.473%	0.822%	2.787%
Panel B: 2000–2016										
Constant	0.0055	0.0204	0.0196	0.1630	0.1347	0.0055	0.0204	0.0196	0.1614	0.1347
	(0.055)	(0.300)	(0.269)	(0.045)	(0.103)	(0.097)	(0.284)	(0.292)	(0.068)	(0.139)
$R_{m,t}$						−0.0026	0.0481	0.0035	0.0357	−0.0022
						(0.979)	(0.656)	(0.971)	(0.744)	(0.982)
$Sk_{vw,t}$	−0.2147		−0.2021		−0.1801	−0.2154		−0.2012		−0.1806
	(0.003)		(0.014)		(0.033)	(0.004)		(0.012)		(0.025)
$ECON_{PC,t}$		−0.0137	−0.0113				−0.0138	−0.0113		
		(0.332)	(0.384)				(0.313)	(0.404)		
$ECON_{AVG,t}$				0.1929	0.1565				0.1912	0.1565
				(0.045)	(0.112)				(0.068)	(0.150)
Adjusted R^2	3.579%	1.073%	4.161%	3.102%	5.371%	3.098%	0.812%	3.681%	2.746%	4.896%

Note: This table reports results of the 1-month-ahead predictive regressions of the S&P 500 Futures Index return $r_{m,t+1}$. $Sk_{vw,t}$ is the value-weighted average skewness. The other predictors include $\text{ECON}_{PC,t}$: the first principle component and $\text{ECON}_{AVG,t}$: the equal-weighted average of 14 economic predictors described in Section 4.3. Rows without brackets show the parameter estimates. Rows with brackets show the two-sided p values based on Newey-West adjusted t statistics. The last row presents the adjusted R^2 values. The sample periods are June 1982 to December 2016 (Panel A) and January 2000 to December 2016 (Panel B).

the sample period from 2000 to 2016 (results in Panel B), we find similar result as in Panel A. The average skewness continues to perform better than the economic variables in predicting index futures return.

In-sample analysis provides a very clear indication that average skewness predicts subsequent index futures return. In the next section, we investigate its performance in terms of out-of-sample prediction and asset allocation.

5 | OUT-OF-SAMPLE EVALUATION

Following Goyal and Welch (2008) and Ferreira and Santa-Clara (2011), we predict the index futures return using a sequence of expanding windows. For the first window, we use the first s_0 observations, $t = 1, ..., s_0$. Then, for the sample ending in month $s = s_0, ..., T - 1$, we run the following predictive regression:

$$r_{m,t+1} = \mu + \vartheta' \quad X_t + \eta_{t+1}, \quad t = 1, ..., s,$$

where X_t denotes a set of predictive variables. By increasing the sample size s from s_0 to $T - 1$, we generate a sequence of $T_{OOS} = T - s_0$ out-of-sample index futures return forecasts based on the information available up to time s:

$$\hat{\mu}_{m,s}^{\{X\}} = E\left[r_{m,s+1} | X_s\right] = \hat{\mu} + \hat{\vartheta}' \quad X_s, \quad s = s_0, ..., T - 1.$$

This process mimics the way in which a sequence of forecasts is achieved in practice. We also denote by $\bar{r}_{m,s} = \frac{1}{s}\sum_{t=1}^{s} r_{m,t}$ the historical mean of market excess returns up to time s.

We evaluate the performance of the competing indicators in the forecasting exercise using several statistics. First, the out-of-sample R^2 compares the predictive power of the regression with the historical sample mean. It is defined as $R_{OOS}^{\{X\}2} = 1 - MSE_P^{\{X\}}/MSE_N$, where $MSE_P^{\{X\}} = (1/T_{OOS})\sum_{t=s_0}^{T-1}(r_{m,t+1} - \hat{\mu}_{m,t}^{\{X\}})^2$ is the mean square error of the out-of-sample predictions based on the model, $MSE_N = (1/T_{OOS})\sum_{t=s_0}^{T-1}(r_{m,t+1} - \bar{r}_{m,t})^2$ is the mean square error based on the sample mean (assuming no predictability). The out-of-sample $R_{OOS}^{\{X\}2}$ takes positive (negative) values when the model predicts returns with higher (lower) accuracy than the historical mean. We also use the encompassing ENC test statistic proposed by Harvey, Leybourne, and Newbold (1998) and Clark and McCracken (2001) and defined as

$$ENC^{\{X\}} = \frac{T_{OOS} - k + 1}{T_{OOS}} \frac{\sum_{t=s_0}^{T-1}\left[(r_{m,t+1} - \bar{r}_{m,t})^2 - (r_{m,t+1} - \bar{r}_{m,t})\left(r_{m,t+1} - \hat{\mu}_{m,t}^{\{X\}}\right)\right]}{MSE_P^{\{X\}}}. \tag{5}$$

Under the null hypothesis, the forecasts based on the historical mean encompass the forecasts based on the model, meaning that the model does not help to predict index futures returns. Because the test statistic has a nonstandard distribution under the null hypothesis in the case of nested models, we rely on the critical values computed by Clark and McCracken (2001).

The performances of the competing predictors are also compared using an out-of-sample trading strategy based on predictive regressions, which combines the index futures and the risk-free asset (1-month Treasury bill; Ferreira & Santa-Clara, 2011). For each period, predictions of market excess returns are used to calculate the Markowitz optimal weight on the index futures:

$$w_{m,s}^{\{X\}} = \frac{\hat{\mu}_{m,s}^{\{X\}}}{\lambda \quad \hat{V}_{m,s}^{\{X\}}}, \tag{6}$$

where λ is the risk aversion and $\hat{V}_{m,s}^{\{X\}}$ is the corresponding sample variance of market return.[8] Portfolio decisions can be made in real time with data available at the time of the decision. The ex post portfolio excess return is then calculated at the end of month $s + 1$ as follows:

$$r_{p,s+1}^{\{X\}} = w_{m,s}^{\{X\}} \quad r_{m,s+1}. \tag{7}$$

After iterating this process until the end of the sample ($T - 1$), we obtain a time series of ex post excess returns for each optimal portfolio. Denoting by $\bar{r}_p^{\{X\}}$ the sample mean and by $\sigma_p^{\{X\}2}$ the sample variance of the portfolio return, we define two statistics to evaluate the performance of the trading strategies: the Sharpe ratio, $\mathrm{SR}^{\{X\}} = \bar{r}_p^{\{X\}}/\sigma_p^{\{X\}}$, which measures the risk-adjusted performance of the strategy, and the certainty equivalent return, $\mathrm{CE}^{\{X\}} = \bar{r}_p^{\{X\}} - (\lambda/2)\sigma_p^{\{X\}2}$, which is the risk-free return that a mean-variance investor (with risk aversion λ) would consider equivalent to investing in the strategy. To test whether the SR of the strategy based on predictor X is equal to the SR of the strategy based on the historical mean of market return, denoted by SR_0, we follow the approach of Jobson and Korkie (1981) and DeMiguel, Garlappi, and Uppal (2009). We proceed in a similar way to test whether the CE of the strategy based on X is equal to the CE of the strategy based on the historical mean of market excess return, denoted by CE_0.[9] Finally, we compute the annual transaction fee generated by each strategy as $Fee^{\{X\}} = \frac{12f}{T_{OOS}}\sum_{t=s_0}^{T-1}|w_{m,t+1}^{\{X\}} - w_{m,t+}^{\{X\}}|$, where f is the fee per dollar and $w_{m,t+}^{\{X\}}$ denotes the market weight just before rebalancing at $t + 1$.

Table 6 reports the results for the out-of-sample predictions based on the variance and skewness measures introduced in Section 3 and the economic predictors introduced in Section 4.3. We consider the January 2000–December

[8]Following Campbell and Thompson (2008), we impose realistic portfolio constraint: $w_{m,s}^{\{X\}}$ lies between 0 and 2 to exclude short sales and to allow for at most 100% leverage. We also use 5-year rolling windows of past monthly returns to estimate $\hat{V}_{m,s}^{\{X\}}$.

[9]To test the null hypothesis that $\mathrm{SR}^{\{X\}} = \mathrm{SR}_0$, we use the statistic given in footnote 16 of DeMiguel et al. (2009). Similarly, to test the null hypothesis that $\mathrm{CE}^{\{X\}} = \mathrm{CE}_0$, we use the statistic for the test of equal CE is given in their footnote 18.

TABLE 6 Out-of-sample performances based on predictive regressions of index futures return

	β	*p* Val	Adj. R^2 (%)	R^2_{OOS} (%)	ENC	SR	SR *t* stat	CE (%)	CE *t* stat	SR gain	SR gain *t* stat	CE gain (%)	CE gain *t* stat	Ann. fee (%)	Ann. ret. (%)
Buy-&-Hold						0.24	−0.13	2.82	−0.18	0.44	1.52	8.66	2.69	0.00	5.53
$r_{m,t}$	0.04	0.62	−0.30	−1.27	−1.06	−0.10	−2.99	−3.28	−2.86	0.11	0.79	2.55	0.96	0.24	−0.18
$Sk_{vw,t}$	−0.21	0.00	3.58	2.78	6.26	0.47	3.62	6.73	4.03	0.67	6.10	12.57	5.05	0.75	9.76
$Sk_{ew,t}$	−0.16	0.02	1.88	−0.98	−0.47	−0.07	−2.11	−3.54	−2.45	0.14	0.75	2.30	0.87	0.37	0.35
$V_{vw,t}$	−0.05	0.48	−0.29	−2.99	0.75	0.08	−1.02	0.71	−0.88	0.29	1.75	6.55	2.28	0.26	2.82
$V_{ew,t}$	−0.05	0.59	−0.37	−0.08	−0.06	−0.27	−2.40	−6.32	−2.42	−0.06	−1.21	−0.48	−0.42	0.06	−3.10
$V_{m,t}$	−20.50	0.19	1.29	0.82	1.97	−0.12	−3.40	−2.51	−2.96	0.09	0.00	3.33	0.85	0.17	−0.12
$Sk_{m,t}$	−0.01	0.11	0.28	0.63	1.17	0.09	−0.88	0.31	−1.26	0.30	1.73	6.14	2.20	0.56	3.23
ECON_{PC}	0.03	0.05	1.68	0.70	1.11	−0.07	−2.15	−2.81	−1.51	0.13	0.87	3.03	4.21	0.04	0.35
ECON_{Avg}	0.19	0.05	3.10	1.54	2.08	−0.10	−2.51	−3.06	−1.62	0.11	0.53	2.77	3.67	0.05	0.00

Note: This table reports the following results: the in-sample regression result, the out-of-sample R^2_{OOS}, the *ENC* statistics, which is the encompassing test of Harvey et al. (1998) and Clark and McCracken (2001) defined as *ENC*. The annualized Sharpe ratio and the annualized certainty equivalence for index futures return forecasts at monthly frequency from predictive regressions with an expanding window. The Sharpe ratio (SR) gain is the Sharpe ratio gain of a trading strategy based on different return forecasts relative to that obtained with the historical mean return. The certainty equivalent (CE) gain is the portfolio gain of a trading strategy based on different return forecasts relative to that obtained with the historical mean return. It also reports t-statistics for SR and CE, and the gains in SR and CE. The annual transaction fee, obtained by assuming an $f = 10$ basis point fee, and the annualized return of each strategy are also reported. The predicted variable is the 1-month-ahead S&P 500 Futures Index return. The risk-aversion parameter λ is equal to 2. The evaluation in- and out-of-sample period is from January 2000 to December 2016. Estimation period for variance is 5 years. Critical values for the encompassing test statistics are from Clark and McCracken (2001) (Table 1).

2016 sample to compute the performance of these alternative predictors. Consistently with individual regressions reported in Table 2, the predictor with the highest out-of-sample R^2 is the value-weighted average skewness, with $R^2_{OOS} = 2.78\%$. The encompassing test *ENC* confirms that the value-weighted average skewness is a statistically significant predictor of index futures return at the 5% significance level. Although the other predictors also generate positive R^2_{OOS}, such as 1.54% for the simple average of 14 economic variables, their predictability fails to pass the encompassing test.

Figure 3 displays the temporal evolution of the out-of-sample performance of the predictors over time. The performance is measured by the difference between the cumulative sum of squared errors (SSE) generated by the sample mean and the cumulative SSE generated by a given set of variables. The out-of-sample performance is measured by $OOS_s^{\{X\}} = \sum_{t=1}^{s}(r_{m,t+1} - \bar{r}_{m,t})^2 - \sum_{t=1}^{s}(r_{m,t+1} - \hat{\mu}_{m,t}^{\{X\}})^2$, $s = s_0, \ldots, T-1$, where the mean $\bar{r}_{m,t}$ and the prediction $\hat{\mu}_{m,t}^{\{X\}}$ are calculated over the period from s_0 to t. An increase in the line indicates that the model provides a better prediction than the prevailing mean. Several results are worth emphasizing. First, as Figure 3 demonstrates, the OOS measures

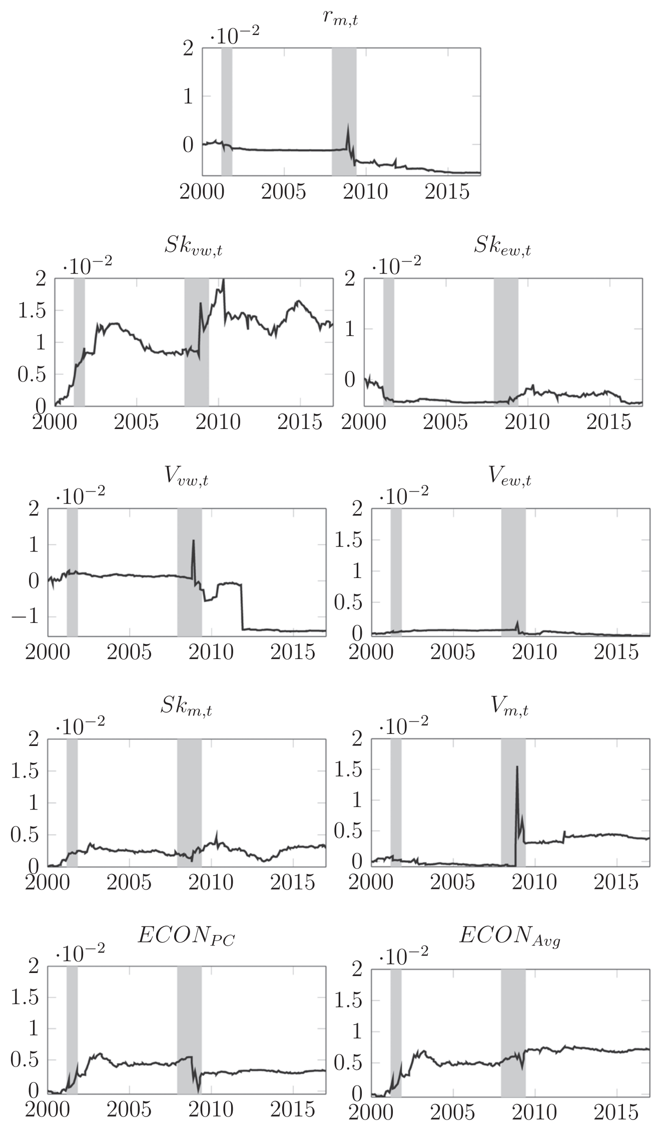

FIGURE 3 Dynamics of cumulative sum of squared errors. This figure presents the out-of-sample predictive performance of the alternative models. The performance is measured by the difference between the cumulative SEE generated by a given set of variables and the cumulative SEE generated by the prevailing sample mean. Out-of-sample forecasts begins from January 2000. Sample: 2000–2016. NBER recessions are represented by shaded bars. SSE, sum of squared errors

involving the index futures return, the average variance, and market variance jump up during the subprime crisis, reflecting some instability in the relation between these predictors and the subsequent index futures return. The out-of-sample performance of the index futures return and the average variance are even negative for relatively long periods of time. In contrast, for the value-weighted average skewness, the OOS measure increases in a smooth way, which reflects the stable relation between this variable and the subsequent index futures return. Especially during the subprime crisis, the value-weighted average skewness generates a higher out-of-sample performance. Based on the evolution of predictive performance, the value-weighted average skewness is clearly the best predictor, with an almost continuously increasing out-of-sample performance.

The table also reports performance measures of the trading strategies, that is the SR, CE, the SR gain, the CE gain, and the annualized return. The annualized SR of the strategy based on the historical mean is equal to −0.2. The SR is increased to 0.47 for the strategy based on the value-weighted average skewness. The increase in SR relative to the strategy based on the historical mean is statistically significant only for this strategy. Most of the annualized CE values of the strategy based on one predictor are in the range [−3%; 1%], and the highest CE value is generated by strategy based on value-weighted average skewness with CE value of 6.73%. The CE gain relative to the strategy based on the historical mean is statistically significant only for the strategy based on the value-weighted average skewness.

We also consider a simple Buy-and-Hold strategy, with a constant weight computed to produce the same volatility of the portfolio return as the strategy based on market return and average skewness. The SR of this strategy is equal to 0.24. The difference between the SR of the strategy based on value-weighted skewness and the SR of the Buy-and-Hold strategy is sizable (0.47 vs. 0.24), although not statistically different from 0. However, the difference in CE (6.73% vs. 2.82%) is statistically positive at the usual 5% significance level.

Finally, the annual fee that an investor would have to pay for implementing the various strategies is moderate, between 0.04% and 0.75% of the value of the portfolio per year. The strategy based on the value-weighted average skewness generates relatively large annual fee because they imply more rebalancing every month. But we also note that the annual return of the strategy based on value-weighted skewness is the highest with a value equal to 9.76%. This result suggests that, although transaction costs should not be neglected, they are unlikely to substantially reduce the relative performance of the trading strategy. To summarize, the strategy with value-weighted average skewness generates superior economic performance in terms of SR, CE values, and annual returns relative to other competing predictors.

6 | CONCLUSION

In this paper, we investigate the ability of the average skewness of the firm's returns to predict S&P 500 index futures returns. We find that the value-weighted average of (standardized) stock skewness is by far the best predictor of monthly index futures returns. The magnitude of the effect of the monthly average skewness on the subsequent index futures return is sizable, as a one standard deviation increase in the average skewness implies, on average, a 0.59% decrease in the index futures return next month. This result is robust to the alternative sample periods or controls that we consider.

The predictability of the average skewness is also economically significant: An investor implementing a mean-variance strategy based on market return forecasts would obtain a higher Sharpe ratio if she predicts the future market return with the current value-weighted average skewness. The annualized returns are equal to 9.76%, with an annualized Sharpe ratio and a certainty equivalent equal to 0.47% and 6.73%, respectively.

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DATA AVAILABILITY STATEMENT

All data that support this study are available from the Datastream (Index Futures data) and CRSP (firm-level stock data and treasury bill rate). The data for replicating the main findings of this study are available at Dryad (doi:10.5061/dryad.866t1g1n1).

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