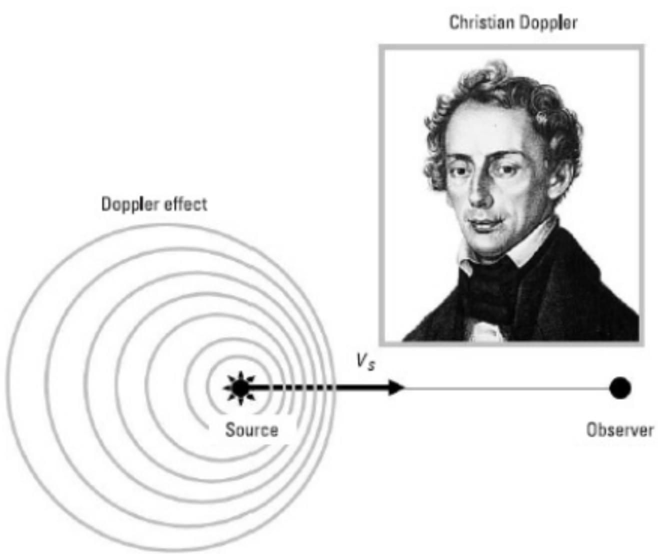
# $2^{nd}$ DSP Challenge

- Often the most relevant targets exhibit relative motion with respect to the radar sensor;
- The motion of the target, the motion of the platform or the combination of both affects the radar return;
- The relative target velocity information is useful in several applications;
- A processing strategy is needed to extract the targets' velocity information.

# Doppler Effect

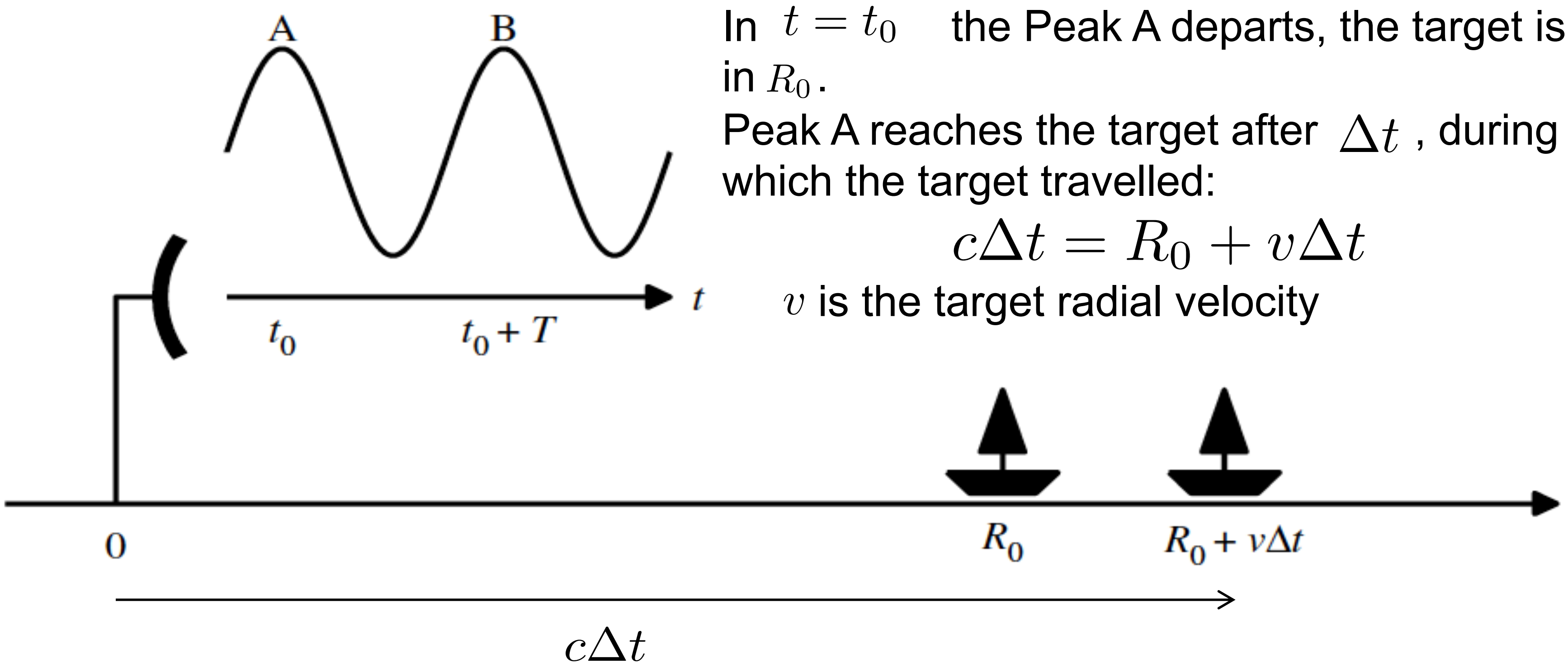



Studied by Christian Doppler in astronomy, in a binary stars system those approaching the Earth appear bluer while those moving away appear redder;

In 1843 the Doppler effect was demonstrated measuring the sound from a train whistle with different velocities.

- A Radar transmits a EM signal and receives the return from an object (target);
- If the object is moving the received signal will be frequency shifted;
- The Doppler shift is determined by the target radial velocity, or rather the velocity component in the Line Of Sight (Los);
- It is generally measured through the use of the Fourier Transform of the received signal, from it then the target radial velocity can be obtained.

# Doppler Effect



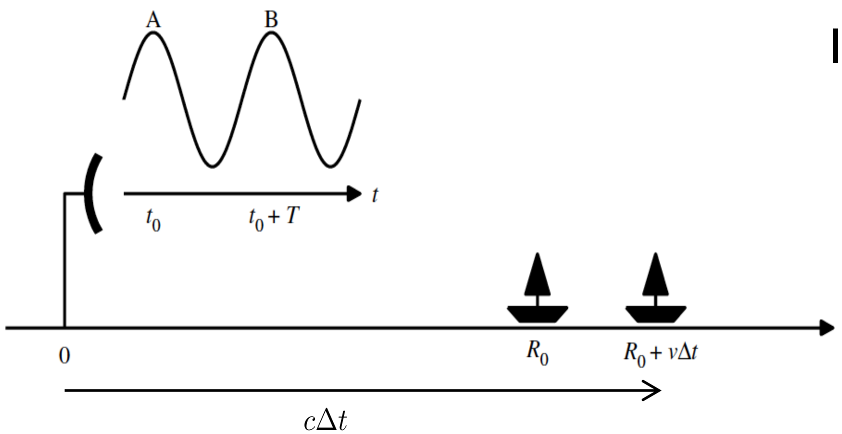

In $t = t_0$ the Peak A departs, the target is in $R_0$.
Peak A reaches the target after $\Delta t$, during which the target travelled:

$$c\Delta t = R_0 + v\Delta t$$

$v$ is the target radial velocity

The signal's travel time is $\Delta t = \dfrac{R_0}{c - v}$

# Doppler Effect




In $t = t_1$ the Peak A returns to the radar

$$t_1 = t_0 + 2\Delta t = t_0 + \frac{2R_0}{c - v}$$

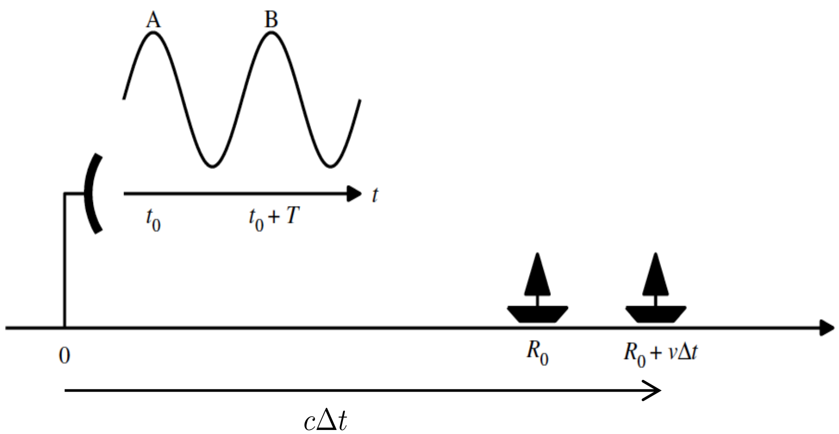
Similar considerations can be done for Peak B, departed after $T$

$$t_2 = t_0 + T + \frac{2R_1}{c - v}$$

$R_1$ is the target location when B leaves the radar and $T$ is the period of the transmitted sinusoidal waveform.

Note that $R_1 = R_0 + vT$

# Doppler Effect



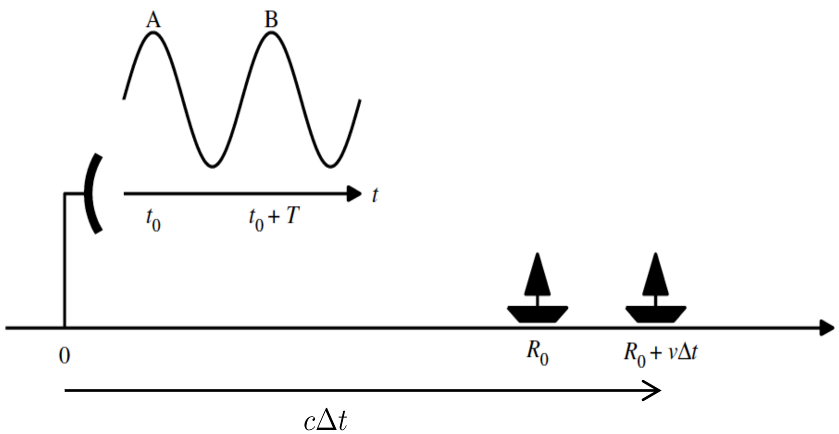

The period of the received waveform is equal to, to the difference between the arrival times of the two peaks.

$$T_R = t_2 - t_1 = t_0 + T + \frac{2(R_0 + vT)}{c - v} - \left( t_0 + \frac{2R_0}{c - v} \right) = T\frac{c + v}{c - v}$$

The ratio between received and transmitted period is:

$$\frac{T_R}{T} = \frac{c + v}{c - v} \longrightarrow \frac{f_R}{f_0} = \frac{c - v}{c + v} = \frac{1 - v/c}{1 + v/c}$$

# Doppler Effect




The received frequency is then

$$f_R = f_0 \frac{1 - v/c}{1 + v/c}$$

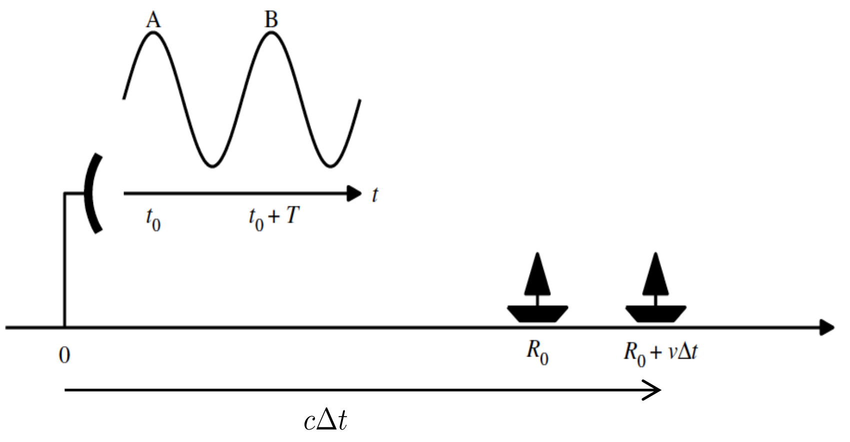
As it is expected $v \ll c$ then the following approximation can be used

$$\frac{1}{1 + v/c} = 1 - \frac{v}{c} + \frac{v^2}{c^2} - \cdots$$

thus

$$f_R = f_0 \left(1 - \frac{v}{c}\right) \left(1 - \frac{v}{c} + \frac{v^2}{c^2} - \cdots\right) = f_0 \left(1 - \frac{2v}{c} + \cdots\right) \approx f_0 \left(1 - \frac{2v}{c}\right)$$

# Doppler Effect




Rewriting we get

$$f_R \approx f_0 - \frac{2v}{c/f_0} = f_0 - \frac{2v}{\lambda}$$

Where $\lambda$ is the transmitted wavelength

Finally the Doppler shift is given by

$$f_D = f_R - f_0 \approx -\frac{2v}{\lambda}$$

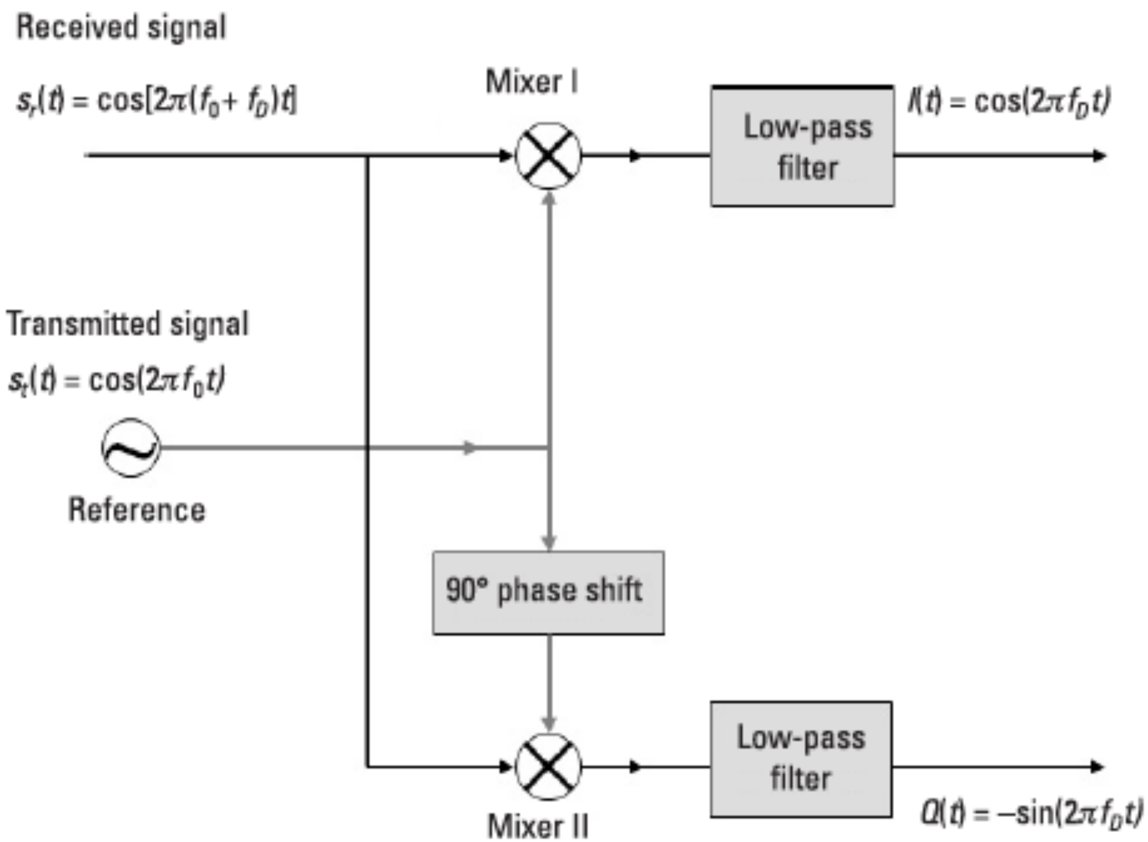
The sign depends on the relative direction of travel

# Doppler Effect and Phase retrieval

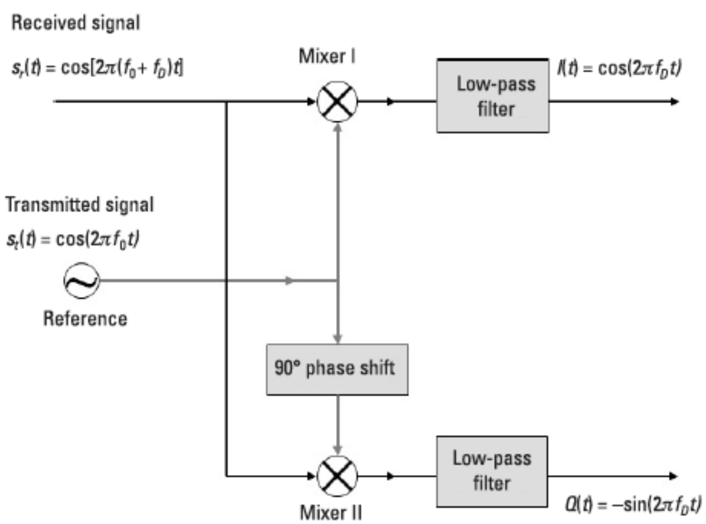
- The relative motion of a target introduces the Doppler effect in the received radar echo;
- The Doppler shift information resides in the phase of the received signal;
- Before doing any other processing on the signal, the phase information must be retrieved.

# Quadrature Detector

The coherent receiver is used to measure correctly both amplitude and phase of the received signal.
It leads to the formulation of the canonical form of the received signal



# Quadrature Detector



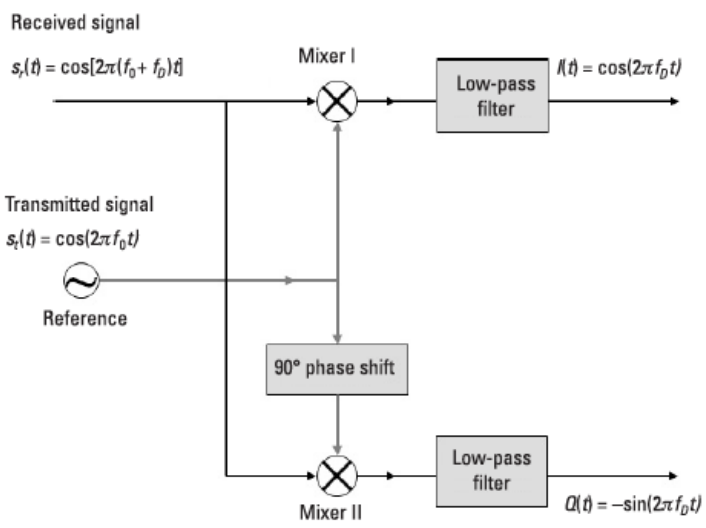

In the quadrature detector, the received signal is split into two mixers called synchronous detectors. In the synchronous detector I, the received signal is mixed with a reference signal, the transmitted signal; in the other channel it is mixed with a 90 degree shift of the transmitted signal

The received signal is:

$$s_r(t) = a\cos[2\pi(f_0 + f_D)t] = a\cos[2\pi f_0 t + \phi(t)]$$

$$\phi(t) = 2\pi f_D t$$

# Quadrature Detector




By mixing with the transmitted signal

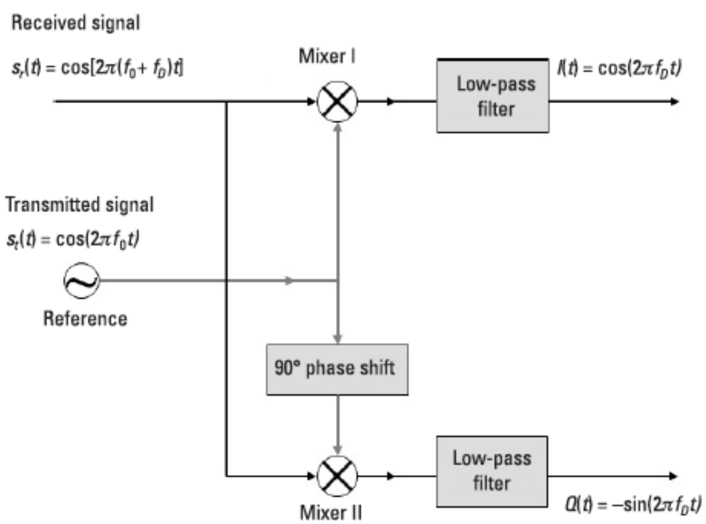
$$s_t(t) = \cos(2\pi f_0 t)$$

$$s_r(t)s_t(t) = \frac{a}{2}\cos[4\pi f_0 t + \phi(t)] + \frac{a}{2}\cos\phi(t)$$

After Low Pass Filtering:

$$I(t) = \frac{a}{2}\cos\phi(t)$$

# Quadrature Detector




By mixing with the shifted transmitted signal

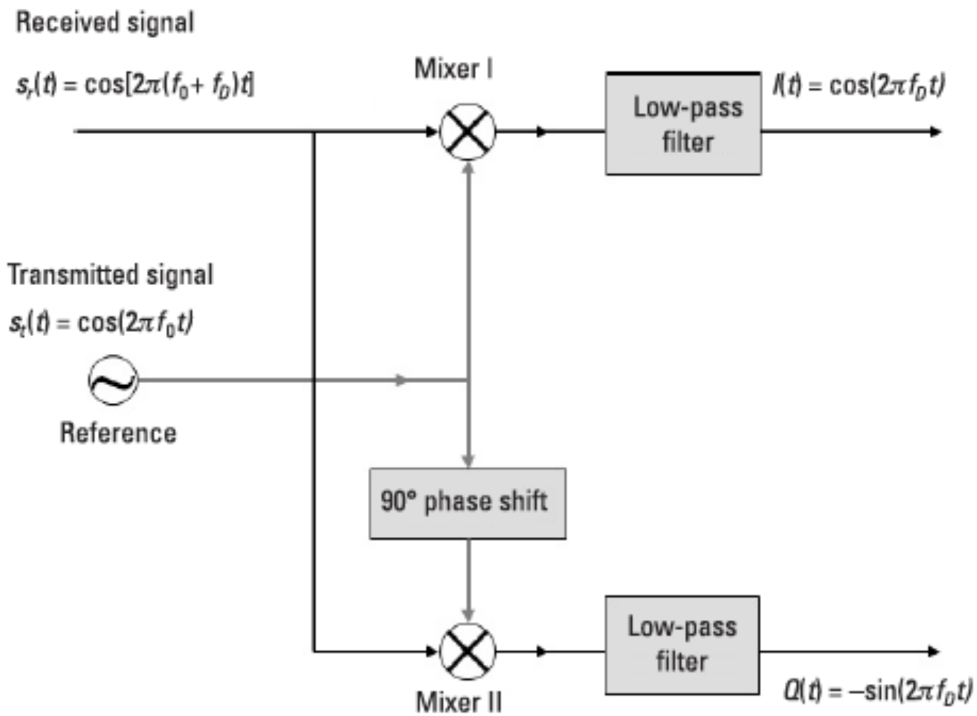
$$s_t^{90} = \sin(2\pi f_0 t)$$

$$s_r(t)s_t^{90} = \frac{a}{2}\sin[4\pi f_0 t + \phi(t)] - \frac{a}{2}\sin\phi(t)$$

After Low Pass Filtering:

$$Q(t) = -\frac{a}{2}\sin\phi(t)$$

# Quadrature Detector




Combining I and Q we are able to reconstruct the Doppler shifted signal

$$s_D(t) = I(t) + jQ(t) = \frac{a}{2}\exp[-j\phi(t)] = \frac{a}{2}\exp(-j2\pi f_D t)$$

Using the Fourier transform now is possible to estimate the Doppler shift

# Matched Filter for Doppler shifted returns

- A signal reflected from a moving target is Doppler affected
- Without exact knowledge of the Doppler shift, the radar receiver cannot modify its matched receiver to the new carrier frequency exactly, and mismatch occurs.
- We analyze the output of the matched filter $u_o[n]$ when the input complex envelope contains a Doppler frequency shift, $\nu$ .
- The Doppler-shifted complex envelope $u_D[n]$ is therefore

$$u_D[n] = u[n]exp(j2\pi\nu n)$$

# Matched Filter for Doppler shifted returns

We can write the output of the matched filter as function of the Doppler shift.

$$u_o[n,\nu] = \sum_{-\infty}^{\infty} u[k]exp(j2\pi\nu k)u^*[n-k]$$

And reversing the role of $n$ and $k$

$$u_o[k,\nu] = \sum_{-\infty}^{\infty} u[n]exp(j2\pi\nu n)u^*[k-n]$$

This function is known as ambiguity function.

# Ambiguity Function

The ambiguity function (AF) has an important practical meaning,  it describes the output of a matched filter when the input signal is delayed by k  and Doppler shifted by v  relative to nominal values for which the matched  filter was designed.

$$\chi_o[k,\nu] = \sum_{-\infty}^{\infty} u[n] exp(j2\pi\nu n) u^*[k-n]$$

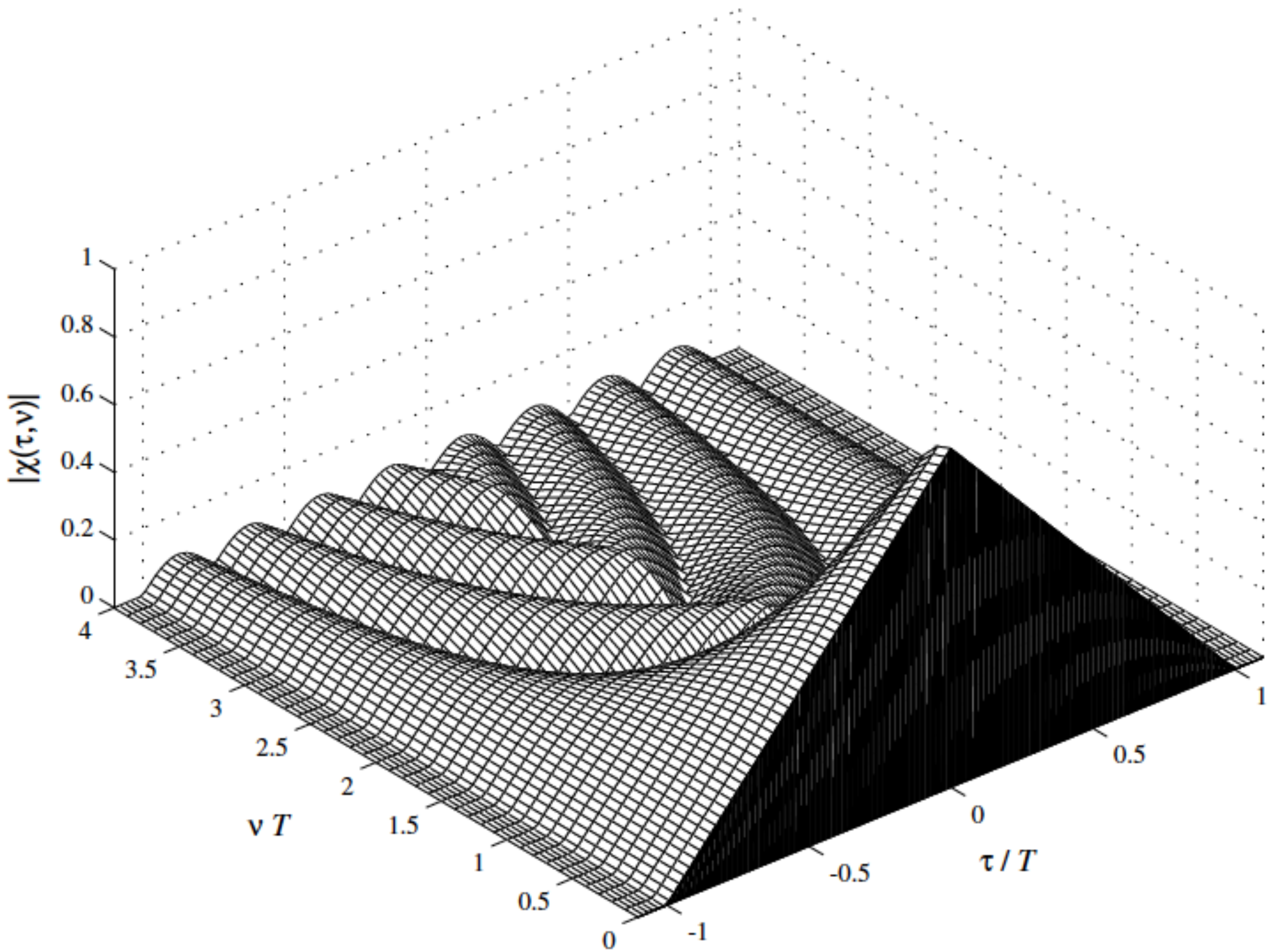
The standard form,  is

$$|\chi_o[k,\nu]|^2 = \left| \sum_{-\infty}^{\infty} u[n] exp(j2\pi\nu n) u^*[k-n] \right|^2$$

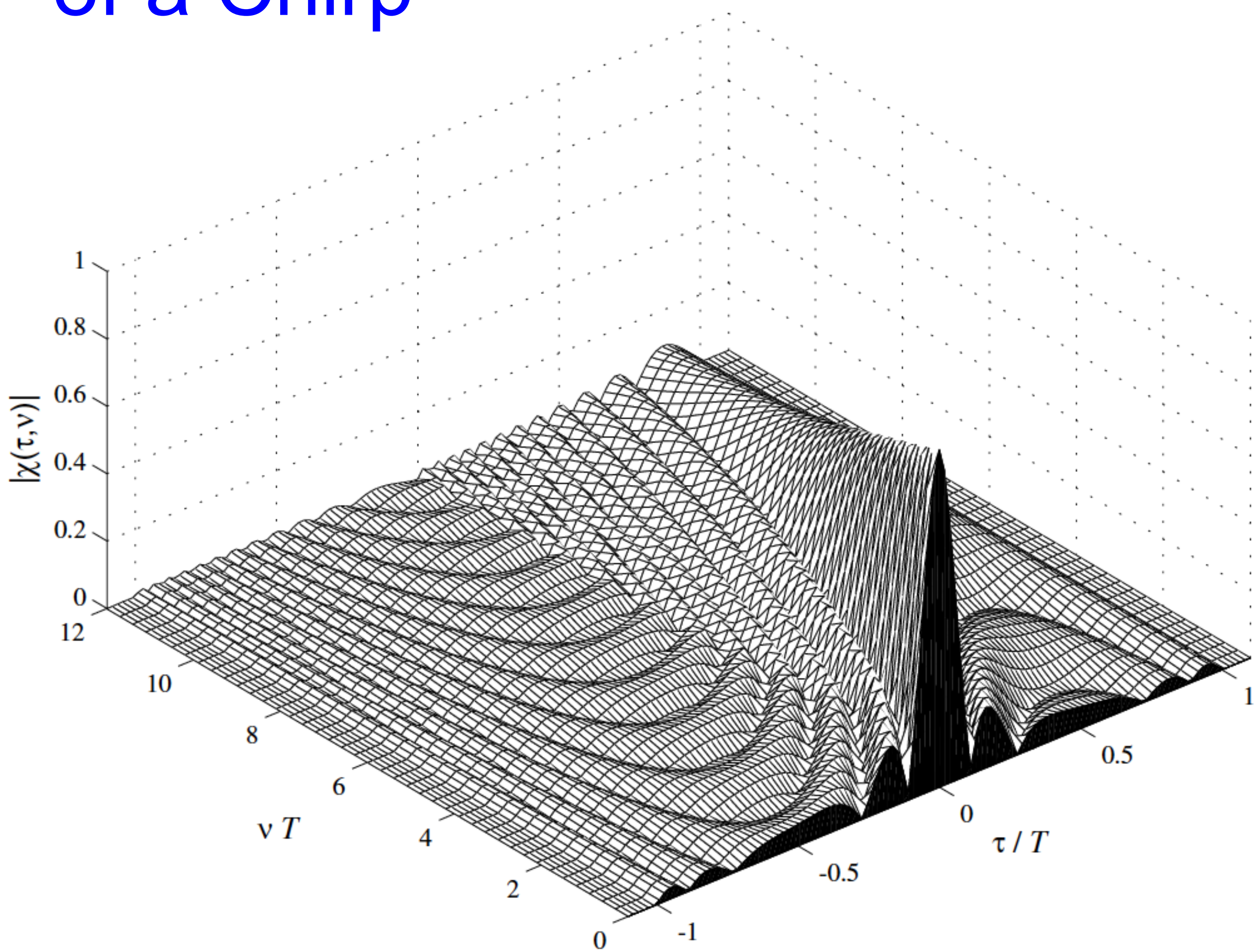
The form without the squared value is also used for better representation

# AF of a Square Pulse

Constant Frequency Pulse



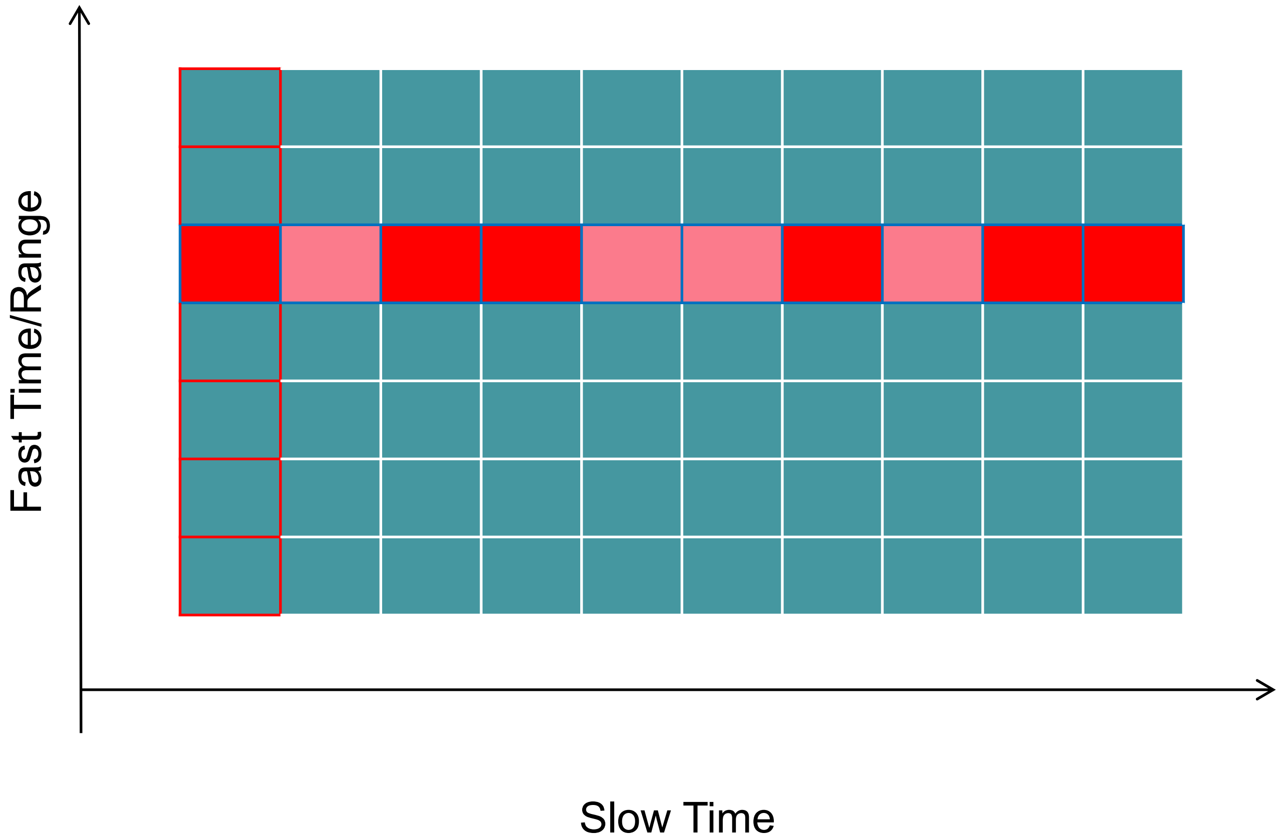
# AF of a Chirp



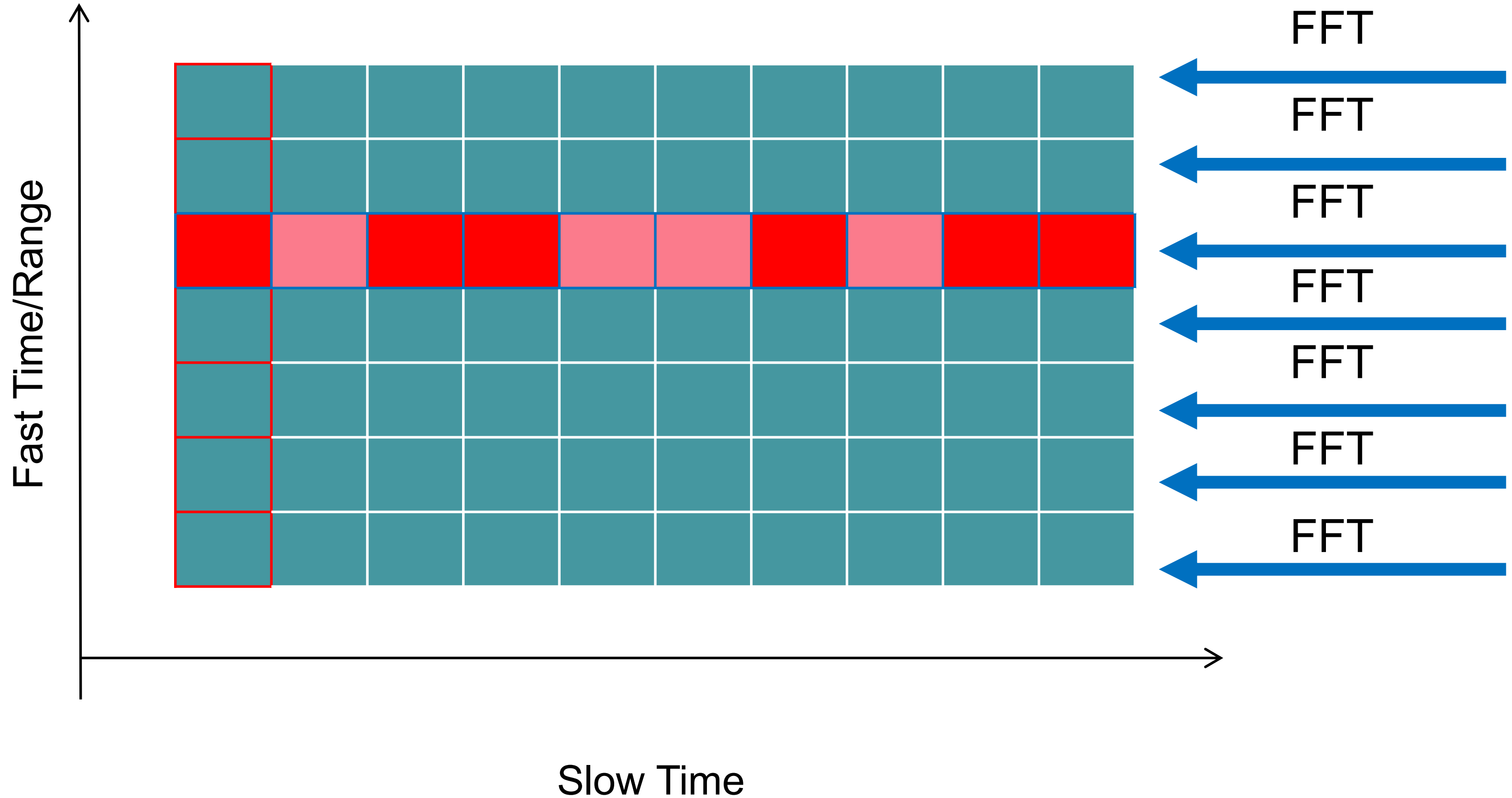

Range-Doppler coupling is a drawback of this technique, however in many applications the the delay error is acceptable and makes the chirp a Doppler tolerant technique

# Range-Doppler Maps

- In a moving target the phase information appears in each received pulse.
- Different returns can be separated in the Doppler domain

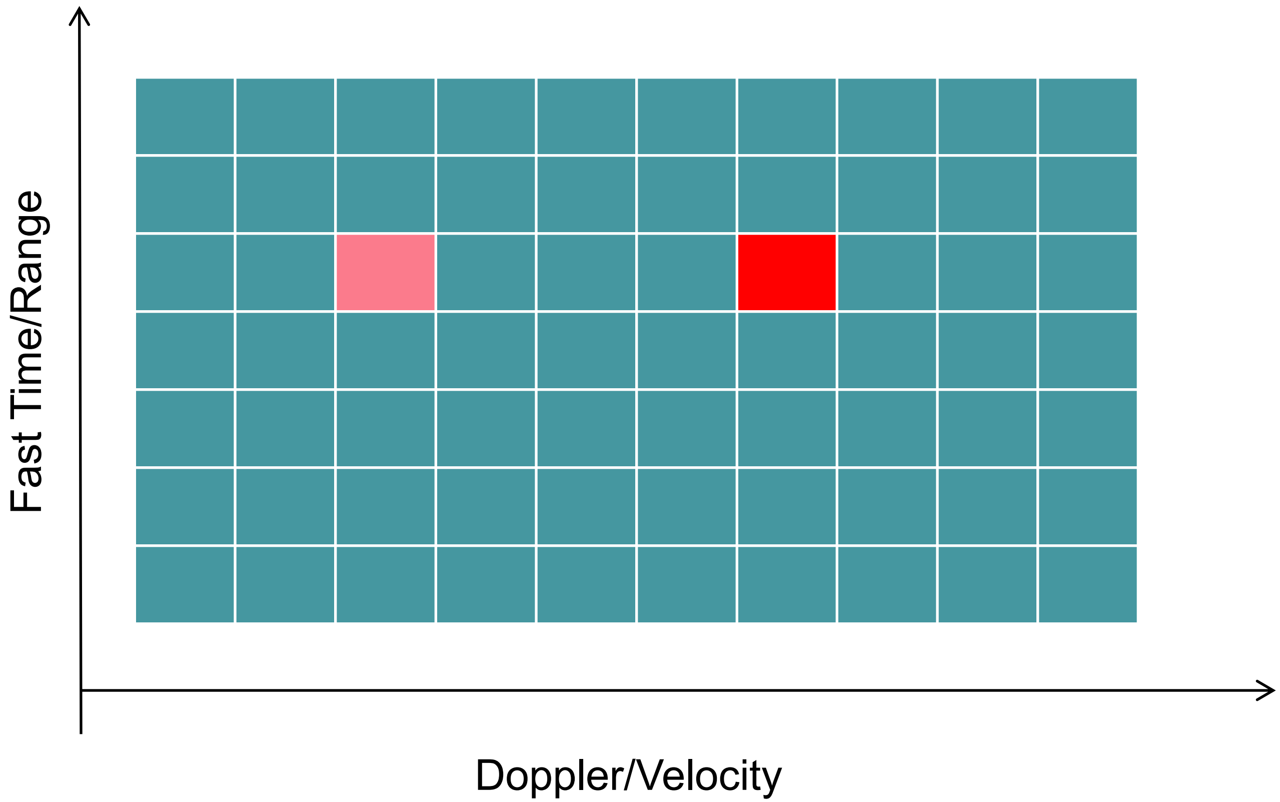


# Range-Doppler Maps



# Range-Doppler Maps

- Range-Doppler maps provide a powerful tool to separate targets in both range and velocity domains.




Two targets at the same range can be separated in the Doppler domain

# 2nd DSP Challenge – Solved!

- The Doppler effect can be measured and allows us to extract the relative radial velocity of a target;
- Coherent demodulation makes feasible the phase retrieval and thus the Doppler estimation;
- The ambiguity function provides waveform performance to Doppler shifted returns;
- The data matrix can be used to generate Range-Doppler maps to separate targets both in Range and Doppler domains;
- Mismatched filters can be used to maximize the SNR and filtering can be used to remove clutter from the received signal.

# Worked Example – Speed Resolution

A radar working at a carrier frequency of 10 GHz uses a PRF of 2 kHz. It performs Doppler processing (FFT in slow time) with a Coherent Pulse Interval of 256 samples. What is the maximum unambiguous detectable speed by the radar ? What is the speed resolution of the radar ?

- In order to compute the maximum unambiguous detectable speed depends on the maximum unambiguous Doppler. The sampling frequency in slow time is the PRF, thus according to Nyquist theorem the maximum unambiguous Doppler is PRF/2 = 1kHz. The maximum unambiguous speed is then:

$$\lambda = \frac{c}{f_0} = 3cm$$

$$f_{d_{max}} = \pm 1000Hz; \quad f_{d_{max}} = \pm \frac{-2v_{max}}{\lambda}$$

$$v_{max} = \frac{\pm f_{d_{max}} \lambda}{2} = \pm 15m/s$$

# Worked Example – Speed Resolution

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- The speed resolution depends on the DFT resolution:

$$DFT_{res} = PRF/N = 2000/256 = 7.8Hz$$

$$\Delta v = \frac{DFT_{res}\lambda}{2} = 0.1172m/s$$

So the resolution in speed is 0.1172 m/s. The same result could be obtained as

$$\Delta v = \frac{2v_{max}}{256} = 0.1172m/s$$