FORMULA SHEETS

Discrete Time Fourier Transforms

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\Omega}$$

$$x(n) = \frac{1}{2\pi} \int_{\Omega=-\pi}^{\pi} X(e^{j\Omega}) e^{jn\Omega}$$

$$x(n) --------- > X(\Omega)$$

$$\delta(n) --------- > 1$$

$$\delta(n-n_0) ---- > e^{-jn_0\Omega}$$

$$1 --------- > 2\pi\delta(\Omega)$$

$$e^{jnw_0} ------- > 2\pi\delta(\Omega - w_0)$$

$$a^n u(n), |a| < 1 \quad ---- > \quad \frac{1}{1 - ae^{-j\Omega}}$$

$$\cos(nw_0) ------- > \pi\delta(\Omega + w_0) + \pi\delta(\Omega - w_0)$$

$$\sin(nw_0) ------- > \frac{\pi}{j}\delta(\Omega - w_0) - \frac{\pi}{j}\delta(\Omega + w_0)$$

X(Ω) is periodic with period 2π
i.e. X(Ω)=X(Ω+2π)

SOME DTFT PROPERTIES

$$ax[n] + by[n] ----------- > aX(\Omega) + bY(\Omega)$$

$$x[n-n_0] ------------- > e^{-jn_0\Omega} X(\Omega)$$

$$x[-n] ---------------- > X(-\Omega)$$

$$e^{jnw_0} x[n] ------------- > X(\Omega - \omega_0)$$

$$x[n] * y[n] ------------ > X(\Omega)Y(\Omega)$$

$$nx[n] --------------- > je^{j\Omega} \frac{dX(\Omega)}{d\Omega}$$

$$x[n]y[n] ------------- > \frac{1}{2\pi} \int_{\theta=-\pi}^{\pi} X(\theta)Y(\Omega - \theta) d\theta$$

Table of one-sided Z-Transforms

$$X(z)=\sum_{n=0}^{\infty}x(n)z^{-n}$$

$x[n] \quad --> X(z)$

$x[n-i]--> z^{-i}X(z)$

$\delta[n] \quad --> \ 1$

$\delta[n-i] --> z^{-i}$

$u[n] \quad --> z/(z-1)$

$a^{n}u[n] \quad --> z/(z-a)$

$a^{n}x[n] \quad --> X(z/a)$

$nx[n] \quad --> -z\dfrac{d[X(z)]}{dz}$

$a^{n}\,\mathrm{Sin}(\beta n)\,u[n] \ --> \dfrac{za\,\mathrm{Sin}(\beta)}{z^{2}-2az\,\mathrm{Cos}(\beta)+a^{2}}$

$a^{n}\,\mathrm{Cos}(\beta n)u[n] \ --> \dfrac{z^{2}-za\,\mathrm{Cos}(\beta)}{z^{2}-2az\,\mathrm{Cos}(\beta)+a^{2}}$

Solving Difference Equations with initial conditions

y[n-1] → $z^{-1}Y(z) + y[-1]$

y[n-2] → $z^{-2}Y(z) + y[-2] + y[-1]z^{-1}$

Linear Convolution

$$\mathrm{y[n]}=\sum_{\mathrm{k}=-\infty}^{\infty}\mathrm{x[k]h[n-k]}$$

Linear Correlation

$$\mathrm{R_{x,h}[j]}=\sum_{\mathrm{k}=-\infty}^{\infty}\mathrm{x[k]h[k-j]}$$

Cyclic Convolution

$$\mathrm{y[n]}=\sum_{\mathrm{k}=0}^{\mathrm{N}-1}\mathrm{x[k]h[n-k]}$$

Cyclic Correlation

$$\mathrm{R_{x,h}[j]}=\sum_{\mathrm{k}=0}^{\mathrm{N}-1}\mathrm{x[k]h[(k-j)Mod\,N]}$$

Discrete Fourier Transform Pair

$$X[k]=\sum_{n=0}^{N-1}x[n]W_N^{nk} \qquad k=0,1,2...N-1$$

$$x[n]=\frac{1}{N}\sum_{k=0}^{N-1}X[k]W_N^{-nk} \qquad n=0,1,2....N-1$$

$$W_N=e^{-j\frac{2\pi}{N}}$$

N-point Windows:

$0 \le n \le N-1$

$$w_r[n] = 1$$

$$w_{hann}[n] = 0.5 - 0.5\cos(\frac{2\pi n}{N-1})$$

$$w_{hamm}[n] = 0.54 - 0.46\cos(\frac{2\pi n}{N-1})$$

$$w_{bl}[n] = 0.42 - 0.5\cos(\frac{2\pi n}{N-1}) + 0.08\cos(\frac{4\pi n}{N-1})$$

$$w_{kai}[n] = \frac{I_0[\beta(1-\{(n-\alpha)/\alpha\}^2)^{1/2}]}{I_0[\beta]}$$

Additional Formulae:

$$\sum_{i=0}^{n} \alpha^i = \frac{1-\alpha^{n+1}}{1-\alpha}$$

cos(a+b)= cos(a) cos(b) – sin(a) sin(b)
cos(a-b)= cos(a) cos(b) + sin(a) sin(b)
sin(a+b) = sin(a) cos(b) + cos(a) sin(b)
sin(a-b) = sin(a) cos(b) - cos(a) sin(b)
$\cos^2(a) = 0.5 [1 + \cos(2a)]$

N: Number of Coefficients, Δf: Normalised Transition BW A_s : Stopband Attenuation

Window	N(Δf) Hz	A_s dBs
rectangular	0.9	-21
Hanning	3.1	-44
Hamming	3.3	-53
Blackman	5.5	-74
Kaiser b=2	1.5	-29
b=4	2.6	-45
b=6	3.8	-63
b=10	5.1	-81
b=12	6.4	-99