Block 4: Tutorials: Digital Filters

Q1: A major problem in the recording of electrcardiograms (ECGs) is the appearance of unwanted 60-Hz interference in the output. The causes of this power line interference include magnetic induction, displacement currents in the leads on the body of the patient and equipment interconnections. It is known that the signal of interest is band limited such that $X(f)=0$ for $|f| > 1000Hz$ (maximum frequency of interest is 1000Hz).

The analog signal is converted into a discrete time signal with an ideal A/D converter using a sampling frequency of f_s Hz. The resulting signal x[n] is then processed by 2 different digital filters to produce an output y[n].

Design the two Digital Filters for removing the 60-Hz interference by specifying values for f_s, the 'a' and 'b' vectors so that the 60-Hz signal of the form '$A\ sin(120\pi t)$' will not appear in the output y[n].

(i) Digital 1 (FIR)

2 zeros at $\Omega=\pm\ \Omega_1$ to eliminate the interference and 2 poles at z=0 to ensure the filter is Causal.

(ii) Digital Filter 2 (IIR)

2 zeros at $\Omega=\pm\ \Omega_1$ to eliminate the interference and 2 poles at $\Omega=\pm\ \Omega_1$ of length 'r' to narrow the transition BW.

Comment on the advantages and disadvantages of each Digital Filter.

Q2: Use the Impulse Invariance method to design digital filters for the following two continuous systems using sampling intervals of (a) $T_1=1$ sec and (b) $T_2=0.1$ sec: $H_1(s)=1/(s+1)$, $H(w) = 1/(jw+1)$, $h(t) = exp(-t)\ u(t)$ and $H_2(s)= 1/(s^2+3s+2)$, $H(w) = 1/(2-w^2 + 3jw)$, $h(t) = exp(-t)\ u(t) + exp(-2t)\ u(t)$

Make an approximate sketch of the magnitude of the frequency spectrum for the digital filters. In your design you are requested to produce the appropriate difference equation and a realisation of the digital filter.

Q3: Ignoring the frequency warping design two digital filters from the system functions described in Q27 using the Bilinear Transformation mapping and a sampling interval T=0.1 seconds.

Q4: Given a Low Pass RC filter with RC=1. Design a low pass discrete time filter using the bilinear transformation method such that its 3-dB bandwidth (cut-off frequency) is $\pi/4$ i.e $\Omega_{3dB}=\pi/4$.

Confirm your design by evaluating the frequency response of the digital filter at $\Omega=0$ and $\pi/4$.

Draw the canonical realisation of the digital filter.

Q5: A second order Butterworth prototype filter having a -3dB low pass cut-off at 1 rad/sec has a system function

$$H(s) = \frac{1}{s^2 + \sqrt{2}\,s + 1}$$

(i) Using the Bilinear Transformation show that an equivalent sampled data digital filter sampling at 10^3 rad/sec with cut off frequency at 100 rad/s has a transfer function

$$H(z) = \frac{0.067z^{-2} + 0.135z^{-1} + 0.067}{0.413z^{-2} - 1.141z^{-1} + 1}$$

(ii) Write a difference equation for this digital filter and construct a canonical realisation of the difference equation.

Q6: A Linear Phase FIR digital band Pass Filter is to be designed using the FIR windowing design method. The specifications are

fs=1024Hz
Lower StopBand 3dB cutoff 128Hz Upper StopBand 3dB cutoff 384Hz
3dB Passband cutoff frequencies 192 Hz and 320Hz.
StopBand attenuation must be lower than 80dBs.

(i) Draw the two sided specifications on the Hz axis, Discrete Frequency axis and normalised frequency axis (max freq occurs at ±1).
(ii) Using the normalised frequency axis compute the transition bandwidths Δf_{lower} and Δf_{upper}
(iii) Consulting the characteristics of various windows, determine which window will be appropriate for the FIR design and estimate the number of coefficients that will be required to achieve the specifications.