EE969-469:

Time Frequency Analysis – Part 2

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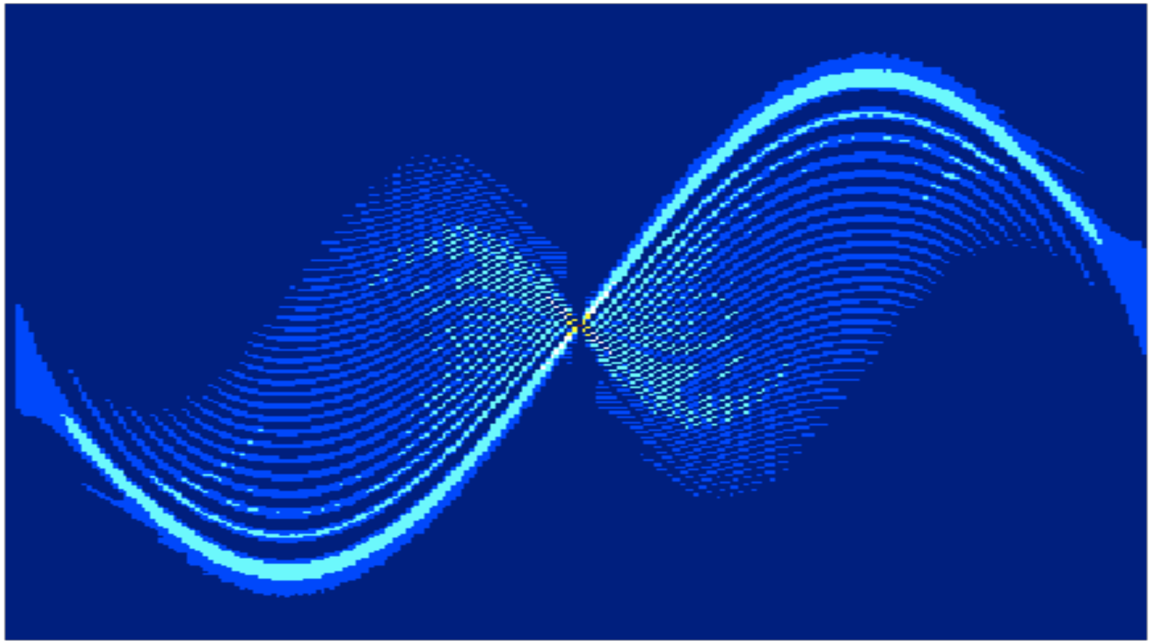
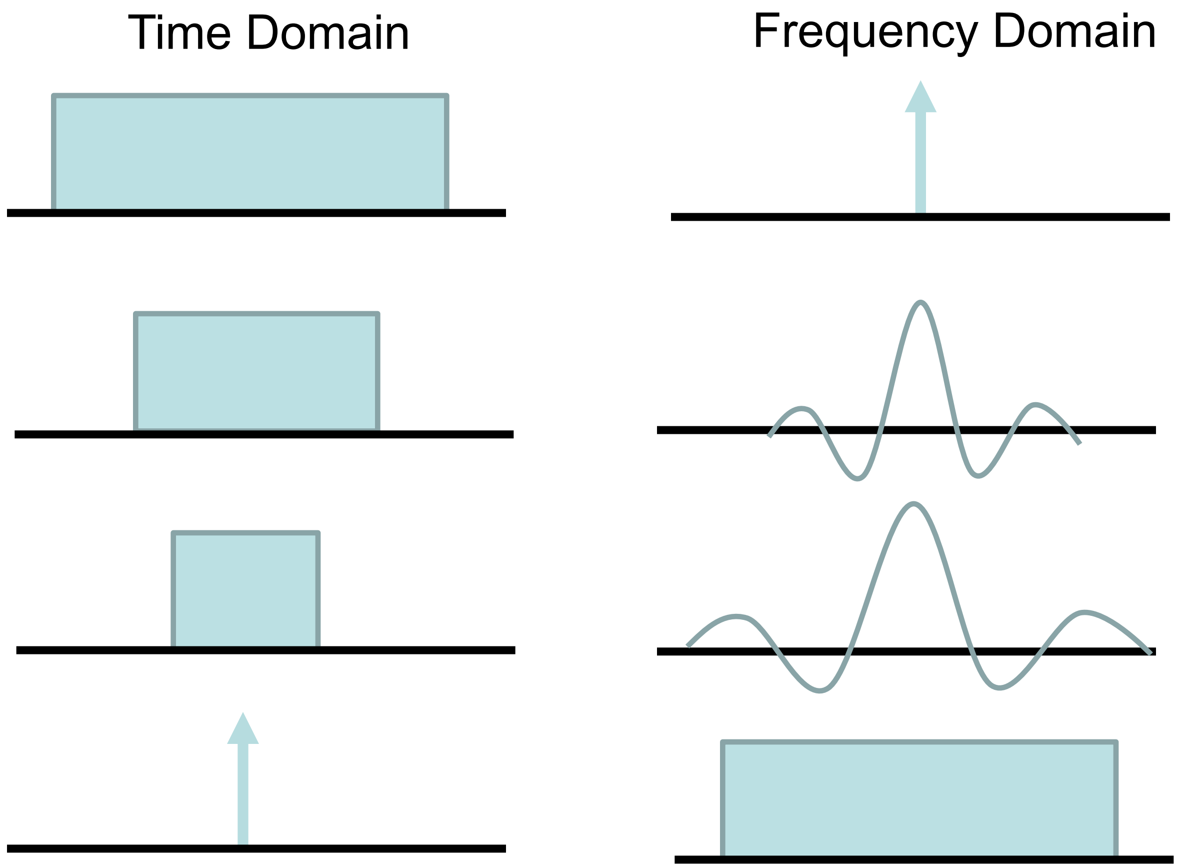
Recap

- Time frequency analysis is used to analyse signals whose frequency content changes with time;
- Time Frequency Resolutions trade-off…
- Today we will look more at this trade off and we will introduce additional TFDs.



Gabor Uncertainty Principle



$$\Delta f \, \Delta t \geq \frac{1}{4\pi}$$

Gabor Transform

The only case in which:

$$\Delta f \Delta t = \frac{1}{4\pi}$$

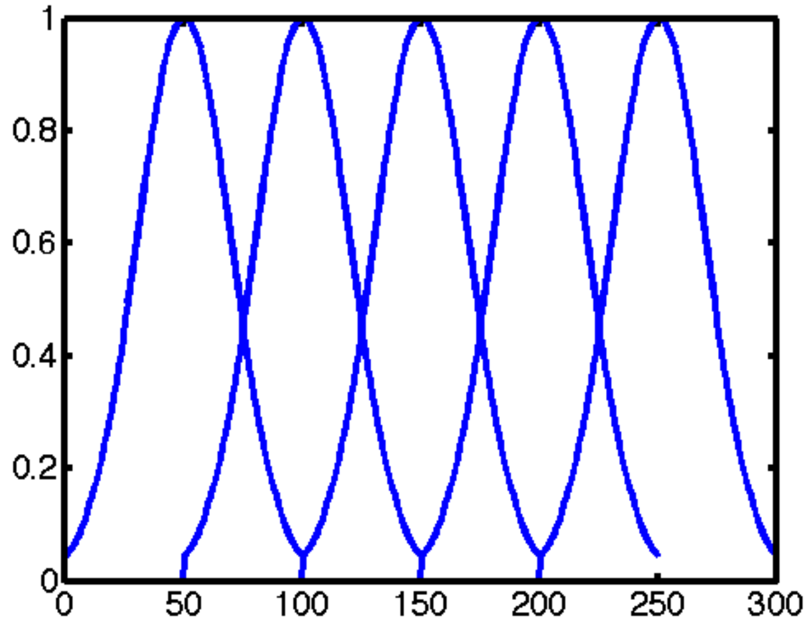
Occurs when a Gaussian window is used:

$$w[n] = exp^{-n^2/2\sigma^2}$$

A Short Time –Fourier Transform that uses Gaussian windows is also called Gabor Transform.

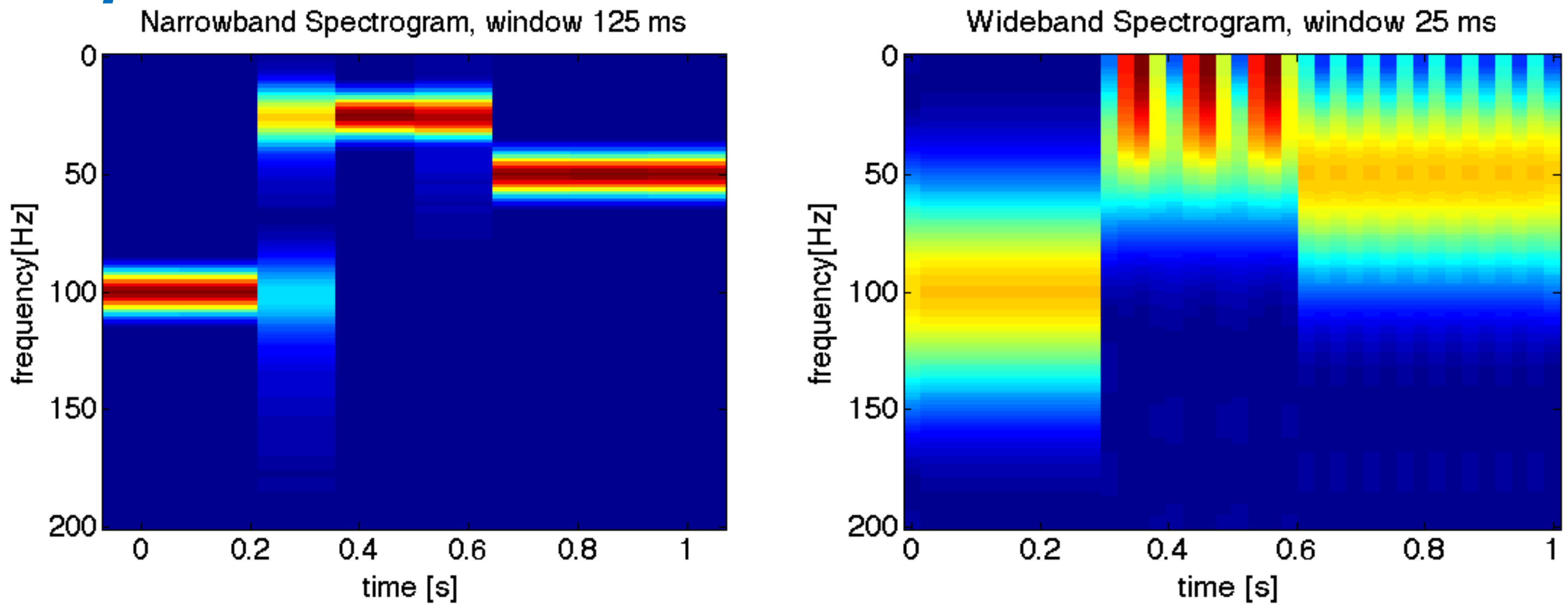
Overlap

- Windows are useful to reduce spectral leakage;
- However some information can be lost due to the transition and the weighting

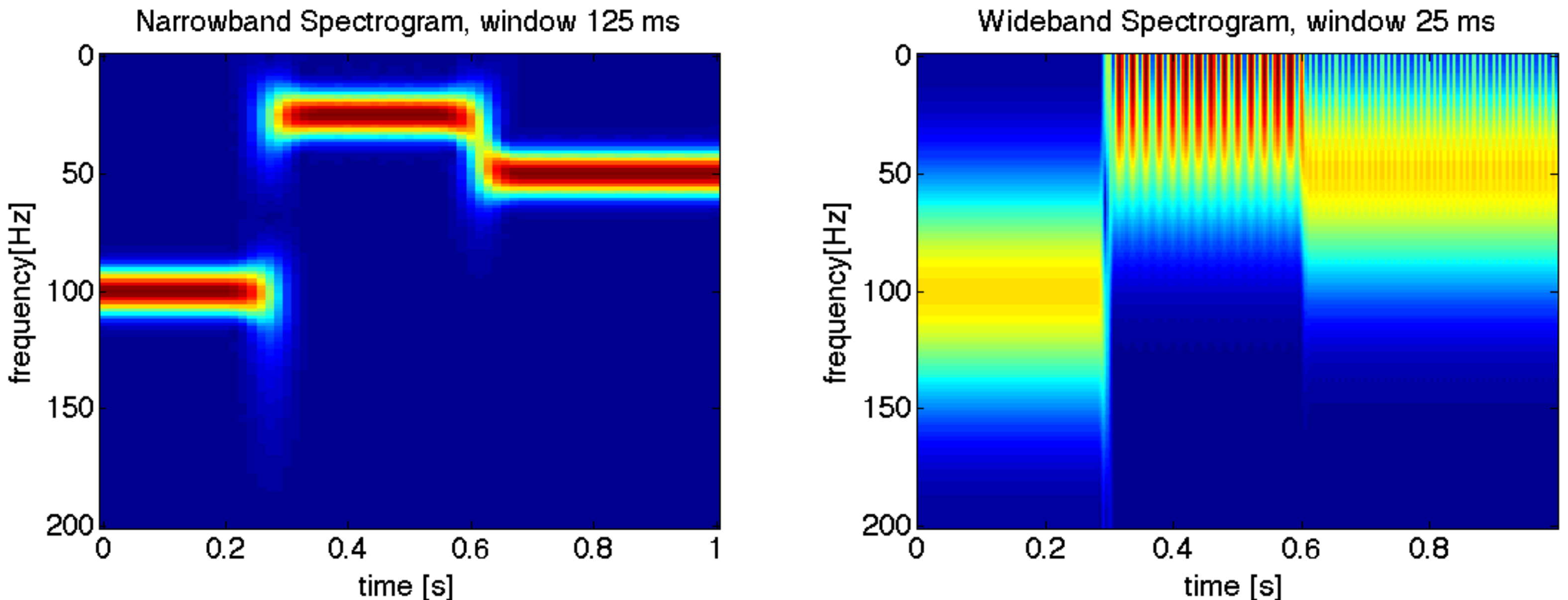


- Applying overlap reduces the information loss;
- The drawback is higher computational cost.

Overlap



With Overlap



Energy Distributions

Energy Distributions

- STFT like transform decompose the signal in atoms, so they are also known as “atomic” distributions;
- Distributions whose aim is to distribute the signal’s energy over time and frequency variables are called “energy” distributions;
- The basic idea is to compute the joint time-frequency energy density of the signal such that:

$$E_s = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho_s(t, \nu) dt d\nu$$

- Where E_s is the energy of the signal

Wigner-Ville Distribution

- Belongs to the class of energy distributions, in particular to the family of energy distributions that shares the time-frequency covariant property (time and frequency shifts of the signal maps on the TFD);

- It is the Fourier transform of the auto-correlation function:

$$WVD_s(m,\omega) = \sum_{n=-\infty}^{\infty} s[m+n]s^*[m-n]exp^{-j2\omega n}$$

- High time and frequency resolution achievable (sample scale);

Wigner-Ville Distribution

- As the WVD is a biliniar function, the quadratic superposition princple applies:

$$W_{s1+s2}(n,\omega) = W_{s1}(n,\omega) + W_{s2}(n,\omega) + 2\mathcal{R}\{W_{s1,s2(n,\omega)}\}$$

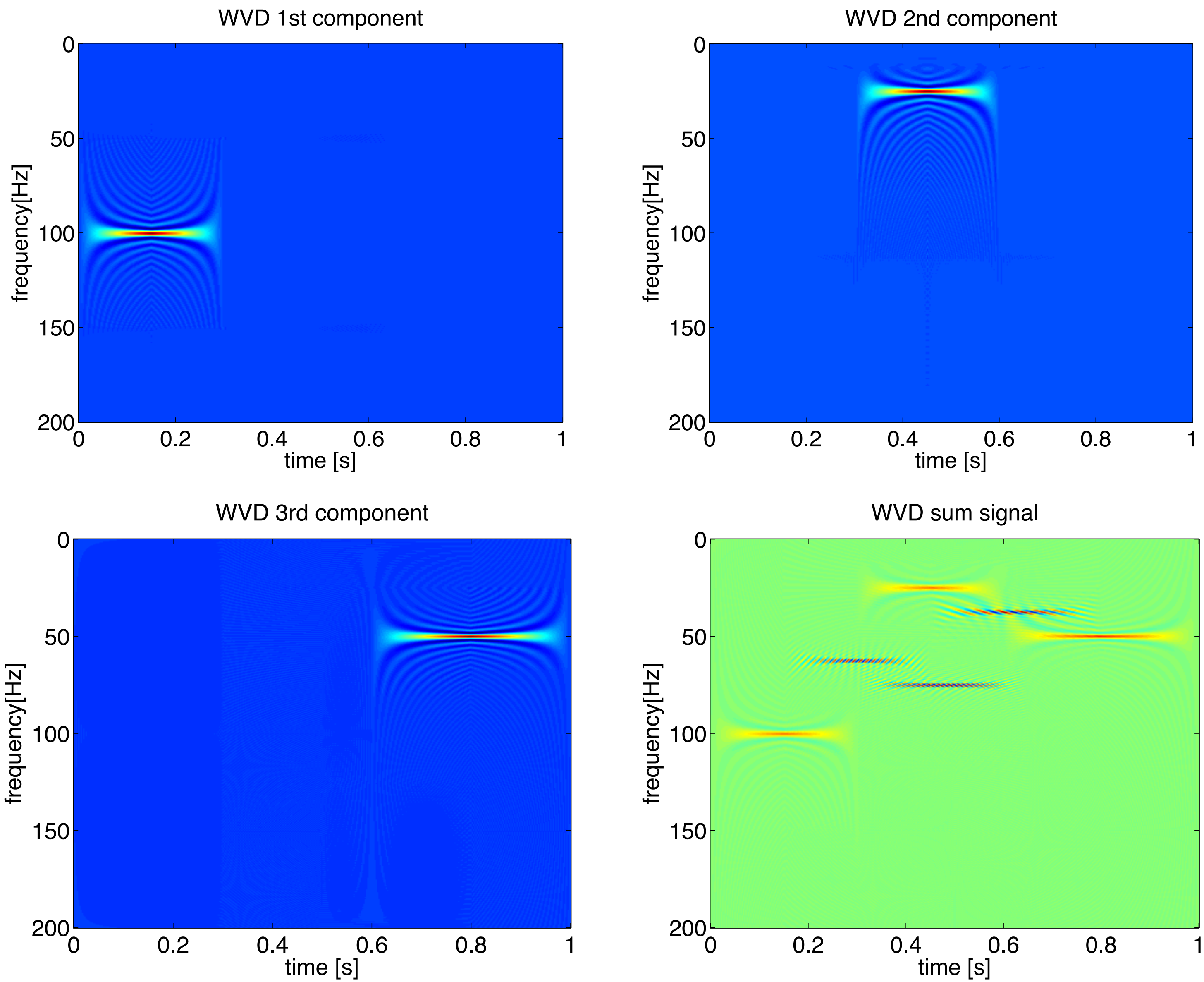
where

$$W_{s1,s2(n,\omega)} = \sum_{n=-\infty}^{\infty} s_1[m+n]s_2^*[m-n]exp^{-j2\omega n}$$

is the cross-WVD.

- So the WVD suffers of non-zero interference terms (cross-terms) that can affect the correct identification of the real signal components

Wigner-Ville Distribution



Pseudo and Smoothed Pseudo Wigner-Ville Distributions

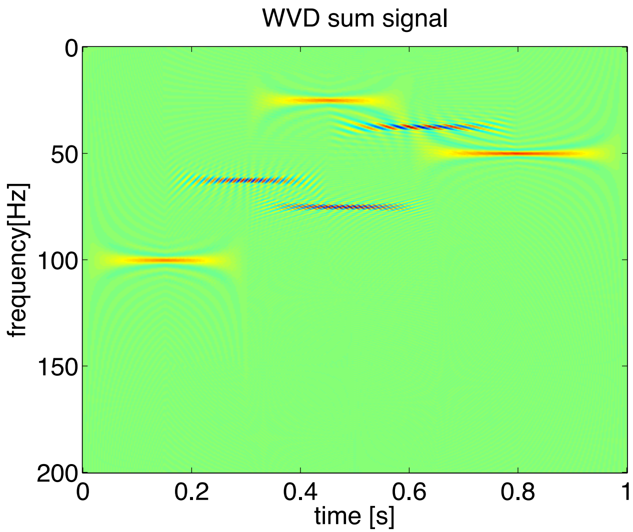
- By applying time and frequency windowing the inteference effect can be mitigated:

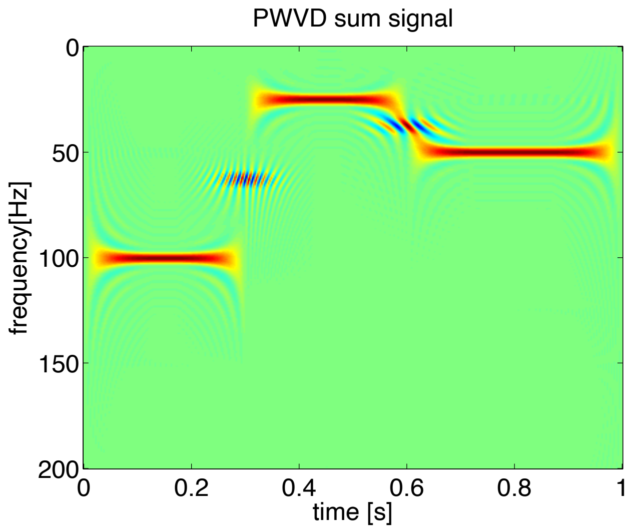
$$PWVD_s(m,\omega) = \sum_{n=-\infty}^{\infty} w[n]s[m+n]s^*[m-n]exp^{-j2\omega n}$$

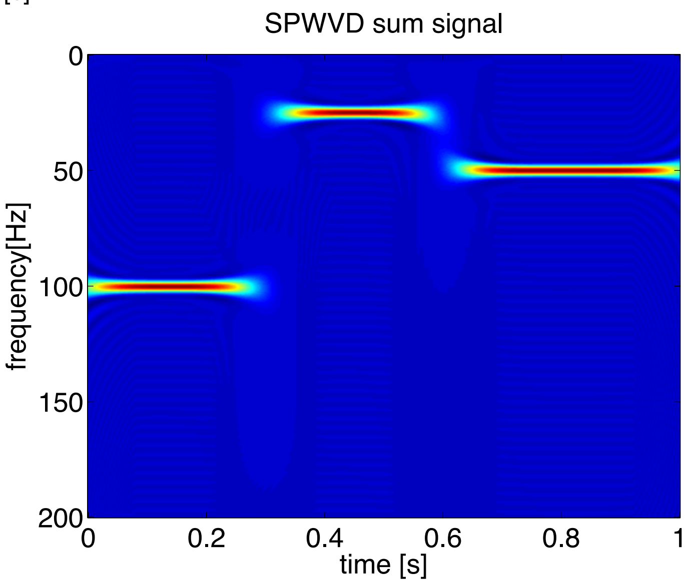
$$SPWVD_s(m,\omega) = \sum_{n=-\infty}^{\infty} w[n] \sum_{l=-\infty}^{\infty} g[l-m]s[m+n]s^*[m-n]exp^{-j2\omega n}$$

- These are known as Pseudo Wigner-Ville Distribution and Smoothed-Pseudo Wigner-Ville Distribution.

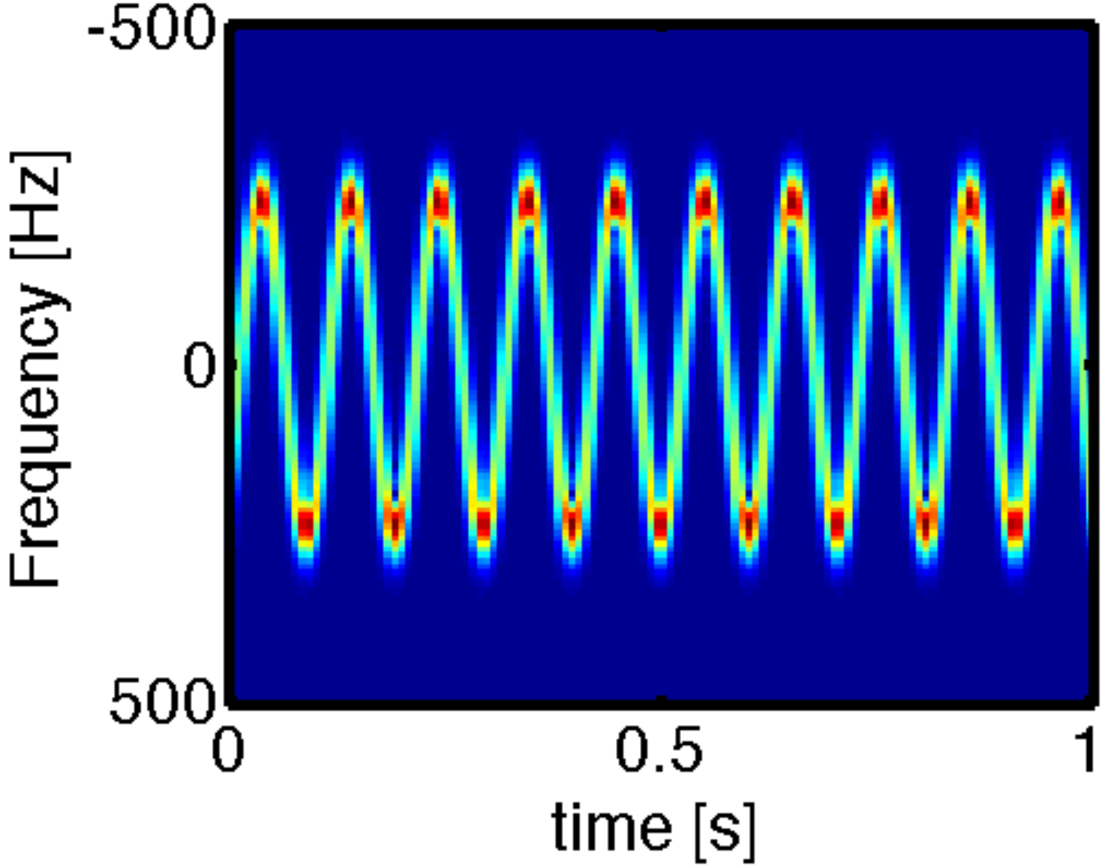
Pseudo and Smoothed Pseudo Wigner-Ville Distributions







Detecting periodicity in the frequency domain



Cepstrum

- Cepstral signal analysis is one out of several methods that enables us to find out whether a signal contains periodic elements.

- The method can also be used to determine the pitch of a signal.

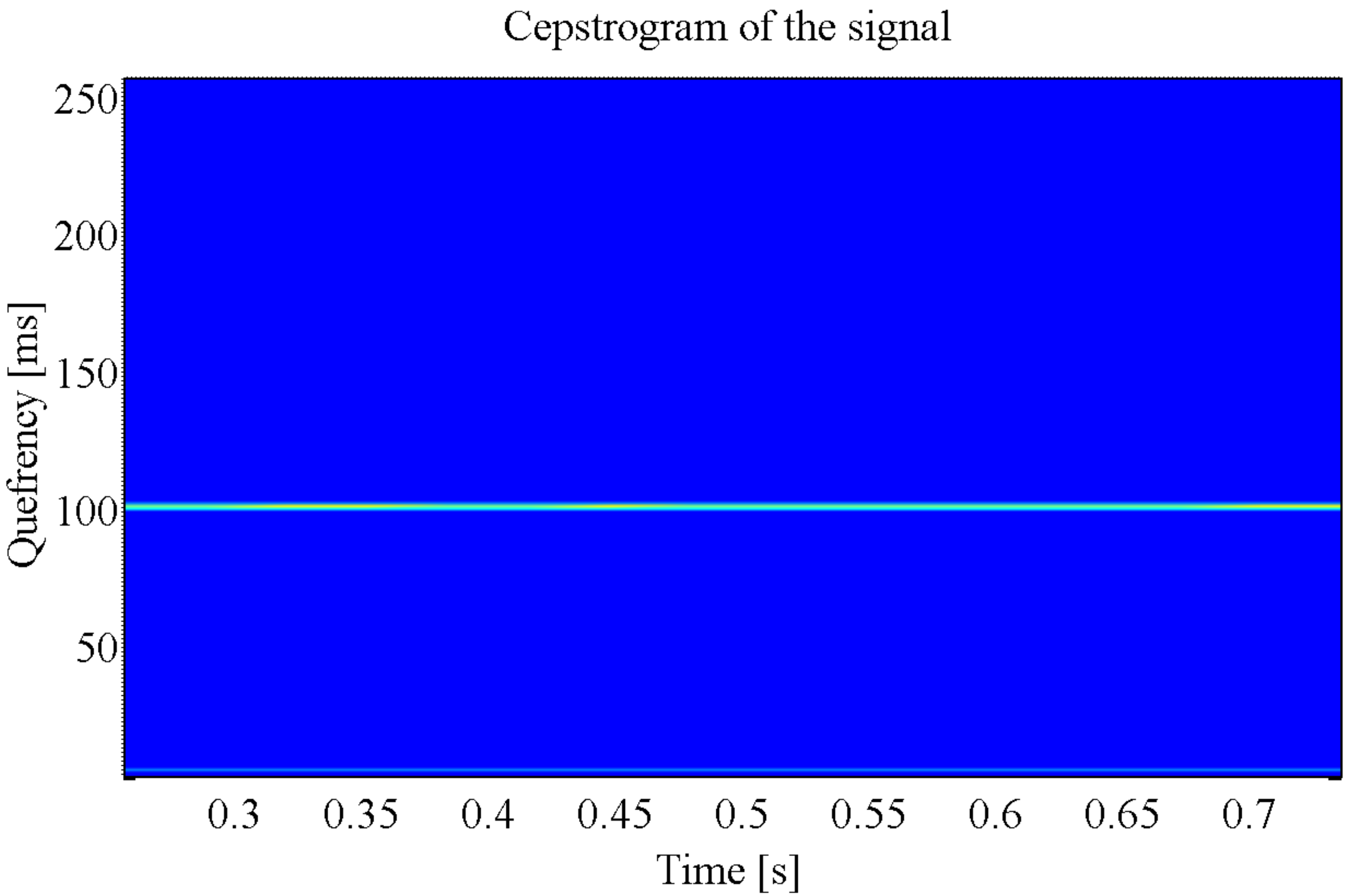
- The cepstral coefficients are found by using the following equation

$$c(\psi) = DFT\left\{\log(|DFT\{x[n]\}|^2\right\}$$

- The variable ψ is called quefrency.

Cepstrogram

- Applying the cepstrum to a spectrogram a CEPSTROGRAM is obtained



Homework – Not Compulsory

Download on your phone a spectrogram app:

- Android
  - https://play.google.com/store/apps/details?id=net.galmiza.android.spectrogram&hl=it
- IOS
  - https://apps.apple.com/us/app/spectrumview/id472662922

Familiarize with the app and its options and try to analyse your own speech, music, environmental noise, varying the offered parameters.