

Mark Scheme uBooNE

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Abstract

Exercise 1

Answered via page 13 of Ole's thesis, but is rather trivial once you know the tricks.

Exercise 2

Neutrinos interact via the weak force. So you get

- Inverse beta decay ($\text{neutrino} + n \rightarrow \text{electron} + \text{proton}$)
- Elastic interactions with nucleus ($\text{neutrino} + N \rightarrow \text{neutrino} + N$) or electrons
- **quali-elastic scattering** $\nu_{\text{lepton}} + n \rightarrow \text{lepton} + p$, massive lepton but generally same as inverse beta decay
- inelastic scattering ($\nu_{\text{lepton}} + N \rightarrow \text{lepton} + X$) where X is additional hadrons.

Neutral Current interactions can occur, and inverse beta decay as well as the quasi-elastic scattering and inelastic scattering which produces many of the particles we see within our distributions ($\nu_{\mu} CC\pi^0$)

Exercise 3

Muon particles appear very "track-like", being straight and not having many showering effects within the detector. This is due to their larger mass (being somewhat equivalent to a proton) meaning that further collisions will have less effect on their overall momenta (comparing their tracks to electrons, for example).

Exercise 5

Interactions:

- Cosmic - cosmic-induced interactions
- Out Fid. Vol. - Interactions at the edge of the fiducial volume of the detector
- EXT - data recorded in the detector when the neutrino beam is off...cosmic activity

Trends and correlations within the data are still quick hard to see, due to the number of data points. Easy solution would be to reduce the number of data points randomly across the sample, but could come with variance due to the reduction in data available. Could also change the format in which your information is plotted (histogram, for example).

Exercise 6

Accuracy should sit around 60-70%. It seems okay, but not great.

More depth to the model increases overfitting (and time to run). increasing more trees doesn't help eventually, in reality what you'd need is either more data or a more complex system for sorting the data out (CNN, etc).

Confusion matrix should show that Cosmics are very often misrepresented as EXT. Not surprising as EXT is the real data from uBooNE when beam is turned off, so would expect a lot of cosmics in the EXT. Shows that these two overlap (so classification of this type will not be helpful). Same case with outer fiducial volume and neutral current neutrinos (although that last one is a bit surprising even to me, assumedly due to EXT saturating most of the data).

Exercise 9

Statistical uncertainty would just be the square root of the number of events in each bin, added in quadrature to the flat rate of 15% for systematic uncertainty.

Exercise 11

Bonus component: Any minimiser should give close to zero values for the two parameters, this makes sense as applying muon neutrino disappearance to your simulated data will only take it further away from the MicroBooNE data (which has an excess).

Consequently, varying your oscillation parameters around the "best fit" point should change the chi squared value very little. This shows that we are not sensitive enough near this range of values and so

Exercise 13

Exercise 14

Next page (thank L^AT_EX formatting)

$$\begin{aligned}
 \sin^2 \theta_{\mu\mu} &= 4(1 - |U_{\mu\mu}|^2)|U_{\mu 4}|^2 = 4 \cos^2 \theta_{14} \sin^2 \theta_{24} (1 - \cos^2 \theta_{14} \sin^2 \theta_{24}) \\
 &\quad \text{From Eqn. provided.} \quad \text{using relations.} \\
 \sin^2 \theta_{ee} &= 4(1 - |U_{ee}|^2)|U_{e4}|^2 = 4(1 - \sin^2 \theta_{14}) \sin^2 \theta_{14} = (2 \cos \theta_{14} \sin \theta_{14})^2 \\
 &\quad = \sin^2 2\theta_{14} // \\
 \sin^2 \theta_{\mu e} &= 4|U_{\mu 4}|^2|U_{e4}|^2 = 4 \cos^2 \theta_{14} \sin^2 \theta_{24} \sin^2 \theta_{14} \\
 &= (2 \cos \theta_{14} \sin \theta_{14})^2 \sin^2 \theta_{24} = \sin^2 2\theta_{14} \sin^2 \theta_{24} //
 \end{aligned}$$

Figure 1: Solutions for exercise 13, probabilities are trivial with these results (hopefully!)

$$\sin^2 \theta_{ee} = 4(1 - \sin^2 \theta_{14}) \sin^2 \theta_{14}$$

Ex 12
13

$$0 = 4s_{14}^2 - 4(s_{14}^2)^2 - c_{ee}^2$$

$\begin{matrix} b x & - & a x^2 & - & c \end{matrix}$

$$s_{14}^2 = \frac{-4 \pm \sqrt{16 - 4(-4)(c_{ee}^2)}}{2(-4)}$$

$$s_{14}^2 = \frac{-4 \pm \sqrt{16 + 16 c_{ee}^2}}{-8}$$

$$= \frac{-4 \pm 4\sqrt{1 + c_{ee}^2}}{-8}$$

$$\sin^2 \theta_{14} = 4 \left(\frac{\sqrt{1 + c_{ee}^2}}{2} \right) = |U_{e4}|^2$$

$$\sin^2 \theta_{\mu\mu} = 4 \cos^2 \theta_{14} \sin^2 \theta_{24} (1 - 2 \cos^2 \theta_{14} \sin^2 \theta_{24})$$

$$4 \cos^2 \theta_{14} \sin^2 \theta_{24}$$

$$\sin^2 \theta_{\mu\mu} = 4(c_{14}^2 s_{24}^2) - 4(c_{14}^2)^2 (s_{24}^2)^2$$

$$0 = -4 \underbrace{(c_{14}^2)^2 (s_{24}^2)^2}_a + 4 \underbrace{(c_{14}^2) (s_{24}^2)}_b - \underbrace{s_{\mu\mu}^2}_c$$

$$a = -4(c_{14}^2)^2 \quad b = 4c_{14}^2 \quad c = -s_{\mu\mu}^2$$

$$\begin{aligned}
 a &= -4(C_{14}^2)^2 & b &= 4(C_{14}^2) & c &= -S_{\mu\mu}^2 \\
 S_{24}^2 &= \frac{-4C_{14}^2 \pm \sqrt{16(C_{14}^2)^2 - 4 \cdot (-4)(C_{14}^2)^2 \cdot (-S_{\mu\mu}^2)}}{-8(C_{14}^2)^2} \\
 S_{24}^2 &= \frac{-4C_{14}^2 \pm 4\sqrt{(C_{14}^2)^2 - (C_{14}^2)^2 (S_{\mu\mu}^2)}}{-8(C_{14}^2)^2} \\
 S_{24}^2 &= \frac{-4(C_{14}^2) \pm (C_{14}^2)4\sqrt{1 - S_{\mu\mu}^2}}{-8(C_{14}^2)^2} \\
 S_{24}^2 C_{14}^2 &= \frac{1 \pm \sqrt{1 - S_{\mu\mu}^2}}{2} = |U_{\mu\psi}|^2 \\
 \sin^2 2\theta_{\mu e} &= 4|U_{\mu\psi}|^2 |U_{e\psi}|^2 \\
 \uparrow &= 4 \left(\frac{1 - \sqrt{1 - S_{\mu\mu}^2}}{2} \right) \cdot \left(\frac{1 - \sqrt{1 - S_{ee}^2}}{2} \right) \\
 \text{new oscill param} & & \text{old theta values} & \\
 \theta: 0.001 \rightarrow 1 & & & \\
 \sin^2 2\theta_{\mu e} &= 2(1 - \sqrt{1 - S_{\mu\mu}^2}) \cdot \left(1 - \sqrt{1 - S_{ee}^2} \right) \\
 & & \uparrow & \\
 & & = 0.24 &
 \end{aligned}$$

Figure 2: Solutions for exercise 14, probabilities are trivial with these results (hopefully!) **THE EXTRA FACTOR OF TWO AT THE END ON RHS IS INCORRECT!**

The picture derivation is correct, but I dont think the (below) \LaTeX version is, so only look at the pictures.

Get to this point via page 20 of Ole's thesis. Here are the explicit derivations as a pdf $\sin^2 2\theta_{\mu e} = \sin^2 2\theta_{14} \sin^2 2\theta_{24}$

$$\sin^2 2\theta_{\mu\mu} = 4\cos^2 \theta_{14}\sin^2 \theta_{24}(1 - \cos^2 \theta_{14}\sin^2 \theta_{24})$$

So you insert your basic oscillation list instead of $\sin^2 \theta_{\mu\mu}$, with the parameter now being $\sin^2 2\theta_{\mu e}$, this will scale your results to (hopefully) be in line with LBSND.

I dont think this is correct, so will ignore $\sin^2 2\theta_{ee} = \sin^2 2\theta_{14}$

$\sin^2 2\theta_{14} = 0.24$ provided by the lab

From page 20 of Ole's thesis, you need:

$$\sin^2 2\theta_{\mu\mu} = 4(1 - |U_{\mu 4}|^2) |U_{\mu 4}|^2 \quad (1)$$

Using values given, can be simplified to:

$$\sin^2 2\theta_{\mu\mu} = 4\cos^2 \theta_{14}\sin^2 \theta_{24}(1 - \cos^2 \theta_{14}\sin^2 \theta_{24}) \quad (2)$$

Rearrange to get in the order of a quadratic where x is $\sin^2 \theta_{24}$

$$0 = -(\sin^2 \theta_{24})^2 (\cos^2 \theta_{14})^2 + \sin^2 \theta_{24} \cos^2 \theta_{14} - \frac{\sin^2 2\theta_{\mu\mu}}{4} \quad (3)$$

Gives:

$$\sin^2 \theta_{24} = \frac{1 \pm \sqrt{1 - \sin^2 2\theta_{\mu\mu}}}{2\cos^2 \theta_{14}} \quad (4)$$

Plug into $\sin^2 2\theta_{\mu e} = \sin^2 2\theta_{14} \sin^2 2\theta_{24}$, with your own values for $\sin^2 \theta_{\mu\mu}$ from your theta values from two flavour oscillation. You ignore the $1 + \sqrt{1 - \sin^2 2\theta_{\mu\mu}}$ component as it creates some unfeasible solutions:

Take $0 < \sin^2 \theta_{\mu\mu} < 1$, and study the range. You get situations within the equation where $\cos^2 \theta_{14} > 1$, which is not allowed.