Kalman Filter Design

In this script we develop the Kalman filter equations

We first define the system states and param_eters as symbolic variables

```
syms theta psi theta_dot psi_dot real
syms I_byy I_fyzz d_x g m_e d1 d2 real
syms K_omega d_t K_mtheta K_ttheta K_mpsi K_tpsi real
syms u_tilde [2 1] real
```

We define the reduced system matrices

```
M = [
    I_byy, 0;
    0, I_fyzz + I_byy*cos(theta)^2;
];
C = [
    0, I_byy*psi_dot*cos(theta)*sin(theta);
    0, -2*I_byy*theta_dot*cos(theta)*sin(theta);
];
gvec = [
    d_x * g * m_e * sin(theta);
    0;
];
D = diag([d1; d2]);
B = K_omega^2 * d_t * [
    K_mtheta, K_ttheta;
    -K_mpsi * cos(theta), K_tpsi * cos(theta);
];
```

We can now find the jacobians

```
x1 = [
    theta;
    psi;
];
x2 = [
    theta_dot;
    psi_dot;
];
x = [
    x1;
    x2;
];
f = [
    x2;
    -M \ (C*x2 + D*x2 + gvec - B*u_tilde);
];

dfdx = jacobian(f, x);

% row 1
simplify(dfdx(1,:))
```

ans = $\begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}$

```
% row 2
simplify(dfdx(2,:))
```

ans = $\begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}$

```
% row 3
simplify(dfdx(3,:))
```

ans =

$$\begin{pmatrix} -\frac{I_{\mathrm{byy}}\,(2\cos(\theta)^2-1)\,\dot{\psi}^2 + d_x\,g\,m_e\cos(\theta)}{I_{\mathrm{byy}}} & 0 & -\frac{d_1}{I_{\mathrm{byy}}} & -\dot{\psi}\sin(2\,\theta) \end{pmatrix}$$

```
% row 4
simplify(dfdx(4,1))
```

ans =

$$\frac{4\,I_{\mathrm{byy}}\,\dot{\psi}\,\dot{\theta}\,\sigma_1 + 2\,K_{\mathrm{mpsi}}\,K_\omega{}^2\,d_t\,u_{\mathrm{tilde1}}\sin(\theta) - 2\,K_\omega{}^2\,K_{\mathrm{tpsi}}\,d_t\,u_{\mathrm{tilde2}}\sin(\theta)}{I_{\mathrm{byy}} + 2\,I_{\mathrm{fyzz}} + I_{\mathrm{byy}}\,\sigma_1} - \frac{2\,I_{\mathrm{byy}}\cos(\theta)\sin(\theta)\,\left(d_2\,\dot{\psi} + K_{\mathrm{mpsi}}\,K_\omega{}^2\,d_t\,u_{\mathrm{tilde1}}\cos(\theta) - K_\omega{}^2\,K_{\mathrm{tpsi}}\,d_t\,u_{\mathrm{tilde2}}\cos(\theta) - 2\,I_{\mathrm{byy}}\,\dot{\psi}\,\dot{\theta}\cos(\theta)\sin(\theta)\right)}{\left(I_{\mathrm{byy}}\cos(\theta)^2 + I_{\mathrm{fyzz}}\right)^2}$$

where

$$\sigma_1 = 2\cos(\theta)^2 - 1$$

```
simplify(dfdx(4,2))
```

ans = 0

```
simplify(dfdx(4,3))
```

ans =

$$\frac{2\, I_{\mathrm{byy}}\, \dot{\psi} \cos(\theta) \sin(\theta)}{I_{\mathrm{byy}} \cos(\theta)^2 + I_{\mathrm{fyzz}}}$$

```
simplify(dfdx(4,4))
```

ans =

$$-\frac{d_2 - I_{\mathrm{byy}}\, \dot{\theta} \sin(2\,\theta)}{I_{\mathrm{byy}} \cos(\theta)^2 + I_{\mathrm{fyzz}}}$$