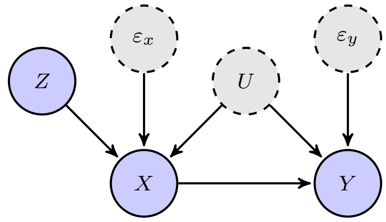

$$\begin{aligned} Z &\sim \mathcal{N}\left(\mu_z\ ,\ v_z\right) \\ U &\sim \mathcal{N}\left(\mu_u\ ,\ v_u\right) \\ \varepsilon_x &\sim \mathcal{N}\left(0\ ,\ \epsilon_x\right) \\ \varepsilon_y &\sim \mathcal{N}\left(0\ ,\ \epsilon_y\right) \end{aligned}$$

$$\begin{aligned} X &= w_{x0} + w_{xz}Z + w_{xu}U + \varepsilon_x \\ Y &= w_{y0} + w_{yu}U + w_{yx}X + \varepsilon_y \end{aligned}$$

As all the exogenous variables $(Z, U, \varepsilon_x, \varepsilon_y)$ are Gaussian and all the functions linear, $\mathrm{P}\left\{Z, U, X, Y\right\}$ and thus $\mathrm{P}\left\{Z, X, Y\right\}$ is a multivarite Gaussian.

$$\mathrm{P}\left\{Z, X, Y\right\} \sim \mathcal{N}\left(\begin{bmatrix} \mu_z \\ w_{x0} + w_{xz}\mu_z + w_{xu}\mu_u \\ (w_{y0} + w_{yx}w_{x0}) + (w_{yu} + w_{yx}w_{xu})\mu_u + w_{yx}w_{xz}\mu_z \end{bmatrix}\ ,\ \Sigma\right)$$

where,

$$\Sigma = \begin{bmatrix} \Sigma_{zz} & \Sigma_{zx} & \Sigma_{zy} \\ . & \Sigma_{xx} & \Sigma_{xy} \\ . & . & \Sigma_{yy} \end{bmatrix} = \begin{bmatrix} v_z & w_{xz}\Sigma_{zz} & w_{yx}\Sigma_{zx} \\ . & w_{xz}^2 v_z + w_{xu}^2 v_u + \epsilon_x & w_{xu}w_{yu}v_u + w_{yx}\Sigma_{xx} \\ . & . & w_{yx}^2\Sigma_{xx} + 2w_{yx}w_{yu}w_{xu}v_u + w_{yu}^2 v_u + \epsilon_y \end{bmatrix}$$

We want to identify,

$$\begin{aligned} Y|do(X=x) &= w_{y0} + w_{yu}U + w_{yx}x + \varepsilon_y \\ \implies \mathrm{P}\left\{Y|do(X=x)\right\} &= \mathcal{N}\left(w_{y0} + w_{yu}\mu_u + w_{yx}x\ ,\ w_{yu}^2 v_u + \epsilon_y\right) \\ &= \mathcal{N}\left(\mu_y + w_{yx}(x - \mu_x)\ ,\ w_{yu}^2 v_u + \epsilon_y\right) \\ &= \mathcal{N}\left(\mu_y + \frac{\Sigma_{zy}}{\Sigma_{zx}}(x - \mu_x)\ ,\ \Sigma_{yy} + \frac{\Sigma_{zy}}{\Sigma_{zx}}\left(\frac{\Sigma_{zy}}{\Sigma_{zx}}\Sigma_{xx} - 2\Sigma_{xy}\right)\right) \end{aligned}$$

The final equation is purely in terms of x and properties of the joint (non interventional) distribution $\mathrm{P}\left\{X, Y, Z\right\}$ so we get unbiased point estimates for both the mean and variance of $\mathrm{P}\left\{Y|do(X=x)\right\}$ provided $\Sigma_{zx} \neq 0$, that is if $v_z \neq 0$ and $w_{xz} \neq 0$.