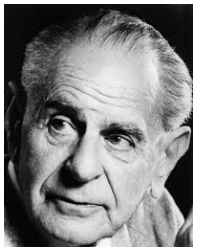
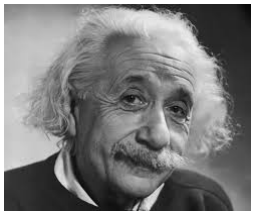
Causality

We call to mind the constant conjunction (of flame and heat) in all past instances. "Without any farther ceremony, we call the one cause and the other effect, and infer the existence of the one from that of the other." **David Hume 1739**

"Beyond such discarded fundamentals as 'matter' and 'force lies still another fetish amidst the inscrutable arcana of modern science, namely, the category of cause and effect." ... "The ultimate scientific statement of description of the relation between two things can always be thrown back upon a contingency table." **Karl Pearson 1911**



"The belief in causality is metaphysical. It is nothing but a typical metaphysical hypostatization of a well-justified methodological rule- the scientist's decision never to abandon his search for laws." **Karl Popper 1934**

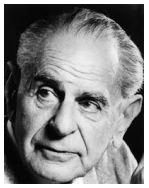


"Development of Western science is based on two great achievements: the invention of the formal logical system (in Euclidean geometry) by the Greek philosophers, and the discovery of the possibility to find out causal relationships by systematic experiment." **Albert Einstein 1953**

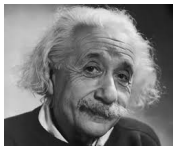
Causality

Causality is an illusion arising from repeated temporal conjunction (Hume)

Causality is outdated, correlation is all that is required (Pearson)



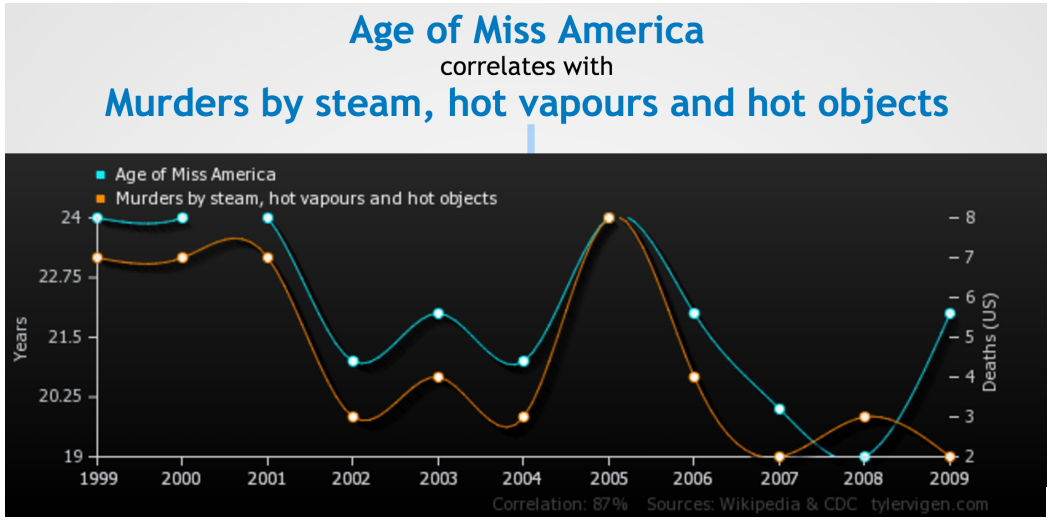
Looking for causal laws seems to work (Popper)



Learning causal relationships is central to science and we do it via systematic experiment (Einstein)

- **A causal model is one that predicts the outcome of intervention in a system**

Correlation is not causation



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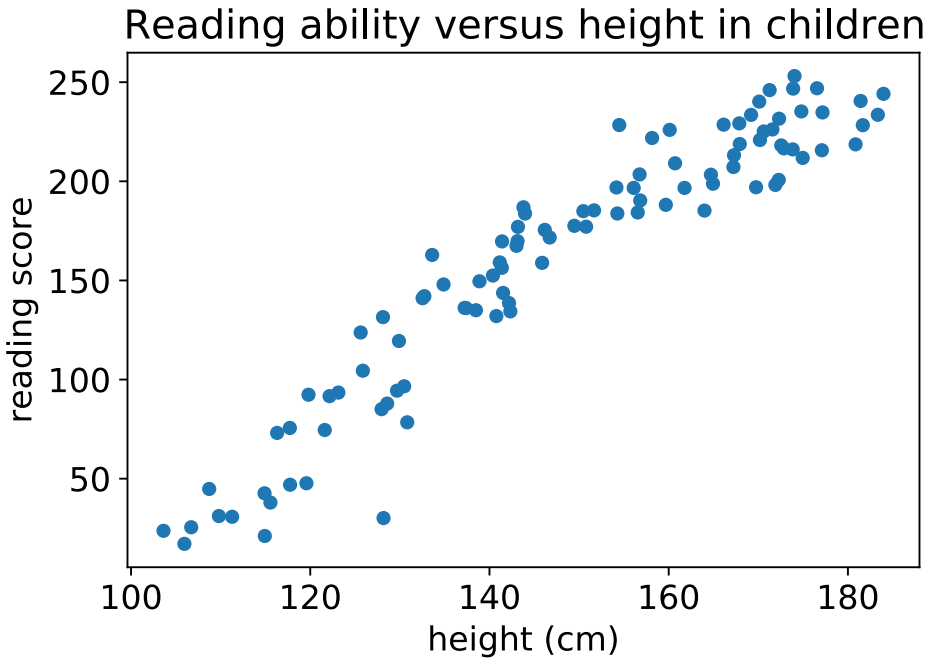


When do we care about causality?

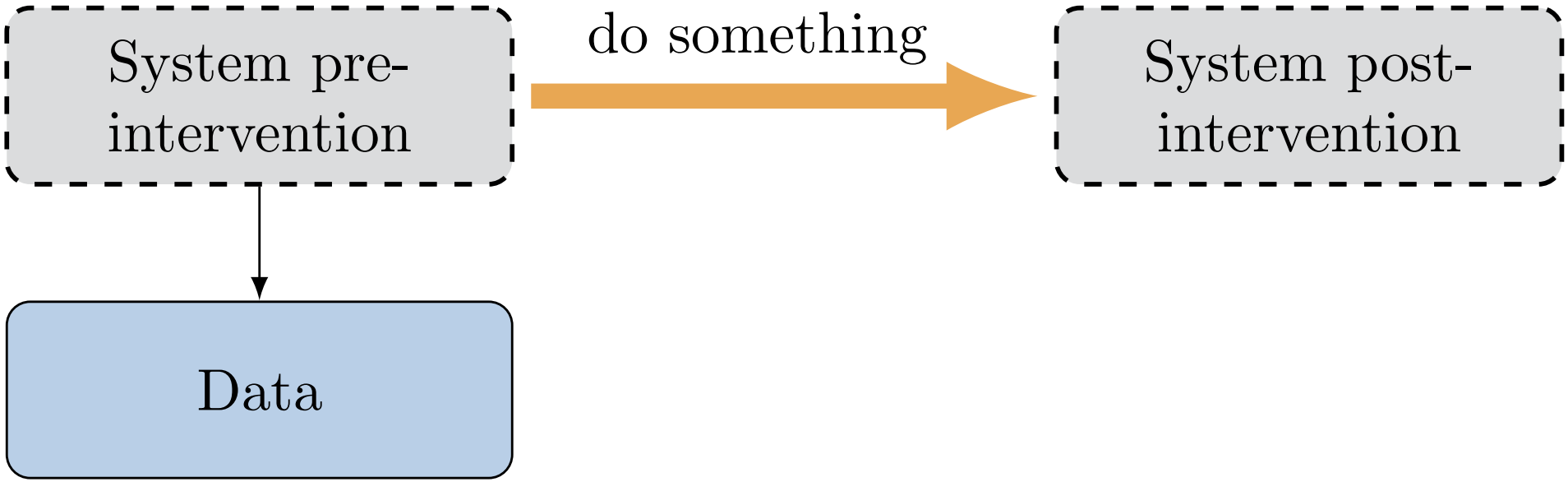
- Forecasting the weather
- Image classification
- Predicting which patients are at risk of death from pneumonia
- Predicting who will offend whilst on parole

The real question is "when don't we care?"

Overview – Causal inference in ML

- What is causality?
- Why should we care in machine learning?
- Observational causal inference
- Interventional causal inference
- A unified approach - Causal Bandits

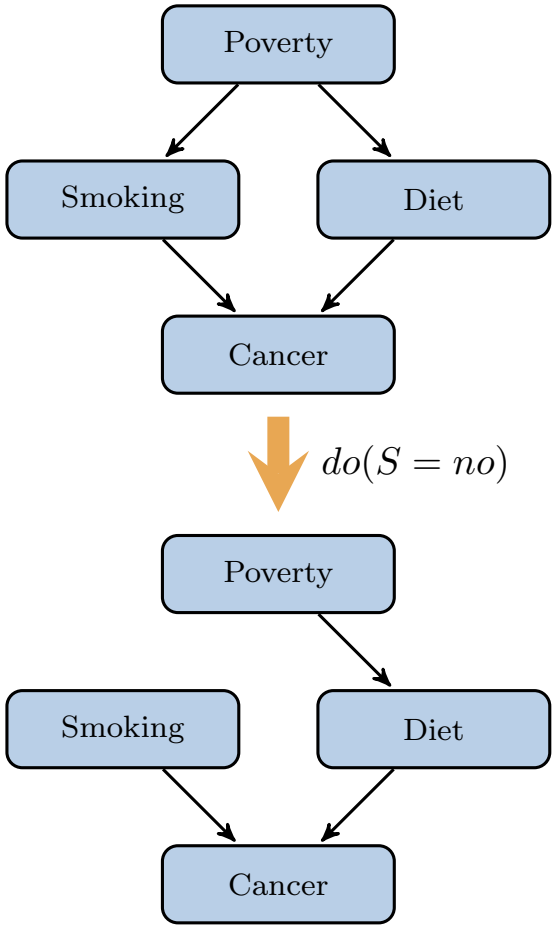
Observational causal inference



Causal Bayesian Networks

$$P(P, D, C, S) = P(P)P(S|P)P(D|P)P(C|S, D)$$

- Truncated product formula: drop terms for intervened variables from the factorisation.
- A CBN represents the set of all possible interventional distributions over its variables

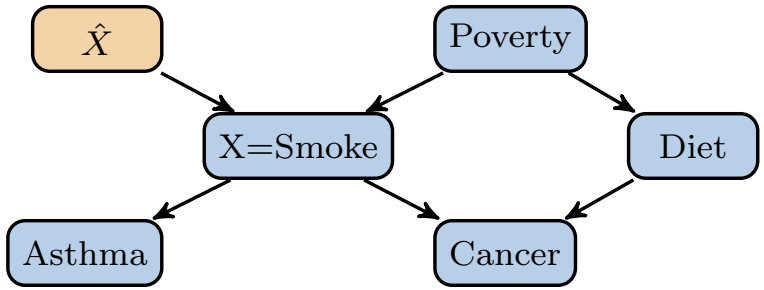


$$P(P, D, C|do(S = no)) = P(P)P(D|P)P(C|S, D)$$

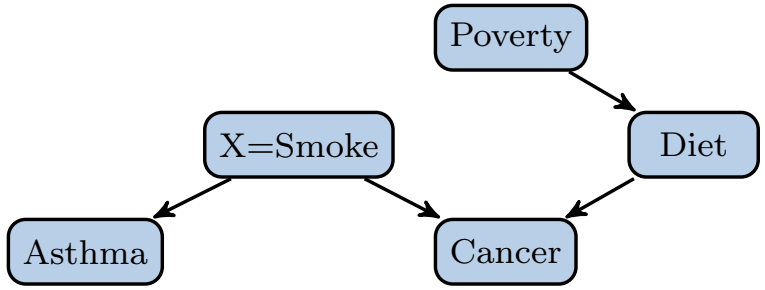
The do calculus (simplified)

1. D-separation still applies after intervention.
   $(Cancer \perp\!\!\!\perp Asthma|Smoke)_{G_{\overline{X}}} \implies P(Cancer|do(Smoke), Asthma) = P(Cancer|do(Smoke)$
2. If there are no backdoor paths from X to Y then intervention≡observation.
   $(\hat{X} \perp\!\!\!\perp Cancer|X, Poverty)_{G^\dagger} \implies P(Cancer|do(Smoke), Poverty) = P(Cancer|Smoke, Poverty)$
3. If there are only backdoor paths from X to Y then intervention doesn't change $P(Y)$.
   $(\hat{X} \perp\!\!\!\perp Diet)_{G^\dagger} \implies P(Diet|do(Smoke)) = P(Diet)$

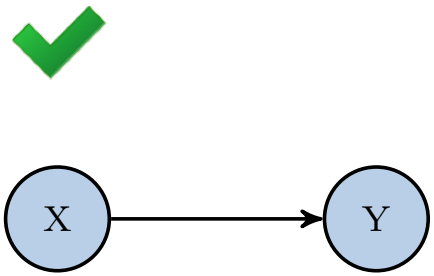
(a) $G^\dagger$

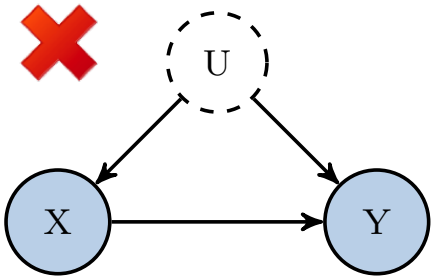


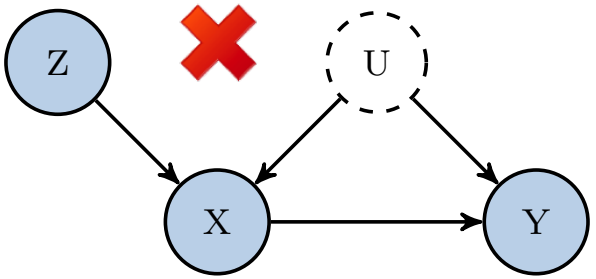
(b) $G_{\overline{X}}$

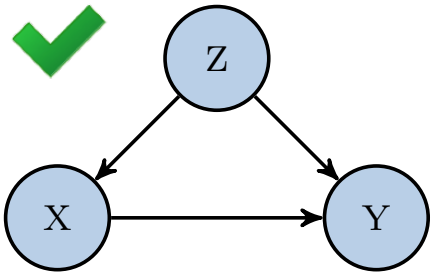


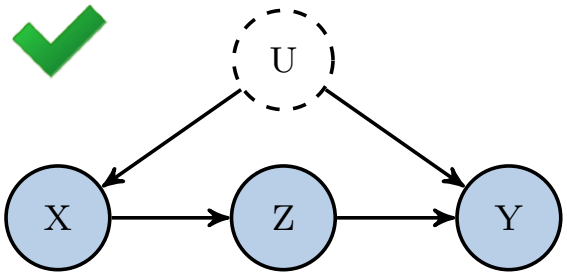
Identifiability

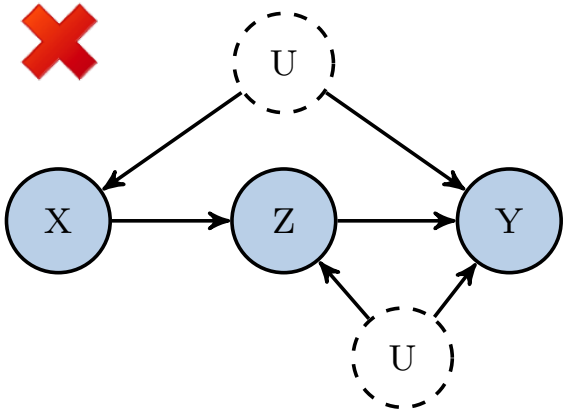




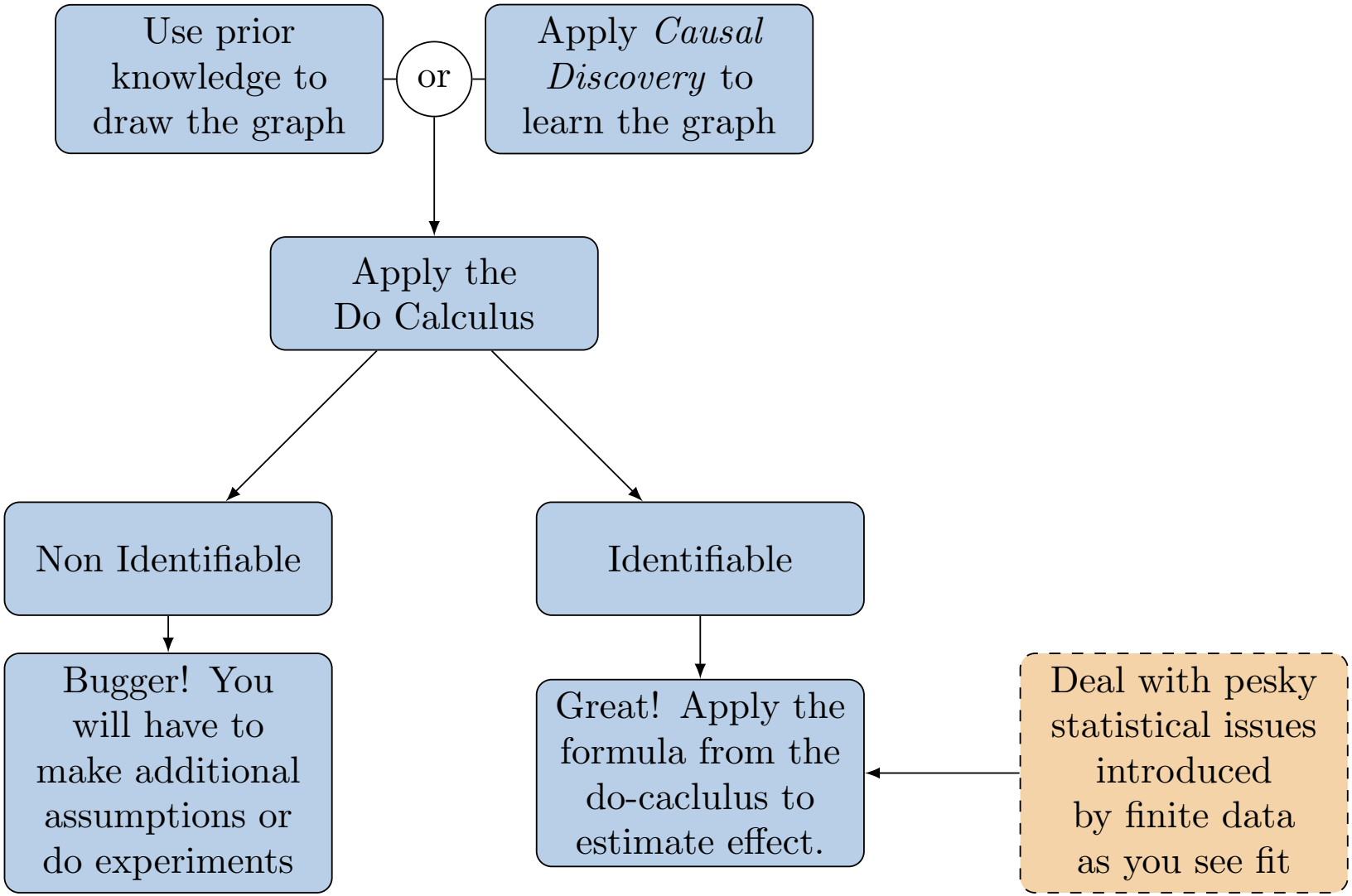




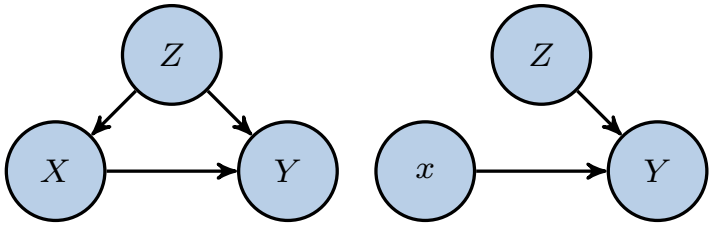




Inference from observational data – aka Pearl

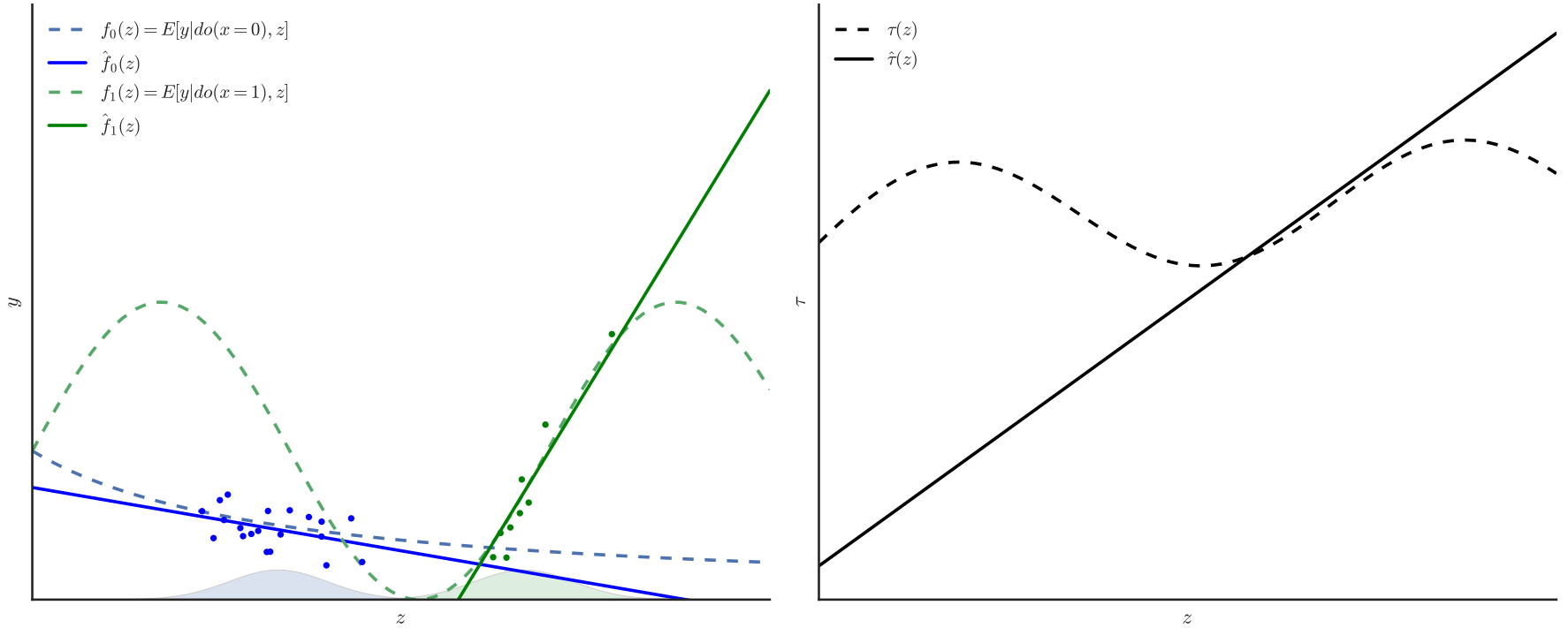


Pesky statistical issues

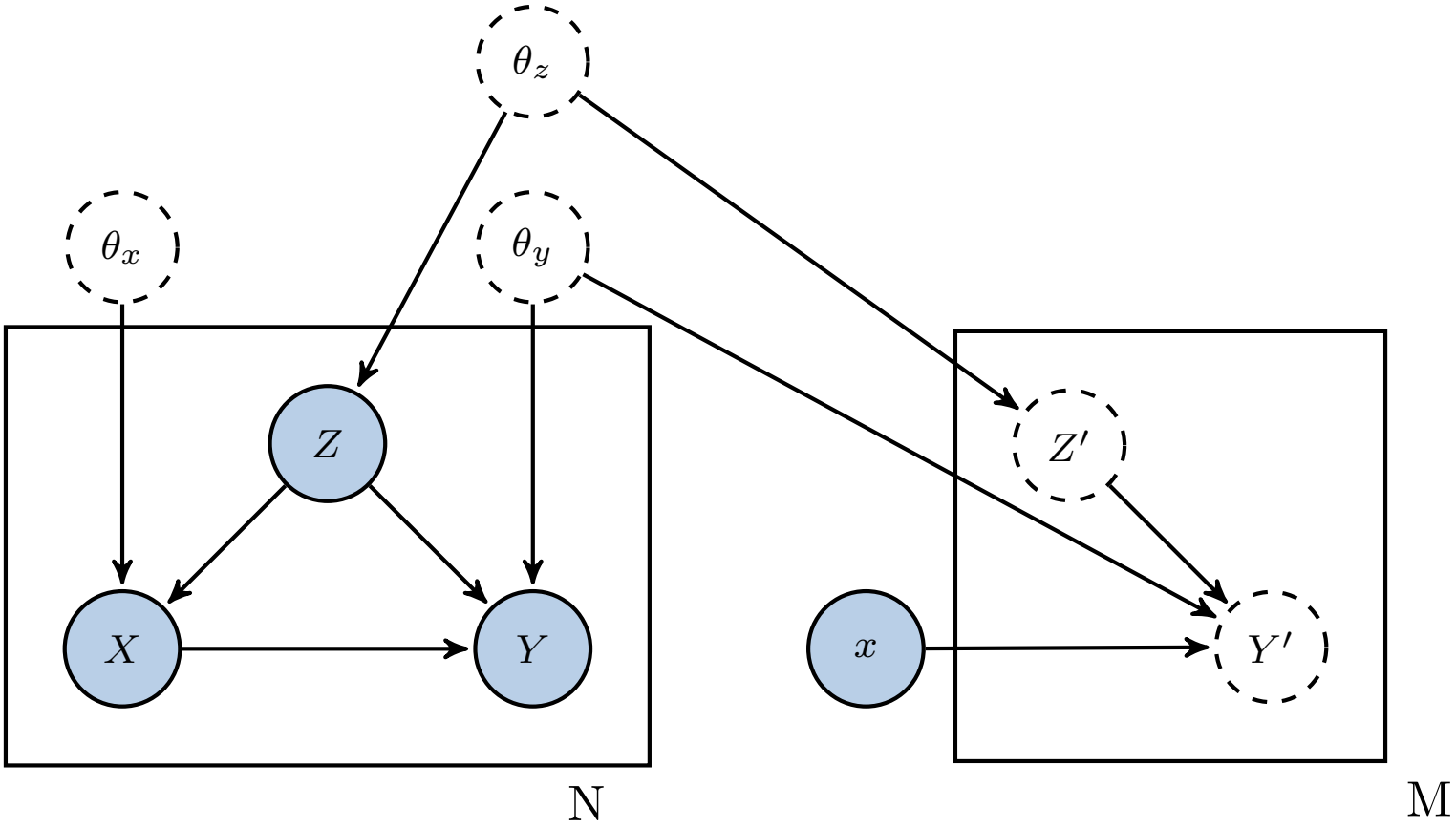


$$\text{Training data} \quad (\boldsymbol{z}_i, \boldsymbol{x}_i, y_i) \sim \mathrm{P}(Z)\,\mathrm{P}(X|Z)\,\mathrm{P}(Y|X,Z)$$

$$\text{Test data} \quad (\boldsymbol{z}_i, \boldsymbol{x}_i, y_i) \sim \mathrm{P}(Z)\,\delta(X=x)\,\mathrm{P}(Y|X,Z)$$



Can’t we just be Bayesian?

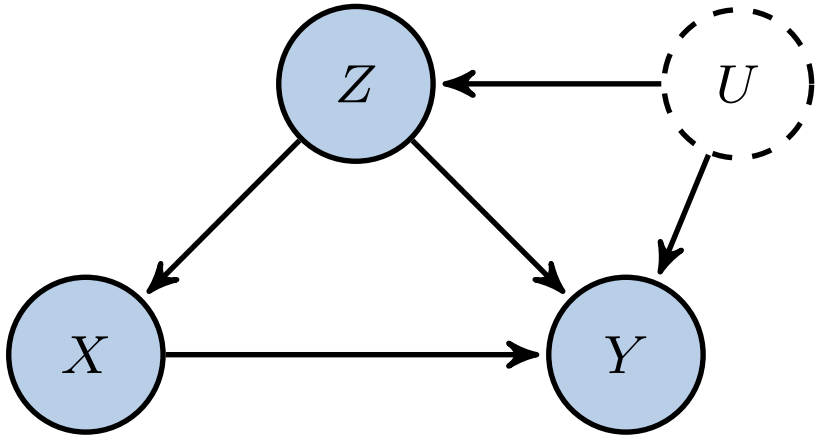


Careful with that prior

- The goal is to estimate the causal effect of X on Y
- It is indefinable via the do-calculus

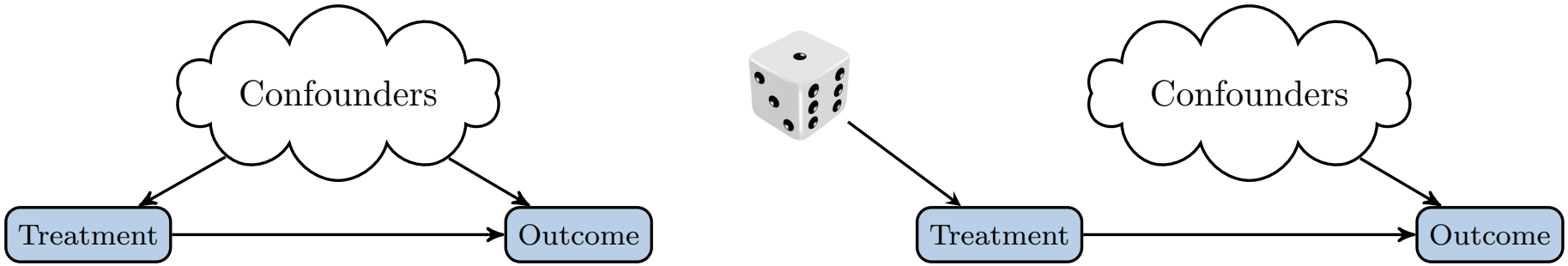
$$\mathrm{P}\left(Y|U,Z,X\right) = N(w_{yu}U + w_{yz}Z + w_{yx}X, v_y)$$

$$\mathrm{P}\left(Y|Z,X\right) = N\left(\left(w_{yz} + \frac{w_{yu}w_{zu}}{w_{zu}^2 + \frac{v_z}{v_u}}\right)Z + w_{yx}X, \varepsilon\right)$$



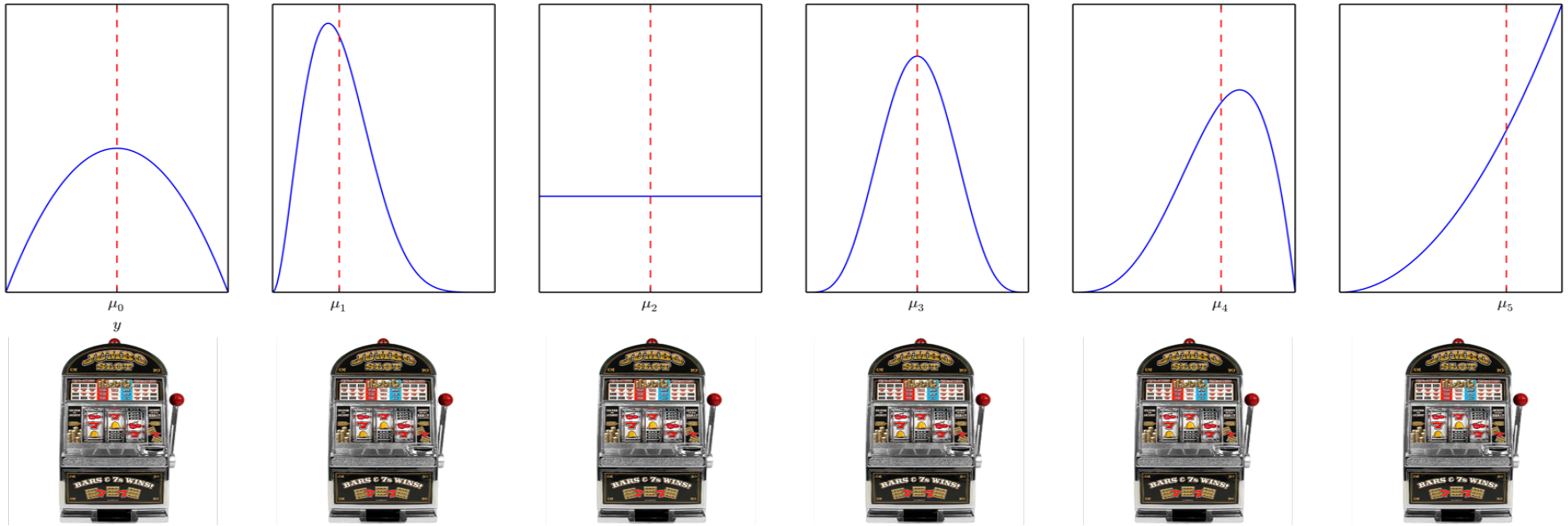
The interventional approach to causality

- Randomized trials are the traditional gold standard for determining causality



Bandits?

Classic Multi-armed Bandits

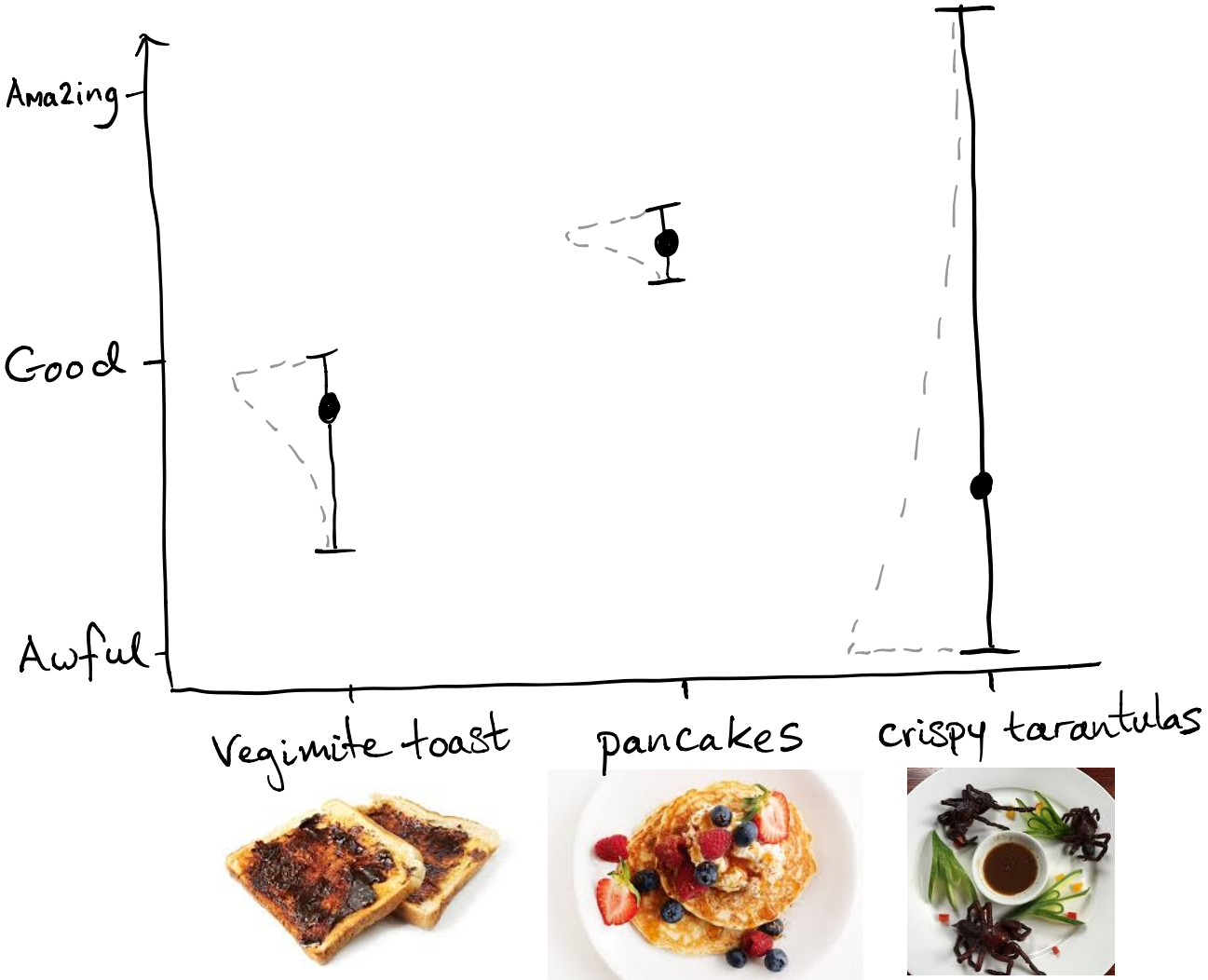


In each round $t \in [1, ...T]$,

1. the learner selects an action $a_t \in \{1, ..., k\}$
2. the world samples rewards for each action, $[Y_t^1, ..., Y_t^k] \sim \mathrm{P}(\boldsymbol{y})$
3. the learner receives the reward for the selected action $Y_t^{a_t}$

The goal is to maximise the total reward

Key Challenges & Approaches



- Explore-exploit trade-off
- Non i.i.d data

Regret

$$\bar{R}_T(\pi) = T\mu_{i^*} - \mathbb{E}\left[\sum_{t=1}^{T} Y_t^{a_t}\right]$$

$$\bar{R}_T \leq \sum_{i:\Delta_i>0}\left(\frac{8\log(T)}{\Delta_i} + 2\right)$$

$$\bar{R}_T \in \mathcal{O}\left(\sqrt{kT}\right)$$

Contextual Bandits

$X = \text{Australian}$

$X = \text{Likes sweets}$

$X = \text{Enjoys Novelty}$

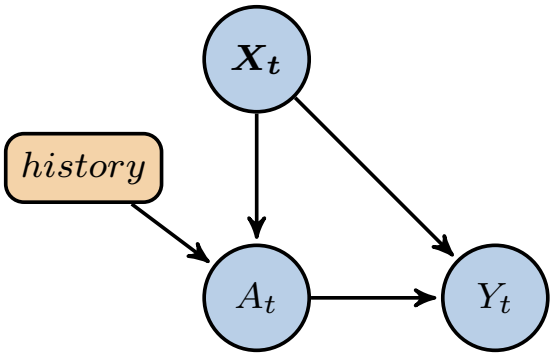
$$\boldsymbol{X} \sim \mathrm{P}\left(x\right) \qquad Y \sim \mathrm{P}\left(y|X,a\right)$$

$$\bar{R}_T = \max_{h \in \mathcal{H}} \mathbb{E}\left[\sum_{t=1}^{T} Y_t^{h(\boldsymbol{X}_t)}\right] - \mathbb{E}\left[\sum_{t=1}^{T} Y_t^{A_t}\right] \qquad \bar{R}_T \in \mathcal{O}\left(\sqrt{kTlog(|\mathcal{H}|)}\right)$$

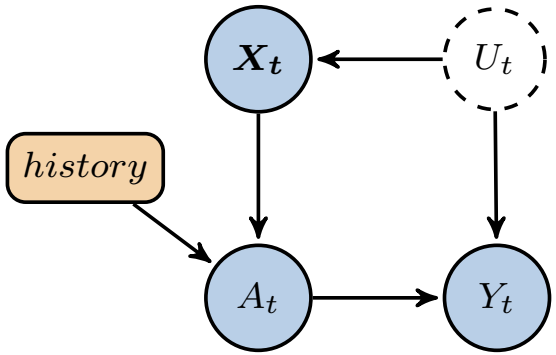
Causal structure of contextual bandit problems

- Contextual bandit algorithms do not require that the context is a cause of the outcome.

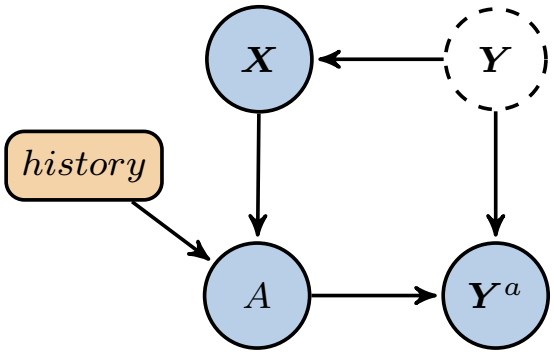
(a) $\boldsymbol{X}$ causes $\boldsymbol{Y}$



(b) $\boldsymbol{X}$ and $\boldsymbol{Y}$ are confounded



(c) $\boldsymbol{Y}$ causes $\boldsymbol{X}$



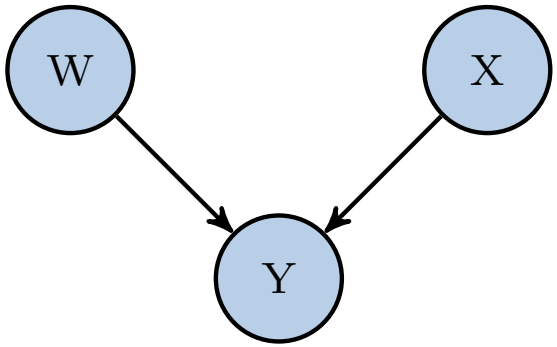
Why unify causal graphs and bandits?

- Regret grows linearly with the number of actions
- There is a lot of data available for which we do not control/know the process selecting actions
- Bandits are a nice subset of general RL problems

Causal Bandit Problems

- Every (allowable) assignment of variables to values is a bandit arm.
- Reward is value of a single specified node in the graph after the action.

Observe $\boldsymbol{X}_c$, $do(\boldsymbol{X_a} = \boldsymbol{x})$, observe $\boldsymbol{X}_o$, obtain reward Y

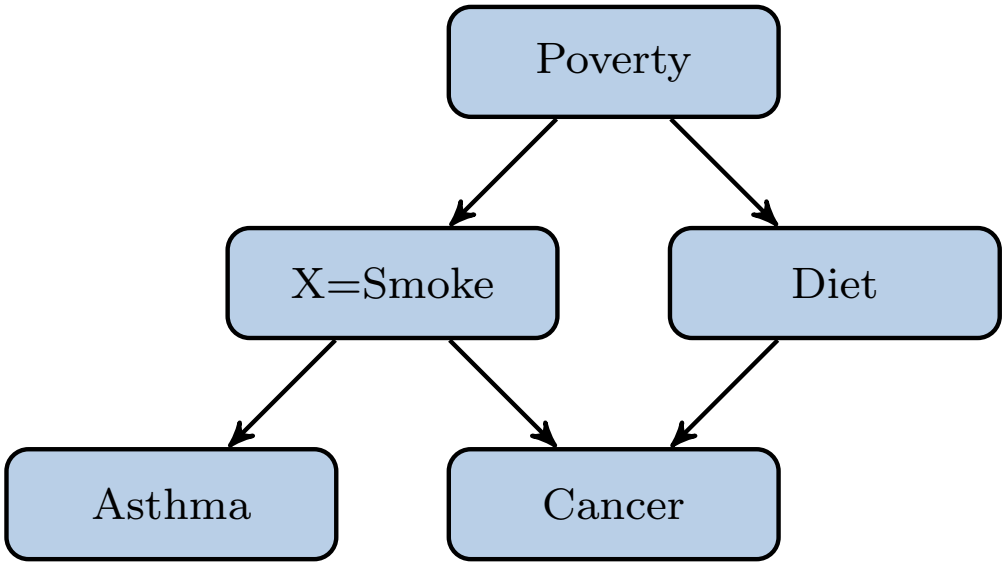


$$\mathcal{A} = \begin{array}{|c|}\hline do(W=0,Z=0) \\ do(W=0,Z=1) \\ do(W=1,Z=0) \\ do(W=1,Z=1) \\ \hline do(W=0) \\ do(W=1) \\ do(Z=0) \\ do(Z=1) \\ do() \\ \hline \end{array}$$

How can causal structure be leveraged

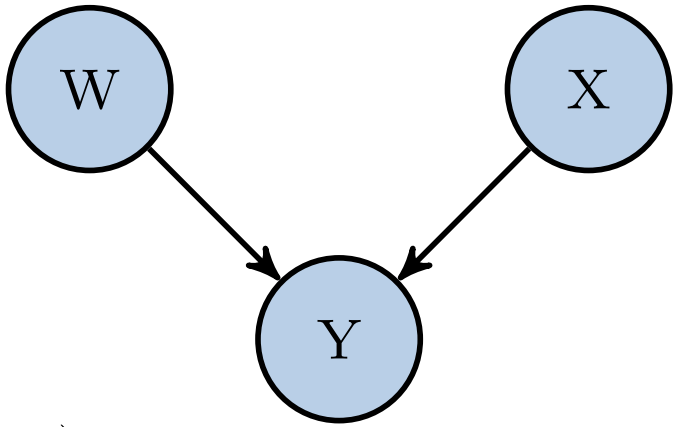
1. Prune actions before commencing

2. Obtain information about the reward for one action by selecting another.



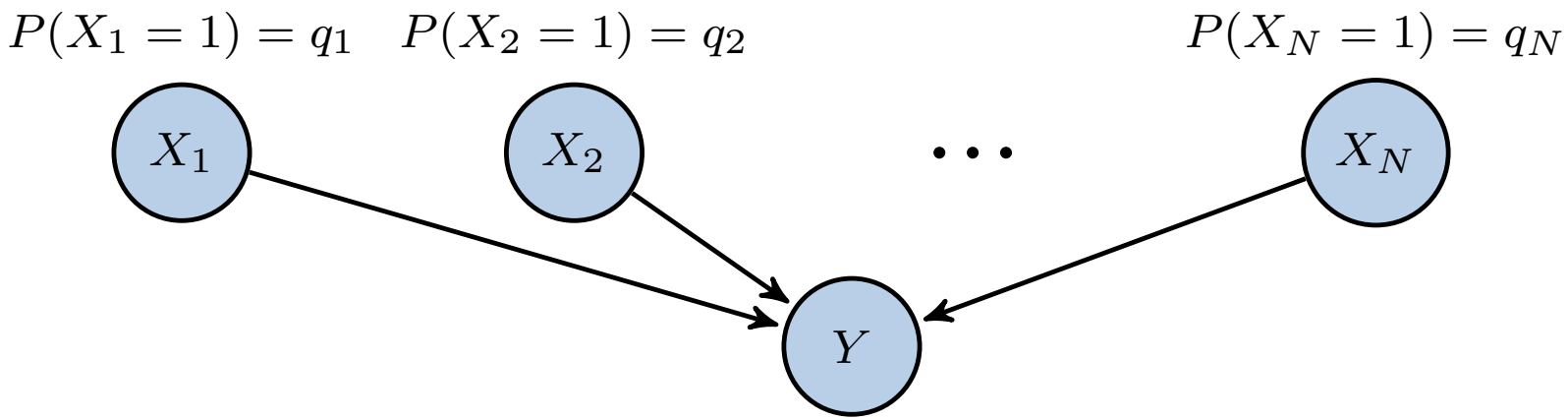
C-Bandit problems with post-action feedback

- Problems for which additional feedback available only **after** action selected (no context).
- Focus is on the simple regret $R_T = \mu_{i*} - \mathbb{E}\left[\mu_{\hat{i}*}\right]$



$$P(Y|do(W=1)) = P(Y|W=1)$$
$$= P(Y|W=1, do(Z=1))P(Z=1) + P(Y|W=1, do(Z=0))P(Z=0)$$

The parallel bandit problem

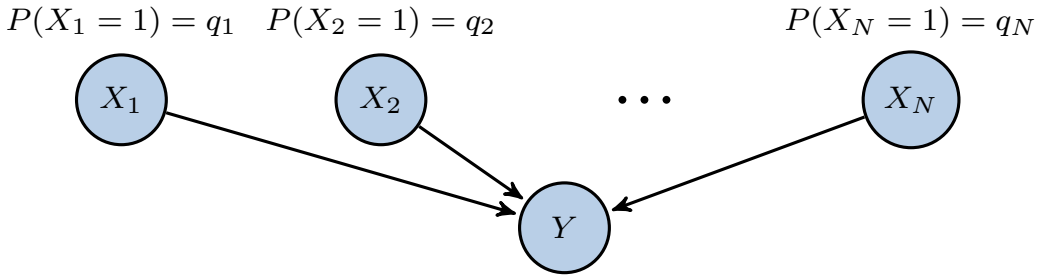


At each timestep $t \in 1, ..., T$:

1. The agent sets the value of zero or one variables, $do()$ or $do(X_i = j)$.
2. Non-intervened variables sampled from underlying distributions.
3. The agent receives reward Y sampled from $P(Y|x_1, ..., x_N)$.
4. The values of all variables, $x_1, ..., x_N$, are revealed to the agent.

The variables are binary $\implies$ there are $2N + 1$ arms.
If the agent opts to observe, $do()$, it will obtain an estimate for half the arms

The parallel bandit problem – algorithm

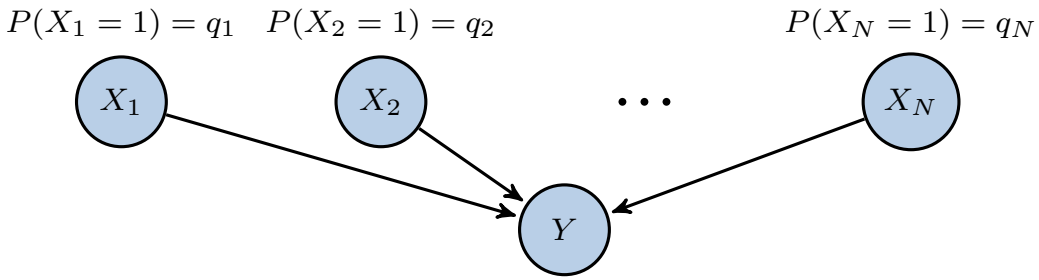


- Observe for first T/2 rounds to estimate reward for actions that occur frequently naturally.
- Uniformly explore infrequent actions in remaining rounds.
- But how do we define infrequent?

$$m(\boldsymbol{q}) = min\left\{m : q_m \geq \frac{1}{m}\right\} \qquad \text{infrequent} \equiv \left\{i : q_i < \frac{1}{m(\boldsymbol{q})}\right\}$$

$$\boldsymbol{q} = [0, 0.01, 0.32, 0.33, 0.33, 0.35, .37, 0.41, 0.45, 0.49, 0.49] \implies m(\boldsymbol{q}) = 4$$

The parallel bandit problem – regret bounds



Parallel bandit $$\Omega\left(\sqrt{\frac{m(\boldsymbol{q})}{T}}\right) \geq R_T \leq \mathcal{O}\left(\sqrt{\frac{m(\boldsymbol{q})}{T}\log\left(\frac{NT}{m(\boldsymbol{q})}\right)}\right)$$

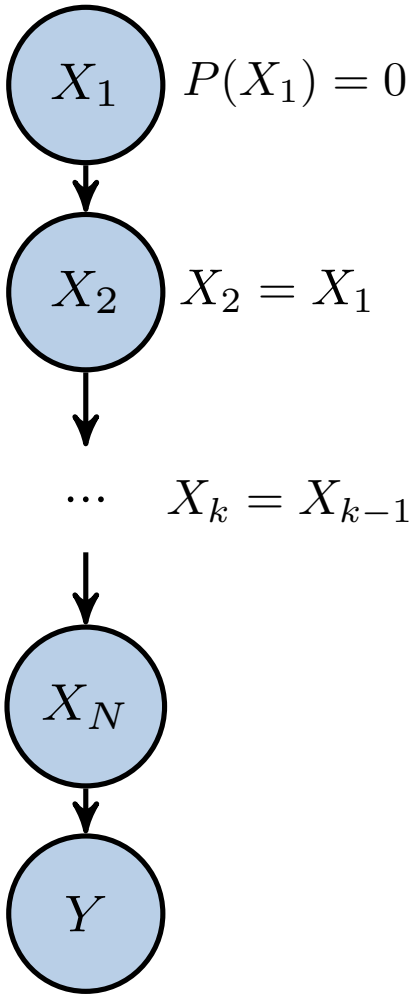
Standard bandit $$\Omega\left(\sqrt{\frac{N}{T}}\right) \geq R_T \leq \mathcal{O}\left(\sqrt{\frac{N}{T}\log T}\right)$$

$m(\boldsymbol{q}) < N$ can be thought of as the *effective* number of arms

General graphs - challenges

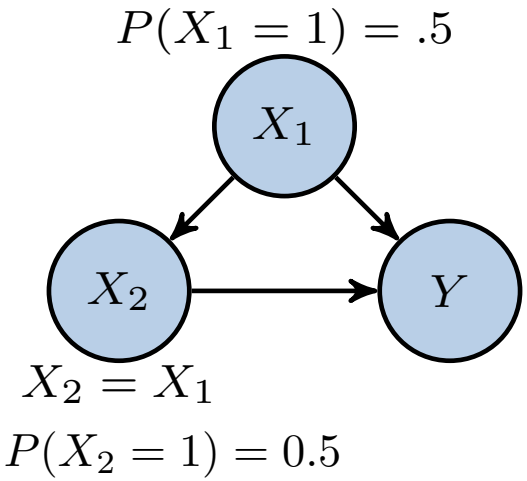
In general, $P(Y|X_i = j) \neq P(Y|do(X_i = j))$

- We could map from observation to intervention via the do-calculus but,
- Its no longer optimal ignore the information intervening on one variable can provide about another.



General graphs - challenges

- The variance of the observational estimate on $P(Y|do(X_i = j))$ no longer depends only on $P(X_i = j)$



$$P(Y|do(X_2 = 1)) = P(X_1 = 1)P(Y|X_1 = 1, X_2 = 1) + P(X_1 = 0)P(Y|X_1 = 0, X_2 = 1)$$

General graphs - solution

We assume the distribution over the parents of Y given each action, $\mathrm{P}\left(\mathcal{P}\mathrm{a}_Y \,|a\right)$, is known for all $a \in \mathcal{A}$

Let η be a distribution of interventions $a \in \mathcal{A}$

for $t \in \{1, ..., T\}$:

Sample action a_t from η

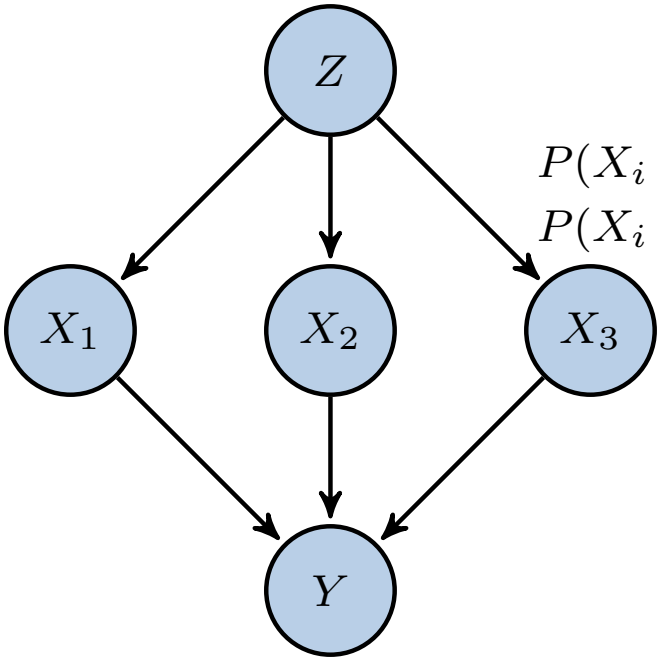
for $a \in \mathcal{A}$:

$$\hat{\mu}_a = \frac{1}{T} \sum_{t=1}^{T} Y_t \frac{\mathrm{P}\left\{\mathcal{P}\mathrm{a}_Y(X_t)|a\right\}}{\sum_{b \in \mathcal{A}} \eta_b \, \mathrm{P}\left(\mathcal{P}\mathrm{a}_Y(X_t)|b\right)}$$

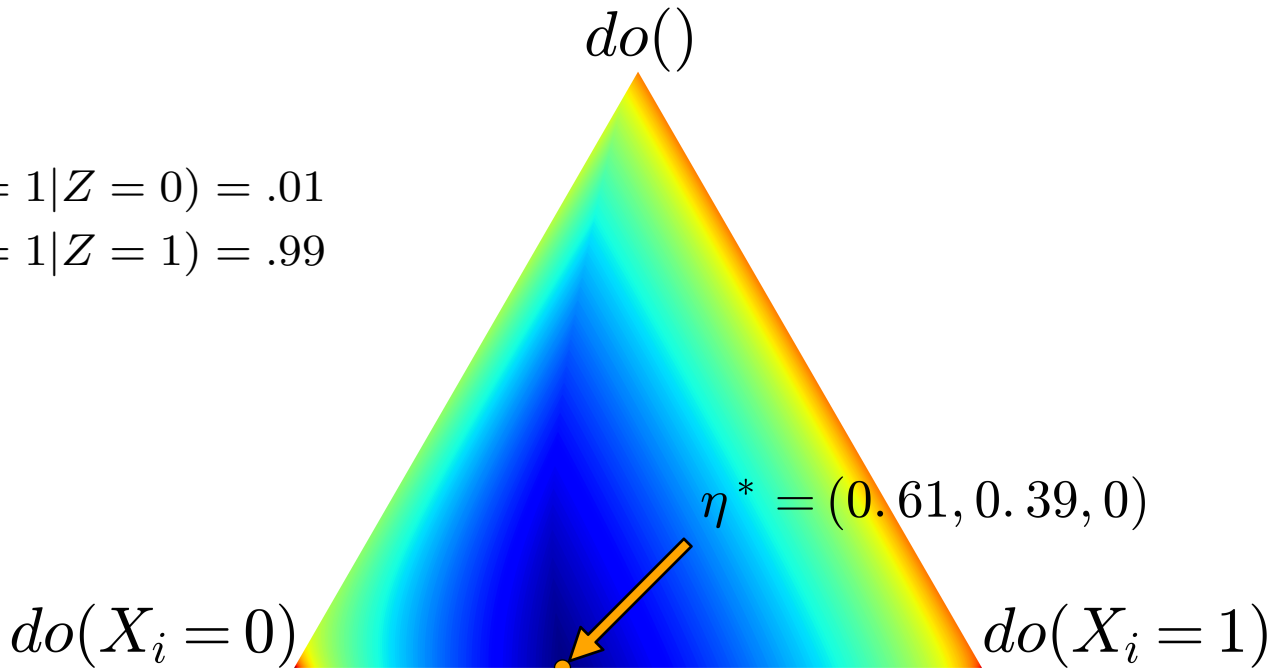
General graphs - solution

$$m(\eta) = \max_{a \in \mathcal{A}} \mathbb{E}_a \left[\frac{\mathrm{P}\left\{\mathcal{P}\mathrm{a}_Y(X)|a\right\}}{\sum_{b \in \mathcal{A}} \eta_b \, \mathrm{P}\left\{\mathcal{P}\mathrm{a}_Y(X)|b\right\}} \right]$$

$P(Z = 1) = 0.7$



$P(X_i = 1|Z = 0) = .01$
$P(X_i = 1|Z = 1) = .99$



General graphs - solution

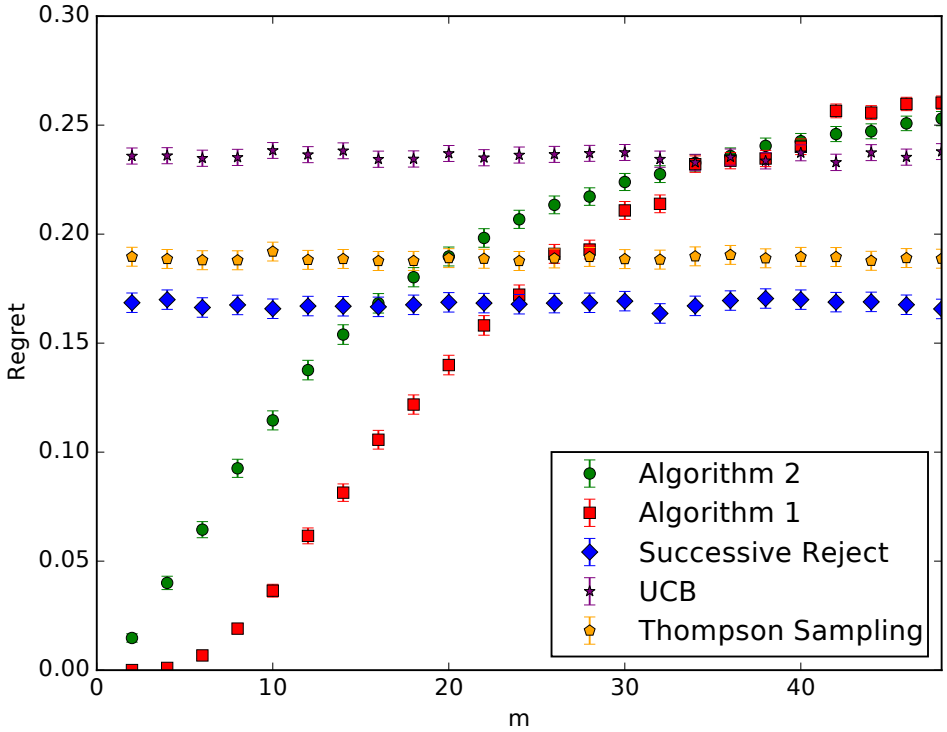
$$m(\eta) = \max_{a \in \mathcal{A}} \mathbb{E}_a \left[\frac{\mathrm{P}\left\{\mathcal{P}\mathrm{a}_Y(X)|a\right\}}{\sum_{b \in \mathcal{A}} \eta_b \mathrm{P}\left\{\mathcal{P}\mathrm{a}_Y(X)|b\right\}} \right]$$

$$\eta^* = \arg\min_{\eta} m(\eta) \qquad m(\eta^*) \leq |\mathcal{A}|$$

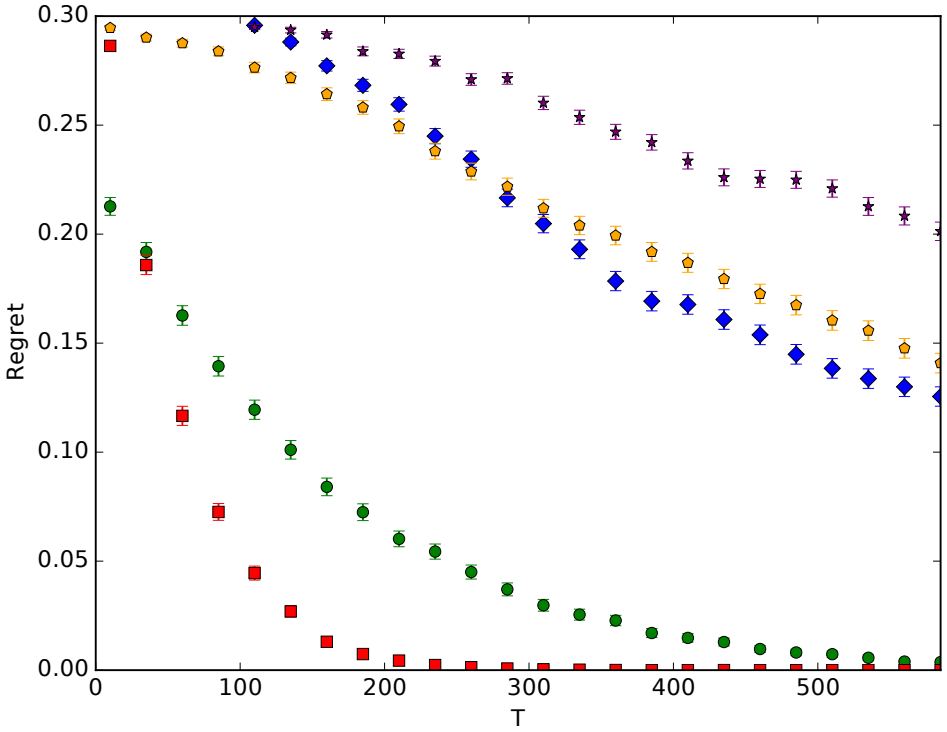
$$R_T \in \mathcal{O}\left(\sqrt{\frac{m(\eta)}{T} \log\left(2T|\mathcal{A}|\right)} \right)$$ ← General CB upper bound

$$R_T \in \Omega\left(\sqrt{\frac{|\mathcal{A}|}{T}} \right)$$ ← Standard lower bound

Experiments



(a) Simple regret vs $m(\boldsymbol{q})$ for fixed horizon $T = 400$ and number of variables $N = 50$

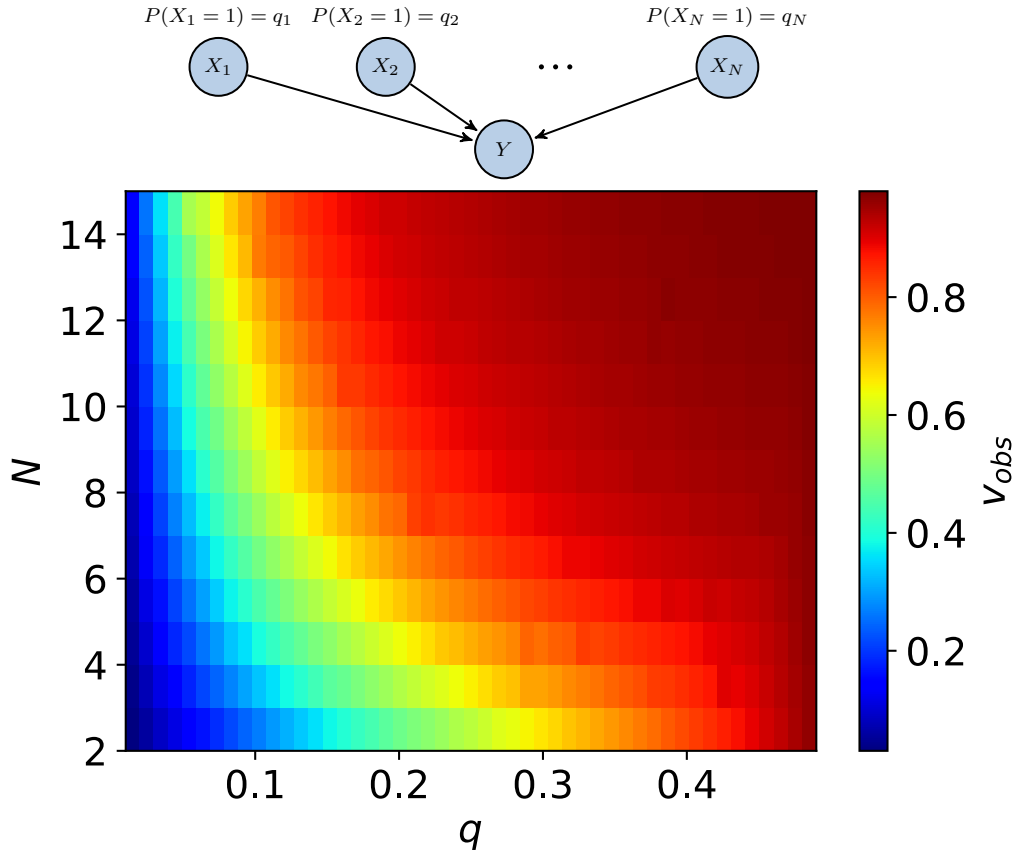


(b) Simple regret vs horizon, T, with $N = 50$, $m = 2$ and fixed $\epsilon = .3$

Quantifying the value of intervention

$$R_T \in \mathcal{O}\left(\sqrt{\frac{m(\eta)}{T}\log\left(2T|\mathcal{A}|\right)}\right) \Longleftarrow \text{holds for all } \eta$$

$$= \frac{m(\eta^*)}{m(\eta_{do})}$$



Summary & Future Work

- Bandits and causal inference fundamentally solve the same problem – learning to act
- Adding structure to bandit problems with causal graphical models allows us to explore large action spaces much more quickly
- Many open questions: relaxing the assumption that the interventional distributions are known, contextual causal bandit problems, cumulative regret, extensions to MDPs, connections between causal effect estimation and off-policy evaluation

Questions