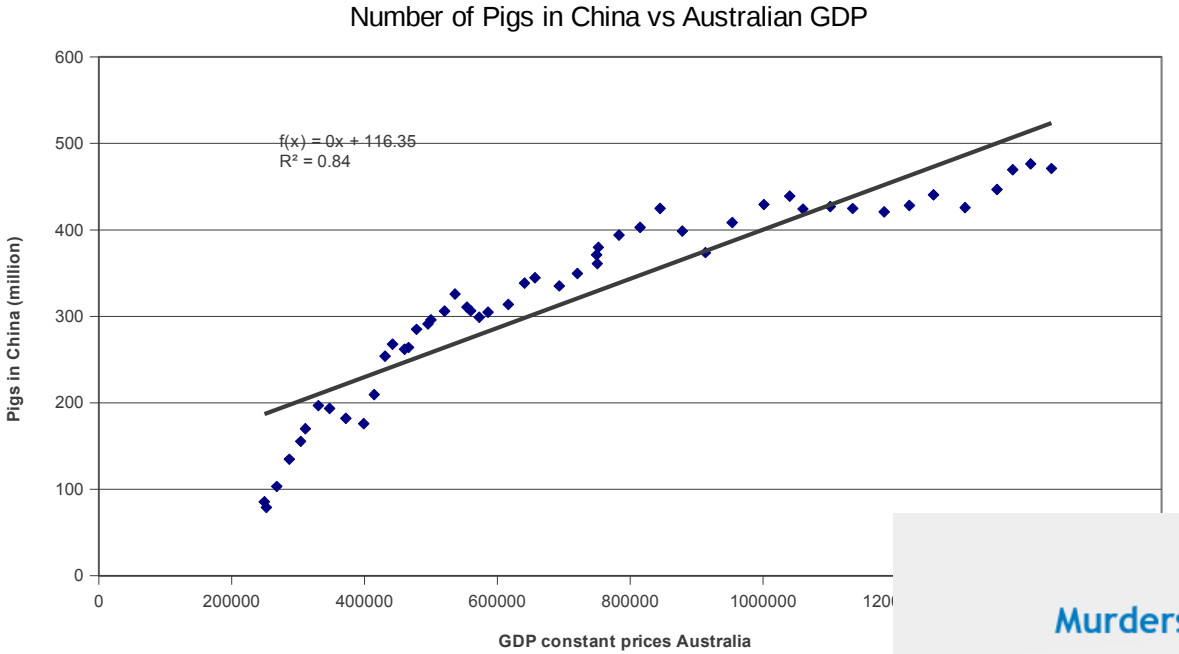
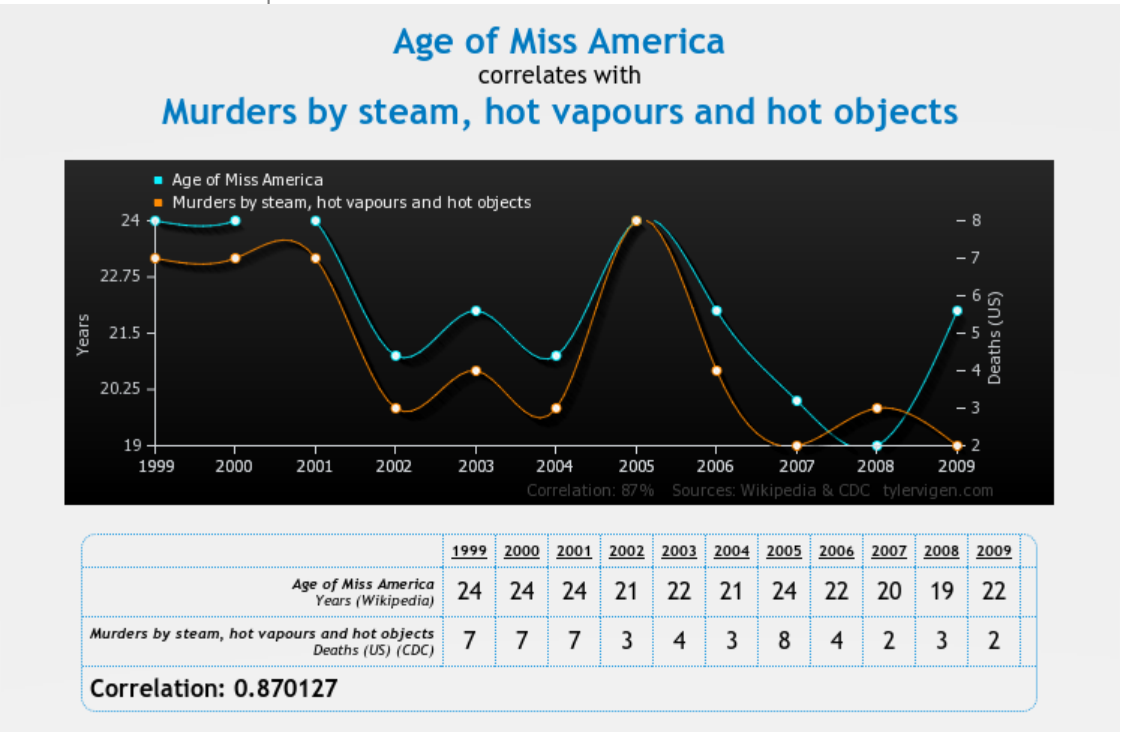
Correlation is not causation

Ways things can go wrong





	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Age of Miss America *Years (Wikipedia)*	24	24	24	21	22	21	24	22	20	19	22
Murders by steam, hot vapours and hot objects *Deaths (US) (CDC)*	7	7	7	3	4	3	8	4	2	3	2

Correlation: 0.870127

Image source: www.tylervigen.com/

We care about causality

Chocolate 'may help keep people slim'

COMMENTS (251)

By Michelle Roberts
Health reporter, BBC News

People who eat chocolate regularly tend to be thinner, new research suggests.

The findings come from a study of nearly 1,000 US people that looked at diet, calorie intake and body mass index (BMI) - a measure of obesity.

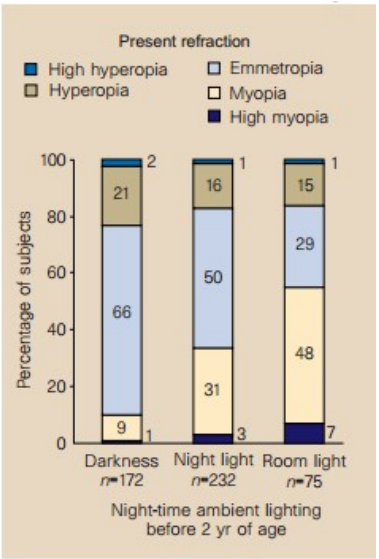
It found those who ate chocolate a few times a week were, on average, slimmer than those who ate it occasionally.

Even though chocolate is loaded with calories, it contains ingredients that may favour weight loss rather than fat synthesis, scientists believe.

Chocolate contains antioxidants but is also high in fat and sugar

Related Stories

http://www.bbc.com/news/health-17511011



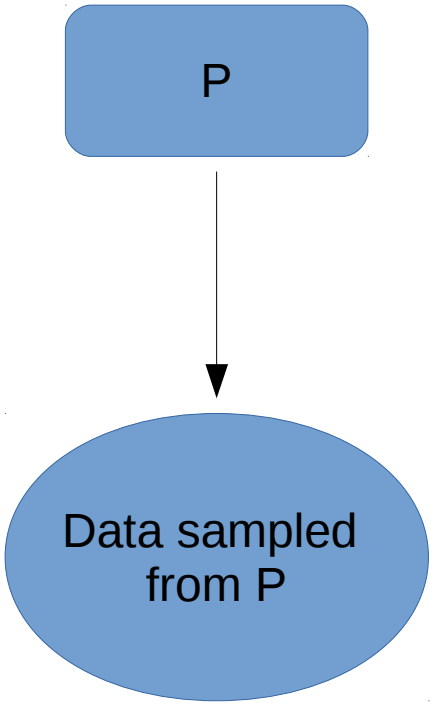
Sleeping with the light on is associated with short-sightedness in kids.
Quinn, Graham E., et al. "Myopia and ambient lighting at night." Nature 399.6732 (1999): 113-114

Childhood obesity partly caused by strict parenting, say scientists

Parents who struck a balance between being strict and kind were less likely to bring up obese children

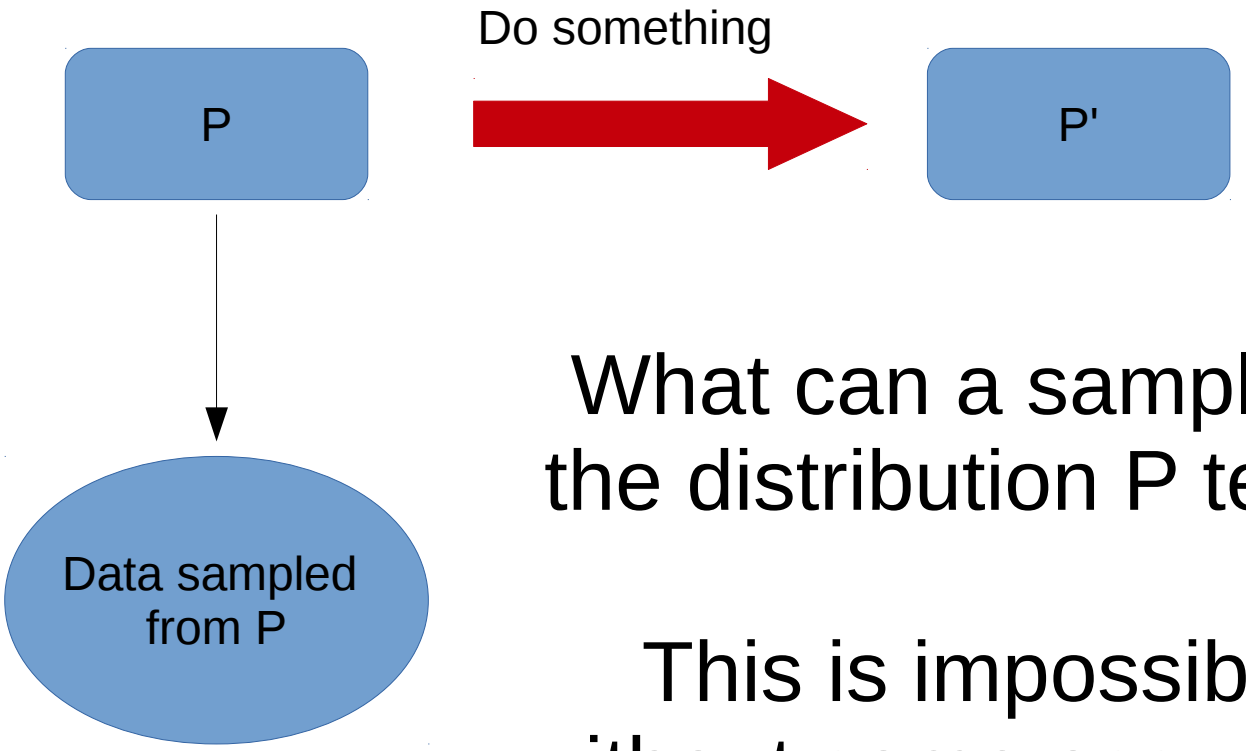
http://www.independent.co.uk/life-style/health-and-families/health-news/childhood-obesity-partly-caused-by-strict-parenting-say-scientists-9206147.html

Machine Learning/Statistics



What can we learn about the distribution P from a sample of data drawn from it?

Causal inference



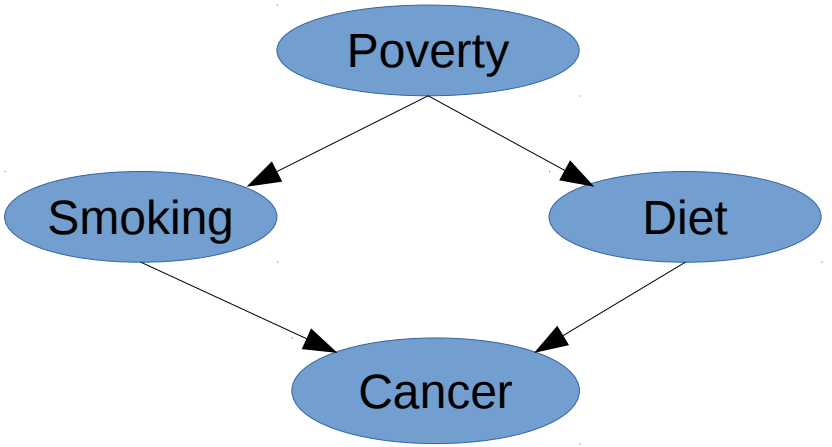
What can a sample of data from the distribution P tell us about P'?

This is impossible to answer without some assumptions on how 'do something' changes P

Causal bayesian networks (causal DAGs)

A bayesian network where A → B is defined to mean A causes B

=> Variables are independent of their non-effects given their direct causes (Causal Markov Property)

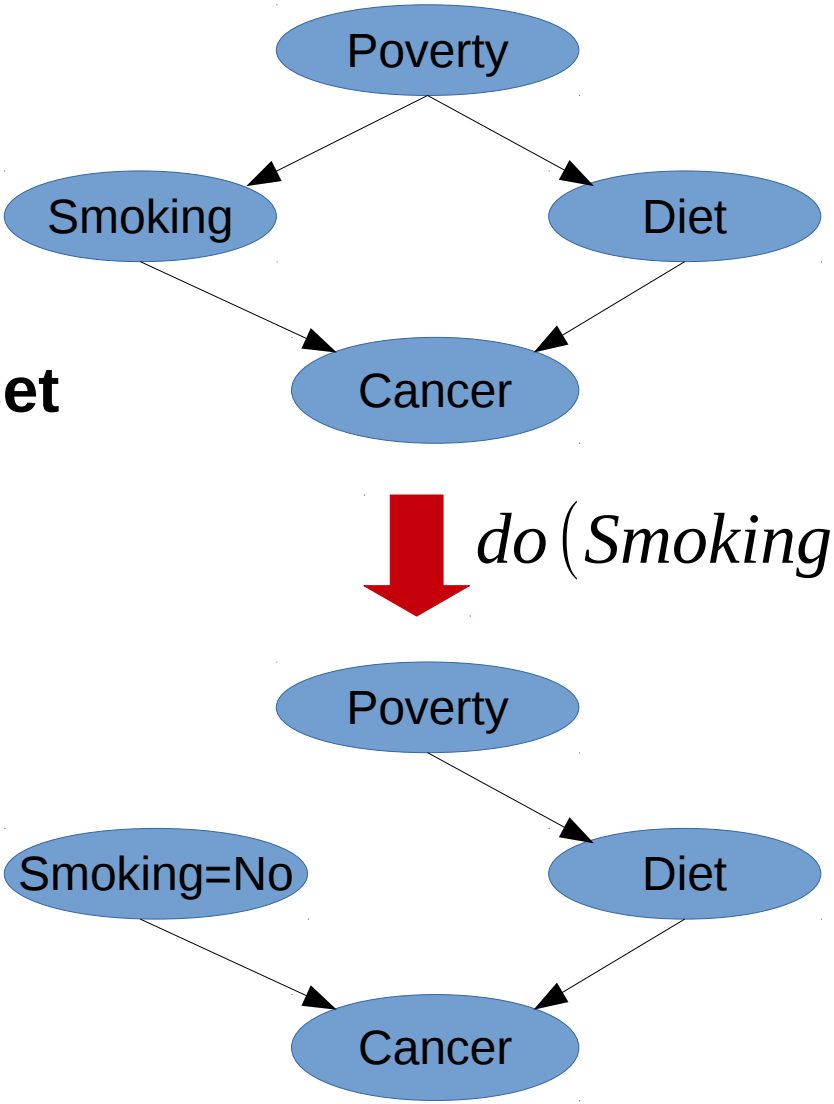


Absent links imply the factorisation of the full distribution can be simplified.

$$P(Po,S,D,C)=P(Po)P(S|Po)P(D|Po,S)P(C|Po,S,D)=P(Po)P(S|Po)P(D|Po)P(C|S,D)$$

Intervention in Causal DAGs

A causal DAG represents the set of all possible interventional distributions over its variables

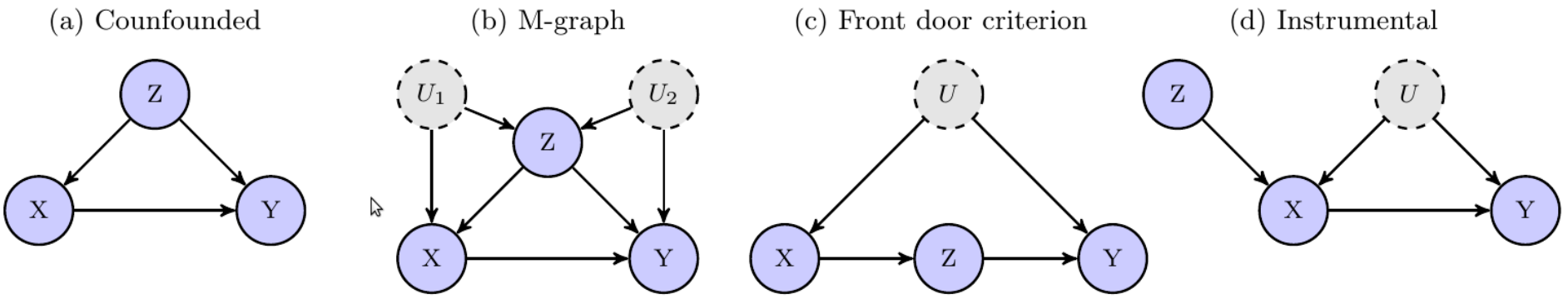


$$P(Cancer|do(Smoking)) = \sum_{Poverty}\sum_{Diet} P(Poverty)\,P(Diet|Poverty)\,P(Cancer|Diet, Smoking)$$

$$\neq P(Cancer|Smoking)$$

Calculating causal effects

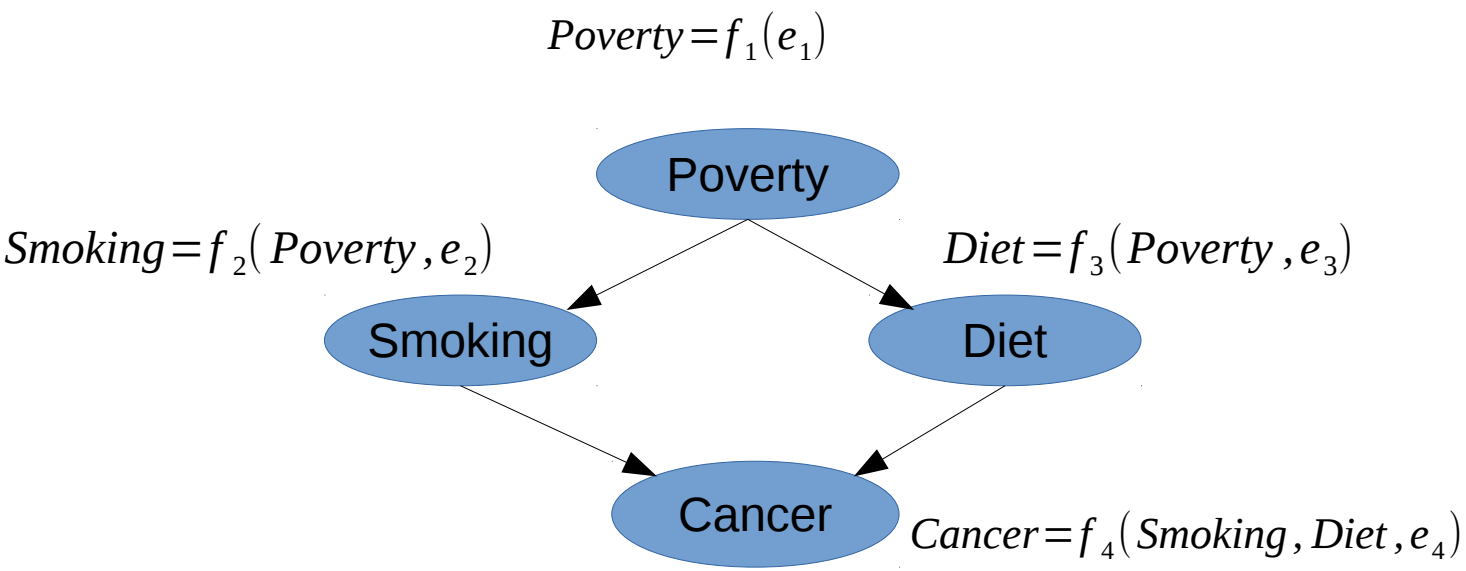
- If you can observe all the variables then you can estimate the effect of any intervention.
- If there are some latent (unobservable) variables, use the do-calculus (see Pearl 2000)



Structural equation models (SEMs)

- Represent each variable as a deterministic function of its direct causes and noise
- Noise terms must be mutually independent

$$x_i = f_i(Parents_i, e_i) \quad \text{where} \quad \{f_1 \ldots f_n\}\ deterministic\ functions$$
$$\{e_1 \ldots e_n\}\ mutually\ independent$$



If the set of equations does not create a cycle then the Causal Markov property holds and the SEM is a causal bayesian network

Causal Discovery

what to do when you don't know the graph

Independence based methods

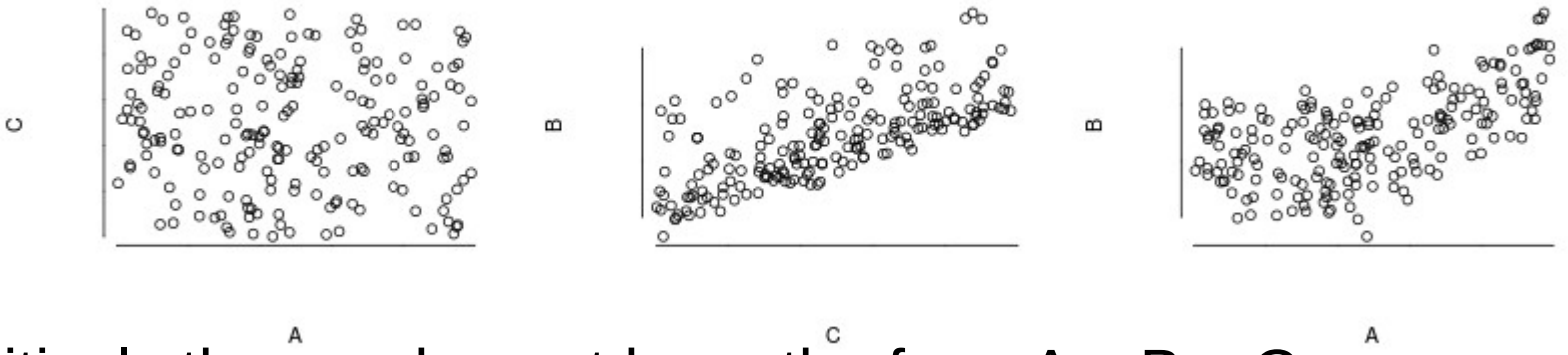
1) We assume our distribution P was generated by some (unknown) causal directed acyclic graph

2) We assume that all the conditional independences in P are implied by d-separation in the true causal network (***faithfulness***).

3) Finding the causal structure equates to finding graphs that imply exactly the conditional independencies observed in our distribution.

Intuition

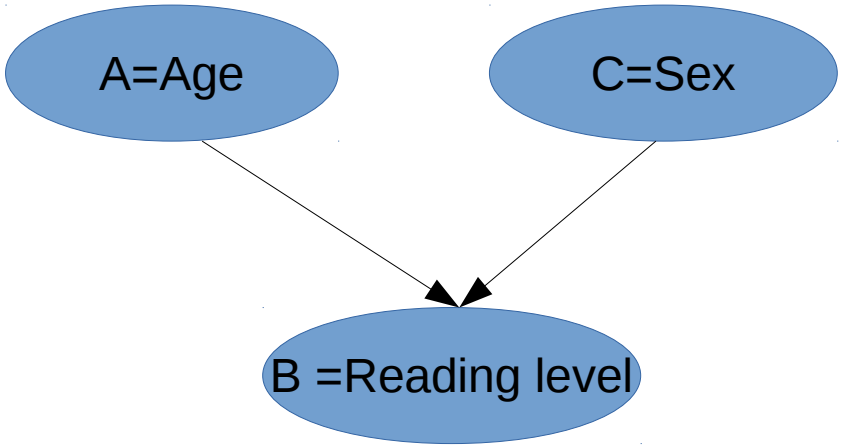
- Suppose I tell you, A and C are indepependent variables both correlated with B.



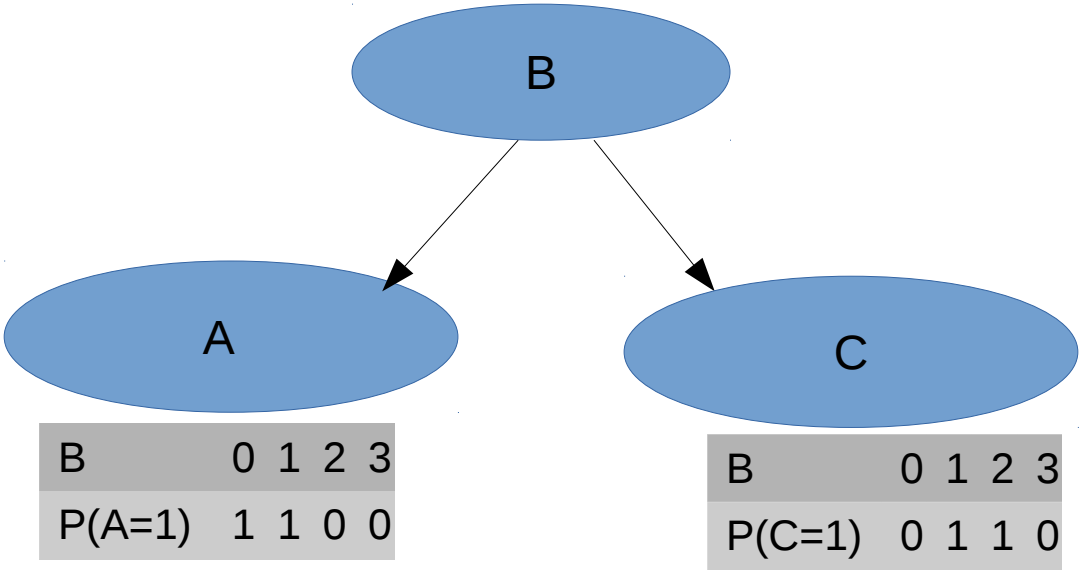
- Intuitively the graph must have the form A – B – C ,
- Can you think of an real world example for A → B ← C,
- What about A ← B → C?

Intuition

A and C are indepependent variables both correlated with B.



B	0	1	2	3
P(B)	0.25	0.25	0.25	0.25



B	0	1	2	3
P(A=1)	1	1	0	0

B	0	1	2	3
P(C=1)	0	1	1	0

$$C \perp A$$

$$C \perp A \mid B$$

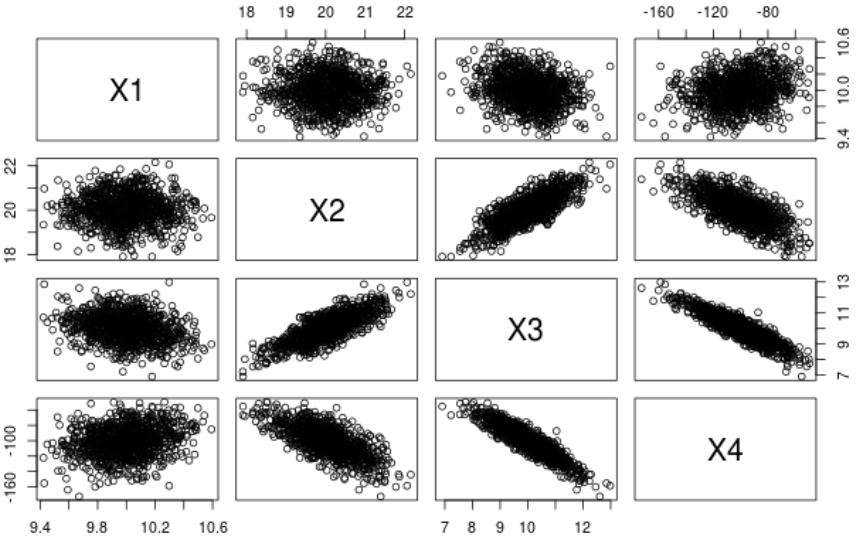
$$P(C=1 \mid A=1) = 0.5 = P(C) \Rightarrow C \perp A$$

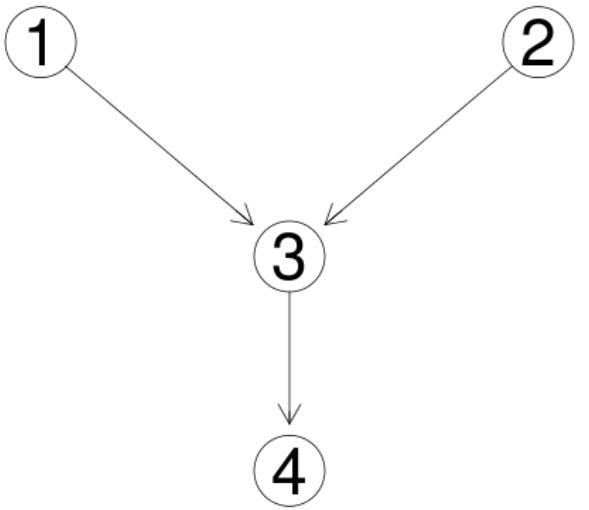
Practicalities

Algorithms: PC, FCI (Sprites 2000),GES (Chickering 2002),RFCI (Colombo, Diego, et al 2012)

Implementations:
Pcalc (R package)
Tedrad

```
library('pcalg')
n = 1000
X1 = rnorm(n,mean=10,sd=.2)
X2 = rnorm(n,mean=20,sd=.7)
X3 = X2-X1+rnorm(n,mean=0,sd=.5)
X4 = -X3^2+rnorm(n,mean=0,sd=8)
df = data.frame(X1,X2,X3,X4)
plot(df)
suffStat <- list(C = cor(df),n=nrow(df))
pc.3var = pc(suffStat,indepTest=gaussCItest,p=ncol(df),alpha=0.01)
plot(pc.3var, main = "")
```





Beyond conditional independence

Additive noise: $y = f(x) + e$

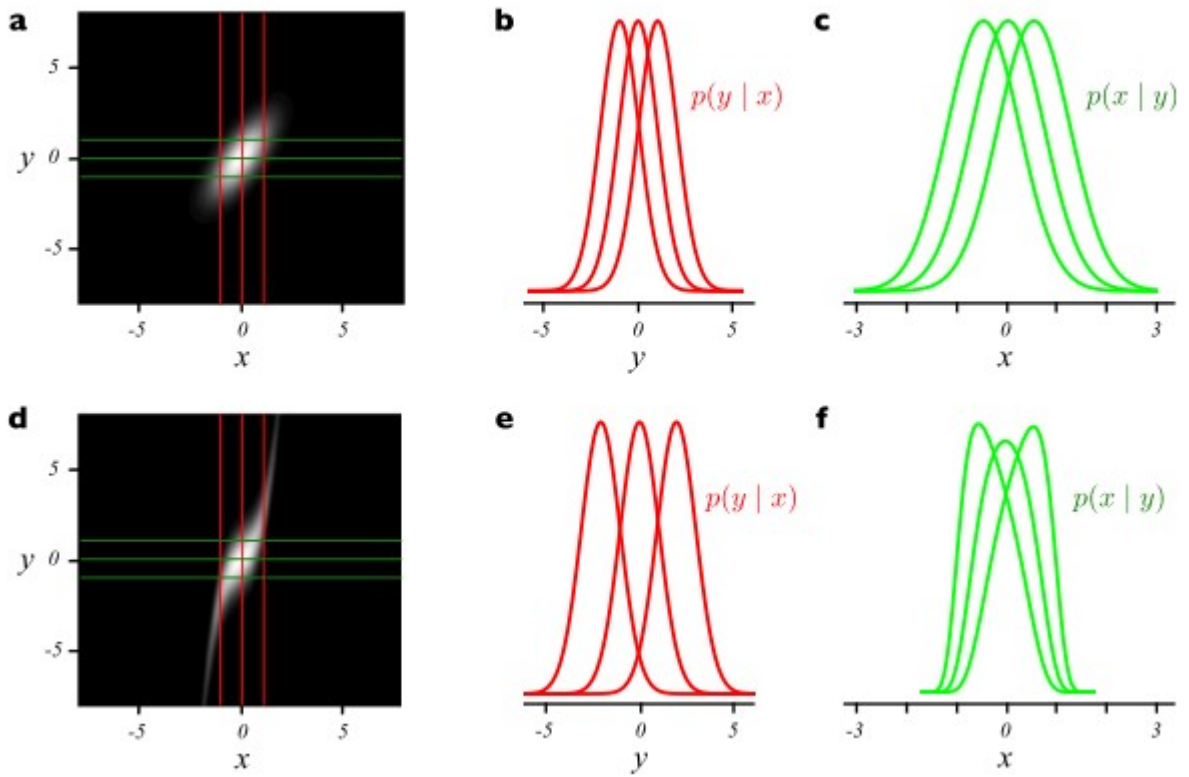


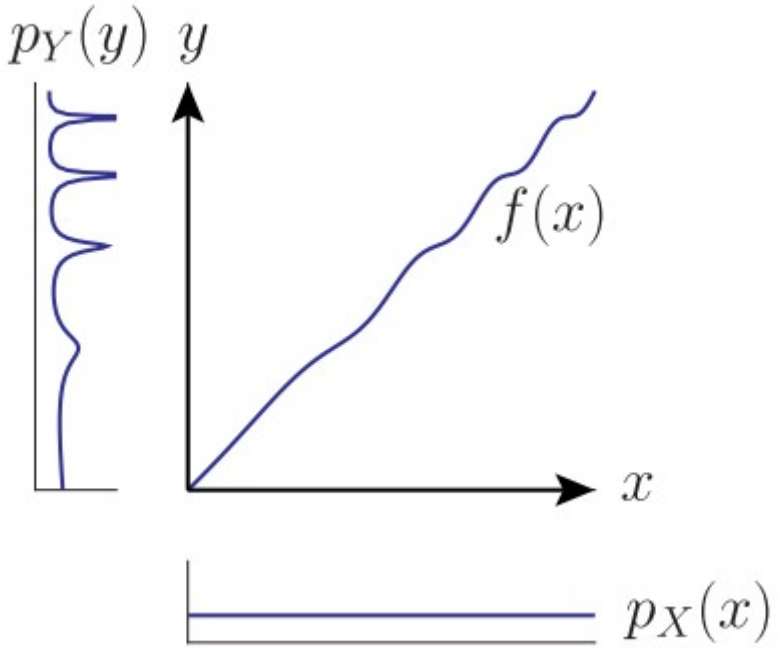
Figure 1, (Hoyer et al 2009)

Can be extended to post-non-linear additive noise $y = h(f(x) + e)$

(Zhang et al 2009)

More asymmetries of cause and effect

Figure 1: Daniusis et al 2012



Independence of function and input:
If X → Y and we have a functional causal model y = f(x,e) then the input distribution P(X) and function f represent independent mechanisms. Changing the input distribution does not modify the function itself.

Causal-Anticausal

- If X → Y, then assuming independence of input and mechanism we would expect P(X) and P(Y|X) should be independent.
- P(Y) and P(X|Y) will generally not be independent.

In semi supervised learning we are given training points from P(X,Y) and additional points sampled from P(X). The goal is to learn P(Y|X). The extra points from P(X) will not help if X → Y.

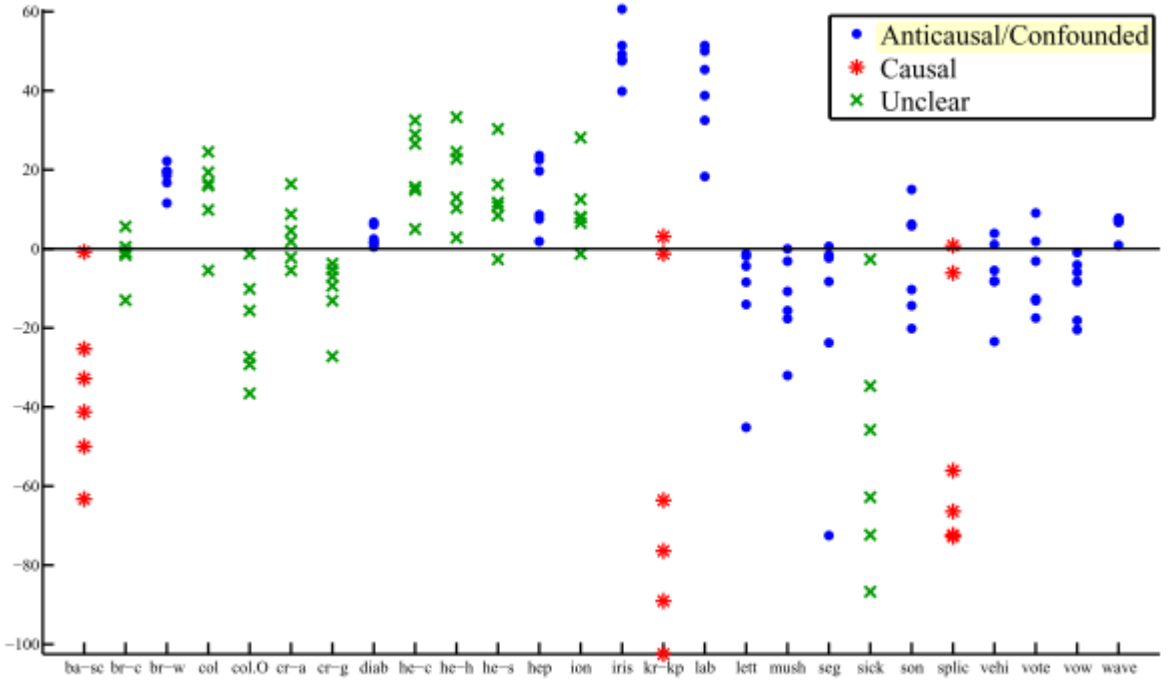


Figure 6, Janzing & Peters 2012

Learning what causality looks like

Suppose we had M different causal pairs data sets.

$$D = \{\{x_j, y_j\}_{j=1}^{N_i}, l_i\}_{i=1}^{M}$$

Where l_i is a binary label that indicates if $X \to Y$ or $Y \to X$ in dataset i.

We expect there to be differences in the relationships between $P(X)$ $P(Y)$ and $P(Y|X)$ for $X \to Y$ and $Y \to X$

Let μ be a kernel mean embedding that maps a distribution P into some Hilbert space.

For each data set $i = 1...M$

Construct a feature vector that approximates $\mu(P(X)), \mu(P(Y)), \mu(P(X,Y))$

Apply a standard classification algorithm

See Lopez-Paz et al 2014

Questions

I may not be able to stay and socialize after the talks (due to morning sickness) but I'm very happy to recieve questions/feedback via email:

finnlattimore@gmail.com

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TETRAD software http://www.phil.cmu.edu/tetrad/

Now with more than two variables!

Bi-variate additive noise model $Y = f(X) + e$

If the triple: $(P(X), P(e), f)$ satisfies a certain condition then model is identifiable

To extend to multivariate case we need to satisfy the condition on the conditional distributions

For each variable j

Pick a single parent, i, and fix all the others

For each subset S that contains the remaining parents, and no descendants of j

The triple $(P(X|S=s), P(e_j), f^{ji}(X_i))$ must satisfy the condition for at least one s.

If we can come up with a condition that guarantees identifiability for the bi-variate case, we can extend that result to get the conditions under which the multivariate case is identifiable.

See Peters et al 2014

Counterfactuals

- Statements about what would have happened in some alternate reality where some specified thing were different.
- A medical trial:

  for an individual, i, $\begin{cases} y_i^1 = outcome\ if\ treated \\ y_i^0 = outcome\ if\ not\ treated \end{cases}$

  we only get to measure one of these.
- Let Y^1 be a random variable, $P(Y^1)$ is the distribution of outcome, Y that would occur if everyone was treated.
- The causal effect is defined as $E[P(Y^1) - P(Y^0)]$
- We can measure $\begin{cases} P(Y|X=0) = P(Y^0|X=0) \\ P(Y|X=1) = P(Y^1|X=1) \end{cases}$

$If\ (X \perp Y^0)\ \&\ (X \perp Y^1)$ ← Ignoreability assumption

$$E[P(Y^1) - P(Y^0)] = E[P(Y|X=1)] - E[P(Y|X=0)]$$

Counterfactuals

Counterfactural queries form a larger set than interventional queries

group	placebo	treatment	probability of group
1	die	die	$\alpha = P(Y^0 = 0, Y^1 = 0)$
2	die	recover	$\beta = P(Y^0 = 0, Y^1 = 1)$
3	recover	die	$\gamma = P(Y^0 = 1, Y^1 = 0)$
4	recover	recover	$\delta = P(Y^0 = 1, Y^1 = 1)$

What is the probability that a patient, who was not treated and died, would have recovered if they had been treated? We know they are in either group 1 or 2 since they died without treatment, so the answer is $\frac{\beta}{\alpha+\beta}$

Can we identify that from $P(Y^0)$ and $P(Y^1)$?

How do the models relate?

- Do type queries can be phrased in terms of counterfactuals

$$Y^0 \equiv P(Y|do(X=0))$$
$$Y^1 \equiv P(Y|do(X=1))$$

- For do type queries, the ignoreability assumption is true if the 'back door criterion' holds.
- Causal bayesian networks don't support full counterfactuals as they only contain information on interventional *distributions*
- SEMs do define counterfactuals $Y = f(X, e) \Rightarrow Y^0 = f(0, e)$
- There is a subtle difference between the assumptions implied by the SEM models and what is actually needed for identification of do-queries (see Richardson & Robins 2013). This makes a difference for identifiability of non-interventional queries.

The Do Calculus: calculating causal effects in a (partially) known network

- The do-calculus rules result from d-separation in a causal DAG
- A causal effect is non-parametrically identifiable if and only if the interventional query can be reduced to an observational one via repeat application of the three rules (see Shpitser&Pearl 2012 for algorithm)

The Do Calculus: calculating causal effects in a (partially) known network

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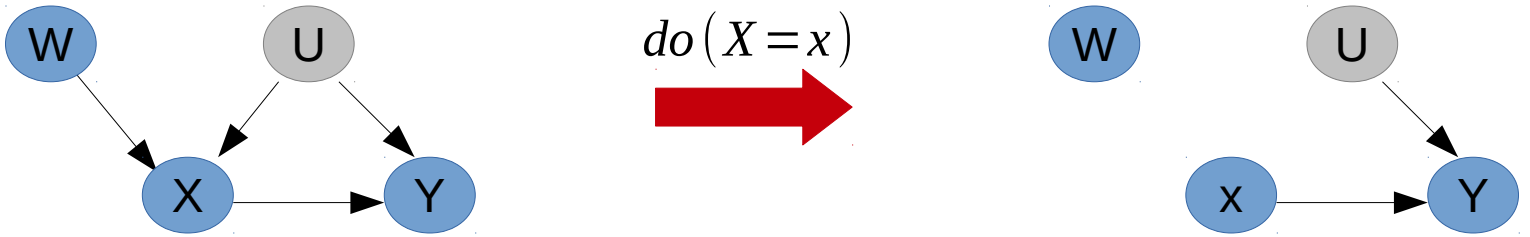
Rule 1: D-separation still applies

The graph $G_{\overline{X}}$ that results from the intervention do(X=x) is still a bayesian network and d-separation applies.

$$(Y \perp W | X)_{G_{\overline{X}}} \Rightarrow P(y|do(x), w) = P(y|do(x))$$

$$(Y \perp W | X, Z)_{G_{\overline{X}}} \Rightarrow P(y|do(x), z, w) = P(y|do(x), z)$$



Rule 2: intervention and observation

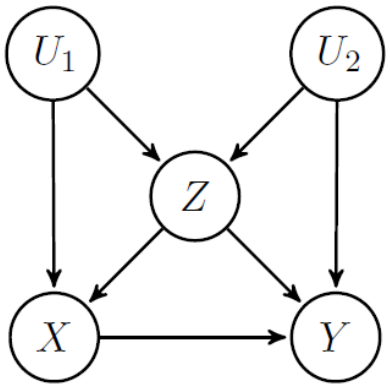
If the target, y, is independent of *how* x is determined intervention is the same as observation.

$$\left(Y \perp \hat{X} \middle| X\right)_{G^+} \Rightarrow P\left(y \middle| do\left(x\right)\right) = P\left(y \middle| x\right)$$

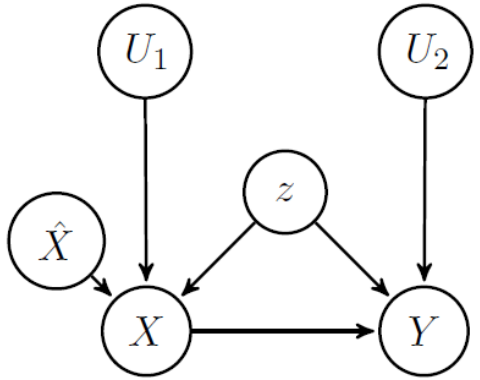
$$\left(Y \perp \hat{X} \middle| X, Z, W\right)_{G^+_{\overline{Z}}}$$

$$\Rightarrow P\left(y \middle| do\left(z\right), do\left(x\right), w\right) = P\left(y \middle| do\left(z\right), x, w\right)$$

(a) original network, G



(b) augmented network after the intervention $do(Z = z)$, $G^{\dagger}_{\overline{Z}}$



Rule 3: Sometimes intervention changes nothing

$$\left(Y \perp \hat{X}\right)_{G^+}$$

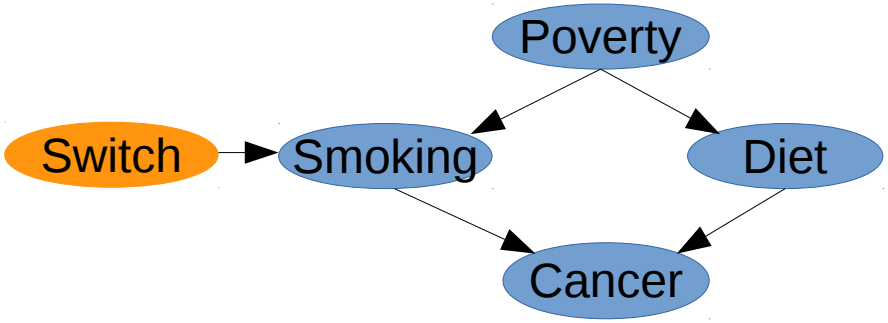
$$\Rightarrow P\left(y|do\left(x\right)\right)=P\left(y\right)$$

$$\left(Y \perp \hat{X}|, Z, W\right)_{G^+_{\bar{Z}}}$$

$$\Rightarrow P\left(y|do\left(z\right), do\left(x\right), w\right)=P\left(y|do\left(z\right), w\right)$$

Smoking switch is d-separated from diet. There is no direct causal path from smoking to diet, so the intervention doesn't change anything.

$$P\left(Diet|do\left(Smoke=N\right)\right)=P\left(Diet\right)$$



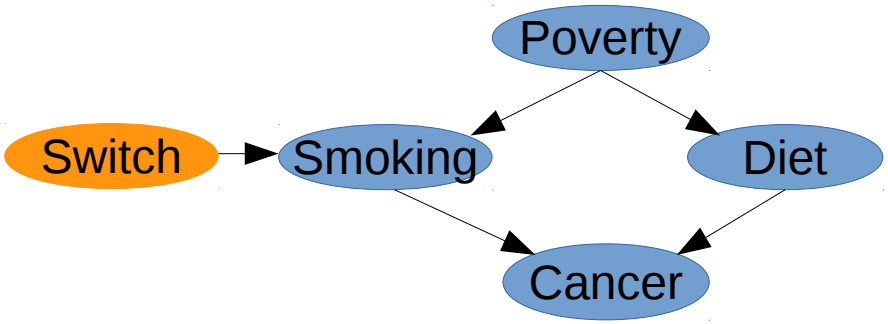
The back door criterion

Just a special case of rule 2. If Z blocks all back door paths from X to Y then the causal effect of X on Y is obtained by adjusting for Z.

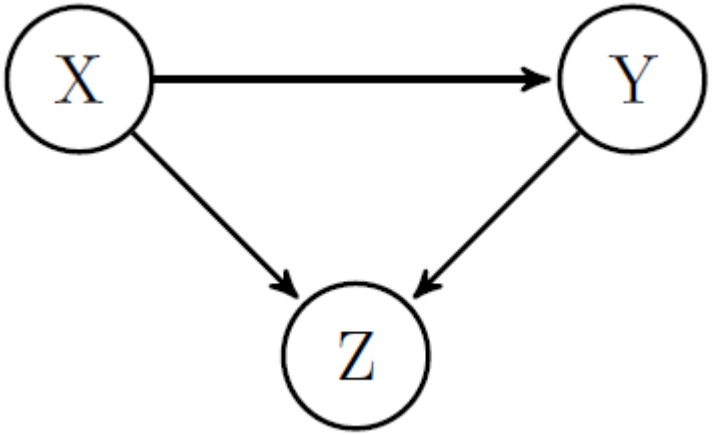
$$\left(Y \perp \hat{X} \middle| X, Z\right)_{G^+}$$

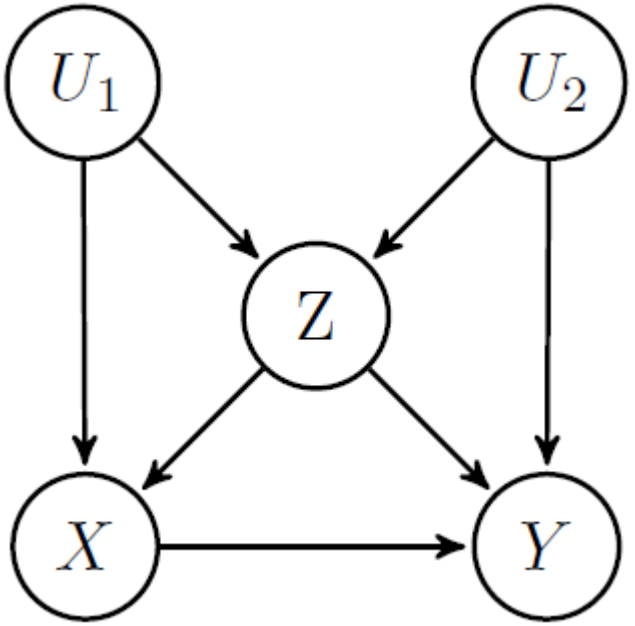
$$\Rightarrow P\left(y \middle| do\left(x\right), z\right) = P\left(y \middle|, x, z\right)$$

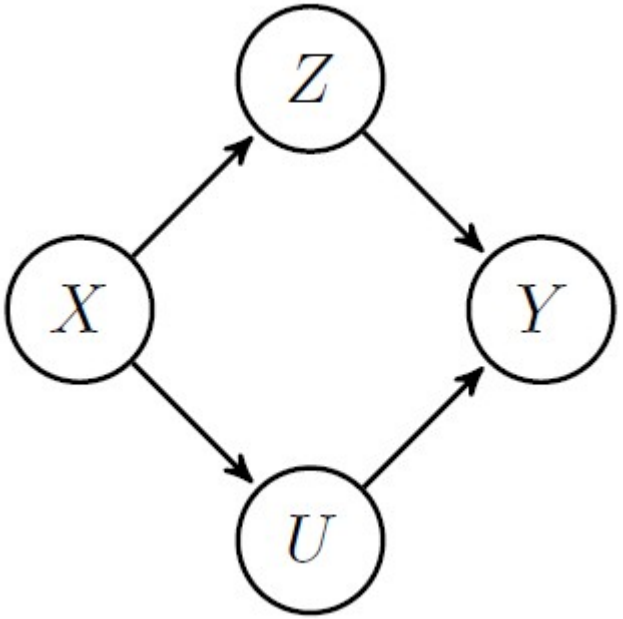
$$\Rightarrow P\left(y \middle| do\left(x\right)\right) = \sum_z P\left(y \middle|, x, z\right) P\left(z\right)$$

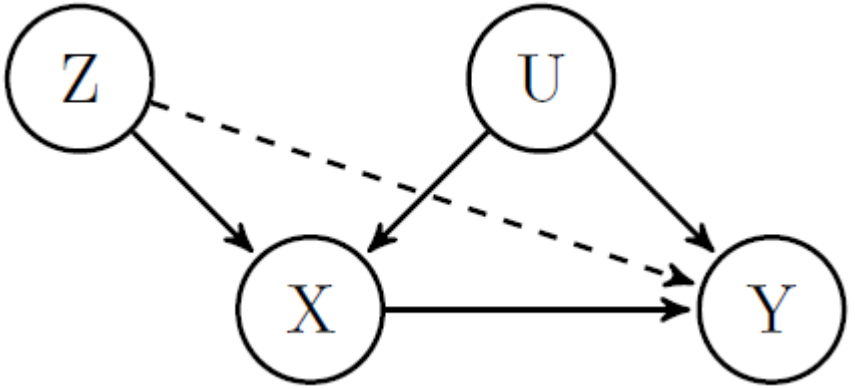


Causal estimation by 'adjusting'









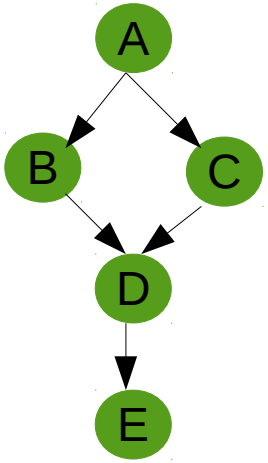
The IC (or SGS) algorithm

1. *for all pairs of variables* a, b *search for a set* S_{ab} *such that* $a \perp b | S_{ab}$.
   If there is no such set, then draw an undirected link between them.

2. *for all pairs of non−linked nodes with a common neighbour,* c*,:*
   If $c \notin S_{ab}$ *direct links towards* c

3. *Orient any undirected edges so as to avoid creating cycles or additional v-structures*

True Causal Model

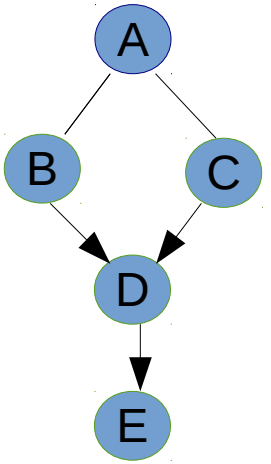


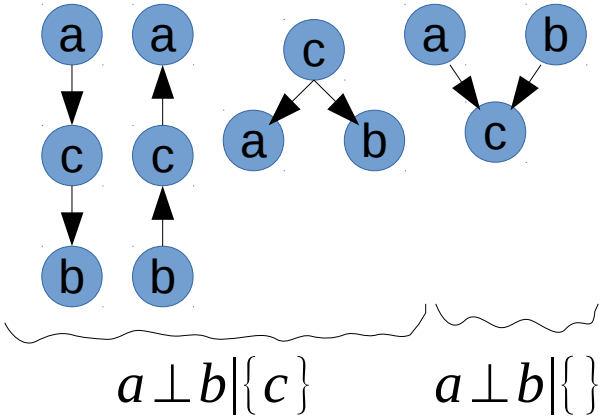
$B \perp C | A$

$D \perp A | B, C$

$E \perp A, B, C | D$

Inferred output





$a \perp b | \{c\}$ $\quad$ $a \perp b | \{\}$

The PC algorithm

A more tractable version of the SGS algorithm

2. **for** each link $a - b$:

    $n = 0$

    $\boldsymbol{A}_{a,b} = \{A_1 ... A_j\}$ be the set of nodes adjacent to a and/or b

    while a and b are connected and $n < j$:

        if any subset of size n of $\boldsymbol{A}$ makes a and b conditionally independent:

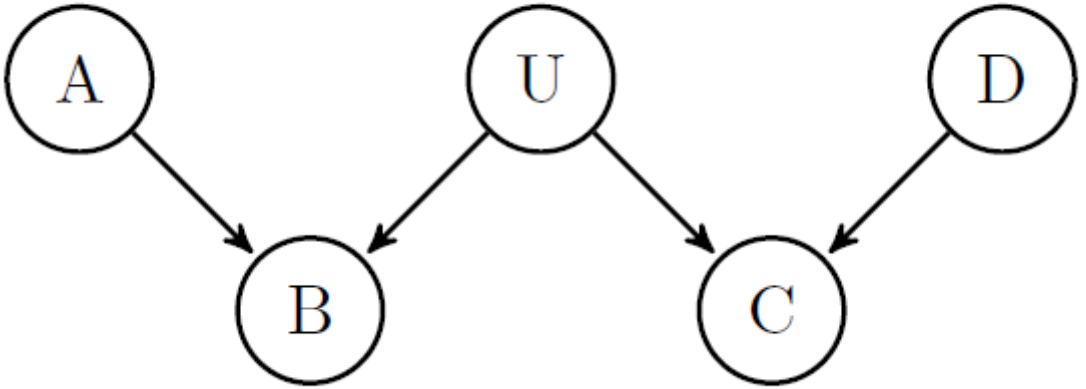
            delete the link

        $n = n + 1$

Latent variables

Problems:

- Infinite search space
- The space of causal DAGs is not closed under marginalization



Latent variables

Theorem (Verma 1993):

for any latent structure there is an equivalent structure such that every latent variable is a root node with exactly 2 children.