In[18]:= `ClearAll["Global`*"]`

In[34]:= `R00 = a (T - a T / 2 (Sqrt[(T / K)] + Sqrt[N] )) + N / 2`

Out[34]= $$\frac{N}{2} + a\left(T - \frac{1}{2} a T \left(\sqrt{N} + \sqrt{\frac{T}{K}}\right)\right)$$

In[35]:= `R0 = a (T - T / 2 Sqrt[-Log[1 - 4 a^2] (T / K + N)] ) + N / 2`

Out[35]= $$\frac{N}{2} + a\left(T - \frac{1}{2} T \sqrt{-\left(N + \frac{T}{K}\right) \mathrm{Log}\left[1 - 4 a^2\right]}\right)$$

In[36]:= `R = a (T - T / K - T / 2 Sqrt[-Log[1 - 4 a^2] (T / K + N)] ) + N / 2`

Out[36]= $$\frac{N}{2} + a\left(T - \frac{T}{K} - \frac{1}{2} T \sqrt{-\left(N + \frac{T}{K}\right) \mathrm{Log}\left[1 - 4 a^2\right]}\right)$$

In[37]:= `R2 = Normal[Series[R0, {a, 0, 2}]]`

Out[37]= $$\frac{N}{2} + a T - a^2 T \sqrt{\frac{K N + T}{K}}$$

In[38]:= `amax = Simplify[Solve[D[R2, a] == 0, a]]`

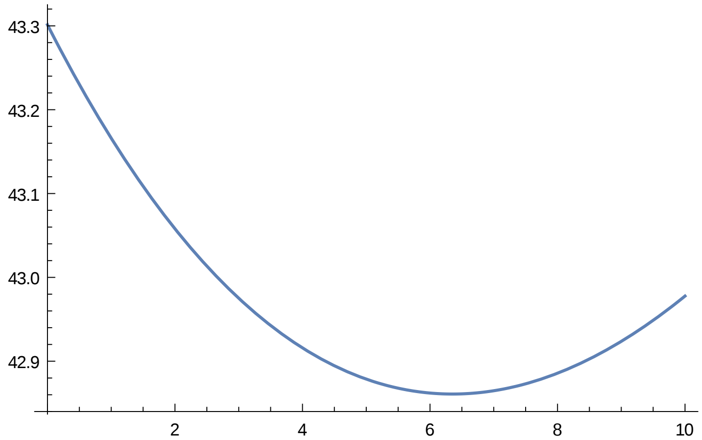
Out[38]= $$\left\{\left\{a \to \frac{1}{2\sqrt{N + \frac{T}{K}}}\right\}\right\}$$

In[39]:= R3 = Simplify$\left[\text{R2 /. a} \to \frac{1}{2\sqrt{N + \frac{T}{K}}}\right]$

Out[39]= $$\frac{N}{2} + \frac{K T \sqrt{N + \frac{T}{K}}}{4 (K N + T)}$$

In[69]:= `Plot[R3 //. {T → 1000, K → 30}, {N, 0, 10}]`

Out[69]=



In[41]:= `Nmin = Simplify[Solve[D[R3, N] == 0, N]]`

Out[41]= $$\left\{\left\{N \to \frac{T^{2/3}}{2 \times 2^{1/3}} - \frac{T}{K}\right\}, \left\{N \to \frac{i \left(i + \sqrt{3}\right) T^{2/3}}{4 \times 2^{1/3}} - \frac{T}{K}\right\}, \left\{N \to -\frac{\left(1 + i \sqrt{3}\right) T^{2/3}}{4 \times 2^{1/3}} - \frac{T}{K}\right\}\right\}$$

In[42]:= $$\text{Reduce}\left[\frac{T^{2/3}}{2 \times 2^{1/3}} - \frac{T}{K} > 0, T\right]$$

Out[42]= $$(K < 0 \;\&\&\; T > 0) \;||\; \left(K > 0 \;\&\&\; 0 < T < \frac{K^3}{16}\right)$$

Lets try minimizing with respect to N first
Ok lets try starting with R2

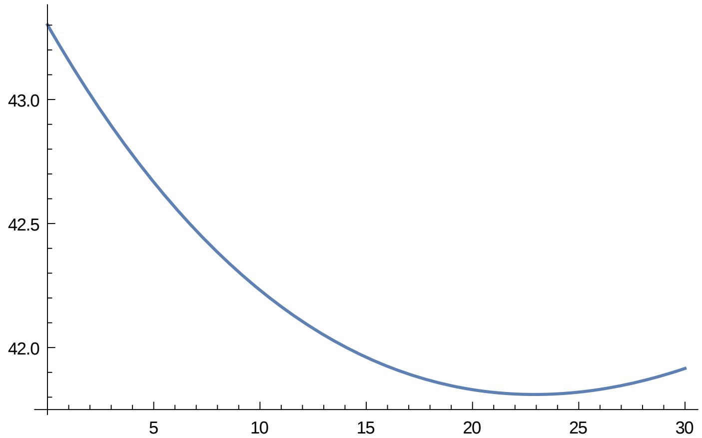
First up, what happens if we just use 1 / 2 Sqrt[K / T] for a ...

In[75]:= `R7 = R2 /. a → 1 / 2 Sqrt[K / T]`

Out[75]= $$\frac{N}{2} + \frac{1}{2} \sqrt{\frac{K}{T}}\, T - \frac{1}{4} K \sqrt{\frac{K N + T}{K}}$$

In[77]:=
```
Plot[R7 //. {T → 1000, K → 30}, {N, 0, 30}]
```

Out[77]=




```
Nmin1 = Apart[Solve[D[R7, N] == 0, N]]
```

Out[91]= $\left\{\left\{N \to \frac{K^2}{16} - \frac{T}{K}\right\}\right\}$

In[89]:= $\text{Reduce}\left[\frac{K^2}{16} - \frac{T}{K} > 0\right]$

Out[89]= $\left(K < 0 \;\&\&\; T > \frac{K^3}{16}\right) \;||\; \left(K > 0 \;\&\&\; T < \frac{K^3}{16}\right)$

```
R8 = Apart[Simplify[R7 /. Nmin1]]
```

Out[98]= $\left\{\frac{K^2}{32} - \frac{K\sqrt{K^2}}{16} - \frac{T}{2K} + \frac{1}{2}\sqrt{\frac{K}{T}}\;T\right\}$

Now,

In[110]:=
```
Nmin2 = Apart[Solve[D[R2, N] == 0, N]]
```

Out[110]= $\left\{\left\{N \to -\frac{T}{K} + a^4 T^2\right\}\right\}$

In[111]:=
```
Nmin3 = Solve[D[R00, N] == 0, N]
```

Out[111]= $\left\{\left\{N \to \frac{a^4 T^2}{4}\right\}\right\}$

In[114]:=
```
R4 = Normal[Series[R0 /. Nmin3, {a, 0, 4}]]
```

Out[114]= $\left\{a\,T - a^2\,T\sqrt{\frac{T}{K}} + a^4\left(\frac{T^2}{8} - T\sqrt{\frac{T}{K}}\right)\right\}$

In[115]:= `R5 = a T - a^2 T Sqrt[T/K] + a^4 (T^2/8)`

$$R5 = a\,T - a^2\,T\sqrt{\frac{T}{K}} + a^4\left(\frac{T^2}{8}\right)$$

Out[115]= $$a\,T + \frac{a^4\,T^2}{8} - a^2\,T\sqrt{\frac{T}{K}}$$

In[166]:= `R9 = Normal[Series[R0 /. Nmin3, {a, 0, 2}]]`

Out[166]= $$\left\{a\,T - a^2\,T\sqrt{\frac{T}{K}}\right\}$$

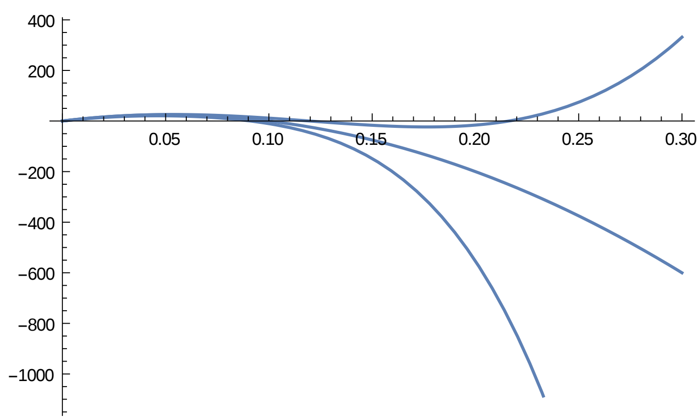
In[174]:= `R10 = R /. Nmin3`

Out[174]= $$\left\{\frac{a^4\,T^2}{8} + a\left(T - \frac{T}{K} - \frac{1}{2}\,T\sqrt{-\left(\frac{T}{K} + \frac{a^4\,T^2}{4}\right)\mathrm{Log}\left[1 - 4\,a^2\right]}\right)\right\}$$

$$\left\{\frac{a^4\,T^2}{8} + a\left(T - \frac{T}{K} - \frac{1}{2}\,T\sqrt{-\left(\frac{T}{K} + \frac{a^4\,T^2}{4}\right)\mathrm{Log}\left[1 - 4\,a^2\right]}\right)\right\}$$

In[173]:= `Plot[{R4, R9, R10} //. {T → 1000, K → 10}, {a, 0, .3}]`

Out[173]=



In[122]:= **amax2 = Simplify[Solve[D[R5, a] == 0, a]]**

Out[122]=
$$\left\{\left\{a \to \frac{4\sqrt{\frac{T}{K}}}{3^{1/3}\left(-9\,T^2+\sqrt{3}\sqrt{T^3\left(27\,T-64\left(\frac{T}{K}\right)^{3/2}\right)}\right)^{1/3}}+\frac{\left(-9\,T^2+\sqrt{3}\sqrt{T^3\left(27\,T-64\left(\frac{T}{K}\right)^{3/2}\right)}\right)^{1/3}}{3^{2/3}\,T}\right\},\right.$$

$$\left\{a \to \left(-4\,3^{1/6}\left(3\,i+\sqrt{3}\right)T\sqrt{\frac{T}{K}}+i\,3^{1/3}\left(i+\sqrt{3}\right)\left(-9\,T^2+\sqrt{3}\sqrt{T^4\left(27-\frac{64\sqrt{\frac{T}{K}}}{K}\right)}\right)^{2/3}\right)\Big/\right.$$

$$\left.\left(6\,T\left(-9\,T^2+\sqrt{3}\sqrt{T^4\left(27-\frac{64\sqrt{\frac{T}{K}}}{K}\right)}\right)^{1/3}\right)\right\},$$

$$\left\{a \to \frac{2\,i\left(i+\sqrt{3}\right)\sqrt{\frac{T}{K}}}{3^{1/3}\left(-9\,T^2+\sqrt{3}\sqrt{T^4\left(27-\frac{64\sqrt{\frac{T}{K}}}{K}\right)}\right)^{1/3}}-\frac{1}{2\times 3^{2/3}\,T}\right.$$

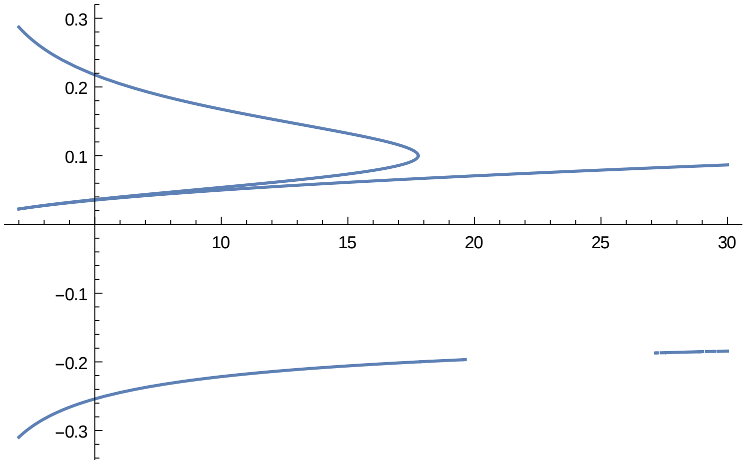
$$\left.\left.\left(1+i\sqrt{3}\right)\left(-9\,T^2+\sqrt{3}\sqrt{T^3\left(27\,T-64\left(\frac{T}{K}\right)^{3/2}\right)}\right)^{1/3}\right\}\right\}$$

Out[156]= $0.167513 - 2.77556\times 10^{-17}\,i$

In[155]:= **N[Values[amax2] //. {T → 1000, K → 10}]**

Out[155]= $\{\{0.167513 - 2.77556\times 10^{-17}\,i\}, \{-0.221432 + 2.77556\times 10^{-17}\,i\}, \{0.0539189 + 5.55112\times 10^{-17}\,i\}\}$

In[140]:= `Plot[Append[Values[amax2], 1 / 2 Sqrt[K / T]] /. T → 1000, {K, 2, 30}]`

Out[140]=


In[135]:= `Plot[Append[Values[amax2], Sqrt[K / T]] /. T → 1000, {K, 2, 20}]`