



Clarification about $\sin(k(1+\varepsilon)\pi)$:

$$\text{For even } k, \sin(k(1+\varepsilon)\pi) = \sin(k(1+\varepsilon)\pi - 2\pi \cdot \frac{k}{2}) \\ = \sin(k\varepsilon\pi)$$

$$\text{For odd } k, \sin(k(1+\varepsilon)\pi) = \sin(k(1+\varepsilon)\pi - 2\pi \frac{k-1}{2}) \\ = \sin(k\varepsilon\pi + \pi) \\ = -\sin(k\varepsilon\pi)$$

Therefore, $|\sin(k(1+\varepsilon)\pi)| = |\sin(k\varepsilon\pi)|$ for all $k=1, 2, 3, \dots$