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ORIGINAL ARTICLE

Performance evaluation of NBA teams: A non-homogeneous DEA approach

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ABSTRACT

National Basketball Association (NBA) is one of the four major sports leagues in North America. The performance evaluation of NBA teams is an important reference for team managers. However, non-homogeneity issues exist on both inputs and outputs sides for NBA teams evaluations. For example, some teams do not have "high-level" players as inputs and some teams are not qualified to play in the playoffs with respect to "wins in the playoffs" as an output. The current paper extends the existing non-homogeneous DEA method to address the non-homogeneous structure of NBA teams in performance evaluation by splitting NBA teams into types of homogeneous sub-units. The sub-unit, in this paper, consists of empirical input subset and output subset which satisfies both atomic property and maximum property. In addition, our method yields a unique efficiency decomposition of sub-units of NBA teams in 2018–2019 season without the need for imposing any additional conditions which are needed in some prior relative researches. As a result, detailed performance improvement directions for all teams can be provided.

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Data envelopment analysis (DEA); National Basketball Association (NBA); non-homogeneous; efficiency decomposition

1. Introduction

As the profit of commercial sports event mainly depend on their results (Moreno & Lozano, 2014), teams managers need to know and commit to improving the performance of their teams. However, due to the elimination mechanism that generally adopted at different stages of sports professional leagues (e.g., regular season and the playoffs), researchers have to deal with some non-homogeneity issues among the sports teams: on the one hand, the situation where some teams cannot participate in the playoffs and have no corresponding outputs results in the non-homogeneity on the output side; On the other hand, the fact that high level of players (All-Stars) as inputs are not equipped in all teams also leads to the non-homogeneity on the input side.

In this study, we focus on analysing the competitive performance of the teams of National Basketball Association (NBA), which is one of the four major sports leagues in North America. NBA consists of 30 franchised member teams which are grouped into two 15-team divisions: eastern division and western division. A NBA season has two stages: the regular season and the playoffs. All teams compete in the regular season, but only the top eight teams of each division are qualified to participate in the playoffs. Therefore, 14 teams, which cannot participate in the playoffs and naturally have no corresponding outputs, are non-homogeneous with other teams on the output side. In addition, NBA players as inputs can be classified into two levels including All-Stars and ordinary players. Players at different levels are viewed as distinct non-homogeneous inputs because of their quite different contributions to wins.

Data envelopment analysis (DEA), initialed by Charnes, Cooper, and Rhodes (1978), is a data-driven approach for performance evaluation and benchmarking of decision making units (DMUs) with multiple indicators. It has been widely applied in performance evaluation of sports events including NBA in the prior literature (Chen, Gong, & Li, 2017; Gutiérrez & Ruiz, 2013; Moreno & Lozano, 2014; Walraven, Koning, Bijmolt, & Los, 2016; Yang, Lin, & Chen, 2014; Yang, Li, & Liang, 2015). The main NBA evaluation researches based on DEA are shown here. Moreno & Lozano (2014) considers the distribution of the budget between first-team players and the rest players to evaluate the performance of NBA teams in regular season. Yang et al. (2014) evaluate the efficiency of NBA teams in regular season under a two-stage DEA framework and decompose overall team efficiency into wage

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efficiency and on-court efficiency. Chen et al. (2017) apply a bounded integer DEA model to evaluate the performance of NBA players.

In summary, all previous researches about NBA evaluation do not take account of the non-homogeneous issues among NBA teams. They evaluate the performance of NBA either by only using the regular season data, or by using the sum of the data from both seasons as the outputs of the whole season. Nevertheless, the performance of NBA teams in the regular season may be not representative of the overall performance of NBA teams. In addition, taking the sum of wins in regular season and playoff as one type of output of whole season may encounter modelling issues, e.g., the wins in the playoffs have greater significance to NBA teams, because only the team who wins the most games in the playoffs wins the NBA champion. Therefore, it is necessary to develop a non-homogeneity DEA approach for the performance evaluation of NBA teams.

In the literature, many researches probe into the non-homogeneity issues. Cook, Harrison, Rouse, and Zhu (2012) study a problem with a non-homogeneous output that is missing from some DMUs. They split DMUs into types of homogeneous sub-units and denote the overall efficiency of DMUs as the weighted aggregation of the Charnes-Cooper-Rhodes efficiency (or CCR, Charnes et al., 1978) of its sub-units. Based on Cook et al. (2012), Cook, Harrison, Imanirad, Rouse, and Zhu (2013), and Imanirad, Cook, and Zhu (2013) present alternative methods for measuring the relative efficiencies of DMUs with non-homogeneous outputs. Li, Liang, Cook, and Zhu (2016) extend the earlier researches to encompass the case where the input mix of some DMUs can be different from the rest.

As far as the construction of the overall efficiency is concerned, the preceding researches propose a weighted aggregation of the CCR efficiency of sub-units in a "Sub-Overall" framework. The weights of sub-units are usually not unique. Cook et al. (2012) proposes three kinds of weighting schemes to obtain the overall efficiency. Different from the above researches, Du, Liang, Chen, Cook and Zhu (2011) evaluate the overall efficiency of non-homogeneous DMU by applying an aggregate model, and then obtain the efficiency decomposition for each sub-unit in an "Overall-Sub" framework. As the efficiency decomposition may not be unique, they propose a subjective ordered sequence to obtain the efficiencies of sub-units by their order.

All abovementioned researches, however, are not applicable to solve these non-homogeneity issues in NBA teams directly for two reasons. On the one hand, they cannot deal with the cases where non-homogeneity exists on both the input and output sides. On the other hand, they cannot give an inartificial unique efficiency aggregation and decomposition simultaneously, i.e., these researches of sub-efficiencies aggregation need to choose a kind of weighting schemes to achieve unique overall efficiency, and the researches of overall efficiency decomposition need to apply a subjective ordered sequence to obtain unique sub-efficiencies.

Therefore, this study aims to propose an improved non-homogeneity DEA approach to deal with the practice problems mentioned above. Specifically, an extended "Overall-Sub" framework DEA approach is proposed to address the non-homogeneous issues of DMUs on both input and output sides, which can evaluate the overall performance of NBA teams that considering both the regular season and the playoffs. Besides, the proposed approach can be applied to yield unique efficiency decomposition of sub-units without imposing any additional conditions which are needed in prior relative researches, then further discover the deep reasons for the ineffectiveness of NBA teams. Furthermore, the immanent causes of uniqueness of efficiency decomposition are also revealed theoretically in this work.

The rest of this paper is organized as follows. Section 2 introduces the proposed approach and Section 3 presents the data of 30 NBA basketball teams in 2018–2019 and the results of performance evaluation and efficiency decomposition. Section 4 concludes the paper. The detailed analyses of the inartificial uniqueness of efficiency decomposition in current study will be present in Appendix.

2. Performance evaluation models for NBA teams

Based on the methods in Cook et al. (2012, 2013) mentioned above, in this section, we propose an alternative non-homogeneity approach to evaluate performance of NBA teams. Before introducing the model, some necessary preliminaries such as the inner structure and variables of NBA teams and the definition of sub-unit are needed to be introduced as follows.

2.1. The inner structure and variables of NBA teams

In this study, we model a NBA team's "production process" as a DMU consisting of two activities (or one if DMU does not participate in the playoffs). The diagram of the two-activity process is provided in Figure 1. In the first activity (i.e., regular season), each team plays all 82 games and aims to win as many games as possible. Then only the top 8 teams in each division (east or west) are qualified to play

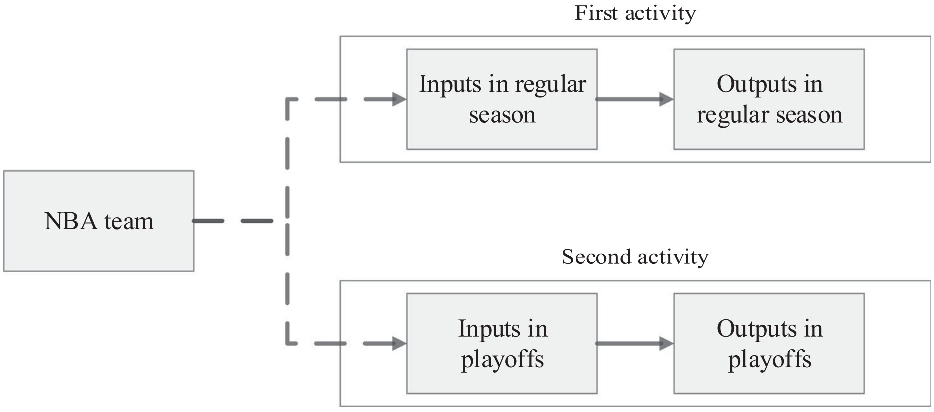

Figure 1. Inner structure of NBA teams.

games in the second activity (i.e., the playoffs). The goal of teams in the second activity is to win the NBA championship.

Then, we need to select appropriate inputs and outputs measures. In terms of inputs, the sum of players' salaries is an important input of NBA teams, which is also used in previous researches (Moreno & Lozano, 2014; Yang, Lin, & Chen, 2014). In addition, the number of players is also considered as important inputs to sports team (Barros & Leach, 2006; Chen & Johnson, 2010). Considering that, in the case of a given output, over investment in players or wages are both inefficient. Therefore, we choose both the number of players and the sum of their salaries as the inputs of NBA teams.

In addition, players are divided into All-Stars and ordinary players in this study. Here, All-Stars are voted on by coaches and fans based on their performance, and non-All-Stars are classified as ordinary players. Here note that, as NBA is a competitive sports association, All-Stars essentially represent the highest level of NBA players in the corresponding NBA season. This can be illustrated by objective data. According to NBA official statistics, All-Stars make up only 4.4% (24/547) of all NBA players. However, these 24 All-Stars account for 17.8% win shares in regular season and 35% win shares in the playoffs (2018–2019). Here, the win shares is a widely used players statistic which attempts to divvy up credit for team success to the individuals on the team. Therefore, players at different levels are viewed as distinct inputs, because they are quite different in terms of contributions to wins.

In terms of outputs, considering that NBA is a competitive sports league, the number of wins will represents the competitive strength of the team. In addition, the number of wins is a common output variable for NBA teams in previous researches (Moreno and Lozano, 2014; Yang, Lin, & Chen, 2014). Therefore, the outputs of each DMU in this work are set as the number of wins in the regular season (the first activity) and the number of wins in the playoffs (the second activity), respectively.

To sum up, the inputs and outputs of DMU_j are as follows.

- Inputs:
  1. All-Stars (x_{1j});
  2. Ordinary players (x_{2j});
  3. The sum of All-Stars players' salaries (x_{3j});
  4. The sum of ordinary players' salaries (x_{4j}).
- Outputs:
  1. Wins in the regular season (y_{1j});
  2. Wins in the playoffs season (y_{2j}).

It is important to note here that we will equalize the weights of x_{3j} and x_{4j} in future processing to respect the fact that they are both financial inputs. However, this setting will not have a substantial impact on the results of this paper. Then, in order to deal with the non-homogeneity on both the input and output sides, we will introduce an appropriate method of decomposing non-homogeneous DMUs into sub-units, to ensure that the same type sub-units decomposed by different decision-making units will be homogeneous.

2.2. The definition of Sub-unit

To define a sub-unit of a DMU, one should: (1) specify input subsets and output subsets for sub-unit. (2) allocate input and output data of DMU to sub-unit.

Before *specifying* input and output subsets for sub-unit, we first divide the inputs and outputs sets (e.g., the aforementioned four inputs and two outputs) into mutually exclusive sets with the following two properties empirically.

- Atomic property: an input or output set satisfies atomic property if one element of the set appears in the bundle of an activity, and then the rest of the set should also be consumed by the activity. One may notice that the definition here is ambiguous because if a set contained in the empirical input subset of an activity is an input set, then any non-empty subset of this set is also an input set.

This is true. Hence, the input set should be defined satisfying a maximum property.

- Maximum property: an input or output set satisfies the maximum property if no input (or output) that is not contained in the set can be added to the set without violating the atomic property.

For any input or output set that are the union of all the DMUs' inputs and outputs, it is straightforward to show that the sets can always be partitioned into unique mutually exclusive sets satisfying the maximum and atomic properties.

Based on the above exposition, we now proceed to *specify* the inputs and outputs subsets by introducing some definitions.

Definition 1. Let A^j and B^j denote the inputs and outputs of DMU$_j$. And we denote $A = \cup_{j=1}^{n} A^j$, $B = \cup_{j=1}^{n} B^j$, respectively. A_c ($c = 1,.,C$) and B_d ($d = 1,.,D$), which both satisfying atomic property and maximum property, respectively denote mutually exclusive subsets of A and B. Below we refer to A_c, B_d as empirical input and output subsets, respectively.

Based on Definition 1, the empirical input and output subsets in this NBA case are formed by different levels of players and their salaries and the number of wins in different activities as follows:

1. Empirical input subset A_1: (All-Stars players and the sum of their salary);
2. Empirical input subset A_2: (ordinary players and the sum of their salary);
3. Empirical output subset B_1: (wins in regular season);
4. Empirical output subset B_2: (wins in the playoffs).

By virtue of empirical input/output subsets, we now present the definition of a sub-unit of a DMU.

Definition 2. Suppose A^j and B^j, respectively, are composed of C empirical input sets, denoted as $\{A_1, A_2, \ldots, A_C\}$ and D empirical output sets, denoted as $\{B_1, B_2 \ldots, B_D\}$. Then DMU$_j$ is decomposed into C^*D sub-unit^k ($k = 1, \ldots, g$, $g = C^*D$), each of which is a process, denoted as (A_c, B_d), that uses an empirical input subset A_c ($c = 1, \ldots, C$) of inputs to produce an empirical output set B_d ($d = 1, \ldots, D$) of outputs.

Notice that there is a one-to-one correspondence between the superscript k and the subscript tuple (c, d). Thus, one may refer to a specific sub-unit using either way interchangeably without causing confusion. Compared with the concept of an activity of a DMU, a sub-unit is a modelling construct to solve the non-homogeneous issues on both input side and output side. We now turn to demonstrate how to *allocate* the DMU-level input and output data to the input and output data of a sub-unit^k.

Definition 3. Suppose DMU$_j$ is decomposed into g ($g = C^*D$) $sub-unit_j^k$ (A_c, B_d) ($c = 1, \ldots, C$, $d = 1, \ldots D$), X_{cj} and Y_{dj} are the vectors of the observational data corresponding to the amount of A_c consumed and the amount of B_d produced by DMU$_j$, respectively. Let us define α_{cdj}^k as the percentage of X_{cj} that is used by $sub-unit_j^k$ and β_{cdj}^k as the percentage of Y_{dj} that is produced by $sub-unit_j^k$. Thus, the input and output data of $sub-unit_j^k$ can be expressed as $(\alpha_{cdj}^k X_{cj}, \beta_{cdj}^k Y_{dj})$ and be denoted as (X_j^k, Y_j^k).

In the evaluation of NBA case, we assume that X_{cj} (number of level c of players and their salaries) are divided into different activities (regular season and the playoffs) by the time that they have spent in each activity. And assume Y_{dj} (number of wins in d^{th} activity) are divided into different empirical input subsets by the contributions they did. Correspondingly, α_{cdj}^k is the percentage of time that X_{cj} spent in $sub-unit_j^k$, and β_{cdj}^k are the percentage of sum of win shares that Y_{dj} got in $sub-unit_j^k$, respectively. For the one-to-one correspondence between the superscript k and the subscript tuple (c, d), the symbols α_j^k, β_j^k and α_{cdj}, β_{cdj} will be used as the same roles as α_{cdj}^k and β_{cdj}^k in the rest part of this paper.

α_{cdj}^k and β_{cdj}^k are very common in the literature about the evaluation of DMUs with shared inputs and outputs (Cook et al., 2012, 2013; Li et al., 2016). These percentages can be decision variables or provided constants. In the current paper, we allocate players and their salaries according to the length of time of the regular season and the playoffs, and we assign wins in the regular season or the playoffs to players at different levels based on their win shares. These two kinds of statistics are available on the professional NBA statistics website.

The detailed calculation process of α_{cdj}^k and β_{cdj}^k is as follows:

Let T_{dj} denote the length of time of season$_d$ and S_{cdj} denote the total win shares that respectively are attributed to the level$_c$ players of team$_j$ in season$_d$ (i.e., regular season and the playoffs). Then, α_{cdj}^k and β_{cdj}^k is calculated as follows:

$$\alpha_{cdj}^k = \frac{T_{dj}}{\sum_d T_{dj}}, (d = 1, 2) \tag{4.1}$$

$$\beta_{cdj}^k = \frac{S_{cdj}}{\sum_c S_{cdj}}, (c = 1, 2) \tag{4.2}$$

In some special cases, e.g., teamj participates in the playoffs but win none of games, we set the percentages of sum of win shares that both types of

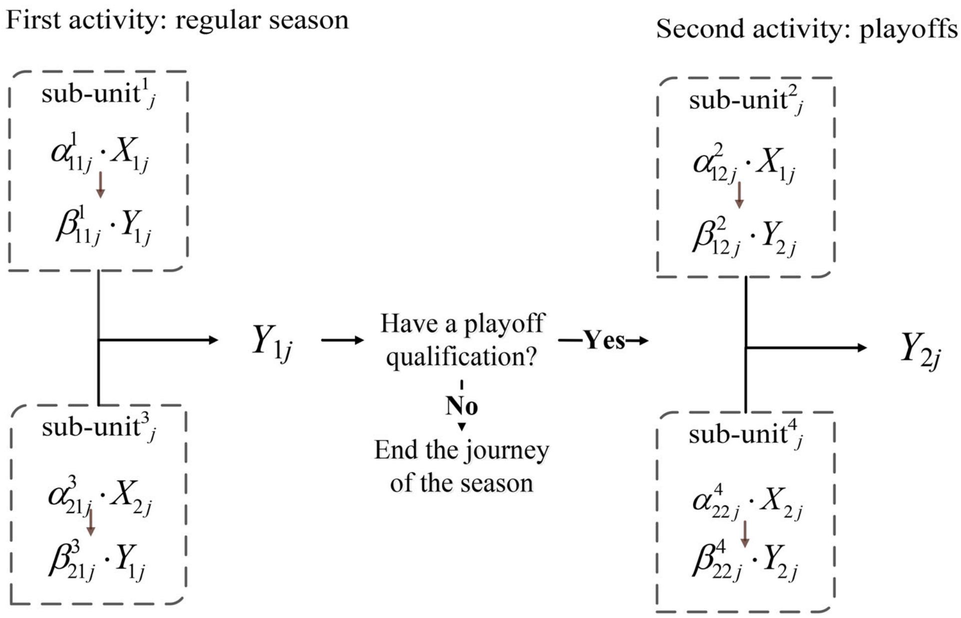

Figure 2. The sub-units of DMU$_j$.

players got in playoffs are equal to 0.5. And it is worth noting that, we can also refer to the practice in Cook et al. (2012) and treat α^k_{cdj} and β^k_{cdj} as decision variables without affecting the following work of this study. However, as we are able to calculate the accurate value of α^k_{cdj} and β^k_{cdj}, we will obtain more reliable efficiency evaluation results than treating them as decision variables.

Subsequently, DMU$_j$ $(j = 1, \dots, 30)$ can be split into at most 4 sub-units as shown in Figure 2.

We think of the NBA team "production process" as an athletic process in which players and salaries are invested and games are won. Where:

- $sub-unit^1_j$ denote the athletic process of All-Stars players in regular season,
- $sub-unit^2_j$ denote the athletic process of All-Stars players in the playoffs,
- $sub-unit^3_j$ denote the athletic process of ordinary players in regular season, and
- $sub-unit^4_j$ denote the athletic process of ordinary players in the playoffs.

According to this specific decomposition of sub-units, we are able to classify the 30 teams into 4 types according to the types of sub-units included in DMU$_j$. Here, we denote $S(j)$ as the index set of the sub-units of DMU$_j$, and $I = \{1, 2, 3, 4\}$ as the overall index set of sub-units of all the DMU$_j$ $(j = 1, \dots, n)$. Table 1 shows the results of classification.

On the base of the decomposition of DMUs into sub-units, we may deal with the non-homogeneity issues by just doing relative comparisons in sub-unit levels. However, treating in this way may not take the following issues into account.

- Sub-unit level comparisons only show the performances of teams in special aspect, but not the overall performance of teams. However, considering that all NBA teams are equally compete in the same association, managers need to know the overall efficiency for evaluating the performance of their teams, although the NBA teams have non-homogeneous inner structures.
- In terms of overall efficiencies, we can improve the performance of NBA teams by not only adjusting their inputs or outputs but also adjusting their inner structures. If we do not evaluate the performance among teams, we do not know which inner structures are most suitable to them.

Therefore, for dealing with these two issues, evaluation in both sub-units levels and DMU level are needed.

2.3. The performance evaluation model for the NBA teams

After we decompose non-homogeneous DMUs into sub-units, two frameworks denoted as "Sub-Overall" and "Overall-Sub" frameworks can be employed to obtain the overall efficiency of DMUs and the sub-efficiency of sub-units. And the details of these two frameworks are displayed as follows.

2.3.1. "Sub-Overall" framework

In the "Sub-Overall" framework, we do the relative comparisons in sub-unit levels at first, then express the overall efficiency of DMUs by aggregating the efficiency of its sub-units.

Before we evaluating the efficiency in sub-unit level, we first define the production probability set (PPS) of a $sub-unit^k$ (Sub-PPS) as follows:

$$T(sub{-}unit^k) = \{(X^k, Y^k) \mid X^k \text{ can produce } Y^k, X^k \in R_+^{m^k}, Y^k \in R_+^{s^k}\},$$

where m^k and s^k is the dimension of A_c and B_d of $sub{-}unit^k$. And the empirical Sub-PPS based on observed data and the assumption of constant returns to scale as follows:

$$\hat{T}(sub{-}unit^k) = \{(X^k, Y^k) | X^k \geq \sum_j \lambda_j^k X_j^k, Y^k \leq \sum_j \lambda_j^k Y_j^k, \lambda_j^k \geq 0, j \in DMU(k)\},$$

where $DMU(k) = \{j \mid sub{-}unit_j^k \text{ exist}\}$.

Based on the Sub-PPS, the envelopment model for $sub{-}unit_0^k$ can be expressed as model (5), and its dual model can be obtained as model (6).

$$\begin{aligned} \min\ & e_0^k = \theta \\ \text{s.t.}\ & \sum_{j \in DMU(k)} \lambda_j^k X_j^k \leq \theta X_0^k \\ & \sum_{j \in DMU(k)} \lambda_j^k Y_j^k \geq Y_0^k \\ & \lambda_j^k \geq 0 \end{aligned} \tag{5}$$

$$\begin{aligned} \max\ & e_0^k = \sum_{r \in B_d} y_{r0} u_r \\ \text{s.t.}\ & \sum_{i \in A_c} x_{i0} v_i = 1 \\ & \sum_{i \in A_c} \alpha_{cdj}^k v_i x_{ij} \geq \sum_{r \in B_d} \beta_{cdj}^k u_r y_{rj}, \forall j \in DMU(k) \\ & u_r, v_i \geq 0 \end{aligned} \tag{6}$$

By solving model (6), we get the efficiency e_j^k and correspondingly optimal solution (U^{k*}, V^{k*}) of each $sub{-}unit_j^k$. Denote w_j^k as the weight of $sub{-}unit_j^k$ to the overall efficiency, where $\sum_{k \in S(j)} w_j^k = 1$, then the overall efficiency can be expressed as $\sum_{k \in S(j)} w_j^k e_j^k$.

One widely used choice of weight w_j^k is the proportion of total inputs devoted to $sub{-}unit_j^k$ (Chen et al., 2009; Du, Chen, & Huo, 2015). For example, based on given multipliers $(U*, V*)$, w_j^k can be expressed mathematically as:

$$\frac{\sum_{i \in A_c} \alpha_{cdj}^k v_i^* x_{ij}}{\sum_{i \in A^j} v_i^* x_{ij}}$$

One issue of this framework is that, the formula of w_j^k is based on one set of multipliers (U^*, V^*). However, (U^{k*}, V^{k*}) obtained by solving model (6) are $sub{-}unit_j^k$ specific (Cook et al., 2012). In other words, it is difficult to naturally obtain an objective set of multipliers (U^*, V^*). For dealing with this problem, Cook et al. (2012) propose to choose one of (U^{k*}, V^{k*}) or use the linear combination of (U^{k*}, V^{k*}) $(k \in S(j))$ as the specific multiplier $(U*, V*)$ to weight the importance of e_j^k to the overall efficiency. However, one can realize another issue that different choice of $(U*, V*)$ may lead to different overall efficiencies, and further result in the instability of the performance in DMU level.

Table 1. The types of NBA teams.

Types	Whether have All-Stars or not	Whether participate in the playoffs or not	S(j)
1	No	No	{3}
2	No	Yes	{3, 4}
3	Yes	No	{1, 3}
4	Yes	Yes	{1, 2, 3, 4}

For the reasons explained above, we do not use the "Sub-Overall" framework in this study. Instead, we will adopt the "Overall-Sub" framework which is introduced in next sub-section.

2.3.2. "Overall-Sub" framework

For the evaluation of the NBA teams in DMU level, we need to develop the PPS in DMU level with non-homogeneity on both input and output side. By viewing a NBA team as a system of sub-units, the vector of inputs and outputs of a DMU$_j$, which are denoted as X_j and Y_j, are the aggregations of some X_j^k and Y_j^k, respectively. Here note that, each type of DMUs in Table 1 has a particular structure which is determined by the kinds of formed sub-units. We denote $S(j)$ as the index set of the sub-units of DMU_j, and $I = \{1, 2, 3, 4\}$ as the overall index set of sub-units of all the DMU$_j$ ($j = 1, \ldots, n$). However, X^k and Y^k have different dimensions among different index k. Therefore, for the feasibility of the aggregation of X^k and Y^k $(k \in S(j))$, we need to augment their dimensions according to the dimensions of A and B, and get the augmented vectors $X_{aug}^k \in R_+^m$ $(0, \ldots, X^k, \ldots, 0)$ and $Y_{aug}^k \in R_+^s$ $(0, \ldots, Y^k, \ldots, 0)$ as shown in Figure 3.

Then, for arbitrary DMUs with special sub-units index set S, we can set the corresponding overall-PPS of them as follows.

$$T(I) = \left\{(X, Y) \mid (X_{aug}, Y_{aug}) = \left(\sum_{k \in I} X_{aug}^k, \sum_{k \in I} Y_{aug}^k\right), X^k \text{ can produce } Y^k\right\},$$

where $X_{aug} \in R_+^m$ and $Y_{aug} \in R_+^s$ are the augmented vectors of X and Y, respectively.

However, the actual $T(I)$ is unknown, thus we propose the empirical PPS based on observed data and the assumption of constant returns to scale as follows.

$$\hat{T}(I) = \left\{(X, Y) | X_{aug} \geq \sum_{k \in I} \sum_j \lambda_j^k X_{augj}^k,\ Y_{aug} \leq \sum_{k \in I} \sum_j \lambda_j^k Y_{augj}^k,\ \lambda_j^k \geq 0,\ j \in DMU(k)\right\}.$$

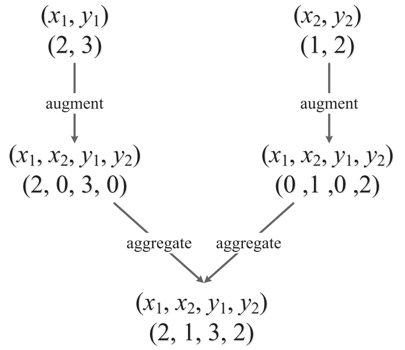

Figure 3. The example of augmented vectors.

Where $DMU(k) = \{j \mid sub\text{--}unit_j^k \text{ exist}\}$. Then, the performance of DMU_0 can be expressed as the optimum solution of the following linear problem.

$$
\begin{aligned}
&\min\ E_0 = \theta \\
&\text{s.t.}\ \sum_{k\in S(0)} \sum_{j\in DMU(k)} \lambda_j^k X_{augj}^k \le \theta X_{aug0} \\
&\qquad \sum_{k\in S(0)} \sum_{j\in DMU(k)} \lambda_j^k Y_{augj}^k \ge Y_{aug0} \\
&\qquad \lambda_j^k \ge 0
\end{aligned}
\tag{7}
$$

The dual model of Model (7) is shown as follows.

$$
\begin{aligned}
&\max\ E_0 = \sum_{r\in B^0} y_{r0} u_r \\
&\text{s.t.}\ \sum_{i\in A^0} x_{i0} v_i = 1 \\
&\sum_{i\in A_c} \alpha_{cdj}^k v_i x_{ij} \ge \sum_{r\in B_d} \beta_{cdj}^k u_r y_{rj}, \forall k \in S(0), \forall j \in DMU(k) \\
&u_r, v_i \ge 0
\end{aligned}
\tag{8}
$$

In real life applications, one can improve the results of model (8) by incorporating weight restrictions. For example, in the performance evaluation of NBA teams, the first one is $v_1 \ge v_2$ considering the gaps of players among two levels in physical talent, skill and experience and so on. The second one is $v_3 = v_4$, for they are both monetary variables and are measured in US dollars. The third one is $u_1 \le u_2$ taking the difference of elimination mechanism and fierce confrontation degree between regular season and the playoffs into account. Model (8) can, of course, provides the unique overall efficiency of each DMU, but does not reveal the internal sources of their inefficiency. To shed light on the sources of inefficiency, the performances of sub-units are defined as follows.

Definition 4. For an element $(\mathbf{u}, \mathbf{v}) \in \Theta_j$, ($\Theta_j$ is the feasible domain of Model (8) where DMU_j is under evaluation, $(\mathbf{u}, \mathbf{v})$ is an element of Θ_j), the weights-based ratio efficiencies under $(\mathbf{u}, \mathbf{v})$ of DMU_j and $sub\text{--}unit_j^k$ are defined as $\varepsilon_{j(\mathbf{u},\mathbf{v})} = \sum_{r\in B^j} u_r y_{rj} / \sum_{i\in A^j} v_i x_{ij}$ and $\varepsilon_{j(\mathbf{u},\mathbf{v})}^k = \beta_j^k \sum_{r\in B_d} u_r y_{rj} / \alpha_j^k \sum_{i\in A_c} v_i x_{ij}$. Let $\Omega_j = \{(\mathbf{u}, \mathbf{v}) \mid (\mathbf{u}, \mathbf{v})$ is an optimal solution of Model (6) where DMU_j is under evaluation$\}$. Then, $E_j = \sum_{r\in B^j} u_r^* y_{rj} / \sum_{i\in A^j} v_i^* x_{ij}$, $E_j^k = \beta_j^k \sum_{r\in B_d} u_r^* y_{rj} / \alpha_j^k \sum_{i\in A_c} v_i^* x_{ij}$ where $(\mathbf{u}^*, \mathbf{v}^*) \in \mathbf{\Omega}_j$, are respectively, dubbed as the overall efficiency of DMU_j and the sub-efficiency of $sub\text{--}unit_j^k$, under $(\mathbf{u}^*, \mathbf{v}^*) \in \mathbf{\Omega}_j$ to highlight that they are evaluated under optimal solutions.

It should be noted that, the sub-efficiency $E_j^k = \beta_j^k \sum_{r\in B_d} u_r^* y_{rj} / \alpha_j^k \sum_{i\in A_c} v_i^* x_{ij}$ is based upon using the multipliers $(\mathbf{u}^*, \mathbf{v}^*) \in \mathbf{\Omega}_j$, which ensure that the evaluation results in DMU level and sub-unit level are based on the same multipliers basis. Therefore, we have the following Proposition 1.

Proposition 1. *For arbitrary DMU_j, the necessary and sufficient condition of its efficient is that all its sub-units are efficient.*

Proof. See Appendix 1.

Then, for testing the stability of evaluation results in sub-unit level, we need to judge that whether the sub-efficiencies are unique. We design an experiment to test the uniqueness of our proposed efficiency decomposition scheme as follows.

$$
\begin{aligned}
&\max\ \varepsilon_j^{k\max} = \frac{\sum_{r\in A_c} u_{rj} y_{rj}}{\sum_{i\in B_d} v_{ij} x_{ij}} \\
&\text{s.t.}\quad (\mathbf{u}^*, \mathbf{v}^*) \in \mathbf{\Omega}_j.
\end{aligned}
\tag{9.1}
$$

$$
\begin{aligned}
&\min\ \varepsilon_j^{k\min} = \frac{\sum_{r\in A_c} u_{rj} y_{rj}}{\sum_{i\in B_d} v_{ij} x_{ij}} \\
&\text{s.t.}\quad (\mathbf{u}^*, \mathbf{v}^*) \in \mathbf{\Omega}_j.
\end{aligned}
\tag{9.2}
$$

To each $sub\text{--}unit_j^k$, we will calculate its maximal and minimal value in the optimal solution set $\mathbf{\Omega}_j$ of DMU_j. If $\varepsilon_j^{k\max} = \varepsilon_j^{k\min}$, then E_j^k has unique value. Otherwise we can refer to the methods proposed in Du et al. (2015), Du et al. (2011), and Kao and Hwang (2008) for choosing an unique decomposition of efficiency.

However, as an empirical result, all NBA teams have unique efficiency decomposition in this work (see Section 3). This result shows that our "Overall-Sub" framework can obtain objective sub-efficiency. And we will explain theoretically why our method can provide this objective results in the Appendix 2.

3. The data and performances of NBA teams

3.1. Data and pretreatment

The data source used in this article is http://www.stat-nba.com, which is a publicly accessible website.

Table 2. The inputs and outputs data of 30 NBA teams in 2018–2019 season.

Team number	Team	x1	x2	x3	x4	y1	y2
1	Bulls	—	20	—	11,279	22	—
2	Cavaliers	—	19	—	12,461	19	—
3	**Pistons**	1	16	3209	9315	41	0
4	**Pacers**	1	20	2100	8976	48	0
5	**Bucks**	2	19	2416	10,173	60	10
6	**Nets**	—	20	—	11,825	42	1
7	**Celtics**	1	15	2010	10,377	49	5
8	New York Nicks	—	18	—	12,277	17	—
9	**76ers**	2	15	3190	8098	51	7
10	**Raptors**	2	15	5431	8180	58	16
11	Hawks	—	23	—	10,771	29	—
12	Hornets	1	14	1200	11,034	39	—
13	Heat	—	16	—	15,315	39	—
14	**Magic**	1	16	1275	10,071	42	1
15	Wizards	1	17	2543	9853	32	—
16	**Nuggets**	1	15	2461	9024	54	7
17	Timberwolves	1	19	784	11,404	36	—
18	**Thunder**	2	14	6621	7936	49	1
19	**Trailblazers**	1	17	2798	10,453	53	8
20	**Jazz**	—	15	—	11,315	50	1
21	**Warriors**	3	13	8645	6165	57	14
22	**Clippers**	—	21	—	11,695	48	2
23	Lakers	1	16	3565	7154	37	—
24	Suns	—	25	—	11,147	19	—
25	Kings	—	18	—	10,097	39	—
26	Mavericks	—	19	—	9062	33	—
27	**Rockets**	1	14	6608	5765	53	6
28	Grizzlies	—	20	—	12,323	33	—
29	Pelicans	1	18	2543	8932	33	—
30	**Spurs**	1	16	2235	9689	48	3
Average of Eastern Division		0.800	17.533	1558.267	10,667	39.200	2.667
Average of Western Division		0.800	17.333	2417.333	9477.400	42.800	2.800

Table 3. T_{dj} and S_{cdj} of 30 NBA teams in 2018–2019 season.

Team number	Team	T_1	T_2	S_{11}	S_{21}	S_{12}	S_{22}
1	Bulls	6	—	—	22	—	—
2	Cavaliers	6	—	—	19	—	—
3	**Pistons**	6	2	8	33	0	0
4	**Pacers**	6	2	2.5	45.5	0	0
5	**Bucks**	6	2	20.5	39.5	3.8	6.2
6	**Nets**	6	2	—	42	—	1
7	**Celtics**	6	2	9.1	39.9	0.4	4.6
8	New York Nicks	6	—	—	17	—	—
9	**76ers**	6	2	16.9	34.1	2.4	4.6
10	**Raptors**	6	2	16.1	41.9	7.7	8.3
11	Hawks	6	—	—	29	—	—
12	Hornets	6	—	7.4	31.6	—	—
13	Heat	6	—	—	39	—	—
14	**Magic**	6	2	10.1	31.9	0	1
15	Wizards	6	—	7.6	24.4	—	—
16	**Nuggets**	6	2	11.8	42.2	3.1	3.9
17	Timberwolves	6	—	10.5	25.5	—	—
18	**Thunder**	6	2	18.7	30.3	0.6	0.4
19	**Trailblazers**	6	2	12.1	40.9	1.6	6.4
20	**Jazz**	6	2	—	50	—	—
21	**Warriors**	6	2	26.5	30.5	6.4	7.6
22	**Clippers**	6	2	—	48	—	2
23	Lakers	6	—	7.2	29.8	—	—
24	Suns	6	—	—	19	—	—
25	Kings	6	—	—	39	—	—
26	Mavericks	6	—	—	33	—	—
27	**Rockets**	6	2	15.2	37.8	1.7	4.3
28	Grizzlies	6	—	—	33	—	—
29	Pelicans	6	—	9.6	23.4	—	—
30	**Spurs**	6	2	9.3	38.7	0.7	2.3
Average of 30 teams				7.303	33.697	0.947	1.753

The inputs and outputs data of 30 NBA teams for 2018–2019 season are shown in Table 2 (the measurement unit of salaries is \$10,000).

Here the bold type in Table 2 indicates that the team is qualified to play in the playoffs for 2018–2019 season. The data of each team can be found in Table 2 as the values of a series of inputs and outputs. In addition, the average indices of each division is also proposed.

The data of T_{dj} (measurement unit: month) and S_{cdj} (measurement unit: match) of 30 teams are shown in Table 3, respectively.

Table 4. α^k_{cdj} and β^k_{cdj} of 30 NBA teams in 2018–2019 season.

Team number	Team	α^1_{11}	α^2_{12}	α^3_{21}	α^4_{22}	β^1_{11}	β^3_{21}	β^2_{12}	β^4_{22}
1	Bulls	—	—	1	—	—	1	—	—
2	Cavaliers	—	—	1	—	—	1	—	—
3	**Pistons**	0.750	0.250	0.750	0.250	0.195	0.805	0.500	0.500
4	**Pacers**	0.750	0.250	0.750	0.250	0.052	0.948	0.500	0.500
5	**Bucks**	0.750	0.250	0.750	0.250	0.342	0.658	0.380	0.620
6	**Nets**	—	—	0.750	0.250	—	1	—	1
7	**Celtics**	0.750	0.250	0.750	0.250	0.186	0.814	0.080	0.920
8	New York Nicks	—	—	1	—	—	1	—	—
9	**76ers**	0.750	0.250	0.750	0.250	0.331	0.669	0.343	0.657
10	**Raptors**	0.750	0.250	0.750	0.250	0.278	0.722	0.481	0.519
11	Hawks	—	—	1	—	—	1	—	—
12	Hornets	1	—	1	—	0.190	0.810	—	—
13	Heat	—	—	1	—	—	1	—	—
14	**Magic**	0.750	0.250	0.750	0.250	0.240	0.760	0.000	1
15	Wizards	1	—	1	—	0.238	0.763	—	—
16	**Nuggets**	0.750	0.250	0.750	0.250	0.219	0.781	0.443	0.557
17	Timberwolves	1	—	1	—	0.292	0.708	—	—
18	**Thunder**	0.750	0.250	0.750	0.250	0.382	0.618	0.600	0.400
19	**Trailblazers**	0.750	0.250	0.750	0.250	0.228	0.772	0.200	0.800
20	**Jazz**	—	—	0.750	0.250	—	1	—	1
21	**Warriors**	0.750	0.250	0.750	0.250	0.465	0.535	0.457	0.543
22	**Clippers**	—	—	0.750	0.250	—	1	—	1
23	Lakers	1	—	1	—	0.195	0.805	—	—
24	Suns	—	—	1	—	—	1	—	—
25	Kings	—	—	1	—	—	1	—	—
26	Mavericks	—	—	1	—	—	1	—	—
27	**Rockets**	0.750	0.250	0.750	0.250	0.287	0.713	0.283	0.717
28	Grizzlies	—	—	1	—	—	1	—	—
29	Pelicans	1	—	1	—	0.291	0.709	—	—
30	**Spurs**	0.750	0.250	0.750	0.250	0.194	0.806	0.233	0.767
Average of 30 teams						0.178	0.822	0.351	0.649

Table 5. The overall efficiency score and the corresponding rank of 30 NBA teams.

Team number	Team	Overall efficiency	Rank	Team number	Team	Overall efficiency	Rank
1	Bulls	0.277	27	16	**Nuggets**	0.796	2
2	Cavaliers	0.238	28	17	Timberwolves	0.401	23
3	**Pistons**	0.494	19	18	**Thunder**	0.543	14
4	**Pacers**	0.535	17	19	**Trailblazers**	0.711	6
5	**Bucks**	0.702	7	20	**Jazz**	0.779	3
6	**Nets**	0.539	15	21	**Warriors**	0.712	5
7	**Celtics**	0.683	8	22	**Clippers**	0.616	11
8	New York Nicks	0.221	29	23	Lakers	0.471	20
9	**76ers**	0.669	9	24	Suns	0.206	30
10	**Raptors**	0.812	1	25	Kings	0.546	13
11	Hawks	0.337	26	26	Mavericks	0.462	21
12	Hornets	0.509	18	27	**Rockets**	0.772	4
13	Heat	0.548	12	28	Grizzlies	0.402	22
14	**Magic**	0.539	16	29	Pelicans	0.386	24
15	Wizards	0.376	25	30	**Spurs**	0.631	10
Average of Eastern Division		0.499		Average of Western Division		0.562	

Subsequently, the values of α^k_{cdj} and β^k_{cdj} of these 30 teams are calculated via formula (4.1) and (4.2) and shown in Table 4.

3.2. Efficiency result and analysis

After introducing the input and output data and corresponding pretreatment, we can start to evaluate the performance of NBA teams in DMU level and sub-unit level, respectively. In addition, corresponding analysis and suggestions for teams' improvement will be proposed in this section.

3.2.1. Overall efficiency

The results of 30 DMUs' overall efficiencies are calculated via model (8) and shown in Table 5.

In summary, we can observe from Table 5 that the average performance of the western division (0.562) are better than the eastern division (0.499), which means that the comprehensive performance of western teams is better than that of the eastern teams. Specifically, the team with the highest efficiency is Raptors, which is also the final champion in 2018–2019 season. The teams ranked from second to fifth are Nuggets, Jazz, Rockets and Warriors. And the five under-performing teams are Hawks, Bulls, Cavaliers, New York Nicks and Suns.

One noted result is that, none team is efficient in Table 5, even the champion team Raptors. According to the Proposition 1, we know that this result means one or some sub-units of Raptors are inefficient. For revealing the deep reasons of DMUs'

Table 6. The sub-efficiency scores of 30 NBA teams' sub-units.

Team number	Team	Overall	E_j^1	E_j^2	E_j^3	E_j^4
1	Bulls	0.277	—	—	0.277	—
2	Cavaliers	0.238	—	—	0.238	—
3	**Pistons**	0.494	0.573	0.000	0.684	0.000
4	**Pacers**	0.535	0.210	0.000	0.822	0.000
5	**Bucks**	0.702	1	0.628	0.710	0.378
6	**Nets**	0.539	—	—	0.693	0.079
7	**Celtics**	0.683	0.776	0.116	0.826	0.323
8	New York Nicks	0.221	—	—	0.221	—
9	**76ers**	0.669	0.769	0.371	0.775	0.354
10	**Raptors**	0.812	0.618	1	0.947	0.637
11	Hawks	0.337	—	—	0.337	—
12	Hornets	0.509	0.542	—	0.501	—
13	Heat	0.548	—	—	0.548	—
14	**Magic**	0.539	0.971	0.000	0.643	0.068
15	Wizards	0.376	0.449	—	0.358	—
16	**Nuggets**	0.796	0.940	0.836	0.922	0.289
17	Timberwolves	0.401	0.832	—	0.330	—
18	**Thunder**	0.543	0.726	0.073	0.698	0.029
19	**Trailblazers**	0.711	0.915	0.411	0.783	0.415
20	**Jazz**	0.779	—	—	1	0.114
21	**Warriors**	0.712	0.661	0.541	0.833	0.705
22	**Clippers**	0.616	—	—	0.770	0.154
23	Lakers	0.471	0.370	—	0.505	—
24	Suns	0.206	—	—	0.206	—
25	Kings	0.546	—	—	0.546	—
26	Mavericks	0.462	—	—	0.462	—
27	**Rockets**	0.772	0.751	0.284	1	0.386
28	Grizzlies	0.402	—	—	0.402	—
29	Pelicans	0.386	0.566	—	0.341	—
30	**Spurs**	0.631	0.766	0.195	0.791	0.160
Average of Eastern Division			0.656	0.302	0.572	0.230
Average of Western Division			0.725	0.390	0.639	0.282

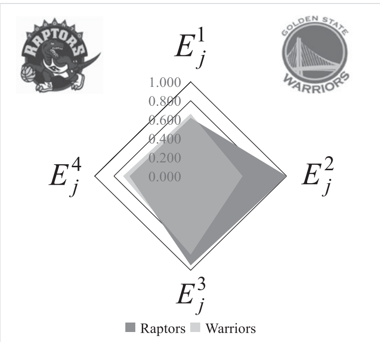

Figure 4. The radar maps of sample teams.

inefficient, we need to decompose their overall efficiency into sub-efficiencies.

3.2.2. Sub-efficiency

The sub-efficiencies of NBA teams' sub-units are given in Definition 4, and the empirical results via model (9.1) and (9.2) show that the efficiency decomposition are naturally unique. The detailed results of efficiency decomposition of 30 teams are shown in Table 6.

According to the sub-efficiencies, we can give the following analysis and suggestions:

- Analysis of the deep reasons for inefficiency.

Specifically, since sub-efficiencies reveal the performance of special level of players in special stage of NBA season, we can find the reason of inefficiency. Taking the champion (Raptors) and runner-up (Warriors) as examples, we can draw the radar map of their sub-efficiency according to the corresponding data in Table 6 as Figure 4 to get a more intuitive impression of their performance.

From Figure 4, we can observe that Raptors' All-Stars player is efficient in the playoffs ($E_{10}^2 = 1$), but he still needs to improve his regular performance($E_{10}^1 = 0.618$). In addition, Raptors' ordinary players perform good in the regular season ($E_{10}^3 = 0.947$), but they need to step up their playoff performance ($E_{10}^4 = 0.637$).

It also can be observed that, Warriors' All-Stars' playoff performances ($E_{21}^1 = 0.541$) have a lot of room for improvement. As the performance of these two teams are close in the other three sub-units, we suggest that Warriors should prioritize the playoffs performance of their All-Stars to improve their overall performance.

- Suggestions for team efficiency improvement.

After revealing the reason of inefficiency, we can make the targeted plan of improvement for each team. The advice may involve whether the performance of team's All-Stars or ordinary players in the regular season or the playoffs should be strengthened.

Take Jazz as an example. Jazz has two sub-units 3 and 4. From Table 6, we can observe that $E_{20}^3 = 1$, which means Jazz's ordinary players are efficient

Table 7. The CCR efficiency scores of 30 NBA teams' sub-units.

Team number	Team	Sub-unit1	Sub-unit2	Sub-unit3	Sub-unit4
1	Bulls	—	—	0.277	—
2	Cavaliers	—	—	0.238	—
3	**Pistons**	0.643	0.000	0.684	0.000
4	**Pacers**	0.219	0.000	0.822	0.000
5	**Bucks**	1	1	0.710	0.558
6	**Nets**	—	—	0.693	0.086
7	**Celtics**	0.809	0.136	0.826	0.524
8	New York Nicks	—	—	0.221	—
9	**76ers**	0.786	0.500	0.775	0.524
10	**Raptors**	0.670	1	0.947	0.947
11	Hawks	—	—	0.337	—
12	Hornets	0.544	—	0.508	—
13	Heat	—	—	0.548	—
14	**Magic**	0.975	0.000	0.643	0.107
15	Wizards	0.480	—	0.358	—
16	**Nuggets**	1	0.881	0.922	0.445
17	Timberwolves	1	—	0.330	—
18	**Thunder**	0.748	0.078	0.724	0.049
19	**Trailblazers**	0.999	0.416	0.783	0.644
20	**Jazz**	—	—	1	0.114
21	**Warriors**	0.726	0.554	0.833	1
22	**Clippers**	—	—	0.770	0.000
23	Lakers	0.425	—	0.505	—
24	Suns	—	—	0.206	—
25	Kings	—	—	0.546	—
26	Mavericks	—	—	0.462	—
27	**Rockets**	1	0.441	1	0.605
28	Grizzlies	—	—	0.402	—
29	Pelicans	0.606	—	0.341	—
30	**Spurs**	0.807	0.217	0.791	0.246

in the regular season. However, they are inefficient in the playoffs ($E_{20}^4 = 0.114$). Therefore, in order to improve the team's overall performance, Jazz need to enhance their ordinary players' competitiveness in the playoffs.

- Suggestions for team's player trading.

Player trading is an important way for NBA teams to enhance their performance. In this respect, the proposed method can provide decision support for their player trading based on the results of sub-efficiency. Team manager can pick target players which got high sub-efficiency. For example, if Jazz want to bring in All-Stars, they can choose the player who can bring more playoff wins to the them, e.g., the All-Star player of Raptors, for his high efficiency in the playoffs.

- Analysis for special data.

There are some special data in Table 6 that need to be interpreted, e.g., the efficiency scores of sub-unit 2 and 4 of Pistons and Pacers are equal to 0. This is because they do not win any competition in the playoffs.

Another noted result is that none of sub-unit4 in Table 6 is efficient. This result can be attributed to the leader-follower relationship between the overall efficiency and the sub-efficiency. This leader-follower relationship is determined by Definition 4. As E_j^k is calculated with the optimal multipliers $(\mathbf{u}^*, \mathbf{v}^*) \in \Omega_j$ of corresponding E_j but not the optimal multipliers of corresponding CCR Model (6), we always have $\sum_{r\in B_d} y_{r0}u_r^* / \sum_{i\in A_c} x_{i0}v_i^* \le e_j^k$. Let we denote Ω_j^k as the optimal solution set of model (6), then when $\Omega_j \cap \Omega_j^k = \emptyset$, we have $E_0^k < e_j^k$, when $\Omega_j \cap \Omega_j^k \ne \emptyset$, we have $E_0^k = e_j^k$.

To better illustrate this leader-follower relationship, we obtain the CCR efficiency scores of sub-units by solving Model (6), and list the results in Table 7.

By comparing the results in Tables 6 and 7, we can draw an intuitive impression of the leader-follower relationship between the overall efficiency and the sub-efficiency (e.g., e_{21}^4 is equal to 1 but E_{21}^4 is equal to 0.661). This kind of leader-follower relationship are common in sub-units, which also can be observed in $sub-unit_7^1$, $sub-unit_5^2$ and $sub-unit_{12}^3$.

4. Conclusions and future research

This study proposes an alternative DEA method for evaluating the competitive performance of NBA teams with non-homogeneity on both input and output sides. By opening the inner structure of DMUs, we split them into types of homogeneous sub-units. Under an "Overall-Sub" framework, we propose an overall evaluation model for obtaining the efficiencies of DMUs and then decompose the overall efficiency for obtaining the sub-efficiencies of sub-units. By applying the proposed method, the overall performance of NBA teams are obtained and the empirical results validate that the efficiency decomposition is naturally unique without any other additional conditions. Therefore, based on this framework, we can not only evaluate the efficiency of non-homogeneous DMUs, but also reveal the deep reasons for their inefficiency.

This work can be extended both practically and theoretically. Practically, the proposed method can be further applied to some other professional sports leagues with non-homogeneous issues. For example, Major League Baseball (MLB), National Football League (NFL) and National Hockey League (NHL), all have regular season and the playoffs in their seasons. Theoretically, both the DMU decompose scheme and the "Overall-Sub" framework can be extended to the network DEA method, for opening the inner structure and decomposing the overall efficiency more systematically.

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