The Phillips and Beveridge Curves in a Multi-Sector Economy*

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Abstract

I develop a New Keynesian model with input-output linkages, search-and-matching frictions, and sticky prices to study how sector-specific shocks propagate through the economy to output and inflation. Firms compete for workers from a common labor pool, creating a labor-market propagation channel: higher demand in one sector raises wages and job-finding rates there, redirects household job search, and increases hiring costs elsewhere. Solving the model nonlinearly shows that a sector's importance for inflation and the effects of monetary policy are state-dependent: sectors with higher tightness raise prices more in response to changes in demand or supply. This generates a Phillips curve that steepens as tightness—measured by job vacancies over unemployment—rises, consistent with recent evidence, and implies that monetary policy has larger effects on inflation but smaller effects on output when tightness in some sectors is high. I explore the implications for the post-COVID inflation surge by calibrating the model to U.S. input-output and labor market data. The model shows that the post-pandemic shift toward demand for goods alone (holding aggregate demand constant) can account for about 5% (0.3pp) of the inflation and most of the reduction in measured aggregate matching efficiency. Because labor markets were tight, particularly in the goods sector, keeping interest rates low led to a larger increase in inflation and but a smaller reduction in unemployment than a linearized model would predict.

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1 Introduction

In this paper, I introduce search-and-matching frictions into a model with a production network and sticky prices. The existing production network literature—which models the economy as an interconnected network of sectors that trade intermediate inputs—shows that input-output linkages are central to the propagation of sector-specific shocks and their aggregate effects. Micro shocks—changes in productivity or demand in one sector—create spillovers to other sectors through prices and quantities. Intuitively, the macroeconomic impact of a micro shock on inflation and output depends on the affected sector's importance as both a supplier and a user of intermediate inputs. Firms, however, interact not only through intermediate inputs but also through competition for workers in a shared labor market. This paper's main innovation is to incorporate search-and-matching frictions into a production-network model, allowing me to capture the consequences of these labor market interactions for inflation and monetary policy.

Incorporating search-and-matching frictions yields three key insights. First, the endogenous reallocation of workers across sectors generates new spillovers through hiring costs. As labor demand in a sector rises, wages increase, and it becomes easier for workers to find jobs there. Households respond by searching more in high-wage, high-opportunity sectors and less in others. This reallocation reduces labor supply in other sectors, raises hiring costs across the economy, and alters how sectoral shocks propagate through the network. In effect, a demand shock in one sector behaves like a negative supply shock in others, reducing labor supply and raising hiring costs. A key implication is that sectoral shocks can lower measured aggregate matching efficiency—which captures how effectively vacancies and unemployed workers are converted into new hires—even when matching efficiency remains constant at the sector level.

Second, search-and-matching frictions generate a nonlinear Phillips curve at the sector level that steepens with labor market tightness—the ratio of vacancies to unemployed workers. As labor market tightness in a sector rises, it becomes increasingly difficult for firms in that sector to hire workers. Hiring costs rise, and firms become progressively more supply constrained, forcing them to respond to further changes in supply or demand by adjusting prices rather than quantities. These dynamics produce a nonlinear Phillips curve—consistent with recent empirical evidence (Benigno & Eggertsson, 2023; Gitti, 2024)—but through a mechanism driven by nonlinearities in hiring costs and the elasticity of labor supply rather than by wage rigidity, a novel mechanism relative to the existing literature.

Third, when labor markets feature search-and-matching frictions, a sector's influence on aggregate inflation becomes state dependent. Sectors with tighter labor markets exert a stronger influence on overall inflation. As a result, shifts in demand across sectors can generate endogenous cost-push shocks even when aggregate demand remains unchanged and sectors are otherwise identical. Rising tightness in sectors experiencing higher demand leads to larger price increases than the price declines in sectors facing lower demand. The result is an increase in aggregate inflation.

These findings have important implications for the conduct and effectiveness of monetary policy. A nonlinear Phillips curve implies that additional positive demand or negative supply shocks have larger effects on inflation when some sectors' labor markets are tight. Furthermore, because changes in aggregate demand have larger effects on inflation and smaller effects on output, monetary policy does as well. Consequently, the effects of monetary policy are also state-dependent: in a tight labor market, policy is a weaker tool for stimulating output but has greater power to affect inflation. While the main contribution of my paper is theoretical, this result has clear implications for the post-COVID inflation surge, when inflation rose to around 8 percent in 2022.

The spike in inflation from early 2021 to mid-2022 coincided with a shift in consumption spending from services to goods and an unprecedented surge in labor market tightness concentrated in the goods sector. This tightness did not manifest in low unemployment (U) but rather in high vacancies (V), motivating the focus on $\frac{V}{U}$ as a key indicator of labor-market conditions. The Federal Reserve kept interest rates low—the first rate hike occured in March 2022—despite rising inflation and a rapidly tightening labor market. Pre-pandemic estimates suggested a flat Phillips curve: changes in unemployment were associated with only small increases in inflation (Hazell et al., 2022). This implied that the risks of keeping interest rates low and running the economy hot were small—unemployment could fall substantially without generating large increases in prices. My model instead suggests that because the goods sector, in particular, was so tight and therefore capacity constrained, this policy choice instead had a large effect on inflation but only a limited impact on unemployment—the opposite of what the Fed hoped to achieve. Had they tightened earlier, they could likely have reduced inflation substantially without causing much unemployment.

I begin by developing a framework that combines three well-established components from the literature: a production network, sticky prices, and search-and-matching in the labor market. I then use this framework to think about the effect of sectoral shocks on inflation and monetary policy, one of many possible applications. The production network allows me to take a sectoral approach to inflation—following Baqaee and Farhi (2022), La'O and Tahbaz-Salehi (2022), and Rubbo (2023)—by tracing how shocks propagate downstream to users of a sector's output and upstream to its input suppliers. Households optimally send more workers to sectors with higher wages and a higher probability of finding a job, triggering cascades in hiring costs, and creating a labor reallocation channel absent from previous models.

The labor reallocation channel is reminiscent of a classic literature debating whether sectoral reallocation caused by sector-specific shocks can generate persistent frictional unemployment (Abraham & Katz, 1986; Lilien, 1982). In my framework, frictional unemployment exists even in the absence of sectoral shocks; it instead arises from search-and-matching frictions as in the Diamond-Mortensen-Pissarides model (Diamond, 1982b; Mortensen, 1982a; Pissarides, 1984). However, sectoral shocks do cause costly reallocation of labor across sectors, altering the propagation of shocks to marginal costs and, ultimately, affecting measured aggregate matching efficiency.[1]

Unlike most prior work on inflation and monetary policy in production networks, I solve the model nonlinearly by simulating perfect-foresight impulse responses to unexpected shocks.[2] Using an illustrative calibration with identical sectors designed to resemble an average U.S. sector, I show that nonlinearities are quantitatively important for both inflation and the effectiveness of monetary policy. For instance, when labor-market tightness is roughly 125% above steady state, a policy-rate cut has nearly ten times the effect on inflation as under steady-state tightness. An 8% shift in relative demand from services to goods—similar in magnitude to the post-pandemic reallocation—raises aggregate inflation by 7 percentage points, reduces measured aggregate matching efficiency by 25%, and amplifies the inflationary impact of a policy-rate cut by roughly 150%, even though aggregate demand remains unchanged.

A linearized version would miss the steepening of the Phillips curve at high levels of tightness, thereby underestimating the effects of aggregate-demand stimulus on inflation when labor markets are tight and failing to capture the inflationary effects of demand reallocation. Furthermore, the nonlinear solution shows that search-and-matching frictions not only alter which sectors are most influential for inflation through the labor-market spillover channel—that would persist even in a linearized model—but also make a sector's importance state dependent.

After establishing the key theoretical mechanisms in the illustrative calibration, I apply an empirical calibration—accounting for differences in size, labor share, and other sectoral characteristics—to the recent inflation surge, focusing on the distinction between goods and services. I calibrate the production network parameters to U.S. input-output tables from the

[1] Aggregate matching efficiency measures the effectiveness of the matching process; it is effectively a residual capturing the wedge between the observed number of hires and the number predicted by the aggregate numbers of unemployed workers and vacancies in the economy.

[2] Although the perfect-foresight assumption abstracts from risk, it allows me to isolate the nonlinearities arising from labor-market frictions.

Bureau of Economic Analysis (BEA) and estimate the remaining labor-market parameters in a linearized version of the model using Bayesian methods and data from 2000-2019—a period when labor-market tightness remained below the exceptionally high post-pandemic levels, making the linearized model a reasonable approximation. The persistent rise in the consumption share of goods relative to services after the pandemic increased inflation by about 0.3 percentage points and explains nearly the entire decline in aggregate matching efficiency. However, the sectoral demand shock alone cannot explain the large and persistent rise in aggregate inflation and generates a counterfactual decline in labor-market tightness in services. Incorporating shocks to the separation rate and aggregate demand yields a much closer fit to the data, and only a modest amount of monetary stimulus leads to large effects on inflation.

My paper relates to three broad strands of the literature. First, it builds a production network model in the tradition of Acemoglu et al. (2012), Baqaee and Farhi (2019), Baqaee and Rubbo (2022), V. M. Carvalho and Tahbaz-Salehi (2019), and Jones (2011). Much of this literature focuses on the effects of sector specific productivity shocks. According to Hulten's theorem—a fundamental benchmark in the literature—the aggregate effect of a productivity shock in a sector is proportional to its Domar weight, the sector's income over GDP (Hulten, 1978). Like other inefficiencies explored in the literature—for instance markups and rigid prices—labor market frictions lead to violations of Hulten's theorem, altering how shocks propagate and which sectors are most important for aggregate outcomes. The literature typically accounts for these violations through a model-informed reweighting of the input-output linkages, leading to altered—but fixed—quasi-Domar weights.

My contribution is twofold. First, I show how incorporating search-and-matching frictions in the labor market alters the propagation of sectoral shocks, extending Schüle and Sheng (2024)[3]. Relative to Schüle and Sheng (2024), I incorporate nominal rigidities and dynamics to analyze inflation and monetary policy, and I solve the model nonlinearly. The presence of labor-market frictions implies that a sector's importance for aggregate inflation depends on how shocks pass through to sectoral tightness, how sectoral tightness affects hiring costs, and how those costs transmit to inflation—necessitating a reweighting of sectoral importance even in the linearized case. Furthermore, I show that sectoral importance for output and inflation is state dependent. When labor markets feature search-and-matching frictions, sectors with tighter labor markets experience larger increases in hiring costs for the same increase in tightness, and pass on more of any change in demand or supply to inflation. Therefore, even the static reweighting is not sufficient to capture a sectors importance for inflation: accurately capturing which sectors are likely to drive inflation requires a dynamic

[3]This research agenda has grown out of collaboration with Haoyu Sheng, a fellow graduate student.

and state-dependent approach.

Second, my paper relates to the extensive literature that uses multi-sector models to study the post-pandemic economy (Amiti et al., 2023; Baqaee & Farhi, 2022; Boehm & Pandalai-Nayar, 2022; Comin et al., 2023; Di Giovanni et al., 2023; Ferrante et al., 2023; Guerrieri et al., 2022). Guerrieri et al. (2022) show that sectoral supply shocks can trigger demand contractions in other sectors, pushing aggregate demand below potential—a phenomenon they term a "Keynesian supply shock". In my model, labor market spillovers reverse this result: an increase in demand in one sector reduces labor supply in others. As a result, even if aggregate demand remains unchanged, shifts in relative demand across sectors reduce potential output through labor reallocation, inducing a positive cost-push shock. These results are complementary: both show that changes in demand or supply in one sector lead to spillovers in others—demand shocks are no longer purely demand shocks, and supply shocks are no longer purely supply shocks. The mechanisms differ, however: Guerrieri et al. (2022) emphasize demand spillovers through income and substitution effects, whereas I focus on labor market spillovers driven by search-and-matching frictions, which are especially important when shocks lead to substantial heterogeneity in tightness across sectors, as during the pandemic recovery.

Like Amiti et al. (2023), Comin et al. (2023), Di Giovanni et al. (2023), and Rubbo (2024), I examine how sectoral shocks affect inflation. My contribution is to show that search-and-matching frictions generate a nonlinear Phillips curve at the sector level and state-dependent effects of sectoral shocks. These frictions also amplify the inflationary effects of changes in productivity or demand when labor markets are tight. Given the unusually tight labor markets of the post-pandemic period, my mechanism implies that even modest changes in sectoral supply or aggregate demand can generate substantial inflationary pressure. Consequently, the risk of keeping interest rates low for too long is greater than standard models suggest.

Within this literature, my paper is most closely related to Ferrante et al. (2023) and Boehm and Pandalai-Nayar (2022). Relative to Ferrante et al. (2023) I microfound hiring costs using standard search-and-matching, which naturally generates larger hiring costs in tighter labor markets and makes hiring costs depend not only on a sector's own hiring decisions but, through labor reallocation, also on the hiring decisions of other sectors. Relative to Boehm and Pandalai-Nayar (2022), I show that search-and-matching frictions can effectively induce capacity constraints at the sector level, amplifying the effects of supply and demand disturbances on inflation and altering the effectiveness of monetary policy.

Third, I contribute to recent work exploring the links between the inflation surge and labor market conditions. I build on Blanchard and Galí (2010), who incorporate search-and-matching into an otherwise standard New Keynesian model, by adding multiple sectors that interact through a production network. This extension allows me to analyze the effects of sectoral shocks and to highlight important cross-sector spillovers that shape aggregate inflation dynamics.

Benigno and Eggertsson (2023) and Michaillat and Saez (2024) propose mechanisms that generate a nonlinear Phillips curve in labor market tightness—an empirically plausible feature (Gitti, 2024)—by assuming downwardly rigid nominal wages. By solving the model nonlinearly, I show that a smooth steepening of the Phillips curve arises from incorporating search-and-matching into a standard New Keynesian model with Nash bargained wages. The nonlinearity emerges from the convexity of hiring costs—it becomes progressively more difficult to fill vacancies as tightness rises. By focusing on a multi-sector economy, I also show how this nonlinearity induces state-dependent sectoral importance and endogenous cost-push shocks when demand shifts across sectors.

Furthermore, I show that sectoral labor reallocation and heterogeneity in tightness across sectors can lead to endogenous declines in aggregate matching efficiency, shifting the Beveridge curve outward, as observed during the pandemic recovery. Workers moving across sectors can be viewed as a form of job-to-job transition, which Bagga et al. (2025) show helps explain the outward shift in the Beveridge curve. Allowing for endogenous separations in response to wage differentials across sectors would strengthen the power of the reallocation channel I highlight. Similarly, Afrouzi et al. (2024) propose a mechanism linking inflation to separations, whereby workers leave their jobs when inflation is high to recoup losses in real earnings. Again, I view my results as complementary: an increase in demand in one sector, which raises wages and prices there, would induce higher separations and greater labor flows across firms, further amplifying the labor spillover channel I highlight.

The paper proceeds as follows. Section 2 introduces the model and outlines the equilibrium conditions. Section 3 presents key predictions. Section 4 applies the model to post-pandemic inflation and labor market dynamics. Section 5 concludes.

2 A Model with Production Networks and Labor-Market Frictions

In this section, I build a New Keynesian model with labor market frictions and production linkages. The model has three key components: (1) sticky prices, (2) a production network, and (3) search-and-matching frictions in the labor market. Sticky prices generate a New Keynesian Phillips curve, linking inflation to firms' marginal costs and expected future inflation. The production network—firms use intermediate inputs from other sectors—links marginal costs in one sector to prices in other sectors. Finally, search-and-matching frictions capture the costly hiring process, linking firms' marginal costs to labor market conditions. Together, these three components generate a sector-level Phillips curve that captures how inflation spreads across sectors through both prices and labor market conditions.

2.1 The Labor Market

I begin with the labor market, modeled using a search-and-matching framework in which firms post vacancies and workers search for jobs (Diamond, 1982a, 1982b; Mortensen, 1982a, 1982b; Pissarides, 1984, 1985). The search-and-matching process generates a probability of a worker finding a job and of a firm filling a vacancy that depend on sectoral labor market tightness—the ratio of vacancy postings to unattached job seekers in sector i. As a result, tightness directly affects firms' hiring costs, and therefore their marginal costs.

At the start of each discrete time period t, firms in sector $i \in \{1, \cdots, J\}$ post vacancies $V_{i,t}$ and hire $H_{i,t}$ workers from a pool of searchers $U_{i,t}$.[4] A hire occurs whenever a searching worker matches with a hiring firm. As is standard in the search-and-matching literature, I represent this matching process using a matching function $m_{i,t}(U_{i,t}, V_{i,t})$.[5] The total number of hires in sector i is

$$H_{i,t} = m_{i,t}(U_{i,t}, V_{i,t}). \tag{1}$$

The matching function exhibits constant returns to scale, $m_{i,t}(\lambda U_{i,t}, \lambda V_{i,t}) = \lambda H_{i,t}$, is increasing in both arguments, $\frac{\partial m_{i,t}}{\partial U_{i,t}}, \frac{\partial m_{i,t}}{\partial V_{i,t}} > 0$, and concave, $\frac{\partial^2 m_{i,t}}{\partial U_{i,t}^2}, \frac{\partial^2 m_{i,t}}{\partial V_{i,t}^2} < 0$. Lastly, the matching function satisfies $m_{i,t}(U_{i,t}, V_{i,t}) \leq \min\{U_{i,t}, V_{i,t}\}$. This condition ensures that the numbers of vacancies and unattached workers at the end of the period t are non-negative. Intuitively, it implies firms cannot conjure workers out of thin air by posting more vacancies.

Because the matching function is constant returns to scale, I can conveniently express the probability of a firm filling a vacancy (or the vacancy-filling rate) in sector i in terms of sector-specific labor market tightness $\theta_{i,t} = \frac{V_{i,t}}{U_{i,t}}$:

$$q_{i,t} = \frac{H_{i,t}}{V_{i,t}} = m_{i,t}\left(\theta_{i,t}^{-1}, 1\right). \tag{2}$$

[4] While I denote total searchers by $U_{i,t}$ for consistency with the notation commonly used in the search-and-matching literature, I think of $U_{i,t}$ as capturing the number of unattached workers searching for a job in sector i.

[5] By allowing the function $m_{i,t}$ to be sector-specific and time varying, I can allow for exogenous variation in the efficiency of the matching process across sectors and over time.

Since $m_{i,t}$ is increasing in both arguments, the vacancy-filling rate falls as tightness rises: it gets harder for firms to hire as tightness rises. Similarly, the probability that a worker finds a job in sector i (or the job-finding rate), conditional on searching in that sector, is

$$f_{i,t} = \frac{H_{i,t}}{U_{i,t}} = m_{i,t}(1, \theta_{i,t}). \tag{3}$$

The job-finding rate is increasing in tightness: it gets easier for workers to find a job as tightness rises. Since $m_{i,t}(U_{i,t}, V_{i,t}) \leq \min\{U_{i,t}, V_{i,t}\}$, both the vacancy-filling rate and the job-finding rate are bounded between 0 and 1: $q_{i,t}, f_{i,t} \in [0, 1]$.

I assume that a vacancy requires $r_{i,t}$ hours of labor to post and maintain—firms pay a recruiting cost in terms of labor, representing time spent on posting vacancies, reviewing applications, interviewing candidates, and other recruiting activities. As tightness rises, firms must post more vacancies, employ more recruiters, and incur higher recruiting costs to achieve a given level of hiring. This creates a link between labor market tightness and marginal costs. Finally, I assume that an exogenous fraction $s_{i,t}$ of workers separate from their jobs at the beginning of each period.

2.2 Firms

The production structure in each sector incorporates the three key model ingredients: (1) sticky prices, (2) a production network, and (3) search-and-matching frictions. To allow for sticky prices, I assume that each sector contains a continuum of monopolistically competitive firms that use labor and intermediate goods to produce output and set prices subject to adjustment costs. The presence of price adjustment costs generates a sector-specific New Keynesian Phillips curve linking inflation to marginal costs and expected future inflation. As in Baqaee (2018),Baqaee and Farhi (2019), and Baqaee and Rubbo (2022), firms use intermediate inputs from other sectors, generating a production network. Finally, firms make hiring decisions subject to the search-and-matching structure described above, linking marginal costs to labor market conditions.

Each sector contains a unit continuum of monopolistically competitive intermediate goods producers indexed by z, and a representative firm that aggregates the intermediate goods to produce sectoral output. The representative firm in each sector produces the sectoral output $Y_{i,t}$ from the outputs of each of the intermediate firms $Y_{i,t}(z)$ using CES technology.

$$Y_{i,t} = \left(\int_0^1 Y_{i,t}(z)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

where ϵ is the elasticity of substitution between the outputs of the different intermediate goods producers in sectoral output. This representative firm chooses how much of each firm z's output to use in production to minimize its costs, resulting in downward-sloping demand functions for each z:

$$Y_{i,t}(z) = \left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\epsilon} Y_{i,t} \tag{5}$$

where $P_{i,t}(z)$ is the price of firm z's output.

As in the standard production networks setup, the monopolistically competitive intermediate goods producers in each sector produce their output $Y_{i,t}(z)$ using labor $N_{i,t}(z)$ and intermediate inputs $X_{ij,t}(z)$.

$$Y_{i,t}(z) = A_{i,t}\left(\omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t}(z)^{\frac{\epsilon_y-1}{\epsilon_y}} + \sum_{j=1}^{J} \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t}(z)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}} \tag{6}$$

where $A_{i,t}$ is a sector-specific productivity shock, which captures, for instance, supply chain disruptions that disproportionately affect one sector. ω_{in} is the share of labor and ω_{ij} is the share of sector j's output in sector i's production process. ϵ_y is the elasticity of substitution between labor and intermediate inputs.

Each firm z chooses how many vacancies $V_{i,t}(z)$ to post, taking the total number of vacancy postings in sector i, $V_{i,t} = \int_0^1 V_{i,t}(z)dz$, and therefore sector level tightness $\theta_{i,t}$, as given. The number of newly hired workers at firms z, $H_{i,t}(z)$ depends on the vacancy-filling rate $q_{i,t}$, implying the following law of motion for employment at firm z:

$$N_{i,t}(z) + N^r_{i,t}(z) = q_{i,t}V_{i,t}(z) + (1 - s_{i,t})\left(N_{i,t-1}(z) + N^r_{i,t-1}(z)\right). \tag{7}$$

Where $N_{i,t}(z)$ is labor hours engaged in production and $N^r_{i,t}(z) = r_{i,t}V_{i,t}(z)$ is labor hours spent recruiting.

Finally, each firm z chooses its price $P_{i,t}(z)$ subject to Rotemberg adjustment costs.[6], the number of vacancy postings $V_{i,t}(z)$, and how much of each intermediate input to use in production $\{X_{ij,t}(z)\}_{j=1}^{J}$ in order to maximize its dividend payments[7],

$$\max_{\{P_{i,t+s}(z), V_{i,t+s}(z), \{X_{ij,t+s}(z)\}_{j=1}^{J}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty}\left[SDF_{t|t+s}\frac{D_{i,t+s}(z)}{P_{t+s}}\right] \tag{8}$$

[6] I choose Rotemberg adjustment costs because they do not lead to price dispersion across firms, and I therefore do not need to track the distribution of prices, output, and therefore employment across firms.

[7] By choosing $V_{i,t}(z)$ the firm is also choosing $N_{i,t}(z)$ through the law of motion for employment.

subject to Eqs. (5), (6), and (7). Where

$$\frac{D_{i,t}(z)}{P_t} = \frac{P_{i,t}(z)}{P_t}\left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\epsilon} Y_{i,t} - \frac{W_{i,t}}{P_t}\left[N_{i,t}(z) + N^r_{i,t}(z)\right]$$
$$- \sum_{j=1}^{J} \frac{P_{j,t}}{P_t} X_{ij,t}(z) - \frac{\psi_{p,i}}{2}\left(\frac{P_{i,t}(z)}{\Pi P_{i,t-1}(z)} - 1\right)^2 Y_{i,t}.$$

Since all firms in a given sector face an identical problem, they all make identical input choices and pricing decisions. I therefore drop the z indexation from the following optimality conditions. I allow the parameter governing the size of price adjustment costs, $\psi_{p,i}$, to vary at the sector level (Ferrante et al., 2023; Pasten et al., 2020).

Firms post vacancies up to the point where the marginal cost of an additional vacancy equals the marginal benefit of an additional vacancy

$$\frac{W_{i,t}}{P_t} r_{i,t} = \mu_{i,t}(q_{i,t} - r_{i,t}) + E_t\left[SDF_{t|t+1}\Pi_{t+1}(1 - s_{i,t+1})\mu_{i,t+1} r_{i,t}\right] \tag{9}$$

where $\mu_{i,t}$ is the marginal value of an additional employee to the firm. The marginal value of an additional employee equals the marginal product of labor today, net of the wage, plus the continuation value of the employee.

$$\mu_{i,t} = \lambda_{i,t} \beta_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y - 1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} - \frac{W_{i,t}}{P_t} + E_t\left[SDF_{t|t+1}\Pi_{t+1}(1 - s_{i,t+1})\mu_{i,t+1}\right] \tag{10}$$

where $\lambda_{i,t}$ is the real marginal cost. Each firm chooses intermediate inputs so that good j's share in total costs satisfies

$$\frac{\frac{P_{j,t}}{P_t} X_{ij,t}}{\lambda_{i,t} Y_{i,t}} = (\beta_{ix}\omega_{ij})^{\frac{1}{\epsilon_y}} \left(A_{i,t}\frac{X_{ij,t}}{Y_{i,t}}\right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \tag{11}$$

Each firm's optimal price setting implies a sector-specific Phillips curve as in Ferrante et al. (2023), La'O and Tahbaz-Salehi (2022), and Rubbo (2023, 2024).

$$\left(\frac{\Pi_{i,t}}{\Pi} - 1\right)\frac{\Pi_{i,t}}{\Pi} = \frac{\epsilon}{\phi_{p,i}}\left(\frac{MC_{i,t}}{P_t} - \frac{\epsilon - 1}{\epsilon}\frac{P_{i,t}}{P_t}\right) + E_t\left[SDF_{t+1|t}\Pi_{t+1}\left(\frac{\Pi_{i,t+1}}{\Pi} - 1\right)\frac{\Pi_{i,t+1}}{\Pi}\frac{Y_{i,t+1}}{Y_{i,t}}\right] \tag{12}$$

where $SDF_{t+1|t} = \beta \frac{U_{c,t+1}}{U_{c,t}} \Pi_{t+1}^{-1}$ is the household's stochastic discount factor.

Finally, each sector's output must satisfy the market clearing condition

$$Y_{i,t} = C_{i,t} + \sum_{j=1}^{J} X_{ji,t} \tag{13}$$

2.3 Households

A unit mass of identical households indexed by z supply labor to firms, consume goods produced by each of the J sectors, and save in the form of bonds.

Households have CES preferences for consumption across the sectors. The aggregate consumption good C_t is therefore:

$$C_t = \left(\sum_{i=1}^{J} \alpha_{i,t}^{\frac{1}{\epsilon_d}} C_{i,t}^{\frac{\epsilon_d - 1}{\epsilon_d}} \right)^{\frac{\epsilon_d}{\epsilon_d - 1}} \tag{14}$$

where ϵ_d is the elasticity of substitution across sectors, and $\alpha_{i,t}$ is a time-varying exogenous preference shifter that captures households' changing preferences for consumption in sector i. Note that $C_{i,t}$ is final household consumption, not total output, because some of the output from each sector is used as an intermediate input in production.

These preferences imply downward-sloping consumption demand functions:

$$C_{i,t} = \alpha_{i,t} \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon_d} C_t \tag{15}$$

where the aggregate consumer price index is

$$P_t = \left(\sum_{i=1}^{J} \alpha_{i,t} P_{i,t}^{1-\epsilon_d} \right)^{\frac{1}{1-\epsilon_d}} \tag{16}$$

Each household maximizes lifetime utility subject to their budget constraint and the law of motion for employment in each sector, which depends on the job-finding rates.

$$\max_{\left\{C_{t+s}(z),\{L_{i,t+s}(z)\}_{i=1}^{J}\right\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \beta^s U_{t+s}\left(C_{t+s}(z), \left\{\tilde{L}_{i,t+s}(z), L_{i,t+s}(z)\right\}_{i=1}^{J}\right)$$
$$\text{s.t. } P_{t+s}C_{t+s}(z) + B_{t+s}(z) = (1+i_{t+s})B_{t+s-1}(z) + \sum_{i=1}^{J} W_{i,t+s}\tilde{L}_{i,t+s}(z) + T_{t+s}(z)$$
$$\tilde{L}_{i,t+s}(z) = (1-f_{i,t+s})(1-s_{i,t+s})\tilde{L}_{i,t+s-1}(z) + f_{i,t+s}L_{i,t+s}(z)$$
$$\text{and } \sum_{i=1}^{J} L_{i,t+s} \leq \overline{L}_{t+s} \tag{17}$$

where $C_t(z)$ is the aggregate consumption bundle, $L_{i,t}(z)$ is labor supplied to sector i (including employed and searching workers), $\tilde{L}_{i,t}(z)$ is employment in sector i, $B_t(z)$ is a one-period risk-free bond, i_t is the nominal interest rate, and $T_{t+s}(z)$ are lump-sum transfers and dividend payments. The model features a form of directed search: households choose how to divide their endowment of unattached workers, $\overline{L}_t - \sum_{i=1}^{J}(1-s_{i,t})\tilde{L}_{i,t-1}(z)$, across the sectors based on both the real wage and the likelihood of finding a job in each sector.

The households' period utility function is:

$$U_{t+s}(\cdot) = Z_{t+s}\left[\frac{C_{t+s}(z)^{1-\sigma}}{1-\sigma} - \sum_{i=1}^{J}\left(\chi_{i,t+s}\frac{\tilde{L}_{i,t+s}(z)^{1+\varphi}}{1+\varphi} + \frac{\psi_{L,i}}{2}\left(\frac{L_{i,t+s}/\overline{L}_{t+s}}{L_{i,t+s-1}/\overline{L}_{t+s-1}} - 1\right)^2 L_{i,t+s}\right)\right]. \tag{18}$$

The final term in the households' utility function, $\frac{\psi_{L,i}}{2}\left(\frac{L_{i,t+s}/\overline{L}_{t+s}}{L_{i,t+s-1}/\overline{L}_{t+s-1}} - 1\right)^2 L_{i,t+s}$, is a quadratic cost of adjusting the fraction of workers who search in a given sector i. This adjustment cost captures frictions in relocating across sectors because of moving costs, retraining costs, or other frictions. These types of frictions are empirically important in determining where workers search for jobs (Humlum, 2021), and within the model will be important for determining the labor-supply response to sectoral shocks. $\psi_{L,i}$ governs the size of reallocation costs while φ governs how responsive labor supply is to changes in real wages.

The households first-order conditions imply the standard Euler equation for consumption:

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left[C_{t+1}^{-\sigma}\frac{Z_{t+1}}{Z_t}\Pi_{t+1}^{-1}\right] \tag{19}$$

The household optimal decisions for how many workers to send in search of a job in each sector equalizes the expected return from searching in each sector, subject to the labor adjustment costs.

$$\delta_t = \Xi_{i,t} f_{i,t} - \psi_{L,i} Z_t \left(\left[\frac{L_{i,t}/\overline{L}_t}{L_{i,t-1}/\overline{L}_{t-1}} - 1 \right] + \frac{1}{2} \left[\frac{L_{i,t}/\overline{L}_t}{L_{i,t-1}/\overline{L}_{t-1}} - 1 \right]^2 \right)$$
$$+ \beta \psi_{L,i} E_t \left[Z_{t+1} \left(\frac{L_{i,t+1}/\overline{L}_{t+1}}{L_{i,t}/\overline{L}_t} - 1 \right) \frac{L^2_{i,t+1}/\overline{L}_{t+1}}{L^2_{i,t}/\overline{L}_t} \right] \tag{20}$$

where $\Xi_{i,t}$ is the value to the household of an additional employed worker in sector i , $f_{i,t}$ is the job-finding rate in sector i, and δ_t marginal value of an additional unit of aggregate labor to the household. Absent labor adjustment costs, the household adjusts search across sectors up to the point where $f_{i,t}\Xi_{i,t} = f_{j,t}\Xi_{j,t}$. $f_{i,t}\Xi_{i,t}$ is the expected return from searching in sector i. It is the probability of finding a job in sector i times the value to the household of an additional employee in sector i.

The value of an additional employed worker in sector i is the value of being employed today—how much the employed worker earns, converted into utils, net of the utility cost of an additional employee—plus the continuation value of being employed tomorrow.

$$\Xi_{i,t} = Z_t \left(C_t^{-\sigma} \frac{W_{i,t}}{P_t} - \chi_{i,t} \tilde{L}^{\varphi}_{i,t} \right) + \beta E_t \left[(1 - f_{i,t+1})(1 - s_{i,t+1}) \Xi_{i,t+1} \right] \tag{21}$$

Finally, the number of unattached workers at the beginning for the period is given by

$$U_{i,t} = L_{i,t} - (1 - s_{i,t}) \tilde{L}_{i,t-1}.$$

2.4 Wages

As in standard search-and-matching models, when workers are matched with firms they face a bilateral monopoly. Wages are therefore not determined by the model's equilibrium conditions,[8] and instead require an assumption about the wage-setting process.

I use Nash bargaining between the household and the firm as the basis for my wage determination process. Let $\mathcal{S}^f_{i,t}$ denote the firm surplus from a match in sector i and let $\mathcal{S}^h_{i,t}$ denote the household surplus from a match in sector i. The Nash bargaining solution implies

$$\mathcal{S}^h_{i,t} = \kappa \mathcal{S}^f_{i,t} \tag{22}$$

where κ is determined by the relative bargaining power of the household.[9]

[8] The household and firm equilibrium conditions define a range of acceptable wages for a given match.

[9] If, for instance, the firm and household split the total surplus equally, then $\kappa = 1$.

Because hiring an additional worker requires $\frac{1}{q_{i,t}}$ vacancy postings at cost $r_{i,t}W_{i,t}$, firms can replace any worker at a cost $\frac{r_{i,t}}{q_{i,t}}W_{i,t}$. By free entry into vacancy postings, the firm surplus from an existing worker is:

$$\mathcal{S}^f_{i,t} = \frac{r_{i,t}}{q_{i,t}}W_{i,t} \tag{23}$$

From the household's first order conditions, the value of an additional employee to the household, measured in utils, is given by $\Xi_{i,t}$ in Eq.(21). The surplus in nominal terms is therefore $\frac{\Xi_{i,t}}{Z_t C_t^{-\sigma}}P_t$. The Nash bargaining solution then implies that the Nash-bargained wage is

$$W_{i,t} = \frac{q_{i,t}}{q_{i,t} - \kappa r_{i,t}}\left(\chi_{i,t}\tilde{L}^{\varphi}_{i,t}C^{\sigma}_t P_t - \kappa E_t\left[SDF_{t+1|t}(1-f_{i,t+1})(1-s_{i,t+1})\frac{r_{i,t+1}}{q_{i,t+1}}W_{i,t+1}\right]\right) \tag{24}$$

As is well understood since Shimer (2005) and R. E. Hall (2005), fully flexible Nash bargaining leads to counterfactually large wage fluctuations and small employment fluctuations over the business cycle. I therefore allow for a degree of wage rigidity by assuming that the final wage is a weighted average of this Nash bargaining solution and last period's wage and I allow for exogenous shocks to the wage, $W^s_{i,t}$, so that the overall wage is given by

$$\begin{aligned} W_{i,t} = &\left[\frac{q_{i,t}}{q_{i,t} - \kappa r_{i,t}}\left(\chi_{i,t}\tilde{L}^{\varphi}_{i,t}C^{\sigma}_t P_t - \kappa E_t\left[SDF_{t+1|t}(1-f_{i,t+1})(1-s_{i,t+1})\frac{r_{i,t+1}}{q_{i,t+1}}W_{i,t+1}\right]\right)\right]^{1-\rho_w} \\ &\times \left(W_{i,t-1}\right)^{\rho_w} W^s_{i,t} \end{aligned} \tag{25}$$

This specification nests fully flexible Nash bargained wages ($\rho_w = 0$) and fully rigid wages ($\rho_w = 1$). This is a reduced-form way to capture that wages may not adjust fully within a month to changes in economic conditions, and as Gertler and Trigari (2009) demonstrate, partial wage rigidity allows the search-and-matching model to generate more realistic fluctuations in wages and unemployment.

2.5 Monetary Policy and Shock Processes

I close the model by assuming that the central bank sets the nominal interest rate according to a Taylor rule in inflation and the output gap.

$$(1+i_t) = (1-i_{t-1})^{\rho_i}\left(\left[\frac{\Pi^{agg}_t}{\Pi}\right]^{\phi_\pi}\left[\frac{Y^{agg}_t}{Y^{agg,flex}_t}\right]^{\phi_y}\right)^{1-\rho_i}\varepsilon^m_t \tag{26}$$

where ε_t^m is a normally distributed monetary policy shock.

Finally, I assume that the natural logarithms of Z_t, $\chi_{i,t}$, $\alpha_{i,t}$, $s_{i,t}$, $r_{i,t}$, $W_{i,t}^s$, follow AR(1) processes with normally distributed innovations.

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z, \qquad \log \chi_{i,t} = \rho_\chi \log \chi_{i,t-1} + \varepsilon_{i,t}^\chi \tag{27}$$

$$\log \alpha_{i,t} = \rho_\alpha \log \alpha_{i,t-1} + \varepsilon_{i,t}^\alpha, \qquad \log s_{i,t} = \rho_S \log s_{i,t-1} + \varepsilon_{i,t}^s \tag{28}$$

$$\log r_{i,t} = \rho_r \log r_{i,t-1} + \varepsilon_{i,t}^r, \qquad \log W_{i,t}^s = \rho_w \log W_{i,t-1}^s + \varepsilon_{i,t}^{W^s} \tag{29}$$

I summarize the full set of equilibrium conditions in Appendix A.[10]

2.6 Equilibrium Definition

An equilibrium is a set of sequences for endogenous variables

$$\left\{C_t, Y_t, \Pi_t, \left\{C_{i,t}, L_{i,t}, L_{i,t}, U_{i,t}, V_{i,t}, N_{i,t}, Y_{i,t}, \{X_{ij,t}\}_{j=1}^J\right\}_{i=1}^J\right\}_{t=0}^\infty$$

Given paths for the exogenous processes, $\left\{Z_t, \varepsilon_t^m \{\alpha_{i,t}, r_{i,t}, s_{i,t}, \chi_{i,t}, W_{i,t}^s\}_{i=1}^J\right\}_{t=0}^\infty$, such that:

1. Firms maximize profits,

2. Households maximize lifetime utility,

3. Markets clear

$$Y_{i,t} = C_{i,t} + \sum_{j=1}^J X_{ji,t}, \qquad N_{i,t} + r_{i,t} V_{i,t} = L_{i,t}, \qquad Y_t = C_t \tag{30}$$

4. The central bank sets the nominal policy rate subject to the Taylor rule in Equation (26).

3 Results: Inflation and labor market dynamics

In this section, I solve the model nonlinearly under perfect foresight and highlight three key predictions. First, accounting for labor market frictions alters the propagation of sectoral shocks. Second, the model features a nonlinear Phillips curve at the sectoral and aggregate levels that steepens at high levels of labor market tightness. Third, as a result, how important

[10] I also impose that $\sum_{i=1}^J \alpha_{i,t} = 1$ for all t so that $\alpha_{i,t}$ shifts relative preferences across sectors without changing aggregate demand.

each sector is for aggregate inflation changes endogenously with labor market conditions. As a result, monetary policy is less effective at stimulating output when some labor markets are tight. I then show how these three features together imply that a shift in relative demand from one sector to another can generate aggregate inflation, a steepening of the aggregate Phillips curve, and a decline in aggregate matching efficiency—an outward shift in the Beveridge curve.

3.1 Solution Method and Calibration

I solve the model nonlinearly by simulating impulse responses to a range of shocks under perfect foresight using the Newton-Raphson method. That is, I write the equilibrium conditions as a system of nonlinear equations.

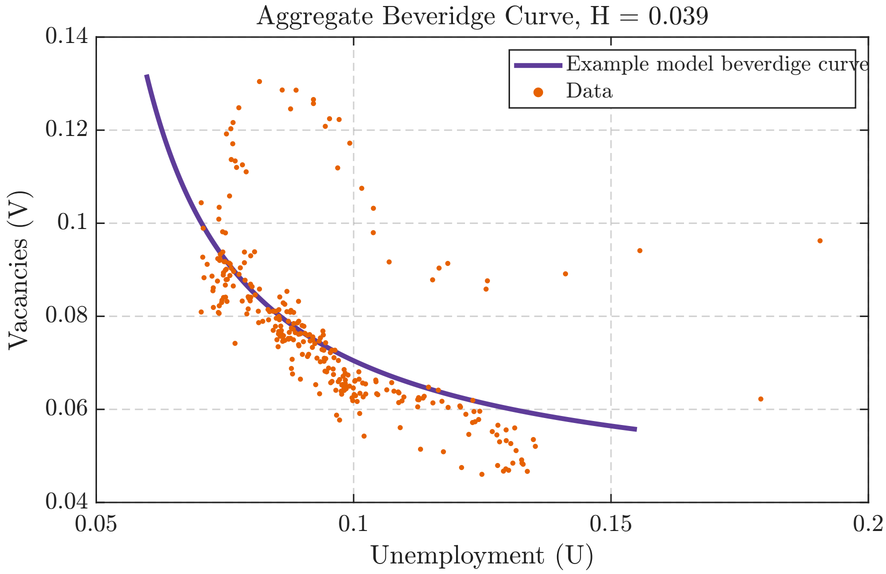
$$F(X) = 0, \tag{31}$$

where X stacks all endogenous variables over the simulation horizon T.[11] I then solve for X such that $F(X) = 0$ holds given the path of exogenous variables, using the steady-state as the initial guess and the Jacobian to update the guess until convergence.

I solve the model nonlinearly for two reasons. First, search-and-matching introduces potentially interesting nonlinear dynamics for large changes in tightness that I want to be able to capture. Second, sector-level shocks can lead to large deviations from steady-state at the sector level, even when aggregate variables remain relatively close to steady-state. The large deviations from steady-state at the sector level necessitate a treatment that accounts for the possible nonlinearities for large shocks. Labor market tightness for instance routinely rises and falls by large amounts over the business cycle, and rose to over 100% above the pre-Covid average in the aftermath of the pandemic.

[11] I use $T = 200$ and find that the results are robust to increasing the horizon T.

Figure 1: Fit of model implied Beveridge curve to JOLTS data (2000–2025)



Note: Orange dots plot monthly data on model consistent measure of aggregate unattached workers ($U+H$ in CPS/JOLTS data) and aggregate vacancies ($U+H$ in JOLTS data). These are the model constent estimates since JOLTs and CPS measures of V and U are vacancies and unemployed worker who still have not matched at the end of the sample period, whereas H is total hires over the entire sample period. The start of sample period U and V in model terms are therefore $U + H$ and $V + H$. The purple line plots an example Beveridge curve implied by the model specification at the average level of hires. In reality, the model beveridge curve is constantly shifting in response to shocks, and is not in fact stable at the drawn curve. The purpose is to demonstrate that the shape of the curve (steeper when labor markets are tight) conforms with the data.

I demonstrate the model's mechanisms in a simulated symetric version of the model where each sector is calibrated to look like an average sector in the U.S. economy. I calibrate the model so that each sector has an intermediate-share of 50%, which roughly matches the intermediates share in the Bureau of Economic Analysis (BEA) input–output (I–O) tables. I assume the share of their own output in intermediates is 70% and the share of the other sector's output in their intermediates is 30%, to reflect the large diagonal elements in the I–O tables.

I set the other parameters of the model using existing estimates from the literature. I set the elasticity of substitution between intermediates and labor (ϵ_y) to 0.2 to match the estimated elasticity of substitution between intermediate inputs in Atalay (2017). I purposefully choose an estimate on the low end of the range of estimates in the litarature because this parameter captures the elasticity of substitution between intermediates and labor in the short run (a period in the model is just one month). I want the model to capture that labor scarcity in a sector can lead to supply constraints because firms find it difficult to substitute between inputs in the short run, a feature highlighted as potentially important in the post-pandemic recovery by Lorenzoni and Werning (2024). I set the sectoral elasticity of substitution between intermediate suppliers (ϵ) to 4.33, which implies a markup of about 30% (R. Hall, 2018). I set ϕ_π, ϕ_y, and ρ_i to 1.39, 1.01, and 0.82, respectively, based on estimates for the Greenspan–Bernanke era in C. Carvalho et al. (2021). I set ψ_p by aggregating the estimates for the frequency of price changes at the sector level in Pasten et al., 2020 using sales shares as weights. I set $\varphi = 3.57$, consistent with Chetty (2012). I set $\psi_L = 31.24$, roughly half the value of a job to the household, to capture the fact that workers face high costs of switching across occupations and sectors (Artuç et al., 2010; Caliendo et al., 2019; Cardoza et al., 2022; Humlum, 2021).

Table 1: Two-symmetric-sectors Calibration

Parameter	Value	Source
ϵ_d	0.60	Atalay (2017)
ϵ_y	0.20	Elasticity of substitution between intermediate inputs, Atalay (2017)
ϵ	4.33	Implied markup of 30%, R. Hall (2018)
ϕ_π	1.39	Estimates for Greenspan–Bernanke Period, C. Carvalho et al. (2021)
ϕ_y	1.01	Estimates for Greenspan–Bernanke Period, C. Carvalho et al. (2021)
ρ_i	0.82	Estimates for Greenspan–Bernanke Period, C. Carvalho et al. (2021)
κ	1	Equal surplus sharing between households and firms
ρ_w	0.8	—
σ	2	—
φ	3.57	Chetty (2012)
η_i	0.93	Estimate using JOLTS (2000-2025)
$\Omega_{d,i}$	0.50	Nominal Consumption Share (Symmetric)
$\Omega_{x,i}$	0.50	Roughly Intermediates Share in BEA I–O tables
Ω_{ii}	0.70	—
Ω_{ij}	0.30	—
$\psi_{p,i}$	60.11	Pasten et al. (2020)
$\psi_{L,i}$	31.24	Half value of a job to household.

Note: Calibrated parameter values in two-sector-symmetric economy. Based on estimates or standard values in the literature.

I assume a matching function of the form $m_{i,t}(U_{i,t}, V_{i,t}) = \left(U_{i,t}^{-\eta_i} + V_{i,t}^{-\eta_i}\right)^{-\frac{1}{\eta_i}}$, which satisfies the properties outline in Section 2. Crucially, it ensures that $H_{i,t} \leq \min\{U_{i,t}, V_{i,t}\}$ while remaining differentiable throughout. I estimate η_i by nonlinear least squares using aggregate data on vacancies, unemployment, and hires from the Job Openings and Labor Turnover Survey (JOLTS) from 2000–2025. Figure 1 demonstrates that the model-implied Beveridge curve at the average hiring rate is a good fit to the JOLTS data. In particular, the functional form I assume can capture both the flatter portion of the Beveridge curve when unemployment is high, and the steepening at low levels of unemployment. I summarize the full parametrization in Table 1.

3.2 The Propagation of Shocks Through the Labor Market

I begin by showing how the presence of labor market frictions alters the propagation of sectoral shocks via a new labor channel. Higher labor demand in one sector leads to a reduction in labor supply elsewhere, raising hiring costs across the network. As a result, positive demand shocks in one sector can effectively create negative supply shocks in other sectors. This is the converse of, but closely related to, the finding in Guerrieri et al. (2022) that negative supply shocks in one sector lead to negative demand shocks in other sectors.

Consider, for instance, a permanent 1% increase in the household consumption preference for goods in sector i, captured by a permanent increase in $\varepsilon^{\alpha}_{i,t}$. The shock raises $\alpha_{i,t}$, and therefore, as Figure 2a shows, household consumption of sector i's output, $C_{i,t}$.

$$C_{i,t} = \alpha_{i,t} \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon_d} C_t, \qquad\qquad C_{j,t} = \alpha_{j,t} \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon_d} C_t$$

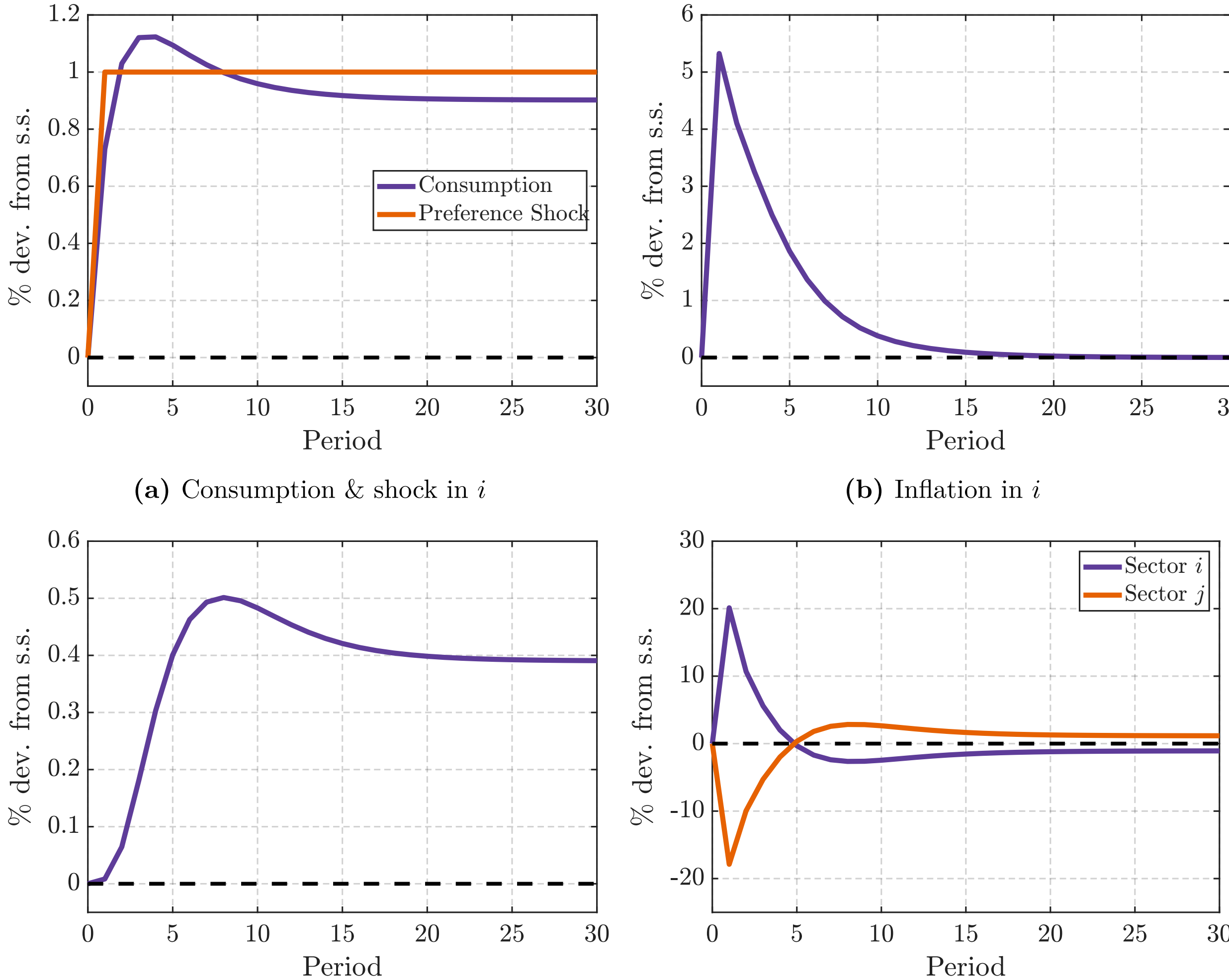
Because $\sum_{j=1}^{J} \alpha_{j,t} = 1$, this implies a decline in $\alpha_{j,t}$ for some $j \neq i$, and therefore a decline in household consumption of sector j's output.[12] For simplicity, I assume in the figures below that the increase in $\alpha_{i,t}$ is offset entirely by an equal decline in $\alpha_{j,t}$ in just one other sector j.

The increase in demand for sector i output leads to an increase in inflation in sector i (see Figure 2b). As is well understood, in any production network economy this increase in prices spills over to other sectors through input–output linkages in production. The increase in inflation in sector i affects the price of intermediate inputs, and therefore marginal costs, in sector j. All else equal, this results in higher sector j inflation than absent the network, raising aggregate inflation.

As firms in sector i increase hiring to meet the increase in consumption demand for their good, both wages and the job-finding rate in sector i rise (see Figure 3b). The job-finding rate increases on impact because the increase in vacancy postings by firms raises labor market tightness, $\theta_{i,t}$, and therefore $q_{i,t}$. Because the households' optimal search strategy implies that the amount of labor searching in sector i rises with wages and the job-finding rate, this in turn, leads to an increase in the labor supply to sector i (See Figure 2c), and, similarly, a decline in labor supply in sector j.

[12] As a result, this shock is pure demand-reallocation shock that does not effect the level of aggregate demand.

Figure 2: Impulse Responses to an increase in $\alpha_{i,t}$



(a) Consumption & shock in i

(b) Inflation in i

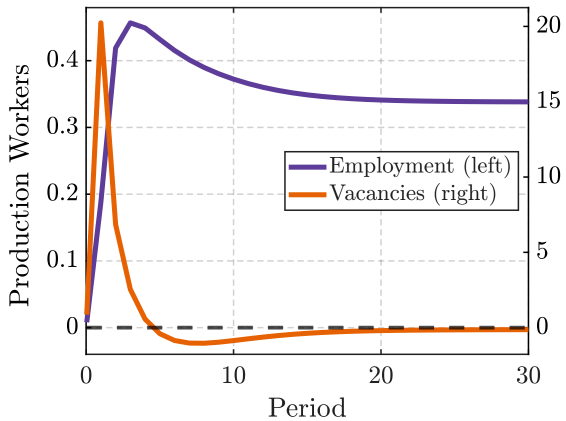
(c) Labor Supply in i

(d) Tightness in i and j

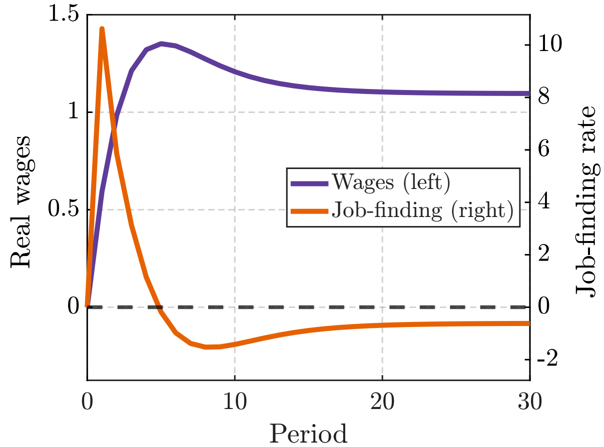
Note: Impulse responses to an increase in relative household preferences for consumption in sector i. Top left panel: Impulse response of real consumption in sector i $C_{i,t}$ (in purple) to the preference shock in sector i $\alpha_{i,t}$ (in orange). Top right panel: Impulse response of inflation in sector i $\Pi_{i,t}$. Bottom left panel: Impulse response of labor supply in sector i $L_{i,t}$. Bottom right panel: Impulse response of tightness in sector i $\theta_{i,t}$ (in purple) and in sector j $\theta_{j,t}$ (in orange). All variables in percentage deviations from steady-state.

The spillovers in labor supply from sector j to sector i increase tightness, and therefore hiring costs and marginal costs in sector j, leading to a new spillover from sector-specific shocks. Indeed, as Figure 2d demonstrates, the labor supply spillovers can be strong enough to eventually fully offset the increase in labor demand in sector i, leading to a decline in tightness in sector i and an increase in tightness in sector j, once the initial spike in labor demand subsides in sector i.

Figure 3: Impulse Responses to an increase in $\alpha_{i,t}$



(a) Employment & vacancies in i



(b) Wages & job-finding rate in i

Note: Impulse responses to an increase in relative household preferences for consumption in sector i. Left panel: Impulse response of employment in sector i $N_{i,t}$ (in purple, left y-axis) and vacancies sector i $V_{i,t}$ (in orange, right y-axis). Right panel: Impulse response of real wages in sector i $\frac{W_{i,t}}{P_t}$ (in purple, left y-axis) and job-finding rate in sector i $F_{i,t}$ (in orange, right y-axis). All variables in percentage deviations from steady-state.

3.3 A nonlinear Sector-Specific Phillips Curve

The presence of labor market frictions leads to a nonlinear Phillips curve that steepens at high labor market tightness. This finding is consistent with the evidence in Gitti (2024) and Benigno and Eggertsson (2023). Benigno and Eggertsson (2023) develop a model to capture this nonlinearity by assuming a kink in the wage-setting process at levels of tightness above 1. I demonstrate that the nonlinearity arises from a standard search-and-matching framework with generalized Nash bargained wages once I solve the model nonlinearly. In addition, in a multi-sector economy, the presence of nonlinearities at the sector level leads to endogenous changes in which sectors are the most important for aggregate inflation as local labor market conditions change.

To build intuition, consider a simplified version of the model outlined above where wages are rigid, $\rho_w = 1$, there are no household labor adjustment costs, $\psi_L = 0$, and firms and households make static labor supply decisions. This last simplification can be rationalized by assuming, as in Benigno and Eggertsson (2023), that all workers separate at the start of each period, before a random fraction $(1 - s_{i,t})$ are reemployed in sector i without needing to go through the matching process. As a result, households and firms no longer account for the dynamic consequences of their labor supply and demand decisions. In this case, to first-order aggregate inflation is given by

$$\pi_t^{agg} = \boldsymbol{\Gamma}_\theta \boldsymbol{\theta}_t + \boldsymbol{\Gamma}_\pi E_t \boldsymbol{\pi}_{t+1} + \upsilon_t. \tag{32}$$

Where $\boldsymbol{\theta}_t$ is a $J \times 1$ vector of sectoral tightness, $\boldsymbol{\pi}_{t+1}$ is a $J \times 1$ vector of sectoral inflation, and υ_t is an endogenous cost push shock as in Rubbo (2023). I derive this first-order approximation of the aggregate New Keynesian Phillips curve by combining Equations (12) and (16). It holds for any constant returns to scale production and matching functions, and does not depend on the specific functional forms assumed above. [13] $\boldsymbol{\Gamma}_\theta$ and $\boldsymbol{\Gamma}_\pi$ are $1 \times J$ coefficient vectors that capture how important each sector is for aggregate inflation. This expression is equivalent to the network Phillips curve derived in Rubbo (2023), adjusted for the presence of labor market frictions.

The coefficient matrices on tightness and expected future inflation are

$$\begin{aligned}
\boldsymbol{\Gamma}_\theta &= \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \boldsymbol{\Omega}_n \boldsymbol{\Gamma}_Q \boldsymbol{\eta} \\
\boldsymbol{\Gamma}_\pi &= \beta \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1}
\end{aligned}$$

where $\boldsymbol{\Omega}_d$ is a $J \times 1$ vector of steady-state nominal consumption shares, $\boldsymbol{\Omega}_x$ is the $J \times J$ input–output matrix capturing the steady-state expenditure shares of each sector i on each intermediate input j, $\boldsymbol{\Omega}_n$ is a $J \times J$ diagonal matrix with the steady-state labor share in each sector on the diagonal. The two new terms relative to a standard production network setup are $\boldsymbol{\eta}$, a $J \times J$ diagonal matrix with the negative of the elasticity of the vacancy-filling rate to changes in tightness along the diagonal, and $\boldsymbol{\Gamma}_Q$, a $J \times J$ matrix capturing the pass-through from changes in the vacancy-filling rate to marginal costs in each sector.

The presence of labor market frictions therefore alters the propagation of sectoral shocks to aggregate inflation in two distinct ways. First, as demonstrated in the previous section, an increase in labor demand in one sector leads to a reduction in labor supply in other sectors, potentially leading to a cascade of labor market tightness throughout the network. As a result, sectors that lead to larger spillovers in tightness to other sectors, and therefore to larger changes in the entire vector of sectoral tightness, $\boldsymbol{\theta}_t$, are more important for aggregate inflation. Second, how changes in tightness pass through to prices depends on the details of

[13] See the appendix for a detailed derivation. The expression for the endogenous cost push shock is

$$\begin{aligned}
\upsilon_t = &\boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{I} + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right] \boldsymbol{p}_{t-1} \\
&+ \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{\Omega}_n \boldsymbol{\Gamma}_Q \boldsymbol{r}_t - \boldsymbol{a}_t\right] \\
&+ \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\mathbf{1}\boldsymbol{\Omega}_d'\right] \boldsymbol{\alpha}_t
\end{aligned}$$

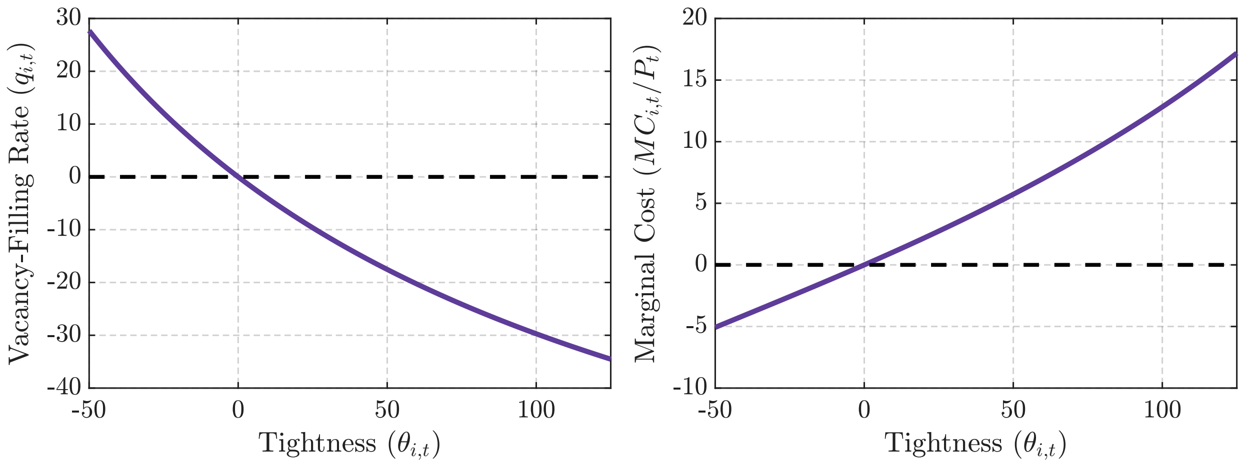
the sector-specific matching process. Labor market conditions in sectors where the vacancy-filling rate changes more in response to changes in tightness, where hiring costs respond more to changes in the vacancy-filling rate, and where marginal costs respond more to changes in labor costs, matter more for aggregate inflation.

To first order, shifting demand into sectors where labor markets are more rigid—where firms have a harder time adjusting their labor input either because increasing vacancy postings leads to fewer additional hires or because the effective hiring costs rise more quickly as tightness increases—leads to an endogenous cost-push shock, triggering aggregate inflation. The intution is similar to Rubbo (2024), who shows that shifting demand into sectors with less elastically supplied inputs leads to higher aggregate inflation. Intuitively, sectors with less elastically supplied inputs have a harder time increasing output in response to a positive demand shock, leading to a smaller increase in quantities and a larger increase in prices when these sectors experience a surge in demand. In Rubbo (2024), all sectors face identically elastic labor supply curves, but vary in terms of their capital and intermediate input usage, leading to differences in the elasticity of input supply curves across sectors. Here, I show that a similar mechanism arises when sectors face frictional labor markets: any variation in how rigid labor markets are across sectors leads to differences in the elasticity of input supply curves, and therefore alters how important each sector is for aggregate inflation. In the appendix, I demonstrate using the BLS's Job Openings and Labor Turnover Survey (JOLTS) data that there is substantial heterogeneity in average vacancy-filling rates across sectors, and therefore in steady-state hiring costs.[14]

In the nonlinear full model, the elasticity of the vacancy-filling rate and the pass-through to hirings costs changes endogenously with local labor market conditions. Figure 4a demonstrates that as tightness in sector i rises, the vacancy-filling rate falls. As a result, firms in sector i need to post more vacancies to achieve the same level of hiring: it gets harder and harder for firms in that sector to hire additional workers. This, in turn, leads to both a direct increase in marginal costs (see Figure 4b), operating both through hiring costs and wages, and reduces the elasticity of the labor supply curve faced by firms in sector i. Both the hiring costs and the elasticity of labor supply change nonlinear as the constraint that $H_{i,t} \leq m_{i,t}(U_{i,t}, V_{i,t})$ approaches, leading to endogenous changes in which sectors are the most important for aggregate inflation.

[14] The average monthly vacancy-filling rate in the finance and insurance industry, for instance, from 2000 to 2025 is about 0.37, while the average vacancy-filling rate in construction over the same horizon is about 0.68. These large differences in average vacancy-filling rates likely reflect differences in how search works across sectors, depending on the skill level of employees and the time each application takes to process.

Figure 4: How Hiring costs and Marginal Costs vary with Tightness



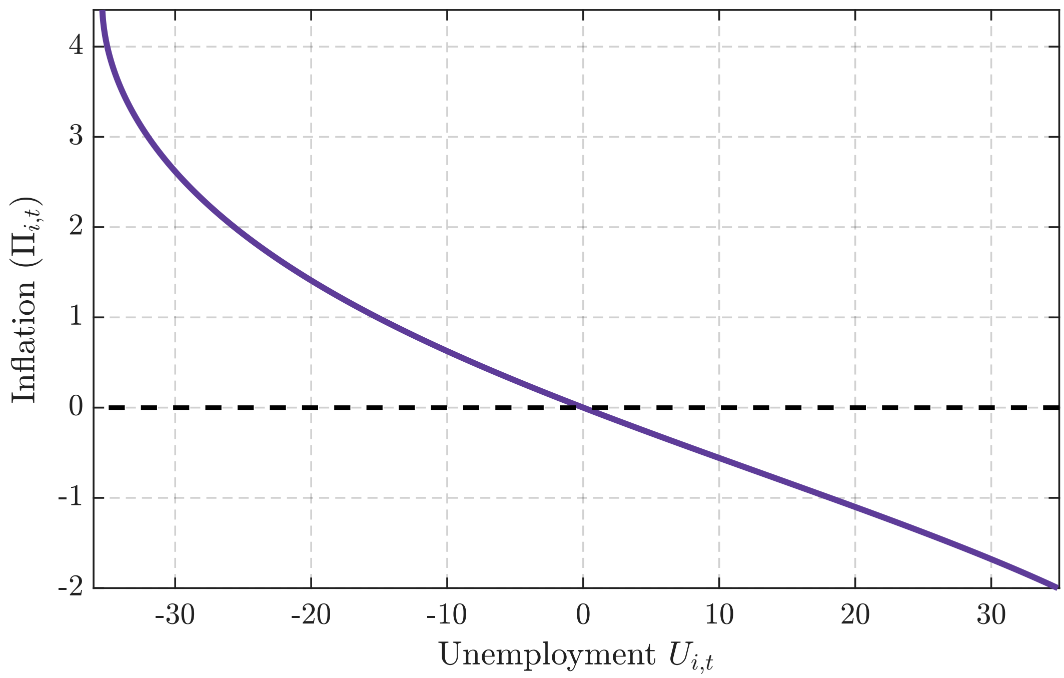
(a) Relationship between $\theta_{i,t}$ and $q_{i,t}$ (b) Relationship between $\theta_{i,t}$ and $MC_{i,t}$

Note: Right panel: Relationship between tightness and vacancy-filling rate in sector i, generated by varying size of demand shock in sector i. Left panel: Relationship between tightness and marginal costs in sector i, generated by varying size of demand shock in sector i. All variables in percentage deviations from steady-state.

As tightness rises, firms become effectively more supply constrained. As a result, they change output by less and inflation by more in response to additional demand or supply shocks. Indeed, as Figure 5 demonstrates, this mechanism results in a nonlinear Phillips curve at the sector level. The solid purple line in Figure 5 plots the relationship between inflation and unemployment in response to sector-specific demand shocks. The inflation-unemployment Phillips curve steepens as unemployment falls precisely because firms become more supply constrained at high levels of tightness, forcing more of the effects of positive demand shocks into prices rather than quantities.

Relatedly, because output becomes more constrained at high levels of tightness, monetary policy has larger effects on inflation and smaller effects on output when tightness is high. To demonstrate this effect, Figure 6a plots $\frac{\partial \Pi}{\partial i^{pol}}(\theta)$, the effect of a change in the nominal policy rate on inflation, by plotting the response of inflation to a 1 percentage point policy rate cut, relative to the effect of a rate cut when tightness is initially at steady-state. Conversely, Figure 6b plots $\frac{\partial Y}{\partial i}(\theta)$, the effect of a change in the nominal policy rate on output, by plotting the response of output to a 1 percentage point policy rate cut, relative to the effect of a rate cut when tightness is initially at steady-state. When tightness in sector i is 125% above steady-state, for instance, monetary policy has nearly 10 times larger effects on inflation in that sector.

Figure 5: Nonlinear sector-specific Phillips curve in Unemployment

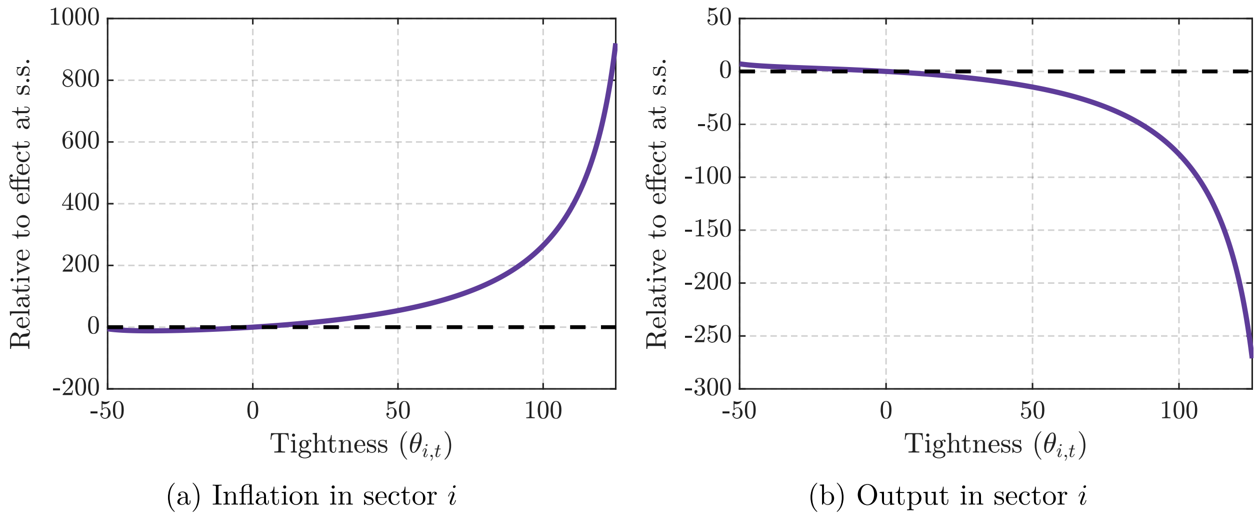


Note: Phillips curve relationship between inflation and unemployment in sector i, generated by varying size of demand shock in sector i. The curve steepens at high levels of tightness, and therefore low levels of unemployment, because firms become more supply constrained as tightness rises.

The nonlinear Phillips curve is consistent with the empirical findings in Gitti (2024) that the Phillips curve is steeper in U.S. in regions with tighter labor markets. In addition, as argued in Benigno and Eggertsson (2023), the presence of a nonlinear Phillips curve can help explain why central banks were surprised by the surge in inflation in 2021. A central bank that assumes a linear Phillips curve underestimates the inflationary pressure in tight labor markets, and will therefore tend to allow inflation to surge more than expected.

However, my findings are distinct in two important ways. First, unlike Benigno and Eggertsson (2023), where the Phillips curve steepens when tightness crosses a certain threshold because of an assumed kink in the wage setting process, I highlight an alternative channel generates a non-linear Phillips curve. The Phillips curve steepens in the standard search-and-matching framework once I solve the model nonlinearly because of nonlinearities in vacancy filling rates and therefore in hiring costs. This suggests that the findings in Benigno and Eggertsson (2023) do not rely on the specific specification for the wage setting and search process, and instead arise more generally in models featuring frictional labor markets.

Figure 6: Effects of monetary policy on output and inflation in sector i



Note: Right panel: Effect of -1% monetary policy shock on inflation in sector i at different levels of tightness in sector i, relative to effect of monetary policy at steady state. Left panel: Effect of -1% monetary policy shock on C_i at different levels of tightness in sector i, relative to effect of monetary policy at steady state.

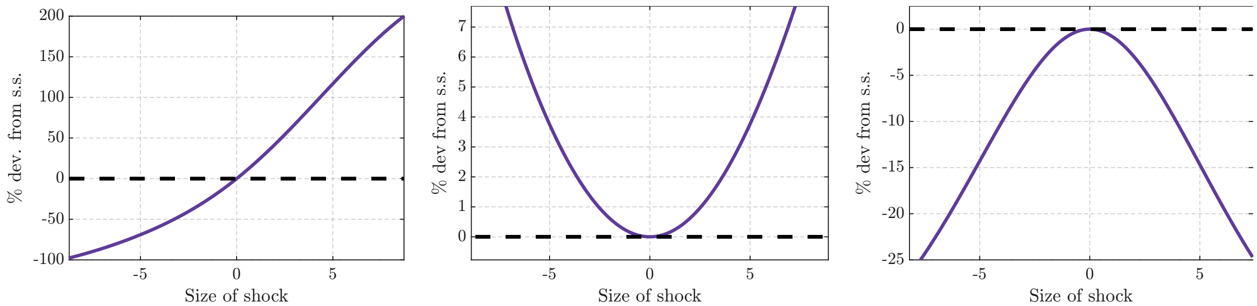
Second, by solving the model nonlinearly instead of assuming a kink, I show that search-and-matching frictions lead to a continuous steepening of the Phillips curve: the curve gets progressively steeper as tightness rises. As a result, assuming a linear Phillips curve becomes a worse approximation as tightness rises substantially above steady-state, as it did in the U.S. in 2021 and 2022. In these circumstances, the monetary authority is especially prone to allowing inflation to surge more than expected if it extrapolates from the relatively flat Phillips curve in normal times.

3.4 Aggregate Effects of Sectoral Demand Shocks

I now demonstrate how a production network, capturing sector-specific shocks, and labor market frictions alter the aggregate effects of demand shocks. In particular, I show that the Phillips curve steepens and monetary policy becomes less effective at stimulating output when just a few sectors experience large increases in tightness.

As demonstrated above, a shock to relative demand in sector i, $\alpha_{i,t}$, increases demand in sector i and reduces demand in sector j. As the size of the relative demand shock increases, so does the increase in labor demand and tightness in sector i. For instance, in the baseline calibration, an 8% increase in $\alpha_{i,t}$ triggers about a 180% increase in tightness.

Figure 7: Aggregate effects of sectoral demand shocks



(a) Tightness in i (b) Aggregate inflation (c) Matching efficiency

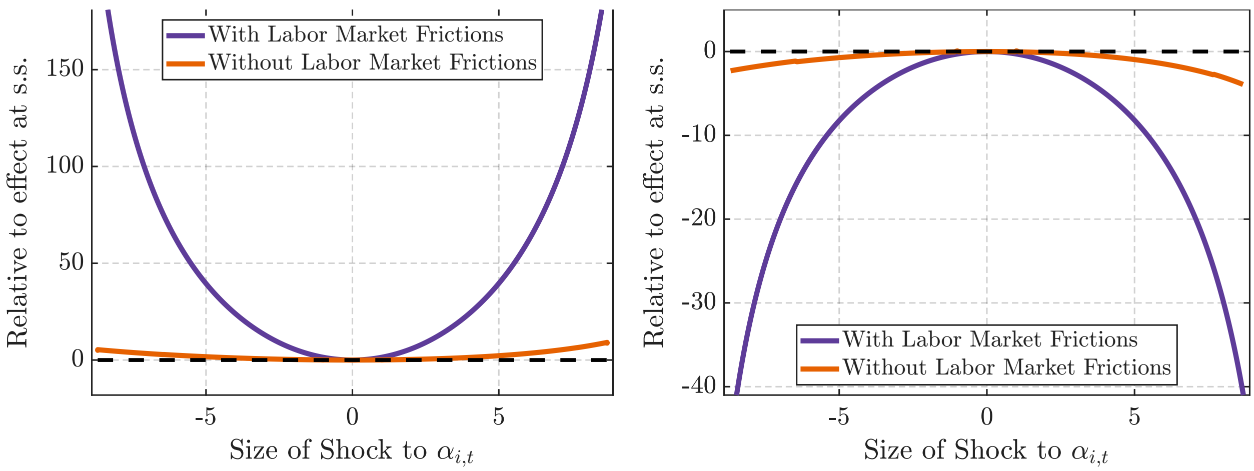
Note: All panels plot response to $\alpha_{i,t}$ shocks of various sizes. Right panel: Effect of shifting consumer preferences on tightness in sector i. Middle panel: Effect of shifting consumer preferences on aggregate inflation. Left panel: Effect of shifting consumer preferences on measured aggregate matching efficiency $\frac{H_t}{\sqrt{U_t V_t}}$.

As tightness rises in sector i, it gets progressively more difficult to hire, leading to both an increase in inflation and a decrease in aggregate matching efficiency, a measure of how many hires are generated for a given number of unemployed workers and vacancies. Aggregate inflation rises because the rise in tightness in sector i causes firms in that sector to become relatively more supply constrained. Firms in sector j, on the other hand, experience a decline in tightness. As a result, prices rise by more in sector i than they fall in sector j. In addition, the rise in $\alpha_{i,t}$ gives sector i a larger weight in the aggregate price index. These two factors combine to produce a significant rise in aggregate inflation: in the illustrative symmetric calibration an 8% increase $\alpha_{i,t}$, in line with the shift from goods to services following the Covid pandemic, generates about a 7% increase in aggregate inflation.

Figure 7c demonstrates that a shift in demand from one sector to the other also results in a decline in the aggregate matching efficiency, which I define as $\phi_t^{agg} = \frac{H_t^{agg}}{\sqrt{U_t^{agg} V_t^{agg}}}$. Aggregate matching efficiency falls because labor demand is concentrated precisely in the sector where it is hardest to hire. It gets harder to hire in sector i for two reasons. First, as in Şahin et al. (2014), sectoral shocks can generate mismatch between where vacancy postings are and where unattached workers search. This mismatch is exacerbated when labor adjustment costs are larger. Second, the number of hires generated per additional vacancy declines endogenously as the constraint $H_{i,t} \leq m_{i,t}(U_{i,t}, V_{i,t})$ approaches. That is, the natural constraint that firms cannot create additional workers out of thin air by posting a larger number of vacancies, naturally implies that a large increase in vacancies in just one sectoral labor market leads to a decline in the number of hires relative to what one would expect given the number of aggregate vacancies and unemployed workers. An 8% increase $\alpha_{i,t}$ generates about a

25% decrease in measured aggregate matching efficiency, despite no change in underlying matching efficiency at the sector level.

Figure 8: Aggregate effects of monetary policy on output and inflation



(a) Aggregate inflation (b) Aggregate output

Note: Right panel: Effect of -1% monetary policy shock on aggregate inflation relative to effect at steady state. Purple line is effect in model with labor market frictions, orange line is effect in model without labor market frictions. Left panel: Effect of -1% monetary policy shock on aggregate output relative to effect at steady state. Purple line is effect in model with labor market frictions, orange line is effect in model without labor market frictions.

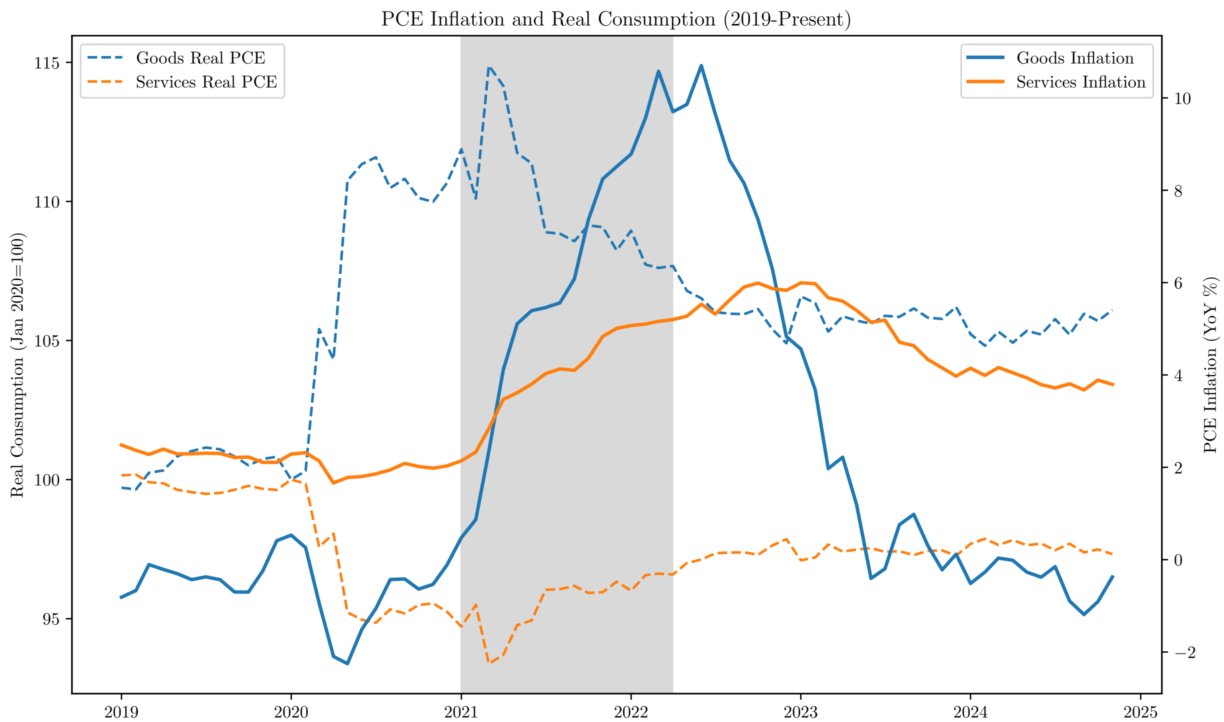
As figure 8 demonstrates, in the presence of labor market frictions, monetary policy has smaller effects on output and larger effects on inflation when just a few sectors experience larger increases in tightness. The solid purple line in the top panel plots the effect of a 1 percentage point cut in the nominal policy rate on aggregate inflation, relative to the effect of a rate cut when $\alpha_{i,t}$ is at steady-state. Conversely, the solid black line in the bottom panel plots the effect of a 1 percentage point cut in the nominal policy rate on aggregate real output, relative to the effect of a rate cut when $\alpha_{i,t}$ is at steady-state. Following an 8% increase in $\alpha_{i,t}$, monetary policy has about 160% larger effects on aggregate inflation than it does at the steady state.

As tightness rises in sector i, and firms in that sector become more supply constrained, they become progressively less able to increase output in response to an additional positive aggregate demand shock, and therefore must raise prices by more instead. As a result, it is enough for some sectors to be supply constrained for the aggregate Phillips curve to steepen and for monetary policy to become less effective at stimulating output. This suggests that central banks must account for sectoral labor market conditions when predicting the effects of their monetary policy actions. When there are exceptional labor market conditions in just a few sectors in the economy, acting as if the aggregate Phillips curve remains flat may lead the central bank to underestimate the inflationary consequences of its actions. Conversely, in an uneven downturn or recovery, monetary policy may be a less effective tool for stimulating aggregate output since constrained sectors respond less to additional demand stimulus.

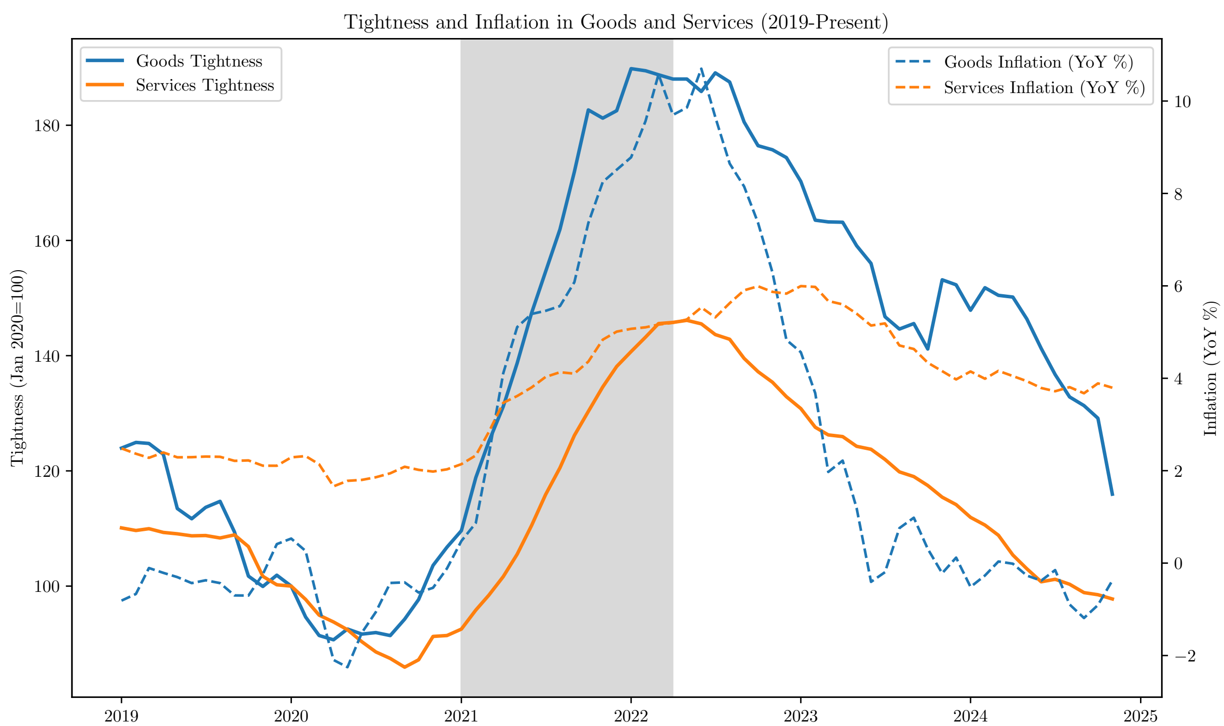
The last prediction is an important insight for when monetary policy is and is not likely to be an effective stabilization tool. For instance, consider two shocks that lead to an identical increase in aggregate demand. Suppose the first shock is a true aggregate demand shock affecting all sectors equally, while the second raises demand in some sectors by significantly more than others. The model's predictions for the effects of sectoral demand shocks suggest that monetary policy will have a larger effect on output in the case of the pure aggregate demand shock, where no sector experiences a spike in tightness and becomes supply constrained, than in the case of an uneven sectoral demand shock. Similarly, if sectoral shocks lead an overall decline in demand, triggered by large declines in demand in some sectors and an increase in demand in others, monetary policy may be unable to stimulate aggregate output, and stimulative monetary policy will instead have larger inflationary effects.

4 Quantitative Application to Post-Pandemic Inflation and Labor Market Dynamics

I now turn to a quantitative application of the model to the recent inflation and labor market dynamics in the United States following the COVID-19 pandemic. I begin by documenting sectoral differences in inflation and labor market dynamics during the recovery. I then demonstrate that a shock to relative demand for goods over services can partially account for both inflation and matching efficiency in an estimated two-sector goods-services version of the model.



(a) Real PCE and PCE Inflation.



(b) Labor Market Tightness and PCE Inflation.

Figure 9: Left Axis (both subplots): Dashed lines are changes in real PCE relative to January 2020, by major expenditure category: goods (blue) and services (orange). Right Axis: Solid lines are year-over-year changes in the PCE price index, by major category: goods (blue) and services (orange). The gray shaded area indicates the period from the start of the inflation surge in early 2021 to the first Federal Reserve rate hike in March 2022.

4.1 Inflation and Labor Market Dynamics in Goods and Services

From 2021 to 2024, the United States experienced two phenomena unprecedented since at least the 1980s: (1) inflation surged to levels not seen since the Volcker disinflation, and (2) labor market tightness reached historic highs, with job openings per unemployed worker at levels unseen since the 1960s. I begin by documenting sectoral differences in these dynamics during the pandemic recovery, focusing on the responses in the goods and services sectors. Real consumption, labor market tightness, and new-hire wages rose more in goods than in services. As a result, inflation rose far more in goods as well. These patterns suggest a link between inflation and labor market dynamics at the sectoral level.

In the figures above, I use the PCE goods and services price and quantity indices from the Bureau of Economic Analysis. I use vacancy data from the Bureau of Labor Statistics' Job Openings and Labor Turnover Survey (JOLTS) roughly at the 2-digit NAICS level, and aggregate to the two-sector goods-services breakdown available from the PCE. I use unemployment data from the Current Population Survey (CPS).

As Figure 9a shows, the surge in U.S. inflation, which began in early 2021 and peaked near 7.2 percent year-over-year in June 2022, was uneven across the economy. Inflation in goods jumped from about 0 to over 10 percent by mid-2022, while services inflation rose more slowly, peaked around 6 percent, and stayed persistently elevated near 4 percent into 2024. This pattern suggests that a mechanism where inflation in one sector, triggered by either sectoral supply or demand shocks, gradually propagates to other sectors through the production network as in Minton and Wheaton (2023).

This initial surge followed a sharp rise in demand for goods, as consumers shifted toward lower-contact spending. The goods share of real PCE rose from 32 to 37 percent early in the pandemic—a 15 percent jump—and has remained elevated since. Alongside this shift, supply chain disruptions and the war in Ukraine created supply shocks that hit goods disproportionately.[15] Recent work emphasizes the importance of these sector-specific demand and supply disturbances for the inflation surge (Amiti et al., 2023; Comin et al., 2023; Di Giovanni et al., 2023; di Giovanni et al., 2023; Ferrante et al., 2023; Guerrieri et al., 2022; Lorenzoni & Werning, 2024; Rubbo, 2024).

Figure 9b shows that inflation rose alongside labor market tightness, measured as the sector-specific ratio of job vacancies to total searchers. I calculate searchers using CPS microdata on unemployment-to-employment (UE), nonparticipation-to-employment (NE), and employment-to-employment (EE) transitions, adjusted following Fujita et al. (2024).

[15] For example, the New York Fed's Global Supply Chain Pressure Index peaked at 4.5 standard deviations above average in December 2021, the highest on record. The ISM Supplier Deliveries Index reached a record 78.8 in May 2021.

Assuming random matching and sector-specific job search, total searchers in sector i are:

$$TS_{i,t} = U_{i,t} + \frac{\rho_{i,t}^{NE}}{\rho_{i,t}^{UE}}\frac{U_{i,t}}{U_t}E_t + \frac{\rho_{i,t}^{NE}}{\rho_{i,t}^{UE}}\frac{U_{i,t}}{U_t}N_t \tag{33}$$

where $U_{i,t}$ is the number of unemployed workers most recently employed in sector i, U_t is the total number of unemployed workers, E_t is the total number of employed workers, N_t is the total number of people not currently in the labor force, $\rho_{i,t}^{NE} = \frac{H_{i,t}^N}{N_t}$ is the NE rate into sector i ($H_{i,t}^N$ is the number of hires in sector i from nonparticipation), $\rho_{i,t}^{UE} = \frac{H_{i,t}^E}{U_t}$ is the UE rate into sector i, and $\rho_{i,t}^{EE} = \frac{H_{i,t}^E}{E_t}$ is the EE rate into sector i.

I use this broader measure rather than the conventional unemployment-based one for three reasons. First, recent work highlights the role of EE transitions in post-pandemic labor markets (Autor et al., 2023; Bagga et al., 2025; Barlevy et al., 2023; Faccini & Melosi, 2025; Moscarini & Postel-Vinay, 2023). Second, Barnichon and Shapiro (2024) show that tightness based on total searchers forecasts inflation better than unemployment alone. Third, the measure aligns with the model introduced above, in which households allocate members to search across labor markets, and some unattached members find a new job within the same period, never showing up in end-of-period unemployment numbers.

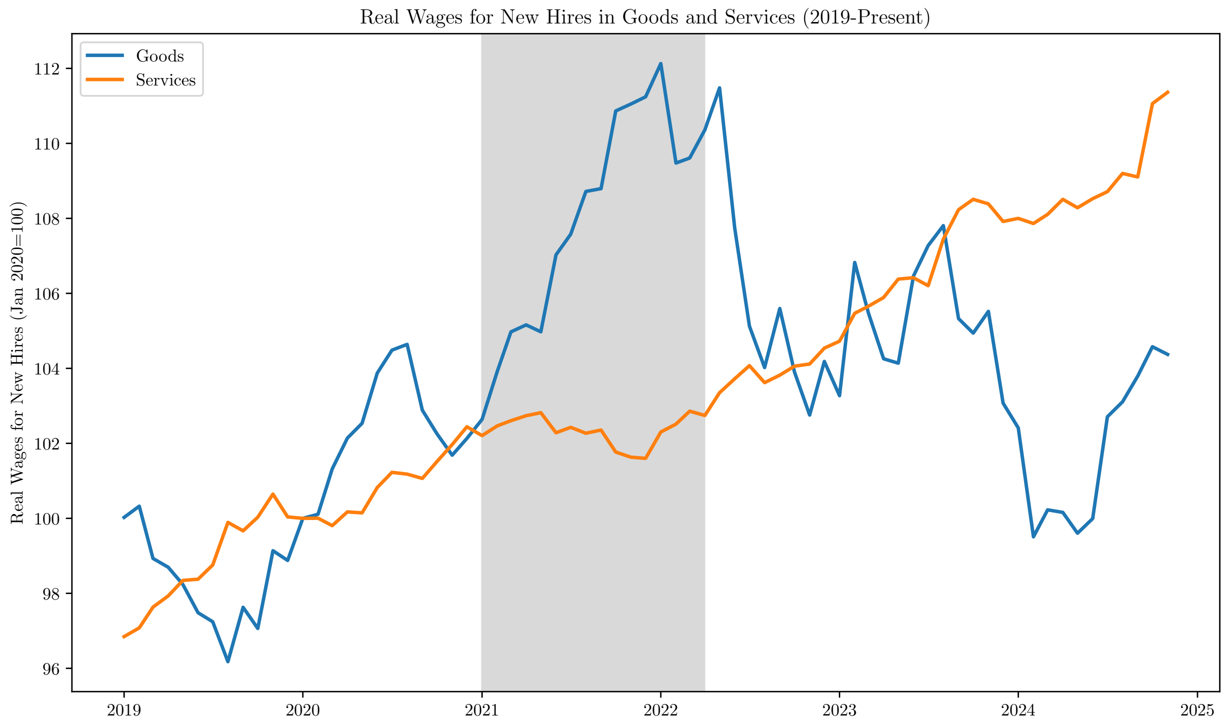
The goods-sector labor market was about 80 percent tighter in early 2022 than in January 2020, when conditions were already historically tight.[16] Services were about 40 percent tighter. Goods-sector tightness closely tracks goods inflation, underscoring the potential importance of accounting for labor market dynamics, even in a multi-sector setting.

Similarly, Figure 10b shows that matching efficiency—the number of hires per vacancy and searcher—declined sharply, and by a comparable amount to the Great Recession. Figure 10b plots the residuals from a regression based on a Cobb-Douglas matching function,

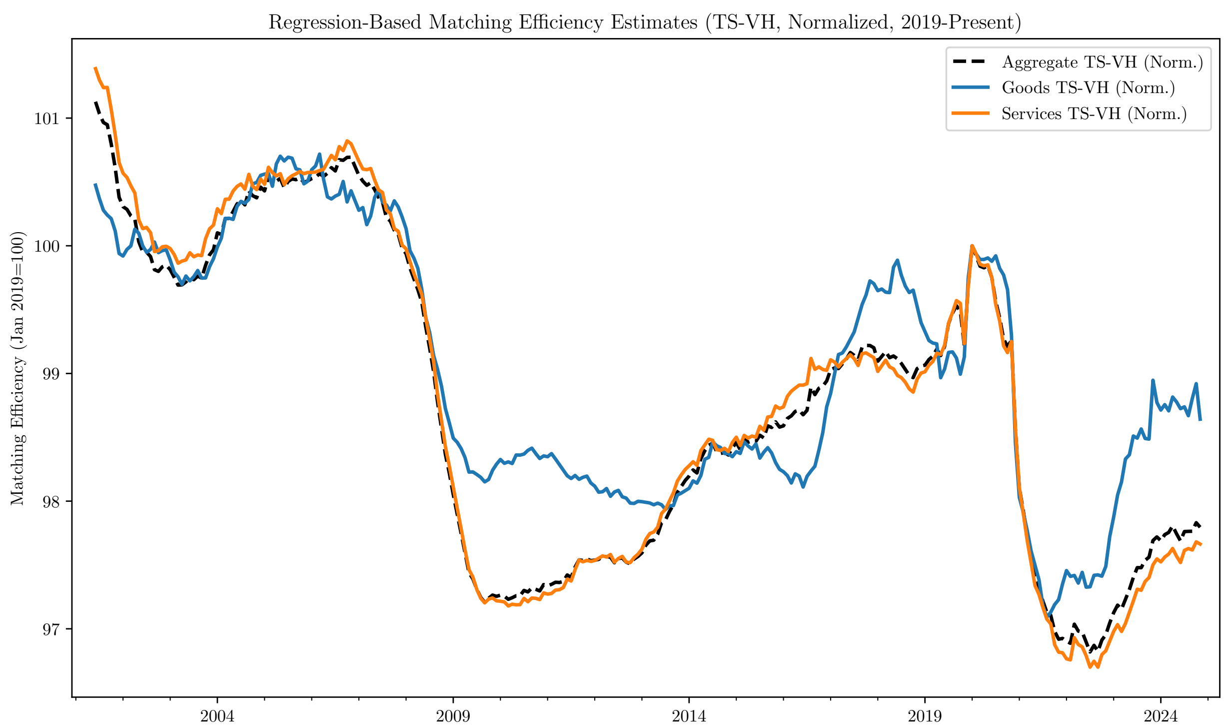
$$H_{i,t} = \phi_{i,t} TS_{i,t}^{\eta} V_{i,t}^{1-\eta} \tag{34}$$

where η is the elasticity of the matching process to total searchers and $\phi_{i,t}$ is the matching efficiency.

[16] I show in Appendix C that the same pattern holds when using the conventional V/U tightness measure. Tightness moves more, though, when using V/U, and by this metric the goods-sector labor market was about 150 percent tighter in early 2022 than in January 2020.



(a) New-Hire Wages.



(b) Matching Efficiency

Figure 10: Top: Real wages for newly hired workers in the goods (blue) and services (orange) sectors, relative to January 2020. Bottom: Matching efficiency in the goods (blue) and services (orange) sectors, relative to January 2020. The gray shaded area indicates the period from the start of the inflation surge in early 2021, to the first Federal Reserve rate hike in March 2022.

I run the following regression separately for each sector i and for the aggregate:

$$\log\left(\frac{H_{i,t}}{TS_{i,t}}\right) = \log\phi + (1-\eta)\log\theta_{i,t} + \epsilon_{i,t} \tag{35}$$

where $\log\phi + \epsilon_{i,t} = \log\phi_{i,t}$ is the log matching efficiency estimated in the data. The decline was larger in services, consistent with goods-sector demand making it harder for services to hire. My model links this fall in matching efficiency, which shifts the Beveridge curve, to higher firm marginal costs and inflation.

Despite the historic rise in labor market tightness and the substantial decline in matching efficiency, a common objection to labor-based explanations is that real wages fell as prices rose. Indeed, aggregate real wages declined in both goods and services. But Figure 10a shows that real wages for new hires—relevant for marginal costs—rose sharply just as goods inflation accelerated. Because services dominate employment, the aggregate masks these sectoral patterns. As the model demonstrates, even without wage growth, rising tightness and falling matching efficiency raise costs by increasing firms' hiring costs.

Rising tightness also reduces the effective elasticity of labor supply to firms, and especially so in the sector experiencing a larger rise in tightness. Cross-sectoral shifts in tightness driven by changes in relative demand can therefore lead to an endogenous decrease in the input elasticity of the goods sector, exerting upward pressure on prices even absent a wage response. Taken together, these facts point to both sectoral heterogeneity and labor market frictions as central to the pandemic recovery, consistent with the model outlined above.

4.2 Can Demand Shocks Explain the Joint Dynamics of Inflation and the Labor Market?

As I demonstrate above, the model's predictions are broadly consistent with the experience during the post-pandemic recovery. In this section, I use Bayesian methods to estimate the parameters of the shock processes in a linearized version of the model on data from 2000 to 2019, a period when shocks were small relative to the COVID-19 period and where the linear approximation is therefore more likely to be accurate. I then use these estimated parameters to calibrate the fully nonlinear model to assess the impacts of different shocks during the post-pandemic recovery.

The first three columns of Table 2 report the prior mean, variance, and distribution for each of the estimated parameters. The last column reports the mode from 3 million draws from the posterior generated with a standard Random-Walk Metropolis–Hastings algorithm. The priors are taken from the literature and are listed in Table 1. I make two minor alteration to the model from above: In addition to the labor reallocation costs on the household side, I allow for a vacancy posting adjustment cost on the firm side to help generate slightly smoother paths for vacancy postings, and partial indexation by firms to past inflation to match smoother movements in inflation, as seen in the data.[17]

I then ask whether the model can quantitatively account for changes in inflation and labor market conditions with shocks to consumer preferences for relative consumption and then with a combination of preference shocks, aggregate demand shocks, and a positive shock to the separation rate. I find that relative preference shocks can broadly match the movements of wages, tightness in the goods sector, and aggregate matching efficiency, but cannot, on their own, account for changes in inflation or tightness in services. Exogenous increases in the separation rate lead to broad-based increases in tightness, but have relatively limited effects on wages, inflation, and matching efficiency.[18] The model cannot match the persistence of inflation without a persistent positive aggregate demand shock, suggesting that all three played an important role in generating the post-pandemic inflation surge.

Table 2: Prior Distributions and Estimated Posterior Parameter Values

Parameter	Mean	Variance	Distribution	Mode
ϕ_π	1.39	0.30	Truncated Normal	1.1456
ϕ_y	1.01	0.10	Gamma	1.1441
ρ_i	0.82	0.15	Beta	0.6736
κ	0.50	0.10	Gamma	0.7232
σ	2.00	0.50	Gamma	1.6686
φ	3.57	0.50	Gamma	3.6303
$\psi_{p,G}$	57.13	10.00	Gamma	48.0468
$\psi_{p,S}$	63.10	10.00	Gamma	60.1747
ψ_L	36.17	10.00	Gamma	38.0123
$\psi_{v,G}$	0.52	0.25	Gamma	0.5245
$\psi_{v,S}$	0.52	0.25	Gamma	0.1917
ρ_w	0.80	0.15	Beta	0.5663

Note: The left panel shows prior distributions taken from the literature; see Table 1.

I calibrate the remaining parameters of the model to match the real consumption share, labor share, and input–output structure of the goods and services sectors. I report the

[17]For details, see the Appendix. I am also planning on adding some price indexation to match the smoother path of inflation as well.

[18]Afrouzi et al. (2024) provide one possible rationale for the observed increase in separation rates, positing that workers are more likely to engage in on-the-job search and accept outside offers when inflation erodes the value of their existing wages. Bagga et al. (2025) provide an alternative explanation based on shifting preferences for job amenities during the pandemic.

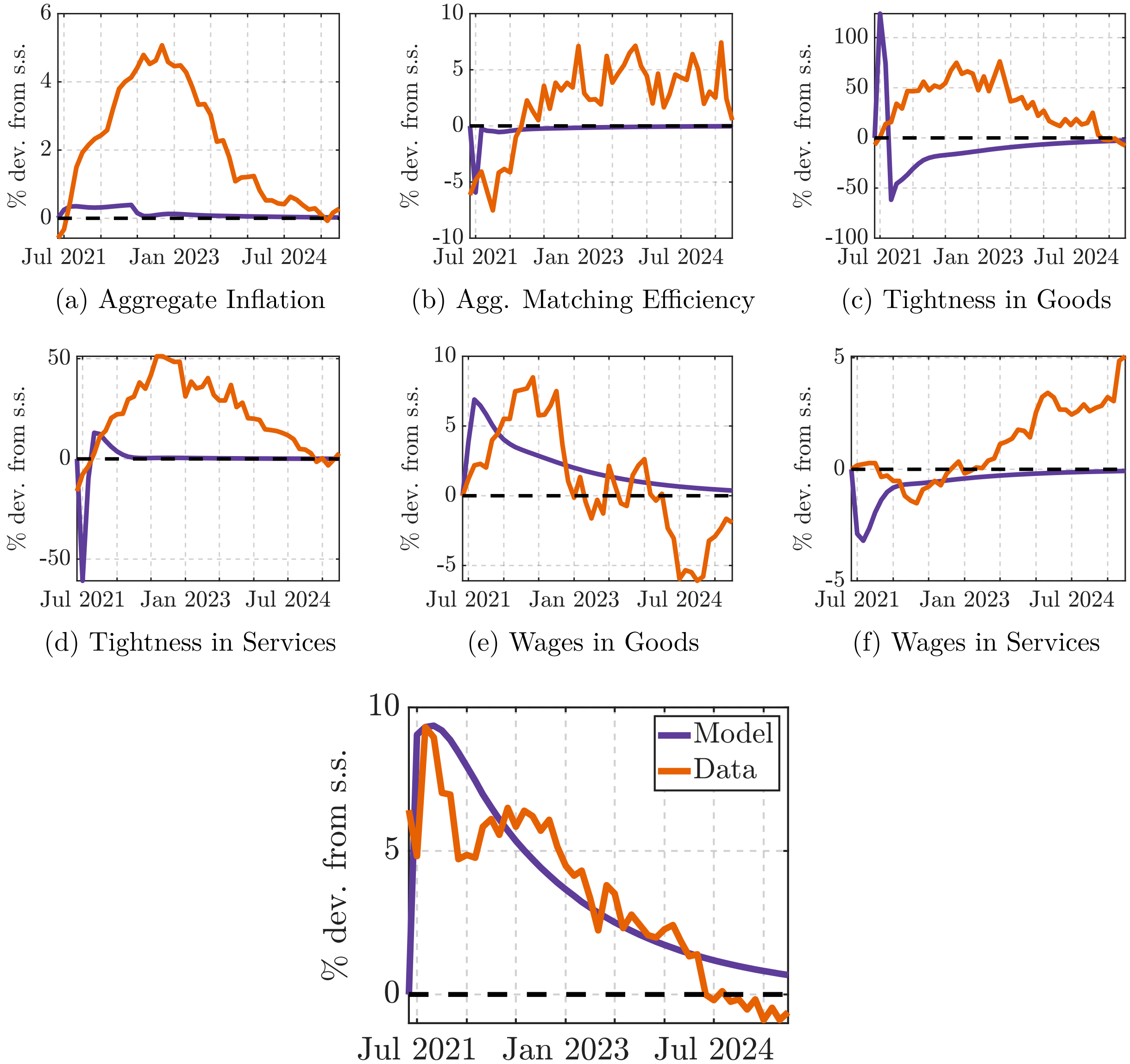
calibrated parameter values in Table 3. I set the values of ϵ_d, ϵ_y, and ϵ to those reported in Table 1.

Table 3: Sectoral Shares and Elasticity Parameters

Category	**Goods**	**Services**
Consumption Share	0.3338	0.6662
Intermediates Share (Total)	0.6210	0.3921
Intermediates Share (Goods)	0.4171	0.0684
Intermediates Share (Services)	0.2039	0.3237

Notes: Goods–services sectoral consumption shares, and intermediate shares, calculated from PCE and BEA input–output tables.

Figure 11: Shock to Consumption Preferences Only



(a) Aggregate Inflation

(b) Agg. Matching Efficiency

(c) Tightness in Goods

(d) Tightness in Services

(e) Wages in Goods

(f) Wages in Services

(g) Nominal Consumption Share in Goods

Note: All panels plot model predicted responses to persistence shock to relative consumption preferences for goods (in purple) and detrended and demeaned data (in orange).

4.2.1 Response of Inflation, Real Wages, and Aggregate Matching Efficiency to Consumption Preference Shocks

I start by introducing a shock to the relative demand for goods over services, $\alpha_{G,t}$, calibrated to increase the real consumption share in goods post-pandemic. I set the persistence of this shock to 0.94 to roughly match the path of the real consumption share in goods from 2021 to the end of 2024. I then solve for the nonlinear impulse responses to this shock under perfect foresight as described in the previous section. Figure 11 reports the impulse responses of key variables to this shock.

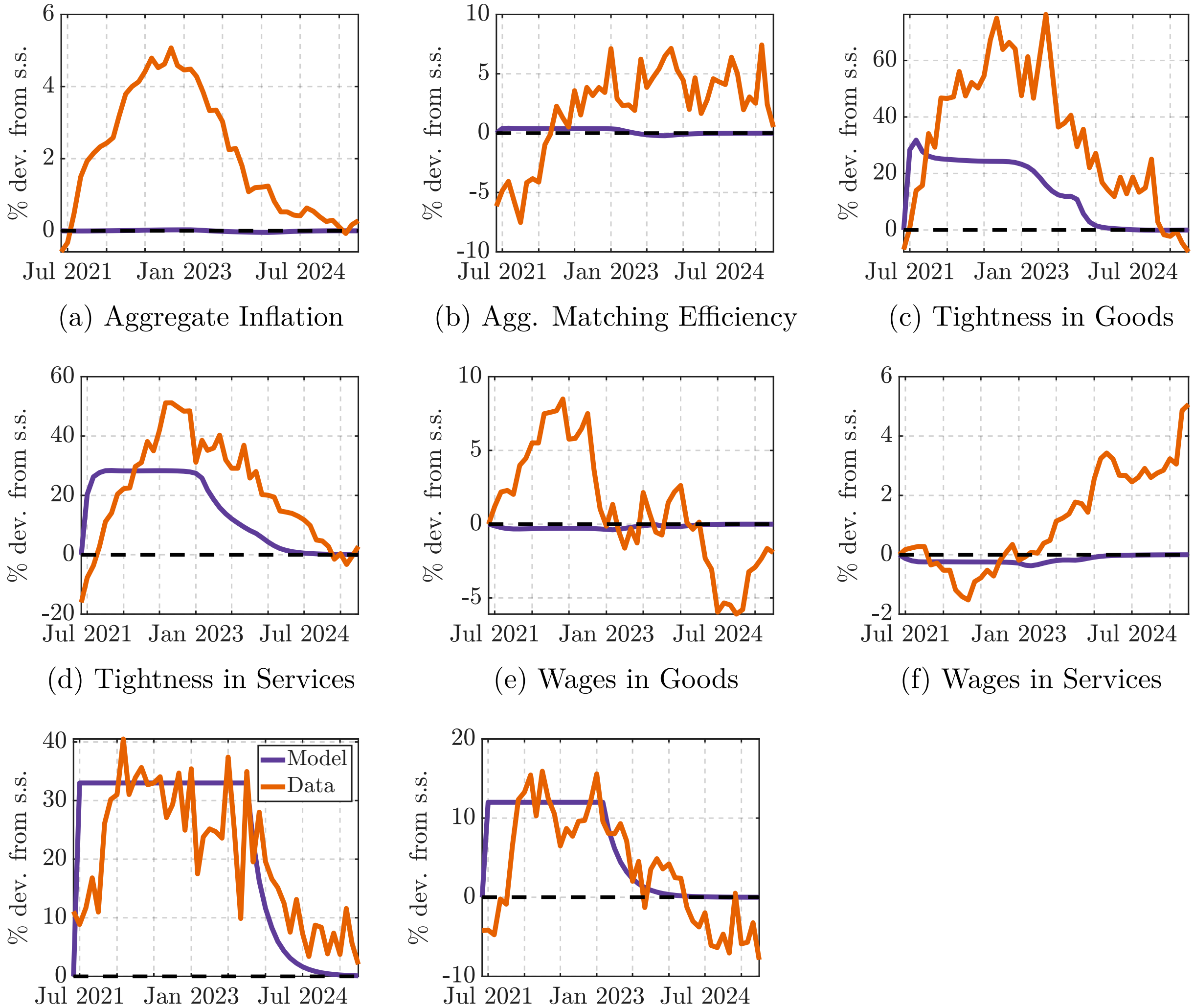
A shock to the relative demand for goods over services generates a spike in inflation and a drop in aggregate matching efficiency, a rise in real wages for new hires in goods, and a decline for new-hire wages in services that are qualitatively in line with the data. Unsurprisingly, given the many additional shocks hitting the economy in the post-COVID period, the relative demand shock alone only explains a small portion of the inflation surge. It does, however, account for a significant portion of the decline in the aggregate matching efficiency, and can therefore explain at least some of the joint behavior of the Beveridge curve and Phillips curve during the recovery. In addition, while the relative demand shock does generate an increase in tightness in the goods sector, it generates a counterfactual decline in tightness in services, absent an additional aggregate demand shock.

4.2.2 Response to a Shock to the Separation Rate

Next, I introduce sector-specific shocks to the separation rates. These shocks capture the increase in the separation rate observed in the data, and which several recent papers tie to the unusual behavior of the Beveridge curve during the COVID recovery (Afrouzi et al., 2024; Bagga et al., 2025). I calibrate the shocks to generate a roughly 35 percent increase in the separation rate in the goods sector and a 12 percent increase in the separation rate in the service sector, in line with the deviations from steady-state in the data for these two separation rates.

As Figure 12 shows, the increase in the separation rate does raise tightness in both the goods and services sectors as firms post additional vacancies to replace the additional unattached workers. An increase in the separation rate alone, though, does not generate much movement in inflation, real wages, or matching efficiency.

Figure 12: Shock to Separation Rates Only



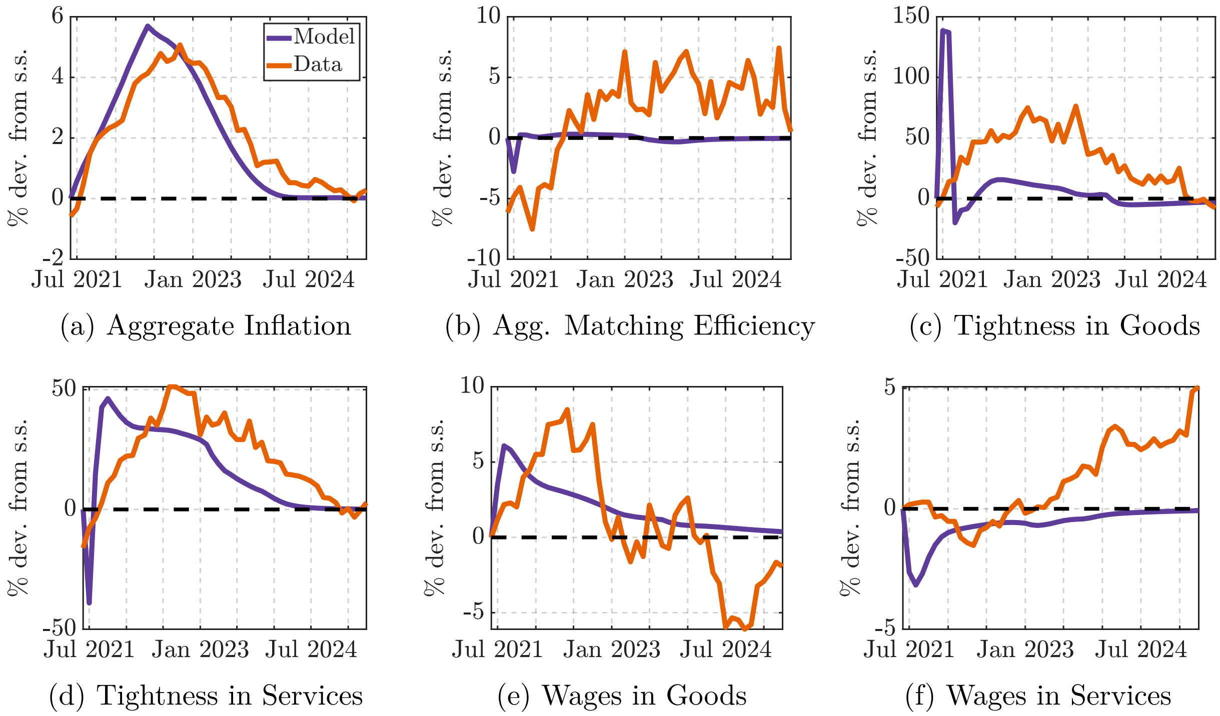
(a) Aggregate Inflation (b) Agg. Matching Efficiency (c) Tightness in Goods
(d) Tightness in Services (e) Wages in Goods (f) Wages in Services
(g) Separations in Goods (h) Separations in Services

Note: All panels plot model predicted responses to persistence shock to separation rates in goods and service sectors (in purple) and detrended and demeaned data (in orange).

4.2.3 Adding an Aggregate Demand Shock

To match the rise in inflation, I introduce an additional aggregate demand shock, a persistent 50-basis-point stimulative monetary-policy shock that lasts for two years from 2021, along with the sectoral demand shock and the separation shock from above. Figure 13 shows the impulse responses to this combination of shocks. The combination of shocks can broadly match the dynamics of inflation, real wages, tightness in both sectors, and aggregate matching efficiency observed in the data during the post-pandemic recovery.

Figure 13: Combination of Shocks to Consumption Preferences, Separation Rates, and Aggregate Demand



(a) Aggregate Inflation

(b) Agg. Matching Efficiency

(c) Tightness in Goods

(d) Tightness in Services

(e) Wages in Goods

(f) Wages in Services

Note: All panels plot model predicted responses to persistence shock to separation rates in goods and service sectors (in purple) and detrended and demeaned data (in orange).

5 Conclusion

The recent post-pandemic inflation surge highlights the potential importance of both sector-specific shocks and labor market frictions for aggregate inflation. Sector-specific shocks that propagate through the production network, can generate large aggregate inflationary pressures, particularly if they hit sectors with inelastic input supply curves; such sectors struggle to expand output and instead raise prices. I show how labor market frictions at the sector level endogenously change the elasticity of the input supply curves faced by sectors. Sectors with tight labor markets find it harder to hire and thus to adjust output when additional shocks hit. As a result, labor market frictions lead to a nonlinear Phillips curve, even if only a few sectors are constrained, making monetary policy less effective at stabilizing output and more effective at combating inflation.

Consequently, uneven demand and supply across the economy can generate aggregate inflation, a decline in aggregate matching efficiency, and a steeper aggregate Phillips curve qualitatively consistent with observed post-pandemic dynamics. In addition, the model predicts that monetary policy, while a useful stabilization tool in normal times, particularly in the face of aggregate disturbances that affect all sectors relatively equally, may be less effective when the economy experiences large shocks or an uneven distribution of demand across sectors. In these circumstances, introducing sector-specific wage subsidies to ease the transition of workers across sectors may be effective at alleviating and equalizing tightness across the economy. The optimal policy response to sector-specific shocks in light of labor market frictions is an interesting avenue for future work.

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A All Equilibrium Conditions

The following conditions hold for each sector $i \in J$:

$$Y_{i,t} = A_{i,t} \left(\omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t}^{\frac{\epsilon_y - 1}{\epsilon_y}} + \left(\sum_{i=1}^{J} \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t}^{\frac{\epsilon_y - 1}{\epsilon_y}} \right) \right)^{\frac{\epsilon_y}{\epsilon_y - 1}} \tag{36}$$

$$r_{i,t} \frac{W_{i,t}}{P_t} = \mu_{i,t} \left(q_{i,t} - r_{i,t} \right) + E_t \left[SDF_{t|t+1} (1 - s_{i,t+1}) \mu_{i,t+1} r_{i,t} \right] \tag{37}$$

$$\mu_{i,t} \frac{q_{i,t}}{r_{i,t}} = \frac{MC_{i,t}}{P_t} \beta_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y - 1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} \tag{38}$$

$$\frac{P_{j,t} X_{ij,t}}{MC_{i,t} Y_{i,t}} = (\beta_{ix} \omega_{ij})^{\frac{1}{\epsilon_y}} \left(A_{i,t} \frac{X_{ij,t}}{Y_{i,t}} \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \quad \forall \quad j \in J \tag{39}$$

$$C_{i,t} = \alpha_{i,t} \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon_d} C_t^{agg} \tag{40}$$

$$\left(\frac{\Pi_{i,t}}{\Pi} - 1 \right) \frac{\Pi_{i,t}}{\Pi} = \frac{\epsilon}{\psi_p} \left(\frac{MC_{i,t}}{P_t} - \frac{\epsilon - 1}{\epsilon} \frac{P_{i,t}}{P_t} \right) + E_t \left[SDF_{t|t+1} \left(\frac{\Pi_{i,t+1}}{\Pi} - 1 \right) \frac{\Pi_{i,t+1}}{\Pi} \frac{Y_{i,t+1}}{Y_{i,t}} \right] \tag{41}$$

$$SDF_{t|t+1} = \beta E_t \left[\left(\frac{C_{t+1}^{agg}}{C_t^{agg}} \right)^{-\sigma} \frac{Z_{t+1}}{Z_t} \right] \tag{42}$$

$$\Pi_{i,t} = \Pi_t^{agg} \left(\frac{P_{i,t}/P_t}{P_{i,t-1}/P_{t-1}} \right) \tag{43}$$

$$Y_{i,t} = C_{i,t} + \sum_{j=1}^{J} X_{ji,t} \tag{44}$$

$$\delta_t = \Xi_{i,t} f_{i,t} - \psi_{L,i} Z_t \left(\left[\frac{L_{i,t}/\overline{L}_t}{L_{i,t-1}/\overline{L}_{t-1}} - 1 \right] + \frac{1}{2} \left[\frac{L_{i,t}/\overline{L}_t}{L_{i,t-1}/\overline{L}_{t-1}} - 1 \right]^2 \right) \tag{45}$$

$$+ \beta \psi_{L,i} E_t \left[Z_{t+1} \left[\frac{L_{i,t+1}/\overline{L}_{t+1}}{L_{i,t}/\overline{L}_t} - 1 \right] \frac{L_{i,t+1}^2/\overline{L}_{t+1}}{L_{i,t}^2/\overline{L}_t} \right] \tag{46}$$

$$\Xi_{i,t} = Z_t \left((C_t^{agg})^{-\sigma} \frac{W_{i,t}}{P_t} - \Xi_{i,t} L_{i,t}^{\varphi} \right) + \beta E_t \left[(1 - f_{i,t+1})(1 - s_{i,t+1}) \Xi_{i,t+1} \right] \tag{47}$$

$$\overline{L}_t = \sum_{i=1}^{J} L_{i,t} \tag{48}$$

$$L_{i,t} = N_{i,t} + r_{i,t} V_{i,t} \tag{49}$$

$$L_{i,t} = (1 - s_{i,t}) L_{i,t-1} + U_{i,t} \tag{50}$$

$$q_{i,t} = \frac{H_{i,t}}{V_{i,t}} \tag{51}$$

$$f_{i,t} = \frac{H_{i,t}}{U_{i,t}} \tag{52}$$

$$\theta_{i,t} = \frac{V_{i,t}}{U_{i,t}} \tag{53}$$

$$H_{i,t} = \zeta_{i,t}\left(U_{i,t}^{-\eta_i} + V_{i,t}^{-\eta_i}\right)^{-\frac{1}{\eta_i}} \tag{54}$$

$$N_{i,t} + r_{i,t}V_{i,t} = H_{i,t} + (1 - s_{i,t})(N_{i,t-1} + r_{i,t-1}V_{i,t-1}) \tag{55}$$

$$\frac{W_{i,t}}{P_t} = \frac{1}{Z_t\left(C_t^{agg}\right)^{-\sigma} - \kappa\frac{r_{i,t}}{q_{i,t}}}\left(Z_t\chi_{i,t}L_{i,t}^{\varphi} - \beta E_t\left[(1 - f_{i,t+1})(1 - s_{i,t+1})\Xi_{i,t+1}\right]\right) \tag{56}$$

Aggregate variables satisfy:

$$C_t^{agg} = \left(\sum_{i=1}^{J}\alpha_{i,t}^{\frac{1}{\epsilon_d}}C_{i,t}^{\frac{\epsilon_d-1}{\epsilon_d}}\right)^{\frac{\epsilon_d}{\epsilon_d-1}} \tag{57}$$

$$H_t^{agg} = \sum_{i=1}^{J}H_{i,t} \tag{58}$$

$$V_t^{agg} = \sum_{i=1}^{J}V_{i,t} \tag{59}$$

$$U_t^{agg} = \sum_{i=1}^{J}U_{i,t} \tag{60}$$

$$\theta_t^{agg} = \frac{V_t^{agg}}{U_t^{agg}} \tag{61}$$

$$1 = \beta(1 + i_t)E_t\left[\left(\frac{C_{t+1}^{agg}}{C_t^{agg}}\right)^{-\sigma}\frac{Z_{t+1}}{Z_t}\frac{1}{\Pi_t^{agg}}\right] \tag{62}$$

$$(1 + i_t) = (1 + i_{t-1})^{\rho_i}\left(\left[\Pi_t^{agg}\right]^{\phi_\pi}\left[Y_t^{agg}\right]^{\phi_y}\right)^{1-\rho_i}M_t \tag{63}$$

$$Y_t^{agg} = C_t^{agg} \tag{64}$$

Finally, the shock processes are given by:

$$s_{i,t} = s_{i,t-1}^{\rho_s}\varepsilon_{s,i,t} \tag{65}$$

$$A_{i,t} = A_{i,t-1}^{\rho_A}\varepsilon_{A,i,t} \tag{66}$$

$$\alpha_{i,t} = \alpha_{i,t-1}^{\rho_\alpha}\varepsilon_{\alpha,i,t} \quad \forall \quad i \in J-1 \tag{67}$$

$$\alpha_{J,t}^{\frac{1}{\epsilon_d}} = 1 - \sum_{i=1}^{J-1}\alpha_{i,t}^{\frac{1}{\epsilon_d}} \tag{68}$$

$$r_{i,t} = r_{i,t-1}^{\rho_r}\varepsilon_{r,i,t} \tag{69}$$

$$\zeta_{i,t} = \zeta_{i,t-1}^{\rho_\zeta}\varepsilon_{\zeta,i,t} \tag{70}$$

$$\chi_{i,t} = \chi_{i,t-1}^{\rho_\chi}\varepsilon_{\chi,i,t} \tag{71}$$

$$Z_t = Z_{t-1}^{\rho_Z}\varepsilon_{Z,t} \tag{72}$$

$$M_t = M_{t-1}^{\rho_M}\varepsilon_{m,t} \tag{73}$$

A.1 Adding vacancy Adjustment Costs and Price Indexation to Past Inflation

$$\begin{aligned}
\mathcal{L}(\cdot) = E_t \sum_{s=0}^{\infty} \Bigg\{ & SDF_{t|t+s}\left[\frac{P_{i,t+s}(z)}{P_{t+s}}\left(\frac{P_{i,t+s}(z)}{P_{i,t+s}}\right)^{-\epsilon} Y_{i,t+s} - \frac{W_{i,t+s}}{P_{t+s}}\left[N_{i,t+s}(z) + r_{i,t+s}V_{i,t+s}(z)\right]\right.\\
& \left. - \sum_j \frac{P_{j,t}}{P_t}X_{ij,t+s}(z) - \frac{\psi_p}{2}\left(\frac{P_{i,t+s}(z)}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}P_{i,t+s-1}(z)} - 1\right)^2 Y_{i,t+s}\right]\\
& + \lambda_{i,t+s}\left[A_{i,t+s}\left(\beta_{ix}^{\frac{1}{\epsilon_y}}\left[\sum_j \omega_{ij}^{\frac{1}{\epsilon_y}}X_{ij,t+s}(z)^{\frac{\epsilon_y-1}{\epsilon_y}}\right] + \beta_{in}^{\frac{1}{\epsilon_y}}N_{i,t+s}(z)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}} - \left(\frac{P_{i,t+s}(z)}{P_{i,t+s}}\right)^{-\epsilon}Y_{i,t+s}\right]\\
& + \mu_{i,t+s}\Big[(1-s_{i,t+s})\left(N_{i,t+s-1}(z) + r_{i,t+s-1}V_{i,t+s-1}(z)\right) + \left(q_{i,t+s} - r_{i,t+s}\right)V_{i,t+s}(z) - N_{i,t+s}(z)\Big]\Bigg\}
\end{aligned}$$

FOCs

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial P_{i,t}(z)} = & (1-\epsilon)\frac{1}{P_t}\left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\epsilon}Y_{i,t} + \epsilon\lambda_{i,t}\left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\epsilon}\frac{Y_{i,t}}{P_{i,t}(z)}\\
& - \psi_p\Bigg\{\left(\frac{P_{i,t}(z)}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}P_{i,t-1}(z)} - 1\right)\frac{1}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}P_{i,t-1}(z)}Y_{i,t}\\
& - E_t\left[SDF_{t|t+s}\left(\frac{P_{i,t+1}(z)}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}P_{i,t}(z)} - 1\right)\frac{P_{i,t+1}(z)}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}P_{i,t}(z)^2}Y_{i,t+1}\right]\Bigg\} = 0
\end{aligned}$$

Now I assume a symmetric equilibrium, where we can drop the z indexation. Starting with (74), this implies

$$\left(\frac{\Pi_{i,t}}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}}-1\right)\frac{1}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}P_{i,t-1}}Y_{i,t}=\frac{1}{\psi_p}\left[(1-\epsilon)\frac{1}{P_t}Y_{i,t}+\epsilon\lambda_{i,t}\frac{Y_{i,t}}{P_{i,t}}\right]$$
$$+E_t\left[SDF_{t|t+s}\left(\frac{\Pi_{i,t+1}}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}P_{i,t}}-1\right)\frac{P_{i,t+1}}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}P_{i,t}^2}Y_{i,t+1}\right]$$

Dividing by $Y_{i,t}$ and multiplying by $P_{i,t}$ gives

$$\left(\frac{\Pi_{i,t}}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}}-1\right)\frac{\Pi_{i,t}}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}}=\frac{\epsilon}{\psi_p}\left[\lambda_{i,t}-\frac{\epsilon-1}{\epsilon}\frac{P_{i,t}}{P_t}\right]$$
$$+E_t\left[SDF_{t|t+s}\left(\frac{\Pi_{i,t+1}}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}}-1\right)\frac{\Pi_{i,t+1}}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}}\frac{Y_{i,t+1}}{Y_{i,t+1}}\right]$$

I add vacancy adjustment costs by modeling $r_{i,t}$ as a quadratic function that depends on the growth of firm level vacancy postings from one period to the next.

B First Order Approximation to the Phillips Curve

$$\max E_t SDF_{t+s|t}\frac{D_{i,t+s}(z)}{P_{t+s}}$$

$$D_{i,t}(z)=\frac{P_{i,t}(z)}{P_t}\left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\epsilon}Y_{i,t}-\frac{W_{i,t}}{P_t}-\frac{W_{i,t}}{P_t}\left[N_{i,t}(z)+N_{i,t}^r(z)\right]\quad-\sum_{j=1}^{J}\frac{P_{j,t}}{P_t}X_{ij,t}(z)-\frac{\psi_p}{2}\Bigg(\frac{}{\Pi}$$

$$N_{i,t}(z)+r_{i,t}V_{i,t}(z)=q_{i,t}V_{i,t}(z)+(1-s_{i,t})(N_{i,t-1}+r_{i,t-1}V_{i,t-1})$$
$$Y_{i,t}(z)=A_{i,t}\left(\omega_{in}^{\frac{1}{\epsilon_y}}N_{i,t}(z)^{\frac{\epsilon_y-1}{\epsilon_y}}+\sum_{j=1}^{J}\omega_{ij}^{\frac{1}{\epsilon_y}}X_{ij,t}(z)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}}$$

Lagrangian:

$$\mathcal{L}(\cdot) = E_t \sum_{s=0}^{\infty} \Bigg\{ SDF_{t|t+s} \Bigg[\frac{P_{i,t+s}(z)}{P_{t+s}} \left(\frac{P_{i,t+s}(z)}{P_{i,t+s}} \right)^{-\epsilon} Y_{i,t+s} - \frac{W_{i,t+s}}{P_{t+s}} \left[N_{i,t+s}(z) + r_{i,t+s} V_{i,t+s}(z) \right]$$

$$- \sum_j \frac{P_{j,t}}{P_t} X_{ij,t+s}(z) - \frac{\psi_p}{2} \left(\frac{P_{i,t+s}(z)}{\Pi P_{i,t+s-1}(z)} - 1 \right)^2 Y_{i,t+s} \Bigg]$$

$$+ \lambda_{i,t+s} \Bigg[A_{i,t+s} \left(\left[\sum_j \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right] + \omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}} - \left(\frac{P_{i,t+s}(z)}{P_{i,t+s}} \right)^{-\epsilon} Y_{i,t+s} \Bigg]$$

$$+ \mu_{i,t+s} \Big[(1 - s_{i,t+s}) \left(N_{i,t+s-1} + r_{i,t+s-1} V_{i,t+s-1} \right) + \left(q_{i,t+s} - r_{i,t+s} \right) V_{i,t+s}(z) - N_{i,t+s}(z) \Big] \Bigg\}$$

FOCs

$$\frac{\partial \mathcal{L}}{\partial P_{i,t}(z)} = (1-\epsilon) \frac{1}{P_t} \left(\frac{P_{i,t}(z)}{P_{i,t}} \right)^{-\epsilon} Y_{i,t} + \epsilon \lambda_{i,t} \left(\frac{P_{i,t}(z)}{P_{i,t}} \right)^{-\epsilon} \frac{Y_{i,t}}{P_{i,t}(z)} \tag{74}$$

$$- \psi_p \left\{ \left(\frac{P_{i,t}(z)}{\Pi P_{i,t-1}(z)} - 1 \right) \frac{1}{\Pi P_{i,t-1}(z)} Y_{i,t} - E_t \left[SDF_{t|t+s} \left(\frac{P_{i,t+1}(z)}{\Pi P_{i,t}(z)} - 1 \right) \frac{P_{i,t+1}(z)}{\Pi P_{i,t}(z)^2} Y_{i,t+1} \right] \right\} = 0$$

$$\frac{\partial \mathcal{L}}{\partial X_{ij,t}(z)} = -\frac{P_{j,t}}{P_t} + \lambda_{i,t} A_{i,t} \frac{\epsilon_y}{\epsilon_y - 1} \Bigg\{ \left(\left[\sum_j \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right] + \omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1} - 1} \tag{75}$$

$$\times \omega_{ij}^{\frac{1}{\epsilon_y}} \frac{\epsilon_y - 1}{\epsilon_y} X_{ij,t}(z)^{\frac{\epsilon_y - 1}{\epsilon_y} - 1} \Bigg\} = 0$$

$$\frac{\partial \mathcal{L}}{\partial N_{i,t}(z)} = -\frac{W_{i,t}}{P_t} + \lambda_{i,t} A_{i,t} \frac{\epsilon_y}{\epsilon_y - 1} \Bigg\{ \left(\left[\sum_j \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right] + \omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1} - 1} \tag{76}$$

$$\times \omega_{in}^{\frac{1}{\epsilon_y}} \frac{\epsilon_y - 1}{\epsilon_y} N_{ij,t}(z)^{\frac{\epsilon_y - 1}{\epsilon_y} - 1} \Bigg\} - \mu_{i,t} = 0 \tag{77}$$

$$\frac{\partial \mathcal{L}}{\partial V_{i,t}(z)} = -\frac{W_{i,t}}{P_t} r_{i,t} + \left(q_{i,t} - r_{i,t} \right) \mu_{i,t} = 0 \tag{78}$$

Implies,

$$\mu_{i,t} = \frac{r_{i,t}}{q_{i,t} - r_{i,t}} \frac{W_{i,t}}{P_t}$$

$$\mu_{i,t} = \lambda_{i,t} \omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y - 1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} - \frac{W_{i,t}}{P_t}$$

Combining implies,

$$\mu_{i,t} = \lambda_{i,t}\omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} - \frac{q_{i,t}-r_{i,t}}{r_{i,t}}\mu_{i,t}$$
$$\Rightarrow \mu_{i,t} + \frac{q_{i,t}-r_{i,t}}{r_{i,t}}\mu_{i,t} = \lambda_{i,t}\omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}}$$
$$\Rightarrow \frac{q_{i,t}}{r_{i,t}}\mu_{i,t} = \lambda_{i,t}\omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}}$$

Or alternatively,

$$\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\frac{W_{i,t}}{P_t} = \lambda_{i,t}\omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}}$$

Similarly,

$$\frac{P_{j,t}}{P_t} = (\omega_{ij})^{\frac{1}{\epsilon_y}} \lambda_{i,t} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} X_{ij,t}^{-\frac{1}{\epsilon_y}}$$

Combining, we can write all $X_{ij,t}$ and $N_{i,t}$ in terms of $X_{ii,t}$:

$$\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\frac{W_{i,t}}{P_t} = \frac{P_{i,t}}{P_t}\left(\frac{\omega_{in}}{\omega_{ii}}\right)^{\frac{1}{\epsilon_y}}\left(\frac{X_{ii,t}}{N_{i,t}}\right)^{\frac{1}{\epsilon_y}}$$
$$\Rightarrow N_{i,t}^{\frac{1}{\epsilon_y}} = \frac{P_{i,t}}{W_{i,t}}\frac{q_{i,t}-r_{i,t}}{q_{i,t}}\left(\frac{\omega_{in}}{\omega_{ii}}\right)^{\frac{1}{\epsilon_y}} X_{ii,t}^{\frac{1}{\epsilon_y}}$$
$$\Rightarrow N_{i,t} = \left(\frac{\omega_{in}}{\omega_{ii}}\right)\left(\frac{P_{i,t}}{W_{i,t}}\frac{q_{i,t}-r_{i,t}}{q_{i,t}}\right)^{\epsilon_y} X_{ii,t}$$

And similarly,

$$X_{ij,t} = \left(\frac{\omega_{ij}}{\omega_{ii}}\right)\left(\frac{P_{i,t}}{P_{j,t}}\right)^{\epsilon_y} X_{ii,t}$$

Now plugging into the production function for optimal input choices,

$$Y_{i,t} = A_{i,t}\left(\omega_{in}^{\frac{1}{\epsilon_y}}\left(\left(\frac{\omega_{in}}{\omega_{ii}}\right)\left(\frac{P_{i,t}}{W_{i,t}}\frac{q_{i,t}-r_{i,t}}{q_{i,t}}\right)^{\epsilon_y} X_{ii,t}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} + \sum_{j=1}^{J}\omega_{ij}^{\frac{1}{\epsilon_y}}\left(\left(\frac{\omega_{ij}}{\omega_{ii}}\right)\left(\frac{P_{i,t}}{P_{j,t}}\right)^{\epsilon_y} X_{ii,t}\right)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}}$$
$$= A_{i,t}\left(\omega_{in}\left(W_{i,t}\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\right)^{1-\epsilon_y} + \sum_{j=1}^{J}\omega_{ij}P_{j,t}^{1-\epsilon_y}\right)^{\frac{\epsilon_y}{\epsilon_y-1}}\frac{1}{\omega_{ii}}P_{i,t}^{\epsilon_y}X_{ii,t}$$

We can rearrange this to

$$X_{ii,t} = \omega_{ii} A_{i,t}^{-1} \left(\frac{P_{i,t}}{\Theta_{i,t}}\right)^{-\epsilon_y} Y_{i,t}$$

Where, $\Theta_{i,t} = \left(\omega_{in}\left(W_{i,t}\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\right)^{1-\epsilon_y} + \sum_{j=1}^{J}\omega_{ij}P_{j,t}^{1-\epsilon_y}\right)^{\frac{1}{1-\epsilon_y}}$. This lets us write

$$N_{i,t} = \omega_{in} A_{i,t}^{-1} \left(\frac{W_{i,t}}{\Theta_{i,t}}\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\right)^{-\epsilon_y} Y_{i,t}$$
$$X_{ij,t} = \omega_{ij} A_{i,t}^{-1} \left(\frac{P_{j,t}}{\Theta_{i,t}}\right)^{-\epsilon_y} Y_{i,t}$$

We can then write the cost in terms of $Y_{i,t}$,

$$\begin{aligned} Cost_{i,t} &= W_{i,t}\left(\frac{q_{i,t}}{q_{i,t}-r_{i,t}}N_{i,t} - \frac{r_{i,t}}{q_{i,t}-r_{i,t}}\left(N_{i,t-1}+r_{i,t-1}V_{i,t-1}\right)\right) + \sum_{j=1}^{J} P_{j,t}X_{ij,t} \\ &= \omega_{in}A_{i,t}^{-1}\left(W_{i,t}\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\right)^{1-\epsilon_y}\Theta_{i,t}^{\epsilon_y}Y_{i,t} + \sum_{j=1}^{J}\omega_{ij}A_{i,t}^{-1}P_{j,t}^{1-\epsilon_y}\Theta_{i,t}^{\epsilon_y}Y_{i,t} + ... \end{aligned}$$

The marginal cost is then,

$$\begin{aligned} MC_{i,t} &= A_{i,t}^{-1}\Theta_{i,t}^{\epsilon_y}\left(\omega_{in}\left(W_{i,t}\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\right)^{1-\epsilon_y} + \sum_{j=1}^{J}\omega_{ij}P_{j,t}^{1-\epsilon_y}\right) \\ &= A_{i,t}^{-1}\Theta_{i,t}^{\epsilon_y}\Theta_{i,t}^{1-\epsilon_y} = A_{i,t}^{-1}\Theta_{i,t} \\ &= A_{i,t}^{-1}\left(\omega_{in}\left(W_{i,t}\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\right)^{1-\epsilon_y} + \sum_{j=1}^{J}\omega_{ij}P_{j,t}^{1-\epsilon_y}\right)^{\frac{1}{1-\epsilon_y}} \end{aligned}$$

All other conditions are the same as in the full model.

On the household side, assume there are no labor rellocation costs, so that the household problem is

$$\max E_t \sum_{s=0}^{\infty} Z_{t+s} \left[\frac{C_{t+s}(z)^{1-\sigma}}{1-\sigma} - \sum_{i=1}^{J} \left(\chi_{i,t+s} \frac{L_{i,t+s}(z)^{1+\varphi}}{1+\varphi} \right) \right]$$

$$\text{s.t. } P_{t+s}C_{t+s}(z) + B_{t+s}(z) = (1+i_{t+s})B_{t+s-1}(z) + \sum_{i=1}^{J} W_{i,t+s}L_{i,t+s}(z) + T_{t+s}(z)$$

$$L_{i,t+s}(z) = (1-f_{i,t+s})(1-s_{i,t+s})L_{i,t+s-1} + f_{i,t+s}L_{i,t+s}(z)$$

$$\text{and, } \sum_{i=1}^{J} L_{i,t+s}(z) \leq \overline{L}_{t+s}$$

The Lagrangian is

$$\begin{aligned}
\mathcal{L} = {} & E_t \sum_{s=0}^{\infty} \beta^s \Bigg(Z_{t+s} \left[\frac{C_{t+s}(z)^{1-\sigma}}{1-\sigma} - \sum_{i=1}^{J} \left(\chi_{i,t+s} \frac{L_{i,t+s}(z)^{1+\varphi}}{1+\varphi} \right) \right] \\
& + \upsilon_{t+s} \left[(1+i_{t+s-1})B_{t+s-1}(z) + \sum_{i=1}^{J} W_{i,t+s}L_{i,t+s}(z) + T_{t+s}(z) - P_{t+s}C_{t+s}(z) - B_{t+s}(z) \right] \\
& + \sum_{i=1}^{J} \Xi_{i,t+s} \left[(1-f_{i,t+s})(1-s_{i,t+s})L_{i,t+s-1} + f_{i,t+s}L_{i,t+s}(z) - L_{i,t+s}(z) \right] \\
& + \delta_{t+s} \left[\overline{L}_{t+s} - \sum_{i=1}^{J} L_{i,t+s} \right]
\end{aligned}$$

And the first order conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_{t+s}(z)} &= \beta^s E_t \left[Z_{t+s}C_{t+s}(z)^{-\sigma} - \upsilon_{t+s}P_{t+s} \right] = 0 \\
\frac{\partial \mathcal{L}}{\partial B_{t+s}(z)} &= E_t \left[\beta^{s+1}\upsilon_{t+s+1}(1+i_{t+s}) - \beta^s \upsilon_{t+s} \right] = 0 \\
\frac{\partial \mathcal{L}}{\partial L_{i,t+s}(z)} &= E_t \left[\beta^s \Xi_{i,t+s} f_{i,t+s} - \beta^s \delta_{t+s} \right] = 0 \\
\frac{\partial \mathcal{L}}{\partial L_{i,t+s}(z)} &= E_t \left[\beta^s \upsilon_{t+s} W_{i,t+s} - \beta^s Z_t \chi_{i,t+s} L_{i,t+s}^{\varphi} - \beta^s \Xi_{i,t+s} \right] = 0
\end{aligned}$$

Which implies,

$$\begin{aligned}
\Xi_{i,t} f_{i,t} &= \delta_t \\
Z_t \left(C_t^{-\sigma} \frac{W_{i,t}}{P_t} - \chi_{i,t} L_{i,t}^{\varphi} \right) &= \Xi_{i,t}
\end{aligned}$$

The Nash bargaining solution is then

$$\kappa \frac{r_{i,t}}{q_{i,t}} W_{i,t} = \frac{P_t}{Z_t C_t^{-\sigma}} Z_t \left(C_t^{-\sigma} \frac{W_{i,t}}{P_t} - \chi_{i,t} L_{i,t}^{\varphi} \right)$$
$$\kappa \frac{r_{i,t}}{q_{i,t}} W_{i,t} = W_{i,t} - \chi_{i,t} L_{i,t}^{\varphi} C_t^{\sigma} P_t$$
$$\Rightarrow \left(1 - \kappa \frac{r_{i,t}}{q_{i,t}}\right) W_{i,t} = \chi_{i,t} L_{i,t}^{\varphi} C_t^{\sigma} P_t$$
$$\Rightarrow \frac{W_{i,t}}{P_t} = \frac{1}{1 - \kappa \frac{r_{i,t}}{q_{i,t}}} \chi_{i,t} L_{i,t}^{\varphi} C_t^{\sigma}$$

Suppose

$$W_{i,t} = \left[\frac{1}{1 - \kappa \frac{r_{i,t}}{q_{i,t}}} \chi_{i,t} L_{i,t}^{\varphi} C_t^{\sigma} P_t \right]^{1-\rho_w} W_{i,t-1}^{\rho_w}$$
$$\Rightarrow \frac{W_{i,t}}{P_t} = \left[\frac{1}{1 - \kappa \frac{r_{i,t}}{q_{i,t}}} \chi_{i,t} L_{i,t}^{\varphi} C_t^{\sigma} \right]^{1-\rho_w} \left(\frac{W_{i,t-1}}{P_{t-1}} \Pi_t^{-1} \right)^{\rho_w}$$

B.1 First order approx

To first order, the NKPC is

$$\pi_{i,t} = \frac{\epsilon - 1}{\psi_{p,i}} \left(mc_{i,t} - p_{i,t} \right) + \beta E_t \pi_{i,t+1}$$

For a general matching function $m(U_i, V_i)$, the job-finding rate is

$$q_{i,t} = m_{i,t} \left(1, \theta_{i,t}\right)$$
$$q_i + \left(q_{i,t} - q_i\right) = q_i + \frac{\partial q_i}{\partial \theta_i} \left(\theta_{i,t} - \theta_i\right)$$
$$\frac{q_{i,t} - q_i}{q_i} = \frac{\partial q_i}{\partial \theta_i} \frac{\theta_i}{q_i} \frac{\theta_{i,t} - \theta_i}{\theta_i}$$
$$q_{i,t} = \mathcal{E}_{\theta_i}^{q_i} \widehat{\theta}_{i,t}$$

Where $\mathcal{E}_{\theta_i}^{q_i}$ is the elasticity of the vacancy-filling rate to changes in tightness.

Now for the marginal cost,

$$\left(\frac{MC_{i,t}}{P_t}\right)^{1-\epsilon_y} = A_{i,t}^{-(1-\epsilon_y)}\left(\omega_{in}\left(\frac{W_{i,t}}{P_t}\frac{q_{i,t}}{q_{i,t}-r_{i,t}}\right)^{1-\epsilon_y} + \sum_{j=1}^{J}\omega_{ij}\left(\frac{P_{j,t}}{P_t}\right)^{1-\epsilon_y}\right)$$

$$\left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y} + (1-\epsilon_y)\left(\frac{MC_i}{P_t}\right)^{-\epsilon_y}\left(\frac{MC_{i,t}}{P_t} - \frac{MC_i}{P}\right) = \left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y} - (1-\epsilon_y)\left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y}\frac{1}{A_i}\left(A_{i,t} - A_i\right.$$

$$+ (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{q_i}{q_i-r_i}\right)^{1-\epsilon_y}\frac{P}{W_i}\left(\frac{W_{i,t}}{P_t} - \frac{W_i}{P}\right) - (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{q_i}{q_i-r_i}\right)^{1-\epsilon_y}\frac{r_i}{(q_i-r_i)^2}\left(q_{i,t} - q_i\right)$$

$$+ (1-\epsilon_y)\frac{1}{A_i}\left(\frac{W_i}{P}\frac{q_i}{q_i-r_i}\right)^{1-\epsilon_y}\frac{q_i}{(q_i-r_i)^2}(r_{i,t} - r_i) + (1-\epsilon_y)\frac{1}{A_i}\sum_{j=1}^{J}\omega_{ij}\left(\frac{P_j}{P}\right)^{1-\epsilon_y}\frac{P}{P_j}\left(\frac{P_{j,t}}{P_t} - \frac{P_j}{P}\right)$$

$$\Rightarrow (1-\epsilon_y)\left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y} mc_{i,t} = -(1-\epsilon_y)\left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y} a_{i,t}$$

$$+ (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{q_i}{q_i-r_i}\right)^{1-\epsilon_y} w_{i,t} - (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{q_i}{q_i-r_i}\right)^{1-\epsilon_y}\frac{r_i q_i}{(q_i-r_i)^2}q_{i,t}$$

$$+ (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{q_i}{q_i-r_i}\right)^{1-\epsilon_y}\frac{r_i q_i}{(q_i-r_i)^2}r_{i,t} + (1-\epsilon_y)\frac{1}{A_i}\sum_{j=1}^{J}\omega_{ij}\left(\frac{P_j}{P}\right)^{1-\epsilon_y} p_{j,t}$$

From the first order conditions,

$$\begin{aligned}
\frac{q_i}{q_i-r_i}\frac{W_i N_i}{MC_i Y_i} &= \omega_{in}^{\frac{1}{\epsilon_y}} A_i^{\frac{\epsilon_y-1}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} = \Omega_{in} \\
\Rightarrow \frac{q_i}{q_i-r_i}\frac{W_i}{MC_i} &= \omega_{in}^{\frac{1}{\epsilon_y}} A_i^{\frac{\epsilon_y-1}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{-\frac{1}{\epsilon_y}} \\
\Rightarrow \left[\frac{q_i}{q_i-r_i}\frac{W_i}{MC_i}\right]^{1-\epsilon_y} &= \omega_{in}^{\frac{1-\epsilon_y}{\epsilon_y}} A_i^{\frac{(1-\epsilon_y)(\epsilon_y-1)}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} \\
\Rightarrow \frac{\omega_{in}}{A_i}\left[\frac{q_i}{q_i-r_i}\frac{W_i}{MC_i}\right]^{1-\epsilon_y} &= \frac{\omega_{in}}{A_i}\omega_{in}^{\frac{1-\epsilon_y}{\epsilon_y}} A_i^{\frac{(1-\epsilon_y)(\epsilon_y-1)}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} \\
&= A_i^{-\epsilon_y}\omega_{in}^{\frac{1}{\epsilon_y}} A_i^{\frac{\epsilon_y-1}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} \\
&= A_i^{-\epsilon_y}\Omega_{in} = \Omega_{in}
\end{aligned}$$

So we can rewrite the marginal cost as

$$mc_{i,t} = -a_{i,t} + \Omega_{in}\left[w_{i,t} - \frac{r_i}{q_i-r_i}\frac{q_i}{q_i-r_i}(q_{i,t} - r_{i,t})\right] + \sum_{j=1}^{J}\Omega_{ij}p_{j,t}$$

Or in terms of tightness,

$$mc_{i,t} = -a_{i,t} + \Omega_{in}\left[w_{i,t} - \frac{r_i}{q_i - r_i}\frac{q_i}{q_i - r_i}(\mathcal{E}_{\theta_i}^{q_i}\widehat{\theta}_{i,t} - r_{i,t})\right] + \sum_{j=1}^{J}\Omega_{ij}p_{j,t}$$

Stacking over sectors, and assuming wages are fully rigid,

$$\boldsymbol{mc}_t = -\boldsymbol{a}_t + \boldsymbol{\Omega}_n \underbrace{\boldsymbol{r}(\boldsymbol{Q}-\boldsymbol{r})^{-1}\boldsymbol{Q}(\boldsymbol{Q}-\boldsymbol{r})^{-1}}_{\boldsymbol{\Gamma}_Q}(\boldsymbol{\eta}\widehat{\boldsymbol{\theta}}_t + \boldsymbol{r}_t) + \boldsymbol{\Omega}_x\boldsymbol{p}_t$$

Where $\boldsymbol{\eta}$ is a diagonal matrix with the negative of the elasticity of the vacancy-filling rate to changes in tightness on the diagonal. Stacking the sector level PC over sectors gives

$$\boldsymbol{\pi}_t = \boldsymbol{\lambda}\left(\boldsymbol{mc}_t - \boldsymbol{p}_t\right) + \beta E_t\boldsymbol{\pi}_{t+1}$$

Where $\boldsymbol{\lambda}$ is a diagonal matrix capturing pricing frictions in each sector. Plugging into for the marginal cost gives

$$\begin{aligned}\boldsymbol{\pi_t} &= \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\boldsymbol{p}_t + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{\eta}\widehat{\boldsymbol{\theta}}_t + \beta E_t\boldsymbol{\pi_{t+1}} \\ &\quad + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{r}_t - \boldsymbol{a}_t\end{aligned}$$

Finally, using $\boldsymbol{p}_t = \boldsymbol{\pi}_t + \boldsymbol{p}_{t-1} - \boldsymbol{1}\pi_t^{agg}$, we can rewrite the sector level NKPC as

$$\begin{aligned}\boldsymbol{\pi_t} &= \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{\pi}_t - \boldsymbol{1}\pi_t^{agg}\right) + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{\eta}\widehat{\boldsymbol{\theta}}_t + \beta E_t\boldsymbol{\pi_{t+1}} \\ &\quad + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{r}_t - \boldsymbol{a}_t + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\boldsymbol{p}_{t-1}\end{aligned}$$

Now from the the aggregate consumption price index,

$$\begin{aligned}\frac{P_t}{P_{t-1}} &= \left(\sum_{i=1}^{J}\alpha_{i,t}\left(\frac{P_{i,t}}{P_{i,t-1}}\frac{P_{i,t-1}}{P_{t-1}}\right)^{1-\epsilon_d}\right)^{\frac{1}{1-\epsilon_d}} \\ \Pi_t^{1-\epsilon_d} &= \sum_{i=1}^{J}\alpha_{i,t}\left(\Pi_{i,t-1}\frac{P_{i,t-1}}{P_{t-1}}\right)^{1-\epsilon_d}\end{aligned}$$

To first order,

$$\Pi^{1-\epsilon_d} + (1-\epsilon_d)\Pi^{1-\epsilon_d}\frac{\Pi_t - \Pi}{\Pi} = \Pi^{1-\epsilon_d} + \sum_{i=1}^{J}\Pi^{1-\epsilon_d}\alpha_i\left(\frac{P_i}{P}\right)^{1-\epsilon_d}\frac{\alpha_{i,t}-\alpha_i}{\alpha_i}$$
$$+\sum_{i=1}^{J}(1-\epsilon_d)\Pi^{1-\epsilon_d}\alpha_i\left(\frac{P_i}{P}\right)^{1-\epsilon_d}\left(\frac{\Pi_{i,t}-\Pi}{\Pi} + \frac{P_{i,t}/P_t - P_i/P}{P_i/P}\right)$$

Which implies,

$$\begin{aligned}\pi_t^{agg} &= \sum_{i=1}^{J}\Omega_{d,i}\left[\frac{1}{1-\epsilon_d}\widehat{\alpha}_{i,t} + \pi_{i,t} + p_{i,t-1}\right] \\ &= \boldsymbol{\Omega}_d'\left[\boldsymbol{\pi}_t + \boldsymbol{p}_{t-1} + \frac{1}{1-\epsilon_d}\boldsymbol{\alpha}_t\right]\end{aligned}$$

Plugging in to the sectoral PC gives

$$\begin{aligned}\boldsymbol{\pi_t} &= \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\boldsymbol{\pi}_t + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{\eta}\widehat{\boldsymbol{\theta}}_t + \beta E_t\boldsymbol{\pi_{t+1}} \\ &\quad + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{r}_t - \boldsymbol{a}_t + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\boldsymbol{p}_{t-1} - \frac{1}{1-\epsilon_d}\boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\mathbf{1}\boldsymbol{\Omega}_d'\boldsymbol{\alpha}_t \\ \Rightarrow \boldsymbol{\pi}_t &= \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1}\left[\boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{\eta}\boldsymbol{\theta}_t + \beta E_t\boldsymbol{\pi}_{t+1}\right] + \boldsymbol{\upsilon}_t^{sec}\end{aligned}$$

Where

$$\begin{aligned}\boldsymbol{\upsilon}_t^{sec} &= \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \\ &\quad\left[\boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{r}_t - \boldsymbol{a}_t + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\boldsymbol{p}_{t-1} - \frac{1}{1-\epsilon_d}\boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\mathbf{1}\boldsymbol{\Omega}_d'\boldsymbol{\alpha}_t\right]\end{aligned}$$

Finally, using the expression for aggregate inflation, the aggregate NKPC is

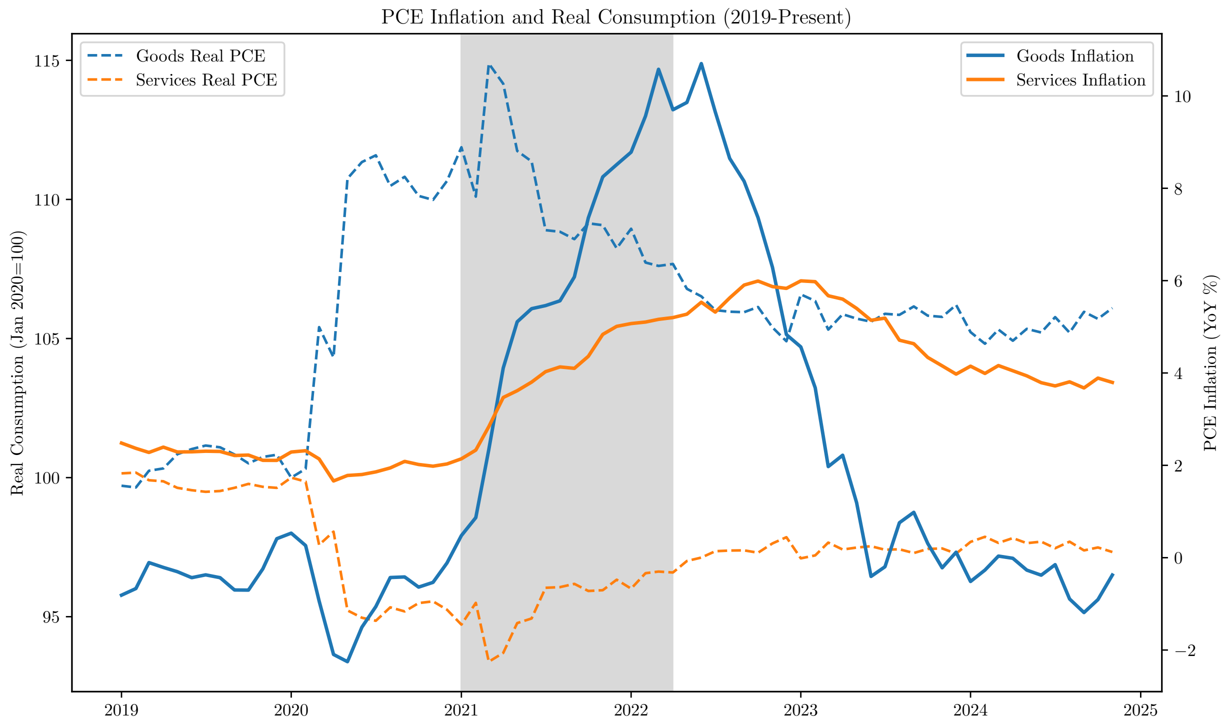
$$\pi_t^{agg} = \boldsymbol{\Gamma}_\theta\boldsymbol{\theta}_t + \boldsymbol{\Gamma}_\pi E_t\boldsymbol{\pi}_{t+1} + \upsilon_t$$

Where,

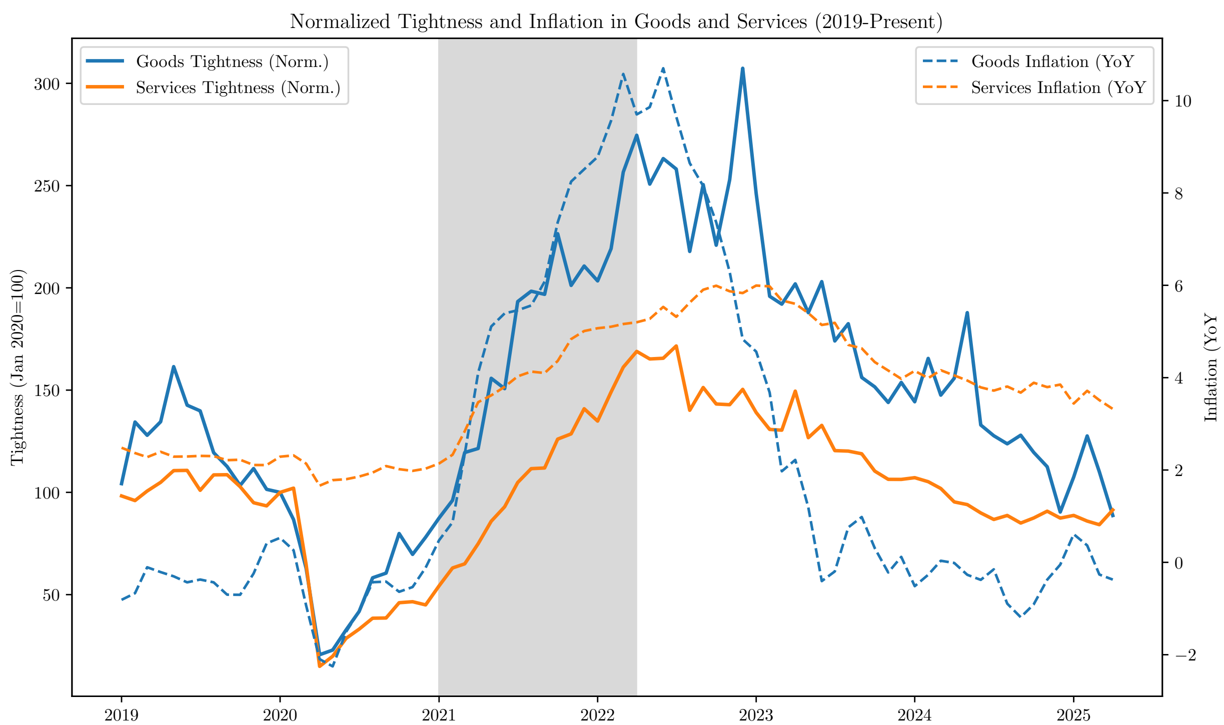
$$
\begin{aligned}
\boldsymbol{\Gamma}_\theta &= \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \boldsymbol{\Omega}_n \boldsymbol{\Gamma}_Q \boldsymbol{\eta} \\
\boldsymbol{\Gamma}_\pi &= \beta \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \\
\upsilon_t &= \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{I} + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right] \boldsymbol{p}_{t-1} \\
&\quad + \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{\Omega}_n \boldsymbol{\Gamma}_Q \boldsymbol{r}_t - \boldsymbol{a}_t\right] \\
&\quad + \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\mathbf{1}\boldsymbol{\Omega}_d'\right] \boldsymbol{\alpha}_t
\end{aligned}
$$

C Unemployment Based Measure of Tightness

In this section, I show that the patterns I highlight in section 4.1 hold when using the more conventional measure of tightness $\frac{V}{U}$.

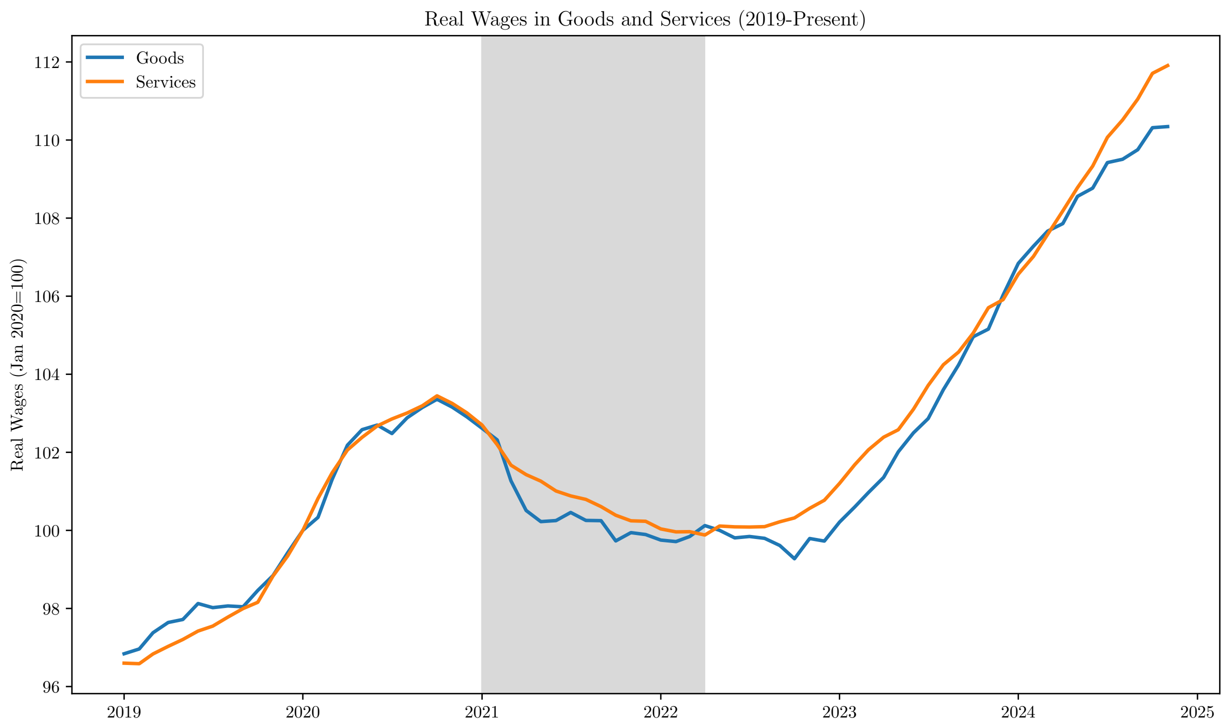


(a) Real PCE and PCE Inflation.

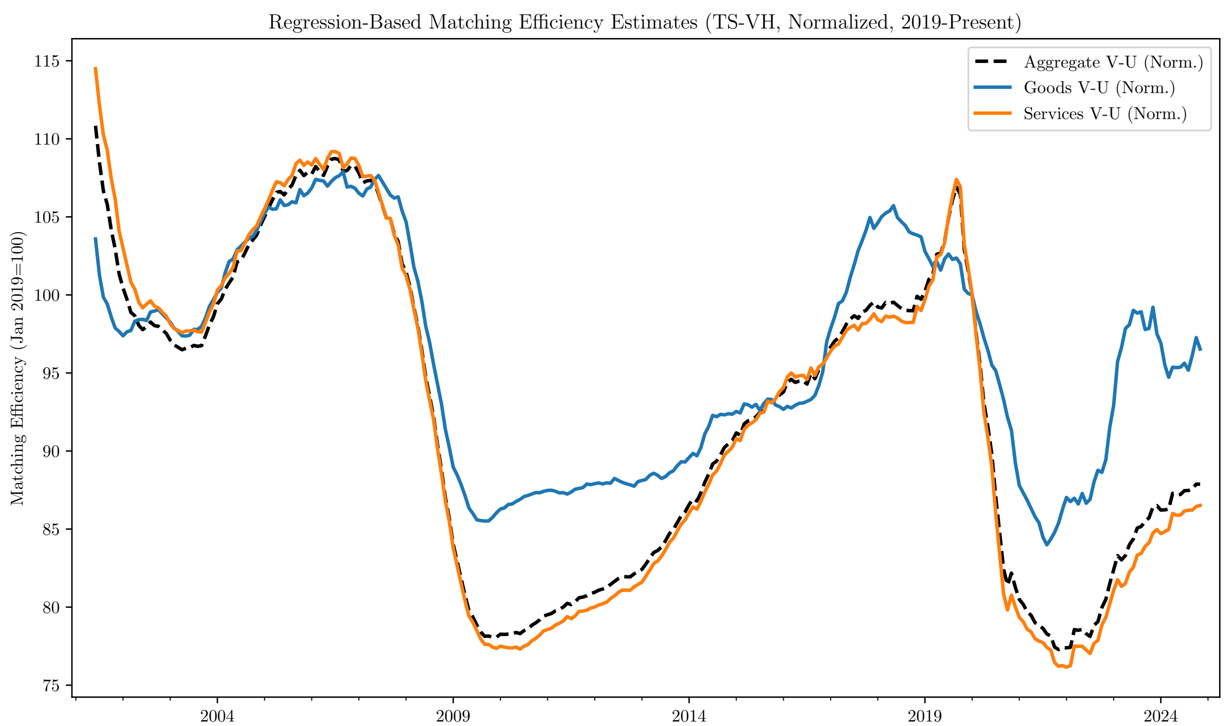


(b) labor market tightness and PCE Inflation.

Figure 14: Left Axis (both subplots): Dashed lines are changes in real PCE relative to January 2020, by major expenditure category: goods (blue) and services (orange). Right Axis: Solid lines are year-over-year changes in the PCE price index, by major category: goods (blue) and services (orange). The gray shaded area indicates the period from the start of the inflation surge in early 2021, to the first Federal Reserve rate hike in March 2022.



(a) All Wages.



(b) Matching Efficiency

Figure 15: Top: Real wages for all workers in the goods (blue) and services (orange) sectors, relative to January 2020. Bottom: Matching efficiency in the goods (blue) and services (orange) sectors, relative to January 2020. The gray shaded area indicates the period from the start of the inflation surge in early 2021, to the first Federal Reserve rate hike in March 2022.