The Phillips Curve and Beveridge Curve in a Multi-Sector Economy*

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Abstract

I develop a New Keynesian model with input–output linkages, search-and-matching frictions, and sticky prices to study how shocks propagate to output and inflation. Hiring costs tie firms' marginal costs to local labor market tightness, creating a labor market propagation channel—distinct from input-price spillovers—by which higher demand in one sector raises wages and job-finding rates in that sector, redirects job search across sectors, and increases hiring costs elsewhere. Solving the model non-linearly yields a Phillips curve that steepens as tightness rises, consistent with recent evidence, implying weaker output effects and stronger inflation responses to monetary policy when some sectors are tight. Calibrated to BEA input–output data and estimated using data from 2000–2019, the model shows that the post-pandemic shift toward demand for goods raised inflation and lowered matching efficiency; additional increases in job separations and aggregate demand are required to match the 2021–2023 inflation surge.

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1 Introduction

Economies feature rich production networks. Firms in one sector use goods produced in other sectors as intermediate inputs in their production processes. These production linkages imply that sector-specific shocks propagate through the economy, with large consequences for aggregate output and inflation. A large recent literature builds on this insight to explore the sectoral sources of the recent surge in U.S. inflation—from an average of just over 2 percent per year from 2000 to 2019 to an average of 5.5 percent per year from 2021 to 2023.

The current generation of production network models, however, generally assumes frictionless labor markets, whereas in reality firms face substantial hiring costs when adjusting labor. This matters for two reasons. First, sectors not only use other sectors' output as an intermediate input in production; they also compete for workers from the same labor pool. As a result, an increase in labor demand in one sector can reduce labor supply to other parts of the economy. When labor markets are frictional, the reduction in labor supply lowers the effective elasticity of labor across the network, altering the propagation of shocks and potentially shifting which sectors are the main drivers of aggregate output and inflation.

Second, the recent inflation surge highlights that the Phillips curve—the relationship between inflation and economic slack—steepens when labor market tightness—the ratio of vacancies to the number of unattached workers—is high. If sectors vary in local labor market tightness, they will also vary in the slope of their local Phillips curve, again altering how sector-specific shocks propagate through the economy. In this paper, I ask how labor market frictions alter the propagation and inflation dynamics resulting from sector-level shocks.

To address this question, I build a model with three key components: a production network, search-and-matching frictions, and sticky prices. The production network models realistic sector-specific shocks, tracking the downstream propagation from the shocked sector to users of its output and upstream to its input suppliers. Search-and-matching frictions capture the costly hiring process firms face and link firms' marginal costs to local labor market conditions. Sticky prices generate a sector-specific New Keynesian Phillips curve linking inflation to current marginal costs and expected future inflation. Marginal costs, in turn, depend on both the prices of intermediate inputs—capturing input-price spillovers through the production network—and on labor costs—capturing the link between labor market conditions and inflationary pressures.

Incorporating search-and-matching into a production network model with sticky prices generates three key insights. First, the model features a new labor market channel through which sectoral shocks propagate across the economy. As labor demand increases in one sector, wages and the likelihood that searching workers find a job in that sector rise. This causes households to increase their labor supply to that sector and reduce their labor supply to other sectors. The reduction in labor supply in sectors that do not directly experience a change in demand raises their hiring costs, causing spillovers to marginal costs through a channel distinct from the standard network channel via prices. Sectors that generate larger labor market spillovers will therefore be more important for aggregate inflation dynamics.

Second, by solving the model nonlinearly, I show that search-and-matching frictions lead to a Phillips curve that steepens as tightness rises. Sector-level Phillips curves steepen at high levels of tightness because firms find it harder to hire when the labor market is already tight. In other words, they face a less elastic labor supply. As a result, when tightness is high, firms are less able to adjust the quantity of output they produce and instead raise prices. This finding is consistent with the empirical evidence in Benigno and Eggertsson (2023) and Gitti (2024). I demonstrate that the nonlinearity arises naturally from a standard search-and-matching framework in a model with sticky prices, without ad hoc kinks in wage setting or search. My findings, therefore, suggest that the conclusions in Benigno and Eggertsson (2023) do not rely on the specific wage mechanism they propose. In addition, in a multi-sector economy, sector-specific shocks can lead to simultaneous high tightness in some sectors and low tightness in others. I show that it is enough for just some sectors to be tight for the aggregate Phillips curve to steepen, demonstrating that the nonlinearity is relevant whenever the economy experiences uneven expansions or contractions across sectors.

Third, the presence of a nonlinear Phillips curve, in turn, implies that monetary policy is less effective at stimulating output and has larger effects on inflation whenever labor markets are tight. Crucially, this is true at both the aggregate and the sector level. It is enough that some sectors are tight to constrain monetary policy's ability to stimulate output. Whenever some sectors are tight and others are slack, divine coincidence no longer holds: Stabilizing inflation no longer automatically stabilizes the output gap. Instead, the central bank faces a tradeoff between raising output in slack sectors and combating inflation in tight sectors.

My model embeds a standard production network model, as in Acemoglu et al. (2012) and Baqaee and Farhi (2019), into a New Keynesian framework with sticky prices, as in Rubbo (2023a). Each sector consists of a unit continuum of monopolistically competitive firms that use intermediate inputs from other sectors and labor in production. Firms set prices subject to quadratic Rotemberg (1982) adjustment costs, which serve as a reduced form for menu or reputational costs firms face when changing prices. The presence of price adjustment costs generates a sector-specific Phillips curve linking inflation to current marginal costs and expected future inflation.

I assume that firms' labor inputs are subject to search-and-matching frictions in sector-specific labor markets, as in Diamond (1982a), Mortensen (1982a), and Pissarides (1984).

To hire additional workers, firms must post vacancies, which take time to maintain. Time spent recruiting is time not spent producing and is therefore a real cost to firms. Crucially, the number of vacancies a firm needs to post to hire an additional worker depends on labor market tightness in its sector: when labor markets are tight, competition for workers is greater and firms must post more vacancies. As a result, firms' hiring costs rise and the elasticity of labor supply falls as tightness rises.

Households choose both where to consume and where to search for work to maximize lifetime utility. Households send more workers to sectors with higher wages and a higher probability of finding a job, subject to reallocation costs. Reallocation costs—the costs workers face when moving across sectors—are empirically large and relevant (Humlum, 2021). When reallocation costs are higher, fewer workers move into sectors with high labor demand. This reduces labor market spillovers to other sectors but means that sectors with high demand become supply-constrained more quickly than they otherwise would. Finally, I assume a wage specification that nests two standard wage-setting assumptions in the search-and-matching literature: Nash bargaining and rigid wages (R. E. Hall, 2005; Shimer, 2005).

I then solve the model nonlinearly by simulating the response of endogenous variables to a range of shocks under perfect foresight. While the perfect-foresight assumption requires abstracting from risk, it allows me to focus on the nonlinearities arising from labor market frictions. I show that these nonlinearities are quantitatively important for both inflation and the effectiveness of monetary policy. A linearized version would miss the steepening of the Phillips curve at high levels of tightness and therefore overestimate the effectiveness of aggregate-demand stimulus at high tightness. Both results depend on the nonlinear relationship between tightness, hiring costs, and labor supply elasticity.

Once I establish the key theoretical mechanisms and predictions of the model, I apply it to the recent inflation surge, focusing on the distinction between goods and services. I calibrate the production network parameters to match the BEA input–output tables and estimate the remaining labor market parameters in a linearized version of the model with Bayesian methods using data from 2000–2019. I then show that the persistent rise in the consumption share of goods relative to services after the pandemic increased inflation and reduced aggregate matching efficiency, consistent with the joint behavior of the Phillips curve and the Beveridge curve over this period. The sectoral demand shock alone, however, is not enough to explain the large and persistent rise in aggregate inflation and generates a counterfactual decline in labor market tightness in services. Adding shocks to the separation rate and to aggregate demand allows for a closer fit to the data.

Related Literature: My paper relates to two broad strands of the literature. First, I build a production network model in the vein of Acemoglu et al. (2012), Baqaee and Farhi (2019), Baqaee and Rubbo (2022), and Jones (2011). My contribution is to show how accounting for frictional labor markets alters the propagation of sectoral shocks in a production network, building on Schüle and Sheng (2024). Second, my paper relates to the extensive literature that uses multi-sector models to study the recent post-pandemic experience. Similar to Guerrieri et al. (2022), who show that sectoral supply shocks can lead to demand shocks, in my model sectoral demand shocks can lead to supply shocks through their effects on labor supply across the production network. Like Amiti et al. (2023), Comin et al. (2023), and Di Giovanni et al. (2023), I explore the implications for the recent inflation surge. I focus on nonlinearities at the sector level generated by search-and-matching frictions in the labor market. Within this literature, my paper is most closely related to Ferrante et al. (2023), who show that sectoral demand shocks played a key role in the surge in a production network model with asymmetric hiring costs. My findings complement theirs by showing that similar conclusions hold in a model where hiring costs are microfounded through standard search-and-matching. In my framework, search-and-matching naturally leads to larger hiring costs in tighter labor markets. I show that this results in a nonlinear sector-level Phillips curve with important implications for aggregate inflation and the effectiveness of monetary policy.

Second, I contribute to recent work exploring the links between the inflation surge and labor market conditions. Benigno and Eggertsson (2023) argue for a nonlinear Phillips curve that steepens at high levels of tightness. They propose a model in which a kink in wage setting generates a piecewise linear Phillips curve that steepens above a certain level of tightness. I show that a similar, but smooth, steepening of the Phillips curve arises naturally from incorporating search-and-matching into a New Keynesian model when I solve the model nonlinearly. Gitti (2024) demonstrate empirically that the Phillips curve steepens at high levels of tightness, supporting the mechanism in my paper. Several papers propose alternative explanations for the Beveridge curve's unusual post-pandemic behavior. Bagga et al. (2025) argue that a shift in preferences for job amenities, resulting in more job-to-job transitions, can partly explain the shift in the Beveridge curve. Similarly, Afrouzi et al. (2024) propose a mechanism from inflation to separations, where workers leave their jobs when inflation is high to recoup the loss in real earnings. Again, I view my results as complementary. My proposed explanation—a decline in matching efficiency resulting from sectoral reallocation—can explain some but not all of the unusual labor market dynamics. An exogenous increase in separations is important for explaining the joint increase in tightness across both goods and services.

The paper proceeds as follows. Section 2 introduces the model and outlines the equilibrium conditions. Section 3 presents key predictions. Section 4 applies the model to post-pandemic inflation and labor market dynamics. Section 5 concludes.

2 A Model with Production Networks and Labor-Market Frictions

In this section, I build a New Keynesian model with labor market frictions and production linkages. The model has three key components: (1) sticky prices, (2) a production network, and (3) search-and-matching frictions in the labor market. Sticky prices generate a New Keynesian Phillips curve, linking inflation to firms' marginal costs and expected future inflation. The production network—firms use intermediate inputs from other sectors—links marginal costs in one sector to prices in other sectors. Finally, search-and-matching frictions capture the costly hiring process, linking firms' marginal costs to labor market conditions. Together, these three components generate a sector-level Phillips curve that captures how inflation spreads across sectors through both prices and labor market conditions.

2.1 The Labor Market

I begin with the labor market, modeled using a search-and-matching framework in which firms post vacancies and workers search for jobs (Diamond, 1982a, 1982b; Mortensen, 1982a, 1982b; Pissarides, 1984, 1985). The search-and-matching process generates a probability of a worker finding a job and of a firm filling a vacancy that depend on sectoral labor market tightness—the ratio of vacancy postings to unattached job seekers in sector i. As a result, tightness directly affects firms' hiring costs, and therefore their marginal costs.

At the start of each discrete time period t, firms in sector $i \in \{1, \cdots, J\}$ post vacancies $V_{i,t}$ and hire $H_{i,t}$ workers from a pool of searchers $U_{i,t}$.[1] A hire occurs whenever a searching worker matches with a hiring firm. As is standard in the search-and-matching literature, I represent this matching process using a matching function $m_{i,t}(U_{i,t}, V_{i,t})$.[2] The total number of hires in sector i is

$$H_{i,t} = m_{i,t}(U_{i,t}, V_{i,t}). \tag{1}$$

The matching function exhibits constant returns to scale, $m_{i,t}(\lambda U_{i,t}, \lambda V_{i,t}) = \lambda H_{i,t}$, is increasing in both arguments, $\frac{\partial m_{i,t}}{\partial U_{i,t}}, \frac{\partial m_{i,t}}{\partial V_{i,t}} > 0$, and concave, $\frac{\partial^2 m_{i,t}}{\partial U_{i,t}^2}, \frac{\partial^2 m_{i,t}}{\partial V_{i,t}^2} < 0$. Lastly, the matching function satisfies $m_{i,t}(U_{i,t}, V_{i,t}) \leq \min\{U_{i,t}, V_{i,t}\}$. This condition ensures that the numbers of vacancies and unattached workers at the end of the period t are non-negative. Intuitively,

[1] While I denote total searchers by $U_{i,t}$ for consistency with the notation commonly used in the search-and-matching literature, I think of $U_{i,t}$ as capturing the number of unattached workers searching for a job in sector i.

[2] By allowing the function $m_{i,t}$ to be sector specific and time varying, I can allow for exogenous variation in the efficiency of the matching process across sectors and over time.

it implies firms cannot conjure workers out of thin air by posting more vacancies.

Because the matching function is constant returns to scale, I can conveniently express the probability of a firm filling a vacancy (or the vacancy-filling rate) in sector i in terms of sector-specific labor market tightness $\theta_{i,t} = \frac{V_{i,t}}{U_{i,t}}$:

$$Q_{i,t} = \frac{H_{i,t}}{V_{i,t}} = m_{i,t}\left(\theta_{i,t}^{-1}, 1\right). \tag{2}$$

Since $m_{i,t}$ is increasing in both arguments, the vacancy-filling rate falls as tightness rises: it gets harder for firms to hire as tightness rises. Similarly, the probability that a worker finds a job in sector i (or the job-finding rate), conditional on searching in that sector, is

$$F_{i,t} = \frac{H_{i,t}}{U_{i,t}} = m_{i,t}(1, \theta_{i,t}). \tag{3}$$

The job-finding rate is increasing in tightness: it gets easier for workers to find a job as tightness rises. Since $m_{i,t}(U_{i,t}, V_{i,t}) \leq \min\{U_{i,t}, V_{i,t}\}$, both the vacancy-filling rate and the job-finding rate are bounded between 0 and 1: $Q_{i,t}, F_{i,t} \in [0, 1]$.

I assume that a vacancy requires $R_{i,t}$ hours of labor to post and maintain—firms pay a recruiting cost in terms of labor, representing time spent on posting vacancies, reviewing applications, interviewing candidates, and other recruiting activities. As tightness rises, firms must post more vacancies, employ more recruiters, and incur higher recruiting costs to achieve a given level of hiring. This creates a link between labor market tightness and marginal costs. Finally, I assume that an exogenous fraction $s_{i,t}$ of workers separate from their jobs at the beginning of each period.

2.2 Firms

The production structure in each sector incorporates the three key model ingredients: (1) sticky prices, (2) a production network, and (3) search-and-matching frictions. To allow for sticky prices, I assume that each sector contains a continuum of monopolistically competitive firms that use labor and intermediate goods to produce output and set prices subject to adjustment costs. The presence of price adjustment costs generates a sector-specific New Keynesian Phillips curve linking inflation to marginal costs and expected future inflation. As in Baqaee (2018),Baqaee and Farhi (2019), and Baqaee and Rubbo (2022), firms use intermediate inputs from other sectors, generating a production network. Finally, firms make hiring decisions subject to the search-and-matching structure described above, linking marginal costs to labor market conditions.

Each sector contains a unit continuum of monopolistically competitive intermediate goods producers indexed by z, and a representative firm that aggregates the intermediate goods to produce sectoral output. The representative firm in each sector produces the sectoral output $Y_{i,t}$ from the outputs of each of the intermediate firms $Y_{i,t}(z)$ using CES technology.

$$Y_{i,t} = \left(\int_0^1 Y_{i,t}(z)^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

where ϵ is the elasticity of substitution between the outputs of the different intermediate goods producers in sectoral output. This representative firm chooses how much of each firm z's output to use in production to minimize its costs, resulting in downward-sloping demand functions for each z:

$$Y_{i,t}(z) = \left(\frac{P_{i,t}(z)}{P_{i,t}} \right)^{-\epsilon} Y_{i,t} \tag{5}$$

where $P_{i,t}(z)$ is the price of firm z's output.

As in the standard production networks setup, the monopolistically competitive intermediate goods producers in each sector produce their output $Y_{i,t}(z)$ using labor $N_{i,t}(z)$ and intermediate inputs $X_{ij,t}(z)$.

$$Y_{i,t}(z) = A_{i,t} \left(\omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t}(z)^{\frac{\epsilon_y-1}{\epsilon_y}} + \sum_{j=1}^{J} \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t}(z)^{\frac{\epsilon_y-1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y-1}} \tag{6}$$

where $A_{i,t}$ is a sector-specific productivity shock, which captures, for instance, supply chain disruptions that disproportionately affect one sector. ω_{in} is the share of labor and ω_{ij} is the share of sector j's output in sector i's production process. ϵ_y is the elasticity of substitution between labor and intermediate inputs.

Each firm z chooses how many vacancies $V_{i,t}(z)$ to post, taking the total number of vacancy postings in sector i, $V_{i,t} = \int_0^1 V_{i,t}(z)dz$, and therefore sector level tightness $\theta_{i,t}$, as given. The number of newly hired workers at firms z, $H_{i,t}(z)$ depends on the vacancy-filling rate $Q_{i,t}$, implying the following law of motion for employment at firm z:

$$N_{i,t}(z) + R_{i,t}V_{i,t}(z) = Q_{i,t}V_{i,t}(z) + (1 - s_{i,t}) \left(N_{i,t-1}(z) + R_{i,t-1}V_{i,t-1}(z) \right). \tag{7}$$

Finally, each firm z chooses its price $P_{i,t}(z)$ subject to Rotemberg adjustment costs.[3], the number of vacancy postings $V_{i,t}(z)$, and how much of each intermediate input to use in

[3] I choose Rotemberg adjustment costs because they do not lead to price dispersion across firms, and I therefore do not need to track the distribution of prices, output, and therefore employment across firms.

production $\{X_{ij,t}(z)\}_{j=1}^{J}$ in order to maximize its dividend payments[4],

$$\max_{\{P_{i,t+s}(z), V_{i,t+s}(z), \{X_{ij,t+s}(z)\}_{j=1}^{J}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \left[SDF_{t|t+s} \frac{D_{i,t+s}(z)}{P_{t+s}} \right] \tag{8}$$

subject to Eqs. (5), (6), and (7). Where

$$\begin{aligned} \frac{D_{i,t}(z)}{P_t} = & \frac{P_{i,t}(z)}{P_t} \left(\frac{P_{i,t}(z)}{P_{i,t}} \right)^{-\epsilon} Y_{i,t} - \frac{W_{i,t}}{P_t} \left[N_{i,t}(z) + R_{i,t} V_{i,t}(z) \right] \\ & - \sum_{j=1}^{J} \frac{P_{j,t}}{P_t} X_{ij,t}(z) - \frac{\psi_{p,i}}{2} \left(\frac{P_{i,t}(z)}{\Pi P_{i,t-1}(z)} - 1 \right)^2 Y_{i,t}. \end{aligned}$$

Since all firms in a given sector face an identical problem, they all make identical input choices and pricing decisions. I therefore drop the z indexation from the following optimality conditions. I allow the parameter governing the size of price adjustment costs, $\psi_{p,i}$, to vary at the sector level (Ferrante et al., 2023; Pasten et al., 2020).

Firms post vacancies up to the point where the marginal cost of an additional vacancy equals the marginal benefit of an additional vacancy

$$\frac{W_{i,t}}{P_t} R_{i,t} = \mu_{i,t}(Q_{i,t} - R_{i,t}) + E_t \left[SDF_{t|t+1} \Pi_{t+1} (1 - s_{i,t+1}) \mu_{i,t+1} R_{i,t} \right] \tag{9}$$

where $\mu_{i,t}$ is the marginal value of an additional employee to the firm. The marginal value of an additional employee equals the marginal product of labor today, net of the wage, plus the continuation value of the employee.

$$\mu_{i,t} = \lambda_{i,t} \beta_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y - 1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} - \frac{W_{i,t}}{P_t} + E_t \left[SDF_{t|t+1} \Pi_{t+1} (1 - s_{i,t+1}) \mu_{i,t+1} \right] \tag{10}$$

where $\lambda_{i,t}$ is the real marginal cost. Each firm chooses intermediate inputs so that good j's share in total costs satisfies

$$\frac{\frac{P_{j,t}}{P_t} X_{ij,t}}{\lambda_{i,t} Y_{i,t}} = (\beta_{ix} \omega_{ij})^{\frac{1}{\epsilon_y}} \left(A_{i,t} \frac{X_{ij,t}}{Y_{i,t}} \right)^{\frac{\epsilon_y - 1}{\epsilon_y}} \tag{11}$$

Each firm's optimal price setting implies a sector-specific Phillips curve as in Ferrante et al.

[4]By choosing $V_{i,t}(z)$ the firm is also choosing $N_{i,t}(z)$ through the law of motion for employment.

(2023), La'O and Tahbaz-Salehi (2022), and Rubbo (2023b, 2024).

$$\left(\frac{\Pi_{i,t}}{\Pi}-1\right)\frac{\Pi_{i,t}}{\Pi}=\frac{\epsilon}{\phi_{p,i}}\left(\frac{MC_{i,t}}{P_t}-\frac{\epsilon-1}{\epsilon}\frac{P_{i,t}}{P_t}\right)+E_t\left[SDF_{t+1|t}\Pi_{t+1}\left(\frac{\Pi_{i,t+1}}{\Pi}-1\right)\frac{\Pi_{i,t+1}}{\Pi}\frac{Y_{i,t+1}}{Y_{i,t}}\right] \tag{12}$$

where $SDF_{t+1|t}=\beta\frac{U_{c,t+1}}{U_{c,t}}\Pi_{t+1}^{-1}$ is the household's stochastic discount factor.

Finally, each sector's output must satisfy the market clearing condition

$$Y_{i,t}=C_{i,t}+\sum_{j=1}^{J}X_{ji,t} \tag{13}$$

2.3 Households

A unit mass of identical households indexed by z supply labor to firms, consume goods produced by each of the J sectors, and save in the form of bonds.

Households have CES preferences for consumption across the sectors. The aggregate consumption good C_t is therefore:

$$C_t=\left(\sum_{i=1}^{J}\alpha_{i,t}^{\frac{1}{\epsilon_d}}C_{i,t}^{\frac{\epsilon_d-1}{\epsilon_d}}\right)^{\frac{\epsilon_d}{\epsilon_d-1}} \tag{14}$$

where ϵ_d is the elasticity of substitution across sectors, and $\alpha_{i,t}$ is a time-varying exogenous preference shifter that captures households' changing preferences for consumption in sector i. Note that $C_{i,t}$ is final household consumption, not total output, because some of the output from each sector is used as an intermediate input in production.

These preferences imply downward-sloping consumption demand functions:

$$C_{i,t}=\alpha_{i,t}\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_d}C_t \tag{15}$$

where the aggregate consumer price index is

$$P_t=\left(\sum_{i=1}^{J}\alpha_{i,t}P_{i,t}^{1-\epsilon_d}\right)^{\frac{1}{1-\epsilon_d}} \tag{16}$$

Each household maximizes lifetime utility subject to their budget constraint and the law of motion for employment in each sector, which depends on the job-finding rates.

$$\max_{\left\{C_{t+s}(z),\{L_{i,t+s}(z)\}_{i=1}^{J}\right\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \beta^s U_{t+s}\left(C_{t+s}, \left\{\tilde{L}_{i,t+s}, L_{i,t+s}\right\}_{i=1}^{J}\right)$$

$$\text{s.t. } P_{t+s}C_{t+s}(z) + B_{t+s}(z) = (1+i_{t+s})B_{t+s-1}(z) + \sum_{i=1}^{J} W_{i,t+s}\tilde{L}_{i,t+s}(z) + T_{t+s}(z)$$

$$\tilde{L}_{i,t+s}(z) = (1-F_{i,t+s})(1-s_{i,t+s})\tilde{L}_{i,t+s-1}(z) + F_{i,t+s}L_{i,t+s}(z)$$

$$\text{and } \sum_{i=1}^{J} L_{i,t+s} \leq \overline{L}_{t+s} \tag{17}$$

where $C_t(z)$ is the aggregate consumption bundle, $L_{i,t}(z)$ is labor supplied to sector i (including employed and searching workers), $\tilde{L}_{i,t}(z)$ is employment in sector i, $B_t(z)$ is a one-period risk-free bond, i_t is the nominal interest rate, and $T_{t+s}(z)$ are lump-sum transfers and dividend payments. The model features a form of directed search: households choose how to divide their endowment of unattached workers, $\overline{L}_t - \sum_{i=1}^{J}(1-s_{i,t})\tilde{L}_{i,t-1}(z)$, across the sectors based on both the real wage and the likelihood of finding a job in each sector.

The households' period utility function is:

$$U_{t+s}(\cdot) = Z_{t+s}\left[\frac{C_{t+s}(z)^{1-\sigma}}{1-\sigma} - \sum_{i=1}^{J}\left(\chi_{i,t+s}\frac{\tilde{L}_{i,t+s}(z)^{1+\varphi}}{1+\varphi} + \frac{\psi_{L,i}}{2}\left(\frac{L_{i,t+s}/\overline{L}_{t+s}}{L_{i,t+s-1}/\overline{L}_{t+s-1}} - 1\right)^2 L_{i,t+s}\right)\right]. \tag{18}$$

The final term in the households' utility function, $\frac{\psi_{L,i}}{2}\left(\frac{L_{i,t+s}/\overline{L}_{t+s}}{L_{i,t+s-1}/\overline{L}_{t+s-1}} - 1\right)^2 L_{i,t+s}$, is a quadratic cost of adjusting the fraction of workers who search in a given sector i. This adjustment cost captures frictions in relocating across sectors because of moving costs, retraining costs, or other frictions. These types of frictions are empirically important in determining where workers search for jobs (Humlum, 2021), and within the model will be important for determining the labor-supply response to sectoral shocks. $\psi_{L,i}$ governs the size of reallocation costs while φ governs how responsive labor supply is to changes in real wages.

The households first-order conditions imply the standard Euler equation for consumption:

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left[C_{t+1}^{-\sigma}\frac{Z_{t+1}}{Z_t}\Pi_{t+1}^{-1}\right] \tag{19}$$

The household optimal decisions for how many workers to send in search of a job in each sector equalizes the expected return from searching in each sector, subject to the labor adjustment costs.

$$\delta_t = \Xi_{i,t} F_{i,t} - \psi_{L,i} Z_t \left(\left[\frac{L_{i,t}/\overline{L}_t}{L_{i,t-1}/\overline{L}_{t-1}} - 1 \right] + \frac{1}{2} \left[\frac{L_{i,t}/\overline{L}_t}{L_{i,t-1}/\overline{L}_{t-1}} - 1 \right]^2 \right)$$
$$+ \beta \psi_{L,i} E_t \left[Z_{t+1} \left(\frac{L_{i,t+1}/\overline{L}_{t+1}}{L_{i,t}/\overline{L}_t} - 1 \right) \frac{L^2_{i,t+1}/\overline{L}_{t+1}}{L^2_{i,t}/\overline{L}_t} \right] \tag{20}$$

where $\Xi_{i,t}$ is the value to the household of an additional employed worker in sector i , $F_{i,t}$ is the job-finding rate in sector i, and δ_t marginal value of an additional unit of aggregate labor to the household. Absent labor adjustment costs, the household adjusts search across sectors up to the point where $F_{i,t}\Xi_{i,t} = F_{j,t}\Xi_{j,t}$. $F_{i,t}\Xi_{i,t}$ is the expected return from searching in sector i. It is the probability of finding a job in sector i times the value to the household of an additional employee in sector i.

The value of an additional employed worker in sector i is the value of being employed today—how much the employed worker earns, converted into utils, net of the utility cost of an additional employee—plus the continuation value of being employed tomorrow.

$$\Xi_{i,t} = Z_t \left(C_t^{-\sigma} \frac{W_{i,t}}{P_t} - \chi_{i,t} \tilde{L}^{\varphi}_{i,t} \right) + \beta E_t \left[(1 - F_{i,t+1})(1 - s_{i,t+1}) \Xi_{i,t+1} \right] \tag{21}$$

Finally, the number of unattached workers at the beginning for the period is given by

$$U_{i,t} = L_{i,t} - (1 - s_{i,t}) \tilde{L}_{i,t-1}.$$

2.4 Wages

As in standard search-and-matching models, when workers are matched with firms they face a bilateral monopoly. Wages are therefore not determined by the model's equilibrium conditions,[5] and instead require an assumption about the wage-setting process.

I use Nash bargaining between the household and the firm as the basis for my wage determination process. Let $\mathcal{S}^f_{i,t}$ denote the firm surplus from a match in sector i and let $\mathcal{S}^h_{i,t}$ denote the household surplus from a match in sector i. The Nash bargaining solution implies

$$\mathcal{S}^h_{i,t} = \kappa \mathcal{S}^f_{i,t} \tag{22}$$

where κ is determined by the relative bargaining power of the household.[6]

[5]The household and firm equilibrium conditions define a range of acceptable wages for a given match.
[6]If, for instance, the firm and household split the total surplus equally, then $\kappa = 1$.

Because hiring an additional worker requires $\frac{1}{Q_{i,t}}$ vacancy postings at cost $R_{i,t}W_{i,t}$, firms can replace any worker at a cost $\frac{R_{i,t}}{Q_{i,t}}W_{i,t}$. By free entry into vacancy postings, the firm surplus from an existing worker is:

$$\mathcal{S}^{f}_{i,t} = \frac{R_{i,t}}{Q_{i,t}} W_{i,t} \tag{23}$$

From the household's first order conditions, the value of an additional employee to the household, measured in utils, is given by $\Xi_{i,t}$ in Eq.(21). The surplus in nominal terms is therefore $\frac{\Xi_{i,t}}{Z_t C_t^{-\sigma}} P_t$. The Nash bargaining solution then implies that the Nash-bargained wage is

$$W_{i,t} = \frac{Q_{i,t}}{Q_{i,t} - \kappa R_{i,t}} \left(\chi_{i,t} \tilde{L}^{\varphi}_{i,t} C^{\sigma}_t P_t - \kappa E_t \left[SDF_{t+1|t} (1 - F_{i,t+1})(1 - s_{i,t+1}) \frac{R_{i,t+1}}{Q_{i,t+1}} W_{i,t+1} \right] \right) \tag{24}$$

As is well understood since Shimer (2005) and R. E. Hall (2005), fully flexible Nash bargaining leads to counterfactually large wage fluctuations and small employment fluctuations over the business cycle. I therefore allow for a degree of wage rigidity by assuming that the final wage is a weighted average of this Nash bargaining solution and last period's wage and I allow for exogenous shocks to the wage, $W^{s}_{i,t}$, so that the overall wage is given by

$$\begin{aligned} W_{i,t} = & \left[\frac{Q_{i,t}}{Q_{i,t} - \kappa R_{i,t}} \left(\chi_{i,t} \tilde{L}^{\varphi}_{i,t} C^{\sigma}_t P_t - \kappa E_t \left[SDF_{t+1|t} (1 - F_{i,t+1})(1 - s_{i,t+1}) \frac{R_{i,t+1}}{Q_{i,t+1}} W_{i,t+1} \right] \right) \right]^{1-\rho_w} \\ & \times \left(W_{i,t-1} \right)^{\rho_w} W^{s}_{i,t} \end{aligned} \tag{25}$$

This specification nests fully flexible Nash bargained wages ($\rho_w = 0$) and fully rigid wages ($\rho_w = 1$). This is a reduced-form way to capture that wages may not adjust fully within a month to changes in economic conditions, and as Gertler and Trigari (2009) demonstrate, partial wage rigidity allows the search-and-matching model to generate more realistic fluctuations in wages and unemployment.

2.5 Monetary Policy and Shock Processes

I close the model by assuming that the central bank sets the nominal interest rate according to a Taylor rule in inflation and the output gap.

$$(1+i_t) = (1-i_{t-1})^{\rho_i} \left(\left[\frac{\Pi_t^{agg}}{\Pi}\right]^{\phi_\pi} \left[\frac{Y_t^{agg}}{Y_t^{agg,flex}}\right]^{\phi_y} \right)^{1-\rho_i} \varepsilon_t^m \tag{26}$$

where ε_t^m is a normally distributed monetary policy shock.

Finally, I assume that the natural logarithms of Z_t, $\chi_{i,t}$, $\alpha_{i,t}$, $S_{i,t}$, $R_{i,t}$, $W_{i,t}^s$, follow AR(1) processes with normally distributed innovations.

$$\log Z_t = \rho_z \log Z_{t-1} + \varepsilon_t^z, \qquad \log \chi_{i,t} = \rho_\chi \log \chi_{i,t-1} + \varepsilon_{i,t}^\chi \tag{27}$$

$$\log \alpha_{i,t} = \rho_\alpha \log \alpha_{i,t-1} + \varepsilon_{i,t}^\alpha, \qquad \log S_{i,t} = \rho_S \log S_{i,t-1} + \varepsilon_{i,t}^S \tag{28}$$

$$\log R_{i,t} = \rho_r \log R_{i,t-1} + \varepsilon_{i,t}^R, \qquad \log W_{i,t}^s = \rho_w \log W_{i,t-1}^s + \varepsilon_{i,t}^{W^s} \tag{29}$$

I summarize the full set of equilibrium conditions in Appendix A.[7]

2.6 Equilibrium Definition

An equilibrium is a set of sequences for endogenous variables

$$\left\{ C_t, Y_t, \Pi_t, \left\{ C_{i,t}, L_{i,t}, \tilde{L}_{i,t}, U_{i,t}, V_{i,t}, N_{i,t}, Y_{i,t}, \{X_{ij,t}\}_{j=1}^J \right\}_{i=1}^J \right\}_{t=0}^\infty$$

Given paths for the exogenous processes, $\left\{ Z_t, \varepsilon_t^m \{\alpha_{i,t}, R_{i,t}, s_{i,t}, \chi_{i,t}, W_{i,t}^s\}_{i=1}^J \right\}_{t=0}^\infty$, such that:

1. Firms maximize profits,

2. Households maximize lifetime utility,

3. Markets clear

$$Y_{i,t} = C_{i,t} + \sum_{j=1}^J X_{ji,t}, \qquad N_{i,t} + R_{i,t} V_{i,t} = \tilde{L}_{i,t}, \qquad Y_t = C_t \tag{30}$$

4. The central bank sets the nominal policy rate subject to the Taylor rule in Equation (26).

[7] I also impose that $\sum_{i=1}^J \alpha_{i,t} = 1$ for all t so that $\alpha_{i,t}$ shifts relative preferences across sectors without changing aggregate demand.

3 Results: Inflation and labor market dynamics

In this section, I solve the model nonlinearly under perfect foresight and highlight three key predictions. First, accounting for labor market frictions alters the propagation of sectoral shocks. Second, the model features a nonlinear Phillips curve at the sectoral and aggregate levels that steepens at high levels of labor market tightness. Third, as a result, how important each sector is for aggregate inflation changes endogenously with labor market conditions, and monetary policy is less effective at stimulating output when labor markets are tight. I then show how these three features together imply that a shift in relative demand from one sector to another can generate aggregate inflation, a steepening of the aggregate Phillips curve, and a decline in aggregate matching efficiency—an outward shift in the Beveridge curve.

3.1 Solution Method and Calibration

I solve the model nonlinearly by simulating impulse responses to a range of shocks under perfect foresight. I solve for the paths of endogenous variables consistent with the nonlinear equilibrium conditions for a given path of exogenous shocks using the Newton-Raphson method. That is, I write the equilibrium conditions as a system of nonlinear equations.

$$F(X) = 0, \tag{31}$$

where X stacks all endogenous variables over the simulation horizon T.[8] I then solve for X such that $F(X) = 0$ holds given the path of exogenous variables, using the steady-state as the initial guess and the Jacobian to update the guess until convergence.

I solve the model nonlinearly for two reasons. First, sector-level shocks can lead to large deviations from steady-state at the sector level, even when aggregate variables remain relatively close to steady-state. The large deviations from steady-state at the sector level necessitate a treatment that accounts for the possible nonlinearities for large shocks. Second, labor market tightness routinely rises and falls by large amounts over the business cycle, and rose to over 100% above the pre-Covid average in the aftermath of the pandemic. The search-and-matching process introduces potentially interesting nonlinear dynamics for large changes in tightness that I want to be able to capture.

I demonstrate the model's mechanisms in a simulated symetric two-sector version of the model. The results extend to economies with more than two sectors. I calibrate the model so that both sectors have an intermediate-share of 50%, which roughly matches the intermediates share in the Bureau of Economic Analysis (BEA) input–output (I–O) tables.

[8] I use $T = 200$ and find that the results are robust to increasing the horizon T.

I assume the share of their own output in intermediates is 70% and the share of the other sector's output in their intermediates is 30%, to reflect the large diagonal elements in the I–O tables.

I set the other parameters of the model using existing estimates from the literature. I set the elasticity of substitution between intermediates and labor (ϵ_y) to 0.2 to match the estimated elasticity of substitution between intermediate inputs in Atalay (2017). I purposefully choose an estimate on the low end of the range of estimates in the litarature because this parameter captures the elasticity of substution between intermediates and labor in the short run (a period in the model is just one month). I want the model to capture that labor scarcity in a sector can lead to supply constraints because firms find it difficult to substitute between inputs in the short run, a feature highlighted as potentially important in the post-pandemic recovery by Lorenzoni and Werning (2024). I set the sectoral elasticity of substitution between intermediate suppliers (ϵ) to 4.33, which implies a markup of about 30% (R. Hall, 2018). I set ϕ_π, ϕ_y, and ρ_i to 1.39, 1.01, and 0.82, respectively, based on estimates for the Greenspan–Bernanke era in Carvalho et al. (2021). I set ψ_p by aggregating the estimates for the frequency of price changes at the sector level in Pasten et al., 2020 using sales shares as weights. I set $\varphi = 3.57$, consistent with Chetty (2012). I set $\psi_L = 31.24$, roughly half the value of a job to the household, to capture the fact that workers face high costs of switching across occupations and sectors (Artuç et al., 2010; Caliendo et al., 2019; Cardoza et al., 2022; Humlum, 2021).

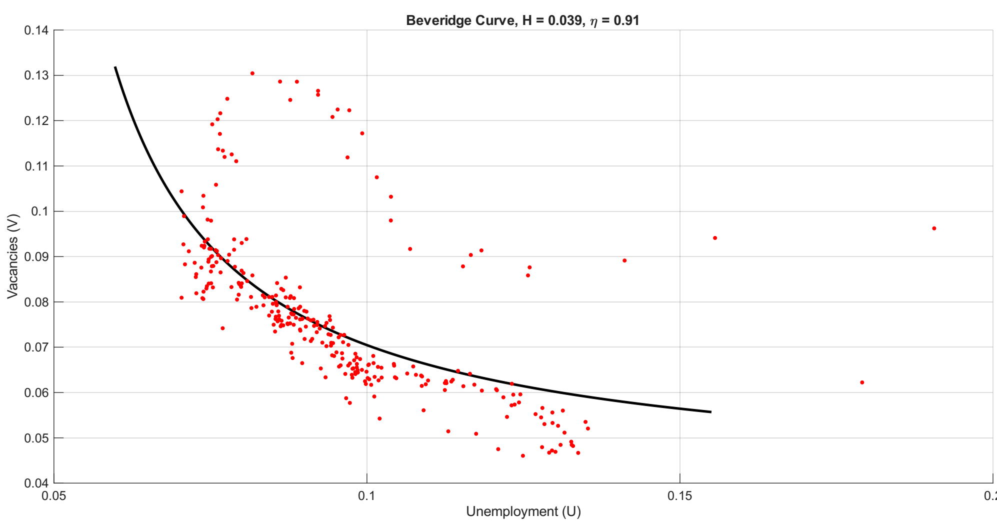


Figure 1: Fit of model implied Beveridge curve (at average hiring rate) to JOLTS data (2000–2025)

Finally, I estimate η_i by nonlinear least squares using aggregate data on vacancies, unemployment, and hires from the Job Openings and Labor Turnover Survey (JOLTS) from

2000–2025. Figure 1 demonstrates that the model-implied Beveridge curve at the average hiring rate that is a good fit to the JOLTS data. In particular, the functional form I assume can capture both the flatter portion of the Beveridge curve when unemployment is high, and the steepening at low levels of unemployment. I summarize the full parametrization in Table 1.

Table 1: Two-symmetric-sectors Calibration

Parameter	Value	Source
ϵ_d	0.60	Atalay (2017)
ϵ_y	0.20	Elasticity of substitution between intermediate inputs, Atalay (2017)
ϵ	4.33	Implied markup of 30%, R. Hall (2018)
ϕ_π	1.39	Estimates for Greenspan–Bernanke Period, Carvalho et al. (2021)
ϕ_y	1.01	Estimates for Greenspan–Bernanke Period, Carvalho et al. (2021)
ρ_i	0.82	Estimates for Greenspan–Bernanke Period, Carvalho et al. (2021)
κ	1	Equal surplus sharing between households and firms
ρ_w	0.8	—
σ	2	—
φ	3.57	Chetty (2012)
η_i	0.93	Estimate using JOLTS (2000-2025)
$\Omega_{d,i}$	0.50	Nominal Consumption Share (Symmetric)
$\Omega_{x,i}$	0.50	Roughly Intermediates Share in BEA I–O tables
Ω_{ii}	0.70	—
Ω_{ij}	0.30	—
$\psi_{p,i}$	60.11	Pasten et al. (2020)
$\psi_{L,i}$	31.24	Half value of a job to household.

Note: Calibrated parameter values in two-sector-symmetric economy. Based on estimates or standard values in the literature.

3.2 The Propagation of Shocks Through the Labor Market

I begin by showing how the presence of labor market frictions alters the propagation of sectoral shocks via a new labor channel. Higher labor demand in one sector leads to a reduction in labor supply elsewhere. As a result, positive demand shocks in one sector can effectively create negative supply shocks in other sectors. This is the converse of, but closely related to, the finding in Guerrieri et al. (2022) that negative supply shocks in one sector lead to negative demand shocks in other sectors.

Consider, for instance, a permanent 1% increase in the household consumption preference for goods in sector i, captured by a permanent increase in $\varepsilon^{\alpha}_{i,t}$. The shock raises $\alpha_{i,t}$, and therefore, as Figure 2a shows, household consumption of sector i's output, $C_{i,t}$.

$$C_{i,t} = \alpha_{i,t} \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon_d} C_t, \qquad\qquad C_{j,t} = \alpha_{j,t} \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon_d} C_t$$

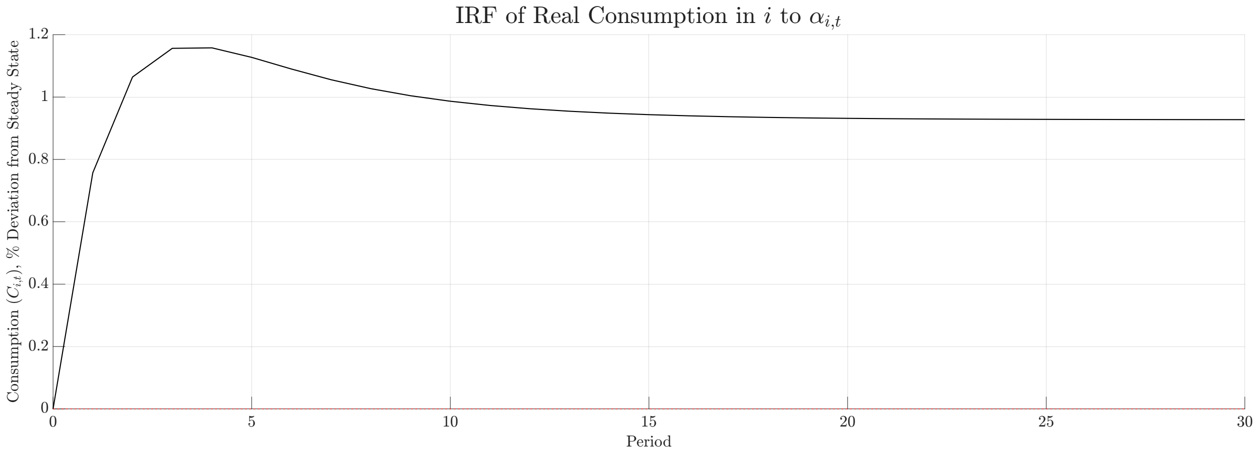
Because $\sum_{j=1}^{J} \alpha_{j,t} = 1$, this implies a decline in $\alpha_{j,t}$ for $j \neq i$, and therefore a decline in household consumption of sector j's output.[9] For simplicity, I assume in the figures below that the increase in $\alpha_{i,t}$ is offset entirely by an equal decline in $\alpha_{j,t}$ in just one other sector j.

The increase in demand for sector i output leads to an increase in inflation in sector i (see Figure 2b). As is well understood, in any production network economy this increase in prices spills over to other sectors through input–output linkages in production. The increase in inflation in sector i affects the price of intermediate inputs, and therefore marginal costs, in sector j. All else equal, this results in higher sector j inflation than absent the network, raising aggregate inflation.

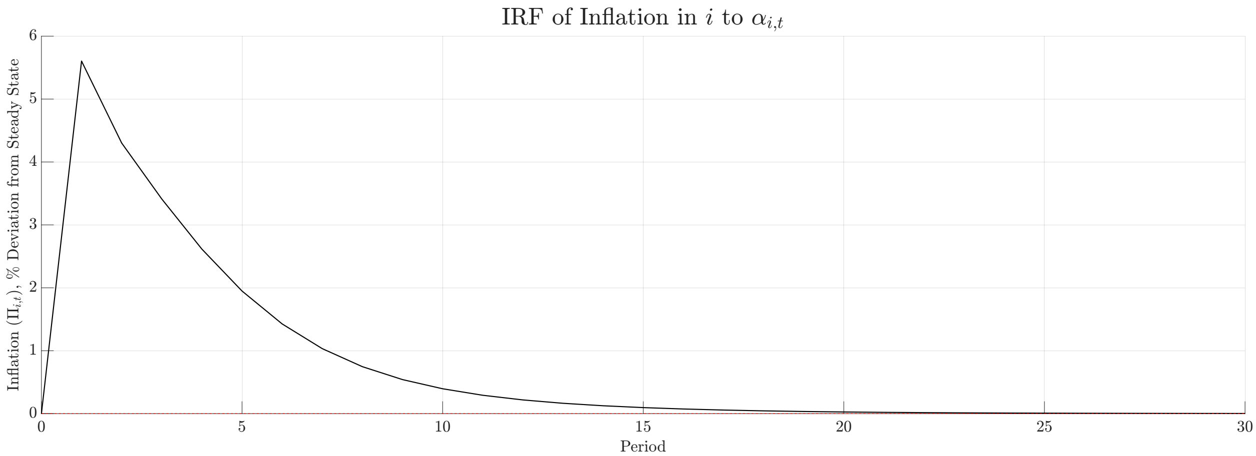
As firms in sector i increase hiring to meet the increase in consumption demand for their good, both wages and the job-finding rate in sector i rise (see Figure 3a). The job-finding rate increases on impact because the increase in vacancy postings by firms raises labor market tightness, $\theta_{i,t}$, and therefore $Q_{i,t}$. Because the households' optimal search strategy implies that the amount of labor searching in sector i rises with wages and the job-finding rate, this in turn, leads to an increase in the labor supply to sector i (See Figure 3b), and, similarly, a decline in labor supply in sector j.

The spillovers in labor supply from sector j to sector i increase tightness, and therefore hiring costs and marginal costs in sector j, leading to a new spillover from sector-specific shocks. Indeed, as Figure 3c demonstrates, the labor supply spillovers can be strong enough to eventually fully offset the increase in labor demand in sector i, leading to a decline in tightness in sector i and an increase in tightness in sector j, once the initial spike in labor demand subsides in sector i.

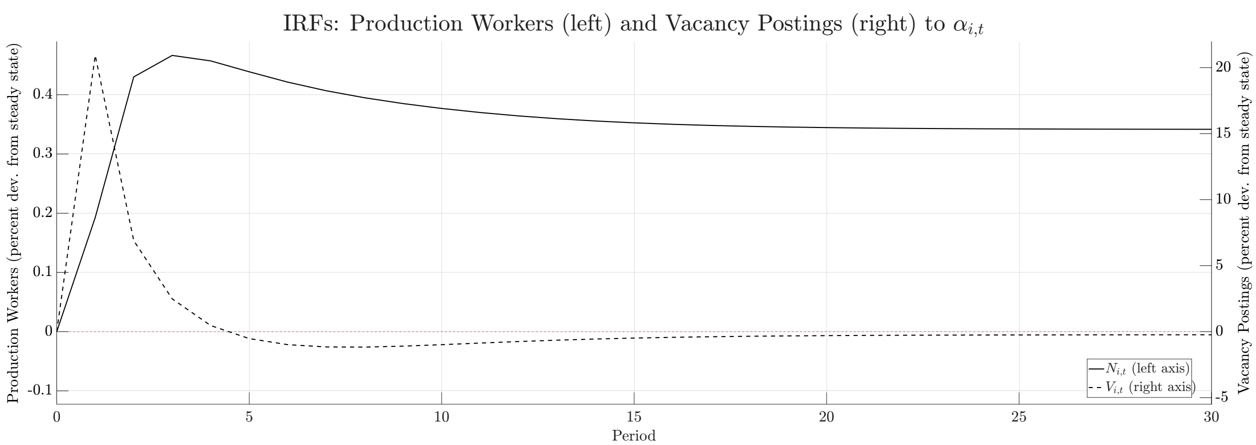
[9] As a result, this shock is pure demand-reallocation shock that does not effect the level of aggregate demand.



(a) Response of consumption in secector i to an increase in $\alpha_{i,t}$

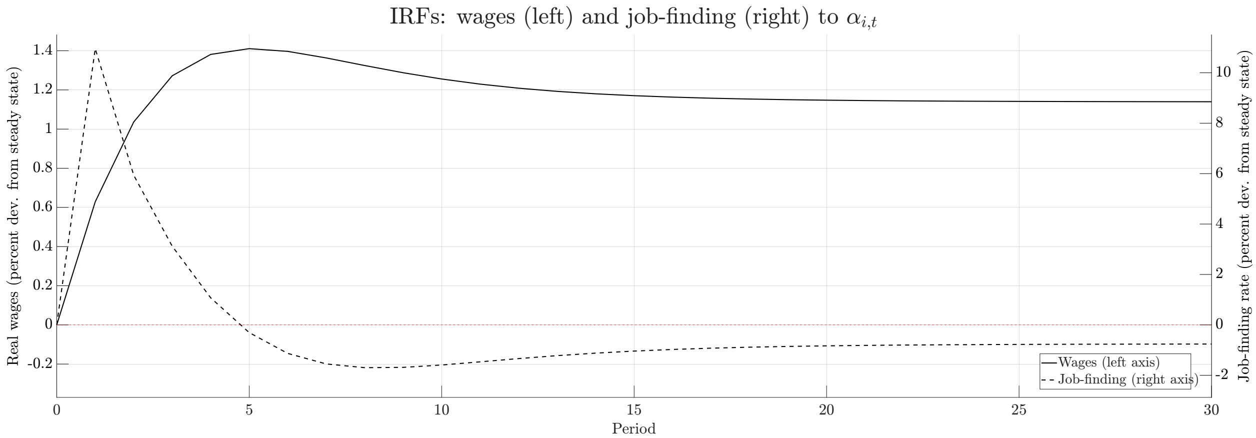


(b) Response of inflation in secector i to an increase in $\alpha_{i,t}$

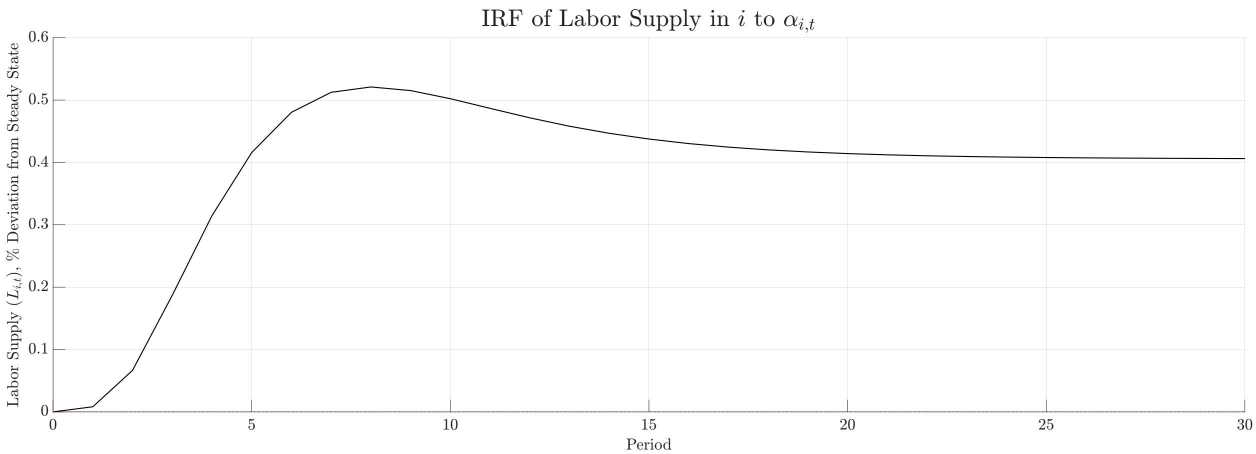


(c) Response of productive employment and vacancy postings in secector i to an increase in $\alpha_{i,t}$

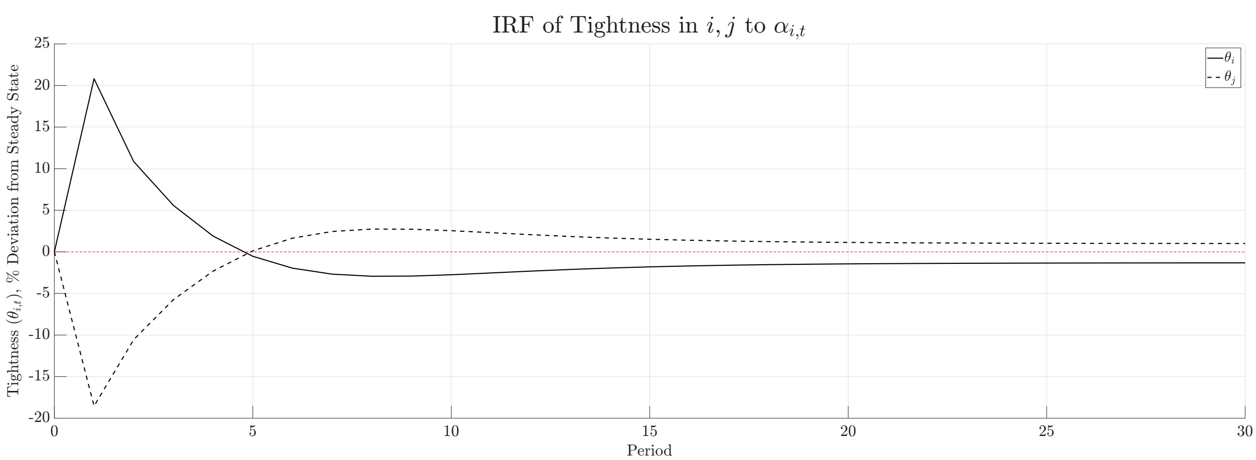
Figure 2: Impulse responses to a shock to α_s.



(a) Wages (left axis) and job-finding rate (right axis) in secector i to an increase in $\alpha_{i,t}$



(b) Response of Labor Supply in secector i to an increase in $\alpha_{i,t}$



(c) Response of Tightness in sectors i and j to an increase in $\alpha_{i,t}$

Figure 3: Impulse responses to a shock to α_s.

3.3 A nonlinear Sector-Specific Phillips Curve

The presence of labor market frictions leads to a nonlinear Phillips curve that steepens at high labor market tightness. This finding is consistent with the evidence in Gitti (2024) and Benigno and Eggertsson (2023). Benigno and Eggertsson (2023) develop a model to capture this nonlinearity by assuming a kink in the wage-setting process at levels of tightness above 1. I demonstrate that the nonlinearity arises naturally from a standard search-and-matching framework with generalized Nash bargained wages once I solve the model nonlinearly, without the need to assume ad hoc kinks in either the wage setting or search process. In addition, in a multi-sector economy, the presence of nonlinearities at the sector level leads to endogenous changes in which sectors are the most important for aggregate inflation as local labor market conditions change.

To build intuition, consider a simplified version of the model outlined above where wages are rigid, $\rho_w = 1$, there are no household labor adjustment costs, $\psi_L = 0$, and firms and households make static labor supply decisions.[10] In this case, to first-order aggregate inflation is given by

$$\pi_t^{agg} = \mathbf{\Gamma}_\theta \boldsymbol{\theta}_t + \mathbf{\Gamma}_\pi E_t \boldsymbol{\pi}_{t+1} + \upsilon_t. \tag{32}$$

This is a first-order approximation of the aggregate New Keynesian Phillips curve from combining Equations (12) and (16), where $\boldsymbol{\theta}_t$ is a $J \times 1$ vector of sectoral tightness, $\boldsymbol{\pi}_{t+1}$ is a $J \times 1$ vector of sectoral inflation, and υ_t is an endogenous cost push shock as in Rubbo (2023b).[11] $\mathbf{\Gamma}_\theta$ and $\mathbf{\Gamma}_\pi$ are $1 \times J$ coefficient vectors that capture how important each sector is for aggregate inflation. This expression is equivalent to the network Phillips curve derived in Rubbo (2023b), adjusted for the presence of labor market frictions.

[10] This last simplification can be rationalized by assuming, as in Benigno and Eggertsson (2023), that all workers separate at the start of each period, before a random fraction $(1 - s_{i,t})$ are reemployed in sector i without needing to go through the matching process. As a result, households and firms no longer account for the dynamic consequences of their labor supply and demand decisions.

[11] See the appendix for a detailed derivation. The expression for the endogenous cost push shock is

$$\begin{aligned}
\upsilon_t &= \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{I} + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right] \boldsymbol{p}_{t-1} \\
&+ \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{\Omega}_n \mathbf{\Gamma}_Q \boldsymbol{r}_t - \boldsymbol{a}_t\right] \\
&+ \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\mathbf{1}\boldsymbol{\Omega}_d'\right] \boldsymbol{\alpha}_t
\end{aligned}$$

The coefficient matrices on tightness and expected future inflation are

$$\Gamma_\theta = \Omega_d' \left[\boldsymbol{I} - \boldsymbol{\lambda} \left(\Omega_x - \boldsymbol{I} \right) \left(\boldsymbol{I} - \mathbf{1}\Omega_d' \right) \right]^{-1} \Omega_n \Gamma_Q \boldsymbol{\eta}$$
$$\Gamma_\pi = \beta \Omega_d' \left[\boldsymbol{I} - \boldsymbol{\lambda} \left(\Omega_x - \boldsymbol{I} \right) \left(\boldsymbol{I} - \mathbf{1}\Omega_d' \right) \right]^{-1}$$

where Ω_d is a $J \times 1$ vector of steady-state nominal consumption shares, Ω_x is the $J \times J$ input–output matrix capturing the steady-state expenditure shares of each sector i on each intermediate input j, Ω_n is a $J \times J$ diagonal matrix with the steady-state labor share in each sector on the diagonal. The two new terms relative to a standard production network setup are $\boldsymbol{\eta}$, a $J \times J$ diagonal matrix with the negative of the elasticity of the vacancy-filling rate to changes in tightness along the diagonal, and Γ_Q, a $J \times J$ matrix capturing the pass-through from changes in the vacancy-filling rate to marginal costs in each sector.

The presence of labor market frictions therefore alters the propation of sectoral shocks to aggregate inflation in two distinct ways. First, as demonstrated in the previous section, an increase in labor demand in one sector leads to a reduction in labor supply in other sectors, potentially leading to a cascade of labor market tightness throughout the network. As a result, sectors that lead to larger spillovers in tightness to other sectors, and therefore to larger changes in the entire vector of sectoral tightness, $\boldsymbol{\theta}_t$, are more important for aggregate inflation. Second, how changes in tightness pass through to prices depends on the details of the sector-specific matching process. labor market conditions in sectors where the vacancy-filling rate changes more in response to changes in tightness, where hiring costs respond more to changes in the vacancy-filling rate, and where marginal costs respond more th changes in labor costs, matter more for aggregate inflation.

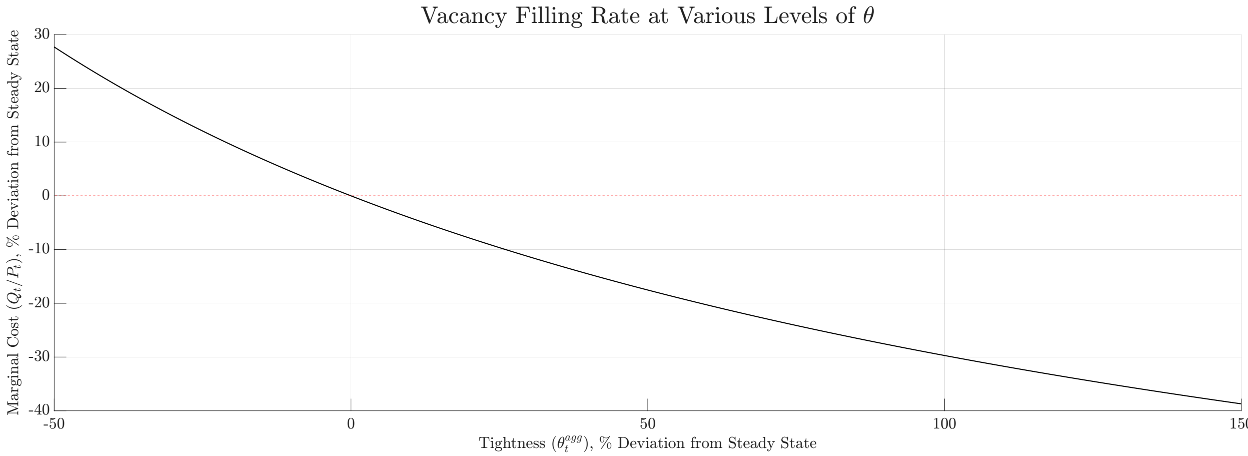
To first order, shifting demand into sectors where labor markets are more rigid—where firms have a harder time adjusting their labor input either because increasing vacancy postings leads to fewer additional hires or because the effective hiring costs rise more quickly as tightness increases—leads to an endogenous cost-push shock, triggering aggregate inflation. The intution is the same as in Rubbo (2024), who shows that shifting demand into sectors with less elastically supplied inputs leads to higher aggregate inflation. Intuitively, sectors with less elastically supplied inputs have a harder time increasing output in response to a positive demand shock, leading to a smaller increase in quantities and a larger increase in prices when these sectors experience a surge in demand. In Rubbo (2024), all sectors face identically elastic labor supply curves, but vary in terms of their capital and intermediate input usage, leading to differences in the elasticity of input supply curves across sectors. Here, I show that a similar mechanism arises when sectors face frictional labor markets: any variation in how rigid labor markets are across sectors leads to differences in the elasticity of input supply curves, and therefore alters how important each sector is for aggregate inflation. In the appendix, I demonstrate using the BLS's Job Openings and Labor Turnover Survey (JOLTS) data that there is substantial heterogeneity in average vacancy-filling rates across sectors, and therefore in steady-state hiring costs.[12]

In the nonlinear full model, the elasticity of the vacancy-filling rate and the pass-through to hirings costs changes endogenously with local labor market conditions. Figure 4a demonstrates that as tightness in sector i rises, the vacancy-filling rate falls. As a result, firms in sector i need to post more vacancies to achieve the same level of hiring: it gets harder and harder for firms in that sector to hire additional workers. This, in turn, leads to both a direct increase in marginal costs (see Figure 4b), operating both through hiring costs and wages, and reduces the elasticity of the labor supply curve faced by firms in sector i. Both the hiring costs and the elasticity of labor supply change nonlinear as the constraint that $H_{i,t} \leq m_{i,t}(U_{i,t}, V_{i,t})$ approaches, leading to endogenous changes in which sectors are the most important for aggregate inflation.

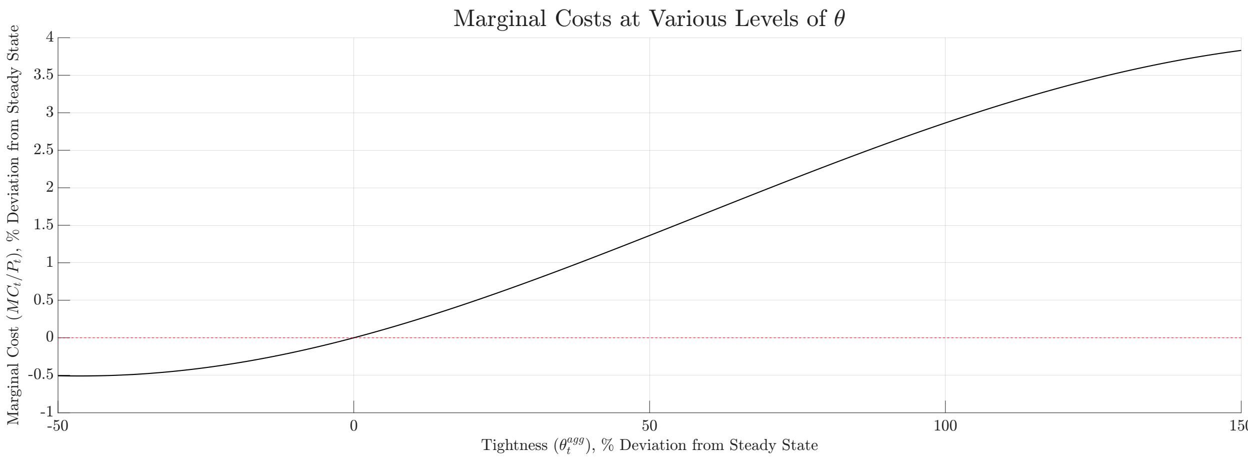
The intuition is the same as above: as tightness rises, firms become effectively more supply constrained. As a result, they change output by less and inflation by more in response to additional demand or supply shocks. Indeed, as Figure 4c demonstrates, this mechanism results in a nonlinear Phillips curve at both the sectoral and aggregate levels. The solid black line in Figure 4c plots the relationship between inflation and the output gap in response to monetary policy shocks. The inflation-output-gap Phillips curve steepens as the output gap rises precisely because firms become more supply constrained at high levels of tightness, forcing more of the effects of positive demand shocks into prices rather than quantities.

Indeed, the relationship between inflation and the output gap steepens if tightness is initially high for reasons unrelated to aggregate demand. If, for instance, tightness in a sector is already high due to a sectoral demand shock, a reduction in household willingness to work in sector i, or a decrease in aggregate labor supply, then additional shocks to aggregate demand lead to larger increases in inflation. To demonstrate this mechanism, the dashed black line in Figure 4c plots the inflation-output gap relationship for the same monetary policy shocks as the solid black line, but assuming that tightness is exogenously higher because of a decline in the aggregate labor supply $\overline{L}_t$. As the figure demonstrates, the higher initial level of tightness results in a steeper inflation-output gap Phillips curve.

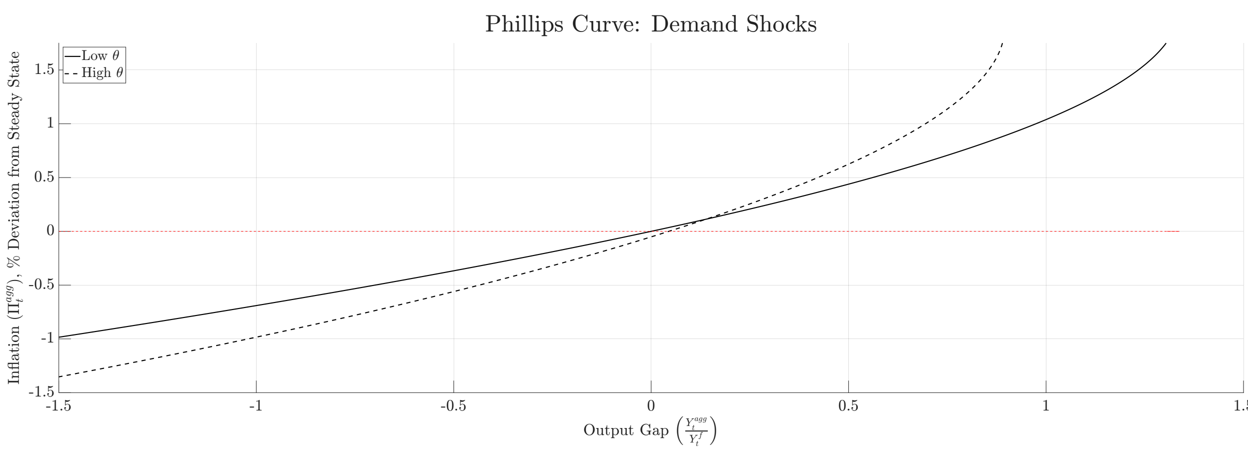
[12] The average monthly vacancy-filling rate in the finance and insurance industry, for instance, from 2000 to 2025 is about 0.37, while the average vacancy-filling rate in construction over the same horizon is about 0.68. These large differences in average vacancy-filling rates likely reflect differences in how search works across sectors, depending on the skill level of employees and the time each application takes to process.



(a) Response of vacancy-filling rate in sector i ($Q_{i,t}$) to changes in tightness in sector i ($\theta_{i,t}$)

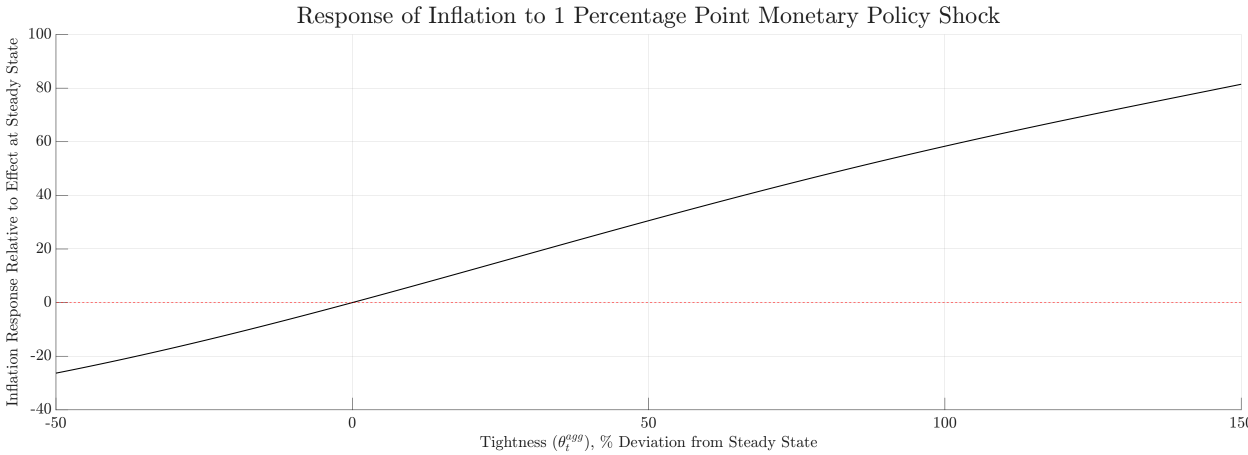


(b) Response of labor supply in sector i to an increase in $\alpha_{i,t}$

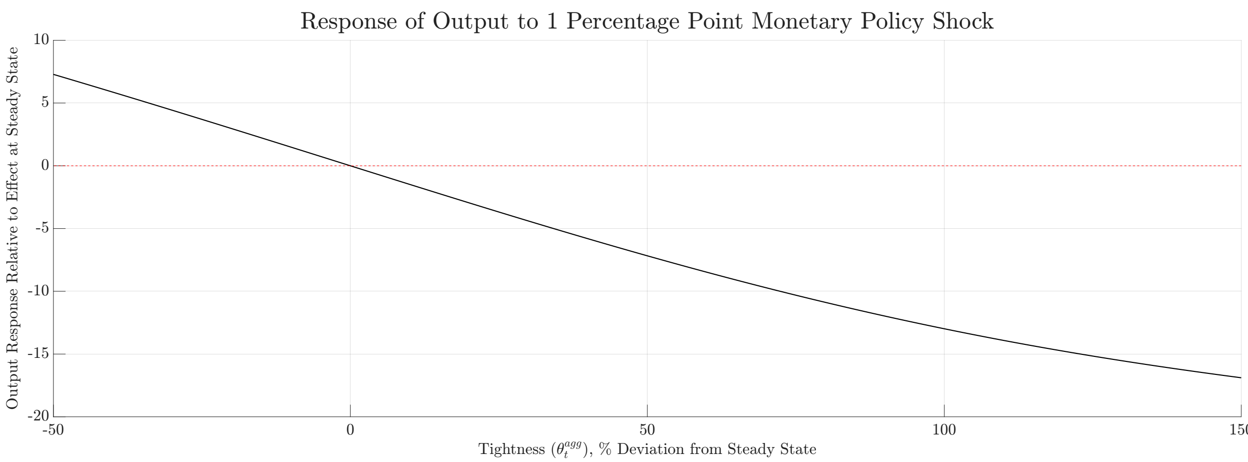


(c) Nonlinear Phillips curve generated by monetary policy shocks

Figure 4: Nonlinear sector-specific Phillips curve



(a) Effect of -1% monetary policy shock on inflation at different levels of initial tightness



(b) Effect of -1% monetary policy shock on output at different levels of initial tightness

Figure 5: Nonlinear sector-specific Phillips curve

Relatedly, because output becomes more constrained at high levels of tightness, monetary policy has larger effects on inflation and smaller effects on output when tightness is high. To demonstrate this effect, Figure 7a plots $\frac{\partial \Pi}{\partial i}(\theta)$, the effect of a change in the nominal policy rate on inflation, by plotting the response of inflation to a 1 percentage point policy rate cut, relative to the effect of a rate cut when tightness is initially at steady-state. Conversely, Figure 7b plots $\frac{\partial Y}{\partial i}(\theta)$, the effect of a change in the nominal policy rate on output, by plotting the response of output to a 1 percentage point policy rate cut, relative to the effect of a rate cut when tightness is initially at steady-state. As the figures demonstrate, monetary policy has a larger effect on inflation and a smaller effect on output when tightness is high.

The nonlinear Phillips curve is consistent with the empirical findings in Gitti (2024) that the Phillips curve is steeper in U.S. in regions with tighter labor markets. In addition, as argued in Benigno and Eggertsson (2023), the presence of a nonlinear Phillips curve can help explain why central banks were surprised by the surge in inflation in 2021. A central bank that assumes a linear Phillips curve underestimates the inflationary pressure in tight labor markets, and will therefore tend to allow inflation to surge more than expected.

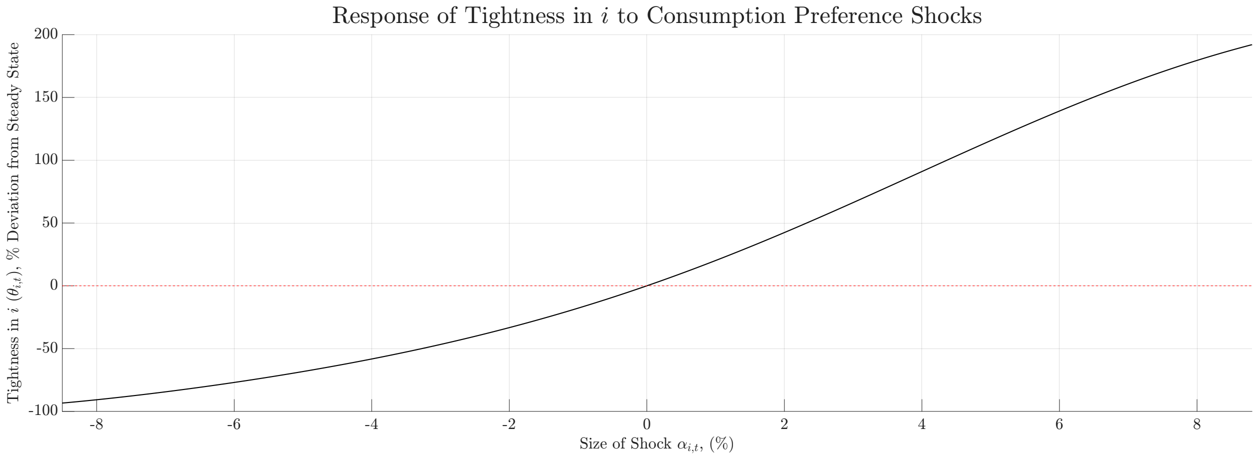
However, my findings are distinct in two important ways. First, unlike Benigno and Eggertsson (2023), where the Phillips curve steepens when tightness crosses a certain threshold because of an assumed kink in the wage setting process, I demonstrate that the Phillips curve steepens naturally in the standard search-and-matching framework once I solve the model nonlinearly. This suggests that the findings in Benigno and Eggertsson (2023) do not rely on the specific specification for the wage setting and search process, and instead arise more generally in models featuring frictional labor markets. Second, by solving the model nonlinearly instead of assuming a kink, I show that search-and-matching frictions lead to a continuous steepening of the Phillips curve: the curve gets progressively steeper as tightness rises. As a result, assuming a linear Phillips curve becomes a worse approximation as tightness rises substantially above steady-state, as it did in the U.S. in 2021 and 2022. In these circumstances, the monetary authority is especially prone to allowing inflation to surge more than expected if it extrapolates from the relatively flat Phillips curve in normal times.

3.4 Aggregate Effects of Sectoral Demand Shocks

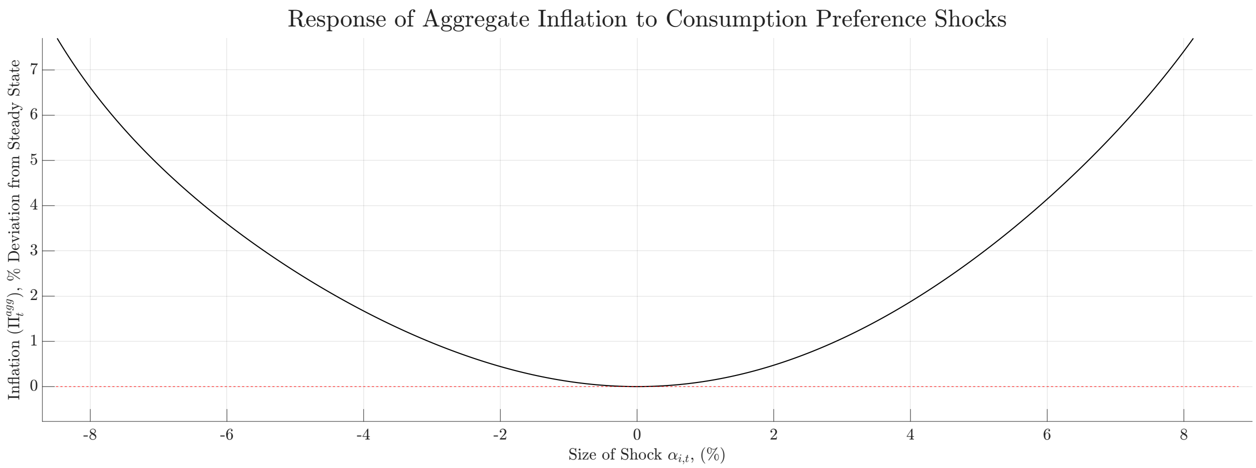
I now demonstrate how a production network, capturing sector-specific shocks, and labor market frictions alter the aggregate effects of demand shocks. In particular, I show that the Phillips curve steepens and monetary policy becomes less effective at stimulating output when just a few sectors experience large increases in tightness.

As demonstrated above, a shock to relative demand in sector i, $\alpha_{i,t}$, increases demand in sector i and reduces demand in sector j. As the size of the relative demand shock increases, so does the increase in labor demand and tightness in sector i. For instance, in the baseline calibration, an 8% increase in $\alpha_{i,t}$ triggers about a 180% increase in tightness.

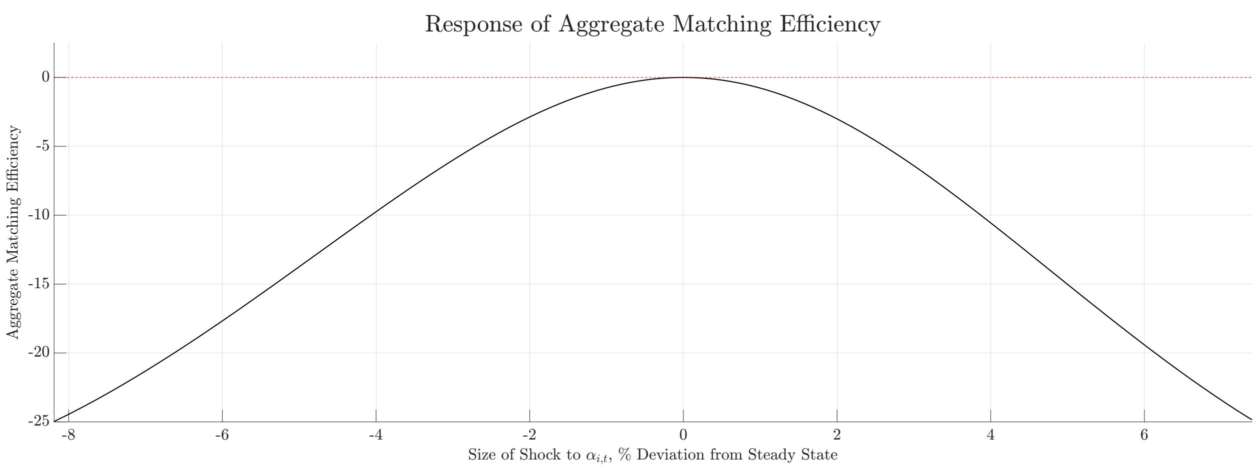
As tightness rises in sector i, it gets progressively more difficult to hire, leading to both an increase in inflation and a decrease in aggregate matching efficiency, a measure of how many hires are generated for a given number of unemployed workers and vacancies. Aggregate inflation rises because the rise in tightness in sector i causes firms in that sector to become relatively more supply constrained. Firms in sector j, on the other hand, experience a decline in tightness. As a result, prices rise by more in sector i than they fall in sector j. In addition, the rise in $\alpha_{i,t}$ gives sector i a larger weight in the aggregate price index. These two factors combine to produce a significant rise in aggregate inflation.



(a) Response of tightness in sector i as the size of the shock to $\alpha_{i,t}$ increases

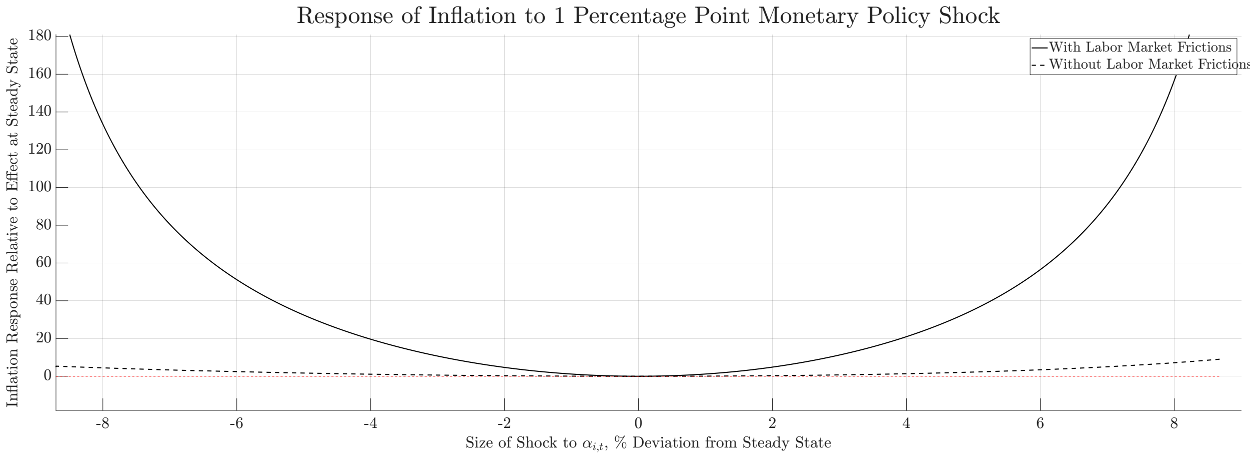


(b) Response of aggregate inflation as the size of the shock to $\alpha_{i,t}$ increases

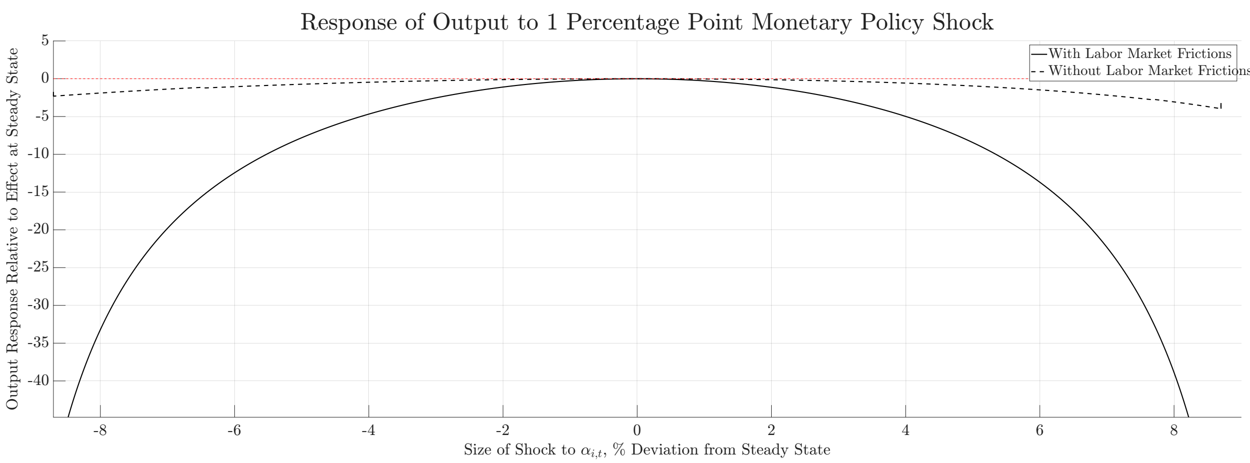


(c) Response of aggregate output as the size of the shock to $\alpha_{i,t}$ increases

Figure 6: Response of tightness in sector i, aggregate inflation, and aggregate matching efficiency on impact as the size of the shock to $\alpha_{i,t}$ increases.



(a) Effect of -1% monetary policy shock on inflation relative to effect at steady state



(b) Effect of -1% monetary policy shock on output relative to effect at steady state

Figure 7: Effects of an additional -1% monetary policy shock on inflation and output as the size of the shock to $\alpha_{i,t}$ increases, relative to effect of -1% monetary policy shock when sectoral preferences are at steady-state. Black dashed line is response in model without labor market frictions. Black solid line is response in model with labor market frictions.

Figure 6c demonstrates that a shift in demand from one sector to the other also results in a decline in the aggregate matching efficiency, which I define as $\phi_t^{agg} = \frac{H_t^{agg}}{\sqrt{U_t^{agg} V_t^{agg}}}$. Aggregate matching efficiency falls because labor demand is concentrated precisely in the sector where it is hardest to hire. It gets harder to hire in sector i for two reasons. First, as in Şahin et al. (2014), sectoral shocks can generate mismatch between where vacancy postings are and where unattached workers search. This mismatch is exacerbated when labor adjustment costs are larger. Second, the number of hires generated per additional vacancy declines endogenously as the constraint $H_{i,t} \leq m_{i,t}(U_{i,t}, V_{i,t})$ approaches. That is, the natural constraint that firms cannot create additional workers out of thin air by posting a larger number of vacancies, naturally implies that a large increase in vacancies in just one sectoral labor market leads to a decline in the number of hires relative to what one would expect given the number of aggregate vacancies and unemployed workers.

As figure 7 demonstrates, in the presence of labor market frictions, monetary policy has smaller effects on output and larger effects on inflation when just a few sectors experience larger increases in tightness. The solid black line in the top panel plots the effect of a 1 percentage point cut in the nominal policy rate on aggregate inflation, relative to the effect of a rate cut when $\alpha_{i,t}$ is at steady-state. Conversely, the solid black line in the bottom panel plots the effect of a 1 percentage point cut in the nominal policy rate on aggregate real output, relative to the effect of a rate cut when $\alpha_{i,t}$ is at steady-state.

As tightness rises in sector i, and firms in that sector become more supply constrained, they become progressively less able to increase output in response to an additional positive aggregate demand shock, and therefore must raise prices by more instead. As a result, it is enough for some sectors to be supply constrained for the aggregate Phillips curve to steepen and for monetary policy to become less effective at stimulating output. This suggests that central banks must account for sectoral labor market conditions when predicting the effects of their monetary policy actions. When there are exceptional labor market conditions in just a few sectors in the economy, acting as if the aggregate Phillips curve remains flat may lead the central bank to underestimate the inflationary consequences of its actions. Conversely, in an uneven downturn or recovery, monetary policy may be a less effective tool for stimulating aggregate output since constrained sectors respond less to additional demand stimulus.

The last prediction is an important insight for when monetary policy is and is not likely to be an effective stabilization tool. For instance, consider two shocks that lead to an identical increase in aggregate demand. Suppose the first shock is a true aggregate demand shock affecting all sectors equally, while the second raises demand in some sectors by significantly more than others. The model's predictions for the effects of sectoral demand shocks suggest that monetary policy will have a larger effect on output in the case of the pure aggregate demand shock, where no sector experiences a spike in tightness and becomes supply constrained, than in the case of an uneven sectoral demand shock. Similarly, if sectoral shocks lead an overall decline in demand, triggered by large declines in demand in some sectors and an increase in demand in others, monetary policy may be unable to stimulate aggregate output, and stimulative monetary policy will instead have larger inflationary effects.

4 Quantitative Application to Post-Pandemic Inflation and Labor Market Dynamics

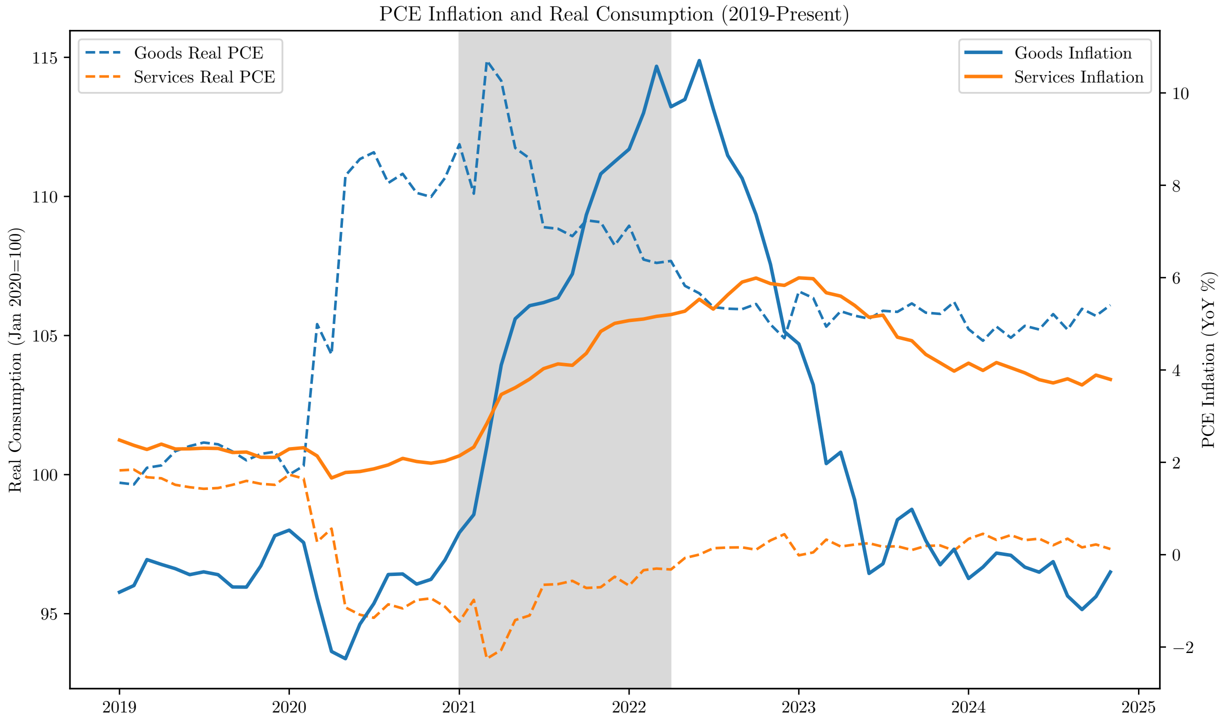
I now turn to a quantitative application of the model to the recent inflation and labor market dynamics in the United States following the COVID-19 pandemic. I begin by documenting sectoral differences in inflation and labor market dynamics during the recovery. I then demonstrate that a shock to relative demand for goods over services can partially account for both inflation and matching efficiency in an estimated two-sector goods-services version of the model.

4.1 Inflation and Labor Market Dynamics in Goods and Services

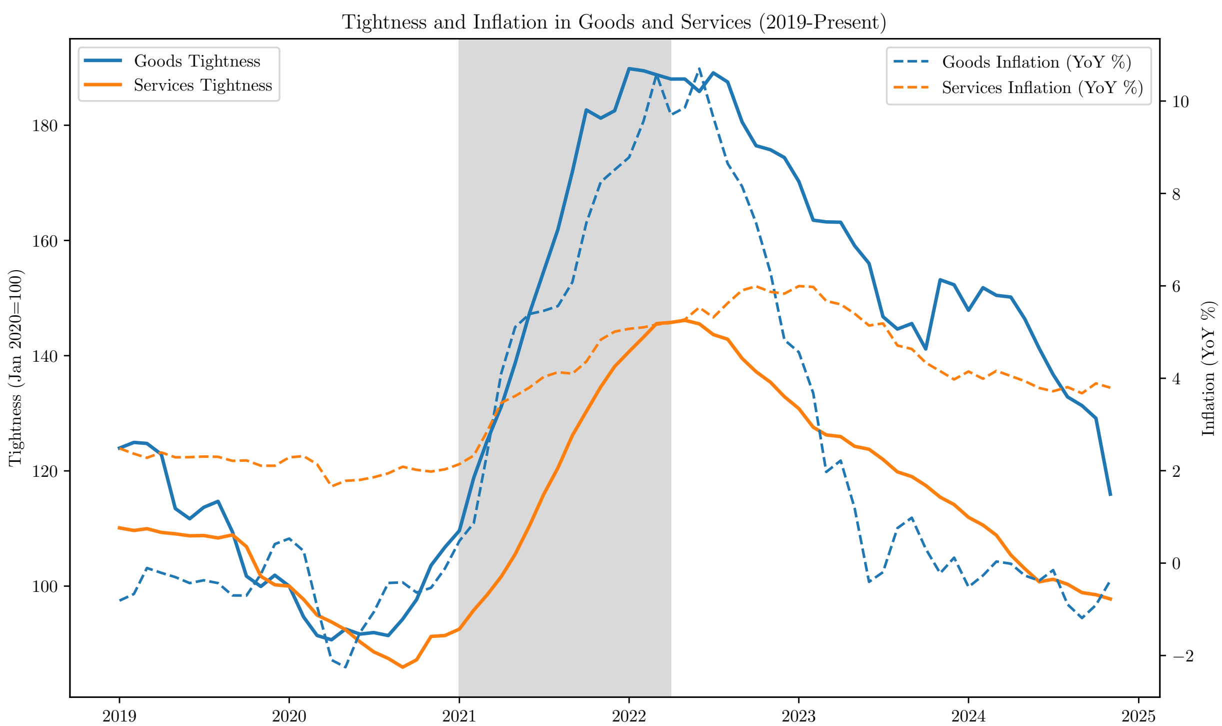
From 2021 to 2024, the United States experienced two phenomena unprecedented since at least the 1980s: (1) inflation surged to levels not seen since the Volcker disinflation, and (2) labor market tightness reached historic highs, with job openings per unemployed worker at levels unseen since the 1960s. I begin by documenting sectoral differences in these dynamics during the pandemic recovery, focusing on the responses in the goods and services sectors. Real consumption, labor market tightness, and new-hire wages rose more in goods than in services. As a result, inflation rose far more in goods as well. These patterns suggest a link between inflation and labor market dynamics at the sectoral level.

In the figures below, I use the PCE goods and services price and quantity indices from the Bureau of Economic Analysis. I use vacancy data from the Bureau of Labor Statistics' Job Openings and Labor Turnover Survey (JOLTS) roughly at the 2-digit NAICS level, and aggregate to the two-sector goods-services breakdown available from the PCE. I use unemployment data from the Current Population Survey (CPS).

As Figure 8a shows, the surge in U.S. inflation, which began in early 2021 and peaked near 7.2 percent year-over-year in June 2022, was uneven across the economy. Inflation in goods jumped from about 0 to over 10 percent by mid-2022, while services inflation rose more slowly, peaked around 6 percent, and stayed persistently elevated near 4 percent into 2024. This pattern suggests that a mechanism where inflation in one sector, triggered by either sectoral supply or demand shocks, gradually propagates to other sectors through the production network as in Minton and Wheaton (2023).



(a) Real PCE and PCE Inflation.



(b) Labor Market Tightness and PCE Inflation.

Figure 8: Left Axis (both subplots): Dashed lines are changes in real PCE relative to January 2020, by major expenditure category: goods (blue) and services (orange). Right Axis: Solid lines are year-over-year changes in the PCE price index, by major category: goods (blue) and services (orange). The gray shaded area indicates the period from the start of the inflation surge in early 2021 to the first Federal Reserve rate hike in March 2022.

This initial surge followed a sharp rise in demand for goods, as consumers shifted toward lower-contact spending. The goods share of real PCE rose from 32 to 37 percent early in the pandemic—a 15 percent jump—and has remained elevated since. Alongside this shift, supply chain disruptions and the war in Ukraine created supply shocks that hit goods disproportionately.[13] Recent work emphasizes the importance of these sector specific demand and supply disturbances for the inflation surge (Amiti et al., 2023; Comin et al., 2023; Di Giovanni et al., 2023; di Giovanni et al., 2023; Ferrante et al., 2023; Guerrieri et al., 2022; Lorenzoni & Werning, 2024; Rubbo, 2024).

Figure 8b shows that inflation rose alongside labor market tightness, measured as the sector-specific ratio of job vacancies to total searchers. I calculate searchers using CPS microdata on unemployment-to-employment (UE), nonparticipation-to-employment (NE), and employment-to-employment (EE) transitions, adjusted following Fujita et al. (2024).[14] Assuming random matching and sector-specific job search, total searchers in sector i are:

$$TS_{i,t} = U_{i,t} + \frac{\rho_{i,t}^{NE}}{\rho_{i,t}^{UE}} \frac{U_{i,t}}{U_t} E_t + \frac{\rho_{i,t}^{NE}}{\rho_{i,t}^{UE}} \frac{U_{i,t}}{U_t} N_t \tag{33}$$

where $U_{i,t}$ is the number of unemployed workers most recently employed in sector i, U_t is the total number of unemployed workers, E_t is the total number of employed workers, N_t is the total number of people not currently in the labor force, $\rho_{i,t}^{NE} = \frac{H_{i,t}^{N}}{N_t}$ is the NE rate into sector i ($H_{i,t}^{N}$ is the number of hires in sector i from nonparticipation), $\rho_{i,t}^{UE} = \frac{H_{i,t}^{E}}{U_t}$ is the UE rate into sector i, and $\rho_{i,t}^{EE} = \frac{H_{i,t}^{E}}{E_t}$ is the EE rate into sector i.

I use this broader measure rather than the conventional unemployment-based one for three reasons. First, recent work highlights the role of EE transitions in post-pandemic labor markets (Autor et al., 2023; Bagga et al., 2025; Barlevy et al., 2023; Faccini & Melosi, 2025; Moscarini & Postel-Vinay, 2023). Second, Barnichon and Shapiro (2024) show that tightness based on total searchers forecasts inflation better than unemployment alone. Third, the measure aligns with the model introduced above, in which households allocate members to search across labor markets, and some unattached members find a new job within the same period, never showing up in end-of-period unemployment numbers.

The goods-sector labor market was about 80 percent tighter in early 2022 than in January 2020, when conditions were already historically tight.[15] Services were about 40 percent

[13]For example, the New York Fed's Global Supply Chain Pressure Index peaked at 4.5 standard deviations above average in December 2021, the highest on record. The ISM Supplier Deliveries Index reached a record 78.8 in May 2021.

[14]Details in Appendix **??**.

[15]I show in Appendix C that the same pattern holds when using the conventional V/U tightness measure. Tightness moves more, though, when using V/U, and by this metric the goods-sector labor market was about

tighter. Goods-sector tightness closely tracks goods inflation, underscoring the potential importance of accounting for labor market dynamics, even in a multi-sector setting.

Similarly, Figure 9b shows that matching efficiency—the number of hires per vacancy and searcher—declined sharply, and by a comparable amount to the Great Recession. Figure 9b plots the residuals from a regression based on a Cobb-Douglas matching function,

$$H_{i,t} = \phi_{i,t} TS_{i,t}^{\eta} V_{i,t}^{1-\eta} \tag{34}$$

where η is the elasticity of the matching process to total searchers and $\phi_{i,t}$ is the matching efficiency. I run the following regression separately for each sector i and for the aggregate:

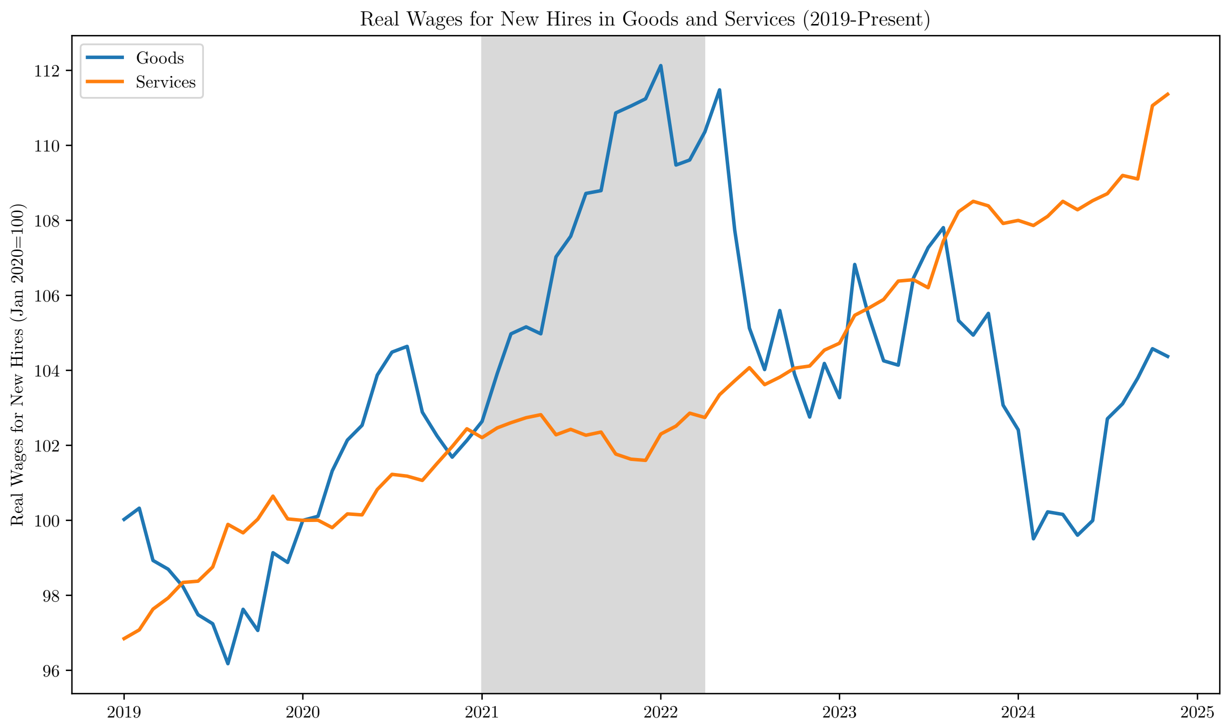
$$\log\left(\frac{H_{i,t}}{TS_{i,t}}\right) = \log\phi + (1-\eta)\log\theta_{i,t} + \epsilon_{i,t} \tag{35}$$

where $\log\phi + \epsilon_{i,t} = \log\phi_{i,t}$ is the log matching efficiency estimated in the data. The decline was larger in services, consistent with goods-sector demand making it harder for services to hire. My model links this fall in matching efficiency, which shifts the Beveridge curve, to higher firm marginal costs and inflation.

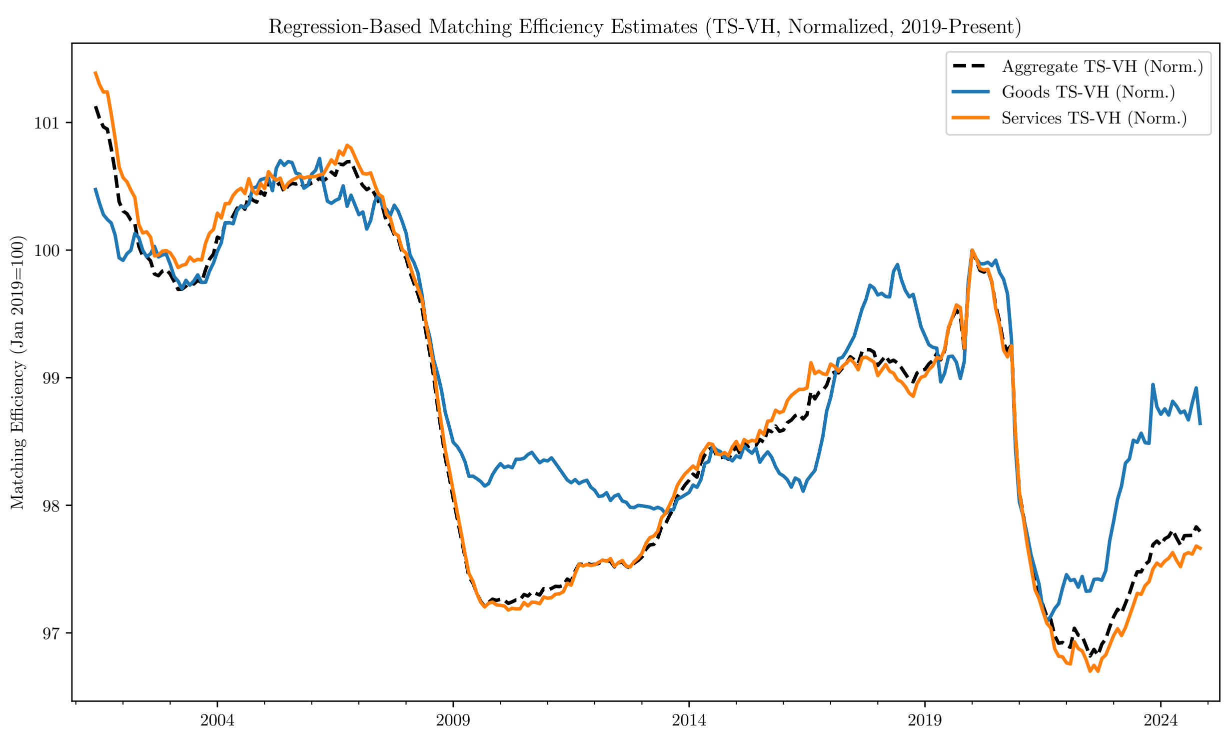
Despite the historic rise in labor market tightness and the substantial decline in matching efficiency, a common objection to labor-based explanations is that real wages fell as prices rose. Indeed, aggregate real wages declined in both goods and services. But Figure 9a shows that real wages for new hires—relevant for marginal costs—rose sharply just as goods inflation accelerated. Because services dominate employment, the aggregate masks these sectoral patterns. As the model demonstrates, even without wage growth, rising tightness and falling matching efficiency raise costs by increasing firms' hiring costs. Rising tightness also reduces the effective elasticity of labor supply to firms, and especially so in the sector experiencing a larger rise in tightness. Cross-sectoral shifts in tightness driven by changes in relative demand can therefore lead to an endogenous decrease in the input elasticity of the goods sector, exerting upward pressure on prices even absent a wage response.

Taken together, these facts point to both sectoral heterogeneity and labor market frictions as central to the pandemic recovery, consistent with the model outlined above.

150 percent tighter in early 2022 than in January 2020.



(a) New-Hire Wages.



(b) Matching Efficiency

Figure 9: Top: Real wages for newly hired workers in the goods (blue) and services (orange) sectors, relative to January 2020. Bottom: Matching efficiency in the goods (blue) and services (orange) sectors, relative to January 2020. The gray shaded area indicates the period from the start of the inflation surge in early 2021, to the first Federal Reserve rate hike in March 2022.

4.2 Can Demand Shocks Explain the Joint Dynamics of Inflation and the Labor Market?

As I demonstrate above, the model's predictions are broadly consistent with the experience during the post-pandemic recovery. In this section, I use Bayesian methods to estimate the parameters of the shock processes in a linearized version of the model on data from 2000 to 2019, a period when shocks were small relative to the COVID-19 period and where the linear approximation is therefore more likely to be accurate. I then use these estimated parameters to calibrate the fully nonlinear model to assess the impacts of different shocks during the post-pandemic recovery.

The first three columns of Table 2 report the prior mean, variance, and distribution for each of the estimated parameters. The last column reports the mode from 3 million draws from the posterior generated with a standard Random-Walk Metropolis–Hastings algorithm. The priors are taken from the literature and are listed in Table 1. I make twi minor alteration to the model from above: In addition to the labor reallocation costs on the household side, I allow for a vacancy posting adjustment cost on the firm side to help generate slightly smoother paths for vacancy postings, and partial indexation by firms to past inflation to match smoother movements in inflation, as seen in the data.[16]

I then ask whether the model can quantitatively account for changes in inflation and labor market conditions with shocks to consumer preferences for relative consumption and then with a combination of preference shocks, aggregate demand shocks, and a positive shock to the separation rate. I find that relative preference shocks can broadly match the movements of wages, tightness in the goods sector, and aggregate matching efficiency, but cannot, on their own, account for changes in inflation or tightness in services. Exogenous increases in the separation rate lead to broad-based increases in tightness, but have relatively limited effects on wages, inflation, and matching efficiency.[17] The model cannot match the persistence of inflation without a persistent positive aggregate demand shock, suggesting that all three played an important role in generating the post-pandemic inflation surge.

[16] For details, see the Appendix. I am also planning on adding some price indexation to match the smoother path of inflation as well.

[17] Afrouzi et al. (2024) provide one possible rationale for the observed increase in separation rates, positing that workers are more likely to engage in on-the-job search and accept outside offers when inflation erodes the value of their existing wages. Bagga et al. (2025) provide an alternative explanation based on shifting preferences for job amenities during the pandemic.

Table 2: Prior Distributions and Estimated Posterior Parameter Values

Parameter	Mean	Variance	Distribution	Mode
ϕ_π	1.39	0.30	Truncated Normal	1.1456
ϕ_y	1.01	0.10	Gamma	1.1441
ρ_i	0.82	0.15	Beta	0.6736
κ	0.50	0.10	Gamma	0.7232
σ	2.00	0.50	Gamma	1.6686
φ	3.57	0.50	Gamma	3.6303
$\psi_{p,G}$	57.13	10.00	Gamma	48.0468
$\psi_{p,S}$	63.10	10.00	Gamma	60.1747
ψ_L	36.17	10.00	Gamma	38.0123
$\psi_{v,G}$	0.52	0.25	Gamma	0.5245
$\psi_{v,S}$	0.52	0.25	Gamma	0.1917
ρ_w	0.80	0.15	Beta	0.5663

Note: The left panel shows prior distributions taken from the literature; see Table 1.

I calibrate the remaining parameters of the model to match the real consumption share, labor share, and input–output structure of the goods and services sectors. I report the calibrated parameter values in Table 3. I set the values of ϵ_d, ϵ_y, and ϵ to those reported in Table 1.

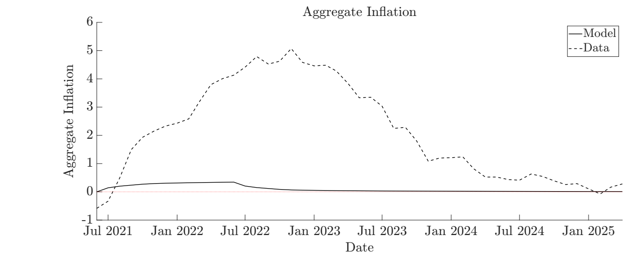
Table 3: Sectoral Shares and Elasticity Parameters

Category	**Goods**	**Services**
Consumption Share	0.3338	0.6662
Intermediates Share (Total)	0.6210	0.3921
Intermediates Share (Goods)	0.4171	0.0684
Intermediates Share (Services)	0.2039	0.3237

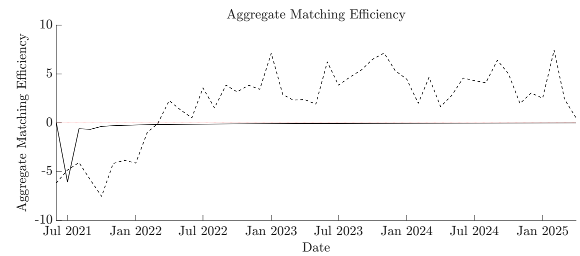
Notes: Goods–services sectoral consumption shares, and intermediate shares, calculated from PCE and BEA input–output tables.

4.2.1 Response of Inflation, Real Wages, and Aggregate Matching Efficiency to Consumption Preference Shocks

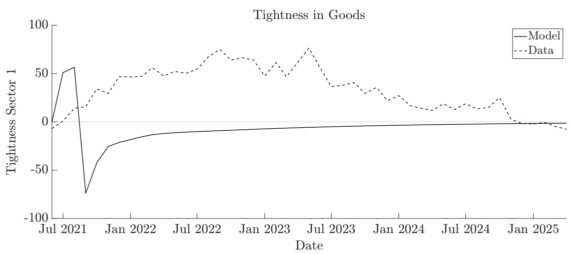
I start by introducing a shock to the relative demand for goods over services, $\alpha_{G,t}$, calibrated to increase the real consumption share in goods post-pandemic. I set the persistence of this shock to 0.94 to roughly match the path of the real consumption share in goods from 2021 to the end of 2024. I then solve for the nonlinear impulse responses to this shock under perfect foresight as described in the previous section. Figure 10 reports the impulse responses of key variables to this shock.



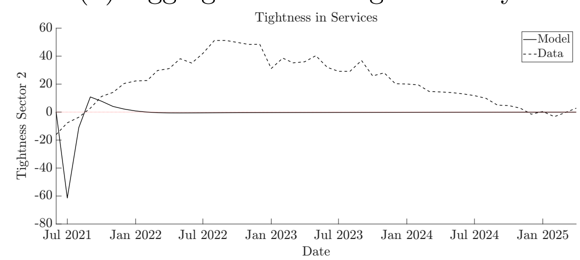
(a) Nominal Consumption Share



(b) Aggregate Matching Efficiency

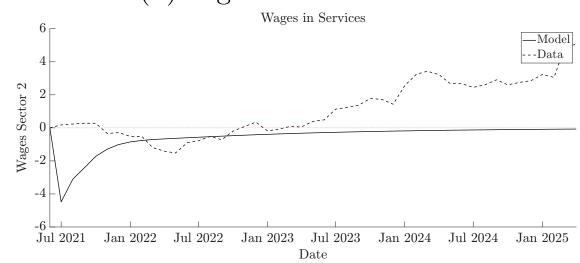


(c) Tightness in Goods

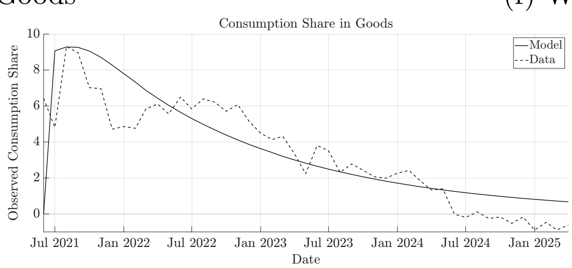


(d) Tightness in Services

(e) Wages in Goods



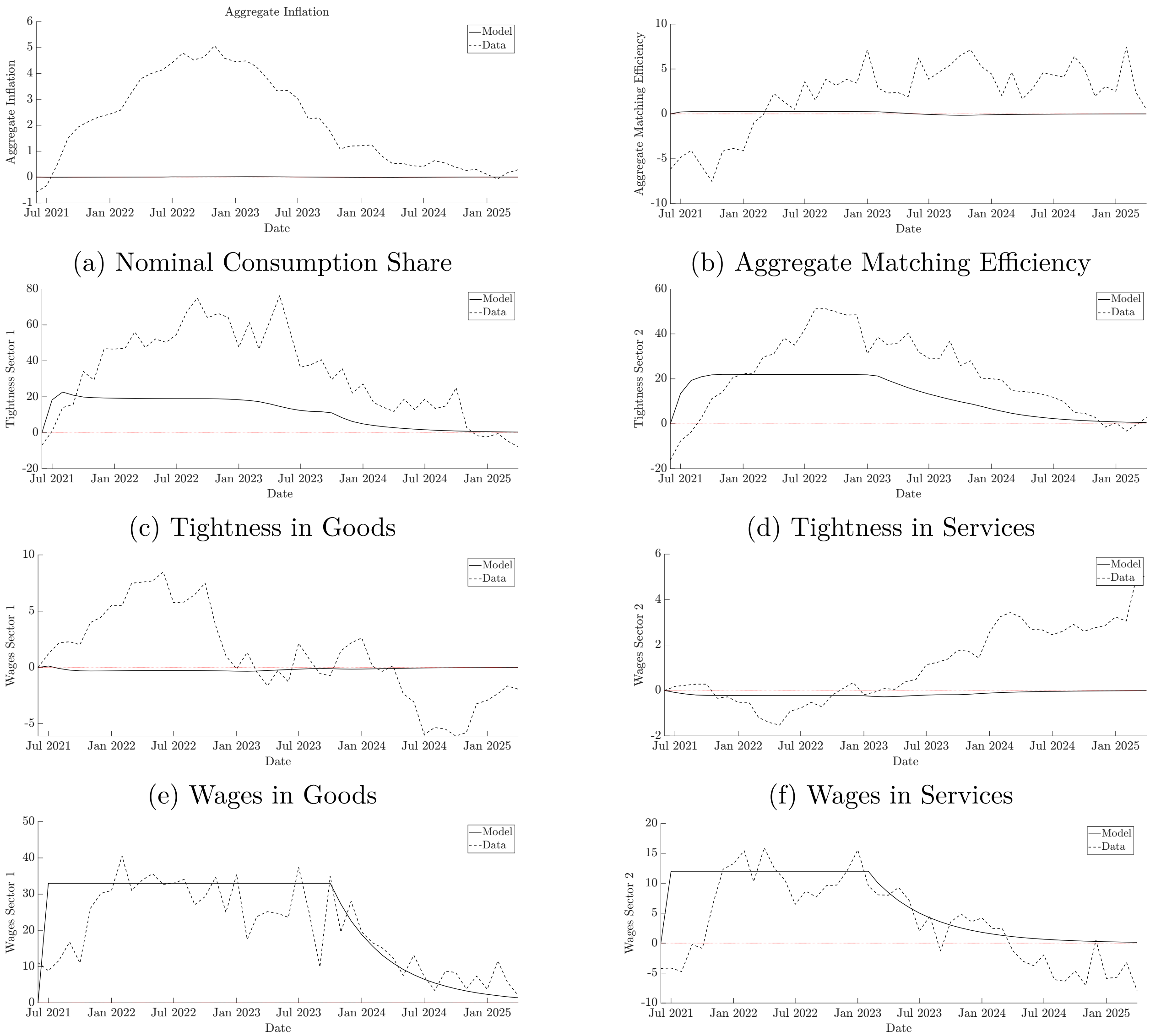
(f) Wages in Services



(g) Aggregate Inflation

Figure 10: Shock to Consumption Preferences Only

A shock to the relative demand for goods over services generates a spike in inflation and a drop in aggregate matching efficiency, a rise in real wages for new hires in goods, and a decline for new-hire wages in services that are qualitatively in line with the data. Unsurprisingly, given the many additional shocks hitting the economy in the post-COVID period, the relative demand shock alone only explains a small portion of the inflation surge. It does, however, account for a significant portion of the decline in the aggregate matching efficiency, and can therefore explain at least some of the joint behavior of the Beveridge curve and Phillips curve during the recovery. In addition, while the relative demand shock does generate an increase in tightness in the goods sector, it generates a counterfactual decline in tightness in services, absent an additional aggregate demand shock.



(a) Nominal Consumption Share

(b) Aggregate Matching Efficiency

(c) Tightness in Goods

(d) Tightness in Services

(e) Wages in Goods

(f) Wages in Services

(g) Separations in Goods

(h) Separations in Services

Figure 11: Combination of Shocks

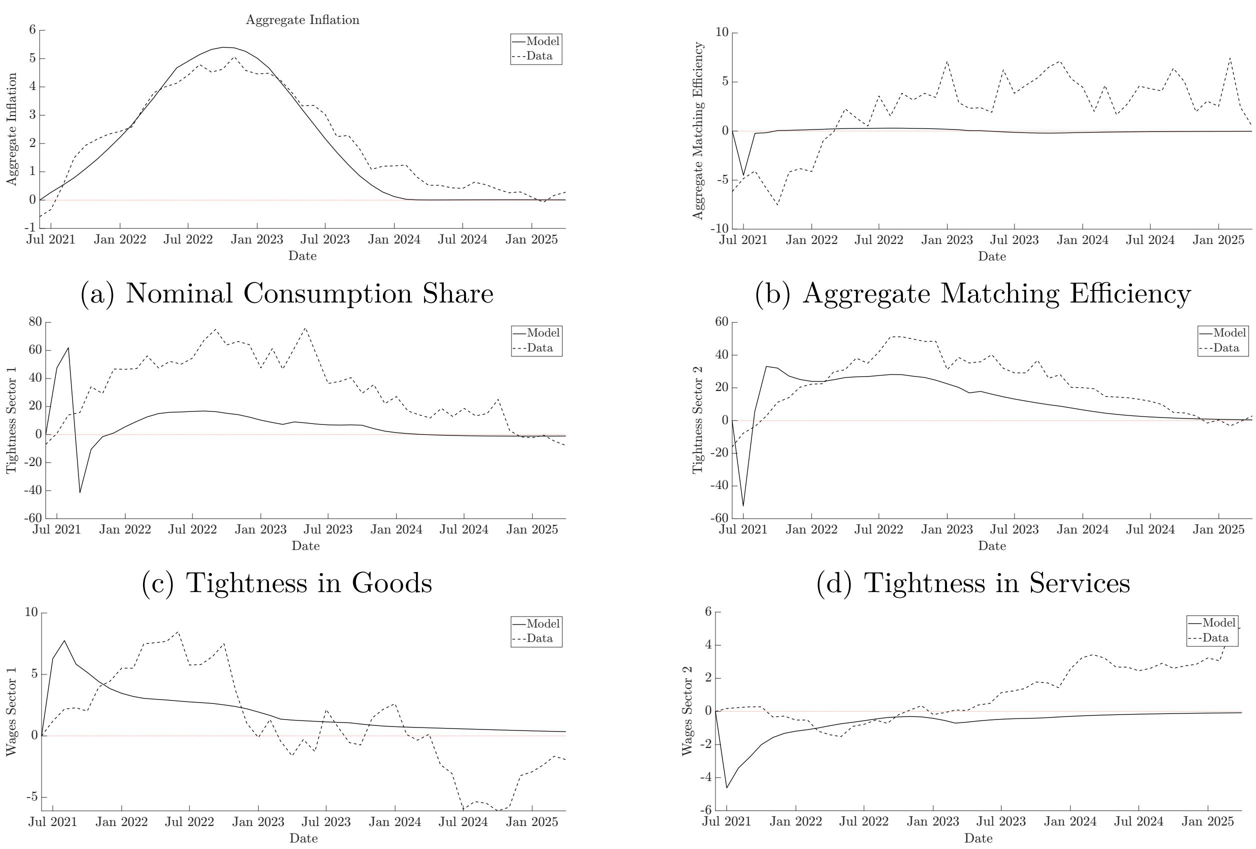
4.2.2 Response to a Shock to the Separation Rate

Next, I introduce sector-specific shocks to the separation rates. These shocks capture the increase in the separation rate observed in the data, and which several recent papers tie to the unusual behavior of the Beveridge curve during the COVID recovery (Afrouzi et al., 2024; Bagga et al., 2025). I calibrate the shocks to generate a roughly 35 percent increase in the separation rate in the goods sector and a 12 percent increase in the separation rate in the service sector, in line with the deviations from steady-state in the data for these two separation rates.

As Figure 11 shows, the increase in the separation rate does raise tightness in both the goods and services sectors as firms post additional vacancies to replace the additional unattached workers. An increase in the separation rate alone, though, does not generate much movement in inflation, real wages, or matching efficiency.

4.2.3 Adding an Aggregate Demand Shock

To match the rise in inflation, I introduce an additional aggregate demand shock, a persistent 50-basis-point stimulative monetary-policy shock that lasts for two years from 2021, along with the sectoral demand shock and the separation shock from above. Figure 12 shows the impulse responses to this combination of shocks. The combination of shocks can broadly match the dynamics of inflation, real wages, tightness in both sectors, and aggregate matching efficiency observed in the data during the post-pandemic recovery.



(a) Nominal Consumption Share (b) Aggregate Matching Efficiency

(c) Tightness in Goods (d) Tightness in Services

(e) Wages in Goods (f) Wages in Services

Figure 12: Combination of Shocks

5 Conclusion

The recent post-pandemic inflation surge highlights the potential importance of both sector-specific shocks and labor market frictions for aggregate inflation. Sector-specific shocks that propagate through the production network, can generate large aggregate inflationary pressures, particularly if they hit sectors with inelastic input supply curves; such sectors struggle to expand output and instead raise prices. I show how labor market frictions at the sector level endogenously change the elasticity of the input supply curves faced by sectors. Sectors with tight labor markets find it harder to hire and thus to adjust output when additional shocks hit. As a result, labor market frictions lead to a nonlinear Phillips curve, even if only a few sectors are constrained, making monetary policy less effective at stabilizing output and more effective at combating inflation.

Consequently, uneven demand and supply across the economy can generate aggregate inflation, a decline in aggregate matching efficiency, and a steeper aggregate Phillips curve qualitatively consistent with observed post-pandemic dynamics. In addition, the model predicts that monetary policy, while a useful stabilization tool in normal times, particularly in the face of aggregate disturbances that affect all sectors relatively equally, may be less effective when the economy experiences large shocks or an uneven distribution of demand across sectors. In these circumstances, introducing sector-specific wage subsidies to ease the transition of workers across sectors may be effective at alleviating and equalizing tightness across the economy. The optimal policy response to sector-specific shocks in light of labor market frictions is an interesting avenue for future work.

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A All Equilibrium Conditions

The following conditions hold for each sector $i \in J$:

$$Y_{i,t} = A_{i,t}\left(\omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} + \left(\sum_{i=1}^{J} \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t}^{\frac{\epsilon_y-1}{\epsilon_y}}\right)\right)^{\frac{\epsilon_y}{\epsilon_y-1}} \tag{36}$$

$$R_{i,t}\frac{W_{i,t}}{P_t} = \mu_{i,t}\left(Q_{i,t} - R_{i,t}\right) + E_t\left[SDF_{t|t+1}(1-s_{i,t+1})\mu_{i,t+1}R_{i,t}\right] \tag{37}$$

$$\mu_{i,t}\frac{Q_{i,t}}{R_{i,t}} = \frac{MC_{i,t}}{P_t}\beta_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} \tag{38}$$

$$\frac{P_{j,t}X_{ij,t}}{MC_{i,t}Y_{i,t}} = (\beta_{ix}\omega_{ij})^{\frac{1}{\epsilon_y}}\left(A_{i,t}\frac{X_{ij,t}}{Y_{i,t}}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} \quad \forall \quad j \in J \tag{39}$$

$$C_{i,t} = \alpha_{i,t}\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_d} C_t^{agg} \tag{40}$$

$$\left(\frac{\Pi_{i,t}}{\Pi} - 1\right)\frac{\Pi_{i,t}}{\Pi} = \frac{\epsilon}{\psi_p}\left(\frac{MC_{i,t}}{P_t} - \frac{\epsilon-1}{\epsilon}\frac{P_{i,t}}{P_t}\right) + E_t\left[SDF_{t|t+1}\left(\frac{\Pi_{i,t+1}}{\Pi} - 1\right)\frac{\Pi_{i,t+1}}{\Pi}\frac{Y_{i,t+1}}{Y_{i,t}}\right] \tag{41}$$

$$SDF_{t|t+1} = \beta E_t\left[\left(\frac{C_{t+1}^{agg}}{C_t^{agg}}\right)^{-\sigma}\frac{Z_{t+1}}{Z_t}\right] \tag{42}$$

$$\Pi_{i,t} = \Pi_t^{agg}\left(\frac{P_{i,t}/P_t}{P_{i,t-1}/P_{t-1}}\right) \tag{43}$$

$$Y_{i,t} = C_{i,t} + \sum_{j=1}^{J} X_{ji,t} \tag{44}$$

$$\delta_t = \Xi_{i,t}F_{i,t} - \psi_{L,i}Z_t\left(\left[\frac{L_{i,t}/\overline{L}_t}{L_{i,t-1}/\overline{L}_{t-1}} - 1\right] + \frac{1}{2}\left[\frac{L_{i,t}/\overline{L}_t}{L_{i,t-1}/\overline{L}_{t-1}} - 1\right]^2\right) \tag{45}$$

$$+ \beta\psi_{L,i}E_t\left[Z_{t+1}\left[\frac{L_{i,t+1}/\overline{L}_{t+1}}{L_{i,t}/\overline{L}_t} - 1\right]\frac{L_{i,t+1}^2/\overline{L}_{t+1}}{L_{i,t}^2/\overline{L}_t}\right] \tag{46}$$

$$\Xi_{i,t} = Z_t\left((C_t^{agg})^{-\sigma}\frac{W_{i,t}}{P_t} - \Xi_{i,t}\tilde{L}_{i,t}^{\varphi}\right) + \beta E_t\left[(1-F_{i,t+1})(1-s_{i,t+1})\Xi_{i,t+1}\right] \tag{47}$$

$$\overline{L}_t = \sum_{i=1}^{J} L_{i,t} \tag{48}$$

$$\tilde{L}_{i,t} = N_{i,t} + R_{i,t}V_{i,t} \tag{49}$$

$$L_{i,t} = (1-s_{i,t})\tilde{L}_{i,t-1} + U_{i,t} \tag{50}$$

$$Q_{i,t} = \frac{H_{i,t}}{V_{i,t}} \tag{51}$$

$$F_{i,t} = \frac{H_{i,t}}{U_{i,t}} \tag{52}$$

$$\theta_{i,t} = \frac{V_{i,t}}{U_{i,t}} \tag{53}$$

$$H_{i,t} = \zeta_{i,t} \left(U_{i,t}^{-\eta_i} + V_{i,t}^{-\eta_i} \right)^{-\frac{1}{\eta_i}} \tag{54}$$

$$N_{i,t} + R_{i,t} V_{i,t} = H_{i,t} + (1 - s_{i,t})(N_{i,t-1} + r_{i,t-1} V_{i,t-1}) \tag{55}$$

$$\frac{W_{i,t}}{P_t} = \frac{1}{Z_t \left(C_t^{agg}\right)^{-\sigma} - \kappa \frac{R_{i,t}}{Q_{i,t}}} \left(Z_t \chi_{i,t} \tilde{L}_{i,t}^{\varphi} - \beta E_t \left[(1 - F_{i,t+1})(1 - s_{i,t+1}) \Xi_{i,t+1} \right] \right) \tag{56}$$

Aggregate variables satisfy:

$$C_t^{agg} = \left(\sum_{i=1}^{J} \alpha_{i,t}^{\frac{1}{\epsilon_d}} C_{i,t}^{\frac{\epsilon_d - 1}{\epsilon_d}} \right)^{\frac{\epsilon_d}{\epsilon_d - 1}} \tag{57}$$

$$H_t^{agg} = \sum_{i=1}^{J} H_{i,t} \tag{58}$$

$$V_t^{agg} = \sum_{i=1}^{J} V_{i,t} \tag{59}$$

$$U_t^{agg} = \sum_{i=1}^{J} U_{i,t} \tag{60}$$

$$\theta_t^{agg} = \frac{V_t^{agg}}{U_t^{agg}} \tag{61}$$

$$1 = \beta(1 + i_t) E_t \left[\left(\frac{C_{t+1}^{agg}}{C_t^{agg}} \right)^{-\sigma} \frac{Z_{t+1}}{Z_t} \frac{1}{\Pi_t^{agg}} \right] \tag{62}$$

$$(1 + i_t) = (1 + i_{t-1})^{\rho_i} \left(\left[\Pi_t^{agg}\right]^{\phi_\pi} \left[Y_t^{agg}\right]^{\phi_y} \right)^{1-\rho_i} M_t \tag{63}$$

$$Y_t^{agg} = C_t^{agg} \tag{64}$$

Finally, the shock processes are given by:

$$s_{i,t} = s_{i,t-1}^{\rho_s}\varepsilon_{s,i,t} \tag{65}$$

$$A_{i,t} = A_{i,t-1}^{\rho_A}\varepsilon_{A,i,t} \tag{66}$$

$$\alpha_{i,t} = \alpha_{i,t-1}^{\rho_\alpha}\varepsilon_{\alpha,i,t} \quad \forall \quad i \in J-1 \tag{67}$$

$$\alpha_{J,t}^{\frac{1}{\epsilon_d}} = 1 - \sum_{i=1}^{J-1}\alpha_{i,t}^{\frac{1}{\epsilon_d}} \tag{68}$$

$$R_{i,t} = r_{i,t-1}^{\rho_r}\varepsilon_{r,i,t} \tag{69}$$

$$\zeta_{i,t} = \zeta_{i,t-1}^{\rho_\zeta}\varepsilon_{\zeta,i,t} \tag{70}$$

$$\chi_{i,t} = \chi_{i,t-1}^{\rho_\chi}\varepsilon_{\chi,i,t} \tag{71}$$

$$Z_t = Z_{t-1}^{\rho_Z}\varepsilon_{Z,t} \tag{72}$$

$$M_t = M_{t-1}^{\rho_M}\varepsilon_{m,t} \tag{73}$$

A.1 Adding vacancy Adjustment Costs and Price Indexation to Past Inflation

$$\begin{aligned}
\mathcal{L}(\cdot) = E_t\sum_{s=0}^{\infty}\Bigg\{ & SDF_{t|t+s}\left[\frac{P_{i,t+s}(z)}{P_{t+s}}\left(\frac{P_{i,t+s}(z)}{P_{i,t+s}}\right)^{-\epsilon}Y_{i,t+s} - \frac{W_{i,t+s}}{P_{t+s}}\left[N_{i,t+s}(z) + r_{i,t+s}V_{i,t+s}(z)\right]\right.\\
& \left. - \sum_j \frac{P_{j,t}}{P_t}X_{ij,t+s}(z) - \frac{\psi_p}{2}\left(\frac{P_{i,t+s}(z)}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}P_{i,t+s-1}(z)} - 1\right)^2 Y_{i,t+s}\right]\\
& + \lambda_{i,t+s}\left[A_{i,t+s}\left(\beta_{ix}^{\frac{1}{\epsilon_y}}\left[\sum_j \omega_{ij}^{\frac{1}{\epsilon_y}}X_{ij,t+s}(z)^{\frac{\epsilon_y-1}{\epsilon_y}}\right] + \beta_{in}^{\frac{1}{\epsilon_y}}N_{i,t+s}(z)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}} - \left(\frac{P_{i,t+s}(z)}{P_{i,t+s}}\right)^{-\epsilon}Y_{i,t+s}\right]\\
& + \mu_{i,t+s}\Big[(1-s_{i,t+s})\left(N_{i,t+s-1}(z) + r_{i,t+s-1}V_{i,t+s-1}(z)\right) + \left(Q_{i,t+s} - r_{i,t+s}\right)V_{i,t+s}(z) - N_{i,t+s}(z)\Big]\Bigg\}
\end{aligned}$$

FOCs

$$\begin{aligned}
\frac{\partial\mathcal{L}}{\partial P_{i,t}(z)} = & (1-\epsilon)\frac{1}{P_t}\left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\epsilon}Y_{i,t} + \epsilon\lambda_{i,t}\left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\epsilon}\frac{Y_{i,t}}{P_{i,t}(z)}\\
& - \psi_p\Bigg\{\left(\frac{P_{i,t}(z)}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}P_{i,t-1}(z)} - 1\right)\frac{1}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}P_{i,t-1}(z)}Y_{i,t}\\
& - E_t\left[SDF_{t|t+s}\left(\frac{P_{i,t+1}(z)}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}P_{i,t}(z)} - 1\right)\frac{P_{i,t+1}(z)}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}P_{i,t}(z)^2}Y_{i,t+1}\right]\Bigg\} = 0
\end{aligned}$$

Now I assume a symmetric equilibrium, where we can drop the z indexation. Starting with (74), this implies

$$\left(\frac{\Pi_{i,t}}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}} - 1\right)\frac{1}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}P_{i,t-1}}Y_{i,t} = \frac{1}{\psi_p}\left[(1-\epsilon)\frac{1}{P_t}Y_{i,t} + \epsilon\lambda_{i,t}\frac{Y_{i,t}}{P_{i,t}}\right]$$
$$+ E_t\left[SDF_{t|t+s}\left(\frac{\Pi_{i,t+1}}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}P_{i,t}} - 1\right)\frac{P_{i,t+1}}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}P_{i,t}^2}Y_{i,t+1}\right]$$

Dividing by $Y_{i,t}$ and multiplying by $P_{i,t}$ gives

$$\left(\frac{\Pi_{i,t}}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}} - 1\right)\frac{\Pi_{i,t}}{\Pi^{1-\rho_\pi}\Pi_{i,t-1}^{\rho_\pi}} = \frac{\epsilon}{\psi_p}\left[\lambda_{i,t} - \frac{\epsilon-1}{\epsilon}\frac{P_{i,t}}{P_t}\right]$$
$$+ E_t\left[SDF_{t|t+s}\left(\frac{\Pi_{i,t+1}}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}} - 1\right)\frac{\Pi_{i,t+1}}{\Pi^{1-\rho_\pi}\Pi_{i,t}^{\rho_\pi}}\frac{Y_{i,t+1}}{Y_{i,t+1}}\right]$$

I add vacancy adjustment costs by modeling $R_{i,t}$ as a quadratic function that depends on the growth of firm level vacancy postings from one period to the next.

B First Order Approximation to the Phillips Curve

$$\max E_t SDF_{t+s|t}\frac{D_{i,t+s}(z)}{P_{t+s}}$$

$$D_{i,t}(z) = \frac{P_{i,t}(z)}{P_t}\left(\frac{P_{i,t}(z)}{P_{i,t}}\right)^{-\epsilon}Y_{i,t} - \frac{W_{i,t}}{P_t} - \frac{W_{i,t}}{P_t}\left[N_{i,t}(z) + R_{i,t}V_{i,t}(z)\right]$$
$$- \sum_{j=1}^{J}\frac{P_{j,t}}{P_t}X_{ij,t}(z) - \frac{\psi_p}{2}\left(\frac{P_{i,t}(z)}{\Pi P_{i,t-1}(z)} - 1\right)^2 Y_{i,t}$$
$$N_{i,t}(z) + R_{i,t}V_{i,t}(z) = Q_{i,t}V_{i,t}(z) + (1-s_{i,t})(N_{i,t-1} + r_{i,t-1}V_{i,t-1})$$
$$Y_{i,t}(z) = A_{i,t}\left(\omega_{in}^{\frac{1}{\epsilon_y}}N_{i,t}(z)^{\frac{\epsilon_y-1}{\epsilon_y}} + \sum_{j=1}^{J}\omega_{ij}^{\frac{1}{\epsilon_y}}X_{ij,t}(z)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}}$$

Lagrangian:

$$\mathcal{L}(\cdot) = E_t \sum_{s=0}^{\infty} \Bigg\{ SDF_{t|t+s} \Bigg[\frac{P_{i,t+s}(z)}{P_{t+s}} \left(\frac{P_{i,t+s}(z)}{P_{i,t+s}} \right)^{-\epsilon} Y_{i,t+s} - \frac{W_{i,t+s}}{P_{t+s}} \left[N_{i,t+s}(z) + r_{i,t+s} V_{i,t+s}(z) \right]$$

$$- \sum_j \frac{P_{j,t}}{P_t} X_{ij,t+s}(z) - \frac{\psi_p}{2} \left(\frac{P_{i,t+s}(z)}{\Pi P_{i,t+s-1}(z)} - 1 \right)^2 Y_{i,t+s} \Bigg]$$

$$+ \lambda_{i,t+s} \Bigg[A_{i,t+s} \left(\left[\sum_j \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right] + \omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1}} - \left(\frac{P_{i,t+s}(z)}{P_{i,t+s}} \right)^{-\epsilon} Y_{i,t+s} \Bigg]$$

$$+ \mu_{i,t+s} \Bigg[(1 - s_{i,t+s}) \left(N_{i,t+s-1} + r_{i,t+s-1} V_{i,t+s-1} \right) + \left(Q_{i,t+s} - r_{i,t+s} \right) V_{i,t+s}(z) - N_{i,t+s}(z) \Bigg] \Bigg\}$$

FOCs

$$\frac{\partial \mathcal{L}}{\partial P_{i,t}(z)} = (1 - \epsilon) \frac{1}{P_t} \left(\frac{P_{i,t}(z)}{P_{i,t}} \right)^{-\epsilon} Y_{i,t} + \epsilon \lambda_{i,t} \left(\frac{P_{i,t}(z)}{P_{i,t}} \right)^{-\epsilon} \frac{Y_{i,t}}{P_{i,t}(z)} \tag{74}$$

$$- \psi_p \left\{ \left(\frac{P_{i,t}(z)}{\Pi P_{i,t-1}(z)} - 1 \right) \frac{1}{\Pi P_{i,t-1}(z)} Y_{i,t} - E_t \left[SDF_{t|t+s} \left(\frac{P_{i,t+1}(z)}{\Pi P_{i,t}(z)} - 1 \right) \frac{P_{i,t+1}(z)}{\Pi P_{i,t}(z)^2} Y_{i,t+1} \right] \right\} = 0$$

$$\frac{\partial \mathcal{L}}{\partial X_{ij,t}(z)} = -\frac{P_{j,t}}{P_t} + \lambda_{i,t} A_{i,t} \frac{\epsilon_y}{\epsilon_y - 1} \Bigg\{ \left(\left[\sum_j \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right] + \omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1} - 1} \tag{75}$$

$$\times \omega_{ij}^{\frac{1}{\epsilon_y}} \frac{\epsilon_y - 1}{\epsilon_y} X_{ij,t}(z)^{\frac{\epsilon_y - 1}{\epsilon_y} - 1} \Bigg\} = 0$$

$$\frac{\partial \mathcal{L}}{\partial N_{i,t}(z)} = -\frac{W_{i,t}}{P_t} + \lambda_{i,t} A_{i,t} \frac{\epsilon_y}{\epsilon_y - 1} \Bigg\{ \left(\left[\sum_j \omega_{ij}^{\frac{1}{\epsilon_y}} X_{ij,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right] + \omega_{in}^{\frac{1}{\epsilon_y}} N_{i,t+s}(z)^{\frac{\epsilon_y - 1}{\epsilon_y}} \right)^{\frac{\epsilon_y}{\epsilon_y - 1} - 1} \tag{76}$$

$$\times \omega_{in}^{\frac{1}{\epsilon_y}} \frac{\epsilon_y - 1}{\epsilon_y} N_{ij,t}(z)^{\frac{\epsilon_y - 1}{\epsilon_y} - 1} \Bigg\} - \mu_{i,t} = 0 \tag{77}$$

$$\frac{\partial \mathcal{L}}{\partial V_{i,t}(z)} = -\frac{W_{i,t}}{P_t} R_{i,t} + \left(Q_{i,t} - R_{i,t} \right) \mu_{i,t} = 0 \tag{78}$$

Implies,

$$\mu_{i,t} = \frac{R_{i,t}}{Q_{i,t} - R_{i,t}} \frac{W_{i,t}}{P_t}$$

$$\mu_{i,t} = \lambda_{i,t} \omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y - 1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} - \frac{W_{i,t}}{P_t}$$

Combining implies,

$$
\begin{aligned}
\mu_{i,t} &= \lambda_{i,t}\omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} - \frac{Q_{i,t}-R_{i,t}}{R_{i,t}}\mu_{i,t} \\
\Rightarrow \mu_{i,t} + \frac{Q_{i,t}-R_{i,t}}{R_{i,t}}\mu_{i,t} &= \lambda_{i,t}\omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}} \\
\Rightarrow \frac{Q_{i,t}}{R_{i,t}}\mu_{i,t} &= \lambda_{i,t}\omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}}
\end{aligned}
$$

Or alternatively,

$$
\frac{Q_{i,t}}{Q_{i,t}-R_{i,t}}\frac{W_{i,t}}{P_t} = \lambda_{i,t}\omega_{in}^{\frac{1}{\epsilon_y}} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} N_{i,t}^{-\frac{1}{\epsilon_y}}
$$

Similarly,

$$
\frac{P_{j,t}}{P_t} = (\omega_{ij})^{\frac{1}{\epsilon_y}} \lambda_{i,t} A_{i,t}^{\frac{\epsilon_y-1}{\epsilon_y}} Y_{i,t}^{\frac{1}{\epsilon_y}} X_{ij,t}^{-\frac{1}{\epsilon_y}}
$$

Combining, we can write all $X_{ij,t}$ and $N_{i,t}$ in terms of $X_{ii,t}$:

$$
\begin{aligned}
\frac{Q_{i,t}}{Q_{i,t}-R_{i,t}}\frac{W_{i,t}}{P_t} &= \frac{P_{i,t}}{P_t}\left(\frac{\omega_{in}}{\omega_{ii}}\right)^{\frac{1}{\epsilon_y}} \left(\frac{X_{ii,t}}{N_{i,t}}\right)^{\frac{1}{\epsilon_y}} \\
\Rightarrow N_{i,t}^{\frac{1}{\epsilon_y}} &= \frac{P_{i,t}}{W_{i,t}}\frac{Q_{i,t}-R_{i,t}}{Q_{i,t}}\left(\frac{\omega_{in}}{\omega_{ii}}\right)^{\frac{1}{\epsilon_y}} X_{ii,t}^{\frac{1}{\epsilon_y}} \\
\Rightarrow N_{i,t} &= \left(\frac{\omega_{in}}{\omega_{ii}}\right)\left(\frac{P_{i,t}}{W_{i,t}}\frac{Q_{i,t}-R_{i,t}}{Q_{i,t}}\right)^{\epsilon_y} X_{ii,t}
\end{aligned}
$$

And similarly,

$$
X_{ij,t} = \left(\frac{\omega_{ij}}{\omega_{ii}}\right)\left(\frac{P_{i,t}}{P_{j,t}}\right)^{\epsilon_y} X_{ii,t}
$$

Now plugging into the production function for optimal input choices,

$$
\begin{aligned}
Y_{i,t} &= A_{i,t}\left(\omega_{in}^{\frac{1}{\epsilon_y}}\left(\left(\frac{\omega_{in}}{\omega_{ii}}\right)\left(\frac{P_{i,t}}{W_{i,t}}\frac{Q_{i,t}-R_{i,t}}{Q_{i,t}}\right)^{\epsilon_y} X_{ii,t}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} + \sum_{j=1}^{J}\omega_{ij}^{\frac{1}{\epsilon_y}}\left(\left(\frac{\omega_{ij}}{\omega_{ii}}\right)\left(\frac{P_{i,t}}{P_{j,t}}\right)^{\epsilon_y} X_{ii,t}\right)^{\frac{\epsilon_y-1}{\epsilon_y}}\right)^{\frac{\epsilon_y}{\epsilon_y-1}} \\
&= A_{i,t}\left(\omega_{in}\left(W_{i,t}\frac{Q_{i,t}}{Q_{i,t}-R_{i,t}}\right)^{1-\epsilon_y} + \sum_{j=1}^{J}\omega_{ij}P_{j,t}^{1-\epsilon_y}\right)^{\frac{\epsilon_y}{\epsilon_y-1}} \frac{1}{\omega_{ii}}P_{i,t}^{\epsilon_y}X_{ii,t}
\end{aligned}
$$

We can rearrange this to

$$X_{ii,t} = \omega_{ii} A_{i,t}^{-1} \left(\frac{P_{i,t}}{\Theta_{i,t}} \right)^{-\epsilon_y} Y_{i,t}$$

Where, $\Theta_{i,t} = \left(\omega_{in} \left(W_{i,t} \frac{Q_{i,t}}{Q_{i,t} - R_{i,t}} \right)^{1-\epsilon_y} + \sum_{j=1}^{J} \omega_{ij} P_{j,t}^{1-\epsilon_y} \right)^{\frac{1}{1-\epsilon_y}}$. This lets us write

$$\begin{aligned}
N_{i,t} &= \omega_{in} A_{i,t}^{-1} \left(\frac{W_{i,t}}{\Theta_{i,t}} \frac{Q_{i,t}}{Q_{i,t} - R_{i,t}} \right)^{-\epsilon_y} Y_{i,t} \\
X_{ij,t} &= \omega_{ij} A_{i,t}^{-1} \left(\frac{P_{j,t}}{\Theta_{i,t}} \right)^{-\epsilon_y} Y_{i,t}
\end{aligned}$$

We can then write the cost in terms of $Y_{i,t}$,

$$\begin{aligned}
Cost_{i,t} &= W_{i,t} \left(\frac{Q_{i,t}}{Q_{i,t} - R_{i,t}} N_{i,t} - \frac{R_{i,t}}{Q_{i,t} - R_{i,t}} \left(N_{i,t-1} + r_{i,t-1} V_{i,t-1} \right) \right) + \sum_{j=1}^{J} P_{j,t} X_{ij,t} \\
&= \omega_{in} A_{i,t}^{-1} \left(W_{i,t} \frac{Q_{i,t}}{Q_{i,t} - R_{i,t}} \right)^{1-\epsilon_y} \Theta_{i,t}^{\epsilon_y} Y_{i,t} + \sum_{j=1}^{J} \omega_{ij} A_{i,t}^{-1} P_{j,t}^{1-\epsilon_y} \Theta_{i,t}^{\epsilon_y} Y_{i,t} + ...
\end{aligned}$$

The marginal cost is then,

$$\begin{aligned}
MC_{i,t} &= A_{i,t}^{-1} \Theta_{i,t}^{\epsilon_y} \left(\omega_{in} \left(W_{i,t} \frac{Q_{i,t}}{Q_{i,t} - R_{i,t}} \right)^{1-\epsilon_y} + \sum_{j=1}^{J} \omega_{ij} P_{j,t}^{1-\epsilon_y} \right) \\
&= A_{i,t}^{-1} \Theta_{i,t}^{\epsilon_y} \Theta_{i,t}^{1-\epsilon_y} = A_{i,t}^{-1} \Theta_{i,t} \\
&= A_{i,t}^{-1} \left(\omega_{in} \left(W_{i,t} \frac{Q_{i,t}}{Q_{i,t} - R_{i,t}} \right)^{1-\epsilon_y} + \sum_{j=1}^{J} \omega_{ij} P_{j,t}^{1-\epsilon_y} \right)^{\frac{1}{1-\epsilon_y}}
\end{aligned}$$

All other conditions are the same as in the full model.

On the household side, assume there are no labor rellocation costs, so that the household problem is

$$
\begin{aligned}
\max E_t \sum_{s=0}^{\infty} Z_{t+s} & \left[\frac{C_{t+s}(z)^{1-\sigma}}{1-\sigma} - \sum_{i=1}^{J} \left(\chi_{i,t+s} \frac{\tilde{L}_{i,t+s}(z)^{1+\varphi}}{1+\varphi} \right) \right] \\
\text{s.t. } P_{t+s}C_{t+s}(z) + B_{t+s}(z) &= (1+i_{t+s})B_{t+s-1}(z) + \sum_{i=1}^{J} W_{i,t+s}\tilde{L}_{i,t+s}(z) + T_{t+s}(z) \\
\tilde{L}_{i,t+s}(z) &= (1-F_{i,t+s})(1-s_{i,t+s})\tilde{L}_{i,t+s-1} + F_{i,t+s}L_{i,t+s}(z) \\
\text{and, } \sum_{i=1}^{J} L_{i,t+s}(z) &\leq \overline{L}_{t+s}
\end{aligned}
$$

The Lagrangian is

$$
\begin{aligned}
\mathcal{L} = E_t \sum_{s=0}^{\infty} \beta^s \Bigg(& Z_{t+s} \left[\frac{C_{t+s}(z)^{1-\sigma}}{1-\sigma} - \sum_{i=1}^{J} \left(\chi_{i,t+s} \frac{\tilde{L}_{i,t+s}(z)^{1+\varphi}}{1+\varphi} \right) \right] \\
& + \upsilon_{t+s} \left[(1+i_{t+s-1})B_{t+s-1}(z) + \sum_{i=1}^{J} W_{i,t+s}\tilde{L}_{i,t+s}(z) + T_{t+s}(z) - P_{t+s}C_{t+s}(z) - B_{t+s}(z) \right] \\
& + \sum_{i=1}^{J} \Xi_{i,t+s} \left[(1-F_{i,t+s})(1-s_{i,t+s})\tilde{L}_{i,t+s-1} + F_{i,t+s}L_{i,t+s}(z) - \tilde{L}_{i,t+s}(z) \right] \\
& + \delta_{t+s} \left[\overline{L}_{t+s} - \sum_{i=1}^{J} L_{i,t+s} \right]
\end{aligned}
$$

And the first order conditions are

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial C_{t+s}(z)} &= \beta^s E_t \left[Z_{t+s}C_{t+s}(z)^{-\sigma} - \upsilon_{t+s}P_{t+s} \right] = 0 \\
\frac{\partial \mathcal{L}}{\partial B_{t+s}(z)} &= E_t \left[\beta^{s+1}\upsilon_{t+s+1}(1+i_{t+s}) - \beta^s \upsilon_{t+s} \right] = 0 \\
\frac{\partial \mathcal{L}}{\partial L_{i,t+s}(z)} &= E_t \left[\beta^s \Xi_{i,t+s}F_{i,t+s} - \beta^s \delta_{t+s} \right] = 0 \\
\frac{\partial \mathcal{L}}{\partial \tilde{L}_{i,t+s}(z)} &= E_t \left[\beta^s \upsilon_{t+s}W_{i,t+s} - \beta^s Z_t \chi_{i,t+s}\tilde{L}_{i,t+s}^{\varphi} - \beta^s \Xi_{i,t+s} \right] = 0
\end{aligned}
$$

Which implies,

$$
\begin{aligned}
\Xi_{i,t}F_{i,t} &= \delta_t \\
Z_t \left(C_t^{-\sigma} \frac{W_{i,t}}{P_t} - \chi_{i,t}\tilde{L}_{i,t}^{\varphi} \right) &= \Xi_{i,t}
\end{aligned}
$$

The Nash bargaining solution is then

$$
\begin{aligned}
\kappa \frac{R_{i,t}}{Q_{i,t}} W_{i,t} &= \frac{P_t}{Z_t C_t^{-\sigma}} Z_t \left(C_t^{-\sigma} \frac{W_{i,t}}{P_t} - \chi_{i,t} \tilde{L}_{i,t}^{\varphi} \right) \\
\kappa \frac{R_{i,t}}{Q_{i,t}} W_{i,t} &= W_{i,t} - \chi_{i,t} \tilde{L}_{i,t}^{\varphi} C_t^{\sigma} P_t \\
\Rightarrow \left(1 - \kappa \frac{R_{i,t}}{Q_{i,t}} \right) W_{i,t} &= \chi_{i,t} \tilde{L}_{i,t}^{\varphi} C_t^{\sigma} P_t \\
\Rightarrow \frac{W_{i,t}}{P_t} &= \frac{1}{1 - \kappa \frac{R_{i,t}}{Q_{i,t}}} \chi_{i,t} \tilde{L}_{i,t}^{\varphi} C_t^{\sigma}
\end{aligned}
$$

Suppose

$$
\begin{aligned}
W_{i,t} &= \left[\frac{1}{1 - \kappa \frac{R_{i,t}}{Q_{i,t}}} \chi_{i,t} \tilde{L}_{i,t}^{\varphi} C_t^{\sigma} P_t \right]^{1-\rho_w} W_{i,t-1}^{\rho_w} \\
\Rightarrow \frac{W_{i,t}}{P_t} &= \left[\frac{1}{1 - \kappa \frac{R_{i,t}}{Q_{i,t}}} \chi_{i,t} \tilde{L}_{i,t}^{\varphi} C_t^{\sigma} \right]^{1-\rho_w} \left(\frac{W_{i,t-1}}{P_{t-1}} \Pi_t^{-1} \right)^{\rho_w}
\end{aligned}
$$

B.1 First order approx

To first order, the NKPC is

$$
\pi_{i,t} = \frac{\epsilon - 1}{\psi_{p,i}} \left(mc_{i,t} - p_{i,t} \right) + \beta E_t \pi_{i,t+1}
$$

For a general matching function $m(U_i, V_i)$, the job-finding rate is

$$
\begin{aligned}
Q_{i,t} &= m_{i,t} \left(1, \theta_{i,t} \right) \\
Q_i + \left(Q_{i,t} - Q_i \right) &= Q_i + \frac{\partial Q_i}{\partial \theta_i} \left(\theta_{i,t} - \theta_i \right) \\
\frac{Q_{i,t} - Q_i}{Q_i} &= \frac{\partial Q_i}{\partial \theta_i} \frac{\theta_i}{Q_i} \frac{\theta_{i,t} - \theta_i}{\theta_i} \\
q_{i,t} &= \mathcal{E}_{\theta_i}^{Q_i} \widehat{\theta}_{i,t}
\end{aligned}
$$

Where $\mathcal{E}_{\theta_i}^{Q_i}$ is the elasticity of the vacancy-filling rate to changes in tightness.

Now for the marginal cost,

$$\left(\frac{MC_{i,t}}{P_t}\right)^{1-\epsilon_y} = A_{i,t}^{-(1-\epsilon_y)}\left(\omega_{in}\left(\frac{W_{i,t}}{P_t}\frac{Q_{i,t}}{Q_{i,t}-R_{i,t}}\right)^{1-\epsilon_y} + \sum_{j=1}^{J}\omega_{ij}\left(\frac{P_{j,t}}{P_t}\right)^{1-\epsilon_y}\right)$$

$$\left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y} + (1-\epsilon_y)\left(\frac{MC_i}{P_t}\right)^{-\epsilon_y}\left(\frac{MC_{i,t}}{P_t} - \frac{MC_i}{P}\right) = \left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y} - (1-\epsilon_y)\left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y}\frac{1}{A_i}(A_{i,t} - A_i$$

$$+ (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{Q_i}{Q_i-r_i}\right)^{1-\epsilon_y}\frac{P}{W_i}\left(\frac{W_{i,t}}{P_t} - \frac{W_i}{P}\right) - (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{Q_i}{Q_i-r_i}\right)^{1-\epsilon_y}\frac{r_i}{(Q_i-r_i)^2}(Q_{i,t} -$$

$$+ (1-\epsilon_y)\frac{1}{A_i}\left(\frac{W_i}{P}\frac{Q_i}{Q_i-r_i}\right)^{1-\epsilon_y}\frac{Q_i}{(Q_i-r_i)^2}(R_{i,t} - r_i) + (1-\epsilon_y)\frac{1}{A_i}\sum_{j=1}^{J}\omega_{ij}\left(\frac{P_j}{P}\right)^{1-\epsilon_y}\frac{P}{P_j}\left(\frac{P_{j,t}}{P_t} - \frac{P_j}{P}\right)$$

$$\Rightarrow (1-\epsilon_y)\left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y} mc_{i,t} = -(1-\epsilon_y)\left(\frac{MC_i}{P_t}\right)^{1-\epsilon_y} a_{i,t}$$

$$+ (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{Q_i}{Q_i-r_i}\right)^{1-\epsilon_y} w_{i,t} - (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{Q_i}{Q_i-r_i}\right)^{1-\epsilon_y}\frac{r_iQ_i}{(Q_i-r_i)^2}q_{i,t}$$

$$+ (1-\epsilon_y)\frac{1}{A_i}\omega_{in}\left(\frac{W_i}{P}\frac{Q_i}{Q_i-r_i}\right)^{1-\epsilon_y}\frac{r_iQ_i}{(Q_i-r_i)^2}R_{i,t} + (1-\epsilon_y)\frac{1}{A_i}\sum_{j=1}^{J}\omega_{ij}\left(\frac{P_j}{P}\right)^{1-\epsilon_y} p_{j,t}$$

From the first order conditions,

$$\begin{aligned}
\frac{Q_i}{Q_i-r_i}\frac{W_iN_i}{MC_iY_i} &= \omega_{in}^{\frac{1}{\epsilon_y}} A_i^{\frac{\epsilon_y-1}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} = \Omega_{in} \\
\Rightarrow \frac{Q_i}{Q_i-r_i}\frac{W_i}{MC_i} &= \omega_{in}^{\frac{1}{\epsilon_y}} A_i^{\frac{\epsilon_y-1}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{-\frac{1}{\epsilon_y}} \\
\Rightarrow \left[\frac{Q_i}{Q_i-r_i}\frac{W_i}{MC_i}\right]^{1-\epsilon_y} &= \omega_{in}^{\frac{1-\epsilon_y}{\epsilon_y}} A_i^{\frac{(1-\epsilon_y)(\epsilon_y-1)}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} \\
\Rightarrow \frac{\omega_{in}}{A_i}\left[\frac{Q_i}{Q_i-r_i}\frac{W_i}{MC_i}\right]^{1-\epsilon_y} &= \frac{\omega_{in}}{A_i}\omega_{in}^{\frac{1-\epsilon_y}{\epsilon_y}} A_i^{\frac{(1-\epsilon_y)(\epsilon_y-1)}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} \\
&= A_i^{-\epsilon_y}\omega_{in}^{\frac{1}{\epsilon_y}} A_i^{\frac{\epsilon_y-1}{\epsilon_y}}\left(\frac{N_i}{Y_i}\right)^{\frac{\epsilon_y-1}{\epsilon_y}} \\
&= A_i^{-\epsilon_y}\Omega_{in} = \Omega_{in}
\end{aligned}$$

So we can rewrite the marginal cost as

$$mc_{i,t} = -a_{i,t} + \Omega_{in}\left[w_{i,t} - \frac{r_i}{Q_i-r_i}\frac{Q_i}{Q_i-r_i}(q_{i,t} - R_{i,t})\right] + \sum_{j=1}^{J}\Omega_{ij}p_{j,t}$$

Or in terms of tightness,

$$mc_{i,t} = -a_{i,t} + \Omega_{in}\left[w_{i,t} - \frac{r_i}{Q_i - r_i}\frac{Q_i}{Q_i - r_i}(\mathcal{E}_{\theta_i}^{Q_i}\widehat{\theta}_{i,t} - R_{i,t})\right] + \sum_{j=1}^{J}\Omega_{ij}p_{j,t}$$

Stacking over sectors, and assuming wages are fully rigid,

$$\boldsymbol{mc}_t = -\boldsymbol{a}_t + \boldsymbol{\Omega}_n \underbrace{\boldsymbol{r}(\boldsymbol{Q}-\boldsymbol{r})^{-1}\boldsymbol{Q}(\boldsymbol{Q}-\boldsymbol{r})^{-1}}_{\boldsymbol{\Gamma}_Q}(\boldsymbol{\eta}\widehat{\boldsymbol{\theta}}_t + \boldsymbol{r}_t) + \boldsymbol{\Omega}_x\boldsymbol{p}_t$$

Where $\boldsymbol{\eta}$ is a diagonal matrix with the negative of the elasticity of the vacancy-filling rate to changes in tightness on the diagonal. Stacking the sector level PC over sectors gives

$$\boldsymbol{\pi}_t = \boldsymbol{\lambda}\left(\boldsymbol{mc}_t - \boldsymbol{p}_t\right) + \beta E_t\boldsymbol{\pi}_{t+1}$$

Where $\boldsymbol{\lambda}$ is a diagonal matrix capturing pricing frictions in each sector. Plugging into for the marginal cost gives

$$\begin{aligned}\boldsymbol{\pi_t} &= \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\boldsymbol{p}_t + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{\eta}\widehat{\boldsymbol{\theta}}_t + \beta E_t\boldsymbol{\pi_{t+1}}\\ &+ \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{r}_t - \boldsymbol{a}_t\end{aligned}$$

Finally, using $\boldsymbol{p}_t = \boldsymbol{\pi}_t + \boldsymbol{p}_{t-1} - \boldsymbol{1}\pi_t^{agg}$, we can rewrite the sector level NKPC as

$$\begin{aligned}\boldsymbol{\pi_t} &= \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{\pi}_t - \boldsymbol{1}\pi_t^{agg}\right) + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{\eta}\widehat{\boldsymbol{\theta}}_t + \beta E_t\boldsymbol{\pi_{t+1}}\\ &+ \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{r}_t - \boldsymbol{a}_t + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\boldsymbol{p}_{t-1}\end{aligned}$$

Now from the the aggregate consumption price index,

$$\begin{aligned}\frac{P_t}{P_{t-1}} &= \left(\sum_{i=1}^{J}\alpha_{i,t}\left(\frac{P_{i,t}}{P_{i,t-1}}\frac{P_{i,t-1}}{P_{t-1}}\right)^{1-\epsilon_d}\right)^{\frac{1}{1-\epsilon_d}}\\ \Pi_t^{1-\epsilon_d} &= \sum_{i=1}^{J}\alpha_{i,t}\left(\Pi_{i,t-1}\frac{P_{i,t-1}}{P_{t-1}}\right)^{1-\epsilon_d}\end{aligned}$$

To first order,

$$\Pi^{1-\epsilon_d} + (1-\epsilon_d)\Pi^{1-\epsilon_d}\frac{\Pi_t - \Pi}{\Pi} = \Pi^{1-\epsilon_d} + \sum_{i=1}^{J}\Pi^{1-\epsilon_d}\alpha_i\left(\frac{P_i}{P}\right)^{1-\epsilon_d}\frac{\alpha_{i,t}-\alpha_i}{\alpha_i}$$
$$+ \sum_{i=1}^{J}(1-\epsilon_d)\Pi^{1-\epsilon_d}\alpha_i\left(\frac{P_i}{P}\right)^{1-\epsilon_d}\left(\frac{\Pi_{i,t}-\Pi}{\Pi} + \frac{P_{i,t}/P_t - P_i/P}{P_i/P}\right)$$

Which implies,

$$\begin{aligned}\pi_t^{agg} &= \sum_{i=1}^{J}\Omega_{d,i}\left[\frac{1}{1-\epsilon_d}\widehat{\alpha}_{i,t} + \pi_{i,t} + p_{i,t-1}\right] \\ &= \boldsymbol{\Omega}_d'\left[\boldsymbol{\pi}_t + \boldsymbol{p}_{t-1} + \frac{1}{1-\epsilon_d}\boldsymbol{\alpha}_t\right]\end{aligned}$$

Plugging in to the sectoral PC gives

$$\begin{aligned}\boldsymbol{\pi_t} &= \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\boldsymbol{\pi}_t + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{\eta}\widehat{\boldsymbol{\theta}}_t + \beta E_t\boldsymbol{\pi_{t+1}} \\ &\quad + \boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{r}_t - \boldsymbol{a}_t + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\boldsymbol{p}_{t-1} - \frac{1}{1-\epsilon_d}\boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\mathbf{1}\boldsymbol{\Omega}_d'\boldsymbol{\alpha}_t \\ \Rightarrow \boldsymbol{\pi}_t &= \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1}\left[\boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{\eta}\boldsymbol{\theta}_t + \beta E_t\boldsymbol{\pi}_{t+1}\right] + \boldsymbol{\upsilon}_t^{sec}\end{aligned}$$

Where

$$\begin{aligned}\boldsymbol{\upsilon}_t^{sec} &= \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \\ &\quad \left[\boldsymbol{\Omega}_n\boldsymbol{\Gamma}_Q\boldsymbol{r}_t - \boldsymbol{a}_t + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\boldsymbol{p}_{t-1} - \frac{1}{1-\epsilon_d}\boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\mathbf{1}\boldsymbol{\Omega}_d'\boldsymbol{\alpha}_t\right]\end{aligned}$$

Finally, using the expression for aggregate inflation, the aggregate NKPC is

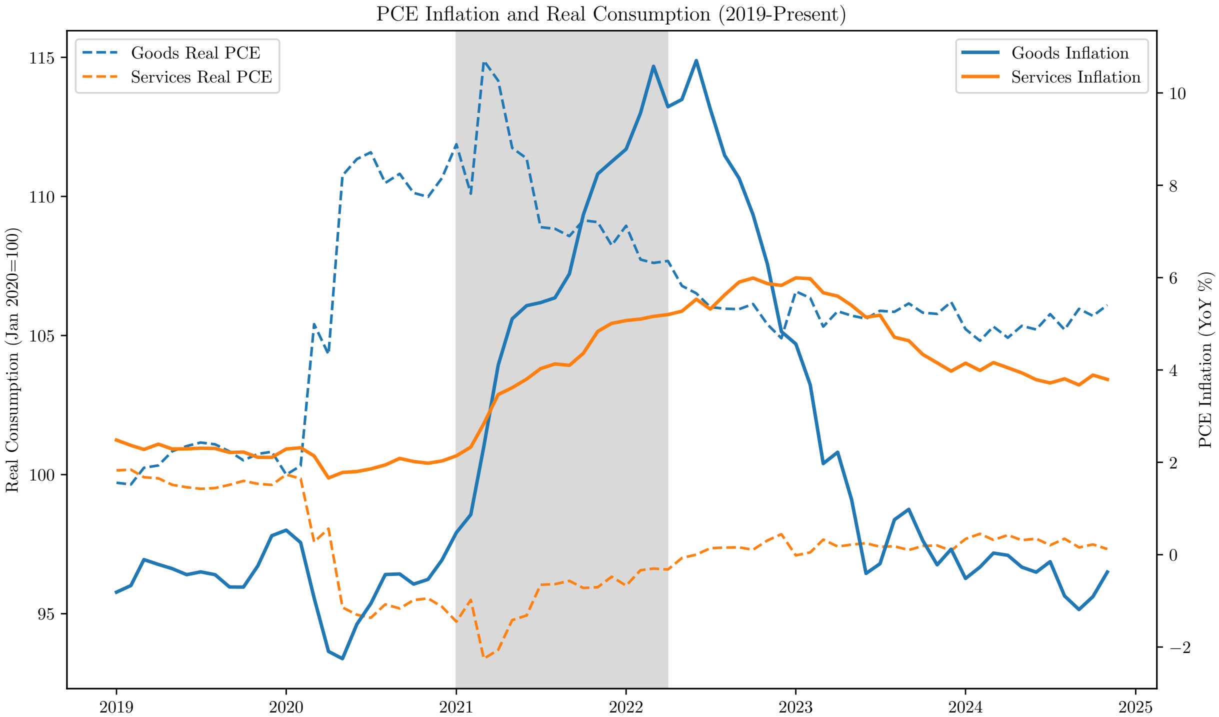
$$\pi_t^{agg} = \boldsymbol{\Gamma}_\theta\boldsymbol{\theta}_t + \boldsymbol{\Gamma}_\pi E_t\boldsymbol{\pi}_{t+1} + \upsilon_t$$

Where,

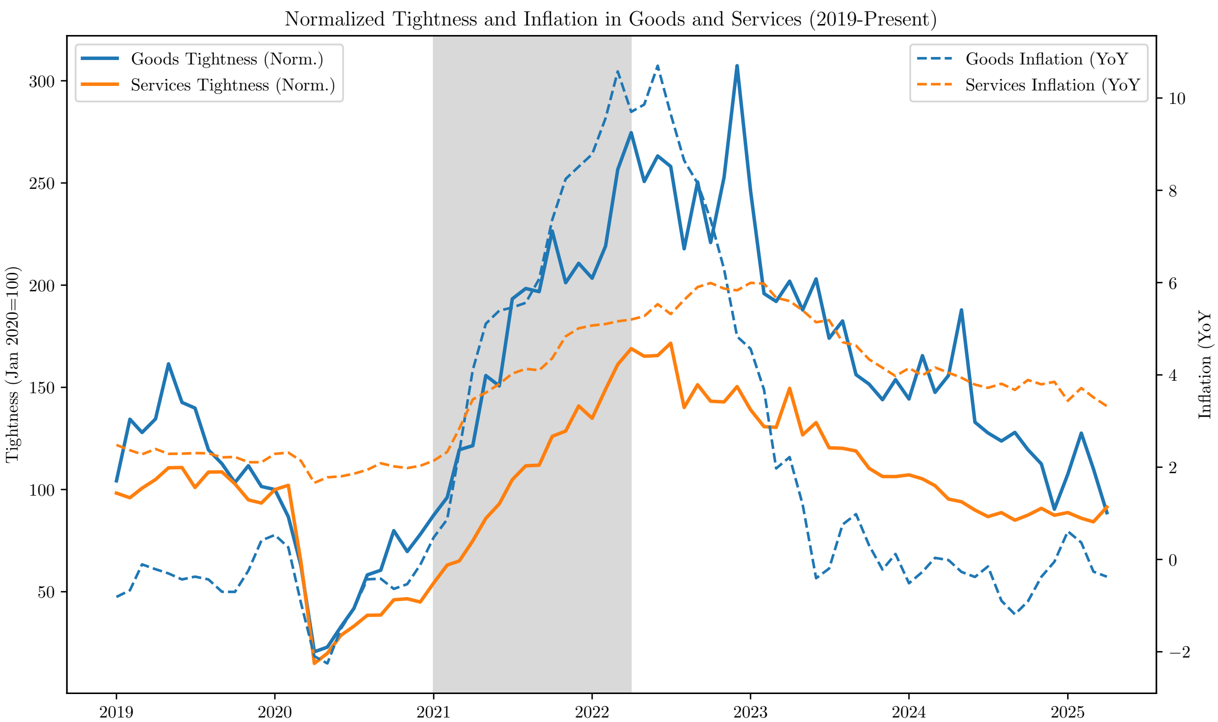
$$
\begin{aligned}
\boldsymbol{\Gamma}_\theta &= \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \boldsymbol{\Omega}_n \boldsymbol{\Gamma}_Q \boldsymbol{\eta} \\
\boldsymbol{\Gamma}_\pi &= \beta \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \\
\upsilon_t &= \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{I} + \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right] \boldsymbol{p}_{t-1} \\
&\quad + \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{\Omega}_n \boldsymbol{\Gamma}_Q \boldsymbol{r}_t - \boldsymbol{a}_t\right] \\
&\quad + \boldsymbol{\Omega}_d' \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\left(\boldsymbol{I} - \mathbf{1}\boldsymbol{\Omega}_d'\right)\right]^{-1} \left[\boldsymbol{I} - \boldsymbol{\lambda}\left(\boldsymbol{\Omega}_x - \boldsymbol{I}\right)\mathbf{1}\boldsymbol{\Omega}_d'\right] \boldsymbol{\alpha}_t
\end{aligned}
$$

C Unemployment Based Measure of Tightness

In this section, I show that the patterns I highlight in section 4.1 hold when using the more conventional measure of tightness $\frac{V}{U}$.

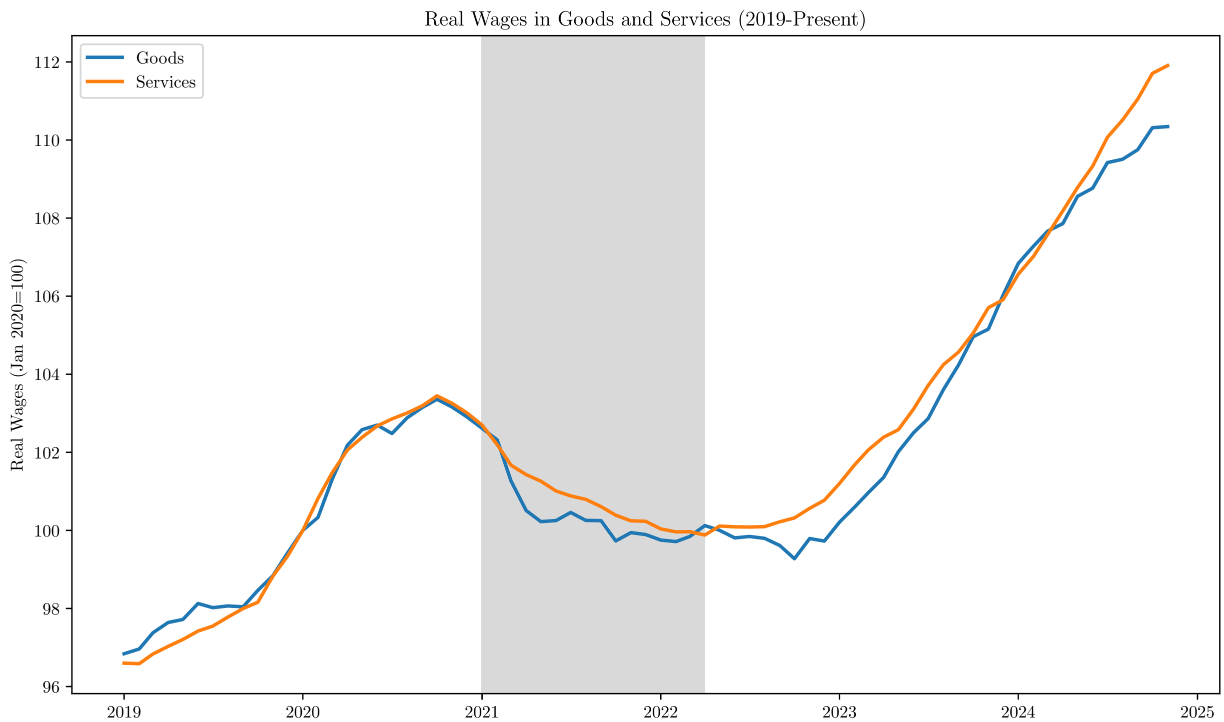


(a) Real PCE and PCE Inflation.

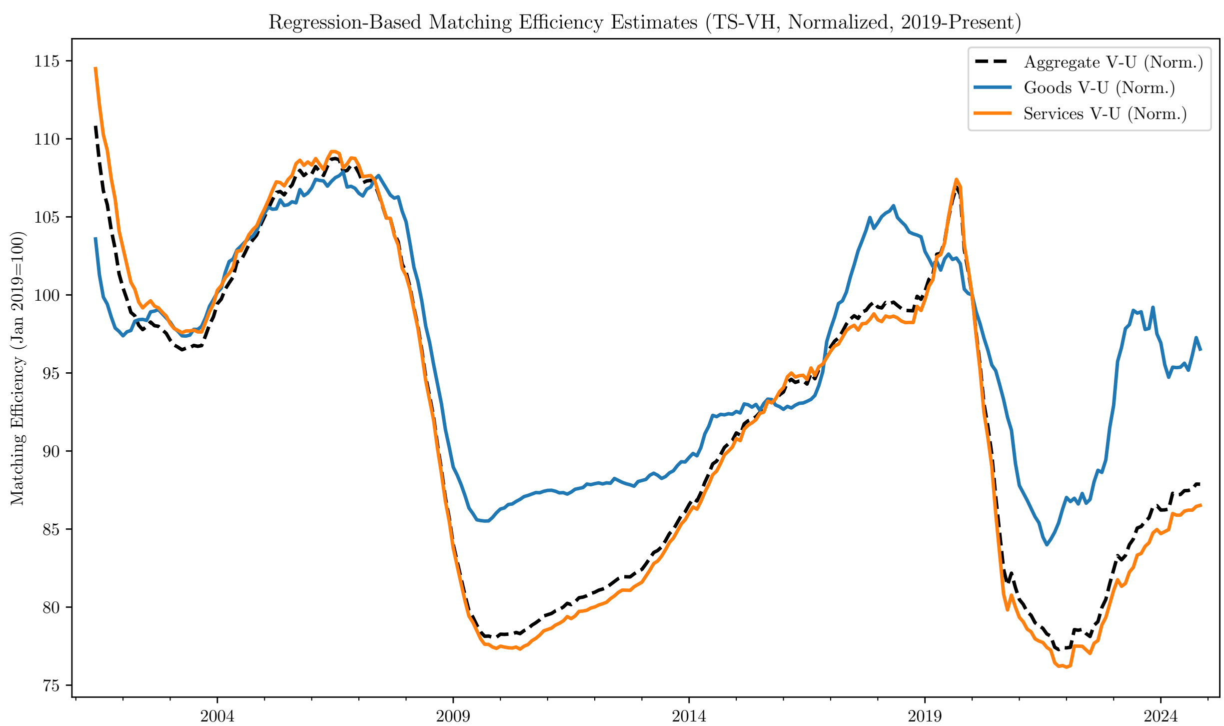


(b) labor market tightness and PCE Inflation.

Figure 13: Left Axis (both subplots): Dashed lines are changes in real PCE relative to January 2020, by major expenditure category: goods (blue) and services (orange). Right Axis: Solid lines are year-over-year changes in the PCE price index, by major category: goods (blue) and services (orange). The gray shaded area indicates the period from the start of the inflation surge in early 2021, to the first Federal Reserve rate hike in March 2022.



(a) All Wages.



(b) Matching Efficiency

Figure 14: Top: Real wages for all workers in the goods (blue) and services (orange) sectors, relative to January 2020. Bottom: Matching efficiency in the goods (blue) and services (orange) sectors, relative to January 2020. The gray shaded area indicates the period from the start of the inflation surge in early 2021, to the first Federal Reserve rate hike in March 2022.