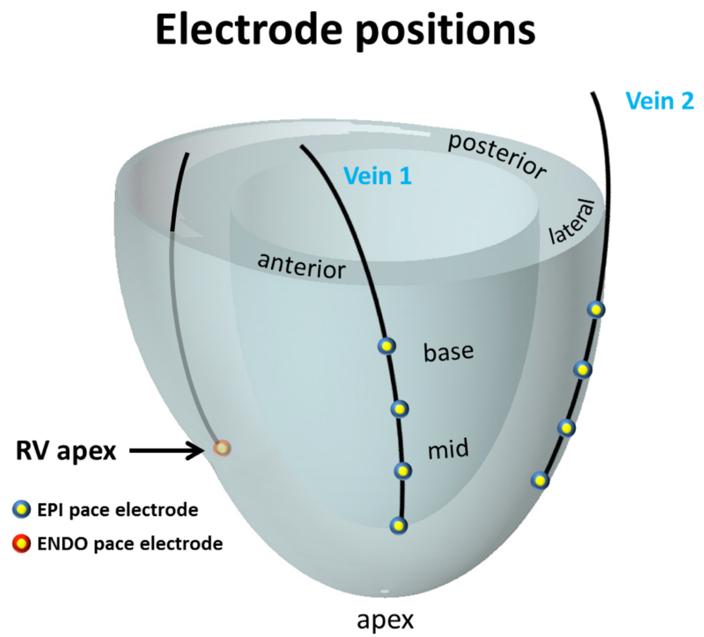
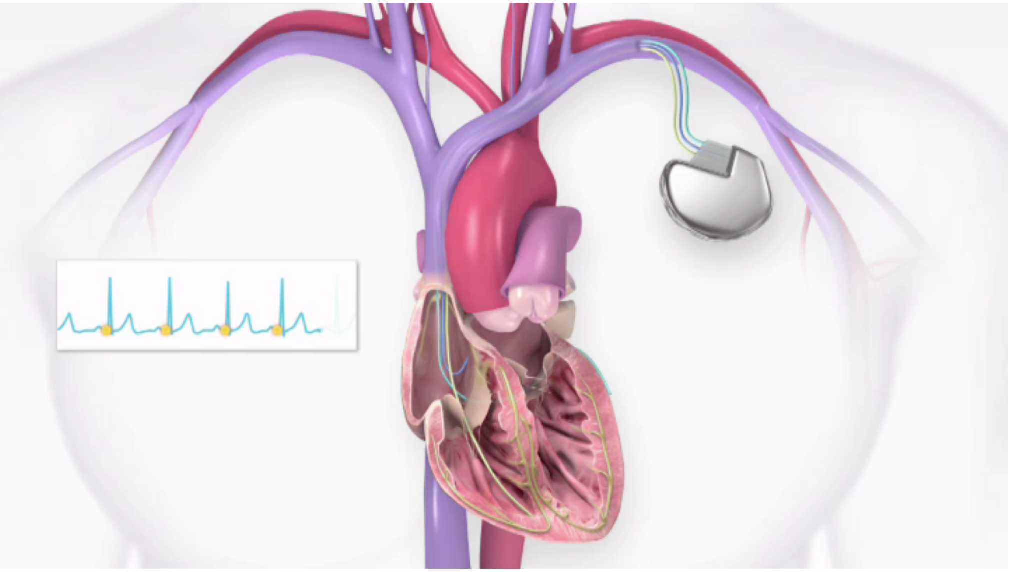
Heart diseases is the no. 1 cause of death in the Western World

- 31 % of all deaths
- 5 % of people between 60 and 70 years live with heart failure

American Heart Asscociation, 2015

Understanding the mechanical events occuring during a cardiac cycle may improve understanding of optimal treatment

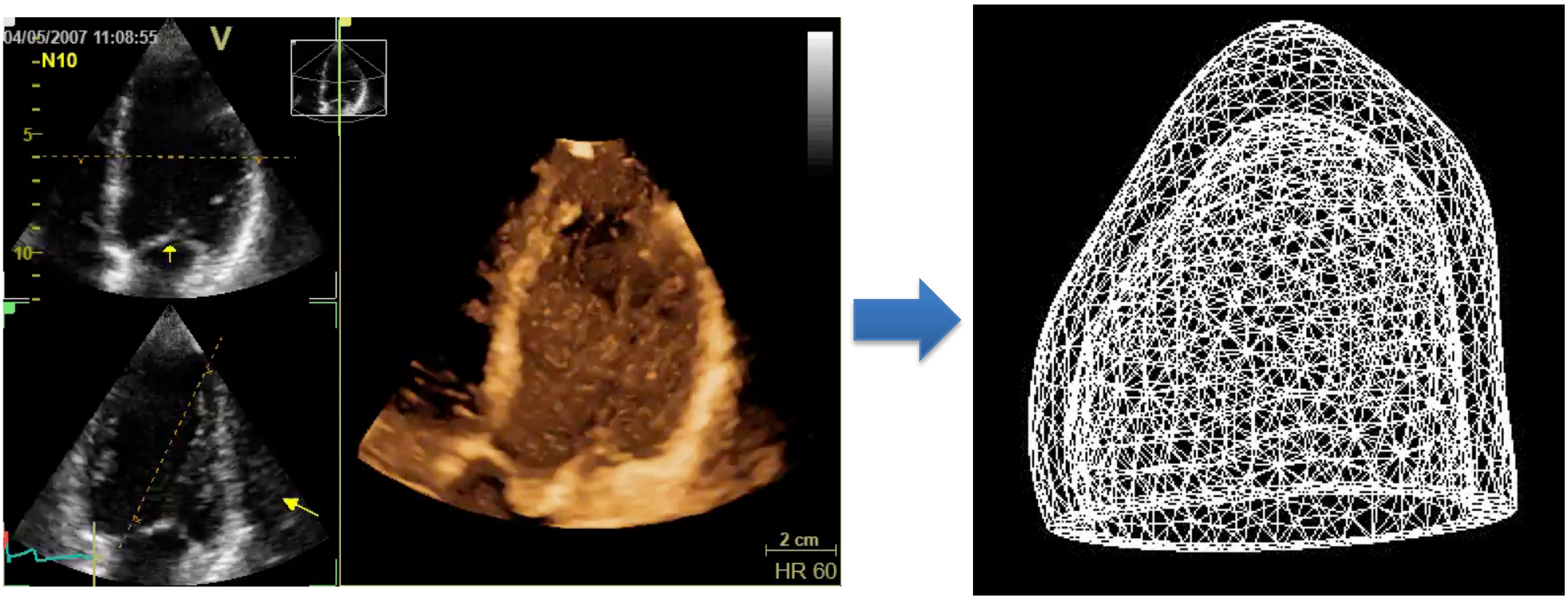




30 – 40 % do not respond

Source:CCI, American Heart Asscociation

Patient Specific modeling of the heart can provide us with insight and optimize patient treatment



FEniCS' 16

Optimization of a Spatially Varying Cardiac Contraction Parameter using the Adjoint Method

Henrik Finsberg*, Gabriel Balaban, Joakim Sundnes, Marie Rognes, Hans Henrik Odland, Stian Ross, Samuel Wall

May 19, 2016

CENTER FOR CARDIOLOGICAL INNOVATION

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The problem is formulated as a PDE-Constrained Optimization problem

$$\begin{aligned} \underset{\gamma}{\text{minimize}} \quad & I(\mathbf{u}, \gamma) \\ \text{subject to} \quad & R(\mathbf{u}, \gamma) = 0 \\ & \gamma \in [0, 1) \end{aligned}$$

I: Misfit functional between observed and simulated data

R: Force-balance equation

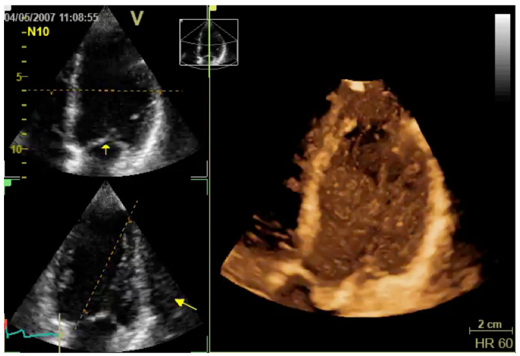
$\mathbf{u}$: displacement field

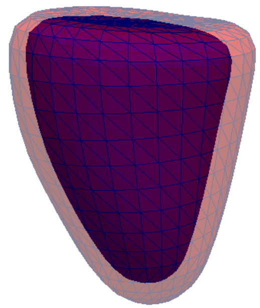
$\gamma = \gamma(\mathbf{X})$: contraction field

dolfin-adjoint

FEniCS Project

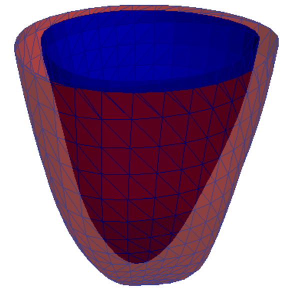
The geometry is extracted from 4D echocardiography, fiber orientations are assigned using a rule-based algorithm



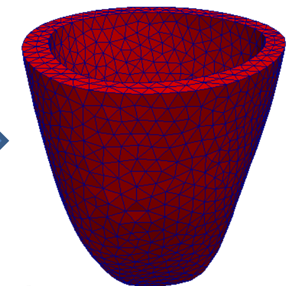


Epi, Endo

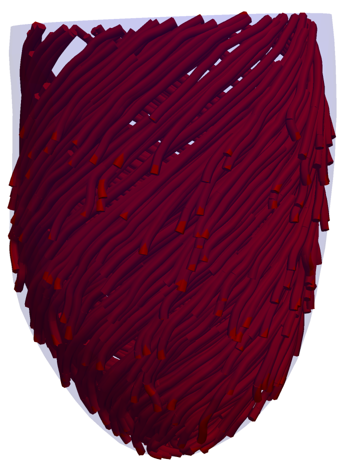
Orient
Cut
Smooth

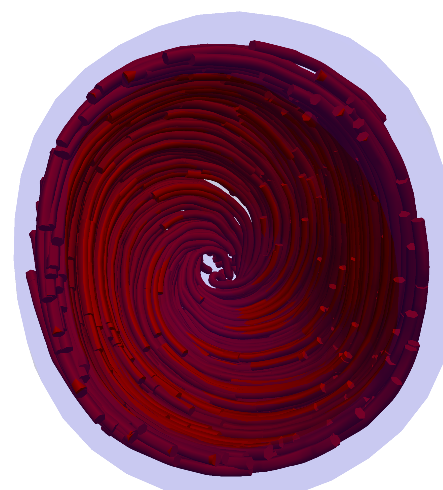


gmsh



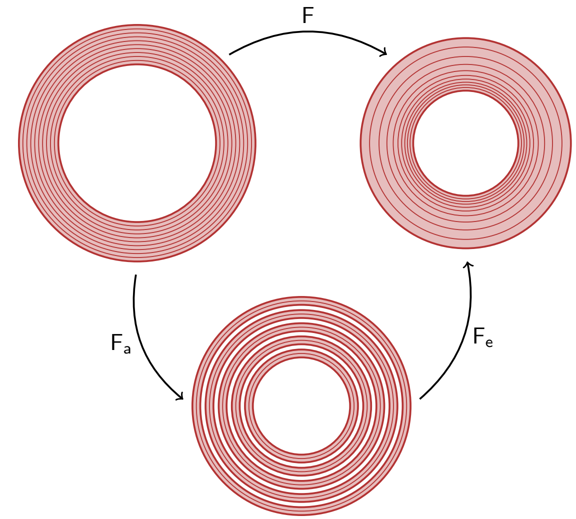
Mesh





Bayer et al 2012

The mechanical model is based on the active strain framework



Active Strain Decomposition:

$$\mathbf{F} = \mathbf{F}_e \mathbf{F}_a$$

4 parameters

Passive Mechanics:

$$\mathcal{W}(\mathbf{C}_e) = \frac{a}{2b}\left(e^{b(I_1 - 3)} - 1\right) + \frac{a_f}{2b_f}\left(e^{b_f (I_{4,\mathbf{f}_0} - 1)_+^2} - 1\right)$$

$$\mathbf{C}_e = \mathbf{F}_e^T \mathbf{F}_e \qquad I_{4,\mathbf{f}_0} = \mathbf{f}_0 \cdot (\mathbf{C}_e \mathbf{f}_0) \qquad I_1 = \mathrm{tr}(\mathbf{C}_e)$$

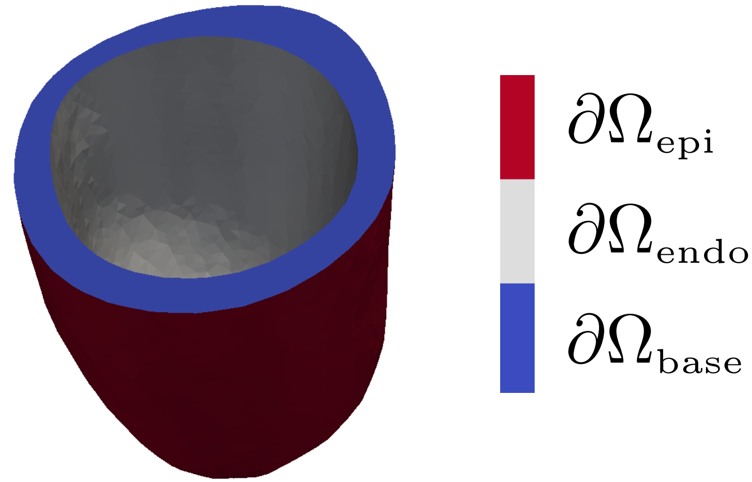
Active Mechanics:

 # parameters = # vertices

$$\mathbf{F}_a = (1 - \gamma)\mathbf{f}_0 \otimes \mathbf{f}_0 + \frac{1}{\sqrt{1-\gamma}}(\mathbf{I} - \mathbf{f}_0 \otimes \mathbf{f}_0)$$

Holzapfel and Ogden 2009, Pezzuto et. al

Boundary Conditions:



Endocardial pressure from clinical data :

$$\mathbf{T} = J p_{\mathrm{LV}} \mathbf{F}^{-T} \mathbf{N}, \ \mathbf{x} \in \partial\Omega_{\mathrm{endo}}$$

Fix base in longitudinal direction. Allow some displacement in the base plane:

$$\mathbf{u} = (u_1, u_2, u_3)$$

$$u_1 = 0, \ \mathbf{x} \in \partial\Omega_{\mathrm{base}}$$

$$\mathbf{T} = -k\mathbf{u}, \ \mathbf{x} \in \partial\Omega_{\mathrm{base}}$$

The force-balance equation represents the minimum potential energy state

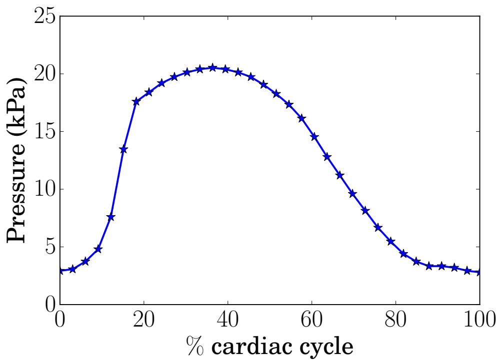
$$\Pi = \int_{\Omega} \mathcal{W}(\mathbf{C}_e) + p(J-1)dV$$

$$+ \text{ boundary conditions}$$

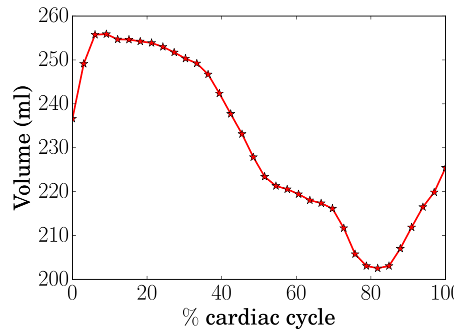
$$R(\mathbf{u}, p) = \begin{pmatrix} D_{\delta\mathbf{u}}\Pi \\ D_{\delta p}\Pi \end{pmatrix} = \mathbf{0}$$

We use measured pressure, volume and regional strain to personalize the mechanics

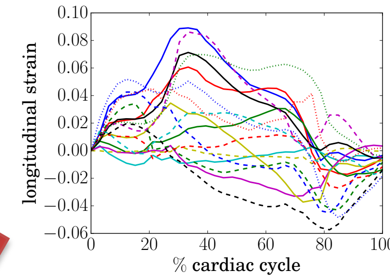
LV Pressure:



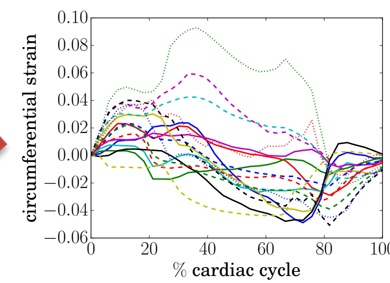
LV Volume:



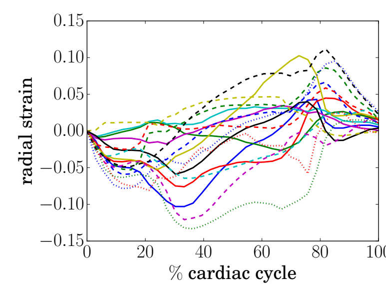
Longitudinal



Circumferential



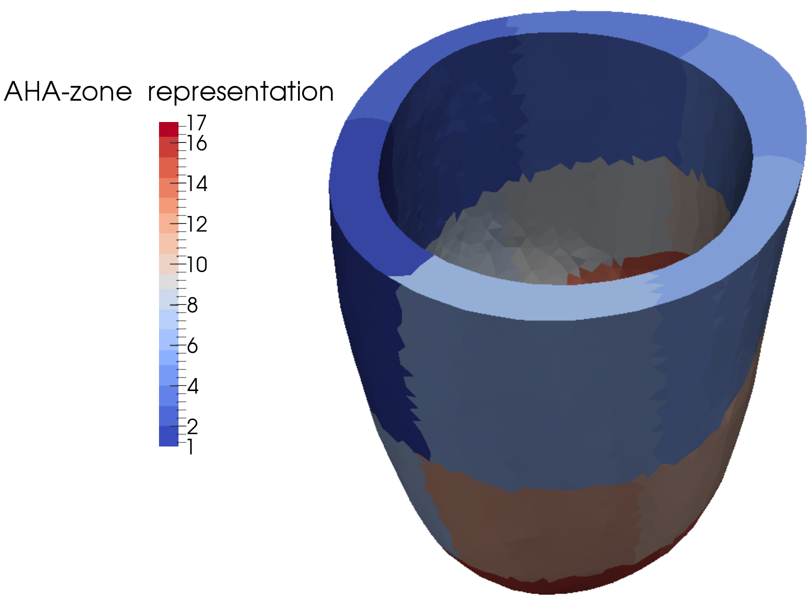
Radial

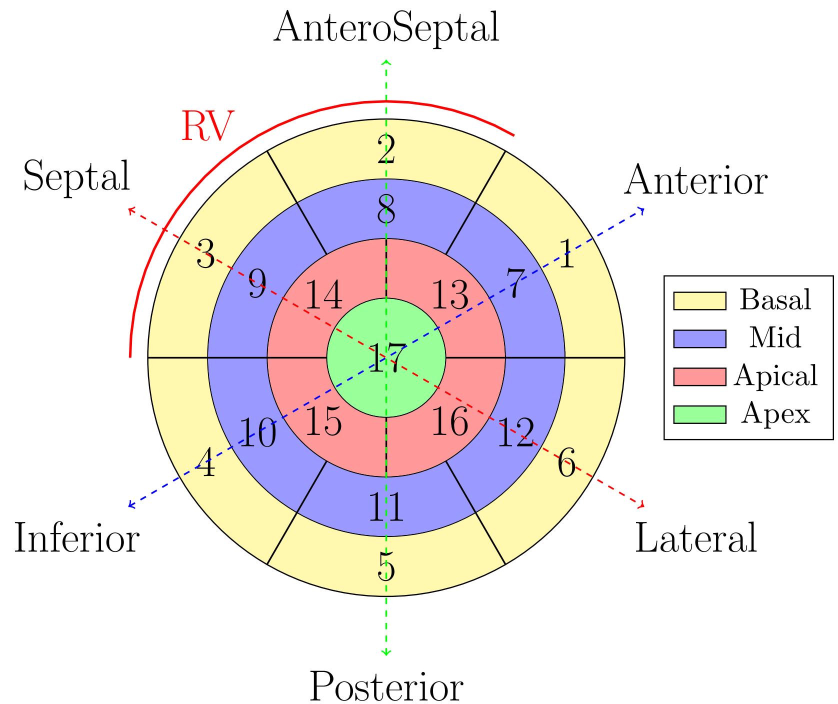


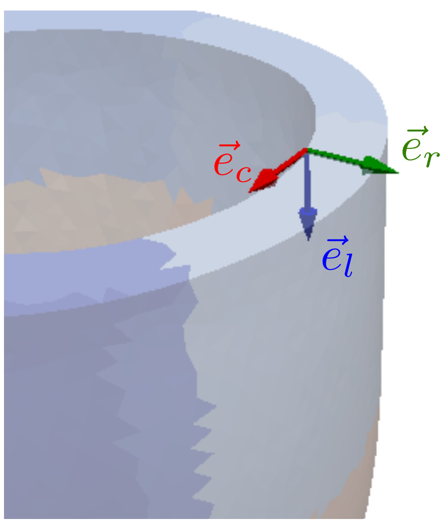
Boundary Condition

Objective function

The average regional strain is computed in three directions in the left ventricle(LV)







$$\Omega_{\mathrm{LV}} = \bigcup_{j=1}^{17} \Omega^j$$

$$\tilde{\varepsilon}_{k,j} = \frac{1}{|\Omega^j|} \int_{\Omega^j} e_k^T \nabla \mathbf{u} \cdot e_k \; dx$$

We minimize the difference between measured and simulated strain and volume

Volume misfit functional:

$$I^i_{\mathrm{vol}} = \left(\frac{V^i - \tilde{V}^i}{V^i}\right)^2, \quad \tilde{V}^i = -\frac{1}{3}\int_{\partial\Omega_{\mathrm{endo}}} (\mathbf{X} + \mathbf{u})\cdot J\mathbf{F}^{-T}\mathbf{N}\; dS,$$

Strain misfit functional:

$$I^i_{\mathrm{strain}} = \sum_{j=1}^{17} \sum_{k\in\{c,r,l\}} \left(\varepsilon^i_{k,j} - \tilde{\varepsilon}^i_{k,j}\right)^2$$

Total misfit functional (weighted):

$$I^i_{\alpha} = \alpha I^i_{\mathrm{vol}} + (1-\alpha) I^i_{\mathrm{strain}}$$

Parameters are estimated by solving a PDE-constrained optimization problem

Passive material parameters:

$$\underset{\mathbf{m}}{\text{minimize}} \quad \sum_{i=0}^{N_{\mathrm{ED}}} I_\alpha^i$$

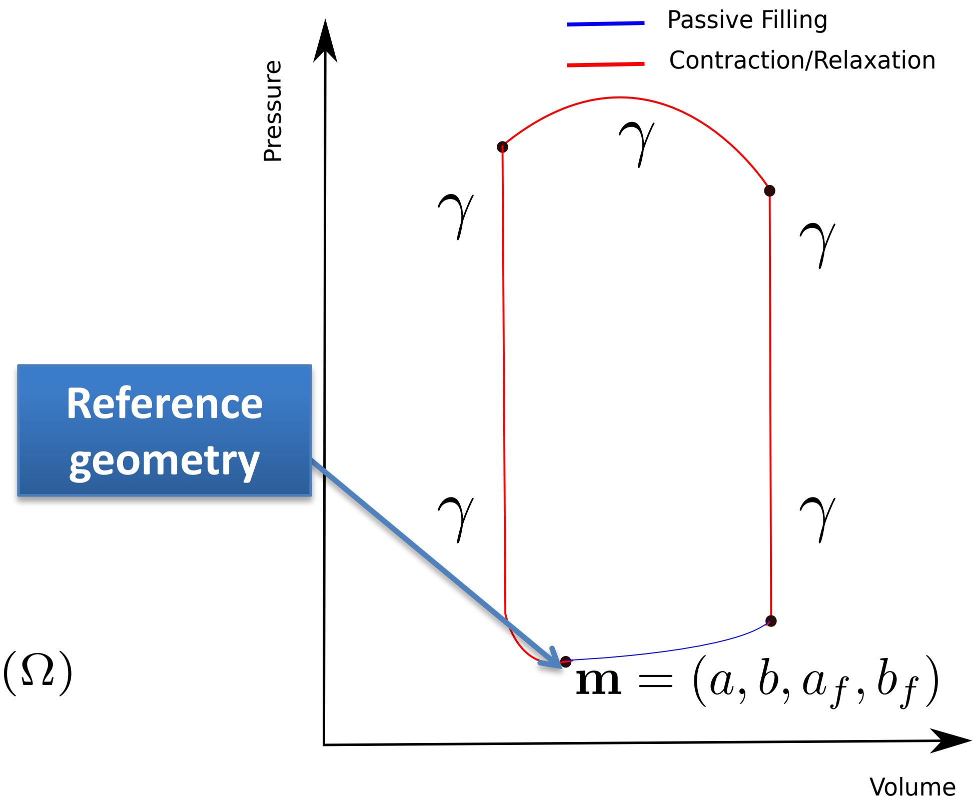
$$\text{subject to} \quad R(\mathbf{u}, p) = 0.$$

Active contraction parameter:

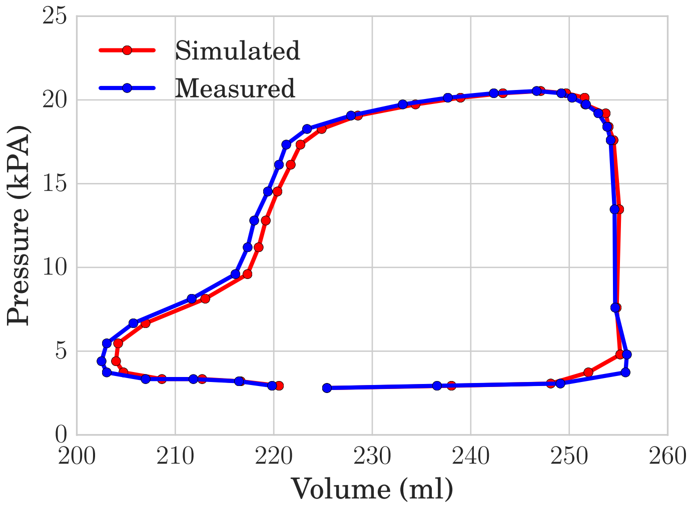
$$\underset{\gamma(\mathbf{x},i)}{\text{minimize}} \quad I_\alpha^i + \lambda \|\nabla \gamma\|^2_{L^2(\Omega)}$$

$$\text{subject to} \quad R(\mathbf{u}, p) = 0,$$

$$\gamma(\mathbf{x}, i) \in [0, 1), \;\; \mathbf{x} \in \Omega, i = N_{\mathrm{ED}} + 1, \cdots, N.$$

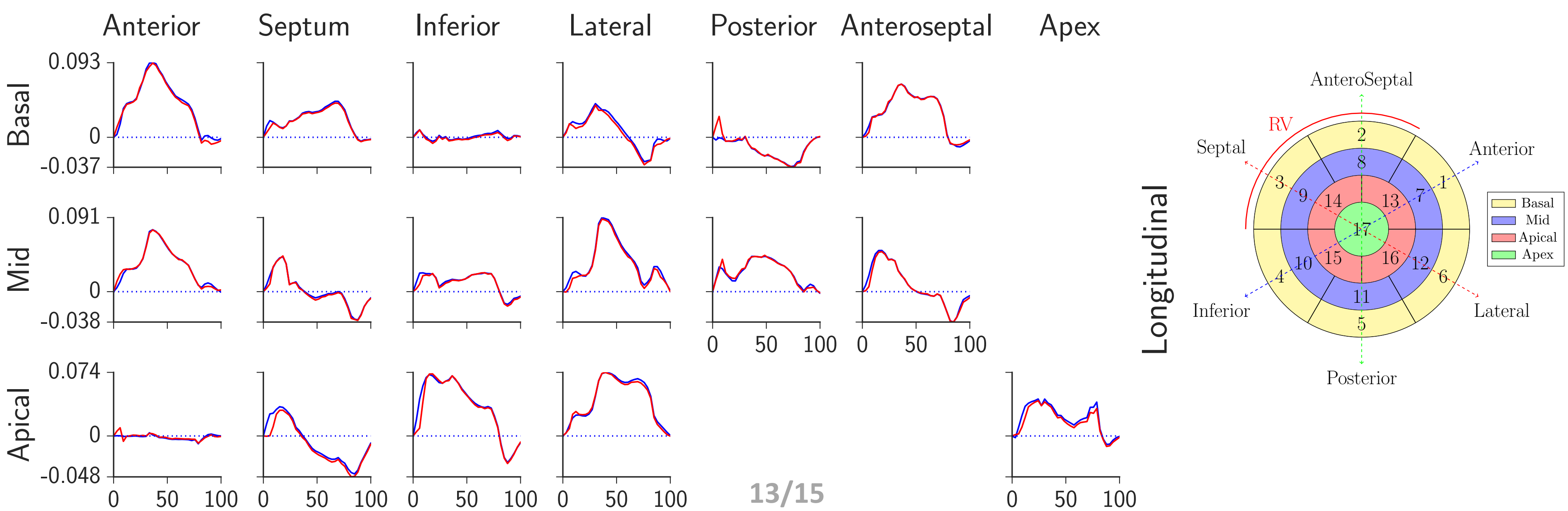


We get excellent match between simulated and measured data, at a low computational cost

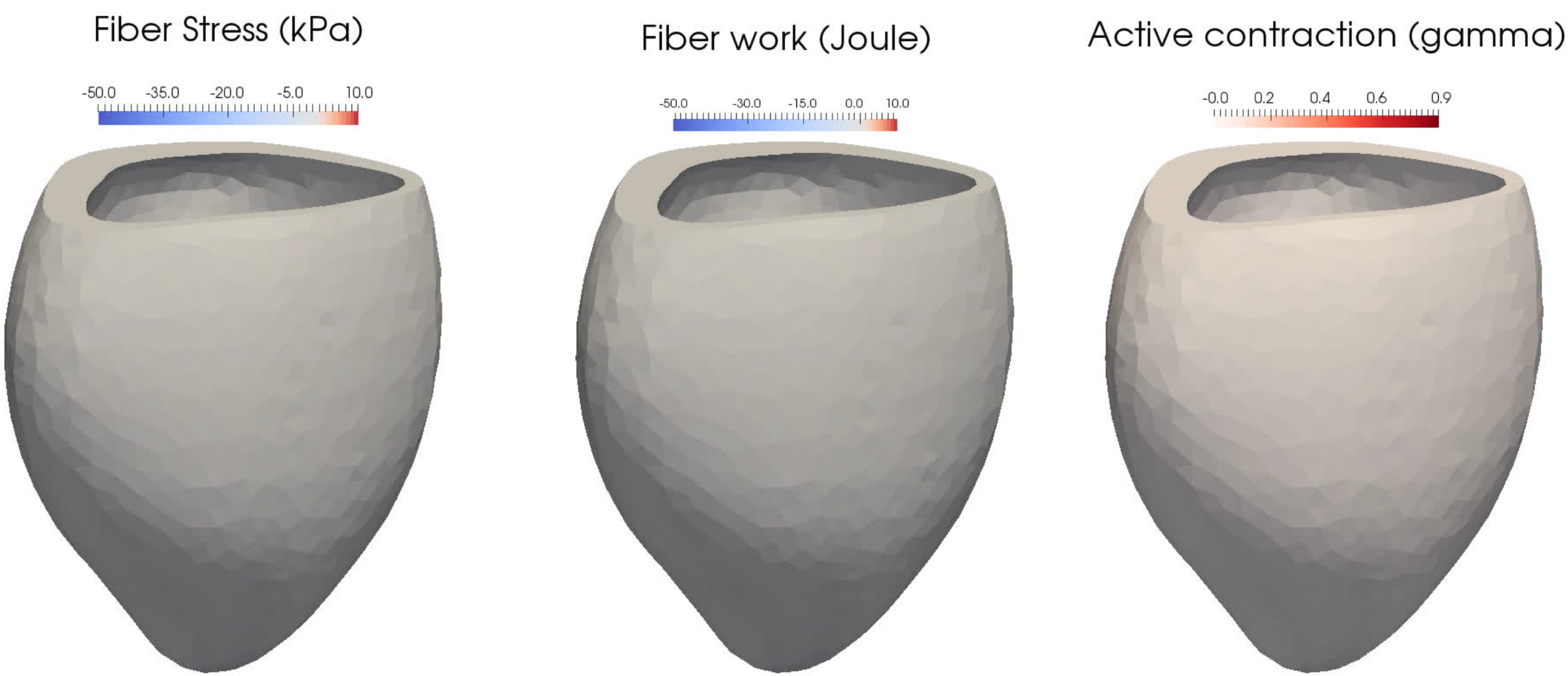


Timings running on 8 cores

Number of vertices	Functional evaluation (sec)	Derivative Evaluation (sec)
1457	3.6	13.0
2661	8.4	13.5
4812	19.3	16.5



We can use the simulation to visualize mechanical features which are impossible to measure



In summary, we are able to fit a model to clinical measurements, a key step towards bringing modeling into the clinic.

Acknowledgements

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- Joakim Sundnes
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- Stian Ross
- Samuel Wall

Thank you!

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