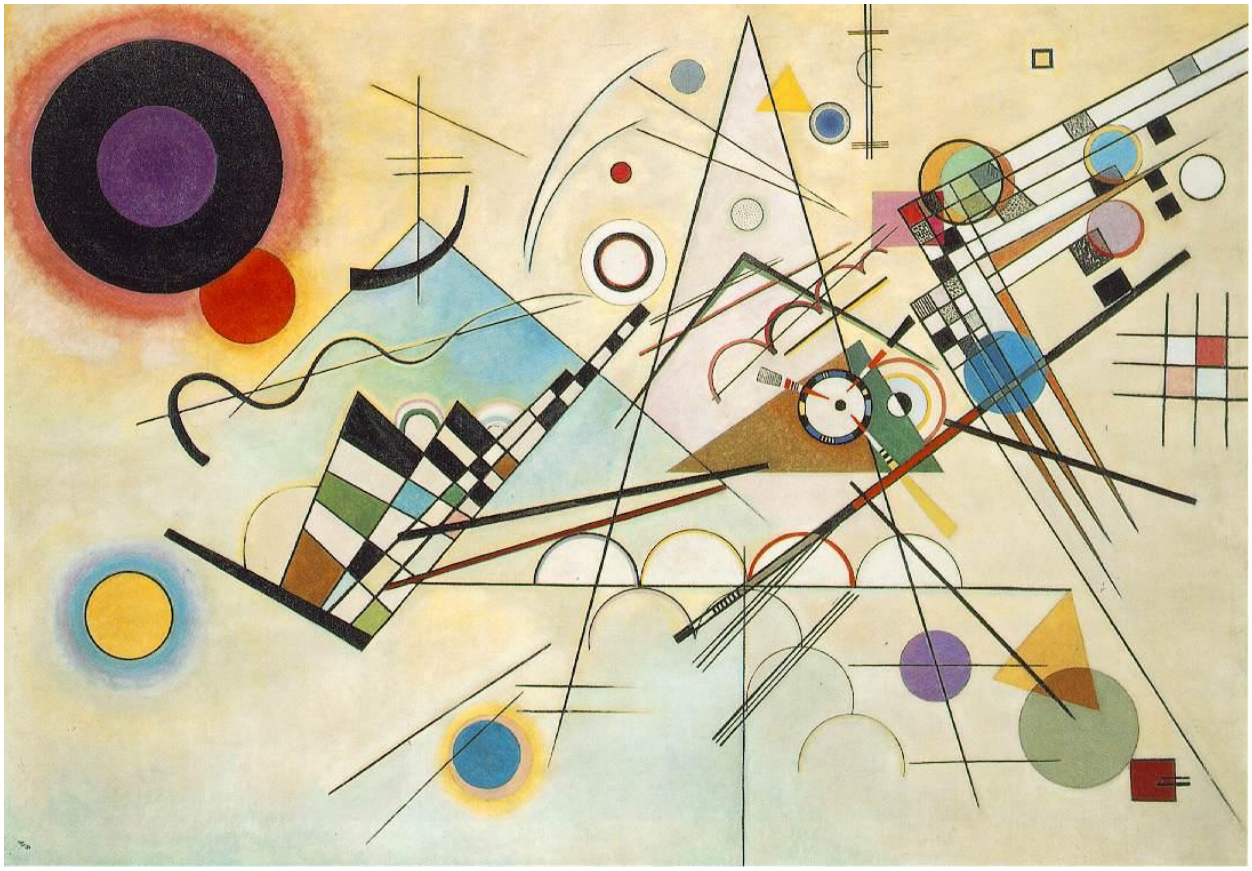

Intensional semantics

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About these lecture notes

These lecture notes have been evolving for many years now, starting with some notes from the early 1990s by Angelika Kratzer, Irene Heim, and Kai von Fintel, which have since been modified and expanded many times by Irene and/or Kai, with feedback and contributions from colleagues and students.

Cite as follows:
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GitHub repository with the current & up-to-date development version:
https://github.com/fintelkai/fintel-heim-intensional-notes

The cover image is "Composition 8" by Vasily Kandinsky:
https://www.guggenheim.org/artwork/1924.

Acknowledgments

Thanks to generations of students in our classes. Thanks to colleagues who have used these notes in their classes. Here's an almost certainly incomplete list of people who have helped us in some way or other with these notes (telling us about typos, asking questions, pointing out issues, etc.): Paul Elbourne, Nick Fleisher, Graeme Forbes, Adam Goodkind, Aaron Jacobs, Magda Kaufmann, Ezra Keshet, Angelika Kratzer, Manfred Kupffer, Eric Mack, Yuliya Manyakina, Liz Smeets, Vesela Tihomirova Simeonova, Ede Zimmermann.

Some advice

1. These notes presuppose familiarity with the material, concepts, and notation of the Heim & Kratzer textbook.
2. There are numerous exercises throughout the notes. It is highly recommended to do all of them and it is certainly necessary to do so if you at all anticipate doing semantics-related work in the future.
3. The notes are designed to go along with explanatory lectures. You should ask questions and make comments as you work through the notes.
4. While most of the object language examples are from English, semantics is a cross-linguistic enterprise. Students (and teachers) should bring in a cross-linguistic perspective throughout.
5. Students with semantic ambitions should also at an early point start reading supplementary material (as for example listed at the end of each chapter of these notes).
6. Prospective semanticists may start thinking about how *they* would teach this material.
7. For more advice, see http://kaivonfintel.org/prerequisites/.

Not a spectator sport

"As someone once said, Mathematics is not a spectator sport. To learn and understand Mathematics you have to get stuck in and get your hands dirty. You have to do the calculations, manipulations, and proofs yourself, not just read the stuff and pretend you understand it."

Harold Simmons, An introduction to category theory, 2011, Cambridge University Press

The same goes for formal semantics.

Contents

PART I

Beyond 3D: Worlds and times

Hypotheticals, 'imaginaries', conditionals, the syntax of counter-factuality and contingency may well be the generative centres of human speech. […] Language is the main instrument of man's refusal to accept the world as it is.

George Steiner, *After Babel* (1975), p.226 & p.228

1 Beginnings

1.1 Displacement

Hockett 1960 presented a list of DESIGN FEATURES OF HUMAN LANGUAGE, which continues to play a role in current discussions of animal communication and the evolution of language. One of the design features is DISPLACEMENT: human language is not restricted to discourse about the *actual here and now*. We use language to speculate about how things might have been different, about what would happen if we don't find dinner soon; we wonder what our friend thinks the world is like; we may want to tell our boss what it would take for us not to resign.

How does natural language untie us from the actual here and now? One degree of freedom is given by the ability to name entities and refer to them even if they are not where we are when we speak:

(1) Rahel is in Hamburg.

This kind of displacement is not something we will explore here. We'll take it for granted.

Consider a sentence with no names of absent entities in it:

Hockett cites this passage from Childe 1936: p.30 to illustrate the utility of displacement:

"[…] parents can, with the aid of language, instruct their offspring how to deal with situations which cannot conveniently be illustrated by actual concrete examples. The child need not wait till a bear attacks the family to learn how to avoid it. Instruction by example alone in such a case is liable to be fatal to some of the pupils. Language, however, enables the elders to forewarn the young of the danger while it is absent, and then demonstrate the appropriate course of action."

(2) It is snowing. [said in Cambridge]

On its own, (2) makes a claim about what is happening right now here in Cambridge. But there are devices at our disposal that can be added to (2), resulting in claims about snow in displaced situations. Displacement can occur in the TEMPORAL dimension:

(3) At noon yesterday, it was snowing.

This sentence makes a claim not about snow now but about snow at noon yesterday, a different time from now. We will look at temporal semantics later in this book.

In the beginning, we will focus on what might be called the MODAL dimension. Here's an example of modal displacement:

(4) COUNTERFACTUAL CONDITIONAL
If the storm system hadn't been deflected by the jet stream, it would have been snowing.

The terms MODAL and MODALITY descend from the Latin *modus*, "way", and are ancient terms pertaining to the way a proposition holds, necessarily, contingently, etc. For more on the history, see Auwera & Aguilar 2015.

This sentence makes a claim not about snow in the actual world but about snow in the world as it would have been if the storm system hadn't been deflected by the jet stream, a world distinct from the actual one (where the system did not hit us), a merely POSSIBLE WORLD.

If we wanted to be more fanciful, we could call this the Fifth Dimension (after the four dimensions of space and time). Take a look at the original intro to *The Twilight Zone*: https://www.youtube.com/watch?v=vB1Ot9MEOOs

Natural language abounds in modal constructions (see Kratzer 1981). Here are some other examples:

(5) MODAL AUXILIARIES
It may be snowing.

(6) MODAL ADVERBS
Possibly, it will snow tomorrow.

(7) PROPOSITIONAL ATTITUDES
Miriam believes that it is snowing.

(8) EVIDENTIALS
It appears that it is snowing.

(9) HABITUALS
Ellen smokes.

(10) GENERICS
Bears like honey.

What's modal about (9) and (10)? Consider an example like *This can opens at the top.* It's well possible that this can (or any of its kind) has not and will not ever be opened. Maybe it's meant to be displayed on a collector's shelf. So, our sentence must be talking about opening in some other possible world(s).

(11) IMPERATIVES
Get your snow shovels ready!

(12) SUFFICIENCY AND EXCESS
Linda is old enough to watch The X-Files.
Klara is too expensive to hire.

(13) INFINITIVAL RELATIVES

I know $\left\{\begin{matrix}\text{an}\\\text{the}\end{matrix}\right\}$ expert to talk to.

Exercise 1.1 *Collect some examples of modal displacement in a language other than English (whether spoken by you or someone else you know). These can serve as a touchstone of understanding as we proceed.* □

In this chapter, we will put in place the basic framework of INTENSIONAL SEMANTICS, the kind of semantics that models displacement of the point of evaluation in temporal and modal dimensions. To do this, we will start with one rather special example of modal displacement:

(14) In the world of Sherlock Holmes, a detective lives at 221B Baker Street.

(14) doesn't claim that a detective lives at 221B Baker Street in the actual world (presumably a false claim), but that in the world as it is described in the Sherlock Holmes stories of Sir Arthur Conan Doyle, a detective lives at 221B Baker Street (a true claim, of course). We choose this example rather than one of the more well-studied displacement constructions because we want to focus on conceptual and technical matters for now before we get distracted by many interesting complications.

The questions we want to answer are: How does natural language achieve this feat of modal displacement? How do we manage to make claims about other possible worlds? And why would we want to?

To make displacement possible and compositionally tractable, we need meanings of natural language expressions, and of sentences in particular, to be displaceable in the first place. They need to be "portable", so to speak, able to make claims about more than just the actual here and now. And we need other natural language expressions that take that portable meaning and apply it to some situation other than the actual here and now. That is what intensionality is all about.

The basic idea of the account we'll develop is this:

- expressions are assigned their semantic values relative to a possible world;
- in particular, sentences have truth-values in possible worlds;
- in the absence of modal displacement, we evaluate sentences with respect to the "actual" world, the world in which we are speaking;
- modal displacement changes the world of evaluation;
- displacement is effected by special operators, whose semantics is our primary concern here.

A terminological note: we will call the sister of the intensional operator its PREJACENT, a useful term introduced by our medieval colleagues.

1.2 An intensional semantics in 10 easy steps

1.2.1 Laying the foundations

Step 1: Possible worlds

Our first step is to introduce possible worlds. This is not the place to discuss the metaphysics of possible worlds in any depth. Instead, we will just start working with them and see what they can do for us. Basically, a possible world is a way that things might have been. In the actual world, there are two coffee mugs on my desk, but there could have been more or less. So, there is a possible world — albeit a rather bizarre one — where there are 17 coffee mugs on my desk. We join Heim & Kratzer in adducing this quote from Lewis (1986: 1f.):

> The world we live in is a very inclusive thing. Every stick and every stone you have ever seen is part of it. And so are you and I. And so are the planet Earth, the solar system, the entire Milky Way, the remote galaxies we see through telescopes, and (if there are such things) all the bits of empty space between the stars and galaxies. There is nothing so far away from us as not to be part of our world. Anything at any distance at all is to be included. Likewise the world is inclusive in time. No long-gone ancient Romans, no long-gone pterodactyls, no long-gone primordial clouds of plasma are too far in the past, nor are the dead dark stars too far in the future, to be part of the same world. …
>
> The way things are, at its most inclusive, means the way the entire world is. But things might have been different, in ever so many ways. This book of mine might have been finished on schedule. Or, had I not been such a commonsensical chap, I might be defending not only a plurality of possible worlds, but also a plurality of impossible worlds, whereof you speak truly by contradicting yourself. Or I might not have existed at all — neither myself, nor any counterparts of me. Or there might never have been any people. Or the physical constants might have had somewhat different values, incompatible with the emergence of life. Or there might have been altogether different laws of nature; and instead of electrons and quarks, there might have been alien particles, without charge or mass or spin but with alien physical properties that nothing in this world shares. There are ever so many ways that a world might be: and one of these many ways is the way that this world is.

Previously, our "metaphysical inventory" included a domain of entities and a set of two truth-values and increasingly complex functions between entities, truth-values, and functions thereof. Now, we will add possible worlds to the inventory. Let's assume we are given a set W, the set of all possible worlds, which is a vast space since there are so many ways that things might have been different from the way they are. Each world has as among its parts entities like you and me and these coffee mugs. Some of them may not exist in other possible worlds. So, strictly speaking each possible worlds has its own, possibly distinctive, domain of entities. What we will use in our system, however, will be the grand union of all these world-specific domains of entities. We will use D to stand for the set of all possible individuals.

It's possible that your previous inventory also included pluralities, events, and/or degrees. We're just adding to the menagerie now. Questions arise about what the limits are and whether the inventory is universal. For some discussion, see Rett 2022.

Among the many possible worlds that there are — according to Lewis, there is a veritable plenitude of them — is the world as it is described in the Sherlock Holmes stories by Sir Arthur Conan Doyle. In that world, there is a famous detective Sherlock Holmes, who lives at 221B Baker Street in London and has a trusted sidekick named Dr. Watson. Our sentence *In the world of Sherlock Holmes, a detective lives at 221B Baker Street* displaces the claim that a famous detective lives at 221B Baker Street from the actual world to the world as described in the Sherlock Holmes stories. In other words, the following holds (until we revise it):

(15) The sentence *In the world of Sherlock Holmes, a detective lives at 221B Baker Street* is true in a world w iff the sentence *a detective lives at 221B Baker Street* is true in the world as it is described in the Sherlock Holmes stories.

What this suggests is that we need to make space in our system for having devices that control in what world a claim is evaluated. This is what we will do now.

Step 2: The evaluation world parameter

Recall from H&K that we were working with a semantic interpretation function that was relativized to an assignment function g, which was needed to take care of pronouns, traces, variables, etc. From now on, we will relativize the semantic values in our system to possible worlds as well. What this means is that from now on, our interpretation function will have two superscripts: a world w and an assignment g: $[\![\cdot]\!]^{w,g}$. For a given expression ϕ, we call $[\![\phi]\!]^{w,g}$ the EXTENSION of ϕ at w (relative to g). The extension of an expression at a world is in fact the central notion of an intensional semantics. We will see that this system built around the extensions of expressions at any given world automatically provides the notion of an intension.

So, the prejacent embedded in (14) will have its truth-conditions described as follows:

(16) For any world w and assignment function g:
$[\![$a famous detective lives at 221B Baker Street$]\!]^{w,g} = 1$
iff a famous detective lives at 221B Baker Street in world w.

Recall from H&K, pp.22f, that what's inside the interpretation brackets is a mention of an object language expression. They make this clear by bold-facing all object language expressions inside interpretation brackets. In this book, we will follow common practice in the field and not use a special typographic distinction, but let it be understood that what is interpreted are object language expressions.

It is customary to refer to the world for which we are calculating the extension of a given expression as the EVALUATION WORLD. In the absence of any shifting devices, we would normally evaluate a sentence in the actual world. But then there are shifting devices such as our *in the world of Sherlock Holmes*. We will soon see how they work. But first some more pedestrian steps: adding lexical entries and composition principles that are formulated relative to a possible world. This will allow us to derive the truth-conditions as stated in (16) in a compositional manner.

Step 3: Lexical entries

Among our lexical items, we can distinguish between items which have a WORLD-DEPENDENT semantic value and those that are world-independent. Let's start with the entry for *famous*:

(17) For any $w \in W$ and any assignment function g:
$[\![\text{famous}]\!]^{w,g} = \lambda x \in D.\ x$ is famous in w.

Of course, "$\lambda x \in D.\ \ldots$" is short for "$\lambda x\colon\ x \in D.\ \ldots$". Get used to semanticists condensing their notation whenever convenient! A further step of condensation is taken below: "$\lambda x\colon\ x \in D_e.\ \ldots$" becomes "$\lambda x_e.\ \ldots$".

Always make sure that you actually understand what the notation means. Here, for example, we are saying that the semantic value of the word *famous* with respect to a given possible world w and a variable assignment g is that function that is defined for an argument x only if x is a member of the domain of individuals and that, if it is defined, yields the truth-value 1 if and only if x is famous in w. (17) does *not* mean that the function maps x to "x is famous in w", which would be very weird: mapping an individual to a meta-language statement!

A couple more predicates:

(18) a. $[\![\text{detective}]\!]^{w,g} = \lambda x \in D.\ x$ is a detective in w.
b. $[\![\text{lives-at}]\!]^{w,g} = \lambda x \in D.\ \lambda y \in D.\ y$ lives-at x in w.

(18) shows another customary condensation of notation: the universal quantification over evaluation worlds and assignment functions is left implicit and on the face of it, the parameters seem unbound. But they are.

The set of detectives will obviously differ from world to world, and so will the set of famous individuals and the set of pairs where the first element lives at the second element.

Other items have semantic values which do not differ from world to world. The most important such items are certain "logical" expressions, such as truth-functional connectives and determiners:

(19) a. $[\![\text{and}]\!]^{w,g} = \lambda u \in D_t.\ \lambda v \in D_t.\ u = v = 1.$
b. $[\![\text{the}]\!]^{w,g} = \lambda f \in D_{\langle e,t\rangle} :\ \exists! x[f(x) = 1].$ the y such that $f(y) = 1.$
c. $[\![\text{every}]\!]^{w,g} = \lambda f_{\langle e,t\rangle}.\ \lambda h_{\langle e,t\rangle}.\ \forall x_e :\ f(x) = 1 \rightarrow h(x) = 1.$
d. $[\![\text{a/some}]\!]^{w,g} = \lambda f_{\langle e,t\rangle}.\ \lambda h_{\langle e,t\rangle}.\ \exists x_e :\ f(x) = 1\ \&\ h(x) = 1.$

Again, note the ruthless condensation of the notation in (c) and (d): variables are subscripted with the type of the domain that their values are constrained to come from.

Note that there is no occurrence of w on the right-hand side of the entries in (19). That's the tell-tale sign of the world-independence of the semantics of these items.

We will also assume that proper names have world-independent semantic values, that is, they refer to the same individual in any possible world.

(20) a. $[\![\text{Noam Chomsky}]\!]^{w,g}$ = Noam Chomsky.
b. $[\![\text{Sherlock Holmes}]\!]^{w,g}$ = Sherlock Holmes.
c. $[\![\text{221B Baker Street}]\!]^{w,g}$ = 221B Baker Street.

This is a popular thing to complain about. Alternatives exist and may in the long run be preferable to the simpler assumption made here. See Matushansky 2008 for an influential contribution from a linguistic perspective. See Abbott 2013 and Cumming 2019 for some philosophical background.

Step 4: Composition principles

The old rules of Functional Application, Predicate Modification, and λ-Abstraction can be retained almost intact. We just need to modify them by adding world-superscripts to the interpretation function. For example:

(21) Functional Application (FA)
If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any world w and assignment g: if $[\![\beta]\!]^{w,g}$ is a function whose domain contains $[\![\gamma]\!]^{w,g}$, then $[\![\alpha]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\gamma]\!]^{w,g})$.

The rule simply passes the world parameter down.

Exercise 1.2 *Formulate the new versions of Predicate Modification and λ-Abstraction.* □

Step 5: Truth

We will want to connect our semantic system to the notion of the truth of an utterance. We first adopt the "Appropriateness Condition" from Heim & Kratzer (p.243):

(22) Appropriateness Condition
A context c is *appropriate* for an LF ϕ only if c determines a variable assignment g_c whose domain includes every index which has a free occurrence in ϕ.

We then intensionalize Heim & Kratzer's definition of truth and falsity of utterances:

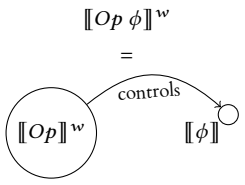
(23) Truth and Falsity Conditions for Utterances
An utterance of a sentence ϕ in a context c in a possible world w is *true* if $[\![\phi]\!]^{w,g_c} = 1$ and *false* if $[\![\phi]\!]^{w,g_c} = 0$.

Exercise 1.3 *Compute under what conditions an utterance in possible world w_7 (which may or may not be the one we are all living in) of the sentence* a famous detective lives at 221B Baker Street *is true.* □

1.2.2 Intensional operators

So far we have merely "redecorated" the system inherited from Heim & Kratzer. We have introduced possible worlds into our inventory, our lexical entries and our old composition principles. But with the tools we have now, all we can do so far is to keep track of the world in which we evaluate the semantic value of an expression, complex or lexical. We will get real mileage once we introduce intensional operators which are capable of shifting the world parameter. We mentioned that there are a number of devices for modal displacement. As advertised, for now, we will just focus on a very particular one: the expression *in the world of Sherlock Holmes*. We will assume, as seems reasonable, that this expression is a sentence-modifier both syntactically and semantically.

Here's an illustration of the basic idea we're trying to implement:



Step 6: A syncategorematic entry

We begin with a heuristic step. We want to derive something like the following truth-conditions for our sentence:

(24) $[\![$in the world of Sherlock Holmes,
a famous detective lives at 221B Baker Street$]\!]^{w,g} = 1$
iff the world w' as it is described in the Sherlock Holmes stories is such that there exists a famous detective in w' who lives at 221B Baker Street in w'.

We would get this if in general we had this rule for *in the world of Sherlock Holmes*:

(25) For any sentence ϕ, any world w, and any assignment g:
$[\![$in the world of Sherlock Holmes, $\phi]\!]^{w,g} = 1$
iff the world w' as it is described in the Sherlock Holmes stories is such that $[\![\phi]\!]^{w',g} = 1$.

This is a so-called syncategorematic treatment of the meaning of this expression. Instead of giving an explicit semantic value to the expression, we specify what effect it has on the meaning of a complex expression that contains it. In (25), we do not compute the meaning for *in the world of Sherlock Holmes, ϕ* from the combination of the meanings of its parts,

since *in the world of Sherlock Holmes* is not given a separate meaning, but in effect triggers a special composition principle. This format is very common in modal logic systems, which usually give a syncategorematic semantics for the two classic modal operators (the necessity operator □ and the possibility operator ◇). When one only has a few closed class expressions to deal with that may shift the world parameter, employing syncategorematic entries is a reasonable strategy. But we are facing a multitude of displacement devices. We will therefore need to make our system more modular.

The diamond ◇ symbol for possibility is due to C.I. Lewis, first introduced in Lewis & Langford 1932, but he made no use of a symbol for the dual combination ¬◇¬. The dual symbol □ ("Box") was later devised by F.B. Fitch and first appeared in print in 1946 in a paper by his doctoral student Barcan (1946). See footnote 425 of Hughes & Cresswell 1968. Another notation one finds is *L* for necessity and *M* for possibility, the latter from the German *möglich* 'possible'.

We want to give *in the world of Sherlock Holmes* its own meaning and combine that meaning with that of its prejacent by a general composition principle. The Fregean slogan we adopted says that all composition is function application (modulo the need for λ-abstraction and the possible need for predicate modification).

See Heim & Kratzer, Section 4.3, pp. 63–72 for a reminder about the status of predicate modification.

What we will want to do is to make (24) be the result of functional application. But we can immediately see that it cannot be the result of our usual rule of functional application, since that would feed to *in the world of Sherlock Holmes* the semantic value of *a famous detective lives in 221B Baker Street* in w, which would be a particular truth-value, 1 if a famous detective lives at 221B Baker Street in w and 0 if there doesn't. And whatever the semantics of *in the world of Sherlock Holmes* is, it is certainly *not* a truth-functional operator.

We need to feed something else to *in the world of Sherlock Holmes*. At the same time, we want the operator to be able to shift the evaluation world of its prejacent. Can we do this?

Exercise 1.4 *How would you show that* in the world of Sherlock Holmes *is not a truth-functional operator?* □

Step 7: Intensions

We will define a richer notion of semantic value, the INTENSION of an expression. This will be a function from possible worlds to the extension of the expression in that world. The intension of a sentence can be applied to any world and give the truth-value of the sentence in that world. Intensional operators take the intension of their prejacent as their argument, that is we will feed the intension of the embedded sentence to the shifting operator. The operator will use that intension and apply it to the world it wants the evaluation to happen in. Voilà.

Now let's spell that account out. Our system actually provides us with two kinds of meanings. For any expression α, we have $[\![\alpha]\!]^{w,g}$, the semantic value of α in w, also known as the EXTENSION of α in w. But since our system delivers an extension (or lack of one) for any expression at any world and assignment, we can also calculate $\lambda w.[\![\alpha]\!]^{w,g}$, the function that assigns to any world w the extension of α in that world. This is usually called the INTENSION of α.

Just like H&K, we make no claim that the semantic values that are attributed to expressions in our framework fully capture what is informally meant by "meaning". But certainly, intensions come closer to "meaning" than extensions.

We will sometimes use an abbreviatory notation for the intension of α:

(26) $[\![\alpha]\!]^g_{\mathfrak{c}} := \lambda w.[\![\alpha]\!]^{w,g}$

The notation with the subscripted cent-sign comes from Montague Grammar. See Dowty, Wall & Peters 1981: p. 147 (it's a bit of mystery to us why the cent-sign was chosen for this. There's a footnote marker ([6]) in the relevant place but no corresponding footnote text in the endnotes to the chapter. In fact, the existing footnotes in that chapter start with Fn.1 on a much later page.

It should be immediately obvious that since the definition of intension abstracts over the evaluation world, intensions are not world-dependent.

Note that strictly speaking, it now makes no sense anymore to speak of "*the* semantic value" of an expression α. What we have is a semantic system that allows us to calculate extensions (for a given possible world w) as well as intensions for all (interpretable) expressions. We will see that when α occurs in a particular bigger tree, it will always be determinate which of the two "semantic values" of α is the one that enters into the compositional semantics. So, that one — whichever one it is, the extension or the intension of α — might then be called "*the* semantic value of α in the tree β".

It should be noted that the terminology of EXTENSION vs. INTENSION is time-honored but that the possible worlds interpretation thereof is more recent. The technical notion we are using is certainly less rich a notion of meaning than traditionally assumed. (For example, Frege's "modes of presentation" are not obviously captured by this possible worlds implementation of extension/intension.)

The definition here is simplified, in that it glosses over the fact that some expressions, in particular those that contain PRESUPPOSITION TRIGGERS, may fail to have an extension in certain worlds. In such a case, the intension has no extension to map such a world to. Therefore, the intension will have to be a partial function. So, the official, more "pedantic", definition will have to be as follows: $[\![\alpha]\!]^g_{\mathfrak{c}} := \lambda w\colon\ \alpha \in \mathrm{dom}([\![\ ]\!]^{w,g}).[\![\alpha]\!]^{w,g}$.

The Port-Royal logicians distinguished EXTENSION from COMPREHENSION. Leibniz preferred the term INTENSION rather than COMPREHENSION. The notion probably goes back even further. See Spencer 1971 for some notes on this. The possible worlds interpretation is due to Carnap 1947.

Step 8: Semantic types and semantic domains

If we want to be able to feed the intensions to lexical items like *in the world of Sherlock Holmes*, we need to have the appropriate types in our system.

Recall that W is the set of all possible worlds. And recall that D is the set of all POSSIBLE INDIVIDUALS and thus contains all individuals existing in the actual world *plus* all individuals existing in any of the merely possible worlds.

We now expand the set of semantic types, to add intensions. Intensions are functions from possible worlds to all kinds of extensions. So, basically we want to add for any kind of extension we have in our system, a corresponding kind of intension, a function from possible worlds to that kind of extension.

We add a new clause, (27c), to the definition of semantic types:

(27) SEMANTIC TYPES

a. e and t are semantic types.
b. If σ and τ are semantic types, then $\langle\sigma,\tau\rangle$ is a semantic type.
c. If σ is a semantic type, then $\langle s,\sigma\rangle$ is a semantic type.
d. Nothing else is a semantic type.

This is as good a place as any to add a sermon about the notation for types. Functional types like $\langle e,t\rangle$ are written with angled brackets because of the mathematical connection between functions and ordered pairs. The basic types like e and t, however are not written with angled brackets, since they are not of the type of functions. Please do not make this mistake. See Rawlins 2018 for more of a disquisition on this.

We also add a fourth clause to the previous definition of semantic domains:

(28) Semantic Domains

a. $D_e = D$, the set of all possible individuals

b. $D_t = \{0, 1\}$, the set of truth-values

c. If σ and τ are semantic types, then $D_{\langle\sigma,\tau\rangle}$ is the set of all functions from D_σ to D_τ.

d. Intensions: If σ is a type, then $D_{\langle s,\sigma\rangle}$ is the set of all functions from W to D_σ.

Clause (d) is the addition to our previous system of types. The functions of the schematic type $\langle s, \ldots\rangle$ are intensions. Here are some examples of intensions:

- The intensions of sentences are of type $\langle s, t\rangle$, functions from possible worlds to truth values. These are usually called propositions. Note that if the function is total, then we can see the sentence as picking out a set of possible worlds, those in which the sentence is true. More often than not, however, propositions will be partial functions from worlds to truth-values, that is functions that fail to map certain possible worlds into either truth-value. This will be the case when the sentence contains a presupposition trigger, such as *the*. The famous sentence *The King of France is bald* has an intension that (at least in the analysis sketched in Heim & Kratzer) is undefined for any world where there fails to be a unique King of France.
- The intensions of one-place predicates are of type $\langle s, \langle e, t\rangle\rangle$, functions from worlds to set of individuals. These are usually called properties.
- The intensions of expressions of type e are of type $\langle s, e\rangle$, functions from worlds to individuals. These are usually called individual concepts.

Note a curious feature of this set-up: there is no type s and no associated domain. This corresponds to the assumption that there are no expressions of English that take as their extension a possible world, that is, there are no pronouns or names referring to possible worlds. We will actually question this assumption later in this book. For now, we will stay with this more conventional set-up.

Exercise: specify the intension of *The King of France is bald* in lambda notation.

Step 9: A lexical entry for a shifter

We are ready to formulate the lexical entry for *in the world of Sherlock Holmes*:

(29) $[\![\text{in the world of Sherlock Holmes}]\!]^{w,g} =$
$\lambda p_{\langle s,t\rangle}$. the world w' as it is described in the Sherlock Holmes stories is such that $p(w') = 1$.

That is, *in the world of Sherlock Holmes* expects as its argument a function of type $\langle s, t\rangle$, a proposition. It yields the truth-value 1 iff the proposition is true in the world as it is described in the Sherlock Holmes stories.

All that's left to do now is to provide *in the world of Sherlock Holmes* with a proposition as its argument. This is the job of a new composition principle.

This is not yet the final semantics, see Section 1.3 for complications. One complication we will not even start to discuss is that obviously it is not a necessity that there are Sherlock Holmes stories in the first place and that the use of this operator *presupposes* that they exist; so a more fully explicit semantics would need to build in that presuppositional component. Also, note again the condensed notation: "$\lambda p_{\langle s,t\rangle}$. ..." stands for the fully official "$\lambda p\colon p \in D_{\langle s,t\rangle}$. ...".

Step 10: Intensional Functional Application

We add the new rule of Intensional Functional Application.

(30) Intensional Functional Application (IFA)
If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any world w and assignment g: if $[\![\beta]\!]^{w,g}$ is a function whose domain contains $[\![\gamma]\!]^g_c$, then $[\![\alpha]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\gamma]\!]^g_c)$.

Note that for the function-denoting expressions β, we use its extension at w. Question: what would go wrong if we tried $[\![\beta]\!]^g_c([\![\gamma]\!]^g_c)$?

This is the crucial move. It makes space for expressions that want to take the intension of their sister as their argument and do stuff to it. Now, everything is in place. Given (29), the semantic argument of *in the world of Sherlock Holmes* will not be a truth-value but a proposition. And thus, *in the world of Sherlock Holmes* will be able to check the truth-value of its prejacent in various possible worlds. To see in practice that we have all we need, please do the following exercise.

Exercise 1.5 *Calculate the conditions under which an utterance in a given possible world w_7 of the sentence* in the world of the Sherlock Holmes stories, a famous detective lives at 221B Baker Street *is true.* □

Exercise 1.6 *What in our system prevents us from computing the extension of* Watson is slow*, for example, by applying the intension of* slow *to the extension of* Watson*? What in our system prevents us from computing the extension of* Watson is slow *by applying the intension of* slow *to the intension of* Watson*?* □

Exercise 1.7 *What is wrong with the following equation:*

(31) (λx. x is slow in w) (Watson) = Watson is slow in w ?

[Hint: there is nothing wrong with the following:

(32) (λx. x is slow in w) (Watson) = 1 iff Watson is slow in w.] □

Please think about this exercise before looking at Section 1.5, which explores this issue.

1.3 Comments and complications

1.3.1 Intensions all the way?

We have seen that to adequately deal with expressions like *in the world of Sherlock Holmes*, we need an intensional semantics, one that gives us access to the extensions of expressions across the multitude of possible worlds. At the same time, we have kept the semantics for items like *and*, *every*, and *a* unchanged and extensional. This is not the only way one can set up an intensional semantics. The following exercise demonstrates this.

Exercise 1.8 *Consider the following "intensional" meaning for* and*:*

(33) $[\![and]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \lambda q_{\langle s,t\rangle}.\ p(w) = q(w) = 1.$

With this semantics, the conjunction and *would operate on the intensions of the two conjoined sentences. In any possible world w, the complex sentence will be true iff the component propositions are both true of that world.*

Compute the truth-conditions of the sentence In the world of Sherlock Holmes, Holmes is quick and Watson is slow *both with the extensional meaning for* and *given earlier and the intensional meaning given here. Is there any difference in the results?* □

There are then at least two ways one could develop an intensional system.

(i) We could "generalize to the worst case" and make the semantics deliver intensions as *the* semantic value of an expression. Such systems are common in the literature (see Lewis 1970, Cresswell 1973).

(ii) We could maintain much of the extensional semantics we have developed so far and extend it conservatively so as to account for non-extensional contexts.

We have chosen to pursue (ii) over (i), because it allows us to keep the semantics of extensional expressions simpler. The philosophy we follow is that we will only move to the intensional sub-machinery when triggered by an expression that creates a non-extensional context. As the exercise just showed, this might be more a matter of taste than a deep scientific decision. We will turn to questions of expressive power later in this book.

Exercise 1.9 *Imagine that for whatever reason, the extension of sentences were a proposition:*

- $[\![\textit{it is snowing}]\!]^{w,g} = \lambda w'.$ *it is snowing in* w'
- $[\![\textit{it is cold}]\!]^{w,g} = \lambda w'.$ *it is cold in* w'

And imagine that and *continues to have its normal world-independent extension:*

- $[\![\textit{and}]\!]^{w,g} = \lambda u.\lambda v.\ u = v = 1$

Devise a new composition principle that would allow us to calculate the truth-value of a conjoined sentence like It is snowing and it is cold.

1.3.2 Why talk about other worlds?

Why would natural language bother having such elaborate mechanisms to talk about other possible worlds? While having devices for spatial and temporal displacement (talking about Hamburg or what happened yesterday) seems eminently reasonable, talking about worlds other than the actual world seems only suitable for poets and the like. So, why?

The solution to this puzzle lies in a fact that our current semantics of the shifter *in the world of Sherlock Holmes* does not yet accurately capture: modal sentences have empirical content, they make CONTINGENT claims, claims that are true or false depending on the circumstances in the actual world.

Our example sentence *In the world of Sherlock Holmes, a famous detective lives at 221B Baker Street* is true in this world but it could easily have been

false. There is no reason why Sir Arthur Conan Doyle could not have decided to locate Holmes' abode on Abbey Road.

To see that our semantics does not yet capture this fact, notice that in the semantics we gave for *in the world of Sherlock Holmes*:

(34) $[\![\text{in the world of Sherlock Holmes}]\!]^{w,g} =$
$\lambda p_{\langle s,t\rangle}$. the world w' as it is described in the Sherlock Holmes stories is such that $p(w') = 1$.

there is no occurrence of w on the right hand side. This means that the truth-conditions for sentences with this shifter would be world-independent. In other words, they are predicted to make non-contingent claims that are either true no-matter-what or false no-matter-what. This needs to be fixed.

The fix is obvious: what matters to the truth of our sentence is the content of the Sherlock Holmes stories as they are in the evaluation world. So, we actually need the following semantics for our shifter:

(35) $[\![\text{in the world of Sherlock Holmes}]\!]^{w,g} =$
$\lambda p_{\langle s,t\rangle}$. the world w' as it is described in the Sherlock Holmes stories *in* w is such that $p(w') = 1$.

We see now that sentences with this shifter do make a claim about the evaluation world: namely, that the Sherlock Holmes stories as they are in the evaluation world describe a world in which such-and-such is true. So, what is happening is that although it appears at first as if modal statements concern other possible worlds and thus couldn't really be very informative, they actually only talk about *certain* possible worlds, those that stand in some relation to what is going on at the ground level in the actual world. As a crude analogy, consider:

(36) My grandmother is sick.

At one level this is a claim about my grandmother. But it is also a claim about me: namely that I have a grandmother who is sick. Thus it is with modal statements. They talk about possible worlds that stand in a certain relation to the actual world and thus they, at least indirectly, make claims about the actual world.

1.3.3 The worlds of Sherlock Holmes

So far, we have played along with colloquial usage in talking of *the* world of Sherlock Holmes. But it is important to realize that this is sloppy talk. Lewis (1978) writes:

> [I]t will not do to follow ordinary language to the extent of supposing that we can somehow single out a single one of the worlds [as the one described by the stories]. Is the world of Sherlock Holmes a world where Holmes has an even or

> an odd number of hairs on his head at the moment when he first meets Watson? What is Inspector Lestrade's blood type? It is absurd to suppose that these questions about the world of Sherlock Holmes have answers. The best explanation of that is that the world*s* of Sherlock Holmes are plural, and the questions have different answers at different ones.

The usual move at this point is to talk about the set of worlds "COMPATIBLE WITH the (content of) Sherlock Holmes stories in w". We imagine that we ask of each possible world whether what is going on in it is compatible with the stories as they were written in our world. Worlds where Holmes lives on Abbey Road are not compatible. Some worlds where he lives at 221B Baker Street are compatible (again not all, because in some such worlds he is not a famous detective but an obscure violinist). Among the worlds compatible with the stories are ones where he has an even number of hairs on his head at the moment when he first meets Watson and there are others where he has an odd number of hairs at that moment.

An equivalent way to phrase this is that we are looking at the worlds *not excluded by the (content of the) stories*.

What the operator *in the world of Sherlock Holmes* expresses is that its complement is true throughout the worlds compatible with the stories. In other words, the operator *universally quantifies* over the compatible worlds. Our next iteration of the semantics for the operator is therefore this:

(37) $[\![\text{in the world of Sherlock Holmes}]\!]^{w,g} =$
$\lambda p_{\langle s,t\rangle}.\ \forall w'$ compatible with the Sherlock Holmes stories *in* w:
$p(w') = 1$.

At a very abstract level, the way we parse sentences of the form *in the world of Sherlock Holmes, ϕ* is that both components, the *in*-phrase and the prejacent, determine sets of possible worlds and that the set of possible worlds representing the content of the fiction mentioned in the *in*-phrase is a subset of the set of possible worlds determined by the prejacent. We will encounter the same rough structure of relating sets of possible worlds in other intensional constructions.

This is where we will leave things. There is more to be said about fiction operators like *in the world of Sherlock Holmes*, but we will just refer to you to the relevant literature. In particular, one might want to make sense of Lewis' idea that a special treatment is needed for cases where the sentence makes a claim about things that are left open by the fiction (no truth-value, perhaps?). One also needs to figure out how to deal with cases where the fiction is internally inconsistent. In any case, for our purposes we're done with this kind of operator.

1.3.4 What's next and a general pattern

With the basic framework of intensional semantics in place, we can now look at a succession of intensional operators. In particular, we will explore

the semantics of propositional attitude predicates such as *believe* or *want*, modal auxiliaries such as *must* or *might*, and conditional sentences.

We will see a general pattern at work. In our Sherlock Holmes example, an intensional operator can be seen as taking an *anchor a* (the Sherlock Holmes stories as they are in actual world) and *project* from it a set of compatible worlds, this can be encapsulated in a "*flavor*" function f from anchors to sets of possible worlds; the operator then makes a claim with a certain quantificational *force M* about the relation between the projected set of worlds and the *prejacent* worlds ϕ. Indirectly then, a claim is thereby made about the anchor.

The latter feature of this kind of analysis is crucial. It may seem cognitively dubious that to evaluate a modal claim (such as an *in Sherlock Holmes* claim), we would mentally travel one by one to all the worlds compatible wth the stories. Rather, we read the stories and reason about propositions entailed by them. We examine the modal anchor here rather than booking flights on Trans-World Airlines to each of an infinite set of possible worlds.

In what's to come, we will look at the non-trivial issues that arise when several intensional operators interact (modals under attitudes, modals in the consequent of a conditional, etc.). We will also see that constituents of the prejacent can sometimes be evaluated with respect to a world that is not the world that the intensional operator is taking us to (so-called *de re* readings). Further, we will move from worlds to times and explore the semantics of tense and aspect. And, for the intrepid, this can all come together by exploring how tense and aspect interact with attitudes, modality, and conditionals.

$M\ [f(a)]\ (\phi)$

M: a quantificational/modal relation between two sets of worlds (propositions)

a: the anchor of the modal claim

f: the flavor function that projects a set of worlds from the anchor

ϕ: the prejacent set of worlds (proposition)

The term "anchor" was used in the discussion of modal semantics by Hacquard 2006 and is a central concept in Kratzer 2013. The term was widespread in adjacent areas before; see for example Enç 1987.

If you'd like to begin thinking about how modal semantics fits into human cognition, you might start with Phillips & Kratzer 2022.

1.4 Two more reasons to go intensional

The main reason we have discussed so far for going intensional is the property of "displacement", the shifting of circumstances of evaluation. We will now briefly touch upon two further reasons.

1.4.1 Propositions as the objects of linguistic interaction

When A tells B that ϕ, what travels between them is not a binary choice between the truth-values 1 and 0. A informs B that the proposition expressed by ϕ is true in the world they both are in. This is a different piece of information than saying ψ is also true.

Stalnaker 1978 proposes that a core component tracked in context is the "common ground" of the participants in the conversation: The common ground of a conversation at any given time is the set of propositions that the participants in that conversation at that time mutually assume to be taken for granted and not subject to (further) discussion.

When uttered assertively, sentences are meant to update the common ground. If the sentence is accepted by the participants, the proposition it expresses is added to the common ground. From then on, the truth of the sentence is part of the common ground, is mutually assumed to be taken for granted and not subject to further discussion.

The common ground describes a set of worlds, the "context set", which are those worlds in which all of the propositions in the common ground are true. The context set is the set of worlds that for all that is currently assumed to be taken for granted, could be the actual world. If a proposition is added to the common ground, the context set is updated by removing the worlds in which this proposition is false and by keeping the worlds in which the proposition is true.

In more sophisticated theories, there may be distinctions between the "ingredient sense" contributed by a sentence in a more complex construction and the "assertoric content" expressed by a sentence that is put forth in a conversation. (The distinction is due to Dummett 1991, and related to the proposal in Lewis 1980 by Stanley 2017). But we will for now stick with having propositions do both jobs.

1.4.2 Propositions in the meaning of only

Only is a generalized negation of sorts. It says that all of the relevant alternatives to its prejacent are false. Focus-marking constrains what the alternatives are.

(38) Only [I invited TIERNA$_F$].

The argument for going intensional in this section is borrowed from class slides by Luka Crnič.

Never mind that the sentence actually surfaces with the subject on the left of *only*.

(38) says that all of the sentences of the form "I invited x" where x is not Tierna are false. This can be made precise by using a recursive calculation of the constraints on alternatives induced by focus: see Katzir 2007, Fox & Katzir 2011.

But non-identity between the prejacent and the alternatives to be negated fails when we consider examples like the following:

(39) Only [I invited [TIERNA and SILVIA]$_F$].

This sentence is not interpreted as negating the non-identical sentences "I invited Tierna" and "I invited Silvia" (which would lead to a contradiction).

The way out is to say that *only* only negates those alternatives that are not entailed by its prejacent. (39) doesn't negate that I invited Tierna because the proposition that I invited Tierna and Silvia entails the proposition that I invited Tierna.

(40) $[\![\text{only } \phi]\!]^{w,g} = 1$ iff $[\![\phi]\!]^{w,g} = 1 \;\&\; \forall \psi \in ALT(\phi):$

$$(\exists w' : [\![\phi]\!]^{w',g} = 1 \;\&\; [\![\psi]\!]^{w',g} = 0) \rightarrow [\![\psi]\!]^{w,g} = 0$$

The entry in (40) "tests" whether there is at least one world where the prejacent is true but the given alternative is false, if so it says that the alternative is in fact false.

So, we need to use propositions in the semantics of *only*, even though *only* is not an intensional shifter the way that attitude predicates, modals, and conditionals are.

The literature on *only* is huge. Just to pick one at random: von Fintel 1997.

1.5 *Issues with an informal meta-language

Thanks to Magda Kaufmann, Angelika Kratzer, and Ede Zimmermann for discussions of the issues explored in this section, which is optional on a first pass, as indicated by the star on the section title.

Exercise 1.7 asks what is wrong with writing something like

(41) (λx. x is slow in w) (Watson) = Watson is slow in w.

Think about it. On the left hand side of the "=" sign is a meta-language expression consisting of a λ-expression (so some kind of function) applied to an individual (contributed by the meta-language name "Watson"). The function is a function from individuals to truth-values that will deliver the truth-value 1 iff the individual is slow in world w. So, what we have on the left hand side is the result of a function from individuals to truth-values applied to an individual. In other words, on the left hand side we have a truth-value, namely the truth-value 1 if Watson is slow in w and the truth-value 0 if Watson is not slow in w.

Now, what do we have on the right hand side of the "="? We have the meta-language sentence "Watson is slow in w". That is not nor does it contribute a truth-value. It is a statement of fact. Truth-values are not the same as statements of fact.

The proper thing to do is to write

(42) (λx. x is slow in w) (Watson) = 1 iff Watson is slow in w.

There are actually two ways to parse the statement in (42), both legitimate it appears.

On one parse, the major connective is the meta-language expression "iff". On its left hand side is a meta-language statement (that applying the function to the individual Watson gives the truth-value 1) and on the right hand side of the "iff" we have another meta-language statement (that Watson is slow in w). So, the whole thing says that these two statements are equivalent: (i) that function applied to that individual gives us the truth-value 1, and (ii) that Watson is slow in w.

The other parse is perhaps more conspicuously represented as follows:

(43) (λx. x is slow in w) (Watson) = $\begin{cases} 1 \text{ if Watson is slow in } w \\ 0 \text{ if Watson is not slow in } w \end{cases}$

Is this weird? It turns out that natural language, not just our semi-formal meta-language, has conditionals that seem very similar: *I fear [the consequences if we fail].* See Lasersohn 1996, Frana 2017, and Blümel 2019 for some discussion.

Here, the "=" sign is the major connective. The left hand side is a meta-language expression that resolves to a truth-value and the right hand side as well contributes a truth-value: 1 if such and such and 0 if such and such.

H&K, of course, introduced a convention that allowed meta-language statements to be used in a place where a truth-value was expected (p.37, (9)):

> Read "[$\lambda\alpha$: ϕ. γ]" as either (i) or (ii), whichever makes sense.
> (i) "the function which maps every α such that ϕ to γ"
> (ii) "the function which maps every α such that ϕ to 1, if γ, and to 0 otherwise"

Since it never makes sense to map anything to a meta-language statement, no ambiguity will ever arise.

So, one might want to extend this leeway and use it in the case of (41) as well. We could say that in general, meta-language statements supply truth-values wherever that makes sense. In that case, (41) is just shorthand for (42).

This is the approach of von Stechow 1991.

Alternatively, one can introduce a new notation that indicates that a meta-language statement is being used to contribute a truth-value:

This is the approach Ede Zimmermann (pc) prefers and has been using in his classes.

$$(44) \quad \vdash \alpha \dashv = \begin{cases} 1 \text{ if } \alpha \\ 0 \text{ if otherwise} \end{cases}$$

Lastly, one could abandon the H&K informal meta-language approach altogether and introduce a rigidly formalized meta-language.

These lecture notes will proceed to follow H&K's approach and will not introduce any further innovations. So, (41) is illicit and only (42) is acceptable.

1.6 Further readings

There is considerable overlap between this chapter and Chapter 12 of Heim & Kratzer 1998. Here, we approach intensional semantics from a slightly different angle. It would probably be beneficial if you read H&K's Chapter 12 in addition to this chapter and if you did the exercises in there. Come to think of it, some other ancillary reading is also recommended. You may want to look at relevant chapters in other textbooks (for example: Dowty, Wall & Peters 1981: Chapters 5&6, Gamut 1991: Volume II: Intensional Logic and Logical Grammar, Chierchia & McConnell-Ginet 2000: Chapter 5: Intensionality, Zimmermann & Sternefeld 2013: Chapters 7&8: Propositions, Intensions, and Winter 2016: Chapter 6: Intensionality and Possible Worlds, and Coppock & Champollion 2022: Chapters 12&13).

Between textbooks and the primary literature, there are handbook articles, encyclopedia entries, and other surveys and reviews. In semantics, the following are all relevant:

- The Stanford encyclopedia of philosophy.
- The first semantics handbook: von Stechow & Wunderlich 1991.
- The handbook of logic and language: van Benthem & ter Meulen 2011.
- The three volumes of Maienborn, von Heusinger & Portner 2011a,b,c.
- The Routledge companion to philosophy of language: Russell & Fara 2013.
- The handbook of contemporary semantic theory: Lappin & Fox 2015.
- The Cambridge handbook of formal semantics: Aloni & Dekker 2016.

- The Wiley companion to philosophy of language: Hale, Wright & Miller 2017.
- The Wiley Blackwell companion to semantics: Gutzmann et al. 2020.
- The three journals: Language and Linguistics Compass, Philosophy Compass, and Annual Review of Linguistics.

It makes sense to familiarize yourself with these resources and consult them regularly when you want to dive into a new area or topic.

A collection of classic papers is Portner & Partee 2002. And a useful collection of modern-day reappraisals of classic papers is McNally & Szabó 2022.

To get a sense of where the field is moving, keep informed about articles in the four leading journals (*Journal of Semantics, Linguistics and Philosophy, Natural Language Semantics, Semantics and Pragmatics*), as well as the proceedings of the big semantics conferences (SALT, Sinn und Bedeutung, Amsterdam Colloquium).

A couple of influential philosophical works on the metaphysics and uses of possible worlds are Kripke 1980 and Lewis 1986. An interesting paper on the origins of the modern possible worlds semantics for modal logic is Copeland 2002.

A must read for students who plan to go on to becoming specialists in semantics, together with a handbook article putting it in perspective: Montague 1973 and Partee & Hendriks 2011.

To learn more about discourse about fiction, read Lewis 1978. Recent reconsiderations: Bonomi & Zucchi 2003, Hanley 2004, Proudfoot 2006. An interesting paper that explores the meaning of fictional texts: Bauer & Beck 2014. Inconsistencies in fictions and elsewhere are discussed in Varzi 1997 and Lewis 1982.

Some other interesting work on stories and pictures and their content: Ross 1997, Zucchi 2001, Blumson 2010. More recently, there's been quite a bit of work on pictorial semantics, see for example Abusch & Rooth 2017, Greenberg 2018, Maier & Bimpikou 2019.

Hockett & Altmann 1968 is a follow-up to Hockett's original article on design features. See Emonds 2011 for a recent re-appraisal. If you're interested in whether displacement really is an exclusive feature of human language and cognition, there are some survey articles: Cheke & Clayton 2010, Redshaw 2014. Two quite recent articles about displacement in humans and other animals are Leahy & Carey 2020 and Redshaw & Suddendorf 2020. There's now also a book for a general audience: Suddendorf, Redshaw & Bulley 2022.

For more on the cognitive science aspects of modality, see Phillips & Knobe 2018, Grigoroglou & Ganea 2022, Phillips & Kratzer 2022.

Astonishingly, Lewis' doctrine of the reality of the plurality of possible worlds is being paralleled (pun absolutely intended) by theoretical physicists in a number of ways. There is a controversial "many worlds" interpretation of quantum mechanics, for example. Other terms found are

the "multiverse" and "parallel universes". See for starters, Kai's blog entry on a popular book on the issue, http://kaivonfintel.org/many-worlds/, MIT physics professor Max Tegmark's page on the topic, http://space.mit.edu/home/tegmark/crazy.html, and a Fresh Air interview with physicist Brian Greene about his book *The Hidden Reality: Parallel Universes and the Deep Laws of the Cosmos*: http://www.npr.org/2011/01/24/132932268/a-physicist-explains-why-parallel-universes-may-exist.

As a follow-up on the section about our meta-language, it might be instructive to read Saka 2017, where a variety of thorny questions about kinds of quotation are raised.

2 *Attitudes, conditionals, modals*

Introduction

Towards the end of the first chapter, we identified a general schema for modal displacement operators. It begins with a "flavor function" that "projects" a set of relevant worlds from an "anchor", and then a quantificational claim is made about those worlds and their relation to the prejacent. We will now see this pattern at work in three kinds of constructions: propositional attitudes, conditionals, and modals. These look superficially quite dissimilar:

$M\ [f(a)]\ (\phi)$

M: a quantificational/modal relation between two sets of worlds (propositions)

a: the anchor of the modal claim

f: the flavor function that projects a set of worlds from the anchor

ϕ: the prejacent set of worlds (proposition)

(1) a. Charlotte believes that Lucy is smart.
b. If Lucy is smart, she will cancel the meeting.
c. Lucy might cancel the meeting.

The intensional operator in (1a) is the lexical verb *believe*, in (1b) it is the subordinating complementizer *if*, and in (1c) it is the auxiliary verb *might*. Despite this surface variety, the core semantic contributions are very similar.

Propositional attitude predicates like *believe* project a set of worlds from the mental state of their subject and relate those worlds to the worlds de-

scribed by the prejacent proposition. Conditionals select a relevant subset of the worlds described by their antecedent and relate them to the worlds described by their consequent. And the modal *might* says that some worlds in a relevant set make the prejacent proposition true.

We will now look at these ideas in more detail and build some nimbleness in deploying the technical notions here.

2.1 Attitudes

We move from the contents of works of fiction to the contents of the minds of people. Instead of worlds compatible with the Sherlock Holmes stories, we look at the worlds compatible with the beliefs or other mental states of a person.

2.1.1 Hintikka's idea

Expressions like *believe*, *know*, *doubt*, *expect*, *want*, *regret*, and so on are usually said to describe PROPOSITIONAL ATTITUDES, expressing relations between individuals (the attitude holder) and propositions (intensions of sentences).

According to Hintikka 1969, the term PROPOSITIONAL ATTITUDE goes back to Russell 1940.

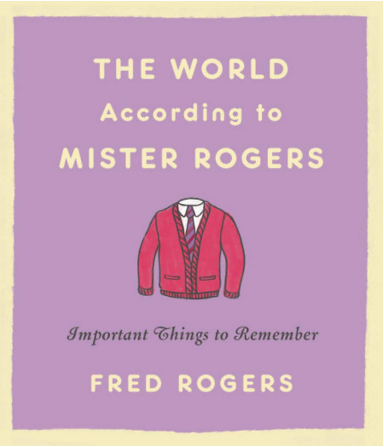
The simple idea is that *Amandine believes that Letícia is a spy* claims that Amandine believes of the proposition that Letícia is a spy that it is true. Note that for the attitude ascription to be true it does not have to hold that Letícia is actually a spy. But where — in which world(s) — does Letícia have to be a spy for it be true that Amandine believes that Letícia is a spy?

The possible worlds semantics for propositional attitudes was in place long before the extension to fiction contexts was proposed. Our discussion here has inverted the historical sequence for pedagogical purposes.

We might want to be inspired by the colloquial phrase "the world according to …" and say that *Amandine believes that Letícia is a spy* is true iff in the world according to Amandine, Letícia is a spy. We immediately recall from Chapter 1 that we need to fix this idea up by making space for multiple worlds compatible with Amandine's beliefs and by tying the truth-conditions to contingent facts about the evaluation world. That is, what Amandine believes is different in different possible worlds.

The following lexical entry thus offers itself:

(2) $[\![\text{believe}]\!]^{w,g} =$
$\lambda p_{\langle s,t\rangle}.\ \lambda x.\ \forall w'$ compatible with x'*s* beliefs in w: $p(w') = 1$.



It is important to realize the modesty of this semantics: we are not trying to figure out what belief systems are and particularly not what their internal workings are. That is the job of psychologists (and philosophers of mind, perhaps). For our semantics, we treat the belief system as a black box that determines for each possible world whether it considers it possible that it is the world it is located in.

What is going on in this semantics? We conceive of Amandine's beliefs as a state of her mind about whose internal structure we will remain agnostic, a matter left to other cognitive scientists. What we require of it is that it embody opinions about what the world she is located in looks like. In other words, if her beliefs are confronted with a particular possible world w', they will determine whether that world may or may not be the world as they think it is. What we are asking of Amandine's mental state is whether any state of affairs, any event, anything in w' is in contradic-

tion with anything that Amandine believes. If not, then w' is compatible with Amandine's beliefs. For all Amandine believes, w' may well be the world where she lives. Many worlds will pass this criterion, just consider as one factor that Amandine is unlikely to have any precise opinions about the number of leaves on the tree in front of my house. Amandine's belief system determines a set of worlds compatible with her beliefs: those worlds that are viable candidates for being the actual world, as far as her belief system is concerned.

Now, Amandine believes a proposition iff that proposition is true in all of the worlds compatible with her beliefs. If there is just one world compatible with her beliefs where the proposition is not true, that means that she considers it possible that the proposition is not true. In such a case, we can't say that she believes the proposition.

Here is the same story in the words of Hintikka (1969), the source for this semantics for propositional attitudes:

> My basic assumption (slightly simplified) is that an attribution of any propositional attitude to the person in question involves a division of all the possible worlds (…) into two classes: into those possible worlds which are in accordance with the attitude in question and into those which are incompatible with it. The meaning of the division in the case of such attitudes as knowledge, belief, memory, perception, hope, wish, striving, desire, etc. is clear enough. For instance, if what we are speaking of are (say) *a*'s memories, then these possible worlds are all the possible worlds compatible with everything he remembers. […]
>
> How are these informal observations to be incorporated into a more explicit semantical theory? According to what I have said, understanding attributions of the propositional attitude in question (…) means being able to make a distinction between two kinds of possible worlds, according to whether they are compatible with the relevant attitudes of the person in question. The semantical counterpart to this is of course a function which to a given individual person assigns a set of possible worlds.
>
> However, a minor complication is in order here. Of course, the person in question may himself have different attitudes in the different worlds we are considering. Hence this function in effect becomes a relation which to a given individual and to a given possible world μ associates a number of possible worlds which we shall call the ALTERNATIVES to μ. The relation will be called the alternativeness relation. (For different propositional attitudes, we have to consider different alternativeness relations.)

We recognize our schema: the anchor is the pair of an individual and a world, the flavor function projects from the anchor a set of worlds compatible with the relevant attitude of the anchor individual in the anchor world.

Note that what Hintikka calls "alternativeness relations" are now more commonly known as ACCESSIBILITY RELATIONS. We will encounter such relations again and again, and we will explore their formal properties later on.

Exercise 2.1 *Let's adopt Hintikka's idea that we can use a function that maps x and w into the set of worlds w' compatible with what x believes in w. Call this function $\mathcal{B}$. That is,*

(3) $\mathcal{B} = \lambda x.\ \lambda w.\ \{w' :\ w' \textit{ is compatible with } x\textit{'s beliefs in } w\}.$

Using this notation, our lexical entry for believe *could look as follows:*

(4) $[\![believe]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \lambda x.\ \mathcal{B}(x)(w) \subseteq p.$

We are here indulging in the usual sloppiness in treating p both as a function from worlds to truth-values and as the set characterized by that function.

Here now are two "alternatives" for the semantics of believe*:*

(5) *Attempt 1 (very wrong)*
$[\![believe]\!]^{w,g} = \lambda p \in D_{\langle s,t\rangle}.\big[\lambda x \in D.\ p = \mathcal{B}(x)(w)\big].$

(6) *Attempt 2 (also very wrong)*
$[\![believe]\!]^{w,g} = \lambda p \in D_{\langle s,t\rangle}.\big[\lambda x \in D.\ p \cap \mathcal{B}(x)(w) \neq \emptyset\big].$

Explain why these do not adequately capture the meaning of believe. □

Exercise 2.2 *Follow-up: The semantics in (6) would have made* believe *into an existential quantifier of sorts: it would say that* some *of the worlds compatible with what the subject believes are such-and-such. You have argued (successfully, of course) that such an analysis is wrong for* believe*. But* are *there attitude predicates with such an "existential" meaning? Discuss some candidates.* □

If you can't find any candidates that survive scrutiny, can you speculate why there might be no existential attitude predicates? [Warning: this is underexplored territory!]

Exercise 2.3 *Propose a semantics for the adjective* alleged *as in* Vera is an alleged kleptomaniac*. Do not assume any hidden structure. Try to relate your semantics to the verb* allege *as in* Romelu alleged that Vera is a kleptomaniac. □

After this little exercise, you might be interested in some really tough questions about intensionality inside noun phrases: Bogal-Allbritten 2013, Bogal-Allbritten & Weir 2017 and Hirsch 2017: especially Chapter 4.

2.1.2 Iterated attitudes

Semantics is a lab science in several ways. The most crucial way is that we learn a lot about our objects of study when we put them together and see how they react to each other.

We expect attitudes to be able to *iterate*: an attitude claim is a sentence with contingent truth-conditions and thus provides a proposition that in turn could be the complement of another attitude claim. In fact, one suspects that much of the fabric of human life involves iterated attitudes: we wonder whether Emma realizes that Caroline believes that Janet has invited Preston for dinner without telling Abby.

Exercise 2.4 *Derive the truth-conditions of* Emma believes that Caroline believes that Preston is from Texas*. [Follow-up question: if Emma's belief is correct, does that mean that Preston is from Texas?]* □

2.2 *Conditionals*

In many ways, conditionals are the archetypal construction of displacement: the consequent is evaluated not against the actual here and now but against the scenario conjured up by the antecedent. Consider a few conditional sentences:

(7) a. If Kim left before 6am, she got there in time.
b. If there's an earthquake tomorrow, this house will collapse.
c. If there had been a massive snowstorm last night, Kai would have stayed home.

These represent the three main subtypes of conditionals (there are more): (7a) is an "indicative" conditional about the past, (7b) is an indicative conditional about the future, and (7c) is a "subjunctive" conditional. For the moment, the differences will be left aside.

The basic idea of how conditionals work is this: the *if*-clause whisks us away to a particular possible world (or maybe a set thereof) and the consequent clause is asserted to be true of that world (or those worlds). But what world(s) are we being taken to? The most obvious requirement is that the antecedent of the conditional needs to be true of the world(s). But there's more.

Given our discussions of how the semantics of fiction operators anchors them in facts about the actual world (the content of the relevant body of fiction) and how the semantics of attitude predicates is anchored in the mental states of an individual in the actual world, it shouldn't come as a surprise that conditionals are similarly anchored. So, look at the examples in (7): what in the actual world are they about?

Here's a first attempt of an answer: (7a) is about the local transportation system, the weather, the traffic, and so on. (7b) is about the sturdiness of this house, facts of geology, laws of physics, and so on. (7c) is about Kai's proclivities (such as avoiding traffic snarls), the local climate, and so on. Since the conditionals are anchored in real world facts, they are no mere flights of fancy and whether they are true depends on those facts. If today's traffic was particularly bad, it may be false that Kim's leaving before 6am would have got her there in time. If the architects went to great lengths to make the house earthquake-safe, (7b) may well be false. And if there was an attendance-mandatory faculty meeting, Kai may well have come in in spite of a massive snowstorm.

So, the outlines of the semantics of conditionals are clear: *if* takes us to worlds where the antecedent is true but that match the actual world in certain relevant features. And the consequent then is evaluated in those worlds. There are many details to work out and we'll keep returning to that task. But for now, we put forward a placeholder analysis.

Lewis used a rather whimsical example to start off his seminal 1973 book on counterfactuals: "If kangaroos had not tails, they would topple over". For another example of counterfactual whimsy, consider this scene from the TV show "Big Bang Theory": https://www.youtube.com/watch?v=0lpY0Kt4bn8. As the examples in the text make clear, conditionals are actually very down-to-earth in real life.

We will treat *if* as a higher-order operator that together with the antecedent creates an intensional operator with a semantics very similar to the final analysis we gave to *in the world of Sherlock Holmes*. But where the fiction operator directly encoded what features of the actual world it's sensitive to (the Sherlock Holmes fiction), conditionals rely on context for this job. Here's a first draft of the proposal:

(8) $[\![\text{if}]\!]^{w,g} = \lambda p \in D_{\langle s,t\rangle}.\ \lambda q \in D_{\langle s,t\rangle}.$
$\forall w' :\ p(w') = 1\ \&\ w'$ is relevantly like $w \rightarrow q(w') = 1.$

The contextual anchoring to features of the evaluation world w is here effected by the placeholder "relevantly like w". This is crucial because otherwise the conditional would talk about any world whatsoever where the antecedent is true. This would make the truth-conditions not just not contingent on the actual world but also far too strong to allow most sensible conditionals to be true ever.

Think about the earthquake conditional (7b): we would derive the absurdly strong truth-conditions that the conditional is true iff *all* of the worlds where there is a major earthquake in Cambridge tomorrow are worlds where my house collapses.

There are some obvious and immediate problems with this analysis. For one, while it's easy to imagine circumstances where the conditional (7b) is judged to be true, there surely are possible worlds where there's an earthquake but my house does not collapse: perhaps, the builders in that world used all the recommended best practices to make the building earthquake-safe, perhaps it's a world where I'm simply unreasonably lucky, or the house is immediately adjacent to much sturdier neighboring buildings which keep it propped up, or Harry Potter flies by and protects the house at the last minute (he owes me a favor, after all). This problem (that the house doesn't in fact collapse in *all* possible worlds where there's an earthquake but that the conditional can still be judged true in some worlds) is accompanied with another problem: whether the conditional is true depends on what the world is like. Was the house built to exacting standards? Is it propped up by its neighbors? Does Harry Potter owe me a favor? That is the problem solved by restricting the quantifier over worlds to world "relevantly like w".

Obviously, this is a semantics with a "placeholder", because what does "relevantly like" mean precisely? Now, just because the semantics is therefore rather vague and context-dependent doesn't mean it is wrong. As Lewis 1973: p.1 writes:

> Counterfactuals are notoriously vague. That does not mean that we cannot give a clear account of their truth conditions. It does mean that such an account must either be stated in vague terms — which does not mean ill-understood terms — or be made relative to some parameter that is fixed only within rough limits on any given occasion of language use.

The insight articulated by Lewis here is very important. Applying mathematical or logical methods to analyzing natural language meaning often arouses severe skepticism, precisely because natural language is often vague and context-dependent. But that just means that an adequate analysis needs to not ignore vagueness and context-dependence and rather be clear about where they enter.

Strawson 1950 famously wrote: "Neither Aristotelian nor Russellian rales give the exact logic of any expression of ordinary language; for ordinary language has no exact logic."

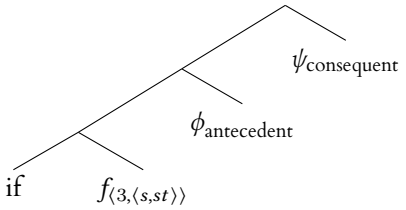
All the more reason to refine our initial draft of the proposal. We put a placeholder for context-dependence in the meta-language ("worlds relevantly like the evaluation world") but that is not really sufficient. We would like to embed the analysis in a general framework for how context enters the semantics. For the purposes of this book, we will adopt an approach that generalizes from the analysis of "free" pronouns in the Heim & Kratzer textbook.

In H&K, chapters 9–11, a technical implementation of context-dependency is developed for pronouns and their referential (and E-Type) readings. Referential pronouns are analyzed there as free variables, appealing to a general principle that free variables in an LF need to be supplied with values from the utterance context. If we want to describe the context-dependency of conditionals (and as we'll soon see, modals) in a technically analogous fashion, we can think of their LF-representations as incorporating or subcategorizing for a kind of invisible pronoun, a free variable that effects the anchoring of the conditional claim to relevant features of the evaluation world.

Concretely, we posit LF-structures where *if* doesn't just take two propositions as its arguments but also an object language variable of type $\langle s, \langle s, t\rangle\rangle$:

We are using the notation for variables of types other than *e* introduced by Heim & Kratzer, p. 213. An index on a variable now is an ordered pair of a natural number and a type. The variable assignments relative to which we calculate semantic values now are functions from ordered pairs of a natural number and a type to elements of the domain of objects of that type.

(9)



We have written the silent pronoun as "f" to remind you of "flavor", the evocative term we used to describe the anchoring of intensional operators. So, we have a variable over flavor functions that will return a set of worlds when applied to a given world. We will give *if* the job of telling f that what we need is the set of worlds that f assigns to the evaluation world:

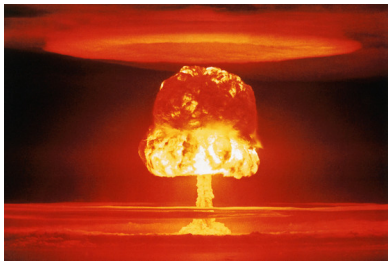
$$(10)\quad [\![\text{if}]\!]^{w,g} = \lambda f \in D_{\langle s,\langle s,t\rangle\rangle}.\ \lambda p \in D_{\langle s,t\rangle}.\ \lambda q \in D_{\langle s,t\rangle}.\ \forall w' :\ f(w)(w') = 1\ \&\ p(w') = 1 \rightarrow q(w') = 1.$$

Together this means that a conditional says about the evaluation world w_0 that among the worlds that are f-related to w_0, the ones where the antecedent is true are all worlds where the consequent is also true.

We get different flavors of conditionals from different contextual resolutions of f. Consider for example the earthquake conditional in (7b). Context might assign to f the function that when given an evaluation world w returns the set of worlds that "agree" with w on how sturdy this house is, what the local geology is like, and what the laws of physics are. Then, the conditional claims that the actual world is such that all the worlds that agree with it via f and where there is an earthquake are worlds where this house collapses.

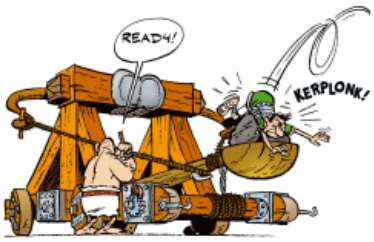
With context-dependency comes the spectre of indeterminacy. A famous example is a pair attributed by Quine 1960: p. 221 to Nelson Goodman (imagine these being said while the Korean War was going on):

(11) a. If Caesar were in command, he would use the atom bomb.
b. If Caesar were in command, he would use catapults.



Both versions seem possible: saying (11a) would talk about worlds where Caesar with all his ruthlessness is in command, while (11b) would talk about worlds where Caesar's own arsenal comes with him.

We will return to conditionals in the next chapter when some additional complications become necessary.



Exercise 2.5 *We continue our experiments and embed a conditional under an attitude. Compute the truth-conditions of the following sentence:*

(12) Allie believes that if there's an earthquake, this house will collapse.

You don't need to analyze the internal make-up of the antecedent and the consequent of the conditional.

Exercise 2.6 *Here's a potentially simpler analysis of conditionals. What if the silent pronoun were of type* $\langle s, t \rangle$*, a set of worlds? The connective* if *would then take the contextually assigned set of worlds and say that all the worlds in that set in which the antecedent is true are worlds where the consequent is true. Spell out the details of this idea. How would it fare when confronted with the embedded conditional (12) from the previous exercise?* □

2.3 Modals

The final empirical addition of this chapter are modal auxiliaries like *may, must, can, have to*, etc. Most of what we say here should carry over straightforwardly to modal adverbs like *maybe, possibly, certainly*, etc. We will make certain syntactic assumptions, which make our work easier but which leave aside many questions that at some point deserve to be addressed.

2.3.1 Syntactic assumptions

We will assume, at least for the time being, that a modal like *may* is a RAISING predicate (rather than a CONTROL predicate), i.e., its subject is not its own argument, but has been moved from the subject-position of its infinitival complement. So, we are dealing with the following kind of structure:

To explore questions about the syntax-semantics interface of modals, a good place to start is a handout by Kai von Fintel and Sabine Iatridou from an LSA class: https://web.mit.edu/fintel/lsa220-class-2-handout.pdf. See also Iatridou & Zeijlstra 2013 and Barbiers & van Dooren 2017.

(13) a. Ann may be smart.
b. [Ann [λ_1 [may [t_1 be smart]]]]

Actually, we will be working here with the even simpler structure below, in which the subject has been reconstructed to its lowest trace position. (E.g., these could be generated by deleting all but the lowest copy in the movement chain.) We will be able to prove that movement of a name or pronoun never affects truth-conditions, so at any rate the interpretation of the structure in (13b) would be the same as that of (14). As a matter of convenience, then, we will take the reconstructed structures, which allow us to abstract away from the (here irrelevant) mechanics of variable binding.

We will talk about reconstruction in more detail later.

(14) may [Ann be smart]

We will assume that even though *Ann be smart* is a non-finite sentence, this will not have any effect on its semantic type, which is that of a sentence, which in turn means that its semantic value is a truth-value. This is hopefully independent of the (interesting) fact that *Ann be smart* on its own cannot be used to make a truth-evaluable assertion.

So, for now at least, we are assuming that modals are expressions that take a full sentence as their semantic argument. Now then, what do modals mean?

2.3.2 Quantification over possible worlds

The basic idea of the possible worlds semantics for modal expressions is that they are quantifiers over possible worlds. Toy lexical entries for *must* and *may*, for example, would look like this:

This idea goes back a long time. It was famously held by Leibniz, but there are precedents in the medieval literature; see Knuuttila 2021. See Copeland 2002 for the modern history of the possible worlds analysis of modal expressions.

(15) $[\![\text{must}]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \forall w' :\ p(w') = 1.$

(16) $[\![\text{may}]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \exists w' :\ p(w') = 1.$

A necessity modal like *must* says that all worlds make its prejacent true, while a possibility modal like *may* says that some worlds make its prejacent true. Note that our previous intensional operators were all universal

Sometimes, people call necessity modals "universal modals" and possibility modals "existential modals", which obviously presupposes this quantificational analysis.

quantifiers (unless you found some existential attitudes in Exercise 2.2), so the existential force of *may* is a new frontier for us.

The analysis in (15)/(16) is too crude (in particular, notice that it would make modal sentences non-contingent — there is no occurrence of the evaluation world on the right hand side!). But it does already have some desirable consequences that we will seek to preserve through all subsequent refinements. It correctly predicts a number of intuitive judgments about the logical relations between *must* and *may* and among various combinations of these items and negations. To start with some elementary facts, we feel that *must* ϕ entails *may* ϕ, but not vice versa:

(17) You must stay.
Therefore, you may stay. VALID

(18) You may stay.
Therefore, you must stay. INVALID

(19) a. You may stay, but it is not the case that you must stay.
b. You may stay, but you don't have to stay. CONSISTENT

The somewhat stilted *it is not the case*-construction is used in (19a) to make certain that negation takes scope over *must*. When modal auxiliaries and negation are together in the auxiliary complex of the same clause, their relative scope seems not to be transparently encoded in the surface order; specifically, the scope order is not reliably negation > modal. (Think about examples with *mustn't*, *can't, shouldn't, may not* etc. What's going on here? This is an interesting topic which we must set aside for now. See the references at the end of the chapter for relevant work.) With modal *main* verbs (such as *have to*), this complication doesn't arise; they are consistently inside the scope of clause-mate auxiliary negation. Therefore we can use (19b) to (unambiguously) express the same scope order as (19a), without having to resort to a biclausal structure.

We judge *must* ϕ incompatible with its "inner negation" *must* [*not* ϕ], but find *may* ϕ and *may* [*not* ϕ] entirely compatible:

(20) You must stay, and/but also, you must leave. (leave = not stay).
CONTRADICTORY

(21) You may stay, but also, you may leave. CONSISTENT

We also judge that in each pair below, the (a)-sentence and the (b)-sentences say the same thing.

(22) a. You must stay.
b. It is not the case that you may leave.
You aren't allowed to leave.
(You may not leave.)
(You can't leave.)

(23) a. You may stay.
b. It is not the case that you must leave.
You don't have to leave.
You don't need to leave.
(You needn't leave.)

The parenthesized variants of the (b)-sentences in (54) are pertinent here only to the extent that we can be certain that negation scopes over the modal. In these examples, apparently it does, but as we remarked above, this cannot be taken for granted in all structures of this form.

Given that *stay* and *leave* are each other's negations (i.e. $[\![\text{leave}]\!]^{w,g} = [\![\text{not stay}]\!]^{w,g}$, and $[\![\text{stay}]\!]^{w,g} = [\![\text{not leave}]\!]^{w,g}$), the LF-structures of these equivalent pairs of sentences can be seen to instantiate the following schemata:

In logicians' jargon, *must* and *may* behave as DUALS of each other. For definitions of "dual", see Barwise & Cooper 1981: p. 197 or Gamut 1991: vol.2, 238.

(24) a. *must* $\phi \equiv$ *not* [*may* [*not* ϕ]]
b. *must* [*not* ψ] $\equiv$ *not* [*may* ψ]

(25) a. *may* $\phi \equiv$ *not* [*must* [*not* ϕ]]
b. *may* [*not* ψ] $\equiv$ *not* [*must* ψ]

Our present analysis of *must*, *have-to*, … as universal quantifiers and of *may*, *can*, … as existential quantifiers straightforwardly predicts all of the above judgments, as you can easily prove.

More linguistic data regarding the "parallel logic" of modals and quantifiers can be found in Horn's dissertation (Horn 1972).

2.3.3 *Contingency, flavors, context-dependency*

We already said that the semantics we started with is too simple-minded. In particular, we have no dependency on the evaluation world, which would make modal statements non-contingent. This is not correct.

If one says *It may be snowing in Cambridge*, that may well be part of useful, practical advice about what to wear on your upcoming trip to Cambridge. It may be true or it may be false. The sentence seems true if said in the dead of winter when we have already heard about a Nor'Easter that is sweeping across New England. The sentence seems false if said by a clueless Australian acquaintance of ours in July.

The contingency of modal claims is not captured by our current semantics. All the *may*-sentence would claim under that semantics is that there is some possible world where it is snowing in Cambridge. And surely, once you have read Lewis' quote in Chapter 1, where he asserts the existence of possible worlds with different physical constants than we enjoy here, you must admit that there have to be such worlds even if it is July. The problem is that in our semantics, repeated here:

(26) $[\![\text{may}]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \exists w' :\ p(w') = 1,$

there is no occurrence of w on the right hand side. This means that the truth-conditions for *may*-sentences are world-independent. In other words, they make non-contingent claims that are either true whatever or false whatever, and because of the plenitude of possible worlds they are more likely to be true than false. This needs to be fixed. But how?

Conversely, the plenitude of possible worlds would make *must*-claims very likely false if they are not reigned in or anchored somehow.

Well, what makes *it may be snowing in Cambridge* seem true when we know about a Nor'Easter over New England? What makes it seem false when we know that it is summer in New England? The idea is that we only consider possible worlds COMPATIBLE WITH THE EVIDENCE AVAILABLE TO US. And since what evidence is available to us differs from world to world, so will the truth of a *may*-statement.

(27) $[\![\text{may}]\!]^{w,g} = \lambda p.\ \exists w'$ compatible w/ the evidence in $w :\ p(w') = 1.$

(28) $[\![\text{must}]\!]^{w,g} = \lambda p.\ \forall w'$ compatible w/ the evidence in $w :\ p(w') = 1.$

From now on, we will leave off type-specifications such as that p has to be of type $\langle s, t\rangle$, whenever it is obvious what they should be and when saving space is aesthetically called for.

Let us consider a different example:

(29) You have to be quiet.

Imagine this sentence being said based on the house rules of the particular dormitory you live in. Again, this is a sentence that could be true or could be false. Why do we feel that this is a contingent assertion? Well, the house rules can be different from one world to the next, and so we might be unsure or mistaken about what they are. In one possible world, they say that all noise must stop at 11pm, in another world they say that all noise must stop at 10pm. Suppose we know that it is 10:30 now, and that the dorm we are in has either one or the other of these two rules, but we have forgotten which. Then, for all we know, *you have to be quiet* may be true or it may be false. This suggests a lexical entry along these lines:

(30) $[\![\text{have-to}]\!]^{w,g} = \lambda p.\ \forall w'$ compatible with the rules in w: $p(w') = 1$.

Again, we are tying the modal statement about other worlds down to certain worlds that stand in a certain relation to the actual world: those worlds where the rules as they are here are obeyed.

A note of caution: it is very important to realize that the worlds compatible with the rules as they are in w are those worlds where nothing happens that violates any of the w-rules. This is not at all the same as saying that the worlds compatible with the rules in w are those worlds where the same rules are in force. Usually, the rules do not care what the rules are, unless the rules contain some kind of meta-statement to the effect that the rules have to be the way they are, i.e. that the rules cannot be changed. So, in fact, a world w' in which nothing happens that violates the rules as they are in w but where the rules are quite different and in fact what happens violates the rules as they are in w' is nevertheless a world compatible with the rules in w. For example, imagine that the only relevant rule in w is that students go to bed before midnight. Take a world w' where a particular student goes to bed at 11:30 pm but where the rules are different and say that students have to go to bed before 11 pm. Such a world w' is compatible with the rules in w (but of course not with the rules in w').

Apparently, there are different flavors of modality, varying in what kind of facts in the evaluation world they are sensitive to. The semantics we gave for *must* and *may* above makes them talk about evidence, while the semantics we gave for *have-to* made it talk about rules. But that was just because the examples were hand-picked. In fact, in the dorm scenario we could just as well have said *You must be quiet*. And, vice versa, there is nothing wrong with using *it has to be snowing in Cambridge* based on the evidence we have. In fact, many modal expressions seem to be multiply ambiguous. The English modal *have to* is probably the world champion in this regard:

(31) a. It has to be raining.
b. Visitors have to leave by six pm.
c. You have to go to bed in ten minutes.
d. I have to sneeze.
e. To get home in time, you have to take a taxi.

Traditional descriptions of modals often distinguish a number of "readings": EPISTEMIC, DEONTIC, ABILITY, CIRCUMSTANTIAL, DYNAMIC, …. Here are some initial illustrations.

Beyond "epistemic" and "deontic," there is a great deal of terminological exuberance. Sometimes all non-epistemic readings are grouped together under the term ROOT MODALITY (nobody knows why).

(32) EPISTEMIC MODALITY

A: Where is John?
B: I don't know. He *may* be at home.

(33) DEONTIC MODALITY

A: Am I allowed to stay over at Janet's house?
B: No, but you *may* bring her here for dinner.

(34) CIRCUMSTANTIAL/DYNAMIC MODALITY

A: I will plant the rhododendron here.
B: That's not a good idea. It *can* grow very tall.

How are *may* and *can* interpreted in each of these examples? What do the interpretations have in common, and where do they differ?

In all three examples, the modal makes an existentially quantified claim about possible worlds. This is usually called the MODAL FORCE of the claim. What differs is what worlds are quantified over, sometimes called the MODAL FLAVOR. In EPISTEMIC modal sentences, we quantify over worlds compatible with the available evidence. In DEONTIC modal sentences, we quantify over worlds compatible with the rules and/or regulations. And in the CIRCUMSTANTIAL modal sentence, we quantify over the set of worlds which conform to the laws of nature (in particular, plant biology). What speaker B in (34) is saying, then, is that there are some worlds conforming to the laws of nature in which this rhododendron grows very tall.

2.3.4 Epistemic vs. Circumstantial

Do flavors of modality correspond to some sorts of signals in the structure of sentences? Read the following famous passage from Kratzer 1991 and think about how the two sentences with their very different modal meanings differ in structure:

In the earlier Kratzer 1981, the hydrangeas were *Zwetschgenbäume* 'plum trees'. The German word *Zwetschge*, by the way, is etymologically derived from the name of the city Damascus (Syria), the center of the ancient plum trade.

Consider sentences (35) and (36):

(35) Hydrangeas can grow here.

(36) There might be hydrangeas growing here.

The two sentences differ in meaning in a way which is illustrated by the following scenario.

> Suppose I acquire a piece of land in a far away country and discover that soil and climate are very much like at home, where hydrangeas prosper everywhere. Since hydrangeas are my favorite plants, I wonder whether they would grow in this place and inquire about it. The answer is (35). In such a situation, the proposition expressed by (35) is true. It is true regardless of whether it is or isn't likely that there are already hydrangeas in the country we are considering. All that matters is climate, soil, the special properties of hydrangeas, and the like. Suppose now that the country we are in has never had any contacts whatsoever with Asia or America, and the vegetation is altogether different from ours. Given this evidence, my utterance of (36) would express a false proposition. What counts here is the complete evidence available. And this evidence is not compatible with the existence of hydrangeas.
>
> (35) together with our scenario illustrates the pure CIRCUMSTANTIAL reading of the modal *can*. […]. (36) together with our scenario illustrates the epistemic reading of modals. […] circumstantial and epistemic conversational backgrounds involve different kinds of facts. In using an epistemic modal, we are interested in what else may or must be the case in our world given all the evidence available. Using a circumstantial modal, we are interested in the necessities implied by or the possibilities opened up by certain sorts of facts. Epistemic modality is the modality of curious people like historians, detectives, and futurologists. Circumstantial modality is the modality of rational agents like gardeners, architects, and engineers. A historian asks what might have been the case, given all the available facts. An engineer asks what can be done given certain relevant facts.

Consider also the very different prominent meanings of the following two sentences, taken from Kratzer as well:

(37) a. Cathy can make a pound of cheese out of this can of milk.
b. Cathy might make a pound of cheese out of this can of milk.

2.3.5 Toward an analysis

How can we account for this variety of readings? One way would be to write a host of lexical entries, basically treating this as a kind of (more or

less principled) ambiguity/polysemy. Another way, which is preferred by many people, is to treat this as a case of context-dependency, as argued in seminal work by Kratzer (1977, 1978, 1981, 1991).

According to Kratzer, what a modal brings with it intrinsically is just a modal force, that is, whether it is an existential (possibility) modal or a universal (necessity) modal. What worlds it quantifies over is determined by context. In essence, the context has to supply a restriction to the quantifier. How can we implement this idea? Well, we just have to transpose the setup we put in place for conditionals.

It is well-known that natural language quantification is in general subject to contextual restriction. See for example von Fintel 1994: Ch.2 and Stanley & Szabó 2000.

We give modals a flavor argument, a silent pronoun of type $\langle s, st\rangle$, just like we did with *if*. So we posit LF-structures like this:

(38) [[must $f_{\langle n,\langle s,st\rangle\rangle}$] [you quiet]]

$f_{\langle n,\langle s,st\rangle\rangle}$, again, is a variable over functions from worlds to (characteristic functions of) sets of worlds, which — like all free variables — needs to receive a value from the utterance context. For example, it may be supplied with the function which, for any world w, yields the set $\{w'$: the house rules that are in force in w are obeyed in $w'\}$. If we apply this function to a world w_1, in which the house rules read "no noise after 10 pm", it will yield a set of worlds in which nobody makes noise after 10 pm. If we apply the same function to a world w_2, in which the house rules read "no noise after 11 pm", it will yield a set of worlds in which nobody makes noise after 11 pm.

The new lexical entries for *must* and *may* that will fit this new structure are these:

Compare these entries for the modals to the entry for *if* in (10). What are the differences?

(39) a. $[\![\text{must}]\!]^{w,g} =$
$\lambda f \in D_{\langle s,st\rangle}.\ \lambda q \in D_{\langle s,t\rangle}.\ \forall w' \in W\ [f(w)(w') = 1 \rightarrow q(w') = 1]$ in set talk: $f(w) \subseteq q$

b. $[\![\text{may}]\!]^{w,g} =$
$\lambda f \in D_{\langle s,st\rangle}.\ \lambda q \in D_{\langle s,t\rangle}.\ \exists w' \in W\ [f(w)(w') = 1\ \&\ q(w') = 1]$ in set talk: $f(w) \cap q \neq \emptyset$

Exercise 2.7 *Let w be a world, and assume that the context supplies an assignment g such that:*

(40) $g(f_{\langle 17,\langle s,st\rangle\rangle}) = \lambda w.\ \lambda w'.$ *the rules in force in w are obeyed in w'.*

Compute the truth-conditions for the LF in (41):

(41) *[must $f_{\langle 17,\langle s,st\rangle\rangle}$ [you quiet]]*

Does truth-value of (41) correctly depend on the evaluation world w? □

On this approach, the epistemic, deontic, etc. "readings" of individual occurrences of modal verbs come about by a combination of two separate things. The lexical semantics of the modal itself encodes just a quantificational force, a *relation* between sets of worlds. This is either the subset-relation (universal quantification; necessity) or the relation of non-

disjointness (existential quantification; possibility). The covert variable next to the modal when applied to the evaluation world yields a set of worlds, and this functions as the quantifier's restrictor. The labels "epistemic", "deontic", "circumstantial" etc. group together certain conceptually natural classes of possible values for this covert restrictor.

Notice that, strictly speaking, there is not just one deontic reading (for example), but many. A speaker who utters

(42) You have to be quiet.

might mean: 'I want you to be quiet,' (i.e., you are quiet in all those worlds that conform to my preferences). Or she might mean: 'unless you are quiet, you won't succeed in what you are trying to do,' (i.e., you are quiet in all those worlds in which you succeed at your current task). Or she might mean: 'the house rules of this dormitory here demand that you be quiet,' (i.e., you are quiet in all those worlds in which the house rules aren't violated). And so on. So the label "deontic" appears to cover a whole open-ended set of imaginable "readings", and which one is intended and understood on a particular utterance occasion may depend on all sorts of things in the interlocutors' previous conversation and tacit shared assumptions. (And the same goes for the other traditional labels.)

Proponents of polysemy accounts of the variety of modal flavors will presumably have to tackle the apparent limitlessness of variation in some principle way. See Viebahn & Vetter 2016 for a polysemy account.

A disappointing feature of our analysis in (39) is that a lot of the work is being done by the modals: they don't just take a restriction as their argument but they have to enforce that this restriction is evaluated in the evaluation world. This is a departure from the ideal that modals are simply quantifiers over possible worlds. It would be preferable to merely present them with a set of worlds to quantify over rather than giving them the responsibility of obtaining this set by applying an accessibility relation to the evaluation world. We will later put in place a different framework (for unrelated reasons) that will make good on this vision.

Exercise 2.8 *Describe two worlds w_1 and w_2 so that* $[\![\textit{must } f_{\langle 17,\langle s,st\rangle\rangle} \textit{ you quiet}]\!]^{w_1,g} = 1$ *and* $[\![\textit{must } f_{\langle 17,\langle s,st\rangle\rangle} \textit{ you quiet}]\!]^{w_2,g} = 0$. □

Exercise 2.9 *In analogy to the deontic relation $g(f_{\langle 17,\langle s,st\rangle\rangle})$ defined in (40), define an appropriate relation that yields an epistemic reading for a sentence like* You may be quiet. □

Exercise 2.10 *Describe the set of worlds that constitutes the understood restrictor of* have to *in each of the examples in (31).* □

Exercise 2.11 *Describe values for the covert $\langle s, st\rangle$-variable that are intuitively suitable for the interpretation of the modals in the following sentences:*

(43) As far as John's preferences are concerned, you may *stay with us.*

(44) *According to the guidelines of the graduate school, every PhD candidate* must *take 9 credit hours outside his/her department.*

(45) *John* can *run a mile in 5 minutes.*

(46) *This* has *to be the White House.*

(47) *This elevator* can *carry up to 3000 pounds.*

For some of the sentences, different interpretations are conceivable depending on the circumstances in which they are uttered. You may therefore have to sketch the utterance context you have in mind before describing the accessibility relation. □

Exercise 2.12 *Collect two naturally occurring examples of modalized sentences (e.g., sentences that you overhear in conversation, or read in a newspaper or novel – not ones that are being used as examples in a linguistics or philosophy paper!), and give definitions of values for the covert $\langle s, st\rangle$-variable which account for the way in which you actually understood these sentences when you encountered them. (If the appropriate interpretation is not salient for the sentence out of context, include information about the relevant preceding text or non-linguistic background.)* □

Exercise 2.13 *Modals can be iterated like other intensional operators:*

(48) *You might have to walk.*

Describe a context in which (48) would be an appropriate utterance. State plausible values for the $\langle s, st\rangle$ flavor functions restricting the two modals. Posit an LF for (48). Derive the truth-conditions compositionally. □

2.4 Explorations and variations

With the basic analyses of attitudes, conditionals, and modals in place, we turn to some technical explorations.

2.4.1 Accessibility relations

In all of the cases we have looked at the intensional operator ranges over worlds that are relevant related to the evaluation world (via some kind of anchor). In the case of conditional and modals, in fact, we made the operator take a relation between worlds as its contextually provided first argument:

(49) $[\![\text{if}]\!]^{w,g} = \lambda f_{\langle s,st\rangle}.\ \lambda p_{\langle s,t\rangle}.\ \lambda q_{\langle s,t\rangle}.$
$\forall w' :\ f(w)(w') = 1\ \&\ p(w') = 1 \rightarrow q(w') = 1.$

(50) $[\![\text{must}]\!]^{w,g} = \lambda f_{\langle s,st\rangle}.\ \lambda q_{\langle s,t\rangle}.\ \forall w' :\ f(w)(w') = 1 \rightarrow q(w') = 1.$

In the Hintikka-semantics for attitudes as well, we think of the worlds quantified over as those that stand in a particular relation to the evaluation world. Which relation that is is lexically specified by the attitude predicate. So, *believe* ranges over worlds compatible with the subject's beliefs in the evaluation world.

It might be useful to rehearse some mathematical basics. A relation is something that two elements stand in. A relation might not be symmetrical (or as we say outside of math, it might not be reciprocated), so the order of elements is part of the notion. The formal model of a relation $\mathcal{R}$ therefore is a set of ordered pairs drawn from two sets A and B:

$$\mathcal{R} = \{\langle x, y\rangle : \ x \in A \ \& \ y \in B\}$$

In the function-centric and Schönfinkeled world we inherit from Heim & Kratzer, this notion of a set of ordered pairs becomes a staggered function that takes first an element from A, then an element from B, and then yields a truth-value:

$$f_{\mathcal{R}} = \lambda x : \ x \in A. \ (\lambda y : \ y \in B. \ \langle x, y\rangle \in \mathcal{R})$$

Since we are used to thinking of a function from some type to truth-values as characterizing a set of things of that type, we can reconceive a relation as a function from elements of the first set to a set of elements from the second set (the ones they stand in the relevant relation to).

Note that in the case of accessibility relations, we are dealing with a relation between worlds. That is, the elements that stand in the relation come from the same set.

One might call this an "endorelation".

Recall now (for example, from Section 6.6 of H&K) that the linguistic study of determiners benefitted quite a bit from an investigation of the formal properties of the relations between sets of individuals that determiners express. We can do the same thing here and ask about the formal properties of the accessibility relation associated with belief versus the one associated with knowledge, etc. The obvious properties to think about are reflexivity, transitivity, and symmetry.

Why do we call reflexivity, transitivity, and symmetry "formal" properties of relations? The idea is that certain properties are "formal" or "logical", while others are more substantial. So, the fact that the relation "have the same birthday as" is symmetric seems a more formal fact about it than the fact that the relation holds between my daughter and my brother-in-law. Nevertheless, one of the most common ways of characterizing formal/logical notions (permutation-invariance, if you're curious) does not in fact make symmetry etc. a formal/logical notion. So, while intuitively these do seem to be formal/logical properties, we do not know how to substantiate that intuition. See MacFarlane 2005 for discussion.

A relation is REFLEXIVE iff for any object in the domain of the relation we know that the relation holds between that object and itself. Which accessibility relations are reflexive? Take a knowledge-induced accessibility relation (let's call it $\mathcal{K}_x$):

(51) $w\mathcal{K}_x w'$ iff w' is compatible with what x knows in w.

We are asking whether for any given possible world w, we know that $\mathcal{K}_x$ holds between w and w itself. It will hold if w is a world that is compatible with what we know in w. And clearly that must be so. Take our body of knowledge in w. The concept of knowledge crucially contains the concept of truth: what we know must be true. So if in w, we know that something is the case then it must be the case in w. So, w must be compatible with all we know in w. $\mathcal{K}_x$ is reflexive.

We talk here about knowledge entailing (or even presupposing) truth but we do not mean to say that knowledge simply equals true belief. For initial overview of the issues, see the Stanford Encyclopedia entry: Ichikawa & Steup 2018.

Now, if an attitude X corresponds to a reflexive accessibility relation, then we can conclude from *a Xs that p* being true in w that p is true in w. This property of an attitude predicate is often called VERIDICALITY. It is to be distinguished from FACTIVITY, which is a property of attitudes which *presuppose* – rather than (merely) entail – the truth of their complement.

In modal logic notation: $\Box p \rightarrow p$. This pattern is sometimes called **T** or **M**, as is the corresponding system of modal logic.

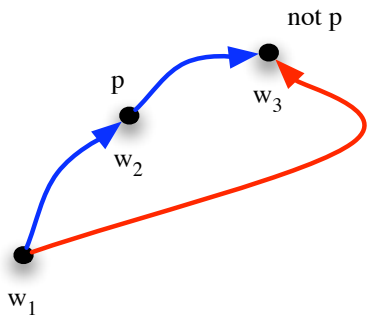
If we consider the relation $\mathcal{B}_x$ pairing with a world w those worlds w' which are compatible with what x *believes* in w, we no longer have reflexivity: belief is not a veridical attitude. It is easy to have false beliefs, which means that the actual world is not in fact compatible with one's beliefs, which contradicts reflexivity. And many other attitudes as well do not involve veridicality/reflexivity: what we hope may not come true, what we remember may not be what actually happened, etc.

The difference between *believe* and *know* in natural discourse is quite delicate, especially when one considers first person uses (*I believe the earth is flat* vs. *I know the earth is flat*).

In modal logic, the correspondence between formal properties of the accessibility relation and the validity of inference patterns is well-studied. What we have just seen is that reflexivity of the accessibility relation corresponds to the validity of $\Box p \rightarrow p$. Other properties correspond to other characteristic patterns. Let's see this for transitivity and symmetry.

TRANSITIVITY of the accessibility relation corresponds to the inference $\Box p \rightarrow \Box\Box p$. The pattern seems not obviously wrong for knowledge: if one knows that p, doesn't one thereby know that one knows that p? But before we comment on that, let's establish the formal correspondence between transitivity and that inference pattern. This needs to go in both directions.

In the literature on epistemic modal logic, the pattern is known as the KK THESIS or POSITIVE INTROSPECTION. In general modal logic, it is the characteristic axiom **4** of the modal logic system **S4**, which is a system that adds **4** to the previous axiom **M**/**T**. Thus, **S4** is the logic of accessibility relations that are both reflexive and transitive.



What does it take for the pattern to be valid? Assume that $\Box p$ holds for an arbitrary world w, i.e. that p is true in all worlds w' accessible from w. Now, the inference is to the fact that p again holds in any world w'' accessible from any of those worlds w' accessible from w. But what would prevent p from being false in some w'' accessible from some w' accessible from w? That could only be prevented from happening if we knew that w'' itself is accessible from w as well, because then we would know from the premiss that p is true in it (since p is true in *all* worlds accessible from w). Ah, but w'' (some world accessible from a world w' accessible from w) is only guaranteed to be accessible from w if the accessibility relation is transitive (if w' is accessible from w and w'' is accessible from w', then transitivity ensures that w'' is accessible from w). This reasoning has shown that validity of the pattern requires transitivity. The other half of proving the correspondence is to show that transitivity entails that the pattern is valid.

The proof proceeds by reductio. Assume that the accessibility relation is transitive. Assume that (i) $\Box p$ holds for some world w but that (ii) $\Box\Box p$ doesn't hold in w. We will show that this situation cannot obtain. By (i), p is true in all worlds w' accessible from w. By (ii), there is some non-p world w'' accessible from some world w' accessible from w. But by transitivity of the accessibility relation, that non-p world w'' must be accessible

from w. And since *all* worlds accessible from w are p worlds, w'' must be a p world, in contradiction to (ii). So, as soon as we assume transitivity, there is no way for the inference not to go through.

Now, do any of the attitudes have the transitivity property? It seems rather obvious that as soon as you believe something, you thereby believe that you believe it (and so it seems that belief involves a transitive accessibility relation). And in fact, as soon as you believe something, you believe that you *know* it. But one might shy away from saying that knowing something automatically amounts to knowing that you know it. For example, many are attracted to the idea that to know something requires that (i) that it is true, (ii) that you believe it, and (iii) that you are justified in believing it: the justified true belief analysis of knowledge. So, now couldn't it be that you know something, and thus (?) that you believe you know it, and thus that you believe that you are justified in believing it, but that you are not justified in believing that you are *justified* in believing it? After all, one's source of knowledge, one's reliable means of acquiring knowledge, might be a mechanism that one has no insight into. So, while one can implicitly trust (believe) in its reliability, and while it is in fact reliable, one might not have any means to have trustworthy beliefs about it. [Further worries about the KK Thesis are discussed in Williamson 2000.]

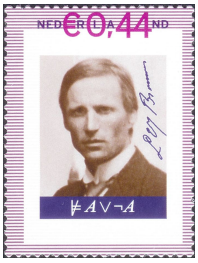
What would the consequences be if the accessibility relation were SYMMETRIC? Symmetry of the accessibility relation $\mathcal{R}$ corresponds to the validity of the following principle:

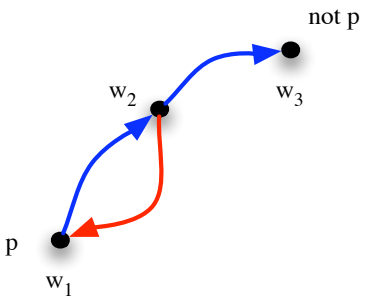
(52) Brouwer's Axiom :
$\forall p \forall w : \ w \in p \rightarrow \left[\forall w' \left[w \mathcal{R} w' \rightarrow \exists w'' \left[w' \mathcal{R} w'' \& \ w'' \in p\right]\right]\right]$

Here's the reasoning: Assume that R is in fact symmetric. Pick a world w in which p is true. Now, could it be that the right hand side of the inference fails to hold in w? Assume that it does fail. Then, there must be some world w' accessible from w in which $\Diamond p$ is false. In other words, from that world w' there is no accessible world w'' in which p is true. But since R is assumed to be symmetric, one of the worlds accessible from w' is w and in w, p is true, which contradicts the assumption that the inference doesn't go through. So, symmetry ensures the validity of the inference.

The other way (validity of the inference requires symmetry): the inference says that from any p-world ("p-world" is a very common way of saying "world of which p is true"), we can only access worlds from which, in turn, there is at least one accessible p-world. But imagine that p is true in w but not true in any other world. So, the only way for the conclusion of the inference to hold automatically is to have a guarantee that w (the only p-world) is accessible from any world accessible from it. That is, we need to have symmetry. QED.

In modal logic notation, (52) looks as follows: $p \rightarrow \Box\Diamond p$, known simply as B in modal logic. The system that combines **T/M** with B is often called Brouwer's System (**B**), after the mathematician L.E.J. Brouwer, not because he proposed it but because it was thought that it had some connections to his doctrines. Brouwer got his own commemorative stamp from the Netherlands:





To see whether a particular kind of attitude is based on a symmetric accessibility relation, we can ask whether Brouwer's Axiom is intuitively valid with respect to this attitude. If it is not valid, this shows that the accessibility relation can't be symmetric. In the case of a knowledge-based accessibility relation (epistemic accessibility), one can argue that *symmetry does not hold* (thanks to Bob Stalnaker (pc to Kai von Fintel) for help with the following reasoning):

> The symmetry condition would imply that if something happens to be true in the actual world, then you know that it is compatible with your knowledge (Brouwer's Axiom). This will be violated by any case in which your beliefs are consistent, but mistaken. Suppose that while p is in fact true, you feel certain that it is false, and so think that you know that it is false. Since you think you know this, it is compatible with your knowledge that you know it. (Since we are assuming you are consistent, you can't both believe that you know it, and know that you do not). So it is compatible with your knowledge that you know that *not* p. Equivalently: you don't know that you don't know that *not* p. Equivalently: you don't know that it's compatible with your knowledge that p. But by Brouwer's Axiom, since p is true, you would have to know that it's compatible with your knowledge that p. So if Brouwer's Axiom held, there would be a contradiction. So Brouwer's Axiom doesn't hold here, which shows that epistemic accessibility is not symmetric.

This and the following step rely on the duality of necessity and possibility: q is compatible with your knowledge iff you don't know that *not* q.

Game theorists and theoretical computer scientists who traffic in logics of knowledge often assume that the accessibility relation for knowledge is an equivalence relation (reflexive, symmetric, and transitive). But this is appropriate only if one abstracts away from any error, in effect assuming that belief and knowledge coincide. One striking consequence of working with an equivalence relation as the accessibility relation for knowledge is that one predicts the principle of Negative Introspection to hold:

All one really needs to make **NI** valid is to have a Euclidean accessibility relation: any two worlds accessible from the same world are accessible from each other. It is a nice little exercise to prove this, if you have become interested in this sort of thing. Note that all reflexive and Euclidean accessibility relations are transitive and symmetric as well — another nice little thing to prove.

(53) Negative Introspection (**NI**)
If one doesn't know that p, then one knows that one doesn't know that p. $(\neg\Box p \rightarrow \Box\neg\Box p)$.

This surely seems rather dubious: imagine that one strongly believes that p but that nevertheless p is false, then one doesn't know that p, but one doesn't seem to believe that one doesn't know that p, in fact one believes that one does know that p.

2.4.2 *Conversational backgrounds*

There is a well-known technical variant on relativizing the semantics of intensional operators to accessibility relations. The basic idea, due to Kratzer, is to make intensional operators work with a set of propositions rather than a set of worlds. For example, *must* would say that a certain set of propositions together entail the prejacent. A conditional *if p, q* would say that a certain set of propositions when augmented with the antecedent proposition p will entail the consequent proposition q. And one could conceive of a belief state as providing a set of propositions (each of which is a belief of the agent), so that the intensional operator *believe* would claim that this set of propositions entails the prejacent.

We need to add the usual layer of contingency, which in this case means that the relevant ingredient is actually a function from evaluation worlds to sets of propositions.

At this point, we will assume that the resulting set of propositions is always consistent, by which we mean that there's at least one world where all of the propositions in the set are true:

(54) A set of propositions $\mathcal{P}$ is CONSISTENT relative to a set of worlds W iff $\exists w \in W\colon\ \forall p \in \mathcal{P}\colon\ p(w) = 1$.

From any consistent set of propositions, we can retrieve the set of worlds characterized by it: those worlds such that each proposition in the set is true in them. If we think of propositions as sets of worlds, this corresponds to the grand intersection of the set of propositions:

(55) For any consistent set of propositions $\mathcal{P}$,
$char(\mathcal{P}) = \{w\colon\ \forall p \in \mathcal{P}\colon\ p(w) = 1\} = \bigcap(\mathcal{P})$

The semantics of the modal *must*, for example, can now be rewritten:

(56) $[\![\text{must}]\!]^{w,g} = \lambda\mathcal{M}_{\langle s,\langle st,t\rangle\rangle}.\ \lambda p.\ \bigcap(\mathcal{M}(w)) \subseteq p$

The context supplies a function $\mathcal{M}$ from worlds to sets of propositions and *must* claims that when applied to the evaluation world, $\mathcal{M}$ yields a set of propositions that jointly entail the prejacent p, or in other words: all the worlds where all the propositions in $\mathcal{M}(w)$ are true are worlds where the prejacent p is true.

Kratzer called functions of type $\langle s, \langle st, t\rangle\rangle$ CONVERSATIONAL BACKGROUNDS and used the term MODAL BASE for the conversational backgrounds that restrict modal operators. She also sometimes calls the resulting sets of propositions PREMISE SETS.

The type of $\mathcal{M}$ in unabbreviated form is $\langle s, \langle\langle s, t\rangle, t\rangle\rangle$. Officially, this is the type of functions from worlds to functions from propositions (themselves functions from worlds to truth-values) to truth-values. Set-talk, as used in the main text, is easier: $\mathcal{M}$ maps worlds to sets of propositions.

Note that we can retrieve accessibility relations from conversational backgrounds:

(57) For any conversational background $\mathcal{M}$ of type $\langle s, \langle st, t\rangle\rangle$, we define the corresponding accessibility relation $f_{\mathcal{M}}$ of type $\langle s, st\rangle$ as follows: $f_{\mathcal{M}} := \lambda w.\, \lambda w'.\, \forall p\, [\mathcal{M}(w)(p) = 1 \;\rightarrow\; p(w') = 1]$.

In words, w' is $f_{\mathcal{M}}$-accessible from w iff all propositions p that are assigned by $\mathcal{M}$ to w are true in w'.

What motivates the complication from accessibility relations to conversational backgrounds? One consideration that we will turn to in the next chapter concerns inconsistent premise sets. But there is also another, perhaps more intuitive, motivation. A conversational background is the sort of thing that is identified by phrases like *what the law provides, what we know*, etc. Take the phrase *what the law provides*. What the law provides is different from one possible world to another. And what the law provides in a particular world is a *set of propositions*. Likewise, what we know differs from world to world. And what we know in a particular world is a set of propositions. The intension of *what the law provides* is then that function which assigns to every possible world the set of propositions p such that the law provides in that world that p. Of course, that doesn't mean that p holds in that world itself: the law can be broken. And the intension of *what we know* will be that function which assigns to every possible world the set of propositions we know in that world. Now, consider:

(58) (In view of what we know,) Brown must have murdered Smith.

The *in view of*-phrase may explicitly signal the intended conversational background. Or, if the phrase is omitted, we can just infer from other clues in the discourse that such an epistemic conversational background is intended.

What follows are some (increasingly technical exercises) on conversational backgrounds.

Exercise 2.14 *Imagine that we model individual x's belief state with a set of propositions $\mathcal{BS}_x$. Now, when x forms a new opinion, we could model this by adding a new proposition p to $\mathcal{BS}_x$. So, $\mathcal{BS}_x$ now contains one further element. There are now more opinions. What happens to the set of worlds compatible with x's beliefs? Does it get bigger or smaller? Is the new set a subset or superset of the previous set of compatible worlds?* □

"$\mathcal{BS}$" stands for "belief state".

Exercise 2.15 *Kratzer calls a conversational background (modal base)* REALISTIC *iff it assigns to* any *world a set of propositions that are all true in that world. The modal base* what we know *is realistic, the modal bases* what we believe *and* what we want *are not.*

Show that a conversational background $\mathcal{M}$ is realistic iff the corresponding accessibility relation $f_{\mathcal{M}}$ (defined as in (57)) is reflexive. □

Exercise 2.16 *Let us call an accessibility relation* TRIVIAL *if it makes every world accessible from every world. f is* TRIVIAL *iff* $\forall w \forall w' : w' \in f(w)$. *What would the conversational background $\mathcal{M}$ have to be like for the accessibility relation $f_{\mathcal{M}}$ to be trivial in this sense?* □

Exercise 2.17 *[For the intrepid only!] The definition in (57) specifies, in effect, a function from $D_{\langle s,\langle st,t\rangle\rangle}$ to $D_{\langle s,st\rangle}$. It maps each function $\mathcal{M}$ of type $\langle s,\langle st,t\rangle\rangle$ to a unique function $f_{\mathcal{M}}$ of type $\langle s,st\rangle$. This mapping is not one-to-one, however. Different elements of $D_{\langle s,\langle st,t\rangle\rangle}$ may be mapped to the same value in $D_{\langle s,st\rangle}$.*

Prove this claim. I.e., give an example of two functions $\mathcal{M}$ and $\mathcal{M}'$ in $D_{\langle s,\langle st,t\rangle\rangle}$ for which (57) determines $f_{\mathcal{M}} = f_{\mathcal{M}'}$.

As you have just proved, if every function of type $\langle s,\langle st,t\rangle\rangle$ qualifies as a 'conversational background', then two different conversational backgrounds can collapse into the same accessibility relation. Conceivably, however, if we imposed further restrictions on conversational backgrounds (i.e., conditions by which only a proper subset of the functions in $D_{\langle s,\langle st,t\rangle\rangle}$ would qualify as conversational backgrounds), then the mapping between conversational backgrounds and accessibility relations might become one-to-one after all. In this light, consider the following potential restriction:

(59) *Every conversational background $\mathcal{M}$ must be "closed under entailment"; i.e., it must meet this condition:*
$\forall w.\forall p\ [\cap\mathcal{M}(w) \subseteq p \rightarrow p \in \mathcal{M}(w)]$.
(In words: if the propositions in $\mathcal{M}(w)$ taken together entail p, then p must itself be in $\mathcal{M}(w)$.)

Show that this restriction would ensure that the mapping defined in (57) will be one-to-one. □

2.5 Further readings

We have just put in place some of the basics of the analysis of attitudes, conditionals, and modals. Much of the work in this area will become accessible to you after the following chapter. For now, we recommend just a few further readings.

Swanson 2011 is a recent survey on attitudes.

Further connections between mathematical properties of accessibility relations and logical properties of various notions of necessity and possibility are studied extensively in modal logic, see Hughes & Cresswell 1996 and Garson 2018, especially section 7 and 8, "Modal Axioms and Conditions on Frames", "Map of the Relationships between Modal Logics". An open access textbook on modal logic is Zach 2019.

We encourage you to visit the admirable Open Logic Project (https://openlogicproject.org/), that the Zach 2019 textbook is part of.

A thorough discussion of the possible worlds theory of attitudes, and some of its potential shortcomings, can be found in Bob Stalnaker's work (1984, 1999).

Two introductory readings on conditionals are von Fintel 2011, 2012.

The most important background readings on modals are the two papers Kratzer 1981, 1991. There are updated versions of Kratzer's classic papers in her volume "Modals and conditionals" (Kratzer 2012). A major resource on modality is Paul Portner's book: Portner 2009. You might also profit from some survey-ish type papers on modals and modality: von Fintel 2005, von Fintel & Gillies 2007, Swanson 2008, Hacquard 2011. Finally, Condoravdi's commentary on Kratzer's "Notional category" paper: Condoravdi 2022.

3 *Restricting and ordering*

3.1 Conditionals and modals

We have a basic theory of conditionals in place as well as a basic theory of modals. Both are treated as intensional operators that move us from the initial evaluation world to another set of worlds: in the case of conditionals, those worlds where the antecedent is true that are otherwise relevantly like the evaluation world; in the case of modals, to whatever worlds the contextually supplied accessibility relation assigns to the evaluation world. This means that if a sentence contains both a conditional and a modal, we expect them to work together to express nested intensional shifting.

Thus continues our "chemistry" experiments, combining expressions we have an initial theory about and seeing what happens when they are put in contact.

3.1.1 First steps

In many cases, the experiments seem to have the expected outcome. Consider the following attempt to embed a modal in the antecedent of a conditional:

(1) If one can get to that beach by bike, Iris did just that.

This seems to have the meaning we predict: the conditional take us to worlds where a certain modal fact holds — that the beach is reachable by

bike — and then we say that in those worlds Iris went to the beach by bike.

Or consider our friend Howard again:

(2) If Howard has to pay a heavy fine, he will be broke.

Note that by itself, being under the obligation to pay a fine doesn't automatically mean that one does. So, (2) indirectly signals that if Howard has to pay a heavy fine, he will comply and thus he will be broke.

In our system, this could be captured by saying that the context supplies an f that assigns to the evaluation world only worlds where Howard does what he is obligated to, at least in as much as paying fines is concerned.

One thing that lots of people think is not straightforwardly possible is epistemic modals in conditionals antecedent. Papafragou 2006, for example, finds the following examples problematic:

(3) a. ?If Max must be lonely, his wife will be worried.
b. ?If Max may be lonely, his wife will be worried.

One possible explanation is that conditionals signal that the antecedent is "iffy". An epistemic modal statement can only be "iffy" if the speaker is not certain about what "the evidence" is. That can only be if "the evidence" is not the evidence that the speaker has full access to. Relevant examples include the "cancer scenarios" of DeRose 1991 and the "Mastermind" cases of von Fintel & Gillies 2007, 2011:

What exactly we might mean by "iffy" and how exactly conditionals signal iffiness is an interesting question. Any ideas?

(4) a. If John might have cancer [the doctors haven't told us], he will have to see an expert in Boston.
b. If there have to be two reds, your next move is obvious.

These seem to have acceptable readings where the conditional takes us to worlds where the embedded epistemic modal claim holds.

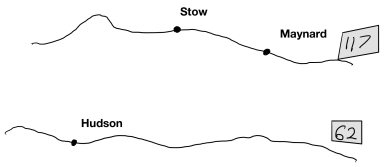
Consider next modals in the consequent of conditionals. On the face of it, examples abound:

(5) a. If jaywalking is illegal here, then that guy has to pay a fine.
b. If Caspar vacuums on Saturday, then Chris has to cook on Sunday.
c. If Britanny drinks Coke, she must drink Coke.
d. If Howard returns his book late, he has to pay a fine.
e. If Cosette has to be home by midnight, she ought to think about leaving now.

(5b) is from a term paper by Moss. (5c) is due to Zvolenszky.

We will see, however, that there are significant challenges for compositional semantics hiding here. We begin with the interaction of epistemic modals and conditionals.

3.1.2 A serious problem



Our friends have been driving in the Massachusetts hinterlands, inexplicably without iPhones or GPS, and are relying entirely on an old-fashioned map. They've just passed through a little town with an iconic New England church and are looking on the map to try to figure out where they are. They have concluded that they are either on Route 117 or on Route 62. There are two plausible candidate towns on Route 117 (Maynard and Stow) and just one on Route 62 (Hudson).

So, given all the evidence available to them, there are three live possibilities: that they are in Hudson (on Rte 62, and they don't know it), that they are in Stow (on Rte 117, and they don't know it), and that they are in Maynard (on Rte 117, and they don't know it).

It's true when they say:

(6) We might be in Maynard.

since there are worlds compatible with their evidence where they are in Maynard.

It's true when they say:

(7) If we're on Route 62, we're in Hudson.

because of the three towns that they know they might be in, only Hudson is on Rte 62.

Our semantics for conditionals has the conditional take us to worlds that are (i) in $f(w)$, here in the set of worlds compatible with their evidence and (ii) are antecedent worlds. Among the worlds compatible with their evidence, all p-worlds (worlds where they are in Rte 62) are worlds where they are in Hudson.

So far so good. But here are some problematic cases (with the intuitive truth-value judgments in the given scenario):

(8)	a.	If we're on Route 117, we might be in Stow.	True
	b.	If we're on Route 117, we might be in Hudson.	False
	c.	If we're on Route 62, we must be in Hudson.	True

We will now see that these cannot be explained in our framework!

Take (8c) first. The conditional takes us to those worlds that are (i) compatible with their evidence or with what they know (which includes their knowledge that they don't know in which of the three towns they are) and (ii) where they are on Rte 62. In all of those worlds, they are in Hudson, but in none of them do they *know* or *have any additional evidence* that they are in Hudson. So, we expect (8c) to be false, contrary to fact.

Now, take (8b). We go to worlds compatible with their evidence where they are on Route 117. All of them are worlds where they are on Route 117 without knowing that they are there, since they know that they don't know where they are. In those worlds, is it true that for all they know,

they are in Hudson? Yes. Therefore, we predict that (8b) is true, again contrary to fact.

Finally, while we do predict that (8a) is true, the reason is simply that because of their ignorance, any world compatible with their evidence is a world where any of the relevant *might*-claims is true. So, the conditional antecedent is predicted to make no difference to the truth of the epistemic possibility claim in its consequent, contrary to fact (our intuitions are different for (8a) vs. (8b)).

Before we turn to one of the dominant ways of accounting for the meanings of the examples in (8), let's consider a tempting idea about (8c): what if the epistemic necessity modal *must* actually scopes over the conditional, even though it appears in its consequent? At first glance, the resulting meaning is not far off the target. The claim would be that in all the epistemically accessible worlds the conditional (7) *if we're on Rte 62, we're in Hudson* is true. We already saw that that conditional is true in the actual world, and there's no reason to say that it isn't true in other epistemically accessible worlds. So far, so good.

Unfortunately, there are several reasons for doubting that this LF with wide-scope for the modal is a convincing solution to our troubles:

1. For the *might*-version, we can't resort to wide-scoping the modal, because the epistemic conditional *if we're on Rte 117, we're in Stow* is false in the actual world and in fact in all epistemically accessible worlds: they have evidence that entails that it doesn't follow from being on Rte 117 that they're in Stow. So, (8a) would be predicted to be false under the wide-scope LF. So, no matter how we scope the modal, we either have trivial truth or falsity, and not what we want: non-trivial truth-conditions.
2. Epistemic *must* carries with it an "evidential signal" (von Fintel & Gillies 2010):

   (9) It must be raining.

   This signals that the speaker inferred the truth of *it is raining* indirectly on the basis of other evidence. The signal is still perceptible when the prejacent is a generalization or conditional:

   (10) a. Elephants must dislike bees.
   b. George must have to be home by midnight.
   c. It must be that if they're on Rte 62, they are in Clinton.

   But (8c) is not felt to have such a signal, other than that the antecedent would be an additional piece of information in the deduction that they're in Hudson.
3. We also need to be able to analyze epistemic conditionals with two modals in a conjunctive consequent (examples like this are discussed in Gillies 2010):

(11) If we're on Rte 62, we must be in Hudson and might be very close to the Horseshoe pub.

A wide-scope analysis of the modals in (11) seems impossible.

Conclusion: our analysis really is in trouble and can't be rescued by wide-scoping.

In the next section, we present the dominant treatment of the interaction of conditionals and modals in current linguistic semantics. There are alternatives that deserve to be considered, but that would lead us beyond the bounds of this textbook.

3.1.3 If-*clauses as restrictors*

The problem we have encountered here with the interaction of an *if*-clause and the modal operator *might* is similar to others that have been noted in the literature. Most influentially, Lewis in his paper "Adverbs of Quantification" (Lewis 1975) showed how hard it is to find an adequate analysis of the interaction of *if*-clauses and ADVERBS OF QUANTIFICATION like *never, rarely, sometimes, often, usually, always* in sentences like these:

(12) { Always / Usually / Often / Sometimes / Rarely / Never } if a driver sees a friend walking, she offers her a ride.

Lewis proposed that in these cases, the adverb is the only operator at work and that the *if*-clause serves to restrict the adverb. Thus, it has much the same function that a common noun phrase has in a determiner-quantification.

> The *if* of our restrictive *if*-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts. The *if* in *always if …, …, sometimes if …, …*, and the rest is on a par with the non-connective *and* in *between …and …*, with the non-connective *or* in *whether …or …*, or with the non-connective *if* in *the probability that …if …*. It serves merely to mark an argument-place in a polyadic construction. (Lewis 1975: p. 11)

Building on Lewis' insight, Kratzer argued for a uniform treatment of *if*-clauses as restrictors. She claimed that

> the history of the conditional is the story of a syntactic mistake. There is no two-place *if …then* connective in the logical forms of natural languages. *If*-clauses are devices for restricting the domains of various operators. (Kratzer 1986)

Let us repeat this:

(13) Kratzer's Thesis

If-clauses are devices for restricting the domains of various operators.

Kratzer's Thesis gives a unified picture of the semantics of conditional clauses. Note that it is not meant to supplant previous accounts of the *meaning* of conditionals. It just says that what those accounts are analyzing is not the meaning of *if* itself but the meaning of the operators that *if*-clauses restrict.

Let us see how this idea helps us with our most problematic case, *(8c) If we're on Route 62, we must be in Hudson*. The idea is to deny that there are two quantifiers over worlds in this sentence. Instead, the *if*-clause merely contributes a further restriction to the modal *must*. In effect, the modal is not quantifying over *all* the worlds compatible with our friends' knowledge but only over those where they are on Route 62. It then claims that all of those worlds are worlds where they are in Hudson. Since that is in fact true, we now correctly predict that (8c) is true.

Note that this is not the same as worlds where they are on Route 62 *and know that they are*. This is made vivid by examples ultimately due to Thomason (as noted in van Fraasen 1980): *If my partner is embezzling, I will never know (because I'm so bad at accounting)*.

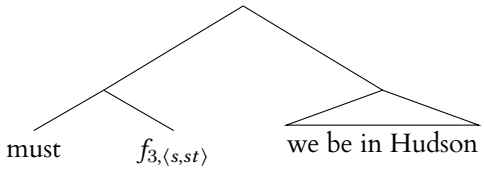
What we don't yet have is a compositional calculation. What does it mean in structural terms for the *if*-clause to be restricting the domain of the modal? We present here a particularly "flat-footed" implementation. The idea is that *if*-clauses serve as modifiers of the modal flavor argument of modals.

Recall the entry we had for *must*:

(14) $[\![\text{must}]\!]^{w,g} = \lambda f_{s,st}.\lambda p_{st}.\ \forall w' :\ w' \in f(w) \longrightarrow\ p(w') = 1.$

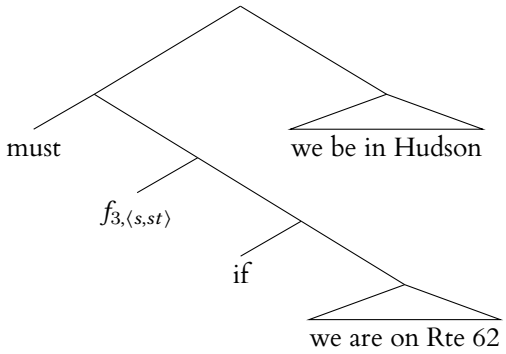
We supplied the modal with a covert "pronoun" of type $\langle s, st\rangle$ as its first argument:

(15)



We assume a structure where the *if*-clause is the sister of the f-pronoun:

(16)



The idea now is that the two restrictive devices work together: we just feed to the modal the *intersection* of (i) the set of worlds that are f_3-accessible from the actual world, and (ii) the set of worlds where they are on Route 62.

Obviously, we don't hear *if*-clauses where they are located in this tree. So, on the way to PF, the *if*-clause must be moved to the periphery. Note that *if*-clauses can appear on the left periphery or on the right.

This implementation of the restrictor theory was considered (and, without much of an argument, dismissed) in Section 3.2 of von Fintel 1994 and is also found in lecture notes from 2004 by von Stechow. In Section 3.3 of von Fintel 1994, a different approach is developed, which is adopted (at least for illustration) by Kratzer 2021. The idea is that *if*-clauses are variable modifiers, something very like variable binders without entirely overwriting the current variable assignment. Further references about the restrictor theory are given at the end of this chapter.

Exercise 3.1 *Write an appropriate lexical entry for* if *to make the structure in (16) interpretable. The idea is that* if *takes its antecedent proposition and an accessibility relation and returns a restricted accessibility relation that a world stands in (relative to the evaluation world) iff it stands in the original accessibility relation plus makes the antecedent true.* □

Exercise 3.2 *Consider the example with two modals in the consequent we gave earlier:*

(11) If we're on Rte 62, we must be in Hudson and might be very close to the Horseshoe pub.

How could this be analyzed? □

What about conditionals that do not contain a modal operator or any other operator that the *if*-clause could be restricting? Kratzer proposed that such bare conditionals contain implicit operators. There's a case for at least two such operators: (i) a covert generic frequency operator (giving rise to what are sometimes called "multi-case" conditionals), and (ii) a covert (epistemic?) necessity operator akin to *must* (giving rise to ordinary "one-case" conditionals):

The one-case vs. multi-case distinction is discussed and so-named in Kadmon 1987.

(17) a. If Polly sees a husky, she pets it.
b. If Kim left before 6am, she got there in time.

3.2 Ordering

Our semantics treats modals as quantifiers over worlds. The set of worlds they quantify over is supplied by context by way of assigning an accessibility function to a covert pronoun, and is optionally subject to being restricted by an *if*-clause, as developed just now.

We have distinguished (at least) epistemic and deontic accessibility functions:

(18) a. The keys must be in the car. epistemic
b. Your guest can't stay past midnight. deontic

For deontic modals, and perhaps more generally what Portner 2018 calls "priority" expressions, there is a strong argument that there is more at play.

3.2.1 The best we can get

Consider our friend Howard:

(19) [We think that Howard forgot to return his library book.]
He has to pay a $5 fine.

According to our analysis, the deontic modal *has to* here claims that he pays a $5 fine in all of the worlds compatible with what the relevant rules (the library regulations, in this case) require. But wait: surely the rules really require everyone to return their books on time! And so, the worlds compatible with the rules are all worlds where all books are returned on time, including Howard's, and thus nobody pays a fine. How can (19) actually be true?

It's clear that (19) is naturally understood in such a way that its truth depends *both* on facts about Howard's actions *and* facts about the library regulations. For instance, it will be judged true if (i) Howard did indeed fail to return his book, and (ii) the regulations mandate a fine in such cases. It may be false either because the regulations are different or because Howard did return the book. Our accessibility relation therefore needs to be more complex, *combining facts and regulations*.

A second attempt at specifying the accessibility relation might thus go something like this:

(20) $\lambda w.\ \lambda w'.$ [what happened in w' up to now is the same as what happened in w and w' conforms to what the rules in w demands].

The problem with (20) is that, unless there were no infractions of the rules at all in w up to now, no world w' will be accessible from w. Therefore, (19) is predicted to follow logically from the premise that Howard broke some rule. This does not represent our intuition about its truth conditions.

A better definition of the appropriate accessibility relation has to be even more complicated:

(21) $\lambda w.\ \lambda w'.$ [what happened in w' up to now is the same as what happened in w and w' conforms at least as well to what the rules in w demands as does any other world in which what happened up to now is the same as in w].

(21) makes explicit that there is an important difference between the ways in which facts about Howard's actions on the one hand, and facts about the rules on the other, enter into the truth conditions of sentences like (19). Worlds in which Howard didn't do what he did are simply excluded from the domain of the modal here. Worlds in which the rules aren't obeyed are not absolutely excluded. Rather, we restrict the domain to those worlds in which the rules are obeyed as well as they can be, considering what has happened. We exclude only those worlds in which there

The proposal in (20) suggests that there's a temporal asymmetry here. But this is not necessarily so. Prakken & Sergot 1996 present the case of a set of formal and informal regulations governing the appearance and use of holiday cottages, which say that there are not to be any fences but that if there are fences, they must be white. In such a case, one could say: *This fence should be white*, expressing the kind of complex flavor we are dealing with.

are infractions above and beyond those that are shared by all the worlds in which Howard has done what he has done. The analysis of (19) thus crucially involves the notion of an ordering of worlds: here they are ordered according to how well they conform to what the rules in *w* demand.

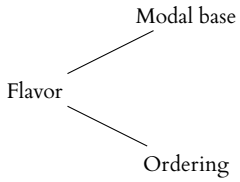
The diagnosis then is that the modal here is not a pure deontic modal. Rather, its flavor is *complex*. We take for granted the fact that Howard did not return his book on time. Consider then just those worlds where Howard did not return the book. None of those worlds are fully compatible with the rules. But among those worlds, the ones where he pays a fine satisfy the rules *as best as possible*. So, the flavor of the modal combines some actual world circumstances (the book was not returned on time) and what the rules require (late books incur a fine). And the flavor is *essentially* complex. Imagine that Howard is a scofflaw who never pays fines. If this fact were part of the flavor of the modal in (21), then we would expect the sentence to be false, but intuitively it is true. And as we already saw, a purely deontic reading would also predict the sentence to be false.

So, the flavor of the modal in (21) is best characterized as the following mixture: it quantifies over worlds where (i) the same relevant things happened as in the evaluation world and (ii) apart from that, things develop as best as possible according to the rules.

This is a very common pattern: intensional operators have complex flavors that combine a set of circumstances taken for granted and some way of identifying the best worlds within the set of worlds characterized by those circumstances.

If we want to stick to our simple semantics, with its flavor function (from evaluation worlds to sets of worlds quantified over), we have to locate the complexity in the pragmatics of determining a salient value to the context-dependent flavor. For (21), the contextual variable assignment would have to assign to the accessibility function variable a function like the one in (21).

Kratzer famously proposed a different diagnosis: modals are *doubly relative*, requiring two separate contextually supplied pieces of information. In addition to the accessibility relation or MODAL BASE (a function that assigns a set of accessible worlds to any evaluation world), modals are also sensitive to an ORDERING of the accessible worlds.



3.2.2 Mathematical interlude on orderings

All order relations are binary relations between the elements of a set. Here we're dealing with sets of worlds, but we can order soccer teams (by how good they are), people (by how rich they are, or by how close to Rome they were born), or race horses (by how they finished in the race). You get the idea.

In some sense, the most general kind of order is a PREORDER, one that you could read as "at least as highly ranked as". It would be REFLEXIVE,

since any element is trivially at least as highly ranked as itself. And it would be TRANSITIVE, since if x is at least as highly ranked as y and y is at least as highly ranked as z, then surely x is at least as highly ranked as z.

When a preorder is also ANTI-SYMMETRIC (no distinct elements can have the same rank), it is called a PARTIAL ORDER. An order is STRICT if it actually doesn't allow elements at the same rank, so it is transitive but asymmetric. An order is TOTAL if it is a COMPLETE relation: any two distinct elements are related in one way or the other.

An order is WELL-FOUNDED if for any subset of the domain of the relation, there are elements in the subset that are "optimal": there is no other element in the subset that is strictly better by the ordering. Any order on a finite set is well-founded, but not every order on an infinite set is.

We provide two handy charts, one list of ours and one from Amartya Sen's influential book *Collective Choice and Social Welfare* (Sen 1970).

R is **irreflexive** on S	$\forall x \in S\colon\ \neg R(x,x)$
R is **reflexive** on S	$\forall x \in S\colon\ R(x,x)$
R is **transitive** on S	$\forall x,y,z \in S\colon\ R(x,y)\ \&\ R(y,z) \rightarrow R(x,z)$
R is **symmetric** on S	$\forall x,y \in S\colon\ R(x,y) \leftrightarrow R(y,x)$
R is **antisymmetric** on S	$\forall x,y \in S\colon\ R(x,y)\ \&\ R(y,x) \rightarrow x = y$
R is **asymmetric** on S	$\forall x,y \in S\colon\ R(x,y) \rightarrow \neg R(y,x)$
R is **complete** on S	$\forall x,y \in S\colon\ x \neq y \rightarrow R(x,y) \vee R(y,x)$
R is **dense** on S	$\forall x,y \in S\colon\ x \neq y\ \&\ R(x,y) \rightarrow$ $\exists z \in S\colon\ z \neq x\ \&\ z \neq y\ \&\ R(x,z)\ \&\ R(z,y)$
R is **well-founded** on S	$\forall S' \subseteq S\ \exists x \in S'\colon$ $\neg\exists y \in S'\colon\ R(y,x)\ \&\ \neg R(x,y)$

Figure 3.1: Properties of relations

The first few chapters of Sen's book are worth reading. A semantic introduction to orders can be found in Landman 1991. If you're curious about the mathematical aspects of this topic, we recommend Davey & Priestley 2002.

	Properties satisfied	Name to be used in this work	Other names used in the literature
1.	reflexivity and transitivity	quasi-ordering	pre-ordering
2.	reflexivity, transitivity and completeness	ordering	complete pre-ordering; complete quasi-ordering; weak ordering
3.	reflexivity, transitivity and anti-symmetry	partial ordering	ordering
4.	reflexivity, transitivity, completeness and anti-symmetry	chain	linear ordering; complete ordering; simply ordering
5.	transitivity and asymmetry	strict partial ordering	
6.	transitivity, asymmetry and completeness	strong ordering	ordering; strict ordering; strict complete ordering

Figure 3.2: Sen 1970's summary of types of orders

3.2.3 *Ordering worlds*

Armed with all this information we can return to the ordering of worlds. When we think about worlds being ordered with respect to how well they obey a law, for example, it's clear that we need to allow different worlds to be "tied", to be at the same rank. So, our order will need to be a preorder. It's also wise not to assume that for two given worlds, we can necessarily order them with respect to each other at all. So, our order will not be total. It will be a useful idealization if we assume that our orders will be well-founded. Then, we can assume that we can always find worlds that are "as good as possible". In sum, we assume that the context will supply a function from evaluation worlds to a well-founded preorder on worlds. So, now we can find for any set of worlds the contextually best worlds among them.

This is essentially the same as what is also known as the "Limit Assumption" in conditional/modal semantics. For discussion, see Lewis 1974 and now Kaufmann 2017.

Given a well-founded preorder $\leq$, we define a function from sets of worlds to subsets thereof that yields the $\leq$-best worlds in the original set:

(22) For any set of worlds p and any well-founded preorder $\leq$ on W:
$$O_{\leq}(p) = \{w \in p\colon\ \neg\exists w'(w' \leq w\ \&\ \neg(w \leq w'))\}$$

Be aware that the literature is hopelessly inconsistent with respect to the direction of the symbols for "at least as good as" or "better than". Here, we use "$w_1 \leq w_2$" for "w_1 is at least as good w_2" because the intuition is that w_1 is less far from the ideal than w_2.

We can now state the semantics for doubly-relative modals. Here's the new entry for *must*:

(23) $[\![\text{must}]\!]^{w,g} = \lambda f_{s,st}.\ \lambda o_{s,sst}\colon\ o(w)$ is a well-founded preorder on W. $\lambda p_{st}.$
$$\forall w' \in O_{o(w)}(f(w))\colon\ p(w') = 1$$

As before, the context supplies a flavor function f that yields for any world as set of f-accessible worlds. The context now also supplies a function that for any world yields a well-founded preorder. We will call this function the ORDERING SOURCE (a term from Kratzer). The semantics of modals uses the preorder to order the set of accessible worlds and to find the best accessible worlds. The modal *must*, as a necessity modal, claims that all of the best accessible worlds are worlds where its prejacent is true.

Exercise 3.3 *Write a doubly-flavored lexical entry for the possibility modal* may. □

Let's see how the analysis applies to (19): *Howard has to pay a fine*.

- The modal base will be a function that assigns to any evaluation world the set of worlds where the same relevant circumstances hold. Since in our stipulated evaluation world, Howard failed to return his book, the modal base will assign to our world a set of worlds that only contains worlds where Howard didn't return the book.
- The ordering source will be a function that assigns to any evaluation world an ordering that represents what the rules in that world prefer. For our cases and for the given evaluation world, the ordering will prefer worlds where no book is returned late to worlds

where books are returned late. It further prefers worlds where fines are paid for late books to worlds where no fines are paid.

- For our simple example then, any world in the modal base where Howard pays a fine will count as better than an otherwise similar world where he doesn't. The very best worlds simpliciter are worlds where there's never any late books, but since there aren't any such worlds in the modal base, the ordering has to make do with what it's given.
- Modals then make quantificational claims about the best worlds in the modal base (those for which there isn't a world in the modal base that is better than them).
- In our case, (19) claims that in the best worlds (among those where Howard failed to return his book), he pays a fine.

3.2.4 Kratzer's way

As we've seen in the previous chapter, Kratzer actually calculates the set of accessible worlds from a function of type $\langle s, \langle st, t\rangle\rangle$ (what she calls a *conversational background*). Recall that the construction is simple: find the worlds where all the propositions in the set are true. The additional power that comes with using sets of propositions remains essentially unused in the case of modal bases. So, to simplify matters, we will stick with our earlier assumption that the domain of modals is a set of worlds resulting from applying a contextually supplied value of a pronoun of type $\langle s, st\rangle$ to the evaluation world.

For the ordering component of the doubly-relative semantics of modals, Kratzer's construction is quite intuitive. We assume that the ordering source argument is a function from evaluation worlds to sets of propositions. The idea is now that such a set $\mathcal{P}$ of propositions can be used to order the worlds in the modal base.

For any pair of worlds w_1 and w_2, we say that w_1 comes closer than w_2 to the ideal set up by $\mathcal{P}$ (in symbols: $w_1 <_{\mathcal{P}} w_2$), iff the set of propositions from $\mathcal{P}$ that are true in w_2 is a proper subset of the set of propositions from $\mathcal{P}$ that are true in w_1.

For our example about Howard's fine, we might imagine that for our evaluation world, the ordering source assigns a set of propositions that contains (among others) the following two propositions: (i) nobody returns books late, (ii) anybody who returns a book late pays a fine. It's crucial here that the second proposition is (vacuously) true when nobody returns books late.

Here's a more precise derivation of the ordering procedure:

(24) For any set of propositions $\mathcal{P}$, we define a strict partial order $<_{\mathcal{P}}$:
$\forall w', w'' :\ w' <_{\mathcal{P}} w''$ iff
$\forall p \in \mathcal{P}\,(w'' \in p \rightarrow w' \in p)$ and $\exists p \in \mathcal{P}\,(w' \in p \wedge w'' \notin p)$
w' is better than w'' according to $\mathcal{P}$ iff all propositions in P that hold in w'' also hold in w' but some hold in w' that do not also hold in w''.

And of course, once we have such a strict partial order and we assume well-foundedness, as before, we can define the selection function that gives us the set of $<_{\mathcal{P}}$-best worlds from any set X of worlds:

(25) $O_{<_\mathcal{P}}(X) = \{w \in X\colon\ \neg\exists w'(w' <_\mathcal{P} w)\}$

Finally, we need to rewrite the semantics of modals so that they are sensitive both to a modal base/accessibility relation and an ordering source of the Kratzerian kind:

(26) $[\![\text{must}]\!]^w = \lambda f_{s,st}.\lambda o_{s,stt}.\lambda p.\ \forall w' \in O_{<_{o(w)}}(f(w))\colon\ p(w') = 1.$

Exercise 3.4 *Give an LF for the sentence* Ashlyn must leave *that will work with the entry for* must *in (26).* □

3.2.5 More cases of modal ordering

The combination of "facts on the ground" and "doing the best with what we have" is pervasive in intensional constructions. We will discuss a succession of brief case-studies but there's still much to explore.

How to get to Harvard Square. Our initial example involved deontic modality. Next is the closely connected case of goal-oriented or teleological modality. You want to get to Harvard Square from MIT. This is your primary goal. You have secondary goals: to not spend a lot of money and to stay reasonably safe. I say:

(27) You ought to take the Red Line.

A two-factor analysis of (27) might look like this:

- The modal base is circumstantial. It assigns to the evaluation world a set of propositions describing the relevant circumstances. Imagine that for our world, the following facts are relevant: (i) you are in Kendall Square, (ii) you can get to Harvard Square on the Red Line, by taxi or Lyft, or on foot, (iii) the Red Line costs $1.25, takes 10 minutes, and is safe, (iv) the taxi/Lyft costs $10, is fast, and is safe, (v) walking is free, slow, and possibly unsafe this time of the night.
- The ordering source describes your goals (often called a *teleological* ordering source). Your relevant goals in our world are: (i) you get to Harvard Square, (ii) you pay less than $2, (iii) you are safe.
- What are the best worlds in the modal base according to the given ordering source? The worlds where you take the Red Line.
- That's why it seems true to say (27).

In von Fintel & Iatridou 2008, a *three*-factor analysis is proposed, distinguishing technically between the primary goal and a secondary ordering source. There's further discussion in work by Rubinstein (2012, 2014).

The weakness of *must*. A more controversial application of the two-factor analysis is found in Kratzer 1991: she argues that apparently strong necessity modals in their epistemic construal are actually weakened by ordering. Consider someone who observes people coming into a windowless classroom with wet outer clothing. They might say:

(28) It must be raining.

Kratzer argues that what underlies (28) is a combination of an epistemic modal base and an ordering source based on not completely exceptionless generalization and assumptions. In more recent work, von Fintel & Gillies 2010 have argued that any weakness in the meaning of (28) has to do with an "evidential signal" (that the evidence for the prejacent is merely indirect) rather than a lack of logical strength.

The debate continues with Lassiter 2016, Del Pinal & Waldon 2019, von Fintel & Gillies 2021.

Wanting what's best. Intriguingly, we can find the facts + ordering phenomenology in the semantics of propositional attitudes as well, most clearly with bouletic attitudes like *want*. Heim 1992 tells the story of someone who in her dream scenario would not teach next semester but knows that she will have to. She does have preferences on what week days to teach. So, she says:

(29) I want to teach Tuesdays and Thursdays next semester.

In terms of the two-factor analysis, the idea here is that the accessibility relation is "doxastic": the modal base contains the worlds compatible with the agent's beliefs. In the case at hand, all worlds in that modal base have her teach. The ordering is based on her preferences: among the worlds in the modal base, she prefers the ones where she teaches Tuesdays and Thursdays.

Heim herself gives a dynamic semantics analysis for attitudes that incorporates these insights. In later work, von Fintel 1999 gives a static version of a two-factor analysis that is closely modeled after Kratzer's analysis of modals. See also Rubinstein 2017 for a recent discussion.

The Good Samaritan. Prior 1958 introduced the following "Paradox of the Good Samaritan". Imagine that someone has been robbed and John is walking by. It is easy to conceive of a code of ethics that would make the following sentence true:

(30) John ought to help the person who was robbed.

In our previous one-factor semantics for modals, we would have said that (30) says that in all of the deontically accessible worlds (those compatible with the code of ethics) John helps the person who was robbed. Prior's point was that under such a semantics, something rather unfortunate holds. Notice that in all of the worlds where John helps the person who was robbed, someone was robbed in the first place. Therefore, it will be true that in all of the deontically accessible worlds, someone was robbed. Thus, (30) will entail:

(31) It ought to be the case that someone was robbed.

It clearly would be good not make such a prediction.

The doubly-relative analysis of modality can successfully avoid this unfortunate prediction. We conceive of (30) as being uttered with respect to a circumstantial modal base that includes the fact that someone was robbed. Among those already somewhat ethically deficient worlds, the relatively best ones are all worlds where John helps the victim.

Note that we still have the problematic fact that among the worlds in the modal base, all are worlds where someone was robbed, and we would thus appear to still make the unfortunate prediction that (31) should be

true. But this can now be fixed. For example, we could say that *ought p* is semantically defective if p is true throughout the worlds in the modal base. This could be a presupposition or some other ingredient of meaning. So, with respect to a modal base which pre-determines that someone was robbed, one couldn't felicitously say (31).

Consequently, saying (31) would only be felicitous if a different modal base is intended, one that contains both p and non-p worlds. And given a choice between worlds where someone was robbed and worlds where nobody was robbed, most deontic ordering sources would probably choose the no-robbery worlds, which would make (31) false, as desired.

3.2.6 Ordering in conditionals

Historically, the use of an ordering of possible worlds in the semantics of intensional operators was first proposed by Stalnaker and Lewis in their work on conditionals (Stalnaker 1968, Lewis 1973). In our attempt at an analysis for conditionals in Section 2.2, we used a one-factor approach where conditionals quantify over *all* contextually accessible antecedent worlds. Stalnaker and Lewis argue that the logic of conditionals, primarily counterfactuals, reveals the effects of an ordering. Consider the case of a perfectly normal match in front of us and the following two counterfactual conditionals:

Even though Lewis' work appeared five years after Stalnaker's, it was independently developed. See Arvan 2015 and Fn.29 in Starr 2021 for further details about the intellectual history of these ideas.

The match makes an early appearance in Goodman 1947.

(32) a. If I had struck this match, it would have lit.
b. If I had dipped this match in water and struck it, it would have lit.

Intuitively, it is clear that (32a) may well be true while (32b) is almost certainly false. But according to our previous analysis, which is a version of what in the trade is called a "strict conditional" analysis, this cannot be. If all of the accessible worlds where I strike this match are worlds where it lights, then this must *a fortiori* also be true of that subset of the accessible worlds where I strike this match after having dipped it in water.

In the Stalnaker-Lewis ordering semantics, (32b) doesn't anymore follow from (32a). The most highly ranked worlds where I strike the match might well be worlds where I don't dip it in water first, and so (32a) could be true while (32b) is false.

Within this book's framework, we would replace the meaning for *if* from (10) in Section 2.2 with the following two-factor meaning:

(33) $[\![\text{if}]\!]^{w,g} = \lambda f_{s,st}.\lambda o_{s,stt}.\lambda p_{st}.\lambda q_{st}.\ \forall w' \in O_{o(w)}(f(w) \cap p):\ q(w') = 1.$

The crucial move here is that the antecedent proposition p is used to find the p-worlds that are among the f-accessible worlds. The resulting set of accessible p-worlds is then ordered and the best accessible worlds p are what the universal quantifier makes a claim about.

But wait. In Section 3.1, we argued that *if*-clauses don't actually do their own modal quantification, so the analysis in (33) can't be quite what we should adopt here. In fact, from the restrictor point of view, the semantics of conditionals and their reliance on an ordering on possible worlds should be a compositional effect: the *if*-clause serves as a restrictor for a (possibly covert) modal and it is that modal that is sensitive to an accessibility relation and an ordering. In the next section, we explore the interplay of conditionals and modals in a theory that involves modal bases and orderings. Things are not entirely straightforward.

3.3 The interplay of ordering and restricting

We return to Howard and the saga of the possibly late library book. Imagine that we don't know whether he returned it late but we're confident we understand the library regulations. So, we say:

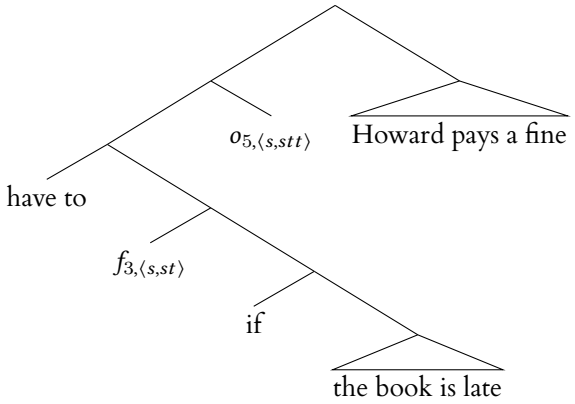
(34) If Howard returned his book late, he has to pay a fine.

There is one analysis that will not work: the restrictor analysis plus the pragmatically sophisticated but semantically simple idea that modals are just sensitive to an accessibility relation even when that relation incorporates considerations of ordering. So, assume that the accessibility relation is the one that delivers the best worlds among those that match the evaluation world with respect to whether the book was late or not. Now, imagine a scenario where unbeknownst to us, Howard actually returned the book on time. It would seem that this would not threaten the intuitive truth of (34). But according to the analysis under consideration, we predict something odd: the worlds that match the evaluation world are all worlds where the book was not late and the best ones among them thus are also not-late worlds (and of course they are worlds without a fine). The *if*-clause would then try to find late worlds in that set of accessible worlds, but since there are none, we would get an empty set of worlds to quantify over, resulting either in vacuous truth or some kind of infelicity. This is not correct.

One might think that instead of starting with the worlds that match the evaluation world in whether the book was returned late, we could start with the worlds compatible with our evidence. Then, we would include both worlds where the book was late and where it wasn't. But even among those, the not-late worlds are the best. So, then again, the *if*-clause could not find any antecedent worlds among the accessible worlds.

The two-factor analysis (modal base f, ordering o) can help with the issue. The idea is that the *if*-clause does not restrict the o-best f-worlds but restricts f before o enters into the picture. In other words, we posit an LF like this one:

(35)



To evaluate this LF, we can use the entry for *if* you developed in Exercise 3.1 and the entry for modals in (26). Note that for the *if*-clause not to be an idle restrictor, the worlds f delivers need to include worlds where the antecedent is true.

Exercise 3.5 *In fact, calculate the truth-conditions for (35) with the ingredients just mentioned.* □

Exercise 3.6 *At this point, it would be very instructive to read Kratzer 1991. In particular, you should study her version of the Samaritan Paradox in her Section 3.2 and her solution in her Section 8.* □

It appears then that we have found vindication both for the two-factor semantics for modals and for the restrictor theory. There's a problem though, illustrated by Zvolenszky 2002 with the following example:

(36) If Britney Spears drinks Coke in public, then she must drink Coke in public.

An intuitively plausible use case for (36) would be a situation where we consider the possibility of seeing Spears drinking Coke in public. We conclude that if we saw her drinking Coke, we would be able to deduce that she is somehow contractually obligated to drink Coke.

Unfortunately, our current analysis predicts that (36) should, if anything, be vacuously true rather than making a contingent claim. The *if*-clause would restrict whatever accessibility relation we're using and ensure that we're only dealing with worlds where Spears drinks Coke in public. Then, the ordering would apply to find the best such worlds. Finally, the modal would say that the best such worlds are worlds where she drinks Coke in public. That is of course trivially true.

Consider what is needed to make the modal claim (that Spears must drink Coke in public) non-trivial. The domain of quantification needs to include worlds where she doesn't drink Coke (perhaps she drinks Pepsi,

While Zvolensky's 2002 SALT paper was instrumental in bringing the issue into the natural language semantics literature, it had been already identified by Frank 1996 and in fact, it had been well-known as the "if p, ought p" problem in the more logico-philosophical world. See Carr 2014 for references and an important recent contribution.

As a matter of fact, Spears was paid to endorse Pepsi.

or Poland Spring Water). Then, the modal's ordering source identifies the best worlds and the modal says that all of those are Coke worlds. This non-triviality can only arise if the *if*-clause doesn't in fact restrict the modal to Coke worlds in the first place. So, is the restrictor theory in trouble?

At this point, we must recall that under the restrictor theory, *if*-clauses can also restrict covert modals. This means that when we have an *if*-clause and an overt modal, we can't automatically conclude that the conditional is restricting that modal. It might be restricting another modal, a covert one, instead.

So, let's look at (36) in that light. Imagine that the *if*-clause is restricting a higher covert epistemic modal. Then, we are taken from the evaluation world to those worlds compatible with our evidence where Spears drinks Coke. We are saying that in all of those worlds Spears is obligated to drink Coke. That will be a non-trivial claim if the worlds the lower modal quantifies over contains both Coke and non-Coke worlds.

More technically, we'd be working with an LF like the one sketched in (37). We use □ for the covert modal. We need to distinguish four modal parameter "pronouns".

(37) □ (f_1 if Coke) (o_1) [must f_2 o_2 Coke]

What values for the four parameter would yield the reading we want? f_1 should give us the worlds compatible with our evidence. o_1 could be trivial or the kind of not entirely reliable evidence that Kratzer 1991 argues for. Note that the higher modal will pass down only Coke-worlds. Crucially, f_2 would give us both Coke and non-Coke worlds when applied to the epistemically accessible worlds passed on by the higer operator. o_2, finally, would encode Spear's contractual obligations.

With this, we have rescued the restrictor theory. The existence of such nested structures is predicted by the framework and now we see that they are empirically attested.

See Geurts 2004 for a discussion of how this kind of structure is expected.

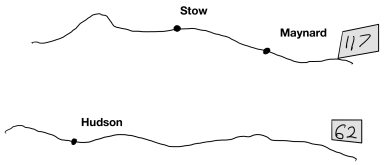
Let's return to the case of Howard and his book again. As it turns out, we could get the correct reading for our sentence *If Howard returned the book late, he has to pay a fine* without having *if* directly restrict the overt modal. Assume that the *if*-clause restricts a higher covert modal, again maybe an epistemic one. And now, in contrast to the Spears example, assume that the lower *f*-relation assigns to any of the worlds passed down (all of which will be late book worlds) a set of worlds matching the facts about whether the book was returned late. So, in effect, the conditional antecedent will end up restricting both modals: the higher one directly and the lower one indirectly since the lower accessibility relation will ensure matching of the relevant fact.

This possibility led Frank 1996 to propose that deontic modals in fact never are directly restricted:

> There are in fact no truly deontically modalized *if*-conditionals. Instead, we assume conditionals with a deontic modal operator in the consequent clause to be analyzed throughout in terms of an implicit or explicit epistemically (or circumstantially) based modal operator. The deontic modal adverb is then to be analyzed within the scope of the 'higher' epistemic modal operator.

Now, if this is in fact feasible, why should we then still maintain the restrictor analysis in the first place? Why not go back to the earlier idea that *if* is a modal operator in its own right?

To evaluate this possibility, let's return once more to our friends who do not know precisely where they are. Imagine that they have reason to think that among the two Rte 117 possibilities, Stow is much more likely than Maynard, perhaps because they know Maynard a bit better and think that if they were there, they would very likely recognize some building or other. They have no reason to think that Stow is more likely than Hudson (which is on Rte 62, remember). So, they say:



(38) If we're on Route 117, we ought to be in Stow.

Now, to be clear: they are still lost, so their evidence is compatible with them being on either of the two routes and in any of the three towns. So, for (38) to have a chance of being true, the modal *ought* here can't simply map the evaluation world to the worlds epistemically accessible from it (and then potentially order the resulting worlds). We need to narrow things down to the Route 117 worlds, even though our friends don't know whether they're there, even if they are. So, as things stand, we still need the restrictor analysis. And we need the restriction to happen *before* the ordering applies. So, for now, (38) provides the strongest argument we have for both the restrictor theory and the two-factor semantics for modals.

At this point, we leave this intriguing and complex inquiry and refer you to the ever-growing literature, some we've already mentioned and more we will list below.

3.4 Further readings

Conditionals. In-depth overviews of the logico-philosophical perspective on conditionals include Nute 1984, Edgington 1995, Bennett 2003, and Starr 2021.

Ideas pertaining to the implementation of the restrictor theory are found in Reich 2009 and Ebert, Ebert & Hinterwimmer 2014. Schlenker 2004 presents an alternative that is at least plausibly in the same orbit. One should really take into account what we know about the syntax of conditionals, some of which is discussed in Chapter 3 of Kai's thesis but much

more is found in Bhatt & Pancheva 2006. Rothschild 2011 explains the restrictor theory to philosophers, might be useful as an additional reading.

Gillies 2010 proposes an ingenious alternative to the restrictor theory. Khoo 2011 files a complaint about the coverage of Gillies' proposal. The interaction of conditionals with probability operators is very tricky, see Egré & Cozic 2011 and von Fintel & Gillies 2015 for some discussion.

Higginbotham 1986 identified the interaction of negative quantifiers like *no student* with conditionals as a compositionality puzzle. The restrictor theory has sometimes been seen as a solution to the puzzle (von Fintel 1998) and sometimes not (von Fintel & Iatridou 2002). See Dekker 2001, Higginbotham 2003, Leslie 2009, Huitink 2010, Klinedinst 2011, Kratzer 2021, Lauer & Nadathur 2016 among others.

To understand *only if*-conditionals, von Fintel 1997 needs to work hard to neutralize the universal force of bare conditionals stemming from Kratzer's implicit necessity modal.

Herburger 2015, 2016, and Bassi & Bar-Lev 2017 consider the possibility that bare conditionals are actually (sometimes or always) existentially quantified.

Attitudes. Linguistic work on attitudes has often been concerned with various co-occurrence patterns, particularly which moods (indicative or subjunctive or infinitive) occur in the complement and whether negative polarity items are licensed in the complement. Mood licensing: Portner 1997. NPI-Licensing: Kadmon & Landman 1993, von Fintel 1999, Giannakidou 1999.

Tamina Stephenson in her MIT dissertation and related work explores the way attitude predicates interact with epistemic modals and taste predicates in their complements: Stephenson 2007a,b.

Jon Gajewski in his MIT dissertation and subsequent work explores the distribution of the NEG-RAISING property among attitude predicates and traces it back to presuppositional components of the meaning of the predicates: Gajewski 2005, 2007.

Interesting work has also been done on presupposition projection in attitude contexts: Asher 1987, Heim 1992, Geurts 1998.

Modals. We recommend Bhatt 1997 and Wurmbrand 1999 on the question of whether modals are raising predicates. A more general survey on the syntax of modals is Barbiers & van Dooren 2017. The following paper explores some issues in the LF-syntax of epistemic modals: von Fintel & Iatridou 2003,

Valentine Hacquard's MIT dissertation is a rich source of cross-linguistic issues in modality, as is Fabrice Nauze's Amsterdam dissertation: Hacquard 2006, Nauze 2008.

Some more recent work by Hacquard deals with deriving and correlating modal flavors with syntactic position of the modal auxiliaries: Hacquard 2010, Hacquard 2013. A recent handbook article by Hacquard on

actuality entailments (involving the interaction of modality with aspect; we'll discuss aspect later in these notes): Hacquard 2020.

The semantics of epistemic modals has become a hot topic recently. Here are some of the main references: Hacking 1967, Teller 1972, DeRose 1991, Egan, Hawthorne & Weatherson 2005, Egan 2007, MacFarlane 2011. Stephenson 2007a, Hawthorne 2007, von Fintel & Gillies 2008, von Fintel & Gillies 2011.

A paper by Pranav Anand and Valentine Hacquard tackles what happens to epistemic modals under attitude predicates: Anand & Hacquard 2013.

Evidentiality is a topic closely related to epistemic modality. Some references: Willett 1988, Aikhenvald 2004, Drubig 2001, Blain & Déchaine 2007, McCready & Ogata 2007, Speas 2008, von Fintel & Gillies 2010.

Modals interact with disjunction and indefinites to generate so-called FREE CHOICE-readings, which are a perennial puzzle. Here is just a very small set of initial references: Kamp 1973, Zimmermann 2000, Schulz 2005, Aloni 2007, Alonso-Ovalle 2006, Fox 2007, van Rooij 2006.

On ability modals, see Hackl 1998, Mandelkern, Schultheis & Boylan 2017.

Restricting and ordering. Very important for the problem we ended the chapter on are these papers: Kaufmann & Schwager 2009 and Condoravdi & Lauer 2016. Recent work includes: Blumberg & Holguín 2019, von Fintel & Pasternak 2022.

4 Beginnings of tense and aspect

4.1 The world is not enough; the times, they are a-changing

This chapter is even more the outcome of collaborative efforts than other chapters. We are very much indebted to Roger Schwarzschild, who has used our notes several times in his teaching and is the source of many edits and additions.

Tense logic, or temporal logic, is a branch of logic first developed by the aptly named Arthur Prior in a series of works, in which he proposed treating tense in a way that is formally quite parallel to the treatment of modality discussed in Chapter 3. Since tense logic (and modal logic) typically is formulated at a high level of abstraction regarding the structure of sentences, it doesn't concern itself with the internal make-up of "atomic" sentences and thus treats tenses as sentential operators (again, in parallel to the way modal operators are typically treated in modal logic). We will begin by integrating a version of Prior's tense logic into our framework.

We remain vague for now about what we mean by "times" (points in time? time intervals?). This will soon need clarification, and we will decide that we should mean "intervals".

The first step is to switch to a version of our intensional semantic system where instead of a world parameter, the evaluation function is sensitive to a parameter that is a pair of a world and a time. Such a pair will also be called an "index". We use metalanguage variables i, i', … for indices, and write w_i and t_i to pick out the world in i and the time in i respectively (i.e., $i = \langle w_i, t_i \rangle$). Predicates will now have lexical entries that incorporate their sensitivity to both worlds and times:

(1) $[\![\text{tired}]\!]^i = \lambda x \in D.\ x \text{ is tired in } w_i \text{ at } t_i$

The composition principles from Heim & Kratzer and the preceding chapters stay the same, except that the (pseudo-)type s now stands for indices (pairs of worlds and times) and not just worlds. For example, the

intension of sentence is now a function from world-time pairs to truth-values. We might call this a "temporal proposition", to distinguish it from a function from just worlds to truth-values, but we will often just call it a "proposition".

This necessitates a slight rewriting of our previous entries for modals and attitude verbs. We will attend to this when we get to relevant examples.

In this framework, we can formulate a very simple-minded first analysis of the present and past tenses and the future auxiliary *will.* As for (LF) syntax let's assume that complete sentences are TPs, headed by T (for "tense"). There are two morphemes of the functional category T, namely PAST (past tense) and PRES (present tense). The complement of T is an MP or a VP. MP is headed by M (for "modal"). Morphemes of the category M include the modal auxiliaries *must*, *can*, etc., which we talked about in previous chapters, the semantically vacuous *do* (in so-called "*do*-support" structures), and the future auxiliary *will*. Evidently, this is a semantically heterogeneous category, grouped together solely because of their common syntax (they are all in complementary distribution with each other). The complement of M is a VP. When the sentence contains none of the items in the category M, we assume that MP isn't projected at all; the complement of T is just a VP in this case. (TP is always projected in a root clause, whether there is an MP or not.) We thus have LF-structures like the following. (The corresponding surface sentences are given below, and we won't be explicit about the derivational relation between these and the LFs. Assume your favorite theories of syntax and morphology here.)

Many subordinate clauses — those we call "finite" — also always have a TP. As for embedded clauses more generally (including infinitives etc.), we don't need to take a stand here.

(2) [$_{TP}$ Svenja [$_{T'}$ PRES [$_{VP}$ t [$_{V'}$ be tired]]]]
= Svenja is tired.

(3) [$_{TP}$ Svenja [$_{T'}$ PAST [$_{VP}$ t [$_{V'}$ be tired]]]]
= Svenja was tired.

(4) [$_{TP}$ Svenja [$_{T'}$ PRES [$_{MP}$ t [$_{M'}$ woll [$_{VP}$ t [$_{V'}$ be tired]]]]]]
= Svenja will be tired.

woll in (4) stands for the underlying uninflected form of the auxiliary which surfaces as *will* in the present tense (and as *would* in the past tense). When we have proper name subjects, we will assume for simplicity that they are reconstructed into their VP-internal base position.

We use "*woll*" as the name of the root underlying *will* and *would*, following Abusch 1988 and Ogihara 1989: p.32; Abusch (1997: fn.14, p.22) attributes the coinage of *woll* to Mats Rooth in class lectures at UT Austin.

What are the meanings of PRES, PAST, and *woll*? For PRES, the simplest assumption that seems to work is that it is semantically vacuous. This means that the interpretation of the LF in (2) is identical to the interpretation of the bare VP *Svenja be tired*:

(5) For any index i: $[\![\text{PRES (Svenja be tired)}]\!]^i = [\![\text{Svenja be tired}]\!]^i = 1$ iff Svenja is tired in w_i at t_i.

Does this adequately capture the intuitive truth-conditions of the sentence *Svenja is tired*? It does if we make the following general assumption:

(6) Utterance Rule:
An utterance of a sentence (= LF) ϕ that is made in a world w at a time t counts as true iff $[\![\phi]\!]^{\langle w,t\rangle} = 1$ (and as false if $[\![\phi]\!]^{\langle w,t\rangle} = 0$).

This assumption ensures that (unembedded) sentences are, in effect, interpreted as claims about the time at which they are uttered ("utterance time" or "speech time"). If we make this assumption and we stick to the lexical entries we have adopted, then we are driven to conclude that the present tense has no semantic job to do. A tenseless VP *Svenja be tired* would in principle be just as good as (2) to express the assertion that Svenja is tired at the utterance time. Apparently, it is just not well-formed as an unembedded structure, but this fact then must be attributed to principles of syntax rather than semantics.

What about PAST? When a sentence like (3) *Svenja was tired* is uttered at a time t, then what are the conditions under which this utterance is judged to be true? A quick answer is: an utterance of (3) at t is true iff there is some time before t at which Svenja is tired. This suggests the following entry:

(7) For any index i: $[\![\text{PAST}]\!]^i = \lambda p \in D_{\langle s,t\rangle}.\ \exists t \text{ before } t_i:\ p(\langle w_i, t\rangle) = 1$

So, the past tense seems to be an existential quantifier over times, restricted to times before the utterance time.

For *will*, we can say something completely analogous:

(8) For any index i: $[\![\text{woll}]\!]^i = \lambda p \in D_{\langle s,t\rangle}.\ \exists t \text{ after } t_i:\ p(\langle w_i, t\rangle) = 1$

Apparently, PAST and *woll* are semantically alike, even mirror images of each other, though they are of different syntactic categories. The fact that PAST is the topmost head in its sentence, while *woll* appears below PRES, is due to the fact that our syntax happens to require a T-node in every complete sentence. Semantically, this has no effect, since PRES is vacuous.

Both (7) and (8) presuppose that the set of times comes with an intrinsic order. For concreteness, assume that the relation 'precedes' (in symbols: $<$) is a strict linear order on the set of all times. The relation 'follows', of course, can be defined in terms of 'precedes' (t follows t' iff t' precedes t).

Are there also tenses with universal force ? Two possible candidates that call for closer examination: gnomic tenses (e.g. in Ancient Greek), and the (universal reading of the) English perfect (as in *I have been tired since yesterday morning)*. Both have been written about in the formal semantics literature (the latter extensively — you could start with Iatridou, Anagnostopolou & Izvorski 2001 and von Fintel & Iatridou 2019.

Strict linear orders are transitive, irreflexive, asymmetric, and connected. See Section 3.2.2 on the basics of order theory. Some rethinking is needed once we move to intervals.

4.2 Time frame adverbials

Let's take a brief look at temporal adverbials, specifically so-called frame adverbials, such as:

(9) Svenja was tired on February 1, 2001.

There are two ideas that come to mind. One is that phrases like *on February 1, 2001* are restrictors of temporal operators (kind of like *if*-clauses are restrictors of modals). The other idea is that they are modifiers of the

proposition in the temporal operator's scope. If we want to go with the first idea, we have to make some changes. Our current PAST and *woll* are unrestricted (1-place) operators, so there is no place for a restrictor. The second idea is easier to implement, and we try that first.

A propositional modifier is a function from propositions to truth-values, where "proposition" for us now means "temporal proposition". Here is an entry for on February 1, 2001. Intuitively, this modifier takes a proposition and returns a proposition that puts an added condition on the time-coordinate of its index-argument.

Technically, the modifier returns a truth-value, not a proposition. We get back a proposition only when we compute the intension of the phrase that includes the modifier.

(10) $[\![\text{on February 1, 2001}]\!]^i = \lambda p \in D_{\langle s,t\rangle}.$
$[p(i) = 1 \;\&\; t_i \text{ is part of February 1, 2001}]$

(11) LF: PAST [$_{VP}$ [$_{VP}$ Svenja be tired][$_{PP}$ on February 1, 2001]]

Exercise 4.1 *Imagine that sentence (9) is not given the LF in (11), but this one, with the PP attached higher:*

(12) LF: [$_{T'}$ PAST [$_{VP}$ *Svenja be tired*]] [$_{PP}$ *on February 1, 2001*]

What would the truth-conditions of this LF be? Does this result correspond at all to a possible reading of this sentence (or any other analogous sentence)? If not, how could we prevent such an LF from being produced? □

The truth conditions that we derive given (10) and (11) look good: the sentence is predicted true as uttered if there is a time which is both before the utterance time and within Feb 1, 2001 and at which Svenja is tired, and it is predicted false if there is no such time. But arguably this is not *exactly* right. Suppose that somebody uttered this sentence at an utterance time that *preceded* the date in the adverbial, say at some time in the year 2000. Our analysis predicts that this utterance is false. But in fact it feels more like a presupposition failure; the speaker is heard to be taking for granted that Feb 1, 2001 is in the past of his speaking. Standard presupposition tests confirm this. For example, the negated sentence (*Svenja wasn't tired on Feb 1, 2001*) and the polar question (*Was Svenja tired on Feb 1, 2001?*) also convey that the speaker assumes he is speaking after Feb 1, 2001.

If we want to account for this more fine-grained intuition, the restrictor approach has an advantage after all. Let's revise the entries for PAST and *woll* so that they denote 2-place operators, and moreover they encode a non-emptiness presupposition.

It also has the virtue of avoiding the potential overgeneration issue that you looked at in the exercise above. Q: How so?

(13) For any index i:
$[\![\text{PAST}]\!]^i = \lambda p\colon\ \exists t[t < t_i \;\&\; p(\langle w_i, t\rangle) = 1].$
$\lambda q.\ \exists t[t < t_i \;\&\; p(\langle w_i, t\rangle) = 1 \;\&\; q(\langle w_i, t\rangle) = 1]$

(14) For any index i:
$[\![\text{woll}]\!]^i = \lambda p\colon\ \exists t[t > t_i\ \&\ p(\langle w_i, t\rangle) = 1].$
$\lambda q.\ \exists t[t > t_i\ \&\ p(\langle w_i, t\rangle) = 1\ \&\ q(\langle w_i, t\rangle) = 1]$

Furthermore, let's change (i.e., simplify) the meaning of the adverbial so that it has a suitable type to serve as the temporal operator's first argument. The LF-structure must be accordingly different as well. Instead of (11) above, we now posit (17), where the adverb forms a constituent with the tense. This requires the surface order to be derived by some reordering, perhaps extraposition of the adverbial.

(16) $[\![\text{on February 1, 2001}]\!]^i = 1$ iff t_i is part of February 1, 2001

(17) LF: [$_{T'}$ PAST [$_{PP}$ on February 1, 2001]][$_{VP}$ Svenja be tired]

The meanings we now derive contain the desired presuppositions: The past tense sentence (9) presupposes that Feb 1, 2001 is at least in part before the utterance time, the future sentence *Svenja will be tired on Feb 1, 2001* presupposes that this date is at least in part after the utterance time, and the present tense sentence *Svenja is tired on Feb 1, 2001* presupposes that the utterance time is on this date. Apart from the presuppositions, the meanings are the same as before. On the down-side, the new analysis posits both more complex meanings for the tenses and a less direct correspondence between LF constituency and surface structure. Furthermore, how is it supposed to apply to simple sentences without adverbials? Not every tensed sentence contains an obligatory frame adverb, after all. We are forced to say there is a covert restrictor whenever there isn't an overt one. But this, upon reflection, turns out to be a virtue, as we will see in the next section.

How about present tense? Should we make this presuppositonal as well — which would imply it is not, after all, completely vacuous? Frame adverbials in present tense sentences do occur. Typically they are adverbials like *today, on this beautiful Monday*, which in virtue of their own meaning already are required to contain the speech time. The following entry would make room for them and duplicate this requirement as a presupposition:

(14) For any index i: $[\![\text{PRES}]\!]^i = \lambda p \in D_{\langle s,t\rangle}\colon\ p(\langle w_i, t_i\rangle).\ \lambda q \in D_{\langle s,t\rangle}.\ q(\langle w_i, t_i\rangle) = 1$

For the purposes of these lecture notes, we leave the matter open and stick with the vacuous meaning for PRES in the discussions of the upcoming sections.

Exercise 4.2 *When a quantifier appears in a tensed sentence, we expect two scope construals. Consider a sentence like this:*

(18) Every professor (in the department) was a teenager in the Sixties.

We can imagine two LFs:

(19) [PAST in the sixties] [every professor be a teenager]

(20) [every professor] 7 [[PAST in the sixties] [t_7 be a teenager]]

Describe the different truth-conditions which our system assigns to the two LFs. Is the sentence ambiguous in this way? If not this sentence, are there analogous sentences that do have the ambiguity? □

Considering the mismatch between the LF in (17) and the surface order, note that we saw a similar issue of apparent mismatch when we decided to treat *if*-clauses as restrictors of modals. Both issues might be addressed by simply letting modal and temporal operators take their arguments in the opposite order (something suggested by Chierchia 1995). Rewriting the lexical entries in this way is a routine exercise. We leave this matter open. The syntax of frame adverbials is a non-trivial object of study.

Exercise 4.3 *Our official entry for* every *makes it a time-insensitive (and world-insensitive) item:*

(21) *For any i,* $[\![every]\!]^i = \lambda f_{\langle e,t\rangle}.\lambda g_{\langle e,t\rangle}.\ \forall x\colon\ f(x) = 1 \rightarrow g(x) = 1$

Consider now two possible variants (we have boxed the portion where they differ):

(22) *For any i,* $[\![every]\!]^i = \lambda f_{\langle e,t\rangle}.\lambda g_{\langle e,t\rangle}.\ \forall x\ \boxed{at\ t_i}\colon\ f(x) = 1 \rightarrow g(x) = 1$

(23) *For any i,* $[\![every]\!]^i = \lambda f_{\langle e,t\rangle}.\lambda g_{\langle e,t\rangle}.\ \forall x\colon\ f(x) = 1\ \boxed{at\ t_i} \rightarrow g(x) = 1$

Does either of these alternative entries make sense? If so, what does it say? Is it equivalent to our official entry? Could it lead to different predictions about the truth-conditions of English sentences? □

4.3 Are tenses referential?

Our first semantics for the past tense, in Section 4.1, treated it as an unrestricted existential quantifier over times. This seems quite adequate for examples like (24), which seem to display the expected unrestricted existential meaning:

(24) Georgia went to a private school.

All we learn from (24) is that at some point in the past, whenever it was that Georgia went to school, she went to a private school.

Partee in her famous paper "Some structural analogies between tenses and pronouns in English" (Partee 1973) presented an example where tense appears to act more "referentially":

(25) I didn't turn off the stove.

"When uttered, for instance, halfway down the turnpike, such a sentence clearly does not mean either that there exists some time in the past at which I did not turn off the stove or that there exists no time in the past at which I turned off the stove. The sentence clearly refers to a particular time — not a particular instant, most likely, but a definite interval whose identity is generally clear from the extralinguistic context, just as the identity of the *he* in [*He shouldn't be in here*] is clear from the context."

Partee argues, in effect, that neither of the two plausible LFs that our system from Section 4.1 derives can correctly capture the meaning of (25). Given that the sentence contains a past tense and a negation, there are two possible scopings of the two operators:

(26) a. PAST NEG I turn off the stove.
b. NEG PAST I turn off the stove.

Exercise 4.4 *Using our old semantics from 4.1, show that neither LF in (26) captures the meaning of (25) correctly.*

In a commentary on Partee's paper (at the same conference it was presented at), Stalnaker pointed out that a minor amendment of the Priorean theory can deal with (25). One just needs to allow the existential quantifier to be contextually restricted to times in a salient interval. Since natural language quantifiers are typically subject to contextual restrictions, this is not a problematic assumption. Note that Partee formulated her observation in quite a circumspect way: "The sentence refers to a particular time"; Stalnaker's suggestion was that the reference to a particular time is part of the restriction to the quantifier over times expressed by tense, rather than tense itself being a referring item.

Ogihara 1995, 1996 argued that the restricted existential quantification view is in fact superior to Partee's analysis, since Partee's analysis needs an existential quantifier anyway. It is clear that the time being referred to in the stove-sentence (25) is a protracted interval (the time during which Partee was preparing to leave her house). But the sentence is not interpreted as merely saying that this interval is not a time at which she turned off her stove. That would only exclude a fairly absurd kind of slow-motion turning-off-of-the-stove (turning off the stove only takes a moment). Instead, the sentence says that in the salient interval there is no time at which she turned off the stove. Clearly, we need an existential quantifier in there somewhere and the Priorean theory provides one.

The alternative is to say that the existential quantifier is not expressed by tense but comes from somewhere else — perhaps from aspect, or from the lexical entry of the verb itself. We will come back to these options.

Ogihara made the point with the following example:

(27) Patricia: Did you see Solène?
Lea: Yes, I saw her, but I don't remember exactly when.

The question and answer in this dialogue concern the issue of whether Lea saw Solène at some time in a contextually salient interval.

Stalnaker's and Ogihara's conclusions converge with what we already ended up with in Section 4.2, after considering the interaction of tenses with time frame adverbials. In order to capture presuppositions of tensed sentences with frame adverbials, we already modified Prior's original proposal and made room for a restrictor in the semantics of the past tense. Given this revised analysis of the past tense as a 2-place existential quantifier, it is unsurprising, in fact expected, that an implicit, contextually salient restrictor should be present when there isn't an overt one. What then about example (24), *Georgia went to a private school*, for which the unrestricted analysis seemed to do well? Let us say that the covert restrictor in this case picks out a very long interval, perhaps Georgia's entire lifetime, or even the entire past from the big bang to the utterance time, or all eternity. (What exactly the right restrictor is in this case, and what makes it contextually available, may be a bit unclear, but we leave it at that.)

Exercise 4.5 *Assuming the restricted existential quantifier analysis of past tense that we adopted in Section 4.2, which of the scope constellations in (26) captures the meaning of (25) correctly?*

4.4 Referential tense and perfective/imperfective aspect

4.4.1 Referential tense after all

Let us return to Partee's stove and the prospects of a "referential" theory of tense. Our discussion of Partee's example (following Stalnaker and Ogihara) just now came to the conclusion that we did need past tense to be an existential quantifier over times, albeit a contextually restricted one. The stove-example is interpreted as a claim about a particular contextually relevant interval. But the speaker's claim is not merely that she didn't turn off the stove at that interval. That in itself would be compatible with her turning off the stove at some smaller interval inside the contextually relevant interval. The speaker's claim is stronger: she did not turn off the stove at any time that is contained in this interval. This is a negative existential claim. So there needs to be an existential quantifier somewhere in the LF and below the scope of *not*, and we concluded that past tense must be supplying it. But this conclusion is not inescapable. Granted that there has to be an existential quantifier somewhere — but couldn't it be somewhere else than in the meaning of tense? One alternative that comes to mind is to locate it in the lexical meaning of the verb (here *turn off*). This means we would abandon the lexical entry in (28) and instead adopt the one in (29).

(28) $[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.\ x$ turns off y in w_i at t_i

(29) $[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.\ x$ turns off y in w_i $\boxed{\text{in}}$ t_i

The difference between 'at' and 'in' looks small at first, but if we reflect on the meaning of 'in', we see the hidden existential quantifier. When something happens in an interval, it happens at some part of the interval. We can make this more transparent in the metalanguage and rewrite (29) as (30).

(30) $[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.\ \exists t \subseteq t_i :\ x$ turns off y in w_i at t

The subset sign here stands for the containment relation between time intervals. A time interval can be defined as a certain kind of set of moments, as in (31), so the subset relation is well defined.

(31) A set of moments S is an interval iff for any two moments that are in S, every moment between them is also in S.

Another way to clarify the distinction between 'at' and 'in' is to use the kind of metalanguage that is familiar from the literature on Davidsonian event semantics.

(32) abbreviations in "event talk":

a. turn-off(e, x, y) = e is an event of turning off y by agent x

b. $\tau(e)$ = the (exact) time-interval occupied by event e
also called the "run-time" or "temporal trace" of e

(33) event-talk formulation of (28), the old entry with 'at':
$[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.\ \exists e[\text{turn-off}(e, x, y)\ \&\ e \text{ is in } w_i\ \&\ \tau(e) = t_i]$

(34) event-talk formulation of (29), the new entry with 'in':
$[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.\ \exists e[\text{turn-off}(e, x, y)\ \&\ e \text{ is in } w_i\ \&\ \tau(e) \subseteq t_i]$

Let us spell out now how Partee's proposal for the meaning of past tense can be upheld after all, once we assume the lexical semantics specified in (29)/(30)/(34). The first task here is to write new lexical entries for the tense morphemes, which encode Partee's idea that tenses refer to specific time intervals and are semantically and pragmatically akin to personal pronouns. We will defer the full execution of this task until later and make do for the time being with a couple of syncategorematic ad hoc rules for the interpretation of TPs.

(35) $[\![\textsc{past}\ \phi]\!]^i = 1$ iff $[\![\phi]\!]^{\langle w_i, t'\rangle} = 1$, where t' is the contextually salient time before t_i (no truth value defined if there is no such time)

(36) $[\![\textsc{woll}\ \phi]\!]^i = 1$ iff $[\![\phi]\!]^{\langle w_i, t'\rangle} = 1$, where t' is the contextually salient time after t_i (no truth value defined if there is no such time)

This contextually salient time is also called the "topic time" (Klein 1994) or the "reference time" (a term which goes back to Reichenbach 1947, but which has various other uses in the literature).

(PRES remains vacuous as before, i.e., $[\![\textsc{pres}\ \phi]\!]^i = [\![\phi]\!]^i$.)

The stove example has the two potential LFs in (37).

(37) a. PAST NEG I turn off the stove.

b. NEG PAST I turn off the stove.

We can compute the truth conditions for both of these under the new semantics for PAST and *turn off*, and it turns out that they are the same.

(38) $[\![(37a)]\!]^i = 1$ iff $[\![(37b)]\!]^i = 1$ iff
$\neg\exists e[\text{turn-off}(e, x, y)\ \&\ e \text{ is in } w_i\ \&\ \tau(e) \subseteq t']$,
where t' is the contextually salient time before t_i
(no truth-value defined if there is no such time)

The fact that both scopal orders yield the same truth conditions is arguably a point in favor of this approach. The English sentence is not in fact perceived as ambiguous. Our earlier approach, on which past tense was a contextually restricted existential quantifier, did not make this prediction — at least not without the help of additional assumptions (such as a syntactic constraint on the position of negation with respect to other heads on the clausal spine). Now that the existential quantifier comes bundled with the lexical verb, its scope is automatically "frozen" below everything that scopes over the verb.

4.4.2 *Event semantics and perfective aspect*

Up to now, we have presupposed a pre-Davidsonian view of lexical meanings, on which verbs take only individuals or propositions as their arguments. Even when we recently inserted some event-talk into the metalanguage of our lexical entries, we still defined the denotation of a verb like *turn off* as a function from two individuals to a truth value. In this section, we switch to a Davidsonian treatment of verbs as predicates of events and integrate this with our conception of sentence-intensions as temporal propositions.

We share the goal of integrating Davidsonian event semantics and traditional intensional semantics with von Stechow & Beck 2015, from whom we borrow a number of ideas.

In an extensional Davidsonian semantics, lexical entries look like (39).

(39) $[\![\text{laugh}]\!] = \lambda x.\lambda e.\ e$ is an event of x laughing
abbreviated: $[\![\text{laugh}]\!] = \lambda x.\lambda e.\ \text{laugh}(e, x)$

Assuming that events are not in D_e, but have their own basic type v, VPs thus are of type $\langle v, t\rangle$. (All the verb's non-event arguments are merged inside the VP.)

Many practitioners of event semantics assume that the event argument is the only real argument of the verb, whereas the subject, object, etc. are arguments of abstract theta-role heads that combine with the verb in the manner of modifiers. Here we remain agnostic on this matter. For concreteness, we assume that verbs take all the traditional arguments in addition to their event-argument, but the other view is equally compatible with everything we will say. We just abstract away from the internal compositional semantics of the VP.

In a semantics that is both Davidsonian and intensional, do we have to rewrite these entries? For example, should we perhaps rewrite (39) as (40)?

(40) $[\![\text{laugh}]\!]^i = \lambda x.\lambda e.\ e$ is an event of x laughing $\boxed{\text{in } w_i \text{ at } t_i}$

That depends. Here we follow Kratzer and assume that each event occurs in just one world and at just one time. It is not possible for a given e to be an event of x laughing in one world and to be some other kind of event in another world. Nor is it possible for one and the same e to be an event of x laughing at one time and something else at another time. Reformulations such as (40) are uncalled for then, and we can essentially stick with (39).

This assumption is made here mostly to keep things simple. It is not innocuous and not uncontroversial. See e.g. Hacquard 2009 for an analysis of root modals that makes crucial use of the idea that an actual event exists in non-actual worlds and has different properties there.

But how then does world and time dependence enter the semantic computation? And how can tenses and modal operators combine with VPs? VPs are now type $\langle v, t\rangle$, which leads to a type-mismatch if we try to combine them directly with a modal operator or with a tense (regardless of whether the tense is a Priorian temporal operator or a Partee-style referential tense). The way out of this problem is to posit a more complex clause structure, with a further functional head that intervenes between T (or M) and V. This is called an "aspect" head (category label "Asp"), and its semantic job is to existentially bind the event argument of the VP and return a world- and time-sensitive denotation of type t.

Strictly speaking, we should now write (41), but since i in (39) does not occur on the right side of =, (39) can be shorthand for (41).

(41) For any index i, $[\![\text{laugh}]\!]^i = \lambda x.\lambda e.\ \text{laugh}(e, x)$

One instance of Asp is the so-called "perfective", for which we posit the following entry.

(42) $[\![\textsc{pfv}]\!]^i = \lambda P_{\langle v,t\rangle}.\exists e[P(e) = 1 \;\&\; \tau(e) \subseteq t_i \;\&\; e \leq w_i]$

$\leq$:= is part of (= occurs in)

τ := the run time of (temporal trace of)

(42) combines the standard formal analysis of perfective aspect (among many others: Klein 1994, Kratzer 1998) with the semantics of von Stechow & Beck 2015's Modl head. It locates the event both in a time interval and in a possible world.

PFV is morphologically zero in English, so we can posit it in the LFs of sentences with simple tensed verbs. Here is an example.

(43) a. Barbara turned off the stove.

b. LF: [$_{\text{TP}}$ PAST [$_{\text{AspP}}$ PFV [$_{\text{VP}}$ Barbara turn-off the stove]]]

Using the syncategorematic rule (35) for referential PAST, our entry (42) for PFV, and a Davidsonian entry for the verb, we compute the following interpretation. (Do this as an exercise.)

(44) $[\![(43b)]\!]^i = 1$ iff $\exists e[\text{turn-off}(e,\ \text{B, the stove}) \;\&\; \tau(e) \subseteq t' \;\&\; e \leq w_i]$,

where t' is the contextually salient time before t_i

(no truth-value defined if there is no such time)

This is the same meaning that we obtained in the previous section, when we had built the existential quantification into the lexical meaning of the verb. What used to be the meaning of VP is now the meaning of AspP. We have located the event-quantifier in its own functional head, but otherwise it is the same analysis.

Exercise 4.6 *What about the negated sentence that was Partee's original example? Where can we now generate negation in an interpretable LF? Does the current analysis still predict that the sentence is not in fact ambiguous?* □

4.4.3 The English progressive

Besides perfective aspect, there is imperfective aspect — or more accurately, there is probably a family of imperfective aspects in different languages that have some shared and some non-shared properties. English has an imperfective aspect known as the "progressive", with an overt morphology that consists of a copula which governs a present-participial form of the VP (*V-ing*). We posit a functional head be-PROG as the aspect head in the English progressive construction.

See e.g. Arregui, Rivero & Salanova 2014 for a recent approach to cross-linguistic semantic variation in imperfective aspects.

We are not serious about morphology here. The meaning may well be carried by an abstract head and the *be* a vacuous element.

The basic intuition behind much work on the perfective-imperfective distinction is that, whereas perfective aspect locates an event within the evaluation time, imperfective aspect does the reverse, i.e., it places the evaluation time within the event time. If we formalize this intuition directly, without introducing any further differences from the perfective, we come up with (45).

(45) First attempt:
$[\![\text{be-PROG}]\!]^i = \lambda P_{\langle v,t\rangle}.\exists e[P(e) = 1 \;\&\; \boxed{t_i \subseteq \tau(e)} \;\&\; e \leq w_i]$

It is well-known, however, that there is also a difference in how the event is related to the evaluation world. While perfective places the event within the actual world, the progressive permits it to be partly in another world, so to speak. This point, which was at the center of Dowty 1977's seminal work on the progressive, is brought home by examples like (46).

(46) John was going to the store when he ran into Svenja.

We can't infer from this sentence that John actually made it to the store, or will ever make it there. The sentence leaves this open. Perhaps John does complete his trip to the store after the encounter, and perhaps he doesn't. The truth-conditions of the sentence (46) are compatible with either scenario. The entry in (45), on the other hand, would require that there be a John-going-to-the-store event which occupies a super-interval of the time of the encounter with Svenja and which *occurs in the actual world*. So (45) can't be quite right.

Dowty's analysis of the progressive says instead that a John-going-to-the-store event occurs in certain *possible* worlds. These possible worlds are related to the actual world in a particular way: they are worlds which share a history with the actual world up to a certain point and then develop (possibly counterfactually) in such a way that no events that were already in progress get interrupted ("inertia worlds"). The idea is, very roughly, that the sentence tells us: either John actually went to the store, or if he didn't, then at least he *would have gone* there if he hadn't been interrupted. Since the publication of Dowty's paper, there has been a succession of sophisticated counterexamples and refinements to his original proposal, but this is beyond the scope of this introduction. Here is a version based on Dowty.

(47) second attempt (and final version for us):

$$[\![\text{be-PROG}]\!]^i = \lambda P_{\langle v,t\rangle}.\forall w\big[w \in \text{Inert}(i) \rightarrow \exists e[P(e) = 1 \;\&\; t_i \subset^{<} \tau(e) \;\&\; e \leq w]\big]$$

where $\subset^{<}$ abbreviates: "is a non-final subinterval of"
(that is: $\tau(e)$ includes every moment in t_i as well as some moment after the end of t_i)

(48) Definition: $w \in \text{Inert}(i)$ iff
w is exactly like w_i up to the end of t_i and then develops in such a way that no events are interrupted.

We will see in a minute that there is a class of VPs for which the truth-conditions predicted by (47) come very close to those predicted by the simpler (45). But examples like (46) show that this must not always hold.

This is similar to the first formal analysis of the progressive, due to Bennett & Partee 1978. They did not work in an event semantics, however. Also, their semantics required t_i to be a *non-final* subinterval of $\tau(e)$, rather than merely $t_i \subseteq \tau(e)$. This requirement seemed too strong in light of examples such as Dowty 1977's *John was watching TV when he fell asleep* (which does not say that TV-watching continued beyond the point of falling asleep). However, as Dowty showed, it turned out to be the right requirement in the context of the modalized analysis that he proposed, see below.

Dowty dubbed this the "imperfective paradox", although it's not really a paradox, just a counterexample to a certain analysis that looked plausible at first.

See among many others, Landman 1992, Portner 1998.

Apart from introducing quantification over other worlds, (47) also differs from (45) in that it strengthens the requirement on the temporal relation between t_i and $\tau(e)$: not only must $\tau(e)$ contain all of t_i, but it must moreover extend into the time after t_i. This is intended to capture the intuition that e.g. (46) is not appropriate if John already reaches the store during his encounter with Svenja; see Dowty 1977 for discussion.

Let's do a simple example.

(49) a. Sari is laughing.
b. LF: [$_{TP}$ PRES [$_{AspP}$ be-PROG [$_{VP}$ Sari laugh]]]
c. $[\![(49b)]\!]^i = 1$ iff
$\forall w\big[w \in \text{Inert}(i) \rightarrow \exists e[t_i \subset^{<} \tau(e) \;\&\; e \leq w \;\&\; \text{laugh}(e, \text{Sari})]\big]$

Just as it stands, (49c) does not logically entail that any laughing happens in the world w_i (i.e., in the utterance world w_u if this is an unembedded assertion). It only talks about the inertia worlds. However, there is a property of the lexical meaning of *laugh* that permits us to draw further inferences. Laughing events are made up of lots of sub-events which themselves are laughing events, down to very little ones that don't last much more than an instant. Given this, consider a world in $\text{Inert}(\langle w_u, t_u\rangle)$, say w. If (49b) is true in w_u at t_u , it follows that w contains an event of Sari laughing whose run-time includes t_u. Among the subevents of this event, which themselves are events of Sari laughing, there will most likely be one that is early enough and small enough to have transpired by the end of t_u. And since up to the end of t_u, the histories of w and w_u are identical, this small Sari-laughing event in w must have a perfectly matching counterpart in w_u. That's why we infer from (49a) that there is actual laughing at the utterance time.

The only condition under which this would not hold is if the laughing starts right at the beginning of t_u and t_u itself is too short to fit even a minimal laughing event. This would have to be a very short utterance time, shorter than it realistically takes to say *Sari laughs*, so we disregard this possibility. But we will later see a problem with this.

This is the kind of example for which (47) and the simpler entry (45) predict almost identical truth conditions. (47) demands something slightly stronger, namely that moreover the laughing continues at least a little bit beyond the utterance time unless it is interrupted (which means it *would* have continued). So they are not quite equivalent, but the difference is very subtle.

Importantly, however, this almost-equivalence depends on the particular property of the meaning of the VP that we just exploited in our reasoning. Had the VP been *Sari go to the store*, it would have been a very different matter. Events of Sari going to the store are *not* made up of lots of smaller events which each are events of Sari going to the store. They are made up of smaller events which are events of Sari going *towards* the store, but since most of these don't end with Sari at the store, they are not events of Sari going *to* the store. So if we are told that every $w \in \text{Inert}(i)$ contains an event of Sari going to the store which occupies a super-interval of t_i, we cannot infer that Sari goes to the store in w_i. We can merely infer that w_i contains an event that is indistinguishable from those parts of the inertia-worldly trips-to-the-store which fall *before the end of* t_i. In other words, we infer that w_i contains the *beginning* of a Sari-go-to-the-store event, but not necessarily anything more.

The attentive reader may have wondered why we used a past tense example to illustrate the perfective in the previous section, but a present tense example for the progressive in the current section. Indeed, it is in-

cumbent upon us to examine what the theory predicts for every possible combination of a tense and an aspect.

4.4.4 Stativity effects

It is well known that non-stative predicates in the simple present tense have a limited range of felicitous uses. Sentences such as those in (50) are spontaneously judged as odd by speakers of English.

(50) a. #Sari laughs.
b. #Sari wakes up.
c. #Sari goes to the store.

The examples in (50) do not have prominent habitual (generic) readings. Ignore such readings if you can get them anyway. The # judgments apply to an intended episodic reading (describing a single event).

Simple present tense on a non-stative verb is systematically grammatical when the sentence has a generic or habitual interpretation, or when it describes the content of a plan or schedule. We don't worry about these cases here, since they very plausibly involve a covert modal operator of some kind that applies to the VP before any tense or (higher) aspect. (See for example, Copley 2008, Thomas 2014, Rullmann et al. 2022.) That modal operator may itself be an aspect head, or it may create a bigger VP which is a predicate of states. In the latter case, whatever explains the acceptability of stative VPs under present tense will also explain the acceptability of present tense generics/habituals. See below. Some kind of modal analysis might also work for some of the cases in the list, like stage directions and plot summaries, but less plausibly to e.g. the sportscaster style or the historical present.

Let us see what our theory predicts. We see no progressive morphology, but there has to be an aspect head for the sentence to express a proposition, so the aspect must be PFV. With present tense semantically vacuous, we then have LFs and predicted meanings like (51) for (50c).

(51) LF: [$_{TP}$ PRES [$_{AspP}$ PFV [$_{VP}$ Sari go to the store]]]
true at i iff $\exists e[\tau(e) \subseteq t_i \;\&\; e \leq w_i \;\&\;$ go-to$(e,$ Sari, the store$)]$

This says that if *Sari goes to the store* is asserted in w_u at t_u, the assertion is true iff there is a Sari-go-to-the store event in w_u whose run-time is contained within t_u. This is a somewhat implausible scenario, given that trips to the store typically take longer than the production of such a short sentence. One may be tempted to attribute the strangeness of (50c) to this fact. But upon reflection, that doesn't look like the right explanation. We can set up a scenario that eliminates the implausibility. Imagine Sari was already very close to the store, and/or she is on a very fast vehicle …. The judgment about (50c) is not really affected by such manipulations, but we would expect it to be if pragmatic plausibility were all that mattered. And the pragmatic explanation looks even less convincing when we consider the other examples in (50). Waking-up events are very short, if not instantaneous, so such events should have no problem fitting inside the utterance time and (50b) should be just fine. As regards (50a), we have already said that longer laughing events are made up of shorter laughing events. So if Sari laughs for any duration that overlaps with the utterance time, there is probably a laughing event within the utterance time, and (50a) should be fine as well.

Friends of pragmatic approaches like to remind us that the examples in (50) are not ungrammatical. Sentences of this sort are acceptable in a variety of special contexts or registers, such as play-by-play sportscasting, the historical or narrative present, newspaper headlines, stage directions, plot summaries, explicit performatives, …, to name some. It is appealing to say that the essence of (at least some of) these special uses is a pretense that the utterance time is something other than what it is, a pretense that one is speaking at a time closer to the events being reported, at a pretend-

There are few formal semantic analyses of the historical present. An exception is Zucchi 2005.

utterance-time that is earlier and/or longer than one's actual utterance. This may or may not be right. At any rate, it does not directly address the question why (50a–c) are unacceptable *outside* of these special registers or contexts. One seems to need a concomitant assumption that the "ordinary" register involves a different pretense, namely that the utterance time is *shorter* than it actually is, in fact, that it is a mere instant in the technical sense (a singleton of one moment), and hence too short to contain even a getting-up event or a minimal laughing-event.

For the sake of the argument, let's see how it may help to stipulate that t_u is always an instant. To get the desired mileage out of this assumption, we must also sharpen some specifics regarding the lexical meanings of verbs. These assumptions are not uncontroversial, but widely accepted in the literature: None of the VPs in (50) describe events that can possibly have run-times that are instants. Any VP that entails a change of state — whether it is a change that takes time (like getting from some place else to the store) or a virtually "instantaneous" change (like from asleep to awake) — because of that applies only to events whose run-time contains at least two moments (one at which the previous state holds and one at which the result state holds). Likewise, any VP that describes an activity or movement or other happening of some sort (like laughing, oscillating, raining, even sleeping) describes events that may have very short run times but never just a single moment. These assumptions about lexical semantics make the predicted truth-conditions for clauses with PFV (as computed in (51)) impossible to satisfy unless t_i is a proper interval, i.e., not a singleton.

From this perspective there is a straightforward account of what makes stative predicates different. Once we change the VPs in (50) to stative ones, the simple present tense becomes perfectly fine (in every register).

(52) a. Sari is tired.
b. Sari is at home.
c. Sari owns a factory.

Suppose the distinguishing semantic feature is precisely that predicates like *tired*, *at home*, and *own a factory* describe eventualities ("states") whose run-times can be instants. A state of Sari being tired may be long or short, but it is necessarily made up of shorter and shorter sub-states which are also states of Sari being tired. And not only that — it is even made up of such sub-states that occupy a single instant. The latter makes *tired* different from *laugh* or even *move*, which apply to eventualities whose run-times may be infinitesimally short but are still always proper intervals. What does this buy us? It lets us say that the sentences in (52) have the exact same parses as those in (50), with a perfective aspect head, and yet they have truth-conditions which can be satisfied by an instant.

Cf. Bennett & Partee 1978: "We regard a speech act as occurring at a moment of time and understand the assertion as being true at that moment. Accordingly, we are inclined to only use the reportive simple present when the act being described seems to be almost instantaneous and to be occurring at the moment of utterance." See also Dowty 1979.

As Milo Philipps-Brown (pc) pointed out, one worry about this assumption is that it undermines our earlier reasoning about the progressive *Sari is laughing*. There we attributed the intuition that this sentence entails the existence of laughter in w_u to the fact that t_u was long enough to contain a minimal laugh. It is not clear how to resolve this tension. Perhaps we can get out of it by convincing ourselves that we judge the utterance true, after all, if all that actually happens before the interruption is an instant sized beginning of a minimal laugh.

These ideas are common in the literature and go back at least to Taylor 1977. See Filip 2012 for a recent and comprehensive survey.

(53) LF: PRES [PFV [Sari *be* tired]]
meaning: true at i iff $\exists e[\tau(e) \subseteq t \;\&e \leq w_i \;\&\; \text{tired}(e, \text{Sari})]$
lexical entry: $[\![\text{tired}]\!] = \lambda x.\lambda e.\ e$ is a state of x being tired

(Type v must be understood in such a way that D_v includes states in addition to "events" in a narrow sense. Bach 1986 coined the term "eventuality" for this broader sense of "event".)

So, together with the stipulation that the utterance time is treated as an instant, this approach to the stative/non-stative distinction provides an explanation for why stativity is required in the simple present tense. We can also reassure ourselves that present progressives are still expected to be uniformly good even if t_u must be an instant. This is because *be*-PROG places the event in a super-interval of t_i.

Whether or not the assumption that t_u is an instant can ultimately be defended, it is important to be aware that the stativity effect we witness in present tense matrix clauses is replicated perfectly in certain environments which are neither matrix nor (morphologically) present. These environments include the complements of epistemic necessity modals and the infinitival complements of verbs like *believe* and *claim*.

(54) a. Sari must sleep/go to the store.
deontic reading only
b. Sari must be at home/ be sleeping.
epistemic reading okay

(55) Sari claimed to *work/*go to work/be at work/be going to work.

(56) Sari believed Svenja to *sleep/*go to the store/be at home/be sleeping.

We may return to this observation later.

It is often said in this context that progressives pattern with statives in the present tense because progressive VPs *are* stative. This is not literally true on our analysis, because *be*-PROG is an aspect head and AspPs are not predicates of states (or of eventualities of any kind). One might, however, entertain a different analysis on which (at least some of) the operators we are used to calling "aspects" have meanings of type $\langle vt, vt \rangle$. (Another head higher in the structure would then have to be responsible for binding the state argument and introducing the world and time.)

The judgment in (a) presupposes an intended episodic reading for the VP. To the extent that the VP can be read habitually, the epistemic reading becomes available. The point here is that the judgments for the constructions in (54)–(56) are parallel to the judgments for the same VPs in the simple present tense.

4.5 *Formalizing the referential analysis*

Above we stated Partee's proposal as follows:

(35) $[\![\text{PAST } \phi]\!]^i = 1$ iff $[\![\phi]\!]^{\langle w_i, t' \rangle} = 1$, where t' is the contextually salient time before t_i (no truth value defined if there is no such time)

Apart from not being fully compositional, this is a bit vague for us to work with when we consider complex sentences with several occurrences of past tense. Let us therefore make it a little more precise.

Partee suggested that past tense was analogous to a pronoun like *he*. We are used to representing pronouns as variables (see e.g. Heim & Kratzer), so Partee-style tenses too should then have denotations that are sensitive to a variable assignment. So let's make two assumptions: First, each occurrence of PAST and *woll* at LF must carry a numerical subscript, like a pronoun. Second, we add a new type i (for "intervals") and we assume

that variable assignments can assign elements of D_i to object-language variables (numerical indices). Notice that this new type i is not the same as our existing type s (world-interval-pairs), though the second member of an element of D_s is always a member of D_i.

We now write the following entries. (PRES remains vacuous for now and therefore needs no entry.)

(57) $[\![\text{PAST}_n]\!]^{i,g} = \lambda p \in D_{st} :\ g(n) \in D_i \ \&\ g(n) < t_i .\ p(w_i, g(n)) = 1$

(58) $[\![\text{woll}_n]\!]^{i,g} = \lambda p \in D_{st} :\ g(n) \in D_i \ \&\ g(n) > t_i .\ p(w_i, g(n)) = 1$

Let's illustrate how this works in a simple example.

(59) Barbara turned off the stove.
new LF: PAST$_7$ [PFV [Barbara turn off the stove]]

(60) $[\![\text{PAST}_7\ [\ \text{PFV}\ [\ \text{Barbara turn off the stove}\ ]]]\!]^{i,g}$ is defined
iff $7 \in \text{dom}(g)\ \&\ g(7) \in D_i\ \&\ g(7) < t_i$
when defined, $[\![\text{PAST}_7\ [\ \text{PFV}\ [\ \text{Barbara turn off the stove}\ ]]]\!]^{i,g} = 1$
iff $\exists e[\tau(e) \subseteq g(7)\ \&\ e \leq w_i\ \&\ \text{turn-off}(e, \text{Barbara, the stove})]$

If this is a matrix sentence, we evaluate it with respect to the utterance world and time, and we also rely on the utterance context to furnish a suitable assignment (call it g_u), which maps the free variable 7 to a time interval that precedes t_u. Intuitively, this is the salient past interval that the speaker has in mind when making this past-tense claim (also called the "topic time"). So we have:

(61) An utterance u of the LF "PAST$_7$[PFV [Barbara turn off the stove]]" is felicitous only if
g_u is such that $7 \in \text{dom}(g_u)\ \&\ g_u(7) \in D_i\ \&\ g_u(7) < t_u$,
and it is moreover true iff
$\exists e[\tau(e) \subseteq g(7)\ \&\ e \leq w_i\ \&\ \text{turn-off}(e, \text{Barbara, the stove})]$

It will also be useful to clarify how frame adverbials might be treated in a Partee-style approach to tense. A natural idea here is that a frame adverb contributes a further presupposition about the intended topic time. Recall our treatment of frame adverbs as having extensions of type t.

(16) $[\![\text{on February 1, 2001}]\!]^i = 1$ iff t_i is part of February 1, 2001

To make room for this in the LF of a sentence with a Partee-style tense, we want to revise the entries from (57) and (58) and give PAST and *woll* a second argument.

(62) $[\![\text{PAST}_n]\!]^{i,g} = \lambda p \in D_{st} :\ g(n) \in D_i\ \&\ g(n) < t_i\ \&\ p(w_i, g(n)) = 1.$
$\lambda q_{st} . q(w_i, g(n)) = 1$

For example:

(63) Barbara turned off the stove on February 1, 2001.
LF: [PAST $_7$ on February 1, 2001 [PFV [Barbara turn off the stove]]]
uttered felicitously only if
$7 \in \mathrm{dom}(g_u)$ & $g_u(7) \in D_i$ & $g_u(7) < t_u$ & $g_u(7) \subseteq$ Feb 1, 2001,
and uttered truly iff $\exists e[\tau(e) \subseteq g(7)$ & $e \leq w_u$ & turn-off$(e$, Barbara, the stove$)]$

Notice that there is still some room for context-dependency, in that the speaker may be referring to either the whole of Feb 1st or to a proper part of it (e.g. the morning of that day). But the role of context is greatly reduced by the contribution of the adverb.

The revised, 2-place, entry for the future is analogous. When we have frame adverbs with present tense, we also need a non-vacuous semantics for PRES, but this is no different in the Partee-approach than it was in the Priorian approach.

To conclude, let us highlight how the Partee-style, "referential", analysis of tenses differs from the Prior-as-modified-by-Stalnaker-style analysis, and also what they have in common. The essential difference is that Partee-style PAST$_n$ and *woll*$_n$ do not express existential quantification over times, but instead rely on a contextually furnished variable assignment to supply a particular time. In the Partee-approach, the denotations of past and future clauses are always context-dependent; in the modified-Prior approach, they only are if there happens be a silent restrictor together with the existential quantifier. Both approaches assume that the extensions of PAST$_{(n)}$ and *woll*$_{(n)}$ are sensitive to the evaluation time, and both assume that these items *shift* the evaluation time for their complements. The two approaches also agree on the treatment of PRES, which, according to both, does not shift evaluation time and makes no semantic contribution other than a presupposition when there is a frame adverb.

4.6 Further readings

To get more background and second opinions on much of what we've discussed here, you could start with: Kusumoto 2005, von Stechow & Beck 2015.

The sidenotes in this chapter include plenty of initial further reading on specific issues raised.

On the perennial question of the nature of future reference: Bochnak 2019, Cariani 2022.

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