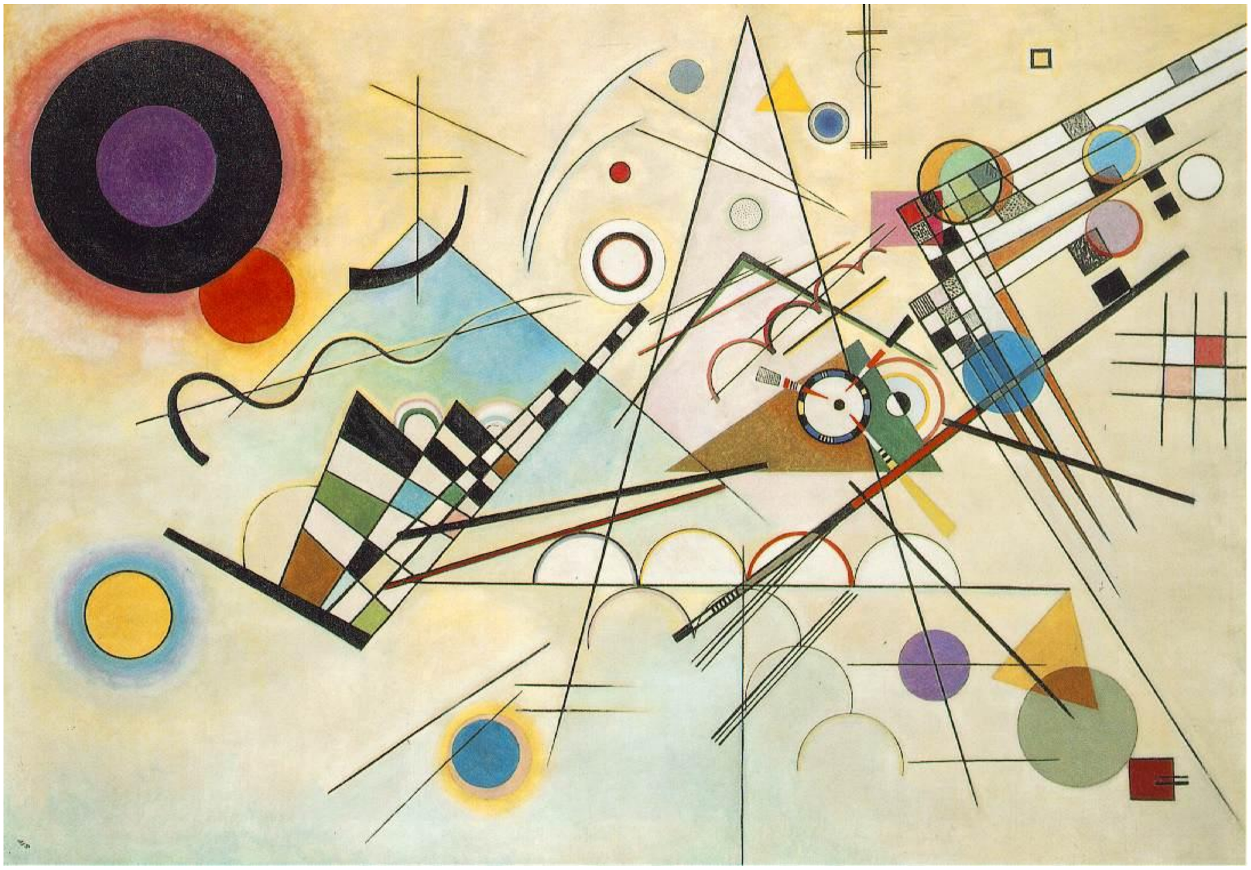


Intensional semantics

KAI VON FINTEL & IRENE HEIM

Kai von Fintel & Irene Heim

Intensional semantics

Massachusetts Institute of Technology • Spring 2021

About these lecture notes

These lecture notes have been evolving for many years now, starting with some notes from the early 1990s by Angelika Kratzer, Irene Heim, and Kai von Fintel, which have since been modified and expanded many times by Irene and/or Kai, with feedback and contributions from colleagues and students.

We encourage the use of these notes in courses at other institutions. Of course, you need to give full credit to the authors and you may not use the notes for any commercial purposes. If you use the notes, we would like to be notified and we would very much appreciate any comments, criticism, and advice on these materials.

Cite as follows:
von Fintel, Kai & Irene Heim. 1997–2021. Intensional semantics. MIT.

GitHub repository with the current & up-to-date development version:
https://github.com/fintelkai/fintel-heim-intensional-notes

Acknowledgments

Thanks to generations of students in our classes. Thanks to colleagues who have used these notes in their classes. Here's an almost certainly incomplete list of people who have helped us in some way or other with these notes (telling us about typos, asking questions, pointing out issues, etc.): Paul Elbourne, Nick Fleisher, Graeme Forbes, Adam Goodkind, Aaron Jacobs, Magda Kaufmann, Ezra Keshet, Angelika Kratzer, Manfred Kupffer, Eric Mack, Yuliya Manyakina, Liz Smeets, Vesela Tihomirova Simeonova, Ede Zimmermann.

Some advice

1. These notes presuppose familiarity with the material, concepts, and notation of the Heim & Kratzer textbook.
2. There are numerous exercises throughout the notes. It is highly recommended to do all of them and it is certainly necessary to do so if you at all anticipate doing semantics-related work in the future.
3. The notes are designed to go along with explanatory lectures. You should ask questions and make comments as you work through the notes.
4. While most of the object language examples are from English, semantics is a cross-linguistic enterprise. Students (and teachers) should bring in a cross-linguistic perspective throughout.
5. Students with semantic ambitions should also at an early point start reading supplementary material (as for example listed at the end of each chapter of these notes).
6. Prospective semanticists may start thinking about how *they* would teach this material.
7. For more advice, see http://kaivonfintel.org/prerequisites/.

Not a spectator sport

"As someone once said, Mathematics is not a spectator sport. To learn and understand Mathematics you have to get stuck in and get your hands dirty. You have to do the calculations, manipulations, and proofs yourself, not just read the stuff and pretend you understand it."

Harold Simmons, An introduction to category theory, 2011, Cambridge University Press

The same goes for formal semantics.

Contents

PART I THE FIFTH DIMENSION

1 Beginnings 3

2 Attitudes, conditionals, modals 21

3 Restricting and ordering 47

PART II ESCAPING OPACITY

4 Specificity and Transparency 71

5 The Third Reading 89

PART III TIME

6 Beginnings of tense and aspect 103

7 Embedded tenses 121

PART IV QUESTIONS

8 Interrogative clauses 149

BIBLIOGRAPHY

PART I

The Fifth Dimension

Hypotheticals, 'imaginaries', conditionals, the syntax of counter-factuality and contingency may well be the generative centres of human speech. […] Language is the main instrument of man's refusal to accept the world as it is.

George Steiner, *After Babel* (1975), p.226 & p.228

1 *Beginnings*

1.1 Displacement 3
1.2 An intensional semantics in 10 easy steps 5
1.2.1 Laying the foundations 6
1.2.2 Intensional operators 10
1.3 Comments and complications 14
1.3.1 Intensions all the way? 14
1.3.2 Why talk about other worlds? 15
1.3.3 The worlds of Sherlock Holmes 16
1.3.4 What's next and a general pattern 17
1.4 *Issues with an informal meta-language 18
1.5 Further readings 19

1.1 Displacement

Hockett 1960 presented a list of DESIGN FEATURES OF HUMAN LANGUAGE, which continues to play a role in current discussions of animal communication and the evolution of language. One of the design features is DISPLACEMENT: human language is not restricted to discourse about the *actual here and now*. We use language to speculate about how things might have been different, about what would happen if we don't find dinner soon; we wonder what our friend thinks the world is like; we may want to tell our boss what it would take for us not to resign.

Hockett cites this passage from Childe 1936: p.30 to illustrate the utility of displacement:

"[…] parents can, with the aid of language, instruct their offspring how to deal with situations which cannot conveniently be illustrated by actual concrete examples. The child need not wait till a bear attacks the family to learn how to avoid it. Instruction by example alone in such a case is liable to be fatal to some of the pupils. Language, however, enables the elders to forewarn the young of the danger while it is absent, and then demonstrate the appropriate course of action."

How does natural language untie us from the actual here and now? One degree of freedom is given by the ability to name entities and refer to them even if they are not where we are when we speak:

(1) Rahel is in Hamburg.

This kind of displacement is not something we will explore here. We'll take it for granted.

Consider a sentence with no names of absent entities in it:

(2) It is snowing. [said in Cambridge]

On its own, (2) makes a claim about what is happening right now here in Cambridge. But there are devices at our disposal that can be added to (2), resulting in claims about snow in displaced situations. Displacement can occur in the TEMPORAL dimension:

(3) At noon yesterday, it was snowing.

This sentence makes a claim not about snow now but about snow at noon yesterday, a different time from now. We will look at temporal semantics in Part 2 of this book.

In Part 1 of this book, we will focus on what might be called the MODAL dimension. Here's an example of modal displacement:

(4) COUNTERFACTUAL CONDITIONAL
If the storm system hadn't been deflected by the jet stream, it would have been snowing.

This sentence makes a claim not about snow in the actual world but about snow in the world as it would have been if the storm system hadn't been deflected by the jet stream, a world distinct from the actual one (where the system did not hit us), a merely POSSIBLE WORLD.

Natural language abounds in modal constructions (see Kratzer 1981). Here are some other examples:

(5) MODAL AUXILIARIES
It may be snowing.

(6) MODAL ADVERBS
Possibly, it will snow tomorrow.

(7) PROPOSITIONAL ATTITUDES
Miriam believes that it is snowing.

(8) EVIDENTIALS
It appears that it is snowing.

(9) HABITUALS
Ellen smokes.

(10) GENERICS
Bears like honey.

(11) IMPERATIVES
Get your snow shovels ready!

(12) SUFFICIENCY AND EXCESS
Linda is old enough to watch The X-Files.
Klara is too expensive to hire.

(13) INFINITIVAL RELATIVES
I know $\left\{ \begin{array}{l} \text{an} \\ \text{the} \end{array} \right\}$ expert to talk to.

The terms MODAL and MODALITY descend from the Latin *modus*, "way", and are ancient terms pertaining to the way a proposition holds, necessarily, contingently, etc. For more on the history, see Auwera & Aguilar 2015.

If we wanted to be more fanciful, we could call this the Fifth Dimension (after the four dimensions of space and time). Take a look at the original intro to *The Twilight Zone*: https://www.youtube.com/watch?v=vB1Ot9MEOOs

Exercise 1.1 *Collect some examples of modal displacement in a language other than English (whether spoken by you or someone else you know). These can serve as a touchstone of understanding as we proceed.* □

In this chapter, we will put in place the basic framework of INTENSIONAL SEMANTICS, the kind of semantics that models displacement of the point of evaluation in temporal and modal dimensions. To do this, we will start with one rather special example of modal displacement:

(14) In the world of Sherlock Holmes, a detective lives at 221B Baker Street.

(14) doesn't claim that a detective lives at 221B Baker Street in the actual world (presumably a false claim), but that in the world as it is described in the Sherlock Holmes stories of Sir Arthur Conan Doyle, a detective lives at 221B Baker Street (a true claim, of course). We choose this example rather than one of the more well-studied displacement constructions because we want to focus on conceptual and technical matters for now before we get distracted by many interesting complications.

The questions we want to answer are: How does natural language achieve this feat of modal displacement? How do we manage to make claims about other possible worlds? And why would we want to?

To make displacement possible and compositionally tractable, we need meanings of natural language expressions, and of sentences in particular, to be displaceable in the first place. They need to be "portable", so to speak, able to make claims about more than just the actual here and now. And we need other natural language expressions that take that portable meaning and apply it to some situation other than the actual here and now. That is what intensionality is all about.

The basic idea of the account we'll develop is this:

- expressions are assigned their semantic values relative to a possible world;
- in particular, sentences have truth-values in possible worlds;
- in the absence of modal displacement, we evaluate sentences with respect to the "actual" world, the world in which we are speaking;
- modal displacement changes the world of evaluation;
- displacement is effected by special operators, whose semantics is our primary concern here.

A terminological note: we will call the sister of the intensional operator its PREJACENT, a useful term introduced by our medieval colleagues.

1.2 An intensional semantics in 10 easy steps

1.2.1 *Laying the foundations*

Step 1: Possible worlds

Our first step is to introduce possible worlds. This is not the place to discuss the metaphysics of possible worlds in any depth. Instead, we will just start working with them and see what they can do for us. Basically, a possible world is a way that things might have been. In the actual world, there are two coffee mugs on my desk, but there could have been more or less. So, there is a possible world — albeit a rather bizarre one — where there are 17 coffee mugs on my desk. We join Heim & Kratzer in adducing this quote from Lewis (1986: 1f.):

> The world we live in is a very inclusive thing. Every stick and every stone you have ever seen is part of it. And so are you and I. And so are the planet Earth, the solar system, the entire Milky Way, the remote galaxies we see through telescopes, and (if there are such things) all the bits of empty space between the stars and galaxies. There is nothing so far away from us as not to be part of our world. Anything at any distance at all is to be included. Likewise the world is inclusive in time. No long-gone ancient Romans, no long-gone pterodactyls, no long-gone primordial clouds of plasma are too far in the past, nor are the dead dark stars too far in the future, to be part of the same world. …
>
> The way things are, at its most inclusive, means the way the entire world is. But things might have been different, in ever so many ways. This book of mine might have been finished on schedule. Or, had I not been such a commonsensical chap, I might be defending not only a plurality of possible worlds, but also a plurality of impossible worlds, whereof you speak truly by contradicting yourself. Or I might not have existed at all — neither myself, nor any counterparts of me. Or there might never have been any people. Or the physical constants might have had somewhat different values, incompatible with the emergence of life. Or there might have been altogether different laws of nature; and instead of electrons and quarks, there might have been alien particles, without charge or mass or spin but with alien physical properties that nothing in this world shares. There are ever so many ways that a world might be: and one of these many ways is the way that this world is.

Previously, our "metaphysical inventory" included a domain of entities and a set of two truth-values and increasingly complex functions between entities, truth-values, and functions thereof. Now, we will add possible worlds to the inventory. Let's assume we are given a set W, the set of all possible worlds, which is a vast space since there are so many ways that things might have been different from the way they are. Each world has as among its parts entities like you and me and these coffee mugs. Some of them may not exist in other possible worlds. So, strictly speaking each possible worlds has its own, possibly distinctive, domain of entities. What we will use in our system, however, will be the grand union of all these world-specific domains of entities. We will use D to stand for the set of all possible individuals.

It's possible that your previous inventory also included pluralities, events, and/or degrees. We're just adding to the menagerie now. Questions arise about what the limits are and whether the inventory is universal. For some discussion, see the manuscript "A typology of semantic entities" (Rett 2019).

Among the many possible worlds that there are — according to Lewis, there is a veritable plenitude of them — is the world as it is described in the Sherlock Holmes stories by Sir Arthur Conan Doyle. In that world, there is a famous detective Sherlock Holmes, who lives at 221B Baker Street in London and has a trusted sidekick named Dr. Watson. Our sentence *In the world of Sherlock Holmes, a detective lives at 221B Baker Street* displaces the claim that a famous detective lives at 221B Baker Street from the actual world to the world as described in the Sherlock Holmes stories. In other words, the following holds (until we revise it):

(15) The sentence *In the world of Sherlock Holmes, a detective lives at 221B Baker Street* is true in a world w iff the sentence *a detective lives at 221B Baker Street* is true in the world as it is described in the Sherlock Holmes stories.

What this suggests is that we need to make space in our system for having devices that control in what world a claim is evaluated. This is what we will do now.

Step 2: The evaluation world parameter

Recall from H&K that we were working with a semantic interpretation function that was relativized to an assignment function g, which was needed to take care of pronouns, traces, variables, etc. From now on, we will relativize the semantic values in our system to possible worlds as well. What this means is that from now on, our interpretation function will have two superscripts: a world w and an assignment g: $[\![\cdot]\!]^{w,g}$. For a given expression ϕ, we call $[\![\phi]\!]^{w,g}$ the EXTENSION of ϕ at w (relative to g). The extension of an expression at a world is the central notion of an intensional semantics.

So, the prejacent embedded in (14) will have its truth-conditions described as follows:

(16) For any world w and assignment function g:
$[\![$a famous detective lives at 221B Baker Street$]\!]^{w,g} = 1$
iff a famous detective lives at 221B Baker Street in world w.

It is customary to refer to the world for which we are calculating the extension of a given expression as the EVALUATION WORLD. In the absence of any shifting devices, we would normally evaluate a sentence in the actual world. But then there are shifting devices such as our *in the world of Sherlock Holmes*. We will soon see how they work. But first some more pedestrian steps: adding lexical entries and composition principles that are formulated relative to a possible world. This will allow us to derive the truth-conditions as stated in (16) in a compositional manner.

Recall from H&K, pp.22f, that what's inside the interpretation brackets is a mention of an object language expression. They make this clear by bold-facing all object language expressions inside interpretation brackets. In this book, we will follow common practice in the field and not use a special typographic distinction, but let it be understood that what is interpreted are object language expressions.

Step 3: Lexical entries

Among our lexical items, we can distinguish between items which have a WORLD-DEPENDENT semantic value and those that are world-independent. Let's start with the entry for *famous*:

(17) For any $w \in W$ and any assignment function g:
$[\![\text{famous}]\!]^{w,g} = \lambda x \in D.\ x$ is famous in w.

Of course, "$\lambda x \in D. \ \ldots$" is short for "$\lambda x\colon\ x \in D. \ \ldots$". Get used to semanticists condensing their notation whenever convenient! A further step of condensation is taken below: "$\lambda x\colon\ x \in D_e. \ \ldots$" becomes "$\lambda x_e. \ \ldots$".

Always make sure that you actually understand what the notation means. Here, for example, we are saying that the semantic value of the word *famous* with respect to a given possible world w and a variable assignment g is that function that is defined for an argument x only if x is a member of the domain of individuals and that, if it is defined, yields the truth-value 1 if and only if x is famous in w. (17) does *not* mean that the function maps x to "x is famous in w", which would be very weird: mapping an individual to a meta-language statement!

A couple more predicates:

(18) a. $[\![\text{detective}]\!]^{w,g} = \lambda x \in D.\ x$ is a detective in w.
b. $[\![\text{lives-at}]\!]^{w,g} = \lambda x \in D.\ \lambda y \in D.\ y$ lives-at x in w.

(18) shows another customary condensation of notation: the universal quantification over evaluation worlds and assignment functions is left implicit and on the face of it, the parameters seem unbound. But they are.

The set of detectives will obviously differ from world to world, and so will the set of famous individuals and the set of pairs where the first element lives at the second element.

Other items have semantic values which do not differ from world to world. The most important such items are certain "logical" expressions, such as truth-functional connectives and determiners:

Again, note the ruthless condensation of the notation in (c) and (d): variables are subscripted with the type of the domain that their values are constrained to come from.

(19) a. $[\![\text{and}]\!]^{w,g} = \lambda u \in D_t.\ \lambda v \in D_t.\ u = v = 1.$

b. $[\![\text{the}]\!]^{w,g} = \lambda f \in D_{\langle e,t\rangle}:\ \exists! x[f(x) = 1].$ the y such that $f(y) = 1.$

c. $[\![\text{every}]\!]^{w,g} = \lambda f_{\langle e,t\rangle}.\ \lambda h_{\langle e,t\rangle}.\ \forall x_e:\ f(x) = 1 \rightarrow h(x) = 1.$

d. $[\![\text{a/some}]\!]^{w,g} = \lambda f_{\langle e,t\rangle}.\ \lambda h_{\langle e,t\rangle}.\ \exists x_e:\ f(x) = 1\ \&\ h(x) = 1.$

Note that there is no occurrence of w on the right-hand side of the entries in (19). That's the tell-tale sign of the world-independence of the semantics of these items.

We will also assume that proper names have world-independent semantic values, that is, they refer to the same individual in any possible world.

(20) a. $[\![\text{Noam Chomsky}]\!]^{w,g}$ = Noam Chomsky.

b. $[\![\text{Sherlock Holmes}]\!]^{w,g}$ = Sherlock Holmes.

c. $[\![\text{221B Baker Street}]\!]^{w,g}$ = 221B Baker Street.

Step 4: Composition principles

The old rules of Functional Application, Predicate Modification, and λ-Abstraction can be retained almost intact. We just need to modify them by adding world-superscripts to the interpretation function. For example:

(21) Functional Application (FA)
If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any world w and assignment g: if $[\![\beta]\!]^{w,g}$ is a function whose domain contains $[\![\gamma]\!]^{w,g}$, then $[\![\alpha]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\gamma]\!]^{w,g})$.

The rule simply passes the world parameter down.

Exercise 1.2 *Formulate the new versions of Predicate Modification and λ-Abstraction.* □

Step 5: Truth

We will want to connect our semantic system to the notion of the truth of an utterance. We first adopt the "Appropriateness Condition" from Heim & Kratzer (p.243):

(22) Appropriateness Condition
A context c is *appropriate* for an LF ϕ only if c determines a variable assignment g_c whose domain includes every index which has a free occurrence in ϕ.

We then intensionalize Heim & Kratzer's definition of truth and falsity of utterances:

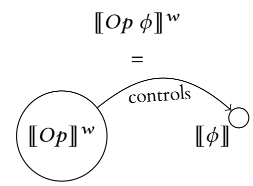
(23) Truth and Falsity Conditions for Utterances
An utterance of a sentence ϕ in a context c in a possible world w is *true* iff $[\![\phi]\!]^{w,g_c} = 1$ and *false* if $[\![\phi]\!]^{w,g_c} = 0$.

Exercise 1.3 *Compute under what conditions an utterance in possible world* w_7 *(which may or may not be the one we are all living in) of the sentence* a famous detective lives at 221B Baker Street *is true.* □

1.2.2 Intensional operators

So far we have merely "redecorated" the system inherited from Heim & Kratzer. We have introduced possible worlds into our inventory, our lexical entries and our old composition principles. But with the tools we have now, all we can do so far is to keep track of the world in which we evaluate the semantic value of an expression, complex or lexical. We will get real mileage once we introduce intensional operators which are capable of shifting the world parameter. We mentioned that there are a number of devices for modal displacement. As advertised, for now, we will just focus on a very particular one: the expression *in the world of Sherlock Holmes*. We will assume, as seems reasonable, that this expression is a sentence-modifier both syntactically and semantically.

Here's an illustration of the basic idea we're trying to implement:



Step 6: A syncategorematic entry

We begin with a heuristic step. We want to derive something like the following truth-conditions for our sentence:

(24) [\![in the world of Sherlock Holmes,
a famous detective lives at 221B Baker Street]\!]$^{w,g} = 1$
iff the world w' as it is described in the Sherlock Holmes stories is such that there exists a famous detective in w' who lives at 221B Baker Street in w'.

We would get this if in general we had this rule for *in the world of Sherlock Holmes*:

(25) For any sentence ϕ, any world w, and any assignment g:
[\![in the world of Sherlock Holmes, ϕ]\!]$^{w,g} = 1$
iff the world w' as it is described in the Sherlock Holmes stories is such that $[\![\phi]\!]^{w',g} = 1$.

This is a so-called syncategorematic treatment of the meaning of this expression. Instead of giving an explicit semantic value to the expression, we specify what effect it has on the meaning of a complex expression that contains it. In (25), we do not compute the meaning for *in the world of Sherlock Holmes, ϕ* from the combination of the meanings of its parts, since *in the world of Sherlock Holmes* is not given a separate meaning, but in effect triggers a special composition principle. This format is very common in modal logic systems, which usually give a syncategorematic semantics for the two classic modal operators (the necessity operator $\Box$ and the possibility operator $\Diamond$). When one only has a few closed class expressions to deal with that may shift the world parameter, employing syncategorematic entries is a reasonable strategy. But we are facing a multitude of displacement devices. We will therefore need to make our system more modular.

The diamond $\Diamond$ symbol for possibility is due to C.I. Lewis, first introduced in Lewis & Langford 1932, but he made no use of a symbol for the dual combination $\neg\Diamond\neg$. The dual symbol $\Box$ ("Box") was later devised by F.B. Fitch and first appeared in print in 1946 in a paper by his doctoral student Barcan (1946). See footnote 425 of Hughes & Cresswell 1968. Another notation one finds is L for necessity and M for possibility, the latter from the German *möglich* 'possible'.

We want to give *in the world of Sherlock Holmes* its own meaning and combine that meaning with that of its prejacent by a general composition principle. The Fregean slogan we adopted says that all composition is function application (modulo the need for λ-abstraction and the possible need for predicate modification).

See Heim & Kratzer, Section 4.3, pp. 63–72 for a reminder about the status of predicate modification.

What we will want to do is to make (24) be the result of functional application. But we can immediately see that it cannot be the result of our usual rule of functional application, since that would feed to *in the world of Sherlock Holmes* the semantic value of *a famous detective lives in 221B Baker Street* in w, which would be a particular truth-value, 1 if a famous detective lives at 221B Baker Street in w and 0 if there doesn't. And whatever the semantics of *in the world of Sherlock Holmes* is, it is certainly *not* a truth-functional operator.

We need to feed something else to *in the world of Sherlock Holmes*. At the same time, we want the operator to be able to shift the evaluation world of its prejacent. Can we do this?

Exercise 1.4 *How would you show that* in the world of Sherlock Holmes *is not a truth-functional operator?* $\Box$

Step 7: Intensions

We will define a richer notion of semantic value, the INTENSION of an expression. This will be a function from possible worlds to the extension of the expression in that world. The intension of a sentence can be applied to any world and give the truth-value of the sentence in that world. Intensional operators take the intension of their prejacent as their argument, that is we will feed the intension of the embedded sentence to the shifting operator. The operator will use that intension and apply it to the world it wants the evaluation to happen in. Voilà.

Now let's spell that account out. Our system actually provides us with two kinds of meanings. For any expression α, we have $[\![\alpha]\!]^{w,g}$, the semantic value of α in w, also known as the EXTENSION of α in w. But we can also calculate $\lambda w.[\![\alpha]\!]^{w,g}$, the function that assigns to any world w the extension of α in that world. This is usually called the INTENSION of α. We will sometimes use an abbreviatory notation for the intension of α:

Just like H&K, we make no claim that the semantic values that are attributed to expressions in our framework fully capture what is informally meant by "meaning". But certainly, intensions come closer to "meaning" than extensions.

(26) $[\![\alpha]\!]^{g}_{\text{¢}} := \lambda w.[\![\alpha]\!]^{w,g}$

It should be immediately obvious that since the definition of intension abstracts over the evaluation world, intensions are not world-dependent.

Note that strictly speaking, it now makes no sense anymore to speak of "*the* semantic value" of an expression α. What we have is a semantic system that allows us to calculate extensions (for a given possible world w) as well as intensions for all (interpretable) expressions. We will see that when α occurs in a particular bigger tree, it will always be determinate which of the two "semantic values" of α is the one that enters into the compositional semantics. So, that one — whichever one it is, the extension or the intension of α — might then be called "*the* semantic value of α in the tree β".

It should be noted that the terminology of EXTENSION vs. INTENSION is time-honored but that the possible worlds interpretation thereof is more recent. The technical notion we are using is certainly less rich a notion of meaning than traditionally assumed. (For example, Frege's "modes of presentation" are not obviously captured by this possible worlds implementation of extension/intension.)

The notation with the subscripted cent-sign comes from Montague Grammar. See Dowty, Wall & Peters 1981: p. 147.

The definition here is simplified, in that it glosses over the fact that some expressions, in particular those that contain PRESUPPOSITION TRIGGERS, may fail to have an extension in certain worlds. In such a case, the intension has no extension to map such a world to. Therefore, the intension will have to be a partial function. So, the official, more "pedantic", definition will have to be as follows: $[\![\alpha]\!]^{g}_{\text{¢}} := \lambda w\colon\ \alpha \in \mathrm{dom}([\![\,]\!]^{w,g}).[\![\alpha]\!]^{w,g}$.

The Port-Royal logicians distinguished EXTENSION from COMPREHENSION. Leibniz preferred the term INTENSION rather than COMPREHENSION. The notion probably goes back even further. See Spencer 1971 for some notes on this. The possible worlds interpretation is due to Carnap 1947.

Step 8: Semantic types and semantic domains

If we want to be able to feed the intensions to lexical items like *in the world of Sherlock Holmes*, we need to have the appropriate types in our system.

Recall that W is the set of all possible worlds. And recall that D is the set of all POSSIBLE INDIVIDUALS and thus contains all individuals existing in the actual world *plus* all individuals existing in any of the merely possible worlds.

We now expand the set of semantic types, to add intensions. Intensions are functions from possible worlds to all kinds of extensions. So, basically we want to add for any kind of extension we have in our system, a corresponding kind of intension, a function from possible worlds to that kind of extension.

We add a new clause, (27c), to the definition of semantic types:

(27) SEMANTIC TYPES

a. e and t are semantic types.
b. If σ and τ are semantic types, then $\langle\sigma,\tau\rangle$ is a semantic type.
c. If σ is a semantic type, then $\langle s,\sigma\rangle$ is a semantic type.
d. Nothing else is a semantic type.

This is as good a place as any to add a sermon about the notation for types. Functional types like $\langle e,t\rangle$ are written with angled brackets because of the mathematical connection between functions and ordered pairs. The basic types like e and t, however are not written with angled brackets, since they are not of the type of functions. Please do not make this mistake.

We also add a fourth clause to the previous definition of semantic domains:

(28) Semantic Domains

a. $D_e = D$, the set of all possible individuals

b. $D_t = \{0, 1\}$, the set of truth-values

c. If σ and τ are semantic types, then $D_{\langle\sigma,\tau\rangle}$ is the set of all functions from D_σ to D_τ.

d. Intensions: If σ is a type, then $D_{\langle s,\sigma\rangle}$ is the set of all functions from W to D_σ.

Clause (d) is the addition to our previous system of types. The functions of the schematic type $\langle s, \ldots\rangle$ are intensions. Here are some examples of intensions:

Note a curious feature of this set-up: there is no type s and no associated domain. This corresponds to the assumption that there are no expressions of English that take as their extension a possible world, that is, there are no pronouns or names referring to possible worlds. We will actually question this assumption in a later chapter. For now, we will stay with this more conventional set-up.

- The intensions of sentences are of type $\langle s, t\rangle$, functions from possible worlds to truth values. These are usually called propositions. Note that if the function is total, then we can see the sentence as picking out a set of possible worlds, those in which the sentence is true. More often than not, however, propositions will be partial functions from worlds to truth-values, that is functions that fail to map certain possible worlds into either truth-value. This will be the case when the sentence contains a presupposition trigger, such as *the*. The famous sentence *The King of France is bald* has an intension that (at least in the analysis sketched in Heim & Kratzer) is undefined for any world where there fails to be a unique King of France.
- The intensions of one-place predicates are of type $\langle s, \langle e, t\rangle\rangle$, functions from worlds to set of individuals. These are usually called properties.
- The intensions of expressions of type e are of type $\langle s, e\rangle$, functions from worlds to individuals. These are usually called individual concepts.

Step 9: A lexical entry for a shifter

We are ready to formulate the lexical entry for *in the world of Sherlock Holmes*:

(29) $[\![\text{in the world of Sherlock Holmes}]\!]^{w,g} =$
$\lambda p_{\langle s,t\rangle}$. the world w' as it is described in the Sherlock Holmes stories is such that $p(w') = 1$.

This is not yet the final semantics, see Section 1.3 for complications. One complication we will not even start to discuss is that obviously it is not a necessity that there are Sherlock Holmes stories in the first place and that the use of this operator *presupposes* that they exist; so a more fully explicit semantics would need to build in that presuppositional component. Also, note again the condensed notation: "$\lambda p_{\langle s,t\rangle}. \ldots$" stands for the fully official "$\lambda p\colon p \in D_{\langle s,t\rangle}. \ldots$".

That is, *in the world of Sherlock Holmes* expects as its argument a function of type $\langle s, t\rangle$, a proposition. It yields the truth-value 1 iff the proposition is true in the world as it is described in the Sherlock Holmes stories.

All that's left to do now is to provide *in the world of Sherlock Holmes* with a proposition as its argument. This is the job of a new composition principle.

Step 10: Intensional Functional Application

We add the new rule of Intensional Functional Application.

(30) Intensional Functional Application (IFA)
If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any world w and assignment g: if $[\![\beta]\!]^{w,g}$ is a function whose domain contains $[\![\gamma]\!]^{g}_{¢}$, then $[\![\alpha]\!]^{w,g} = [\![\beta]\!]^{w,g}([\![\gamma]\!]^{g}_{¢})$.

This is the crucial move. It makes space for expressions that want to take the intension of their sister as their argument and do stuff to it. Now, everything is in place. Given (29), the semantic argument of *in the world of Sherlock Holmes* will not be a truth-value but a proposition. And thus, *in the world of Sherlock Holmes* will be able to check the truth-value of its prejacent in various possible worlds. To see in practice that we have all we need, please do the following exercise.

Exercise 1.5 *Calculate the conditions under which an utterance in a given possible world* w_7 *of the sentence* in the world of the Sherlock Holmes stories, a famous detective lives at 221B Baker Street *is true.* □

Exercise 1.6 *What in our system prevents us from computing the extension of* Watson is slow*, for example, by applying the intension of* slow *to the extension of* Watson*? What in our system prevents us from computing the extension of* Watson is slow *by applying the intension of* slow *to the intension of* Watson*?* □

Exercise 1.7 *What is wrong with the following equation:*

(31) $(\lambda x.\ x \text{ is slow in } w)\ (\text{Watson}) = \text{Watson is slow in } w\ ?$

[Hint: there is nothing wrong with the following:

(32) $(\lambda x.\ x \text{ is slow in } w)\ (\text{Watson}) = 1 \text{ iff Watson is slow in } w.$ *]* □

Please think about this exercise before looking at Section 1.4, which explores this issue.

1.3 Comments and complications

1.3.1 Intensions all the way?

We have seen that to adequately deal with expressions like *in the world of Sherlock Holmes*, we need an intensional semantics, one that gives us access to the extensions of expressions across the multitude of possible worlds. At the same time, we have kept the semantics for items like *and*, *every*, and *a* unchanged and extensional. This is not the only way one can set up an intensional semantics. The following exercise demonstrates this.

Exercise 1.8 *Consider the following "intensional" meaning for* and*:*

(33) $[\![and]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \lambda q_{\langle s,t\rangle}.\ p(w) = q(w) = 1.$

With this semantics, the conjunction and *would operate on the intensions of the two conjoined sentences. In any possible world w, the complex sentence will be true iff the component propositions are both true of that world.*

Compute the truth-conditions of the sentence In the world of Sherlock Holmes, Holmes is quick and Watson is slow *both with the extensional meaning for* and *given earlier and the intensional meaning given here. Is there any difference in the results?* □

There are then at least two ways one could develop an intensional system.

(i) We could "generalize to the worst case" and make the semantics deliver intensions as *the* semantic value of an expression. Such systems are common in the literature (see Lewis 1970, Cresswell 1973).

(ii) We could maintain much of the extensional semantics we have developed so far and extend it conservatively so as to account for non-extensional contexts.

We have chosen to pursue (ii) over (i), because it allows us to keep the semantics of extensional expressions simpler. The philosophy we follow is that we will only move to the intensional sub-machinery when triggered by an expression that creates a non-extensional context. As the exercise just showed, this might be more a matter of taste than a deep scientific decision. We will turn to questions of expressive power later in this book.

1.3.2 Why talk about other worlds?

Why would natural language bother having such elaborate mechanisms to talk about other possible worlds? While having devices for spatial and temporal displacement (talking about Hamburg or what happened yesterday) seems eminently reasonable, talking about worlds other than the actual world seems only suitable for poets and the like. So, why?

The solution to this puzzle lies in a fact that our current semantics of the shifter *in the world of Sherlock Holmes* does not yet accurately capture: modal sentences have empirical content, they make CONTINGENT claims, claims that are true or false depending on the circumstances in the actual world.

Our example sentence *In the world of Sherlock Holmes, a famous detective lives at 221B Baker Street* is true in this world but it could easily have been false. There is no reason why Sir Arthur Conan Doyle could not have decided to locate Holmes' abode on Abbey Road.

To see that our semantics does not yet capture this fact, notice that in the semantics we gave for *in the world of Sherlock Holmes*:

(34) $[\![\text{in the world of Sherlock Holmes}]\!]^{w,g} =$
$\lambda p_{\langle s,t \rangle}$. the world w' as it is described in the Sherlock Holmes stories is such that $p(w') = 1$.

there is no occurrence of w on the right hand side. This means that the truth-conditions for sentences with this shifter would be world-independent.

In other words, they are predicted to make non-contingent claims that are either true no-matter-what or false no-matter-what. This needs to be fixed.

The fix is obvious: what matters to the truth of our sentence is the content of the Sherlock Holmes stories as they are in the evaluation world. So, we actually need the following semantics for our shifter:

(35) $[\![$in the world of Sherlock Holmes$]\!]^{w,g}$ =
$\lambda p_{\langle s,t \rangle}$. the world w' as it is described in the Sherlock Holmes stories *in* w is such that $p(w') = 1$.

We see now that sentences with this shifter do make a claim about the evaluation world: namely, that the Sherlock Holmes stories as they are in the evaluation world describe a world in which such-and-such is true. So, what is happening is that although it appears at first as if modal statements concern other possible worlds and thus couldn't really be very informative, they actually only talk about *certain* possible worlds, those that stand in some relation to what is going on at the ground level in the actual world. As a crude analogy, consider:

(36) My grandmother is sick.

At one level this is a claim about my grandmother. But it is also a claim about me: namely that I have a grandmother who is sick. Thus it is with modal statements. They talk about possible worlds that stand in a certain relation to the actual world and thus they make claims about the actual world, albeit slightly indirectly.

1.3.3 The worlds of Sherlock Holmes

So far, we have played along with colloquial usage in talking of *the* world of Sherlock Holmes. But it is important to realize that this is sloppy talk. Lewis (1978) writes:

> [I]t will not do to follow ordinary language to the extent of supposing that we can somehow single out a single one of the worlds [as the one described by the stories]. Is the world of Sherlock Holmes a world where Holmes has an even or an odd number of hairs on his head at the moment when he first meets Watson? What is Inspector Lestrade's blood type? It is absurd to suppose that these questions about the world of Sherlock Holmes have answers. The best explanation of that is that the world*s* of Sherlock Holmes are plural, and the questions have different answers at different ones.

The usual move at this point is to talk about the set of worlds "COMPATIBLE WITH the (content of) Sherlock Holmes stories in w". We imagine that we ask of each possible world whether what is going on in it is compatible with the stories as they were written in our world. Worlds where Holmes

An equivalent way to phrase this is that we are looking at the worlds *not excluded by the (content of the) stories*.

lives on Abbey Road are not compatible. Some worlds where he lives at 221B Baker Street are compatible (again not all, because in some such worlds he is not a famous detective but an obscure violinist). Among the worlds compatible with the stories are ones where he has an even number of hairs on his head at the moment when he first meets Watson and there are others where he has an odd number of hairs at that moment.

What the operator *in the world of Sherlock Holmes* expresses is that its complement is true throughout the worlds compatible with the stories. In other words, the operator *universally quantifies* over the compatible worlds. Our next iteration of the semantics for the operator is therefore this:

(37) $[\![\text{in the world of Sherlock Holmes}]\!]^{w,g} =$
$\lambda p_{\langle s,t \rangle}.\ \forall w'$ compatible with the Sherlock Holmes stories *in* w:
$p(w') = 1.$

At a very abstract level, the way we parse sentences of the form *in the world of Sherlock Holmes,* ϕ is that both components, the *in*-phrase and the prejacent, determine sets of possible worlds and that the set of possible worlds representing the content of the fiction mentioned in the *in*-phrase is a subset of the set of possible worlds determined by the prejacent. We will encounter the same rough structure of relating sets of possible worlds in other intensional constructions.

This is where we will leave things. There is more to be said about fiction operators like *in the world of Sherlock Holmes*, but we will just refer to you to the relevant literature. In particular, one might want to make sense of Lewis' idea that a special treatment is needed for cases where the sentence makes a claim about things that are left open by the fiction (no truth-value, perhaps?). One also needs to figure out how to deal with cases where the fiction is internally inconsistent. In any case, for our purposes we're done with this kind of operator.

1.3.4 What's next and a general pattern

With the basic framework of intensional semantics in place, we can now look at a succession of intensional operators. In particular, we will explore the semantics of propositional attitude predicates such as *believe* or *want*, modal auxiliaries such as *must* or *might*, and conditional sentences.

We will see a general pattern at work. In our Sherlock Holmes example, an intensional operator can be seen as taking an *anchor a* (the Sherlock Holmes stories as they are in actual world) and *project* from it a set of compatible worlds, this can be encapsulated in a "*flavor*" function f from anchors to sets of possible worlds; the operator then makes a claim with a certain quantificational *force M* about the relation between the projected set of worlds and the *prejacent* worlds ϕ. Indirectly then, a claim is thereby made about the anchor.

$M\ [f(a)]\ (\phi)$

M: a quantificational/modal relation between two sets of worlds (propositions)

a: the anchor of the modal claim

f: the flavor function that projects a set of worlds from the anchor

ϕ: the prejacent set of worlds (proposition)

We will look at the non-trivial issues that arise when several intensional operators interact (modals under attitudes, modals in the consequent of a conditional, etc.). We will also see that constituents of the prejacent can sometimes be evaluated with respect to a world that is not the world that the intensional operator is taking us to (so-called *de re* readings). Further, we will move from worlds to times and explore the semantics of tense and aspect. And, for the intrepid, this can all come together by exploring how tense and aspect interact with attitudes, modality, and conditionals.

1.4 *Issues with an informal meta-language

Exercise 1.7 asks what is wrong with writing something like

(38) ($\lambda x.\ x$ is slow in w) (Watson) = Watson is slow in w.

Thanks to Magda Kaufmann, Angelika Kratzer, and Ede Zimmermann for discussions of the issues explored in this section, which is optional on a first pass, as indicated by the star on the section title.

Think about it. On the left hand side of the "=" sign is a meta-language expression consisting of a λ-expression (so some kind of function) applied to an individual (contributed by the meta-language name "Watson"). The function is a function from individuals to truth-values that will deliver the truth-value 1 iff the individual is slow in world w. So, what we have on the left hand side is the result of a function from individuals to truth-values applied to an individual. In other words, on the left hand side we have a truth-value, namely the truth-value 1 if Watson is slow in w and the truth-value 0 if Watson is not slow in w.

Now, what do we have on the right hand side of the "="? We have the meta-language sentence "Watson is slow in w". That is not nor does it contribute a truth-value. It is a statement of fact. Truth-values are not the same as statements of fact.

The proper thing to do is to write

(39) ($\lambda x.\ x$ is slow in w) (Watson) = 1 iff Watson is slow in w.

There are actually two ways to parse the statement in (39), both legitimate it appears.

On one parse, the major connective is the meta-language expression "iff". On its left hand side is a meta-language statement (that applying the function to the individual Watson gives the truth-value 1) and on the right hand side of the "iff" we have another meta-language statement (that Watson is slow in w). So, the whole thing says that these two statements are equivalent: (i) that function applied to that individual gives us the truth-value 1, and (ii) that Watson is slow in w.

The other parse is perhaps more conspicuously represented as follows:

(40) $(\lambda x.\ x \text{ is slow in } w)(\text{Watson}) = \begin{cases} 1 \text{ if Watson is slow in } w \\ 0 \text{ if Watson is not slow in } w \end{cases}$

Is this weird? It turns out that natural language, not just our semi-formal meta-language, has conditionals that seem very similar: *I fear [the consequences if we fail].* See Lasersohn 1996, Frana 2017, and Blümel 2019 for some discussion.

Here, the "=" sign is the major connective. The left hand side is a meta-language expression that resolves to a truth-value and the right hand side as well contributes a truth-value: 1 if such and such and 0 if such and such.

H&K, of course, introduced a convention that allowed meta-language statements to be used in a place where a truth-value was expected (p.37, (9)):

> Read "$[\lambda\alpha: \phi. \gamma]$" as either (i) or (ii), whichever makes sense.
> (i) "the function which maps every α such that ϕ to γ"
> (ii) "the function which maps every α such that γ to 1, if γ, and to 0 otherwise"

Since it never makes sense to map anything to a meta-language statement, no ambiguity will ever arise.

So, one might want to extend this leeway and use it in the case of (38) as well. We could say that in general, meta-language statements supply truth-values wherever that makes sense. In that case, (38) is just shorthand for (39).

This is the approach of von Stechow 1991.

Alternatively, one can introduce a new notation that indicates that a meta-language statement is being used to contribute a truth-value:

This is the approach Ede Zimmermann (pc) advocates and has been using in his classes.

(41) $\vdash \alpha \dashv = \begin{cases} 1 \text{ if } \alpha \\ 0 \text{ if otherwise} \end{cases}$

Lastly, one could abandon the H&K informal meta-language approach altogether and introduce a rigidly formalized meta-language.

These lecture notes will proceed to follow H&K's approach and will not introduce any further innovations. So, (38) is illicit and only (39) is acceptable.

1.5 *Further readings*

There is considerable overlap between this chapter and Chapter 12 of Heim & Kratzer 1998. Here, we approach intensional semantics from a slightly different angle. It would probably be beneficial if you read H&K's Chapter 12 in addition to this chapter and if you did the exercises in there. Come to think of it, some other ancillary reading is also recommended. You may want to look at relevant chapters in other textbooks (for example: Dowty, Wall & Peters 1981: Chapters 5&6, Gamut 1991: Volume II: Intensional Logic and Logical Grammar, Chierchia & McConnell-Ginet 2000: Chapter 5: Intensionality, and Zimmermann & Sternefeld 2013).

The *Stanford encyclopedia of philosophy* is always a good resource. Menzel 2016 is the entry on possible worlds. A couple of influential philosophical works on the metaphysics and uses of possible worlds are Kripke 1980 and Lewis 1986. An interesting paper on the origins of the modern possible worlds semantics for modal logic is Copeland 2002.

A must read for students who plan to go on to becoming specialists in semantics, together with a handbook article putting it in perspective: Montague 1973 and Partee & Hendriks 1997.

To learn more about discourse about fiction, read Lewis 1978. Recent reconsiderations: Bonomi & Zucchi 2003, Hanley 2004, Proudfoot 2006. An interesting paper that explores the meaning of fictional texts: Bauer & Beck 2014. Inconsistencies in fictions and elsewhere are discussed in: Varzi 1997 and Lewis 1982.

Some other interesting work on stories and pictures and their content: Ross 1997, Zucchi 2001, Blumson 2009. More recently, there's been quite a bit of work on pictorial semantics, see for example Abusch & Rooth 2017, Greenberg 2018, Maier & Bimpikou 2019.

Hockett & Altmann 1968 is a follow-up to Hockett's original article on design features. See Emonds 2011 for a recent re-appraisal. If you're interested in whether displacement really is an exclusive feature of human language and cognition, there are some recent survey articles: Cheke & Clayton 2010, Redshaw 2014. Two very recent articles about displacement in humans and other animals are Leahy & Carey 2020 and Redshaw & Suddendorf 2020.

Astonishingly, Lewis' doctrine of the reality of the plurality of possible worlds is being paralleled (pun absolutely intended) by theoretical physicists in a number of ways. There is a controversial "many worlds" interpretation of quantum mechanics, for example. Other terms found are the "multiverse" and "parallel universes". See for starters, Kai's blog entry on a popular book on the issue, http://kaivonfintel.org/many-worlds/, MIT physics professor Max Tegmark's page on the topic, http://space.mit.edu/home/tegmark/crazy.html, and a Fresh Air interview with physicist Brian Greene about his book *The Hidden Reality: Parallel Universes and the Deep Laws of the Cosmos*: http://www.npr.org/2011/01/24/132932268/a-physicist-explains-why-parallel-universes-may-exist.

2 Attitudes, conditionals, modals

2.1 Attitudes 22
2.1.1 Hintikka's idea 22
2.1.2 Iterated attitudes 24
2.2 Conditionals 25
2.3 Modals 29
2.3.1 Syntactic assumptions 29
2.3.2 Quantification over possible worlds 29
2.3.3 Contingency, flavors, context-dependency . 31
2.3.4 Epistemic vs. Circumstantial 33
2.3.5 Toward an analysis 34
2.4 Explorations and variations 37
2.4.1 Accessibility relations 37
2.4.2 Conversational backgrounds 42

Introduction

Towards the end of the first chapter, we identified a general schema for modal displacement operators. It begins with a "flavor function" that "projects" a set of relevant worlds from an "anchor", and then a quantificational claim is made about those worlds and their relation to the prejacent. We will now see this pattern at work in three kinds of constructions: propositional attitudes, conditionals, and modals. These look superficially quite dissimilar:

$M\ [f(a)]\ (\phi)$

M: a quantificational/modal relation between two sets of worlds (propositions)

a: the anchor of the modal claim

f: the flavor function that projects a set of worlds from the anchor

ϕ: the prejacent set of worlds (proposition)

(1) a. Charlotte believes that Lucy is smart.
b. If Lucy is smart, she will cancel the meeting.
c. Lucy might cancel the meeting.

The intensional operator in (1a) is the lexical verb *believe*, in (1b) it is the subordinating complementizer *if*, and in (1c) it is the auxiliary verb *might*. Despite this surface variety, the core semantic contributions are very similar.

Propositional attitude predicates like *believe* project a set of worlds from the mental state of their subject and relate those worlds to the worlds described by the prejacent proposition. Conditionals select a relevant subset of the worlds described by their antecedent and relate them to the worlds described by their consequent. And the modal *might* says that some worlds in a relevant set make the prejacent proposition true.

We will now look at these ideas in more detail and build some nimbleness in deploying the technical notions here.

2.1 *Attitudes*

We move from the contents of works of fiction to the contents of the minds of people. Instead of worlds compatible with the Sherlock Holmes stories, we look at the worlds compatible with the beliefs or other mental states of a person.

2.1.1 *Hintikka's idea*

Expressions like *believe*, *know*, *doubt*, *expect*, *want*, *regret*, and so on are usually said to describe PROPOSITIONAL ATTITUDES, expressing relations between individuals (the attitude holder) and propositions (intensions of sentences).

According to Hintikka 1969, the term PROPOSITIONAL ATTITUDE goes back to Russell 1940.

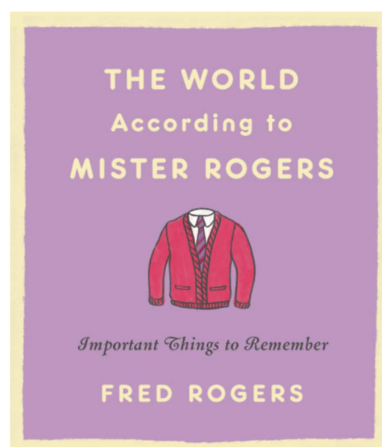
The simple idea is that *Amandine believes that Letícia is a spy* claims that Amandine believes of the proposition that Letícia is a spy that it is true. Note that for the attitude ascription to be true it does not have to hold that Letícia is actually a spy. But where — in which world(s) — does Letícia have to be a spy for it be true that Amandine believes that Letícia is a spy?

The possible worlds semantics for propositional attitudes was in place long before the extension to fiction contexts was proposed. Our discussion here has inverted the historical sequence for pedagogical purposes.

We might want to be inspired by the colloquial phrase "the world according to …" and say that *Amandine believes that Letícia is a spy* is true iff in the world according to Amandine, Letícia is a spy. We immediately recall from Chapter 1 that we need to fix this idea up by making space for multiple worlds compatible with Amandine's beliefs and by tying the truth-conditions to contingent facts about the evaluation world. That is, what Amandine believes is different in different possible worlds.

The following lexical entry thus offers itself:

(2) $[\![\text{believe}]\!]^{w,g} =$
$$\lambda p_{\langle s,t \rangle}.\ \lambda x.\ \forall w' \text{ compatible with } x\text{'s beliefs in } w\text{: } p(w') = 1.$$



What is going on in this semantics? We conceive of Amandine's beliefs as a state of her mind about whose internal structure we will remain agnostic, a matter left to other cognitive scientists. What we require of it is that it embody opinions about what the world she is located in looks like. In other words, if her beliefs are confronted with a particular possible world w', they will determine whether that world may or may not be the world as they think it is. What we are asking of Amandine's mental state is whether any state of affairs, any event, anything in w' is in contradiction with anything that Amandine believes. If not, then w' is compatible

It is important to realize the modesty of this semantics: we are not trying to figure out what belief systems are and particularly not what their internal workings are. That is the job of psychologists (and philosophers of mind, perhaps). For our semantics, we treat the belief system as a black box that determines for each possible world whether it considers it possible that it is the world it is located in.

with Amandine's beliefs. For all Amandine believes, w' may well be the world where she lives. Many worlds will pass this criterion, just consider as one factor that Amandine is unlikely to have any precise opinions about the number of leaves on the tree in front of my house. Amandine's belief system determines a set of worlds compatible with her beliefs: those worlds that are viable candidates for being the actual world, as far as her belief system is concerned.

Now, Amandine believes a proposition iff that proposition is true in all of the worlds compatible with her beliefs. If there is just one world compatible with her beliefs where the proposition is not true, that means that she considers it possible that the proposition is not true. In such a case, we can't say that she believes the proposition.

Here is the same story in the words of Hintikka (1969), the source for this semantics for propositional attitudes:

> My basic assumption (slightly simplified) is that an attribution of any propositional attitude to the person in question involves a division of all the possible worlds (…) into two classes: into those possible worlds which are in accordance with the attitude in question and into those which are incompatible with it. The meaning of the division in the case of such attitudes as knowledge, belief, memory, perception, hope, wish, striving, desire, etc. is clear enough. For instance, if what we are speaking of are (say) *a*'s memories, then these possible worlds are all the possible worlds compatible with everything he remembers. […]
>
> How are these informal observations to be incorporated into a more explicit semantical theory? According to what I have said, understanding attributions of the propositional attitude in question (…) means being able to make a distinction between two kinds of possible worlds, according to whether they are compatible with the relevant attitudes of the person in question. The semantical counterpart to this is of course a function which to a given individual person assigns a set of possible worlds.
>
> However, a minor complication is in order here. Of course, the person in question may himself have different attitudes in the different worlds we are considering. Hence this function in effect becomes a relation which to a given individual and to a given possible world μ associates a number of possible worlds which we shall call the ALTERNATIVES to μ. The relation will be called the alternativeness relation. (For different propositional attitudes, we have to consider different alternativeness relations.)

We recognize our schema: the anchor is the pair of an individual and a world, the flavor function projects from the anchor a set of worlds compatible with the relevant attitude of the anchor individual in the anchor world.

Note that what Hintikka calls "alternativeness relations" are now more commonly known as ACCESSIBILITY RELATIONS. We will encounter such relations again and again, and we will explore their formal properties later on.

Exercise 2.1 *Let's adopt Hintikka's idea that we can use a function that maps x and w into the set of worlds w' compatible with what x believes in w. Call this function $\mathcal{B}$. That is,*

(3) $\mathcal{B} = \lambda x.\ \lambda w.\ \{w' :\ w' \text{ is compatible with } x\text{'s beliefs in } w\}$.

Using this notation, our lexical entry for believe *could look as follows:*

(4) $[\![\textit{believe}]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \lambda x.\ \mathcal{B}(x)(w) \subseteq p$.

We are here indulging in the usual sloppiness in treating p both as a function from worlds to truth-values and as the set characterized by that function.

Here now are two "alternatives" for the semantics of believe*:*

(5) *Attempt 1 (very wrong)*
$[\![\textit{believe}]\!]^{w,g} = \lambda p \in D_{\langle s,t\rangle}.[\lambda x \in D.\ p = \mathcal{B}(x)(w)]$.

(6) *Attempt 2 (also very wrong)*
$[\![\textit{believe}]\!]^{w,g} = \lambda p \in D_{\langle s,t\rangle}.[\lambda x \in D.\ p \cap \mathcal{B}(x)(w) \neq \emptyset]$.

Explain why these do not adequately capture the meaning of believe. □

Exercise 2.2 *Follow-up: The semantics in (6) would have made* believe *into an existential quantifier of sorts: it would say that* some *of the worlds compatible with what the subject believes are such-and-such. You have argued (successfully, of course) that such an analysis is wrong for* believe. *But* are *there attitude predicates with such an "existential" meaning? Discuss some candidates.* □

If you can't find any candidates that survive scrutiny, can you speculate why there might be no existential attitude predicates? [Warning: this is underexplored territory!]

Exercise 2.3 *Propose a semantics for the adjective* alleged *as in* Vera is an alleged kleptomaniac. *Do not assume any hidden structure. Try to relate your semantics to the verb* allege *as in* Romelu alleged that Vera is a kleptomaniac. □

After this little exercise, you might be interested in some really tough questions about intensionality inside noun phrases: Bogal-Allbritten 2013, Bogal-Allbritten & Weir 2017 and Hirsch 2017: especially Chapter 4.

2.1.2 Iterated attitudes

Semantics is a lab science in several ways. The most crucial way is that we learn a lot about our objects of study when we put them together and see how they react to each other.

We expect attitudes to be able to *iterate*: an attitude claim is a sentence with contingent truth-conditions and thus provides a proposition that in turn could be the complement of another attitude claim. In fact, one suspects that much of the fabric of human life involves iterated attitudes: we wonder whether Emma realizes that Caroline believes that Janet has invited Preston for dinner without telling Abby.

Exercise 2.4 *Derive the truth-conditions of* Emma believes that Caroline believes that Preston is from Texas. *[Follow-up question: if Emma's belief is correct, does that mean that Preston is from Texas?]* □

2.2 *Conditionals*

In many ways, conditionals are the archetypal construction of displacement: the consequent is evaluated not against the actual here and now but against the scenario conjured up by the antecedent. Consider a few conditional sentences:

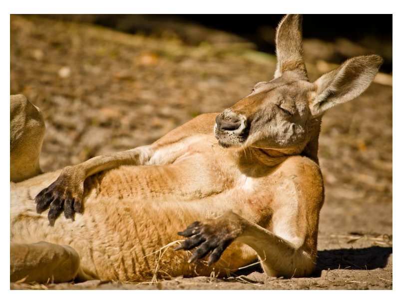
(7) a. If Kim left before 6am, she got there in time.
b. If there's an earthquake tomorrow, this house will collapse.
c. If there had been a massive snowstorm last night, Kai would have stayed home.

These represent the three main subtypes of conditionals (there are more): (7a) is an "indicative" conditional about the past, (7b) is an indicative conditional about the future, and (7c) is a "subjunctive" conditional. For the moment, the differences will be left aside.

The basic idea of how conditionals work is this: the *if*-clause whisks us away to a particular possible world (or maybe a set thereof) and the consequent clause is asserted to be true of that world (or those worlds). But what world(s) are we being taken to? The most obvious requirement is that the antecedent of the conditional needs to be true of the world(s). But there's more.

Given our discussions of how the semantics of fiction operators anchors them in facts about the actual world (the content of the relevant body of fiction) and how the semantics of attitude predicates is anchored in the mental states of an individual in the actual world, it shouldn't come as a surprise that conditionals are similarly anchored. So, look at the examples in (7): what in the actual world are they about?

Lewis used a rather whimsical example to start off his seminal 1973 book on counterfactuals: "If kangaroos had not tails, they would topple over". For another example of counterfactual whimsy, consider this scene from the TV show "Big Bang Theory": https://www.youtube.com/watch?v=0lpY0Kt4bn8. As the examples in the text make clear, conditionals are actually very down-to-earth in real life.



Here's a first attempt of an answer: (7a) is about the local transportation system, the weather, the traffic, and so on. (7b) is about the sturdiness of this house, facts of geology, laws of physics, and so on. (7c) is about Kai's proclivities (such as avoiding traffic snarls), the local climate, and so on. Since the conditionals are anchored in real world facts, they are no mere flights of fancy and whether they are true depends on those facts. If today's traffic was particularly bad, it may be false that Kim's leaving before 6am would have got her there in time. If the architects went to great lengths to make the house earthquake-safe, (7b) may well be false. And if there was an attendance-mandatory faculty meeting, Kai may well have come in in spite of a massive snowstorm.

So, the outlines of the semantics of conditionals are clear: *if* takes us to worlds where the antecedent is true but that match the actual world in certain relevant features. And the consequent then is evaluated in those worlds. There are many details to work out and we'll keep returning to that task. But for now, we put forward a placeholder analysis.

We will treat *if* as a higher-order operator that together with the antecedent creates an intensional operator with a semantics very similar to the final analysis we gave to *in the world of Sherlock Holmes*. But where the fiction operator directly encoded what features of the actual world it's sensitive to (the Sherlock Holmes fiction), conditionals rely on context for this job. Here's a first draft of the proposal:

(8) $$[\![\text{if}]\!]^{w,g} = \lambda p \in D_{\langle s,t\rangle}.\ \lambda q \in D_{\langle s,t\rangle}.\ \forall w'\colon\ p(w') = 1\ \&\ w' \text{ is relevantly like } w \rightarrow q(w') = 1.$$

The contextual anchoring to features of the evaluation world w is here effected by the placeholder "relevantly like w". This is crucial because otherwise the conditional would talk about any world whatsoever where the antecedent is true. This would make the truth-conditions not just not contingent on the actual world but also far too strong to allow most sensible conditionals to be true ever.

Think about the earthquake conditional (7b): we would derive the absurdly strong truth-conditions that the conditional is true iff *all* of the worlds where there is a major earthquake in Cambridge tomorrow are worlds where my house collapses.

There are some obvious and immediate problems with this analysis. For one, while it's easy to imagine circumstances where the conditional (7b) is judged to be true, there surely are possible worlds where there's an earthquake but my house does not collapse: perhaps, the builders in that world used all the recommended best practices to make the building earthquake-safe, perhaps it's a world where I'm simply unreasonably lucky, or the house is immediately adjacent to much sturdier neighboring buildings which keep it propped up, or Harry Potter flies by and protects the house at the last minute (he owes me a favor, after all). This problem (that the house doesn't in fact collapse in *all* possible worlds where there's an earthquake but that the conditional can still be judged true in some worlds) is accompanied with another problem: whether the conditional is true depends on what the world is like. Was the house built to exacting standards? Is it propped up by its neighbors? Does Harry Potter owe me a favor? That is the problem solved by restricting the quantifier over worlds to world "relevantly like w".

Obviously, this is a semantics with a "placeholder", because what does "relevantly like" mean precisely? Now, just because the semantics is therefore rather vague and context-dependent doesn't mean it is wrong. As Lewis 1973: p.1 writes:

> Counterfactuals are notoriously vague. That does not mean that we cannot give a clear account of their truth conditions. It does mean that such an account must either be stated in vague terms — which does not mean ill-understood terms — or be made relative to some parameter that is fixed only within rough limits on any given occasion of language use.

The insight articulated by Lewis here is very important. Applying mathematical or logical methods to analyzing natural language meaning often arouses severe skepticism, precisely because natural language is often vague and context-dependent. But that just means that an adequate analysis needs to not ignore vagueness and context-dependence and rather be clear about where they enter.

Strawson 1950 famously wrote: "Neither Aristotelian nor Russellian rales give the exact logic of any expression of ordinary language; for ordinary language has no exact logic."

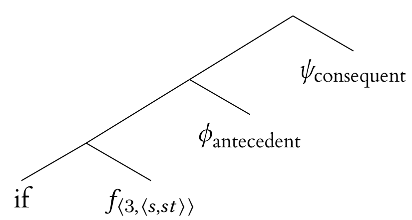
All the more reason to refine our initial draft of the proposal. We put a placeholder for context-dependence in the meta-language ("worlds relevantly like the evaluation world") but that is not really sufficient. We would like to embed the analysis in a general framework for how context enters the semantics. For the purposes of this book, we will adopt an approach that generalizes from the analysis of "free" pronouns in the Heim & Kratzer textbook.

In H&K, chapters 9–11, a technical implementation of context-dependency is developed for pronouns and their referential (and E-Type) readings. Referential pronouns are analyzed there as free variables, appealing to a general principle that free variables in an LF need to be supplied with values from the utterance context. If we want to describe the context-dependency of conditionals (and as we'll soon see, modals) in a technically analogous fashion, we can think of their LF-representations as incorporating or subcategorizing for a kind of invisible pronoun, a free variable that effects the anchoring of the conditional claim to relevant features of the evaluation world.

Concretely, we posit LF-structures where *if* doesn't just take two propositions as its arguments but also an object language variable of type $\langle s, \langle s, t\rangle\rangle$:

We are using the notation for variables of types other than e introduced by Heim & Kratzer, p. 213. An index on a variable now is an ordered pair of a natural number and a type. The variable assignments relative to which we calculate semantic values now are functions from ordered pairs of a natural number and a type to elements of the domain of objects of that type.

(9)



We have written the silent pronoun as "f" to remind you of "flavor", the evocative term we used to describe the anchoring of intensional operators. So, we have a variable over flavor functions that will return a set of worlds when applied to a given world. We will give *if* the job of telling f that what we need is the set of worlds that f assigns to the evaluation world:

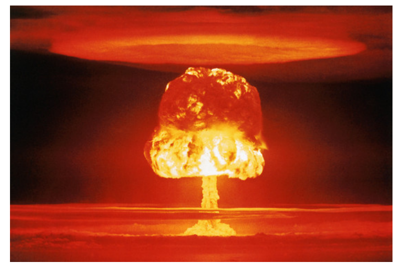
(10) $$[\![\text{if}]\!]^{w,g} = \lambda f \in D_{\langle s,\langle s,t\rangle\rangle}.\ \lambda p \in D_{\langle s,t\rangle}.\ \lambda q \in D_{\langle s,t\rangle}.\ \forall w'\colon\ f(w)(w') = 1\ \&\ p(w') = 1 \rightarrow q(w') = 1.$$

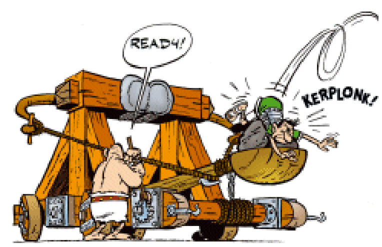
Together this means that a conditional says about the evaluation world w_0 that among the worlds that are f-related to w_0, the ones where the antecedent is true are all worlds where the consequent is also true.

We get different flavors of conditionals from different contextual resolutions of f. Consider for example the earthquake conditional in (7b). Context might assign to f the function that when given an evaluation world w returns the set of worlds that “agree” with w on how sturdy this house is, what the local geology is like, and what the laws of physics are. Then, the conditional claims that the actual world is such that all the worlds that agree with it via f and where there is an earthquake are worlds where this house collapses.

With context-dependency comes the spectre of indeterminacy. A famous example is a pair attributed by Quine 1960: p. 221 to Nelson Goodman (imagine these being said while the Korean War was going on):

(11) a. If Caesar were in command, he would use the atom bomb.
b. If Caesar were in command, he would use catapults.





Both versions seem possible: saying (11a) would talk about worlds where Caesar with all his ruthlessness is in command, while (11b) would talk about worlds where Caesar’s own arsenal comes with him.

We will return to conditionals in the next chapter when some additional complications become necessary.

Exercise 2.5 *We continue our experiments and embed a conditional under an attitude. Compute the truth-conditions of the following sentence:*

(12) Allie believes that if there’s an earthquake, this house will collapse.

Exercise 2.6 *Here’s a potentially simpler analysis of conditionals. What if the silent pronoun were of type $\langle s, t \rangle$, a set of worlds?* If *would then take the contextually assigned set of worlds and say that all the worlds in that set in which the antecedent is true are worlds where the consequent is true. Spell out the details of this idea. How would it fare when confronted with the embedded conditional (12) from the previous exercise?* □

2.3 Modals

The final empirical addition of this chapter are modal auxiliaries like *may, must, can, have to*, etc. Most of what we say here should carry over straightforwardly to modal adverbs like *maybe, possibly, certainly*, etc. We will make certain syntactic assumptions, which make our work easier but which leave aside many questions that at some point deserve to be addressed.

2.3.1 Syntactic assumptions

We will assume, at least for the time being, that a modal like *may* is a RAISING predicate (rather than a CONTROL predicate), i.e., its subject is not its own argument, but has been moved from the subject-position of its infinitival complement. So, we are dealing with the following kind of structure:

(13) a. Ann may be smart.
  b. [Ann [λ_1 [may [t_1 be smart]]]]

Actually, we will be working here with the even simpler structure below, in which the subject has been reconstructed to its lowest trace position. (E.g., these could be generated by deleting all but the lowest copy in the movement chain.) We will be able to prove that movement of a name or pronoun never affects truth-conditions, so at any rate the interpretation of the structure in (13b) would be the same as that of (14). As a matter of convenience, then, we will take the reconstructed structures, which allow us to abstract away from the (here irrelevant) mechanics of variable binding.

We will talk about reconstruction in more detail later.

We will assume that even though *Ann be smart* is a non-finite sentence, this will not have any effect on its semantic type, which is that of a sentence, which in turn means that its semantic value is a truth-value. This is hopefully independent of the (interesting) fact that *Ann be smart* on its own cannot be used to make a truth-evaluable assertion.

(14) may [Ann be smart]

So, for now at least, we are assuming that modals are expressions that take a full sentence as their semantic argument. Now then, what do modals mean?

2.3.2 Quantification over possible worlds

The basic idea of the possible worlds semantics for modal expressions is that they are quantifiers over possible worlds. Toy lexical entries for *must* and *may*, for example, would look like this:

This idea goes back a long time. It was famously held by Leibniz, but there are precedents in the medieval literature; see Knuuttila 2003. See Copeland 2002 for the modern history of the possible worlds analysis of modal expressions.

(15) $[\![\text{must}]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \forall w':\ p(w') = 1.$

(16) $[\![\text{may}]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \exists w':\ p(w') = 1.$

A necessity modal like *must* says that all worlds make its prejacent true, while a possibility modal like *may* says that some worlds make its prejacent true. Note that our previous intensional operators were all universal

Sometimes, people call necessity modals "universal modals" and possibility modals "existential modals", which obviously presupposes this quantificational analysis.

quantifiers (unless you found some existential attitudes in Exercise 2.2), so the existential force of *may* is a new frontier for us.

The analysis in (15)/(16) is too crude (in particular, notice that it would make modal sentences non-contingent — there is no occurrence of the evaluation world on the right hand side!). But it does already have some desirable consequences that we will seek to preserve through all subsequent refinements. It correctly predicts a number of intuitive judgments about the logical relations between *must* and *may* and among various combinations of these items and negations. To start with some elementary facts, we feel that *must* ϕ entails *may* ϕ, but not vice versa:

(17) You must stay.
Therefore, you may stay. VALID

(18) You may stay.
Therefore, you must stay. INVALID

(19) a. You may stay, but it is not the case that you must stay.
b. You may stay, but you don't have to stay. CONSISTENT

The somewhat stilted *it is not the case*-construction is used in (19a) to make certain that negation takes scope over *must*. When modal auxiliaries and negation are together in the auxiliary complex of the same clause, their relative scope seems not to be transparently encoded in the surface order; specifically, the scope order is not reliably negation > modal. (Think about examples with *mustn't*, *can't, shouldn't, may not* etc. What's going on here? This is an interesting topic which we must set aside for now. See the references at the end of the chapter for relevant work.) With modal *main* verbs (such as *have to*), this complication doesn't arise; they are consistently inside the scope of clause-mate auxiliary negation. Therefore we can use (19b) to (unambiguously) express the same scope order as (19a), without having to resort to a biclausal structure.

We judge *must* ϕ incompatible with its "inner negation" *must* [*not* ϕ], but find *may* ϕ and *may* [*not* ϕ] entirely compatible:

(20) You must stay, and/but also, you must leave. (leave = not stay).
CONTRADICTORY

(21) You may stay, but also, you may leave. CONSISTENT

We also judge that in each pair below, the (a)-sentence and the (b)-sentences say the same thing.

(22) a. You must stay.
b. It is not the case that you may leave.
You aren't allowed to leave.
(You may not leave.)
(You can't leave.)

(23) a. You may stay.
b. It is not the case that you must leave.
You don't have to leave.
You don't need to leave.
(You needn't leave.)

The parenthesized variants of the (b)-sentences in (54) are pertinent here only to the extent that we can be certain that negation scopes over the modal. In these examples, apparently it does, but as we remarked above, this cannot be taken for granted in all structures of this form.

Given that *stay* and *leave* are each other's negations (i.e. $[\![\text{leave}]\!]^{w,g} = [\![\text{not stay}]\!]^{w,g}$, and $[\![\text{stay}]\!]^{w,g} = [\![\text{not leave}]\!]^{w,g}$), the LF-structures of these equivalent pairs of sentences can be seen to instantiate the following schemata:

In logicians' jargon, *must* and *may* behave as DUALS of each other. For definitions of "dual", see Barwise & Cooper 1981: p. 197 or Gamut 1991: vol.2, 238.

(24) a. $must\ \phi \equiv not\ [may\ [not\ \phi]]$
b. $must\ [not\ \psi] \equiv not\ [may\ \psi]$

(25) a. $may\ \phi \equiv not\ [must\ [not\ \phi]]$
b. $may\ [not\ \psi] \equiv not\ [must\ \psi]$

Our present analysis of *must*, *have-to*, … as universal quantifiers and of *may*, *can*, … as existential quantifiers straightforwardly predicts all of the above judgments, as you can easily prove.

More linguistic data regarding the "parallel logic" of modals and quantifiers can be found in Horn's dissertation (Horn 1972).

2.3.3 Contingency, flavors, context-dependency

We already said that the semantics we started with is too simple-minded. In particular, we have no dependency on the evaluation world, which would make modal statements non-contingent. This is not correct.

If one says *It may be snowing in Cambridge*, that may well be part of useful, practical advice about what to wear on your upcoming trip to Cambridge. It may be true or it may be false. The sentence seems true if said in the dead of winter when we have already heard about a Nor'Easter that is sweeping across New England. The sentence seems false if said by a clueless Australian acquaintance of ours in July.

The contingency of modal claims is not captured by our current semantics. All the *may*-sentence would claim under that semantics is that there is some possible world where it is snowing in Cambridge. And surely, once you have read Lewis' quote in Chapter 1, where he asserts the existence of possible worlds with different physical constants than we enjoy here, you must admit that there have to be such worlds even if it is July. The problem is that in our semantics, repeated here:

(26) $[\![\text{may}]\!]^{w,g} = \lambda p_{\langle s,t\rangle}.\ \exists w' \colon\ p(w') = 1,$

there is no occurrence of w on the right hand side. This means that the truth-conditions for *may*-sentences are world-independent. In other words, they make non-contingent claims that are either true whatever or false whatever, and because of the plenitude of possible worlds they are more likely to be true than false. This needs to be fixed. But how?

Conversely, the plenitude of possible worlds would make *must*-claims very likely false if they are not reigned in or anchored somehow.

Well, what makes *it may be snowing in Cambridge* seem true when we know about a Nor'Easter over New England? What makes it seem false when we know that it is summer in New England? The idea is that we only consider possible worlds COMPATIBLE WITH THE EVIDENCE AVAILABLE TO US. And since what evidence is available to us differs from world to world, so will the truth of a *may*-statement.

(27) $[\![\text{may}]\!]^{w,g} = \lambda p.\ \exists w'$ compatible w/ the evidence in $w\colon\ p(w') = 1.$

(28) $[\![\text{must}]\!]^{w,g} = \lambda p.\ \forall w'$ compatible w/ the evidence in $w\colon\ p(w') = 1.$

From now on, we will leave off type-specifications such as that p has to be of type $\langle s,t\rangle$, whenever it is obvious what they should be and when saving space is aesthetically called for.

Let us consider a different example:

(29) You have to be quiet.

Imagine this sentence being said based on the house rules of the particular dormitory you live in. Again, this is a sentence that could be true or could be false. Why do we feel that this is a contingent assertion? Well, the house rules can be different from one world to the next, and so we might be unsure or mistaken about what they are. In one possible world, they say that all noise must stop at 11pm, in another world they say that all noise must stop at 10pm. Suppose we know that it is 10:30 now, and that the dorm we are in has either one or the other of these two rules, but we have forgotten which. Then, for all we know, *you have to be quiet* may be true or it may be false. This suggests a lexical entry along these lines:

(30) $[\![\text{have-to}]\!]^{w,g} = \lambda p.\ \forall w'$ compatible with the rules in w: $p(w') = 1$.

Again, we are tying the modal statement about other worlds down to certain worlds that stand in a certain relation to the actual world: those worlds where the rules as they are here are obeyed.

A note of caution: it is very important to realize that the worlds compatible with the rules as they are in w are those worlds where nothing happens that violates any of the w-rules. This is not at all the same as saying that the worlds compatible with the rules in w are those worlds where the same rules are in force. Usually, the rules do not care what the rules are, unless the rules contain some kind of meta-statement to the effect that the rules have to be the way they are, i.e. that the rules cannot be changed. So, in fact, a world w' in which nothing happens that violates the rules as they are in w but where the rules are quite different and in fact what happens violates the rules as they are in w' is nevertheless a world compatible with the rules in w. For example, imagine that the only relevant rule in w is that students go to bed before midnight. Take a world w' where a particular student goes to bed at 11:30 pm but where the rules are different and say that students have to go to bed before 11 pm. Such a world w' is compatible with the rules in w (but of course not with the rules in w').

Apparently, there are different flavors of modality, varying in what kind of facts in the evaluation world they are sensitive to. The semantics we gave for *must* and *may* above makes them talk about evidence, while the semantics we gave for *have-to* made it talk about rules. But that was just because the examples were hand-picked. In fact, in the dorm scenario we could just as well have said *You must be quiet*. And, vice versa, there is nothing wrong with using *it has to be snowing in Cambridge* based on the evidence we have. In fact, many modal expressions seem to be multiply ambiguous. The English modal *have to* is probably the world champion in this regard:

(31) a. It has to be raining.
b. Visitors have to leave by six pm.
c. You have to go to bed in ten minutes.
d. I have to sneeze.
e. To get home in time, you have to take a taxi.

Traditional descriptions of modals often distinguish a number of "readings": EPISTEMIC, DEONTIC, ABILITY, CIRCUMSTANTIAL, DYNAMIC, ... Here are some initial illustrations.

Beyond "epistemic" and "deontic," there is a great deal of terminological exuberance. Sometimes all non-epistemic readings are grouped together under the term ROOT MODALITY (nobody knows why).

(32) EPISTEMIC MODALITY

A: Where is John?
B: I don't know. He *may* be at home.

(33) DEONTIC MODALITY

A: Am I allowed to stay over at Janet's house?
B: No, but you *may* bring her here for dinner.

(34) CIRCUMSTANTIAL/DYNAMIC MODALITY

A: I will plant the rhododendron here.
B: That's not a good idea. It *can* grow very tall.

How are *may* and *can* interpreted in each of these examples? What do the interpretations have in common, and where do they differ?

In all three examples, the modal makes an existentially quantified claim about possible worlds. This is usually called the MODAL FORCE of the claim. What differs is what worlds are quantified over, sometimes called the MODAL FLAVOR. In EPISTEMIC modal sentences, we quantify over worlds compatible with the available evidence. In DEONTIC modal sentences, we quantify over worlds compatible with the rules and/or regulations. And in the CIRCUMSTANTIAL modal sentence, we quantify over the set of worlds which conform to the laws of nature (in particular, plant biology). What speaker B in (34) is saying, then, is that there are some worlds conforming to the laws of nature in which this rhododendron grows very tall.

2.3.4 Epistemic vs. Circumstantial

Do flavors of modality correspond to some sorts of signals in the structure of sentences? Read the following famous passage from Kratzer 1991 and think about how the two sentences with their very different modal meanings differ in structure:

In the earlier Kratzer 1981, the hydrangeas were *Zwetschgenbäume* 'plum trees'. The German word *Zwetschge*, by the way, is etymologically derived from the name of the city Damascus (Syria), the center of the ancient plum trade.

> Consider sentences (35) and (36):
>
> (35) Hydrangeas can grow here.

(36) There might be hydrangeas growing here.

> The two sentences differ in meaning in a way which is illustrated by the following scenario.
>
> Suppose I acquire a piece of land in a far away country and discover that soil and climate are very much like at home, where hydrangeas prosper everywhere. Since hydrangeas are my favorite plants, I wonder whether they would grow in this place and inquire about it. The answer is (35). In such a situation, the proposition expressed by (35) is true. It is true regardless of whether it is or isn't likely that there are already hydrangeas in the country we are considering. All that matters is climate, soil, the special properties of hydrangeas, and the like. Suppose now that the country we are in has never had any contacts whatsoever with Asia or America, and the vegetation is altogether different from ours. Given this evidence, my utterance of (36) would express a false proposition. What counts here is the complete evidence available. And this evidence is not compatible with the existence of hydrangeas.
>
> (35) together with our scenario illustrates the pure CIRCUMSTANTIAL reading of the modal *can*. [...]. (36) together with our scenario illustrates the epistemic reading of modals. [...] circumstantial and epistemic conversational backgrounds involve different kinds of facts. In using an epistemic modal, we are interested in what else may or must be the case in our world given all the evidence available. Using a circumstantial modal, we are interested in the necessities implied by or the possibilities opened up by certain sorts of facts. Epistemic modality is the modality of curious people like historians, detectives, and futurologists. Circumstantial modality is the modality of rational agents like gardeners, architects, and engineers. A historian asks what might have been the case, given all the available facts. An engineer asks what can be done given certain relevant facts.

Consider also the very different prominent meanings of the following two sentences, taken from Kratzer as well:

(37) a. Cathy can make a pound of cheese out of this can of milk.
b. Cathy might make a pound of cheese out of this can of milk.

2.3.5 Toward an analysis

How can we account for this variety of readings? One way would be to write a host of lexical entries, basically treating this as a kind of (more or less principled) ambiguity/polysemy. Another way, which is preferred by many people, is to treat this as a case of context-dependency, as argued in seminal work by Kratzer (1977, 1978, 1981, 1991).

According to Kratzer, what a modal brings with it intrinsically is just a modal force, that is, whether it is an existential (possibility) modal or a universal (necessity) modal. What worlds it quantifies over is determined by context. In essence, the context has to supply a restriction to the quantifier. How can we implement this idea? Well, we just have to transpose the setup we put in place for conditionals.

It is well-known that natural language quantification is in general subject to contextual restriction. See for example von Fintel 1994: Ch.2 and Stanley & Szabó 2000.

We give modals a flavor argument, a silent pronoun of type $\langle s, st\rangle$, just like we did with *if*. So we posit LF-structures like this:

(38) [[must $f_{\langle n,\langle s,st\rangle\rangle}$] [you quiet]]

$f_{\langle n,\langle s,st\rangle\rangle}$, again, is a variable over functions from worlds to (characteristic functions of) sets of worlds, which — like all free variables — needs to receive a value from the utterance context. For example, it may be supplied with the function which, for any world w, yields the set $\{w'$: the house rules that are in force in w are obeyed in $w'\}$. If we apply this function to a world w_1, in which the house rules read "no noise after 10 pm", it will yield a set of worlds in which nobody makes noise after 10 pm. If we apply the same function to a world w_2, in which the house rules read "no noise after 11 pm", it will yield a set of worlds in which nobody makes noise after 11 pm.

The new lexical entries for *must* and *may* that will fit this new structure are these:

Compare these entries for the modals to the entry for *if* in (10). What are the differences?

(39) a. $[\![\text{must}]\!]^{w,g} =$
$\lambda f \in D_{\langle s,st\rangle}.\ \lambda q \in D_{\langle s,t\rangle}.\ \forall w' \in W\ [f(w)(w') = 1 \rightarrow q(w') = 1]$

in set talk: $f(w) \subseteq q$

b. $[\![\text{may}]\!]^{w,g} =$
$\lambda f \in D_{\langle s,st\rangle}.\ \lambda q \in D_{\langle s,t\rangle}.\ \exists w' \in W\ [f(w)(w') = 1\ \&\ q(w') = 1]$

in set talk: $f(w) \cap q \neq \emptyset$

Exercise 2.7 *Let w be a world, and assume that the context supplies an assignment g such that:*

(40) $g(f_{\langle 17,\langle s,st\rangle\rangle}) = \lambda w.\ \lambda w'.$ *the rules in force in w are obeyed in w′.*

Compute the truth-conditions for the LF in (41):

(41) [must $f_{\langle 17,\langle s,st\rangle\rangle}$ *[you quiet]]*

Does truth-value of (41) correctly depend on the evaluation world w? □

On this approach, the epistemic, deontic, etc. "readings" of individual occurrences of modal verbs come about by a combination of two separate things. The lexical semantics of the modal itself encodes just a quantificational force, a *relation* between sets of worlds. This is either the subset-relation (universal quantification; necessity) or the relation of nondisjointness (existential quantification; possibility). The covert variable next to the modal when applied to the evaluation world yields a set of worlds, and this functions as the quantifier's restrictor. The labels "epistemic", "deontic", "circumstantial" etc. group together certain conceptually natural classes of possible values for this covert restrictor.

Notice that, strictly speaking, there is not just one deontic reading (for example), but many. A speaker who utters

(42) You have to be quiet.

might mean: 'I want you to be quiet,' (i.e., you are quiet in all those worlds that conform to my preferences). Or she might mean: 'unless you are quiet, you won't succeed in what you are trying to do,' (i.e., you are quiet in all those worlds in which you succeed at your current task). Or she might mean: 'the house rules of this dormitory here demand that you be quiet,' (i.e., you are quiet in all those worlds in which the house rules aren't violated). And so on. So the label "deontic" appears to cover a whole open-ended set of imaginable "readings", and which one is intended and understood on a particular utterance occasion may depend on all sorts of things in the interlocutors' previous conversation and tacit shared assumptions. (And the same goes for the other traditional labels.)

Proponents of polysemy accounts of the variety of modal flavors will presumably have to tackle the apparent limitlessness of variation in some principle way. See Viebahn & Vetter 2016 for a polysemy account.

A disappointing feature of our analysis in (39) is that a lot of the work is being done by the modals: they don't just take a restriction as their argument but they have to enforce that this restriction is evaluated in the evaluation world. This is a departure from the ideal that modals are simply quantifiers over possible worlds. It would be preferable to merely present them with a set of worlds to quantify over rather than giving them the responsibility of obtaining this set by applying an accessibility relation to the evaluation world. We will later put in place a different framework (for unrelated reasons) that will make good on this vision.

Exercise 2.8 *Describe two worlds* w_1 *and* w_2 *so that*
$[\![\textit{must } f_{\langle 17,\langle s,st\rangle\rangle} \textit{ you quiet}]\!]^{w_1,g} = 1$
and $[\![\textit{must } f_{\langle 17,\langle s,st\rangle\rangle} \textit{ you quiet}]\!]^{w_2,g} = 0$. □

Exercise 2.9 *In analogy to the deontic relation* $g(f_{\langle 17,\langle s,st\rangle\rangle})$ *defined in (40), define an appropriate relation that yields an epistemic reading for a sentence like* You may be quiet. □

Exercise 2.10 *Describe the set of worlds that constitutes the understood restrictor of* have to *in each of the examples in (31).* □

Exercise 2.11 *Describe values for the covert* $\langle s, st\rangle$*-variable that are intuitively suitable for the interpretation of the modals in the following sentences:*

(43) *As far as John's preferences are concerned, you* may *stay with us.*

(44) *According to the guidelines of the graduate school, every PhD candidate* must *take 9 credit hours outside his/her department.*

(45) *John* can *run a mile in 5 minutes.*

(46) *This* has *to be the White House.*

(47) *This elevator* can *carry up to 3000 pounds.*

For some of the sentences, different interpretations are conceivable depending on the circumstances in which they are uttered. You may therefore have to sketch the utterance context you have in mind before describing the accessibility relation. □

Exercise 2.12 *Collect two naturally occurring examples of modalized sentences (e.g., sentences that you overhear in conversation, or read in a newspaper or novel – not ones that are being used as examples in a linguistics or philosophy paper!), and give definitions of values for the covert* $\langle s, st\rangle$*-variable which account for the way in which you actually understood these sentences when you encountered them. (If the appropriate interpretation is not salient for the sentence out of context, include information about the relevant preceding text or non-linguistic background.)* □

Exercise 2.13 *Modals can be iterated like other intensional operators:*

(48) *You might have to walk.*

Describe a context in which (48) would be an appropriate utterance. State plausible values for the $\langle s, st\rangle$ *flavor functions restricting the two modals. Posit an LF for (48). Derive the truth-conditions compositionally.* □

2.4 Explorations and variations

With the basic analyses of attitudes, conditionals, and modals in place, we turn to some technical explorations.

2.4.1 Accessibility relations

In all of the cases we have looked at the intensional operator ranges over worlds that are relevant related to the evaluation world (via some kind of anchor). In the case of conditional and modals, in fact, we made the operator take a relation between worlds as its contextually provided first argument:

(49) $$[\![\text{if}]\!]^{w,g} = \lambda f_{\langle s,st\rangle}.\ \lambda p_{\langle s,t\rangle}.\ \lambda q_{\langle s,t\rangle}.\ \forall w'\colon\ f(w)(w') = 1\ \&\ p(w') = 1 \rightarrow q(w') = 1.$$

(50) $$[\![\text{must}]\!]^{w,g} = \lambda f_{\langle s,st\rangle}.\ \lambda q_{\langle s,t\rangle}.\ \forall w'\colon\ f(w)(w') = 1 \rightarrow q(w') = 1.$$

In the Hintikka-semantics for attitudes as well, we think of the worlds quantified over as those that stand in a particular relation to the evaluation world. Which relation that is is lexically specified by the attitude predicate. So, *believe* ranges over worlds compatible with the subject's beliefs in the evaluation world.

It might be useful to rehearse some mathematical basics. A relation is something that two elements stand in. A relation might not be symmetrical (or as we say outside of math, it might not be reciprocated), so the order of elements is part of the notion. The formal model of a relation $\mathcal{R}$ therefore is a set of ordered pairs drawn from two sets A and B:

$$\mathcal{R} = \{\langle x, y\rangle :\ x \in A \ \&\ y \in B\}$$

In the function-centric and Schönfinkeled world we inherit from Heim & Kratzer, this notion of a set of ordered pairs becomes a staggered function that takes first an element from A, then an element from B, and then yields a truth-value:

$$f_{\mathcal{R}} = \lambda x :\ x \in A.\ (\lambda y :\ y \in B.\ \langle x, y\rangle \in \mathcal{R})$$

Since we are used to thinking of a function from some type to truth-values as characterizing a set of things of that type, we can reconceive a relation as a function from elements of the first set to a set of elements from the second set (the ones they stand in the relevant relation to).

Note that in the case of accessibility relations, we are dealing with a relation between worlds. That is, the elements that stand in the relation come from the same set.

One might call this an "endorelation".

Recall now (for example, from Section 6.6 of H&K) that the linguistic study of determiners benefitted quite a bit from an investigation of the formal properties of the relations between sets of individuals that determiners express. We can do the same thing here and ask about the formal properties of the accessibility relation associated with belief versus the one associated with knowledge, etc. The obvious properties to think about are reflexivity, transitivity, and symmetry.

Why do we call reflexivity, transitivity, and symmetry "formal" properties of relations? The idea is that certain properties are "formal" or "logical", while others are more substantial. So, the fact that the relation "have the same birthday as" is symmetric seems a more formal fact about it than the fact that the relation holds between my daughter and my brother-in-law. Nevertheless, one of the most common ways of characterizing formal/logical notions (permutation-invariance, if you're curious) does not in fact make symmetry etc. a formal/logical notion. So, while intuitively these do seem to be formal/logical properties, we do not know how to substantiate that intuition. See MacFarlane 2005 for discussion.

A relation is REFLEXIVE iff for any object in the domain of the relation we know that the relation holds between that object and itself. Which accessibility relations are reflexive? Take a knowledge-induced accessibility relation (let's call it $\mathcal{K}_x$):

(51) $w\mathcal{K}_x w'$ iff w' is compatible with what x knows in w.

We are asking whether for any given possible world w, we know that $\mathcal{K}_x$ holds between w and w itself. It will hold if w is a world that is compatible with what we know in w. And clearly that must be so. Take our body of knowledge in w. The concept of knowledge crucially contains the concept of truth: what we know must be true. So if in w, we know that something is the case then it must be the case in w. So, w must be compatible with all we know in w. $\mathcal{K}_x$ is reflexive.

We talk here about knowledge entailing (or even presupposing) truth but we do not mean to say that knowledge simply equals true belief. For initial overview of the issues, see the Stanford Encyclopedia entry: Ichikawa & Steup 2018.

Now, if an attitude X corresponds to a reflexive accessibility relation, then we can conclude from *a Xs that p* being true in w that p is true in w. This property of an attitude predicate is often called VERIDICALITY. It is to be distinguished from FACTIVITY, which is a property of attitudes which *presuppose* – rather than (merely) entail – the truth of their complement.

In modal logic notation: $\Box p \rightarrow p$. This pattern is sometimes called **T** or **M**, as is the corresponding system of modal logic.

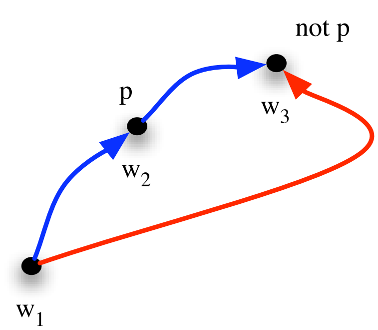
If we consider the relation $\mathcal{B}_x$ pairing with a world w those worlds w' which are compatible with what x *believes* in w, we no longer have reflexivity: belief is not a veridical attitude. It is easy to have false beliefs, which means that the actual world is not in fact compatible with one's beliefs, which contradicts reflexivity. And many other attitudes as well do not involve veridicality/reflexivity: what we hope may not come true, what we remember may not be what actually happened, etc.

The difference between *believe* and *know* in natural discourse is quite delicate, especially when one considers first person uses (*I believe the earth is flat* vs. *I know the earth is flat*).

In modal logic, the correspondence between formal properties of the accessibility relation and the validity of inference patterns is well-studied. What we have just seen is that reflexivity of the accessibility relation corresponds to the validity of $\Box p \rightarrow p$. Other properties correspond to other characteristic patterns. Let's see this for transitivity and symmetry.

TRANSITIVITY of the accessibility relation corresponds to the inference $\Box p \rightarrow \Box\Box p$. The pattern seems not obviously wrong for knowledge: if one knows that p, doesn't one thereby know that one knows that p? But before we comment on that, let's establish the formal correspondence between transitivity and that inference pattern. This needs to go in both directions.

In the literature on epistemic modal logic, the pattern is known as the KK THESIS or POSITIVE INTROSPECTION. In general modal logic, it is the characteristic axiom **4** of the modal logic system **S4**, which is a system that adds **4** to the previous axiom **M/T**. Thus, **S4** is the logic of accessibility relations that are both reflexive and transitive.



What does it take for the pattern to be valid? Assume that $\Box p$ holds for an arbitrary world w, i.e. that p is true in all worlds w' accessible from w. Now, the inference is to the fact that p again holds in any world w'' accessible from any of those worlds w' accessible from w. But what would prevent p from being false in some w'' accessible from some w' accessible from w? That could only be prevented from happening if we knew that w'' itself is accessible from w as well, because then we would know from the premiss that p is true in it (since p is true in *all* worlds accessible from w). Ah, but w'' (some world accessible from a world w' accessible from w) is only guaranteed to be accessible from w if the accessibility relation is transitive (if w' is accessible from w and w'' is accessible from w', then transitivity ensures that w'' is accessible from w). This reasoning has shown that validity of the pattern requires transitivity. The other half of proving the correspondence is to show that transitivity entails that the pattern is valid.

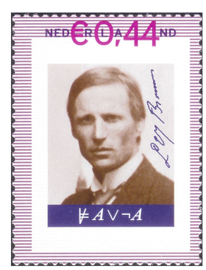
The proof proceeds by reductio. Assume that the accessibility relation is transitive. Assume that (i) $\Box p$ holds for some world w but that (ii) $\Box\Box p$ doesn't hold in w. We will show that this situation cannot obtain. By (i), p is true in all worlds w' accessible from w. By (ii), there is some non-p world w'' accessible from some world w' accessible from w. But by transitivity of the accessibility relation, that non-p world w'' must be accessible from w. And since *all* worlds accessible from w are p worlds, w'' must be a p world, in contradiction to (ii). So, as soon as we assume transitivity, there is no way for the inference not to go through.

Now, do any of the attitudes have the transitivity property? It seems rather obvious that as soon as you believe something, you thereby believe that you believe it (and so it seems that belief involves a transitive accessibility relation). And in fact, as soon as you believe something, you believe that you *know* it. But one might shy away from saying that knowing something automatically amounts to knowing that you know it. For example, many are attracted to the idea that to know something requires that (i) that it is true, (ii) that you believe it, and (iii) that you are justified in believing it: the justified true belief analysis of knowledge. So, now couldn't it be that you know something, and thus (?) that you believe you know it, and thus that you believe that you are justified in believing it, but that you are not justified in believing that you are *justified* in believing it? After all, one's source of knowledge, one's reliable means of acquiring knowledge, might be a mechanism that one has no insight into. So, while one can implicitly trust (believe) in its reliability, and while it is in fact reliable, one might not have any means to have trustworthy beliefs about it. [Further worries about the KK Thesis are discussed in Williamson 2000.]

What would the consequences be if the accessibility relation were SYMMETRIC? Symmetry of the accessibility relation $\mathcal{R}$ corresponds to the validity of the following principle:

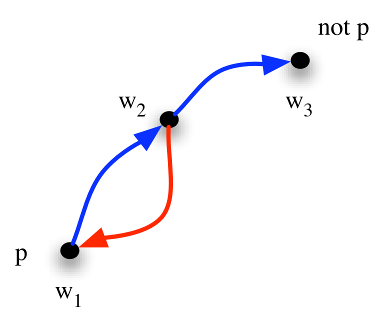
(52) Brouwer's Axiom :
$$\forall p \forall w : \ w \in p \rightarrow \Big[\forall w' \big[w \mathcal{R} w' \rightarrow \exists w'' \ [w' \mathcal{R} w'' \& \ w'' \in p]\big]\Big]$$

In modal logic notation, (52) looks as follows: $p \rightarrow \Box\Diamond p$, known simply as B in modal logic. The system that combines **T/M** with B is often called Brouwer's System (**B**), after the mathematician L.E.J. Brouwer, not because he proposed it but because it was thought that it had some connections to his doctrines. Brouwer got his own commemorative stamp from the Netherlands:



Here's the reasoning: Assume that R is in fact symmetric. Pick a world w in which p is true. Now, could it be that the right hand side of the inference fails to hold in w? Assume that it does fail. Then, there must be some world w' accessible from w in which $\Diamond p$ is false. In other words, from that world w' there is no accessible world w'' in which p is true. But since R is assumed to be symmetric, one of the worlds accessible from w' is w and in w, p is true, which contradicts the assumption that the inference doesn't go through. So, symmetry ensures the validity of the inference.

The other way (validity of the inference requires symmetry): the inference says that from any p-world ("p-world" is a very common way of saying "world of which p is true"), we can only access worlds from which, in turn, there is at least one accessible p-world. But imagine that p is true in w but not true in any other world. So, the only way for the conclusion of the inference to hold automatically is to have a guarantee that w (the only p-world) is accessible from any world accessible from it. That is, we need to have symmetry. QED.



To see whether a particular kind of attitude is based on a symmetric accessibility relation, we can ask whether Brouwer's Axiom is intuitively valid with respect to this attitude. If it is not valid, this shows that the accessibility relation can't be symmetric. In the case of a knowledge-based accessibility relation (epistemic accessibility), one can argue that *symmetry does not hold* (thanks to Bob Stalnaker (pc to Kai von Fintel) for help with the following reasoning):

> The symmetry condition would imply that if something happens to be true in the actual world, then you know that it is compatible with your knowledge (Brouwer's Axiom). This will be violated by any case in which your beliefs are consistent, but mistaken. Suppose that while p is in fact true, you feel certain that it is false, and so think that you know that it is false. Since you think you know this, it is compatible with your knowledge that you know it. (Since we are assuming you are consistent, you can't both believe that you know it, and know that you do not). So it is compatible with your knowledge that you know that *not* p. Equivalently: you don't know that you don't know that *not* p. Equivalently: you don't know that it's compatible with your knowledge that p. But by Brouwer's Axiom, since p is true, you would have to know that it's compatible with your knowledge that p. So if Brouwer's Axiom held, there would be a contradiction. So Brouwer's Axiom doesn't hold here, which shows that epistemic accessibility is not symmetric.

This and the following step rely on the duality of necessity and possibility: q is compatible with your knowledge iff you don't know that *not* q.

Game theorists and theoretical computer scientists who traffic in logics of knowledge often assume that the accessibility relation for knowledge is an equivalence relation (reflexive, symmetric, and transitive). But this is appropriate only if one abstracts away from any error, in effect assuming that belief and knowledge coincide. One striking consequence of working with an equivalence relation as the accessibility relation for knowledge is that one predicts the principle of NEGATIVE INTROSPECTION to hold:

All one really needs to make **NI** valid is to have a EUCLIDEAN accessibility relation: any two worlds accessible from the same world are accessible from each other. It is a nice little exercise to prove this, if you have become interested in this sort of thing. Note that all reflexive and Euclidean accessibility relations are transitive and symmetric as well — another nice little thing to prove.

(53) NEGATIVE INTROSPECTION (**NI**)
If one doesn't know that p, then one knows that one doesn't know that p. ($\neg\Box p \rightarrow \Box\neg\Box p$).

This surely seems rather dubious: imagine that one strongly believes that p but that nevertheless p is false, then one doesn't know that p, but one doesn't seem to believe that one doesn't know that p, in fact one believes that one does know that p.

2.4.2 *Conversational backgrounds*

There is a well-known technical variant on relativizing the semantics of intensional operators to accessibility relations. The basic idea, due to Kratzer, is to make intensional operators work with a set of propositions rather than a set of worlds. For example, *must* would say that a certain set of propositions together entail the prejacent. A conditional *if p, q* would say that a certain set of propositions when augmented with the antecedent proposition p will entail the consequent proposition q. And one could conceive of a belief state as providing a set of propositions (each of which is a belief of the agent), so that the intensional operator *believe* would claim that this set of propositions entails the prejacent.

We need to add the usual layer of contingency, which in this case means that the relevant ingredient is actually a function from evaluation worlds to sets of propositions.

At this point, we will assume that the resulting set of propositions is always consistent, by which we mean that there's at least one world where all of the propositions in the set are true:

(54) A set of propositions $\mathcal{P}$ is CONSISTENT relative to a set of worlds W iff $\exists w \in W\colon\ \forall p \in \mathcal{P}\colon\ p(w) = 1)$.

From any consistent set of propositions, we can retrieve the set of worlds characterized by it: those worlds such that each proposition in the set is true in them. If we think of propositions as sets of worlds, this corresponds to the grand intersection of the set of propositions:

(55) For any consistent set of propositions $\mathcal{P}$,
$char(\mathcal{P}) = \{w\colon\ \forall p \in \mathcal{P}\colon\ p(w) = 1\} = \bigcap(\mathcal{P})$

The semantics of the modal *must*, for example, can now be rewritten:

(56) $[\![\text{must}]\!]^{w,g} = \lambda \mathcal{M}_{\langle s,\langle\langle st,t\rangle\rangle\rangle}.\ \lambda p.\ \bigcap(\mathcal{M}(w)) \subseteq p$

The context supplies a function $\mathcal{M}$ from worlds to sets of propositions and *must* claims that when applied to the evaluation world, $\mathcal{M}$ yields a set of propositions that jointly entail the prejacent p, or in other words: all the worlds where all the propositions in $\mathcal{M}(w)$ are true are worlds where the prejacent p is true.

Kratzer called functions of type $\langle s, \langle\langle st, t\rangle\rangle\rangle$ CONVERSATIONAL BACKGROUNDS and used the term MODAL BASE for the conversational backgrounds that restrict modal operators. She also sometimes calls the resulting sets of propositions PREMISE SETS.

Note that we can retrieve accessibility relations from conversational backgrounds:

(57) For any conversational background $\mathcal{M}$ of type $\langle s, \langle st, t\rangle\rangle$, we define the corresponding accessibility relation $f_{\mathcal{M}}$ of type $\langle s, st\rangle$ as follows: $f_{\mathcal{M}} := \lambda w.\ \lambda w'.\ \forall p\ [\mathcal{M}(w)(p) = 1\ \rightarrow\ p(w') = 1]$.

In words, w' is $f_{\mathcal{M}}$-accessible from w iff all propositions p that are assigned by $\mathcal{M}$ to w are true in w'.

What motivates the complication from accessibility relations to conversational backgrounds? One consideration that we will turn to in the next chapter concerns inconsistent premise sets. But there is also another, perhaps more intuitive, motivation. A conversational background is the sort of thing that is identified by phrases like *what the law provides, what we know*, etc. Take the phrase *what the law provides*. What the law provides is different from one possible world to another. And what the law provides in a particular world is a *set of propositions*. Likewise, what we know differs from world to world. And what we know in a particular world is a set of propositions. The intension of *what the law provides* is then that function which assigns to every possible world the set of propositions p such that the law provides in that world that p. Of course, that doesn't mean that p holds in that world itself: the law can be broken. And the intension of *what we know* will be that function which assigns to every possible world the set of propositions we know in that world. Now, consider:

(58) (In view of what we know,) Brown must have murdered Smith.

The *in view of*-phrase may explicitly signal the intended conversational background. Or, if the phrase is omitted, we can just infer from other clues in the discourse that such an epistemic conversational background is intended.

What follows are some (increasingly technical exercises) on conversational backgrounds.

Exercise 2.14 *Imagine that we model individual x's belief state with a set of propositions $\mathcal{BS}_x$. Now, when x forms a new opinion, we could model this by adding a new proposition p to $\mathcal{BS}_x$. So, $\mathcal{BS}_x$ now contains one further element. There are now more opinions. What happens to the set of worlds compatible with x's beliefs? Does it get bigger or smaller? Is the new set a subset or superset of the previous set of compatible worlds?* □

"$\mathcal{BS}$" stands for "belief state".

Exercise 2.15 *Kratzer calls a conversational background (modal base)* REALISTIC *iff it assigns to* any *world a set of propositions that are all true in that world. The modal base* what we know *is realistic, the modal bases* what we believe *and* what we want *are not.*

Show that a conversational background $\mathcal{M}$ is realistic iff the corresponding accessibility relation $f_{\mathcal{M}}$ (defined as in (57)) is reflexive. □

Exercise 2.16 *Let us call an accessibility relation* TRIVIAL *if it makes every world accessible from every world. f is* TRIVIAL *iff $\forall w \forall w'$: $w' \in f(w)$. What would the conversational background $\mathcal{M}$ have to be like for the accessibility relation $f_{\mathcal{M}}$ to be trivial in this sense?* □

Exercise 2.17 *[For the intrepid only!] The definition in (57) specifies, in effect, a function from $D_{\langle s,\langle st,t\rangle\rangle}$ to $D_{\langle s,st\rangle}$. It maps each function $\mathcal{M}$ of type $\langle s, \langle st, t\rangle\rangle$ to a unique function $f_{\mathcal{M}}$ of type $\langle s, st\rangle$. This mapping is not one-to-one, however. Different elements of $D_{\langle s,\langle st,t\rangle\rangle}$ may be mapped to the same value in $D_{\langle s,st\rangle}$.*

Prove this claim. I.e., give an example of two functions $\mathcal{M}$ and $\mathcal{M}'$ in $D_{\langle s,\langle st,t\rangle\rangle}$ for which (57) determines $f_{\mathcal{M}} = f_{\mathcal{M}'}$.

As you have just proved, if every function of type $\langle s, \langle st, t\rangle\rangle$ qualifies as a 'conversational background', then two different conversational backgrounds can collapse into the same accessibility relation. Conceivably, however, if we imposed further restrictions on conversational backgrounds (i.e., conditions by which only a proper subset of the functions in $D_{\langle s,\langle st,t\rangle\rangle}$ would qualify as conversational backgrounds), then the mapping between conversational backgrounds and accessibility relations might become one-to-one after all. In this light, consider the following potential restriction:

(59) Every conversational background $\mathcal{M}$ must be "closed under entailment"; i.e., it must meet this condition:
$\forall w.\forall p\ [\cap\mathcal{M}(w) \subseteq p \ \rightarrow\ p \in \mathcal{M}(w)]$.
(In words: if the propositions in $\mathcal{M}(w)$ taken together entail p, then p must itself be in $\mathcal{M}(w)$.)

Show that this restriction would ensure that the mapping defined in (57) will be one-to-one. □

Further readings

We have just put in place some of the basics of the analysis of attitudes, conditionals, and modals. Much of the work in this area will become accessible to you after the following chapter. For now, we recommend just a few further readings.

Swanson 2011 is a recent survey on attitudes.

Further connections between mathematical properties of accessibility relations and logical properties of various notions of necessity and possibility are studied extensively in modal logic, see Hughes & Cresswell 1996 and Garson 2018, especially section 7 and 8, "Modal Axioms and Conditions on Frames", "Map of the Relationships between Modal Logics". An open access textbook on modal logic is Zach 2019.

We encourage you to visit the admirable Open Logic Project (https://openlogicproject.org/), that the Zach 2019 textbook is part of.

A thorough discussion of the possible worlds theory of attitudes, and some of its potential shortcomings, can be found in Bob Stalnaker's work (1984, 1999).

Two introductory readings on conditionals are von Fintel 2011, 2012.

The most important background readings on modals are the two papers Kratzer 1981, 1991. There are updated versions of Kratzer's classic papers in her volume "Modals and conditionals" (Kratzer 2012). A major resource on modality is Paul Portner's book: Portner 2009. You might also profit from some survey-ish type papers on modals and modality: von Fintel 2005, von Fintel & Gillies 2007, Swanson 2008, Hacquard 2009a.

3 Restricting and ordering

3.1 Conditionals and modals 47
3.1.1 First steps 47
3.1.2 A serious problem 49
3.1.3 *If*-clauses as restrictors 51
3.2 Ordering 53
3.2.1 The best we can get 54
3.2.2 Mathematical interlude on orderings 55
3.2.3 Ordering worlds 57
3.2.4 Kratzer's way 58
3.2.5 More cases of modal ordering 59
3.2.6 Ordering in conditionals 61
3.3 The interplay of ordering and restricting 62

3.1 Conditionals and modals

We have a basic theory of conditionals in place as well as a basic theory of modals. Both are treated as intensional operators that move us from the initial evaluation world to another set of worlds: in the case of conditionals, those worlds where the antecedent is true that are otherwise relevantly like the evaluation world; in the case of modals, to whatever worlds the contextually supplied accessibility relation assigns to the evaluation world. This means that if a sentence contains both a conditional and a modal, we expect them to work together to express nested intensional shifting.

Thus continues our "chemistry" experiments, combining expressions we have an initial theory about and seeing what happens when they are put in contact.

3.1.1 First steps

In many cases, the experiments seem to have the expected outcome. Consider the following attempt to embed a modal in the antecedent of a conditional:

(1) If one can get to that beach by bike, Iris did just that.

This seems to have the meaning we predict: the conditional take us to worlds where a certain modal fact holds—that the beach is reachable by bike — and then we say that in those worlds Iris went to the beach by bike.

Or consider our friend Howard again:

(2) If Howard has to pay a heavy fine, he will be broke.

Note that by itself, being under the obligation to pay a fine doesn't automatically mean that one does. So, (2) indirectly signals that if Howard has to pay a heavy fine, he will comply and thus he will be broke.

In our system, this could be captured by saying that the context supplies an f that assigns to the evaluation world only worlds where Howard does what he is obligated to, at least in as much as paying fines is concerned.

One thing that lots of people think is not straightforwardly possible is epistemic modals in conditionals antecedent. Papafragou 2006, for example, finds the following examples problematic:

(3) a. ?If Max must be lonely, his wife will be worried.
b. ?If Max may be lonely, his wife will be worried.

One possible explanation is that conditionals signal that the antecedent is "iffy". An epistemic modal statement can only be "iffy" if the speaker is not certain about what "the evidence" is. That can only be if "the evidence" is not the evidence that the speaker has full access to. Relevant examples include the "cancer scenarios" of DeRose 1991 and the "Mastermind" cases of von Fintel & Gillies 2007, 2011:

What exactly we might mean by "iffy" and how exactly conditionals signal iffiness is an interesting question. Any ideas?

(4) a. If John might have cancer [the doctors haven't told us], he will have to see an expert in Boston.
b. If there have to be two reds, your next move is obvious.

These seem to have acceptable readings where the conditional takes us to worlds where the embedded epistemic modal claim holds.

Consider next modals in the consequent of conditionals. On the face of it, examples abound:

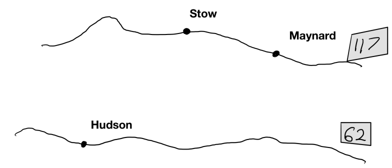
(5) a. If jaywalking is illegal here, then that guy has to pay a fine.
b. If Caspar vacuums on Saturday, then Chris has to cook on Sunday.
c. If Britanny drinks Coke, she must drink Coke.
d. If Howard returns his book late, he has to pay a fine.
e. If Cosette has to be home by midnight, she ought to think about leaving now.

(5b) is from a term paper by Moss. (5c) is due to Zvolenszky.

We will see, however, that there are significant challenges for compositional semantics hiding here. We begin with the interaction of epistemic modals and conditionals.

3.1.2 *A serious problem*

Our friends have been driving in the Massachusetts hinterlands, inexplicably without iPhones or GPS, and are relying entirely on an old-fashioned map. They've just passed through a little town with an iconic New England church and are looking on the map to try to figure out where they are. They have concluded that they are either on Route 117 or on Route 62. There are two plausible candidate towns on Route 117 (Maynard and Stow) and just one on Route 62 (Hudson).



So, given all the evidence available to them, there are three live possibilities: that they are in Hudson (on Rte 62, and they don't know it), that they are in Stow (on Rte 117, and they don't know it), and that they are in Maynard (on Rte 117, and they don't know it).

It's true when they say:

(6) We might be in Maynard.

since there are worlds compatible with their evidence where they are in Maynard.

It's true when they say:

(7) If we're on Route 62, we're in Hudson.

because of the three towns that they know they might be in, only Hudson is on Rte 62.

Our semantics for conditionals has the conditional take us to worlds that are (i) in $f(w)$, here in the set of worlds compatible with their evidence and (ii) are antecedent worlds. Among the worlds compatible with their evidence, all p-worlds (worlds where they are in Rte 62) are worlds where they are in Hudson.

So far so good. But here are some problematic cases (with the intuitive truth-value judgments in the given scenario):

(8) a. If we're on Route 117, we might be in Stow. True
b. If we're on Route 117, we might be in Hudson. False
c. If we're on Route 62, we must be in Hudson. True

We will now see that these cannot be explained in our framework!

Take (8c) first. The conditional takes us to those worlds that are (i) compatible with their evidence or with what they know (which includes their knowledge that they don't know in which of the three towns they are) and (ii) where they are on Rte 62. In all of those worlds, they are in Hudson, but in none of them do they *know* or *have any additional evidence* that they are in Hudson. So, we expect (8c) to be false, contrary to fact.

Now, take (8b). We go to worlds compatible with their evidence where they are on Route 117. All of them are worlds where they are on Route 117 without knowing that they are there, since they know that they don't know where they are. In those worlds, is it true that for all they know, they are in Hudson? Yes. Therefore, we predict that (8b) is true, again contrary to fact.

Finally, while we do predict that (8a) is true, the reason is simply that because of their ignorance, any world compatible with their evidence is a world where any of the relevant *might*-claims is true. So, the conditional antecedent is predicted to make no difference to the truth of the epistemic possibility claim in its consequent, contrary to fact (our intuitions are different for (8a) vs. (8b)).

Before we turn to one of the dominant ways of accounting for the meanings of the examples in (8), let's consider a tempting idea about (8c): what if the epistemic necessity modal *must* actually scopes over the conditional, even though it appears in its consequent? At first glance, the resulting meaning is not far off the target. The claim would be that in all the epistemically accessible worlds the conditional (7) *if we're on Rte 62, we're in Hudson* is true. We already saw that that conditional is true in the actual world, and there's no reason to say that it isn't true in other epistemically accessible worlds. So far, so good.

Unfortunately, there are several reasons for doubting that this LF with wide-scope for the modal is a convincing solution to our troubles:

1. For the *might*-version, we can't resort to wide-scoping the modal, because the epistemic conditional *if we're on Rte 117, we're in Stow* is false in the actual world and in fact in all epistemically accessible worlds: they have evidence that entails that it doesn't follow from being on Rte 117 that they're in Stow. So, (8a) would be predicted to be false under the wide-scope LF. So, no matter how we scope the modal, we either have trivial truth or falsity, and not what we want: non-trivial truth-conditions.
2. Epistemic *must* carries with it an "evidential signal" (von Fintel & Gillies 2010):

   (9) It must be raining.

   This signals that the speaker inferred the truth of *it is raining* indirectly on the basis of other evidence. The signal is still perceptible when the prejacent is a generalization or conditional:

   (10) a. Elephants must dislike bees.
   b. George must have to be home by midnight.
   c. It must be that if they're on Rte 62, they are in Clinton.

   But (8c) is not felt to have such a signal, other than that the antecedent would be an additional piece of information in the deduction that they're in Hudson.
3. We also need to be able to analyze epistemic conditionals with two modals in a conjunctive consequent (examples like this are discussed in Gillies 2010):

(11) If we're on Rte 62, we must be in Hudson and might be very close to the Horseshoe pub.

A wide-scope analysis of the modals in (11) seems impossible.

Conclusion: our analysis really is in trouble and can't be rescued by wide-scoping.

In the next section, we present the dominant treatment of the interaction of conditionals and modals in current linguistic semantics. There are alternatives that deserve to be considered, but that would lead us beyond the bounds of this textbook.

3.1.3 If-*clauses as restrictors*

The problem we have encountered here with the interaction of an *if*-clause and the modal operator *might* is similar to others that have been noted in the literature. Most influentially, Lewis in his paper "Adverbs of Quantification" (Lewis 1975) showed how hard it is to find an adequate analysis of the interaction of *if*-clauses and adverbs of quantification like *never, rarely, sometimes, often, usually, always* in sentences like these:

(12) $$\left\{\begin{array}{l}\text{Always}\\ \text{Usually}\\ \text{Often}\\ \text{Sometimes}\\ \text{Rarely}\\ \text{Never}\end{array}\right\} \text{ if a driver sees a friend walking, she offers her a ride.}$$

Lewis proposed that in these cases, the adverb is the only operator at work and that the *if*-clause serves to restrict the adverb. Thus, it has much the same function that a common noun phrase has in a determiner-quantification.

> The *if* of our restrictive *if*-clauses should not be regarded as a sentential connective. It has no meaning apart from the adverb it restricts. The *if* in *always if …, …, sometimes if …, …*, and the rest is on a par with the non-connective *and* in *between …and …*, with the non-connective *or* in *whether …or …*, or with the non-connective *if* in *the probability that …if …*. It serves merely to mark an argument-place in a polyadic construction. (Lewis 1975: p. 11)

Building on Lewis' insight, Kratzer argued for a uniform treatment of *if*-clauses as restrictors. She claimed that

> the history of the conditional is the story of a syntactic mistake. There is no two-place *if …then* connective in the logical forms of natural languages. *If*-clauses are devices for restricting the domains of various operators. (Kratzer 1986)

Let us repeat this:

(13) Kratzer's Thesis

If-clauses are devices for restricting the domains of various operators.

Kratzer's Thesis gives a unified picture of the semantics of conditional clauses. Note that it is not meant to supplant previous accounts of the *meaning* of conditionals. It just says that what those accounts are analyzing is not the meaning of *if* itself but the meaning of the operators that *if*-clauses restrict.

Let us see how this idea helps us with our most problematic case, *(8c) If we're on Route 62, we must be in Hudson*. The idea is to deny that there are two quantifiers over worlds in this sentence. Instead, the *if*-clause merely contributes a further restriction to the modal *must*. In effect, the modal is not quantifying over *all* the worlds compatible with our friends' knowledge but only over those where they are on Route 62. It then claims that all of those worlds are worlds where they are in Hudson. Since that is in fact true, we now correctly predict that (8c) is true.

Note that this is not the same as worlds where they are on Route 62 *and know that they are*. This is made vivid by examples ultimately due to Thomason (as noted in van Fraasen 1980): *If my partner is embezzling, I will never know (because I'm so bad at accounting)*.

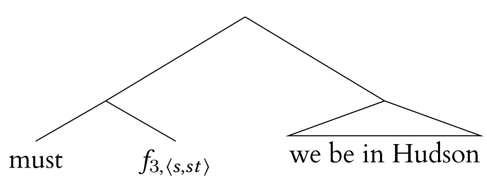
What we don't yet have is a compositional calculation. What does it mean in structural terms for the *if*-clause to be restricting the domain of the modal? We present here a particularly "flat-footed" implementation. The idea is that *if*-clauses serve as modifiers of the modal flavor argument of modals.

Recall the entry we had for *must*:

(14) $[\![\text{must}]\!]^{w,g} = \lambda f_{s,st}.\lambda p_{st}.\ \forall w':\ w' \in f(w) \longrightarrow\ p(w') = 1.$

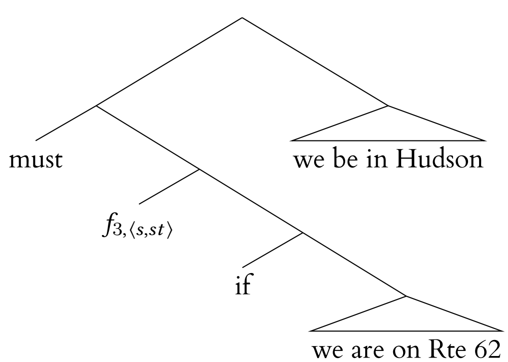
We supplied the modal with a covert "pronoun" of type $\langle s, st\rangle$ as its first argument:

(15)



We assume a structure where the *if*-clause is the sister of the f-pronoun:

(16)



The idea now is that the two restrictive devices work together: we just feed to the modal the *intersection* of (i) the set of worlds that are f_3-accessible from the actual world, and (ii) the set of worlds where they are on Route 62.

This implementation of the restrictor theory was considered (and, without much of an argument, dismissed) in Section 3.2 of von Fintel 1994 and is also found in lecture notes from 2004 by von Stechow. In Section 3.3 of von Fintel 1994, a different approach is developed, which is adopted (at least for illustration) by Kratzer 2015. The idea is that *if*-clauses are variable modifiers, something very like variable binders without entirely overwriting the current variable assignment. Further references about the restrictor theory are given at the end of this chapter.

Obviously, we don't hear *if*-clauses where they are located in this tree. So, on the way to PF, the *if*-clause must be moved to the periphery. Note that *if*-clauses can appear on the left periphery or on the right.

Exercise 3.1 *Write an appropriate lexical entry for* if *to make the structure in (16) interpretable. The idea is that* if *takes its antecedent proposition and an accessibility relation and returns a restricted accessibility relation that a world stands in (relative to the evaluation world) iff it stands in the original accessibility relation plus makes the antecedent true.* □

Exercise 3.2 *Consider the example with two modals in the consequent we gave earlier:*

(11) *If we're on Rte 62, we must be in Hudson and might be very close to the Horseshoe pub.*

How could this be analyzed? □

What about conditionals that do not contain a modal operator or any other operator that the *if*-clause could be restricting? Kratzer proposed that such bare conditionals contain implicit operators. There's a case for at least two such operators: (i) a covert generic frequency operator (giving rise to what are sometimes called "multi-case" conditionals), and (ii) a covert (epistemic?) necessity operator akin to *must* (giving rise to ordinary "one-case" conditionals):

The one-case vs. multi-case distinction is discussed and so-named in Kadmon 1987.

(17) a. If Polly sees a husky, she pets it.
b. If Kim left before 6am, she got there in time.

3.2 Ordering

Our semantics treats modals as quantifiers over worlds. The set of worlds they quantify over is supplied by context by way of assigning an accessibility function to a covert pronoun, and is optionally subject to being restricted by an *if*-clause, as developed just now.

We have distinguished (at least) epistemic and deontic accessibility functions:

(18) a. The keys must be in the car. — epistemic
b. Your guest can't stay past midnight. — deontic

For deontic modals, and perhaps more generally what Portner 2017 calls "priority" expressions, there is a strong argument that there is more at play.

3.2.1 The best we can get

Consider our friend Howard:

(19) [We think that Howard forgot to return his library book.]
He has to pay a $5 fine.

According to our analysis, the deontic modal *has to* here claims that he pays a $5 fine in all of the worlds compatible with what the relevant rules (the library regulations, in this case) require. But wait: surely the rules really require everyone to return their books on time! And so, the worlds compatible with the rules are all worlds where all books are returned on time, including Howard's, and thus nobody pays a fine. How can (19) actually be true?

It's clear that (19) is naturally understood in such a way that its truth depends *both* on facts about Howard's actions *and* facts about the library regulations. For instance, it will be judged true if (i) Howard did indeed fail to return his book, and (ii) the regulations mandate a fine in such cases. It may be false either because the regulations are different or because Howard did return the book. Our accessibility relation therefore needs to be more complex, *combining facts and regulations*.

A second attempt at specifying the accessibility relation might thus go something like this:

(20) $\lambda w. \lambda w'$. [what happened in w' up to now is the same as what happened in w and w' conforms to what the rules in w demands].

The proposal in (20) suggests that there's a temporal asymmetry here. But this is not necessarily so. Prakken & Sergot 1996 present the case of a set of formal and informal regulations governing the appearance and use of holiday cottages, which say that there are not to be any fences but that if there are fences, they must be white. In such a case, one could say: *This fence should be white*, expressing the kind of complex flavor we are dealing with.

The problem with (20) is that, unless there were no infractions of the rules at all in w up to now, no world w' will be accessible from w. Therefore, (19) is predicted to follow logically from the premise that Howard broke some rule. This does not represent our intuition about its truth conditions.

A better definition of the appropriate accessibility relation has to be even more complicated:

(21) $\lambda w. \lambda w'$. [what happened in w' up to now is the same as what happened in w and w' conforms at least as well to what the rules in w demands as does any other world in which what happened up to now is the same as in w].

(21) makes explicit that there is an important difference between the ways in which facts about Howard's actions on the one hand, and facts about the rules on the other, enter into the truth conditions of sentences like (19). Worlds in which Howard didn't do what he did are simply excluded from the domain of the modal here. Worlds in which the rules aren't obeyed are not absolutely excluded. Rather, we restrict the domain to those worlds in which the rules are obeyed as well as they can be, considering what has happened. We exclude only those worlds in which there

are infractions above and beyond those that are shared by all the worlds in which Howard has done what he has done. The analysis of (19) thus crucially involves the notion of an ordering of worlds: here they are ordered according to how well they conform to what the rules in *w* demand.

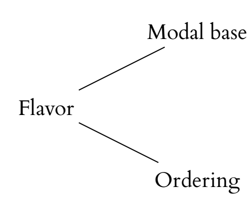
The diagnosis then is that the modal here is not a pure deontic modal. Rather, its flavor is *complex*. We take for granted the fact that Howard did not return his book on time. Consider then just those worlds where Howard did not return the book. None of those worlds are fully compatible with the rules. But among those worlds, the ones where he pays a fine satisfy the rules *as best as possible*. So, the flavor of the modal combines some actual world circumstances (the book was not returned on time) and what the rules require (late books incur a fine). And the flavor is *essentially* complex. Imagine that Howard is a scofflaw who never pays fines. If this fact were part of the flavor of the modal in (21), then we would expect the sentence to be false, but intuitively it is true. And as we already saw, a purely deontic reading would also predict the sentence to be false.

So, the flavor of the modal in (21) is best characterized as the following mixture: it quantifies over worlds where (i) the same relevant things happened as in the evaluation world and (ii) apart from that, things develop as best as possible according to the rules.

This is a very common pattern: intensional operators have complex flavors that combine a set of circumstances taken for granted and some way of identifying the best worlds within the set of worlds characterized by those circumstances.

If we want to stick to our simple semantics, with its flavor function (from evaluation worlds to sets of worlds quantified over), we have to locate the complexity in the pragmatics of determining a salient value to the context-dependent flavor. For (21), the contextual variable assignment would have to assign to the accessibility function variable a function like the one in (21).

Kratzer famously proposed a different diagnosis: modals are *doubly relative*, requiring two separate contextually supplied pieces of information. In addition to the accessibility relation or modal base (a function that assigns a set of accessible worlds to any evaluation world), modals are also sensitive to an ordering of the accessible worlds.



3.2.2 *Mathematical interlude on orderings*

All order relations are binary relations between the elements of a set. Here we're dealing with sets of worlds, but we can order soccer teams (by how good they are), people (by how rich they are, or by how close to Rome they were born), or race horses (by how they finished in the race). You get the idea.

In some sense, the most general kind of order is a preorder, one that you could read as "at least as highly ranked as". It would be reflexive, since any element is trivially at least as highly ranked as itself. And it would be TRANSITIVE, since if x is at least as highly ranked as y and y is at least as highly ranked as z, then surely x is at least as highly ranked as z.

When a preorder is also ANTI-SYMMETRIC (no distinct elements can have the same rank), it is called a PARTIAL ORDER. An order is STRICT if it actually doesn't allow elements at the same rank, so it is transitive but asymmetric. An order is TOTAL if it is a COMPLETE relation: any two distinct elements are related in one way or the other.

An order is WELL-FOUNDED if for any subset of the domain of the relation, there are elements in the subset that are "optimal": there is no other element in the subset that is strictly better by the ordering. Any order on a finite set is well-founded, but not every order on an infinite set is.

We provide two handy charts, one list of ours and one from Amartya Sen's influential book *Collective Choice and Social Welfare* (Sen 1970).

R is **irreflexive** on S	$\forall x \in S:\ \neg R(x,x)$
R is **reflexive** on S	$\forall x \in S:\ R(x,x)$
R is **transitive** on S	$\forall x,y,z \in S:\ R(x,y)\ \&\ R(y,z) \rightarrow R(x,z)$
R is **symmetric** on S	$\forall x,y \in S:\ R(x,y) \leftrightarrow R(y,x)$
R is **antisymmetric** on S	$\forall x,y \in S:\ R(x,y)\ \&\ R(y,x) \rightarrow x = y$
R is **asymmetric** on S	$\forall x,y \in S:\ R(x,y) \rightarrow \neg R(y,x)$
R is **complete** on S	$\forall x,y \in S:\ x \neq y \rightarrow R(x,y) \vee R(y,x)$
R is **dense** on S	$\forall x,y \in S:\ x \neq y\ \&\ R(x,y) \rightarrow$ $\exists z \in S:\ z \neq x\ \&\ z \neq y\ \&\ R(x,z)\ \&\ R(z,y)$
R is **well-founded** on S	$\forall S' \subseteq S\ \exists x \in S':$ $\neg\exists y \in S':\ R(y,x)\ \&\ \neg R(x,y)$

Figure 3.1: Properties of relations

The first few chapters of Sen's book are worth reading. A semantic introduction to orders can be found in Landman 1991. If you're curious about the mathematical aspects of this topic, we recommend Davey & Priestley 2002.

	Properties satisfied	Name to be used in this work	Other names used in the literature
1.	reflexivity and transitivity	quasi-ordering	pre-ordering
2.	reflexivity, transitivity and completeness	ordering	complete pre-ordering; complete quasi-ordering; weak ordering
3.	reflexivity, transitivity and anti-symmetry	partial ordering	ordering
4.	reflexivity, transitivity, completeness and anti-symmetry	chain	linear ordering; complete ordering; simply ordering
5.	transitivity and asymmetry	strict partial ordering	
6.	transitivity, asymmetry and completeness	strong ordering	ordering; strict ordering; strict complete ordering

Figure 3.2: Sen 1970's summary of types of orders

3.2.3 *Ordering worlds*

Armed with all this information we can return to the ordering of worlds. When we think about worlds being ordered with respect to how well they obey a law, for example, it's clear that we need to allow different worlds to be "tied", to be at the same rank. So, our order will need to be a preorder. It's also wise not to assume that for two given worlds, we can necessarily order them with respect to each other at all. So, our order will not be total. It will be a useful idealization if we assume that our orders will be well-founded. Then, we can assume that we can always find worlds that are "as good as possible". In sum, we assume that the context will supply a function from evaluation worlds to a well-founded preorder on worlds. So, now we can find for any set of worlds the contextually best worlds among them.

This is essentially the same as what is also known as the "Limit Assumption" in conditional/modal semantics. For discussion, see Lewis 1974 and now S. Kaufmann 2017.

Given a well-founded preorder $\leq$, we define a function from sets of worlds to subsets thereof that yields the $\leq$-best worlds in the original set:

Be aware that the literature is hopelessly inconsistent with respect to the direction of the symbols for "at least as good as" or "better than". Here, we use "$w_1 \leq w_2$" for "w_1 is at least as good w_2" because the intuition is that w_1 is less far from the ideal than w_2.

(22) For any set of worlds p and any well-founded preorder $\leq$ on W:
$$O_{\leq}(p) = \{w \in p : \neg\exists w'(w' \leq w \;\&\; \neg(w \leq w'))\}$$

We can now state the semantics for doubly-relative modals. Here's the new entry for *must*:

(23) $$[\![\text{must}]\!]^{w,g} = \lambda f_{s,st}.\ \lambda o_{s,sst} : o(w) \text{ is a well-founded preorder on } W.\ \lambda p_{st}.\ \forall w' \in O_{o(w)}(f(w)) : p(w') = 1$$

As before, the context supplies a flavor function f that yields for any world as set of f-accessible worlds. The context now also supplies a function that for any world yields a well-founded preorder. We will call this function the ORDERING SOURCE (a term from Kratzer). The semantics of modals uses the preorder to order the set of accessible worlds and to find the best accessible worlds. The modal *must*, as a necessity modal, claims that all of the best accessible worlds are worlds where its prejacent is true.

Exercise 3.3 *Write a doubly-flavored lexical entry for the possibility modal* may. □

Let's see how the analysis applies to (19): *Howard has to pay a fine.*

- The modal base will be a function that assigns to any evaluation world the set of worlds where the same relevant circumstances hold. Since in our stipulated evaluation world, Howard failed to return his book, the modal base will assign to our world a set of worlds that only contains worlds where Howard didn't return the book.
- The ordering source will be a function that assigns to any evaluation world an ordering that represents what the rules in that world prefer. For our cases and for the given evaluation world, the ordering will prefer worlds where no book is returned late to worlds

where books are returned late. It further prefers worlds where fines are paid for late books to worlds where no fines are paid.

- For our simple example then, any world in the modal base where Howard pays a fine will count as better than an otherwise similar world where he doesn't. The very best worlds simpliciter are worlds where there's never any late books, but since there aren't any such worlds in the modal base, the ordering has to make do with what it's given.
- Modals then make quantificational claims about the best worlds in the modal base (those for which there isn't a world in the modal base that is better than them).
- In our case, (19) claims that in the best worlds (among those where Howard failed to return his book), he pays a fine.

3.2.4 Kratzer's way

As we've seen in the previous chapter, Kratzer actually calculates the set of accessible worlds from a function of type $\langle s, \langle st, t\rangle\rangle$ (what she calls a *conversational background*). Recall that the construction is simple: find the worlds where all the propositions in the set are true. The additional power that comes with using sets of propositions remains essentially unused in the case of modal bases. So, to simplify matters, we will stick with our earlier assumption that the domain of modals is a set of worlds resulting from applying a contextually supplied value of a pronoun of type $\langle s, st\rangle$ to the evaluation world.

For the ordering component of the doubly-relative semantics of modals, Kratzer's construction is quite intuitive. We assume that the ordering source argument is a function from evaluation worlds to sets of propositions. The idea is now that such a set $\mathcal{P}$ of propositions can be used to order the worlds in the modal base.

For our example about Howard's fine, we might imagine that for our evaluation world, the ordering source assigns a set of propositions that contains (among others) the following two propositions: (i) nobody returns books late, (ii) anybody who returns a book late pays a fine. It's crucial here that the second proposition is (vacuously) true when nobody returns books late.

For any pair of worlds w_1 and w_2, we say that w_1 comes closer than w_2 to the ideal set up by $\mathcal{P}$ (in symbols: $w_1 <_{\mathcal{P}} w_2$), iff the set of propositions from $\mathcal{P}$ that are true in w_2 is a proper subset of the set of propositions from $\mathcal{P}$ that are true in w_1.

Here's a more precise derivation of the ordering procedure:

(24) For any set of propositions $\mathcal{P}$, we define a strict partial order $<_P$:
$\forall w', w'' :\ w' <_P w''$ iff
$$\forall p \in \mathcal{P}\,(w'' \in p \rightarrow w' \in p) \text{ and } \exists p \in \mathcal{P}\,(w' \in p \wedge w'' \notin p)$$
w' is better than w'' according to $\mathcal{P}$ iff all propositions in P that hold in w'' also hold in w' but some hold in w' that do not also hold in w''.

And of course, once we have such a strict partial order and we assume well-foundedness, as before, we can define the selection function that gives us the set of $<_{\mathcal{P}}$-best worlds from any set X of worlds:

(25) $O_{<_{\mathcal{P}}}(X) = \{w \in X : \neg\exists w'(w' <_{\mathcal{P}} w)\}$

Finally, we need to rewrite the semantics of modals so that they are sensitive both to a modal base/accessibility relation and an ordering source of the Kratzerian kind:

(26) $[\![\text{must}]\!]^w = \lambda f_{s,st}.\lambda o_{s,stt}.\lambda p.\ \forall w' \in O_{o(w)}(f(w)) : p(w) = 1.$

Exercise 3.4 *Give an LF for the sentence* Ashlyn must leave *that will work with the entry for* must *in (26).* □

3.2.5 More cases of modal ordering

The combination of "facts on the ground" and "doing the best with what we have" is pervasive in intensional constructions. We will discuss a succession of brief case-studies but there's still much to explore.

How to get to Harvard Square. Our initial example involved deontic modality. Next is the closely connected case of goal-oriented or teleological modality. You want to get to Harvard Square from MIT. This is your primary goal. You have secondary goals: to not spend a lot of money and to stay reasonably safe. I say:

(27) You ought to take the Red Line.

A two-factor analysis of (27) might look like this:

In von Fintel & Iatridou 2008, a *three*-factor analysis is proposed, distinguishing technically between the primary goal and a secondary ordering source. There's further discussion in work by Rubinstein (2012, 2014).

- The modal base is circumstantial. It assigns to the evaluation world a set of propositions describing the relevant circumstances. Imagine that for our world, the following facts are relevant: (i) you are in Kendall Square, (ii) you can get to Harvard Square on the Red Line, by taxi or Lyft, or on foot, (iii) the Red Line costs \$1.25, takes 10 minutes, and is safe, (iv) the taxi/Lyft costs \$10, is fast, and is safe, (v) walking is free, slow, and possibly unsafe this time of the night.
- The ordering source describes your goals (often called a *teleological* ordering source). Your relevant goals in our world are: (i) you get to Harvard Square, (ii) you pay less than \$2, (iii) you are safe.
- What are the best worlds in the modal base according to the given ordering source? The worlds where you take the Red Line.
- That's why it seems true to say (27).

The weakness of *must*. A more controversial application of the two-factor analysis is found in Kratzer 1991: she argues that apparently strong necessity modals in their epistemic construal are actually weakened by ordering. Consider someone who observes people coming into a windowless classroom with wet outer clothing. They might say:

(28) It must be raining.

Kratzer argues that what underlies (28) is a combination of an epistemic modal base and an ordering source based on not completely exceptionless generalization and assumptions. In more recent work, von Fintel & Gillies 2010 have argued that any weakness in the meaning of (28) has to do with an "evidential signal" (that the evidence for the prejacent is merely indirect) rather than a lack of logical strength.

The debate continues with Lassiter 2016, Del Pinal & Waldon 2019, von Fintel & Gillies 2020.

Wanting what's best. Intriguingly, we can find the facts + ordering phenomenology in the semantics of propositional attitudes as well, most clearly with bouletic attitudes like *want*. Heim 1992 tells the story of someone who in her dream scenario would not teach next semester but knows that she will have to. She does have preferences on what week days to teach. So, she says:

(29) I want to teach Tuesdays and Thursdays next semester.

In terms of the two-factor analysis, the idea here is that the accessibility relation is "doxastic": the modal base contains the worlds compatible with the agent's beliefs. In the case at hand, all worlds in that modal base have her teach. The ordering is based on her preferences: among the worlds in the modal base, she prefers the ones where she teaches Tuesdays and Thursdays.

Heim herself gives a dynamic semantics analysis for attitudes that incorporates these insights. In later work, von Fintel 1999 gives a static version of a two-factor analysis that is closely modeled after Kratzer's analysis of modals. See also Rubinstein 2017 for a recent discussion.

The Good Samaritan. Prior 1958 introduced the following "Paradox of the Good Samaritan". Imagine that someone has been robbed and John is walking by. It is easy to conceive of a code of ethics that would make the following sentence true:

(30) John ought to help the person who was robbed.

In our previous one-factor semantics for modals, we would have said that (30) says that in all of the deontically accessible worlds (those compatible with the code of ethics) John helps the person who was robbed. Prior's point was that under such a semantics, something rather unfortunate holds. Notice that in all of the worlds where John helps the person who was robbed, someone was robbed in the first place. Therefore, it will be true that in all of the deontically accessible worlds, someone was robbed. Thus, (30) will entail:

(31) It ought to be the case that someone was robbed.

It clearly would be good not make such a prediction.

The doubly-relative analysis of modality can successfully avoid this unfortunate prediction. We conceive of (30) as being uttered with respect to a circumstantial modal base that includes the fact that someone was robbed. Among those already somewhat ethically deficient worlds, the relatively best ones are all worlds where John helps the victim.

Note that we still have the problematic fact that among the worlds in the modal base, all are worlds where someone was robbed, and we would thus appear to still make the unfortunate prediction that (31) should be true. But this can now be fixed. For example, we could say that *ought p* is semantically defective if p is true throughout the worlds in the modal base. This could be a presupposition or some other ingredient of meaning. So, with respect to a modal base which pre-determines that someone was robbed, one couldn't felicitously say (31).

Consequently, saying (31) would only be felicitous if a different modal base is intended, one that contains both p and non-p worlds. And given a choice between worlds where someone was robbed and worlds where nobody was robbed, most deontic ordering sources would probably choose the no-robbery worlds, which would make (31) false, as desired.

3.2.6 Ordering in conditionals

Historically, the use of an ordering of possible worlds in the semantics of intensional operatores was first proposed by Stalnaker and Lewis in their work on conditionals (Stalnaker 1968, Lewis 1973). In our attempt at an analysis for conditionals in Section 2.2, we used a one-factor approach where conditionals quantify over *all* contextually accessible antecedent worlds. Stalnaker and Lewis argue that the logic of conditionals, primarily counterfactuals, reveals the effects of an ordering. Consider the case of a perfectly normal match in front of us and the following two counterfactual conditionals:

Even though Lewis' work appeared five years after Stalnaker's, it was independently developed. See Arvan 2015 and Fn.29 in Starr 2019 for further details about the intellectual history of these ideas.

The match makes an early appearance in Goodman 1947.

(32) a. If I had struck this match, it would have lit.
b. If I had dipped this match in water and struck it, it would have lit.

Intuitively, it is clear that (32a) may well be true while (32b) is almost certainly false. But according to our previous analysis, which is a version of what in the trade is called a "strict conditional" analysis, this cannot be. If all of the accessible worlds where I strike this match are worlds where it lights, then this must *a fortiori* also be true of that subset of the accessible worlds where I strike this match after having dipped it in water.

In the Stalnaker-Lewis ordering semantics, (32b) doesn't anymore follow from (32a). The most highly ranked worlds where I strike the match might well be worlds where I don't dip it in water first, and so (32a) could be true while (32b) is false.

Within this book's framework, we would replace the meaning for *if* from (10) in Section 2.2 with the following two-factor meaning:

(33) $[\![\text{if}]\!]^{w,g} = \lambda f_{s,st}.\lambda o_{s,stt}.\lambda p_{st}.\lambda q_{st}.\ \forall w' \in O_{o(w)}(f(w) \cap p):\ q(w') = 1.$

The crucial move here is that the antecedent proposition p is used to find the p-worlds that are among the f-accessible worlds. The resulting set of accessible p-worlds is then ordered and the best accessible worlds p are what the universal quantifier makes a claim about.

But wait. In Section 3.1, we argued that *if*-clauses don't actually do their own modal quantification, so the analysis in (33) can't be quite what we should adopt here. In fact, from the restrictor point of view, the semantics of conditionals and their reliance on an ordering on possible worlds should be a compositional effect: the *if*-clause serves as a restrictor for a (possibly covert) modal and it is that modal that is sensitive to an accessibility relation and an ordering. In the next section, we explore the interplay of conditionals and modals in a theory that involves modal bases and orderings. Things are not entirely straightforward.

3.3 *The interplay of ordering and restricting*

We return to Howard and the saga of the possibly late library book. Imagine that we don't know whether he returned it late but we're confident we understand the library regulations. So, we say:

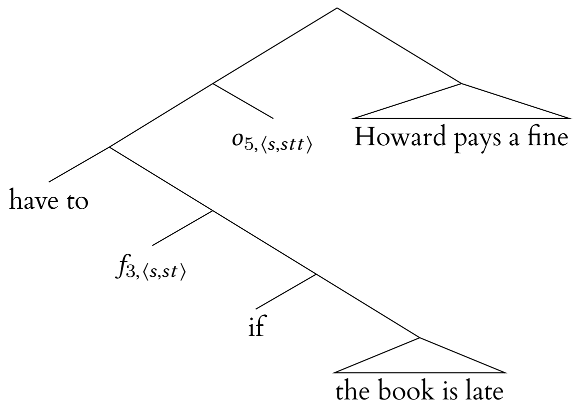
(34) If Howard returned his book late, he has to pay a fine.

There is one analysis that will not work: the restrictor analysis plus the pragmatically sophisticated but semantically simple idea that modals are just sensitive to an accessibility relation even when that relation incorporates considerations of ordering. So, assume that the accessibility relation is the one that delivers the best worlds among those that match the evaluation world with respect to whether the book was late or not. Now, imagine a scenario where unbekownst to us, Howard actually returned the book on time. It would seem that this would not threaten the intuitive truth of (34). But according to the analysis under consideration, we predict something odd: the worlds that match the evaluation world are all worlds where the book was not late and the best ones among them thus are also not-late worlds (and of course they are worlds without a fine). The *if*-clause would then try to find late worlds in that set of accessible worlds, but since there are none, we would get an empty set of worlds to quantify over, resulting either in vacuous truth or some kind of infelicity. This is not correct.

One might think that instead of starting with the worlds that match the evaluation world in whether the book was returned late, we could start with the worlds compatible with our evidence. Then, we would include both worlds where the book was late and where it wasn't. But even among those, the not-late worlds are the best. So, then again, the *if*-clause could not find any antecedent worlds among the accessible worlds.

The two-factor analysis (modal base f, ordering o) can help with the issue. The idea is that the *if*-clause does not restrict the o-best f-worlds but restricts f before o enters into the picture. In other words, we posit an LF like this one:

(35)



To evaluate this LF, we can use the entry for *if* you developed in Exercise 3.1 and the entry for modals in (26). Note that for the *if*-clause not to be an idle restrictor, the worlds f delivers need to include worlds where the antecedent is true.

Exercise 3.5 *In fact, calculate the truth-conditions for (35) with the ingredients just mentioned.* □

Exercise 3.6 *At this point, it would be very instructive to read Kratzer 1991. In particular, you should study her version of the Samaritan Paradox in her Section 3.2 and her solution in her Section 8.* □

It appears then that we have found vindication both for the two-factor semantics for modals and for the restrictor theory. There's a problem though, illustrated by Zvolenszky 2002 with the following example:

(36) If Britney Spears drinks Coke in public, then she must drink Coke in public.

An intuitively plausible use case for (36) would be a situation where we consider the possibility of seeing Spears drinking Coke in public. We conclude that if we saw her drinking Coke, we would be able to deduce that she is somehow contractually obligated to drink Coke.

Unfortunately, our current analysis predicts that (36) should, if anything, be vacuously true rather than making a contingent claim. The *if*-clause would restrict whatever accessibility relation we're using and ensure that we're only dealing with worlds where Spears drinks Coke in public. Then, the ordering would apply to find the best such worlds. Finally, the modal would say that the best such worlds are worlds where she drinks Coke in public. That is of course trivially true.

Consider what is needed to make the modal claim (that Spears must drink Coke in public) non-trivial. The domain of quantification needs to include worlds where she doesn't drink Coke (perhaps she drinks Pepsi,

While Zvolensky's 2002 SALT paper was instrumental in bringing the issue into the natural language semantics literature, it had been already identified by Frank 1996 and in fact, it had been well-known as the "if p, ought p" problem in the more logico-philosophical world. See Carr 2014 for references and an important recent contribution.

As a matter of fact, Spears was paid to endorse Pepsi.

or Poland Spring Water). Then, the modal's ordering source identifies the best worlds and the modal says that all of those are Coke worlds. This non-triviality can only arise if *if*-clause doesn't in fact restrict the modal to Coke worlds in the first place. So, is the restrictor theory in trouble?

At this point, we must recall that under the restrictor theory, *if*-clauses can also restrict covert modals. This means that when we have an *if*-clause and an overt modal, we can't automatically conclude that the conditional is restricting that modal. It might be restricting another modal, a covert one, instead.

So, let's look at (36) in that light. Imagine that the *if*-clause is restricting a higher covert epistemic modal. Then, we are taken from the evaluation world to those worlds compatible with our evidence where Spears drinks Coke. We are saying that in all of those worlds Spears is obligated to drink Coke. That will be a non-trivial claim if the worlds the lower modal quantifies over contains both Coke and non-Coke worlds.

More technically, we'd be working with an LF like the one sketched in (37). We use $\Box$ for the covert modal. We need to distinguish four modal parameter "pronouns".

(37) $\Box$ (f_1 if Coke) (o_1) [must f_2 o_2 Coke]

What values for the four parameter would yield the reading we want? f_1 should give us the worlds compatible with our evidence. o_1 could be trivial or the kind of not entirely reliable evidence that Kratzer 1991 argues for. Note that the higher modal will pass down only Coke-worlds. Crucially, f_2 would give us both Coke and non-Coke worlds when applied to the epistemically accessible worlds passed on by the higer operator. o_2, finally, would encode Spear's contractual obligations.

With this, we have rescued the restrictor theory. The existence of such nested structures is predicted by the framework and now we see that they are empirically attested.

See Geurts 2004 for a discussion of how this kind of structure is expected.

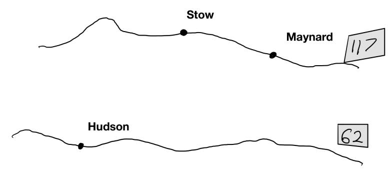
Let's return to the case of Howard and his book again. As it turns out, we could get the correct reading for our sentence *If Howard returned the book late, he has to pay a fine* without having *if* directly restrict the overt modal. Assume that the *if*-clause restricts a higher covert modal, again maybe an epistemic one. And now, in contrast to the Spears example, assume that the lower f-relation assigns to any of the worlds passed down (all of which will be late book worlds) a set of worlds matching the facts about whether the book was returned late. So, in effect, the conditional antecedent will end up restricting both modals: the higher one directly and the lower one indirectly since the lower accessibility relation will ensure matching of the relevant fact.

This possibility led Frank 1996 to propose that deontic modals in fact never are directly restricted:

> There are in fact no truly deontically modalized *if*-conditionals. Instead, we assume conditionals with a deontic modal operator in the consequent clause to be analyzed throughout in terms of an implicit or explicit epistemically (or circumstantially) based modal operator. The deontic modal adverb is then to be analyzed within the scope of the 'higher' epistemic modal operator.

Now, if this is in fact feasible, why should we then still maintain the restrictor analysis in the first place? Why not go back to the earlier idea that *if* is a modal operator in its own right?

To evaluate this possibility, let's return once more to our friends who do not know precisely where they are. Imagine that they have reason to think that among the two Rte 117 possibilities, Stow is much more likely than Maynard, perhaps because they know Maynard a bit better and think that if they were there, they would very likely recognize some building or other. They have no reason to think that Stow is more likely than Hudson (which is on Rte 62, remember). So, they say:



(38) If we're on Route 117, we ought to be in Stow.

Now, to be clear: they are still lost, so their evidence is compatible with them being on either of the two routes and in any of the three towns. So, for (38) to have a chance of being true, the modal *ought* here can't simply map the evaluation world to the worlds epistemically accessible from it (and then potentially order the resulting worlds). We need to narrow things down to the Route 117 worlds, even though our friends don't know whether they're there, even if they are. So, as things stand, we still need the restrictor analysis. And we need the restriction to happen *before* the ordering applies. So, for now, (38) provides the strongest argument we have for both the restrictor theory and the two-factor semantics for modals.

At this point, we leave this intriguing and complex inquiry and refer you to the ever-growing literature, some we've already mentioned and more we will list below.

Further readings

Conditionals. In-depth overviews of the logico-philosophical perspective on conditionals include Nute 1984, Edgington 1995, Bennett 2003, and Starr 2019.

Ideas pertaining to the implementation of the restrictor theory are found in Reich 2009 and Ebert, Ebert & Hinterwimmer 2014. Schlenker 2004 presents an alternative that is at least plausibly in the same orbit. One should really take into account what we know about the syntax of conditionals, some of which is discussed in Chapter 3 of Kai's thesis but much more is found in Bhatt & Pancheva 2006. Rothschild 2011 explains the restrictor theory to philosophers, might be useful as an additional reading.

Gillies 2010 proposes an ingenious alternative to the restrictor theory. Khoo 2011 files a complaint about the coverage of Gillies' proposal. The interaction of conditionals with probability operators is very tricky, see Egré & Cozic 2011 and von Fintel & Gillies 2015 for some discussion.

Higginbotham 1986 identified the interaction of negative quantifiers like *no student* with conditionals as a compositionality puzzle. The restrictor theory has sometimes been seen as a solution to the puzzle (von Fintel 1998) and sometimes not (von Fintel & Iatridou 2002). See Dekker 2001, Higginbotham 2003, Leslie 2009, Huitink 2010, Klinedinst 2011, Kratzer 2015, Lauer & Nadathur 2016 among others.

To understand *only if*-conditionals, von Fintel 1997 needs to work hard to neutralize the universal force of bare conditionals stemming from Kratzer's implicit necessity modal.

Herburger 2015, 2016, and Bassi & Bar-Lev 2017 consider the possibility that bare conditionals are actually (sometimes or always) existentially quantified.

Attitudes. Linguistic work on attitudes has often been concerned with various co-occurrence patterns, particularly which moods (indicative or subjunctive or infinitive) occur in the complement and whether negative polarity items are licensed in the complement. Mood licensing: Portner 1997. NPI-Licensing: Kadmon & Landman 1993, von Fintel 1999, Giannakidou 1999.

Tamina Stephenson in her MIT dissertation and related work explores the way attitude predicates interact with epistemic modals and taste predicates in their complements: Stephenson 2007a,b.

Jon Gajewski in his MIT dissertation and subsequent work explores the distribution of the NEG-RAISING property among attitude predicates and traces it back to presuppositional components of the meaning of the predicates: Gajewski 2005, 2007.

Interesting work has also been done on presupposition projection in attitude contexts: Asher 1987, Heim 1992, Geurts 1998.

Modals. On the syntax of modals, there are only a few papers of uneven quality: Bhatt 1997, Wurmbrand 1999, Cormack & Smith 2002, Butler 2003. Follow up on older references from the bibliographies in these papers. The following paper explores some issues in the LF-syntax of epistemic modals: von Fintel & Iatridou 2003,

Valentine Hacquard's MIT dissertation is a rich source of cross-linguistic issues in modality, as is Fabrice Nauze's Amsterdam dissertation: Hacquard 2006, Nauze 2008.

Some more recent work by Hacquard deals with deriving and correlating modal flavors with syntactic position of the modal auxiliaries: Hacquard 2010, Hacquard 2013. A recent handbook article by Hacquard on actuality entailments (involving the interaction of modality with aspect; we'll discuss aspect later in these notes): Hacquard 2016.

The semantics of epistemic modals has become a hot topic recently. Here are some of the main references: Hacking 1967, Teller 1972, DeRose 1991, Egan, Hawthorne & Weatherson 2005, Egan 2007, MacFarlane 2006. Stephenson 2007a, Hawthorne 2007, von Fintel & Gillies 2008a, von Fintel & Gillies 2008b.

A paper by Pranav Anand and Valentine Hacquard tackles what happens to epistemic modals under attitude predicates: Anand & Hacquard 2013.

Evidentiality is a topic closely related to epistemic modality. Some references: Willett 1988, Aikhenvald 2004, Drubig 2001, Blain & Déchaine 2007, McCready & Ogata 2007, Speas 2008, von Fintel & Gillies 2010.

Modals interact with disjunction and indefinites to generate so-called FREE CHOICE-readings, which are a perennial puzzle. Here is just a very small set of initial references: Kamp 1973, Zimmermann 2000, Schulz 2005, Aloni 2007, Alonso-Ovalle 2006, Fox 2007, van Rooij 2006.

Restricting and ordering. Very important for the problem we ended the chapter on are these papers: Kaufmann & Schwager 2009 and Condoravdi & Lauer 2016.

PART II

Escaping opacity

4 *Specificity and Transparency*

4.1 Predictions of our framework 71
4.2 Raised subjects 76
4.2.1 Non-specific readings for raised subjects . . . 76
4.2.2 Syntactic "Reconstruction" 80
4.2.3 Some Alternatives to Syntactic Reconstruction 83
4.3 Further readings 88

4.1 *Predictions of our framework*

When a DP occurs in the scope of an intensional operator, our framework makes clear predictions. Consider, for example:

(1) Emma wanted Taylor to buy a book about soccer.

Imagine that we give the following simplified meaning to *want*:

We know from the previous chapter exactly in what sense this meaning is simplified: it doesn't capture the interplay of beliefs and preferences.

(2) $[\![\text{want}]\!]^{w,g} = \lambda p.\lambda x.\ \forall w'\colon\ x\text{'s wants in } w \text{ are satisfied in } w' \rightarrow p(w') = 1.$

In other words, x *wants* p is true iff p is true in all worlds where x's wants are satisfied. Further, assume that the DP *a book about soccer* is interpreted within the embedded clause. Then, we claim, (1) will be true iff in all of the worlds that satisfy all of Emma's wants, there is a book about soccer that Taylor buys. (You prove this claim in the following exercise.)

Exercise 4.1 *Draw the obvious, if simplified, LF for (1) and calculate its truth-conditions.* □

Now, consider what happens if the object of the lower verb QRs and adjoins to the matrix clause:

(3) [a book about soccer] (1 [Emma wanted Taylor to buy t_1])

When you calculate the truth-conditions of (3) [please do so], you will get a result that is very different from the previous exercise. Now what is claimed is that there is a book about soccer, call it x, such that in all of the worlds satisfying all of Emma's wants Taylor buys x.

There are two important differences between the truth-conditions that our framework assigns to these two LFs.

Quantifier scope: Since *want* is a universal quantifier over worlds and *a book about soccer* is an existential quantifier over individuals, there's a question about the relative scope of the two quantifiers. In the first truth-conditions we sketched, the existential quantifier scopes under the universal quantifier: for every world there is an individual such that bla-bla. In the second LF, the existential quantifier scopes over the universal one: there is an individual such that in every world yadda-yadda. The most common terminology for this difference involves the pair *specific/non-specific*: the sentence (or the object DP) is interpreted specifically if the DP takes scope over the intensional operator, and it is interpreted non-specifically if the DP takes scope under the intensional operator.

Predicate evaluation: When the existential quantifier scopes over the intensional operator, this also has the effect that the predicate contained in it, *book about soccer*, is evaluated in the matrix evaluation world. And when the quantifier takes lower scope, its predicate is evaluated in the worlds that the intensional operator shifts to. One evocative terminology for whether the predicate is evaluated with respect to the matrix evaluation world or the worlds shifted to by the intensional operator is *transparent/opaque*. A predicate evaluated relative to the matrix world is called *transparent*. A predicate evaluated in the worlds shifted to by an intensional operator is receiving an *opaque* interpretation.

There's another terminological pair that is very common: *de re/de dicto*. One way to conceive of that distinction in our framework is that it stands for a particular combination of the two distinctions we just introduced: *de re* means specific and transparent, and *de dicto* means non-specific and opaque.

Caution about terminology: terminological confusion and exuberance is rampant in this area (and many others). In a way, terminology is just a short-hand way to pick out salient properties of LFs (or their denotation). It's the latter that truly matters. One particular problematic aspect of the terminology is its binary nature, while the relevant distinctions are actually more complex, especially as soon as we are dealing with nested intensionality.

Exercise 4.2 *Consider the sentence* Emma must want Taylor to buy a book about soccer*. One can imagine using this to describe a scenario where we are seeing Taylor enter a bookstore known to cater to soccer aficionados. For some reason we won't go into, we come to the conclusion that there is a specific book about soccer that Emma must have asked Taylor to buy. But at the same time, we have no idea what that book might be, so there's not a specific book about which we made our deduction. This suggest that we may want to give the object DP intermediate scope. So, draw an LF that corresponds to this idea and calculate its truth-conditions.* □

The notion of *specificity* is particularly laden with complexity. We use it here to indicate the scope of a DP with respect to an intensional operator. There are quite a few other conceptions, usefully surveyed by von Heusinger 2011. One that is very common is the notion that specific DPs indicate that the speaker has a referent or referents "in mind", something that may even be relevant in the absence of intensional operators. Consider:

(4) A man is knocking on the door.

The speaker might base such a claim on circumstantial evidence (hearing a particularly masculine knock on the door) or on clear visual evidence (seeing the man through the window). In the second case, we're tempted to say that the speaker has a specific referent "in mind" while in the first case there is non-specific existential quantification.Whether this is merely a pragmatic distinction about what is grounding the speaker's claim or whether the LF reflects this difference in "specificity" is a debatable issue. If one adopted the idea that all matrix sentences contain an epistemic operator, such as Meyer's "Matrix K" operator (Meyer 2013), then we would have a genuine LF ambiguity if DPs such as *a man* could take scope over or under that operator.

For discussion, see for example Ludlow & Neale 1991.

Our recommendation is to use the terms *specific/non-specific*, *transparent/opaque*, *de re/de dicto* only with extreme caution. They are sometimes useful shorthands, but unless it is crystal-clear what properties of LFs/denotations you are using them to pick out, they are more likely to be a source of obfuscation and confusion.

Let's look at some more examples of the ambiguity predicted by our framework as soon as we allow for the possibility of an embedded DP to take scope either under or over a relevant intensional operator. A classic kind of example is (5), which is of the same form as our initial example and contains the DP *a plumber* inside the infinitival complement of *want*.

(5) Mesut wants to marry a plumber.

According to the non-specific reading, every possible world in which Mesut gets what he wants is a world in which there is a plumber whom he marries. According to the specific reading, there is a plumber in the actual world whom Mesut marries in every world in which he gets what he wants. We can imagine situations in which one of the readings is true and the other one false.

For example, suppose Mesut thinks that plumbers make ideal spouses, because they can fix things around the house. He has never met one so far, but he definitely wants to marry one. In this scenario, the non-specific reading is true, but the specific reading is false. What all of Mesut's desire-worlds have in common is that they have a plumber getting married to Mesut in them. But it's not the same plumber in all those worlds. In fact, there is no particular individual (actual plumber or other) whom he marries in every one of those worlds.

For a different scenario, suppose that Mesut has fallen in love with Robin and wants to marry Robin. Robin happens to be a plumber, but Mesut doesn't know this; in fact, he wouldn't like it and might even call off the engagement if he found out. Here the specific reading is true, because there is an actual plumber, viz. Robin, who gets married to Mesut in every world in which he gets what he wants. The non-specific reading is false, however, because the worlds which conform to Mesut's wishes actually do not have him marrying a plumber in them. In his favorite worlds, he marries Robin, who is not a plumber in those worlds.

When confronted with this second scenario, you might, with equal justification, say 'Mesut wants to marry a plumber', or 'Mesut *doesn't* want to marry a plumber'. Each can be taken in a way that makes it a true description of the facts — although, of course, you cannot assert both in the same breath.This intuition fits well with the idea that we are dealing with a genuine ambiguity.

Actually, *why* wouldn't one be able to assert both sentences in the same breath, if both have a true reading?

Let's look at another example:

(6) Sari believes that your abstract will be accepted.

Here the relevant DP in the complement clause of the verb *believe* is *your abstract*. Again, we detect an ambiguity, which is brought to light by constructing different scenarios.

(i) Sari's belief may be about an abstract that she reviewed, but since the abstract is anonymous, she doesn't know who wrote it. She told me that there was a wonderful abstract about subjacency in Hindi that is sure to be accepted. I know that it was your abstract and inform you of Sari's opinion by saying (6). This is the specific reading. In the same situation, the non-specific reading is false: Among Sari's belief worlds, there are many worlds in which *your abstract will be accepted* is not true or even false. For all she knows, you might have written, for instance, that terrible abstract about Antecedent-Contained Deletion, which she also reviewed and is positive will be rejected.

(ii) For the other scenario, imagine that you are a famous linguist, and Sari doesn't have a very high opinion about the fairness of the abstract selection process. She thinks that famous people never get rejected, however the anonymous reviewers judge their submissions. She believes (correctly or incorrectly — this doesn't matter here) that you submitted a (unique) abstract. She has no specific information or opinion about the abstract's content and quality, but given her general beliefs and her knowledge that you are famous, she nevertheless believes that your abstract will be accepted. This is the non-specific reading. Here it is true in all of Sari's belief worlds that you submitted a (unique) abstract and it will be accepted. The specific reading of (6), though, may well be false in this scenario. Suppose—to flesh it out further—the abstract you actually submitted is that terrible one about ACD. That one surely doesn't get accepted in every one of Sari's belief worlds. There may be some where it gets in (unless Sari is certain it can't be by anyone famous, she has to allow at least the possibility that it will get in despite its low quality). But there are definitely also belief-worlds of hers in which it doesn't get accepted.

We have taken care here to construct scenarios that make one of the readings true and the other false. This establishes the existence of two distinct readings. We should note, however, that there are also many possible and natural scenarios that simultaneously support the truth of *both* readings. Consider, for instance, the following third scenario for sentence (6).

(iii) Sari is your adviser and is fully convinced that your abstract will be accepted, since she knows it and in fact helped you when you were writing it. This is the sort of situation in which both the non-specific and the specific reading are true. It is true, on the one hand, that the sentence *your abstract will be accepted* is true in every one of Sari's belief worlds (non-specific reading). And on the other hand, if we ask whether the abstract which you actually wrote will get accepted in each of Sari's belief worlds, that is likewise true (specific reading).

In fact, this kind of "doubly verifying" scenario is very common when we look at actual uses of attitude sentences in ordinary conversation. There may even be many cases where communication proceeds smoothly without either the speaker or the hearer making up their minds as to which of the two readings they intend or understand. It doesn't matter, since the possible circumstances in which their truth-values would differ are unlikely and ignorable anyway. Still, we *can* conjure up scenarios in which the two readings come apart, and our intuitions about those scenarios do support the existence of a semantic ambiguity.

Exercise 4.3 *For the two examples just discussed, we can explain their non-specific (and opaque) interpretation via LFs where the relevant DP remains inside the scope of the intensional operator at LF:*

(7) *Mesut wants [[a plumber]$_1$ [PRO$_2$ to marry t_1]]*

(8) *Sari believes [the abstract-by-you will-be-accepted]*

To obtain the specific (and transparent) readings, we apparently have to QR the DP to a position above the intensional predicate, minimally the VP headed by want *or* believe*.*

(9) *[a plumber]$_1$ [Mesut wants [PRO_2 to marry t_1]]*

(10) *[the abstract-by-you]$_1$ [Sari believes t_1 will-be-accepted]*

Calculate the interpretations of the four structures in (7) — (10), and determine their predicted truth-values in each of the (types of) possible worlds that we described above in our introduction to the ambiguity.

Some assumptions to make the job easier: (i) Assume that (7) and (9) are evaluated with respect to a variable assignment that assigns Mesut to the number 2. This assumption takes the place of a worked out theory of how controlled PRO is interpreted. (ii) Assume that abstract-by-you *is an unanalyzed one-place predicate. This takes the place of a worked out theory of how genitives with a non-possessive meaning are to be analyzed.* □

4.2 Raised subjects

In the examples of ambiguities that we have looked at so far, the surface position of the DP in question was inside the modal predicate's clausal or VP-complement. We saw that if it stays there at LF, a non-specific opaque reading results, and if it covertly moves up above the modal operator, we get a specific transparent reading. In the present section, we will look at cases in which a DP that is superficially *higher* than a modal operator can still be read non-specifically. In these cases, it is the specific reading which we obtain if the LF looks essentially like the surface structure, and it is the non-specific reading for which we apparently have to posit a non-trivial covert derivation.

4.2.1 Non-specific readings for raised subjects

Suppose I come to my office one morning and find the papers and books on my desk in different locations than I remember leaving them the night before. I say:

(11) Somebody must have been here (since last night).

On the assumptions we have been making, *somebody* is base-generated as the subject of the VP *be here* and then moved to its surface position above the modal. So (11) has the following S-structure, which is also an interpretable LF.

(12) somebody [2 [[must f] [t_2 have-been-here]]]

What does (12) mean? The appropriate reading for *must* here is epistemic, so suppose the variable f is mapped to the relation $[\lambda w.\lambda w'.\ w'$ is compatible with the evidence in $w]$. Let w_0 be the utterance world. Then the truth-condition calculated by our rules is as follows.

(13) (12) is true iff $\exists x[x$ is a person in w_0 & $\forall w'[w'$ is compatible with the evidence in $w_0 \rightarrow$ x was here in $w']]$

But this is not the intended meaning. For (13) to be true, there has to be a person who in every world compatible with the evidence was in my office. In other words, all the relevant worlds have to have one and the same person coming to my office. But this is not what you intuitively understood me to be saying about the evidence when I said (11). The context we described suggests that I do not know (nor have any opinion about) which person it was that was in my office. For all I know, it might have been John, or it might have been Mary, or it have been this stranger here, or that stranger there. In each of the relevant worlds , somebody or other was in my office, but no one person was there in all of them. I do not believe of anyone in particular that he or she was there, and you did not understand me to be saying so when I uttered (11). What you did understand me to be claiming, apparently, was not (13) but (14).

(14) $\forall w'[w'$ is compatible with the evidence in w_0 $\rightarrow \exists x\ [x$ is a person in w' & x was here in $w']]$

In other words—to use the terminology we introduced in the last section—the DP *somebody* in (11) appears to have a non-specific reading.

How can sentence (11) have the meaning in (14)? The LF in (12), as we saw, means something else; it expresses a specific reading, which typically is false when (11) is uttered sincerely. So there must be another LF. What does it look like and how is it derived? One way to capture the intended reading, it seems, would be to generate an LF that's essentially the same as the underlying structure we posited for (11), i.e., the structure *before* the subject has raised:

(15) $[_{\text{IP}}$ e $[_{\text{I}'}$ [must f] [somebody have-been-here]]]

(15) means precisely (14) (assuming that the unfilled Spec-of-IP position is semantically vacuous), as you can verify by calculating its interpretation by our rules. So is (15) (one of) the LF(s) for (11), and what assumption about syntax allow it to be generated? Or are there other—perhaps less obvious, but easier to generate—candidates for the non-specific LF-structure of (11)?

Before we get into these question, let's look at a few more examples. Each of the following sentences, we claim, has a non-specific reading for the subject, as given in the accompanying formula. The modal operators in the examples are of a variety of syntactic types, including modal auxiliaries, main verbs, adjectives, and adverbs.

(16) Everyone in the class may have received an A.
$\exists w'[w'$ conforms to what I believe in w &
$\forall x[x$ is in this class in $w' \rightarrow x$ received an A in $w']]$.

(17) At least two semanticists have to be invited.
$\forall w'[w'$ conforms to what is desirable in w
$\rightarrow \exists_2 x\ [x$ is a semanticist in w' & x is invited in $w']]$.

(18) Somebody from New York is expected to win the lottery.
$\forall w'[w'$ conforms to what is expected in w
$\rightarrow \exists x[x$ is a person from NY in w' & x wins the lottery in $w']]$

(19) Somebody from New York is likely to win the lottery.
$\forall w'[w'$ is as likely as any other world, given what I know in w
$\rightarrow \exists x[x$ is a person from NY in w' & x wins the lottery in $w']]$

(20) One of these two people is probably infected.
$\forall w'[w'$ is as likely as any other world, given what I know in w
$\rightarrow \exists x[x$ is one of these two people & x is in infected in $w']]$

To bring out the intended non-specific reading of the last example (to pick just one) imagine this scenario: We are tracking a dangerous virus infection and have sampled blood from two particular patients. Unfortunately, we were sloppy and the blood samples ended up all mixed up in one container. The virus count is high enough to make it quite probable that one of the patients is infected but because of the mix-up we have no evidence about which one of them it may be. In this scenario, (20) appears to be true. It would not be true under a specific reading, because neither one of the two people is infected in every one of the likely worlds.

Excursus. Hopefully the exact analysis of the modal operators *likely* and *probably* is not too crucial for the present discussion, but you may still be wondering about it. As you see in our formula, we are thinking of *likely* (*probably*) as a kind of epistemic necessity operator, i.e., a universal quantifier over a set of worlds that is somehow determined by the speaker's knowledge. (We are focussing on the "subjective probability" sense of these words. Perhaps there is also an "objective probability" reading that is circumstantial rather than epistemic.) What is the difference then between *likely* and e.g. epistemic *must* (or *necessary* or *I believe that*)? Intuitively, 'it is likely that p' makes a weaker claim than 'it must be the case that p'. If both are universal quantifiers, then, it appears that *likely* is quantifying over a smaller set than *must*, i.e., over only a proper subset of the worlds that are compatible with what I believe. The difference concerns those worlds that I cannot strictly rule out but regard as remote possibilities. These worlds are included in the domain for *must*, but not in the one for *likely*. For example, if there was a race between John and Mary, and I am willing to bet that Mary won but am not completely sure she did, then those worlds where John won are remote possibilities for me. They are included in the domain of *must*, and so I will not say that Mary *must* have won, but they are not in the domain quantified over by *likely*, so I do say that Mary is *likely* to have won.

This is only a very crude approximation, of course. For one thing, probability is a gradable notion. Some things are more probable than others, and where we draw the line between what's probable and what isn't is a vague or context-dependent matter. Some people even believe that *must*, *necessary* etc. arguably don't really express complete certainty (because in practice there is hardly anything we are completely certain of), but rather just a very high degree of probability. For more discussion of *likely*, *necessary*, and other graded modal concepts in a possible worlds semantics, see e.g. Kratzer 1981, Yalcin 2010, Lassiter 2017.

A different approach may be that *likely* quantifies over the same set of worlds as *must*, but with a weaker, less than universal, quantificational force. I.e., 'it is likely that p' means something like p is true in *most* of the worlds conforming to what I know. A *prima facie* problem with this idea is that presumably every proposition is true in infinitely many possible worlds, so how can we make sense of cardinal notions like 'more' and 'most' here? But perhaps this can be worked out somehow. **End of excursus.**

A word of clarification about our empirical claim: We have been concentrating on the observation that non-specific readings are *available*, but have not addressed the question whether they are the *only* available readings or coexist with equally possible specific readings. Indeed, some of the sentences in our list appear to be ambiguous: For example, it seems that (18) could also be understood to claim that there is a particular New Yorker who is likely to win (e.g., because he has bribed everybody). Others arguably are not ambiguous and can only be read non-specific. This is what von Fintel & Iatridou (2003) claim about sentences like (16). They note that if (16) also allowed a specific reading, it should be possible to make coherent sense of (21).

(21) Everyone in the class may have received an A. But not everybody did.

In fact, (21) sounds contradictory, which they show is explained if only the non-specific reading is permitted by the grammar. They conjecture that this is a systematic property of epistemic modal operators (as opposed to deontic and other types of modalities). Epistemic operators always have widest scope in their sentence.

Some follow up on von Fintel & Iatridou 2003 can be found in Tancredi 2008, Swanson 2010

So there are really two challenges here for our current theory. We need to account for the existence of non-specific readings, and also for the absence, in at least some of our examples, of specific readings. We will be concerned here exclusively with the first challenge and will set the second aside. We will aim, in effect, to set up the system so that all sentences of this type are in principle ambiguous, hoping that additional constraints that we are not investigating here will kick in to exclude the specific readings where they are missing.

To complicate the empirical picture further, there are also examples where raised subjects are unambiguously specific. Such cases have been around in the syntactic literature for a while, and they have received renewed attention in the work of Lasnik and others. To illustrate just one of the systematic restrictions, negative quantifiers like *nobody* seem to permit only surface scope (i.e., wide scope) with respect to a modal verb or adjective they have raised over.

(22) Nobody from New York is likely to win the lottery.

(22) does not have a non-specific reading parallel to the one for (19) above, i.e., it cannot mean that it is likely that nobody from NY will win. It can only mean that there is nobody from NY who is likely to win. This too is an issue that we set aside.

For a thorough investigation of low scope readings of negative DPs, see Iatridou & Sichel 2011.

In the next couple of sections, all that we are trying to do is find and justify a mechanism by which the grammar is able to generate both specific and non-specific readings for subjects that have raised over modal operators. It is quite conceivable, of course, that the nature of the additional constraints which often exclude one reading or the other is ultimately relevant to this discussion and that a better understanding of them may undermine our conclusions. But this is something we must leave for further research.

4.2.2 Syntactic "Reconstruction"

Given that the non-specific reading of (11) we are aiming to generate is equivalent to the formula in (14), an obvious idea is that there is an LF which is essentially the pre-movement structure of this sentence, i.e., the structure prior to the raising of the subject above the operator. There are a number of ways to make such an LF available.

One option, most defended in Sauerland & Elbourne (2002), is to assume that the raising of the subject can happen in a part of the derivation which only feeds PF, not LF. In that case, the subject simply stays in its underlying VP-internal position throughout the derivation from DS to LF. (Recall that quantifiers are interpretable there, as they generally are in subject positions.)

Another option is a version of the so-called Copy Theory of movement introduced in Chomsky (1993). This assumes that movement generally proceeds in two separate steps, rather than as a single complex operation as we have assumed so far. Recall that in H&K, it was stipulated that every movement effects the following four changes:

(i) a phrase α is deleted,

(ii) an index i is attached to the resulting empty node (making it a so-called trace, which the semantic rule for "Pronouns and Traces" recognizes as a variable),
(iii) a new copy of α is created somewhere else in the tree (at the "landing site"), and
(iv) the sister-constituent of this new copy gets another instance of the index i adjoined to it (which the semantic rule of Predicate Abstraction recognizes as a binder index).

If we adopt the Copy Theory, we assume instead that there are three distinct operations:

"Copy": Create a new copy of α somewhere in the tree, attach an index i to the original α, and adjoin another instance of i to the sister of the new copy of α. (= steps (ii), (iii), and (iv) above)
"Delete Lower Copy": Delete the original α. (= step (i) above)
"Delete Upper Copy": Delete the new copy of α and both instances of i.

The Copy operation is part of every movement operation, and can happen anywhere in the syntactic derivation. The Delete operations happen at the end of the LF derivation and at the end of the PF deletion. We have a choice of applying either Delete Lower Copy or Delete Upper Copy to each pair of copies, and we can make this choice independently at LF and at PF. (E.g., we can do Copy in the common part of the derivation and than Delete Lower Copy at LF and Delete Upper Copy at PF.) If we always choose Delete Lower Copy at LF, this system generates exactly the same structures and interpretations as the one from H&K. But if we exercise the Delete Upper Copy option at LF, we are effectively undoing previous movements, and this gives us LFs with potentially new interpretations. In the application we are interested in here, we would apply the Copy step of subject raising before the derivation branches, and then choose Delete Lower Copy at PF but Delete Upper Copy at LF. The LF will thus look as if the raising never happened, and it will straightforwardly get the desired non-specific reading.

If the choice between the two Delete operations is generally optional, we in principle predict ambiguity wherever there has been movement. Notice, however, first, that the two structures will often be truth-conditionally equivalent (e.g. when the moved phrase is a name), and second, that they will not always be both interpretable. (E.g., if we chose Delete Upper Copy after QRing a quantifier from object position, we'd get an uninterpretable structure, and so this option is automatically ruled out.) Even so, we predict lots of ambiguity. Specifically, since raised subjects are always interpretable in both their underlying and raised locations, we predict all raising structures where a quantificational DP has raised over a modal operator (or over negation or a temporal operator) to be ambiguous. As we have already mentioned, this is not factually correct, and so there must be various further constraints that somehow restrict the choices. (Similar comments apply, of course, to the option we mentioned first, of applying raising only on the PF-branch.)

Yet another solution was first proposed by May (1977): May assumed that QR could in principle apply in a "downward" fashion, i.e., it could adjoin the moved phrase to a node that doesn't contain its trace. Exercising this option with a raised subject would let us produce the following structure, where the subject has first raised over the modal and then QRed below it.

(23) t_j λ_i [must f [someone λ_j [t_i have been here]]]

As it stands, this structure contains at least one free variable (the trace t_j) and can therefore not possibly represent any actual reading of this sentence. May further assumes that traces can in principle be deleted, when their presence is not required for interpretability. This is not yet quite enough, though to make (23) interpretable, at least not within our framework of assumptions, for (24) is still not a candidate for an actual reading of (11).

(24) λ_i [must f [someone λ_j [t_i have been here]]]

We would need to assume further that the topmost binder index could be deleted along with the unbound trace, and also that the indices i and j can be the same, so that the raising trace t_j is bound by the binding-index created by QR. If these things can be properly worked out somehow, then this is another way to generate the non-specific reading. Notice that the LF is not exactly the same as on the previous two approaches, since the subject ends up in an adjoined position rather than in its original argument position, but this difference is presumably without semantic import.

What all of these approaches have in common is that they place the burden of generating the non-specific reading for raised subjects on the syntactic derivation. Somehow or other, they all wind up with structures in which the subject is lower than it is on the surface and thereby falls within the scope of the modal operator. They also have in common that they take the modal operator (here the auxiliary, in other cases a main predicate or an adverb) to be staying put. I.e., they assume that the non-specific readings are not due to the modal operator being covertly higher than it seems to be, but to the subject being lower. Approaches with these features will be said to appeal to "syntactic reconstruction" of the subject.

This is a very broad notion of "reconstruction", where basically any mechanism which puts a phrase at LF in a location nearer to its underlying site than its surface site is called "reconstruction". In some of the literature, the term is used more narrowly. For example, May's downward QR is sometimes explicitly contrasted with genuine reconstruction, since it places the quantifier somewhere else than exactly where it has moved from.

4.2.3 *Some Alternatives to Syntactic Reconstruction*

Besides (some version of) syntactic reconstruction, there are many other ways in which one can try to generate non-specific readings for raised subjects. Here are some other possibilities that have been suggested and/or readily come to mind. We will see that some of them yield exactly the non-specific reading as we have been describing it so far, whereas others yield a reading that is very similar but not quite the same. We will confine ourselves to analyses which involve no or only minor changes to our system of syntactic and semantic assumptions. Obviously, if one departed from these further, there would be even more different options, but even so, there seem to be quite a few.

Let us look again at (11) and its apparent surface structure (12), both repeated here:

(11) Somebody must have been here (since last night).

(12) somebody [2 [[must f] [t_2 have-been-here]]]

1. Raising the modal operator, variant 1: no trace

Conceivably, an LF for the non-specific reading of (11) might be derived from the S-structure (=(12)) by covertly moving *must* (and its covert *R*-argument) up above the subject. This would have to be a movement which leaves no (semantically non-vacuous) trace. Given our inventory of composition rules, the only type that the trace could have to make the structure containing it interpretable would be the type of the moved operator itself (i.e. $\langle st, t\rangle$). If it had that type, however, the movement would be semantically inconsequential, i.e., the structure would mean exactly the same as (12). So this would not be a way to provide an LF for the non-specific reading. If there was no trace left however (and also no binder index introduced), we indeed would obtain the non-specific reading.

Exercise 4.4 *Prove the claims we just made in the previous paragraph. Why is no type for the trace other than $\langle st, t\rangle$ possible? Why is the movement semantically inert when this type is chosen? How does the correct intended meaning arise if there is no trace and binder index?* □

2. Raising the modal operator, variant 2: trace of type s

[Requires slightly modified inventory of composition rules. Derives an interpretation that is not quite the same as the non-specific opaque reading we have assumed so far. Rather, it is the non-specific transparent "third" reading discussed in the next chapter.]

3. Higher type for trace of raising, variant 1: type $\langle et, t\rangle$

[Before reading this section, read and do the exercise on p.212/3 in H&K]

So far in our discussion, we have taken for granted that the LF which corresponds to the surface structure, viz. (12), gives us the specific reading. This, however, is correct only on the tacit assumption that the trace of raising is a variable of type e. If it is part of our general theory that all variables, or at least all interpretable binder indices (hence all bound variables), in our LFs are of type e, then there is nothing more here to say. But it is not *prima facie* obvious that we must or should make this general assumption, and if we don't, then the tree in (12) is not really one single LF, but the common structure for many different ones, which differ in the type chosen for the trace. Most of the infinitely many semantic types we might assign to this trace will lead to uninterpretable structures, but there turns out to be one other choice besides e that works, namely $\langle et, t\rangle$:

(25) somebody $\lambda_{2,\langle et,t\rangle}$ [[must f] [$t_{2,\langle et,t\rangle}$ have-been-here]]

(25) is interpretable in our system, but again, as in the previous approach, the predicted interpretation is not exactly the non-specific reading as we have been describing it so far, but the non-specific transparent third reading.

That a trace of type $\langle et, t\rangle$ does not in fact yield the targeted non-specific opaque reading had not been noticed until we bothered to calculate the meaning of (25). For example, Fox 2000, which derives from a dissertation supervised by us, is unaware of the fact that a high-type but extensional trace gives a scope-reconstructed but transparent reading.

Exercise 4.5 *Using higher-type traces to "reverse" syntactic scope-relations is a trick which can be used quite generally. It is useful to look at a non-intensional example as a first illustration. (26) contains a universal quantifier and a negation, and it is scopally ambiguous between the readings in (a) and (b).*

(26) Everything that glitters is not gold.

a. $\forall x[x \text{ glitters} \rightarrow \neg x \text{ is gold}]$ *"surface scope"*

b. $\neg\forall x[x \text{ glitters} \rightarrow x \text{ is gold}]$ *"inverse scope"*

We could derive the inverse scope reading for (26) by generating an LF (e.g. by some version of syntactic reconstruction") in which the every-*DP is below* not. *Interestingly, however, we can also derive this reading if the* every-*DP is in its raised position above* not *but its trace has the type* $\langle\langle e, t\rangle, t\rangle$.

Spell out this analysis. (I.e., draw the LF and show how the inverse-scope interpretation is calculated by our semantic rules.) □

Exercise 4.6 *Convince yourself that there are no other types for the raising trace besides e and* $\langle et, t\rangle$ *that would make the structure in (12) interpretable. (At least not if we stick exactly to our current composition rules.)* □

4. Higher type for trace of raising, variant 2: type $\langle s, \langle et, t\rangle\rangle$

If we want to get *exactly* the non-specific reading that results from syntactic reconstruction out of a surface-like LF of the form (12), we must use an even higher type for the raising trace, namely $\langle s, \langle\langle e, t\rangle, t\rangle\rangle$, the type of the intension of a quantifier. As you just proved in the exercise, this is not possible if we stick to exactly the composition rules that we have currently available. The problem is in the VP: the trace in subject position is of type $\langle s, \langle\langle e, t\rangle, t\rangle\rangle$ and its sister is of type $\langle e, t\rangle$. These two cannot combine by either FA or IFA, but it works if we employ another variant of functional application.

(27) *Extensionalizing Functional Application* (EFA)
If α is a branching node and $\{\beta, \gamma\}$ the set of its daughters, then, for any world w and assignment g:
if $[\![\beta]\!]^{w,g}(w)$ is a function whose domain contains $[\![\gamma]\!]^{w,g}$,
then $[\![\alpha]\!]^{w,g} = [\![\beta]\!]^{w,g}(w)([\![\gamma]\!]^{w,g})$.

Notice that the problem here is kind of the mirror image of the problem that led to the introduction of "Intensional Functional Application" in H&K, ch. 12. There, we had a function looking for an argument of type $\langle s, t\rangle$, but the sister node had an extension of type t. IFA allowed us to, in effect, construct an argument with an added "s" in its type. This time around, we have to get rid of an "s" rather than adding one; and this is what EFA accomplishes.

So we now have three different "functional application"-type rules altogether in our system: ordinary FA simply applies $[\![\beta]\!]^{w}$ to $[\![\gamma]\!]^{w}$; IFA applies $[\![\beta]\!]^{w}$ to $\lambda w'.[\![\gamma]\!]^{w'}$; and EFA applies $[\![\beta]\!]^{w}(w)$ to $[\![\gamma]\!]^{w}$. At most one of them will be applicable to each given branching node, depending on the type of $[\![\gamma]\!]^{w}$.

Think about the situation. Might there be other variant functional application rules?

Exercise 4.7 *Calculate the truth-conditions of (12) under the assumption that the trace of the subject quantifier is of type $\langle s, \langle\langle e, t\rangle, t\rangle\rangle$.* □

Can we choose between all these options?

Two of the methods we tried derived readings in which the raised subject's *quantificational determiner* took scope below the world-quantifier in the modal operator, but the raised subject's *restricting NP* still was evaluated in the utterance world (or the evaluation world for the larger sentence, whichever that may be), in other words: a non-specific but transparent interpretation. It is difficult to assess whether such readings are actually available for the particular sentences under consideration, and we will postpone this question to the next chapter. We would like to argue here, however, that even if these readings are available, they cannot be the *only* readings that are available for raised subjects besides their wide-scope readings. In other words, even if we allowed one of the mechanisms that generated these sort of hybrid readings, we would still need another mechanism that gives us, for at least some examples, the "real" non-specific opaque readings that we obtain e.g. by syntactic reconstruction. The relevant examples that show this most clearly involve DPs with more descriptive content than *somebody* and whose NPs express clearly contingent properties.

(28) A neat-freak must have been here.

If I say this instead of our original (11) when I come to my office in the morning and interpret the clues on my desk, I am saying that every world compatible with the evidence is such that someone who is a neat-freak *in that world* was here in that world. Suppose there is a guy, Bill, whom I know slightly but not well enough to have an opinion on whether or not he is neat. He may or not be, for all I know. So there are worlds among the relevant worlds where he is a neat-freak and worlds where he is not. I also don't have an opinion on whether he was or wasn't the one who came into my office last night. He did in some of the relevant worlds and he didn't in others. I am implying with (28), however, that if Bill isn't a neat-freak, then it wasn't him in my office. I.e., (28) *is* telling you that, even if there are relevant worlds in which Bill is a slob and worlds in which (only) he was in my office, there aren't any relevant worlds in which Bill is a slob *and* the only person who was in my office. This is correctly predicted if (28) expresses the "genuine" non-specific reading in (29), but not if it expresses the "hybrid" reading in (30).

(29) $\forall w'[w' \text{ is compatible with the evidence in } w_0 \rightarrow \exists x[x \text{ is a neatfreak } \textit{in } w' \text{ and } x \text{ was here in } w']]$

(30) $\forall w'[w' \text{ is compatible with the evidence in } w_0 \rightarrow \exists x[x \text{ is a neatfreak } \textit{in } w_0 \text{ and } x \text{ was here in } w']]$

We therefore conclude the mechanisms 2 and 3 considered above (whatever their merits otherwise) cannot supplant syntactic reconstruction or some other mechanism that yields readings like (29).

This leaves only the first and fourth options that we looked at as potential competitors to syntactic reconstruction, and we will focus the rest of the discussion on how we might be able to tease apart the predictions that these mechanisms imply from the ones of a syntactic reconstruction approach.

As for moving the modal operator, there are no direct bad predictions that we are aware of with this. But it leads us to expect that we might find not only scope ambiguities involving a modal operator and a DP, but also scope ambiguities between two modal operators, since one of them might covertly move over the other. It seems that this never happens. Sentences with stacked modal verbs seem to be unambiguous and show only those readings where the scopes of the operators reflect their surface hierarchy.

(31) a. I have to be allowed to graduate.
b. #I am allowed to have to graduate.

Of course, this might be explained by appropriate constraints on the movement of modal operators, and such constraints may even come for free in the right syntactic theory. Also, we should have a much more comprehensive investigation of the empirical facts before we reach any verdict. If it is true, however, that modal operators only engage in scope interaction with DPs and never with each other, then a theory which does not allow any movement of modals at all could claim the advantage of having a simple and principled explanation for this fact.

See Lechner 2007 for an early discussion of semantic effects of head movement. See McCloskey 2016 for a recent re-assessment.

What about the "semantic reconstruction" option, where raised subjects can leave traces of type $\langle s, \langle et, t\rangle\rangle$ and thus get narrow scope semantically without ending up low syntactically? This type of approach has been explored quite thoroughly and defended with great sophistication. The main consideration against semantic reconstruction and in favor of syntactic reconstruction comes from binding theoretic concerns. We give some crucial examples from Fox 2000 here.

Scope reconstruction and Condition C.

Consider:

(32) a. [A first year student] seems to David t to be at the party.
b. [Someone from NY] is very likely t to win the lottery.

These examples are a reminder that we can get non-specific readings of raised subjects. Fox argues that Binding Theory Condition C can be used to distinguish syntactic and semantic reconstruction. Assuming that Condition C applies at LF, syntactic reconstruction (where the full DP is interpreted in a lower position) predicts that an R-expression in that DP should not be allowed if there's a higher coreferential pronoun. The semantic reconstruction account has no simple way of making the same prediction.

Here are the crucial tests:

(33) a. [A student of his_1] seems to David_1 to be at the party. ^{OK}specific, ^{OK}non-specific
b. [A student of David's_1] seems to him_1 to be at the party. ^{OK}specific, *non-specific

(34) a. [Someone from his_1 city] seems to David_1 t to be likely to win the lottery. ^{OK}specific, ^{OK}non-specific
b. [Someone from David_1's city] seems to him_1 t to be likely to win the lottery. ^{OK}specific, *non-specific

Fox claims that the (b) cases do not have a non-specific reading. If syntactic reconstruction is the mechanism that gives us non-specific readings of A-moved subjects, the explanation is straightforward.

Exercise 4.8 *Use the following observations to bolster Fox's argument.*

(35) a. The cat seems to be out of the bag.
b. Advantage might have been taken of them.

(36) a. For these issues to be clarified, many new papers about his_1 philosophy seem to Quine t to be needed.
b. #For these issues to be clarified, many new papers about Quine's_1 philosophy seem to him t to be needed. □

If Fox's argument is correct, then (some form of) syntactic reconstruction is the mechanism by which we get non-specific readings of A-movement subjects. This means that syntactic reconstruction is possible, but it also means that semantic reconstruction must not be available. Otherwise, there would be a way of getting non-specific readings that would not be sensitive to Condition C. So, the question arises *why* semantic reconstruction is unavailable. Fox (2000: p. 171, fn. 41) discusses two ways of ruling out the high type traces that would give rise to semantic reconstruction:

(i) "traces, like pronouns, are always interpreted as variables that range over individuals (type *e*)",
(ii) "the semantic type of a trace is determined to be the lowest type compatible with the syntactic environment (as suggested in Beck 1996)".

We will return to this issue in a later chapter when we can raise it again in a slightly different framework.

4.3 Further readings

In addition to the references about reconstruction in the text, you might want to look at Boeckx 2001 and Baltin 2010.

5 *The Third Reading*

5.1 A Problem 89
5.2 The Standard Solution: Overt World Variables 91
5.3 The third reading with conditionals and modals . . . 96
5.4 Binding Theory for World Variables 97
5.5 Excursus: Semantic reconstruction revisited 98
5.6 Further reading 99

5.1 A Problem

Janet Dean Fodor discussed examples like (1) in her dissertation (1970).

(1) Mary wanted to buy a hat just like mine.

Fodor observes that (1) has three readings, which she labels "specific transparent", "non-specific transparent", and "non-specific opaque."

(i) On the "specific transparent" reading, the sentence says that there is a particular hat which is just like mine such that Mary has a desire to buy it. Say, I am walking along Newbury Street with Mary. Mary sees a hat in a display window and wants to buy *it*. She tells me so. I don't reveal that I have one just like it. But later I tell *you* by uttering (1).

(ii) On the "non-specific opaque" reading, the sentence says that Mary's desire was to buy some hat or other which fulfills the description that it is just like mine. She is a copycat.

(iii) On the "non-specific transparent" reading, finally, the sentence will be true, e.g., in the following situation: Mary's desire is to buy some hat or other, and the only important thing is that it be a Red Sox cap. Unbeknownst to her, my hat is one of those as well.

The existence of three different readings appears to be problematic for the scopal account of specific/non-specific ambiguities that we have been assuming. It seems that our analysis allows just two semantically distinct types of LFs: Either the DP *a hat just like mine* takes scope below *want*, as in (2), or it takes scope above *want*, as in (3).

(2) Mary wanted [[a hat-just-like-mine]$_1$ [PRO to buy t_1]]

(3) [a hat-just-like-mine]$_1$ [Mary wanted [PRO to buy t_1]]

In the system we have developed so far, (2) says that in every world w' in which Mary gets what she wants, there is something that she buys in w' that's a hat in w' and like my hat in w'. This is Fodor's "non-specific opaque" reading. (3), on the other hand, says that there is some thing x which is a hat in the actual world and like my hat in the actual world, and Mary buys x in every one of her desire worlds. That is Fodor's "specific transparent." But what about the "non-specific transparent" reading? To obtain this reading, it seems that we would have to evaluate the predicate *hat just like mine* in the actual world, so as to obtain its actual extension (in the scenario we have sketched, the set of all Red Sox caps). But the existential quantifier expressed by the indefinite article in the *hat*-DP should not take scope over the modal operator *want*, but below it, so that we can account for the fact that in different desire-worlds of Mary's, she buys possibly different hats.

There is a tension here: one aspect of the truth-conditions of this reading suggests that the DP *a hat just like mine* should be *outside* of the scope of *want*, but another aspect of these truth-conditions compels us to place it *inside* the scope of *want*. We can't have it both ways, it would seem, which is why this has been called a "scope paradox"

Another example of this sort, due to Bäuerle 1983, is (4):

(4) Georg believes that a woman from Stuttgart loves every member of the VfB team.

Bäuerle describes the following scenario: Georg has seen a group of men on the bus. This group happens to be the VfB team (Stuttgart's soccer team), but Georg does not know this. Georg also believes (Bäuerle doesn't spell out on what grounds) that there is some woman from Stuttgart who loves every one of these men. There is no particular woman of whom he believes that, so there are different such women in his different belief-worlds. Bäuerle notes that (4) can be understood as true in this scenario. But there is a problem in finding an appropriate LF that will predict its truth here. First, since there are different women in different belief-worlds of Georg's, the existential quantifier *a woman from Stuttgart* must be inside the scope of *believe*. Second, since (in each belief world) there aren't different women that love each of the men, but one that loves them all, the *a*-DP should take scope over the *every*-DP. If the *every*-DP is in the scope of the *a*-DP, and the *a*-DP is in the scope of *believe*, then it follows that the *every*-DP is in the scope of *believe*. But on the other hand, if we want to capture the fact that the men in question need not be VfB-members in Georg's belief-worlds, the predicate *member of the VfB team* needs to be outside of the scope of *believe*. Again, we have a "scope paradox".

Before we turn to possible solutions for this problem, let's have one more example:

(5) Mary hopes that a friend of mine will win the race.

This again seems to have three readings. In Fodor's terminology, the DP *a friend of mine* can be "non-specific opaque," in which case (5) is true iff in every world where Mary's hopes come true, there is somebody who is my friend and wins. It can also have a "specific transparent" reading: Mary wants John to win, she doesn't know John is my friend, but I can still report her hope as in (5). But there is a third option, the "non-specific transparent" reading. To bring out this rather exotic reading, imagine this: Mary looks at the ten contestants and says *I hope one of the three on the right wins - they are so shaggy - I like shaggy people*. She doesn't know that those are my friends. But I could still report her hope as in (5).

5.2 The Standard Solution: Overt World Variables

The scope paradoxes we have encountered can be traced back to a basic design feature of our system of intensional semantics: the relevant "evaluation world" for each predicate in a sentence is strictly determined by its LF-position. All predicates that occur in the (immediate) scope of the same modal operator must be evaluated in the same possible worlds. E.g. if the scope of *want* consists of the clause *a friend of mine (to) win*, then every desire-world w' will be required to contain an individual that wins in w' and is also my friend *in* w'. If we want to quantify over individuals that are my friends in the actual world (and not necessarily in all the subject's desire worlds), we have no choice but to place *friend of mine* outside of the scope of *want*. And if we want to accomplish this by means of QR, we must move the entire DP *a friend of mine*.

Not every kind of intensional semantics constrains our options in this way. One way to visualize what we might want is to write down an LF that looks promising:

(6) Mary wanted$_{w_0}$ [$\lambda w'$[a hat-just-like-mine$_{w_0}$] λx_1[PRO to buy$_{w'}$ x_1]]

We have annotated each predicate with the world in which we wish to evaluate it. w_0 is the evaluation world for the entire sentence and it is the world in which we evaluate the predicates *want* and *hat-just-like-mine*. The embedded sentence contributes a function from worlds to truth-values and we insert an explicit λ-operator binding the world where the predicate *buy* is evaluated. The crucial aspect of (6) is that the world in which *hat-just-like-mine* is evaluated is the matrix evaluation world and not the same world in which its clause-mate predicate *buy* is evaluated. This LF thus looks like it might faithfully capture Fodor's third reading.

Logical forms with overt world variables such as (6) are in fact the standard solution to the problem presented by the third reading. Let us spell out some of the technicalities. Later, we will consider a couple of alternatives.

We return to the basic system used in Heim & Kratzer up to chapter 11. The interpretation function is relativized only to an assignment function, not to any other evaluation parameters such as a world, a time, or an index. The semantic rules are Functional Application, Predicate Abstraction, and Predicate Modification, in their formulations from the earlier part of H&K. There is no rule of Intensional Functional Application. The only ingredient of intensional semantics that we do retain is the expanded type system and ontology. We have a third basic type besides e and t, the type s. D_s is the set of all indices, for now possible worlds (later: world-time pairs).

There are a number of innovations in the lexicon and in the syntax. As for the lexicon, the main change concerns the treatment of predicates (verbs, nouns, adjectives). They now all get an additional argument, of type s.

The decision to make the world-argument the predicate's first (lowest) argument is arbitrary, and nothing hinges on it. For all we know, it could be the highest argument, or somewhere in between.

(7) a. $[\![\text{smart}]\!] = \lambda w \in D_s.\ \lambda x \in D_e.\ x$ is smart in w

b. $[\![\text{likes}]\!] = \lambda w \in D_s.\ \lambda x \in D_e.\ \lambda y \in D_e.\ y$ likes x in w

c. $[\![\text{teacher}]\!] = \lambda w \in D_s.\ \lambda x \in D_e.\ x$ is a teacher in w

d. $[\![\text{friend}]\!] = \lambda w \in D_s.\ \lambda x \in D_e.\ \lambda y \in D_e.\ y$ is x's friend in w

This also applies to attitude predicates, modals, and tenses. We illustrate with *believe* and *must*:

(8) a. $[\![\text{believe}]\!] = \lambda w \in D_s.\ \lambda p \in D_{\langle s,t\rangle}.\ \lambda x \in D.$
$\forall w'\ [w'$ conforms to what x believes in $w \rightarrow p(w') = 1]$

b. $[\![\text{must}]\!] = \lambda w \in D_s.\ \lambda R \in D_{\langle s,st\rangle}.\ \lambda p \in D_{\langle s,t\rangle}.$
$\forall w'\ [R(w)(w') = 1 \rightarrow p(w') = 1]$

Note that predicates (ordinary ones and modal ones), like the ones in (7) and (8) now have as their semantic values what used to be their *intensions*.

There is no change to the entries of proper names, determiners, or truth-functional connectives; these keep their purely extensional ("s-free") types and meanings:

(9) a. $[\![\text{Ann}]\!]$ =Ann

b. $[\![\text{and}]\!] = \lambda u \in D_t.\ [\lambda v \in D_t.\ u = v = 1]$

c. $[\![\text{the}]\!] = \lambda f \in D_{\langle e,t\rangle} :\ \exists! x.\ f(x) = 1.$ the y such that $f(y) = 1$.

d. $[\![\text{every}]\!] = \lambda f \in D_{\langle e,t\rangle}.\ \lambda g \in D_{\langle e,t\rangle}.\ \forall x[f(x) = 1 \rightarrow g(x) = 1]$

So, let's start analyzing a simple sentence.

(10) $[_{VP}$ John leaves$]$

The verb's type is $\langle s, et \rangle$, so it's looking for a sister node which denotes a *world*. *John*, which denotes an individual, is not a suitable argument.

We get out of this problem by adding a couple of items to our lexicon, which are abstract (unpronounced) morphemes. One is a series of pronouns of type s ("index pronouns" or, for now, "world pronouns"). In this chapter, we will write them as w_n, with a numerical subscript n, or even as w, w', w''. (Later, we sometimes might write them as pro_n and rely on context to make clear we are not referring to an individual.) Their semantics is what you expect: they get values from the assignment function.

We will stipulate that a complete (matrix) sentence must not contain any free variables of type s and must receive a denotation of type $\langle s, t\rangle$. This means that we need binders of world pronouns. Many proposals in this line of thought help themselves to freely inserted covert binders. We will follow H&K in not doing that. Instead we posit one more lexical item, analogous to the covert vacuous operator *PRO* of type e in H&K (pp.227–228): a semantically vacuous operator, *OP*, which moves and leaves a trace of type s. Its syntactic properties are such that it must end up in C or right below a functional head in the "clausal spine" between C and V, and it must get there by a very short movement, a kind of "head movement". We are leaving this rather vague.

In the 2016 edition of this class, Suzana Fong noted that this stipulation is prima facie less appealing than the alternative assumption that type-s pronouns are exactly like type-e pronoun in every respect, including the ability to remain free and get values from a contextually supplied assignment. Irene tried to sketch some principled reason why it might not be possible to refer to a specific world other than the world one is in. But as Mitya Privoznov pointed out, a similar idea is not plausible for times, given the existence of temporal deictics like *then*. So at best there might be a principled reason why the world-coordinate of a free index-pronoun would always have to be w_u. Irene had to concede therefore that the ban against free index-pronouns was just a stipulation. We want to think more about (a) whether we really need it, and (b) if we do, what might explain it.

So, our sentence *John leaves* contains *OP*, generated as the first sister of the verb and then moved to the "top" of the sentence:

(11) *OP* 1 [John [leaves t_1]]

Our system generates the following denotation for (11): "λw_s. John leaves in w", a proposition. We rewrite the definition of truth/falsity of an utterance as follows:

(12) An utterance of a sentence (=LF) ϕ in world w is true iff $[\![\phi]\!](\mathrm{w}) = 1$.

So, if we utter our sentence in this world (call it $w_@$), then the utterance was true iff John leaves in $w_@$.

Now, we have to look at more complex sentences. First, a simple case of embedding. The sentence is *John wants to leave*, which now has an LF like this:

(13) [*OP* 1 [John [wants t_1 [*OP* 2 [PRO(= John) [leave t_2]]]]]]

Exercise 5.1 *Calculate the semantic value of (13).* □

Next, look at an example involving a complex subject, such as *the teacher left*. The verb will need a world argument as before. The noun *teacher* will likewise need one, so that *the* can get the required argument of type $\langle e, t\rangle$ (not $\langle s, et\rangle$!). Now, our system makes an interesting prediction: one of the world arguments has to be *OP* and one of them has to be a pronoun. (Why?) We have free choice, it appears, as to which predicate gets which kind of world argument. Let's assume, for now, that we insert *OP* as the sister of the verb and a world pronoun $w_?$ as the sister of the noun. Since we have stipulated that a complete sentence cannot contain any free world pronouns, the operator and the pronoun have to be co-indexed. So, after *OP*-movement, we will have this LF:

(14) *OP* 1 [[The [teacher w_1]] [left t_1]]

This will denote the correct proposition (true of a world w iff the unique individual who is a teacher in w left in w).

Now comes the payoff. Consider what happens when the sentence contains both a modal operator and a complex DP in its complement.

(15) Mary wants a friend of mine to win.

There are now three predicates that need world arguments. Furthermore, since *want* needs a proposition as its second argument (after its world argument), there needs to be an *OP* on top of the embedded clause. There also needs to be an *OP* on top of the matrix clause. As before, *a friend of mine* can stay in the embedded clause or QR into the matrix clause. When it moves into the matrix clause, the only way to not leave its world argument free is to co-index it with the matrix *OP*. But when it stays below, we can choose to co-index it with either *OP*, which is how we generate two non-specific readings, one opaque and one transparent. Here are the three LFs (to make the structures more readable, we leave off most of the bracketing and start writing the world arguments as subscripts to the predicates):

So, instead of writing t_1 for a trace of type *s* that serves as the first argument of *leaves*, say, we write "leaves$_{w_1}$"

(16) a. non-specific opaque:
OP 1 Mary wants$_{w_1}$ [*OP* 2 a friend-of-mine$_{w_2}$ leave$_{w_2}$]
b. specific transparent:
OP 1 a friend-of-mine$_{w_1}$ 3 Mary wants$_{w_1}$ [*OP* 2 t_3 leave$_{w_2}$]
c. non-specific transparent:
OP 1 Mary wants$_{w_1}$ [*OP* 2 a friend-of-mine$_{w_1}$ leave$_{w_2}$]

Notice that the third reading is minimally different from the first reading: all that happened is the choice to co-index the world argument of *friend of mine* with the matrix *OP*.

In this new framework, then, we have a way of resolving the apparent "scope paradoxes" and of acknowledging Fodor's point that there are two separate distinctions to be made when DPs interact with modal operators. First, there is the scopal relation between the DP and the operator; the DP may take wider scope (Fodor's "specific" reading) or narrower scope ("non-specific" reading) than the operator. Second, there is the choice of binder for the world-argument of the DP's restricting predicate; this may be cobound with the world-argument of the embedded predicate (Fodor "opaque") or with the modal operator's own world-argument ("transparent"). So the transparent/opaque distinction in the sense of Fodor is not *per se* a distinction of scope; but it has a principled connection with scope in one direction: Unless the DP is within the modal operator's scope, the opaque option (= co-binding the world-pronoun with the embedded predicate's world-argument) is in principle unavailable. (Hence "specific" implies "transparent", and "opaque" implies "non-specific".) But there is no implication in the other direction: if the DP has narrow scope w.r.t. to the modal operator, either the local or the long-distance binding option for its world-pronoun is in principle available. Hence "non-specific" readings may be either "transparent" or "opaque".

Exercise 5.2 *For DPs with extensions of type e (specifically, DPs headed by the definite article), there is a truth-conditionally manifest transparent/opaque distinction, but no truth-conditionally detectable specific/non-specific distinction. In other words, if we construct LFs analogous to (16)[a-c] above for an example with a definite DP, we can always prove that the first option (wide scope DP) and the third option (narrow scope DP with distantly bound world-pronoun) denote identical propositions. In this exercise, you are asked to show this for the example in (17).*

(17) John believes that your abstract will be accepted. □

5.3 *The third reading with conditionals and modals*

So, far our examples of the third reading have all been with attitude predicates but the phenomenon can also be observed in conditionals and with modals. A famous example is due to Abusch 1994:

(18) Things would be different if every senator had grown up to be a rancher instead.

What makes conditionals different is that the *if*-clause is a scope island for quantifiers so that *every senator* cannot QR scope out of the *if*-clause in (18). But the question of whether its predicate, *senator*, is interpreted in the matrix evaluation world ("transparent") or in the worlds that *if* takes us to ("opaque") remains open. Abusch's example is constructed to heavily favor the transparent reading.

Percus 2000 provides a clever minimal pair that shows the expected ambiguity:

(19) a. If every semanticist owned a villa in Tuscany, there would be no field at all.
b. If I were a syntactician and if every semanticist owned a villa in Tuscany, I would be quite envious.

We can also see the ambiguities at work in modal sentences:

(20) a. It could have been that everyone inside was outside.
b. Everyone inside is permitted to be outside.

These latter examples are from Yalcin 2015, who proceeds to discuss the very puzzling fact that transparent readings do not seem to be available in certain "epistemic" contexts, neither with indicative conditionals nor modals.

5.4 *Binding Theory for World Variables*

One could in principle imagine some indexings of our LFs that we have not considered so far. In a system (unlike ours) where one freely inserts "λw" operators on top of every clause, one could generate the following LF, which indexes the predicate of the complement clause to the matrix λ-operator rather than to the one on top of its own clause.

(21) λw_0 John wants$_{w_0}$ [λw_1 PRO leave$_{w_0}$]

Of course, the resulting semantics would be pathological: what John would be claimed to stand in the wanting relation to is a set of worlds that is either the entire set W of possible worlds (if the evaluation world is one in which John leaves) or the empty set (if the evaluation world is one in which John doesn't leave). Clearly, the sentence has no such meaning. Would we need to restrict our system to not generate such an LF? Perhaps not, if the meaning is so absurd that the LF would be filtered out by some overarching rules distinguishing sense from nonsense. Nevertheless, it is gratifying to note that this kind of LF is unavailable in our system: the only place where the lower world-binder *OP* could originate is as a sister to *leave* and moving it to the lower CP (where we need a proposition to feed to *want*) would by necessity make it bind the world argument of *leave*.

But there are real problems when we look at more complex examples. Here is one discussed by Percus in important work (Percus 2000):

(22) Mary thinks that my brother is Canadian.

Since the subject of the lower clause is a type *e* expression, we expect at least two readings: opaque and transparent, cf. Exercise 5.2. The two LFs are as follows:

(23) a. opaque
OP 0 Mary thinks$_{w_0}$ [(that) *OP* 1 my brother$_{w_1}$ (is) Canadian$_{w_1}$]
b. transparent
OP 0 Mary thinks$_{w_0}$ [(that) *OP* 1 my brother$_{w_0}$ (is) Canadian$_{w_1}$]

But as Percus points out, there is another indexing that might be generated:

(24) *OP* 0 Mary thinks$_{w_0}$ [(that) *OP* 1 my brother$_{w_1}$ (is) Canadian$_{w_0}$]

In (24), we have co-indexed the main predicate of the lower clause with the matrix λ-operator and co-indexed the nominal predicate *brother* with the embedded λ-operator. That is, in comparison with the transparent reading in (23b), we have just switched around the indices on the two predicates in the lower clause.

Note that this LF will not lead to a pathological reading. So, is the predicted reading one that the sentence actually has? No. For the transparent reading, we can easily convince ourselves that the sentence does have that reading. Here is Percus' scenario: "My brother's name is Allon. Suppose Mary thinks Allon is not my brother but she also thinks that Allon is Canadian." In such a scenario, our sentence can be judged as true, as predicted if it can have the LF in (23b). But when we try to find evidence that (24) is a possible LF for our sentence, we fail. Here is Percus:

> If the sentence permitted a structure with this indexing, we would take the sentence to be true whenever there is some *actual* Canadian who *Mary thinks* is my brother — even when this person is not my brother in actuality, and *even when Mary mistakenly thinks that he is not Canadian*. For instance, we would take the sentence to be true when Mary thinks that Pierre (the Canadian) is my brother and naturally concludes — since she knows that *I* am American — that Pierre too is American. But in fact we judge the sentence to be *false* on this scenario, and so there must be something that makes the indexing in (24) impossible.

Percus then proposes the following descriptive generalization:

(25) GENERALIZATION X: The situation pronoun that a verb selects for must be coindexed with the nearest λ above it.

Percus works with situation pronouns rather than world pronouns, an immaterial difference for our purposes here.

We expect that there will need to be a lot of work done to understand the deeper sources of this generalization. But note that we could implement the constraint in our system by brute force: the *OP* operator can only be generated as the sister of a main predicate, not as the sister of a predicate inside an argument nominal.

5.5 *Excursus: Semantic reconstruction revisited*

Let us look back at the account of non-specific readings of raised subjects that we sketched earlier in Section 4.2.3. We showed that you can derive such readings by positing a high type trace for the subject raising, a trace of type $\langle s, \langle et, t\rangle\rangle$. Before the lower predicate can combine with the trace, the semantic value of the trace has to be extensionalized by being applied to the lower evaluation world (done via the EFA composition principle). Upstairs the raised subject has to be combined with the λ-abstract (which will be of type $\langle\langle s, \langle et, t\rangle\rangle, t\rangle$) via its intension.

We then saw data suggesting that syntactic reconstruction is actually what is going on. This, of course, raises the question of why semantic reconstruction is unavailable (otherwise we wouldn't expect the data that we observed).

In this excursus, we will briefly consider whether our new framework has something to say about this issue. Let's figure out what we would have to do in the new framework to replicate the account in the section on semantics reconstruction.

Downstairs, we would have a trace of type $\langle s, \langle et, t \rangle\rangle$. To calculate its extension, we do not need recourse to a special composition principle, but can simply give it a world-argument (co-indexed with the abstractor resulting from the movement of the *w-OP* in the argument position of the lower verb).

Now, what has to happen upstairs? Well, there we need the subject to be of type $\langle s, \langle et, t \rangle\rangle$, the same type as the trace, to make sure that its semantics will enter the truth-conditions downstairs. But how can we do this?

We need the DP *somebody from New York* to have as its semantic value an intension, the function from any world to the existential quantifier over individuals who are people from New York in that world. This is actually hard to do in our system. It *would* be possible if (i) the predicate(s) inside the DP received *w*-PRO as their argument, and if (ii) that *w*-PRO were allowed to moved to adjoin to the DP. If we manage to rule out at least one of the two preconditions on principled grounds, we would have derived the impossibility of semantic reconstruction as a way of getting non-specific readings of raised subjects.

(i) may be ruled out by the Binding Theory for world pronominals, when it gets developed.
(ii) may be ruled out by principled considerations as well. Perhaps, world-abstractors are only allowed at sentential boundaries.

See Larson 2002 for some discussion of recalcitrant cases, one of which is the object position of so-called intensional transitive verbs, a topic for another occasion.

5.6 Further reading

There is an interesting literature spawned by Percus 2000: Schwager 2009, Romoli & Sudo 2009, Keshet 2010, Keshet 2011, Schwarz 2012, Keshet & Schwarz 2014.

Fodor also discussed a fourth reading, specific opaque, which is hard to fit into our framework. Whether it really exists is a question discussed in some recent work: Szabó 2010 and Francez 2017.

PART III

Time

6 Beginnings of tense and aspect

6.1 A first proposal for tense 103
6.2 Time frame adverbials 105
6.3 Are tenses referential? 108
6.4 Referential tense and perfective/imperfective aspect . 109
6.4.1 Referential tense after all 110
6.4.2 Event semantics and perfective aspect 111
6.4.3 The English progressive 113
6.4.4 Stativity effects 116
6.5 Formalizing the referential analysis 118

6.1 A first proposal for tense

Tense logic, or temporal logic, is a branch of logic first developed by the aptly named Arthur Prior in a series of works, in which he proposed treating tense in a way that is formally quite parallel to the treatment of modality discussed in Chapter 3. Since tense logic (and modal logic) typically is formulated at a high level of abstraction regarding the structure of sentences, it doesn't concern itself with the internal make-up of "atomic" sentences and thus treats tenses as sentential operators (again, in parallel to the way modal operators are typically treated in modal logic). We will begin by integrating a version of Prior's tense logic into our framework.

This chapter is even more the outcome of collaborative efforts than other chapters. We are very much indebted to Roger Schwarzschild, who has used our notes several times in his teaching and is the source of many edits and additions.

The first step is to switch to a version of our intensional semantic system where instead of a world parameter, the evaluation function is sensitive to a parameter that is a pair of a world and a time. Such a pair will also be called an "index". We use metalanguage variables i, i', … for indices, and write w_i and t_i to pick out the world in i and the time in i respectively (i.e., $i = \langle w_i, t_i \rangle$). Predicates will now have lexical entries that incorporate their sensitivity to both worlds and times:

We remain vague for now about what we mean by "times" (points in time? time intervals?). This will soon need clarification, and we will decide that we should mean "intervals".

(1) $[\![\text{tired}]\!]^i = \lambda x \in D.\ x$ is tired in w_i at t_i

The composition principles from Heim & Kratzer and the preceding chapters stay the same, except that type s is now the type of indices, and intensions are functions from indices to extensions. For example, the intension of sentence is now a function from world-time pairs to truth-values. We might call this a "temporal proposition", to distinguish it from a function from just worlds to truth-values, but we will often just call it a "proposition".

This necessitates a slight rewriting of our previous entries for modals and attitude verbs. We will attend to this when we get to relevant examples later on.

In this framework, we can formulate a very simple-minded first analysis of the present and past tenses and the future auxiliary *will.* As for (LF) syntax let's assume that complete sentences are TPs, headed by T (for "tense"). There are two morphemes of the functional category T, namely PAST (past tense) and PRES (present tense). The complement of T is an MP or a VP. MP is headed by M (for "modal"). Morphemes of the category M include the modal auxiliaries *must*, *can*, etc., which we talked about in previous chapters, the semantically vacuous *do* (in so-called "*do*-support" structures), and the future auxiliary *will*. Evidently, this is a semantically heterogeneous category, grouped together solely because of their common syntax (they are all in complementary distribution with each other). The complement of M is a VP. When the sentence contains none of the items in the category M, we assume that MP isn't projected at all; the complement of T is just a VP in this case. (TP is always projected in a root clause, whether there is an MP or not.) We thus have LF-structures like the following. (The corresponding surface sentences are given below, and we won't be explicit about the derivational relation between these and the LFs. Assume your favorite theories of syntax and morphology here.)

Many subordinate clauses — those we call "finite" — also always have a TP. As for embedded clauses more generally (including infinitives etc.), we don't need to take a stand here.

(2) $[_{\text{TP}}$ Svenja $[_{\text{T}'}$ PRES $[_{\text{VP}}$ t $[_{\text{V}'}$ be tired]]]]
= Svenja is tired.

(3) $[_{\text{TP}}$ Svenja $[_{\text{T}'}$ PAST $[_{\text{VP}}$ t $[_{\text{V}'}$ be tired]]]]
= Svenja was tired.

(4) $[_{\text{TP}}$ Svenja $[_{\text{T}'}$ PRES $[_{\text{MP}}$ t $[_{\text{M}'}$ woll $[_{\text{VP}}$ t $[_{\text{V}'}$ be tired]]]]]]
= Svenja will be tired.

woll in (4) stands for the underlying uninflected form of the auxiliary which surfaces as *will* in the present tense (and as *would* in the past tense). When we have proper name subjects, we will assume for simplicity that they are reconstructed into their VP-internal base position.

We use "*woll*" as the name of the root underlying *will* and *would*, following Abusch 1988 and Ogihara 1989: p.32; Abusch (1997: fn.14, p.22) attributes the coinage of *woll* to Mats Rooth in class lectures at UT Austin.

What are the meanings of PRES, PAST, and *woll*? For PRES, the simplest assumption that seems to work is that it is semantically vacuous. This means that the interpretation of the LF in (2) is identical to the interpretation of the bare VP *Svenja be tired*:

(5) For any index i: $[\![\text{PRES (Svenja be tired)}]\!]^i = [\![\text{Svenja be tired}]\!]^i = 1$ iff Svenja is tired in w_i at t_i.

Does this adequately capture the intuitive truth-conditions of the sentence *Svenja is tired*? It does if we make the following general assumption:

(6) Utterance Rule:
An utterance of a sentence (= LF) ϕ that is made in a world w at a time t counts as true iff $[\![\phi]\!]^{\langle w,t\rangle} = 1$ (and as false if $[\![\phi]\!]^{\langle w,t\rangle} = 0$).

This assumption ensures that (unembedded) sentences are, in effect, interpreted as claims about the time at which they are uttered ("utterance time" or "speech time"). If we make this assumption and we stick to the lexical entries we have adopted, then we are driven to conclude that the present tense has no semantic job to do. A tenseless VP *Svenja be tired* would in principle be just as good as (2) to express the assertion that Svenja is tired at the utterance time. Apparently, it is just not well-formed as an unembedded structure, but this fact then must be attributed to principles of syntax rather than semantics.

What about PAST? When a sentence like (3) *Svenja was tired* is uttered at a time t, then what are the conditions under which this utterance is judged to be true? A quick answer is: an utterance of (3) at t is true iff there is some time before t at which Svenja is tired. This suggests the following entry:

(7) For any index i: $[\![\text{PAST}]\!]^i = \lambda p \in D_{\langle s,t\rangle}.\ \exists t \text{ before } t_i :\ p(\langle w_{i,}, t\rangle) = 1$

So, the past tense seems to be an existential quantifier over times, restricted to times before the utterance time.

For *will*, we can say something completely analogous:

(8) For any index i: $[\![\text{woll}]\!]^i = \lambda p \in D_{\langle s,t\rangle}.\ \exists t \text{ after } t_i :\ p(\langle w_{i,}, t\rangle) = 1$

Apparently, PAST and *woll* are semantically alike, even mirror images of each other, though they are of different syntactic categories. The fact that PAST is the topmost head in its sentence, while *woll* appears below PRES, is due to the fact that our syntax happens to require a T-node in every complete sentence. Semantically, this has no effect, since PRES is vacuous.

Both (7) and (8) presuppose that the set or times comes with an intrinsic order. For concreteness, assume that the relation 'precedes' (in symbols: <) is a strict linear order on the set of all times. The relation 'follows', of course, can be defined in terms of 'precedes' (t follows t' iff t' precedes t).

Are there also tenses with universal force ? Two possible candidates that call for closer examination: gnomic tenses (e.g. in Ancient Greek), and the (universal reading of the) English perfect (as in *I have been tired since yesterday morning)*. Both have been written about in the formal semantics literature (the latter extensively — you could start with Iatridou, Anagnostopolou & Izvorski 2001 and von Fintel & Iatridou 2019.

Strict linear orders are transitive, irreflexive, asymmetric, and connected. See Section 3.2.2 on the basics of order theory. Some rethinking is needed once we move to intervals.

6.2 *Time frame adverbials*

In this section, we take a brief look at temporal adverbials, specifically so-called frame adverbials, such as:

(9) Svenja was tired on February 1, 2001.

There are two ideas that come to mind. One is that phrases like *on February 1, 2001* are restrictors of temporal operators (kind of like *if*-clauses are restrictors of modals). The other idea is that they are modifiers of the proposition in the temporal operator's scope. If we want to go with the first idea, we have to make some changes. Our current PAST and *woll* are unrestricted (1-place) operators, so there is no place for a restrictor. The second idea is easier to implement, and we try that first.

A propositional modifier is a function from propositions to truth-values, where "proposition" for us now means "temporal proposition". Here is an entry for on February 1, 2001. Intuitively, this modifier takes a proposition and returns a proposition that puts an added condition on the time-coordinate of its index-argument.

Technically, the modifier returns a truth-value, not a proposition. We get back a proposition only when we compute the intension of the phrase that includes the modifier.

(10) $[\![\text{on February 1, 2001}]\!]^i = \lambda p \in D_{\langle s,t\rangle}.$
$[p(i) = 1 \;\&\; t_i \text{ is part of February 1, 2001}]$

(11) LF: PAST [$_{VP}$ [$_{VP}$ Svenja be tired][$_{PP}$ on February 1, 2001]]

Exercise 6.1 *Imagine that sentence (9) is not given the LF in (11), but this one, with the PP attached higher:*

(12) LF: [$_{T'}$ PAST [$_{VP}$ Svenja be tired]] [$_{PP}$ on February 1, 2001]

What would the truth-conditions of this LF be? Does this result correspond at all to a possible reading of this sentence (or any other analogous sentence)? If not, how could we prevent such an LF from being produced? □

The truth conditions that we derive given (10) and (11) look good: the sentence is predicted true as uttered if there is a time which is both before the utterance time and within Feb 1, 2001 and at which Svenja is tired, and it is predicted false if there is no such time. But arguably this is not *exactly* right. Suppose that somebody uttered this sentence at an utterance time that *preceded* the date in the adverbial, say at some time in the year 2000. Our analysis predicts that this utterance is false. But in fact it feels more like a presupposition failure; the speaker is heard to be taking for granted that Feb 1, 2001 is in the past of his speaking. Standard presupposition tests confirm this. For example, the negated sentence (*Svenja wasn't tired on Feb 1, 2001*) and the polar question (*Was Svenja tired on Feb 1, 2001?*) also convey that the speaker assumes he is speaking after Feb 1, 2001.

If we want to account for this more fine-grained intuition, the restrictor approach has an advantage after all. Let's revise the entries for PAST and *woll* so that they denote 2-place operators, and moreover they encode a non-emptiness presupposition.

It also has the virtue of avoiding the potential overgeneration issue that you looked at in the exercise above. Q: How so?

(13) For any index i:
$[\![\text{PAST}]\!]^i = \lambda p\colon \exists[t < t_i \;\&\; p(w_i, t) = 1].$
$\lambda q.\exists t[t < t_i \;\&\; p(w_i, t) = 1 \;\&\; q(w_i, t) = 1]$

(14) For any index i:
$[\![\text{woll}]\!]^i = \lambda p\colon \exists[t > t_i \;\&\; p(w_i, t) = 1].$

How about present tense? Should we make this presuppositonal as well—which would imply it is not, after all, completely vacuous? Frame adverbials in present tense sentences do occur. Typically they are adverbials like *today, on this beautiful Monday*, which in virtue of their own meaning already are required to contain the speech time. The following entry would make room for them and duplicate this requirement as a presupposition:

$$\lambda q.\exists t[t > t_i \ \&\ p(w_i, t) = 1 \ \&\ q(w_i, t) = 1]$$

Furthermore, let's change (i.e., simplify) the meaning of the adverbial so that it has a suitable type to serve as the temporal operator's first argument. The LF-structure must be accordingly different as well. Instead of (11) above, we now posit (17), where the adverb forms a constituent with the tense. This requires the surface order to be derived by some reordering, perhaps extraposition of the adverbial.

(16) $[\![$on February 1, 2001$]\!]^i = 1$ iff t_i is part of February 1, 2001

(17) LF: $[_{T'}$ PAST $[_{PP}$ on February 1, 2001 $]][_{VP}$ Svenja be tired $]$

The meanings we now derive contain the desired presuppositions: The past tense sentence (9) presupposes that Feb 1, 2001 is at least in part before the utterance time, the future sentence *Svenja will be tired on Feb 1, 2001* presupposes that this date is at least in part after the utterance time, and the present tense sentence *Svenja is tired on Feb 1, 2001* presupposes that the utterance time is on this date. Apart from the presuppositions, the meanings are the same as before. On the down-side, the new analysis posits both more complex meanings for the tenses and a less direct correspondence between LF constituency and surface structure. Furthermore, how is it supposed to apply to simple sentences without adverbials? Not every tensed sentence contains an obligatory frame adverb, after all. We are forced to say there is a covert restrictor whenever there isn't an overt one. But this, upon reflection, turns out to be a virtue, as we will see in the next section.

Considering the mismatch between the LF in (17) and the surface order, note that we saw a similar issue of apparent mismatch when we decided to treat *if*-clauses as restrictors of modals. Both issues might be addressed by simply letting modal and temporal operators take their arguments in the opposite order (something suggested by Chierchia 1995). Rewriting the lexical entries in this way is a routine exercise. We leave this matter open. The syntax of frame adverbials is a non-trivial object of study.

Exercise 6.2 *When a quantifier appears in a tensed sentence, we expect two scope construals. Consider a sentence like this:*

(18) Every professor (in the department) was a teenager in the Sixties.

We can imagine two LFs:

(19) [PAST in the sixties] [every professor be a teenager]

(20) [every professor] 7 [[PAST in the sixties] [t_7 be a teenager]]

Describe the different truth-conditions which our system assigns to the two LFs. Is the sentence ambiguous in this way? If not this sentence, are there analogous sentences that do have the ambiguity? □

Exercise 6.3 *Our official entry for* every *makes it a time-insensitive (and world-insensitive) item:*

(21) For any i, $[\![\textit{every}]\!]^i = \lambda f_{\langle e,t\rangle}.\lambda g_{\langle e,t\rangle}.\ \forall x\colon\ f(x) = 1 \rightarrow g(x) = 1$

Consider now two possible variants (we have boxed the portion where they differ):

(22) *For any* i, $[\![\textit{every}]\!]^i = \lambda f_{\langle e,t\rangle}.\lambda g_{\langle e,t\rangle}.\ \forall x \boxed{\textit{at } t_i}\!:\ f(x) = 1 \rightarrow g(x) = 1$

(23) *For any* i, $[\![\textit{every}]\!]^i = \lambda f_{\langle e,t\rangle}.\lambda g_{\langle e,t\rangle}.\ \forall x\!:\ f(x) = 1 \boxed{\textit{at } t_i} \rightarrow g(x) = 1$

Does either of these alternative entries make sense? If so, what does it say? Is it equivalent to our official entry? Could it lead to different predictions about the truth-conditions of English sentences? □

6.3 Are tenses referential?

Our first semantics for the past tense, in Section 6.1, treated it as an unrestricted existential quantifier over times. This seems quite adequate for examples like (24), which seem to display the expected unrestricted existential meaning:

(24) Georgia went to a private school.

All we learn from (24) is that at some point in the past, whenever it was that Georgia went to school, she went to a private school.

Partee in her famous paper "Some structural analogies between tenses and pronouns in English" (Partee 1973) presented an example where tense appears to act more "referentially":

(25) I didn't turn off the stove.

"When uttered, for instance, halfway down the turnpike, such a sentence clearly does not mean either that there exists some time in the past at which I did not turn off the stove or that there exists no time in the past at which I turned off the stove. The sentence clearly refers to a particular time — not a particular instant, most likely, but a definite interval whose identity is generally clear from the extralinguistic context, just as the identity of the *he* in [*He shouldn't be in here*] is clear from the context."

Partee argues, in effect, that neither of the two plausible LFs that our system from Section 6.1 derives can correctly capture the meaning of (25). Given that the sentence contains a past tense and a negation, there are two possible scopings of the two operators:

(26) a. PAST NEG I turn off the stove.
b. NEG PAST I turn off the stove.

Exercise 6.4 *Using our old semantics from 6.1, show that neither LF in (26) captures the meaning of (25) correctly.*

In a commentary on Partee's paper (at the same conference it was presented at), Stalnaker pointed out that a minor amendment of the Priorean theory can deal with (25). One just needs to allow the existential quantifier to be contextually restricted to times in a salient interval. Since natural language quantifiers are typically subject to contextual restrictions, this is not a problematic assumption. Note that Partee formulated her observation in quite a circumspect way: "The sentence refers to a particular time"; Stalnaker's suggestion was that the reference to a particular time is part of the restriction to the quantifier over times expressed by tense, rather than tense itself being a referring item.

Ogihara 1995, 1996 argued that the restricted existential quantification view is in fact superior to Partee's analysis, since Partee's analysis needs an existential quantifier anyway. It is clear that the time being referred to in the stove-sentence (25) is a protracted interval (the time during which Partee was preparing to leave her house). But the sentence is not interpreted as merely saying that this interval is not a time at which she turned off her stove. That would only exclude a fairly absurd kind of slow-motion turning-off-of-the-stove (turning off the stove only takes a moment). Instead, the sentence says that in the salient interval there is no time at which she turned off the stove. Clearly, we need an existential quantifier in there somewhere and the Priorean theory provides one.

The alternative is to say that the existential quantifier is not expressed by tense but comes from somewhere else — perhaps from aspect, or from the lexical entry of the verb itself. We will come back to these options.

Ogihara made the point with the following example:

(27) Patricia: Did you see Solène?
Lea: Yes, I saw her, but I don't remember exactly when.

The question and answer in this dialogue concern the issue of whether Lea saw Solène at some time in a contextually salient interval.

Stalnaker's and Ogihara's conclusions converge with what we already ended up with in Section 6.2, after considering the interaction of tenses with time frame adverbials. In order to capture presuppositions of tensed sentences with frame adverbials, we already modified Prior's original proposal and made room for a restrictor in the semantics of the past tense. Given this revised analysis of the past tense as a 2-place existential quantifier, it is unsurprising, in fact expected, that an implicit, contextually salient restrictor should be present when there isn't an overt one. What then about example (24), *Georgia went to a private school*, for which the unrestricted analysis seemed to do well? Let us say that the covert restrictor in this case picks out a very long interval, perhaps Georgia's entire lifetime, or even the entire past from the big bang to the utterance time, or all eternity. (What exactly the right restrictor is in this case, and what makes it contextually available, may be a bit unclear, but we leave it at that.)

Exercise 6.5 *Assuming the restricted existential quantifier analysis of past tense that we adopted in Section 6.2, which of the scope constellations in (26) captures the meaning of (25) correctly?*

6.4 Referential tense and perfective/imperfective aspect

6.4.1 *Referential tense after all*

Let us return to Partee's stove and the prospects of a "referential" theory of tense. Our discussion of Partee's example (following Stalnaker and Ogihara) just now came to the conclusion that we did need past tense to be an existential quantifier over times, albeit a contextually restricted one. The stove-example is interpreted as a claim about a particular contextually relevant interval. But the speaker's claim is not merely that she didn't turn off the stove at that interval. That in itself would be compatible with her turning off the stove at some smaller interval inside the contextually relevant interval. The speaker's claim is stronger: she did not turn off the stove at any time that is contained in this interval. This is a negative existential claim. So there needs to be an existential quantifier somewhere in the LF and below the scope of *not*, and we concluded that past tense must be supplying it. But this conclusion is not inescapable. Granted that there has to be an existential quantifier somewhere—but couldn't it be somewhere else than in the meaning of tense? One alternative that comes to mind is to locate it in the lexical meaning of the verb (here *turn off*). This means we abandon the lexical entry in (28) and instead adopt the one in (29).

(28) $[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.x$ turns off y in w_i at t_i

(29) $[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.x$ turns off y in w_i $\boxed{\text{in}}$ t_i

The difference between 'at' and 'in' looks small at first, but if we reflect on the meaning of 'in', we see the hidden existential quantifier. When something happens in an interval, it happens at some part of the interval. We can make this more transparent in the metalanguage and rewrite (29) as (30).

(30) $[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.\exists t \subseteq t_i :\ x$ turns off y in w_i at t

The subset sign here stands for the containment relation between time intervals. A time interval can be defined as a certain kind of set of moments, as in (31), so the subset relation is well defined.

(31) A set of moments S is an interval iff for any two moments that are in S, every moment between them is also in S.

Another way to clarify the distinction between 'at' and 'in' is to use the kind of metalanguage that is familiar from the literature on Davidsonian event semantics.

(32) abbreviations in "event talk":

a. turn-off(e, x, y) = e is an event of turning off y by agent x

b. $\tau(e)$ = the (exact) time-interval occupied by event e
also called the "run-time" or "temporal trace" of e

(33) event-talk formulation of (28), the old entry with 'at':
$[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.\exists e[\text{turn-off}(e, x, y) \ \&\ e \text{ is in } w_i \ \&\ \tau(e) = t_i]$

(34) event-talk formulation of (29), the new entry with 'in':
$[\![\text{turn-off}]\!]^i = \lambda y.\lambda x.\exists e[\text{turn-off}(e, x, y) \ \&\ e \text{ is in } w_i \ \&\ \tau(e) \subseteq t_i]$

Let us spell out now how Partee's proposal for the meaning of past tense can be upheld after all, once we assume the lexical semantics specified in (29)/(30)/(34). The first task here is to write new lexical entries for the tense morphemes, which encode Partee's idea that tenses refer to specific time intervals and are semantically and pragmatically akin to personal pronouns. We will defer the full execution of this task until later and make do for the time being with a couple of syncategorematic ad hoc rules for the interpretation of TPs.

(35) $[\![\textsc{past}\ \phi]\!]^i = 1$ iff $[\![\phi]\!]^{\langle w_i, t'\rangle} = 1$, where t' is the contextually salient time before t_i (no truth value defined if there is no such time)

(36) $[\![\text{woll}\ \phi]\!]^i = 1$ iff $[\![\phi]\!]^{\langle w_i, t'\rangle} = 1$, where t' is the contextually salient time after t_i (no truth value defined if there is no such time)

This contextually salient time is also called the "topic time" (Klein 1994) or the "reference time" (a term which goes back to Reichenbach 1947, but which has various other uses in the literature).

(PRES remains vacuous as before, i.e., $[\![\textsc{pres}\ \phi]\!]^i = [\![\phi]\!]^i$.)

The stove example has the two potential LFs in (37).

(37) a. PAST NEG I turn off the stove.
b. NEG PAST I turn off the stove.

We can compute the truth conditions for both of these under the new semantics for PAST and *turn off*, and it turns out that they are the same.

(38) $[\![(37\text{a})]\!]^i = 1$ iff $[\![(37\text{b})]\!]^i = 1$ iff
$\neg\exists e[\text{turn-off}(e, x, y) \ \&\ e \text{ is in } w_i \ \&\ \tau(e) \subseteq t']$,
where t' is the contextually salient time before t_i
(no truth-value defined if there is no such time)

The fact that both scopal orders yield the same truth conditions is arguably a point in favor of this approach. The English sentence is not in fact perceived as ambiguous. Our earlier approach, on which past tense was a contextually restricted existential quantifier, did not make this prediction — at least not without the help of additional assumptions (such as a syntactic constraint on the position of negation with respect to other heads on the clausal spine). Now that the existential quantifier comes bundled with the lexical verb, its scope is automatically "frozen" below everything that scopes over the verb.

6.4.2 *Event semantics and perfective aspect*

Up to now, we have presupposed a pre-Davidsonian view of lexical meanings, on which verbs take only individuals or propositions as their arguments. Even when we recently inserted some event-talk into the metalan-

We share the goal of integrating Davidsonian event semantics and traditional intensional semantics with von Stechow & Beck 2015, from whom we borrow a number of ideas.

guage of our lexical entries, we still defined the denotation of a verb like *turn off* as a function from two individuals to a truth value. In this section, we switch to a Davidsonian treatment of verbs as predicates of events and integrate this with our conception of sentence-intensions as temporal propositions.

In an extensional Davidsonian semantics, lexical entries look like (39).

(39) $[\![\text{laugh}]\!] = \lambda x.\lambda e.\ e$ is an event of x laughing
abbreviated: $[\![\text{laugh}]\!] = \lambda x.\lambda e.\ \text{laugh}(e, x)$

Many practitioners of event semantics assume that the event argument is the only real argument of the verb, whereas the subject, object, etc. are arguments of abstract theta-role heads that combine with the verb in the manner of modifiers. Here we remain agnostic on this matter. For concreteness, we assume that verbs take all the traditional arguments in addition to their event-argument, but the other view is equally compatible with everything we will say. We just abstract away from the internal compositional semantics of the VP.

Assuming that events are not in D_e, but have their own basic type v, VPs thus are of type $\langle v, t\rangle$. (All the verb's non-event arguments are merged inside the VP.)

In a semantics that is both Davidsonian and intensional, do we have to rewrite these entries? For example, should we perhaps rewrite (39) as (40)?

(40) $[\![\text{laugh}]\!]^i = \lambda x.\lambda e.\ e$ is an event of x laughing $\boxed{\text{in } w_i \text{ at } t_i}$

That depends. Here we follow Kratzer and assume that each event occurs in just one world and at just one time. It is not possible for a given e to be an event of x laughing in one world and to be some other kind of event in another world. Nor is it possible for one and the same e to be an event of x laughing at one time and something else at another time. Reformulations such as (40) are uncalled for then, and we can essentially stick with (39).

This assumption is made here mostly to keep things simple. It is not innocuous and not uncontroversial. See e.g. Hacquard 2009b for an analysis of root modals that makes crucial use of the idea that an actual event exists in non-actual worlds and has different properties there.

But how then does world and time dependence enter the semantic computation? And how can tenses and modal operators combine with VPs? VPs are now type $\langle v, t\rangle$, which leads to a type-mismatch if we try to combine them directly with a modal operator or with a tense (regardless of whether the tense is a Priorian temporal operator or a Partee-style referential tense). The way out of this problem is to posit a more complex clause structure, with a further functional head that intervenes between T (or M) and V. This is called an "aspect" head (category label "Asp"), and its semantic job is to existentially bind the event argument of the VP and return a world- and time-sensitive denotation of type t.

Strictly speaking, we should now write (41), but since i in (39) does not occur on the right side of =, (39) can be shorthand for (41).

(41) For any index i, $[\![\text{laugh}]\!]^i = \lambda x.\lambda e.\ \text{laugh}(e, x)$

One instance of Asp is the so-called "perfective", for which we posit the following entry.

(42) $[\![\text{PFV}]\!]^i = \lambda P_{\langle v,t\rangle}.\exists e[P(e) = 1\ \&\ \tau(e) \subseteq t_i\ \&\ e \leq w_i]$
$\leq$:= is part of (= occurs in)
τ := the run time of (temporal trace of)

(42) combines the standard formal analysis of perfective aspect (among many others: Klein 1994, Kratzer 1998) with the semantics of von Stechow & Beck 2015's Modl head. It locates the event both in a time interval and in a possible world.

PFV is morphologically zero in English, so we can posit it in the LFs of sentences with simple tensed verbs. Here is an example.

(43) a. Barbara turned off the stove.

b. LF: $[_{\text{TP}}$ PAST $[_{\text{AspP}}$ PFV $[_{\text{VP}}$ Barbara turn-off the stove]]]

Using the syncategorematic rule (35) for referential PAST, our entry (42) for PFV, and a Davidsonian entry for the verb, we compute the following interpretation. (Do this as an exercise.)

(44) $[\![(43\text{b})]\!]^i = 1$ iff $\exists e[\text{turn-off}(e,\ \text{B, the stove})\ \&\ \tau(e) \subseteq t'\ \&\ e \leq w_i]$,
where t' is the contextually salient time before t_i
(no truth-value defined if there is no such time)

This is the same meaning that we obtained in the previous section, when we had built the existential quantification into the lexical meaning of the verb. What used to be the meaning of VP is now the meaning of AspP. We have located the event-quantifier in its own functional head, but otherwise it is the same analysis.

Exercise 6.6 *What about the negated sentence that was Partee's original example? Where can we now generate negation in an interpretable LF? Does the current analysis still predict that the sentence is not in fact ambiguous?* □

6.4.3 The English progressive

Besides perfective aspect, there is imperfective aspect—or more accurately, there is probably a family of imperfective aspects in different languages that have some shared and some non-shared properties. English has an imperfective aspect known as the "progressive", with an overt morphology that consists of a copula which governs a present-participial form of the VP (*V-ing*). We posit a functional head be-PROG as the aspect head in the English progressive construction.

See e.g. Arregui, Rivero & Salanova 2014 for a recent approach to cross-linguistic semantic variation in imperfective aspects.

We are not serious about morphology here. The meaning may well be carried by an abstract head and the *be* a vacuous element.

The basic intuition behind much work on the perfective-imperfective distinction is that, whereas perfective aspect locates an event within the evaluation time, imperfective aspect does the reverse, i.e., it places the evaluation time within the event time. If we formalize this intuition directly, without introducing any further differences from the perfective, we come up with (45).

(45) First attempt:
$[\![\text{be-PROG}]\!]^i = \lambda P_{\langle v,t\rangle}.\exists e[P(e) = 1\ \&\ \boxed{t_i \subseteq \tau(e)}\ \&\ e \leq w_i]$

This is similar to the first formal analysis of the progressive, due to Bennett & Partee 1978. They did not work in an event semantics, however. Also, their semantics required t_i to be a *non-final* subinterval of $\tau(e)$, rather than merely $t_i \subseteq \tau(e)$. This requirement seemed too strong in light of examples such as Dowty 1977's *John was watching TV when he fell asleep* (which does not say that TV-watching continued beyond the point of falling asleep). However, as Dowty showed, it turned out to be the right requirement in the context of the modalized analysis that he proposed, see below.

It is well-known, however, that there is also a difference in how the event is related to the evaluation world. While perfective places the event within the actual world, the progressive permits it to be partly in another world, so to speak. This point, which was at the center of Dowty 1977's seminal work on the progressive, is brought home by examples like (46).

(46) John was going to the store when he ran into Svenja.

We can't infer from this sentence that John actually made it to the store, or will ever make it there. The sentence leaves this open. Perhaps John does complete his trip to the store after the encounter, and perhaps he doesn't. The truth-conditions of the sentence (46) are compatible with either scenario. The entry in (45), on the other hand, would require that there be a John-going- to-the-store event which occupies a super-interval of the time of the encounter with Svenja and which *occurs in the actual world*. So (45) can't be quite right.

Dowty dubbed this the "imperfective paradox", although it's not really a paradox, just a counterexample to a certain analysis that looked plausible at first.

Dowty's analysis of the progressive says instead that a John-going-to-the-store event occurs in certain *possible* worlds. These possible worlds are related to the actual world in a particular way: they are worlds which share a history with the actual world up to a certain point and then develop (possibly counterfactually) in such a way that no events that were already in progress get interrupted ("inertia worlds"). The idea is, very roughly, that the sentence tells us: either John actually went to the store, or if he didn't, then at least he *would have gone* there if he hadn't been interrupted. Since the publication of Dowty's paper, there has been a succession of sophisticated counterexamples and refinements to his original proposal, but this is beyond the scope of this introduction. Here is a version based on Dowty.

See among many others, Landman 1992, Portner 1998.

(47) second attempt (and final version for us):

$$[\![\text{be-PROG}]\!]^i = \lambda P_{\langle v,t\rangle}.\forall w\big[w \in \text{Inert}(i) \rightarrow \exists e[P(e) = 1\ \& \\ t_i \subset^{<} \tau(e)\ \&\ e \leq w]\big]$$

where $\subset^{<}$ abbreviates: "is a non-final subinterval of"
(that is: $\tau(e)$ includes every moment in t_i as well as some moment after the end of t_i)

Apart from introducing quantification over other worlds, (47) also differs from (45) in that it strengthens the requirement on the temporal relation between t_i and $\tau(e)$: not only must $\tau(e)$ contain all of t_i, but it must moreover extend into the time after t_i. This is intended to capture the intuition that e.g. (46) is not appropriate if John already reaches the store during his encounter with Svenja; see Dowty 1977 for discussion.

(48) Definition: $w \in \text{Inert}(i)$ iff
w is exactly like w_i up to the end of t_i and then develops in such a way that no events are interrupted.

We will see in a minute that there is a class of VPs for which the truth-conditions predicted by (47) come very close to those predicted by the simpler (45). But examples like (46) show that this must not always hold.

Let's do a simple example.

(49) a. Sari is laughing.

b. LF: [$_{\text{TP}}$ PRES [$_{\text{AspP}}$ be-PROG [$_{\text{VP}}$ Sari laugh]]]

c. $[\![(49\text{b})]\!]^{i} = 1$ iff
$\forall w\big[w \in \text{Inert}(i) \rightarrow \exists e[t_i \subset^{<} \tau(e) \;\&\; e \leq w \;\&\; \text{laugh}(e, \text{Sari})]\big]$

Just as it stands, (49c) does not logically entail that any laughing happens in the world w_i (i.e., in the utterance world w_u if this is an unembedded assertion). It only talks about the inertia worlds. However, there is a property of the lexical meaning of *laugh* that permits us to draw further inferences. Laughing events are made up of lots of sub-events which themselves are laughing events, down to very little ones that don't last much more than an instant. Given this, consider a world in $\text{Inert}(w_u, t_u)$, say w. If (49b) is true in w_u at t_u , it follows that w contains an event of Sari laughing whose run-time includes t_u. Among the subevents of this event, which themselves are events of Sari laughing, there will most likely be one that is early enough and small enough to have transpired by the end of t_u. And since up to the end of t_u, the histories of w and w_u are identical, this small Sari-laughing event in w must have a perfectly matching counterpart in w_u. That's why we infer from (49a) that there is actual laughing at the utterance time.

The only condition under which this would not hold is if the laughing starts right at the beginning of t_u and t_u itself is too short to fit even a minimal laughing event. This would have to be a very short utterance time, shorter than it realistically takes to say *Sari laughs*, so we disregard this possibility. But we will later see a problem with this.

This is the kind of example for which (47) and the simpler entry (45) predict almost identical truth conditions. (47) demands something slightly stronger, namely that moreover the laughing continues at least a little bit beyond the utterance time unless it is interrupted (which means it *would* have continued). So they are not quite equivalent, but the difference is very subtle.

Importantly, however, this almost-equivalence depends on the particular property of the meaning of the VP that we just exploited in our reasoning. Had the VP been *Sari go to the store*, it would have been a very different matter. Events of Sari going to the store are *not* made up of lots of smaller events which each are events of Sari going to store. They are made up of smaller events which are events of Sari going *towards* the store, but since most of these don't end with Sari at the store, they are not events of Sari going *to* the store. So if we are told that every $w \in \text{Inert}(i)$ contains an event of Sari going to the store which occupies a super-interval of t_i, we cannot infer that Sari goes to the store in w_i. We can merely infer that w_i contains an event that is indistinguishable from those parts of the inertia-worldly trips-to-the-store which fall *before the end of* t_i. In other words, we infer that w_i contains the *beginning* of a Sari-go-to-the-store event, but not necessarily anything more.

The attentive reader may have wondered why we used a past tense example to illustrate the perfective in the previous section, but a present tense example for the progressive in the current section. Indeed, it is incumbent upon us to examine what the theory predicts for every possible combination of a tense and an aspect.

6.4.4 *Stativity effects*

It is well known that non-stative predicates in the simple present tense have a limited range of felicitous uses. Sentences such as those in (50) are spontaneously judged as odd by speakers of English.

(50) a. #Sari laughs.
b. #Sari wakes up.
c. #Sari goes to the store.

The examples in (50) do not have prominent habitual (generic) readings. Ignore such readings if you can get them anyway. The # judgments apply to an intended episodic reading (describing a single event).

Simple present tense on a non-stative verb is systematically grammatical when the sentence has a generic or habitual interpretation, or when it describes the content of a plan or schedule. We don't worry about these cases here, since they very plausibly involve a covert modal operator of some kind that applies to the VP before any tense or (higher) aspect. (See for example, Copley 2008 or Thomas 2014.) That modal operator may itself be an aspect head, or it may create a bigger VP which is a predicate of states. In the latter case, whatever explains the acceptability of stative VPs under present tense will also explain the acceptability of present tense generics/habituals. See below. Some kind of modal analysis might also work for some of the cases in the list, like stage directions and plot summaries, but less plausibly to e.g. the sportscaster style or the historical present.

Let us see what our theory predicts. We see no progressive morphology, but there has to be an aspect head for the sentence to express a proposition, so the aspect must be PFV. With present tense semantically vacuous, we then have LFs and predicted meanings like (51) for (50c).

(51) LF: $[_{\text{TP}}$ PRES $[_{\text{AspP}}$ PFV $[_{\text{VP}}$ Sari go to the store $]]]$
true at i iff $\exists e[\tau(e) \subseteq t_i \;\&\; e \leq w_i \;\&\; \text{go-to}(e, \text{Sari}, \text{the store})]$

This says that if *Sari goes to the store* is asserted in w_u at t_u, the assertion is true iff there is a Sari-go-to-the store event in w_u whose run-time is contained within t_u. This is a somewhat implausible scenario, given that trips to the store typically take longer than the production of such a short sentence. One may be tempted to attribute the strangeness of (50c) to this fact. But upon reflection, that doesn't look like the right explanation. We can set up a scenario that eliminates the implausibility. Imagine Sari was already very close to the store, and/or she is on a very fast vehicle .... The judgment about (50c) is not really affected by such manipulations, but we would expect it to be if pragmatic plausibility were all that mattered. And the pragmatic explanation looks even less convincing when we consider the other examples in (50). Waking-up events are very short, if not instantaneous, so such events should have no problem fitting inside the utterance time and (50b) should be just fine. As regards (50a), we have already said that longer laughing events are made up of shorter laughing events. So if Sari laughs for any duration that overlaps with the utterance time, there is probably a laughing event within the utterance time, and (50a) should be fine as well.

Friends of pragmatic approaches like to remind us that the examples in (50) are not ungrammatical. Sentences of this sort are acceptable in a variety of special contexts or registers, such as play-by-play sportscasting, the historical or narrative present, newspaper headlines, stage directions, plot summaries, explicit performatives, ..., to name some. It is appealing to say that the essence of (at least some of) these special uses is a pretense that the utterance time is something other than what it is, a pretense that one is speaking at a time closer to the events being reported, at a pretend-

There are few formal semantic analyses of the historical present. An exception is Zucchi 2005.

utterance-time that is earlier and/or longer than one's actual utterance. This may or may not be right. At any rate, it does not directly address the question why (50a–c) are unacceptable *outside* of these special registers or contexts. One seems to need a concomitant assumption that the "ordinary" register involves a different pretense, namely that the utterance time is *shorter* than it actually is, in fact, that it is a mere instant in the technical sense (a singleton of one moment), and hence too short to contain even a getting-up event or a minimal laughing-event.

Cf. Bennett & Partee 1978: "We regard a speech act as occurring at a moment of time and understand the assertion as being true at that moment. Accordingly, we are inclined to only use the reportive simple present when the act being described seems to be almost instantaneous and to be occurring at the moment of utterance." See also Dowty 1979.

For the sake of the argument, let's see how it may help to stipulate that t_u is always an instant. To get the desired mileage out of this assumption, we must also sharpen some specifics regarding the lexical meanings of verbs. These assumptions are not uncontroversial, but widely accepted in the literature: None of the VPs in (50) describe events that can possibly have run-times that are instants. Any VP that entails a change of state — whether it is a change that takes time (like getting from some place else to the store) or a virtually "instantaneous" change (like from asleep to awake) — because of that applies only to events whose run-time contains at least two moments (one at which the previous state holds and one at which the result state holds). Likewise, any VP that describes an activity or movement or other happening of some sort (like laughing, oscillating, raining, even sleeping) describes events that may have very short run times but never just a single moment. These assumptions about lexical semantics make the predicted truth-conditions for clauses with PFV (as computed in (51)) impossible to satisfy unless t_i is a proper interval, i.e., not a singleton.

As Milo Philipps-Brown (pc) pointed out, one worry about this assumption is that it undermines our earlier reasoning about the progressive *Sari is laughing*. There we attributed the intuition that this sentence entails the existence of laughter in w_u to the fact that t_u was long enough to contain a minimal laugh. It is not clear how to resolve this tension. Perhaps we can get out of it by convincing ourselves that we judge the utterance true, after all, if all that actually happens before the interruption is an instant sized beginning of a minimal laugh.

These ideas are common in the literature and go back at least to Taylor 1977. See Filip 2012 for a recent and comprehensive survey.

From this perspective there is a straightforward account of what makes stative predicates different. Once we change the VPs in (50) to stative ones, the simple present tense becomes perfectly fine (in every register).

(52) a. Sari is tired.
b. Sari is at home.
c. Sari owns a factory.

Suppose the distinguishing semantic feature is precisely that predicates like *tired*, *at home*, and *own a factory* describe eventualities ("states") whose run-times can be instants. A state of Sari being tired may be long or short, but it is necessarily made up of shorter and shorter sub-states which are also states of Sari being tired. And not only that — it is even made up of such sub-states that occupy a single instant. The latter makes *tired* different from *laugh* or even *move*, which apply to eventualities whose run-times may be infinitesimally short but are still always proper intervals. What does this buy us? It lets us say that the sentences in (52) have the exact same parses as those in (50), with a perfective aspect head, and yet they have truth-conditions which can be satisfied by an instant.

(53) LF: PRES [PFV [Sari *be* tired]]
meaning: true at i iff $\exists e[\tau(e) \subseteq t \,\&e \leq w_i \,\&\, \text{tired}(e, \text{Sari})]$
lexical entry: $[\![\text{tired}]\!] = \lambda x.\lambda e.\ e$ is a state of x being tired

(Type v must be understood in such a way that D_v includes states in addition to "events" in a narrow sense. Bach 1986 coined the term "eventuality" for this broader sense of "event".)

So, together with the stipulation that the utterance time is treated as an instant, this approach to the stative/non-stative distinction provides an explanation for why stativity is required in the simple present tense. We can also reassure ourselves that present progressives are still expected to be uniformly good even if t_u must be an instant. This is because *be*-PROG places the event in a super-interval of t_i.

It is often said in this context that progressives pattern with statives in the present tense because progressive VPs *are* stative. This is not literally true on our analysis, because *be*-PROG is an aspect head and AspPs are not predicates of states (or of eventualities of any kind). One might, however, entertain a different analysis on which (at least some of) the operators we are used to calling "aspects" have meanings of type $\langle vt, vt\rangle$. (Another head higher in the structure would then have to be responsible for binding the state argument and introducing the world and time.)

Whether or not the assumption that t_u is an instant can ultimately be defended, it is important to be aware that the stativity effect we witness in present tense matrix clauses is replicated perfectly in certain environments which are neither matrix nor (morphologically) present. These environments include the complements of epistemic necessity modals and the infinitival complements of verbs like *believe* and *claim*.

(54) a. Sari must sleep/go to the store.
deontic reading only
b. Sari must be at home/ be sleeping.
epistemic reading okay

The judgment in (a) presupposes an intended episodic reading for the VP. To the extent that the VP can be read habitually, the epistemic reading becomes available. The point here is that the judgments for the constructions in (54)–(56) are parallel to the judgments for the same VPs in the simple present tense.

(55) Sari claimed to *work/*go to work/be at work/be going to work.

(56) Sari believed Svenja to *sleep/*go to the store/be at home/be sleeping.

We will return to this observation in a later section, in connection with the discussion of so-called "Sequence of Tense".

6.5 *Formalizing the referential analysis*

Above we stated Partee's proposal as follows:

(35) $[\![\text{PAST}\ \phi]\!]^i = 1$ iff $[\![\phi]\!]^{\langle w_i, t'\rangle} = 1$, where t' is the contextually salient time before t_i (no truth value defined if there is no such time)

Apart from not being fully compositional, this is a bit vague for us to work with when (in the next chapter) we consider complex sentences with several occurrences of past tense. Let us therefore make it a little more precise.

Partee suggested that past tense was analogous to a pronoun like *he*. We are used to representing pronouns as variables (see e.g. Heim & Kratzer), so Partee-style tenses too should then have denotations that are sensitive to a variable assignment. So let's make two assumptions: First, each occurrence of PAST and *woll* at LF must carry a numerical subscript, like a pronoun. Second, we add a new type i (for "intervals") and we assume that variable assignments can assign elements of D_i to object-language variables (numerical indices). Notice that this new type i is not the same as our existing type s (world-interval-pairs), though the second member of an element of D_s is always a member of D_i.

We now write the following entries. (PRES remains vacuous for now and therefore needs no entry.)

(57) $[\![\text{PAST}_n]\!]^{i,g} = \lambda p \in D_{st}:\ g(n) \in D_i \ \&\ g(n) < t_i.\ p(w_i, g(n)) = 1$

(58) $[\![\text{woll}_n]\!]^{i,g} = \lambda p \in D_{st}:\ g(n) \in D_i \ \&\ g(n) > t_i.\ p(w_i, g(n)) = 1$

Let's illustrate how this works in a simple example.

(59) Barbara turned off the stove.
new LF: PAST_7 [PFV [Barbara turn off the stove]]

(60) $[\![\text{PAST}\ _7$ [PFV [Barbara turn off the stove]]$]\!]^{i,g}$ is defined
iff $7 \in \text{dom}(g)\ \&\ g(7) \in D_i\ \&\ g(7) < t_i$
when defined, $[\![\text{PAST}\ _7$ [PFV [Barbara turn off the stove]]$]\!]^{i,g} = 1$
iff $\exists e[\tau(e) \subseteq g(7)\ \&\ e \leq w_i\ \&\ \text{turn-off}(e, \text{Barbara, the stove})]$

If this is a matrix sentence, we evaluate it with respect to the utterance world and time, and we also rely on the utterance context to furnish a suitable assignment (call it g_u), which maps the free variable 7 to a time interval that precedes t_u. Intuitively, this is the salient past interval that the speaker has in mind when making this past-tense claim (also called the "topic time"). So we have:

(61) An utterance u of the LF "PAST $_7$[PFV [Barbara turn off the stove]]" is felicitous only if
g_u is such that $7 \in \text{dom}(g_u)\ \&\ g_u(7) \in D_i\ \&\ g_u(7) < t_u$,
and it is moreover true iff
$\exists e[\tau(e) \subseteq g(7)\ \&\ e \leq w_i\ \&\ \text{turn-off}(e, \text{Barbara, the stove})]$

It will also be useful to clarify how frame adverbials might be treated in a Partee-style approach to tense. A natural idea here is that a frame adverb contributes a further presupposition about the intended topic time. Recall our treatment of frame adverbs as having extensions of type t.

(16) $[\![\text{on February 1, 2001}]\!]^{i} = 1$ iff t_i is part of February 1, 2001

To make room for this in the LF of a sentence with a Partee-style tense, we want to revise the entries from (57) and (58) and give PAST and *woll* a second argument.

(62) $[\![\text{PAST}_n]\!]^{i,g} = \lambda p \in D_{st}:\ g(n) \in D_i \ \&\ g(n) < t_i \ \&\ p(w_i, g(n)) = 1.$
$\lambda q_{st}.q(w_i, g(n)) = 1$

For example:

(63) Barbara turned off the stove on February 1, 2001.
LF: [PAST $_7$ on February 1, 2001 [PFV [Barbara turn off the stove]]]
uttered felicitously only if
$7 \in \text{dom}(g_u)$ & $g_u(7) \in D_i$ & $g_u(7) < t_u$ & $g_u(7) \subseteq$ Feb 1, 2001,
and uttered truly iff $\exists e[\tau(e) \subseteq g(7)$ & $e \leq w_u$ & turn-off(e, Barbara, the stove)]

Notice that there is still some room for context-dependency, in that the speaker may be referring to either the whole of Feb 1st or to a proper part of it (e.g. the morning of that day). But the role of context is greatly reduced by the contribution of the adverb.

The revised, 2-place, entry for the future is analogous. When we have frame adverbs with present tense, we also need a non-vacuous semantics for PRES, but this is no different in the Partee-approach than it was in the Priorian approach.

To conclude, let us highlight how the Partee-style, "referential", analysis of tenses differs from the Prior-as-modified-by-Stalnaker-style analysis, and also what they have in common. The essential difference is that Partee-style PAST$_n$ and *woll*$_n$ do not express existential quantification over times, but instead rely on a contextually furnished variable assignment to supply a particular time. In the Partee-approach, the denotations of past and future clauses are always context-dependent; in the modified-Prior approach, they only are if there happens be a silent restrictor together with the existential quantifier. Both approaches assume that the extensions of PAST$_{(n)}$ and *woll*$_{(n)}$ are sensitive to the evaluation time, and both assume that these items *shift* the evaluation time for their complements. The two approaches also agree on the treatment of PRES, which, according to both, does not shift evaluation time and makes no semantic contribution other than a presupposition when there is a frame adverb.

7 *Embedded tenses*

[what follows are the lecture notes from 2018; to be revised and retypeset]

4. Tense in embedded clauses: relative clauses

In this section we consider predictions that our analyses of tense from the previous sections make about multiclausal sentences in which an embedded tense is in the scope of another tense. Specifically, we will look at the interpretation of tenses in relative clauses[39].

For most purposes of this discussion, we can and will remain neutral about the choice between a Partee-style ("referential") analysis of tenses and a more traditional Priorian ("existential operator") analysis. We will cast the main discussion in the latter framework – specifically, we will take *PAST* and *woll* to be contextually restricted existential quantifiers (as spelled out in section 1 above), and assume that *PAST* and *woll* combine directly with a VP whose intension is of type <s,t>. Footnotes or excursions will be added to show how the relevant issues play out under the referential analysis and the division of labor between aspect and lexical verb meaning that we introduced in section 3. In the text, we generally stick to stative VPs and assume that a stative VP is true at an evaluation time if the state holds *throughout* that time.

Throughout this section, we will also reduce clutter by disregarding the world-component of the evaluation index. I.e., we will pretend here that type s is just times (intervals) rather than world-time pairs. (But this simplification will have to be undone again when we turn to tensed complement-clauses in section 5.)

4.1. A predicted correlation between DP-scope and temporal interpretation

Surface sentences of the form $[_{\text{matrix-clause}}$ .. *tense* $[_{\text{VP}}$ . ..... $[_{\text{DP}}$ .. $[_{\text{relative-clause}}$ ... *tense* ....]]]] generally should allow two different LFs, depending on where the DP is QRed: either within the matrix VP or above the matrix tense. Since the relative clause and its tense are contained in the DP, the scope of the DP determines whether or not the embedded tense is in the scope of the matrix tense. Let us look at some concrete examples to see how the relative scopes of the tenses affect the truth-conditions of the sentence.

We begin with an example with matrix future and embedded present.

(1) John will be married to someone who is famous.

To simplify matters, we disregard the covert restrictor that should come with each tense.[40] The two LFs then are (2a) and (3a), and the truth conditions that we compute for them, evaluating the sentence at the utterance time t_u, are (2b) and (3b).[41]

(2) (a) LF: *PRES* woll [[some one[42] 7[*PRES* t_7 be famous]] 8[John be married to t_8]]
(b) $\exists t\ [t > t_u\ \&\ \exists x\ [\text{famous}(x, t)\ \&\ \text{married}(j, x, t)]\ ]$

---

[39] We will also discuss tenses embedded in complement clauses, in a later section.

[40] But below we consider variants of the example that have added frame adverbs.

[41] We use some hopefully self-explanatory abbreviations in the metalanguage, e.g., *married(x, y, t)* for *x is married to y at t*.

[42] W simplilify here by disregarding the pro-head-noun *one* (i.e, treating it as semantically vacuous). Strictly speaking, this contributes a predicate like 'person' or 'human', which is also evaluated at different

(3) (a) LF: [some one 7[*PRES* t_7 be famous]] 8[*PRES* woll [John be married to t_8]]
(b) $\exists$x [famous(x, t_u) & $\exists$t [t > t_u & married(j, x, t)]]

The scopal relations between the two existential quantifiers don't make a logical difference, but what does differ between the two LFs is where the predicate *famous* is evaluated: In (2), *famous* is evaluated at the same future time as the predicate *married-to*, whereas in (3), *famous* is evaluated at the utterance time. So (2) says that at some future time, John will be married to a person who will be famous *then* (a so-called "simultaneous reading"), whereas (3) says that he will be married to a person who is famous *now* – let's call this a "*now* reading". (Neither LF, of course, denies additional fame at other times than those required for its truth.) The two predicted readings are logically independent, i.e., there are scenarios that make (2) true and (3) false, and scenarios that make (3) true and (2) false. The English sentence can indeed be judged as true under either one of these types of scenarios, so our predictions look satisfactory.[43]

Our theory also makes correct predictions about what frame adverbs we can coherently add to this kind of sentence and how their addition narrows down the range of verifying scenarios. Consider, for example, the sentences in (4) - (6). (# marks a judgment of incoherence.)

(4) In 2020, John will be married to someone who is in jail then (in 2020).

(5) In 2020, John will be married to someone who is in jail now (in 2018).

(6) #In 2020, John will be married to someone who is in jail in 2019.

In order to interpret the adverbials, we need the 2-place meaning for *woll* and the non-vacuous, 2-place meaning for *PRES* (see sec 1.2. and fn 12). Spell out the details as an exercise. (4) has to have an LF where the *someone*-DP scopes below the matrix *woll*, and (5) an LF where it scopes above it. The opposite scope constellations can also be generated, but they receive unsatisfiable truth conditions (or presuppositions) due to the adverbs. (Assume that t_u is in 2018.)

Next we have an example with past tenses in both matrix and relative clause, (7). The two LFs and their truth conditions are in (8) and (9).

---

times dependiing on the DP's scope. See below for an example with a more contentful head noun.

[43] In a Partee-style theory with a division of labor between tense and aspect, the predicted truth conditions also depend on the choices of (silent) aspect heads. We assume that, for type-reasons, the *someone*-DP cannot be QRed to the edge of VP (below aspect), but it can scope either just above AspP or above *woll*. If the aspects in both matrix and relative clause are imperfective, the predicted truth-conditions will be essentially the same as in the Priorian theory, modulo the difference that the speaker is referring to a specific future time. Also, if the DP scopes above *woll*, then no matter what aspects we have we predict a *now*-reading, in the sense that fame must hold at t_u for the LF to be true. When the DP scopes below *woll*, however, this isn't necessarily a "simultaneous" reading. If both clauses are parsed with perfective aspect, then the truth-conditions can be satisfied if the speaker's intended future topic time contains a period marriage and a period of fame, but not necessarily overlapping or in any specific order. We may wonder if such a truth-condition is attested, and if not, may want to constrain the system in such a way that at least the embedded stative must be parsed with imperfective. Be that as it may – we do still make the prediction that a *now*-interpretation for the embedded clause is contingent on scoping the DP above *woll*. This prediction is not affected by the switch from Priorian to Partee-style framework. Neither are the predictions regarding compatible frame adverbs.

(7) John was married to someone who was famous.

(8) (a) LF: *PAST* [[some one 7[*PAST* t_7 be famous]] 8[John be married to t_8]]
(b) $\exists t\, [t < t_u\ \&\ \exists x\, [\, \exists t'[t' < t\ \&\ \text{famous}(x, t')] \,\&\, \text{married}(j, x, t)]\,]$

(9) (a) LF: [some one 7[*PAST* t_7 be famous]] 8[*PAST* [John be married to t_8]]
(b) $\exists x\, [\, \exists t'\, [t' < t_u\ \&\ \text{famous}(x, t')] \,\&\, \exists t\, [t < t_u\ \&\ \text{married}(j, x, t)]\,]$

Again there is a difference in how the time of fame relates to the time of marriage. (8b) requires for its truth that the person be famous (at least) for some time *before* the marriage. (Though it does not deny that the person is also famous during or after the marriage.) (9b), by contrast, leaves the temporal relation between fame and marriage completely open. Both must obtain somewhere before the utterance time, but the onset of fame could be before, during, or after the marriage.[44] In this example, one of the predicted readings entails the other; i.e., any scenario that makes (8) true makes (9) true as well – but not the other way round. Again, speaker's judgments about the sentence (7) are consistent with our predictions. In particular, (7) can be judged true in a variety of scenarios, including a scenario where the (first) onset of fame is after the end of the marriage (but before the speech time). Following Kusumoto (2005), we call this type of scenario a "later-than-matrix" scenario.

Again we can add frame adverbs to sharpen intuitions and further test our predictions.

(10) In 2016, John was married to someone who was rich then (in 2016).

(11) In 2016, John was married to someone who was rich in 2015.

(12) In 2016, John was married to someone who was rich in 2017.

All these sentences are coherent, and we can account for this – provided again that we assume suitable scopes for the DP. (12), in particular, receives satisfiable truth conditions only if the DP scopes above the matrix *PAST*.

---

[44] What about the Partee-style framework? Here we have two past tenses, each with its own variable subscript, which in principle could be two occurrences of the same free variable or two different variables. The speaker accordingly must have either just one or else two different salient ("topic") times in mind. It turns out, however, that coindexing is only an option if neither past is in the scope of the other; otherwise we generate a self-contradictory presupposition that the embedded topic time precedes itself. If the embedded past is in the LF-scope of the matrix past, it must refer to a different, and earlier, interval than the matrix past. The predicted truth-conditions for the LF with narrow DP-scope are then essentially the same as in the Priorian analysis (and not detectably affected by choice of aspects).

In the LF where the DP scopes outside the matrix tense, the two past tenses can be coindexed or not and can effectively refer to any two topic times as long as both of them are before t_u. Even if they corefer, the option of construing both predicates as perfective (if available for statives) implies that marriage and fame need not stand in any specific temporal relation to each other. Overall, the range of possible verifying scenarios for the wide-scope-DP LF ends up being the same as in the Priorian analysis. The crucial prediction we are aiming to highlight here – namely that later-than-matrix scenarios can *only* verify the wide-scope-DP LF – is made in the Partee-framework just as much as in the Priorian one.

4.2. An overgeneration problem for present under past

There are a number of other combinations of a matrix tense and an embedded tense that we could consider at this point, but mostly this will not teach us much more. The one exception concerns sentences with a matrix past and an embedded present or *will*.

(13) John was married to someone who is famous.

(14) John was married to someone who will be famous.

Given that *will* spells out *PRES* + *woll*, both (13) and (14) really are instances of *PRES* embedded under *PAST*. We focus our discussion on (13), and you should be able to work out how it extends to (14).

As with the earlier examples, our theory predicts a scopal ambiguity due to multiple possible landing sites for QR of the *someone*-DP.

(15) (a) LF: *PAST* [[some one 7[*PRES* t_7 be famous]] 8[John be married to t_8]]
(b) $\exists t\ [t < t_u\ \&\ \exists x\ [\text{famous}(x, t)\ \&\ \text{married}(j, x, t)]\ ]$

(16) (a) LF: [some one 7[*PRES* t_7 be famous]] 8[*PAST* [John be married to t_8]]
(b) $\exists x\ [\text{famous}(x, t_u)\ \&\ \exists t\ [t < t_u\ \&\ \text{married}(j, x, t)]\ ]$

In (15), *famous* is evaluated at the same (past) time as the predicate *married-to*,[45] and in (16), *famous* is evaluated at the utterance time. Unfortunately for our theory, only (16) corresponds to an attested reading of the English sentence. Present under past can *only* have a "*now* reading". (13) doesn't have an additional "simultaneous" reading with the truth-conditions (15b) that are expressed by LF (15a). This fact is further highlighted by the deviance that results from adding simultaneity-forcing adverbs as in (17).

(17) # In 2016, John was married to someone who is famous at that time/then (in 2016).

So here we have a problem of overgeneration. How might we fix it? For the time being, we will just hint at three general directions for a solution, without trying to make any one of them precise. We will return to this task after we have gathered a bit more evidence that tells us which directions are more likely to be correct.

Since the problem stems from the fact that we generate the LF in (15), one idea is to block the generation of this LF. In (15), *PRES* is in the scope of *PAST*. Perhaps this scope constellation is for some reason not allowed. Stowell (1993, 1995) suggested an analogy with polarity-sensitive items such as *any* and *some*, which mean the same thing but are subject to different distributional constraints at LF. *any* is a negative polarity item (NPI) and as such is required to be in the scope

---

[45] Again, the Partee-framework yields essentially the same prediction. In particular, since present tense in that framework too is vacuous (except for possible presuppositions), a present AspP within the scope of the matrix past tense will be evaluated at the past topic time that the speaker intends for the matrix predicate. Whether this amounts to a genuinely simultaneous reading depends again on the choice of embedded aspect, but either way, it is a reading according to which fame holds in the past and need not hold at t_u. So the overgeneration problem carries over.

of a negation;[46] *some* is a positive polarity item (PPI) and as such is required to be *outside of* the scope of negation. Stowell called the present tense an "anti-past-polarity item", meaning an item that is barred from occurring in the scope of *PAST* at LF – just as *some* is barred from occurring in the LF-scope of *not*. If we place *some* in the surface c-command domain of *not*, as in *John didn't solve some problems*, QR must apply to give it wide scope at LF, and only an inverse-scope reading is therefore attested. Similarly, Stowell suggested, if *PRES* is in the surface c-command domain of *PAST*, as in our sentence (13), the only way to satisfy its anti-past polarity requirement is to change scope relations by means of covert movement – in this case by QRing the DP that contains the *PRES* above the matrix *PAST*. So the only licit LF for (13) will be (16) – which indeed is the LF that captures the attested meaning.

A second, completely different, approach to the problem says that (15a) is basically a fine LF – except it doesn't get pronounced as (13). Rather, it gets pronounced as *John was married to someone who was famous*. In other words, when *PRES* is in the LF-scope of *PAST*, it gets pronounced like a past tense. (This is reminiscent of the "Sequence of Tense" rules of traditional grammar.) If a *PRES* that is in the LF-scope of *PAST* can't be pronounced as a morphological present, then a morphological present can't be parsed as a *PRES* that is in the scope of *PAST* at LF. So again we get rid of the overgeneration problem and predict correctly that the surface sentence (13) doesn't have the reading in (15b). This second approach places the burden on a theory of morphological spell-out which is far from trivial to work out.

Yet another direction to take is to question our assumptions about the semantics of present tense. So far we have assumed that *PRES* is either vacuous or, if restricted by an adverb, introduces a presupposition about the evaluation time. Either way, *PRES* doesn't shift the evaluation time, and therefore the sister-VP of a *PRES* is evaluated at the same time as the material right above it, whatever time that may be. Our semantics recognizes no intrinsic connection between *PRES* and the utterance time t_u. Only when a *PRES* happens to be topmost in a matrix clause (or embedded, but not under any operators that shift evaluation time), will its sister-VP be interpreted at t_u. This is then due to the Utterance Rule (see (6) in sec 1.1.) and not to the meaning of *PRES*. But perhaps what the data in (13) and (17) should be teaching us is that this was wrong. Even when embedded under an evaluation-time shifter such as the matrix *PAST* in LF (15a), *PRES* seems to select t_u as the evaluation time for its VP. So perhaps we must refer to t_u in the semantics of *PRES* itself rather than just in the Utterance Rule. On this type of approach, (15a) would be a well-formed LF and also a possible parse of the surface sentence (13), but it wouldn't have the meaning in (15b).

4.3. Counterexamples to the correlation of DP-scope and temporal interpretation

Even though the overgeneration problem we identified in section 4.2 is the most glaring shortcoming of our current theory, we will set it aside for a while. Instead, we now take a closer

---

[46] This is grossly oversimplified. (For one thing, NPIs can also be in the scope of downward-entailing operators other than negation.) We are just pointing to Stowell's analogy here, not being serious about NPIs and PPIs.

look at the predictions that we drew out in section 4.1. concerning the systematic connection between DP-scope and temporal interpretation in relative clauses. Let's remind ourselves of the predictions.

(18) Prediction 1: In a sentence of the (surface) form

$[_{\text{matrix-clause}}$ .. *past/will* $[_{\text{VP}}$ ..... $[_{\text{DP}}$ .. $[_{\text{relative-clause}}$ ... *present* ....]]]],

the DP must scope higher than the matrix *PAST* or *woll* in order to get a *now* reading for the VP in the relative clause.

(19) Prediction 2: In a sentence of the (surface) form

$[_{\text{matrix-clause}}$ .. *past* $[_{\text{VP}}$ . ..... $[_{\text{DP}}$ .. $[_{\text{relative-clause}}$ ... *past* ....]]]],

the DP must scope higher than the matrix *PAST* for the sentence to be true in a later-than-matrix scenario.

We will consider two types of examples that turn out to violate these predictions. One is a kind of example discussed by Keshet (2010)[47], in which the head noun of the relative clause gets evaluated at a different time than the predicate in the relative clause.

(20) Five years ago, Jill married a 30-year-old who made partner two years later.

It is easy to understand (20) as saying that Jill's spouse was 30 at the time of marriage and that two years later, at the age of 32, the spouse made partner. Given the adverb *five years ago,* we can conclude that the spouse is 35 at the time of utterance. The later-than-matrix reading of the relative clause requires the object DP to take scope above the tense, allowing it to be interpreted relative to t_u. But then the noun phrase *30 year old* will also be interpreted relative to t_u and we'll get the set of individuals who are 30 years old at the time of utterance. Jill's spouse is not in that set. Summarizing, the head noun interpretation requires the object DP to be in the scope of matrix past tense. But the relative clause needs to be outside the scope of the matrix tense, given the theorem in (19). This contradiction reveals a flaw in the system of interpretation.

The second type of example, due to Kusumoto (2005)[48], contains a negative polarity item in the DP whose licenser is below the matrix tense, as in (21).[49]

(21) Next year, he will try not to rent to anybody who lives in this building now.

(22) They failed to write a single law that was signed by the Governor.

The point of using NPIs (here *anybody*, *a single*) is to confine scope of the DP. Since the NPI must be in the scope of its licencer (here *not*, *fail*), it is *ipso facto* in the scope of the matrix tense. In (21), NPI *anybody* must be in the scope of negation and hence it must be in the scope of *woll.* But *lives* is interpreted relative to the time of utterance, which as the theorem in (18) says,

---

[47]Keshet, E. (2010) "Situation Economy," *Natural Language Semantics* **18.4**:385-434

Keshet, E. (2011) "Split Intensionality: A New Scope Theory of De re and De dicto," *Linguistics and Philosophy* **33.4**:251-283

[48] Kusumoto, K. (2005) "On the Quantification over Times in Natural Language," Natural Language Semantics 13.4:317–357

[49] (21) and (22) were provided by Roger Schwarzschild. See below for Kusumoto's own examples.

requires the DP to have scope above tense.

(22) describes a dysfunctional state government in which the legislature writes laws and then nothing happens to them because when they reach the governor's desk, they don't get signed. NPI *a single* must be in the scope of *fail* and hence it must be in the scope of **PAST** on *fail*. The potential signings are later than the writing, hence the relative clause has a later-than-matrix interpretation, which as the theorem in (19) says, requires the DP headed by *a single* to have scope above the tense.

<u>Homework</u>: Kusumoto uses *failed to* rather than *didn't* in order to make sure that the semantic negation is clearly within the scope of the matrix PAST and not e.g. right above it. (If the negation were above the tense, the NPI could be licensed by scoping the DP between them, and this way we could capture the desired reading that's true in the later-than-matrix scenario.) However, this makes sense only within a Partee-style referential approach to tense, not in the modified-Prior framework we have employed here in the main text.[50] To see the issue, consider a simple *fail*-sentence such as (i).

(i) She failed to turn off the stove.

A reasonable semantics for *fail* is (ii).

(ii) $[\![\textbf{fail}]\!]^{i} = \lambda u \in D_t.\ u = 0$

This says that *fail* is means the same as *not*, it just has a different syntax.[51]

(a) Show that, in the context of the Partee approach to tense (as spelled out above in the Appendix to section 3), this meaning for *fail* allows us to assign a reasonable syntactic analysis and intuitively appropriate interpretation to example (i). Explain why the same is not true for the modified Priorian approach.

(b) Analyze example (22) within the Partee approach and spell out how it makes Kusumoto's point.

(c) [To be done after you have read to the end of section 4.4.2.] Show how the problem is solved in the "extensional" framework (Partee-style version) that we introduce in the next section below (incl. footnote). □

Kusumoto's original examples are the past-under-past sentences (23) and (24), to which we can also add present-under-past variants (25) and (26).

(23) (At the audition last month), I tried not to hire anybody who put on a terrible performance (tonight).

(24) (At the Open House last week), she failed to talk to any prospective student who (later) decided to come to UMass.

---

[50] Kusumoto's paper does employ a version of the Partee approach, so this is not a criticism of her argumentation.

[51] Perhaps there is an additional modal component to the meaning of *fail*, something like 'didn't, but should have'. Even so, this won't affect the argument regarding the scope of the negation in *fail*.

(25) (At the Open House last week), she failed to talk to any prospective student who is still undecided (today).

(26) (At the audition last month), I tried not to hire anybody who is putting on a terrible performance (tonight).

The logic of the argument is the same as above. The (only available) licenser for the NPI *any* is clearly within the scope of the matrix past tense. So if the *any*-DP wanted to QR above the matrix tense, it would also have to QR above its NPI-licenser. So our theory predicts that none of these examples should be verifiable by later-than-matrix scenarios, and with the parenthesized adverbs, they should be incoherent. (On Stowell's proposal, the examples with embedded present tense (i.e., (25), (26)) should even be plainly ungrammatical.) But (at least for Kusumoto's informants[52]), these sentences are coherent descriptions of later-than-matrix scenarios. Kusumoto offers the following verifying scenarios. "Suppose we are watching a play with a casting director. Some of the cast members are very bad and the play is a failure. The casting director can truthfully say something like [(23)], claiming no responsibility for the failure of the play." (Kusumoto 2005, p. 327)[53] For (24), "..., suppose that ten prospective students showed up at the UMass open house, all of whom had not decided whether to come to UMass yet. A faculty member talked to only five of them, and none of them decided to come. Among those who she failed to talk to, four decided to come to UMass. In this situation, sentence [(24)] is judged true." (loc.cit.)

Kusumoto's conclusion is that, to avoid these false predictions, we need a theory which allows embedded tenses to be evaluated with respect to the utterance time, even when these embedded tenses are in the LF-scope of higher tenses that shift the evaluation time for their sisters. This situation should remind you of a problem we discussed earlier in the semester: the "third readings" of DPs in the scope of modal operators. Indeed we will join a widespread consensus in the field that the two problems are the same and have a single solution.

4.4. Relative clauses in a framework with index variables in the object language

4.4.1. The extensional system (mostly review[54])

We return to the basic system used in Heim & Kratzer up to chapter 11. The interpretation function is relativized only to an assignment function, not to any other evaluation parameters such as a world, a time, or an index. The semantic rules are Functional Application, Predicate Abstraction, and Predicate Modification, in their formulations from the earlier part of H&K. There is no rule of Intensional Functional Application. The only ingredient of intensional semantics that we do retain is the expanded type system and ontology. We have a third basic

---

[52] Roger Schwarzschild reports disagreement with these judgments.

[53] Since Kusumoto's example (23) has past tenses in both clauses, she imagines the casting director to be speaking after the performance. For the pres-under-past version (26), he would have to be speaking during the play.

[54] from ch. 8 of 2011 lecture notes

type besides e and t, the type s. D_s is the set of all indices, i.e., world-time pairs. A new assumption (different from both our previous extensional and intensional systems) is that assignment functions are now (partial) functions from object-language variables (natural numbers) into $D_e \cup D_s$, i.e., their possible values include world-time pairs as well as individuals.

There are a number of innovations in the lexicon and in the syntax. As for the lexicon, the main change concerns the treatment of predicates (verbs, nouns, adjectives). They now all get an additional argument, of type s.[55]

(10) $[\![\textbf{tired}]\!] = \lambda i \in D_s.\ \lambda x \in D_e.$ x is tired in w_i at t_i
$[\![\textbf{like}]\!] = \lambda i \in D_s.\ \lambda x \in D_e.\ \lambda y \in D_e.$ y likes x in w_i at t_i
$[\![\textbf{person}]\!] = \lambda i \in D_s.\ \lambda x \in D_e.$ x is a person in w_i at t_i
etc.

This also applies to attitude predicates, modals, and tenses. We illustrate with *PAST*[56] and the counterfactual modal *would*[57].

(11) $[\![\textbf{would}]\!] = \lambda i \in D_s.\ \lambda p \in D_{<s,t>}.\ \lambda q \in D_{<s,t>}.$
$\forall w\ [\ p(w,t_i) = 1\ \&\ w \text{ is otherwise similar to } w_i \rightarrow q(w,t_i) = 1\ ]$

(12) $[\![\textbf{PAST}]\!] = \lambda i \in D_s.\ \lambda p \in D_{<s,t>}.\ \lambda q \in D_{<s,t>} : \exists t\ [\ t < t_i\ \&\ p(w_i,t) = 1\ ].$
$\exists t\ [\ t < t_i\ \&\ p(w_i,t) = 1\ \&\ q(w_i,t) = 1\ ]$

There is no change to the entries of proper names, determiners, or truth-functional connectives; these keep their purely extensional ("s-free") types and meanings.

Finally, there are two new kinds of abstract (i.e., unpronounced) morphemes. One is a series of pronouns of type s ("index pronouns" or "world-time pronouns"). We write them as $\textbf{pro}_\textbf{n}$, with a numerical subscript **n**. (This makes them look just like covert pronouns of type e, but the environment will disambiguate. Besides, we won't be using examples with both types of pronouns in them.) Their semantics is what you expect: they get values from the assignment function.

---

[55] The decision to make the index-argument the predicate's first (lowest) argument is arbitrary, and nothing hinges on it. For all we know, it could be the highest argument, or somewhere in between.

[56] (12) is the version for the Priorian framework (as modified to accommodate frame adverbs). A Partee-style tense now gets an entry like (i):

(i)(a) full version that accommodates frame adverbs:
$[\![\textbf{PAST}_\textbf{n}]\!]^g = \lambda i \in D_s.\lambda p \in D_{\langle s,t\rangle}: g(\textbf{n}) \text{ is a time interval } \& \ g(\textbf{n}) < t_i\ \&\ p(w_i, g(\textbf{n})) = 1.$
$\lambda q \in D_{\langle s,t\rangle}.\ q(w_i, g(\textbf{n})) = 1.$

(b) simplified (no adverb):
$[\![\textbf{PAST}_\textbf{n}]\!]^g = \lambda i \in D_s: g(\textbf{n}) \text{ is a time interval } \& \ g(\textbf{n}) < t_i.$
$\lambda q \in D_{\langle s,t\rangle}.\ q(w_i, g(\textbf{n})) = 1.$

Rewriting the entries for aspect heads is also routine, e.g.:

(ii) $[\![\textbf{PFV}]\!] = \lambda i \in D_s.\ \lambda P_{<v,t>}.\ \exists e\ [P(e) = 1\ \&\ \tau(e) \subseteq t_i\ \&\ e \leq w_i].$

[57] We take this modal to be accidentally homophonous with the spell-out of *PAST+woll*. That's probably not right, but this is a complex research area beyond the scope of this class. (See Iatridou's 2000 LI-article and current work by Kai and Sabine.)

The other new item is a semantically vacuous operator, *OP*, which moves and leaves a trace of type s. Its syntactic properties are such that it must end up in C or right below a functional head in the "clausal spine" between C and V, and it must get there by a very short movement, a kind of "head movement". We are leaving this rather vague.[58] – We also stipulate that a complete (matrix) sentence must not contain any free variables of type s and must receive a denotation of type <s,t>. This implies that there is a CP layer in matrix clauses and there is always an instance of *OP* in the matrix C.[59]

We have everything in place now to return to the discussion of tense. Let's just finish our review session with a quick reminder of the analysis for third readings in modal contexts.

(13) If everyone in here were outside, Building 56 would be empty.
LF (ignoring tense):
OP$_{\mathbf{1}}$ [[would-$\mathbf{t_1}$ [*if OP*$_{\mathbf{2}}$. every*one* in-here-$\mathbf{pro_1}$ *be* outside-$\mathbf{t_2}$]] [*OP*$_{\mathbf{3}}$. Bg56 *be* empty-$\mathbf{t_3}$]]

To make interpretable the node immediately above each lexical predicate or modal, some pronoun or trace must occupy its innermost (type-s) argument position. We can't just use pronouns everywhere, because then they would all remain free. There is no free insertion of lambda-binders in this theory. All binding depends on movement, so we need to generate operators in at least some of the argument positions and we must move them. The semantic type of the modal *would* furthermore demands two arguments of type <s,t>, and constituents of this semantic type can only be created by moving an operator to their edges. The complete sentence also must be of type <s,t> and therefore have an operator at the very top. These strictures together determine almost everything in the LF in (13) – except the fact that the *if*-clause must have a pronoun in the subject and an operator in the predicate and not the other way round. We attribute the latter fact to (vaguely stated) syntactic constraints on the operator's movement path and landing site (pending a deeper explanation).

4.4.2. Relative clauses and their tenses

The rule of Predicate Modification (PM) has not changed from the original H&K system. It applies to two phrases of type <e,t>. Therefore a relative clause must have this type in order to be able to combine with its head noun. This means that we don't want an *OP* at the edge of a relative clause, neither below nor above the moved relative pronoun. (We would get the wrong semantic type either way, be it <e,st> or <s,et>.) Instead we have to use a pronoun in the argument structure of the highest predicate/modal/tense within the relative clause. This pronoun is free in the constituent to which we apply PM. (Though, of course, it can be – in fact, must be – bound eventually in the higher structure.) If the relative clause is past or future, this free

[58] As noted in the lectures on the "third reading", there is a substantial recent literature exploring various ways to give principled explanations for Percus's "Generalization X" and similar constraints on the distribution and binding of world variables. Here we just assume that some principles or other are in place to prevent overgeneration (e.g., the unattested reading of Percus's *Mary thinks that my brother is Canadian*).

[59] It also requires rewriting the definition of truth/falsity of an utterance. Instead of (6) in section 1.1, we now need this: An utterance of a sentence ϕ that is made in a world w at a time t is true iff $[\![\phi]\!](w,t) = 1$.

pronoun is the argument of *PAST* or *woll*; if the relative clause is in the present tense (which we still treat as vacuous), the free pronoun is the argument of the verb (or the adjective/noun after a copula)[60]. On the other hand, at the edge of the VP-complement to *PAST* or *woll* we always need a moved *OP*, because these temporal operators select for arguments of type <s,t>.[61] (Furthermore we need to merge an *OP* in the inner argument position of the matrix tense, so that we can move this to the matrix C and satisfy our requirement that the matrix clause denote a proposition of type <s,t>.)

Let us illustrate all this with one of our examples from section 2.1.[62]

(14) John will be married to someone who is famous.

(15) LF1: narrow DP scope, local binding:
$\mathit{OP}_{\mathbf{1}}$ woll-$\mathbf{t_1}$
$\mathit{OP}_{\mathbf{2}}$ [[some *who*$_6$[*PRES* t_6 *be* famous-$\mathbf{pro_2}$]] 7[John *be* married-$\mathbf{t_2}$ *to* t_7]]
$\lambda i.\ \exists t\ [t > t_i\ \ \&\ \exists x\ [\mathrm{famous}(x, t)\ \&\ \mathrm{married}(j, x, t)]\ ]$
("simultaneous" reading, i.e., famous when married)

(16) LF2: narrow DP scope, non-local binding:
$\mathit{OP}_{\mathbf{1}}$ woll-$\mathbf{t_1}$
$\mathit{OP}_{\mathbf{2}}$ [[some *who*$_6$[*PRES* t_6 *be* famous-$\mathbf{pro_1}$]] 7[John *be* married-$\mathbf{t_2}$ *to* t_7]]
$\lambda i.\ \exists t\ [t > t_i\ \ \&\ \exists x\ [\mathrm{famous}(x, t_i)\ \&\ \mathrm{married}(j, x, t)]\ ]$
("famous now" reading)

(17) LF3: wide DP scope:
$\mathit{OP}_{\mathbf{1}}$ [[some *who*$_6$[*PRES* t_6 *be* famous-$\mathbf{pro_1}$]]
7[woll-$\mathbf{t_1}$ $\mathit{OP}_{\mathbf{2}}$ [John *be* married-$\mathbf{t_2}$ *to* t_7]]
$\lambda i.\ \exists x\ [\mathrm{famous}(x, t_i)\ \&\ \exists t\ [t > t_i\ \ \&\ \mathrm{married}(j, x, t)]\ ]$
("famous now" reading)

We are still considering both scopal relations between the *some*-DP and the matrix tense (for all we know, nothing rules one out). But the reading we obtain by scoping the DP high (LF3) is now equivalent to one of the LFs in which the DP scopes low, namely LF2, where the world-time pronoun in the relative clause is non-locally bound from the matrix C.

---

[60] Or the argument of Asp, if we use the Davidsonian framework in which VPs are type <v,t>.

[61] Could we avoid this *OP* and instead move the tense operator itself from the argument position of the verb"? That kind of syntax has indeed been explored by Junri Shimada http://research.nii.ac.jp/~kanazawa/semantics/2007/0817/Head_Movement_Binding_Theory_Phrase_Structure.pdf, who credits the idea to Kai von Fintel http://web.mit.edu/fintel/choicepoints.pdf.

[62] Here we simplify again in various ways. First, we omit the world-component of the index. Second, we ignore the tenses' restrictors. Third, we disregard the dummy head noun 'one'. Notice that the relative clause nevertheless needs to be of type <e,t>, since this is the type that the determiner *some* combines with.

Please draw yourself some trees, they will be more readable than the bracketed strings. Some notational conventions I have used to improve readability at least a little: low numbers (1, 2, ...) and boldface for variables of type s, higher numbers (6, 7, ...) and plain font for variables of type e; hyphenating type-s arguments with the predicates they saturate; small italics for semantically vacuous items.

This equivalence is the key to the solution of Kusumoto's problems. What the system achieves is the freedom to evaluate the embedded tense with respect to the utterance time even when it is in the scope of the matrix tense. Therefore, we can trap the DP's scope below the matrix tense – for example, by introducing an NPI determiner and a suitably low licenser for it – and still evaluate the relative clause at the matrix index.

4.4.3. Present under past again: the overgeneration problem is still with us

Switching to the extensional framework has addressed Kusumoto's argument against the scopal theory of later-than-matrix interpretations, but it has not done anything yet to fix our overgeneration problem. We still predict an unattested simultaneous reading for present embedded under past. Here is its new LF.

(18) John was married to someone who is famous.

(19) LF with narrow scope DP and local binding:
OP$_1$ PAST-$\mathbf{t_1}$
OP$_2$ [[some *who*$_6$[*PRES* t_6 *be* famous-$\mathbf{pro_2}$]] 7[John *be* married-$\mathbf{t_2}$ *to* t_7]]
$\lambda i.\ \exists t\, [t < t_i\ \&\ \exists x\, [\,\text{famous}(x, t)\ \&\ \text{married}(j, x, t)]\,]$
(unattested "simultaneous" reading, i.e., famous when married)

But we have made some progress. For one thing, we have learned something about how *not* to fix the problem. We don't want to legislate against the *scopal relations* that we see in (19). Whatever we do must not rule out the scopally isomorphic LF in (20).

(20) LF with narrow DP scope and non-local binding:
OP$_1$ PAST-$\mathbf{t_1}$
OP$_2$ [[some *who*$_6$[*PRES* t_6 *be* famous-$\mathbf{pro_1}$]] 7[John *be* married-$\mathbf{t_2}$ *to* t_7]]
$\lambda i.\ \exists t\, [\, t < t_i\ \&\ \exists x\, [\,\text{famous}(x, t_i)\ \&\ \text{married}(j, x, t)]\,]$
(attested "famous now" reading)

This means that Stowell's analogy with polarity sensitivity is coming to look unhelpful. What about the traditional idea that a Sequence of Tense rule governs morphological spell-out and ensures that the LF in (19) is paired with the PF *John was married to someone who <u>was</u> famous*? This still also looks rather unappealing, given the lack of locality between the affected verb form and the environment that must trigger the rule. (Again, the rule must not apply indiscriminately to both (19) and (20), yet these structures are indistinguishable in the immediate local vicinity of the affected verb.) Nevertheless, something like a Sequence of Tense rule, a non-local morphological agreement mechanism, turns out to be the favored solution in much of the recent literature.[63] We present this in the next section. And we will see that the switch to our current extensional framework, while not by itself the solution to the problem, was a necessary prequisite for it. The solution to be presented is one that could not have gotten off the ground without a syntax that posits world-time pronouns and traces as part of syntactic representations.

---

[63] For a dissenting view and counterproposal, see two recent papers by Altshuler & Schwarzschild (Amsterdam Colloquium 2013, Sinn und Bedeutung 2013).

4.5. Sequence of Tense as feature-agreement under semantic binding

Kratzer (1998, SALT) proposed that the phenomenon of Sequence of Tense has the same nature and explanation as a certain phenomenon in the morphosemantics of personal pronouns that she dubbed "fake indexicality".[64] We take a brief excursion into this topic before we return to tense.

4.5.1. Fake indexicals and feature transmission

Fake indexicals are first or second person pronouns that are interpreted as bound variables. (This makes them "fake" because a genuine indexical refers, by definition, to a specific contextually determined individual.) An example is (21), on the prominent "sloppy" reading that denies that other people brushed *their* teeth.

(21) Only I brushed my teeth.

The challenge such examples pose emerges, for example, if one tries to extend the presuppositional account of gender features[65] to other phi-features including person. A presuppositional semantics for 1st-person presumably would say that $[\![my_{\mathbf{n}}]\!]^g$ is undefined unless g(**n**) is the speaker. But this would make it impossible for *my* in (21) to take on a range of alternative values, as it has to if it is to be bound by the quantifier *only I*.

We will not engage here in a serious discussion of fake indexicality, just sketch the analysis developed in Kratzer (1998) and related work. This assumes that grammar does not produce a perfect match between semantically interpreted and phonologically realized phi-features. In particular, what we witness in (21) is a 1st-person feature on *my* that is present at PF but absent at LF – hence not contributing to the meaning of this sentence (whatever the actual semantics of 1st person may be). Implementations use either a mechanism that deletes certain base-generated features in the LF-branch of the derivation while retaining them in the PF-branch[66], or else a mechanism that adds (copies) features in the PF-branch onto nodes that are feature-less underlyingly and at LF. Either way, the mechanism is crucially sensitive to a syntactic representation of semantic binding relations (such as coindexing) and it (probably[67]) operates non-locally. For concreteness we state the following rule (22), which employs the concept of "binding" defined in (23).[68]

---

[64] Philippe Schlenker pursued a related but distinct approach to SOT-phenomena in his 1999 MIT thesis. For Schlenker, the essential analogy was between SOT and indexical shifting (in languages like Amharic) – or rather, actually, between indexical shifting and the *absence* of SOT (in non-SOT languages such as Japanese and Russian). How different the two approaches ultimately are depends on how one views the relation between fake indexicals and shifted indexicals, a question on which both Kratzer and Schlenker, as well as a number of later authors, have taken evolving positions over the years.

[65] Cooper, Heim & Kratzer. If one doesn't treat 1st-person pronouns as variables in the first place, but as indexicals in the sense of Kaplan, one has an even more basic problem. They cannot be bound variables then.

[66] see von Stechow 2003

[67] Kratzer (1998, 2005) argues that it obeys certain locality constraints after all, but this is controversial. Most other authors (Schlenker 1999, von Stechow 2003, Heim 2005, 2008, Wurmbrand 2015) assume or argue for non-local versions.

[68] See also H&K p. 263.

(22) Feature transmission under semantic binding:
In the derivation of PF, copy the features of a phrase onto any pronouns and traces it binds.

(23) α binds $\beta_{\mathbf{n}}$ iff α's sister node (not counting semantically vacuous material[69]) has the daughters **n** (a binder index) and γ, such that γ dominates $\beta_{\mathbf{n}}$ (and does not dominate any other occurrence of **n** that c-commands $\beta_{\mathbf{n}}$).

Applying this rule to examples requires a few more ancillary assumptions about feature traffic. E.g., for (21) we must assume that the 1st-person feature base-generated on *I* first percolates to the quantifier *only I*, from whence it then can be transmitted down to the possessive pronoun by rule (22). So the derivation of (21) goes like this.

(24) base-generate: [only [pro$_7$ 1st]] brush pro$_8$'s teeth
subject moves spec-V to spec-I: [only [pro$_7$ 1st]] 8[t$_8$ brush pro$_8$'s teeth]
derivation to LF: no further changes
derivation to PF: percolate in *only*-DP: [only [pro$_7$ 1st]]-*1st* 8[t$_8$ brush pro$_8$'s teeth]
transmit under binding: [only [pro$_7$ 1st]]-*1st* 8[t$_8$-*1st* brush pro$_8$-*1st*'s teeth]

The point is that the trace and possessive pronoun bound by *only I* have 1st-person features at PF, but not at LF. At LF, the only 1st-person feature is on the subject *I*, where indeed it is interpreted and constrains the reference of the free variable 7 to pick out the speaker. The trace and bound pronoun are feature-less variables and thus have well-defined denotations under any assignment that assigns something to the variable 8.

4.5.2. Interpreted and uninterpreted tense features

Let us return to tense now and begin to spell out Kratzer's analogy between fake indexicals and Sequence of Tense. The first step is to clarify the relation between abstract tense morphemes (such as **PAST**) and tense morphology (such as an *-ed* suffix or a suppletive form like *was*). The idea is that this is similar to the relation between interpreted and uninterpreted phi features. There is the item, or – as we will now say – the "(interpreted) feature" **PAST**, which is part of the underlying structure and of the LF, and which is semantically contentful. (Its meaning is what we have been assuming, e.g., (12) in section 4.4.1). And there is an uninterpreted twin of it – we'll write it *PAST* in little italics – which shows up in various places at PF and informs the spell-out of verbs in its vicinity. The actual spell-out rules (e.g. *go PAST* → *went*) can work on very local configurations (and we won't say much more about them here – that's what we have morphologists for). But the mechanism by which the uninterpreted tense features get to be where they are is sensitive to a not necessarily local structural configuration with abstract ingredients like variables and binders. It is, in fact, the very mechanism that is behind fake indexicals. Let us see how our rule (22) "feature transmission under semantic binding" can apply to tense features.

[69] This proviso becomes relevant later: the rule should apply also if there is a vacuous operator together with the binder index. For the moment you can ignore it.

First some mono-clausal examples.

(25) [**PAST-**_OP_ C] [John like-_OP_ Mary]
move operators: _OP_$_1$ [[**PAST-t$_1$** C] _OP_$_2$ [John like-**t$_2$** Mary]]
percolate feature: _OP_$_1$ [[**PAST-t$_1$** C]-_PAST_ _OP_$_2$ [John like-**t$_2$** Mary]]
transmit feature: _OP_$_1$ [[**PAST-t$_1$** C]-_PAST_ _OP_$_2$ [John like-[**t$_2$**-_PAST_] Mary]]
spell out: John liked Mary.

(26) [**PAST-**_OP_ C] [John _be_ rich-_OP_]
move operators: _OP_$_1$ [[**PAST-t$_1$** C] _OP_$_2$ [John _be_ rich-**t$_2$**]]
percolate feature: _OP_$_1$ [[**PAST-t$_1$** C]-_PAST_ _OP_$_2$ [John _be_ rich-**t$_2$**]]
transmit feature: _OP_$_1$ [[**PAST-t$_1$** C]-_PAST_ _OP_$_2$ [John _be_ rich-[**t$_2$**-_PAST_]]]
spell out: John was rich.

We included the tense's restrictor (C) here to be fully explicit about the structure, but will go right back to ignoring it below. In the copular example, the uninterpreted tense feature has ended up a bit lower perhaps than we would want it. Tense gets spelled out on the copula after all, not on the adjective. We won't fret over this here.[70]

What about present tense? As long as we are treating it as vacuous, it isn't binding any variables and thus can't be a source of transmitted features. The most natural move at this point is to abolish the item or feature PRES altogether, and leave it to the morphology to spell out verbs without a tense feature in the form that we traditionally call their "present tense" form[71]. Given that present tense morphology is actually zero (once we factor out subject agreement), this is also reasonable from a morphologist's perspective. But bear in mind that we are not really wedded to the vacuous treatment of the present, but have entertained a non-vacuous version too (when we considered frame adverbs). There might then also be an uninterpreted twin _PRES_ of a non-vacuous **PRES**, and the the analysis of _John likes Mary_ might be (28). But here we go with (27) for simplicity.

(27) John like-_OP_ Mary
move operator: _OP_$_2$ [John like-**t$_2$** Mary]]
spell out: John likes Mary.

---

[70] The issue disappears, or at least changes, when we integrate aspect into the structure. The _PAST_ feature will then end up no lower than the type-s argument of the Asp head – in both copular sentences and those with regular main verbs. When Asp is morphologically zero, the tense feature next to Asp gets spelled out on the verb immediately below. For a recent discussion of verbal morphology and the role of semantically vacuous auxiliaries like _do_ and _be_, see Bjorkman's 2011 MIT thesis.

[71] At least it does this in finite environments. We must assume that the morphology somehow knows whether it is dealing with e.g. an infinitive or a participle, where all tense distinctions are systematically neutralized.

(28) [**PRES-**_OP_ C] [John like-_OP_ Mary]
move operators: _OP_$_1$ [[**PRES-t**$_1$ C] _OP_$_2$ [John like-**t**$_2$ Mary]]
percolate: _OP_$_1$ [[**PRES-t**$_1$ C]-_PRES_ _OP_$_2$ [John like-**t**$_2$ Mary]]
transmit: _OP_$_1$ [[**PRES-t**$_1$ C]-_PRES_ _OP_$_2$ [John like-[**t**$_2$-_PRES_] Mary]]
spell out: John likes Mary.

2.4.3. Sequence of Tense explained

Now we reap the benefits of this new morpho-semantic theory. Look again at our problematic sentence *John was married to someone who is famous*. First, here is a derivation for the attested "famous now" reading.

(29)(a) base-generate:
PAST-_OP_ [John _be_ married-_OP_ _to_ [some _one_ [_who_ _be_ famous-**pro**$_1$]]]

(b) move operators (note non-local binding of pronoun!):
_OP_$_1$ PAST-**t**$_1$
_OP_$_2$ [John _be_ married-**t**$_2$ _to_ some _one_ _who_$_6$[t$_6$ _be_ famous-**pro**$_1$]]]

(c) from (b), by percolation and transmission:
_OP_$_1$ [PAST-**t**$_1$]-_PAST_
_OP_$_2$ [John _be_ married-**t**$_2$-_PAST_ _to_ some _one_ _who_$_6$[t$_6$ _be_ famous-**pro**$_1$]]]

(d) from (b), by QR:
_OP_$_1$ PAST-**t**$_1$
_OP_$_2$ [[some _one_ _who_$_6$[t$_6$ _be_ famous-**pro**$_1$]] 7[John _be_ married-**t**$_2$ _to_ t$_7$]]

The only trace or pronoun that receives a _PAST_ feature by transmission is the argument of the matrix predicate *be married*, so the embedded verb remains without a tense feature and surfaces as present tense.

Now let's convince ourselves that we no longer generate the unattested simultaneous reading. An LF that expresses this reading must have the argument of the embedded predicate semantically bound by the matrix tense, as in the LF in (30).

(30) _OP_$_1$ PAST-**t**$_1$
_OP_$_2$ [[some _one_ _who_$_6$[t$_6$ _be_ famous-**pro**$_2$]] 7[John _be_ married-**t**$_2$ _to_ t$_7$]]

But with semantic binding comes feature transmission, and therefore the embedded predicate would then have to surface as a past tense form. We can't have semantic binding without a morphological reflex. This is what is behind the phenomenon called "Sequence of Tense".

The system makes further predictions. One (closely related to the above) pertains to sentences with embedded future. Consider (31).

(31) John was married to someone who will be famous.

Our old theory predicted, falsely, that this sentence, as spoken in 2016, could be verified by a scenario in which John and Mary were married from 2004 to 2006 and Mary was famous from

2008 to 2010. (In other words, the temporal order in the scenario is marriage < fame < utterance.) When the sentence was given an LF where the *some*-DP scoped below the matrix past, the old theory made that LF true in the described scenario. As a matter of fact, however, the sentence unambiguously entails the existence of a person who is famous sometime *after the utterance time*.

The new theory generates the attested reading by coindexing the pronoun in the argument position of *woll* with the topmost operator in the matrix. (Exercise: Show this.) (It doesn't matter in this case how high the *some*-DP scopes.) For the unattested reading (that would be true in the above scenario), the inner argument of *woll* would have to be bound by the matrix PAST. In that configuration, however, the transmission rule applies and drops a *PAST* feature onto the argument of *woll*. For morphology, this means that *woll* spells out to *would*, not to *will*. So this is the LF of a different sentence, namely (32).

(32) John was married to someone who would be famous.

Indeed (32) sentence has a different meaning from (31) and *can* describe our scenario. (Perhaps it's most felicitous with added adverbials, e.g., *John was married to someone who would later be famous*, or *In 2005, John was married to someone who would be famous in 2009*.)

The new theory does not affect predictions for any of the sentences that have a matrix present. It also does not affect predictions for sentences with matrix future, i.e. *will* – provided we assume that **woll** (unlike **PAST**) is not a feature in the first place and has no uninterpreted twin-feature. (So if there is a feature at all that enters into spelling out *will*, it's just *PRES*.)

As for configurations with matrix past, we have already looked at embedded present and embedded future. What about matrix past embedding another past?

(33) John was married to someone who was famous.

We already know that one of the derivations for this surface configuration involves an underlying structure that has no tense (the equivalent of vacuous present) in the lower clause. This gave a simultaneous reading. (We call this the "SOT derivation".) But is that the only derivation which outputs (33) at PF? Suppose we generate an embedded PAST in the base structure. Then, after the operators have moved, we have a structure of the form in (34).

(34) *OP*$_{\mathbf{1}}$ PAST-$\mathbf{t_1}$
OP$_{\mathbf{2}}$ [John *be* married-$\mathbf{t_2}$ *to* some *one* *who*$_6$[PAST-$\mathbf{pro_n}$ *OP*$_{\mathbf{3}}$[t_6 *be* famous-$\mathbf{t_3}$]]]

I left the subscript on the pronoun unspecified, because we have choices there. We can have **n=1** or **n=2**.

First consider **n=1**. Then the pronoun is bound by the highest *OP* and no features get transmitted onto it. Each of the PAST's, however, transmits a *PAST* to the trace it binds, so we get *PAST* on $\mathbf{t_2}$ and $\mathbf{t_3}$, hence *was married* and *was famous*. The meaning has two independent quantifiers over times before g(**1**), i.e., before the utterance time if this is used as a matrix sentence. So this is a reading where the verifying marriage and fame can in principle be in any temporal order with respect to each other. (Adverbs can be added to constrain it further.) We are happy to generate this derivation, since the surface sentence does allow this kind of interpretation.

Now consider **n=2** in (35). Meaning-wise, this imposes a requirement that the fame held before the marriage (i.e., either the fame began and ended before the marriage or it continued from before the marriage into it or beyond). Since that kind of scenario is already allowed as a special case of the truth conditions of the **n=1** LF, it is hard to determine with truth-value judgments whether the grammar ought to generate this as a separate reading. But it won't hurt, it seems. Now let's look at the PF side for this case. Given what we have made explicit about percolation and transmission, we get the following pre-spell-out structure in the lower clause.

(35) lower clause for (34) with **n=2**, after percolation and transmission:
... *who*$_6$ [[PAST-[**pro$_2$**-*PAST*]]-*PAST* *OP*$_3$ [t$_6$ *be* famous-[**t$_3$**-*PAST*]]]

The left-most *PAST* (the one on **pro$_2$**) has been transmitted from the higher clause. The second *PAST*, the one attached to the complex phrase [PAST-[**pro$_2$**-*PAST*]], got there by percolation from the PAST-head of that phrase itself. And the third *PAST* (on **t$_3$**) has been transmitted from that complex phrase. How does all this get spelled out? Presumably the only place where morphology does something is in the predicate *(be) famous*, so we expect *was famous*. The structure in the complex phrase headed by PAST presumably must stay silent, because there is no verbal element there to carry tense inflection. So this is homophonous with the outputs of the other two derivations – an okay prediction, if not one that we can distinguish empirically at this point from another possibility that might be entertained as well, namely that the structure in (35) is not well-formed or not spell-out-able at all (for some reason to be identified).

A question that tends to come up at this point is whether we should work out a morphology which spells (35) out as a pluperfect, i.e., *who had been famous*. After all, this is intuitively the English sentence which best matches the interpretation we computed for the **n=2**-version of (34): *John was married to someone who had been famous* does entail fame before the marriage. Morphologically, however, it would seem to take a bit of *ad hoc* machinery to get from (35) to *had been famous*. We may have a more elegant way to generate the pluperfect. Assume (as was briefly mentioned earlier) that our lexicon contains an item that is synonymous with PAST, but syntactically different in that it is generated lower (below M, if any) and qualifies as "verbal" in the sense that's relevant to whether morphology can realize tense inflection on it.[72] Then we can generate a minimal variant of (35):

(36) ... *who*$_6$ [[have-[**pro$_2$**-*PAST*]] *OP*$_3$ [t$_6$ *be* famous-**t$_3$**]]
(matrix same as in (34))

The meaning of (36) is the same as with (35), but the morphology is straightforward and outputs *had*.[73] (The copula then surfaces as a past participle form, but this is among the further morphological details that we are not attending to here.)

[72] In the latter respect it is thus like *woll*.

[73] We assume, as we did for **woll**, that **have** is not a feature and ipso facto doesn't have an uninterpreted twin.

5. Embedded tense in complement clauses

We have only looked at relative clauses so far, but there are other multiclausal constructions to consider in which tenses occur in the scope of other tenses. In this section, we look at complement clauses to attitude and speech verbs.

A prerequisite for this discussion is the semantic analysis of verbs such as *believe*, *think*, *know*, *hope*, *tell*, *say*. We rely on earlier in the semester, where we developed lexical entries like (1).

(1) For any world w,
$[\![\text{think}]\!]^w = \lambda p_{\langle s,t\rangle}.\ \lambda x.\ \forall w'$ [w' is compatible with x's beliefs in w $\rightarrow p(w') = 1$]

We adapt this to our current framework as follows.[74]

(2) $[\![\text{think}]\!] = \lambda i.\ \lambda p_{\langle s,t\rangle}.\ \lambda x.\ \forall w$ [w is compatible with x's beliefs in w_i at $t_i \rightarrow p(w, t_i) = 1$]

5.1 Simple cases: non-past matrix

Let's warm up with a sentence that has present tense in both clauses.

(3) John thinks that Mary has the key.

Since we have abolished PRES, the structure of (3) is quite simple. *believe* calls for an argument of type <s,t>, so we must generate an *OP* in the lower clause. The representation after *OP*-movement will be (4).

(4) *OP*$_{\mathbf{1}}$[John think-$\mathbf{t_1}$ *OP*$_{\mathbf{2}}$[Mary have-$\mathbf{t_2}$ the key]]

This structure is interpretable as it stands, so it can be our LF and we compute the meaning in (5a). It is also ready to be spelled out by the morphology, since there are no relevant features here to percolate and transmit. Recall that verbs with no tense feature are spelled out (in finite environments) as present tense.

(5) (a) $\lambda i.\ \forall w$ [w compatible with J's beliefs in w_i at $t_i \rightarrow$ M has the key in w at t_i]
(b) morphology: [think-$\mathbf{t_1}$] $\rightarrow$ *thinks*, [have-$\mathbf{t_2}$] $\rightarrow$ *has*

Examples with a non-present tense in the embedded clause but a present in the matrix are also straightforward, as you can verify yourself. Let's move on to the interesting cases, with non-present tenses in the matrix.

The first case we examine is a future embedding a present.

(6) John will think that Mary has the key.

Again the structure after operator movement can serve without further derivation as LF and as input to morphology (there are still no features to copy around).

[74] There is another natural way to go, which is what you find in most of the contemporary literature (Ogihara, Abusch, Kratzer, von Stechow, Kusumoto, etc).

(i) $[\![\text{think}]\!] = \lambda i.\ \lambda p_{\langle s,t\rangle}.\ \lambda x.\ \forall i'$ [i' is compatible with x's beliefs in w_i at $t_i \rightarrow p(i') = 1$]

(i), unlike (2), quantifies over both worlds and times. The difference between (2) and (i) becomes important in the analysis of so-called *de se* readings, but it won't matter for the modest purposes of this introduction.

(7) $_{OP1}$[woll-$\mathbf{t_1}$ $_{OP2}$[John think-$\mathbf{t_2}$ $_{OP3}$[Mary have-$\mathbf{t_3}$ the key]]]

(a) $\lambda i.\ \exists t\ [\ t > t_i\ \&$
$\forall w\ [w \text{ compatible with J's beliefs in } w_i \text{ at } t \rightarrow \text{M has the key in w at t}]\ ]$

(b) morphology[75]: [woll-$\mathbf{t_1}$] $\rightarrow$ *will*, [have-$\mathbf{t_2}$] $\rightarrow$ *has*

The meaning we computed in (7a) amounts to a "simultaneous" reading. The worlds compatible with John's beliefs at the future time we are talking about are worlds in which Mary has the keys *then*, at the time of his thinking. This is indeed what the English sentence (6) means. In fact, it is its only possible reading. In distinction to the case of a present *relative* clause in a future matrix, there is not an additional reading here on which the present in the complement clause evaluates at the utterance time. The theory predicts this unambiguity. Because the attitude verb selects an argument of type <s,t>, we must generate an operator with the lower verb and move it to the embedded C. We cannot put a pronoun there and bind it non-locally. That would result in a type-mismatch, because the sister of *think* would be type t.

If we do want to talk about a belief that someone will hold in the future about our current utterance time, how do we express this in English? We have to use an embedded past.

(8) scenario: Arriving at the office one morning, Mary realizes that she left her keys at home. But as luck would have it, someone forgot to lock up the night before, so she can get in anyway. As she is entering the office, she thinks about the boss (John), who will arrive later and, not knowing any of the above, will not be surprised to find her sitting at her desk. She says:
John will think that I had my key.

(The analysis of this sentence in our theory is straightforward; do it as an exercise.) These examples provide an illustration of Ogihara's (1996) principle of "temporal directionality isomorphism", i.e., the generalization that tenses in an attitude complement must always reflect the *attitude holder*'s temporal perspective on the embedded event (rather than the speaker's). Kratzer (1998) and Kusumoto (2005) proposed to derive temporal directionality isomorphism from the semantic type of attitude verbs, and we have implemented this approach.

5.2. Past matrix and sequence of tense

Next we ask ourselves what underlying structures we can build with a matrix PAST and what will happen to them in their LF and PF derivations and their semantic and morphological interpretation. First let's base-generate PAST operators in both clauses.

(9) $_{OP1}$[PAST-$\mathbf{t_1}$ $_{OP2}$[John think-$\mathbf{t_2}$ $_{OP3}$[PAST-$\mathbf{t_3}$ $_{OP4}$[Mary have-$\mathbf{t_4}$ the key]]]]

Interpretability requirements once again determine the presence and landing sites of the *OP*'s. (9) is interpretable and expresses what is called a "back-shifted" ("earlier-than-matrix") reading.

---

[75] *think* does not spell out as *thinks*, because it is not in a finite VP. *woll* (like the other auxiliaries of syntactic category M) governs infinitival morphology on its complement VP, and it is irrelevant in this case what tense features, if any, might have been transmitted to the type-s-argument of V. Even when there is a *PAST* on the inner argument of an M (as in 'John could see the ocean', or 'John thought he would see the ocean'), the VP still spells out as a bare infinitive.

(10) $\lambda i.\ \exists t\ [\ t < t_i\ \&$
$\forall w\ [w \text{ compatible with J's beliefs in } w_i \text{ at } t \rightarrow \exists t'\ [\ t' < t\ \ \&\ \text{M has the key in w at } t']\ ]$

What about morphology? Each PAST operator certainly will transmit a feature to its trace, and the outcome will be past tense morphology in both clauses.

(11) $\textit{OP}_{\mathbf{1}}[\underline{[\text{PAST-}\mathbf{t_1}]\text{-}\textit{PAST}}\ \textit{OP}_{\mathbf{2}}[\text{J think-}\underline{\mathbf{t_2}\text{-}\textit{PAST}}\ \textit{OP}_{\mathbf{3}}[\ \underline{[\text{PAST-}\mathbf{t_3}]\text{-}\textit{PAST}}\ \textit{OP}_{\mathbf{4}}[\text{M have-}\underline{\mathbf{t_4}\text{-}\textit{PAST}}\ \text{key}]\ ]\ ]\ ]$
spell out: John thought that Mary had the key.

So we generate a back-shifted reading for this surface sentence of English, and this is good. The sentence can be understood in this way. Imagine, e.g., that we are talking about the story in (8) at a later time. We could then say *John thought that Mary had her key* meaning that when John arrived in the late morning, he thought that Mary had her key at the earlier time of her arrival.

But the same surface sentence also has a "simultaneous" reading. In fact, this may be the most prominent reading out of context. The simultaneous reading is standardly attributed to Sequence of Tense. Let's see how our implementation of SOT as feature-transmission-under-binding might cover this case. To get the simultaneous truth-conditions, our LF must *not* have a PAST operator in the lower clause, so we don't base-generate one there. The structure needs to be as in (12).

(12) $\textit{OP}_{\mathbf{1}}[\ \text{PAST-}\mathbf{t_1}\ \textit{OP}_{\mathbf{2}}[\ \text{John think-}\mathbf{t_2}\ \textit{OP}_{\mathbf{3}}[\text{Mary have-}\mathbf{t_3}\ \text{the key}]\ ]\ ]$
$\lambda i.\ \exists t\ [\ t < t_i\ \&\ \forall w\ [w \text{ compatible with J's beliefs in } w_i \text{ at } t \rightarrow \text{M has the key in w at t}]\ ]$

Does this structure wind up at PF with a transmitted *PAST* feature on the argument of the embedded verb *have*? That actually doesn't quite yet follow from the assumptions we have in place. Percolating the feature in the phrase headed by PAST and then transmitting it down to variables bound by that phrase only gets us as far as (13).

(13) $\textit{OP}_{\mathbf{1}}[\underline{[\text{PAST-}\mathbf{t_1}]\text{-}\textit{PAST}}\ \textit{OP}_{\mathbf{2}}[\text{John think-}\underline{[\mathbf{t_2}\text{-}\textit{PAST}]}\ \textit{OP}_{\mathbf{3}}[\text{Mary have-}\mathbf{t_3}\ \text{the key}]\ ]\ ]$

We need a further assumption here, namely that there is percolation in the phrase headed by the verb *think*, in such a way that the tense feature from the argument of *think* gets passed up.

(14) Percolation in the verbal complex:[76]
[V [arg F]] becomes [V [arg F]]-F

If we apply (14) in (13), this will then feed another application of the transmission rule, and we arrive at (15).

(15) $\textit{OP}_{\mathbf{1}}[[\text{PAST-}\mathbf{t_1}]\text{-}\textit{PAST}\ \textit{OP}_{\mathbf{2}}[\text{John }\underline{[\text{think-}[\mathbf{t_2}\text{-}\textit{PAST}]]\text{-}\textit{PAST}}\ \textit{OP}_{\mathbf{3}}[\text{Mary have-}\underline{\mathbf{t_3}\text{-}\textit{PAST}}\ \text{the key}]\ ]\ ]$
spell out: John thought that Mary had the key.

So we generate the simultaneous reading for this surface sentence.

Two remarks are in order. First, the system does *not* generate any reading for the past-under-past sentence that could describe a "later-than-matrix" scenario. Suppose that someone at some time in the past had a thought about a time which was still in their future at the time of their thinking,

[76] For this to work as intended, it is actually crucial that the index-argument be the innermost argument of the verb – a decision which up to now was an arbitrary matter of exposition.

but which is now past in relation to our utterance time. This sort of thought cannot be reported by a sentence with a simple past in the lower clause, but requires instead an embedded *would* (another difference between complement clauses and relative clauses, and another illustration of Ogihara's "temporal directionality isomorphism"). The following minimal pairs make the point.

(16) (a) * We knew long before the test that you flunked.
(b) We knew long before the test that you would flunk.

(17) I wanted to go shopping before the stores closed last night,
(a) * because I was afraid you were hungry this morning.
(b) because I was afraid you would be hungry this morning.

(18) By the 1970s, people no longer expected
(a) * that Germany was reunified before the end of the century.
(b) that Germany would be reunified before the end of the century.

Examples with *would* (such as the (b)-cases above or *John thought that Mary would have the keys*) are generated with appropriate meanings by our current analysis, as the reader is invited to work out.

A second remark is that the analysis as it now stands does not generate the sentence in (19) at all.

(19) John thought that Mary has the keys.

If we base-generate no temporal operator in the lower clause, the only possible derivation is the one we saw above in (12) - (15), which leads to past tense morphology in both clauses. Interpretability constrains us to generate a locally bound operator in the embedded clause, not a pronoun that might be bound from higher up, i.e., from the matrix C. There is simply no way to derive (19), and the prediction is therefore that present tense in the complement of a past tense matrix attitude verb should be ungrammatical. Unfortunately the empirical facts are not so simple – a complication that we discuss in the following section.

5.3. Present under past and double-access readings

Present-under-past attitude and speech reports are not simply another way to express a simultaneous reading (in which case we could simply have dealt with them by making the new percolation rule in (14) optional). Nor do they simply report someone's past thought about the present time which was in the future of their thinking (which would make them counterexamples to temporal directionality isomorphism). Rather these sentences have a distinctive and peculiar meaning of their own, known in the literature as a "double access" reading. We introduce the phenomenon by quoting from a paper by Altshuler & Schwarzschild[77]:

"Suppose that ... at the mall, I ask Sylvia where her friend, Mary, is. She replies: “Mary is at home today”. Later that day, when I’m at the beach and asked for Mary’s whereabouts, I can truthfully say:

(9) Sylvia said that Mary is at home.

[77] (2013, Amsterdam Colloquium)

Taking our cue from Abusch's (1997) discussion of present complements of *believed,* we observe that (9) is true on the so-called *double access* reading because two conditions are met. To describe those conditions, we will need to refer to the time and world at which Sylvia replied to me at the mall. We symbolize those as $w_@$ and t_{mall} respectively. Below, in (10) - (11), we first give each condition in descriptive terms ...

(10) Relative Present Condition
a. If Sylvia's utterance was true, then Mary is at home in $w_@$ at t_{mall}.
[...]

(11) Deictic Present Condition
a. If Sylvia's utterance was true, then Mary is at home throughout an interval that includes the time at which (9) is uttered.
[...]

We call (10) 'Relative Present Condition' because it makes the present tense look like a relative present – a sort of Priorian present tense (Prior 1967) [...] We call (11) 'Deictic Present Condition' because it makes the present tense look like a deictic present – a tense that in any context picks out the utterance time of that context. [...] This would explain why (9) is true if uttered on the same day as t_{mall} but not if uttered on the following day.

In sum we propose that the present tense in English is an amalgam of both a relative and a deictic present. More concretely, we propose that the English present demands truth at the local evaluation time (relative tense component) and at or after the speech time (deictic tense component). In a simple present tense clause [...], the local evaluation time is the speech time so the two components cannot be told apart."

A&S propose, in effect, that "double access" is hard-wired into the meaning of the English present tense: this tense shifts to a new evaluation time that is constrained by its relations to *two* other times. We adopt a related idea[78], but without actually departing from our assumption that the present tense is vacuous and therefore does not shift evaluation time at all. We instead draw a parallel to a phenomenon with personal plural pronouns known as 'split antecedents' or 'split binding'.[79] An example is (20a), which has an LF like (20b), in which the **+**-sign denotes a function (of type <e,<e,e>>) which maps two individuals to the smallest (plural) individual that contains them both as parts. The truth-conditions of (20a) on this parse are fulfilled by a scenario in which everyone told someone "You and I should get together".

(20) (a) Everyone told someone that they should get together.
(b) everyone **1**[someone **2**[$\mathbf{t_1}$ told $\mathbf{t_2}$ that [$\mathbf{pro_1 + pro_2}$] should get together]]

The idea that we want to spell ou here is that the topmost type-s argument in a double-access complement is a world-time pair in which the time is kind of a plural time. This requires a special sum-operator for world-time pairs, whose definition is admittedly a little funny, since it uses both input times but effectively "throws away" one of the worlds.

[78] Among other differences from A&S's proposal, we are not requiring either of the two arguments to be the utterance index. Empirical consequences of this difference will show up only in sentences with multiple levels of embedding, whose examination we leave to another occasion. There are several other differences too.

[79] refs

(21) $[\![+]\!] = \lambda i \in D_s.\ \lambda j \in D_s.\ \langle w_i, [t_i,t_j]\rangle$,
where $[t_i,t_j]$ is the interval from t_i to t_j, which is the smallest interval that contains both t_i and t_j

What we have in mind for the example sentence *Sylvia said that Mary is at home* is a representation of the form in (22).

(22) *OP*$_1$[[**PAST t$_1$**] *OP*$_2$[Sylvia say-**t$_2$** *OP*$_3$[Mary *be* at-home-[_**+**_]]]]

We have filled in the *OP*'s and traces that are required for the usual reasons of interpretability. Note that these include an *OP* (*OP*$_3$) at the edge of the complement of *say*, to meet the latter's requirement for a proposition. One of the two blanks in the "plural" argument of the embedded verb thus has be occupied by the trace of *OP*$_3$.[80] The other one is up for grabs and could in principle be bound by any one of the three higher *OP*'s (*OP*$_1$, *OP*$_2$, or *OP*$_3$). There is, however, one more principled constraint that we can identify before we examine the remaining options one by one. The trace of the operator *OP*$_3$ will have to go to the *left* of **+**, not to the right. This has to do with the asymmetry regarding the worlds in definition (21). Let's compute what would happen if *OP*$_3$ were to bind (only) a variable on the right. (23) shows the result of interpreting the *say*-VP, assuming $\mathbf{n \neq 3}$.

(23) [[Sylvia say-**t$_2$** *OP*$_3$. Mary *be* at-home-(**pro$_n$ + t$_3$**)]]^g = 1 iff
$\forall$w [w is compatible with what Sylvia says in $w_{g(2)}$ at $t_{g(2)}$
$\rightarrow$ Mary is at home in $w_{g(n)}$ at $[t_{g(n)}, t_{g(2)}]$]

The universal quantifier over worlds binds vacuously here, so this is a pathological meaning.

We are down to three ways of filling the blanks in (22).

(24) *OP*$_1$[[**PAST t$_1$**] *OP*$_2$[Sylvia say-**t$_2$** ...
(a) ... *OP*$_3$[Mary *be* at-home-[**t$_3$ + pro$_3$**]]]]
(b) ... *OP*$_3$[Mary *be* at-home-[**t$_3$ + pro$_2$**]]]]
(c) ... *OP*$_3$[Mary *be* at-home-[**t$_3$ + pro$_1$**]]]]

(24a), with the two arguments of **+** coindexed, amounts to the same meaning as if we had simply put **t$_3$** instead of **t$_3$ + pro$_3$**. (Definition (20) implies that $[\![+]\!](i)(i) = i$.) This expresses a simultaneous reading. (24b) turns out to be equivalent with this as well (prove as exercise). (24c) is the only interesting choice. We compute the proposition in (25).

(25) $\lambda i.\ \exists t\ [\ t < t_i\ \&\ \forall w$ [w is compatible with what Sylvia says in w_i at t
$\rightarrow$ Mary is at home in w at $[t, t_i]$]]

This represents our desired double-access reading. It implies that, according to what Sylvia said at t_{mall}, Mary was at home at the interval from t_{mall} to the utterance time.

[80] It is worth noting that this is another illustration of Ogihara's "temporal directionality isomorphism". Ogihara (1996) actually introduced that principle in the context of discussing the double access phenomenon. He thereby drew attention to the fact that even in this case – which may superficially look as if an embedded tense were chosen to reflect *solely* the *utterer's* perspective – we have a tense that upon careful examination turns out to relate the embedded event *also* to the subjective "now" of the subject in the reported past thought/speech act.

What about morphology? It is one thing to have a semantics that generates the double-access reading, and another to predict correctly that this reading – and *only* this reading – surfaces with a present tense in the embedded clause. Here the story that we need in order to get the facts right is not yet determined by our existing assumptions, unfortunately. Given those assumptions, including Percolation in the Verbal Complex from (14) above, the lower clause under the three disambiguations in (24a - c) will receive the transmitted features shown in (25).

(25) *OP*$_{\mathbf{1}}$[[PAST-$\mathbf{t_1}$]-*PAST* *OP*$_{\mathbf{2}}$[Sylvia [say-[$\mathbf{t_2}$-*PAST*]]-*PAST* ...
- (a) ... *OP*$_{\mathbf{3}}$[Mary *be* at-home-[$\mathbf{t_3}$-*PAST* **+** $\mathbf{pro_3}$-*PAST*]]]]
- (b) .... *OP*$_{\mathbf{3}}$[Mary *be* at-home-[$\mathbf{t_3}$-*PAST* **+** $\mathbf{pro_2}$-*PAST*]]]]
- (c) ... *OP*$_{\mathbf{3}}$[Mary *be* at-home-[$\mathbf{t_3}$-*PAST* **+** $\mathbf{pro_1}$]]]]

In the two structures that express the simultaneous reading, (25a) and (25b), both arguments of **+** have a transmitted *PAST*. We would want to make sure that in this case, *PAST* percolates to the whole "plural" argument and is seen by the morphology and expressed on the verb, giving us *was at home*. (If that didn't happen and we got *is at home*, we would wrongly pair this morphology with an unattested simultaneous reading!) In the structure (25c), on the other hand, which expresses the double access reading, only one of the arguments of **+** is marked *PAST* and the other one is unmarked. In this case, we want to say that *PAST* does *not* percolate to the plural argument and is not spelled out, so that the embedded verb surfaces as *is at home*. Our structures are rich enough for us to be able to make the desired distinction, though the assumptions required don't look as principled as we might have hoped. More research is needed here. If this approach to double access is on the right track, the morphological mechanisms should ultimately fall together with independently attested mechanisms for assigning phi-features to plural personal pronouns and coordinated DPs.[81]

[81] refs include Podobryaev 2014 MIT PhD

PART IV

Questions

8 *Interrogative clauses*

8.1 A little pragmatics 150
8.2 Compositional computation of semantic values for interrogative clauses 155
8.2.1 An updated version of Karttunen 1977 155
8.2.2 Back to pragmatics: Mapping interrogative denotations to partitions 158
8.2.3 Syntax: the role of the [WH] feature 161
8.3 Embedded questions and question-embedding verbs . 163
8.3.1 Responsive verbs, a type-mismatch, and Dayal's strategy 163
8.3.2 An ANS operator inspired by Karttunen . . . 165
8.3.3 Problems, and an ANS operator inspired by Groenendijk & Stokhof 166
8.3.4 ANS in matrix questions? 168
8.3.5 Embedding under rogative verbs 170
8.4 Wh-movement with pied-piping and reconstruction 170
8.4.1 Background on plurals and *many* 171
8.4.2 *How many*-questions 173
8.4.3 More on reconstruction 175

So far, we have developed a semantic system that compositionally derives semantic values for "declarative" clauses, whether standing on their own as main or matrix sentences or embedded as constituent clauses (relative clauses, complement clauses, adjunct clauses). Extensionally speaking, declaratives are of type t and their denotation is determined relative to an evaluation index (a world, or a world-time pair) and a variable assignment: $[\![\phi]\!]^{i,g}$. The system also supplies (possibly relative to a variable assignment) an intension of type $\langle s,t\rangle$: a proposition, or a function from indices to truth-values, $[\![\phi]\!]^{g}_{\mathfrak{c}} = \lambda i.[\![\phi]\!]^{i,g}$.

In set talk: a set of indices. In the simplest system: a set of worlds.

In this chapter, we will expand the system to deal with a central kind of non-declarative clause: interrogatives. These also can stand on their own:

Another important kind of non-declarative are imperatives, if you're interested in which, you should consult Portner 2007, M. Kaufmann 2012, von Fintel & Iatridou 2017.

(1) a. Did Sakina see Emily?
    b. Did Sakina see Emily or did Sakina see Julie?
    c. Who did Sakina see?

And they can appear as embedded clauses:

(2) a. Melanie wonders whether Sakina saw Emily.
  b. Inessa is surprised who Sakina saw.
  c. Whether Sakina saw Emily or not, she definitely saw Julie.

The semanticist's task in the analysis of interrogative clauses is to propose suitable types of semantic values for them and to show how these semantic values are built up compositionally. But before we apply ourselves to this task, it is useful to say a little bit about speech acts, and about the relation between semantic values and speech acts.

8.1 *A little pragmatics*

Leaving semantics out of the picture for the moment, interrogative clauses differ from declarative clauses in their syntax and in their pragmatics. In English, interrogative clauses can be told apart syntactically from declarative clauses by the presence of a clause-initial wh-phrase and/or the inverted order of subject and auxiliary. The terminology "interrogative"/"declarative", however, alludes to a distinction not in grammatical form but in communicative function. When uttered as main clauses (i.e., not embedded in a larger structure), declarative sentences typically serve to make assertions, whereas interrogative sentences serve to ask questions. These two clause-types serve to perform different kinds of speech acts.

Stalnaker 1978 outlined an influential formal model of what happens in a conversation and what it means to make an assertion. The central concept is that of a body of publicly shared information, or "common ground", which evolves as the conversation proceeds. A proposition is in the common ground if each interlocutor is "disposed to act as if he assumes or believes the proposition is true, and as if he assumes or believes that his audience assumes or believes that it is true as well". The common ground can be characterized by the set of worlds in which every proposition in the common ground is true; Stalnaker calls this set of worlds the "context set". The act of making an assertion is a proposal to update — in fact, to shrink — the context set. The particular way in which the context set is to be shrunk depends on the semantic value of the asserted sentence, more specifically, on its intension. To assert a sentence ϕ is to propose that the current context set c be replaced by a new context set which is the intersection of c with the intension of ϕ, i.e., $c \cap [\![\phi]\!]_{\mathbf{c}}$.

Not "shrink" in the sense of reducing cardinality. The cardinality of the context set may never become less than uncountably infinite.

We're leaving off the variable assignment parameter, which is legitimate if there are no free variables in ϕ.

Against this backdrop, how might we think about the speech act of asking a question? What is the point of this speech act? Questions, unlike assertions, don't provide information about the world and therefore do not in themselves lead to updates of the common ground. Their purpose rather is to constrain the future course of the conversation in a certain way. In the absence of any particular question under consideration,

At least this is true if we stick to questions that don't have presuppositions (or to questions whose presuppositions are already common ground at the point when they are asked). When a question that has a presupposition is asked in a context where the presupposition is not yet in the common ground, a listener can get new information by accommodating this presupposition.

a speaker might assert whatever they find interesting or useful. But if someone asks a question, they thereby express an expectation that the next conversational move will be one out of a very limited set of possible assertions — intuitively, the next assertion will "address the question" rather than provide some random other information.

Here is a way to formally model this intuition. A question partitions the context set in a certain way, and it amounts to a request for assertions which, so to speak, "carve up" the context set along the lines determined by this partition. The requested next assertion should not make any distinctions between worlds which the partition lumps together.

Here's the formal definition of a partition:

Definition 1 (Partition) A set $\mathcal{P}$ is a *partition* of a set A iff

(i) every element of $\mathcal{P}$ is a non-empty subset of A,
(ii) any two non-identical elements of $\mathcal{P}$ are disjoint from each other, and
(iii) the union of all the elements of $\mathcal{P}$ is A.

The elements of a partition are also called its cells.

With this definition in hand, we refine Stalnaker's model of conversation. Each stage in a conversation is now characterized not by its context set, but by its "partitioned context set", i.e., by a partition of some set of possible worlds. The union of this partition corresponds to Stalnaker's old context set, i.e., it contains the worlds compatible with every proposition in the common ground. The partitioning represents the interlocutors' shared commitment not to make distinctions between worlds that are "cell-mates" — at least not for the time being. This is a renegotiable commitment and, as we will see, it only stays in force until someone raises a new question. The definition of "relevant assertion" in Definition 3 below spells out precisely what the commitment amounts to. In Definition 2, we adapt Stalnaker's characterization of the context-changing role of assertions to our new, more elaborate set-up.

The idea of refining the Stalnakerian model of the context by moving from the context set to a partitioned context set goes back to Groenendijk 1999. A formally equivalent way is to model the context as an equivalence relation (see, for example, Jäger 1996 and Hulstijn 1997). We will later see approaches that give a partition semantics to interrogatives. It's important to distinguish the two moves. As we'll see in this section, one can have a partition pragmatics without a partition semantics. This theme is explored in much more sophisticated detail in Fox 2018.

Starr 2020 adds to the partitioned context set model a way of tracking the speech act effect of imperatives.

Our proposed context model is simple compared to some other well-known systems, such as Roberts 2012 and Farkas & Bruce 2010.

Definition 2 (Update by assertion) To assert a sentence ϕ is to propose that the current partitioned context set C be replaced by a new partitioned context set which is constructed by intersecting each cell of C with the intension of ϕ. More precisely, C is to be replaced by

$$\{p : p \neq \emptyset \,\&\, \exists p' \in C.\, p = p' \cap [\![\phi]\!]_{\mathfrak{c}}\}$$

Definition 3 (Relevance) An assertion is *relevant* w.r.t. to a partitioned context set C iff the update it proposes results in a subset of C.

The notion of "relevance" that is formalized here calls for a bit of clarification. First, here are a couple of more commonly used (and more transparent) definitions.

Definition 4 (Relevance′) A proposition p (i.e. set of worlds) is relevant w.r.t. to a partition Q of a set of worlds iff p is identical to a cell in Q or to a union of cells in Q.

Definition 5 (Relevance″) A proposition p is relevant w.r.t. to a partition Q of a set of worlds iff there is no pair of worlds w and w' such that both

(i) there is a cell $c \in Q$ such $w \in c$ & $w' \in c$
(i.e., w and w' are cell-mates),
(ii) and $w \in p$ & $w' \in p$
(i.e. p has different truth values in w and w').

These two definitions differ slightly in that Definition 5 (unlike Definition 4) allows p to include arbitrary worlds that are not in any cell of Q at all. But both express the essential requirement that a relevant proposition must not discriminate among cell-mates. It must not "cut up" the space of worlds (in the union of the partition) in any way that does not coincide with the boundaries between cells.

Definitions 4 and 5 apply to a proposition, whereas our Definition 3 applies to an assertion and presupposes the rule for update-by-assertion. But apart from this difference, Definition 3 captures the same notion of relevance. (To make the connections fully precise: If C is a partitioned context set and ϕ is an asserted sentence, then the assertion of ϕ is "relevant" w.r.t. C in the sense of Definition 3 iff $\bigcup C \cap [\![\phi]\!]_{\mathfrak{c}}$ is "relevant" w.r.t. C in the sense of Definition 4, which holds, in turn, iff $[\![\phi]\!]_{\mathfrak{c}}$ is "relevant" w.r.t. C in the sense of Definition 5. Prove these equivalences as an exercise.)

To get an intuitive appreciation of this concept, it is important to see that "relevance" in this technical sense is a purely negative requirement, so to speak: it is just the absence of any irrelevant (i.e., not specifically asked for) information. By this criterion, a "relevant" answer may be totally uninformative; it could be a tautology, or any other proposition that is already entailed by the common ground. So evidently, relevance is not a sufficient condition for appropriate answers; there are additional conditions, notably that an answer — or any assertion, for that matter — should be informative. It is also worth pointing out that the formal requirement of relevance is *prima facie* violated by many answers which in actual fact are judged felicitous, e.g., when somebody responds to the question *Is it raining?* by saying: *Well, Becky just came in dripping wet*, or *Maybe*, or *I don't know*. To make sense of this wider range of intuitively felicitous answers, a proponent of the present formal model needs to say that in these cases, the answerer is implicitly changing or amending the question that was explicitly asked, and instead is addressing a different (though related) question that they don't formulate explicitly but expect their audience to infer ("accommodate").

Now the only piece of our story that is missing is a recipe for "update by question". The idea, as we have said, is that asking a question amounts to proposing a specific replacement of the current partitioned context set by a new one. As in the case of assertions, the construction of the new partitioned context set should be determined by the semantic value of the sentence that was uttered. We know what we have in mind for particular examples. E.g., when someone asks *Is it raining?*, the intended outcome is a two-membered partition, with one cell consisting of worlds in which it is raining and the other cell of worlds in which it is not raining. If the question is a so-called "alternative question" like *Is Julie in Barcelona or did Emily call them on the phone?*, the result should be a four-celled partition corresponding to the four possible configurations of truth-values for the two component sentences. If the question asked is *Who among these two (Carli and Jodie) came to the party?*, the result should be a partition with four cells: one with worlds in which Carli and Jodie both came, one with worlds in which Carli but not Jodie came, one with worlds in which Jodie but not Carli came, and one with worlds in which neither Carli nor Jodie came.

What is the general recipe by which these outcomes are obtained as a function of the semantic value of each interrogative sentence? To answer this question, we have to carry out the semanticist's task, i.e., assign semantic values to interrogative clauses. We will do so in the next section, but for now we will articulate a basic intuition about the semantic value of interrogatives and explore how they can be used to construct a partitioned context set.

Perhaps the most widespread idea about the semantic value of interrogatives is that they denote *sets of propositions*. The particular structural features of interrogatives are taken to serve to both "lift" the type of the denotation from the type of propositions (the denotation of declaratives) to the type of sets of propositions, and to generate the individual propositions in the set. We will look at the mechanics of this in the next section. For now, let's assume the following semantic value for the sample *wh*-questions *Who among these two (Carli and Jodie) came to the party?*:

(3) $[\![$Who among these two (Carli and Jodie) came to the party?$]\!]$ =
$\{\lambda w.$ Carli came to the party in w, $\lambda w.$ Jodie came to the party in $w\}$

Now, notice that the propositions in this set are not disjoint (they can both be true of the same world) and that their union leaves out the worlds where neither Carli nor Jodie came to the party. So, this set of propositions is not a partition of the set of all possible worlds and also not a partition of any context set with minimal information about who came to the party. So, there's apparently not a seamless link between our partition pragmatics for question acts and our semantic values for interrogatives.

Should we therefore redo our semantics so that it does, after all, map interrogative clauses to semantic values that are partitions? This can be done and has been argued for, and we will return to a discussion of this possibility. For now, however, we observe that it is not necessary. An equally reasonable take on the discrepancy between our semantics and our pragmatics is that it simply reflects the division of labor between semantics and pragmatics. We don't necessarily need a semantics that directly delivers partitions as semantic values. All we need is to make precise how the semantic values delivered by the semantics help determine the partitions that figure in the pragmatics. In other words, we need to formulate our pragmatic rule for "update by question" in such a way that it spells out how the partition effected by a question-act is determined by the semantic value of the uttered sentence. We will work up to this formulation in a few steps. As we will see, there is a straightforward and general recipe for converting an arbitrary set of propositions into a partition of a given set of worlds, and our update principle will make use of that recipe.

Informally speaking, each proposition in the question-denotation defines a "dividing line" across the space of worlds in the context set, and as we use multiple propositions to draw multiple lines across this space, we get increasingly fine-grained partitions as the number of propositions goes up. The first proposition cuts the space into two cells (the region where it is true and the region where it is false). The second proposition subdivides these two cells further into four; the third proposition leaves us with eight cells; and so on.

This assumes that the propositions are all "logically independent" of each other and of the propositions in the original common ground. That is, none of them contradicts or is entailed by any other (or any conjunction or disjunction of others). If there are logical relations between the propositions, then the total number of cells is smaller. Note that the definition of "partition" requires all cells to be non-empty, so the intersection of two incompatible propositions is not a cell.

Another way of describing this process is that each new proposition introduces a new condition that cell-mates must satisfy. At the beginning, before any line has been drawn, all of the context set is one big cell and every world counts as a cell-mate of every other one. Then we draw a line for the first proposition (call it p_1), and if two worlds differ with respect to the truth-value of p_1, they now are no longer cell-mates. w and w' are cell-mates now only if $p_1(w) = p_1(w')$. With the second proposition, p_2, we draw a second line and "break up" yet more of the original cell- mate relations. At this point, only those pairs of worlds remain cell-mates which are treated alike by both p_1 and p_2. I.e., w and w' are cell-mates now iff $p_1(w) = p_1(w')$ & $p_2(w) = p_2(w')$. And so on for the third and all other propositions in the given set of propositions.

So a set of propositions can be used to define a "cell-mate relation", and thus a partition.

(4) Let $\mathcal{P}$ be a set of propositions and c a set of worlds.

a. $\sim_{\mathcal{P},c}$, the cellmate relation in c based on $\mathcal{P}$, is defined as follows:
$\forall w, w'\colon\ w \sim_{\mathcal{P},c} w'$ iff $w \in c\ \&\ w' \in c\ \&\ \forall p \in \mathcal{P}\colon\ p(w) = p(w')$.

b. PARTS$(\mathcal{P}, c)$, the partition of c based on $\mathcal{P}$, is defined as follows:
PARTS$(\mathcal{P}, c) = \{p\colon\ \exists w \in c.p = \{w'\colon\ w \sim_{\mathcal{P},c} w'\}\}$

The cellmate-relation is an equivalence relation (reflexive, symmetric, and transitive). It holds between any two worlds that give the same truth-value to each proposition in the "seed set". The corresponding partition consists of the subsets of c that are made up of worlds that are cell-mates.

We are ready to propose a preliminary rule of "Update by question":

Definition 6 (Update by question (draft)) To ask a question by uttering a sentence ϕ that denotes a set of propositions $\mathcal{P}$ is to propose that the current partitioned context set C be replaced by the new partitioned context set PART$(\mathcal{P}, \bigcup C)$.

Note that this update rule implies that each new question "wipes out" the current partitioning of the context set and replaces it by a new one. This may not be right and isn't the only way we can go. We may want a system where successive questions can serve to *refine* the partition. Even more complex possibilities exists, as for example in Roberts 2012.

In the next section, we turn to the task of compositionally deriving the semantic value of an interrogative.

8.2 *Compositional computation of semantic values for interrogative clauses*

8.2.1 *An updated version of Karttunen 1977*

Interrogative clauses are built largely with the same lexicon and syntactic rules as declarative clauses; but they also contain certain interrogative-specific morphemes, syntactic features, or functional heads that make crucial contributions to their non-declarative meanings. Following Karttunen 1977, our syntax for English posits an abstract (i.e., silent) complementizer that has a non-vacuous semantics, and a feature on wh-phrases that is semantically vacuous but subject to a certain distributional constraint.

The implementation here differs from Karttunen 1977 in not using the Montague Grammar framework; instead we presuppose a division of labor between syntax and semantics in line with Heim & Kratzer 1998. Aside from implementation, there are some small substantive differences as well. Most saliently, the extensions we will assign to interrogative clauses will include both true and false propositions. (In Hagstrom 2003's terminology, they will correspond to ANSPOSS rather than ANSTRUE.) Moreover, we will depart from Karttunen in the treatment of polar questions.

Karttunen's motivation for including a restriction to true propositions had to do with the semantics of question-embedding constructions. So the time to discuss it is when we get to embedded questions. Right now we focus on matrix questions.

(5) Karttunen's "proto-question" operator, syntactically a C-head:
$[\![?]\!]^w = \lambda p_{st}.\lambda q_{st}.\ p = q.$

The official denotation in (5) maps propositions to functions from propositions to truth-values, or equivalently, it maps propositions to characteristic functions of sets of propositions. If we replaced these characteristic functions by the corresponding sets, (5) would read as follows:

(5') $[\![?]\!]^w = \lambda p.\{p\}$

In other words, it would be a function that maps each proposition to the singleton set that has it as its only member.

(6) lexical entries for interrogative words, e.g.:
$[\![\text{who}^{[\text{WH}]}]\!]^w = \lambda f_{et}.\ \exists x\ [x \text{ is human in } w\ \&\ f(x) = 1]$

The meaning of *who* is exactly the same as the meaning of *somebody*. We will address the role of the feature [WH] in Section 8.2.3.

In the syntactic derivation of a constituent question, wh-movement applies and puts the wh-word above the operator in C. For the example *Who did Sakina see?* or *who Sakina saw* (we will ignore tense and auxiliary *do*, and thereby ignore the difference between matrix and embedded versions), this gives rise to a structure like (7):

(7) who 2 [? Sakina see t_2]

Unfortunately, when we examine the semantic types at each node in (7), we discover a type-mismatch that prevents us from interpreting the topmost node. We fix this problem by positing a slightly more complex C-head: We base-generate *?* together with another, covert operator as its sister. This covert operator is semantically vacuous, but it can move and leave a trace of type $\langle s, t\rangle$. The full syntactic derivation for a constituent question then proceeds as in (8).

Other vacuous covert operators of this sort are H&K's relative pronouns (and their PRO in ch. 8).

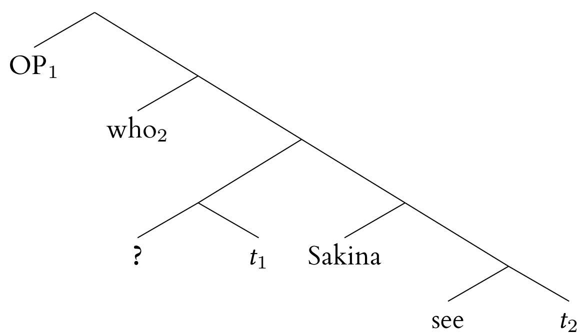
(8) Who did Sakina see?

[$_C$? OP] Sakina see who
who 2 [[? OP] Sakina see t_{2_e}] (wh-movement)
OP 1 [who 2 [[? $t_{1_{st}}$] Sakina see t_{2_e}]] (operator movement)

The movement of who is evidently overt (pre-SS) movement (in this example). For the movement of the operator, which is not pronounced, we can't tell whether it happens before SS or between SS and LF.

The resulting LF looks thus like this in tree format:

(9)



We will usually drop the type-labels on the traces, but keep in mind that the operator leaves a trace of type $\langle s, t\rangle$ (i.e., a variable over propositions, not individuals). Accordingly the topmost application of Predicate Abstraction will yield a function from propositions to truth-values (type $\langle st, t\rangle$, the characteristic function of a set of propositions). Let's compute the meaning of this LF:

We leave out the vacuous operator OP and just interpret the predicate abstract it creates. ∅ in the superscript stands for the empty variable assignment, see H&K ch.5

(10) Computation for LF (9):
$[\![1.\ \text{who}\ 2.\ [?\ t_1]\ \text{Sakina see}\ t_2]\!]^{w,\emptyset}$
= (by Predicate Abstraction)
$\lambda p.\ [\![\text{who}\ 2.\ [?\ t_1]\ \text{Sakina see}\ t_2]\!]^{w,[1\to p]}$
= (by entry for *who* and lambda reduction)
$\lambda p.\ \exists x\ [\ x \text{ is human in } w\ \&\ [\![2.\ [?\ t_1]\ \text{Sakina see}\ t_2]\!]^{w,[1\to p]}(x) = 1\ ]$
= (by Predicate Abstraction and lambda reduction)
$\lambda p.\ \exists x\ [\ x \text{ is human in } w\ \&\ [\![\,[?\ t_1]\ \text{Sakina see}\ t_2]\!]^{w,[1\to p,2\to x]} = 1\ ]$
= (by IFA)
$\lambda p.\ \exists x\ [\ x \text{ is human in } w\ \&\ [\![?\ t_1]\!]^{w,[1\to p,2\to x]}(\lambda w'.\ [\![\text{Sakina see}\ t_2]\!]^{w',[1\to p,2\to x]}) = 1\ ]$

= (by FA and dropping irrelevant superscripts)
$\lambda p.\ \exists x\ [\ x$ is human in w & $[\![?]\!]([\![t_1]\!]^{[1\to p,2\to x]})(\lambda w'.\ [\![\text{Sakina see } t_2]\!]^{w',[1\to p,2\to x]}) = 1\ ]$
= (by Traces rule)
$\lambda p.\ \exists x\ [\ x$ is human in w & $[\![?]\!](p)(\lambda w'.\ [\![\text{Sakina see } t_2]\!]^{w',[1\to p,2\to x]}) = 1\ ]$
= (by entry for *?*)
$\lambda p.\ \exists x\ [\ x$ is human in w & $p = \lambda w'.\ [\![\text{Sakina see } t_2]\!]^{w',[1\to p,2\to x]}\ ]$
= (by FA, entries for *Sakina*, *see*, Traces Rule)
$\lambda p.\ \exists x\ [\ x$ is human in w & $p = \lambda w'.$ Sakina sees x in $w'\ ]$

This characterizes a set of propositions that contains one proposition per human-in-w: the proposition that that human was seen by Sakina.

This is the ANSPOSS in Hagstrom 2003's terminology.

How about polar and alternative questions? Can we just posit the same operators in the C-head? Let's try.

Here we show an LF and computation that includes the vacuous operator. As an exercise, convince yourself that in the case of polar questions, a simpler structure without OP is likewise interpretable, with equivalent results.

(11) Did Sakina see Emily?

$[_C$? OP$]$ Sakina see Emily
OP 1 [? t_1] Sakina see Emily

(12) Computation for LF of the polar question (11):
$[\![1.\ [?\ t_1]\ \text{Sakina see Emily}]\!]^{w,\emptyset}$
= (by Predicate Abstraction)
$\lambda p_{st}.[\![\,[?\ t_1]\ \text{Sakina see Emily}]\!]^{w,[1\to p]}$
= (by IFA)
$\lambda p_{st}.[\![?\ t_1]\!]^{w,[1\to p]}(\lambda w'.[\![\text{Sakina see Emily}]\!]^{w',[1\to p]})$
= (by FA)
$\lambda p_{st}.[\![?]\!]^{w,[1\to p]}([\![t_1]\!]^{w,[1\to p]})(\lambda w'.[\![\text{Sakina see Emily}]\!]^{w',[1\to p]})$
= (by Traces Rule and dropping irrelevant assignment superscripts)
$\lambda p_{st}.[\![?]\!](p)(\lambda w'.[\![\text{Sakina see Emily}]\!]^{w'})$
= (by entry for ? and lambda reduction)
$\lambda p_{st}.\ [p = \lambda w'.[\![\text{Sakina see Emily}]\!]^{w'}]$
= (by FA and entries for *Sakina*, *Emily*, *see*)
$\lambda p_{st}.\ [p = \lambda w'.$ Sakina see Emily in $w']$

This is the characteristic function of a singleton set containing one proposition. Is this a good result? If we have in mind that our denotations for interrogative clauses should directly correspond to an intuitive notion of "possible answer", then this is problematic. There is certainly more than one possible answer to a polar question! But we think of the relation between the semantics and the pragmatics of interrogative clauses in a less simple-minded way. A singleton set of a proposition is used to induce a two-way partition of the context set: one cell containing worlds where the proposition is true and another where it isn't. Any relevant response needs to eliminate one of the cells.

To complete the current section, we consider an alternative question:

The slashes are intended as a crude representation of the distinctive intonational contour that characterizes the alternative-question reading.

(13) Did Sakina see Emily /or did he see Julie\?
DS: [[$_C$? OP] Sakina see Emily] or [[$_C$? OP] Sakina see Julie]
LF: OP 1 [[[? t_1] Sakina see Emily] or [[? t_1] Sakina see Julie]]

In the alternative question in (13), operator movement must be "across the board" (ATB), with the result that a single binder binds two coindexed traces. This is the only way to obtain an interpretable structure, given the semantic type of *or*, which is $\langle t, \langle t, t\rangle\rangle$.

(14) Computation for LF of the alternative question in (13):
$[\![1.\ [?\ t_1]\ \text{Sakina see Emily or } [?\ t_1]\ \text{Sakina see Julie}]\!]^{w,\emptyset}$
= (by Predicate Abstraction)
$\lambda p_{st}.\ [\![[?\ t_1]\ \text{Sakina see Emily or } [?\ t_1]\ \text{Sakina see Julie}]\!]^{w,[1\to p]}$
= (by FA twice and entry for or)
$\lambda p_{st}.\ [\![[?\ t_1]\ \text{Sakina see Emily}]\!]^{w,[1\to p]} \vee [\![[?\ t_1]\ \text{Sakina see Julie}]\!]^{w,[1\to p]}$
= (by IFA, FA, entry for ?, etc — see previous computations)
$\lambda p_{st}.\ [p = \lambda w'.[\![\text{Sakina see Emily}]\!]^{w',[1\to p]} \vee p = \lambda w'.[\![\text{Sakina see Julie}]\!]^{w',[1\to p]}]$
= (by FA and entries for *see* etc.)
$\lambda p_{st}.\ [p = \lambda w'.\ \text{Sakina see Emily in } w' \vee p = \lambda w'.\ \text{Sakina see Julie in } w']$

This is the characteristic function of a set containing two propositions — the same set, in fact, that is denoted by the constituent question *who did Sakina see* if the set of humans happens to be just {Emily, Julie}.

8.2.2 Back to pragmatics: Mapping interrogative denotations to partitions

We can now make good on the promise to provide a bridge between the compositional semantics of interrogatives and their use in updating a context. We gave a draft version earlier in Definition 6, repeated here:

Definition 6 (Update by question (draft)) To ask a question by uttering a sentence ϕ that denotes a set of propositions $\mathcal{P}$ is to propose that the current partitioned context set C be replaced by the new partitioned context set $\textsc{part}(\mathcal{P}, \bigcup C)$.

To finalize the definition, we need to use our semantics to supply a set of propositions to serve as the seed for the partition. This is straightforward in the case of polar and alternative questions, each of which has as its semantic value a set of propositions that is evaluation-world independent. But in the case of wh-questions, an issue arises: their semantic value depends non-trivially on the evaluation world.

(15) $[\![\text{Who did Sakina see?}]\!]^w$
$= \lambda p.\ \exists x\colon\ x \text{ is human in } w.\ p = \lambda w'.\ \text{Sakina saw } x \text{ in } w'$

What set of propositions the interrogative denotes in a world w depends on who the humans in that world are. In case this seems esoteric, consider a more substantively restricted wh-phrase:

(16) $[\![\text{Which students called?}]\!]^w$
$= \lambda p.\ \exists x\colon\ x$ is a student in w. $p = \lambda w'$. Sakina saw x in w'

The semantic value of (16) depends on who the students in the evaluation world are. It is easy to imagine circumstances where it is not common ground who the students are. So, how should we update the context with a question like (16)?

Option 1: The world of utterance. We could say that the context gets partitioned based on the set of propositions denoted by the interrogative relative to the world in which the question is asked. However, this would miss the fact that the participants in the conversation do not know which world they are in. The context set is precisely meant to model that they have some common ground on what world they're in but no more: there are always multiple candidates for what the actual world is. Furthermore, it is presumably inescapable that participants make some false presuppositions, in which case the actual world in which the utterance occurs isn't even part of the context set (even though everyone is acting as if the common ground contains the actual world).

Option 1 would say something like: To ask a question by uttering a sentence ϕ in world w is to propose that the current partitioned context set C be replaced by the new partitioned context set $\text{PART}([\![\phi]\!]^w, \bigcup C)$.

So, it seems clear that we need to interpret the interrogative with respect to the worlds in the context set and use the resulting set of propositions to partition the context set. But now we need to face the possibility that the participants in the conversation where (16) is uttered are uncertain about who the (relevant) students are. To set up a minimal test scenario, assume that everyone knows that a is a student but there's uncertainty about whether b is. Now, assume that (16) is uttered against that background. There are worlds in the context set where a and b are the students, and other worlds where only a is a student. Our semantics will deliver two different sets of propositions as the semantic value of the interrogative for the two different kinds of worlds in the context set: for the a, b-worlds, we get the set containing the proposition that a called and the proposition that b called; while for the a-worlds, we get the set that contains only the proposition that a called. How should the interrogative partition the context set?

Option 2: Collecting all the propositions. To obtain the set of propositions that is used to partition the context set, we could collect all the propositions that for *some* world in the context set are in the semantic value of the interrogative:

(17) The set of propositions that is used to partition a context set c upon the utterance of an interrogative ϕ is

$$\{p\colon\ \exists w \in c.\ p \in [\![\phi]\!]^w\}$$

Equivalently, form the big union of all the sets of propositions that are the semantic value of the interrogative in worlds of the context sets:

$$\bigcup_{w \in c} [\![\phi]\!]^w$$

For our minimal test scenario, that means that the context set will be partitioned by the the set containing the proposition that *a* called and the proposition that *b* called. By our definition of relevance, this means that the answer "*b* called" will be relevant. But notice that when the context set is updated by the assertion that *b* called, there will still be worlds left in the context set where *b* is not a student (while it is now accepted that *b* called). This is incorrect: answering that *b* called when what was asked is *Which students called?* surely commits one to *b* being a student.

Option 3: Presupposition of consensus. Stalnaker 1978 in his analysis of the speech act of assertion formulated three principles that he views as "essential conditions of rational communication":

1. A proposition asserted is always true in some but not all of the possible worlds in the context set.
2. Any assertive utterance should express a proposition, relative to each possible world in the context set, and that proposition should have a truth-value in each possible world in the context set.
3. The same proposition is expressed relative to each possible world in the context set.

What's relevant for us here is the third principle, which ensures that there is common ground on which proposition the speaker is proposing to add to the common ground. We suggest that essentially the same principle applies to the speech act of asking a question: the same set of propositions needs to be expressed by the interrogative relative to each possible world in the context set. This then amounts to the assumption that for each wh-phrase that is used, the interlocutors agree on a specific set of individuals as its intended range. This means that in our test scenario, the question *Which students called?* is a pragmatic error: asking the question presupposes that it is common ground who the students are.

Hence, the final version of our definition is as follows:

Definition 7 (Update by question (final)) Asking a question by uttering a sentence ϕ is only felicitous in a partitioned context set C if $\forall w, w' \in \bigcup C\colon\ [\![\phi]\!]^w = [\![\phi]\!]^{w'}$.

If felicitous, asking a question by uttering a sentence ϕ is to propose that the current partitioned context set C be replaced by the new partitioned context set $\text{PART}(\llbracket \phi \rrbracket^w, \bigcup C)$, where w is an arbitrary world in $\bigcup C$.

8.2.3 Syntax: the role of the [WH] feature

In the theory as it stands so far, the semantic identification of wh-words with indefinites (e.g. $\llbracket \text{who} \rrbracket^w = \llbracket \text{somebody} \rrbracket^w$) leads to overgeneration of readings. Consider the minimal pair in (18).

(18) a. Who did Sakina see?
  b. Did Sakina see somebody?

We have shown in Section 8.2.1 how the Karttunen theory derives the intended denotation for (18a) (= (10)). But from the Deep Structure we posited in (10), we could have derived more than one interpretable structure, namely not just (19a) (which we considered in (10) above) but also (19b).

(19) a. LF-high: OP 1 [who 2 [[? t_1] Sakina see t_2]]
  b. LF-low: OP 1 [[? t_1] who 2 [Sakina see t_7]]

Similarly we can derive two interpretable LFs for (18b).

(20) DS: [$_C$? OP] Sakina see somebody
  a. LF-high: OP 1 [somebody 2 [[? t_1] Sakina see t_2]]
  b. LF-low: OP 1 [[? t_1] somebody 2 [Sakina see t_2]]

In all of these potential LFs, the IP-sister of C, as well as the mother-node of C, are semantically of type t. Therefore, both should be suitable adjunction sites for the generalized quantifiers *who* and *somebody* as far as type-compatibility goes. (The mnemonic labels 'high' and 'low' indicate the difference in the quantifier's scope.)

But what do these LFs mean? We have already computed (19a) and seen that it expresses an appropriate meaning for (18a). Specifically, if we combine our semantics with our pragmatics, and if the domain of people that who ranges over consists of just Emily and Julie, (19a) determines a 4-cell partition. This matches the intuition that the possible fully exhaustive answers to (18a) are that Sakina saw only Emily, that he saw only Julie, that he saw both Emily and Julie, and that he saw neither. But such a 4-cell partition is clearly not what an utterance of (18b) sets up. (18b) is a polar question and elicits answers such as Yes or No, expressing the propositions that Sakina saw somebody and that he saw nobody. So here we want our semantics and pragmatics to determine a 2-cell partition. The LF-low in (20b) does deliver precisely this. (Compute this as an exercise.)

So LF-high in (19a) captures the attested meaning of the wh-question in (18a), and LF-low in (20b) captures the attested meaning of the polar question in (18b). In each case, this is the only attested reading for the English sentence: the wh-question cannot be read as a polar question, and the polar question cannot be read as a wh-question. Our theory so far fails to derive this un-ambiguity. The LF-low in (19b) is semantically equivalent to the LF-low in (20b) and thus represents an unattested polar reading for the wh-question. Similarly, the LF-high in (20a) is equivalent to the LF-high in (19a) and thus represents an unattested wh-question reading for the polar question with *somebody*. We need to amend our theory so that only one of the LFs is generated for each of the sentences.

A standard solution—effectively Karttunen's—is to invoke a syntactic constraint that regulates scopal relations between existential DPs (*who*, *somebody*) and the interrogative complementizer. As our example teaches us, *who* apparently can only scope above *?* (more specifically, between *?* and its associated *OP*), whereas *somebody* is barred from scoping there and must instead scope below *?*. The following stipulation enforces this generalization.

(21) Wh-Licensing Principle:
At LF, a phrase α occupies a specifier position of *?* if and only if α has the feature [WH].

(21) relies on appropriate assumptions about which phrases have the feature [WH]. For the time being, assume that certain words such as *who*, *what*, *how* are marked as [WH] in the lexicon. These then will be the only phrases that can be located right above *?* in well-formed LFs, and moreover, they cannot be located anywhere else. LF-high for the wh-question in (19a) complies with (21), as does LF-low for the polar question in (20b). LF-low in (19b) is ruled out because it has a phrase marked [WH] in a location other than spec-of-*?*, and LF-high in (20a) is prohibited because it has a phrase in spec-of-*?* which lacks [WH].

Exercise 8.1 *Consider the multiple wh-question (22).*

(22) Who likes who?

Propose an LF, say how it is derived in the syntax, compute its semantic interpretation, and then compute the partition it imposes. For simplicity, assume in this last part that the domain of people that who ranges over is just {s, e}, and that the common ground before the question is totally uninformative, i.e, it is a partitioned context set whose union is W (the set of all worlds whatsoever). How many cells does the partition have and what are its cells? Regarding syntax, attend in particular to the satisfaction of the Wh-Licensing Principle.

Exercise 8.2 *In written language, a question containing* or*, such as (23), can be ambiguous.*

(23) Did you talk to David or Norvin?

The questioner may want to know which of the two professors you talked to ("alternative-question reading") or just whether you talked to at least one of them ("polar-question reading"). Perhaps the alternative reading is more salient out of the blue, but the polar reading can be facilitated by an appropriate context. (E.g. imagine that your squib is on multiple-wh-constructions, and you have previously been told that you ought to consult a faculty member who has published on this topic, namely David or Norvin.)

Your task in the exercise is to analyze the two readings of (23), by proposing an appropriate LF for each reading and discussing its syntactic derivation as well as its semantic and pragmatic interpretation. You should assume that English has only one unambiguous word or*, and its semantic type is* $\langle t, \langle t, t\rangle\rangle$. *This means that, for both readings, you must posit some amount of elided material in the right disjunct, since* or *can only coordinate constituents of type* t.

8.3 Embedded questions and question-embedding verbs

A first superficial survey of English verbs that take complement clauses turns up three groups. One group, exemplified by the verb *believe*, consists of verbs that take *that*-clauses but are ungrammatical with an embedded interrogative clause.

White 2021 argues that things are not that simple.

(24) a. Sakina believes that Becky called.
b. *Sakina believes who called.

Another rather large second group consists of verbs that can take either. This includes *know*, *remember*, and *tell*.

(25) a. Sakina knows/remembers/told Emily that Becky called.
b. Sakina knows/remembers/told Emily who called.

Third, there are verbs which take interrogative complements but are ungrammatical with *that*-clauses.

(26) a. *Sakina asked/is wondering that Becky called.
b. Sakina asked/is wondering who called.

We will focus at first on the middle group, which Lahiri 2002 dubbed the class of "responsive" verbs.

8.3.1 Responsive verbs, a type-mismatch, and Dayal's strategy

Let's look at the responsive verb *know*. We have at least a preliminary analysis for verbs like *know* in sentences like *John knows that Ann called*. Such verbs take a proposition and a person as their arguments, and their meaning encodes universal quantifcation over a certain set of possible worlds. Since *know* is a factive verb, it also triggers the presupposition that its complement is true. For simplicity, we assume here that apart from the factive presupposition, the meaning of *know* is the same as the meaning of *believe*, and so we can write the lexical entry in (27).

(27) $[\![\text{know}]\!]^{w} = \lambda p_{st}:\ p(w) = 1.\ \lambda x_e.\ \forall w'[w' \in \textsc{dox}(x, w) \rightarrow p(w') = 1]$

Here, DOX maps an individual x and a world w into the set of worlds compatible with what x believes in w.

This entry was designed to work for *know* with a *that*-clause. What would happen if we tried to interpret an LF with an interrogative clause as the sister of *know*? Evidently, we would run into a type-mismatch. On our current analysis of interrogative clauses, their extensions are of type $\langle st, t\rangle$ and their intensions of type $\langle s, \langle st, t\rangle\rangle$. Neither type can compose with the type of know in (27) by means of any of our semantic rules.

What shall we do? Older literature on question-embedding made verbs like *know* lexically ambiguous, with distinct (though not unrelated) lexical entries for declarative-taking *know* and interrogative-taking *know*. More recent work has pursued the strategy of positing a single unambiguous verb and readjusting the semantic type of the interrogative complement. Groenendijk & Stokhof 1982 did this first, and another influential version of this approach originates with Dayal 1996. Dayal proposed that the combination of the verb with the Karttunen-denotation of its complement is mediated by an "answer operator", which maps sets of propositions to propositions. We will follow Dayal's general strategy in these notes. We will entertain a couple of possible meanings for the answer operator and talk about the empirical considerations that bear on the question which meaning is correct.

The rough intuition to be implemented is that "to know who called" means something like "to know the answer to the question 'who called?'". (Paraphrases of this form work for all the verbs in this group, hence Lahiri's term "responsive verbs".) The answer to a question is a proposition, hence an object of a suitable semantic type to feed to the meaning of *know* in (27). If the LFs of sentences like (25b) contain an operator that maps a question-denotation to the proposition that's the answer to that question, we have a solution to the type-mismatch problem.

Since our syntax for interrogative clauses already happens to posit a silent operator at the top edge of the clause (albeit one that we have so far treated as semantically vacuous) we need not actually make the structure more complex. Instead we can assume (following a suggestion by Danny Fox) that our new answer operator appears instead of the previous vacuous one. This means that we base-generate it inside C as the sister of *?* and move it up for interpretability, leaving a type-$\langle s, t\rangle$ trace as before. Our LF-structure for a sentence with know and an interrogative complement then looks as in (28b).

(28) a. Sakina knows who called.
b. Sakina [$_{VP}$ knows [$_{CP}$ ANS 1 [who 2 [[$_C$? t_1] t_2 called]]]]

We will now focus on the task of proposing a meaning for ANS which not only fixes the type-mismatch, but also yields reasonable truth conditions for the *know*-sentence.

8.3.2 An ANS *operator inspired by Karttunen*

Our initial proposal for the semantics of ANS effectively follows Karttunen 1977 in terms of the truth conditions it predicts for the *know*-sentence as a whole (although the compositional implementation is different). The ANS operator defined in (29) maps the set of propositions denoted by the interrogative to the single proposition which is the conjunction of all its true elements.

(29) $[\![\text{ANS}]\!]^w = \lambda Q_{\langle st,t\rangle}.\ \forall p[Q(p) = 1\ \&\ p(w) = 1 \rightarrow p(w') = 1]$

(29') $[\![\text{ANS}]\!]^w = \lambda Q_{\langle st,t\rangle}.\ \bigcap\{p \in Q:\ w \in p\}$
(the intersection of all members of Q that are true in w)

equivalent formulation (modulo sets vs. characteristic functions)

Let's put this entry to work in a computation for the example sentence.

(30) a. computation of presupposition:
Let w be a world. Then
$[\![\text{Sakina knows who called}]\!]^w$ is defined
iff (by FA twice)
$[\![\text{know}]\!]^w([\![\text{ANS 1 who 2 ?-}t_1\ t_2\text{ called}]\!]^w)(s)$ is defined
iff (by entry for *know*)
$[\![\text{ANS 1 who 2 ?-}t_1\ t_2\text{ called}]\!]^w(w) = 1$
iff (by FA)
$[\![\text{ANS}]\!]^w([\![\text{1 who 2 ?-}t_1\ t_2\text{ called}]\!]^w)(w) = 1$
iff by entry for ANS
$\forall p[[\![\text{1 who 2 ?-}t_1\ t_2\text{ called}]\!]^w(p) = 1\ \& p(w) = 1 \rightarrow p(w) = 1]$
This is a tautology, so we know that $[\![\text{Sakina knows who called}]\!]^w$ is defined for all w.

Observe that the mother node of *know* is interpreted by plain Functional Application (not by Intensional Functional Application, which would have applied if *know* were taking a declarative complement). This is because the *extension* of the constituent headed by ANS is of type $\langle s, t\rangle$.

b. computation of truth condition:
Let w be a world. Then
$[\![\text{Sakina knows who called}]\!]^w = 1$
iff (by FA twice)
$[\![\text{know}]\!]^w([\![\text{ANS 1 who 2 ?-}t_1\ t_2\text{ called}]\!]^w)(s) = 1$
iff (by entry for *know* and truth of presupposition)
$\forall w'[w' \in \text{DOX}(s, w) \rightarrow [\![\text{ANS 1 who 2 ?-}t_1\ t_2\text{ called}]\!]^w(w') = 1]$

c. We interrupt for an embedded computation:
$[\![\text{ANS 1 who 2 ?-}t_1\ t_2\text{ called}]\!]^w(w') = 1$
small iff (by entry for ANS)
$\forall p[[\![\text{1 who 2 ?-}t_1\ t_2\text{ called}]\!]^w(p) = 1\ \&\ p(w) = 1 \rightarrow p(w') = 1]$

iff (by earlier computations)
$\forall p[\exists x[x \text{ is a human in } w \;\&\; p = \lambda w''.\, x \text{ called in } w''] \;\&\; p(w) = 1 \rightarrow p(w') = 1]$
iff (by logic of quantifiers)
$\forall p \forall x[x \text{ is a human in } w \;\&\; p = \lambda w''.\, x \text{ called in } w'' \;\&\; p(w) = 1 \rightarrow p(w') = 1]$
iff (by logic of identity)
$\forall x[x \text{ is a human in } w \;\&\; [\lambda w''.\, x \text{ called in } w''](w) = 1 \rightarrow [\lambda w''.\, x \text{ called in } w''](w') = 1]$
iff (by λ-reduction)
$\forall x[x \text{ is a human in } w \;\&\; x \text{ called in } w \rightarrow x \text{ called in } w']$

d. resuming main computation: …
iff (by plugging in result of embedded computation)
$\forall w'[w' \in \textsc{dox}(s, w) \rightarrow \forall x[x \text{ is a human in } w \;\&\; x \text{ called in } w \rightarrow x \text{ called in } w']]$
iff (by logic of quantifiers)
$\forall x[x \text{ is a human in } w \;\&\; x \text{ called in } w \rightarrow \forall w'[w' \in \textsc{dox}s, w \rightarrow x \text{ called in } w']]$

In other words, the sentence *Sakina knows who called* is true in w if and only if, for every person who in fact called in w, Sakina believes (in w) that this person called.

8.3.3 Problems, and an ANS *operator inspired by Groenendijk & Stokhof*

The essence of ANS in our present analysis is that for any evaluation world w, when applied to the semantic value Q of an interrogative (which is a set of proposition), it gives the proposition that is the conjunction of all the propositions in Q that are true in w. This is well known to make some troublesome predictions. Let's start with the most glaring case, then move to subtler ones, before we diagnose the general problem and proceed to solve it.

The example we analyzed above involved an embedded constituent question. What if we embedded a polar question instead?

(31) a. Sakina knows whether Emily called.
b. LF: Sakina knows $[_{CP}$ ANS 1[$[_C ?\ t_1]$ Emily called]]

Recall that our denotation for a polar question is a singleton set. The sister of ANS in (31b) denotes the set whose only member is the proposition that Emily called. If we complete the calculation, we get the following truth condition.

(32) presupposition of (31b): tautological
$[\![(31b)]\!]^w = 1$ iff
Emily called in $w \rightarrow \forall w'[w' \in \textsc{dox}(s, w) \rightarrow \text{Emily called in } w']$

This says that the sentence (31a) is true in w if either one of the following two conditions is met: either (i) Emily did not call in w, or else (ii) she did call in w and Sakina believes in w that she did. This is not satisfactory. What it gets right is that, if Emily called but Sakina is unaware of this, then the sentence is false. But it also predicts that, if Emily didn't call, then the sentence is true no matter what Sakina believes—even if she wrongly believes that she did call.

A gut reaction to this problem is that the culprit is our semantics for polar questions, not our ANS operator. This is what Karttunen would have said. Indeed, he gave a different semantics for polar questions and did not have this problem. In our variant of his theory, if we minimally changed the semantics of polar questions so that the sister of ANS were to denote the 2-membered set {that Emily called, that Emily didn't call}, the truth conditions would come out correct without any revision to our entry for ANS. (Exercise: Convince yourself of this.) This looks like a good way out—at least at first. But when we look at further problem cases, we will come to see it is a move that is neither sufficient nor necessary.

How might we do that? Perhaps by giving a meaning to *whether*, letting it denote a function that maps a singleton set $\{p\}$ to the set $\{p, \neg p\}$.

Let's return to the case of the embedded constituent question in (28) and scrutinize the truth conditions we derived in (30) a bit more carefully. Suppose that w is a world in which nobody called. Then the universal quantification we computed in (30b) is trivially true: Whatever Sakina's beliefs in w may be, the material conditional '[x called in $w \to$ Sakina believes in w that x called]' is true for every x (since the antecedent is always false). So the sentence (28) is predicted to be true, for example, in a world where nobody called but Sakina falsely believes that Emily and Julie called. This does not conform to our intuitions.

The problem we're discussing here was noticed by Karttunen 1977 in a footnote, and he fixed it by complicating his lexical entry for interrogative-taking *know*. He ended up stating the truth condition in the form of a disjunction, with a special clause for the case where the question-denotation only contains false propositions. Heim 1994 showed how to generalize Karttunen's special clause to a general solution for all the problem cases we consider in this section. The solution we will present in these lecture notes is not quite the same as Heim's. See papers by Rullmann & Beck 1998, Beck & Rullmann 1999, Sharvit 2002, and Sharvit & Guerzoni 2003 for discussion and comparison.

Finally, as Groenendijk & Stokhof 1982 forcefully pointed out, even if we only consider worlds in which some people did in fact call, the truth conditions imposed by our current (and Karttunen's) semantics are too lax. Suppose that only Emily called, but Sakina thinks that Emily, Julie, and Delphine all called. Would we say that Sakina knows who called? We'd be reluctant to. But our semantics deems the sentence *Sakina knows who called* to be true in this scenario. After all, Sakina does believe of every person who in fact called (namely, of Emily), that that person called.

This is all that our predicted truth conditions require. If our analysis were right, it simply shouldn't matter how many false beliefs Sakina has about people calling who did not in fact call.

Groenendijk & Stokhof argued that the correct semantics for *Sakina knows who called* is what they dubbed "strongly exhaustive"—i.e., the *know*-sentence is true only when Sakina is fully informed about who called and who didn't. She believes that they called of all the people who did in fact call, and she believes that they didn't call of all the ones who didn't. Can we revise our entry for the answer operator so that it delivers this more stringent truth condition? Yes, here is how.

(33) strongly exhaustive answer operator:
$[\![\text{ANS}]\!]^w = \lambda Q_{\langle st,t\rangle}.\lambda w'.\ \forall p[p \in Q \rightarrow p(w) = p(w')]$

(33') $[\![\text{ANS}]\!]^w = \lambda Q_{\langle st,t\rangle}.\ \{w' :\ w' \sim_{Q,W} w\}$

equivalent formulation, using the definition of "cell-mate" from (4a). Here, capital W in the subscript to the ~-relation is the set of all possible worlds.

What does this semantics for ANS do? It takes the set of propositions Q denoted by the underlying interrogative and maps the evaluation world to the worlds that agree with it on the truth-value of all propositions in Q. In other words, it maps the evaluation world to its cell-mates relative to Q.

This new semantics solves all the problems that we saw in this section. For embedded polar questions, it delivers the prediction that if Emily didn't call, then *Sakina knows whether Emily called* is only true if Sakina knows that Emily didn't call. For embedded constituent questions, it predicts that if x didn't call, then *Sakina knows who called* is not true unless Sakina knows that x didn't call. As a special case of this latter prediction, we derive that if nobody called, then *Sakina knows who called* is only true if Sakina knows about each person that they didn't call.

Exercise 8.3 *Verify these claims, by doing the computations.*

8.3.4 ANS in matrix questions?

Our latest, strongly exhaustive answer operator bears an obvious logical relation to the pragmatic rule "update by question" (Definition 7) that we posited earlier to link the semantic values of matrix interrogative clauses to the speech acts that they serve to perform. In both cases, we use the set of propositions Q to lump together any evaluation world with those worlds that agree with it on the truth-values of all propositions in Q. In the case of interrogatives embedded under responsive predicates like *know*, this set of worlds that are cell-mates with the evaluation world is fed to the attitude predicate as its prejacent proposition. In the case of matrix questions, we use the set of disjoint but exhaustive cells to partition the context set.

Given this similarity, we may want to consider positing ANS in matrix questions as well, instead of the vacuous covert operator *OP*. This would mean rewriting the "update by question" rule. The idea would be that the pragmatic rule can use the intension of the uttered sentence to construct the cells of the new partition. The recipe, informally, is to apply this intension to each world in the current context set and intersect each result with the current context set.

Definition 8 (Update by question, draft revision) To ask a question by uttering a sentence ϕ is to propose that the current partitioned context set C be replaced by the new partitioned context set

$$\{p :\ \exists w \in \bigcup C.\ p = [\![\phi]\!]^w \cap \bigcup C\}.$$

Note that the relevant sentences ϕ are of the form "ANS Q" and relative to any world w denote the proposition that is true of any world that agrees with w on all the propositions in the set denoted by Q relative to w.

We take each world w in the prior context set in turn. We evaluate the matrix question ϕ (= ANS Q) in w and intersect the resulting proposition with the context set, thus finding those worlds in the context set that agree with w on all the propositions in $[\![Q]\!]^w$. We collect the resulting set of sets of worlds to serve as the new partitioned context set.

There is a tricky issue here, which is a reprise of what we discussed on page 159ff. Take again a test scenario where everyone knows that a is a student but there's uncertainty about whether b is. And there's maximal uncertainty about who called. Now, assume that (34) is uttered against that background:

(34) Which students called?

There are worlds in the context set where a and b are the students, and other worlds where only a is a student. We saw earlier that in such a context, it is best to rule (34) out as pragmatically infelicitous because the set of propositions denoted by the interrogative varies among the worlds in the context set. This was easy enough to do in the system we were working with at that point. Now, however, we are considering the possibility that what is being uttered is really "ANS which students called". That structure has different semantic values across the worlds in the context set simply because there are different true answers in those worlds (otherwise why ask the question?). So, we can't enforce a presupposition that the interrogative have the same value (the same strong true answer) across the context set.

In an earlier version of these lecture notes, it was erroneously claimed that we could evade the problem this way.

The set of sets of worlds we would get from Definition 8 in our test scenario would not in fact be a partition of the context set:

(i) a world w where a called and b is not a student will be lumped with any world where a called and b is a student, no matter whether b called (since the proposition that b called is not in the denotation in w of the underlying interrogative);
(ii) a world w' where a called and b is a student who called will be lumped only with other worlds where a called and b is a student who called;
(iii) the two sets generated by w and w' are not disjoint, since they both contain worlds where a called and b is a student who called.

The only way we can see to prevent this situation is to state directly in the "update by question" rule that the update is only felicitous if it results in a partition:

Definition 9 (Update by question, final revision) To ask a question by uttering a sentence ϕ is to propose that the current partitioned context set C be replaced by the new context set

$$\{p\colon\ \exists w \in \bigcup C.\ p = [\![\phi]\!]^w \cap \bigcup C\}$$

If the resulting set of sets of worlds is not a partition of $\bigcup C$, the utterance is infelicitous.

We conclude this subsection by noting that we have now encountered a kind of presupposition that is not grounded in the denotational semantics of the sentences uttered but emerges from the rules and principles of pragmatics.

In particular, note that we have derived a condition on any wide-scope restriction on wh-phrases that requires the set characterized by that restriction to be "settled" in the prior context set. It would be interesting to compare this result to ideas about "d-linking" that are found in the literature on questions.

8.3.5 Embedding under rogative verbs

Access to the intension of the LF headed by ANS also figures plausibly in the semantics of sentences with non-responsive ("rogative") question-embedders, such as the verbs *ask* and *wonder*. These verbs have lexical meanings that suggest rough paraphrases in which *know* or *tell* is in the scope of another intensional operator, e.g., *ask* = 'request to be told', *wonder* = 'want to know'. Following Groenendijk & Stokhof, let's hypothesize that these verbs differ in semantic type from responsive verbs. Their (internal) argument is of type $\langle s, \langle s, t\rangle\rangle$ rather than $\langle s, t\rangle$. A concrete entry for *wonder* along these lines is (35).

DES maps an individual and a world to those worlds that satisfy the individual's desires as they are in the given world.

(35) $[\![\text{wonder}]\!]^w = \lambda q_{s,st}.\lambda x_e.$
$\forall w'[w' \in \text{DES}(x, w) \rightarrow \forall w''[w'' \in \text{DOX}(x, w') \rightarrow q(w')(w'') = 1]]$

The higher semantic type straightforwardly gives us the prediction that *wonder* cannot combine with a *that*-clause. If we attempted to interpret such an LF, we would encounter a type-mismatch. A *that*-clause has an extension of type t and an intension of type $\langle s, t\rangle$. Whether we used plain FA or IFA, we wouldn't obtain the type-$\langle s, st\rangle$-function that wonder is looking for. But if we embed an interrogative clause (headed by ANS) under *wonder*, we will succeed. The extension of the mother-node of ANS is type $\langle s, t\rangle$, and its intension is type $\langle s, st\rangle$. So IFA allows us to interpret the structure.

Exercise 8.4 *Convince yourself that, given entry (35), the predicted truth condition for* Sakina wonders who called *matches the informal paraphrase that we gave in the text.*

8.4 Wh-movement with pied-piping and reconstruction

As an example of the phenomenon of so-called pied-piping, we will analyze the sentence *How many cats did you adopt?* Before we get to the point that makes the example interesting for our purposes, we must fill in a rudimentary account of plurals and gradability.

8.4.1 Background on plurals and many

The basic idea in most current semantic treatments of plural DPs is that plural definites and pronouns denote entities in D_e, just like singular definites, pronouns, and proper names. The only difference is that the entities denoted by plurals are more complex (and typically spatially discontinuous). A distinction is made within the domain D_e, between so-called "atoms" or "atomic individuals" (the referents of singular DPs) and "pluralities" or "non-atomic individuals" (the referents of plural DPs). Non-atomic individuals contain atomic individuals as (proper) parts; e.g., if Lucy is one of the players, then $[\![\text{Lucy}]\!]$ (i.e., Lucy) is an atomic part of $[\![\text{the players}]\!]$. An atomic individual, on the other hand, has no atomic parts other than itself. (An atom counts as an atomic part of itself.)

Given that D_e contains pluralities along with atoms, predicate extensions of type $\langle e, t\rangle$ are functions that apply to both atoms and pluralities. In the case of common nouns, English has a morphological number distinction which seems to have semantic import:

(36) a. $[\![\text{cat}]\!]^w = \lambda x.$ is a cat in w
b. $[\![\text{cats}]\!]^w = \lambda x.$ every atomic part of x is a cat in w

Being a cat entails being an atomic individual (this is how we agree to understand our metalanguage). Therefore, the denotation of the singular noun as defined in (36a) maps every plurality to 0. The pluralized noun in (36b), on the other hand, maps to 1 those pluralities whose atomic parts are all cats.

Notice that an entity that $[\![\text{cats}]\!]^w$ maps to 1 is not *necessarily* a plurality. As interpreted in (36b), the plural noun *cats* is also true of a single cat, because of the fact that an atom is an atomic part of itself. It would be possible to revise (36b) so that it requires x to be non-atomic. But as we will see below, the current formulation actually works better. The reason is, in a nutshell, that "One." is a perfectly good answer to a *how-many* question.

Verbs can show morphological number too, but we assume that this is always due to morphological agreement with a number-marked subject, and that the number morphology on the verb is not interpreted itself. As far as semantics is concerned, verbs are "number neutral" and typically can be true indiscriminately of both atoms and pluralities. This is reflected, for example, in a lexical entry like (37).

(37) $[\![\text{meow}]\!]^w = \lambda x.$ every atomic part of x meows in w

The condition in (37) can be met by both pluralities and atoms. A plurality is mapped to 1 iff all its atomic parts meow, and an atom is mapped to 1 if it itself meows. (Recall that every atom counts as an atomic part of itself.)

We can count the atomic parts of a plural individual. For example, the plural individual composed of Lucy, Kathellen, and Shanice has 3 atomic parts. Let's have a concise notation for this.

(38) Let x be an element of D_e. Then
$\#(x) :=$ the cardinality of the set $\{y\colon\ y$ is an atomic part of $x\}$.

With this little bit of plural semantics in place, we can now introduce Hackl 2001's semantics for *many* and an appropriate semantics for interrogative *how* that will go with it. Hackl proposes that *many* is not by itself a quantificational determiner of type $\langle et, \langle et, t\rangle\rangle$. Rather it is looking for an argument which denotes a number, and only after it has been saturated with such an argument, the resulting phrase is a quantificational determiner. So the type of *many* is type $\langle e, \langle et, \langle et, t\rangle\rangle\rangle$—assuming that numbers are abstract individuals of some kind, hence members of D_e—and its entry is as in (39).

Actually, Hackl assumes (with most of the literature on adjectives and gradability) that there is an additional basic type d (for "degrees") separate from type e. The number argument of *many* is a special case of a degree argument, and the type for *many* is then $\langle d, \langle et, \langle et, t\rangle\rangle\rangle$.

Hackl's analysis implies that the superficially simplest uses of *many*, as in *Many cats meowed*, are actually more complex at LF: the argument position of *many* is bound by a covert POS ("positive operator"), which is a quantifier over numbers (degrees) and means something like 'a number (degree) above the contextually specified threshold'.

(39) $[\![\text{many}]\!] = \lambda n\colon\ n$ is a number. $\lambda f_{et}.\lambda g_{et}.\ \exists x[\#(x) = n\ \&\ f(x) = 1\ \&\ g(x) = 1]$

When building a sentence with *many*, in the simplest case we would fill the first argument slot of *many* with a word that refers to a number. This might be an anaphoric demonstrative pronoun *that*, which in appropriate discourse contexts can refer to a previously mentioned number, say the number 3. Or it could be a numeral word, like *three*, which we take here to have a meaning of type e and be a proper name of the number 3. These options give us interpretable syntactic representations like (40a,b).

(40) a. [[that_7 many] cats] meowed
b. [[three many] dogs] barked

(40a) is straightforwardly pronounced as it stands, whereas for (40b), Hackl assumes that *many* is unpronounced after numeral words, so this structure surfaces as *Three dogs barked*. Let us compute truth-conditions, using (39). Suppose we have a contextually given assignment for (40a) which maps the variable 7 to the number 3, and we evaluate the sentence in the actual world @. Then, by using FA three times to apply $[\![\text{many}]\!]$ to its three arguments, we obtain the truth-condition in (41).

(41) $\exists x[\#(x) = 3\ \&\ [\![\text{cats}]\!]^{@}(x) = 1\ \&\ [\![\text{meow}]\!]^{@}(x) = 1]$

Now we use our entries for *cats* and *meow*, and this becomes (42).

(42) $\exists x[\#(x) = 3$
& every atomic part of x is a cat in @
& every atomic part of x meows in @]

In other words, there is a plurality composed of three meowing cats. Which is loud but correct.

8.4.2 How many-*questions*

In a *how-many* question, the argument slot that was saturated by that or three in (40) is instead occupied by the wh-word *how*. In Karttunen's theory, this will be an existential quantifier, equivalent to *some number*.

The main source for the argument in this section is Arnim von Stechow's paper "Against LF pied-piping" (von Stechow 1996).

(43) $[\![\text{how}]\!] = \lambda f_{et}.\ \exists n[n \text{ is a number } \& \ f(n) = 1]$
(type $\langle et, t\rangle$, a generalized quantifier)

For Hackl, it would be type $\langle dt, t\rangle$, a generalized quantifier over degrees.

This semantic type is not interpretable in situ as the sister of *many*, and must undergo (covert) movement for interpretability. With this in mind, let's attempt a syntactic derivation for the question *How many cats did you adopt?*

(44) a. $[_C$? OP] [you adopted how many cats]
b. OP 5 [[? t_5] [you adopted how many cats]]
c. OP 5[how many cats 1 [[? t_5] you adopted t_1]
d. OP 5[how 2[t_2 many cats 1[[? t_5] you adopted t_1]]]

We have three movements that derive the LF from the base generated structure in (44a): the movement of the empty OP in (44b), the wh-movement of the wh-phrase *how many cats* in (44c), and the QR of the quantifier *how* in (44d).

We can check the semantic types to confirm that we have derived an interpretable structure (do this as an exercise). Let's compute what (44d) means. (The details are left as an exercise. Here we conflate sets of propositions with their characteristic functions.)

(45) $[\![(44\text{d})]\!]^@ =$
$\ldots$
$= \{p\colon\ \exists n\ [n \text{ is a number } \& \ \exists x[\#(x) = n \ \& \ [\![\text{cats}]\!]^@(x) = 1] \ \&$
$p = \lambda w.[\![\text{adopt}]\!]^w(x)(\text{you})]\}$

With a little bit of Predicate Logic reasoning, we can rewrite this equivalently as follows:

We are exploiting the equivalence of various scopal arrangements in a formula with existential quantifiers and conjunctions. The following four are all equivalent.
$\exists x[Fx \ \& \ \exists y[Rxy \ \& \ Gy]]$
$\exists x \exists y[Fx \ \& \ Rxy \ \& \ Gy]$
$\exists y \exists x[Fx \ \& \ Rxy \ \& \ Gy]$
$\exists y[\exists x[Fx \ \& \ Rxy] \ \& \ Gy]$

(46) $\{p\colon\ \exists x[\boxed{\exists n[n \text{ is a number } \& \ \#(x) = n]} \ \& \ [\![\text{cats}]\!]^@(x) = 1 \ \&$
$p = \lambda w.[\![\text{adopt}]\!]^w(x)(\text{you})]\}$

We can now contemplate the underlined part and convince ourselves that this part is a tautology. It just says that x has some number or other of atomic parts, which cannot fail to be true. So we might as well drop this conjunct and rewrite (46) as (47).

(47) $\{p\colon\ \exists x[[\![\text{cats}]\!]^@(x) = 1 \ \& \ p = \lambda w.[\![\text{adopt}]\!]^w(x)(\text{you})]\}$

Interestingly now, this is precisely the meaning we would have derived for the question *Which cats did you adopt?* In other words, if (44d) really were the LF (or one of the LFs) for the *how- many*-question *How many cats did you adopt?*, then we would be making the bad prediction that this question is synonymous with (or at least shares a reading with) the *which*-question *Which cats did you adopt?* This would be unfortunate for our theory.

Upon closer inspection of (44d), however, it turns out that our theory does not actually generate this LF. We have neglected to check whether (44d) conforms to the Wh-Licensing Principle, repeated here.

(48) At LF, a phrase α occupies a specifier position of *?* if and only if α has the feature [WH].

In (44d), we have two phrases that are scoped above ?, namely *how* and t_2 *many cats*. Let's say that the positions they occupy both count as specifier positions of ? (similar to what one has to say for multiple questions like *who ate what?*—see Exercise 8.1). Then (48) would require that both of these phrases have the feature [WH]. But only *how* actually does, at least on our current assumption that [WH] is a lexical property possessed by only a small set of words. The other phrase that is scoped above ? in (44d) is t_2 *many cats*, which does not carry the feature [WH]. So the structure (44d) is filtered out by the Wh-Licensing principle as syntactically ill-formed. And this is a good thing, because it means we don't generate the unwelcome reading in (47).

We still have to worry, however, about how we generate the reading that our example actually does have. Is there a second, well-formed, LF for our example? The answer is yes if our syntax allows for "reconstruction", i.e. some mechanism by which overtly moved phrases can be restored to (one of) their pre-moved positions at LF. In order to satisfy the Wh-Licensing Principle, reconstruction must apply to the phrase t_2 *many cats* (though not, of course, to *how*). Where can this phrase be reconstructed to? Well, if we restored it all the way down to its original base position as the object of *adopt*, it wouldn't be interpretable there, because quantifiers are not interpretable in object positions. But if we can assume that wh-movement proceeds (or at least is allowed to proceed) successive cyclically, and that an object wh-phrase can stop over e.g. at the edge of VP before it moves on to Spec-CP, we can target this intermediate landing site for reconstruction. (Assuming the VP-internal subject hypothesis, VPs are semantically of type *t* and hence quantifiers are interpretable at their edges.) By means of reconstruction, we can thus obtain another LF that is both interpretable and in compliance with the WH-Licensing principle.

(49) OP 5[how 2[[? t_5] t_2 many cats 1[you adopted t_1]]]

The denotation of (49) (which you should compute as an exercise!) is (50).

(50) $\{p\colon \exists n[n \text{ is a number } \& \ p = \lambda w.\ \exists x[\#(x) = n \ \& \ [\![\text{cats}]\!]^w(x) = 1 \ \& \ [\![\text{adopt}]\!]^w(x)(\text{you})]\}$

This set contains one proposition per number. It contains the proposition that you adopted 1 cat, the proposition that you adopted 2 cats, the proposition that you adopted 3 cats, etc. This prediction accords well with what we feel are expected answers to the question *how many cats did you adopt?*

8.4.3 More on reconstruction

Earlier in these notes in Section 4.2.2, we briefly introduced a version of the "copy theory of movement", in which reconstruction can be implemented as deletion of a higher copy. We reproduce the relevant passage here:

> This assumes that movement generally proceeds in two separate steps, rather than as a single complex operation as we have assumed so far. Recall that in H&K, it was stipulated that every movement effects the following four changes:
>
> (i) a phrase α is deleted,
>
> (ii) an index i is attached to the resulting empty node (making it a so-called trace, which the semantic rule for "Pronouns and Traces" recognizes as a variable),
>
> (iii) a new copy of α is created somewhere else in the tree (at the "landing site"), and
>
> (iv) the sister-constituent of this new copy gets another instance of the index i adjoined to it (which the semantic rule of Predicate Abstraction recognizes as a binder index).
>
> If we adopt the Copy Theory, we assume instead that there are three distinct operations:
>
> *"Copy":* Create a new copy of α somewhere in the tree, attach an index i to the original α, and adjoin another instance of i to the sister of the new copy of α. (= steps (ii), (iii), and (iv) above)
>
> *"Delete Lower Copy":* Delete the original α. (= step (i) above)
>
> *"Delete Upper Copy":* Delete the new copy of α and both instances of i.
>
> The Copy operation is part of every movement operation, and can happen anywhere in the syntactic derivation. The Delete operations happen at the end of the LF derivation and at the end of the PF deletion. We have a choice of applying either Delete Lower Copy or Delete Upper Copy to each pair of copies, and we can make this choice independently at LF and at PF. (E.g., we can do Copy in the common part of the derivation and than Delete Lower Copy at LF and Delete Upper Copy at PF.)

When we try to apply this machinery to reconstruction of pied-piped material in wh-movement, there is a detail that needs attention: After having created two copies (or more) of *how many cats*, what happens when we “move” *how*? Presumably we create another, higher, copy of *how* and coindex it with the lower copy of *how*. With which lower copy of *how*? With the one in the uppermost copy of *how many cats*? If so, then after deletion of that uppermost copy of *how many cats*, the binder index next to the top copy of *how* will no longer bind any trace and the structure will not be interpretable. What we want is to end up with is an LF in which the top copy of *how* binds a variable in a lower (retained) copy of *how many cats*. There may be various ways to achieve this. Perhaps we can simply choose a derivation in which *how* “moves” out of a lower copy of *how many cats*, not out the highest copy. (Richards 1998’s Principle of Minimal Compliance might explain why this long movement is legitimate and yet relies for its legitimacy on the previous movement of the larger phrase.) Alternatively, we may posit a general principle “Copies must remain copies”, which says that, as long as a structure contains more than one copy of a given phrase, every alteration to one of these copies must identically affect them all. In our case, this would mean that when we create the top copy of *how*, we must coindex it with all its existing copies, in the highest as well as all lower instances of *how many cats*. This principle could hopefully be made to fall out from a suitable formalization of copy theory, perhaps in a multi-dominance framework.

Bibliography

Abusch, Dorit. 1988. Sequence of tense, intensionality and scope. *West Coast Conference on Formal Linguistics (WCCFL)* 7. 1–14.

Abusch, Dorit. 1994. The scope of indefinites. *Natural Language Semantics* 2(2). 83–135. https://doi.org/10.1007/BF01250400.

Abusch, Dorit. 1997. Sequence of tense and temporal *de re*. *Linguistics and Philosophy* 20(1). 1–50. https://doi.org/10.1023/A:1005331423820.

Abusch, Dorit & Mats Rooth. 2017. The formal semantics of free perception in pictorial narratives. *Amsterdam Colloquium* 21. https://semanticsarchive.net/Archive/DVlMGQwY/abusch_rooth_ams_2017.pdf.

Aikhenvald, Alexandra Y. 2004. *Evidentiality*. Oxford: Oxford University Press.

Aloni, Maria. 2007. Free choice, modals, and imperatives. *Natural Language Semantics* 15(1). 65–94. https://doi.org/10.1007/s11050-007-9010-2.

Alonso-Ovalle, Luis. 2006. *Disjunction in alternative semantics*. University of Massachusetts at Amherst PhD thesis. http://alonso-ovalle.net/index.php?page_id=28.

Anand, Pranav & Valentine Hacquard. 2013. Epistemics and attitudes. *Semantics and Pragmatics* 6(8). 1–59. https://doi.org/10.3765/sp.6.8.

Arregui, Ana, María Luisa Rivero & Andrés Salanova. 2014. Cross-linguistic variation in imperfectivity. *Natural Language and Linguistic Theory* 32(2). 307–362. https://doi.org/10.1007/s11049-013-9226-4.

Arvan, Marcus. 2015. The Lewis-Stalnaker analysis of counterfactuals: What about Sprigge? https://philosopherscocoon.typepad.com/the_campaign_for_better_c/2015/03/the-lewis-stalnaker-analysis-of-counterfactuals-what-about-sprigge.html.

Asher, Nicholas. 1987. A typology for attitude verbs and their anaphoric properties. *Linguistics and Philosophy* 10(2). 125–197. https://doi.org/10.1007/BF00584317.

Auwera, Johan Van Der & Alfonso Zamorano Aguilar. 2015. The history of modality and mood. In Jan Nuyts & Johan Van Der Auwera (eds.), *The Oxford handbook of modality and mood*. Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199591435.013.4.

Bach, Emmon. 1986. The algebra of events. *Linguistics and Philosophy* 9(1). 5–16. https://doi.org/10.1007/BF00627432.

Baltin, Mark. 2010. The copy theory of movement and the binding-theoretic status of A-traces: You can't get there from here. http://as.nyu.edu/content/dam/nyu-as/linguistics/documents/nyuwpl/NYUWPL_vol_3_Baltin.pdf.

Barcan, Ruth C. 1946. A functional calculus of first order based on strict implication. *Journal of Symbolic Logic* 11(1). 1–16. https://doi.org/10.2307/2269159.

Barwise, John & Robin Cooper. 1981. Generalized quantifiers and natural language. *Linguistics and Philosophy* 4(2). 159–219. https://doi.org/10.1007/BF00350139.

Bassi, Itai & Moshe E. Bar-Lev. 2017. A unified existential semantics for bare conditionals. *Sinn und Bedeutung* 21. https://sites.google.com/site/sinnundbedeutung21/proceedings-preprints/SuB%2021%20Bar-Lev%20and%20Bassi%20final.pdf.

Bauer, Matthias & Sigrid Beck. 2014. On the meaning of fictional texts. In Daniel Gutzmann, Jan Köpping & Cécile Meier (eds.), *Approaches to meaning: Composition, values, and interpretation* (Current Research in the Semantics / Pragmatics Interface 32), 250–275. Brill. https://doi.org/10.1163/9789004279377_012.

Bäuerle, Rainer. 1983. Pragmatisch-Semantische Aspekte der NP-Interpretation. In M. Faust, R. Harweg, W. Lehfeldt & G. Wienold (eds.), *Allgemeine Sprachwissenschaft, Sprachtypologie und Textlinguistik: Festschrift für Peter Hartmann*, 121–131. Tübingen: Narr.

Beck, Sigrid. 1996. Wh-*constructions and transparent Logical Form*. Universität Tübingen PhD Thesis.

Beck, Sigrid & Hotze Rullmann. 1999. A flexible approach to exhaustivity in questions. *Natural Language Semantics* 7(3). 249–298. https://doi.org/10.1023/A:1008373224343.

Bennett, Jonathan. 2003. *A philosophical guide to conditionals*. Oxford University Press.

Bennett, Michael & Barbara Partee. 1978. *Toward the logic of tense and aspect in English*. Indiana University Linguistics Club.

Bhatt, Rajesh. 1997. Obligation and possession. In Heidi Harley (ed.), *Papers from the upenn/mit roundtable on argument structure and aspect*, vol. 32 (MIT Working Papers in Linguistics), 21–40. http://people.umass.edu/bhatt/papers/bhatt-haveto.pdf.

Bhatt, Rajesh & Roumyana Pancheva. 2006. Conditionals. In *The Blackwell companion to syntax*, vol. 1, 638–687. Blackwell. http://www-rcf.usc.edu/~pancheva/bhatt-pancheva_syncom.pdf.

Blain, Eleanor M. & Rose-Marie Déchaine. 2007. Evidential types: Evidence from Cree dialects. *International Journal of American Linguistics (IJAL)* 73(3). 257–291. https://doi.org/10.1086/521728.

Blümel, Andreas. 2019. Adnominal conditionals in German. *Linguistics Vanguard* 5. https://doi.org/10.1515/lingvan-2019-0004.

Blumson, Ben. 2009. Pictures, perspective and possibility. *Philosophical Studies*. https://doi.org/10.1007/s11098-009-9337-2.

Boeckx, Cedric. 2001. Scope reconstruction and A-movement. *Natural Language and Linguistic Theory* 19(3). 503–548. https://doi.org/10.1023/a:1010646425448.

Bogal-Allbritten, Elizabeth. 2013. Modification of DPs by epistemic modal adverbs. *Proceedings of the 19th Amsterdam Colloquium*. 51–58. http://blogs.umass.edu/eba/files/2009/05/BogalAllbritten-2013-AC.pdf.

Bogal-Allbritten, Elizabeth & Andrew Weir. 2017. Sentential and possibly subsentential modification: The ambiguity of Collins conjunctions. *North East Linguistics Society (NELS)* 47. https://elizabethba.files.wordpress.com/2017/04/eba-aw-nels.pdf.

Bonomi, Andrea & Sandro Zucchi. 2003. A pragmatic framework for truth in fiction. *Dialectica* 57(2). Preprint http://filosofia.dipafilo.unimi.it/~bonomi/Pragmatic.pdf, 103–120. https://doi.org/10.1111/j.1746-8361.2003.tb00259.x.

Butler, Jonny. 2003. A minimalist treatment of modality. *Lingua* 113(10). 967–996. https://doi.org/10.1016/S0024-3841(02)00146-8.

Carnap, Rudolf. 1947. *Meaning and necessity: A study in semantics and modal logic*. Chicago: University of Chicago Press.

Carr, Jennifer. 2014. The *if p, ought p* problem. *Pacific Philosophical Quarterly* 95(4). 555–583. https://doi.org/10.1111/papq.12048.

Cheke, Lucy G. & Nicola S. Clayton. 2010. Mental time travel in animals. *Wiley Interdisciplinary Reviews: Cognitive Science* 1(6). 915–930. https://doi.org/10.1002/wcs.59.

Chierchia, Gennaro. 1995. *Dynamics of meaning: Anaphora, presupposition, and the theory of grammar*. University of Chicago Press.

Chierchia, Gennaro & Sally McConnell-Ginet. 2000. *Meaning and grammar: An introduction to semantics (2nd edition)*. MIT Press.

Childe, V. Gordon. 1936. *Man makes himself*. London: C.A. Watts & Co.

Chomsky, Noam. 1993. A minimalist program for linguistic theory. In Kenneth Hale & Samuel Jay Keyser (eds.), *The view from Building 20: Essays in linguistics in honor of Sylvain Bromberger*, 1–52. Cambridge, MA: MIT Press.

Condoravdi, Cleo & Sven Lauer. 2016. Anankastic conditionals are just conditionals. *Semantics and Pragmatics* 9(8). 1–69. https://doi.org/10.3765/sp.9.8.

Copeland, B. Jack. 2002. The genesis of possible worlds semantics. *Journal of Philosophical Logic* 31(2). 99–137. https://doi.org/10.1023/A:1015273407895.

Copley, Bridget. 2008. The plan's the thing: Deconstructing futurate meanings. *Linguistic Inquiry* 39(2). 261–274. https://doi.org/10.1162/ling.2008.39.2.261.

Cormack, Annabel & Neil Smith. 2002. Modals and negation in English. In Sjef Barbiers, Frits Beukema & Wim van der Wurff (eds.), *Modality and its interaction with the verbal system*, 133–163. Benjamins.

Cresswell, Max. 1973. *Logics and languages*. London: Methuen.

Davey, B. A. & H. A. Priestley. 2002. *Introduction to lattices and order*. 2nd edn. Cambridge University Press. https://doi.org/10.1017/CBO9780511809088.

Dayal, Veneeta. 1996. *Locality in WH quantification: Questions and relative clauses in Hindi* (Studies in Linguistics and Philosophy). Springer Netherlands. https://doi.org/10.1007/978-94-011-4808-5.

Dekker, Paul. 2001. On *if* and *only*. *Semantics and Linguistic Theory (SALT)* 11. 114–133. https://doi.org/10.3765/salt.v11i0.3097.

Del Pinal, Guillermo & Brandon Waldon. 2019. Modals under epistemic tension. *Natural Language Semantics* 27(2). 135–188. https://doi.org/10.1007/s11050-019-09151-w.

DeRose, Keith. 1991. Epistemic possibilities. *The Philosophical Review* 100(4). 581–605. https://doi.org/10.2307/2185175.

Dowty, David R. 1977. Toward a semantic analysis of verb aspect and the english 'imperfective' progressive. *Linguistics and Philosophy* 1(1). 45–77. https://doi.org/10.1007/BF00351936.

Dowty, David R. 1979. *Word meaning and Montague grammar: The semantics of verbs and times in generative semantics and in Montague's PTQ*. Dordrecht: Reidel.

Dowty, David R., Robert Wall & Stanley Peters. 1981. *Introduction to Montague semantics*. Kluwer.

Drubig, Hans Bernhard. 2001. On the syntactic form of epistemic modality. ms, Universität Tübingen. http://www.sfb441.uni-tuebingen.de/b2/papers/DrubigModality.pdf.

Ebert, Christian, Cornelia Ebert & Stefan Hinterwimmer. 2014. A unified analysis of conditionals as topics. *Linguistics and Philosophy* 37(5). 353–408. https://doi.org/10.1007/s10988-014-9158-4.

Edgington, Dorothy. 1995. On conditionals. *Mind* 104(414). 235–329. 10.1093/mind/104.414.235.

Egan, Andy. 2007. Epistemic modals, relativism, and assertion. *Philosophical Studies* 133(1). 1–22. https://doi.org/10.1007/s11098-006-9003-x.

Egan, Andy, John Hawthorne & Brian Weatherson. 2005. Epistemic modals in context. In Gerhard Preyer & Georg Peter (eds.), *Contextualism in philosophy: Knowledge, meaning, and truth*, 131–170. Oxford: Oxford University Press.

Egré, Paul & Mikaël Cozic. 2011. *If*-clauses and probability operators. *Topoi* 30(1). 17–29. https://doi.org/10.1007/s11245-010-9087-y.

Emonds, Joseph. 2011. From primate to human in two easy steps. In Nicholas Saul & Simon J. James (eds.), *The evolution of literature: Legacies of Darwin in European cultures* (Internationale Forschungen zur Allgemeinen und Vergleichenden Literaturwissenschaft 152), 51–72. Brill | Rodopi. https://doi.org/10.1163/9789401206846_006.

Farkas, Donka F. & Kim B. Bruce. 2010. On reacting to assertions and polar questions. *Journal of Semantics* 27(1). 81–118. https://doi.org/10.1093/jos/ffp010.

Filip, Hana. 2012. Lexical aspect. In Robert I. Binnick (ed.), *The Oxford handbook of tense and aspect*. Oxford University Press. https://doi.org/10.1093/oxfordhb/9780195381979.013.0025.

von Fintel, Kai. 1994. *Restrictions on quantifier domains*. Amherst, MA: University of Massachusetts PhD thesis. http://semanticsarchive.net/Archive/jA3N2IwN/fintel-1994-thesis.pdf.

von Fintel, Kai. 1997. Bare plurals, bare conditionals, and *only*. *Journal of Semantics* 14(1). 1–56. https://doi.org/10.1093/jos/14.1.1.

von Fintel, Kai. 1998. Quantifiers and *if*-clauses. *The Philosophical Quarterly* 48(191). 209–214. https://doi.org/10.1111/1467-9213.00095.

von Fintel, Kai. 1999. NPI licensing, Strawson entailment, and context dependency. *Journal of Semantics* 16(2). 97–148. https://doi.org/10.1093/jos/16.2.97.

von Fintel, Kai. 2005. Modality and language. In Donald M. Borchert (ed.), *Encyclopedia of philosophy – second edition*. MacMillan. http://mit.edu/fintel/fintel-2005-modality.pdf.

von Fintel, Kai. 2011. Conditionals. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics: An international handbook of meaning*, vol. 2, 1515–1538. de Gruyter. https://doi.org/10.1515/9783110255072.1515. http://mit.edu/fintel/fintel-2011-hsk-conditionals.pdf.

von Fintel, Kai. 2012. Subjunctive conditionals. In Gillian Russell & Delia Graff Fara (eds.), *The Routledge companion to philosophy of language*, 466–477. New York: Routledge. https://doi.org/1721.1/95784. http://mit.edu/fintel/fintel-2012-subjunctives.pdf.

von Fintel, Kai & Anthony S. Gillies. 2007. An opinionated guide to epistemic modality. In Tamar Szabó Gendler & John Hawthorne (eds.), *Oxford studies in epistemology: Volume 2*, 32–62. Oxford University Press. http://mit.edu/fintel/fintel-gillies-2007-ose2.pdf.

von Fintel, Kai & Anthony S. Gillies. 2008a. CIA leaks. *The Philosophical Review* 117(1). 77–98. https://doi.org/10.1215/00318108-2007-025.

von Fintel, Kai & Anthony S. Gillies. 2008b. *MIGHT* made right. To appear in a volume on epistemic modality, edited by Andy Egan and Brian Weatherson, Oxford University Press. http://mit.edu/fintel/fintel-gillies-2008-mmr.pdf.

von Fintel, Kai & Anthony S. Gillies. 2010. *Must* …stay …strong! *Natural Language Semantics* 18(4). 351–383. https://doi.org/10.1007/s11050-010-9058-2.

von Fintel, Kai & Anthony S. Gillies. 2011. *MIGHT* made right. In Andy Egan & Brian Weatherson (eds.), *Epistemic modality*, 108–130. Oxford: Oxford University Press. http://mit.edu/fintel/fintel-gillies-2011-mmr.pdf.

von Fintel, Kai & Anthony S. Gillies. 2015. Hedging your *ifs* and vice versa. http://mit.edu/fintel/fintel-gillies-2015-hedging.pdf.

von Fintel, Kai & Anthony S. Gillies. 2020. Still going strong. *Natural Language Semantics*. to appear.

von Fintel, Kai & Sabine Iatridou. 2002. If and when *if*-clauses can restrict quantifiers. ms, MIT. http://mit.edu/fintel/fintel-iatridou-2002-ifwhen.pdf.

von Fintel, Kai & Sabine Iatridou. 2003. Epistemic containment. *Linguistic Inquiry* 34(2). 173–198. https://doi.org/10.1162/002438903321663370.

von Fintel, Kai & Sabine Iatridou. 2008. How to say *ought* in Foreign: The composition of weak necessity modals. In Jacqueline Guéron & Jacqueline Lecarme (eds.), *Time and modality* (Studies in Natural Language and Linguistic Theory 75), 115–141. Springer. https://doi.org/10.1007/978-1-4020-8354-9.

von Fintel, Kai & Sabine Iatridou. 2017. A modest proposal for the meaning of imperatives. In Ana Arregui, María Luisa Rivero & Andrés Salanova (eds.), *Modality across syntactic categories*, 288–319. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780198718208.003.0013. http://mit.edu/fintel/fintel-iatridou-2017-modest.pdf.

von Fintel, Kai & Sabine Iatridou. 2019. Since ~~since~~. In Daniel Altshuler & Jessica Rett (eds.), *The semantics of plurals, focus, degrees, and times: Essays in honor of Roger Schwarzschild*, 305–333. Springer. https://doi.org/10.1007/978-3-030-04438-1_15. http://mit.edu/fintel/fintel-iatridou-2019-since.pdf.

Fodor, Janet Dean. 1970. *The linguistic description of opaque contexts*. Cambridge, MA: Massachusetts Institute of Technology PhD thesis. https://doi.org/1721.1/12970.

Fox, Danny. 2000. *Economy and semantic interpretation*. MIT Press.

Fox, Danny. 2007. Free choice and the theory of scalar implicatures. In Uli Sauerland & Penka Stateva (eds.), *Presupposition and implicature in compositional semantics* (Palgrave Studies in Pragmatics, Language and Cognition), 71–120. London: Palgrave Macmillan. https://doi.org/10.1057/9780230210752_4. http://web.mit.edu/linguistics/people/faculty/fox/free_choice.pdf.

Fox, Danny. 2018. Partition by exhaustification: Comments on Dayal 1996. *Sinn und Bedeutung* 22(1). 403–434. https://ojs.ub.uni-konstanz.de/sub/index.php/sub/article/download/98/41/.

van Fraasen, Bas C. 1980. Critical notice of Brian Ellis, *Rational Belief Systems*. *Canadian Journal of Philosophy* 10(3). 497–511.

Frana, Ilaria. 2017. Modality in the nominal domain: The case of adnominal conditionals. In Ana Arregui, María Luisa Rivero & Andrés Salanova (eds.), *Modality across syntactic categories*, 49–69. Oxford University Press.

Francez, Itamar. 2017. Summative existentials. To appear in *Linguistic Inquiry*. https://lucian.uchicago.edu/blogs/ifrancez/files/2017/05/Summex-preprint.pdf.

Frank, Anette. 1996. *Context dependence in modal constructions*. Universität Stuttgart PhD thesis. http://www.cl.uni-heidelberg.de/~frank/papers/header.pdf.

Gajewski, Jon. 2005. *Neg-raising: Polarity and presupposition*. Massachusetts Institute of Technology PhD thesis. https://doi.org/1721.1/33696.

Gajewski, Jon. 2007. Neg-raising and polarity. *Linguistics and Philosophy*. https://doi.org/10.1007/s10988-007-9020-z.

Gamut, L. T. F. 1991. *Logic, language, and meaning*. Chicago University Press.

Garson, James. 2018. Modal logic. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*. https://plato.stanford.edu/archives/fall2018/entries/logic-modal/.

Geurts, Bart. 1998. Presuppositions and anaphors in attitude contexts. *Linguistics and Philosophy* 21(6). 545–601. https://doi.org/10.1023/A:1005481821597.

Geurts, Bart. 2004. On an ambiguity in quantified conditionals. ms, University of Nijmegen. http://ncs.ruhosting.nl/bart/papers/conditionals.pdf.

Giannakidou, Anastasia. 1999. Affective dependencies. *Linguistics and Philosophy* 22(4). 367–421. https://doi.org/10.1023/A:1005492130684.

Gillies, Anthony S. 2010. Iffiness. *Semantics and Pragmatics* 3(4). 1–42. https://doi.org/10.3765/sp.3.4.

Goodman, Nelson. 1947. The problem of counterfactual conditionals. *Journal of Philosophy* 44(5). 113–128. https://doi.org/10.2307/2019988.

Greenberg, Gabriel. 2018. Content and target in pictorial representation. *Ergo, an Open Access Journal of Philosophy* 5(33). https://doi.org/10.3998/ergo.12405314.0005.033.

Groenendijk, Jeroen. 1999. The logic of interrogation: Classical version. *Semantics and Linguistic Theory (SALT)* 9. 109–126. https://doi.org/10.3765/salt.v9i0.2835.

Groenendijk, Jeroen & Martin Stokhof. 1982. Semantics analysis of *wh*-complements. *Linguistics and Philosophy* 5(2). 175–233. https://doi.org/10.1007/BF00351052.

Hacking, Ian. 1967. Possibility. *The Philosophical Review* 76(2). 143–168. https://doi.org/10.2307/2183640. http://www.jstor.org/stable/2183640.

Hackl, Martin. 2001. *Comparative quantifiers*. Massachusetts Institute of Technology PhD thesis. https://doi.org/1721.1/8765.

Hacquard, Valentine. 2006. *Aspects of modality*. Massachusetts Institute of Technology PhD thesis. http://people.umass.edu/hacquard/hacquard_thesis.pdf.

Hacquard, Valentine. 2009a. Modality. ms, prepared for *Semantics: An international handbook of meaning*, edited by Klaus von Heusinger, Claudia Maienbon, and Paul Portner. http://ling.umd.edu/~hacquard/papers/HoS_Modality_Hacquard.pdf.

Hacquard, Valentine. 2009b. On the interaction of aspect and modal auxiliaries. *Linguistics and Philosophy* 32(3). 279–315. https://doi.org/10.1007/s10988-009-9061-6.

Hacquard, Valentine. 2010. On the event relativity of modal auxiliaries. *Natural Language Semantics* 18(1). 79–114. https://doi.org/10.1007/s11050-010-9056-4.

Hacquard, Valentine. 2013. The grammatical category of modality. *Proceedings of the 19th Amsterdam Colloquium*. http://www.illc.uva.nl/AC/AC2013/uploaded_files/inlineitem/03_Hacquard.pdf.

Hacquard, Valentine. 2016. Actuality entailments. to appear in L. Matthewson, C. Meier, H. Rullmann, T. E. Zimmermann (eds.) Companion to Semantics. Wiley. http://ling.umd.edu/~hacquard/papers/Hacquard_Actuality%20Entailments_July%202016.pdf.

Hagstrom, Paul. 2003. What questions mean. *Glot International* 7(7/8). 188–201.

Hanley, Richard. 2004. As good as it gets: Lewis on truth in fiction. *Australasian Journal of Philosophy* 82(1). 112–128. https://doi.org/10.1080/713659790.

Hawthorne, John. 2007. Eavesdroppers and epistemic modals. ms, Rutgers University, to appear in the proceedings of the 2007 Sofia Conference in Mexico, in a supplement to *Noûs*.

Heim, Irene. 1992. Presupposition projection and the semantics of attitude verbs. *Journal of Semantics* 9(3). 183–221. https://doi.org/10.1093/jos/9.3.183.

Heim, Irene. 1994. Interrogative semantics and Karttunen's semantics for *know*. In Rhonna Buchalla & Anita Mittwoch (eds.), *The proceedings of the conference of the israel association for theoretical linguistics (iatl 1)*, 128–144. Hebrew University of Jerusalem. http://semanticsarchive.net/Archive/jUzYjk1O.

Heim, Irene & Angelika Kratzer. 1998. *Semantics in generative grammar*. Oxford: Blackwell.

Herburger, Elena. 2015. *Only if*: If only we understood it. *Sinn und Bedeutung* 19. 284–301. https://www.uni-goettingen.de/de/document/download/139914c001e533bc9ce6f1f01127253e.pdf/herburger_sub19.pdf.

Herburger, Elena. 2016. Conditional perfection: The truth and the whole truth. *Semantics and Linguistic Theory (SALT)* 25. 615–635. https://doi.org/10.3765/salt.v25i0.3079.

von Heusinger, Klaus. 2011. Specificity. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics: An international handbook of meaning*, vol. 2 (Handbücher zur Sprach- und Kommunikationswis-

senschaft (HSK) 33.2), 1025–1058. de Gruyter. https://doi.org/10.1515/9783110255072.1025.
Higginbotham, James. 1986. Linguistic theory and Davidson's program in semantics. In Ernest LePore (ed.), *Truth and interpretation: Perspectives on the philosophy of Donald Davidson*, 29–48. Basil Blackwell.
Higginbotham, James. 2003. Conditionals and compositionality. *Philosophical Perspectives* 17(1). 181–194. https://doi.org/10.1111/j.1520-8583.2003.00008.x.
Hintikka, Jaako. 1969. Semantics for propositional attitudes. In J.W. Davis, D.J. Hockney & W.K. Wilson (eds.), *Philosophical logic*, 21–45. Dordrecht: Reidel.
Hirsch, Aron. 2017. *An inflexible semantics for cross-categorial operators*. Massachusetts Institute of Technology PhD. https://hirsch.mit.edu/papers.
Hockett, Charles F. 1960. Logical considerations in the study of animal communication. In W.E. Lanyon & W.N. Tavolga (eds.), *Animal sounds and communication*, 392–430. Washington, DC: American Institute of Biological Sciences.
Hockett, Charles F. & Stuart A. Altmann. 1968. A note on design features. In Thomas A. Sebeok (ed.), *Animal communication: Techniques of study and results of research*, 61–72. Indiana University Press.
Horn, Laurence R. 1972. *On the semantic properties of the logical operators in English*. UCLA PhD thesis.
Hughes, G.E. & M.J. Cresswell. 1968. *An introduction to modal logic*. London: Methuen.
Hughes, G.E. & M.J. Cresswell. 1996. *A new introduction to modal logic*. London: Routledge.
Huitink, Janneke. 2010. Quantified conditionals and compositionality. *Language and Linguistics Compass* 4(1). 42–53. https://doi.org/10.1111/j.1749-818X.2009.00175.x.
Hulstijn, Joris. 1997. Structured information states: Raising and resolving issues. In Anton Benz & Gerhard Jäger (eds.), *MunDial'97: Munich workshop on formal semantics and pragmatics of dialogue, University of Munich, March 10–12, 1997, Proceedings* (CIS-Berichte), 99–117. Centrum für Informations- und Sprachverarbeitung, University of Munich. http://www.sfs.uni-tuebingen.de/~gjaeger/conferences/volume.pdf.
Iatridou, Sabine, Elena Anagnostopolou & Roumyana Izvorski. 2001. Observations about the form and meaning of the perfect: A life in language. In Michael Kenstowicz (ed.), *Ken Hale: A life in language*. MIT Press.
Iatridou, Sabine & Ivy Sichel. 2011. Negative DPs, A-movement, and scope diminishment. *Linguistic Inquiry* 42(4). 595–629. https://doi.org/10.1162/LING_a_00062.

Ichikawa, Jonathan Jenkins & Matthias Steup. 2018. The analysis of knowledge. In Edward N. Zalta (ed.), *The stanford encyclopedia of philosophy*, Summer 2018. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/sum2018/entries/knowledge-analysis/.

Jäger, Gerhard. 1996. Only updates: On the dynamics of the focus particle *only*. *Amsterdam Colloquium* 10. https://semanticsarchive.net/Archive/DgwMDZjZ/gjAC95.pdf.

Kadmon, Nirit. 1987. *On unique and non-unique reference and asymmetric quantification*. University of Massachusetts at Amherst PhD thesis.

Kadmon, Nirit & Fred Landman. 1993. *ANY*. *Linguistics and Philosophy* 16(4). 353–422. https://doi.org/10.1007/BF00985272.

Kamp, Hans. 1973. Free choice permission. *Proceedings of the Aristotelian Society, New Series* 74. 57–74. http://www.jstor.org/stable/4544849.

Karttunen, Lauri. 1977. Syntax and semantics of questions. *Linguistics and Philosophy* 1(1). 3–44. https://doi.org/10.1007/BF00351935.

Kaufmann, Magdalena. 2012. *Interpreting imperatives* (Studies in Linguistics and Philosophy (SLAP) 88). Dordrecht: Springer. https://doi.org/10.1007/978-94-007-2269-9.

Kaufmann, Stefan. 2017. The Limit Assumption. *Semantics and Pragmatics* 10(18). https://doi.org/10.3765/sp.10.18.

Kaufmann, Stefan & Magdalena Schwager. 2009. A unified analysis of conditional imperatives. *Semantics and Linguistic Theory (SALT)* 19. 239–256. https://doi.org/10.3765/salt.v19i0.2545.

Keshet, Ezra. 2010. Situation economy. *Natural Language Semantics* 18(4). 385–434. https://doi.org/10.1007/s11050-010-9059-1.

Keshet, Ezra. 2011. Split intensionality: A new scope theory of *de re* and *de dicto*. *Linguistics and Philosophy* 33(4). 251–283. https://doi.org/10.1007/s10988-011-9081-x.

Keshet, Ezra & Florian Schwarz. 2014. De re / de dicto. http://florianschwarz.net/wp-content/uploads/papers/De_Re___De_Dicto.pdf.

Khoo, Justin. 2011. Operators or restrictors? A reply to Gillies. *Semantics and Pragmatics* 4(4). 1–25. https://doi.org/10.3765/sp.4.4.

Klein, Wolfgang. 1994. *Time in language*.

Klinedinst, Nathan. 2011. Quantified conditionals and conditional excluded middle. *Journal of Semantics* 28(1). 149–170. https://doi.org/10.1093/jos/ffq015.

Knuuttila, Simo. 2003. Medieval theories of modality. In Edward N. Zalta (ed.), *The stanford encyclopedia of philosophy*. http://plato.stanford.edu/archives/fall2003/entries/modality-medieval/.

Kratzer, Angelika. 1977. What *must* and *can* must and can mean. *Linguistics and Philosophy* 1(3). 337–355. https://doi.org/10.1007/BF00353453.

Kratzer, Angelika. 1978. *Semantik der Rede: Kontexttheorie – Modalwörter – Konditionalsätze*. Königstein/Taunus: Scriptor.

Kratzer, Angelika. 1981. The notional category of modality. In Hans-Jürgen Eikmeyer & Hannes Rieser (eds.), *Words, worlds, and contexts: New approaches in word semantics* (Research in Text Theory 6), 38–74. Berlin: de Gruyter.

Kratzer, Angelika. 1986. Conditionals. *Chicago Linguistic Society (CLS)* 22(2). 1–15. https://semanticsarchive.net/Archive/ThkMjYxN/Conditionals.pdf.

Kratzer, Angelika. 1991. Modality. In Arnim von Stechow & Dieter Wunderlich (eds.), *Semantics: An international handbook of contemporary research*, 639–650. Berlin: de Gruyter. https://udrive.oit.umass.edu/kratzer/kratzer-modality.pdf.

Kratzer, Angelika. 1998. More structural analogies between pronouns and tenses. *Semantics and Linguistic Theory (SALT)* 8. 92–110. https://doi.org/10.3765/salt.v8i0.2808.

Kratzer, Angelika. 2012. *Modals and conditionals: New and revised perspectives*. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780199234684.001.0001.

Kratzer, Angelika. 2015. Chasing hook: Quantified indicative conditionals. To appear in a festschrift for Dorothy Edgington. http://semanticsarchive.net/Archive/WU3ODIwZ/kratzer-chasing-hook.pdf.

Kripke, Saul. 1980. *Naming and necessity*. Oxford: Blackwell.

Lahiri, Utpal. 2002. *Questions and answers in embedded contexts* (Oxford Studies in Theoretical Linguistics 2). Oxford University Press.

Landman, Fred. 1991. *Structures for semantics* (Studies in Linguistics and Philosophy 45). Springer. https://doi.org/10.1007/978-94-011-3212-1.

Landman, Fred. 1992. The progressive. *Natural Language Semantics* 1(1). 1–32. https://doi.org/10.1007/BF02342615.

Larson, Richard. 2002. The grammar of intensionality. In Gerhard Preyer (ed.), *On logical form*. Oxford University Press. https://semanticsarchive.net/Archive/WVhMGRhN/larson.intens.pdf.

Lasersohn, Peter. 1996. Adnominal conditionals. *Semantics and Linguistic Theory (SALT)* 6. 154–166. https://doi.org/10.3765/salt.v6i0.2759.

Lassiter, Daniel. 2016. *Must*, knowledge, (in)directness. *Natural Language Semantics* 24. 117–163. https://doi.org/10.1007/s11050-016-9121-8.

Lassiter, Daniel. 2017. *Graded modality: Qualitative and quantitative perspectives* (Oxford Scholarship Online). Oxford University Press. https://doi.org/10.1093/oso/9780198701347.001.0001.

Lauer, Sven & Prerna Nadathur. 2016. Quantified indicative conditionals and the relative reading of *most*. *Sinn und Bedeutung* 21. https://sites.google.com/site/sinnundbedeutung21/proceedings-preprints/LauerNadathur-most.pdf.

Leahy, Brian P. & Susan E. Carey. 2020. The acquisition of modal concepts. *Trends in Cognitive Sciences* 24(1). 65–78. https://doi.org/10.1016/j.tics.2019.11.004.

Lechner, Winfried. 2007. Interpretive effects of head movement. http://ling.auf.net/lingBuzz/000178.

Leslie, Sarah-Jane. 2009. *If*, *unless*, and quantification. In Robert J. Stainton & Christopher Viger (eds.), *Compositionality, context and semantic values: Essays in honour of Ernie Lepore*, 3–30. Springer. https://doi.org/10.1007/978-1-4020-8310-5_1.

Lewis, Clarence Irving & Cooper Harold Langford. 1932. *Symbolic logic*. New York: Century.

Lewis, David. 1970. General semantics. *Synthese* 22(1-2). 18–67. https://doi.org/10.1007/BF00413598.

Lewis, David. 1973. *Counterfactuals*. Oxford: Blackwell.

Lewis, David. 1974. Semantic analyses for dyadic deontic logic. In Sören Stenlund (ed.), *Logical theory and semantic analysis: Essays dedicated to Stig Kanger on his fiftieth birthday*, 1–14. Dordrecht: Reidel.

Lewis, David. 1975. Adverbs of quantification. In Edward Keenan (ed.), *Formal semantics of natural language*, 3–15. Cambridge University Press.

Lewis, David. 1978. Truth in fiction. *American Philosophical Quarterly* 15(1). 37–46. http://www.jstor.org/stable/20009693.

Lewis, David. 1982. Logic for equivocators. *Noûs* 16(3). 431–441. https://doi.org/10.2307/2216219.

Lewis, David. 1986. *On the plurality of worlds*. Oxford: Blackwell.

Ludlow, Peter & Stephen Neale. 1991. Indefinite descriptions: In defense of Russell. *Linguistics and Philosophy* 14(2). 171–202. https://doi.org/10.1007/bf00627402.

MacFarlane, John. 2005. Logical constants. In Edward N. Zalta (ed.), *The stanford encyclopedia of philosophy*. http://plato.stanford.edu/archives/win2005/entries/logical-constants/.

MacFarlane, John. 2006. Epistemic modals are assessment-sensitive. ms, University of California, Berkeley, forthcoming in an OUP volume on epistemic modals, edited by Brian Weatherson and Andy Egan. http://sophos.berkeley.edu/macfarlane/epistmod.pdf.

Maier, Emar & Sofia Bimpikou. 2019. Shifting perspectives in pictorial narratives. *Proceedings of Sinn und Bedeutung* 23(2). 91–106. https://doi.org/10.18148/sub/2019.v23i2.600.

May, Robert. 1977. *The grammar of quantification*. Massachusetts Institute of Technology PhD thesis. https://doi.org/1721.1/16287.

McCloskey, Jim. 2016. Interpretation and the typology of head movement: A re-assessment. Handout for a presentation at the Workshop on the Status of Head Movement in Linguistic Theory, Stanford University, September 16th and 17th 2016. https://drive.google.com/file/d/0BzSn1AcNdRX2VmpWZHJwRXRaZEk/view.

McCready, Eric & Norry Ogata. 2007. Evidentiality, modality and probability. *Linguistics and Philosophy* 30(2). 147–206. https://doi.org/10.1007/s10988-007-9017-7.

Menzel, Christopher. 2016. Possible worlds. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Spring 2016. http://plato.stanford.edu/entries/possible-worlds/.

Meyer, Marie-Christine. 2013. *Ignorance and grammar*. Massachusetts Institute of Technology PhD thesis. https://doi.org/1721.1/84420.

Montague, Richard. 1973. The proper treatment of quantification in ordinary English. In Jaako Hintikka, Julius Moravcsik & Patrick Suppes (eds.), *Approaches to natural language*, 221–242. Dordrecht: Reidel. http://www.blackwellpublishing.com/content/BPL_Images/Content_store/Sample_chapter/9780631215417/Portner.pdf.

Nauze, Fabrice. 2008. *Modality in typological perspective*. Universiteit van Amsterdam PhD thesis. http://www.illc.uva.nl/Publications/Dissertations/DS-2008-08.text.pdf.

Nute, Donald. 1984. Conditional logic. In Dov Gabbay & Franz Guenthner (eds.), *Handbook of philosophical logic. volume ii*, 387–439. Dordrecht: Reidel.

Ogihara, Toshiyuki. 1989. *Temporal reference in English and Japanese*. Austin, TX: University of Texas PhD Thesis.

Ogihara, Toshiyuki. 1995. The semantics of tense in embedded clauses. *Linguistic Inquiry* 26(4). 663–679. http://www.jstor.org/stable/4178918.

Ogihara, Toshiyuki. 1996. *Tense, attitudes, and scope*. Springer. https://doi.org/10.1007/978-94-015-8609-2.

Papafragou, Anna. 2006. Epistemic modality and truth conditions. *Lingua* 116(10). 1688–1702. https://doi.org/10.1016/j.lingua.2005.05.009.

Partee, Barbara H. 1973. Some structural analogies between tenses and pronouns in English. *The Journal of Philosophy* 70(18). 601–609. https://doi.org/10.2307/2025024.

Partee, Barbara H. & Herman L.W. Hendriks. 1997. Montague grammar. In Johan van Benthem & Alice ter Meulen (eds.), *Handbook of logic and language*, 5–91. Elsevier.

Percus, Orin. 2000. Constraints on some other variables in syntax. *Natural Language Semantics* 8(3). 173–229. https://doi.org/10.1023/A:1011298526791.

Portner, Paul. 1997. The semantics of mood, complementation, and conversational force. *Natural Language Semantics* 5(2). 167–212. https://doi.org/10.1023/A:1008280630142.

Portner, Paul. 1998. The progressive in modal semantics. *Language* 74(4). 760–787. https://doi.org/10.2307/417002.

Portner, Paul. 2007. Imperatives and modals. *Natural Language Semantics* 15(4). 351–383. https://doi.org/10.1007/s11050-007-9022-y.

Portner, Paul. 2009. *Modality*. Oxford University Press.

Portner, Paul. 2017. Mood. To appear as a OUP book.

Prakken, Henry & Marek Sergot. 1996. Contrary-to-duty obligations. *Studia Logica* 57(1). 91–115. https://doi.org/10.1007/BF00370671.

Prior, Arthur N. 1958. Escapism: The logical basis of ethics. In A. I. Melden (ed.), *Essays in moral philosophy*, 135–146. Seattle: University of Washington Press.

Proudfoot, Diane. 2006. Possible worlds semantics and fiction. *Journal of Philosophical Logic* 35(1). 9–40. https://doi.org/10.1007/s10992-005-9005-8.

Quine, Willard Van Orman. 1960. *Word and object*. Cambridge, MA: MIT Press.

Redshaw, Jonathan. 2014. Does metarepresentation make human mental time travel unique? *Wiley Interdisciplinary Reviews: Cognitive Science* 5(5). 519–531. https://doi.org/10.1002/wcs.1308.

Redshaw, Jonathan & Thomas Suddendorf. 2020. Temporal junctures in the mind. *Trends in Cognitive Sciences* 24(1). 52–64. https://doi.org/10.1016/j.tics.2019.10.009.

Reich, Ingo. 2009. What asymmetric coordination in German tells us about the syntax and semantics of conditionals. *Natural Language Semantics* 17(3). 219–244. https://doi.org/10.1007/s11050-009-9041-y.

Reichenbach, Hans. 1947. *Elements of symbolic logic*.

Rett, Jessica. 2019. A typology of semantic entities. To appear in Daniel Altshuler (ed.) *Linguistics meets Philosophy*, Cambridge. https://linguistics.ucla.edu/people/Rett/Rett%202019%20typology%20semantic%20entities.pdf.

Richards, Norvin. 1998. The principle of minimal compliance. *Linguistic Inquiry* 29(4). 599–629. https://doi.org/10.1162/002438998553897.

Roberts, Craige. 2012. Information structure: Towards an integrated formal theory of pragmatics. *Semantics and Pragmatics* 5(6). Reissue of a 1998 amendation of the original 1996 manuscript, 1–69. https://doi.org/10.3765/sp.5.6.

Romoli, Jacopo & Yasutada Sudo. 2009. *De re/de dicto* ambiguity and presupposition projection. *Proceedings of Sinn und Bedeutung* 13. 426–438. http://semanticsarchive.net/Archive/DhhOTI2Z/sub13proc.pdf#page=447.

van Rooij, Robert. 2006. Free choice counterfactual donkeys. *Journal of Semantics* 23(4). 383–402. https://doi.org/10.1093/jos/ffl004.

Ross, Jeff. 1997. *The semantics of media* (Studies in Linguistics and Philosophy (SLAP) 64). Dordrecht: Kluwer.

Rothschild, Daniel. 2011. A note on conditionals and restrictors. Paper written for a conference in honor of Dorothy Edgington at the Institute of Philosophy, London in May 2011. http://semanticsarchive.net/Archive/WY1M2MxM/conditionals.pdf.

Rubinstein, Aynat. 2012. *Roots of modality*. University of Massachusetts at Amherst PhD thesis. http://semanticsarchive.net/Archive/zg2ZWFhN/Rubinstein2012Diss.pdf.

Rubinstein, Aynat. 2014. On necessity and comparison. *Pacific Philosophical Quarterly* 95(4). 512–554. https://doi.org/10.1111/papq.12047.

Rubinstein, Aynat. 2017. Straddling the line between attitude verbs and necessity modals. In Ana Arregui, María Luisa Rivero & Andrés Salanova (eds.), *Modality across syntactic categories*, 109–131. Oxford University Press. https://doi.org/10.1093/acprof:oso/9780198718208.003.0007.

Rullmann, Hotze & Sigrid Beck. 1998. Presupposition projection and the interpretation of *which*-questions. *Semantics and Linguistic Theory (SALT)* 8. 215–232. https://doi.org/10.3765/salt.v8i0.2811.

Russell, Bertrand. 1940. *An inquiry into meaning and truth*. London: George Allen & Unwin.

Sauerland, Uli & Paul Elbourne. 2002. Total reconstruction, PF movement, and derivational order. *Linguistic Inquiry* 33(2). 283–319. https://doi.org/10.1162/002438902317406722.

Schlenker, Philippe. 2004. Conditionals as definite descriptions (A referential analysis). *Research on Language and Computation* 2(3). 417–462. https://doi.org/10.1007/s11168-004-0908-2.

Schulz, Katrin. 2005. A pragmatic solution for the paradox of free choice permission. *Synthese* 147(2). 343–377. https://doi.org/10.1007/s11229-005-1353-y.

Schwager, Magdalena. 2009. Speaking of qualities. *Semantics and Linguistic Theory (SALT)* 19. 395–412. https://doi.org/10.3765/salt.v19i0.2534.

Schwarz, Florian. 2012. Situation pronouns in determiner phrases. *Natural Language Semantics* 20(4). 431–475. https://doi.org/10.1007/s11050-012-9086-1.

Sen, Amartya K. 1970. *Collective choice and social welfare*. San Francisco, CA: Holden-Day.

Sharvit, Yael. 2002. Emedded questions and *de dicto* readings. *Natural Language Semantics* 10(2). 97–123. https://doi.org/10.1023/a:1016573907314.

Sharvit, Yael & Elena Guerzoni. 2003. Reconstruction and its problems. *Amsterdam Colloquium* 14. 205–210.

Speas, Peggy. 2008. On the syntax and semantics of evidentials. *Language and Linguistics Compass* 2(5). 940–965. https://doi.org/10.1111/j.1749-818X.2008.00069.x.

Spencer, Mary. 1971. Why the “s” in “intension”? *Mind* 80(317). 114–115. https://doi.org/10.1093/mind/LXXX.317.114.

Stalnaker, Robert. 1968. A theory of conditionals. In Nicholas Rescher (ed.), *Studies in logical theory* (American Philosophical Quarterly Monograph Series 2), 98–112. Oxford: Blackwell.

Stalnaker, Robert. 1978. Assertion. In Peter Cole (ed.), *Syntax and semantics*, vol. 9, 315–332. New York: Academic Press.

Stalnaker, Robert. 1984. *Inquiry*. MIT Press.

Stalnaker, Robert. 1999. *Context and content*. Oxford: Oxford University Press.

Stanley, Jason & Zoltán Gendler Szabó. 2000. On quantifier domain restriction. *Mind and Language* 15(2/3). 219–261. https://doi.org/10.1111/1468-0017.00130.

Starr, William. 2019. Counterfactuals. In Edward N. Zalta (ed.), *The Stanford encyclopedia of philosophy*, Fall 2019. Metaphysics Research Lab, Stanford University. https://plato.stanford.edu/archives/fall2019/entries/counterfactuals/.

Starr, William. 2020. A preference semantics for imperatives. *Semantics and Pragmatics* 13(6). https://doi.org/10.3765/sp.13.6.

von Stechow, Arnim. 1991. Intensionale Semantik: Eingeführt anhand der Temporalität. http://www.sfs.uni-tuebingen.de/~astechow/Aufsaetze/Int.SemTemp91.pdf.

von Stechow, Arnim. 1996. Against LF pied-piping. *Natural Language Semantics* 4(1). 57–110. https://doi.org/10.1007/BF00263537.

von Stechow, Arnim & Sigrid Beck. 2015. Events, times and worlds: An LF architecture. In Christian Fortmann, Anja Lübbe & Irene Rapp (eds.), *Situationsargumente im Nominalbereich* (Linguistische Arbeiten 562), 13–46. De Gruyter. https://doi.org/10.1515/9783110432893-002.

Steiner, George. 1975. *After Babel: Aspects of language and translation*. Oxford University Press.

Stephenson, Tamina. 2007a. Judge dependence, epistemic modals, and predicates of personal taste. *Linguistics and Philosophy* 30(4). 487–525. https://doi.org/10.1007/s10988-008-9023-4.

Stephenson, Tamina. 2007b. *Towards a theory of subjective meaning*. Massachusetts Institute of Technology PhD thesis. http://semanticsarchive.net/Archive/2QxMjk0O/Stephenson-2007-thesis.pdf.

Strawson, P. F. 1950. On referring. *Mind* 59(235). 320–344. https://doi.org/10.1093/mind/lix.235.320.

Swanson, Eric. 2008. Modality in language. *Philosophy Compass* 3(6). 1193–1207. https://doi.org/10.1111/j.1747-9991.2008.00177.x.

Swanson, Eric. 2010. On scope relations between quantifiers and epistemic modals. *Journal of Semantics* 27(4). 529–540. https://doi.org/10.1093/jos/ffq010.

Swanson, Eric. 2011. Propositional attitudes. In Claudia Maienborn, Klaus von Heusinger & Paul Portner (eds.), *Semantics: An international handbook of meaning*, vol. 2, 1538–1561. de Gruyter. https://doi.org/10.1515/9783110255072.1538.

Szabó, Zoltán Gendler. 2010. Specific, yet opaque. In Maria Aloni, Harald Bastiaanse, Tikitu de Jager & Katrin Schulz (eds.), *Logic, language and meaning: 17th Amsterdam Colloquium, Amsterdam, The Netherlands, December 16-18, 2009, revised selected papers* (Lecture Notes in Computer Science 6042), 32–41. Springer. https://doi.org/10.1007/978-3-642-14287-1_4. https://campuspress.yale.edu/zoltanszabo/files/2015/10/Specific-yet-Opaque-uhcqrh.pdf.

Tancredi, Chris. 2008. Multiple models. Unpublished report for The Center for Advanced Research on Logic and Sensibility.

Taylor, Barry. 1977. Tense and continuity. *Linguistics and Philosophy* 1. 199–220. https://doi.org/10.1007/BF00351103.

Teller, Paul. 1972. Epistemic possibility. *Philosophia* 2(4). 302–320. https://doi.org/10.1007/BF02381591.

Thomas, Guillaume. 2014. Nominal tense and temporal implicatures: Evidence from Mbyá. *Natural Language Semantics* 22(4). 357–412. https://doi.org/10.1007/s11050-014-9108-2.

Varzi, Achille. 1997. Inconsistency without contradiction. *Notre Dame Journal of Formal Logic* 38(4). 621–638. https://doi.org/10.1305/ndjfl/1039540773.

Viebahn, Emanuel & Barbara Vetter. 2016. How many meanings for *may*?: The case for modal polysemy. *Philosophers' Imprint* 16(10). 1–26. https://doi.org/2027/spo.3521354.0016.010.

White, Aaron Steven. 2021. On believing and hoping whether. Accepted with minor revision for publication in *Semantics and Pragmatics*. https://ling.auf.net/lingbuzz/005665/current.pdf.

Willett, Thomas. 1988. A cross-linguistic survey of the grammaticalization of evidentiality. *Studies in Language* 12(1). 51–97.

Williamson, Timothy. 2000. *Knowledge and its limits*. Oxford: Oxford University Press.

Wurmbrand, Susi. 1999. Modal verbs must be raising verbs. *West Coast Conference on Formal Linguistics (WCCFL)* 18. 599–612. http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.35.7442&rep=rep1&type=pdf.

Yalcin, Seth. 2010. Probability operators. *Philosophy Compass* 5(11). 916–937. https://doi.org/10.1111/j.1747-9991.2010.00360.x.

Yalcin, Seth. 2015. Epistemic modality *de re*. *Ergo*: An open access journal of philosophy 2(19). https://doi.org/10.3998/ergo.12405314.0002.019.

Zach, Richard. 2019. *Boxes and diamonds: An open introduction to modal logic*. https://bd.openlogicproject.org/bd-screen.pdf.

Zimmermann, Thomas Ede. 2000. Free choice disjunction and epistemic possibility. *Natural Language Semantics* 8(4). 255–290. https://doi.org/10.1023/A:1011255819284.

Zimmermann, Thomas Ede & Wolfgang Sternefeld. 2013. *Introduction to semantics: An essential guide to the composition of meaning*. de Gruyter Mouton.

Zucchi, Sandro. 2001. Tense in fiction: Reference, anaphora and aspect. In Carlo Cecchetto, Gennaro Chierchia & Maria Teresa Guasti (eds.), *Semantic interfaces: Reference, anaphora and aspect*, 320–355. CSLI Publications. http://tinyurl.com/5ulwxwg.

Zucchi, Sandro. 2005. The present mode. In Gregory N. Carlson & Francis Jeffry Pelletier (eds.), *Reference and quantification: The Partee effect*. CSLI.

Zvolenszky, Zsófia. 2002. Is a possible-worlds semantics of modality possible? A problem for Kratzer's semantics. *Semantics and Linguistic Theory (SALT)* 12. 339–358. https://doi.org/10.3765/salt.v12i0.2866.