

A square of necessities: x-marking weak and strong necessity modals*

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Abstract

Many languages express the modal concept known as weak necessity by adding a conventional marking to their strong necessity modals (von Stechow and Iatridou 2008). This extra marking has been dubbed x-marking in von Stechow and Iatridou (2020) who have also shown that it is the same marking that often appears on so-called subjunctive/counterfactual conditionals as well as on attitude verbs expressing unattainable desires. We discuss Portuguese weak and strong necessity modals (*dever* and *ter que*) and claim that both can be x-marked, although no weakening in their modal force is attested. We propose a parametrized version of x-marking formalized in terms of set difference which can account for the observed crosslinguistic variation brought to light by the Portuguese data.

Keywords: *weak necessity; strong necessity; Portuguese; modal verbs; subjunctive conditionals; counterfactuality*

1 Introduction

English modal verbs *ought* and *should* express a modal concept known as WEAK NECESSITY. The core intuition is that the modal force they express is weaker than the (strong) necessity expressed by *must* and *have to* but stronger than the (mere) possibility expressed by auxiliaries *may* and *can*:

- (1) In order to get to the airport, ...
- a. You have to take a cab.
 - b. You should take a cab.
 - c. You can take a cab.

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Roughly put, (1a) conveys that taking a cab is the only way to get to the airport, (1b) conveys that taking a cab is the best (not necessarily the only) way to get to the airport, and (1c) only conveys that taking a cab is a possible (not necessarily the only or the best) way to get to the airport. The example illustrates the teleological (goal oriented) reading of the modal verbs, but the intuition is stable across different modal flavors:

- (2) Concerning the COVID-19 protocol for this building ...
 - a. Adults have to wear a mask.
 - b. Children ought to wear a mask.
- (3) Children ought to wear a mask, but they don't have to.

In this deontic setting, wearing a mask is mandatory for adults, and advisable for children.

- (4) Where is John?
 - a. He must be in his office.
 - b. He ought to be in his office.
 - c. He may be in his office.

Here we entered the domain of epistemic modality. *Grosso modo*, the proposition that John is in his office is presented as an inevitable conclusion by (4a), as a likely conclusion by (4b), and as a mere possibility, something not to be discarded, by (4c). In other words, given the available evidence, the *ought* sentence express an epistemic bias by the speaker towards the truth of John being in his office, a bias which is boosted by replacing *ought* by *must* (and which vanishes with the use of *may*):

- (5) John ought to be in his office. In fact, he must be.

While English expresses weak necessity by dedicated modal verbs *ought* and *should*, many other languages express a similar notion by a morphological marking on their strong necessity modals. This was one of the core findings in von Fintel and Iatridou (2008) seminal paper on the crosslinguistic encoding of weak necessity, followed up more recently by von Fintel and Iatridou (2020). It can be nicely illustrated by comparing languages as different from each other as Hungarian, Greek, and Spanish:

- (6) Péter-nek el **kell-ene** mosogat-ni-a az edény-ek-et, de senki nem
 Péter-dat part **must-nA** wash-inf-3sg the dish-pl-acc but noone not
 követeli meg tőle
 require-3sg.subj-3.obj part 3.sg.abl
 'Péter ought to do the dishes, but he is not obliged to.'
[Hungarian, von Fintel and Iatridou (2020:18)]
- (7) **Tha eprepe** na plinis ta piata ala dhen ise ipexreomenos na to kanis
FUT must-past NA wash the dishes but NEG are obliged NA it do
 'You ought to do the dishes but you are not obliged to do it.'
[Greek, von Fintel and Iatridou (2008:120)]
- (8) **Deberia** limpiar los platos, pero no estoy obligado
must+cond clean the dishes but not am obliged
 'I ought to do the dishes but I am not obliged.'

Thus we have a crosslinguistic contrast between English tripartite modal system comprising separate lexical roots expressing possibility, weak and strong necessity, and bipartite systems of several languages, comprising possibility and strong necessity modal roots, to which weak necessity can be added via morphological marking on strong necessity verbal roots.

Although the nature of this morphological marking may differ from language to language, another major point made by von Fintel and Iatridou is that the same marking often appears on the consequent of so-called counterfactual conditionals, as in (9)-(11), and also on bouletic verbs expressing unattainable desires, as in (12)-(14):

- (9) Ha János tudná a választ, Mari is **tudná** a választ.
 if János know-nA the answer-acc Mari too **know-nA** the answer-acc
 ‘If János knew the answer, Mari would know the answer.’
 [*Hungarian*, von Fintel and Iatridou (2020:10)]
- (10) An efevge simera **tha eftane** tin ali evdhomadha
 if left today **FUT arrive/PAST/IMP** the other week
 ‘If he left today, he would arrive next week’
 [*Greek*, von Fintel and Iatridou (2008:121)]
- (11) Si fuera más alto **sería** un jugador de baloncesto.
 If be.3.sg.past.subj more tall **be.3.sg.cond** a player of basketball
 ‘If s/he was taller, s/he would be a basketball player.’
 [*Spanish*, von Fintel and Iatridou (2020:12)]
- (12) **Szeretném** ha Marcsi tudná a választ.
like-nA-1sg if Marcsi know-3sg-nA the answer-acc
 ‘I wish Marcsi knew the answer.’
 [*Hungarian*, von Fintel and Iatridou (2020:11)]
- (13) **Tha ithela** na imun psiloteri.
FUT want+Past NA was taller
 ‘I wish I was taller’
 [*Greek*, von Fintel and Iatridou (2008:133)]
- (14) **Querría** que fuera más alto de lo que es.
Want.3.sg.cond that be.3.sg.past.subj more tall than it that be.3.sg
 ‘I wish s/he was taller than s/he is.’
 [*Spanish*, von Fintel and Iatridou (2020:11)]

von Fintel and Iatridou (2020) dubbed this extra marking which can show up on strong necessity modals, conditionals and desire verbs *x-marking* and have set a new research agenda aimed at understanding the semantics of x-marking and envisaging a possible common core underlying the combination of X and the roots to which they attach.

In this paper, I would like to broaden the empirical landscape of x-marking and the expression of weak necessity by bringing up data from Portuguese which I believe can shed light on theoretical issues connected to this agenda. On the one hand Portuguese is like English (and different from closely related languages like Spanish) in displaying a tripartite modal system with a dedicated verbal root (*dever*) expressing weak necessity

which coexist with both a possibility modal root (*poder*) and a strong necessity one (*ter que*). On the other hand, like many of the languages discussed by von Fintel and Iatridou, Portuguese provides what looks like prototypical instances of x-marking on its modal system, that is, a morphological marking (past imperfect, in this case) which also shows up on conditionals and desire reports and which do not seem to carry their usual meaning (temporal, in this case). However, the crucial point, and the one which will be the focus of this paper, is that x-marking can show up not only on the strong necessity modal *ter que* but also on the weak necessity modal *dever*. Moreover, neither does x-marking on the strong necessity modal *ter que* yields an interpretation akin to weak necessity, nor does x-marking on the (already) weak necessity modal *dever* yields an even weaker necessity.

In trying to make sense of the data (which will be presented in detail in the first sections of the paper) we will be contrasting the meaning and use of Portuguese weak necessity modal *dever* (in its plain and x-marked versions) with both English *ought* and other languages x-marked strong necessity modals. Besides broadening the empirical landscape of the morphosemantics of weak/strong necessity, my goal in engaging in this exercise of crosslinguistic, contrastive semantics is two fold: (a) to help elucidate some controversies around certain uses of *ought* in English, epistemic *ought* in particular, and its apparently discrepant behavior when compared with epistemic *may* and *must* (cf. Copley (2006), Yalcin (2016), and Hawthorne (2021), *inter alia*); (b) to provide some new perspective on a unified view on x-marking across different grammatical constructions along the lines of von Fintel and Iatridou’s (2020) prolegomena to a theory of x-marking and their prospective remarks on associating x-marking and a general notion of suspension of default assumptions.

The paper is organized as follows: in section 2, I introduce Portuguese modal verb *dever* and show that it carries the characteristic semantic and pragmatic marks of weak necessity. In section 3, we bring English *ought* to the scene, highlighting that when we translate English examples into Portuguese, we immediately notice a morphological split: some occurrences of *ought* translate into non-x-marked *dever* while others translate into x-marked *dever*. We conclude that there is more than weak necessity in the meaning of some occurrences of *ought*. In this section we focus on epistemic uses of these modals and propose that x-marking is closely related to Stalnaker’s (1975) and von Fintel and Iatridou’s (2020) idea that languages may have conventional devices which indicate suspension of some default assumption. In section 4, we extend the discussion to non-epistemic uses of Portuguese x-marked modals and show how they fit the ‘suspension of a default’ insight. In section 5, we follow the lead of von Fintel and Iatridou (2020) and speculate on a possible common denominator for x-marking across different constructions, in particular for the x-marking on strong necessity modals which result in a weakened modal force in the modal system of several languages and the x-marking attested in Portuguese, which does not. We will see that Portuguese x-marking on both strong and weak necessity modals are in fact semantically very close to the x-marking found on conditionals and desire ascriptions, and we propose a formal parametrized implementation which accounts for variation concerning x-marked modal operators. Section 6 offers a brief summary.

2 Portuguese *dever*

We start by introducing Portuguese modal verb *dever*, which we claim is a typical weak necessity modal operator, and its companions *poder* and *ter que*, which express possibility and strong necessity, respectively.¹ We begin with epistemic readings: having analyzed all the evidence relating to a man’s body found in a dark alley, a criminal investigator could announce his findings about the case in the following ways:²

- (15) a. Este homem **tem que** ter sido assassinado.
 This man SN.PRES have been murdered
 “This man must have been murdered.”
 b. Este homem **deve** ter sido assassinado.
 This man WN.PRES have been murdered
 “This man ought to have been murdered.”
 c. Este homem **pode** ter sido assassinado.
 This man POS.PRES have been murdered
 “This man may have been murdered.”

Intuitions are very clear. (15a) leaves no room for an alternative conclusion: given all the evidence, it cannot be the case that the man died from an accident, a heart attack, etc. (15b) implies that the most likely cause of death was murder, but does not dismiss the possibility of alternatives, and (15c) presents murdering as a mere possibility. Thus, while (16) below sounds contradictory, both (17) and (18) sound perfectly fine and consistent:³

- (16) **ter que p & poder $\neg p$**
 #Este homem **tem que** ter sido assassinado, mas ele **pode** não ter sido.
 This man SN.PRES have been murdered, but he POS.PRES not have been
 (17) **dever p & poder $\neg p$**
 Este homem **deve** ter sido assassinado, mas ele **pode** não ter sido.
 This man WN.PRES have been murdered, but he POS.PRES not have been
 (18) **poder p & poder $\neg p$**
 Este homem **pode** ter sido assassinado, mas ele **pode** não ter sido.
 This man POS.PRES have been murdered, but he POS.PRES not have been

Along with these intuitions come judgments about meaning relations among (15a)-(15c), suggesting that (15a) asymmetrically entails (15b) and (15c), and that (15b) asymmetrically entails (15c):

- (19) **ter que p & \neg dever p**
 #Este homem **tem que** ter sido assassinado, mas ele não **deve** ter sido.
 This man SN.PRES have been murdered, but he not WN.PRES have been

¹The forms *poder*, *dever*, and *ter que* are infinitival forms, also used as citation forms for Portuguese verbs. For a comprehensive reference grammar of Portuguese, see Raposo et al. (2013).

²The following abbreviations will be used in the glosses: WN: weak necessity modal; SN: strong necessity modal; POS: possibility modal; PRES: present; PST: past; IMP: imperfective; SUBJ: subjunctive.

³We use # to mark examples which are grammatically correct, but semantically and/or pragmatically anomalous.

- (20) **dever p & \neg ter que p**
 Este homem **deve** ter sido assassinado, mas ele não **tem que** ter sido.
 This man WN.PRES have been murdered, but he not SN.PRES have been
- (21) **dever p & \neg poder p**
 #Este homem **deve** ter sido assassinado, mas ele não **pode** ter sido.
 This man WN.PRES have been murdered, but he not POS.PRES have been
- (22) **poder p & \neg dever p**
 Este homem **pode** ter sido assassinado, mas não **deve** ter sido.
 This man POS.PRES have been murdered, but not WN.PRES have been

Moreover, while possibility *poder* can be consistently conjoined with a minimal pair containing a negated prejacent, neither *dever* nor *ter que* can, the resulting sentences being blatantly contradictory:

- (23) **poder p & poder $\neg p$**
 Este homem **pode** ter sido assassinado, mas ele **pode** não ter sido.
 This man POS.PRES have been murdered, but he POS.PRES not have been.
- (24) **dever p & dever $\neg p$**
 #Este homem **deve** ter sido assassinado, mas ele **deve** não ter sido.
 This man WN.PRES have been murdered, but he WN.PRES not have been
- (25) **ter que p & ter que $\neg p$**
 #Este homem **tem que** ter sido assassinado, mas ele **tem que** não ter sido.
 This man SN.PRES have been murdered, but he SN.PRES not have been

As for non-epistemic readings, we illustrate the relevant facts with a teleological, goal-oriented reading. Suppose Mary is downtown and needs to go to the airport to catch a flight that departs in a few hours. Since she is not familiar with the local transportation system, she asks some local person, who offers her some help. (26a)-(26c) illustrate three possible relevant answers that she may get:

- (26) a. Você **tem que** pegar um taxi.
 You SN.PRES take a cab
 b. Você **deve** pegar um taxi.
 You WN.PRES take a cab
 c. Você **pode** pegar um taxi.
 You POS.PRES take a cab.

Here too intuitions are very clear: (26a) conveys that taking a cab is the only way to get to her destination. (26b) sounds like a piece of advice: although there are other means to get to the airport, taking a cab is the best option. And (26c) only says that taking a cab is one possible way (among others) to get there.

Judgments parallel to the ones we have seen for epistemic readings in (16)-(25) are also very clear. Assuming that taking a cab and taking a bus are mutually exclusive alternatives, we have:

- (27) For mutually exclusive alternatives p and q :
- a. **ter que p & poder q**
 #Você **tem que** pegar um taxi, mas você **pode** pegar um ônibus.
 You SN.PRES take a cab, but you POS.PRES take a bus
 - b. **dever p & poder q**
 Você **deve** pegar um taxi, mas você **pode** pegar um ônibus.
 You WN.PRES take a cab, but you POS.PRES take a bus
 - c. **poder p & poder q**
 Você **pode** pegar um taxi, mas você **pode** pegar um ônibus.
 You POS.PRES take a cab, but you POS.PRES take a bus
- (28) For mutually exclusive alternatives p and q :
- a. **ter que p & \neg dever p**
 #Você **tem que** pegar um taxi, mas você não **deve** pegar.
 You SN.PRES take a cab, but you not WN.PRES take
 - b. **dever p & \neg ter que p**
 Você **deve** pegar um taxi, mas você não **tem que** pegar.
 You WN.PRES take a cab, but you not SN.PRES take
 - c. **dever p & \neg poder p**
 #Você **deve** pegar um taxi, mas você não **pode** pegar.
 You WN.PRES take a cab, but you not POS.PRES take
 - d. **poder p & \neg dever p**
 Você **pode** pegar um taxi, mas você não **deve** pegar.
 You POS.PRES take a cab, but you not WN.PRES take
- (29) For any proposition p and its negation $\neg p$:
- a. **poder p & poder $\neg p$**
 Você **pode** pegar um taxi, mas você **pode** não pegar
 You POS.PRES take a cab, but you POS.PRES not take
 - b. **dever p & dever $\neg p$**
 #Você **deve** pegar um taxi, mas você **deve** não pegar
 You WN.PRES take a cab, but you WN.PRES not take
 - c. **ter que p & ter que $\neg p$**
 #Você **tem que** pegar um taxi, mas você **tem que** não pegar.
 You SN.PRES take a cab, but you SN.PRES not take

These paradigms reveal that *dever* fits the test proposed in Rubinstein (2021) as a working definition for weak necessity modals:

- (30) *Weak Necessity (WN)* [Rubinstein 2021:3156]
 A modal word o is a **WN modal** iff for any proposition q and holding the type of modality constant:
- i. $o(q)$ is entailed by $n(q)$ but not vice versa, for some necessity modal n , and
 - ii. the conjunction $o(q)$ and $o(\neg q)$ is a contradiction as the conclusion of a deliberation.

The emerging picture is the familiar scale of ascending modal force as we move from *poder* to *dever* to *ter que*. Fixing the conversational backgrounds and whatever contextual

parameters which determine the modal flavor, the modal sentences seem to be related by (asymmetric) entailment:

(31) *Ascending scale of modal force:*
 $\text{poder } p < \text{dever } p < \text{ter que } p$

(32) *Entailments:*
 $\text{ter que } p \models \text{dever } p \models \text{poder } p$
 $\text{poder } p \not\models \text{dever } p \not\models \text{ter que } p$

Finally, judgments typically found in the literature about scalar implicatures are easily reproduced with our triplet of modals, with *poder* p conversationally implying $\neg \text{dever } p$ and *dever* p conversationally implying $\neg \text{ter que } p$. Conversational implicatures, as widely assumed, can be reinforced, canceled, and suspended. We illustrate the relevant facts with deontic modality this time:

- (33) *Concerning the COVID-19 protocol of this establishment, ...*
- a. Clientes **devem** usar máscara, mas eles não **têm que**.
 Clients WN.PRES use mask, but they not SN.PRES.
 - b. Clientes **devem** usar máscara. Na verdade, eles **têm que**.
 Clients WN.PRES use mask. In fact, they SN.PRES
 - c. Clientes **devem** usar máscara. Talvez eles **tenham que**.
 clients WN.PRES use mask. Maybe they SN.PRES.SUBJ.

Before concluding this section, it is worth mentioning that the Portuguese modal system differs in crucial respects from the modal systems of closely related romance languages, including Spanish, which has three modal verbs etymologically related to *poder*, *dever* and *ter que*. The core observation is that Spanish *deber* and *tener que*, the morphological counterparts of Portuguese *dever* and *ter que*, both behave as strong necessity modals, as can be seen in data such as (34), taken from von Fintel and Iatridou (2020):

- (34) *Spanish* [von Fintel and Iatridou (2020:18)]
- a. #Tengo que limpiar los platos, pero no estoy obligado
 I.have to clean the dishes but not am obliged
 - b. #Debo limpiar los platos, pero no estoy obligado
 I.must clean the dishes but not am obliged

Transposing the examples into Portuguese produces a contrast, suggesting once again that *dever* is weaker than *ter que*:

- (35) *Portuguese*
- a. #Tenho que limpar os pratos, mas não estou obrigado
 I.have to clean the dishes but not am obliged
 - b. Devo limpar os pratos, mas não estou obrigado
 I.lought clean the dishes but not am obliged

3 Epistemic necessity: English and Portuguese

The data and discussion from the previous section put Portuguese *dever* on a par with English *ought* in terms of modal strength. At first, *ought* is a typical weak necessity modal verb, flexible in terms of modal flavors, weaker than *must/have to* and stronger than *may/can* in terms of modal force. In this section I would like to focus on so-called epistemic *ought* and *should* which can be used to express some (tentative) conclusion or expectation based on available evidence. Epistemic *ought/should* have been shown to be particularly complex, generating recalcitrant data and standing out in their intricacies from their epistemic companions *may/might/must/have to*. As we will discuss later, a comparison with epistemic uses of Portuguese *dever* will suggest some new prospects for the analysis of their modal flavor and force.

We begin with von Stechow and Iatridou’s (2008:126) quick pass on epistemic uses of English *ought*. They provide the following context and example:

- (36) Let’s say you are on your way to Morris’s office, which is down the hall from mine, and ask me whether I think that Morris is in his office. Neither of us knows whether he is, in fact, there.
- (37) It’s 3pm. Given what I know about Morris’s habits, **he ought to be in his office**. Why don’t you go check?

Appropriate paraphrases for cases like this include:

- (38) a. He is likely to be in his office.
 b. He is more likely to be there than not to be there.
 c. It is probable that he is in his office.

But things get more complicated and interesting when we widen our dataset:

- (39) Morris ought to be here by now, but he isn’t.
- (40) The beer should be cold by now, but it isn’t. [Copley 2006]
- (41) They left an hour ago, and there isn’t any traffic. So they should be here by now. But they’re not. [Swanson 2008]

The examples show that the conjunction of *ought/should p* and *not p* isn’t always inconsistent. This is not the case with other epistemic modals:

- (42) a. #The beer must be cold by now, but it isn’t.
 b. #The beer may be cold by now, but it isn’t. [Copley 2006]
- (43) a. #They left an hour ago, and there isn’t any traffic. So they might be here by now. But they’re not.
 b. #They left an hour ago, and there isn’t any traffic. So they probably are here by now. But they’re not. [Yalcin 2016]

As has been frequently remarked in the literature, this is unexpected if *ought/should* merely express weak epistemic necessity of the same kind that *may* and *must* do.

Let us keep this in mind and shift back to Portuguese and its weak necessity modal *dever*. The paradigm we will try to replicate is (44), which sets *ought* apart from both

may and *must* and displays the possibility of consistently conjoining *ought p* and *not p*:

- (44) a. ✓ **ought p & $\neg p$**
 He ought to be here by now, but he isn't.
 b. ✗ **may p & $\neg p$**
 #He may be here by now, but he isn't.
 c. ✗ **must p & $\neg p$**
 #He must be here by now, but he isn't.

When we turn to Portuguese, we notice that the contrast between possibility and strong necessity on one side and weak necessity on the other vanishes if we control for tense marking. First, all present tense versions of the three sentences above sound incoherent and pragmatically inadequate under an epistemic reading:

- (45) a. #Ele **deve** estar aqui agora, mas não está.
 he WN.PRES be here now, but not is
 b. #Ele **pode** estar aqui agora, mas não está.
 he POS.PRES be here now, but not is
 c. #Ele **tem que** estar aqui agora, mas não está.
 he SN.PRES be here now, but not is

However, switching to past tense modals makes all three sentences sound coherent and perfectly fine:

- (46) a. Ele **devia** estar aqui agora, mas não está.
 he WN.PST.IMP be here now, but not is
 b. Ele **podia** estar aqui agora, mas não está.
 he POS.PST.IMP be here now, but not is
 c. Ele **tinha que** estar aqui agora, mas não está.
 he SN.PST.IMP be here now, but not is

Comparing (44) and (45)-(46), we see that epistemic *may* and *must/have to* side with present tense *poder* and *ter que*, while epistemic *ought* sides with past tense *dever*.⁴

However, this isn't always the case. Returning to von Stechow and Iatridou's example from the beginning of this section, we have (47) uttered in a context in which you ask me about Morris' whereabouts and neither of us knows for sure where he is:

- (47) He ought to be in his office.

In this case, present tense *dever* (as well as present tense *poder* and *ter que*) is fine whereas past tense *dever* sounds awkward:

- (48) [I suspect Morris is in his office, but I don't know for sure]
 a. Ele deve estar na sala dele.
 he WN.PRES be in_the office his

⁴The past tense forms *podia*, *devia*, and *tinha que* are traditionally called 'pretérito imperfeito' (past imperfect) forms. Verb forms in the so-called 'conditional tense' (also called 'futuro do pretérito' (future of the past) in some traditional grammars) would also be possible in these examples, with no obvious shift in meaning.

- b. #Ele devia estar na sala dele.
 he WN.PST.IMP be in_the office his

What is behind the present/past tense split in Portuguese and the corresponding flexibility of epistemic *ought* in English? To begin addressing this question it might be useful to consider the presumably related fact that two tokens of epistemic *ought* can occur close to each other in a discourse but with apparently different meanings, as in the following passage from Thomson (2008), quoted in Yalcin (2016:233):

Consider Rasputin. He was hard to kill. First his assassins poisoned him, then they shot him, then they finally drowned him. Let us imagine that we were there. Let us suppose that the assassins fed him pastries dosed with a powerful, fast-acting poison, and then left him alone for a while, telling him they would be back in half an hour. Half an hour later, one of the assassins said to the others, confidently, “He ought to be dead by now.” The others agreed, and they went to look. Rasputin opened his eyes and glared at them. “He ought to be dead by now!” they said, astonished. It might be thought that when they first said the words, they meant that it was then probable that he was dead. Not so when they second said the words. By the time they second said the words, they knew perfectly well that he wasn’t dead. (Thomson 2008:202-3)

... what it calls for is simply that we distinguish: if I say “The car keys ought to be on the hall table,” then I assert different propositions, according as my state of knowledge is different. If (i) I don’t know that the car keys are, or that they aren’t, on the hall table, then if I say “They ought to be on the hall table,” what I mean is that it is probable that they are there. If (ii) I know that they aren’t there, then if I say “They ought to be on the hall table,” what I mean is that it was probable that they would be there. (Thomson 2008:203)

Here too we observe a split when we switch from English to Portuguese: the first occurrence of *ought* is translated into present tense *deve* whereas the second one translates into past tense *devia*:

- (49) *not knowing whether Rasputin is dead*
 a. He ought to be dead.
 b. Ele **deve**/#**devia** estar morto.
- (50) *having found out that Rasputin is alive*
 a. He ought to be dead.
 b. Ele **devia**/#**deve** estar morto.

Thus, Portuguese provides an overt case for the idea voiced by Thomson for two different propositions being expressed by the *ought* sentences in the Rasputin scenario. Moreover, as also proposed by Thomson for English, the difference between the two *oughts* seem to be related to the temporal perspective associated with the modality they express: (49) is based on the interlocutors present epistemic state, while (50) reports on a past epistemic state that no longer matches the speaker’s present state of knowledge.

Contrasts of this sort can be easily replicated with Portuguese possibility and strong necessity modals. Suppose, for instance, that a nurse has given a patient a sedative which takes effect between one and two hours. Having left the patient alone in his room, one hour later she says (51) to herself:

- (51) Ele já **pode** estar dormindo ... Eu vou checar
 he already POS.PRES be sleeping. ... I will check

Then she goes to his room to check and notices that he is still awake. Now she says (52) to herself:

- (52) Ele já **podia** estar dormindo. Vou voltar em quinze minutos.
 he already POS.PST.IMP be sleeping I will return in fifteen minutes

As for strong necessity modals, a minimally different story could be told about a very powerful sedative which is guaranteed to take effect in no longer than one hour. The nurse's words to herself would now be as in (53) and (54), before and after she checks the patient, respectively:

- (53) Ele já **tem que** estar dormindo.
 he already SN.PRES sleeping.
- (54) Ele já **tinha que** estar dormindo. Há algo errado.
 he already SN.PST.IMP be sleeping there is something wrong

Assuming that this flexibility in temporal perspective attributed to English *ought* does not extend to *may* and *must* would then explain why *ought p and not p* sounds consistent whereas *may/must p and not p* does not. That this contrast is related to temporal perspective and not to weak necessity *per se* is evidenced by the Portuguese data, in which possibility, weak necessity and strong necessity modals can all be marked for tense and express either a past or a present modal perspective. Present tense perspective yields inconsistent conjunctions, but past tense perspective does not.

At this point it is natural to assume that from a grammatical point of view there are two *oughts* in English, sharing the same modal core, a root morpheme expressing weak necessity modality (WN), and differing in their temporal perspective, either present or past. Portuguese would then be minimally contrasting, with each morphological complex being spelled out differently:

- (55) First pass on English *ought* vs. Portuguese *dever*
- a. *English*
 ought₁: WN+present tense
 ought₂: WN+past tense
 - b. *Portuguese*
 deve: WN+present tense
 devia: WN+past tense

However, as is so often the case with the interaction between tense and modality, things are more complicated than they appear to be and as we will see now tense marking might not be doing its usual job of temporal location in these examples. The point I would like

to make is that reference to a past epistemic state expressing a previous expectation or a bias towards the truth of its prejacent *p* is neither necessary nor sufficient for the felicity and truth of an utterance of a past tense weak necessity sentence *devia p*. That it is not sufficient is made clear by contrasts such as the following:

- (56) A: Where is Peter?
B: Probably in his office.
A: But today is a holiday!
B: Oh, I didn't know it was. #That's why **ele devia** **estar lá.**
 he WN.PST.IMP be there
[cf. ✓*That's why I expected him to be there.*]
- (57) A: Where is Peter?
B: Probably in his office.
A: I have just checked and he isn't there.
B: Strange ... ✓ **Ele devia** **estar lá.**
 he WN.PST.IMP be there

In both situations, there was a point prior to B's second utterance at which speaker B expected Pedro to be in his office and might even have uttered a modal sentence with present tense *deve*. However, only in (57) is past tense *devia* acceptable. Notice moreover that in both situations at the moment B makes his final remarks he no longer expects the prejacent to be true. If contextual salience of a past epistemic state which favored the truth of the prejacent and which no longer holds were sufficient for the licensing of the past tense necessity modal, there would have been no contrast between (56) and (57).

That a past expectation is not necessary can be shown with the following scenario: a nurse is starting her shift and is about to enter a room to check on one of her patients. Without knowing the patient's conditions, she enters the room. After opening the door, she notices that he is awake and greets him. She then reads his medical records which says that he has taken a powerful sedative one hour before and which normally takes effect in about 50-60 minutes. She then says (58) to him:

- (58) Você **devia** estar dormindo.
you WN.PST.IMP be sleeping

Notice that at no point prior to her utterance she expected the patient to be sleeping. At first, she was not opinionated and then she knew he was not sleeping. In other words, at no point prior to the moment of her utterance she was entitled to expect that he was or would be sleeping. Nevertheless her utterance with past tense *devia* is felicitous and intuitively true.

It might be argued that the major piece of evidence on which the modal claim in (58) was made is the past occurrence of an event (the patient took the sedative an hour before) and that (58) is a case of metaphysical or circumstantial modality expressing that a past event determined a normal future course of events according to which the patient would be sleeping in an hour or so. But this is questionable, since a slight variation on the example can be constructed in which the relevant evidence is tied to the current state of the patient. For instance, the nurse might be looking at a sophisticated monitor next to the patient's bed which is displaying some sort of brain wave typical of sleeping periods.

Knowing that he is awake, she utters (58). In this modified scenario there is no salient past event nor any prior expectation that the patient would be sleeping, and yet (58) is still appropriate and true.

What then is the licensing factor for the use of the past tense in (58)? And what is behind the contrast between (56) and (57)? My answer to these questions is that the past tense is indicating that some salient evidence bearing on the speaker's actual current epistemic state is being intentionally ignored and that the prejacent is being inferred based on this smaller premise set (plus some normality presumption which we will discuss later). This smaller premise set might match some past epistemic state of the speaker towards the prejacent, but, as we have seen, it does not have to. In the scenarios discussed above in connection to (58), the evidence that is being suspended is that the patient is awake. The speaker (the nurse, in those cases) is looking at the patient, talking to him, and it is absolutely clear that he is not sleeping. Were it not for this direct evidence against the prejacent p , the speaker would be entitled to expect or to have an epistemic bias towards the truth of the prejacent p . And back to the contrast between (56) and (57), notice that while (57) is similar to (58) in all relevant respects, (56) is not. In (56) suspension of the relevant evidence (that it is a holiday) did not provide a premise set which entitled the speaker to expect that the prejacent was true. Lack of knowledge about that day being a holiday or a working day removes the bias the speaker had favoring the truth of the prejacent.

We will formalize these ideas by means of a classic Kratzerian framework for modality, according to which modal verbs introduce restricted quantification over possible worlds. The nature of the quantification (the modal force) is determined lexically, and the restriction is obtained by two contextually salient conversational backgrounds: a modal base f and an ordering source g . Technically, f and g are functions from worlds (the world of evaluation) to sets of propositions. When applied to a world w , f and g deliver the sets of propositions f_w and g_w , respectively. The job of the modal base is to pre-select a set of worlds ($\bigcap f_w$) which will then be ranked by the ordering source. For a modal statement to be true, the prejacent p (the propositional argument of the modal verb) has to be true in all/some/Q of the top-ranked worlds ($\text{BEST}_{g_w} \bigcap f_w$). For a (strong) necessity modal verb we have:⁵

- (59) For any modal base f , ordering source g , and worlds u, v, w :
- a. $\llbracket \text{Necessity Modal} \rrbracket^{f,g} = \lambda p. \lambda w. \forall w' \in \text{BEST}_{g_w} \bigcap f_w : w' \in p$
 - b. $\text{BEST}_{g_w} \bigcap f_w = \{w' \mid \neg \exists w'' \in \bigcap f_w : w'' \leq_{g_w} w' \wedge w' \not\leq_{g_w} w''\}$
 - c. $u \leq_{g_w} v$ iff $[\forall p \in g_w : \text{if } v \in p, \text{ then } u \in p]$ ⁶

In the examples we have been discussing in this section, the modal base is epistemic, encoding the speaker's knowledge (or all the evidence available to him) at utterance time in the world of evaluation. As for the ordering source it encodes an ideal of normality or plausibility.⁷

⁵For simplicity, we are making the so-called *limit assumption*, taking for granted that there are top-ranked worlds, that is, worlds in $\bigcap f_w$ which are not outranked by any other world in this set according to g_w .

⁶Technically, \leq_{g_w} is pre-order (a reflexive and transitive binary relation) induced by g_w .

⁷The exact nature of this ordering source has been the subject of some controversies (see, for instance Yalcin (2016), and some of our remarks on section 3.2 below). See also von Stechow and Gillies (2010),

Having set this minimal background, we can formally state our proposal for Portuguese necessity modals *deve/devia* and *tem que/tinha que* as follows:

- (60) For any modal base f , ordering source g , and proposition q :
- a. $\llbracket \text{devia} \rrbracket^{f,g} = \lambda p. \lambda w. \llbracket \text{deve} \rrbracket^{f^{-q},g}(p)(w)$
 - b. $\llbracket \text{tinha que} \rrbracket^{f,g} = \lambda p. \lambda w. \llbracket \text{tem que} \rrbracket^{f^{-q},g}(p)(w)$
 - c. For any world w , $f_w^{-q} = f_w - \{q\}$

When compared to the present tense modals *deve* and *tem que*, the past tense modals *devia* and *tinha que* operate on a smaller modal base. In other words, adding the past imperfect to a necessity modal root signals a modal reasoning based on a smaller premise set obtained by suppression of some salient fact with which the speaker has become acquainted.⁸

We will return shortly to a formal account of the difference in modal force between weak and strong necessity modals, but, as can already be noticed in (60), the past imperfect modals are not semantically weaker or stronger than their respective present tense versions. Although it is true that suppressing a member q of f_w leads to a smaller set (f_w^{-q}), which in turn induces a larger set of worlds which will be passed on to the ordering source ($\bigcap f_w^{-q}$), selection of the best worlds from these different sets delivered by the modal bases need not yield identical or even overlapping sets. As a consequence, there is no entailment relation between our pairs of present (*deve/tem que*) and past (*devia/tinha que*) modals:

- (61) a. *deve p* $\not\models$ *devia p*
 devia p $\not\models$ *deve p*
 b. *tem que p* $\not\models$ *tinha que p*
 tinha que p $\not\models$ *tem que p*

We will return to this issue of entailment relations when we discuss a formal proposal for weak necessity modals.

3.1 Epistemics and counterfactuality

Notice that in all Portuguese examples we have discussed so far, past tense epistemics *devia/tinha que p* were used in contexts in which p was known to be false, which raises the question: does the past tense marking on *devia/tinha que* entail counterfactuality of the prejacent? The answer, I suggest, is negative. Consider, for instance, the following dialogs:

- (62) A: Where is Peter?
 B: In his office, I suppose. Go there and check!
 A: I have just been there and the lights are off.
 B: Strange. ✓ **Ele devia estar lá.** ...
 he WN.PST.IMP be there ...

2021) for arguments against the presence of an ordering source in the semantics of some epistemic strong necessity modals.

⁸This, of course, does not exclude the possibility of real tense marking on the modal, in which case the tense morpheme would carry its primary temporal meaning which also shows up on non-modal verbs.

- (63) B: ... Go there again and knock on the door. I still think he might be there,
although that now seems unlikely to me.
- A: Where is Peter?
- B: In his office, I suppose. Go there and check!
- A: I have just been there and knocked on the door, but noone answered.
- B: Strange. ✓ **Ele devia estar lá.** ...
 he WN.PST.IMP be there ...
- B: ... Go there again and knock louder on the door. I am almost sure he is
there.

As can be judged from B's final remarks after his use of past tense *devia*, Pedro being in his office is still a lively possibility for him. A slight possibility, in (62); a good one, in (63). In these cases too, the use of past tense morphology on the weak necessity modal might be explained as suspension of some relevant evidence for the sake of reasoning: after A has presented some *prima facie* evidence against Pedro being in his office, B signals that putting aside this new piece of information, the remaining evidence supports the conclusion that Pedro is in his office, and that we should not yet give up the possibility that he is indeed there.

Also concerning counterfactuality, it is important not to confuse unembedded uses of past tense modals expressing modality anchored in real world facts or situations from past tense modals embedded under a conditional structure, and expressing modality projected from a counterfactual scenario. Still confining ourselves to epistemic necessity, consider (64) and (65), both uttered in a scenario where I don't know for sure if Pedro is his office, although I expect him to be there. Then I go there to check but what I see is an empty room:

- (64) O Pedro **devia** estar aqui.
the Pedro WN.PST.IMP be here
- (65) ??Se eu não estivesse vendo esta sala vazia, o Pedro **devia** estar aqui.
If I not were seeing this empty room the Pedro WN.PST.IMP be here

(64) is a perfectly fine WN statement and, as we have seen already, it signals that the premise set on which the inference is being drawn excludes some piece of evidence or knowledge which is available to the speaker. (65) is an unsuccessful attempt to paraphrase (64) with a conditional structure based on a counterfactual antecedent expressing the negation of some salient piece of evidence or knowledge.⁹ Notice that this could be easily achieved if we used a propositional attitude verb in the main clause:

⁹This remark is based on von Fintel and Iatridou's (2020) analogous observations for WN in English and Greek. However, as they noticed (see their fn. 23 on page 20), it is not easy to express weak modality in a counterfactual scenario in these languages. English *ought*, accompanied or not by *would* produces ungrammatical results:

- (i) *If you wanted to please your roommate, you would have to/ought/would ought to do the dishes.
[von Fintel and Iatridou 2020:20]

As for Greek (and some other languages discussed by them), WN modals are already morphologically marked and do not allow for a second layer of marking. As we have shown here, Portuguese is more generous with a dedicated WN modal root which can be marked for past tense or not.

- (66) Se eu não estivesse vendo a sala vazia, eu ia achar que o Pedro
 If I not were seeing the empty room I would think that the Pedro
 estava aqui.
 was here

With a modal verb, however, this cannot be done. (65) sounds pretty awkward, contrasting sharply with (64). Epistemic strong necessity modal *ter que* also provides contrasting pairs analogous to (64)-(65):

- (67) O Pedro **tinha que** estar aqui.
 the Pedro SN.PST.IMP be here
- (68) ??Se eu não estivesse vendo essa sala vazia, o Pedro **tinha que** estar aqui.
 If I not were seeing this empty room the Pedro SN.PST.IMP be here

The upshot is that none of the instances of necessity modals marked for past tense which we discussed in this section stand for a conditional structure with an implicit antecedent and a dominating modal. We will return to the relation between Portuguese necessity modals and so-called counterfactual conditionals in the next section.

3.2 Necessity modals, conditionals, and x-marking

The idea of a conventional device to indicate the suspension of some factual assumption or premise while preforming a modal assertion is familiar from Stalnaker's (1975; 2014) work on (so-called) indicative and subjunctive/counterfactual conditionals. According to Stalnaker, *if A, (then) C* always express a proposition which is true in a possible world w if, and only if, the consequent C is true in the world(s) in which the antecedent A is true which is (are) most similar to w . Thus, both (69) and (70), when uttered in the actual world w_0 express that the worlds in which John is in the building which are most similar to w_0 are worlds in which he is in his office:

- (69) If John is in the building, he is in his office.
- (70) If John were in this building, he would be in his office.

The difference between (69) and (70) comes from a pragmatic defeasible presumption according to which the selected worlds at which the truth of the consequent is to be evaluated should belong to the context set of a conversation at the moment the conditional is uttered. The context set is the set of worlds compatible to everything the participants are presupposing (the common ground) at a given moment. Indicative conditionals such as (69) are unmarked and understood as complying to this pragmatic presumption. Its assertion would be suitable for instance in a context in which John's whereabouts is an open issue, and the speaker attempts to eliminate from the context set the possibility that John is the building but not in his office. Subjunctive conditionals such as (70) are marked structures. This extra marking indicates that the pragmatic default is being suspended and that the selected worlds may reach outside the context set. Asserting (70) would be suitable for instance in a context in which the participants are taking for granted that John is not in the building.

If we follow Stalnaker and model the common ground of a conversation as a set cg

of propositions, we may say that subjunctive conditionals carry a conventional marking signaling that some proposition p belonging to this common ground is being temporarily suspended for the evaluation of the consequent. von Fintel and Iatridou (2020) dubbed this extra marking *x-marking*, avoiding associating it with specific grammatical categories. Following von Fintel and Iatridou (2020) and von Fintel (1998), we opt here for a formal implementation of Stalnaker’s idea using a Kratzerian framework for conditionals, according to which *if*-clauses interact with a modal base and an ordering source to restrict a (possibly null) modal quantifier. In the examples we are discussing, the quantifier would be universal (as in the case of necessity modals), the modal base would be the common ground cg , and the ordering source would be totally realistic, ranking worlds based on how similar they are to the actual world (SIM_w):

- (71) *Non-x-marked* (*‘indicative’*) *conditionals*
 $\llbracket \text{if } A, C \rrbracket^w = 1 \text{ iff } \forall w' \in \text{BEST}_{\text{SIM}_w} \bigcap (cg \cup \{A\}) : w' \in C$
- (72) *x-marked* (*‘subjunctive’*) *conditionals*
 $\llbracket \text{if}_X A, C \rrbracket^w = 1 \text{ iff } \forall w' \in \text{BEST}_{\text{SIM}_w} \bigcap (cg^{-p} \cup \{A\}) : w' \in C$
for some p in cg at the moment the conditional is uttered

As for the proposition p which is being suspended in the case of subjunctive conditionals, it may be the antecedent of the conditional, in which case we would have a bona fide counterfactual hypothesis, as in (73) below, but it can also be another proposition, as in cases in which the truth of the antecedent is under discussion and the speaker may be arguing either in support of it, as in (74), or against it, as in (75):

- (73) Unfortunately John isn’t here ...
If he were here, we would be happy.
- (74) If the butler had done it, we would have found just the clues which we in fact found.
[Stalnaker (1975), example adapted from Anderson (1951)]
- (75) The murderer used an ice pick. But if the butler had done it, he wouldn’t have used an ice pick. So, the murderer must have been someone else.
[Stalnaker (1975), example credited to John Watling]

Whereas in (73) the antecedent of the conditional is indeed presupposed to be false, in (74) and (75) it does not seem to be. As pointed out by Stalnaker, presupposing that a proposition is false while arguing for it would be self-defeating, and presupposing that it is false while arguing against it would be begging the question. Thus what (73), (74), and (75) have in common and which seem to motivate the use of an *x*-marked conditional is the suspension of some p belonging to the common ground, i.e, some proposition p that the participants are taking for granted at the moment the conditional is uttered. It is to this derived context, modeled after a premise set which excludes p that the antecedent is added, and the hypothetical reasoning expressed by the conditional proceeds by checking the truth of the consequent. In (73), p would be the antecedent of the conditional. In (74), it would be the proposition that we have found the clues we did. And in (75), p would be the proposition that the murderer used an ice-pick.¹⁰

¹⁰This line of analysis may be extended to cases involving so-called future less vivid conditionals, which

Outside conditionals, Stalnaker (2014) briefly entertained possible instantiations of x-marking (not his terminology though) on English modal verbs. He noticed that (76) can be said to a child in a context in which it is known or presupposed that she did not start a fire:

- (76) You shouldn't have been playing with matches; **you might have started a fire.** [Stalnaker 2014:186]

He also pointed out that tense differences play a role, and that English *may* would not be appropriate in this context. (77) conveys that it is an open issue whether a fire started:

- (77) You shouldn't have been playing with matches; **you may have started a fire.** [Stalnaker 2014:187]

This, of course, is particularly relevant for our purposes in this paper since modal verbs are the empirical domain we have been probing in the previous sections and will continue to explore in the rest of the paper, always taking advantage of the morphologically richer Portuguese data. In this regard it is worth mentioning that the very same morphological marking (past imperfect) which we saw on necessity verbs and which we analyzed as signaling suspension of some factual assumption appears on the inflected verb of the consequent of subjunctive/counterfactual conditionals in Portuguese, inviting for a unified analysis:

- (78) Se Pedro estivesse aqui, ele **estava** feliz.
if Pedro BE.PST.SUBJ here he BE.PST.IMP happy
'If Pedro were here, he would be happy.'
- (79) Se eu ganhasse na loteria, eu **comprava** um carro novo.
if I WIN.PST.SUBJ in_the lottery I BUY.PST.IMP a car new
'If I won the lottery, I would buy a new car.'

Our proposal for Portuguese past imperfect weak and strong necessity modals *devia* and *tinha que* can then be restated as follows:

- (80) *Portuguese x-marked necessity modals:*
a. **devia:** WN_X (weak necessity+x-marking)
b. **tinha que:** SN_X (strong necessity+x-marking)
- (81) For any modal base f , ordering source g , and proposition q :
a. $\llbracket WN_X \rrbracket^{f,g} = \lambda p. \lambda w. \llbracket WN \rrbracket^{f^{-q},g}(p)(w)$
b. $\llbracket SN_X \rrbracket^{f,g} = \lambda p. \lambda w. \llbracket SN \rrbracket^{f^{-q},g}(p)(w)$

are future oriented conditionals with an antecedent describing an eventuality whose occurrence is taken to be unlikely:

- (i) If he took the medicine, he would get better. (Iatridou 2000)

X-marking here might be indicating the suspension of a bias against the truth of the antecedent. It would signal that the modal quantification is reaching outside some privileged zone of the common ground or the epistemic alternatives of the speaker (the most likely worlds). Implementing such an analysis would require a sort of context probabilism (Yalcin 2012) which would assign probabilities to subsets of the worlds selected by the context set, but I do not intend to pursue it here.

- c. For any world w , $f_w^{-q} = f_w - \{q\}$

3.3 A note on epistemics, x-marking, and likelihood

Before leaving this section, I would like to add a brief remark about the very notion of weak epistemic necessity we have been discussing so far. The idea that English *ought* can be an epistemic modal that is flexible with respect to its temporal perspective, being able to convey likelihood with respect to both present and past states of knowledge is questioned by Yalcin (see also Hawthorne 2021) based on contrasts such as (82) uttered in situations like the following (p. 233): an urn has five marbles, one black, four white. A marble is selected at random, and we observe it is black.

- (82) a. It was probable that the marble selected would be white.
b. *The marble selected ought to be white.

This contrast between *be probable* and *ought* remains if the sentences are uttered before the marble is revealed and the temporal perspective is present/non-past:

- (83) a. It is probable that the marble will be white.
b. *The marble ought to be white.

Yalcin concludes that *ought* does not side with ‘true’ epistemic operators, which express inferences based on evidence/knowledge plus likelihood, and among which we find *probable*, *likely*, and also some instances of modal auxiliaries *must* and *may*. According to Yalcin, the ‘pseudo-epistemic’ reading of *ought* is in fact a combination of evidence/knowledge and a notion of normality (Veltman 1986). In many cases, it may be hard to tease normality and likelihood apart, but scenarios based on games of chance show it clearly: randomly selecting the unique white marble from an urn with five or even a hundred marbles is unlikely, but by no means abnormal.

Comparing English to Portuguese reveals a partial match between *ought* and *dever*:

- (84) [*before observing the result*]
a. Deve ser branca.
WN.PRES be white
b. É provável que seja branca.
it_is likely that it_is white
- (85) [*before or after observing the result*]
a. ??Devia ser branca, mas é preta.
WN.PST.IMP be white but it_is black
b. Era provável que fosse branca. Mas é preta.
it_was likely that it_was white but it_is black

The fact that (84a) sounds adequate and true, meaning what (84b) does, indicates there are ‘true’ epistemic WN modals, and that the unavailability of this reading for English *ought* is an idiosyncratic feature of this particular modal verb. The fact that (85a) sounds awkward and cannot mean what (85b) means indicates that x-marking on true epistemics (evidence/knowledge + likelihood) is not allowed for Portuguese *dever* (just as it wasn’t

for English *ought*). There might be a more robust crosslinguistic generalization behind this fact, although I will leave it as it is since I do not have more data to back up a more conclusive statement.¹¹

4 Non-epistemic necessity

We now turn to non-epistemic readings of necessity modals, showing how the ‘x-marking as suspension of factual assumptions’ proposal extends to other modal domains. We start with an example of teleological modality with a future-oriented prejacents:

- (86) Pedro ainda está tossindo. Ele **devia** tomar o remédio.
 Pedro still is coughing he WN.PST.IMP take the medicine

The modalized sentence expresses that given the actual circumstances, the best thing Pedro can do to get better is to take the medicine. The use of past tense *devia* would sound natural in contexts in which Pedro is reluctant to take the medicine or even in which he has told people that he would not take any medicine. Although the contrast is admittedly subtle, this sense of modal remoteness (it is unlikely that Pedro will take the medicine) vanishes if we substitute the present tense *deve* for the past tense *devia*:

- (87) Ele deve tomar o remédio.
 he WN.PRES take the medicine

The contrast is parallel to what we have seen with respect to so-called future less vivid conditionals. From a semantic-pragmatic perspective, (86) is to (87) as (88) is to (89):

- (88) Se Pedro tomasse o remédio, ele melhorava rapidamente.
 if Pedro TAKE.PST.SUBJ the medicine, he HEAL.PST.IMP quickly
- (89) Se Pedro tomar o remédio, ele melhorará rapidamente.
 if Pedro TAKE.PRES.SUBJ the medicine, he HEAL.FUT quickly

Similar remarks apply to cases with strong necessity modal *ter que*:

- (90) Pedro ainda está tossindo. Ele **tinha que** tomar o remédio.
 Pedro still is coughing he SN.PST.IMP take the medicine
- (91) Pedro ainda está tossindo. Ele **tem que** tomar o remédio.
 Pedro still is coughing he SN.PRES take the medicine

Both (90) and (91) say that taking the medicine is necessary for Pedro to get better. Only (90) conveys that his taking the medicine is a remote possibility.

As for non-future oriented prejacent, these meaning contrasts become easier to detect:

¹¹Note, incidentally, that (85) also shows that epistemic *dever* cannot be associated with a past perspective expressing a past state of knowledge/expectation. This might be related to a more general fact about the way epistemic modals interact with tense, although this remains a controversial issue, with some authors claiming that epistemics are never under the scope of past tense (cf. Cinque 1999, Hacquard 2006, 2010) and others claiming they can be outscoped by past tense (cf. von Stechow and Gillies 2008, Rullmann and Matthewson 2018).

- (92) Para fazer esse exame de sangue agora, você **devia** ter
 In_order_to make this exam of blood now you WN.PST.IMP have
 começado seu jejum doze horas atrás.
 started your fast twelve hours ago
- (93) Para fazer esse exame de sangue agora, você **deve** ter começado
 In_order_to make this exam of blood now you WN.PRES have started
 seu jejum 12 horas atrás.
 your fast twelve hours ago

In (92), the prejacent is understood either as counterfactual or very unlikely, with remarks such as *but you didn't* or *but you didn't, right?* being natural follow-ups. In (93) the prejacent is understood as an open issue, and a natural follow-up would be a neutral question such as *did you?*.

As for examples with a more deontic flavor, consider (94) said by a lawyer to a client who did not appear at his hearing with the judge:

- (94) Você devia ter comparecido à audiência.
 you WN.PST.IMP have appeared at_the hearing

The prejacent is counterfactual and the lawyer's words convey that the client's behavior was not in his best interest. Deontic strong necessity is similar. Consider (95) uttered by a judge to a defendant who did not appear at his hearing, justifying a penalty he is about to announce:

- (95) Você tinha que ter comparecido à audiência.
 you SN.PST.IMP have appeared at_the hearing

In these examples, we have past oriented prejacentes and the most natural scenarios which instantiate the truth of the modal claims involve counterfactual prejacentes which are known to be false. Nevertheless, even in cases like these it is possible to envisage scenarios in which the sentences are adequate and true and yet the prejacent is not taken to be false:

- (96) É meio-dia e João está em casa. Mas ele **tinha que** ter comparecido
 it's noon and John is at home but he SN.PST.IMP have attended
 à audiência com o juiz às 11h30. Acho pouco provável que ele tenha
 the hearing with the judge at 11:30. I think little likely that he has
 ido e já tenha chegado de volta, mas vamos perguntar pra ele.
 gone and already has arrived back, but let's ask to_him
 "It's noon and John is home. But he had to have attended the hearing with the
 judge at 11:30. I think it's unlikely that he has gone and has already arrived
 back, but let's ask him."
- (97) É meio-dia e João está em casa. Mas ele **tinha que** ter comparecido
 it's noon and John is at home but he SN.PST.IMP have attended
 à audiência com o juiz às 11h30. Sendo João um cara responsável, é
 the hearing with the judge at 11:30. Being John a man responsible it's

bem provável que ele tenha ido, que a audiência tenha durado pouco e
quite probable that he had gone that the hearing had last short and
o trânsito de volta para cá estava bom.
that the traffic back to here was good
‘‘It’s noon and John is home. But he had to have attended the hearing with the
judge at 11:30. Being John a responsible guy, it is quite likely that he has gone
there, that the hearing didn’t last long and that the traffic back here was good.’’

The modal claims appear in the middle of a reasoning whose conclusion supports the possibility or even the likelihood of the prejacent. As in the cases of conditionals discussed by Stalnaker, it would be awkward to presuppose that a proposition is false while arguing in favor of its possibility or likelihood. The exact role of x-marking in these contexts is not easy to pin down. I would just like to suggest that suspension of presuppositions might be seen as a way of unbiasing the context, detaching the modal claim from some salient circumstantial evidence (John is at home only half an hour after a scheduled hearing far away downtown) which makes it unlikely (though not impossible) that the prejacent is true. It would highlight a potential conflict between an obligation and what is being observed by the speaker and which may lead someone to believe that the obligation has not been fulfilled. Replacing the x-marked, past tense modals by their unmarked, present tense versions would preserve the respective deontic necessities but would not express any sort of modal remoteness, and the prejacent would be understood as unbiased open issues.

As we remarked in the case of epistemic necessity, it is important to tease apart uses of x-marked WN modals, expressing modality anchored in real world situations and non-x-marked WN modals embedded under a CF structure, and expressing modality projecting from a counterfactual scenario:

- (98) A: Pedro needs to be at the airport in less than an hour.
B: Então ele **devia** pegar um Uber.
then he WN.PST.IMP take an Uber
- (99) A: Pedro doesn’t need to be at the airport soon.
B: OK. Se ele precisasse, ele **devia** pegar um Uber.
ok if he needed he WN.PST.IMP take an Uber

(98)-(99) makes the point for teleological weak necessity. What (98) expresses is that given the actual circumstances, the best alternative (though not the only one) is to take an Uber to the airport. The use of *devia* is justified by the fact that Fred seems to be or might be inclined towards the use of a different means of transportation. Contrastingly, what (99) expresses is a counterfactual reasoning: taking an Uber would be the best alternative if Pedro needed to get to the airport in less than an hour. Similar remarks apply to x-marked instances of SN modal *ter que*:

- (100) A: Pedro needs to be at the airport in less than an hour.
B: Então ele **tinha que** pegar um Uber.
then he SN.PST.IMP take an Uber
- (101) A: Pedro doesn’t need to be at the airport soon.

- B: OK. Se ele precisasse, ele **tinha que** pegar um Uber.
 ok if he needed he SN.PST.IMP take an Uber

According to (100), given the actual circumstances, the only way to get to the airport is to take an Uber. Contrastingly, (101) expresses a counterfactual reasoning: taking an Uber would be the only alternative if Pedro needed to get to the airport in less than an hour.

With all this in mind, we can import into the formal analysis of Portuguese x-marked non-epistemic necessities the same Kratzerian framework we used for epistemic necessities. The only difference is that instead of an epistemic modal base and a normal/stereotypical ordering source, we are now dealing with a circumstantial modal base, which encodes some relevant facts holding in the actual world, and ordering sources encoding ideals of various types (teleological, deontic, bouletic, etc.). X-marking on the modal verb signals suppression of some salient circumstance, just like it did for pieces of knowledge or evidence in the case of epistemic modals.

An additional bonus comes from the fact that our proposal aligns perfectly with von Fintel and Iatridou’s cross-linguistic morpho-semantic analysis of attitude reports expressing what they called ‘unattainable desires’. English has a dedicated verb (*wish*) for this particular type of attitude:

- (102) John wishes Mary were happy.

This sentence expresses John’s desire that Mary be happy, but it also conveys that he believes she is not. Portuguese, as several languages discussed by von Fintel and Iatridou, uses past imperfect morphology on its *want*-type verb on the main clause and past subjunctive morphology on its finite complement clause:

- (103) Pedro **quer** que Maria **esteja** feliz.
 Pedro WANT.PRES that Maria BE.PRES.SUBJ happy
- (104) Pedro **queria** que Maria **estivesse** feliz.
 Pedro WANT.PST.IMP that Maria BE.PST.SUBJ happy

Both sentences express Pedro’s desire that Maria be happy. However, only (104) implies that Pedro believes she is not. (103), at least as a default, express that Maria being happy is consistent with his beliefs about her current state. It should be emphasized that the desire expressed by (104) is not counterfactual nor located in the past. (103) and (104) are about desires that Pedro has in the actual world at the utterance time. Facts like these have led von Fintel and Iatridou to analyze this type of desire reports as another instance of x-marking, a conventional extra marking indicating departure from a default assumption. Further support for this assimilation is the fact that we also find the same morphosyntactic profile of (104) in subjunctive/counterfactual conditionals, with past imperfect tense on the main clause and past subjunctive on the subordinate clause:

- (105) Se Pedro **estivesse** aqui, ele **estava** feliz.
 if Pedro BE.PST.SUBJ here he BE.PST.IMP happy
 ‘If Pedro were here, he would be happy.’

This sameness of form was shown by von Fintel and Iatridou to hold in several unrelated

languages, even when the pieces of morphology are not borrowed from the tense/aspect/mood domain.

As for a formal implementation, their point of departure is a Kratzerian modal analysis for desire verbs such as English *want*, according to which it introduces restricted universal quantification over possible worlds. They assume that, as a default, the domain of quantification is formed by a doxastic modal base (the set $B_{a,w}$ of propositions believed by the attitude holder a in the world of evaluation w) and a bouletic ordering source $D_{a,w}$, which ranks those pre-selected worlds based on the agent's desires/preferences in the world of evaluation.^{12 13}

$$(106) \quad \llbracket a \text{ wants } q \rrbracket = \lambda w. \forall w' \in \text{BEST}_{D_{a,w}} \bigcap B_{a,w} : w' \in q$$

X-marking the desire verb would then indicate the suspension of some belief p of agent a , yielding an enlargement of his doxastic alternatives:

$$(107) \quad \llbracket a \text{ wants}_X q \rrbracket = \lambda w. \forall w' \in \text{BEST}_{D_{a,w}} \bigcap B_{a,w}^{-p} : w' \in q, \text{ for some } p \text{ in } B_{a,w}$$

The emerging idea is that both English (102) and Portuguese (104) instantiate (107) and could be used in a scenario in which Pedro believes (knows) that Mary is not happy.¹⁴

We are then left with a good prospect for a unifying analysis for x-marking encompassing conditionals and desire verbs, as discussed and analysed by von Stechow and Iatridou. And if our proposal about past imperfect marking on Portuguese necessity modals are on the right track, they too fit nicely into the picture, given what we discussed in detail in the last two sections.

4.1 A note on sneezes and x-marking

We finish this section on x-marked non-epistemic necessity modals with an intriguing case of what looks like a prototypical example of circumstantial modality in the literature (see, for instance, Kratzer 1991:p.640):

- (108) a. I must sneeze
b. I have to sneeze

- (109) a. I ought to sneeze
b. I should sneeze

Yalcin (2016) has raised doubts about the availability of a circumstantial reading for English *ought*. His comments on these examples are worth quoting:

¹²For simplicity, we will ignore the possibility of agent a having inconsistent beliefs in a world w .

¹³For an alternative analysis, see Heim (1992).

¹⁴Since desires are often future-oriented, examples of what might be called future less vivid desires can be easily constructed in analogy to future less vivid conditionals:

- (i) Eu queria que Maria chegasse a tempo para o almoço, mas eu acho
I WANT.PST.IMP that Maria ARRIVE.PST.SUBJ in time for the lunch but I consider
isso improvável.
it unlikely.

It is just strange to say these [(109)] in the relevant kind of context. Why? Suppose you sense a sneeze coming on, but you are not convinced it is inevitable. Why don't [(109a)] and [(109b)] seem like natural words of warning, slightly weaker than the warning conveyed by [(108a)] and [(108b)]? (As deontic ought is thought to be weaker than deontic must.) We have the intuition that the flavor of modality is qualitatively different—not just weaker—when we move from the strong necessity modals here to the weak ones. This is surprising. Even if [...] [(109a)] and [(109b)] are marked without some additional setup in this kind of scenario, we naively might have thought that there should be a pure circumstantial reading of ought and should available, such that [(109a)] and [(109b)] can be appropriate when you feel a sneeze approaching, in the way [(108a)] and [(108b)] are. But such a reading seems not to be available. This requires explanation.

[Yalcin (2016:242)]

Indeed, the sentences in (109) do not sound as merely weaker versions of (108). They would sound natural, for instance, if I have inhaled some sort of sneezing powder which normally makes people sneeze, but which surprisingly did not take any effect in my case. A comparative analysis with Portuguese weak (and strong) necessity modals is enlightening:

- (110) Eu tenho que espirrar.
I SN.PRES sneeze
- (111) Eu devo espirrar (a qualquer momento)
I WN.PRES sneeze (at any moment)
- (112) Eu tinha que espirrar (estar espirrando).
I SN.PST.IMP sneeze (be sneezing)
- (113) Eu devia espirrar (estar espirrando)
I WN.PST.IMP sneeze (be sneezing)

(110) with present tense SN modal *ter que* conveys what its English *must/have-to* counterparts in (108) do: in view of the circumstances (the state of my nose), it is inevitable that I sneeze. (111) with present tense WN modal *dever* is the missing piece in Yalcin's English puzzle, a sentence that does sound like a weaker version of (110), conveying the reading Yalcin was looking for in a *ought/should* sentence but couldn't find. When we translate into Portuguese the *ought/should* sentences in (109), what we get is a past tense necessity modal, as in (113). Completing the paradigm, we have (112) with past tense SN *ter que* which, as expected, is just like (113) except for its stronger force.

We now have a much better prospect for circumstantial weak necessity which was masked by English impoverished tensed modal system: all examples seen above – English (108)-(109) and Portuguese (110)-(113) – are cases of circumstantial modality: in classic Kratzerian terms, they express inferences based on a realistic modal base and a normality-based ordering source. In the case of English *must/have to* in (108) and Portuguese present tensed modals in (110) and (111), nothing needs to be added. As for the English *ought/should* examples in (109) and the Portuguese past tensed modals in (112) and (113), they can be analyzed as x-marked circumstantial necessity, signaling that some factual

premise is being suspended and the domain of quantification might include worlds which are epistemically inaccessible. Natural contexts for these utterances would include cases in which the prejacent is taken as false (I should be sneezing, but I am not; I should sneeze at any moment, but I think I won't).

The upshot is that English *ought* and *should* are idiosyncratic in that they cannot express non-x-marked circumstantial necessity the way *have to* and *must* do. One shouldn't go deeper than this since the full paradigm of circumstantial necessity, weak and strong, x-marked or not, are displayed by Portuguese necessity modals *dever* and *ter que*

5 A unifying perspective: the square of necessities

Having made a proposal for Portuguese past imperfect necessity modals, and having shown how they fit into von Fintel and Iatridou's x-marking (re)analysis of so-called subjunctive conditionals and unattainable desire reports, we now turn our attention to the third case discussed by von Fintel and Iatridou as a possible host for x-marking, namely, strong necessity modals. In the languages they discuss (Portuguese not included), x-marking has the semantic effect of softening the modal force, turning strong necessity into weak necessity. We repeat an example from Spanish, a language in which so-called conditional tense is used as the exponent of x-marking as we saw before in conditionals and desire verbs:

- (114) #Debo limpiar los platos, pero no estoy obligado
I_xmust clean the dishes but not am obliged
 [von Fintel and Iatridou (2020:18)]
- (115) Deberia limpiar los platos, pero no estoy obligado
must+cond clean the dishes but not am obliged
 'I ought to do the dishes but I am not obliged.'
 [von Fintel and Iatridou (2020:18)]

According to von Fintel and Iatridou (2020:39), this is the place where "the theory of x-marking has serious trouble to provide a unified analysis". We will review the issues behind these difficulties shortly, but given our previous discussion of x-marking on Portuguese necessity modals, it should be clear from the outset that we have put ourselves in a even harder position. X-marking on strong necessity modals in Portuguese does not yield a weak necessity modal, nor x-marking on an (already) weak necessity modal yields an even weaker modal. There seems to be a clash between our proposal for x-marking on Portuguese modals and von Fintel and Iatridou's proposal associating x-marked necessities with weakening of modal force, and which was based on empirical evidence coming from several languages, as we saw in the introduction of this paper.

In this section, we present von Fintel and Iatridou's (2020) prospects for extending their ideas about x-marking to necessity modals, and tentatively suggest that a slight change of perspective on von Fintel and Iatridou's (2008) proposal about the relation between weak and strong necessity might shed some light on this search for a unifying formal account, which should also include Portuguese x-marked necessity modals.

We start with an outline of von Fintel and Iatridou's proposal for WN modals. According to them, both strong and weak necessity modals express universal, restricted

quantification over possible worlds. As in the standard Kratzerian framework for modal verbs, this quantification is parametrized by contextually supplied conversational backgrounds: modal bases and ordering sources. The difference between strong and weak necessities is that the former employs one ordering source whereas the latter employs two. Since the job of an ordering source is to rank the worlds pre-selected by a modal base and extract the top-ranked, best elements, WN is viewed as selecting the best of the best, with its secondary ordering applying on top of the primary ordering which is part of SN. In formal terms:

- (116) For any modal base f and ordering sources g^1, g^2 :
- a. $\llbracket \text{SN} \rrbracket^{f, g^1} = \lambda p. \lambda w. \forall w' \in \text{BEST}_{g^1_w}(\bigcap f_w) : w' \in p$
 - b. $\llbracket \text{WN} \rrbracket^{f, g^1, g^2} = \lambda p. \lambda w. \forall w' \in \text{BEST}_{g^2_w}(\text{BEST}_{g^1_w}(\bigcap f_w)) : w' \in p$

As can be seen in (116a) and (116b), for any given choice of modal base and ordering sources, the worlds over which a WN modal universally quantifies is a subset of the worlds over which a SN modal quantifies, making WN modals semantically weaker than SN modals.

von Fintel and Iatridou (2008) provide intuitions across different modal flavors hinting at what might be behind the primary/secondary ordering source split:¹⁵

“In the goal-oriented case, the first ordering source is simply the goal proposition designated by an (in order) to-adjunct or an if you want to-anankastic conditional. The second, subsidiary ordering source contains considerations such as how fast, how comfortable, how cheap, ... the means for achieving the goal are.”

“Epistemic ought differs from epistemic must/have to in being sensitive not just to the hard and fast evidence available in a situation but also to a set of propositions that describe what is normally the case”

“And in the deontic case, ought might be sensitive to less coercive sets of rules and principles in addition to the laws and regulations that strong necessity modals would be interpreted with respect to.”

[von Fintel and Iatridou 2008:119]

With this much in mind, it is natural to see the x-marking on a SN modal as signaling the addition of a secondary ordering source, which in turn leads to a modal statement weaker than the original strong necessity. This is in contrast with the x-marking on conditionals and desire ascriptions, as well as on Portuguese necessity modals discussed in the previous sections. As we have seen there, x-marking on these other constructions signaled suspension of some default assumption, which can be modeled as subtraction from the modal base f_w to which a modal quantifier is parametrized. This subtraction leads to domain widening obtained by enlarging the set of worlds delivered by the modal

¹⁵Rubinstein (2012) refines these intuitions, trying to ground the primary/secondary ordering source split on more solid pragmatic notions. In particular, she ties the split to a distinction between negotiable and non-negotiable priorities among conversational participants. See also Rubinstein (2021) for an insightful overview of this and related issues concerning weak necessity as well as for pointers to the relevant literature.

base $(\bigcap f_w)$. As von Fintel and Iatridou (2020) put it, “[a]n additional ordering source results in weakening, while a widened modal base does not.”. And they add: “We admit that we have no idea whether a formal implementation of this picture is in reasonable reach. We leave this as a challenge.” I now would like to offer some prospect for a formal implementation which faces this challenge.

Let us recap briefly: the set of propositions delivered by a modal base f (f_w) induces a set of worlds $(\bigcap f_w)$. Suppressing a member p of f_w leads to a smaller set (f_w^{-p}) , which in turn induces a larger set of worlds $(\bigcap f_w^{-p})$. According to von Fintel and Iatridou, in subjunctive/counterfactual conditionals and unattainable desire reports, x-marking signals subtraction from a default f_w , understood as a set of propositions. To these empirical domains, we have added Portuguese weak and strong necessity modals *dever* and *ter que*, respectively. As we saw in previous sections, x-marking these necessity modals would affect a realistic modal base suppressing some salient fact with which the speaker has become acquainted. This can be formally accomplished by eliminating some proposition p from f_w , as we have seen before:

$$(117) \quad f_w^{-p} = f_w - \{p\}$$

In none of these cases, the resulting x-marked modal operator leads to semantic weakening, since selection of the best worlds from the default and the enlarged sets delivered by the modal bases may yield different sets which do not stand in the subset relation.

What about the x-marking on SN modals which does lead to a semantically weaker form of quantification? As reviewed above, von Fintel and Iatridou (2008) (and also Rubinstein (2012)) interpret this type of x-marking as signaling the presence of a secondary ordering source, which acts upon the worlds top-ranked by a primary ordering. We notice that within the standard Kratzerian framework, an ordering source g delivers a set of propositions g_w which induces a pre-order o , a reflexive and transitive binary relation:¹⁶

$$(118) \quad o_{g_w} = \{\langle u, v \rangle \mid \forall p \in g_w : \text{if } v \in p, \text{ then } u \in p\}$$

If we focus on this induced set, call it simply o_w , the addition of a secondary ordering source can be modeled as the elimination of some ordered pairs, resulting in a subset of o_w . Let me illustrate with a toy example related to goal oriented, teleological modality, adding some details along the way:

- (119) In order to get to the airport, ...
- a. You have to take route 1.
 - b. You should take route 1.

Assume there are only four possible worlds w_1 - w_4 compatible with the circumstances holding in the actual/evaluation world w (same locations, same traffic conditions, same means of transportation, ...), so that $\bigcap f_w = \{w_1, w_2, w_3, w_4\}$.

Suppose now that your goal in w is to get to the airport, and that w_1 and w_2 are the only worlds in which you get there. Then, the pre-order o_w will be as follows:

$$(120) \quad o_w = \{\langle w_1, w_1 \rangle, \langle w_1, w_2 \rangle, \langle w_1, w_3 \rangle, \langle w_1, w_4 \rangle, \langle w_2, w_1 \rangle, \langle w_2, w_2 \rangle, \langle w_2, w_3 \rangle, \langle w_2, w_4 \rangle, \langle w_3, w_3 \rangle, \langle w_3, w_4 \rangle, \langle w_4, w_3 \rangle, \langle w_4, w_4 \rangle\}$$

¹⁶See Kratzer (1991:644)

This pre-order ranks w_1 and w_2 on top of w_3 and w_4 , with no distinctions between w_1 and w_2 or between w_3 and w_4 , as shown graphically in (121):

$$(121) \quad \begin{array}{c} w_1, w_2 \\ | \\ w_3, w_4 \end{array}$$

The best worlds, the ones which are not outranked by any world, are w_1 and w_2 . The strong necessity expressed by sentence (119a) will be true just in case you take route 1 in both w_1 and w_2 .

Suppose now that we add a subsidiary goal p : you traveling with comfort. Assume that only in w_1 and w_3 this goal is satisfied. We want a new pre-order o_w^{-p} which encodes this new priority without neglecting the previous one. This can be achieved by inspecting pairs $\langle u, v \rangle$ of equivalent worlds in o_w , eliminating the ones in which v is a world where you travel with comfort, but u isn't:¹⁷

$$(122) \quad o_w^{-p} = \{ \langle w_1, w_1 \rangle, \langle w_1, w_2 \rangle, \langle w_1, w_3 \rangle, \langle w_1, w_4 \rangle, \langle w_2, w_2 \rangle, \langle w_2, w_3 \rangle, \langle w_2, w_4 \rangle, \langle w_3, w_3 \rangle, \langle w_3, w_4 \rangle, \langle w_4, w_4 \rangle \}$$

As can be noticed, o_w^{-p} is a proper subset of o_w . The pairs $\langle w_2, w_1 \rangle$ and $\langle w_4, w_3 \rangle$ have been discarded. o_w^{-p} still ranks w_1 and w_2 above w_3 and w_4 , preserving the information that getting to the airport is still a goal. What makes o_w^{-p} a refinement of o_w is that it also ranks w_1 above w_2 , and w_3 above w_4 , encoding the new, subsidiary goal of traveling with comfort:¹⁸

$$(123) \quad \begin{array}{c} w_1 \\ | \\ w_2 \\ | \\ w_3 \\ | \\ w_4 \end{array}$$

The best world now is w_1 . The weak necessity expressed by sentence (119b) will be true just in case you take route 1 in w_1 , a weaker requirement when compared to the strong necessity expressed by (119a), which required you to take route 1 in both w_1 and w_2 .

In more general terms, we can express the relation between a pre-order o_w and its refinement by a subsidiary priority p , o_w^{-p} as follows:

$$(124) \quad o_w^{-p} = o_w - \{ \langle u, v \rangle \mid u \sim_{o_w} v \wedge v \in p \wedge u \notin p \}^{19}$$

¹⁷Two worlds u, v are equivalent with respect to a pre-order PO ($u \sim_{PO} v$) if, and only if, both $\langle u, v \rangle$ and $\langle v, u \rangle$ are members of PO .

¹⁸The notion of refinement is taken from Veltman (1986:234), which defined them in the context of his Update Semantics and his notion of default. Pre-orders were called *patterns* by Veltman.

¹⁹Being conversational backgrounds, o and o^{-p} themselves are functions from possible worlds to sets (pre-orders), and are related as follows:

- (i) For any possible world w : $o^{-p}(w) = o(w) - \{ \langle u, v \rangle \mid u \sim_{o_w} v \wedge v \in p \wedge u \notin p \}$

That being the case, we may view both types of x-marking as an operation on a modal set (f_w or o_w) which yields a subset thereof. We are then left with the following semantic profile of necessity modals: weak necessity (WN) is universal, restricted modal quantification, just like strong necessity (SN). They are both based on two contextually salient modal parameters, f and o . However, while SN operates directly on these parameters, WN operates on f_w and some subset of o_w :

- (125) For any modal base f and pre-order o :
- a. $\llbracket \text{SN} \rrbracket^{f,o} = \lambda q. \lambda w. \forall w' \in \text{BEST}_{o_w}(\bigcap f_w) : w' \in q$
 - b. $\llbracket \text{WN} \rrbracket^{f,o} = \lambda q. \lambda w. \forall w' \in \text{BEST}_{o_w^{-p}}(\bigcap f_w) : w' \in q$, for some proposition p

The mapping from SN to WN can be viewed as the replacement of the modal parameter o by o^{-p} , for some proposition p . Let us refer to this mapping from SN to WN as X_o -marking and to the result of the mapping (WN) as a X_o -marked SN, which we will represent as SN_{X_o} :

$$(126) \quad \llbracket \text{SN}_{X_o} \rrbracket^{f,o} = \lambda q. \lambda w. \llbracket \text{SN} \rrbracket^{f,o^{-p}}(q)(w), \text{ for some proposition } p$$

Analogously, we can define a mapping from SN affecting the modal base f , replacing f by f^{-p} , for some proposition p . Let us refer to this mapping as X_f -marking and to the result of the mapping as X_f -marked SN, which we will represent as SN_{X_f} :

$$(127) \quad \text{For any modal base } f \text{ and pre-order } o:$$

$$\llbracket \text{SN}_{X_f} \rrbracket^{f,o} = \lambda q. \lambda w. \forall w' \in \text{BEST}_{o_w}(\bigcap f_w^{-p}) : w' \in q$$

$$(128) \quad \text{For any modal base } f \text{ and pre-order } o:$$

$$\llbracket \text{SN}_{X_f} \rrbracket^{f,o} = \lambda q. \lambda w. \llbracket \text{SN} \rrbracket^{f^{-p},o}(q)(w), \text{ for some proposition } p$$

Our proposal is that the x-marking instantiated by Portuguese necessity modals (and also by subjunctive conditionals and unattainable desire reports, as discussed by von Fintel and Iatridou) is X_f -marking.

Finally, we may also combine the two mappings and obtain a new necessity operator $\text{SN}_{X_{f,o}}$ with both original parameters, f and o being replaced by f^{-p_1} and o^{-p_2} , respectively:

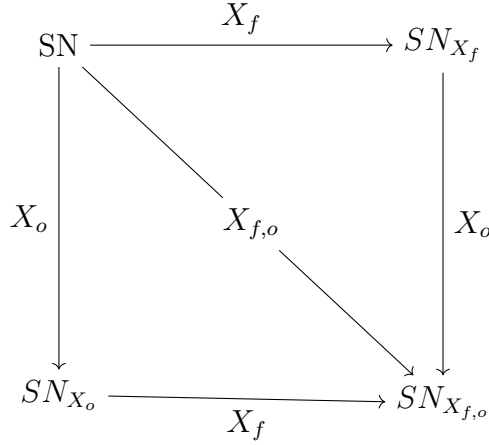
$$(129) \quad \text{For any modal base } f \text{ and pre-order } o:$$

$$\llbracket \text{SN}_{X_{f,o}} \rrbracket^{f,o} = \lambda q. \lambda w. \llbracket \text{SN} \rrbracket^{f^{-p_1},o^{-p_2}}(q)(w), \text{ for propositions } p_1 \text{ and } p_2$$

von Fintel and Iatridou's x-marking can then be viewed as a cover concept encompassing the three mappings SN_{X_f} , SN_{X_o} , and $\text{SN}_{X_{f,o}}$, as defined above, and which are related as follows:

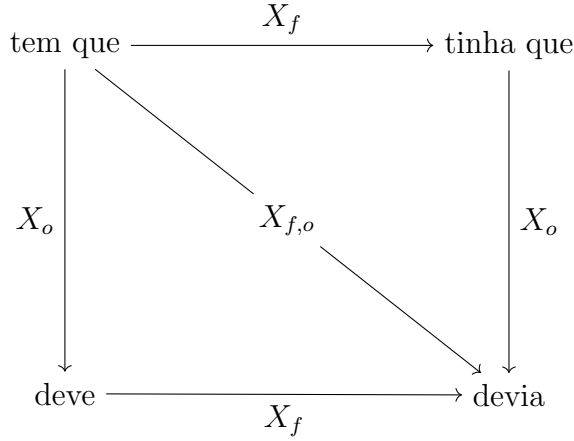
(130) Necessity operators and x-marking

We will stick to our convention of representing a conversational background cb applied to its world argument w as cb_w instead of $cb(w)$.



If our proposals for Portuguese necessity modals are on the right track, they instantiate all three x-markings and occupy the four corners of (130):

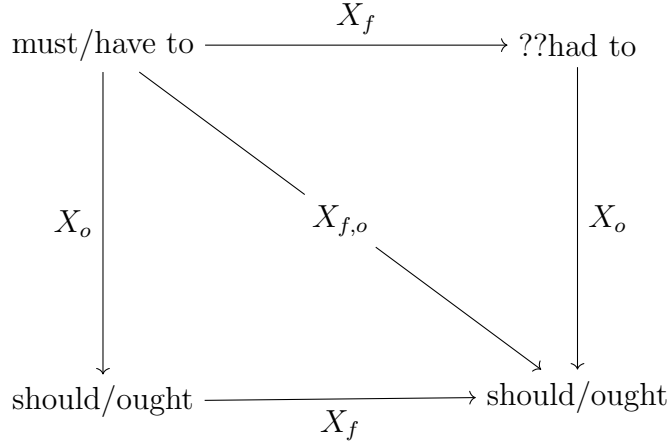
(131) **Portuguese necessity modals and x-marking**



As von Stechow and Iatridou have documented, x-marking can manifest crosslinguistically either lexically or morphologically. In the case of Portuguese necessity modals, X_o is a lexical operation applying to a verb root (*ter que*) and resulting in another verb root (*dever*), there bearing no overt morphophonological relation between them. X_f is a morphological (affixal) operation adding past imperfect morphology to a (weak or strong) necessity verb root. $X_{f,o}$ is just a combination of X_f and X_o . And if our remarks from section 3 about English *ought* are also on the right track, *ought* is ambiguous, expressing weak necessity with or without x-marking. In our formal setting, this means that *ought* can be the lexicalization of both SN_{X_o} and $SN_{X_{f,o}}$:

(132) **English necessity modals and x-marking**²⁰

²⁰We leave a discussion of the status of *had to* as SN_{X_f} for another occasion.



Finally, languages like Greek and Spanish, also discussed by von Fintel and Iatridou obtain SN_{X_o} via morphological marking on their SN modals. It remains to be seen if or how these languages fill the remaining corners of our ‘square of necessities’.

6 Conclusion

In this paper we have discussed instances of Portuguese necessity modals *dever* and *ter que* which carry a morphological marking expressing something akin to what Kai von Fintel and Sabine Iatridou dubbed x-marking. On the one hand, we have assimilated the semantic effect of this marking to the one showing up on so-called counterfactual conditionals and unattainable desire ascriptions, analyzed in von Fintel and Iatridou (2008, 2020) as suspension of some premise or assumption on which a modal claim is based. On the other hand, we have contrasted the x-marking on Portuguese necessity modals with the x-marking on strong necessity modals showing up in many languages which downgrades the modal force from strong to weak necessity, also analyzed by von Fintel and Iatridou. No such weakening is observed in the case of Portuguese x-marked necessity modals. We have also compared Portuguese *dever* to English *ought* with particular attention to some recalcitrant data concerning its epistemic uses, and proposed that *ought* is ambiguous between x-marked and non x-marked weak necessity. This ambiguity is absent in Portuguese, a language in which x-marking on its WN modal produces a different form.

Finally, in searching for a common core underlying all uses of x-marking, we have proposed that it can be viewed as signaling set difference which may apply to a kratzerian modal base (understood as a set of propositions), to a pre-order among possible worlds (understood as a set of ordered pairs), or to both. As a result, a language may have up to four related necessity modals which occupy the vertices of what we called ‘the square of necessities’. Portuguese is such a language.

As we said in the introduction, this paper aimed at shedding light on empirical and theoretical issues connected to the expression of weak and strong necessities by modal verbs. Ideally it will serve to trigger semantic (re)-analyses of morphologically marked necessity modals in other languages, perhaps as x-marking along the lines we have discussed above. Just as an illustrative data point, take Italian, for instance, which has been shown to apply past imperfect morphology to its necessity modal *dovere*. It was

dubbed ‘imperfetto potenziale’ and characterized as ‘mainly related to modal verbs’ and expressing ‘a sort of supposition’ in Bazzanella (1990:443):

- (133) Vincenzo **doveva** essere qui; non capisco cosa gli sia successo.
 Vincenzo dovere-IMP be here; not understand what has to-him happened
 “Vincenzo should have been here; I can’t understand what has happened to
 him.” [Bertinetto (1986:374) *apud* Bazzanella (1990)]

This looks very much like the core data we have presented here. Hopefully this piece of data as well as data and/or analyses coming from other languages and modals will also benefit from our discussion of Portuguese necessity modals.

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