8 Conditionals and Modality

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1. Introduction

It is often considered a characteristic property of human languages that they allow speakers to abstract away from their actual situation by taking into consideration states of affairs at other points in time or in the realm of what are conceivable alternatives. The study of modality in natural languages focuses on expressions that help to encode the latter sort of displacement. Much consideration has been given to modal verbs like "*can*" and "*must*," but modality is by no means confined to such expressions: a variety of lexical elements and syntactic constructions carries modal meaning. Nevertheless, their formal semantic treatment does not in general require an extension of the techniques employed in the investigation of modal verbs. Conditional clauses constitute a notable exception to this: on the currently predominant view in formal semantics, they are a particular type of modal expression which requires an extension of the basic framework. The exact nature of this extension is still under debate. We will therefore focus on the study of modals and conditionals, with the understanding that this provides the necessary backdrop to investigate the semantics of modality in natural language in general.

Modal expressions are generally considered sentential operators that relate the proposition expressed by their argument (the *prejacent*) to a body of information specifiying some set of background assumptions. The particular type of relation is called the *modal force* of the linguistic expression, and the basic distinction is between compatibility (possibility) and consequence (necessity). Furthermore, different *modal flavors* are distinguished according to the criterion used to select the relevant body of information: *epistemic modality* relates the prejacent to a body of information available to the relevant agent(s), *deontic modality* relates the prejacent to a body of rules, *bouletic modality* to what is desirable, *teleological modality* to what the relevant goals are, and *dynamic* or *dispositional modality* relates it to the inherent dispositions of an agent or system (see Portner, 2009, for a recent overview and further categories).

The majority of formal semantic work on modal expressions is couched in possible worlds semantics and builds on insights from modal logic. In section 2 we introduce the relevant logical tools and their applications in the study of natural language expressions. In section 3 we discuss different proposals of modal analyses of conditional sentences. In section 4 we conclude with a discussion of some specific implications for aspects of the analysis of conditional clauses, as well as references to current issues in modality that are not discussed in this chapter for reasons of space but which pose further questions about the standard approach introduced here.

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2. Formal Frameworks

The formal semantic analysis of modality and conditionals owes much to the philosophical tradition of *modal logic* (Hintikka, 1961; Kripke, 1963). We begin this section with a brief discussion of that tradition. The framework that is most commonly encountered in linguistic practice includes some variations and extensions of this basic apparatus, which we introduce in the second part of this section.

2.1 *Modal logic*

We focus on three aspects of the philosophical tradition of modal logic that were highly influential in the development of the linguistic approach discussed below: the standard *language* of modal logic, its model-theoretic interpretation in terms of *possible worlds*, and the study of *systems* of modal logic. In focusing on these elements, we largely ignore other topics that philosophers would consider just as important, such as proof theory and metaphysics. In addition, we restrict our attention to the *propositional* language of modal logic. All of these decisions are made for reasons of space rather than (lack of) thematic germaneness.

2.1.1 Language: syntax and semantics We assume basic familiarity with the standard language of propositional logic, called $\mathcal{L}^0_{\mathcal{A}}$ below. The language $\mathcal{L}_{\mathcal{A}}$ of *propositional modal logic* is obtained from $\mathcal{L}^0_{\mathcal{A}}$ by introducing the two unary sentential operators $\Box$ and $\Diamond$.[1]

Definition 1 (Languages). *Let $\mathcal{A} = \{p, q, r, \ldots\}$ be a set of propositional variables.*

- *The language $\mathcal{L}^0_{\mathcal{A}}$ of* propositional logic *is the smallest set containing $\mathcal{A}$ and such that for all $\phi, \psi \in \mathcal{L}^0_{\mathcal{A}}$, $(\neg\phi), (\phi \wedge \psi), (\phi \vee \psi), (\phi \rightarrow \psi) \in \mathcal{L}^0_{\mathcal{A}}$.*
- *The language $\mathcal{L}_{\mathcal{A}}$ of* propositional modal logic *is the smallest set containing $\mathcal{L}^0_{\mathcal{A}}$ and such that for all $\phi \in \mathcal{L}_{\mathcal{A}}$, $(\Box\varphi), (\Diamond\varphi) \in \mathcal{L}_{\mathcal{A}}$.*

In the following, we omit parentheses when there is no danger of confusion.

Intuitively, the modal operators $\Box$ and $\Diamond$ are intended to form statements of *necessity* and *possibility*, respectively: thus, for instance, the intended truth conditions of the expression in (1a) can be paraphrased as in (1b):

(1) a. $\Box p \rightarrow p$

b. If p is necessary then p is true.

Different interpretations of the modal operators correspond to different modal flavors. For instance, on a *deontic* reading, $\Box p$ states that the truth of p is *required*; on an *epistemic* interpretation, it states that p is *known* by the agent whose epistemic state is being modeled. Similarly, under these two interpretations, $\Diamond p$ states that p is *allowed* and *considered possible*, respectively.

Semantically, the sentences of $\mathcal{L}_{\mathcal{A}}$ are interpreted relative to *possible worlds*. We sidestep all metaphysical issues surrounding this notion. The model is defined as follows.

Definition 2 (Model for $\mathcal{L}_{\mathcal{A}}$). *A model for the interpretation of $\mathcal{L}_{\mathcal{A}}$ is a triple $M = \langle W, R, V\rangle$, where W is a non empty set of possible worlds, $R \subseteq W \times W$ is an accessibility relation, and $V : \mathcal{L}_{\mathcal{A}} \mapsto \wp(W)$ is a* *function from sentences of $\mathcal{L}_\mathcal{A}$ to sets of possible worlds, subject to the following constraints:*

$$V(\neg\phi) = W - V(\phi)$$
$$V(\phi \wedge \psi) = V(\phi) \cap V(\psi)$$
$$V(\phi \vee \psi) = V(\phi) \cup V(\psi)$$
$$V(\phi \to \psi) = W - (V(\phi) - V(\psi))$$
$$V(\Box\phi) = \{w \mid \textit{for all } v,\ \textit{if } wRv \textit{ then } v \in V(\phi)\}$$
$$V(\Diamond\phi) = \{w \mid \textit{for some } v, wRv \textit{ and } v \in V(\phi)\}$$

Following standard practice in natural-language semantics, we refer to sets of possible worlds as *propositions*. In the following, we sometimes use propositional letters as stand ins for the propositions they denote, writing "p" instead of "$V(p)$." No confusion should arise from this slight abuse of notation. We say that a proposition $X \subseteq W$ is *true* at a world w just in case $w \in X$. We also call a sentence ϕ true at $w \in W$ just in case $w \in V(\phi)$.

The last two clauses in Definition 2 show that the interpretation of modalized sentences depends crucially on the *accessibility relation* R. The choice of R determines not only which modal flavor is being represented—epistemic, deontic, and so forth—but more specifically a concrete *instance* of this modal flavor. Thus for example, the requirements of two separate bodies of rules (say, state law and federal law) would be represented by two distinct deontic accessibility relations. If one wants to consider the interactions between different notions of possibility and necessity, additional modal operators have to be added to the language and a suitable model has to have accessibility relations for each of them. In the following, we will sometimes use expressions of $\mathcal{L}_\mathcal{A}$ to represent the meanings of natural language expressions. In such cases, we distinguish modal operators for different modal flavors by using superscripts indicating which accessibility relations they depend on.

2.1.2 Systems: axioms and frame properties Modal logic is concerned with inferences about necessity and possibility, and with semantic relations between statements about these notions. This is a natural point of contact with the linguistic study of modal language.

Syntactically, a classical (nonmodal) propositional logic can be axiomatized in a variety of ways. A commonly encountered characterization, attributed to Jan Łukasiewicz, consists of all substitution instances of the three schemata in (Ł) together with the rule of *modus ponens* (MP).

(Ł) a. $\phi \to (\psi \to \phi)$

b. $(\phi \to (\psi \to \theta)) \to ((\phi \to \psi) \to (\phi \to \theta))$

c. $(\neg\phi \to \neg\psi) \to ((\neg\phi \to \psi) \to \phi)$

(MP) $\dfrac{\phi, \phi \to \psi}{\psi}$

Modal logics generally validate all tautologies of propositional logic, thus their characterization will include an axiom system like that in (Ł) and (MP). In addition, all modal logics contain (K) as an axiom schema and *Necessitation* (N) as an inference rule:[2]

(K) $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$

(N) $\dfrac{\phi}{\Box\phi}$

The most basic system of propositional modal logic is called **K** (named after Saul Kripke) and can be characterized by the axioms in (Ł) and (K) together with the inference rules (MP) and (N). Stronger systems are obtained by adding further axioms. In applications of modal logic to a particular domain, the choice of axioms becomes an empirical question that ultimately concerns the semantic properties of statements about the modality in question. For instance, the statement we gave as an example in (1) above, $\Box p \rightarrow p$, is an instance of a schema commonly called (T):

(T) $\Box\phi \rightarrow \phi$

It is generally assumed that (T) is a reasonable axiom if the modality in question is *knowledge*: recall that under this interpretation, "$\Box\phi$" is the statement that ϕ is *known*, thus the axiom captures the *factivity* of knowledge, i.e., the fact that a knowledge attribution cannot be true unless the content of the attributed knowledge is true. In contrast, under a deontic interpretation, where "$\Box\phi$" is the statement that ϕ is *required* according to some body of norms, the axiom is not plausible if (and since) one wants to be able to model situations in which not everything that is required is true.

These considerations make it clear that the axiomatic study of modalities is of central importance in modeling the semantic properties of linguistic expressions of modality. Other common axioms include the following:

(D) $\Box\phi \rightarrow \Diamond\phi$

(4) $\Box\phi \rightarrow \Box\Box\phi$

(5) $\Diamond\phi \rightarrow \Box\Diamond\phi$

Axiom (D), the condition that what is necessary is also possible, receives its name from *deontic* logic, but it has also been assumed in theorizing about epistemic states and the common ground (Stalnaker, 2002). Generally it amounts to a *consistency* requirement, stating that necessity statements are not vacuously true. Axioms (4) and (5) are well known from the epistemic domain, where they require that epistemic agents are in a sense "aware of" the information they have. In this domain, the assumptions imposed by (4) and (5) are usually referred to as *positive and negative introspection*, respectively. See S. Kaufmann, Condoravdi, and Harizanov (2006) for more discussion of the underlying intuitions.

Some systems that have applications in linguistic theory are listed, along with the associated axioms, in Table 8.1. The axioms are not independent of each other: for instance, (T) implies (D), and (T) and (5) jointly imply (4). For proofs and more details on these and other systems, the reader is referred to Hughes and Cresswell (1996) or the other references below.

An alternative means of characterizing different modalities is in terms of properties of the accessibility relation. Given a model $M = \langle W, R, V \rangle$ (see Definition 2 above), the *frame* of M is the

Table 8.1. Common systems of modal logic.

Name	*Axioms*
T	**K** + (T)
S4	**K** + (T) + (4)
S5	**K** + (T) + (5)
KD45	**K** + (D) + (4) + (5)

structure $\langle W, R \rangle$—that is, the frame includes the set of possible worlds and the accessibility relation, but not the interpretation function V mapping sentences to propositions. It turns out that there is no guarantee that just *any* interpretation function could be defined for any given frame. The accessibility relation may impose dependencies between propositions which rule out some interpretations.

Before we consider a simple example of such a dependency, we fix some more terminology. In addition to the truth of a sentence at a world in a model, we introduce the related notions of *truth in a model* and *validity on a frame*.

Definition 3 (Truth and validity). *A sentence ϕ is*

a. true at a world w in a model $\langle W, R, V \rangle$ iff $w \in V(\phi)$.

b. true in a model $\langle W, R, V \rangle$ iff $W \subseteq V(\phi)$

c. valid on a frame $\langle W, R \rangle$ iff ϕ is true in all models based on $\langle W, R \rangle$.

In this picture, the relationship between properties of frames and possible interpretations that we alluded to above concerns the validity of formulas on frames. For a simple example, consider a frame $\langle W, R \rangle$ in which the accessibility relation is *reflexive*— i.e., wRw for all $w \in W$—and let ϕ be an arbitrary sentence of $\mathcal{L}_{\mathcal{A}}$. It is then impossible to build a model $\langle W, R, V \rangle$ such that at some world, $\Box\phi$ is true while ϕ is false. Put differently, this means that the material conditional $\Box\phi \rightarrow \phi$ is valid on the frame $\langle W, R \rangle$. Moreover, it turns out that the reflexivity of R is not only sufficient but also necessary for the validity of this sentence.

Although this finding is straightforward and familiar, we state it formally because we discuss similar but less familiar correspondences in detail below. It is instructive to start with this simple case. Proofs for this and all subsequent propositions are collected in the appendix.

Proposition 1. *$\Box\phi \rightarrow \phi$ is valid on a frame $\langle W, R \rangle$ iff R is reflexive.*

This result assures us that we have at our disposal two distinct ways of modeling the validity of the inference from $\Box\phi$ to ϕ (for any ϕ): either syntactically, by adding (T) to our system of axioms, or semantically, by stipulating that the admissible models of our language must be built on frames whose accessibility relations are reflexive.

Generally, the relationship between axioms and frame properties constitutes a widely studied and fascinating topic in modal logic, usually subsumed under the heading *correspondence theory* (van Benthem, 1986; Hughes and Cresswell, 1996). Some more common examples are given (without proof) in the following.

(2) a. Axiom (D) is valid on $\langle W, R \rangle$ iff R is *serial*
(i.e., for all worlds w, there is some world v such that wRv).

b. Axiom (4) is valid on $\langle W, R \rangle$ iff R is *transitive*
(i.e., for all worlds w, v, u, if wRv and vRu, then wRu).

c. Axiom (5) is valid on $\langle W, R \rangle$ iff R is *euclidian*
(i.e., for all worlds w, v, u, if wRv and wRu, then vRu).

The correspondences for a few other properties will become relevant in connection with conditionals (see section 3).

2.2 *Kratzer semantics*

As modal logic was developed to model our reasoning about necessity and possibility, it is not surprising that its formal apparatus has also been applied in the linguistic analysis of modal expressions in natural language. The now-standard approach in linguistic semantics started in the 1970s with Angelika Kratzer's (1977; 1978; 1981; 2012) groundbreaking work, which was in turn directly influenced by work in philosophical logic by David Lewis (1973) and others. In this section we give a brief outline of the basic ingredients of this standard approach.

Like standard modal logic, Kratzer's analysis treats necessity and possibility in terms of quantification over possible worlds. There are some significant differences, however, which we indicate in this section.

2.2.1 Conversational backgrounds In modal logic, the accessibility relation determining, for each world w of evaluation, which worlds are relevant to modal quantification is a free parameter of the model. It may be subject to certain structural constraints (such as Axiom (D) in the case of deontic modality), but it is not tied to any goings-on in w. Two worlds w and v can differ only in what worlds are accessible from them, without displaying any factual differences. In contrast, Kratzer seeks to *derive* accessability from facts about w. For instance, for deontic modality, what worlds need to be considered to determine what is required and what is permitted depends on the content of the law as it is written down in w. Roughly speaking, a world v is in the domain of quantification for a modal expression evaluated at a world w if the propositions listed in the law at w are all true at v.[3] Deriving the accessibility relation systematically from what is the content of the law (or for other modalities from what is known, what is desired, etc.) at the world of evaluation aims at modeling entailments between different modal and non modal sentences that cannot be predicted if accessibility relations are considered primitive and independent of what atomic propositions are true at the world of evaluation.

To this end, Kratzer introduces the formal device of a *conversational background*.

Definition 4 (Conversational background). *A conversational background is a function from possible worlds to sets of propositions.*

Different modal flavors involve different such backgrounds. A few examples are given in (3).

(3) a. The *epistemic* background of agent α is the conversational background f s.t. for all $w \in W$, $f(w) = \{p \subseteq W \mid p \text{ is known to } \alpha \text{ in } w\}$.

b. The *deontic* background of agent α is the conversational background f s.t. for all $w \in W$, $f(w) = \{p \subseteq W \mid p \text{ is required of } \alpha \text{ in } w\}$.

c. The *bouletic* background of agent α is the conversational background f s.t. for all $w \in W$, $f(w) = \{p \subseteq W \mid p \text{ is desired by } \alpha \text{ in } w\}$.

Without the fact being discussed explicitly, in Kratzer's own work and in the literature that employs her framework, it is normally assumed that conversational backgrounds contain only sentences that do not themselves contain modal expressions, and therefore do not depend on conversational backgrounds for their own interpretation. Recently, problems arising from the interaction of various modal flavors (e.g., of knowledge with desires or goals), have inspired proposals that depart from that custom (Büring, 2003; von Fintel, 2012; Angelika Kratzer personal communication with the authors, December 2011). The implications of this move for the formal framework have, to the best of our knowledge, not been studied systematically, and while this strikes us as an important matter of investigation, we stop short of pursuing it here.

Conversational backgrounds can be characterized not only in terms of what modal flavor they encode but also according to formal properties much like the frame properties discussed above. Definition 5 lists some properties that are standardly discussed in connection with Kratzer's theory.

Definition 5 (Properties of conversational backgrounds). *Let f be a conversational background.*

a. f *is* consistent *iff for all* $w \in W$, $\bigcap f(w) \neq \emptyset$ *.*

b. f *is* realistic *iff for all* $w \in W$, $w \in \bigcap f(w)$*.*

c. f *is* totally realistic *iff for all* $w \in W$, $\bigcap f(w) = \{w\}$*.*

These properties correspond to the frame properties of *seriality*, *reflexivity*, and *identity*, respectively. We discuss these and other properties of conversational backgrounds and accessibility relations further in section 3 (see especially Table 8.2).

Conversational backgrounds can be used to play an analogous role to that of accessibility relations in modal logic. Indeed, as Kratzer (1991b) pointed out, each conversational background determines a unique accessibility relation, a fact that we make use of for ease of exposition in section 3 below.

Definition 6 (Kratzer accessibility relation). *Let f be a conversational background. The accessibility relation determined by f, R_f, is defined as follows: for all $w, v \in W$, $wR_f v$ iff $v \in \bigcap f(w)$.*

It is worth pointing out that the correspondence between conversational backgrounds and accessibility relations is many-to-one, i.e., a given accessibility relation may be induced by more than one conversational background.[4] In this sense, conversational backgrounds constitute a strictly more expressive formal device. However, little use has been made of the additional expressive power in linguistic analyses. Two examples are von Fintel and Gillies (2010) in the analysis of epistemic "*must*" and S. Kaufmann (2013) for counterfactuals. We do not enter a detailed discussion of these approaches here. In the larger picture, more use has been made of the internal structure of the *ordering source* (see below).

Conversational backgrounds are introduced into the interpretation either as parameters of the interpretation of modal expressions (Kratzer, 1978, 1981, 2012), or are represented syntactically by covert pronouns (Heim and von Fintel, 2011).[5] Expressions like "*in view of*" or "*according to*" are often argued to explicitly name the conversational background under consideration. We stick to Kratzer's original version in the following, representing them as part of the contextual "index" of evaluation alongside the world of evaluation.

2.2.2 Simple modality: modal bases The first step towards an implementation of the Kratzer-style framework employs a conversational background exactly as modal logic employs an accessibility relation: to determine, for each world of evaluation, a set of worlds with respect to which the crucial notions of necessity and possibility are then defined. The conversational background used in this way is called the *modal base*.

Definition 7 (Simple necessity and possibility). *Let w be a possible world, f a conversational background, and p a proposition.*

a. p *is a* necessity *at* w, f *iff for all* $v \in \bigcap f(w), v \in p$*.*

b. p *is a* possibility *at* w, f *iff for some* $v \in \bigcap f(w), v \in p$*.*

We can now spell out the meaning of "*can*" and "*must*" as in Definition 8.

Definition 8 (One-parameter interpretation of modals). *Let ϕ be a sentence, w a possible world, and f a conversational background.*

a. "must ϕ" *is true at w, f iff $V(\phi)$ is a necessity at w, f.*

b. "can ϕ" *is true at w, f iff $V(\phi)$ is a possibility at w, f.*

This approach in terms of simple relative modality mimicks the interpretation of $\Box$ and $\Diamond$ from classical modal logic that we discussed above.

There is a potentially confusing terminological variation in the literature with regard to the term "modal base." One can find it used to refer to f (a conversational background); $f(w)$ (a set of propositions, the value of f at w); or $\bigcap f(w)$ (a set of possible worlds, those at which all propositions in $f(w)$ are true). In this chapter we frequently refer to the first and third of these notions. We reserve the term "modal base" for the conversational background f, and we refer to $\bigcap f(w)$ as the *modal background* at w.

Kratzer herself points out that the approach does not offer a handle on more fine-grained distinctions in necessity or possibility, as for instance in "*there is a good/slight possibility*" or "*p is more likely than q*." Moreover, simple modality falters in the face of inconsistency: for any w and f and propositional expression φ, if $f(w)$ is inconsistent (hence the modal background at w is empty), "*must* φ" is predicted to be true and "*can* φ" is predicted to be false. Arguably, at least some of the bodies of information represented by conversational backgrounds, such as laws or desires, can be inconsistent. But this does not imply that claims about what is necessary or possible with respect to them are trivial.

2.2.3 Graded modality Kratzer (1981, 1991b) suggests that the background information relevant to the interpretation of a modal expression be split into two parts: a *modal base* and an *ordering source*. Both of them are conversational backgrounds, but they play quite different roles: the modal base specifies a necessarily consistent body of background information that singles out a set of possible worlds which are then ranked according to the ordering source. Intuitively, only the best (or comparatively better) worlds among the ones compatible with the modal base are relevant for the truth of the modal statement. Ordering sources typically specify violable and possibly inconsistent information like stereotypical assumptions, preferences, or rules and regulations. According to Kratzer (2012), any realistic conversational background can serve as the modal base. In her earlier work, she distinguishes between epistemic and non epistemic modality: epistemic modality involves as its modal base an epistemic conversational background (encoding what is known by the relevant agent(s)); non epistemic modality involves a so-called circumstantial conversational background that contains all the propositions that are true at the world of evaluation and are relevant to the application of laws, desires, and so forth, in determining if the prejacent is possible or necessary in the relevant sense.

The set of propositions $g(w)$ assigned to the world of evalution w is used to induce a preorder on the set of possible worlds.[6]

Definition 9 (Induced preorder). *Let w be a possible world and g a conversational background. Define a binary relation $\leq_{g(w)}$ between possible worlds as follows: $x \leq_{g(w)} y$ iff $\{p \in g(w) \mid y \in p\} \subseteq \{p \in g(w) \mid x \in p\}$.*

Necessity and possibility are defined relative to this order. The underlying idea is simple: what counts as necessary or possible is not determined by all worlds in the modal background but only by those among them at which as many ordering-source propositions are true as possible. The definitions are a bit cumbersome, however, because if the set of ordering-source propositions is infinite, there may be no set of "best" worlds in this sense but rather an infinite sequence of better and better worlds.

Definition 10 (Two-parameter necessity and possibility). *Let w be a possible world, f, g two conversational backgrounds, and p a proposition.*

a. *p is a* necessity *at w,f,g iff for all $x \in \bigcap f(w)$, there is a $y \in \bigcap f(w)$ s.t. $y \leq_{g(w)} x$ and for all $z \in \bigcap f(w)$ s.t. $z \leq_{g(w)} y$, $z \in p$.*

b. *p is a* possibility *at w,f,g iff there is an $x \in \bigcap f(w)$ s.t. for all $y \in \bigcap f(w)$ s.t. $y \leq_{g(w)} x$, there is a $z \in \bigcap f(w)$ s.t. $z \leq_{g(w)} y$ and $z \in p$.*

With these definitions in place, modals like "*must*" and "*can*" are defined just as above, safe for the addition of the ordering source parameter.

Definition 11 (Two-parameter interpretation of modals). *Let ϕ be a sentence, w a possible world, and f, g two conversational backgrounds.*

a. "must ϕ" *is true at w,f,g iff $V(\phi)$ is a necessity at w,f,g.*

b. "can ϕ" *is true at w,f,g iff $V(\phi)$ is a possibility at w,f,g.*

In the literature one usually encounters a simplified version of the clauses in Definition 10: one that directly refers to the set of possible worlds that matter for necessity and possibility. Technically, these are the *minimal* worlds under the induced pre order in the modal background:

Definition 12 (Minimal worlds). *Let w be a possible world and f, g two conversational backgrounds. The set of* minimal worlds *at w,f,g is defined as* $\mathbf{O}(w,f,g) := \{v \in \bigcap f(w) \mid \forall u \in \bigcap f(w) \left[u \leq_{g(w)} v \rightarrow v \leq_{g(w)} u\right]\}$

In general, if $g(w)$ is infinite then the set of minimal worlds may be empty even if the modal background is not. Moreover, even if there are minimal worlds, they may coexist with infinite sequences of better and better worlds with which they are incomparable under $\leq_{g(w)}$. To forestall the technical complications that these possibilities would raise, one can impose the following stipulation, generally without adverse consequences for linguistic analyses.

Definition 13 (Limit assumption). *A pair f, g of conversational backgrounds satisfies the* Limit Assumption *iff for all possible worlds w, for all $v \in \bigcap f(w)$ there is a $u \in \mathbf{O}(w,f,g)$ such that $u \leq_{g(w)} v$.*

We call this condition the *Limit Assumption* after Lewis (1973, 1981).[7] It is guaranteed to hold if $g(w)$ is a *finite* set of propositions at all worlds w (although the assumption holds for some combinations of modal bases with infinite ordering sources as well). With the Limit Assumption, the following definitions of necessity and possibility can be substituted for the ones in Definition 10 above.

Proposition 2. *Let w be a possible world, f, g two conversational backgrounds, and p a proposition. If f, g meet the Limit Assumption, then*

a. *p is a necessity at w,f,g iff for all $v \in \mathbf{O}(w,f,g)$, $v \in p$.*

b. *p is a possibility at w,f,g iff for some $v \in \mathbf{O}(w,f,g)$, $v \in p$.*

The interpreations of "*must ϕ*" and "*can ϕ*" remain as in Definition 11 above.

2.2.4 Realism We conclude this section with a comment on a constraint Kratzer (1991b, 2012) explicitly imposes on all modal bases: she requires that they be *realistic* conversational backgrounds—formally, that for all modal bases f and worlds w, $w \in \bigcap f(w)$. This is a rather strong constraint, for it implies that modal bases cannot be used to model information or beliefs that are false at the world of evaluation. For an interpretation of modal expressions within the framework of simple modality (i.e., without the additional parameter of an ordering source), this amounts to the assumption that all modals that are formally analyzed in terms of necessity are factive. Kratzer does not expand on the motivation for the constraint; however, we take it to be self-evident that an adequate semantic theory must allow for the non factivity of attitudes like belief, for example.

In this connection, it is important to note that even if one were to adopt Kratzer's realism constraint, the availability of ordering sources as part of the formal toolbox once again makes it possible to capture non-factive interpretations of necessity modals. This is because realism merely requires that $w \in \bigcap f(w)$ for all w; it is consistent with the situation that $w \notin \mathbf{O}(w, f, g)$, for a suitable g. Thus while Kratzer's proposal, as far as we can see, does commit her to an analysis of non factivity in terms of the ordering source rather than the modal base, it does not put such an analysis altogether beyond reach. We will return to this issue repeatedly in section 3.

3. Conditionals

Throughout the history of philosophical logic, the predominant analysis of the conditional "*if-then*" construction has been as a binary sentential operator. Following Gillies (2010), we dub this the *iffy operator* approach, highlighting the fact that on this view the two-place operator is associated with the morpheme "*if*." If one assumes that this is the correct analysis, the remaining question is how to interpret this connective, i.e., how the meaning of "*if p then q*" depends on the meanings of p and q.

The iffy operator analysis is not the only game in town, however. The past several decades have seen the rise of an alternative view on the semantics of the conditional, growing out of the work of Lewis (1975) and, most importantly, Kratzer (1978 and subsequent works). On this view, the "*if-then*" construction really consists of two independently moving parts: the matrix clause is headed by a *modal operator* (often covertly given), which has the consequent q as its prejacent, and the "*if*"-clause is a *restrictor* affecting the interpretation of that operator. The main open questions then are (i) which (overt or covert) modal operators can head conditionals and how their semantic properties affect the interpretation, and (ii) how exactly the restriction by the "*if*"-clause works. We take up both of these questions below.

Nowadays, the restrictor approach is the standard one in the linguistic literature. However, the operator approach is by no means on the retreat, let alone defeated. Debates about the feasibility and relative strengths of either *vis-à-vis* the other are continuing. These issues will come up repeatedly in this chapter and while we stop short of claiming a decisive advantage of one over the other, the total balance of the evidence seems to be in favor of the restrictor approach.

3.1 *Iffy operators and the Import-Export Principle*

3.1.1 Truth-functionality If "*if-then*" is to be analyzed as a two-place sentential operator, then how should it be interpreted? One long-standing approach is to assume that "*if-then*" is a *truth-functional* operator alongside conjunction, disjunction, negation, etc. Now, just a few basic assumptions are sufficient to show that "*if-then*" has to be the material conditional if it is to be truth-functional at all. One such argument was given by Dorothy Edgington:

Proposition 3 (Edgington, 1986). *Assume that*

(Ea) "if-then" *denotes a truth function, call it* $\mathcal{F}_{\text{if}}$*;*

(Eb) *sentences of the form* "if (p and q) then p" *are tautologous;*

(Ec) *conditionals can be false.*

Then $\mathcal{F}_{\text{if}}$ *is the truth function of the material conditional.*

We take the assumptions that underlie Edgington's conclusion to be quite uncontroversial, thus the conclusion that a truth-functional interpretation of "*if-then*" must be the material conditional seems unavoidable. This conclusion is generally considered unwelcome news, for it gives rise to a number of well known counterintuitive predictions about entailments from and to conditionals. (We do not rehearse these problems in detail here, but we do give examples where appropriate below.) It is worth mentioning, however, that despite these unwelcome consequences, one school of thought resolves to bite the bullet and accept the material-conditional analysis, typically in combination with a pragmatic story about the reasons why the truth of the material conditional is not always sufficient for the assertability of the corresponding "*if-then*" sentence. Prominent proponents of such theories include Jackson (1979; 1984; 1987; and following him David Lewis, at least for indicative conditionals), and Abbott (2004, 2010). This is not the place to explore the advantages and disadvantages of this line of research in more detail. It departs from the topic of this chapter—modality and conditionals—and the issues and challenges that it involves would lead us too far afield.

3.1.2 Propositionality Edgington's argument led from the premise that "*if-then*" denotes a truth function to the conclusion that this truth function is the material conditional. The additional assumptions in (Eb) and (Ec) are weak and uncontroversial; for those who reject the material conditional analysis, the upshot therefore is that English "*if-then*", if it is to be a binary sentential connective, cannot receive a truth-functional interpretation.

What kind of operator, then, does "*if-then*" denote? In an influential paper, Gibbard (1981) argues that as long as "*if-then*" is taken to be *propositional* (i.e., that "*if p then q*" for arbitrary p and q denotes a proposition), it must nonetheless be the material conditional. The argument is relevant in the present context for at least two reasons. The first is that it has been cited—most prominently by Kratzer (1986, reprinted in 2012)—as part of the justification for abandoning the iffy operator approach altogether. But the exact implications of Gibbard's proof for the approach have not been explored in detail in this connection, and a careful examination of the issue reveals that they are not as damning as they may seem at first glance.

The second reason why Gibbard's proof is relevant in the present context has to do with its premises. Propositionality is a much weaker condition than truth-functionality,[8] yet Gibbard arrived at a conclusion similar to Edgington's. He did so by introducing additional premises that go beyond Edgington's and that are, arguably, less uncontroversial. Those premises are worth exploring in their own right.

Gibbard's argument goes as follows. We adopt the exposition of Kratzer (1986), slightly adjusting the notation.

Proposition 4 (Gibbard, 1981). *Let* $\textsc{if}(\cdot\,,\,\cdot)$ *be a binary propositional operator with the following properties:*

(Ga) $\textsc{if}(p\,,\,\textsc{if}(q\,,\,r))$ *and* $\textsc{if}(p \wedge q\,,\,r)$ *are logically equivalent.*

(Gb) $\textsc{if}(p\,,\,q)$ *logically implies the material conditional* $p \rightarrow q$.

(Gc) If p logically implies q, then $\text{IF}(p\,,\,q)$ *is a logical truth.*

Then $\text{IF}(\cdot\,,\,\cdot)$ *is the material conditional.*

The conclusion is unavoidable: if we are to uphold Gibbard's assumptions, then $\text{IF}(p\,,\,q)$ must be the material conditional. For Gibbard, this was not hard to swallow. He saw independent grounds for rejecting a propositional approach in the (apparent) incompatibility of the latter with a probabilistic analysis in the sense of Adams (1965, 1975).[9] The most plausible reason he saw for maintaining that conditionals denote propositions was a desire to account for embedded conditionals in a non stipulative way. He suspected, however, that there was not much point in doing so, as he believed that embedded conditionals in natural language, though grammatically possible, were either reinterpreted as sentences not embedding conditionals, or plainly incomprehensible. He saw reinterpretation at work in what is perhaps the most easily accessible and widely attested variety of such sentences: conditionals with conditional consequents. Those are directly taken care of by his assumption (Ga), the *Import-Export Principle*.

Kratzer, unlike Gibbard, maintains that conditionals denote propositions, but denies that they involve a binary sentential operator. We discuss her approach in more detail in the next subsection. Staying for now within the operator approach, we take a closer look at Gibbard's assumptions and some directions in which a viable analysis has been argued to be available.

3.1.3 Conditionals as modal operators Of course, there is in principle much flexibility with regard to *what kind of* propositional operator "*if-then*" might be. But in practice there is little disagreement on the broad underlying intuitions. It is generally agreed that, viewed as a propositional operator, "*if-then*" expresses a relation of *necessary connection* between its constituent propositions. The motivation for this stance has to do largely with the role of conditionals in inference (especially in *modus ponens*-like arguments) and communication (cf. Grice's analysis of conditionals as conveying the information that *modus ponens* is safe), but also with intuitions concerning what situations count as "verifying" or "falsifying" with regard to conditionals (see for instance van Benthem, 1986; Gillies, 2010; Veltman, 1985). As this chapter is particularly concerned with the modal perspective, we adopt this general stance and consider ways of developing an analysis of "*if-then*" as a *relativized modal necessity* operator.

Note that the discussion of conditionals is partly complicated by the distinction between indicative conditionals (characterized by indicative morphology) and counterfactual conditionals (characterized by subjunctive or irrealis morphology in many languages, by the auxiliary "*would*" in the consequent in English, and suggesting the implausibility or even falsity of the antecedent).[10] While we will occasionally draw attention to this distinction, we generally focus on indicative conditionals, assuming that the analysis of counterfactual conditionals requires a conservative extension that involves a series of intricacies unrelated to the issues of interest here.

Informally, under the interpretation of "*if-then*" as expressing relativized modal necessity, the statement "*if p then q*" is true just in case at all relevant worlds at which p is true, q is also true. This paraphrase is simple enough and widely agreed upon. But it is only a skeletal sketch of the general form that an analysis should take. The real work lies in spelling out the details: which worlds are the "relevant" ones, and how are the constituents p and q to be evaluated at those worlds?

To tackle these questions one by one, let us start with a simple formal implementation of the basic idea. We stay within the general (Kratzer-style) framework for the analysis of modality outlined in section 2.2 above, but ignore for now the ordering-source parameter.[11] Conditionals, under this conception, are interpreted as modal operators—specifically, modal *necessity* operators, in line with the universal quantification in the informal paraphrase above. The role of the "*if*"-clause is to assist in the identification of the set of worlds that the modal operator quantifiers over. More technically, the "*if*"-clause modifies the modal base relative to which the sentence is evaluated. This is spelled out in Definition 14.

Definition 14 (Relativized strict necessity): *"*if p then q*" is true at a world w relative to a conversational background f iff for all worlds $v \in \bigcap f(w)$ at which p is true, q is also true.*

Against this backdrop, we return to Gibbard's proof and examine it in some more detail. $\text{IF}(\cdot\,,\,\cdot)$ is a binary sentential operator which we have equipped with a concrete truth definition in terms of a modal base, along the lines familiar from Kratzer's approach to modality. Gibbard's result shows that if our operator validates his premises, then it must be the material conditional. It is instructive to see why this is so, therefore we devote some discussion to the argument. The general roadmap for this investigation is familiar from the discussion of *correspondence theory* in section 2 above. In the Kratzer-style framework, the analog of validity for a frame is *validity for a modal base f*. As might be expected, a formula has this property just in case it is impossible to define an interpretation function under which modal operators are interpreted relative to f in such a way that there is a world at which the formula in question comes out false. With regard to the present investigation, then, the question is, what must the modal base be like in order for the premises of Gibbard's proof to hold?

As a consequence of the way our operator $\text{IF}(\cdot\,,\,\cdot)$ is interpreted, its truth at a world w relative to a modal base f depends only on the modal background $\bigcap f(w)$, not the set of propositions $f(w)$ whose intersection it is. This gives us license to simplify the exposition by stating the relevant constraints in terms of the accessibility relation R_f (see Definition 6 in section 2).

Also as a consequence of the interpretation of $\text{IF}(\cdot\,,\,\cdot)$, we sidestep the question of whether Gibbard's third premise (Gc)—that $\text{IF}(p\,,\,q)$ is a logical truth if p logically implies q—is plausible. Our truth definition for the conditional is set up in such a way that it could not possibly be false in that case. As long we stay in the general framework of modal logic and possible worlds, the assumption is built in.

What, then, do Gibbard's premises amount to in terms of restrictions on the accessibility relation, and hence the modal base?

3.1.4 The operators $\text{IF}(\cdot\,,\,\cdot)$ *and* $\rightarrow$ As a relatively simple first case, consider the second premise (Gb), the statement that the conditional logically implies the corresponding material conditional. Formally, this amounts to the claim that for any modal base f, (4) is a logical truth (like modal operators, we annotate $\text{IF}(\cdot\,,\,\cdot)$ with a superscript to indicate the intended conversational background).

(4) $\text{IF}^f(p\,,\,q) \rightarrow (p \rightarrow q)$

Proposition 5. $\text{IF}^f(p\,,\,q) \rightarrow (p \rightarrow q)$ *is valid iff R_f is reflexive.*

As we mentioned in section 2, the corresponding requirement on Kratzer-style modal bases is that they be *realistic*. How plausible is this as an assumption about conditionals? Gibbard apparently took it to be self-evident; so does Kratzer. But it is far from obvious to us; even more to the point, it is not valid in Kratzer's own two-parameter analysis. This is an issue that merits some discussion after the rest of Gibbard's proof has been considered. We return to it below.

Reflexivity is a relatively simple and straightforward case. We now turn to the more interesting investigation of two distinct versions of the Import-Export Principle, one "pure" and one "shifty."

3.1.5 Pure Import-Export The Import-Export Principle (Ga) states of two formulas that they are logically equivalent. Put differently, this means that each of the material conditionals in (5a,b) is a logical truth.

(5) a. $\text{IF}^f\big(p\,,\,\text{IF}^f(q\,,\,r)\big) \rightarrow \text{IF}^f(p \wedge q\,,\,r)$

b. $\text{IF}^f(p \wedge q\,,\,r) \rightarrow \text{IF}^f\big(q\,,\,\text{IF}^f(q\,,\,r)\big)$

It turns out that these two conditions jointly rule out all but a very restricted class of modal bases. What (5a) states, as a condition on accessibility relations, is that any world that is accessible in one step is accessible to itself. Technically, this means that the accessibility relation is *shift reflexive*.

Definition 15 (Shift reflexivity). *An accessibility relation R is* shift reflexive *iff for all worlds w, v, if wRv then vRv.*

Proposition 6. $\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big) \rightarrow \textsc{if}^f(p \wedge q\,,\,r)$ *is valid iff R_f is shift reflexive.*

The standard axiom characterizing this property can be given as in (6). Interpreting necessity as belief, this says that the agent beliefs that all her beliefs are true (although she may be wrong about that, unbeknown to herself):

(6) **Shift reflexivity**
$\Box(\Box\phi \rightarrow \phi)$

Moving on to the other direction of the Import-Export Principle, (5b), the corresponding requirement is that if a world can be reached by following two accessibility links, the second step is a reflexive loop.

Definition 16 (Shift coreflexivity). *An accessibility relation R is* shift coreflexive *iff for all worlds w, v, u, if wRv and vRu then $v = u$.*

Proposition 7. $\textsc{if}^f(p \wedge q\,,\,r) \rightarrow \textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big)$ *is valid iff R_f is shift coreflexive.*

Again, it is useful to consider the intuitive notion embodied by this constraint, and for this we turn to the standard form of the axiom given in (7):

(7) **Shift coreflexivity**
$\Box(\phi \rightarrow \Box\phi)$

In words, (7) states that the agent believes that she believes everything that is true—put differently, each world v consistent with her beliefs is such that at v, she knows everything that is the case at v. To properly understand the scope of this claim, notice that it does not include the statement that she believes *only* what is the case at v (she may have inconsistent beliefs at v), nor that there is any particular world v of which she believes that it is the actual one.

Jointly, the two conditions of shift reflexivity and shift coreflexivity amount to the statement that each accessible world is related to itself and no other world.

Definition 17 (Shift identity). *An accessibility relation R is* shift identical *iff for all worlds w, v, u, if wRv, then vRu iff $v = u$.*

Theorem 1. $\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big) \leftrightarrow \textsc{if}^f(p \wedge q\,,\,r)$ *is valid iff R_f is shift identical.*

This property is not commonly discussed in modal logic, but an axiom in the usual format can easily be written down for it:

(8) **Shift identity**
$\Box(\varphi \leftrightarrow \Box\varphi)$

Shift identity has rather counterintuitive implications. Imagine an agent who, at world w, considers two distinct worlds possible and whose belief state meets this constraint. Then the agent beliefs that she knows all the truths in the world, but does not know which of the two worlds she inhabits.

Now, adding to this Gibbard's second premise, which as we saw in Proposition 5 implies that the accessibility relation is reflexive, we arrive at the conclusion that the relation is the identity relation.

Definition 18 (Identity). *An accessibility relation R is the* identity *relation iff for all worlds w, v, wRv iff $w = v$.*

Theorem 2. *The following are jointly valid iff R_f is the identity relation.*

a. $\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big) \leftrightarrow \textsc{if}^f(p \wedge q\,,\,r)$

b. $\textsc{if}^f(p\,,\,q) \rightarrow (p \rightarrow q)$

In Kratzer's terms, R_f is the identity relation iff the modal base f is *totally realistic*—i.e., for any world w, the modal background at w is the singleton set containing w. It is easy to see that relative to this modal base, the conditional $\textsc{if}^f(p\,,\,q)$ is equivalent to the corresponding material conditional. This is what Gibbard's result amounts to in the particular framework in which we have defined our conditional operator.

What options are available if we insist on upholding the Import-Export Principle but find the commitment to an analysis in terms of the material conditional unpalatable? One step one might take is to give up assumption (Gb) that conditionals entail the corresponding material conditional, which as we saw above corresponds to *reflexivity* of the accessibility relation. In fact, this move may be attractive independently, as we discuss below.

However, even if we were to give up reflexivity, we would be left with shift identity. This is still far too restrictive a condition if we expect the Import-Export Principle to hold for the kinds of modality that are most commonly associated with indicative conditionals, such as epistemic, doxastic, or circumstantial modality. The relation representing epistemic modality, for instance, is typically taken to be euclidean (see (2c) in section 2). But shift-identity rules this out in all but the most trivial cases: it is easy to show that if R_f is both shift identical and euclidean, then for any world w, there can be at most one world v such that $wR_f v$.

Is there any other way to interpret our conditional operator that circumvents these restrictive consequences while preserving the intuition behind the Import-Export Principle?

3.1.6 Shifty Import-Export Gibbard himself considered the idea that the conditional operator, while expressing a binary propositional function on each of its occurrences, may not express the same function in all contexts. Gibbard did not reject this option, but was doubtful about its usefulness. The idea is not discredited, however; far from it. In fact, Gillies (2010) proposes a "doubly shifty" operator theory which treats "*if*" clauses as shifting both the *index* of evaluation (represented as quantification over possible worlds in our framework) and the *context* (corresponding to the accessibility relation).

Formally, in our present framework, this move comes down to the following: our definition of strict necessity above (Definition 14) calls for the evaluation of the consequent at all accessible worlds at which the antecedent is true. Nothing was said there about the interpretation of conditionals that are embedded in the consequent. In the subsequent proofs we assumed implicitly that those embedded conditionals were interpreted relative to the same modal base as the matrix conditionals containing them. This is made explicit in the formula "$\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big)$," which we used throughout in our discussion of the Import-Export Principle. Gillies's proposal is to introduce instead a change in the modal base for the consequent that is effected by the conditional operator. The change consists in the addition of the antecedent to the modal base. We call this operation an *update* of the modal base, in recognition of the affinity of this operation with the corresponding notion from dynamic semantics. It should be kept in mind, however, that Gillies's definition is still in essence static, in the sense that the shift arises only in the course of evaluating the conditional and does not represent a persistent discourse effect.

Definition 19 (Update). *Let f be a conversational background and p a proposition. The result of the* update *of f with p is a conversational background $f[p]$ defined as follows: for all worlds w, $f[p]\,(w) := f(w) \cup \{p\}$.*

Gillies's proposal can then be cast into the following formal definition. Notice the close similarity to Definition 14, safe for the added specification of the udpated modal base.[12]

Definition 20 (Shifty relativized strict necessity). *"*if p then q*" is true at w relative to f iff for all worlds v such that $wR_f v$ and p is true at v, q is true at v relative to $f[p]$.*

Gillies's motivation for this proposal came from observations on the interpretation of epistemic modals in the consequents of epistemic conditionals. We are interested in how the idea, adapted to our interpretation of right-nested conditionals, affects the resulting constraints on the modal base.

(9) "*if p then if q then r*" is true at w relative to f

a. iff for all worlds v such that $wR_f v$ and p is true at v, "*if q then r*" is true at v relative to $f[p]$;

b. iff for all worlds v such that $wR_f v$ and p is true at v, for all worlds u such that $vR_{f[p]} u$ and q is true at u, r is true at u relative to $f[p][q]$.

Thus in effect, under this new interpretation the Import-Export Principle for conditionals with conditional consequents amounts to the conjunction of (10a,b).

(10) **Shifty Import-Export**

a. $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big) \rightarrow \textsc{if}^f(p \wedge q\,,\,r)$

b. $\textsc{if}^f(p \wedge q\,,\,r) \rightarrow \textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big)$

This interpretation amounts to partly different constraints on the modal base. Regarding (10a) there is no change: just like for (5a), the property is shift reflexivity.

Proposition 8. $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big) \rightarrow \textsc{if}^f(p \wedge q\,,\,r)$ *is valid iff R_f is shift reflexive.*

However, the other direction of the shifty Import-Export Principle, (10b), differs from Import-Export for relativized strict necessity as in (5b). Instead of shift coreflexivity, it characterizes the rather familiar property of transitivity: If a world can be reached in two steps, it can also be reached in one.

Proposition 9. $\textsc{if}^f(p \wedge q\,,\,r) \rightarrow \textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big)$ *is valid iff R_f is transitive.*

On the doxastic interpretation of necessity, transitivity corresponds to *positive introspection*, briefly discussed in section 2 as a key ingredient in the logics **KD45**, **S4**, and **S5** (the latter as a consequence of the standard axiomatization which does not include (4) but implies it).

Thus both parts of the shifty Import-Export Principle together impose the following constraint.

Theorem 3. *$\text{IF}^f\big(p\,,\,\text{IF}^{f[p]}(q\,,\,r)\big) \leftrightarrow \text{IF}^f(p \wedge q\,,\,r)$ is valid iff R_f is transitive and shift reflexive.*

In asking whether this pair of conditions is plausible, particularly noteworthy is the fact that they are consistent with euclidity in non trivial cases; indeed, euclidity *implies* shift-reflexivity (though not *vice versa*). Consequently, the shifty Import-Export Principle is valid in all systems commonly used for the representation of epistemic states or the common ground, notably **KD45**[13] **S4**, and **S5**. If one insists on adding the condition of reflexivity (but recall that we are not inclinced towards this assumption), the resulting logic is **S4**.

Theorem 4. *The following are jointly valid iff R_f is reflexive and transitive.*

a. $\text{IF}^f\big(p\,,\,\text{IF}^{f[p]}(q\,,\,r)\big) \leftrightarrow \text{IF}^f(p \wedge q\,,\,r)$

b. $\text{IF}^f(p\,,\,q) \rightarrow (p \rightarrow q)$

These are welcome results. They suggest that an interpretation of right-nested conditionals along the lines of the shifty Import-Export Principle opens up the option of an iffy operator analysis without dire consequences for the interpretation of the conditional. As long as the accessibility relation has the right properties—shift reflexivity and transitivity, and optionally reflexivity—there are no untoward consequences. Gillies (2010) has recently proposed an operator analysis along these lines. The results of this subsection are summarized in Table 8.2.

Table 8.2. Some properties of accessibility relations referred to in this chapter.

Property of relations	*Property of modal bases*	*Axiom*
Seriality $\exists v[wR_fv]$	$\bigcap f(w) \neq \emptyset$	$\Box\phi \rightarrow \Diamond\phi$
Reflexivity $w = v \rightarrow wR_fv$	$\bigcap f(w) \supseteq \{w\}$	$\Box\phi \rightarrow \phi$
Coreflexivity $wR_fv \rightarrow w = v$	$\bigcap f(w) \subseteq \{w\}$	$\phi \rightarrow \Box\phi$
Reflexivity Identity $wR_fv \leftrightarrow w = v$	$\bigcap f(w) = \{w\}$	$\phi \leftrightarrow \Box\phi$
Euclidity $wR_fu \rightarrow (wR_fv \rightarrow uR_fv)$	$\bigcap f(w) \subseteq \bigcap\{\bigcap f(v) \mid v \in \bigcap f(w)\}$	$\Diamond\phi \rightarrow \Box\Diamond\phi$
Transitivity $wR_fu \rightarrow (uR_fv \rightarrow wR_fv)$	$\bigcap f(w) \supseteq \bigcup\{\bigcap f(v) \mid v \in \bigcap f(w)\}$	$\Box\phi \rightarrow \Box\Box\phi$
Shift reflexivity $wR_fv \rightarrow vR_fv$	$v \in \bigcap f(w) \rightarrow \bigcap f(v) \supseteq \{v\}$	$\Box(\Box\phi \rightarrow \phi)$
Shift coreflexivity $wR_fv \rightarrow (vR_fu \rightarrow v = u)$	$v \in \bigcap f(w) \rightarrow \bigcap f(v) \subseteq \{v\}$	$\Box(\phi \rightarrow \Box\phi)$
Shift identity $wR_fv \rightarrow (vR_fu \leftrightarrow v = u)$	$v \in \bigcap f(w) \rightarrow \bigcap f(v) = \{v\}$	$\Box(\phi \leftrightarrow \Box\phi)$

The upshot of the above discussion is that an iffy operator analysis, which respects (some version of) the Import-Export Principle and avoids the collapse of "*if-then*" into the material conditional, is possible. But the implementation of this idea, at least the one we presented here, based largely on Gillies's work, comes at the cost of making the operator "shifty." Now, strictly speaking, in classical modal logic, there is no sense in which two necessity operators whose interpretations depend on different accessibility relations are "the same operator." With this in mind, does the shifty analysis interpret "*if-then*" as one operator that is sensitive to a contextual parameter, as Gillies would have it, or as a potentially different operator depending on its embedding environment, as Gibbard saw it? As we see it, ultimately this is a question of theory design and philosophical predilections. Such considerations should not stop anyone from adopting this solution to Gibbard's problem. If there is to be a decisive criterion for the choice, it has to be found elsewhere.

3.2 *The restrictor analysis*

Building on work by Lewis (1975), Kratzer (1986; 1991a; reprinted in Kratzer, 2012) took an approach that differs from the one of the previous subsection in an important respect. We refer to it as the "restrictor analysis," following much of the literature, since it assumes that "*if*" clauses restrict the quantificational domains of modal operators. This is not strictly speaking its most distinguishing feature, however: as we have just seen, it is also possible to define iffy binary operators whose semantics is defined in terms of modal operators restricted by the antecedent.

Rather, what sets Kratzer's approach apart from the alternative is that she does not assume that each "*if*" clause introduces its own modal operator. Thus while each conditional contains at least one modal operator, and each "*if*" clause restricts exactly one modal operator, there is nonetheless no one-to-one correspondence between them. Instead, Kratzer treats the two main ingredients of the anlaysis separately: the semantic import of "*if*" clauses is reduced to their role as restrictors, whereas the operators they restrict are taken to be introduced independently. Thus the interpretation of a conditional can be given as follows.

Definition 21 (Restrictedly strict necessity): *"*if *p, q" is true at w relative to f iff q is true at w relative to $f[p]$.*

Definition 21 assumes implicitly that the consequent q contains a modal whose modal base is targeted for modification by the "*if*"-clause. As a simple schematic example, a conditional headed by "*must*" is interpreted simply by evaluating the matrix clause, including the modal, relative to the shifted modal base.

(11) "*if p, Must q*" is true at world w relative to f

a. iff "*Must q*" is true at w relative to $f[p]$;

b. iff for all worlds v such that $wR_{f[p]}v$, q is true.

So far this looks like little more than a notational variant of the shifty relativized strict necessity from Definition 20 above. After all, the worlds such that $wR_{f[p]}v$ are just the worlds such that $wR_f v$ at which p is true. However, the crucial difference is that the operator which quantifies over those worlds is not introduced by the "*if*" clause. Staying with the right-nested conditionals we discussed above in connection with the Import-Export Principle, this allows Kratzer to assume that (12a) has only one modal operator which is restricted by both "*if*"-clauses:

(12) a. If you are back before eight, then, if the roast is ready, we will have dinner together.

b. If you are back before eight and the roast is ready, we will have dinner together.

Schematically, we can give the interpretation as in (13):

(13) "*if p, (if q, Must r)*" is true at w relative to f

a. iff "*if q, Must r*" is true at w relative to $f[p]$;

b. iff "*Must r*" is true at w relative to $f[p][q]$;

c. iff for all worlds v such that $wR_{f[p][q]}v$, q is true at v.

It is easy to show that the successive update of f with two propositions assumed in (13) is equivalent to a single update with the both of those propositions:

(14)
$$\begin{aligned} f[p][q] &= \lambda w[f[p](w) \cup \{q\}] \\ &= \lambda w[\lambda v[f(v) \cup \{p\}](w) \cup \{q\}] \\ &= \lambda w[[f(w) \cup \{p\}] \cup \{q\}] \\ &= \lambda w[f(w) \cup [\{p\} \cup \{q\}]] \\ &= \lambda w[f(w) \cup \{p, q\}] \end{aligned}$$

Thus it is a built-in feature of Kratzer's analysis that right-nested conditionals like (12a) are interpreted as single conditionals with conjunctive antecedents, as in (12b).

Furthermore, if we follow Kratzer in assuming that right-nested conditionals like (12a) do not involve two modal operators to begin with, the questions we were grappling with in the last subsection in connection with Gillies-style iffy operators do not arise. The Import-Export Principle loses its bite as a constraint on admissible interpretations for the modal operators involved.

That said, the restrictor analysis is not without its own conceptual and empirical problems. We discuss some of them below in section 4. For now, we proceed with a discussion of some issues affecting both iffy operator and restrictor analyses.

3.3 Ordering sources

So far our discussion has been limited to the one-parameter interpretation relative to a modal base, without reference to an ordering source. This was largely for simplicity of exposition, but also because some of our formal results could not be as straightforwardly proven if an ordering source were involved. In fact, as we mentioned above, it is quite common in the philosophical literature to assume that the ordering source is absent or semantically inert in the case of epistemic modality. We find this assumption ultimately unsatisfactory, but a full-fledged exploration of the formal consequences of adding the ordering source would go beyond the scope of this survey.

Above, we implemented an interpretation of conditionals in terms of universal quantification over a restricted set of possible worlds. While most authors find this basic idea intuitively appealing, it does not solve some fundamental problems afflicting the material conditional analysis. In particular this concerns the validity of certain inference patterns involving conditionals. As an example, consider *strengthening of the antecedent*, given in (15):

(15)
$$\frac{\text{“}\textit{if p, Must q}\text{”}}{\text{“}\textit{if (p and r), Must q}\text{”}}$$

This inference is valid for the material conditional as well as for all variants of the variably strict analysis we discussed earlier in this section. It is easy to see why the latter is the case: Both schemata in (15) call for the evaluation of q at all accessible worlds at which their respective antecedents are true. The relevant subset of accessible worlds in the premise is a superset of the corresponding set in the conclusion, hence a sentence that is true at all worlds in the former is *a fortiori* true at all worlds in the latter.

Antecedent strengthening is not valid for English conditionals, however. For instance, the sentences in (16) can both be true. This would be unexpected if the inference in (15) were valid.

(16) a. If the windows are dark, John will be asleep.

b. If the power is out and the windows are dark, John may not be asleep.

The Kratzer-style framework offers a straightforward solution to this problem: if we assume that the interpretation of conditionals depends on an *ordering source* in addition to the modal base, then the inference in (15) is no longer predicted to be valid. To see this, it is sufficient to note that while the set of accessible "*p*" worlds is necessarily a superset of the accessible "*p and q*" worlds, the *best* "*p*" worlds under the ordering source may not contain any "*q*" worlds at all, thus the two sets of best worlds may in fact be disjoint.

For illustration, consider the addition of a simple *stereotypical* ordering source that encodes an expectation that there is no power outage. Then under the induced preorder, worlds at which there is a power outage are strictly outranked by (or more far-fetched than) worlds at which there is none. Assuming that the windows can be dark for other reasons, the interpretation of (16a) in effect involves quantification only over worlds at which the windows are dark and there is no power outage. This is clearly distinct from the interpretation of (16b), whose antecedent explictly restricts the interpretation to worlds at which there is a power outage.

The ultimate reason why an ordering source can invalidate the inference of antecedent strengthening is that the restriction by the antecedent affects the modal base, not the ordering source. Intuitively, one can think of the role of the ordering source in the interpretation of a conditional as implicitly and defeasibly strengthening the antecedent. Most importantly, the interpretation rules for modals (see section 2) ensure that the implicit information contributed by the ordering source is *overridden* by conflicting information in the modal base, including conflicting information contributed by the antecedent itself. It is for this reason that the simultaneous truth of (16a,b) does not even involve a shift in context: Both sentences may be true relative to the very same modal base and ordering source.

Similar points can be made about other, similarly problematic inference patterns involving conditionals and modals, notably contraposition (17) and a modalized version of vacuous truth (18):

(17) $\dfrac{\text{“}\textit{if p, Must q}\text{”}}{\text{“}\textit{if not q, Must not p}\text{”}}$

(18) $\dfrac{\text{“}\textit{Must not p}\text{”}}{\text{“}\textit{if p, Must q}\text{”}}$

For reasons of space, we do not discuss these problematic inferences, or the reason why the use of ordering sources helps in circumventing them, any further. The example of antecedent strengthening illustrates the general kind of solution that this framework affords: The ordering source may shift the relevant domains of quantification in such a way that pairs of sentences which instantiate the patterns in syntactic form can nonetheless have logically independent truth conditions.

As a final comment, notice that the discussion in this subsection has been neutral on the choice between the iffy operator approach and the restrictor approach. Both face similar challenges with regard to the inference patterns discussed, and the introduction of ordering sources works similarly for both.

3.3.1 The operators IF$(\cdot,\cdot)$ *and* $\rightarrow$ *again* In the last subsection we considered patterns of inference between conditionals. A separate set of questions is posed by inferences between conditionals and sentences that do not involve any modal expressions, at least according to standard analyses. Specifically, this concerns the validity of inferences by *modus ponens* (also known as detachment), shown schematically in (19):

(19) *"if p, then q"*
"p"
———————
"q"

Recall that one of Gibbard's premises, (Gb), was that the conditional operator entails the corresponding material conditional. This ensures the validity of (19). As we showed above (Proposition 5), under the modal analysis of the conditional operator, (Gb) amounts to the claim that the accessibility relation is *reflexive* (alternatively, that the corresponding conversational background is *realistic* in Kratzer's terms). While we do not intend to give a full-fledged argument in favor or against (Gb) for natural language conditionals, we want to raise two issues that deserve consideration in this respect.

Firstly, as we have seen in section 2.2, there is nothing intrinsically wrong with pairing modals with non reflexive accessibility relations. The modal flavor could be doxastic, rather than epistemic, in the sense that it is made up from beliefs that can be mistaken. If we allow for this possibility, the modal analysis of conditionals (both in the iffy operator version and in the restrictor version) does not warrant (Gb), *pace* Kratzer's (2012) claim that Gibbard's principles (Gb) and (Gc) are "rather obvious" (p. 87) and "generally accepted" (p. 88).

For the graded modality framework, Kratzer (2012) proposes that modal bases have to be realistic, which, applying it to conditionals, might lead one to expect that (Gb) holds for the restrictor semantics. But this is not the case: even if the modal base f is realistic and, hence, the world of evaluation w is always contained in $\bigcap f(w)$, the ordering source may be such that w is outranked by other worlds in $\bigcap f(w)$. In that case, w is not in the domain of the quantification that gets restricted by the antecedent and is said to entail the consequent, and we can easily construe a counterexample to (19) and hence the validity of (Gb).

Secondly, from an empirical point of view, the validity of (19) has been questioned in particular for conditional clauses that contain deontic modals or conditionals in their consequent. For example, consider nested conditionals like (20) (from McGee, 1985):

(20) a. If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

b. A Republican will win the election.

c. If it's not Reagan who wins the election, it will be Anderson.

McGee discusses the examples in (20) against the background of an opinion poll just before the 1980 US presidential election that showed Reagan, one of the Republican candidates, decisively ahead of the Democrat Carter, who, in turn, was decisively ahead of Anderson, the second Republican in the race. He argues that the poll result gives us good reason to believe (20a) and (20b), but not (20c). We obtain, therefore, a counterexample to the validity of conditional detachment. Although the status of these examples is a matter of ongoing discussion, they suggest that (Gb) is not as obviously valid as it might seem at first glance.

These findings tie in nicely with the results obtained for the modal analysis in the absence of reflexivity of the accessibility relation, and, in particular, as resulting from a possibly non realistic ordering source.

4. Current Debates and Open Issues

4.1 Covert operators

The restrictor analysis does not assume that "*if*" clauses introduce their own modal operators, assuming instead that those modal operators are introduced independently. In one version of the restrictor analysis, proposed by S. Kaufmann (2005b), all tensed clauses contain (possibly covert) epistemic modal operators, which play a role in the temporal interpretation. Under this view, it is these modals that are targeted by the antecedent, hence conditionals do not require any special treatment. Kratzer's own approach, in contrast, calls for a special mechanism for conditionals lacking an overt modal. For such cases, Kratzer assumes that a *covert epistemic necessity modal* is inserted by default.

Now, once the option of a covert epistemic necessity modal is allowed into the theory, unless its distribution is constrained in some way, conditional sentences with overt modals in the consequent become ambiguous: the antecedent can restrict either the overt modal in the consequent, or a covert epistemic necessity modal that outscopes it. We call these two readings the *Overt* and *Covert Operator* construal (terms from M. Kaufmann in Schwager, 2006). They are illustrated in (21) and (22) for necessity and possibility modals, respectively (the superscript "*m*" stands for whichever modal flavor is associated with the overt modal in the consequent, and "*e*" marks the epistemic covert operator):

(21) If φ, {must, have to, ought, should, ...} ψ.

a. $[\text{IF}\ \varphi]\ \Box^m \psi$ — Overt Conditional Operator (OCO)

b. $[\text{IF}\ \varphi]\ \Box^e [\Box^m \psi]$ — Covert Conditional Operator (CCO)

(22) If φ, {might, may, can, ...} ψ.

a. $[\text{IF}\ \varphi]\ \Diamond^m \psi$ — Overt Conditional Operator (OCO)

b. $[\text{IF}\ \varphi]\ \Box^e [\Diamond^m \psi]$ — Covert Conditional Operator (CCO)

In the absence of further assumptions, the CCO even allows for an arbitrary number of epistemic necessity operators stacked on top of the overt modal:

(23) $[\text{IF}\ \varphi]\ \Box^e \ldots \Box^e [\text{Modal}^m \psi]$ — Covert Conditional Operator (CCO)

In contrast, the iffy operator analysis does not predict such an ambiguity. Instead, each *if*-clause introduces, in virtue of its denotation, exactly one operator of epistemic necessity. This means that the restrictor analysis and the iffy operator analysis make the same predictions whenever the former postulates a logical form in which the antecedent restricts a single covert epistemic necessity modal. But the iffy operator analysis cannot generate a reading that would correspond to what is obtained from the overt operator construal, nor does it allow for a sequence of covert epistemic operators as in (23). In order to compare the two analyses, the question is therefore whether there is evidence for or against one of the readings that is predicted either by the overt conditional operator construal only, or by a sequence of covert epistemic modal operators.

The variability in the accessibility relations (or, on the graded modality account, modal bases and ordering sources) that the modal operators in a particular structure can draw on, gives rise to a wide range of possible interpretations for each of the construals. This makes it hard to present definite arguments for or against a particular construal. Nevertheless, the following two generalizations strike us as valid.

Firstly, stacks of multiple epistemic operators are semantically inert, as we demonstrate below: they do not give rise to additional readings over and above their lowest element. This is good news for the iffy operator approach, which invariably stacks an additional operator of epistemic necessity on top of even an overt epistemic modal in the consequent; it is also good news for the restrictor analysis, which can freely generate sequences of covert epistemic necessity modals.

Secondly, there seem to be clear cases of conditionals involving overt modals in the consequent that are not being modified by the "*if*" clause. Again, this is good news for the iffy-operator approach, which predicts that no overt modal can ever be restricted by the *if* clause (directly). It is favorable for the restrictor analysis, which at least predicts these readings to exist. However, the restrictor analysis would also predict the existence of the corresponding OCO reading. The question then is whether any readings are attested that specifically call for a OCO analysis. It turns out that the evidence in this respect is less clearcut.

In the remainder of this subsection we discuss each of these two observations in turn.

4.1.1 Stacking epistemic modals Conditionals with overt epistemic modals in the consequent do not necessarily furnish evidence for or against the presence of an additional covert epistemic modal operator. If we assume that the relevant epistemic modal bases have the properties of positive and negative introspection (i.e., the associated accessibility relations are transitive and euclidean), a silent epistemic modal outscoping an overt one (or another covert one) has no semantic effect. To see this, consider (24a,b):

(24) a. If the lights are on, John must be at home.

b. If the lights are on, John may be at home.

Suppose in each case the overt modal is outscoped by a covert one as in (25a,b):

(25) a. $[\text{IF } \textit{lights}]\, \Box^e [\Box^e \textit{home}]$

b. $[\text{IF } \textit{lights}]\, \Box^e [\Diamond^e \textit{home}]$

The sentences in (25) are interpreted as in (26a) and (26b), respectively:

(26) a. $[\text{IF } \textit{lights}]\, \Box^e [\Box^e \textit{home}]$ is true at w relative to f iff for all worlds v such that $wR_{f[lights]}v$, for all worlds u such that $vR_{f[lights]}u$, h is true at u.

b. $[\text{IF } \textit{lights}]\, \Box^e [\Diamond^e \textit{home}]$ is true at w relative to f iff for all worlds v such that $wR_{f[lights]}v$, for some world u such that $vR_{f[lights]}u$, h is true at u.

Without the covert epistemic operator, the right-hand sides in (26a,b) would lack the outer universal quantification over worlds v such that $wR_{f[lights]}v$. But this would not affect the truth conditions. For whenever an accessibility relation R is transitive and euclidean, then for any world v such that wRv, the worlds accessible from v are just those accessible from w. Moreover, the properties of transitivity and euclidity are preserved under the shift from R_f to $R_{f[lights]}$ (see Lemmata 3 and 4 in the Appendix). But this means that, quite generally, $\Box^e\Box^e p$ and $\Box^e\Diamond^e p$ are true just in case $\Box^e p$ and $\Diamond^e p$ are, respectively.

Thus conditionals with overt epistemic operators in the consequent do not constitute evidence against the presence of a covert epistemic operator outscoping them. By the same argument, however, they do not constitute evidence against an iffy operator analysis, either. We have to conclude, then, that these cases do not yield decisive arguments for or against either approach.

4.1.2 OCO and CCO with non epistemic overt modals To date, the most comprehensive discussion of the different predictions resulting from the OCO and CCO construals is due to Frank (1996). She adduces a series of arguments that are meant to show that at least deontic modals in the consequent of a conditional cannot be restricted (directly) by the *if* clause. According to her, a conditional has to be interpreted along the lines of the CCO construal unless the widest scoping operator in the consequent expresses epistemic necessity or possibility (in which case that operator can be directly modified by the "*if*" clause).

Frank's first piece of evidence is that the OCO construal predicts sentences like (28) to be necessarily true (see also Zvolenszky 2002, after whom this observation is sometimes dubbed the *Zvolenszky problem*):

(27) If Jesse robbed the bank, Jesse should rob the bank.
(Frank, 1996: 30 (32b))

(28) If Britney Spears drinks Coke in public, then she must drink Coke in public.
(Zvolenszky, 2002: her (10))

Clearly, if the domain of quantification is restricted to antecedent worlds, the ones among them that are preferred by the ordering source must also be antecedent worlds, and the conditionals in (27) and (28) should be trivially true. This prediction is at odds with speaker intuitions.

Secondly, Frank notes (pp. 31f) that the interpretation of deontic modals in the consequent can depend on the value of the deontic ordering source at worlds other than the world of evaluation. The truth of an example like (29) is independent of whether the new laws are passed in the actual world: it is sufficient to consider (sufficiently similar) worlds at which they are passed and ask what deontic statements would be true at those worlds.

(29) If the new laws for opening hours of shops go through, salespeople will have to work longer. (Frank, 1996: p199 (51))

Thirdly, Frank points out that the interpretation of conditionals with overt deontic modals (of necessity or possibility, cf. 30) does not change if a quantificational adverbial like "*necessarily*" is inserted into the consequent (she offers similar examples for dispositional modals):

(30) a. If Max stays at his Grandma's, he is (*necessarily*) allowed to walk the dog.
(Frank, 1996: 50 (68a))

b. If Max stays with his Grandma, he (*necessarily*) must walk the dog. (Frank, 1996: 50 (68b))

Frank argues that this intuition cannot be captured if, in the absence of the adverb, a conditional like (30a) is interpreted as expressing that among the best worlds at which Max stays at his Grandma's there are some at which he walks the dog. Since this latter interpretation is in fact predicted by the OCO analysis, Frank concludes that the OCO analysis generally does not apply to non epistemic modals, thus *if* clauses always restrict an epistemic modal. She restricts the OCO construal to conditionals with epistemic modals in their consequent.

On the other hand, the OCO construal does seem to make better predictions for cases where the actual rules and regulations enforce the consequent for situations as described in the antecedent. Consider (31) (modified from Frank):

(31) If Max buys a car, he will have to pay car taxes.

Intuitively, (31) does not make a claim about what is in general compatible with the rules and regulations that are in force in each of the worlds in which Max buys a car. Firstly, these rules could be quite different from the ones at the actual world. This effect was needed for (29), but it seems irrelevant to and potentially problematic for (31). Secondly, as long as the laws at a world w at which Max buys a car do not *require* Max to buy a car, there will be deontically accessible worlds v (from w) at which he does not buy a car and does not pay car taxes. Assuming that v does not violate the law in any other ways, even if the laws at w require car owners to pay taxes, world v is deontically accessible and (31) is predicted to be false.

The OCO construal faces no such problem: there is only one modal operator, "*have to*", whose modal base is updated with the information that there is an instance of Max buying a car, and (31) is predicted to be true if, among the worlds compatible with the modal background, those that are deontically best according to the laws in the actual world are such that all car owners (hence, also Max) pay car taxes.

Frank (1996: 54) suggests that the predictions of the CCO construal can be improved for such cases if one assumes that modal bases are anaphoric (see also Geurts, 1999). Another way of obtaining a similar result would be to supply the modal in the consequent with a modal base that takes into account either all the facts up to Max's buying of a car (*historical alternatives*, Thomason, 1984; for an application to conditionals with imperative consequents, see Kaufmann and Schwager, 2011), or at least the ones relevant for the evaluation of what follows from the tax laws: in our case, arguably the fact that Max has a car and does not fall under any particular exceptions concerning tax paying would have to be among them). Nevertheless, improving the predictions of the CCO analysis for sentences like (31) along these lines comes at the expense of reintroducing the Zvolenszky problem, which, originally, had only besieged the OCO analysis. For either construal, we would now have to argue that deontic conditionals are infelicitous if they are true independently of the restriction imposed by the ordering source (Frank, 1996: 54).

4.1.3 If *clauses and non modal quantifiers* Summarizing the discussion so far, we saw that an epistemic operator which outscopes an epistemic modal has no discernible semantic effect. We further saw arguments—albeit somewhat inconclusive—that deontic modals in the consequent tend to be outscoped by epistemic necessity modals as a general rule. These arguments are equally good news for the iffy operator analysis and for the restrictor analysis. The one construal that is *only* available through the restrictor approach—non epistemic modals directly modified by the "*if*" clause—is of dubious empirical status. All of this may be seen as an argument in favor of the iffy operator analysis, which is conceptually simpler, and easy to implement compositionally.

In fact, the best evidence that "*if*" clauses do not always introduce their own epistemic necessity operators comes from data *not* involving modal operators at all. Following Lewis (1975), the restrictor analysis is applied not only to modal operators but also to quantificational adverbials like those in (32a) or quantifiers over individuals (32b):

(32) a. If a student tries, she usually succeeds. $\approx$
Normal situations of a student trying are situations of the student succeeding.

b. No student will fail if he tries. $\approx$
No student who tries will fail.

Such cases fall outside the scope of the present chapter but it would seem that their apparent equivalence with the corresponding paraphrases in terms of explict restriction of the domain of quantification provides strong evidence in favor of the restrictor analysis and against the iffy operator analysis. Unfortunately, things are more complicated than that. Von Fintel and Iatridou (2002) point out that, contrary to what might be expected under the restrictor analysis, by far not just any restrictive relative clause can be freely replaced with the corresponding *if* clause. Rather, the replacement seems to be licensed only in cases that involve a *generic* flavor; but this may once again be introduced by modal elements, rather than the quantifiers themselves.

4.2 Further readings: some questions about homogeneity

The original version of Kratzer's framework assumes that the quantificational force of a modal expression is determined lexically, whereas its modal flavor is determined contextually (Kratzer, 1978, 1981, 1991b). Specifically, different modal flavors are taken to differ only in the particular value assigned to the two parameters: the modal base and the ordering source. Despite their intuitive appeal, these assumptions have been challenged on both empirical and theoretical grounds.

Empirically, cross linguistic investigations show that modal expressions that are open to a wide range of modal flavors may be less pervasive than Kratzer's assumptions lead us to expect (cf. Nauze, 2008). For instance, Rullmann *et al.* (2008) discuss the situation of Salish, where it seems to be the modal flavor that is determined lexically, while the quantificational force is determined contextually.

Theoretically, many modal flavors have been shown to raise problems for a treatment in terms of a modal base and an ordering source the standard framework. For some of them, the problems become particularly obvious in connection with conditionals.

Individuating the knowledge of a particular individual or group of individuals as the background relevant to the interpretation of epistemic modals proves difficult (DeRose, 1991; Egan, 2007; Egan *et al.*, 2005; von Fintel and Gillies, 2007; von Fintel and Gillies, 2008; Hacking, 1967; MacFarlane, 2011; Stephenson, 2007, among others). Proposals to solve this problem range from dependence on a more fine-grained point of evaluation (containing a point of view in addition to world and time) to assuming that epistemic modals can contribute to the discourse an entire set of propositions (von Fintel and Gillies, 2011). Concerns about the interpretation of embedded occurrences give rise to proposals of epistemic modals as sensitive to information states (Yalcin, 2007). If some, or maybe all conditionals rely on quantification over epistemically or doxastically accessible worlds, these questions are certainly highly relevant for conditionals as well.

Dispositional possibility modals are problematic in that they seem to express that an agent is in a position to enforce a certain outcome rather than that the outcome is merely compatible with her dispositions (see in particular Thomason, 2005 drawing on Austin, 1956). For partly orthogonal concerns regarding the occurrence of actuality entailments and their interaction with aspectual oppositions, see Bhatt (1999); Hacquard (2006).

Teleological modality cannot be modeled by a naïve analysis that combines a circumstantial modal base with an ordering source that specifies the goals of the relevant agent. Typically, a specific goal α (possibly made explicit as *in order to* α) is considered inviolable together with the circumstance under which it has to be achieved. Again, for possibility modals it is a point of controversy whether compatibility is strong enough: intuitively, a sentence like (33) expresses that taking the bus is a (reliable) means to get to campus, and is not merely compatible with your getting there.

(33) (To go to campus) you can take the bus.

This issue carries over to the treatment of *anankastic conditionals*, which specify the relevant goal in an antecedent clause and contain the teleological modal in the consequent.

(34) If you want to go to UConn, you should take Storrs Road.

For discussion, see von Fintel and Iatridou (2005); Lauer and Condoravdi (2012); Nissenbaum (2005); Sæbø (2002); von Stechow *et al.* (2005); Werner (2006). Finlay (2009) proposes to reduce all prioritizing modality to teleological modality.

Prioritizing modality as occuring in the process of practical deliberation has been shown to interact with the knowledge of the relevant agents in a way that cannot be captured straightforwardly by the standard framework of Kratzerian graded modality (see discussion above, and in particular Cariani *et al.* 2013; Charlow 2010; von Fintel 2012; Kolodny and MacFarlane 2010).

For discussion of more general concerns with predictions for, in particular deontic and bouletic modality and the upward monotonicity resulting from Kratzer's theory of modals, see Büring (2003); von Fintel (2012); Heim (1992); Lassiter (2011); Levinson (2003). For discussions of lexically triggered non assertive discourse effects of modals and the relation between modals and imperative clauses, see Han (1999); Ninan (2005); M. Kaufmann (2012); Portner (2007).

In addition to possible inadequacies for single modal flavors, the recent literature also notes problems with the quantificational force assigned to particular lexical items and argues that *ought* and *have to* should be treated as expressing weak versus strong necessity. For various cross linguistic patterns and typological observations see von Fintel and Iatridou (2008); Rubinstein (2012).

Besides these semantic concerns, the standard uniform account is challenged by a series of observations that concern the syntax-semantics interface. This includes the debate as to whether epistemic and deontic modals (or also, different types of deontic modals) differ in whether they are raising or control predicates (Bhatt, 1998; Brennan, 1993; Wurmbrand, 1999), or why modal flavors may influence the scope of the modal operator with respect to tense or negation (see Hacquard, 2006, for an attempt to derive these distinctions within a variant of Kratzer's unified account).

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Appendix: Proofs

Proposition 1. *$\Box\phi \rightarrow \phi$ is valid on a frame $\langle W, R\rangle$ iff R is reflexive.*

Proof. ($\Rightarrow$) Suppose R is not reflexive. Thus there is a world $w \in W$ such that $\neg wRw$. Let V be such that (i) for all worlds v, if wRv then $v \in V(\phi)$; and (ii) $w \notin V(\phi)$. By (i), $w \in V(\Box\phi)$; by (ii), $w \notin V(\phi)$. Thus the formula is false at w, hence not valid on $\langle W, R\rangle$.

($\Leftarrow$) Suppose R is reflexive and the formula is not valid. Thus there is a world $w \in W$ such that (i) $w \in V(\Box\phi)$ and (ii) $w \notin V(\phi)$. By (i) and the reflexivity of R, $w \in V(\phi)$, contradicting (ii). $\Box$

Proposition 2. *Let w be a possible world, f, g two conversational backgrounds, and p a proposition. If f, g meet the Limit Assumption, then*

a. p is a necessity at w, f, g iff for all $v \in \mathbf{O}(w, f, g)$, $v \in p$.

b. p is a possibility at w, f, g iff for some $v \in \mathbf{O}(w, f, g)$, $v \in p$.

Proof. We only give the proof for the case of necessity.

($\Rightarrow$) Suppose (i) p is a necessity at w, f, g and (ii) there is a world $v \in \mathbf{O}(w, f, g)$ such that $v \notin p$. By (i) and (ii) there is a world $u \in \bigcap f(w)$ and $u \leq_{g(w)} v$ such that for all $z \leq_{g(w)} u$, $z \in p$. Since $v \in \mathbf{O}(w, f, g)$ and $u \leq_{g(w)} v$, by the definition of $\mathbf{O}(w, f, g)$ (Def. 12), $v \leq_{g(w)} u$. However, by assumption, $v \notin p$. So contrary to (i), p is not a necessity at w, f, g.

($\Leftarrow$) Suppose p is not a necessity at w, f, g. Thus for some world $v \in \bigcap f(w)$, for all $u \in \bigcap f(w)$ such that $u \leq_{g(w)} v$, there is a $t \in \bigcap f(w)$ such that $t \leq_{g(w)} u$ and $t \notin p$. By the Limit Assumption (Def. 13), this implies that there is a world $u^* \in \mathbf{O}(w, f, g)$ such that $u^* \leq_{g(w)} v$ and there is a $t^* \in \bigcap f(w)$ such that $t^* \leq_{g(w)} u^*$ and $t^* \notin p$. By the definition of $\mathbf{O}(w, f, g)$ (Def. 12), $t^* \in \mathbf{O}(w, f, g)$, hence it is not the case that all worlds in $\mathbf{O}(w, f, g)$ are in p. $\Box$

Proposition 3 (Edgington, 1986). *Assume that*

(Ea) "if-then" *denotes a truth function, call it* $\mathcal{F}_{\text{if}}$*;*

(Eb) *sentences of the form* "if (p and q) then p" *are tautologous;*

(Ec) *conditionals can be false.*

Then $\mathcal{F}_{\text{if}}$ *is the truth function of the material conditional.*

Proof. By Assumption (Eb), the sentence "*if p and q then p*" is a tautology, i.e., $\mathcal{F}_{\text{if}}(\text{“}p \text{ and } q\text{”}, p) \equiv 1$. There are four cases:

	p	q	"p and q"	$\mathcal{F}_{\text{if}}(\text{“}p \text{ and } q\text{”}, p) = 1 = \ldots$
(a)	1	1	1	$\mathcal{F}_{\text{if}}(1,1)$.
(b)	1	0	0	$\mathcal{F}_{\text{if}}(0,1)$.
(c)	0	1	0	$\mathcal{F}_{\text{if}}(0,0)$.
(d)	0	0	0	$\mathcal{F}_{\text{if}}(0,0)$.

Cases (a–d) exhaust three of the four possible combinations of arguments of $\mathcal{F}_{\text{if}}$. By Assumption (Ec), conditionals can be false, hence $\mathcal{F}_{\text{if}}(1,0) = 0$. □

Proposition 4 (Gibbard, 1981). *Let* $\textsc{if}(\cdot\,,\cdot)$ *be a binary propositional operator with the following properties:*

(Ga) $\textsc{if}(p\,, \textsc{if}(q\,,r))$ *and* $\textsc{if}(p \wedge q\,, r)$ *are logically equivalent.*

(Gb) $\textsc{if}(p\,,q)$ *logically implies the material conditional* $p \rightarrow q$*.*

(Gc) If p logically implies q, then $\textsc{if}(p\,,q)$ *is a logical truth.*

Then $\textsc{if}(\cdot\,,\cdot)$ *is the material conditional.*

Proof. From (Ga) it follows immediately that (35a) and (35b) are equivalent:

(35) a. $\textsc{if}(p \rightarrow q\,, \textsc{if}(p\,,q))$

b. $\textsc{if}((p \rightarrow q) \wedge p\,, q)$

Next, by standard propositional logic (35b) is equivalent to $\textsc{if}(p \wedge q\,, q)$, which according to (Gc) is a logical truth; hence, by the equivalence, (35a) is a logical truth as well. Finally, from (Gb) it follows that (35a) implies the material conditional in (36a); and since (35a) is a logical truth, so is (36a). But we also know from (Gb) that (36b) is a logical truth.

(36) a. $(p \rightarrow q) \rightarrow \textsc{if}(p\,,q)$

b. $\textsc{if}(p\,,q) \rightarrow (p \rightarrow q)$

This establishes the equivalence of $\textsc{if}(p\,,q)$ and $p \rightarrow q$. □

Proposition 5. $\textsc{if}^f(p\,,q) \rightarrow (p \rightarrow q)$ *is valid iff* R_f *is reflexive.*

Proof. ($\Rightarrow$) Suppose R_f is not reflexive. Thus there is a world w such that $\neg wR_f w$. Let V be such that (i) for all worlds v, if $wR_f v$ then $v \in V(p \rightarrow q)$; and (ii) $w \in V(p \wedge \neg q)$. By (i), $\textsc{if}^f(p\,,q)$ is true at w. By (ii), $(p \rightarrow q)$ is false at w. Thus the formula is not valid.

($\Leftarrow$) Suppose R_f is reflexive and the formula is not valid. Thus there is a world w at which (i) $\textsc{if}^f(p\,,q)$ is true and (ii) $(p \rightarrow q)$ is false. By (i) and the reflexivity of R_f, $(p \rightarrow q)$ is true at w, contradicting (ii). □

Proposition 6. *$\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big) \rightarrow \textsc{if}^f(p \wedge q\,,\,r)$ is valid iff R_f is shift reflexive.*

Proof. ($\Rightarrow$) Suppose R_f is not shift reflexive. Thus there are worlds w, v such that wR_fv but not vR_fv. Now let V be such that (i) $v \in V(p \wedge (q \wedge \neg r))$; (ii) for all worlds u, if wR_fu and $u \in V(p)$, then for all worlds t, if uR_ft then $t \in V(q \rightarrow r)$. By (ii), $\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big)$ is true at w; but by (i), $\textsc{if}^f(p \wedge q\,,\,r)$ is false at w. Thus the formula is not valid.

($\Leftarrow$) Suppose R_f is shift reflexive and the formula is invalid. Thus there is a world w such that (i) $\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big)$ is true at w and (ii) $\textsc{if}^f(p \wedge q\,,\,r)$ is false at w. By (ii), there is a world v such that wR_fv and $v \in V(p \wedge (q \wedge \neg r))$. Since wR_fv and $v \in V(p)$, by (i), for all u such that vR_fu, $u \in V(q \rightarrow r)$. Since R_f is shift reflexive, vR_fv, so $v \in V(q \rightarrow r)$. But this contradicts (ii). □

Lemma 1. *For any conversational background f and proposition p, $R_{f[p]} \subseteq R_f$.*

Proof. wR_fv iff $v \in q$ for all $q \in f(w)$. $f[p] = \lambda w.f(w) \cup \{p\}$, so for all v such that $wR_{f[p]}v$, $v \in q$ for all $q \in f(w)$, hence wR_fv . □

Lemma 2. *If R_f is shift reflexive, then $R_{f[p]}$ is shift reflexive for any proposition p.*

Proof. Assume that $R_{f[p]}$ is not shift-reflexive. This means that there are $w, v \in W$ such that $wR_{f[p]}v$ but $\neg vR_{f[p]}v$. Now, wR_fv by Lemma 1, and vR_fv since R_f is shift-reflexive. Furthermore, (i) $v \in p$ since $wR_{f[p]}v$; and (ii) $v \in \bigcap f(v)$ since vR_fv . By (i) and (ii), $v \in \bigcap f[p]\,(v)$, hence $vR_{f[p]}v$, contrary to assumption. □

Lemma 3. *If R_f is transitive, then $R_{f[p]}$ is transitive for any proposition p.*

Proof. Assume that $R_{f[p]}$ is not transitive. This means that there are w, v, u, such that $wR_{f[p]}v$ and $vR_{f[p]}u$, but $\neg wR_{f[p]}u$. By Lemma 1, wR_fv and vR_fu , thus wR_fu since R_f is transitive. Furthermore, (i) $u \in p$ since $vR_{f[p]}u$; and (ii) $u \in \bigcap f(w)$ since wR_fu. By (i) and (ii), $u \in \bigcap f[p]\,(w)$, hence $wR_{f[p]}u$, contrary to assumption. □

Lemma 4. *If R_f is euclidean, then $R_{f[p]}$ is euclidean for any proposition p.*

Proof. Assume that $R_{f[p]}$ is not euclidean. Thus there are w, v, u such that $wR_{f[p]}v$ and $wR_{f[p]}u$, but $\neg vR_{f[p]}u$. By Lemma 1, wR_fv and wR_fu , thus vR_fu since R_f is euclidean. Furthermore, (i) $u \in p$ since $wR_{f[p]}u$; and (ii) $u \in \bigcap f(v)$ since vR_fu . By (i) and (ii), $u \in \bigcap f[p]\,(v)$, hence $vR_{f[p]}u$, contrary to assumption. □

Proposition 7. *$\textsc{if}^f(p \wedge q\,,\,r) \rightarrow \textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big)$ is valid iff R_f is shift coreflexive.*

Proof. ($\Rightarrow$) Suppose R_f is not shift coreflexive. Thus there are worlds w, v, u such that wR_fv , vR_fu , and $v \neq u$. Let V be such that (i) $v \in V(p \wedge q)$; (ii) $u \in V(q \wedge \neg r)$; and (iii) for all t such that wR_ft and $t \in V(p \wedge q)$, $t \in V(r)$. By (iii), $\textsc{if}^f(p \wedge q\,,\,r)$ is true at w; but by (i) and (ii), $\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big)$ is false at w, hence the formula is not valid.

($\Leftarrow$) Suppose R_f is shift coreflexive and the formula is not valid. Let w be such that (i) $\textsc{if}^f(p \wedge q\,,\,r)$ is true at w, and (ii) $\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big)$ is false at w. By (ii), there are v, u such that wR_fv , vR_fu , $v \in V(p)$ and $u \in V(q \wedge \neg r)$. Since f is shift coreflexive, $v = u$. Thus $v \in V(p \wedge (q \wedge \neg r))$, contradicting (i). □

Theorem 1. *$\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big) \leftrightarrow \textsc{if}^f(p \wedge q\,,\,r)$ is valid iff R_f is shift identical.*

Proof. The formula is valid iff R_f is shift reflexive and shift coreflexive (by Propositions 6 and 7) iff R_f is shift identical (by set theory). □

Theorem 2. *The following are jointly valid iff R_f is the identity relation.*

a. $\textsc{if}^f\big(p\,,\,\textsc{if}^f(q\,,\,r)\big) \leftrightarrow \textsc{if}^f(p \wedge q\,,\,r)$

b. $\textsc{if}^f(p\,,\,q) \rightarrow (p \rightarrow q)$

Proof. From Theorem 1 and Proposition 5. □

Proposition 8. $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big) \rightarrow \textsc{if}^f(p \wedge q\,,\,r)$ *is valid iff R_f is shift reflexive.*

Proof. ($\Rightarrow$) Suppose R_f is not shift reflexive. Thus there are w, v such that wR_fv and $\neg vR_fv$. Let V be such that (i) $v \in V(p \wedge (q \wedge \neg r))$; and (ii) for all u, if wR_fu and $u \in V(p)$, then for all t, if uR_ft then $t \in V((p \wedge q) \rightarrow r)$. By (ii), $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big)$ is true at w. By (i), $\textsc{if}^f(p \wedge q\,,\,r)$ is false at w. Hence the formula is not valid.

($\Leftarrow$) Suppose R_f is shift reflexive and the formula is not valid. Thus there is a world w at which (i) $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big)$ is true and (ii) $\textsc{if}^f(p \wedge q\,,\,r)$ is false. By (ii) there is a world v such that wR_fv and $v \in V(p \wedge (q \wedge \neg r))$. Furthermore, since $v \in V(p)$, $wR_{f[p]}v$, hence $vR_{f[p]}v$ since $R_{f[p]}$ inherits shift reflexivity by Lemma 2. But then $\textsc{if}^{f[p]}(q\,,\,r)$ is false at v, hence $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big)$ is false at w, contrary to (i). □

Proposition 9. $\textsc{if}^f(p \wedge q\,,\,r) \rightarrow \textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big)$ *is valid iff R_f is transitive.*

Proof. ($\Rightarrow$) Suppose R_f is not transitive. Thus there are w, v, u such that wR_fv and vR_fu but $\neg wR_fu$. Let V be such that (i) for all t, if wR_ft then $t \in V((p \wedge q) \rightarrow r)$; (ii) $v \in V(p \wedge (q \wedge r))$; and (iii) $u \in V(p \wedge (q \wedge \neg r))$. Then $\textsc{if}^f(p \wedge q\,,\,r)$ is true at w by (i) and $\textsc{if}^p(p, \textsc{if}^{f[p]}\ (q, r))$ is false at w by (ii) and (iii), hence the formula is not valid.

($\Leftarrow$) Suppose R_f is transitive and the formula is not valid. Thus there is a world w at which (i) $\textsc{if}^f(p \wedge q\,,\,r)$ is true and (ii) $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big)$ is false. By (ii) there are worlds v, u such that wR_fv, vR_fu, $v \in V(p)$, and $u \in V(p \wedge (q \wedge \neg r))$. By transitivity, wR_fu. But then $\textsc{if}^f(p \wedge q\,,\,r)$ is false at w, contrary to (i). □

Theorem 3. $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big) \leftrightarrow \textsc{if}^f(p \wedge q\,,\,r)$ *is valid iff R_f is transitive and shift reflexive.*

Proof. From Propositions 8 and 9. □

Theorem 4. *The following are jointly valid iff R_f is reflexive and transitive.*

a. $\textsc{if}^f\big(p\,,\,\textsc{if}^{f[p]}(q\,,\,r)\big) \leftrightarrow \textsc{if}^f(p \wedge q\,,\,r)$

b. $\textsc{if}^f(p\,,\,q) \rightarrow (p \rightarrow q)$

Proof. From Theorem 3 and Proposition 5. □

NOTES

1 It is customary in logic expositions to treat a proper subset of the operators we introduce in (1) as basic, defining others in terms of them. We are not primarily concerned with formal parsimony, however.

2 Note that (N) says that if ϕ is *provable*, then ϕ is necessarily true. This must be distinguished from the statement "$\phi \rightarrow \Box\phi$," which says that if ϕ is *true* then it is necessarily true. The latter is not a theorem of all systems of modal logic.
3 See the next subsection for the missing "only if" direction of the foregoing statement and Kratzer's treatment of inconsistent laws.
4 This is because the set of worlds accessible from w under R_f is determined by the intersection $\bigcap f(w)$, thus any set of propositions with the same intersection gives rise to the same set of accessible worlds.
5 So far, there is little discussion in the literature as to whether one of them is preferable. One possible argument in favor of a syntactic representation of conversational backgrounds could be the apparent possibility of quantification and binding involving conversational backgrounds (e.g., M. Kaufmann, 2012).
6 A binary relation is a preorder iff it is reflexive and transitive. Both properties follow directly from the definition of $\leq_{g(w)}$ in terms of the subset relation.
7 Lewis's formulation of the Limit Assumption just required that the set of best worlds be non empty. Our version is needed to secure the desired consequences in the Kratzer-style framework, in which worlds may be incomparable under $\leq_{g(w)}$.
8 Modal operators, for instance, are propositional but not truth functional—that is, $\Box p$ denotes a proposition, but its truth value is not a function of the truth value of p.
9 We do not discuss the probabilistic line of research here for reasons of space. See S. Kaufmann (2001, 2005c, 2004, 2005a, 2009) and references therein.
10 We confine our discussion to so-called *hypothetical conditionals*. See Bhatt and Pancheva (2006) for a brief survey of additional types that appear to express relations at the speech act level rather than the logical one. The question as to whether these two classes can or should be unified is controversial and goes beyond the scope of our current survey.
11 It is common for authors on this subject to assume that the ordering-source parameter is inert in the case of epistemic modality (e.g., Gillies, 2010; Kolodny and MacFarlane, 2010), thus this simplification is in line with at least some of the literature. We do not in fact endorse it as a general principle but we adopt it in this subsection for purposes of exposition.
12 Here and below we slightly abuse notation by writing "$f[p]$ ' instead of '$f[V(p)]$." The former is strictly speaking incorrect if p stands for a sentence rather than a proposition, as is the case in Definition 20.
13 Stalnaker (2002) proposes deriving a representation of the common ground between a group of agents by taking the transitive closure of the union of their respective belief relations. He assumes that their individual beliefs obey **KD45**, but the property of euclidity is not guaranteed to be preserved in the common ground under his definition. However, shift reflexivity (implied by euclidity) *is* preserved, along with seriality and transitivity.

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