Joint distribution simulation results - augSIR

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Let $\mathbf{x}$ denote a population trajectory, $\mathbf{x}_j$ denote the trajectory for subject j, and $\mathbf{y}$ denote the binomial samples at observation times. Take the initial distribution for infection at time 0 to be such that all individuals are infected at time 0 with probability 1. All simulations performed with population of size four, infectivity parameter 0.5 (obviously not relevant since individuals only recover), and recovery parameter 1.

Simulation #1 - Redraw subject j

Details

For $k = 1, \ldots, K$,

1. Draw $\mathbf{x}$ via Gillespie, and $\mathbf{y}|\mathbf{x}$. Save $(\mathbf{x}, \mathbf{y})_k$.
2. Discard $\mathbf{x}_j$
3. Draw $\mathbf{x}_j^\star$ conditional on $\mathbf{x}_{-j}$, $\mathbf{y}$ using augSIR.
4. Draw $\mathbf{y}^\star|\mathbf{x}^\star$. Save $(\mathbf{x}^\star, \mathbf{y}^\star)_k$

Results

Simulation #2 - Redraw subject j repeatedly, with binomial re-sampling after each redraw

Details

For $k = 1, \dots, K$,

1. Draw $\mathbf{x}^{(1)}$ via Gillespie, and $\mathbf{y}^{(1)}|\mathbf{x}^{(1)}$. Save $(\mathbf{x}^{(1)}, \mathbf{y}^{(1)})_k$
2. For $n = 2, \dots, N$
   (a) Discard $\mathbf{x}_j^{(n-1)}$.
   (b) Draw $\mathbf{x}_j^{(n)}$ conditional on $\mathbf{x}_{-j},\ \mathbf{y}^{(n-1)}$ using augSIR.
   (c) Draw $\mathbf{y}^{(n)}|\mathbf{x}^{(n)}$
3. Save $(\mathbf{x}^{(N)}, \mathbf{y}^{(N)})_k$

Results

Simulation #3 - Redraw subject j repeatedly, no binomial resampling after each redraw

Details

For $k = 1, \ldots, K$,

1. Draw $\mathbf{x}^{(1)}$ via Gillespie, and $\mathbf{y}^{(1)}|\mathbf{x}^{(1)}$. Save $(\mathbf{x}^{(1)}, \mathbf{y})_k$
2. For $n = 2, \ldots, N$
   (a) Discard $\mathbf{x}_j^{(n-1)}$.
   (b) Draw $\mathbf{x}_j^{(n)}$ conditional on $\mathbf{x}_{-j},\ \mathbf{y}$ using augSIR.
3. Save $(\mathbf{x}^{(N)}, \mathbf{y})_k$

Results

Simulation #4 - Redraw all subjects once

Details

Let $x'_j = (\mathbf{x}^\star_1, \ldots, \mathbf{x}^\star_j, \mathbf{x}_{j+1}, \mathbf{x}_M)$ and $x'_{-j} = (\mathbf{x}^\star_1, \ldots, \mathbf{x}^\star_{j-1}, \mathbf{x}_{j+1}, \mathbf{x}_M)$, where M is the population size. For $k = 1, \ldots, K$,

1. Draw $\mathbf{x}$ via Gillespie, and $\mathbf{y}|\mathbf{x}$. Save $(\mathbf{x}, \mathbf{y})_k$
2. For $j = 1, \ldots, 4$:
   (a) Discard $\mathbf{x}_j$
   (b) Draw $\mathbf{x}^\star_j$ conditional on $\mathbf{x}'_{-j},\ \mathbf{y}$ using augSIR
3. Draw $\mathbf{y}^\star|\mathbf{x}^\star$
4. Save $(\mathbf{x}^\star, \mathbf{y}^\star)_k$

Results

1 Simulation #5 - Redraw all subjects repeatedly, with binomial re-sampling after redrawing all subjects once

Let $x'_j = (\mathbf{x}^\star_1, \ldots, \mathbf{x}^\star_j, \mathbf{x}_{j+1}, \mathbf{x}_M)$ and $x'_{-j} = (\mathbf{x}^\star_1, \ldots, \mathbf{x}^\star_{j-1}, \mathbf{x}_{j+1}, \mathbf{x}_M)$, where M is the population size. For $k = 1, \ldots, K$,

1. Draw $\mathbf{x}^{(1)}$ via Gillespie, and $\mathbf{y}^{(1)}|\mathbf{x}^{(1)}$. Save $(\mathbf{x}^{(1)}, \mathbf{y}^{(1)})_k$
2. For $n = 2, \ldots, N$:
   (a) For $j = 1, \ldots, 4$
      i. Discard $\mathbf{x}_j^{(n-1)}$.
      ii. Draw $\mathbf{x}_j^{(n)}$ conditional on $\mathbf{x}_{-j}^{(n-1)\prime}$, $\mathbf{y}^{(n-1)}$ using augSIR.
   (b) Draw $\mathbf{y}^{(n)}|\mathbf{x}^{(n)}$
3. Save $(\mathbf{x}^{(N)}, \mathbf{y}^{(N)})_k$

Results

Simulation #6 - Redraw all subjects repeatedly, no binomial resampling

Details

Let $x'_j = (\mathbf{x}^\star_1, \dots, \mathbf{x}^\star_j, \mathbf{x}_{j+1}, \mathbf{x}_M)$ and $x'_{-j} = (\mathbf{x}^\star_1, \dots, \mathbf{x}^\star_{j-1}, \mathbf{x}_{j+1}, \mathbf{x}_M)$, where M is the population size. For $k = 1, \dots, K$,

1. Draw $\mathbf{x}^{(1)}$ via Gillespie, and $\mathbf{y}|\mathbf{x}^{(1)}$. Save $(\mathbf{x}^{(1)}, \mathbf{y})_k$
2. For $n = 2, \dots, N$:
   (a) For $j = 1, \dots, 4$
      i. Discard $\mathbf{x}_j^{(n-1)}$.
      ii. Draw $\mathbf{x}_j^{(n)}$ conditional on $\mathbf{x}_{-j}^{(n-1)\prime}$, $\mathbf{y}$ using augSIR.
3. Save $(\mathbf{x}^{(N)}, \mathbf{y})_k$