

## CLASS NOTES: INTRO TO DERIVATIVES

Topic: *The Power Rule & Basic Differentiation*

Subject: Calculus I

### I. DEFINITION OF A DERIVATIVE

*Formal Definition (Limit Definition)*

The fundamental mathematical definition of a derivative is based on limits:

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Concept: The derivative measures the instantaneous rate of change of a function.

Geometrically: It is the slope of the tangent line to the curve at a specific point  $(x)$ .

Notation:

Lagrange:  $f'(x)$  or  $y'$

Leibniz:  $\frac{dy}{dx}$  or  $\frac{d}{dx}[f(x)]$

### II. THE POWER RULE (Shortcut for $x^n$ )

Instead of using the limit definition  $\lim_{h \rightarrow 0}$ , we use the Power Rule for polynomials.

The Formula

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}$$

Steps

Bring the exponent ( $n$ ) down to the front (multiply).

Subtract 1 from the exponent.

### III. EXAMPLES

#### A. Basic Polynomials

Rule: Multiply coeff by power, drop power by 1.

Example 1:  $f(x) = x^4$

$$f'(x) = 4x^{4-1} = 4x^3$$

Example 2:  $y = 5x^3$

$$y' = 5 \cdot (3x^2) = \mathbf{15x^2}$$

*B. Special Cases (Constants & Linear)*

Constant Rule: Derivative of a constant number is 0.

$$y = 7 \rightarrow y' = 0 \quad (\text{Horizontal line has slope 0})$$

Linear Rule: Derivative of  $x$  is 1.

$$y = x \rightarrow y' = 1$$

$$y = 8x \rightarrow y' = 8$$

*C. Rewriting Required (Negatives & Fractions)*

Tip: Always rewrite the function as  $x^n$  before deriving.

1. Negative Exponents (Fractions)

Problem:  $f(x) = \frac{1}{x^3}$

Rewrite:  $f(x) = x^{-3}$

Differentiate:

$$f'(x) = -3x^{-3-1} = -3x^{-4}$$

Simplify:  $f'(x) = -\frac{3}{x^4}$

2. Rational Exponents (Roots)

Problem:  $y = \sqrt{x}$

Rewrite:  $y = x^{1/2}$

Differentiate:

$$y' = \frac{1}{2}x^{1/2-1} = \frac{1}{2}x^{-1/2}$$

Simplify:  $y' = \frac{1}{2\sqrt{x}}$

## IV. SUM AND DIFFERENCE RULE

If you have multiple terms, take the derivative of each one separately.

Example:

$$f(x) = 2x^5 - 4x^3 + 7x - 10$$

$$f'(x) = \underbrace{10x^4}_{2\cdot 5} - \underbrace{12x^2}_{4\cdot 3} + \underbrace{7}_{7\cdot 1} - \underbrace{0}_{const}$$

Final Answer:

$$f'(x) = 10x^4 - 12x^2 + 7$$

## V. KEY TAKEAWAYS / CHEAT SHEET

$\frac{d}{dx}(c) = 0$  (Constants disappear)

$\frac{d}{dx}(cx) = c$  (Linear terms leave just the coefficient)

$\frac{d}{dx}(x^n) = nx^{n-1}$  (Power Rule)

Rewrite roots as fractions ( $\sqrt{x} \rightarrow x^{1/2}$ )

Rewrite denominators as negative powers ( $\frac{1}{x} \rightarrow x^{-1}$ )

## VI. MIXED PRACTICE EXAMPLES

Here are some slightly harder problems combining the rules above.

### 1. Polynomial with Fraction Coefficients

$$y = \frac{1}{3}x^6 - x^2 + \pi$$

Step 1: Apply Power Rule. Note that  $\pi$  is just a number (constant).

$$y' = \frac{1}{3}(6x^5) - 2x + 0$$

Answer:  $y' = 2x^5 - 2x$

### 2. Expansion First

$$f(x) = x(x^2 + 4)$$

Step 1: Distribute the  $x$  first. (It's easier than using Product Rule later). Rewrite:  
 $f(x) = x^3 + 4x$

Step 2: Differentiate.

Answer:  $f'(x) = 3x^2 + 4$

### 3. Complex Rewrite

$$g(x) = \frac{3}{x^2} - 2\sqrt[3]{x}$$

Step 1: Rewrite as powers.

$$g(x) = 3x^{-2} - 2x^{1/3}$$

Step 2: Power Rule.

$$g'(x) = 3(-2x^{-3}) - 2\left(\frac{1}{3}x^{-2/3}\right)$$

$$g'(x) = -6x^{-3} - \frac{2}{3}x^{-2/3}$$

$$\text{Answer: } g'(x) = -\frac{6}{x^3} - \frac{2}{3\sqrt[3]{x^2}}$$