相対論的ハミルトニアンのレゾナンスについて

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We consider the following operators:

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\begin{aligned}
L_{\pm}(c): &= \pm\sqrt{c^2(\sigma\cdot D^b)^2+m^2c^4}+v(x)I_2,\\
H(c): &= c\alpha\cdot D^b+mc^2\beta+v(x)I_4,\\
P_{\pm}: &= \pm\frac{1}{2m}(\sigma\cdot D^b)^2+v(x)I_2.
\end{aligned}
$$

Two operators $L_{\pm}(c)$ and $P_{\pm}$ act in $L^2(\boldsymbol{R}^3)^2$, and $H(c)$ in $L^2(\boldsymbol{R}^3)^4$. Here, $b:\boldsymbol{R}^3\to\boldsymbol{R}^3$ a magnetic potential and $v:\boldsymbol{R}^3\to\boldsymbol{R}$ an electric potential, $c>0$ is the velocity of light, $m>0$ the rest mass of a relativistic particle, and $\sigma=(\sigma_1,\sigma_2,\sigma_3)$, where σ_k's are the Pauli matrices, and $D_b=D-b(x)=-i\nabla-b(x)$. α_k's and β are Dirac matrices. Roughly speaking, we assume that $b(x)$ is bounded and $v(x)$ behaves like $|x|^M$ as $|x|\to\infty$ for some $M>0$. Moreover, we assume that $b(x)$ and $v(x)$ are dilation analytic. Then, resonances of them are defined as eigenvalues of the complex scaled operators for them.

We show that $L_+(c)$ and P_+ have purely discrete spectra and there is no nonreal resonance of them, and that the spectra of P_- (for $M\le 2$), $L_-(c)$ and $H(c)$ coincide with $\boldsymbol{R}$. Next we give resonance-free regions for $L_-(c)$ and $H(c)$. Finally, we investigate nonrelativistic limits ($c\to\infty$) of resonances of $L_-(c)$ and $H(c)$.

References

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