→ /・Some examples of successive integrations・/

(%i2)
```
/・
1. change of order of integrations
・/
/・Use vertical cross-sections・/
integrate(integrate(x・y,y,0,x),x,0,1);
/・Use horizontal cross-sections・/
integrate(integrate(x・y,x,y,1),y,0,1);
```

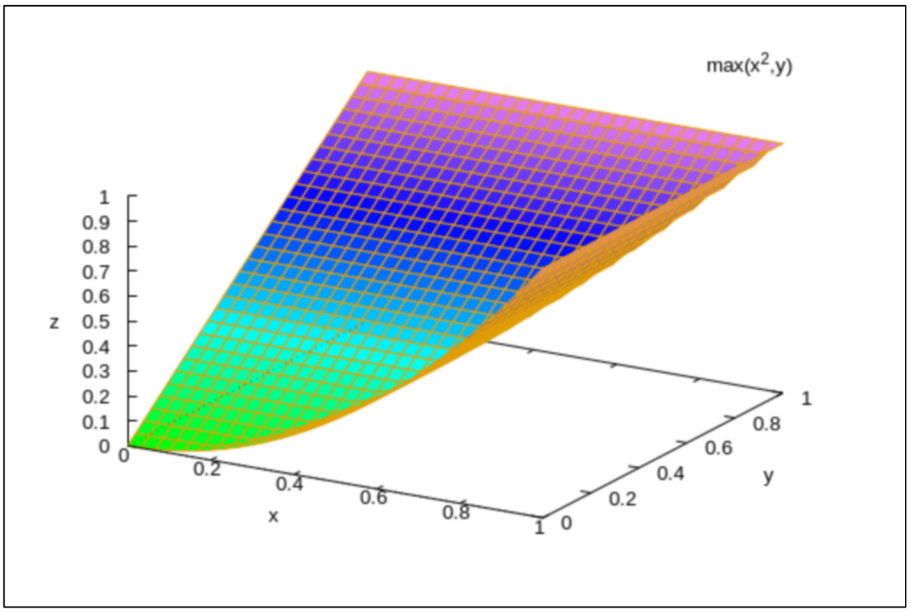
(%o1) $\frac{1}{8}$

(%o2) $\frac{1}{8}$

(%i5)
```
/・
2-1. Maxima does not compute a successive integration
・/
f(x,y):=max(x^2,y);
wxplot3d(f(x,y),[x,0,1],[y,0,1]);
integrate(integrate(f(x,y),y,0,1),x,0,1);
```

(%o3) $f(x,y):=\max(x^2,y)$

(%t4)



(%o4)

(%o5) $\int_0^1 \int_0^1 \max(x^2,y)\,dy\,dx$

(%i7)
```
/*
2-2. Compute the above by splitting the domain
*/
/* Use vertical cross-sections */
integrate(integrate(x^2,y,0,x^2),x,0,1)
+
integrate(integrate(y,y,x^2,1),x,0,1);
/* Use horizontal cross-sections */
integrate(integrate(x^2,x,sqrt(y),1),y,0,1)
+
integrate(integrate(y,x,0,sqrt(y)),y,0,1);
```

(%o6) $\frac{3}{5}$

(%o7) $\frac{3}{5}$

(%i9)
```
/* 3. an example for which one needs to consider the order of integrations 1.
/* One cannot compute by using vertical cross-sections,
and Maxima can compute this. */
integrate(integrate(exp(y^2),y,x,1),x,0,1);
/* Use horizontal cross-sections */
integrate(integrate(exp(y^2),x,0,y),y,0,1);
```

(%o8) $\frac{\%e-1}{2}$

(%o9) $\frac{\%e}{2}-\frac{1}{2}$

(%i11)
```
/* 4. an example for which one needs to consider the order of integrations 2.
/* One cannot compute by using vertical cross-sections.
Maxima cannot compute this too. */
integrate(integrate(x*exp(x*y),x,0,1),y,0,1);
/* Use vertical cross-sections */
integrate(integrate(x*exp(x*y),y,0,1),x,0,1);
```

(%o10) $$\int_0^1 \frac{(y-1)\,\%e^y}{y^2}+\frac{1}{y^2}\,dy$$

(%o11) $\%e-2$

(%i12)
```
/* 5-1. an example for which the polar coordinates are useful. */
/* Use vertical cross-sections */
integrate(integrate(x/(x^2+y^2)^2,y,sqrt(1-x^2),x),x,1/sqrt(2),1)
+
integrate(integrate(x/(x^2+y^2)^2,y,0,x),x,1,sqrt(2))
+
integrate(integrate(x/(x^2+y^2)^2,y,0,sqrt(4-x^2)),x,sqrt(2),2);
Is x positive, negative or zero?positive;
```

(%o12)
$$-\frac{\sqrt{2}\,\pi+2^{3/2}-8}{2^{7/2}}+\frac{\sqrt{2}\,\pi-2^{3/2}}{16}+\frac{\left(1-\frac{1}{\sqrt{2}}\right)(\pi+2)}{8}$$

(%i13)
```
/* 5-2. an example for which the polar coordinates are useful. */
/* Use horizontal cross-sections */
integrate(integrate(x/(x^2+y^2)^2,x,sqrt(1-y^2),sqrt(4-y^2)),y,0,1/sqrt(2))
+
integrate(integrate(x/(x^2+y^2)^2,x,y,sqrt(4-y^2)),y,1/sqrt(2),sqrt(2));
```

(%o13)
$$\frac{1}{2^{3/2}}$$