On the complete integrability of the geodesic flow on pseudo-H type Lie groups

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Pseudo-H type groups form a subclass of all 2-step nilpotent Lie groups. They generalize the notion of Heisenberg and H-type groups and were introduced by P. Ciatti in 2000. More recently, the classification problem for such groups and the question whether they admit integral lattices have gained attention (see [2,3]). In a natural way such groups can be equipped with a left-invariant (pseudo)-Riemannian metric.

In this talk we discuss the problem of complete integrability of the geodesic flow on pseudo-H type groups. There are several sources for the explicit construction of sets of Poisson commuting first integrals. In particular, we explain how to obtain a maximal set which then proves the complete integrability. We mention that in the case of the Heisenberg group this problem has been previously solved in [4]. The problem of how to descend the first integrals to the quotient of the group by a standard lattice constructed in [3] is discussed. Finally, we address the question whether *"one can hear the complete integrability of the geodesic flow"*. More precisely, we ask whether the complete integrability of the geodesic flow of a closed Riemannian manifold is determined by the spectral data. In this framework two examples (not of pseudo-H-type) by D. Schueth are presented.

This talk is based on the joint work [1] with Daisuke Tarama (Ritsumeikan University, Japan).

References

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[4] A. Kocsard, G.P. Ovando, S. Reggiani, *The geodesic flow on nilmanifolds*, Differential Geom. Appl. 49 (2016), 496 - 509.