Spectral theory of Neumann–Poincaré operators and its applications

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The Neumann–Poincaré operator (abbreviated by NP) is a boundary integral operator naturally arising when solving classical boundary value problems using layer potentials. If the boundary of the domain, on which the NP operator is defined, is $C^{1,\alpha}$ smooth, then the NP operator is compact. Thus, the Fredholm integral equation, which appears when solving Dirichlet or Neumann problems, can be solved using the Fredholm index theory. If the domain has corners, the NP operator is not a compact operator any more, but a singular integral operator. The solvability of the corresponding integral equation was established by Verchota. Regarding spectral properties of the NP operator, the spectrum consists of eigenvalues converging to 0 for $C^{1,\alpha}$ smooth boundaries. The NP operator, not self-adjoint, generally, in L^2; can be however realized as a self-adjoint operator in the $H^{-1/2}$-space, provided a new inner product is introduced, and therefore the NP spectrum is real and may consist of a continuous spectrum and a discrete spectrum (and possibly limit points of the discrete spectrum). If the domain has corners, the corresponding NP operator, in fact, possess a continuous spectrum (as well as eigenvalues).

Our main purpose here is to introduce the spectral properties of NP operators. Then we discuss inverse problems and plasmon eigenvalues as applications.