Double fibration transforms with conjugate points

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Double fibration

- Let $\mathcal{G}$ and X be oriented smooth manifolds without boundaries. $N := \dim(\mathcal{G})$ and $n := \dim(X)$. Denote by $d\mathcal{G}$ and dX the orientation forms of $\mathcal{G}$ and X respectively.
- Let Z be an oriented embedded submanifold of $\mathcal{G} \times X$, and let dZ be the orientation form.
- Assume that $N + n > \dim(Z) > N \geqq n \geqq 2$, and set $n' := \dim(Z) - N$ and $n'' := n - n'$. Then $\dim(Z) = N + n'$, $n = n' + n''$ and $n', n'' = 1, \ldots, n-1$.

$$\begin{tikzcd} & Z \arrow[dl, "\pi_{\mathcal{G}}"'] \arrow[dr, "\pi_X"] & \\ \mathcal{G} & & X \end{tikzcd}$$

- We assume that Z is a double fibration, that is, the natural projections $\pi_{\mathcal{G}} : Z \to \mathcal{G}$ and $\pi_X : Z \to X$ are submersions respectively.
- Then $G_z := \pi_x \circ \pi_{\mathcal{G}}^{-1}(z)$ becomes an n'-dim submanifold of X for any $z \in \mathcal{G}$, and $H_x := \pi_{\mathcal{G}} \circ \pi_X^{-1}(x)$ forms an $(N - n'')$-dim submanifold of $\mathcal{G}$ for any $x \in X$.

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References

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