Banking I – Risk

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4. Value at Risk in the Trading Book (Aggregation of Risk)

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Overview

Multiple Sources of Risk

- A typical bank faces multiple sources of risk.
- Interest rate risk is not only coming from one interest rate but from several interest rates along the interest rate curve.
- Moreover, there are different interest rate curves for different instruments.
- Also, there is risk from foreign exchange rates (FX), equity prices, commodity prices and so on.

- **Key question is: How is TOTAL risk measured?**
  I.e.: How can risks be aggregated into a single measure of total risk.

Value at Risk in the Trading Book
Application of / interested parties in total risk measures

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Economic capital	Capital requirements for trading risk
• Risk adjusted performance measurement aims at allocating banks' capital in an optimal way • The risk involved needs to be expressed in a single risk figure. • The risk figure is equal to the economic capital (as opposed to regulatory capital) which needs to be held to cover the risk. • For each business unit, a risk figure needs to be determined. • Also the trading units require a single risk figure.	• The Basel I (1988) regulation was focused on credit risk. • Regulators wanted then to add a complement to the accord to cover trading risk. • There was again the need for a measure of total risk when banks are involved in many markets.

Stylized example of trading book: Setup

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Simple example setup

- Two positions without cash flow risk both maturing after 2 years:
  - Asset
  - Liability
- Cash flows from these positions are:

Position	Year 1	Year 2
Asset ≡ long	+ 100	+ 80
Liability ≡ short	- 80	- 100
Net	+ 20	- 20

Stylized example of trading book: Risk

Simple example setup

- Assume risk-free interest rates of $\boldsymbol{i_{01} = 9}\%$ and $\boldsymbol{i_{02} = 12}\%$.

Stylized example of trading book: Risk

Simple example setup

- Assume risk-free interest rates of $\boldsymbol{i_{01} = 9\%}$ and $\boldsymbol{i_{02} = 12\%}$.

- The current value of the trading book V then is:

PV of first years cash flow

PV of second years cash flow

$$\boldsymbol{V} = \frac{\mathbf{+20}}{\mathbf{1{,}09}} + \frac{\mathbf{-20}}{\mathbf{(1{,}12)^2}} = 18{,}349 - 15{,}944 = 2{,}405$$

- Since cash flow is riskless in the example, risk associated with the value of the trading book comes from potential changes in the interest rates:
    - Changes in the one-year-to-maturity rate $\boldsymbol{i_{01}}$
    - Changes in the two-years-to-maturity rate $\boldsymbol{i_{02}}$

Stylized example of trading book: Risk measure #1 (1/3)

Risk measure #1: Sum of Key VaRs

- Cash flow in t=1 is positive so there is risk from increasing one-year interest rate.
- To measure risk from this source:
  - Analyze historical volatility of the one-year rate.
  - Determine 99% confidence interval: critical interest rate move.
  - Consider the key rate duration as an estimate of marginal change in value due to a marginal change in the considered interest rate (here: one-year). Or explicitly compute new value of position.
  - Then key VaR is the impact on value due to the critical interest rate change considered

$\Delta i_{01} = +1.5\%$

- In our example, we might find the following:
  - Critical change in the one-year interest rate is 1,5%.

$9\% + 1.5\% = 10.5\%$

  - In this case, the one-year rate increases from 9% to 10,5%.
  - Then the value of the first cash flow V_1 is $\boldsymbol{V_1}$=(**+20**)/(**1,105**)=18,1.
  - Then "Key VaR"$_1$=|18,1−18,349|=0,249.

Stylized example of trading book: Risk measure #1 (2/3)

Risk measure #1: Sum of Key VaRs

- To analyze the risk from the two year rate, we have to recognize that it is a liability (negative cash flow). There is risk from decreasing two-years interest rates.
- Accordingly, in our example we might find for the two-years rate:
  - The 99% confidence critical interest rate move is 0,5%. $\Delta i_{02} = -0.5\%$
  - In this case, the two-years rate decreases from 12% to 11,5%. $= 12\% - 0.5\%$
  - Then the value of the second cash flow V_2 is $\boldsymbol{V_2} = \frac{\mathbf{-20}}{(\mathbf{1{,}115})^2}$=-16,087.
  - Then $\text{Key VaR}_2 = |-16{,}087 - (-15{,}944)| = 0{,}143$.
- Our first proposal is to take the sum of the key VaRs as a risk measure:
  - $\text{Total VaR} = \text{Key VaR}_1 + \text{Key VaR}_2 = 0{,}249 + 0{,}143 = 0{,}392$.
  - This represents 0,392/2,405 = approx. 16% of the original value of the position.
- Economically, this would imply a potential loss from a change in the one-year rate, augmented by a potential loss from an adverse change in the two-years rate.
- Therefore, proposal 1 is a very conservative, worst case risk measure.

Stylized example of trading book: Risk measure #1 (3/3)

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$$\text{Total VaR} = \overbrace{\text{Key VaR}_1 + \text{Key VaR}_2}^{\text{sum of Key VaR}}$$

$$= \underbrace{\left(-\text{price}_1 \cdot \frac{D}{1+i_{01}} \cdot \Delta i_{01}\right)}_{\text{Key VaR}_1} + \underbrace{\left(-\text{price}_2 \cdot \frac{D}{1+i_{02}} \cdot \Delta i_{02}\right)}_{\text{Key VaR } 2}$$

$$= -18{,}349 \cdot \frac{1}{1{,}09} \cdot 1.5\% + \left(-(-15{,}944) \cdot \frac{2}{1{,}12} \cdot (-0.5\%)\right)$$

$$= -0.2525 + (-0.1424)$$

$$= -0.3949$$

Stylized example of trading book: Risk measure #2 (1/3)

Risk measure #2: Vector duration

- This method is a generalization of duration analysis, now not only applicable to flat interest rate changes.
- It is also called „twist-in-the-yield-curve“ approach.
- According to the introduced duration measures, we find:

$$TotalVaR =$$
$$\left(-price_1 \cdot \frac{Duration\ of\ first\ cash\ flow}{1+i_{01}} \cdot \Delta i_{01}\right)$$
$$+\left(-price_2 \cdot \frac{Duration\ of\ second\ cash\ flow}{1+i_{02}} \cdot \Delta i_{02}\right)$$
$$= (-p_1 \cdot D_1^* \cdot \Delta i_{01}) + (-p_2 \cdot D_2^* \cdot \Delta i_{02})$$

- In our example, this is:

$$TotalVaR = \left(-18{,}349 \cdot \frac{1}{1{,}09} \cdot \Delta i_{01}\right) + \left(-(-15{,}944) \cdot \frac{2}{1{,}12} \cdot \Delta i_{02}\right).$$

- We now define: $\Delta i_{02} = \alpha \cdot \Delta i_{01}$

Stylized example of trading book: Risk measure #2 (2/3)

Risk measure #2: Vector duration

- The elasticity parameter α allows to model the type of change in the interest rate curve to be modelled, for example:
  - $\alpha = 1$: This is a parallel shift (as discussed with the introduction of the simple flat interest rate curve duration measure).
  - $0 \leq \alpha < 1$: Magnitude of change in two-years rate smaller than in one-year rate, however, same algebraic sign.
  - $\alpha \leq 0$: Interest rate move in opposite directions. Possibly, market anticipates recession / future drop in interest rates. Inversion of the interest rate curve is a special case.

    In the initial example, we assumed $\alpha = \frac{-0,5}{1,5} = -0,33$.

Stylized example of trading book: Risk measure #2 (3/3)

Risk measure #2: Vector duration

- Substituting into the formula yields for our example:

$$TotalVaR = \left|\left[\left(-18{,}349 \cdot \frac{1}{1{,}09}\right) + \left(15{,}944 \cdot \frac{2}{1{,}12} \cdot \alpha\right)\right] \cdot \Delta i_{01}\right|$$
$$= |[-16{,}834 + 28{,}470 \cdot \alpha] \cdot \Delta i_{01}|$$

- TotalVaR is minimal and equal to 0 with an $\alpha = 59{,}13\%$ and positive with alternative α.

- This approach does not make use of correlation. Rather, explicit scenarios are calculated.

Stylized example of trading book: Risk measure #3

$-p_1 \cdot D_1^*$ $-p_2 \cdot D_2^*$

Risk measure #3: „Modern portfolio theory“

- This risk measure applies the Markowitz-/Tobin-portfolio selection theory to our problem of modelling interest rate risk.
- In our example we found: $TotalVaR = |-16{,}834 \cdot \Delta \tilde{i}_{01} + 28{,}470 \cdot \Delta \tilde{i}_{02}|$.
- Thus we have two random variables with some correlation.
- Recall that the Variance of a linear function of two random variables x and y is: $Variance(a \cdot \tilde{x} + b \cdot \tilde{y}) = a^2 \cdot \sigma_x^2 + b^2 \cdot \sigma_y^2 + \left(2 \cdot a \cdot b \cdot \underbrace{\rho_{xy} \cdot \sigma_x \cdot \sigma_y}_{COV(x,y)}\right)$.
- If we assume $\sigma_{i_{01}} = 0{,}006$ and $\sigma_{i_{02}} = 0{,}002$ and $\rho_{i_{01} i_{02}} = 0{,}6$ then we find for the Variance of our example: $\text{Variance}(-16{,}834 \cdot \Delta i_{01} + 28{,}470 \cdot \Delta i_{02}) = \cdots = 0{,}0065$. $Var(..) = (-16.834)^2 \cdot 0{,}006^2 + (28{,}470)^2 \cdot 0{,}002^2 + 2 \cdot (-16.834) \cdot 28.470 \cdot 0{,}006 \cdot 0{,}002 \cdot 0.6$
- From this, we calculate the standard deviation as the square root of the variance.
- Assuming a normal distribution, we can apply the scaling factor of 2,33 to receive the 99% confidence VaR:
  $TotalVaR = 2{,}33 \cdot \sqrt{0{,}0065} = 2{,}33 \cdot \underset{SD}{0{,}081} = 0{,}189$.

0.006 · 2,33 ≈ 0.014 ≅ 1,5% ≡ 1% quantile

0.002 · 2,33 ≈ 0.048 ≅ 0.5%

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Exercise 4.01 for „modern portfolio theory“

t_1 t_2

+20 −20

$i_{01} = 9\%$ $i_{02} = 12\%$

- Consider the simple example discussed so far.
- Further assume $\sigma_{i_{01}}$=0,006 and $\sigma_{i_{02}}$=0,002 and $\rho_{i_{01},i_{02}}$=−1.
- Finally, assume a normal distribution for the considered interest rate changes.
- (1) Compute TotalVaR at a confidence level of 99%.
- (2) Compare your result to risk measure#1.

(1) $Var(-16.834 \cdot \Delta\tilde{i}_{01} + 28.471 \cdot \Delta\tilde{i}_{02})$

$= -16.834^2 \cdot 0.006^2 + 28.471^2 \cdot 0.002^2 + 2 \cdot (-16.834) \cdot 28.471 \cdot 0.006 \cdot 0.002 \cdot (-1)$

$= \dots = 0.025$

$SD(-16.834 \dots) = \sqrt{0.025} = 0.158$

$|VaR| = 2.33 \cdot 0.158 = 0.368$

(2) $\rho = -1$

⇒ interest rates always move in opposite directions

Statistical methods to evaluate Value at Risk (1/2)

Variance-Covariance-Method

- Specify duration for each instrument according to each key rate
- From historical data estimate interest rate volatilities and correlations for all interest rates
- Calculate the portfolio VaR using the durations and assuming normal distribution (as in method #3)
- Underlying assumption is that payoffs are a linear function of interest rates (which is fair for bonds and futures but not for options)

Variance → SD $\xrightarrow{2.33}$ VaR

$$dP_1 = \underbrace{-P_1 \cdot D_1^*}_{\text{constant}} \cdot \tilde{\Delta r}$$

$$dP_2 = -P_2 \cdot D_2^* \cdot \tilde{\Delta r}$$

...

$$dP_{PF} = -P_{PF} \cdot D_{PF}^* \cdot \tilde{\Delta r}$$

$\tilde{\sigma}_{\Delta r_1}$

$\tilde{\sigma}_{\Delta r_2}$

$\rho_{\tilde{\Delta r_1}, \tilde{\Delta r_2}}$

Statistical methods to evaluate Value at Risk (2/2)

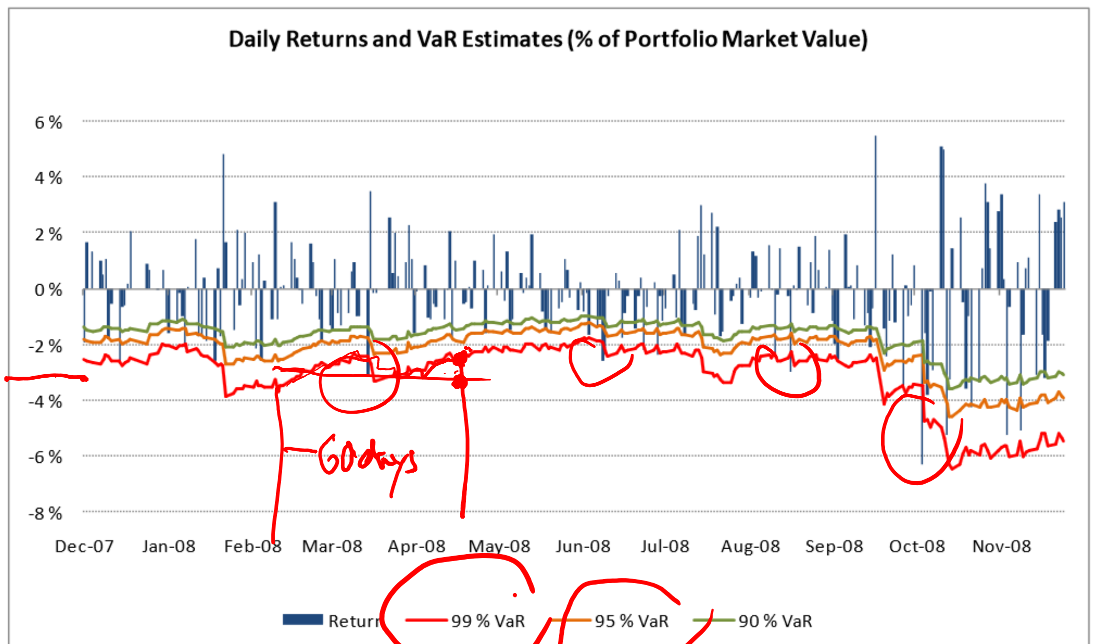
Historical Simulation	• From historical data calculate vector of changes in interest rates (and other risk factors) for each point in time • Then apply all historically observed relative change vectors to current risk factor values • Each time, evaluate the current portfolio value using price formulas with the changed risk factor sets • This yields a vector of portfolio values • Sort and identify VaR • This method is also valid for non-linear instruments such as options
Monte Carlo Simulation	• Assume a Variance-Covariance-Matrix • Make correlated drawings of random variables (interest rates) • Determine vector of relative risk factor changes for each drawing • Then apply all vectors to current risk factor values • Each time, evaluate the current portfolio value using price formulas with the changed risk factor sets • This again yields a vector of portfolio values • Sort and identify VaR • This method is also valid for non-linear instruments

Exercise 4.02 for statistical VaR methods

- Consider a portfolio of two positions:
  - A 10 m€ (nominal) long position in a two year coupon bond with 5% coupon rate.
  - A 5 m€ (nominal) short position in a one year coupon bond with 4% coupon rate.
- Find from Bundesbank.de time serieses (daily values) of the appropriate interest rates for the past 250 trading days.
- Determine the current portfolio value.
- Confidence level is at 99%.
- Determine the portfolio VaR using the Variance-Covariance-Method.
- Determine the portfolio VaR using the Historical simulation method.

Backtesting VaR

- Banks are required to backtest their VaR estimates by ex post confronting these with the actual portfolio returns.
- If models are calibrated correctly, the number of VaR-exceeding losses should be determined by the confidence level.
- For a 250day period, one would expect 2-3 exceeding losses.
- In this case, a „traffic light approach" would change to „yellow" or „red". The latter could have regulatory consequences.



Source: Nieppola, O.: Backtesting VaR-models, Working paper Helsinki School of Economics, 2009, p.56.

Expected shortfall (1/2)

quantile

Expected shortfall as alternative risk measure to VaR

- The VaR only informs about the loss that is not exceeded with a given level of confidence. It does not inform about the magnitude of those losses exceeding the VaR.
- Two portfolios that have identical loss distributions „left“ of the VaR but different loss distribution „right“ of the VaR would have the same VaR-risk measure. Portfolio 2 could have „heavier tails“ than portfolio 1. In principle, a trader could easily make use of this feature by short selling way out of the money options in large quantities. This would increase the profit while the VaR is potentially unaffected. This is because losses from the options potentially are either not captured by cases considered in VaR-simulation routines or only worsen cases beyond the VaR.
- The expected shortfall (ES) measure (or conditional VaR) recognizes also the losses beyond the VaR. It is the expected loss conditional on losses being beyond VaR.
- Other than VaR, the ES fully fulfills a set of theoretically desirable properties („coherence properties“), particularly „subadditivity“. This requires that the risk measure of two portfolios after they are merged should not be greater than the sum of risk measures before they were merged.

Expected shortfall (2/2)

Fundamental Review of the Trading Book (FRTB)

Basel IV

Expected shortfall as alternative risk measure to VaR

- By nature, the expected shortfall gives a risk measure greater than the VaR associated with the specified confidence level.

$$\int_{x_q}^{\infty} f(x)\, x \, dx$$

$$\sum_{x_q}^{\infty} p(x) \cdot x = E(x)$$

x_q

Picture taken from: https://aim.em-lyon.com/2020/10/08/large-financial-networks-via-conditional-autoregressive-expected-shortfall/

Regulatory treatment of market risk (1/3)

Regulatory treatment of market risk: history

- Capital requirements were initially (1988) only on credit risk, no market risk.
- This included losses from a default of a counterparty of a traded security.
- Market risk – defined as losses caused by movements in market prices, interest rates, foreign exchange rates and commodity prices – was included in capital requirements regulation in 1996 by the Basel Committee on Banking Supervision (BCBS) via the market risk amendment.
- The capital charge for market risk remained in force in the packages of Basel II and Basel III.

2011

2004

Bank for Intl. Settlements (BIS)
↳ Basel (CH)

$\boxed{\text{BCBS}}$

„Level playing field"

Regulatory treatment of market risk (2/3)

Regulatory treatment of market risk: standardized model & internal model

- There are two approaches to measure capital requirements for market risk in Basel II / Basel III.
  - In the **standardized approach**, the regulator prescribes the risk measuring procedure and parameters.
  - In the **internal model approach**, the regulator allows banks to choose their methodology themselves. However, there are some mandatory features to be respected when choosing the methodology:
    - VaR must be measured with a confidence level of at least 99%.
    - Historical fluctuations must be recorded using at least one year of data.
    - Holding period (defeasance period) can be extended from 1 day to 10 days at the request of the regulator. In this case, scaling by square root of time can be used.
    - Banks must have a backtesting procedure. If the results exceed the VaR too often, banks will be penalized for not being able to measure risk correctly.
    - In the internal model approach, there is a moving floor of average VaR over the past 60 days.
    - Also, the so determined VaR measure is multiplied by 3 to cover instability of correlations that are used to determine VaR and to cover intraday trading risk.

Regulatory treatment of market risk (3/3)

Regulatory treatment of market risk: adjustments to historical measures

- During the financial market crisis from 2007-2009, the usage of historical data from the period before the measurement date has lead to actual losses continuously exceeding the corresponding VaR estimates.
- Market fluctuations had been low for a significant period of time before the crisis. During the crisis, market fluctuations continuously increased. VaR-estimates increased only with a time-lag.
- Regulators have reacted and since 2011 require banks to use **historical data from stressed market conditions** that have to fit the actual bank portfolio. This requirement is also part of the Basel III package.

„Basel II.5"