Recitation Module 1

Basic terms/definitions to know!!

- The addition rule (disjoint)
- The multiplication rule (independence)
- Odd and evens
- Prime numbers
- Subset vs. proper subset
- Set Builder: A = {x | P(x)}
- Intersection vs. Union
- Powerset

$3 \times 3 \times 5 = 45$

$\{1, 2, 3\}$

$A = \{\{1, 2\}, \{2, 1\}\}$

$\emptyset$

$\{\emptyset\}$

$\{z \mid z \in \mathbb{Z}, z \geq 1, z \leq 1{,}000{,}000\}$

$2^A = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$

Note that this is a dense module with many important concepts. This is by no means an exhaustive summary.

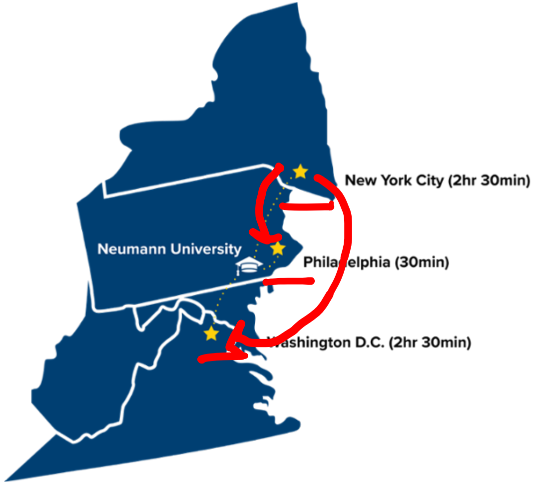
$2^{2^A} = \{\emptyset, \{\emptyset\}, \{\{1\}\}, \{\{2\}\}, \ldots, \{\emptyset, \{1\}\}, \ldots, \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}\}$

What we’re looking for in a solution

- Correct Answer
- Valid Argument
- Correct Citation of Material
- Correct Justification of Applicability of Citation

A First Proof

Prove that New York City is closer to Philadelphia than to Washington DC



Theorem: City A is *closer to* City B than City C if there are less miles between City A and City B than there are between City A and City C.

A First Proof

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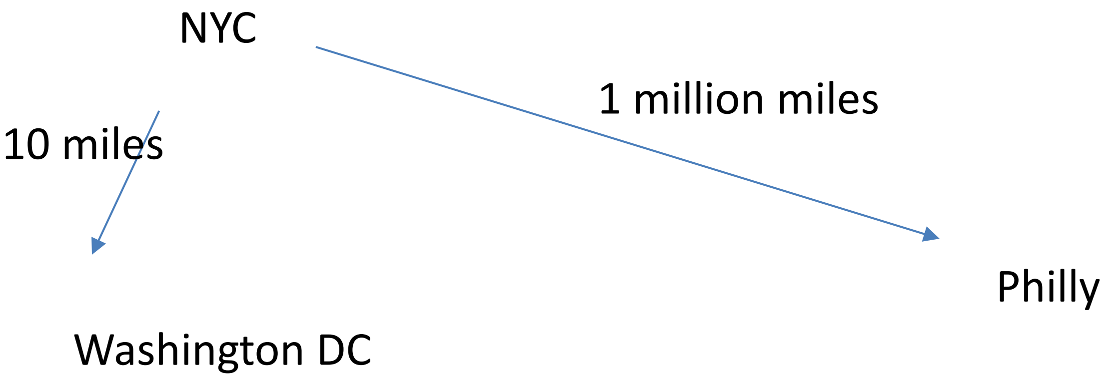

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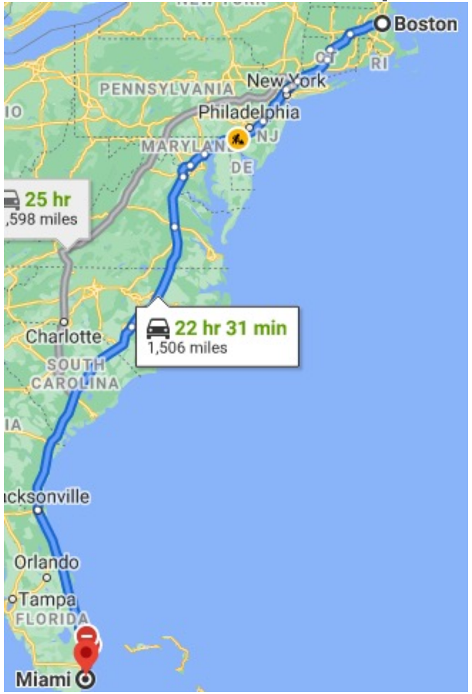

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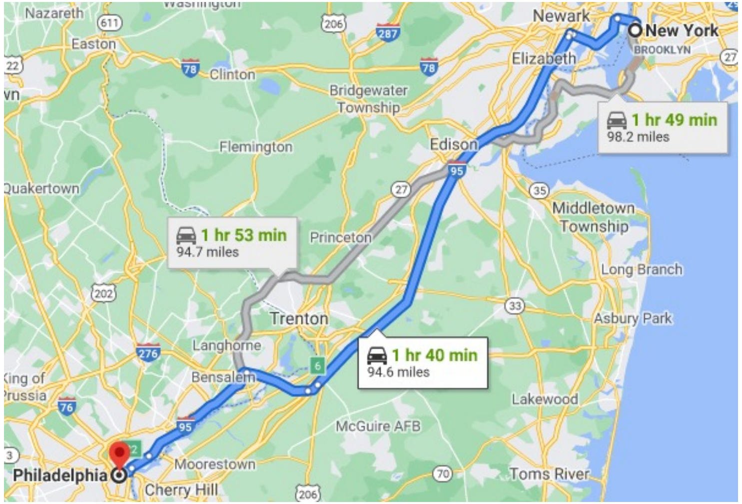
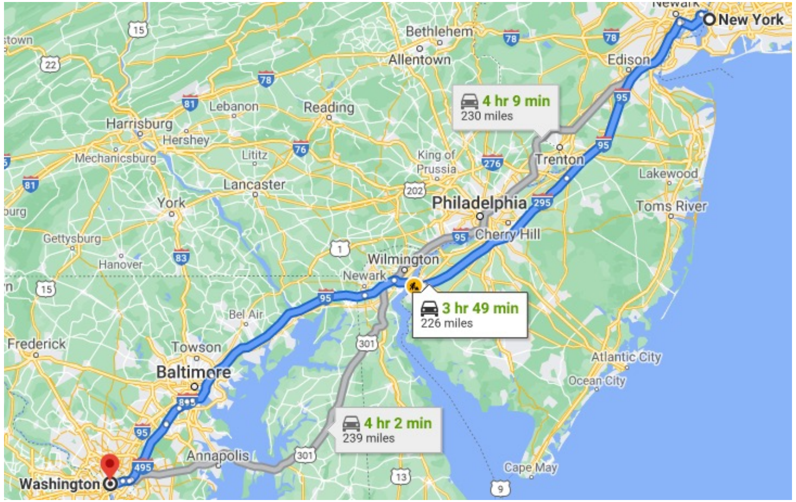




A First Proof

Prove that New York City is closer to Philadelphia than to Washington DC

Proof:
First, note that since the maps that we were given were from Google Maps and they did show the distances between the relevant cities, we can use them. By the maps that we looked up on Google Maps, we know that the distance between NYC and Philly is 94.6 miles and that the distance between NYC and DC is 226 miles. Additionally, per the theorem we were given, City A is *closer to* City B than City C if there are less miles between City A and City B than there are between City A and City C. Therefore, because 94.6 is less than 226, we know that NYC is closer to Philly than to DC.

Question 1

Prove the following: If m and n have different parities, then mn is even.

Answer to Question 1

WLOG (without loss of generality), assume *m* is odd and *n* is even.

$$\begin{aligned} m &= 2k+1,\ \ k \in \mathbb{Z} \\ n &= 2p,\ p \in \mathbb{Z} \\ mn &= (2k+1) \times (2p) \\ &= 4kp + 2p \\ &= 2(2kp + p) \end{aligned}$$

2kp + *p* is an integer, therefore *mn* is even by the definition of even. The proof is exactly the same as if *m* were even and *n* were odd, so we omit it.

Question 2

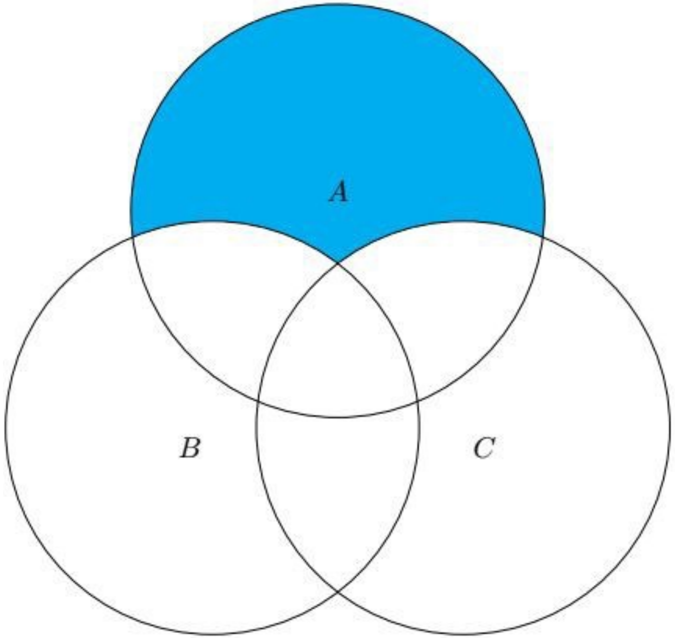
Use an Euler Venn diagram to describe which portion of the diagram corresponds to:

a) $(A \setminus B) \cap (A \setminus C)$

b) $(A \Delta B) \cap C$

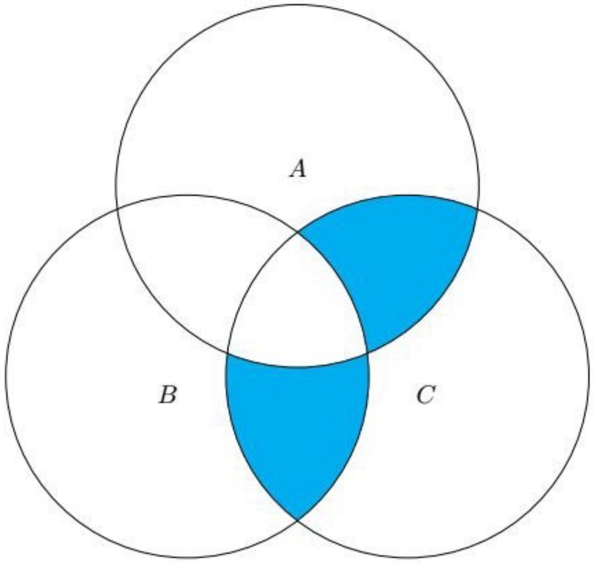
Answer to Question 2

a)



Answer to Question 2 (continued)

b)



Question 3

Prove that, for any integer x, $x^2 + x$ is even.

Answer to Question 3

Using algebra, we can see that $x^2 + x = x(x + 1)$. Now x is an integer, which means it must be either even or odd.

Consider the case in which x is even. In this case, by definition, $x = 2k$ for some integer k. Therefore $x(x + 1) = 2k(2k + 1) = 2(2k^2 + k)$. Since k is an integer, $2k^2 + k$ is an integer. Thus $x^2 + x = 2n$ for some integer n, namely $n = 2k^2 + k$, which means $x^2 + x$ is even.

Now consider the case in which x is odd. In this case, by definition, $x = 2k + 1$ for some integer k. Therefore $x(x + 1) = (2k + 1)(2k + 2) = 4k^2 + 6k + 2 = 2(2k^2 + 3k + 1)$. Since k is an integer, $2k^2 + 3k + 1$ is an integer. Thus $x^2 + x = 2n$ for some integer n, namely $n = 2k^2 + 3k + 1$, which means $x^2 + x$ is even.

We have shown that, in both possible cases, $x^2 + x$ is even. Thus we have proven that $x^2 + x$ is even.