



Recitation Module 12

Lecture Review

- Isomorphic graphs

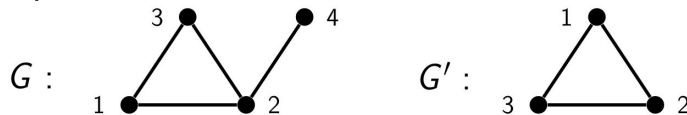
Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are **isomorphic**, notation $G_1 \simeq G_2$, when there is a **bijection** $\beta : V_1 \rightarrow V_2$ such that for any $u_1, v_1 \in V_1$ we have $u_1 - v_1 \in E_1$ **iff** $\beta(u_1) - \beta(v_1) \in E_2$.

- Subgraphs and induced subgraphs

A graph $G_1 = (V_1, E_1)$ is a **subgraph** of the graph $G_2 = (V_2, E_2)$ when $V_1 \subseteq V_2$ and $E_1 \subseteq E_2$. (Beware: not all pairs of such subsets form graphs!)

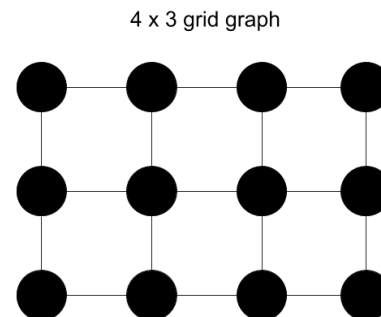
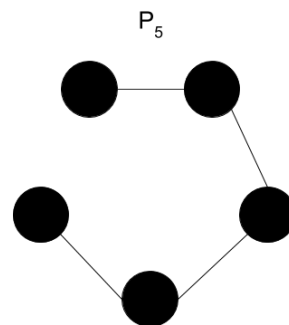
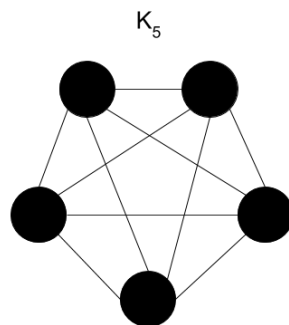
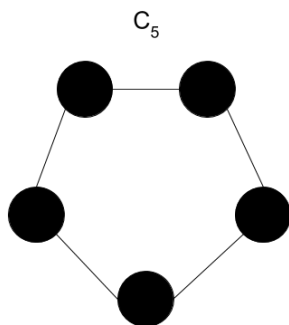
If $G = (V, E)$ is a graph and $V' \subseteq V$ is a set consisting of some of G 's nodes, the subgraph of G **induced** by V' is the graph $G' = (V', E')$ where E' consists of all the edges of G whose endpoints are both in V' .

Example:



Lecture Review

- Cycle, complete, path, and grid graphs



- Closed walks and cycles

A **closed walk** is a walk in which the first and the last vertex are the same.

A **cycle** is a closed walk **of length at least 3** in which all nodes are pairwise distinct, except for the last and the first.

The **length** of the cycle is the length of the closed walk.

Basic terms/definitions to know!!

- Acyclic graphs, trees, and forests

A graph in which there are no cycles is called **acyclic**. The cc's of an acyclic graph are also acyclic.

A graph that is both connected and acyclic is called a **tree**.

Consequently, an acyclic graph is also called a **forest** since all its cc's are trees!

- Cut edges

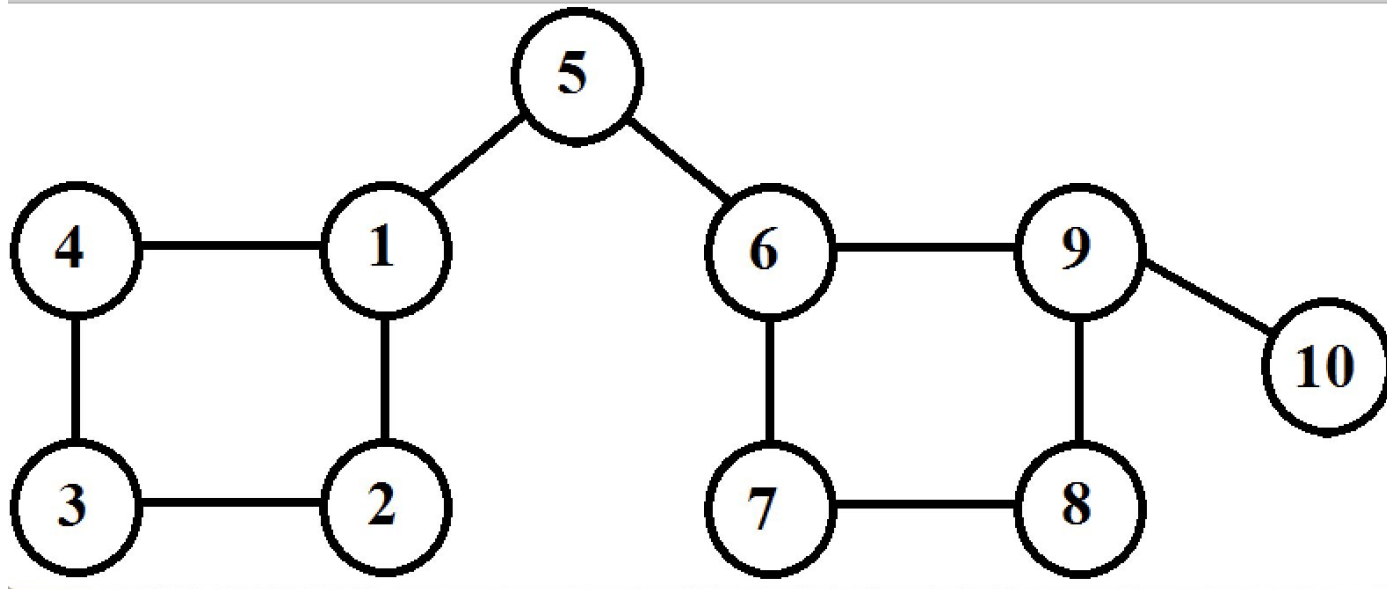
Let $G = (V, E)$ be a graph. An edge of G is a **cut edge** if by removing it we obtain a graph with strictly **more** connected components (cc's) than G .

Basic terms/definitions to know!!

- Properties of trees
 - Every tree is minimally connected:
 - i.e. removing any edge in a tree disconnects it
 - Every tree is maximally acyclic:
 - Adding an edge between **any** two non adjacent vertices in a tree creates a cycle
 - Adding an edge to an acyclic graph creates at most one cycle
 - $|E| = |V| - 1$ (In a forest, $|E| = |V| - |CC|$)
 - Any two distinct vertices of a tree are connected by a **unique** path
- Every tree has at least one leaf which is vertex of degree 1.

Question 1

Identify the cut edges in the following graph:



Question 2

Consider a tree with 5 nodes. Remove 2 edges.

- a. How many CCs will the resulting graph have?
- b. Why must at least one of the resulting CCs have at least 2 nodes?

Question 2 (solution)

Question 3

How many paths of length at least 1 are in C_{100} and K_{100} ?

Question 3 (solution)

Before you go...

Do you have any general questions about this week's assignment? We can't go into minute details about any specific questions, but this is the time to ask any general clarifying/confusing questions!

As always, if you can't talk to us during recitation or office hours, post any questions/anything course related on the forums or email mcitonline@seas.upenn.edu. Ask questions that might be beneficial to other students on the forums, while emailing about more personal questions (regrade requests, etc).