Module 1.5: Subsets and Set-builder Notation

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Lecture Notes

Sets and their elements

A **set** is an unordered collection of distinct **elements**.

Define by enumeration: $V = \{a, e, i, o, u\}$

The elements of V are the vowels of the English alphabet.

Same set : $V = \{u, a, i, e, o\}$ (order does not matter).

Not a set: $\{o, e, a, e\}$ (elements must be distinct).

Notation for "element of" (membership in a set): $\in$.

Letter e is an element of V: $e \in V$.

Letter z is not an element of V: $z \notin V$.

Subset and proper (strict) subset

We say that the set A is a **subset** of the B
and we write $A \subseteq B$
when every element of A is also an element of B

Example: a subset of $V = \{a, e, i, o, u\}$? $\{o, u, i\} \subseteq V$.

Example: not a subset? $\{o, u, a, i, s\} \not\subseteq V$.

A set is its own subset: $V \subseteq V$.

Proper (strict) subset: a subset that is not itself.

Notation for proper subset: $\{o, u, i\} \subsetneq V$.

Empty set

The **empty set** has no elements.

Notation for the empty set: $\emptyset$

The empty set is a subset of *any* set: $\emptyset \subseteq A$.

The empty set is a proper subset of any *non-empty* set! $\emptyset \subsetneq V$

Standard sets of numbers

The **integers** are a proper subset of the **real numbers**: $\mathbb{Z} \subsetneq \mathbb{R}$.

Reminder: $\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

Others: the **rational numbers** $\mathbb{Q}$; the **complex numbers** $\mathbb{C}$.

These sets are **infinite**. We will work mostly with **finite** sets.

We also use:

The **positive integers**: $\mathbb{Z}^+ = \{1, 2, \ldots\}$.

The **natural numbers**: $\mathbb{N} = \{0, 1, 2, \ldots\}$.

0 is a natural number, but it is not a positive integer (in this course!).

Set-builder notation

$$A = \{x \mid P(x)\}$$

This defines A as the set consisting of those elements x that have the property $P(x)$.

Often used in the form: $B = \{x \in X \mid P'(x)\}.$

This is the same as $B = \{x \mid x \in X \text{ and } P'(x)\}.$

Defines B as the subset of X consisting of those elements x that have the property $P'(x)$.

The set-builder notation is also called **set comprehension** notation.

Set-builder examples

The consonants

$$C = \{\ell \mid \ell \text{ is a Latin alphabet letter but not a vowel}\}$$

The positive integers can be defined, for example, in at least two ways:

$$\mathbb{Z}^+ = \{x \in \mathbb{Z} \mid x \geq 1\} \qquad \mathbb{Z}^+ = \{x \mid x \in \mathbb{N} \text{ but } x \neq 0\}$$

The rational numbers can be defined as

$$\mathbb{Q} = \{r \mid \text{there exists } x \in \mathbb{Z} \text{ and there exists } y \in \mathbb{Z}^+ \text{ such that } r = \tfrac{x}{y}\}$$