Questions

This assignment is due in about one week from when the assignment opens. The exact deadline and full instructions for submission are provided in Coursera. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

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1. **[10 pts]** There exists a group of users on LinkedIn with $n \geq 3$ people. This group has a clique of size $n-2$, but does *not* have a clique of size $n-1$. Prove that this group has two distinct independent sets of size 2.

2. **[10 pts]** Show that there is a graph G with exactly 5 nodes where both G and its complement, $\overline{G}$, have chromatic number ≥ 3.

3. **[10 pts]** Suppose that G is a connected graph. It contains *exactly one* spanning tree. Prove that G itself is a tree.

4. **[10 pts]** Let G be a connected graph with at least one cycle. Prove the following statement:
We can remove some edges from G such that the resulting subgraph is bipartite and connected.

5. **[10 pts]** Suppose G be a connected graph with $n \geq 3$ vertices such that $\chi(G) = 3$. Consider a proper 3-coloring of G with colors, purple, yellow, and orange. Prove that there exists a orange node that has both a purple neighbor and a yellow neighbor.