CIT 592 Spring 2022 Homework 10

Questions

This assignment is due in about one week from when the assignment opens. The exact deadline and full instructions for submission are provided in Coursera. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

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1. [10 pts] Let X and Y be independent random variables such that $[X] = 5.3$ and $[Y] = 8.9$. What is the standard deviation of $3X + 2Y$?

2. [10 pts] Suppose James has a garden. Let X be the random variable representing the heights of the flowers in the garden, and let Y be the random variable representing the number of petals the flowers have. Suppose that X and Y are non-negative and independent. Help James prove that $X^2 \perp Y^2$.

3. [10 pts] Suppose you roll $n \geq 1$ fair dice. Let X be the random variable for the sum of their values, and let Y be the random variable for the number of times an odd number comes up. Prove or disprove: X and Y are independent.

4. [10 pts] Suppose that you generate a 12-character password by selecting each character independently and uniformly at random from $\{\mathtt{a}, \mathtt{b}, \ldots, \mathtt{z}\} \cup \{\mathtt{A}, \mathtt{B}, \ldots, \mathtt{Z}\} \cup \{\mathtt{0}, \mathtt{1}, \ldots, \mathtt{9}\}$.

(a) What is the probability that exactly 6 of the characters are digits?

(b) What is the expected number of digits in a password?

(c) What is the variance of the number of digits in a password?

5. **[10 pts]** Suppose Jay shoots a basketball. Let X be the Bernoulli random variable that returns 1 if he makes the shot, and 0 if he misses. Let Y be the Bernoulli random variable (independent of X) that returns 1 if he hits the backboard, and 0 if he does not hit it. X and Y both have parameter 1/2. Let Z be the random variable that returns the remainder of the division of $X + Y$ by 2.

(a) Prove that Z is also a Bernoulli random variable, also with parameter 1/2.

(b) Prove that X, Y, Z are pairwise independent but not mutually independent.

(c) By computing $[X + Y + Z]$ according to the alternative formula for variance and using the variance of Bernoulli r.v.'s, verify that $[X + Y + Z] = [X] + [Y] + [Z]$ (observe that this also follows from the proposition on slide 5 of the lecture segment entitled "Binomial distribution").