Questions

This assignment is due in about one week from when the assignment opens. The exact deadline and full instructions for submission are provided in Coursera. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

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1. [10 pts] Show that each of the following functions is not a bijection by giving either

- an element of the codomain that is not in the range, or
- two elements of the domain that map to the same element in the range.

Be sure to explain why each one is not a bijection!

(a) $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = 8x$

(b) $g : \mathbb{N} \to \mathbb{N}$ given by $g(x) = x + 7$

(c) $h : [11..16] \to [12..16]$ given by $h(x) = \begin{cases} 16 & \text{if } x = 11 \\ x & \text{otherwise} \end{cases}$

(d) $j : \mathbb{N} \to \mathbb{N}$ given by $j(x) = \begin{cases} (x+1)^2 & \text{if } x \text{ is even} \\ 2x + 1 & \text{if } x \text{ is odd} \end{cases}$

(e) $k : [-7..10] \to [0..12]$ given by $k(x) = |x + 2|$

2. [10 pts] Recall that a derangement is a permutation where no element ends up in its original position. In this problem we consider a different, related concept: *deranged anagrams*. We say that an anagram is deranged if no letter ends up in its original position and no letter ends up in the original position of an identical letter. For example, `ffeeco` is a deranged anagram of `coffee`, but `eefcof` is not.

There are $\frac{(2+2+1)!}{2!2!1!} = 30$ anagrams of `radar`. How many of them are deranged?

3. [**10 pts**] Let there be a room that is 8 feet by 8 feet. Suppose that there are 5 people who sit in this room. For simplicity, assume these people are just points. Prove that, among these people, there is some pair that is seated at most $4\sqrt{2}$ feet from each other.

4. [**10 pts**] Alex wants to renew his wardrobe by buying a new item every day. He buys three types of items; shoes, shirts, and pants, and the store never runs out of an item. Alex, being the diligent TA he is, wants to plan out his buying options for the next $n \geq 1$ days, and decides that the easiest way to do this is to create a function f such that for every day i, he will buy item $f(i)$ (where the codomain is $\{shoes, shirt, pants\}$, and he buys one item per day). Because he wants to have full outfits, Alex will only consider functions f that allow him to buy each type of item at least once. How many such functions f can Alex devise?

5. [**10 pts**] After an important and successful meeting, employees of a company shake hands with each other. There are $n \geq 2$ employees at the meeting. Any of these employees can shake hands with any number of the other employees (including zero), but two employees can only shake each others' hands once. A handshake is a mutual event between exactly two employees.

Prove or disprove the claim that there must always be at least two employees who shake the same number of hands.