Questions

This assignment is due in about one week from when the assignment opens. The exact deadline and full instructions for submission are provided in Coursera. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

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1. **[10 pts]** Tautologies are logical expressions that are always true. Decide if the following proposition forms are tautologies using a truth table. Make sure your truth table shows **all** intermediate logical expressions — for example, in showing the truth table for $(p \vee \neg q) \wedge p$, your table should contain separate columns for p, q, $\neg q$, $p \vee \neg q$, as well as the final expression. You should also clearly state your final answer to the question.

(a) $[(\neg p \implies q) \implies (\neg p \wedge q)] \wedge (p \vee q)$

(b) $[p \wedge (q \implies r)] \implies (q \implies r)$

2. **[10 pts]** Michael is a manager for his company, which covers multiple regions of the country.

(a) In region 1, there are 5 unique office buildings to which he would like to assign 10 indistinguishable senior workers and 40 indistinguishable entry-level workers. How many ways are there to assign workers to office buildings, such that each office has at least 1 senior worker and 4 entry-level workers?

(b) After some time, region 1 is idle (no jobs) and is looking to steal employees from the 7 other distinguishable regions. Suppose that all 7 other regions are overworked, each having a large number of indistinguishable senior employees. How many ways can region 1 take exactly 23 senior workers from the other regions, such that there are at least 3 regions from which 6 or more senior employees are taken?

3. [**10 pts**] Three integers are *consecutive* if they immediately follow each other in enumerating the integers. For example, $-13, -12, -11$; or $5, 6, 7$; or $-2, -1, 0$. Prove that if a, b, c are consecutive integers then $a + b + c$ is divisible by 3 but $a^2 + b^2 + c^2$ is *not* divisible by 3.

4. [**10 pts**] How many anagrams of `raspberries` are there that have at least two consecutive `r`'s?

5. [**10 pts**] Consider the following statement.

> There exist integers a and c such that for all integers x if $x \geq a$ then $x^2 < c \cdot x$.

Disprove this statement. (Hint: first write the negation of this statement then prove this negation.)