Module 1.4: Two proofs and primes

MCIT Online - CIT592 - Professor Val Tannen

Lecture Notes

A proof about primes

Proposition. If p, r, s are positive integers such that $p = r \cdot s$ and p is prime then one of r and s is 1 and the other one equals p.

Proof. Assume that $p = r \cdot s$ and p is prime.

Then r is a factor of p $\quad (r \mid p)$.

Since p is prime, $r = 1$ or $r = p$.

In the first case $r = 1$ therefore $p = 1 \cdot s$ and thus $s = p$.

In the second case $r = p$ therefore $p = p \cdot s$.

Dividing both sides of $p = p \cdot s$ by p we get $1 = s$.

Done.

Another proof about primes

Proposition. For all integers x, if $x > 1$, then $x^3 + 1$ is *not* prime.

Proof. Let x be any integer such that $x > 1$ and let's denote $x^3 + 1$ by n.

We are going to show that n has a factor that is neither 1 nor equal to n and therefore n cannot be a prime.

First observe that $x^3 + 1 = (x + 1)(x^2 - x + 1)$. (Multiply and check!)

Let's also denote $x + 1$ by r and $x^2 - x + 1$ by s.
Note that both r and s are factors of n, since $n = r \cdot s = s \cdot r$.

Now, because $x > 1$ we have $r = x + 1 > 1$.

Another proof about primes (continued)

We just derived $r > 1$.

Now, multiply both sides of $r > 1$ with s. We get $r \cdot s > s$.

However $r \cdot s = n$. Therefore $n > s$. underline

We can also show $s > 1$ underline by the following reasoning:

$$
\begin{aligned}
x &> 1 && \text{(Recall assumption)} \\
x^2 &> x && \text{(Multiplying both sides by x.)} \\
x^2 - x &> 0 && \text{(Subtracting x from both sides.)} \\
x^2 - x + 1 &> 1 && \text{(Adding 1 to both sides.)}
\end{aligned}
$$

To summarize, we have shown $1 < s < n$.underline Therefore n has a factor, namely s, that is neither 1 nor equal to n. Done.