Questions

This assignment is due in about one week from when the assignment opens. The exact deadline and full instructions for submission are provided in Coursera. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

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1. **[10 pts]** The 10 decimal digits, $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ are arranged in a uniformly random permutation. We denote by a the integer formed in base 10 by the first five positions in this permutation and by b the integer formed in base 10 by the last five positions in this permutation (either a or b may begin with 0 which in such a case is ignored). For example, if the random permutation is 8621705394 then $a = 86217$ and $b = 5394$. Consider the probability space whose outcomes are these random permutations and a random variable X defined on this probability space such $X = 1$ when the product ab is even and $X = 0$ when that product is odd. Calculate $[X]$.

2. **[10 pts]** The digits 1, 4, and 7 are randomly arranged to form a one digit number and a two-digit number. Each digit can only be used once; for example, if the one-digit number is 7, then the two-digit number is either 14 or 41. What is the expected value of the product of the two numbers?

3. **[10 pts]** You have a standard deck of cards (see the preamble to hw08). You divide this deck in half by selecting uniformly at random 26 of the deck's 52 cards and placing them in your left hand. You hold the remaining 26 cards in your right hand. Prove that the expected number of aces in your right hand is 2.

4. **[10 pts]** Jay has \$500 in the bank when he decides to try a savings experiment. On each day $i \in [1..30]$, Jay flips a fair coin. If it comes up heads, he deposits i dollars into the bank; if it

comes up tails, he withdraws \$10. How many dollars should he expect to have in the bank after 30 days?

5. **[10 pts]** Let A be a set of $n \geq 2$ distinct numbers and let $a_1 a_2 \cdots a_n$ be a permutation of A. For $i = 2, 3, \ldots, n$ we say that position i in the permutation is a *step* if $a_{i-1} < a_i$. We also go ahead and just consider position 1 a step. What is the expected number of steps in a random permutation of A?