Questions

This assignment is due in about one week from when the assignment opens. The exact deadline and full instructions for submission are provided in Coursera. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

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1. **[10 pts]** Matrix Metro is a city with m rows and n columns of buildings, with roads connecting these houses to form a grid. Amy is visiting and wants to walk around the city. Help Amy find the length of the longest path that she can walk (i.e. she never walks to the same building twice). Provide a brief explanation as to why it is the maximum. You can assume $m, n \geq 2$.

2. **[10 pts]** Suppose there are a series of islands connected by bridges. A d-coalition is a group of islands in which every island has exactly d bridges connected to it. Prove that there is no 7-coalition with 39 bridges.

3. **[10 pts]** Suppose there exists a Facebook group with n people, where $n \geq 2$. Each of them has p *or more* connections to other members of the group. Prove that if $p > \frac{n-2}{2}$, then the group is connected (i.e. they are all linked together through some traversal of friendships).

4. **[10 pts]** Suppose there is a shipping network with n shipping locations, where $n \geq 2$. They are connected by roads, but a shipping location could be isolated (i.e. there are no roads to it). Prove that, no matter how these roads are organized, there are at least two shipping locations with the same number of roads.

5. **[10 pts]** Consider a graph G with $2k$ vertices and k edges, where k is a positive integer. Prove that if each vertex in G has degree at least 1, then it has exactly k connected components.