CIT 592 Spring 2022 Homework 2

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1. [10 pts] Let p, q, and r be the following propositions.

p: Comedies are the best type of movie.

q: Everyone loves comedies.

r: Amy will only watch comedies.

Express the following propositions using p, q, r and logical operators.

For this question specifically, a line or two explaining your answer may help, but don't worry too much about providing justification.

(a) Everyone loves comedies and Amy will only watch comedies.

(b) If comedies are not the best type of movie or not everyone loves comedies, then Amy will not watch only comedies.

(c) It is necessary for everyone to love comedies for comedies to be the best type of movie. It is also necessary for everyone to love comedies for Amy to only watch comedies.

(d) Amy will only watch comedies if and only if comedies are the best type of movie.

(e) Amy will not only watch comedies and comedies are the best type of movie.

(f) Comedies being the best type of movie is sufficient for everyone to love comedies and Amy to only watch comedies.

Solution.

(a) $q \wedge r$

(b) $(\neg p \vee \neg q) \implies \neg r$

(c) $p \vee r \implies q$

(d) p iff r

(e) $\neg r \wedge p$

(f) $p \implies (q \wedge r)$

2. [**10 pts**] Let A, B, C be three finite sets.

(a) Prove that if $A \subseteq C$ and $B \subseteq C$ then $A \cup B \subseteq C$.

(b) Prove that if $A \subseteq B$ and $A \subseteq C$ then $A \subseteq B \cap C$.

Solution.

(a) Logical structure: $(A \subseteq C) \wedge (B \subseteq C) \implies A \cup B \subseteq C$
To show an inclusion, $A \cup B \subseteq C$, we consider an arbitrary element x and using the implication proof pattern we prove:
if $x \in A \cup B$, then $x \in C$

We know $A \cup B = \{x | x \in A \vee x \in B\}$
therefore, in the first case where $x \in A$:
$A \subseteq C \implies x \in C$

In the second case where $x \in B$:
$B \subseteq C \implies x \in C$

From above two cases we have concluded $x \in A \vee x \in B \implies x \in C$.
By the definition of union, it follows that $x \in A \cup B \implies x \in C$ and the proof of the inclusion $A \cup B \subseteq C$ is finished.

(b) Logical structure: $(A \subseteq B) \wedge (A \subseteq C) \implies A \subseteq B \cap C$
To show an inclusion, $A \subseteq B \cap C$, we consider an arbitrary element x and using the implication proof pattern we prove:
if $x \in A$, then $x \in B \cap C$

We know $B \cap C = \{x | x \in B \wedge x \in C\}$

In the first case where $x \in A$:
$A \subseteq B \implies \text{x} \in B$

In the second case where $x \in A$:
$A \subseteq C \implies \text{x} \in C$

From above two cases we have concluded $x \in A \implies x \in B \wedge x \in C$.
By the definition of intersection, it follows that $x \in A \implies x \in B \cap C$ and the proof of the inclusion $A \subseteq B \cap C$ is finished.

3. **[10 pts]** Thirteen distinguishable families want to participate in the TV game show "Family Rivalry". Each family consists of a mommy, a daddy, and a child. The game show host must select 20 *people* for the show. The host also wants to see the full family dynamic at play, and thus maximizes the number of times all three members of a family are selected. How many different selections can the host make for this game show?

Solution.

The integer division of 20/3 is 6 and the remainder is 2, which means among the 20 people selected, there are at most 6 full family, and the other 2 are not going to be a full family.

First step: select 6 families from 13 participating families. Since the order here is not relevant, we should calculate the number of combination of 6 out of 13 instead of the number of permutations of 6 out of 13. Therefore there would be $\binom{13}{6}$ ways.

Second step: select 2 people from the remaining 7 families. There are 21 people from 7 family so there would be $\binom{21}{2}$ ways.

Final step: By the multiplication rule the total number of different selections is
$\binom{13}{6} \cdot \binom{21}{2}$

4. [**10 pts**] Instructor Tiny and the 14 TAs of an online course are having lunch at a round table (they are all vaccinated and boosted). Two seatings are the same when everybody has the same left neighbor and the same right neighbor. How many seatings are possible such that two of the TAs, Biggie and Tupac, do *not* sit next to each other?

Solution.

Step1: Since the 15 people are sitting at a round table and only their relative location is relevant, we need to have one person to sit first as a start (probably will be the instructor). Then we can sit the 14 TAs. so the number of total ways to sit them is 14! ways.
Step2: Pick 2 consecutive positions for Bigge and Tupac, and there would be 2 ways.
Step3: Consider Bigge and Tupac as one person, similar to Step 1, we can conclude that the number of total ways to sit 14 people is (14-1)!, aka 13! ways.
Step4: The number of total ways to sit Bigge and Tupac together is 2*13!
Step5: The number of ways such that Bigge and Tupac do not sit together is 14! - 2*13!

5. **[10 pts]** A group of children consists of m families, each family consisting of n siblings. The children decide to arrange themselves in a formation of n rows, each row being a sequence of m children from *different* families. In how many different ways can such a formation be arranged?

Solution.

For every row there are m children and each children from n siblings.
Row 1: Firstly we select first child from his/her n siblings, and then select second child from his/her n siblings... so Row 1 has n^m ways to select m children.
Then we arrange the sequence of the m children: we have $m!$ ways.
Thus the number of ways for Row1 is $n^m * m!$

Row 2: Similar as Row1, except we select first child from his/her remaining (n-1) siblings, and then select the second child from his/her remaining (n-1) siblings .... so Row 2 has $(n-1)^m$ ways to select m children.
Then we arrange the sequence of the m children: we have $m!$ ways.
Thus the number of ways for Row2 is $(n-1)^m * m!$

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Row m: When it comes to Row m there should be only m children left and each one of them is from a different family. Thus we only need to arrange the sequence of them in $m!$ ways.

Conclusion: By the multiplication rule, the number of different ways such a formation can be arranged is $(n^m * m!)$ * $((n-1)^m * m!)$ * $((n-2)^m * m!)$* ... * $m!$, which can be written as $(n!)^m * (m!)^n$