

Questions

This assignment is due in about one week from when the assignment opens. The exact deadline and full instructions for submission are provided in Coursera. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

This problem set will refer to a *standard deck of cards*. Each card in such a deck has a *rank* and a *suit*. The 13 ranks, ordered from lowest to highest, are 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king, and ace. The 4 suits are *clubs* (\clubsuit), *diamonds* (\diamondsuit), *hearts* (\heartsuit), and *spades* (\spadesuit). Cards with clubs or spades are *black*, while cards with diamonds or hearts are *red*. The deck has exactly one card for each rank–suit pair: 4 of diamonds, queen of spades, etc., for a total of $13 \cdot 4 = 52$ cards.

1. [10 pts] Suppose you select a card uniformly at random from a standard deck, and then without putting it back, you select a second card uniformly at random from the remaining cards. What is the probability that both cards have rank no lower than 6, and at least one of the cards is red?
2. [10 pts] Mary and Emily have two distinguishable gardens. Initially, each of the gardens contains four roses and five daisies. Mary first picks a flower uniformly at random from the left garden and moves it to the right garden. Then, Emily picks a flower uniformly at random from the right garden.

What is the probability that Emily picks a rose?

3. [10 pts] For each of three golfers g_1, g_2, g_3 the probabilities of hitting a ball on the green (the desired play!), in a bunker (sand trap), or in a water hazard are given by the following table

(we make the simplifying assumption that these are the only three results of a hit):

golfer	green	bunker	water
g_1	$1/2$	$1/3$	$1/6$
g_2	$1/3$	$1/6$	$1/2$
g_3	$1/6$	$1/2$	$1/3$

The three golfers, playing together, hit one ball each, mutually independently.

- (a) [4 pts] What is the probability that all three balls end up in the water?
- (b) [6 pts] What is the probability that at least one of the balls ends up on the green?
4. [10 pts] 25 participants enter a sushi-making competition! Each of them must make 4 rolls with different proteins: a salmon roll, a tuna roll, a shrimp roll, and a crab roll, for a total of 100 rolls made in the competition. All of the rolls are distinguishable, even if they have the same protein (ex. a salmon roll is distinguishable from all other salmon rolls, as well as all the other rolls with different protein). According to the competition rules, salmon and tuna rolls are *always* made with white rice, while shrimp and crab rolls are *always* made with brown rice. The judges select a sushi roll uniformly at random from the 100 rolls. Without putting it back, they select a second roll uniformly at random from the remaining 99 rolls. What is the probability that both of the rolls are made by the same participant, or both have the same protein, or one has white rice and one has brown rice?
5. [10 pts] Alice and Bob each have a standard deck of cards. First, Alice selects uniformly at random a card from her deck, adds it to Bob's deck and then shuffles his deck. Second, Bob selects uniformly at random a card from his deck, adds it to Alice's deck and then shuffles her deck. Finally, Alice selects uniformly at random a card from her deck. What is the probability that this card's suit is diamond?