Module 8.4: Sketching the Monty Hall Problem

MCIT Online - CIT592 - Professor Val Tannen

Lecture Notes

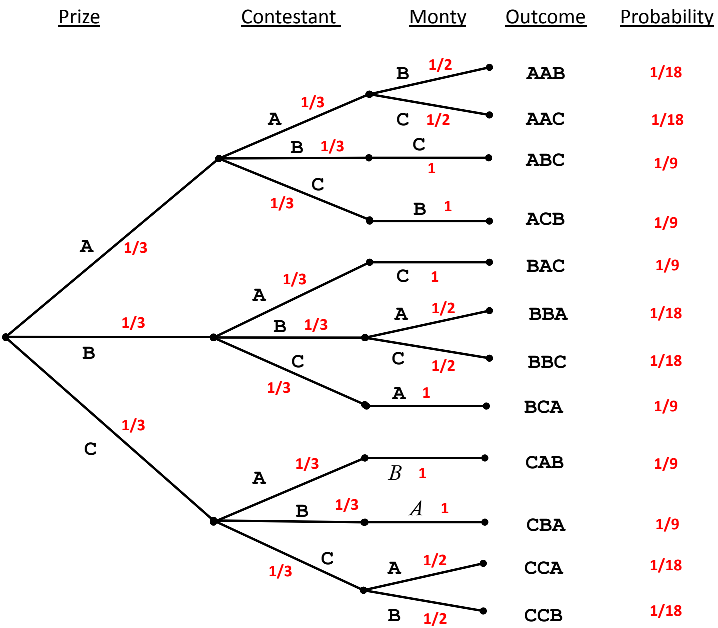
A car and two goats, again!

The Monty Hall Problem. On a game show (hosted by Monty Hall) there are three doors. There is a car behind one of the doors and goats behind the others. The contestant (Ann) chooses a door. Then Monty opens a **different** door behind which there is **always a goat**. Ann can choose whether to **switch** to the **third** door. Is it to Ann's benefit to switch doors?

Answer. We only sketch the answer. For full details see the segment entitled "The Monty Hall problem". Let the doors be A,B and C. The probability space that Ann analyzes has outcomes of the form $D_C D_A D_M$ where D_C is the door that hides the car, D_A is the door Ann chose, and D_M is the door opened by Monty. Because Monty never opens D_C only the following 12 outcomes are possible:

AAB, AAC, ABC, ACB, BAC, BBA, BBC, BCA, CAB, CBA, CCA, CCB

Tree of all possibilities



Assumptions

(a) The car is placed by game staff behind one of the three doors with **equal likelihood**.

(b) Ann does not know where the car is, she chooses her door **independently** of that.

Moreover, we assume that she chooses one of the three doors with **equal likelihood**.

(c) Monty must never reveal the car, so his action does **depend** on where the car was placed and on which door Ann opens.

Moreover, if it turns out that Monty has a choice among two doors, we assume that he opens one of them with **equal likelihood**.

Should the contestant switch doors?

Problem (continued). What is the probability that Ann wins the car if she switches doors? What is the probability that Ann wins the car if she stays with the door she chose first?

Answer (continued). The outcomes contain all the information. For example, in outcome ABC Ann wins the car if she switches doors, while in outcome AAB she wins the car if she stays with her first choice.

The event of interest is $E =$ "win if switch." From the tree:

$$E = \{\mathtt{ABC}, \mathtt{ACB}, \mathtt{BAC}, \mathtt{BCA}, \mathtt{CAB}, \mathtt{CBA}\}$$

$$\Pr[E] = 1/9 + 1/9 + 1/9 + 1/9 + 1/9 + 1/9 = 6 \cdot (1/9) = 2/3$$

On the other hand $\Pr[\overline{E}] = 1/3$. Therefore Ann improves her chances of winning the car (from $1/3$ to $2/3$) if she switches doors.