

Recitation Module 2



Concept Review

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Cardinality of Power Set

Complementary Counting 5 coin flips 2^{5} - 1 = 32 - 1 = 31
  Number of Binary Strings
Partial Permutations \{0, \alpha, x\} \emptyset\alpha, \emptysetx, \alpha\delta, x\delta, x\alpha
 Logical Connectives and A, or V, if then =7, not 7
  Implication, conditional, and equivalence

Combinations

if P, then P2 else P3

Quantifiers

(P_1 \Rightarrow P_2) = (P_1 \Rightarrow P_3)

Quantifiers
  Quantifiers
existential Ix: there exists anx
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Consider the following set of Elves: Haldir, Arwen, Legolas, Galadriel, Tauriel, Luthien, Elrond

- 1. How many groups of these Elves contain Arwen but do not contain Tauriel?
- 2. How many groups of these Elves contain Haldir and Legolas OR contain Elrond?
- 3. How many different permutations can we construct out of the Elves whose names contain at least one letter "a"?

Elves = {Haldir, Arwen, Legolas, Galadriel, Tauriel, Luthien, Elrond}

1. How many groups of these Elves contain Arwen but do not contain Tauriel?

Elves = {Haldir, Arwen, Legolas, Galadriel, Tauriel, Luthien, Elrond}

2. How many groups of these Elves contain Haldir and Legolas OR contain complementary counting Elrond?

- 1) groups that contain H but not Le and $E \rightarrow 2^{4} = 16$ 11) groups that contain Le but not H and $E \rightarrow 2^{4} = 16$ 111) groups w/o H, Le, and $E \rightarrow 2^{4} = 16$ $16 \times 3 = 48$
 - 27 = 128 128 48 = 1807

Elves = {Haldir, Arwen, Legolas, Galadriel, Tauriel, Luthien, Elrond}

3. How many different <u>permutations</u> can we construct out of the Elves whose names contain at least one letter "a"?

- 1. Write the following statement using logical notation: *"If n and m are both odd, then nm is odd"*
- 2. Then, prove the statement.
- 3. Finally, use proof by cases to prove that *"If n and m are not both odd, then nm is even"*

1. Write the following statement using logical notation:

"If n and m are both odd, then nm is odd"
$$(odd(n) \land odd(m)) \Rightarrow odd(nm)$$

2. Then, prove the statement.

3. Finally, use proof by cases to prove that

"If n and m are not both odd, then nm is even"

Case 1: n is even, mis odd

$$n = 2k$$
 $m = 2g + 1$
 $nm = (2k)(2g + 1) = 4gk + 2k$
 $= 2(2gk + k)$
 $= 2(2gk + k)$

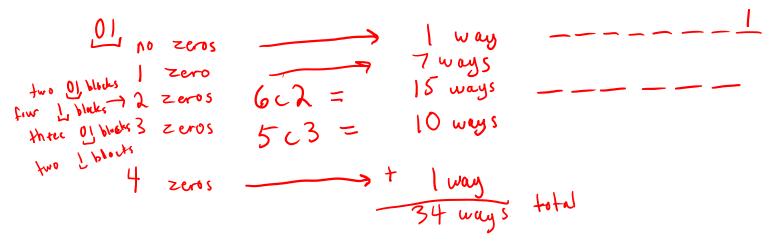
Case 3: n is even, m is even

 $= 2(2gk + k)$
 $= 2(2gk + k)$

In how many ways can we arrange the digits 1-9 in a permutation so that either 5 appears in the middle or 9 appears at the end?

$$8! + 8! - 7! = 28! - 7!
 7! (8+8-1)
 = 75,600
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 - 7!$$

Count the number of sequences of bits of length 8 in which every 0 is followed immediately by a 1. (*Note:* this means the sequence cannot end with a 0!)



Question 5 (time permitting)

7. [6 pts] EXTRA CREDIT CHALLENGE PROBLEM

To cheer up your friend who is safely social distancing at home, you decide to put together and send them a box of chocolates! Let j, k, l, mbe natural numbers such that $1 \le k \le j$ and $1 \le 2l \le m$. You have a total of j dark chocolates and m milk chocolates for you to choose from. All of the chocolates (even the same type) have different fillings and are therefore all distinguishable. However, you remember that l of the milk chocolates have nuts inside of them and that your friend isn't the biggest fan of nuts with milk chocolate. Thus, you decide to include at most one of these l milk chocolates with nuts (yes, you can include none). How many different chocolate boxes can you form consisting of exactly k dark chocolates and *l*_milk chocolates? For full credit, your answer must be in closed form.

durk chocolate

ou
$$\begin{pmatrix}
j \\
k
\end{pmatrix}
\begin{pmatrix}
m-l \\
l
\end{pmatrix}$$
e a

no mik choc w/ nut

$$\frac{\binom{j}{k}\binom{m-l}{l-1}\binom{l}{l}}{\binom{m-l}{k}\binom{m-l}{l-1}\binom{m-l}{l-1}}$$

$$\binom{j}{k}\binom{m-l}{k}\binom{m-l}{l-1}\binom{m-l}{l-1}$$

$$\binom{j}{k}\binom{m-l}{l-1}\binom{m-l}{l-1}$$