Questions

This assignment is due in about one week from when the assignment opens. The exact deadline and full instructions for submission are provided in Coursera. To receive full credit all your answers should be carefully justified. Each solution must be written independently by yourself - **no collaboration is allowed**.

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1. **[10 pts]** Prove by induction that for all positive integers n, we have:

$$1 + 3 + 5 + \cdots + (2n-1) = n^2$$

2. **[10 pts]** Use the recursion tree method or the telescopic method to solve the recurrence relation $f(0) = 7$ and, for all $n \in \mathbb{Z}^+$ $\; f(n) = f(n-1) - 2n$.

3. **[10 pts]** Use strong induction to prove that $C(n) = 2^n + 3$ is a solution to the recurrence $C(0) = 4$, $C(1) = 5$, and, for all $n \in \mathbb{Z}^+$, $n > 1$
$C(n) = 3 \cdot C(n-1) - 2 \cdot C(n-2)$.

4. **[10 pts]** Recall the Fibonacci sequence, where every number in the sequence is the sum of the previous two numbers (except for the first and second positions, which are 0 and 1 respectively). Let F_n represent the nth number in the Fibonacci sequence. Use strong induction to prove that for Fibonacci numbers $F_{n+1} - F_{n-1} < 2^n$ for all positive integers n.

5. **[10 pts]** Use ordinary induction to prove that for every positive integer n, $n^3 - n$ is a multiple of 6. Only proofs by induction are accepted.