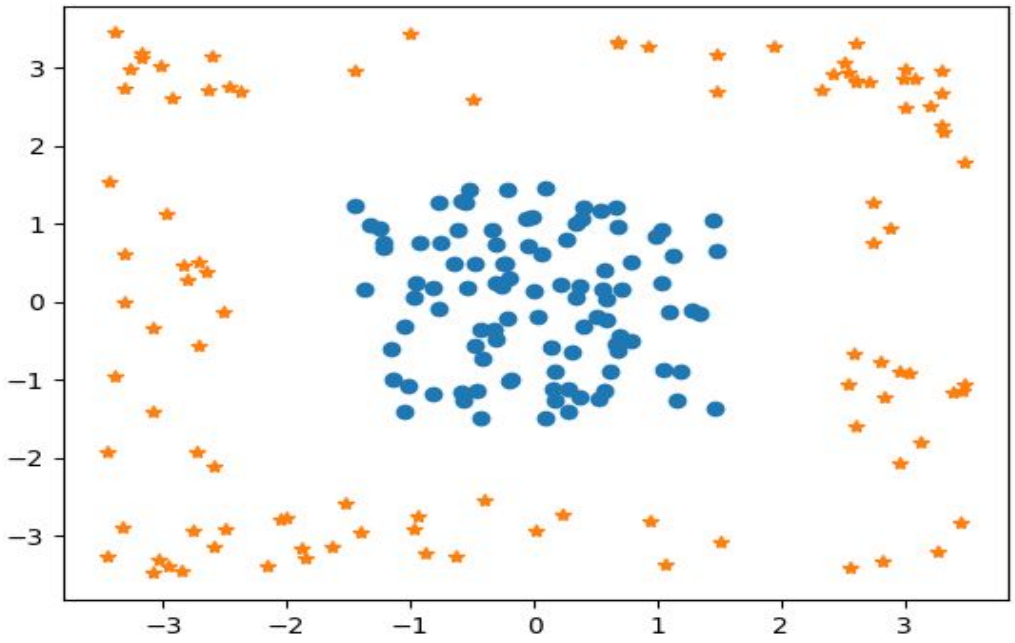
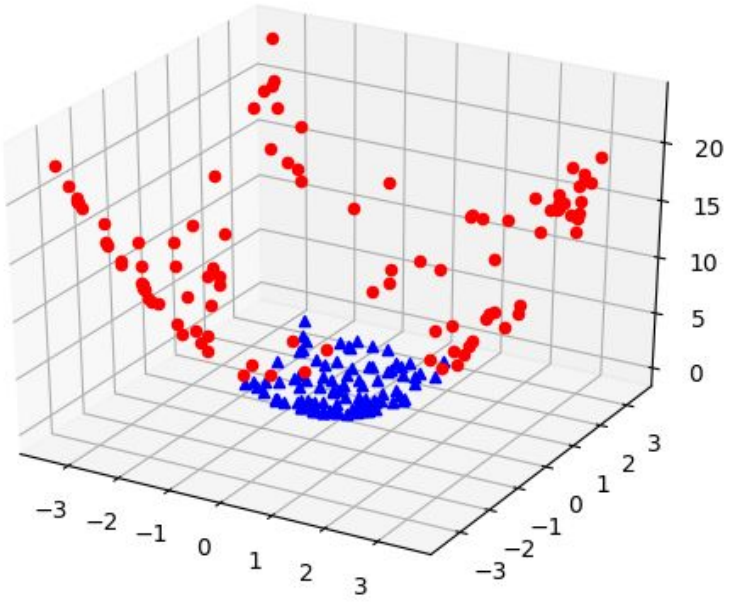
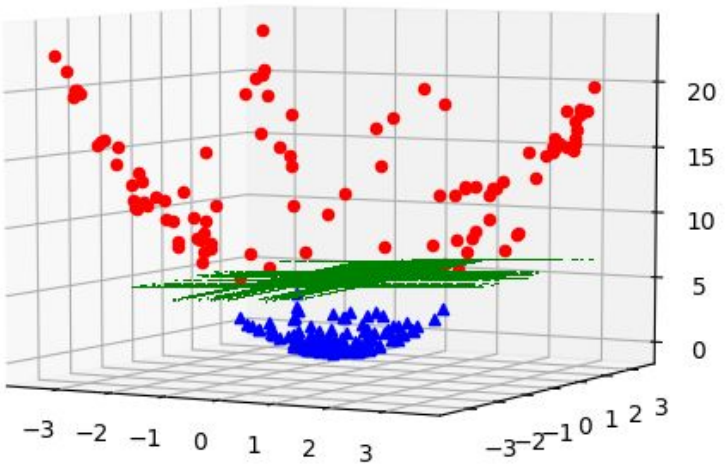
Question 1

Part 1 - Kernel Trick



- The image above is a representation of the provided data in 2D.
- The input variable is two dimensional (from this point referred to as x1 and x2)
- As we can see, the data is zero centered in both the variables.
- Also, the data isn't linearly separable in its current form as data points of one type surround the data points of the other type.
- The previous two points lead to the observation that a kernel function with the third dimension being equal to sum of squares of x1 and x2 would form a paraboloid with its minima at (0, 0).
- Hence, I used the kernel function $\Phi(x1,\ x2) :\ [x1,\ x2,\ x1^2\ +\ x2^2]$, where x1 and x2 are the respective values of the two dimensions of each input and $x1^2 + x2^2$ is the added dimension, leading to a data distribution as shown below.


20
15
10
5
0
−3 −2 −1 0 1 2 3
−3 −2 −1 0 1 2 3


20
15
10
5
0
−3 −2 −1 0 1 2 3
−3 −2 −1 0 1 2 3

- The green area symbolizes the plane which is able to separate the data in this 3 Dimensional space.
- The fact that the data is linearly separable is also proved by the fact that we are able to achieve a Training Accuracy of 100% while trying to separate the data with a perceptron.

Part 2 - Letter Classification

The following are the results of the combinations of Kernels and Hyperparameters that I tried.
The Degree is mentioned wherever the Kernel is polynomial.

Output format:
Line-1: Kernel Type , C, Gamma, Degree (if applicable)
Line-2: Accuracy, Precision, Recall, F1 value

Kernel: poly
('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 0.1, 'degree: ', 2)
(0.8007, 0.8447479412595769, 0.7985145583280193, 0.8083913743437805)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 0.1, 'degree: ', 3)
(0.8245000000000001, 0.8865850699583022, 0.8229290491605299, 0.8413753823747385)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 0.1, 'degree: ', 4)
(0.7682999999999999, 0.90506030653023, 0.7668351006391548, 0.8099551354369184)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 1, 'degree: ', 2)
(0.9449000000000002, 0.9456018874508935, 0.944525478809816, 0.9446085887186454)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 1, 'degree: ', 3)
(0.9467000000000001, 0.9476845281779637, 0.9464934436235184, 0.946609770444109)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 1, 'degree: ', 4)
(0.9134, 0.9153090093017872, 0.9130960931724431, 0.9131652520013152)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 10, 'degree: ', 2)
(0.9423, 0.9428539515976, 0.9419723873599632, 0.9420134685291277)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 10, 'degree: ', 3)
(0.9469, 0.9477502140244191, 0.9466160667996244, 0.9467178282547026)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 10, 'degree: ', 4)
(0.9094, 0.9121518442336889, 0.9089919129107699, 0.9091619371500042)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 100, 'degree: ', 2)
(0.9394, 0.9399550367149562, 0.9390557467651464, 0.9390651060415465)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 100, 'degree: ', 3)
(0.9440999999999999, 0.945295478095009, 0.9437717857083705, 0.9438876978657097)

('kernel: ', 'poly', 'C: ', 0.1, 'gamma: ', 100, 'degree: ', 4)
(0.9132999999999999, 0.9159537120599834, 0.913131381043583, 0.9132942036039655)
('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 0.1, 'degree: ', 2)
(0.9179999999999999, 0.921300150253176, 0.9173674396406992, 0.9179360108520817)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 0.1, 'degree: ', 3)
(0.9304, 0.93667559666232, 0.9298006046089176, 0.9314038856582153)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 0.1, 'degree: ', 4)
(0.8959999999999999, 0.9274312626292047, 0.895443123891299, 0.9046204711357391)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 1, 'degree: ', 2)
(0.9442, 0.9446854757244235, 0.9437821259745707, 0.9438038782161545)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 1, 'degree: ', 3)
(0.9431, 0.9437586927646583, 0.9427511011898574, 0.94278661689412)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 1, 'degree: ', 4)
(0.9132, 0.9156354199553058, 0.9130421055214839, 0.9129970899939238)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 10, 'degree: ', 2)
(0.9427, 0.9431828178550841, 0.9422440837540129, 0.942318644419398)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 10, 'degree: ', 3)
(0.943, 0.9441628176133049, 0.9427216911249351, 0.9429259843027051)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 10, 'degree: ', 4)
(0.9127000000000001, 0.9152554092631385, 0.9124832762879608, 0.912511713936366)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 100, 'degree: ', 2)
(0.9404, 0.9408710897872851, 0.9399872680137747, 0.9400577305173682)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 100, 'degree: ', 3)
(0.9415000000000001, 0.9421939870018405, 0.9413672378238511, 0.9413739784288444)

('kernel: ', 'poly', 'C: ', 1, 'gamma: ', 100, 'degree: ', 4)
(0.9200000000000002, 0.9219062172816266, 0.9196836000021879, 0.9197133277560592)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 0.1, 'degree: ', 2)
(0.9480999999999999, 0.9488031772018818, 0.9476400921190674, 0.9477901132327039)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 0.1, 'degree: ', 3)
(0.9501, 0.9511799469969793, 0.9498169980576993, 0.9499814730718276)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 0.1, 'degree: ', 4)
(0.9224, 0.9258471952958125, 0.9218350187984724, 0.9223762041321564)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 1, 'degree: ', 2)
(0.9412, 0.9418558356968685, 0.9407243154918848, 0.9407819866924866)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 1, 'degree: ', 3)
(0.942, 0.9430682595179032, 0.9417115577328253, 0.9418898384990655)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 1, 'degree: ', 4)
(0.9118999999999999, 0.9138059363181809, 0.9116676536278382, 0.9115714265398381)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 10, 'degree: ', 2)
(0.9418, 0.9426337206088125, 0.9415051233944591, 0.9416107301264297)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 10, 'degree: ', 3)
(0.9479, 0.9487742026675837, 0.9476256675713804, 0.9476907001609739)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 10, 'degree: ', 4)
(0.9149, 0.9171856236486582, 0.9145645137323528, 0.9146893204023219)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 100, 'degree: ', 2)
(0.9384, 0.9393621564030747, 0.9380892158330523, 0.938197316024252)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 100, 'degree: ', 3)
(0.9462999999999999, 0.9470853920494504, 0.9460921902660322, 0.9462326512157595)

('kernel: ', 'poly', 'C: ', 10, 'gamma: ', 100, 'degree: ', 4)
(0.9116000000000002, 0.914081200841582, 0.911415294575009, 0.9115171965537165)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 0.1, 'degree: ', 2)
(0.9423999999999999, 0.9434365354249407, 0.9420608335215469, 0.942212845288911)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 0.1, 'degree: ', 3)
(0.9479, 0.9488847654165031, 0.9475826689197125, 0.9477500350917227)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 0.1, 'degree: ', 4)
(0.9262, 0.9275176639486945, 0.9257976701381961, 0.9258739289309734)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 1, 'degree: ', 2)
(0.9427, 0.943380384985109, 0.9424672068372884, 0.9425418897773087)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 1, 'degree: ', 3)
(0.9431, 0.9440436764100086, 0.942758389886307, 0.9428813614485646)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 1, 'degree: ', 4)
(0.9194000000000001, 0.9214613626308914, 0.9191649434624122, 0.9192812949407736)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 10, 'degree: ', 2)
(0.9421999999999999, 0.9427519410568875, 0.9418958647260256, 0.9418675241422492)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 10, 'degree: ', 3)
(0.943, 0.9440824430325959, 0.9427053878638864, 0.9428333085991399)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 10, 'degree: ', 4)
(0.9168, 0.9193193425394792, 0.9163479201697928, 0.9164340230579967)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 100, 'degree: ', 2)
(0.9397, 0.9407496593915228, 0.939379818732778, 0.9395230474753475)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 100, 'degree: ', 3)
(0.9482000000000002, 0.9490547739990918, 0.9479968406854906, 0.948105755849061)

('kernel: ', 'poly', 'C: ', 100, 'gamma: ', 100, 'degree: ', 4)
(0.9103999999999999, 0.9131218333177623, 0.9101466216805951, 0.9103308201764371)

Kernel: Linear

('kernel: ', 'linear', 'C: ', 0.1, 'gamma: ', 0.1)
(0.8471, 0.8502014644009064, 0.8457060925270566, 0.8459657189637559)

('kernel: ', 'linear', 'C: ', 0.1, 'gamma: ', 1)
(0.8459, 0.8493649614157199, 0.8442186515912408, 0.8449036189587907)

('kernel: ', 'linear', 'C: ', 0.1, 'gamma: ', 10)
(0.8439, 0.8471404116173424, 0.8421017617980757, 0.8427192412675263)

('kernel: ', 'linear', 'C: ', 0.1, 'gamma: ', 100)
(0.8436, 0.8473267914607436, 0.8422022052148757, 0.8429012440664281)

('kernel: ', 'linear', 'C: ', 1, 'gamma: ', 0.1)
(0.8510000000000002, 0.8534425921230971, 0.8497425890259042, 0.8499625511288553)

('kernel: ', 'linear', 'C: ', 1, 'gamma: ', 1)
(0.8541000000000001, 0.8556340969358807, 0.8527572179845866, 0.8530519836750263)

('kernel: ', 'linear', 'C: ', 1, 'gamma: ', 10)
(0.8519, 0.8540487302234169, 0.8505308893086715, 0.8509231355260596)

('kernel: ', 'linear', 'C: ', 1, 'gamma: ', 100)
(0.8478999999999999, 0.850278971684688, 0.8465353960420918, 0.8469292068520273)

('kernel: ', 'linear', 'C: ', 10, 'gamma: ', 0.1)
(0.8524999999999998, 0.8535053883474607, 0.8511367990836712, 0.8509888421127292)

('kernel: ', 'linear', 'C: ', 10, 'gamma: ', 1)
(0.8510000000000002, 0.8530932174388705, 0.8496597522795108, 0.84986954500914)

('kernel: ', 'linear', 'C: ', 10, 'gamma: ', 10)
(0.8496, 0.8516394666553536, 0.8481204534524316, 0.8484555483287476)

('kernel: ', 'linear', 'C: ', 10, 'gamma: ', 100)
(0.8515, 0.852551421471319, 0.8500988400859063, 0.8501550311634614)

('kernel: ', 'linear', 'C: ', 100, 'gamma: ', 0.1)
(0.8518000000000001, 0.854262757962285, 0.8504820682363171, 0.8506482382145466)

('kernel: ', 'linear', 'C: ', 100, 'gamma: ', 1)
(0.8488, 0.8506134139823336, 0.8472658552363728, 0.8476881575808631)

('kernel: ', 'linear', 'C: ', 100, 'gamma: ', 10)
(0.8535999999999999, 0.855180880636472, 0.8521933957946499, 0.8524017267779079)

('kernel: ', 'linear', 'C: ', 100, 'gamma: ', 100)
(0.8522000000000001, 0.8532189971117429, 0.850668366685697, 0.8505799545893806)

Kernel: rbf

('kernel: ', 'rbf', 'C: ', 0.1, 'gamma: ', 0.1)
(0.8541000000000001, 0.8670915842808776, 0.8525285096773372, 0.8548762262758383)

('kernel: ', 'rbf', 'C: ', 0.1, 'gamma: ', 1)
(0.6636, 0.9323169183035815, 0.6597407557772476, 0.7408106171126473)

('kernel: ', 'rbf', 'C: ', 0.1, 'gamma: ', 10)
(0.0427, 0.04002266565590293, 0.04074844074844075, 0.007305333320930259)

('kernel: ', 'rbf', 'C: ', 0.1, 'gamma: ', 100)
(0.04190000000000001, 0.040021415127552286, 0.03991683991683992, 0.005785454007100755)

('kernel: ', 'rbf', 'C: ', 1, 'gamma: ', 0.1)
(0.9587, 0.9598730480453668, 0.9582875071570424, 0.958586536901227)

('kernel: ', 'rbf', 'C: ', 1, 'gamma: ', 1)
(0.9391, 0.9515703748271053, 0.9388199197218918, 0.942417148014262)

('kernel: ', 'rbf', 'C: ', 1, 'gamma: ', 10)
(0.27, 0.9591655291654553, 0.26807550071073594, 0.36468811055490935)

('kernel: ', 'rbf', 'C: ', 1, 'gamma: ', 100)
(0.12810000000000002, 0.9401284674285245, 0.12765830602982012, 0.1572734276376658)

('kernel: ', 'rbf', 'C: ', 10, 'gamma: ', 0.1)
(0.9726000000000001, 0.9730294494602155, 0.9722626532465577, 0.9724013662321243)

('kernel: ', 'rbf', 'C: ', 10, 'gamma: ', 1)
(0.9347000000000001, 0.9483093691716785, 0.9343846507791789, 0.9384604371558511)

('kernel: ', 'rbf', 'C: ', 10, 'gamma: ', 10)
(0.2934, 0.9614538907460352, 0.29148146330831587, 0.3945500318744746)

('kernel: ', 'rbf', 'C: ', 10, 'gamma: ', 100)
(0.1263, 0.955509800711717, 0.12562671119737315, 0.15598375223852864)

('kernel: ', 'rbf', 'C: ', 100, 'gamma: ', 0.1)
(0.9734, 0.9738181926226757, 0.9731998610651506, 0.9732965367092742)

('kernel: ', 'rbf', 'C: ', 100, 'gamma: ', 1)
(0.9442999999999999, 0.9542020077471587, 0.9440038474757, 0.946962264832156)

('kernel: ', 'rbf', 'C: ', 100, 'gamma: ', 10)
(0.2888, 0.9616908300485101, 0.2868618362849921, 0.39062308076583785)

('kernel: ', 'rbf', 'C: ', 100, 'gamma: ', 100)
(0.12510000000000002, 0.9632039634688665, 0.12435930466178484, 0.15361054784137734)

I was able to achieve maximum accuracy with kernel type rbf**,** C value set to 100 and gamma set to 0.1.

Maximum Accuracy: 97.34 %
Maximum Precision: 0.9738181926226757
Maximum Recall: 0.9731998610651506
Maximum F1: 0.9732965367092742