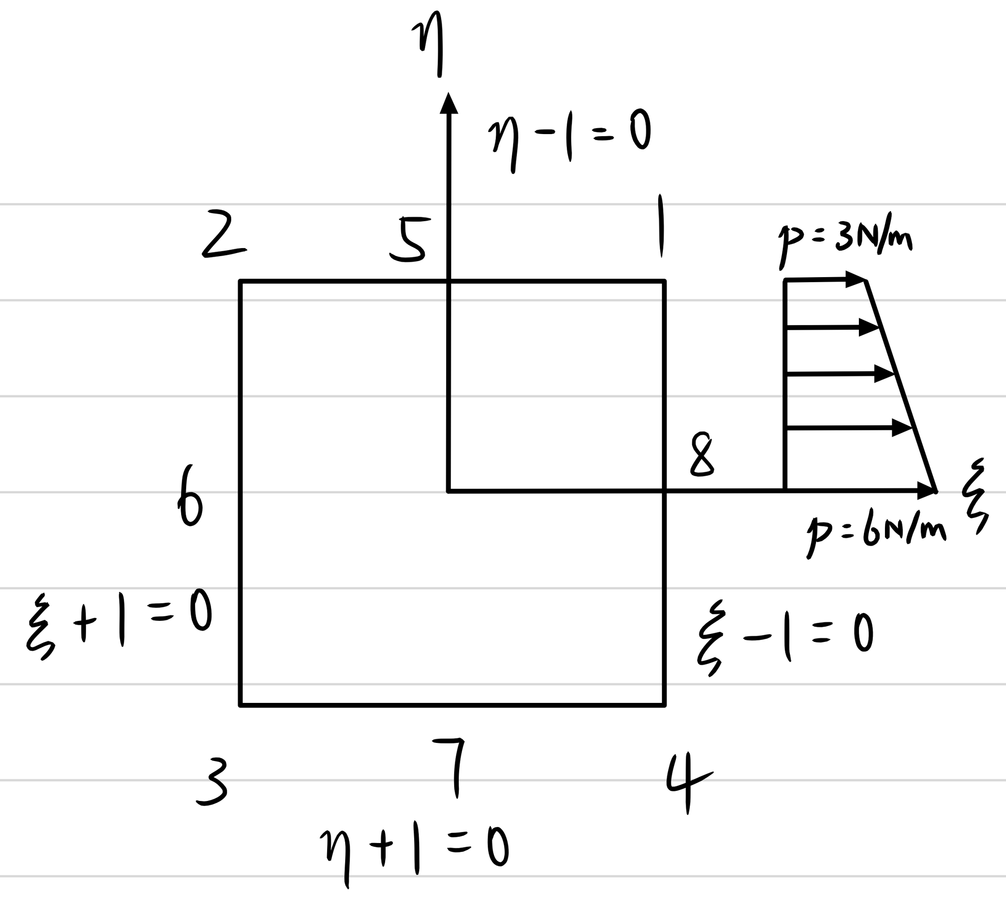
1、




(1) 采用划线法构造8节点插值函数：

$$\begin{cases} N_i=\frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta)(\xi_i\xi+\eta_i\eta-1) & (i=1,2,3,4) \\ N_i=\frac{1}{2}(1-\xi^2)(1+\eta_i\eta) & (i=5,7) \\ N_i=\frac{1}{2}(1+\xi_i\xi)(1-\eta^2) & (i=6,8) \end{cases}$$

(2)

$\bar{t}=\begin{Bmatrix} q_x \\ q_y \end{Bmatrix}=\begin{Bmatrix} 6-3\eta \\ 0 \end{Bmatrix}$ 插值函数用局部坐标表达为：

$N_1=\frac{1}{4}(1+\xi)(1+\eta)(\xi+\eta-1)$

$N_8=\frac{1}{2}(1+\xi)(1-\eta^2)$ 其它均为0 $\xi=1$

$P_s^e=\int_{l_e^s} N^T\bar{t}\,t\,dl$ $\quad P_1^e=\begin{pmatrix} \int_0^1 \frac{1}{2}(\eta+\eta^2)(6-3\eta)t\,d\eta \\ 0 \end{pmatrix}=\begin{pmatrix} \frac{13}{8}t \\ 0 \end{pmatrix}$

$P_8^e=\begin{pmatrix} \int_0^1 (1-\eta^2)(6-3\eta)t\,d\eta \\ 0 \end{pmatrix}=\begin{pmatrix} \frac{13}{4}t \\ 0 \end{pmatrix}$

$P_s^e=\left(\frac{13}{8}t\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \frac{13}{4}t\ \ 0\right)^T$

2、(1) 插值形式为 $\varphi(\xi)=\sum_{i=1}^{4}H_iQ_i$，其中 $H_1=H_1^{(0)}(\xi)$，$H_2=H_1^{(1)}(\xi)$，$H_3=H_2^{(0)}(\xi)$，$H_4=H_2^{(1)}(\xi)$

$Q_1=\varphi_1$，$Q_2=\left(\frac{d\varphi}{d\xi}\right)_1$，$Q_3=\varphi_2$，$Q_4=\left(\frac{d\varphi}{d\xi}\right)_2$ 假设 $H_i$ 为三次多项式

$H_i=a_i+b_i\xi+c_i\xi^2+d_i\xi^3$ $\quad \xi_1=0$，$\xi_2=1$ $\quad H_1(\xi_1)=H_1(0)=a_1=1$

$H_1(\xi_2)=a_1+b_1+c_1+d_1=0$ $\quad \frac{dH_1}{d\xi}(\xi_1)=\frac{dH_1}{d\xi}(0)=b_1=0$ $\quad \frac{dH_1}{d\xi}(\xi_2)=b_1+2c_1+3d_1=0$

解得：$a_1=1$，$b_1=0$，$c_1=-3$，$d_1=2$

$H_2(\xi_1)=H_2(0)=a_2=0$ $\quad H_2(\xi_2)=a_2+b_2+c_2+d_2=0$ $\quad \frac{dH_2}{d\xi}(\xi_1)=b_2=1$ $\quad \frac{dH_2}{d\xi}(\xi_2)=b_2+2c_2+3d_2=0$

解得：$a_2=0$，$b_2=1$，$c_2=-2$，$d_2=1$

$H_3(\xi_1)=a_3=0$ $\quad H_3(\xi_2)=a_3+b_3+c_3+d_3=1$ $\quad \frac{dH_3}{d\xi}(\xi_1)=b_3=0$ $\quad \frac{dH_3}{d\xi}(\xi_2)=b_3+2c_3+3d_3=0$

解得：$a_3=0$，$b_3=0$，$c_3=3$，$d_3=-2$

$H_4(\xi_1)=a_4=0$，$H_4(\xi_2)=a_4+b_4+c_4+d_4=0$ $\quad \frac{dH_4}{d\xi}(\xi_1)=b_4=0$ $\quad \frac{dH_4}{d\xi}(\xi_2)=b_4+2c_4+3d_4=1$

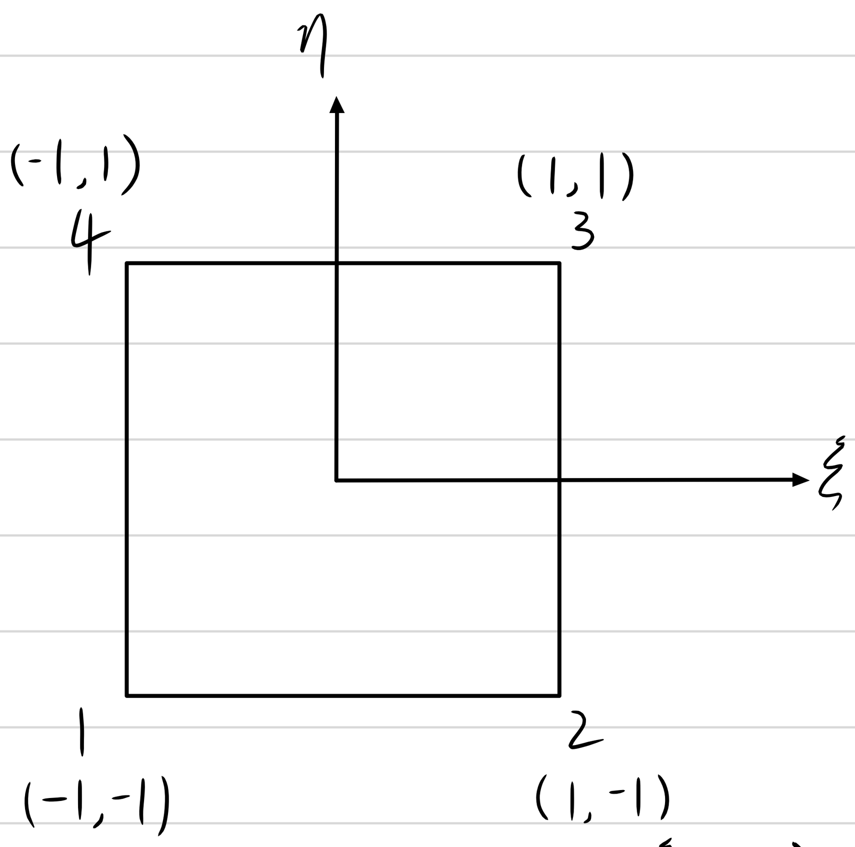
解得：$a_4=0$，$b_4=0$，$c_4=-1$，$d_4=1$

插值函数为：$H_1=1-3\xi^2+2\xi^3$ $\quad H_2=\xi-2\xi^2-\xi^3$

$H_3=3\xi^2-2\xi^3$ $\quad H_4=-\xi^2+\xi^3$

(2) $Q_1=\begin{pmatrix} 0 \\ -\int_0^1 H_1\cdot 3\,d\xi \end{pmatrix}=\begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}$ $\quad M_1=\begin{pmatrix} 0 \\ -\int_0^1 H_2\cdot 3\,d\xi \end{pmatrix}=\begin{pmatrix} 0 \\ \frac{5}{4} \end{pmatrix}$

$Q_2=\begin{pmatrix} 0 \\ -\int_0^1 H_3\cdot 3\,d\xi \end{pmatrix}=\begin{pmatrix} 0 \\ -\frac{3}{2} \end{pmatrix}$ $\quad M_2=\begin{pmatrix} 0 \\ -\int_0^1 H_4\cdot 3\,d\xi \end{pmatrix}=\begin{pmatrix} 0 \\ \frac{1}{4} \end{pmatrix}$




3、局部坐标如图所示

插值函数为 $N_i=\frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta)$ $(i=1,2,3,4)$

$$J=\begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{pmatrix}=\begin{pmatrix} \sum_{i=1}^{4}\frac{\partial N_i}{\partial \xi}x_i & \sum_{i=1}^{4}\frac{\partial N_i}{\partial \xi}y_i \\ \sum_{i=1}^{4}\frac{\partial N_i}{\partial \eta}x_i & \sum_{i=1}^{4}\frac{\partial N_i}{\partial \eta}y_i \end{pmatrix}$$

$$=\begin{pmatrix} \sum_{i=1}^{4}\frac{1}{4}(1+\eta_i\eta)\xi_i x_i & \sum_{i=1}^{4}\frac{1}{4}(1+\eta_i\eta)\xi_i y_i \\ \sum_{i=1}^{4}\frac{1}{4}(1+\xi_i\xi)\eta_i x_i & \sum_{i=1}^{4}\frac{1}{4}(1+\xi_i\xi)\eta_i y_i \end{pmatrix}$$

$(\xi_1,\eta_1)=(-1,-1)$，$(\xi_2,\eta_2)=(1,-1)$，$(\xi_3,\eta_3)=(1,1)$，$(\xi_4,\eta_4)=(-1,1)$

$\sum_{i=1}^{4}\frac{1}{4}(1+\eta_i\eta)\xi_i x_i=-\frac{1}{4}(1-\eta)+\frac{1}{4}(1-\eta)\times 3+\frac{1}{4}(1+\eta)\times 2-\frac{1}{4}(1+\eta)\times 0=\frac{1}{2}(1-\eta)+\frac{1}{2}(1+\eta)=1$

$\sum_{i=1}^{4}\frac{1}{4}(1+\eta_i\eta)\xi_i y_i=-\frac{1}{4}(1-\eta)\times 0+\frac{1}{4}(1-\eta)\times 0+\frac{1}{4}(1+\eta)\times 2-\frac{1}{4}(1+\eta)\times 2=0$

$\sum_{i=1}^{4}\frac{1}{4}(1+\xi_i\xi)\eta_i x_i=-\frac{1}{4}(1-\xi)-\frac{1}{4}(1+\xi)\times 3+\frac{1}{4}(1+\xi)\times 2+\frac{1}{4}(1-\xi)\times 0=-\frac{1}{4}(1-\xi)-\frac{1}{4}(1+\xi)=-\frac{1}{2}$

$\sum_{i=1}^{4}\frac{1}{4}(1+\xi_i\xi)\eta_i y_i=-\frac{1}{4}(1-\xi)\times 0-\frac{1}{4}(1+\xi)\times 0+\frac{1}{4}(1+\xi)\times 2+\frac{1}{4}(1-\xi)\times 2=1$

$J=\begin{pmatrix} 1 & 0 \\ -\frac{1}{2} & 1 \end{pmatrix}$ $\quad |J|=1$