SECPH Session 2023/2024

SECI1013-02 (Discrete Structure)

Assignment 2

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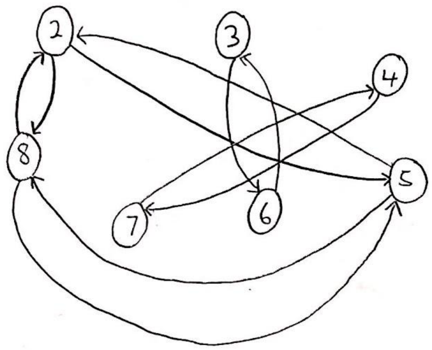
SEC12013 : DISCRETE STRUCTURES

ASSIGNMENT 2 (CHAPTER 2)

Q1. Relation

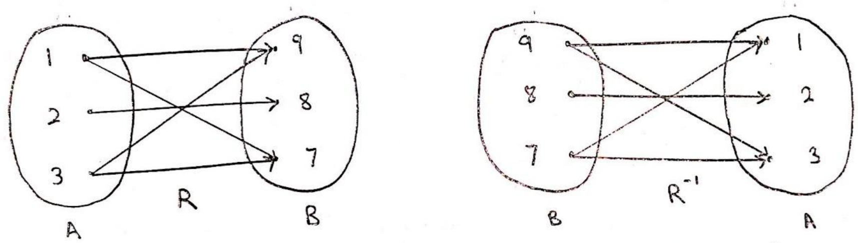
1. $R = \{(5,2),(6,3),(7,4),(8,5),(2,5),(3,6),(4,7),(5,8),(8,2),(2,8)\}$



2. a) $R = \{(1,9),(1,7),(2,8),(3,9),(3,7)\}$

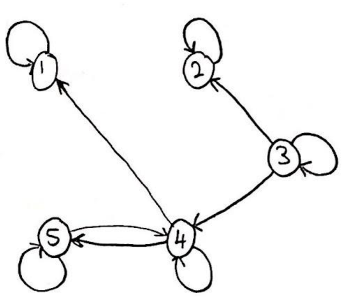
$R^{-1} = \{(9,1),(7,1),(8,2),(9,3),(7,3)\}$

b)



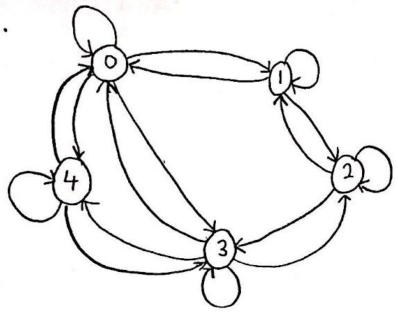
c) R^{-1} is the opposite of R. For $(x,y) \in R$, $(y,x) \in R^{-1}$

3. $R = \{(1,1),(2,2),(3,2),(3,3),(3,4),(4,1),(4,4),(4,5),(5,4),(5,5)\}$



	1	2	3	4	5
In-degree	2	2	1	3	2
Out-degree	1	1	3	3	2

4.



$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$M_R = M_R^T$$

$$M_R \otimes M_R$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- Reflexive
- Symmetric
- Not transitive

5. $R = \{(1,3), (2,6), (3,9), (4,12)\}$

a. The relation is irreflexive because $(x,x) \notin R$ for every $x \in A$

b. The relation is antisymmetric because for all $(x,y) \in R$, $(y,x) \notin R$

c. The relation is not transitive because $(1,3) \in R$ and $(3,9) \in R$ but $(1,9) \notin R$

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6. $R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

a. RS

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

b. SR

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Q2

7. Difference between Relation and Function

A relation is a method that connect sets of values such as integers or words. While a function is a type of relation that produces only one output for each input. This means every function is a relation but not every relation is a function.

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8. (i) Function. Because it is one-to-one function

(ii) Function. Because it is onto function

(iii) Not a function. Because $f(2)$ and $f(3)$ don't have any element assigned to them in A.

(iv) Not a function. Because there is an element in A is not assigned to a unique value. It is assigned to two different values in A that are 2 and 3.

9. $R = \{(x,y) \mid y = x+5, x \text{ is } Z^+ \text{ less than } 6\}$

$R = \{(1,6), (2,7), (3,8), (4,9), (5,10)\}$

Domain $= \{1,2,3,4,5\}$

Range $= \{6,7,8,9,10\}$

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10 (v) $f = \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 1-2x$

$f(a) = f(b)$

$1-2a = 1-2b$

$-2a = -2b$

$a = b$ ← one to one #

$f(x) = 1-2x$

$f(x) = y$

$1-2x = y$

$-2x = y-1$

$2x = 1-y$

$x = \frac{1-y}{2}$

This is a bijective function since it is one-to-one and onto #

this proves that for every y in the codomain, there exists an x in the domain which $f(x) = y$. This proves that this function is onto. #

(vi) $f = \mathbb{R} \rightarrow \mathbb{R}$, $f(x) = 5x^2 - 1$

$f(x) = f(y)$

$5x^2 - 1 = 5y^2 - 1$

$5x^2 = y^2$

$x^2 = y^2$

~~$x = y$, one-to-one #~~

$x = \pm y$ ← not one-to-one #

$f(x) = 5x^2 - 1$

$f(x) = y$

$5x^2 - 1 = y$

$5x^2 = y+1$

$x^2 = \frac{y+1}{5}$

$x = \pm\sqrt{\frac{y+1}{5}}$

#

This proves for every y in the codomain, there exist an x in the domain in which $f(x) = y$. This shows that this is an onto function.

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(vii) $f = \mathbb{R} \to \mathbb{R}, f(x) = x^4$

$f(x) = f(y)$

$x^4 = y^4$

$x = \pm y$ ← not one-to-one #

let $y < 0$.

No x values can map onto negative values of y.

The fourth power of any number is always positive.

Hence, this function is not surjective. #

(viii) $f = \mathbb{R} \to \mathbb{R}, f(x) = \left(\dfrac{x-2}{x-3}\right)$

$f(x) = f(y)$

$\dfrac{x-2}{x-3} = \dfrac{y-2}{y-3}$

$(x-2)(y-3) = (y-2)(x-3)$

$xy - 3x - 2y + 6 = xy - 3y - 2x + 6$

$-x = -y$

$x = y$ ← one-to-one #

let $x = 3$

$f(3) = \dfrac{3-2}{3-3}$

$=$ undefined

there exist an x value that does not map to its codomain. Therefore this function is not surjective. #

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(11) (ix) $f(x) = 3x - 1$; $g(x) = x^2 - 1$

$f(g(x)) = 3(x^2 - 1) - 1$

$= 3x^2 - 3 - 1$

$= 3x^2 - 4$

when $x = 0$

$\cancel{fg}$ $f(g(0)) = 3(0)^2 - 4$

$= -4$

when $x = 1$

$\cancel{fg}$ $f(g(1)) = 3(1)^2 - 4$

$= -1$

when $x = 2$

$f(g(2)) = 3(2)^2 - 4$

$= 8$

when $x = 3$

$f(g(3)) = 3(3)^2 - 4$

$= 23$

(x) $f(x) = x^2$; $g(x) = 5x - 6$

$f(g(x)) = (5x-6)^2$

$= 25x^2 - 60x + 36$

when $x = 0$

$f(g(0)) = 25(0)^2 - 60(0) + 36$

$= 36$

when $x = 1$

$f(g(1)) = 25(1)^2 - 60(1) + 36$

$= 26$

when $x = 2$

$f(g(2)) = 25(2)^2 - 60(2) + 36$

$= 16$

when $x = 3$

$f(g(3)) = 25(3)^2 - 60(3) + 36$

$= 81$

(xi) $f(x) = x-1$; $g(x) = x^3+1$

$f(g(x)) = (x^3+1)-1$

$= x^3$

when $x = 0$

$f(g(0)) = 0^3$

$= 0$

when $x = 1$

$f(g(1)) = 1^3$

$= 1$

when $x = 2$

$f(g(2)) = 2^3$

$= 8$

when $x = 3$

$f(g(3)) = 3^3$

$= 27$

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(12) (XII) $a_n = 6a_{n-1} - 9a_{n-2}$; $a_0 = 1$ and $a_1 = 6$

first few sequence are :

$\cancel{a_k = 6a_{k-k=0} + 9a_k}$

$a_2 = 6a_{2-1=1} - 9a_{2-2=0}$

$= 6a_1 - 9a_0$

$= 6(6) - 9(1)$

$= 27$

$a_3 = 6a_{3-1=2} - 9a_{3-2=1}$

$= 6a_2 - 9a_1$

$= 6(27) - 9(6)$

$= 108$

$a_4 = 6a_{4-1=3} - 9a_{4-2=2}$

$= 6a_3 - 9a_2$

$= 6(108) - 9(27)$

$= 405$

$1, 6, 27, 108, 405 \ldots$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; $a_0 = 2, a_1 = 5, a_2 = 15$

first term sequence are :

$a_3 = 6a_{3-1=2} - 11a_{3-2=1} + 6a_{3-3=0}$
$= 6a_2 - 11a_1 + 6a_0$
$= 6(15) - 11(5) + 6(2)$
$= 47$

$a_4 = 6a_{4-1=3} - 11a_{4-2=2} + 6a_{4-3=1}$
$= 6a_3 - 11a_2 + 6a_1$
$= 6(47) - 11(15) + 6(5)$
$= 147$

$a_5 = 6a_{5-1=4} - 11a_{5-2=3} + 6a_{5-3=2}$
$= 6a_4 - 11a_3 + 6a_2$
$= 6(147) - 11(47) + 6(15)$
$= 455$

$2, 5, 15, 47, 147, 455 \cdots$

(XIV) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; $a_0 = 1$, $a_1 = -2$, $a_2 = -1$

the first few sequence are :

$a_3 = -3a_{3-1=2} - 3a_{3-2=1} + a_{3-3=0}$
$= -3a_2 - 3a_1 + a_0$
$= -3(-1) - 3(-2) + (1)$
$= 10$

$a_4 = -3a_{4-1=3} - 3a_{4-2=2} + a_{4-3=1}$
$= -3a_3 - 3a_2 + a_1$
$= -3(10) - 3(-1) + (-2)$
$= -29$

$a_5 = -3a_{5-1=4} - 3a_{5-2=3} + a_{5-3=2}$
$= -3a_4 - 3a_3 + a_2$
$= -3(-29) - 3(10) + (-1)$
$= 56$

$1, -2, -1, 10, -29, 56$

⑬ (i) $a_1 , a_2 , a_3 , a_4 \ldots$

$$a_{n+1} = 5a_n - 3 \; ; \; a_1 = k$$

$a_1 = k$

$a_2 = a_{1+1} = 5a_1 - 3$

$a_2 = 5k - 3$

$a_3 = a_{2+1} = 5a_2 - 3$

$= 5(5k-3) - 3$

$= 25k - 15 - 3$

$a_3 = 25k - 18$

$a_4 = a_{3+1} = 5a_3 - 3$

$= 5(25k - 18) - 3$

$= 125k - 90 - 3$

$a_4 = 125k - 93$ #

(ii) $a_4 = 7$

$7 = 125k - 93$

$100 = 125k$

$k = 0.8$ #