Exercise Chapter 3

Exercise #7

$P(A) = \frac{1}{2}$ $P(B) = \frac{3}{10}$ $P(C) = \frac{1}{5}$

W : Wrong Size

D : Defects

K : Change mind

$P(W|A) = \frac{3}{5}$ $P(W|B) = \frac{1}{3}$ $P(W|C) = \frac{3}{8}$

$P(D|A) = \frac{1}{10}$ $P(D|B) = \frac{1}{2}$ $P(D|C) = \frac{1}{4}$

$P(K|A) = \frac{3}{10}$ $P(K|B) = \frac{1}{6}$ $P(K|C) = \frac{3}{5}$

i) $P(A|K) = \dfrac{P(K|A) \cdot P(A)}{P(K|A) \cdot P(A) + P(K|B) \cdot P(B) + P(K|C) \cdot P(C)}$

$= \dfrac{\left(\frac{3}{10}\right)\left(\frac{1}{2}\right)}{\left(\frac{3}{10}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{6}\right)\left(\frac{3}{10}\right) + \left(\frac{3}{8}\right)\left(\frac{1}{5}\right)}$

$= \dfrac{\frac{3}{20}}{\frac{11}{40}}$

$= \dfrac{6}{11}$

ii) $P(C|W) = \dfrac{P(W|C) \cdot P(C)}{P(W|C) \cdot P(C) + P(W|A) \cdot P(A) + P(W|B) \cdot P(B)}$

$= \dfrac{\left(\frac{3}{8}\right)\left(\frac{1}{5}\right)}{\left(\frac{3}{8}\right)\left(\frac{1}{5}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{3}\right)\left(\frac{3}{10}\right)}$

$= \dfrac{\frac{3}{40}}{\frac{19}{40}}$

$= \dfrac{3}{19}$

iii) $P(B|D) = \dfrac{P(D|B) \cdot P(B)}{P(D|A) \cdot P(A) + P(D|B) \cdot P(B) + P(D|C) \cdot P(C)}$

$\qquad = \dfrac{\left(\frac{1}{2}\right)\left(\frac{3}{10}\right)}{\left(\frac{1}{10}\right)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{3}{10}\right) + \left(\frac{1}{4}\right)\left(\frac{1}{5}\right)}$

$\qquad = \dfrac{\frac{3}{20}}{\frac{1}{4}}$

$\qquad = \dfrac{3}{5}$

Exercise #8

a) $P(M) = 0.51$

$P(\bar{M}) = 1 - 0.51$

$\qquad = 0.49 = 49\%$

b)i) $P(M \cap R) = 0.095 \qquad P(\bar{M} \cap R) = 0.017$

$P(R|M) = \dfrac{P(R \cap M)}{P(M)}$

$\qquad = \dfrac{0.095}{0.51}$

$\qquad = 0.186$

b)ii) $P(M|R) = \dfrac{P(R|M) \cdot P(M)}{P(R|M) \cdot P(M) + P(R|\bar{M}) \cdot P(\bar{M})}$

$P(R\|\bar{M}) = \dfrac{P(R \cap \bar{M})}{P(\bar{M})}$	$P(M\|R) = \dfrac{(0.186)(0.51)}{(0.186)(0.51) + (0.035)(0.49)}$
$\qquad = \dfrac{0.017}{0.49}$	$\qquad = 0.847$
$\qquad = 0.035$	

STANDARD

b iii) $P(\bar{M}|R) = \dfrac{P(R|\bar{M}) \cdot P(\bar{M})}{P(R|\bar{M}) \cdot P(\bar{M}) + P(R|M) \cdot P(M)}$

$= \dfrac{(0.035)(0.49)}{(0.035)(0.49) + (0.186)(0.51)}$

$= 0.153$

$= 15.3\%$

$0.153 \times 100\,000 = 15300$

Exercise # 9

F : who go to prison

G : who chose to plead guilty

$P(F) = 0.45$ $P(G|F) = 0.4$ $P(G|F') = 0.55$

i) $P(F) = 0.45$

$P(F') = 1 - 0.45$

$= 0.55$

ii) $P(F|G) = \dfrac{P(G|F) \cdot P(F)}{P(G|F) \cdot P(F) + P(G|F') \cdot P(F')}$

$= \dfrac{(0.4)(0.45)}{(0.4)(0.45) + (0.55)(0.55)}$

$= 0.373$

iii) $P(F'|G) = \dfrac{P(G|F') \cdot P(F')}{P(G|F') \cdot P(F') + P(G|F) \cdot P(F)}$

$= \dfrac{(0.55)(0.55)}{(0.55)(0.55) + (0.4)(0.45)}$

$= 0.627$

iv) $P(G \cap F) = 0.4$

$P(G \cap F) = P(G) \cdot P(F)$

$0.4 = P(G)(0.45)$

$P(G) = \dfrac{0.4}{0.45}$

$= 0.89$