

# Exercise 5.8

$$(E): 7x - 13y = 1$$

$$7x - 13y = 1$$

$$(2, 1)$$

$$7x - 13y = 1$$

$$7(x-2) - 13(y-1) = 0$$

$$7x - 13y = 7x - 14 + 13y - 13 = 0$$

$$7(x-2) - 13(y-1) = 0$$

$$7(x-2) = 13(y-1)$$

$$7 \mid 13(y-1) \Rightarrow 7 \mid y-1$$

$$7 \mid 13 = 1$$

$$y = 7k + 1$$

$$7(x-2) = 13(7k)$$

$$x = 13k + 2$$

$$S_{\mathbb{Z} \times \mathbb{Z}} = \{(13k+2); (7k+1); k \in \mathbb{Z}\}$$

$$7x - 13y = 35 - 39 = -4$$

$$(5, 3) \text{ is a solution of } (E')$$

$$(b) 7x - 13y = -4$$

$$7x - 13y = -4$$

$$7(x-5) - 13(y-3) = 0$$

$$7(x-5) = 13(y-3)$$

$$7 \mid 13(y-3) \Rightarrow 7 \mid y-3$$

$$7 \mid 13 = 1$$

$$y = 7k + 3$$

$$7(x-5) = 13(7k)$$

$$x-5 = 13k$$

$$x = 13k + 5$$

$$S_{\mathbb{Z} \times \mathbb{Z}} = \{(13k+5); (7k+3); k \in \mathbb{Z}\}$$

(5)

$$m \equiv 5 (13)$$

$$7m \equiv 35 (13)$$

$$13 \mid 7m - 35$$

$$13$$

$$m \equiv 3 (7)$$

$$7m \equiv 35 (7)$$

$$7 \mid 7m - 35$$

$$13 \mid 7 = 1$$

$$\Rightarrow 13 \mid 7m - 35$$

$$91 \mid 7m - 35$$

$$7m \equiv 35 (91)$$

$$13m \equiv 65 (13)$$

$$13m \equiv 39 (13)$$

$$13 \mid 13m - 39$$

$$m \equiv 3 (7)$$

$$13m \equiv 39 (7)$$

$$7 \mid 13m - 39$$

$$13 \mid 7 = 1$$

$$91 \mid 13m - 39$$

$$13m \equiv 39 (91)$$

$$S \subset (5') \left\{ \begin{array}{l} m \equiv 5 (13) \\ m \equiv 3 (7) \end{array} \right.$$

$$(5): \left\{ \begin{array}{l} m \equiv 5 (13) \\ m \equiv 3 (7) \end{array} \right.$$

$$\Rightarrow (5'): \left\{ \begin{array}{l} 7m \equiv 35 (91) \\ 13m \equiv 39 (91) \end{array} \right.$$