BME2102:

Introduction to Biomechanics

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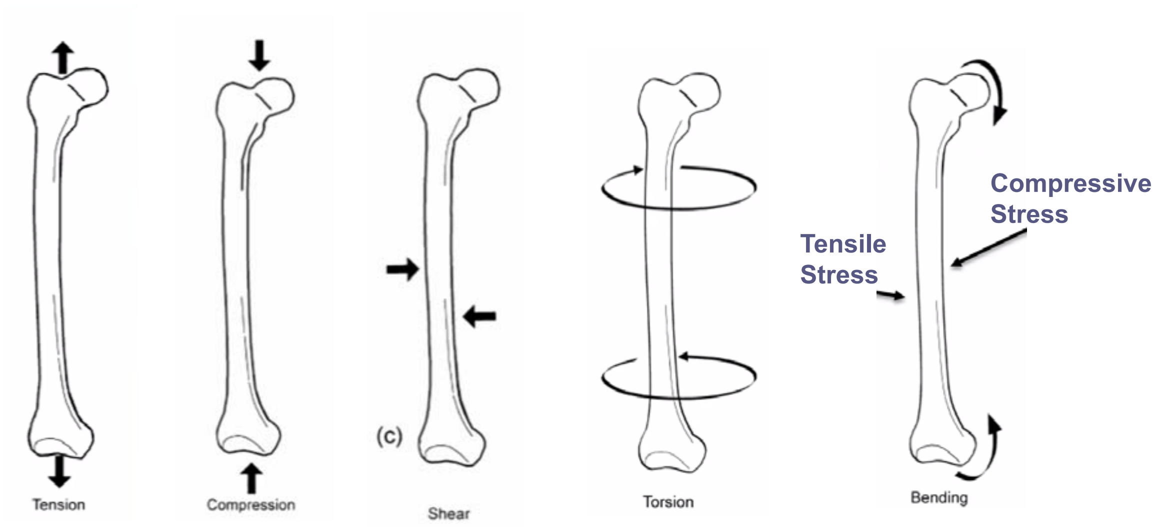
Labs

- Safety issues
  - Wear long pants please. No shorts, no skirts,—A make-up opportunity may be provided if you could not attend the lab due to this but will NOT for future cases.
- Being absent
  - With reasonable excuses: arrange to other sessions
  - Without reasonable excuses: no makeup
- For questions about the lab and your report grades, please contact your TA.
  - **YU Zejie**: zejieyu2-c@my.cityu.edu.hk
  - **LIAO Junchen**: junchliao2-c@my.cityu.edu.hk
  - **CAO Hui**: huicao8-c@my.cityu.edu.hk
  - **WANG Shuideng**: sdwang8-c@my.cityu.edu.hk
- Marking schemes
  - posted before the end of the first deadline of all Labs and reminded during Lecture 5.
  - not for telling you what should be included in the reports--the components have been listed in the Lab instructions
  - just for telling you the marks for each part of your report.

IV. Mechanics of Biomaterials

Tension/Tensile, Compression, Shear, Torsion, and Bending

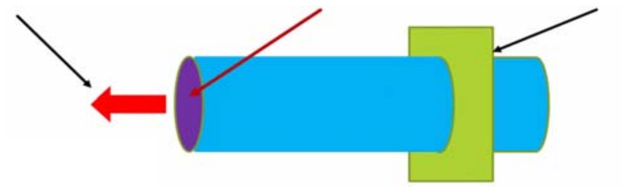


Standardizing Load

- Stress
  - Force has to be normalized – Termed: “Stress”
  - Force per square meter

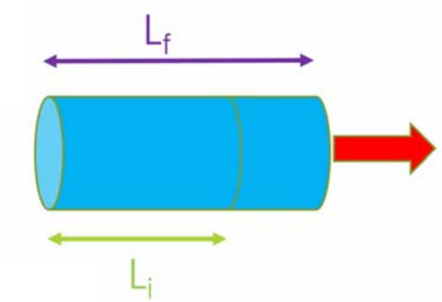
$$\sigma = \frac{F}{A}\,(\mathrm{N/m^2})$$

- Where F is internal force and A is internal area (analysis plane)



Stress to Strain

- As forces act on materials (Stress), the material may deform
- The measure of this deformation is termed: Strain
- Measurement of deformation
  - Strain



$$\varepsilon = \frac{L_f - L_i}{L_i} = \frac{\Delta L}{L_i}$$

$$\varepsilon = \frac{L_f - Li}{L_i} = \frac{\Delta L}{L_i} \times 100 = \%\, Strain$$

Tension: Tensile Stress

$$\sigma = \frac{F}{A}$$

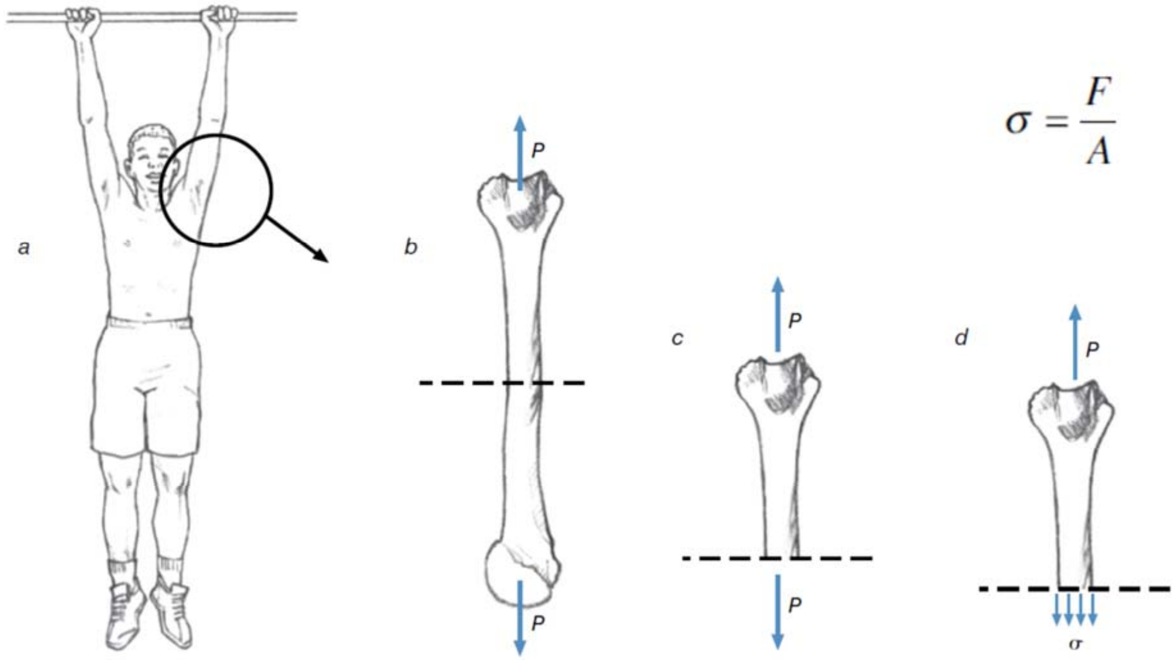


Figure 9.2 The humerus is loaded axially in tension when you do a chin-up.

Compression: Compressive Stress

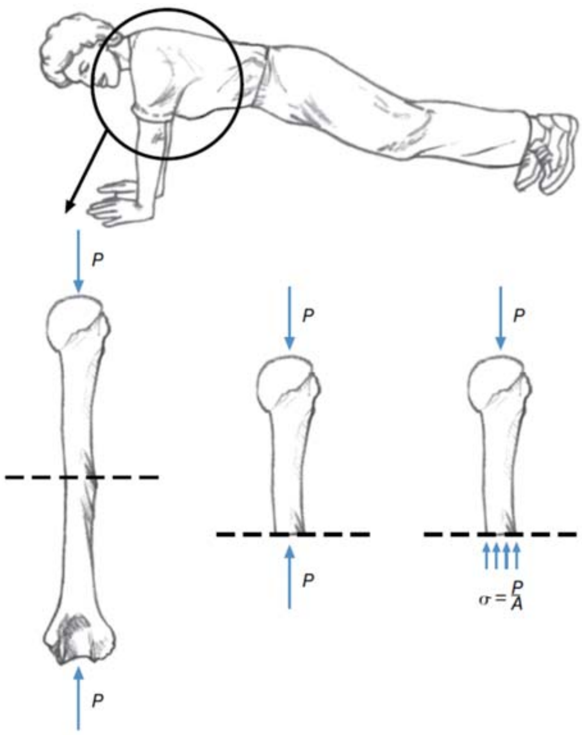
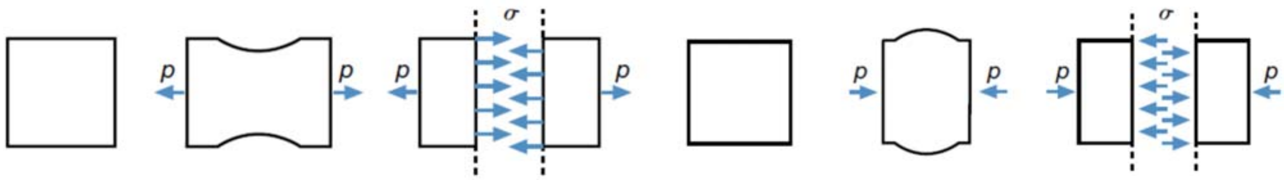


Figure 9.3 The humerus is loaded axially in compression when you do a push-up.

Poisson's Ratio

- Poisson effect



- Poisson's ratio
  - A specific ratio of strain in the axial direction to strain in the transverse direction exists for each different type of material.
  - Values of Poisson's ratio can be as low as 0.1 and as high as 0.5, but for most materials they are between 0.25 and 0.35.

Shear: Shear Stress

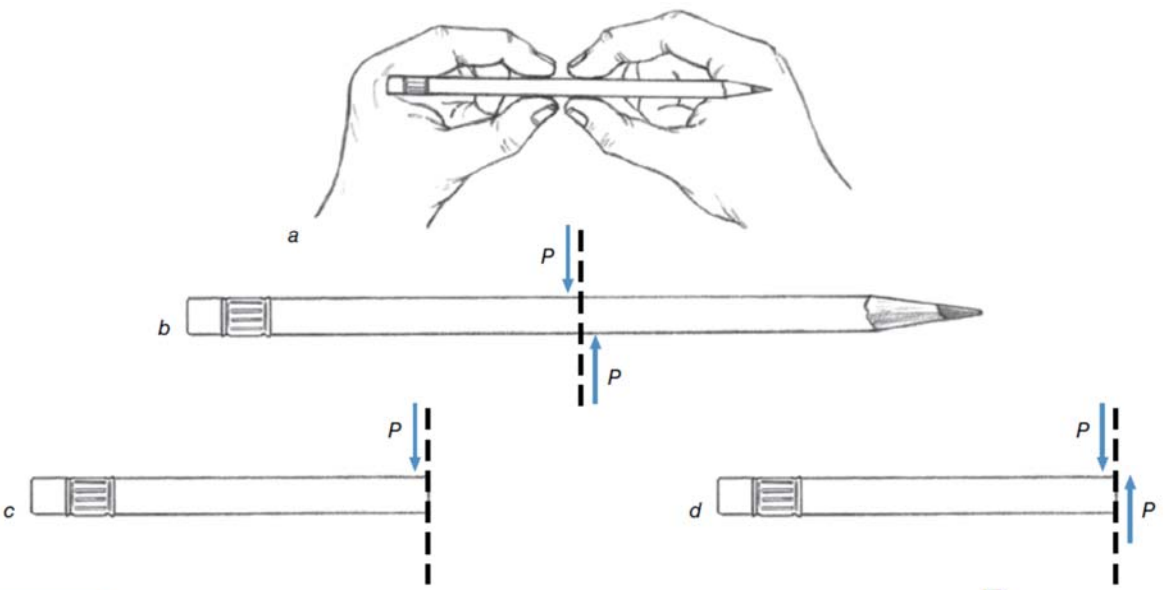


Figure 9.4 Analysis of a pencil withstanding a shear load.

$$\tau = \frac{F}{A}$$

Shear Strain

- Shear strain occurs with a change in orientation of adjacent molecules as a result of these molecules slipping past each other.

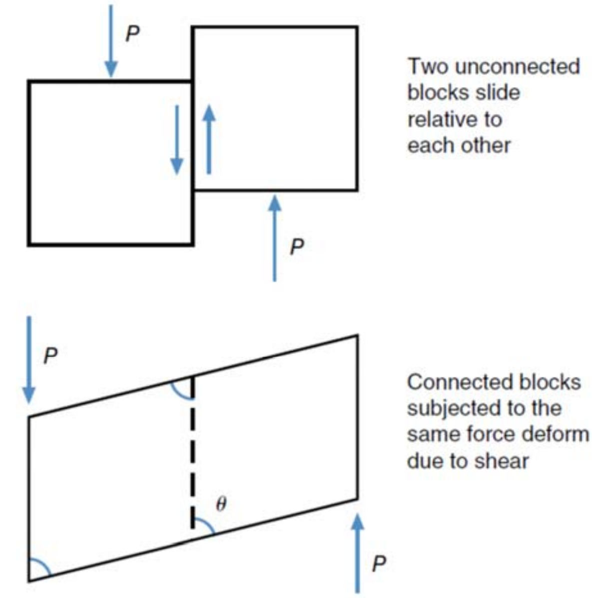
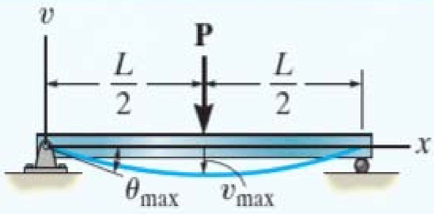
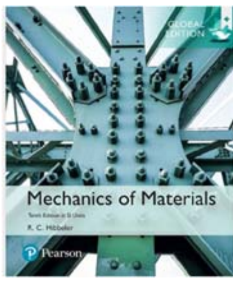


Figure 9.14 Illustration of deformation caused by shear. The change in the angle (θ) indicates the shear strain.

Beam Bending

Simply Supported Beam Slopes and Deflections

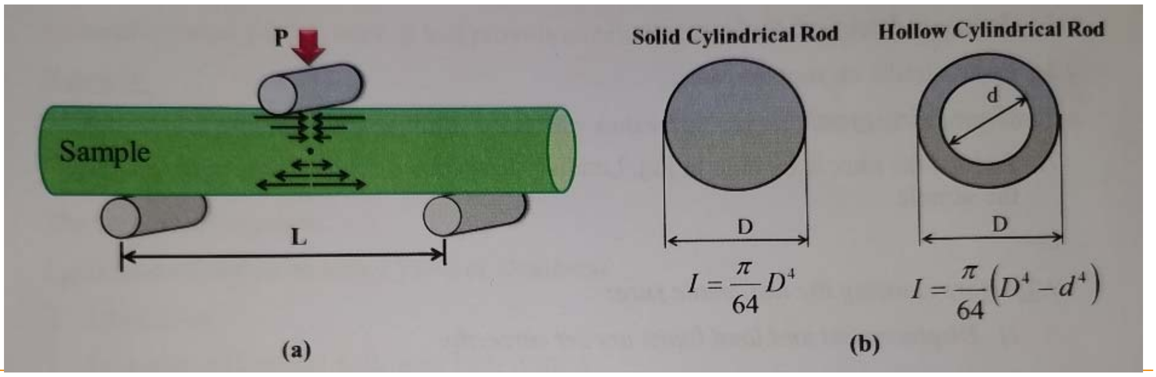
Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \leq x \leq L/2$



3-Point Bending Test

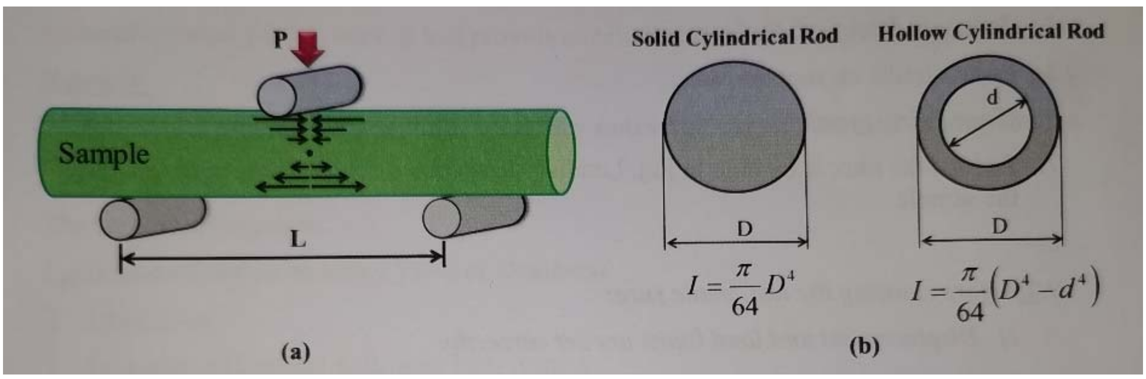
– In this lab, a Tinius Olsen H50KT system is used to perform 3-point bending tests under displacement control at a constant displacement rate at room temperature in air.
– Flexural stress, σ_f, at the out surface of the center section of the beam can be calculated based on simple beam theory by equation

$$\sigma_f = \frac{PLD}{8I}$$



3-Point Bending Test

– Flexural stress $\sigma_f = \frac{PLD}{8I}$
– where, *P* is the value of concentration load, *L* is the supporting span, *D* is the outer diameter of the cross section of the beam, and *I* is the geometry dependent moment of inertia which could be calculated by the equations shown in Figure (b).


P
Sample
L
(a)
Solid Cylindrical Rod
Hollow Cylindrical Rod
d
D
D
$I = \frac{\pi}{64} D^4$
$I = \frac{\pi}{64}(D^4 - d^4)$
(b)

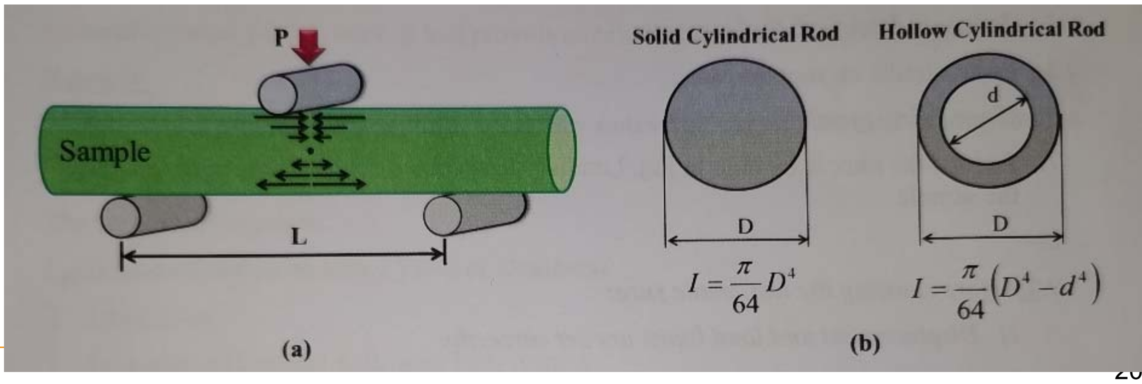
3-Point Bending Test

– The elastic flexural deflection $\delta_{elastic}$ at the center of the beam can be calculated by

$$\delta_{elastic} = \frac{PL^3}{48EI}$$

where, E is elastic modulus which can be obtained from this equation once $\delta_{elastic}$ is measured.

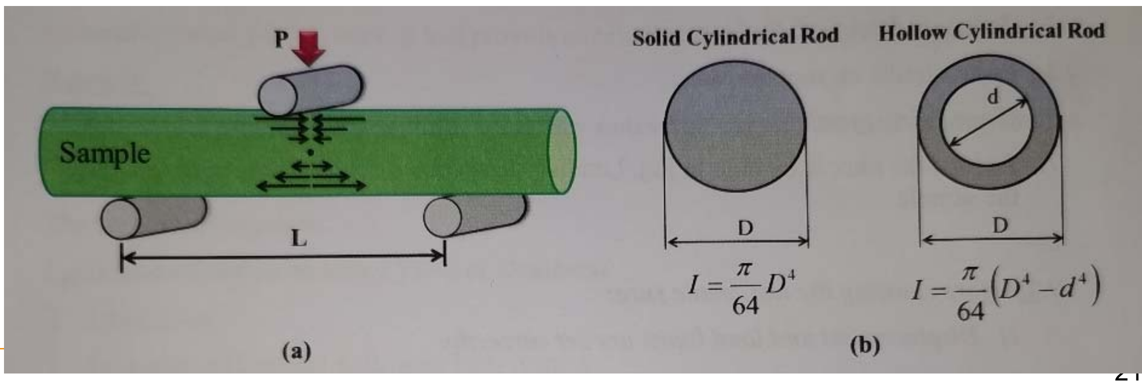
$$E = \frac{PL^3}{48I\delta_{elastic}}$$



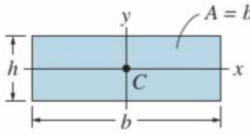
3-Point Bending Test

– The flexural strain ε_f can then be calculated by

$$\varepsilon_f = \frac{\sigma_f}{E} = \frac{\frac{PLD}{8I}}{\frac{PL^3}{48I\delta_{elastic}}} = \frac{6D\delta_{elastic}}{L^2}$$

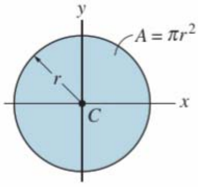


Moment of inertia *I*



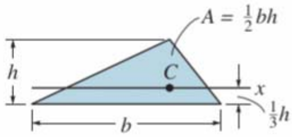
Rectangular area

$$I_x = \tfrac{1}{12} bh^3$$
$$I_y = \tfrac{1}{12} hb^3$$



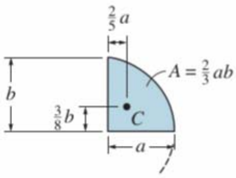
Circular area

$$I_x = \tfrac{1}{4} \pi r^4$$
$$I_y = \tfrac{1}{4} \pi r^4$$

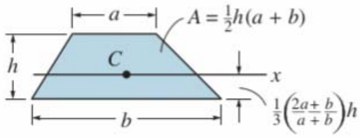


Triangular area

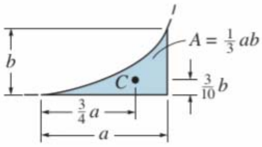
$$I_x = \tfrac{1}{36} bh^3$$



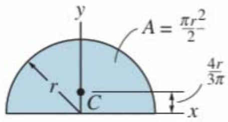
Semiparabolic area



Trapezoidal area

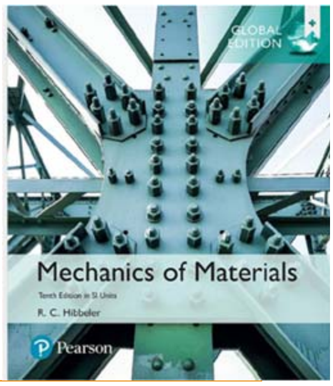


Exparabolic area

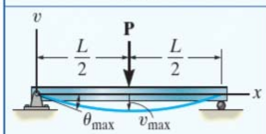
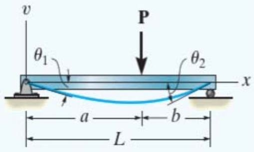
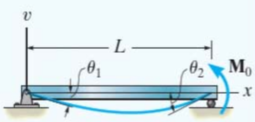
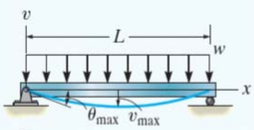
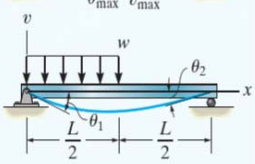
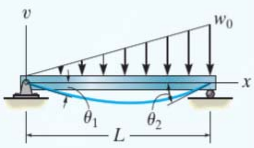


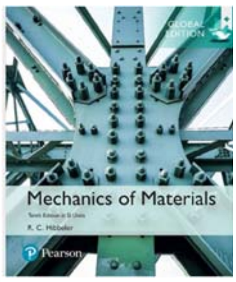
Semicircular area

$$I_x = \tfrac{1}{8} \pi r^4$$
$$I_y = \tfrac{1}{8} \pi r^4$$

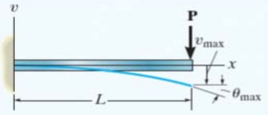
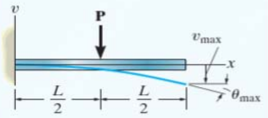
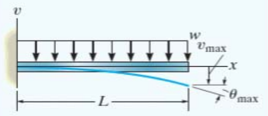
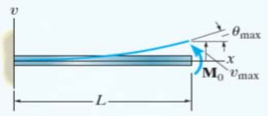
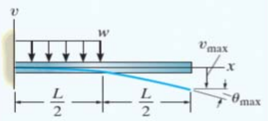
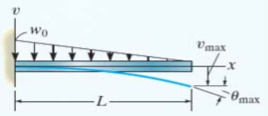


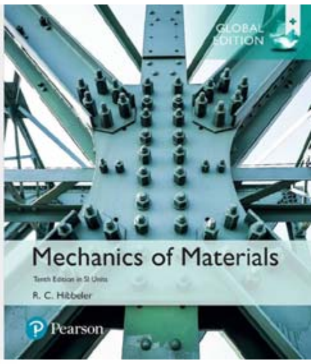
Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{16EI}$	$v_{max} = \dfrac{-PL^3}{48EI}$	$v = \dfrac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \dfrac{-Pab(L + b)}{6EIL}$ $\theta_2 = \dfrac{Pab(L + a)}{6EIL}$	$v\Big\rvert_{x=a} = \dfrac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \dfrac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \dfrac{-M_0 L}{6EI}$ $\theta_2 = \dfrac{M_0 L}{3EI}$	$v_{max} = \dfrac{-M_0 L^2}{9\sqrt{3}\,EI}$ at $x = 0.5774L$	$v = \dfrac{-M_0 x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \dfrac{-wL^3}{24EI}$	$v_{max} = \dfrac{-5wL^4}{384EI}$	$v = \dfrac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \dfrac{-3wL^3}{128EI}$ $\theta_2 = \dfrac{7wL^3}{384EI}$	$v\Big\rvert_{x=L/2} = \dfrac{-5wL^4}{768EI}$ $v_{max} = -0.006563\dfrac{wL^4}{EI}$ at $x = 0.4598L$	$v = \dfrac{\text{[illegible]}}{384EI}(16x^3 - 24Lx^2 + 9L^3)$ $0 \le x \le L/2$ $v = \dfrac{-wL}{384EI}(8x^3 - 24Lx^2 + 17L^2 x - L^3)$ $L/2 \le x < L$
	$\theta_1 = \dfrac{-7w_0 L^3}{360EI}$ $\theta_2 = \dfrac{w_0 L^3}{45EI}$	$v_{max} = -0.00652\dfrac{w_0 L^4}{EI}$ at $x = 0.5193L$	$v = \dfrac{-w_0 x}{360EIL}(3x^4 - 10L^2x^2 + 7L^4)$



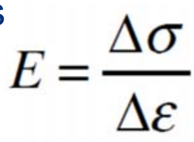
Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \dfrac{-PL^2}{2EI}$	$v_{max} = \dfrac{-PL^3}{3EI}$	$v = \dfrac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \dfrac{-PL^2}{8EI}$	$v_{max} = \dfrac{-5PL^3}{48EI}$	$v = \dfrac{-Px^2}{12EI}(3L - 2x) \quad 0 \le x \le L/2$ $v = \dfrac{-PL^2}{48EI}(6x - L) \quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-wL^3}{6EI}$	$v_{max} = \dfrac{-wL^4}{8EI}$	$v = \dfrac{-wx^2}{24EI}(x^2 - 4Lx + 6L^2)$
	$\theta_{max} = \dfrac{M_0 L}{EI}$	$v_{max} = \dfrac{M_0 L^2}{2EI}$	$v = \dfrac{M_0 x^2}{2EI}$
	$\theta_{max} = \dfrac{-wL^3}{48EI}$	$v_{max} = \dfrac{-7wL^4}{384EI}$	$v = \dfrac{-wx^2}{24EI}\left(x^2 - 2Lx + \tfrac{3}{2}L^2\right) \quad 0 \le x \le L/2$ $v = \dfrac{-wL^3}{384EI}(8x - L) \quad L/2 \le x \le L$
	$\theta_{max} = \dfrac{-w_0 L^3}{24EI}$	$v_{max} = \dfrac{-w_0 L^4}{30EI}$	$v = \dfrac{-w_0 x^2}{120EIL}(10L^3 - 10L^2 x + 5Lx^2 - x^3)$



Stress & Strain Relationship

- As forces act on a material (Stress) the material may deform (Strain)
- Depending on the "Stiffness" of a material the relationship curve may differ
- Stiffness AKA Young's Modulus or Elastic Modulus



$$E = \frac{\Delta\sigma}{\Delta\varepsilon}$$

E = elastic modulus,

$\Delta\sigma$ = change in stress, and

$\Delta\varepsilon$ = change in strain.

Material Toughness

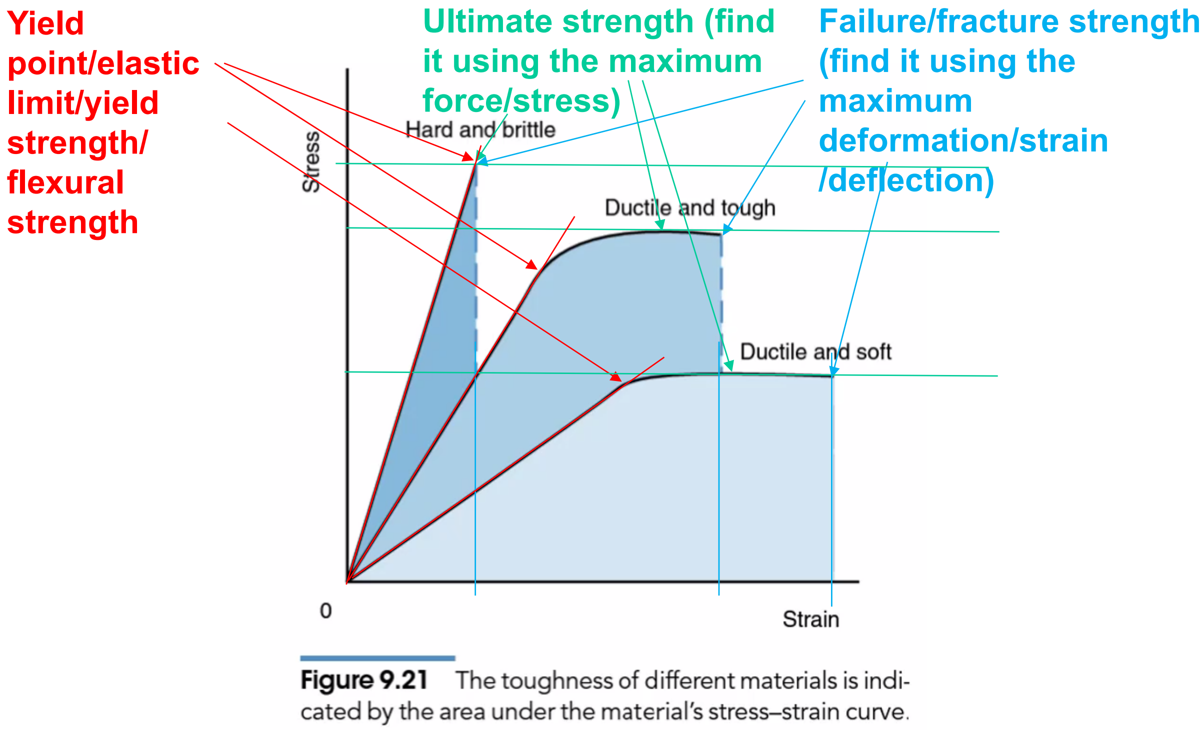
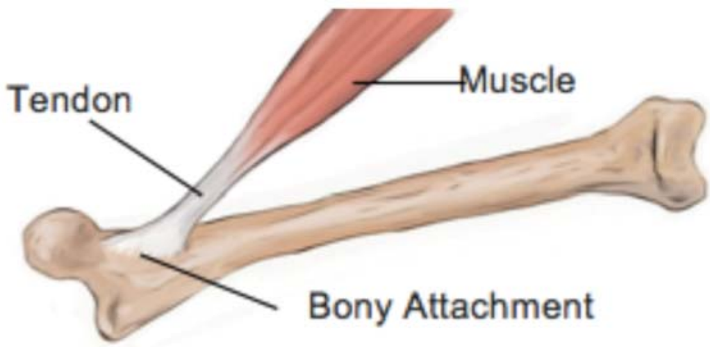
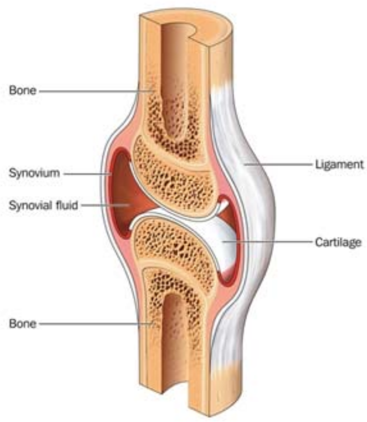


Figure 9.21 The toughness of different materials is indicated by the area under the material's stress–strain curve.

Mechanical Properties of the Musculoskeletal System: Connective Tissues in the Body

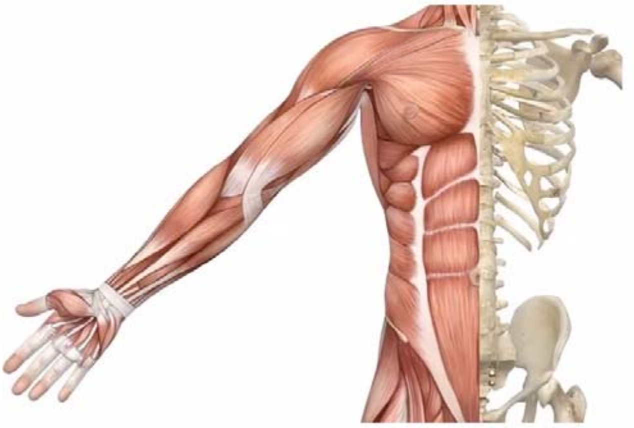
- Bone
- Cartilage
- Tendon
  - Muscle to Bone
- Ligament
  - Bone to Bone





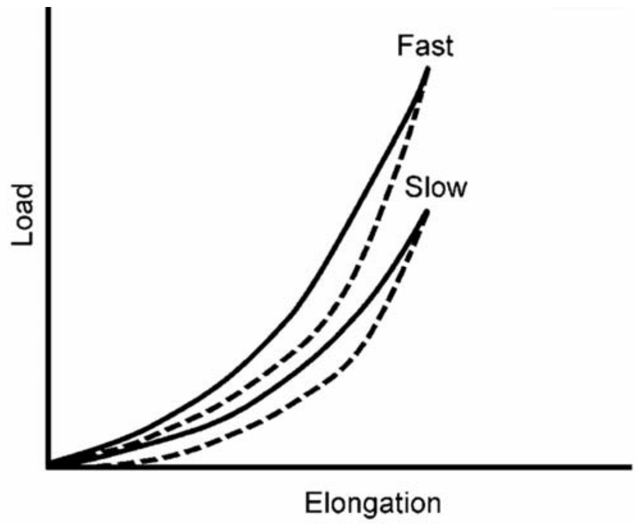
Collagen & Water Tissue Content

- Collagen is the main structural protein in the extracellular matrix in the various connective tissues in the body.
- Bone
  - 30% collagen & 20% water (45% mineral)
- Cartilage
  - 20% collagen & 70% water
- Tendon & Ligament
  - 25% collagen & 70% water



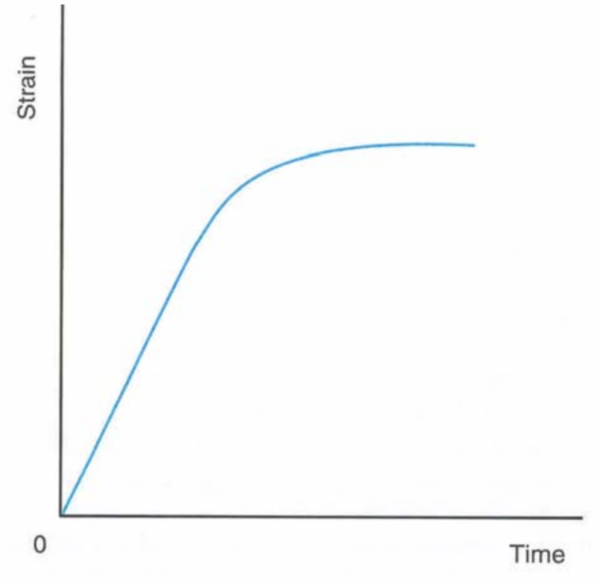
Viscoelasticity

- Viscoelastic properties occur when the stress and strain on a materials are dependent on how quickly or slowly the load applied



Viscoelastic Creep

- Constant compressive stress
  - Increases strain as water content is squeezed out
  - Strain reaches a maximum
- Ex
  - Gravity & Spinal Cord



Viscoelastic Stress Relaxation

- Constant strain results in
  - Stress increases as water content is squeezed out
  - Stress maxes out
  - Stress then decreases
- Ex
  - A long static stretch

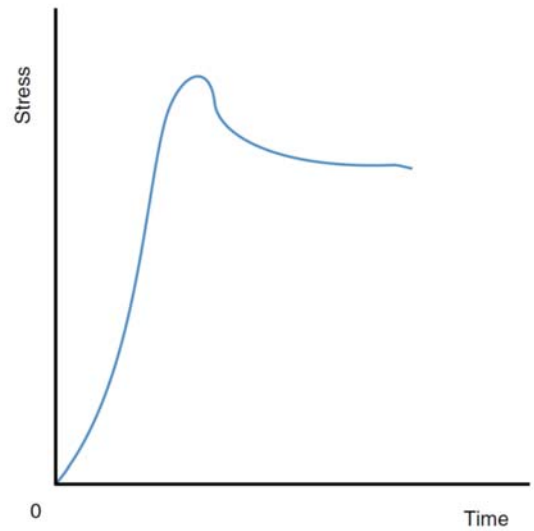
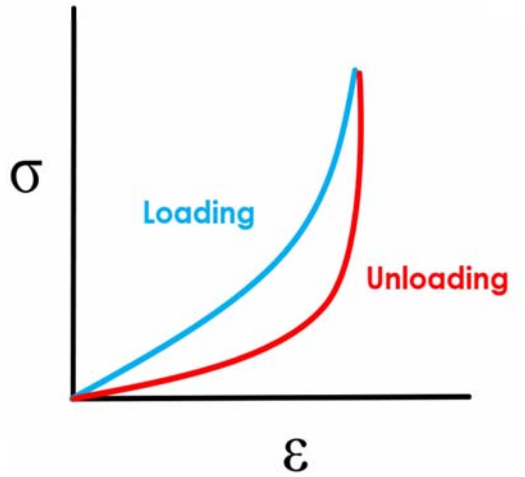


Figure 9.26 Stress relaxation in articular cartilage under constant compressive strain.

Hysteresis

- Hysteresis is an elastic properly describing a material that has a different stress-strain curve when being uploaded compared to loaded
- Area under curved is energy stored/released
- Difference between loaded/unloaded energy lost
  - Energy lost because few materials are perfectly elastic
  - Less area = more elastic or more efficient



Isotropic vs Anisotropic Behavior

- Isotropic
  - A material that has the same mechanical properties regardless of loading direction
- Anisotropic
  - A material that has different mechanical properties dependent on how it is loaded

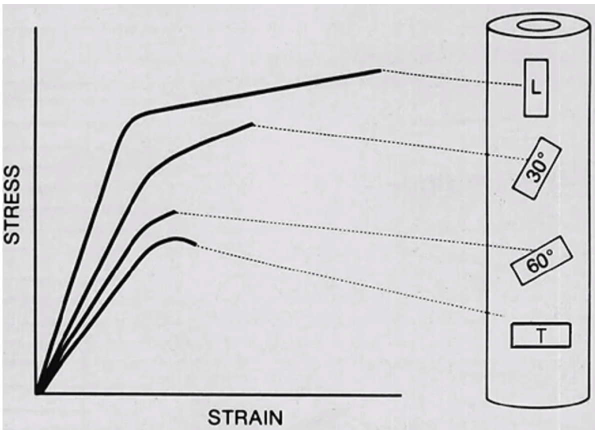
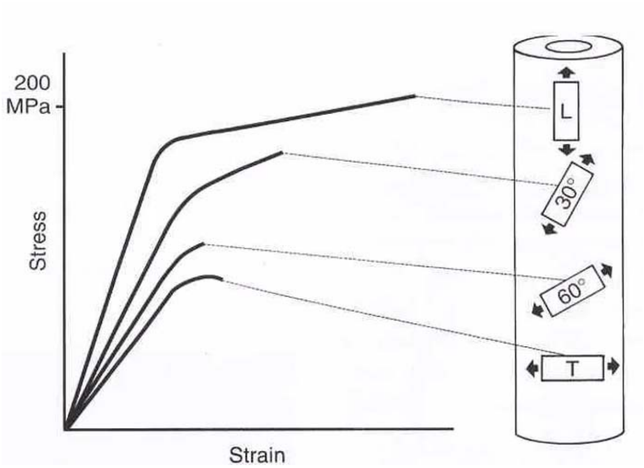


FIG. 12-8 Anisotropic behavior of cortical bone specimens machined from a human femoral shaft and tested in tension. The orientation of load application—longitudinal (*L*), tilted 30° with respect to the bone axis, tilted 60°, and transverse (*T*)—strongly influences both the stiffness and the ultimate strength. (Frankel VH, Nordin M: Basic Biomechanics of the Skeletal System, p. 22. Philadelphia, Lea & Febiger, 1980)

Bone Loading

- Ultimate strength of bone

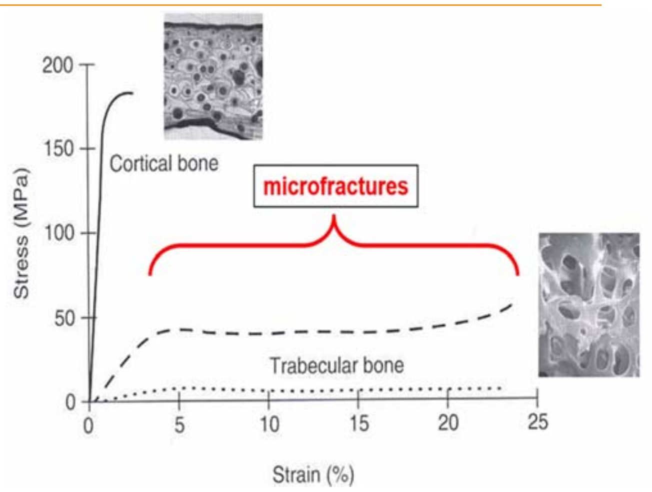


Loading Type	Ultimate Strength
Compression	200 MPa (29,000lb/in^2)
Tension	125 MPa
Shear	65 MPa (9,425lb/in^2)

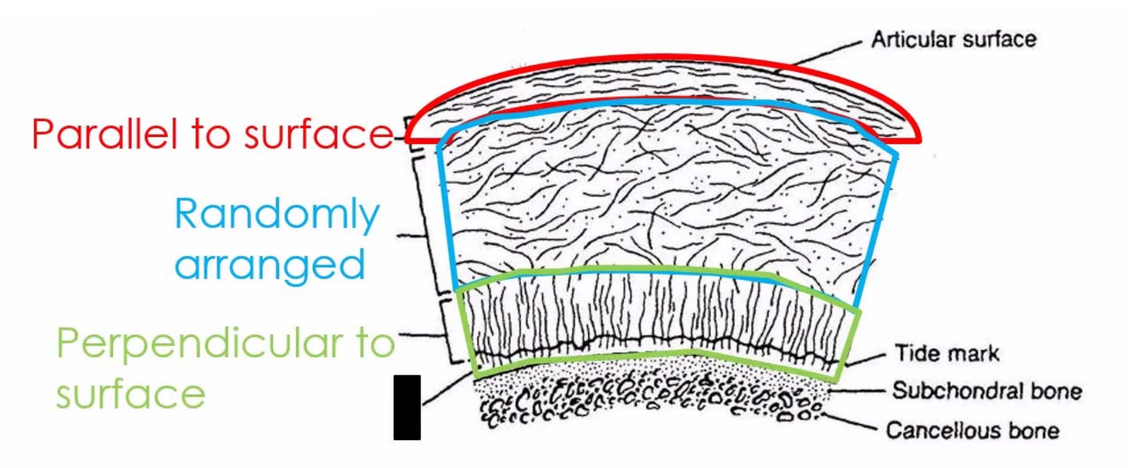
Hayes, 1986

Bone

- Tested in compression
- The hard outer layer of bones is composed of **cortical bone**, which is also called compact bone as it is much denser than cancellous bone. It forms the hard exterior (cortex) of bones.
- **Cancellous bone**, also called **trabecular** or **spongy bone**, is the internal tissue of the skeletal bone and is an open cell porous network.



Collagen in Cartilage



Collagen in Tendon & Ligaments

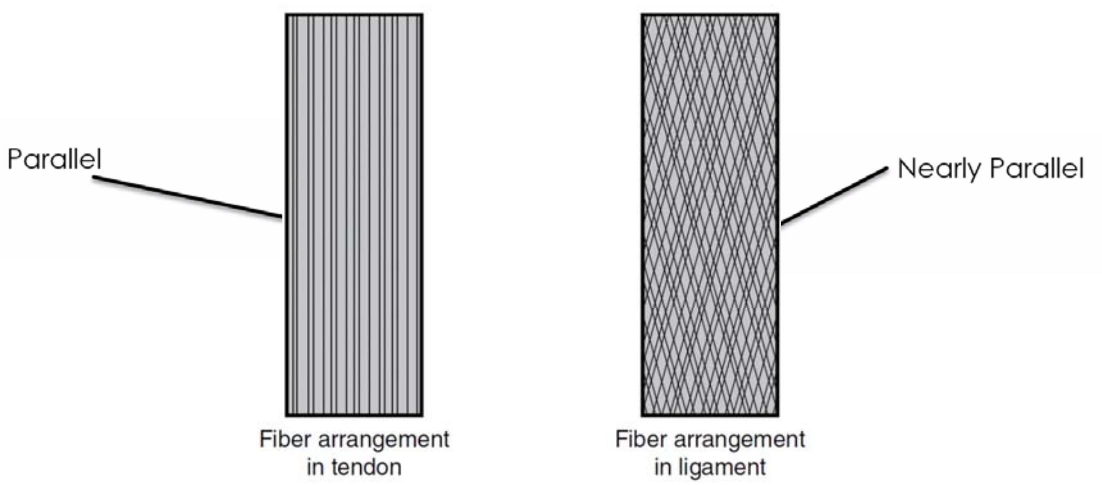
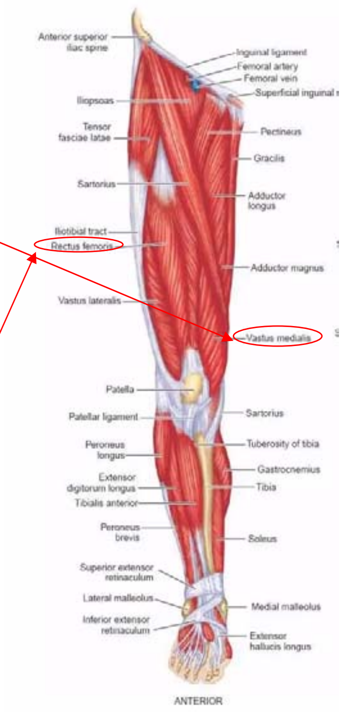
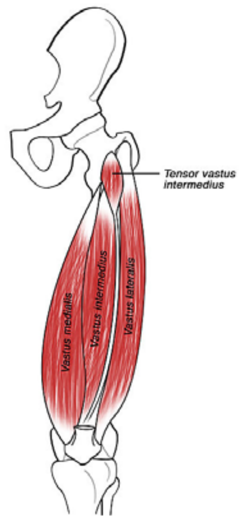


Figure 9.27 Parallel arrangement of collagen fibers in tendon, and nearly parallel arrangement of collagen fibers in ligament.

Muscle Architecture

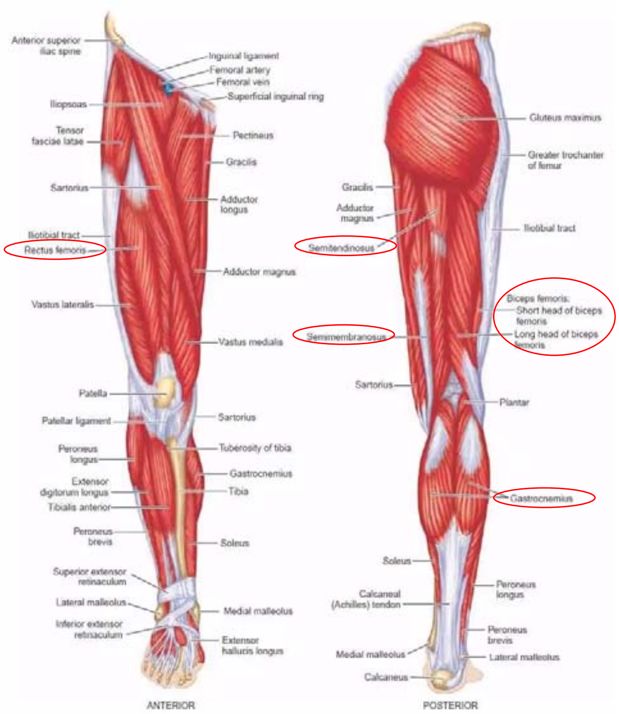
- Single Joint Muscles
  - Muscles that cross one single joint
  - Ex: Vastus Group
- Biarticular Muscles
  - Muscle that cross/span two joints
  - More complex movement
  - Ex: Rectus Femoris





Muscle Architecture

- 3 Leg Biarticulate Muscles
  - Hamstrings: Semitendinosus, Semimembranosus, & Biceps Femoris
    - ➢ Joints: Hip & Knee
    - ➢ Action: Hip Extension & Knee Flexion
  - Rectus Femoris
    - ➢ Joints: Hip & Knee
    - ➢ Action: Hip Flexion & Knee Extension
  - Gastrocnemius
    - ➢ Joints: Knee & Ankle
    - ➢ Action: Knee Flexion & Ankle Plantar Flexion



Muscle Contraction Process

https://www.youtube.com/watch?v=ousfIrOzQHc

Muscle Anatomy

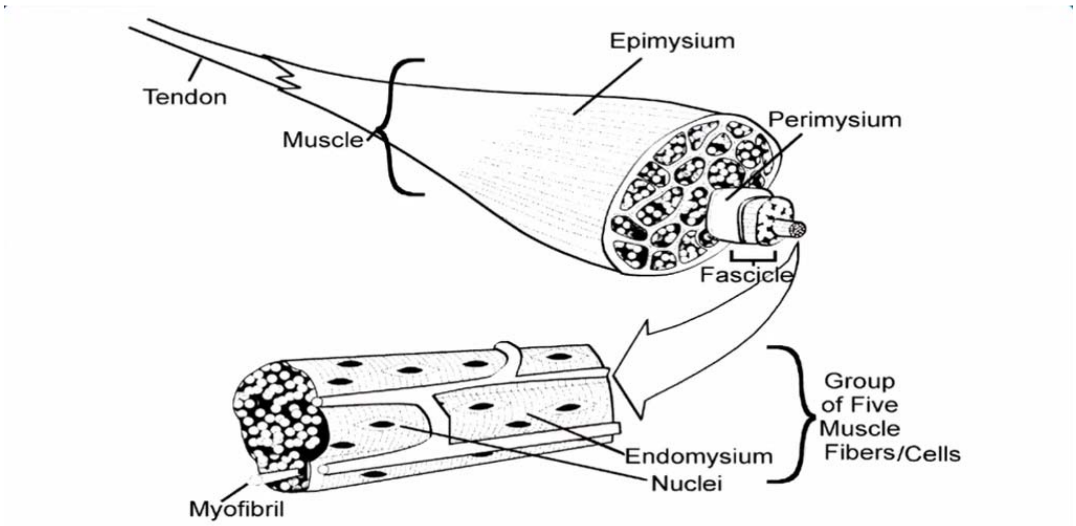
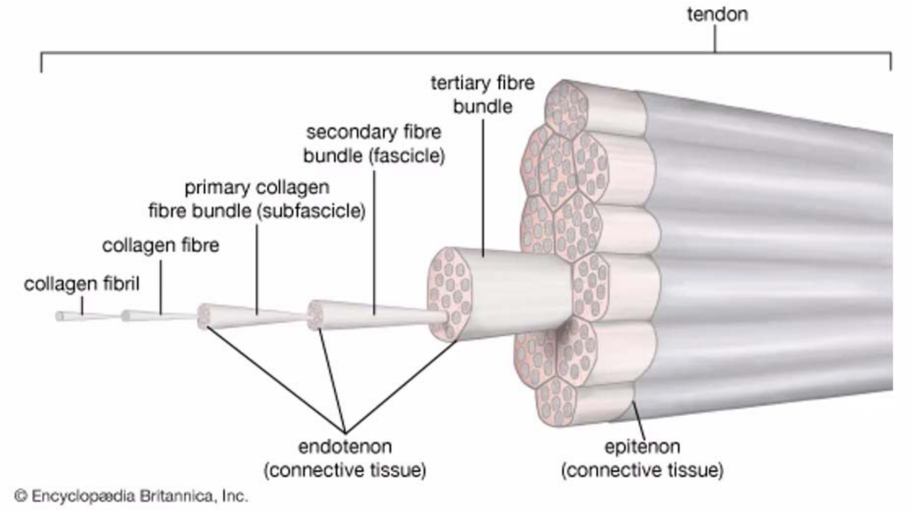


Figure 3.6. The macroscopic structure of muscle includes several layers of connective tissue and bundles of muscle fibers called fascicles. Muscle fibers (cells) are multinucleated and composed of many myofibrils.

Connective Tissues

- Cell
- Collagen
- Mineral
- Water
- Elastin



Muscle Anatomy

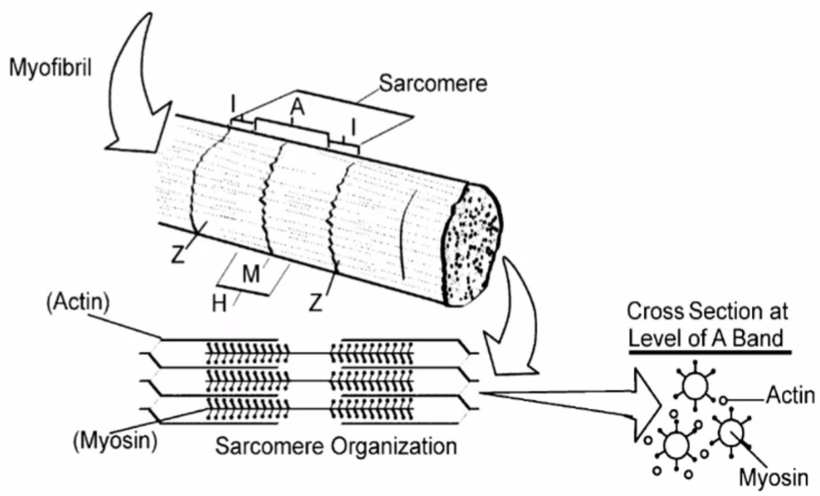


Figure 3.8. The microscopic structure of myofibril components of muscle fibers. Schematics of the sarcomere, as well as of the actin and myosin filaments are illustrated.

Sliding Filament Theory

- Myosin will create a crossbridge with Actin and pull forwards the center
- Sarcomere Shortens = Muscle Contraction

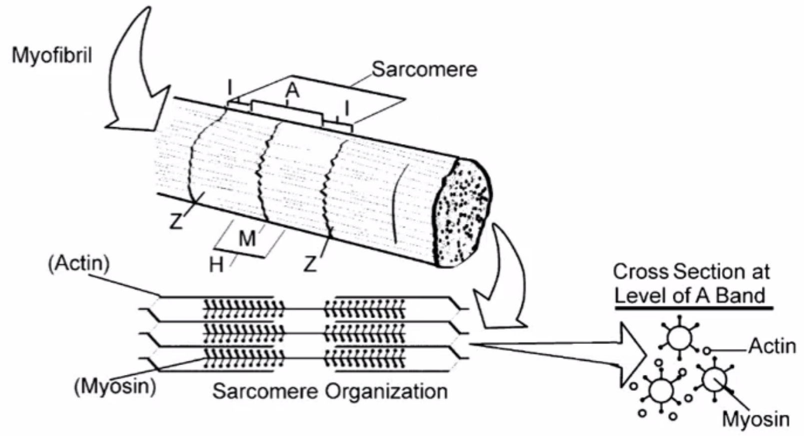


Figure 3.8. The microscopic structure of myofibril components of muscle fibers. Schematics of the sarcomere, as well as of the actin and myosin filaments are illustrated.

Muscle Types

- Type I
  - Slow Twitch
  - Slow Oxidative
- Type II
  - Fast Twitch
  - Fast Glycolytic

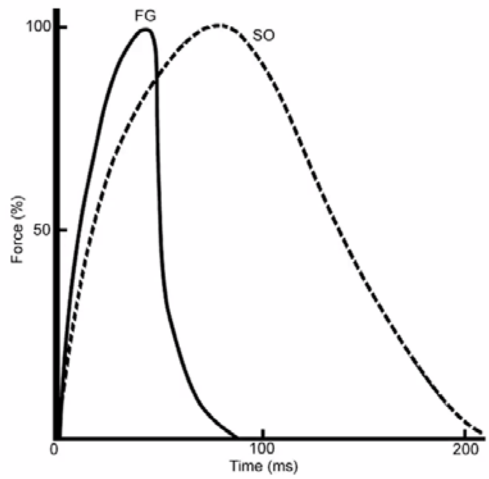
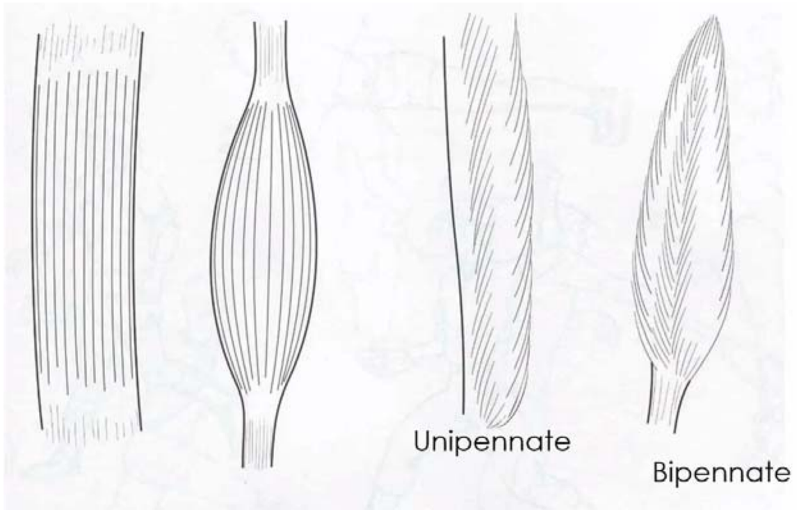


Figure 4.9. The twitch response of fast-twitch (FG) and slow-twitch (SO) muscle fibers. Force output is essentially identical for equal cross-sectional areas, but there are dramatic differences in the rise and decay of tension between fiber types that affect the potential speed of movement.

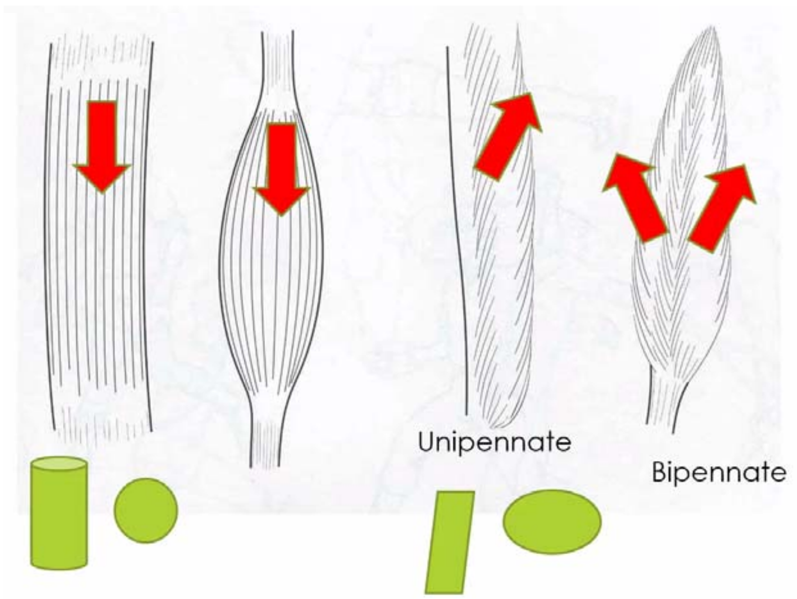
Muscle Architecture

- Parallel
  - Greater ROM
  - Less tension
- Pennate
  - Less ROM
  - Greater tension
    - Cross-sectional area

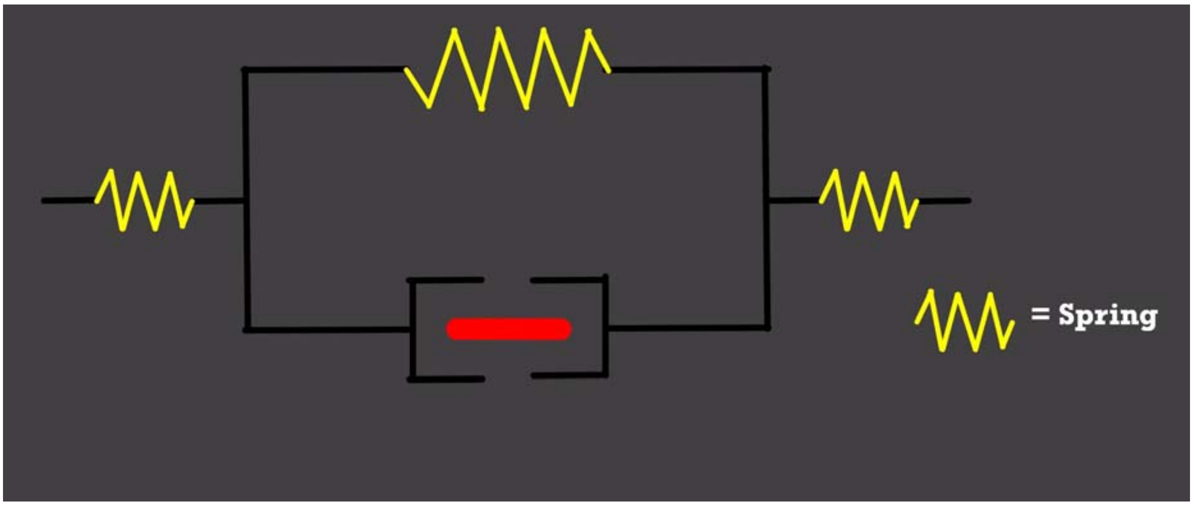


Muscle Architecture

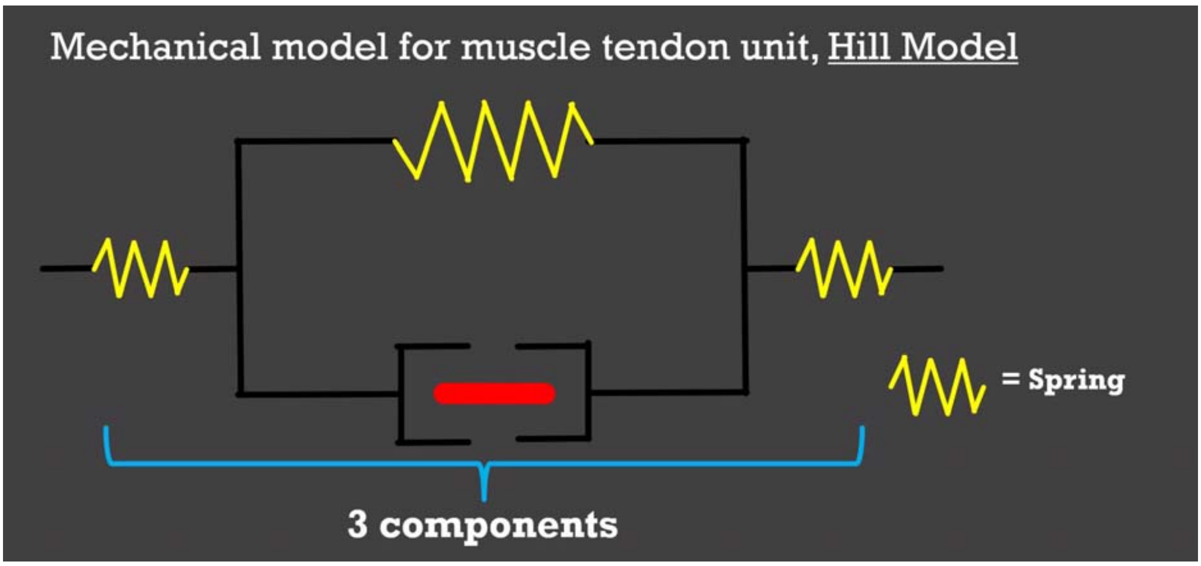
- Parallel
  - Greater ROM
  - Less tension
- Pennate
  - Less ROM
  - Greater tension
    - Cross-sectional area



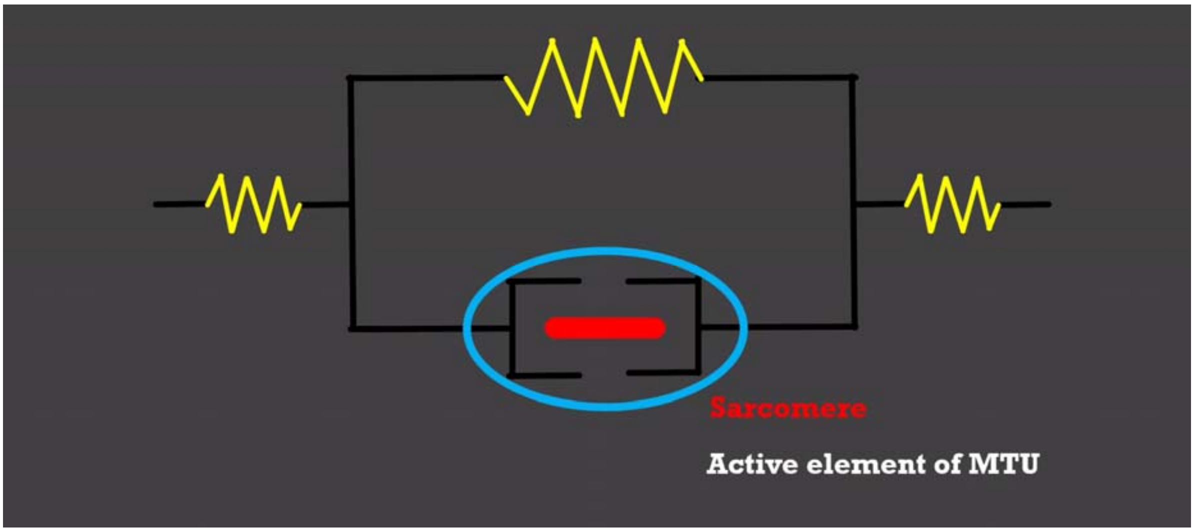
Mechanical Muscle Factors



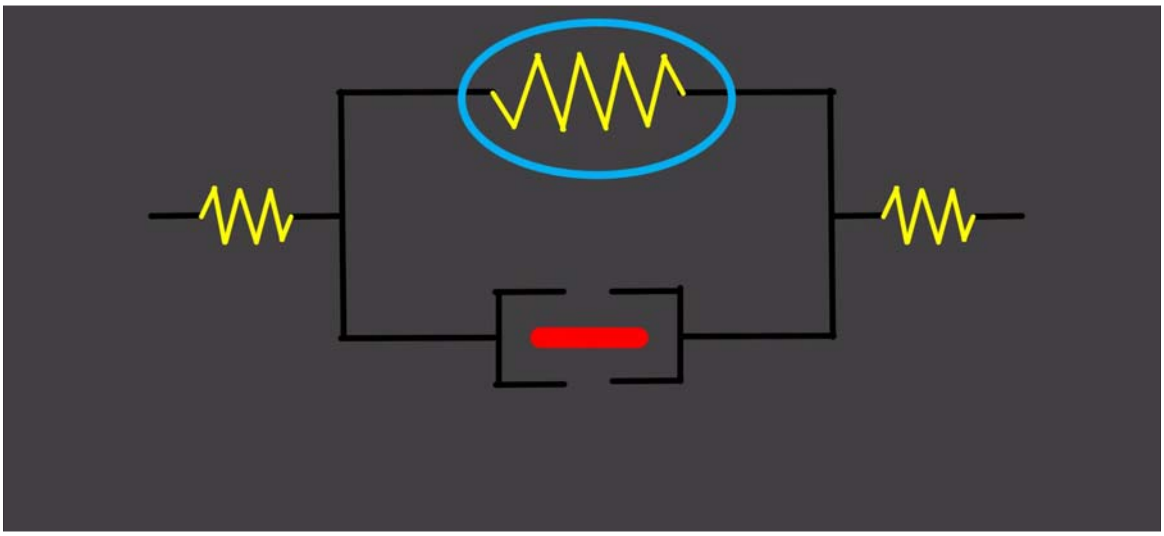
Mechanical Muscle Factors



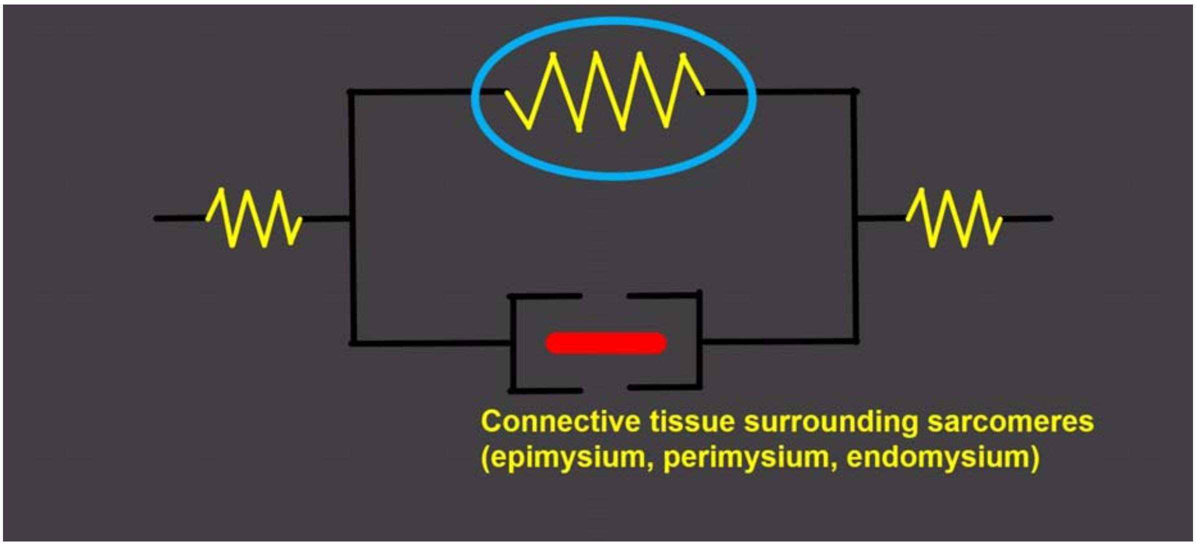
Contractile Component



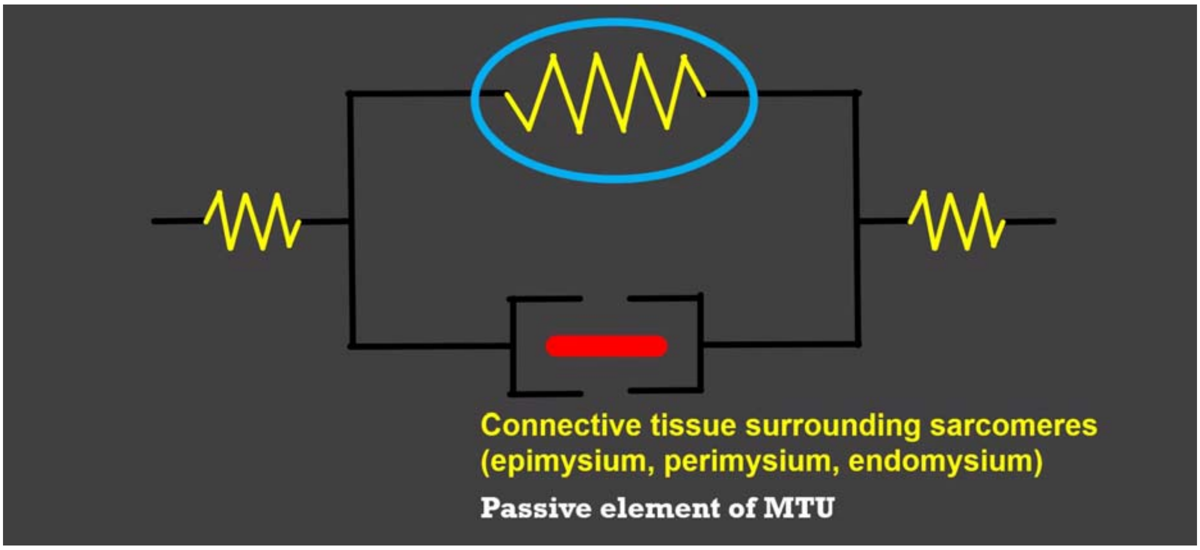
Parallel Elastic Component



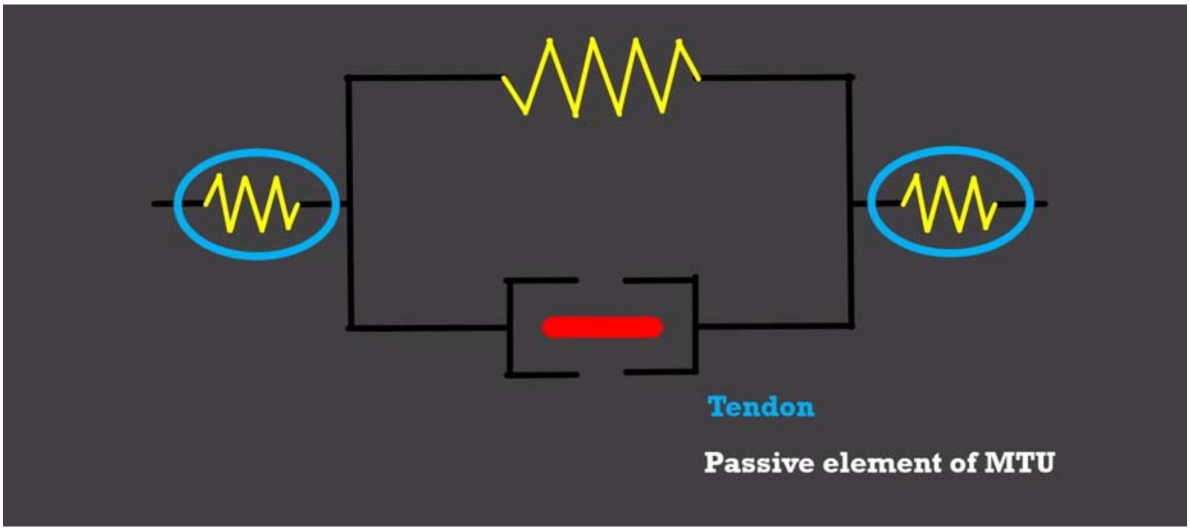
Parallel Elastic Component



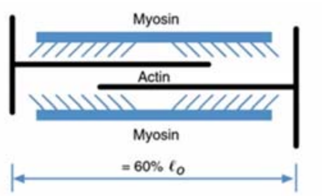
Parallel Elastic Component



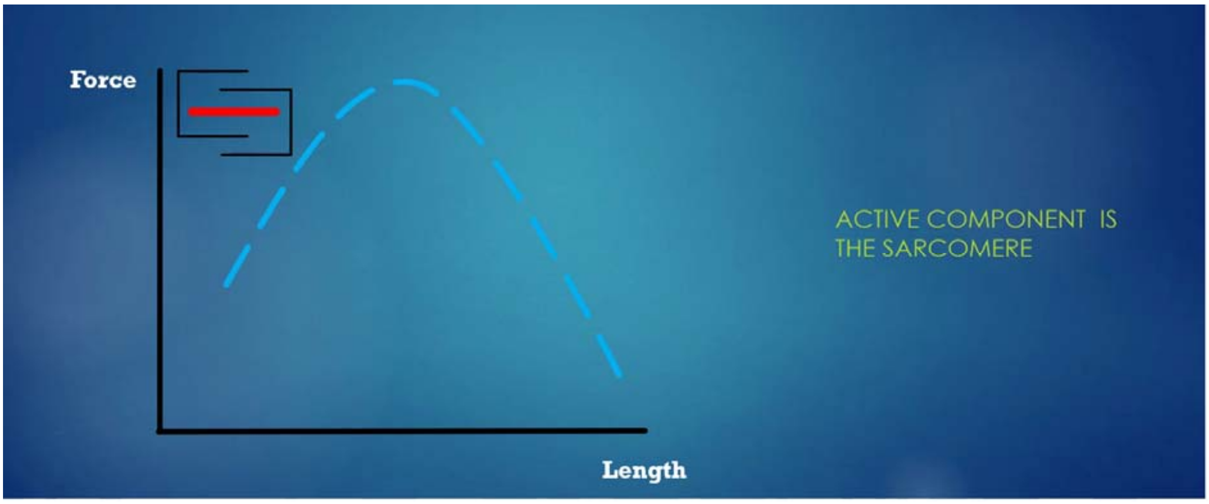
Series Elastic Component



Force-Length Principle

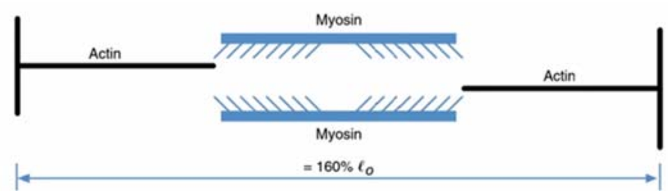


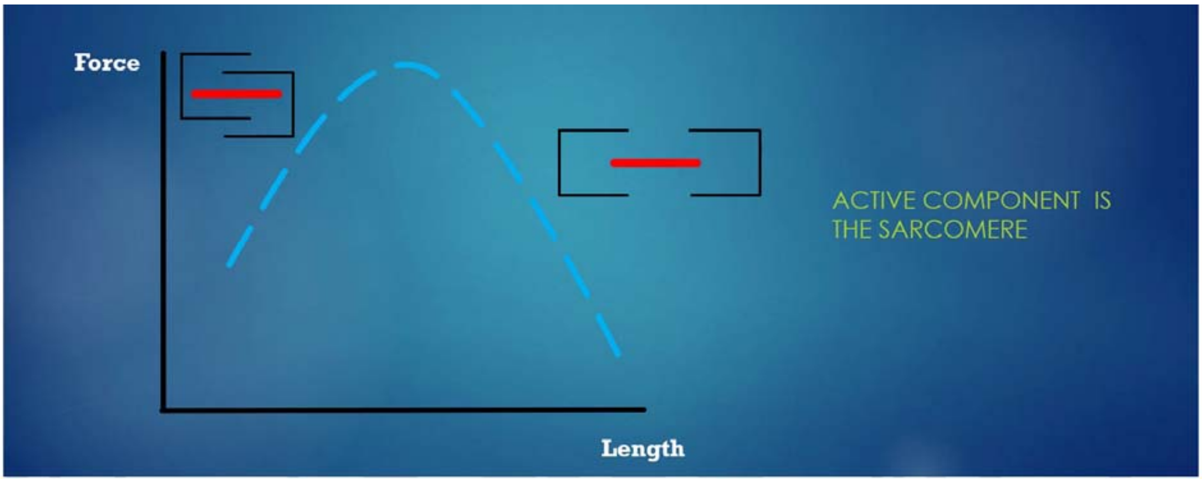
Minimum length for producing active tension



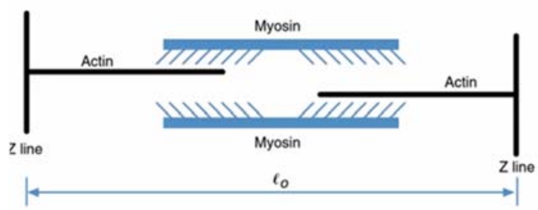
Force-Length Principle

Maximum length for producing active tension

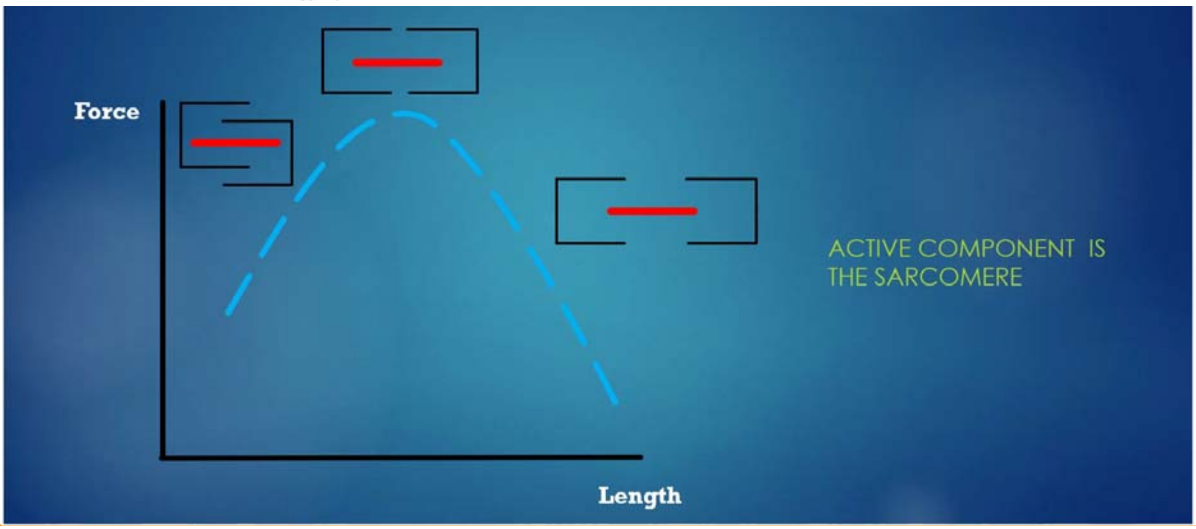




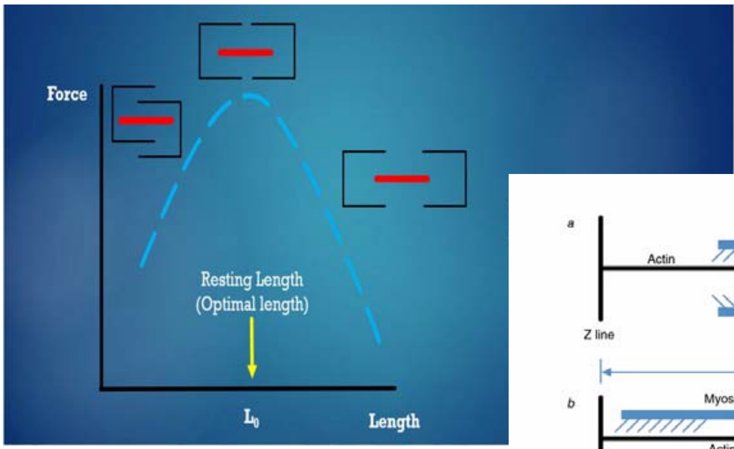
Force-Length Principle

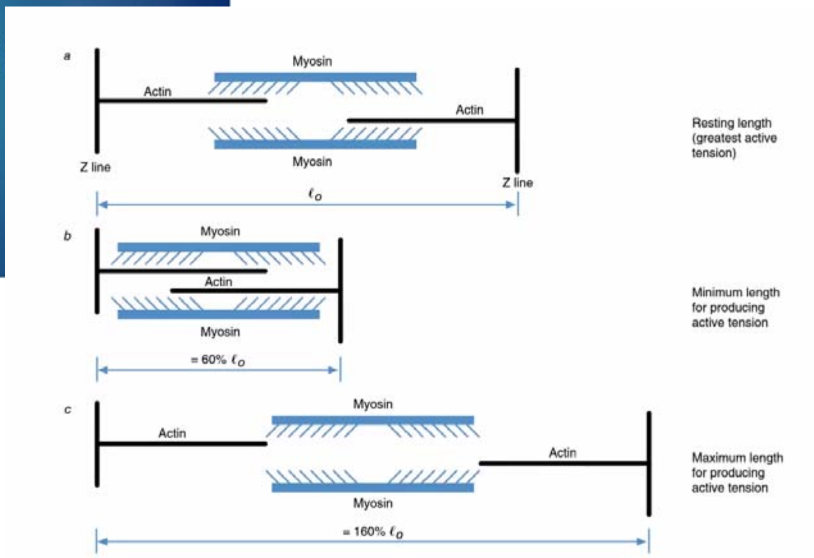


Resting length (greatest active tension)

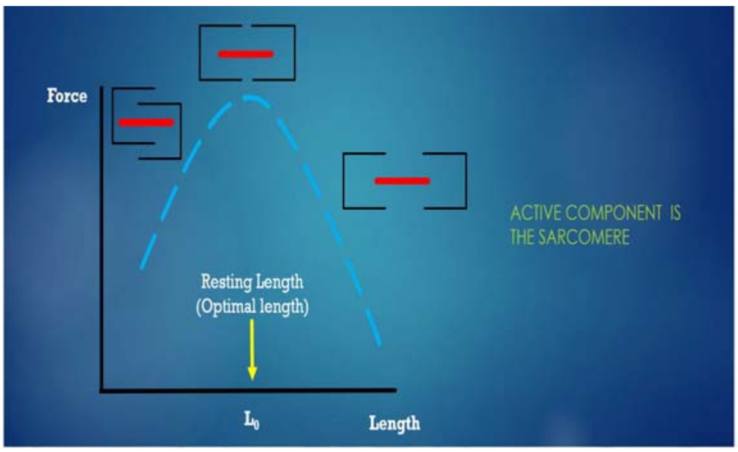


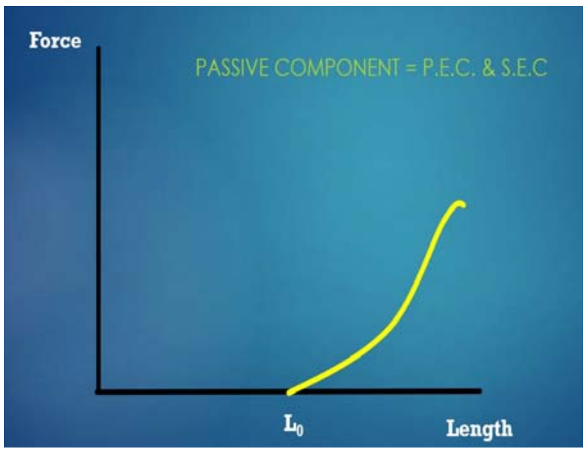
Force-Length Principle





Force-Length Principle





Force-Length Principle

- Single joint muscle
  - Can be stretched roughly 160%
  - Ex: Vastus Lateralis
  - Maximum Tension: ~120% L_o

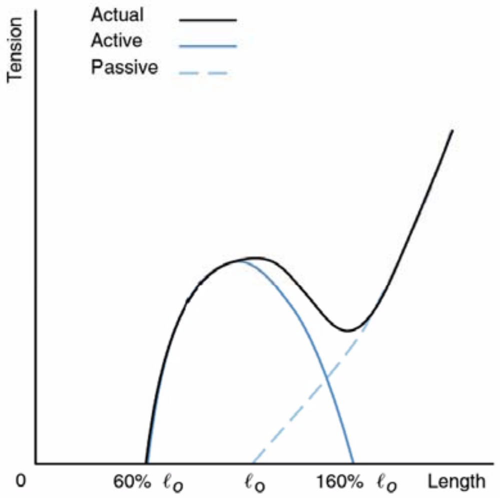
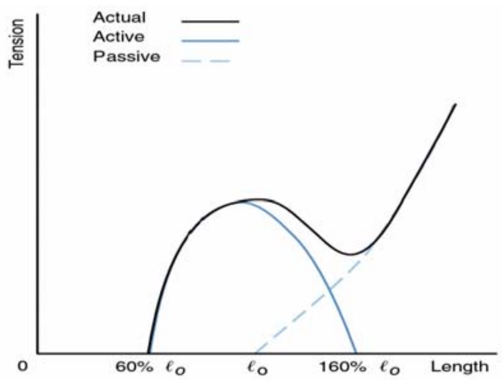


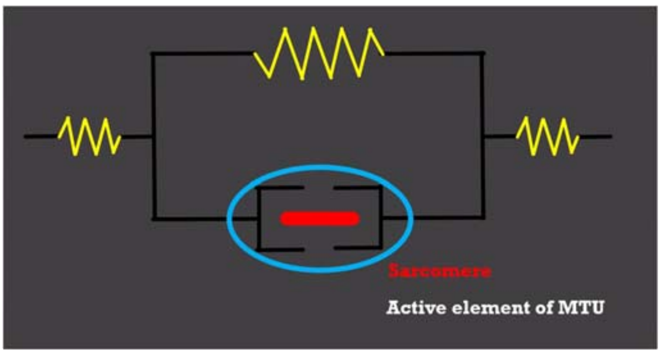
Figure 11.14 The relationship between muscle length and tension.

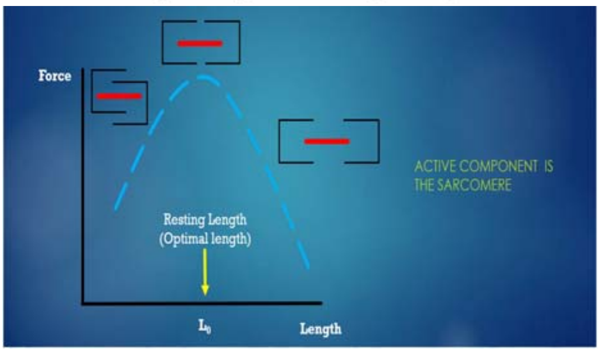
- Biarticular muscle
  - Can be stretched beyond 160%
  - Ex: Rectus femoris (Hip & Knee)
  - Ex: Hamstrings – Semimembranosus, Semitendinosus, & Biceps Femoris (Hip & Knee)

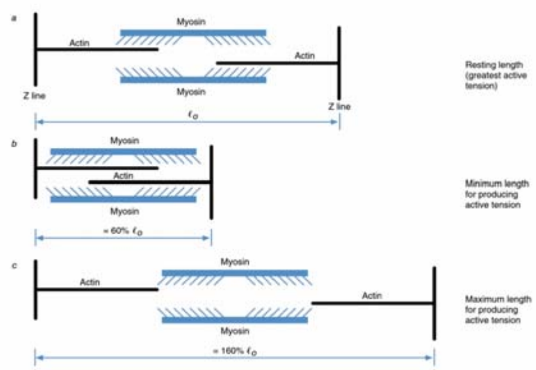
Maximum Tension: >160% L_o

Summary









A simple one-dimensional model of a skeletal muscle

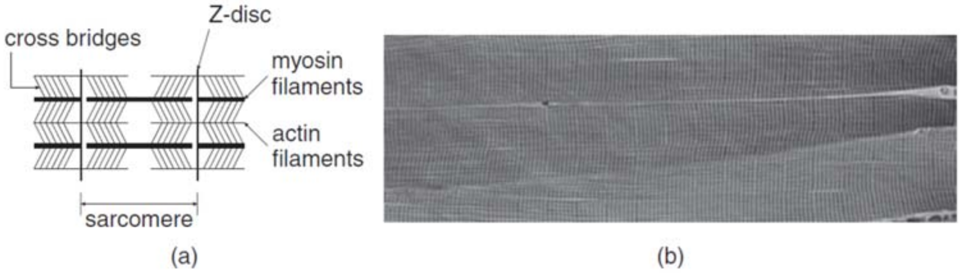
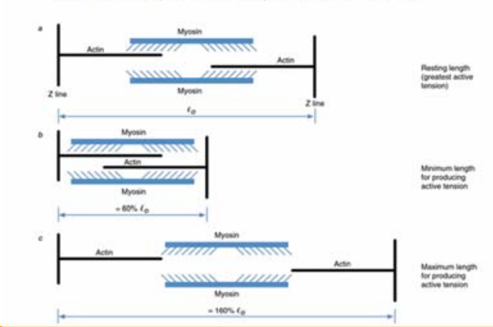


Figure 4.5

(a) Basic structure of a contractile element (sarcomere) of a muscle (b) Cross section of a muscle, vertical stripes correspond to Z-discs.



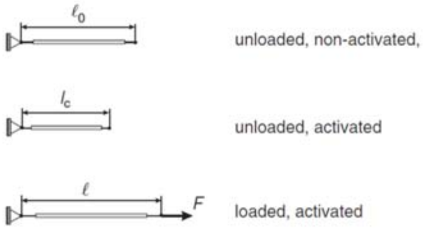


Figure 4.6

Different reference and current lengths of a muscle.

A simple one-dimensional model of a skeletal muscle

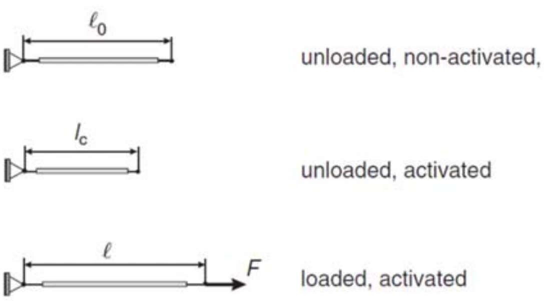


Figure 4.6

Different reference and current lengths of a muscle.

- l_0: the length of the muscle in the non-activated state
- l_c: the length of the muscle in the activated or contracted but unloaded state.
- l: the length of the muscle in the activated and loaded state.

- Now, in contrast with a simple elastic spring, the contracted length l_c serves as the reference length, such that the force in the muscle may be expressed as:

$$F = c\left(\frac{\ell}{\ell_c} - 1\right) \qquad \lambda_c = \frac{\ell_c}{\ell_0} \qquad \lambda = \frac{\ell}{\ell_0}$$

activation or contraction stretch

A simple one-dimensional model of a skeletal muscle

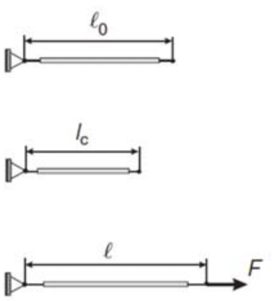


Figure 4.6

Different reference and current lengths of a muscle.

- l_0: the length of the muscle in the non-activated state
- l_c: the length of the muscle in the activated or contracted but unloaded state.
- l: the length of the muscle in the activated and loaded state.

- Now, in contrast with a simple elastic spring, the contracted length l_c serves as the reference length, such that the force in the muscle may be expressed as:

$$F = c\left(\frac{\lambda}{\lambda_c} - 1\right) \quad \text{with} \quad \lambda = \frac{\ell}{\ell_0}$$

More complicated models of a skeletal muscle

- A large group of models is based on experimental work by Hill and supply a phenomenological description of the non-linear activated muscle. These models account for the effect of contraction velocity and for the difference in activated and passive state of the muscle.
  - Hill, A. V. (1938) The heat of shortening and the dynamic constants in muscle. Proc. Roy. Soc. London 126, 136–65.
- Microstructural models were developed based on the sliding filament theories of Huxley. These models can even account for the calcium activation of the muscle.
  - Huxley, A. F. (1957). Muscle structure and theory of contraction. Prog. Biochem. Biophysic. Chem., 255–318.
- A discussion of these models is beyond the scope of this course but may be a good topic for the project.

Elastic fibers in three dimensions

$$\ell_0 = |\vec{x}_{0,B} - \vec{x}_{0,A}| \qquad \lambda = \frac{\ell}{\ell_0} \qquad \ell = |\vec{x}_{\mathrm{B}} - \vec{x}_{\mathrm{A}}|$$

$$\vec{a}_0 = \frac{\vec{x}_{0,\mathrm{B}} - \vec{x}_{0,\mathrm{A}}}{|\vec{x}_{0,\mathrm{B}} - \vec{x}_{0,\mathrm{A}}|} \qquad \vec{a} = \frac{\vec{x}_{\mathrm{B}} - \vec{x}_{\mathrm{A}}}{|\vec{x}_{\mathrm{B}} - \vec{x}_{\mathrm{A}}|}$$

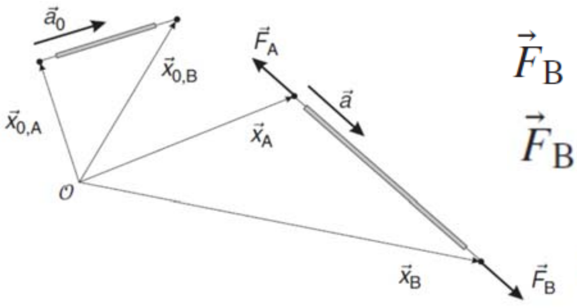


Figure 4.7
Spring in three-dimensional space.

$$\vec{F}_{\mathrm{B}} = -\vec{F}_{\mathrm{A}}$$

$$\vec{F}_{\mathrm{B}} = F\vec{a}$$

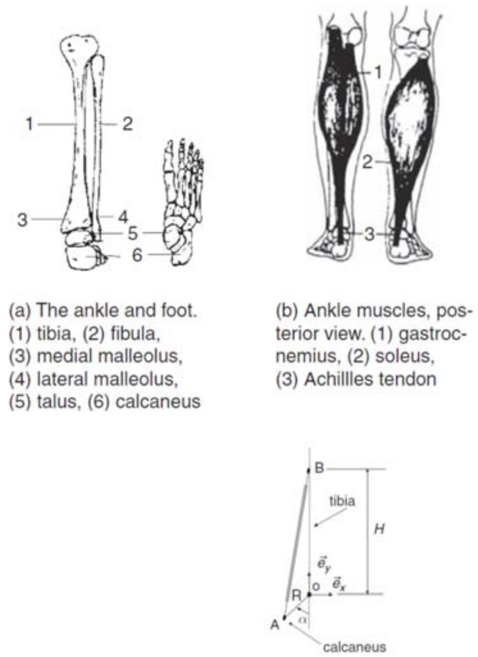
$$F = c(\lambda - 1)$$

$$\vec{F}_{\mathrm{B}} = c(\lambda - 1)\vec{a}$$

$$\vec{F}_{\mathrm{A}} = -c(\lambda - 1)\vec{a}$$

Example

- The Achilles tendon is attached to the rear of the ankle (the calcaneus) and is connected to two muscle groups: the gastrocnemius and the soleus, which, in turn, are connected to the tibia, see Fig. 4.12(a, b). A schematic drawing of this, using a lateral view, is given in Fig. 4.12(c). If the ankle is rotated with respect to the pivot point O, i.e. the origin of the coordinate system, the attachment point A is displaced causing a length change of the muscle system.



(a) The ankle and foot. (1) tibia, (2) fibula, (3) medial malleolus, (4) lateral malleolus, (5) talus, (6) calcaneus

(b) Ankle muscles, posterior view. (1) gastrocnemius, (2) soleus, (3) Achillles tendon

(c) Location of muscle

Figure 4.12
Muscle attached to tibia and calcaneus.

Example

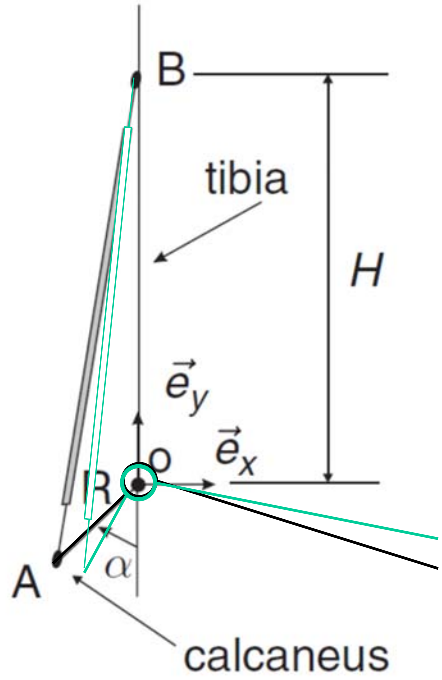
- $\vec{F}_{\mathrm{B}} = ?$
- The position of the attachment point *A* is given by

$$\vec{x}_A = -R\sin(\alpha)\vec{e}_x - R\cos(\alpha)\vec{e}_y,$$

- where *R* is the constant distance of the attachment point *A* to the pivot point. The angle *α* is defined in clockwise direction. The muscles are connected to the tibia at point *B*, hence:

$$\vec{x}_{\mathrm{B}} = H\vec{e}_y,$$

- with *H* the distance of point *B* to the pivot point.



Example

- The positions in the undeformed, unstretched configuration of these points are

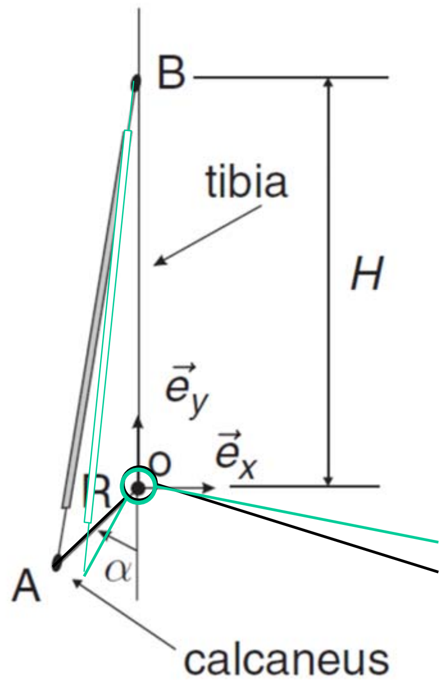
$$\vec{x}_{0,\mathrm{A}} = -R\sin(\alpha_0)\vec{e}_x - R\cos(\alpha_0)\vec{e}_y$$

- and

$$\vec{x}_{0,\mathrm{B}} = H\vec{e}_y.$$

- Hence, the stretch of the muscle follows from

$$\lambda = \frac{|\vec{x}_{\mathrm{A}} - \vec{x}_{\mathrm{B}}|}{|\vec{x}_{0,\mathrm{A}} - \vec{x}_{0,\mathrm{B}}|} = \frac{\sqrt{(R\sin(\alpha))^2 + (R\cos(\alpha) + H)^2}}{\sqrt{(R\sin(\alpha_0))^2 + (R\cos(\alpha_0) + H)^2}},$$



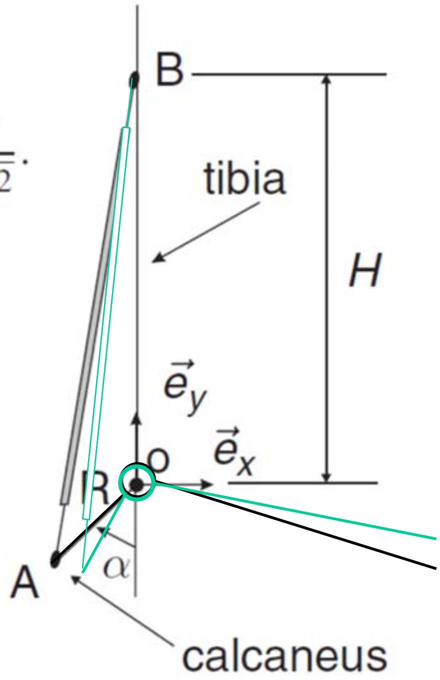
Example

- while the orientation of the muscle is given by and

$$\vec{a} = \frac{R\sin(\alpha)\vec{e}_x + (R\cos(\alpha) + H)\vec{e}_y}{\sqrt{(R\sin(\alpha))^2 + (R\cos(\alpha) + H)^2}}.$$

- From these results the force acting on the muscle at point B may be computed:

$$\vec{F}_B = c(\lambda - 1)\vec{a}.$$



Example

$$\vec{a} = \frac{R\sin(\alpha)\vec{e}_x + (R\cos(\alpha) + H)\vec{e}_y}{\sqrt{(R\sin(\alpha))^2 + (R\cos(\alpha) + H)^2}}$$

$$\lambda = \frac{\sqrt{(R\sin(\alpha))^2 + (R\cos(\alpha) + H)^2}}{\sqrt{(R\sin(\alpha_0))^2 + (R\cos(\alpha_0) + H)^2}}$$

$$\vec{F}_B = c(\lambda - 1)\vec{a}.$$

- The force components in the *x*- and *y*-direction, scaled by the constant *c*, are depicted in Fig. 4.13 in case R = 5 [cm], H = 40 [cm] and an initial angle $\alpha_0 = \pi/4$.

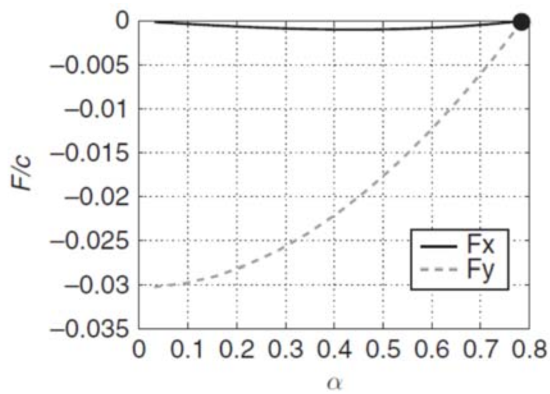


Figure 4.13

Force in muscle.

Biomaterials

- Biomaterials is a term used to indicate materials that constitute parts of medical implants, extracorporeal devices, and disposables that have been utilized in medicine, surgery, dentistry, and veterinary medicine as well as in every aspect of patient health care.

Mechanical Properties of Some Implant Materials and Tissues

	Elastic modulus (GPa)	Yield strength (MPa)	Tensile strength (MPa)	Elongation to failure (%)
Al_2O_3	350	—	1000 to 10,000	0
CoCr Alloy[a]	225	525	735	10
316 S.S.[b]	210	240 (800)[c]	600 (1000)[c]	55 (20)[c]
Ti–6Al–4V	120	830	900	18
Bone (cortical)	15 to 30	30 to 70	70 to 150	0–8
PMMA	3.0	—	35 to 50	0.5
Polyethylene[d]	0.6–1.8	—	23 to 40	200–400
Cartilage	[e]	—	7 to 15	20

[a] 28% Cr, 2% Ni, 7% Mo, 0.3% C (max), Co balance.
[b] Stainless steel, 18% Cr, 14% Ni, 2 to 4% Mo, 0.03 C (max), Fe balance.
[c] Values in parentheses are for the cold-worked state.
[d] High density polyethylene (HDPE) and ultrahigh molecular weight polyethylene (UHMWPE)
[e] Strongly viscoelastic.


BIOMATERIALS SCIENCE

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Average Mechanical Properties of Typical Engineering Materials[a] (SI Units)

Materials		Density ρ (Mg/m^3)	Modulus of Elasticity E (GPa)	Modulus of Rigidity G (GPa)	Yield Strength (MPa) σ_Y Tens.	Yield Strength (MPa) σ_Y Comp.[b]	Yield Strength (MPa) σ_Y Shear	Ultimate Strength (MPa) σ_u Tens.	Ultimate Strength (MPa) σ_u Comp.[b]	Ultimate Strength (MPa) σ_u Shear	% Elongation in 50 mm specimen	Poisson's Ratio ν	Coef. of Therm. Expansion α $(10^{-6})/°C$
Metallic													
Aluminum Wrought Alloys	2014-T6	2.79	73.1	27	414	414	172	469	469	290	10	0.35	23
	6061-T6	2.71	68.9	26	255	255	131	290	290	186	12	0.35	24
Cast Iron Alloys	Gray ASTM 20	7.19	67.0	27	–	–	–	179	669	–	0.6	0.28	12
	Malleable ASTM A-197	7.28	172	68	–	–	–	276	572	–	5	0.28	12
Copper Alloys	Red Brass C83400	8.74	101	37	70.0	70.0	–	241	241	–	35	0.35	18
	Bronze C86100	8.83	103	38	345	345	–	655	655	–	20	0.34	17
Magnesium Alloy	[Am 1004-T61]	1.83	44.7	18	152	152	–	276	276	152	1	0.30	26
Steel Alloys	Structural A-36	7.85	200	75	250	250	–	400	400	–	30	0.32	12
	Structural A992	7.85	200	75	345	345	–	450	450	–	30	0.32	12
	Stainless 304	7.86	193	75	207	207	–	517	517	–	40	0.27	17
	Tool L2	8.16	200	75	703	703	–	800	800	–	22	0.32	12
Titanium Alloy	[Ti-6Al-4V]	4.43	120	44	924	924	–	1,000	1,000	–	16	0.36	9.4
Nonmetallic													
Concrete	Low Strength	2.38	22.1	–	–	–	12	–	–	–	–	0.15	11
	High Strength	2.37	29.0	–	–	–	38	–	–	–	–	0.15	11
Plastic Reinforced	Kevlar 49	1.45	131	–	–	–	–	717	483	20.3	2.8	0.34	–
	30% Glass	1.45	72.4	–	–	–	–	90	131	–	–	0.34	–
Wood Select Structural Grade	Douglas Fir	0.47	13.1	–	–	–	–	2.1[c]	26[d]	6.2[d]	–	0.29[e]	–
	White Spruce	0.36	9.65	–	–	–	–	2.5[c]	36[d]	6.7[d]	–	0.31[e]	–

[a] Specific values may vary for a particular material due to alloy or mineral composition, mechanical working of the specimen, or heat treatment. For a more exact value reference books for the material should be consulted.

[b] The yield and ultimate strengths for ductile materials can be assumed equal for both tension and compression.

[c] Measured perpendicular to the grain.

[d] Measured parallel to the grain.

[e] Deformation measured perpendicular to the grain when the load is applied along the grain.

Hibbeler, Russell C.. Mechanics of Materials in SI Units, Pearson Education Limited, 2017.