**Tutorial 2 (Chapter 2)**

1. There are two children in a family. Given one of them is a girl, what is the probability that the other is also a girl (suppose a child being a boy or a girl is equally likely)?

2. 9 persons randomly enter 3 different rooms. What is the probability that

   (a) the first room has 3 person?

   (b) every room has 3 persons?

   (c) the first room has 4 person, second room 3 persons, third room 2 persons?

   (d) the three rooms have 4,3,2 persons respectively (but we don't know which room has 4 persons, for example)?

3. (a) n persons randomly enter n different rooms, what is the probability that each room has one person?

   (b) n persons randomly enter $k$ different rooms $(n \geq k)$, what is the probability that each room has at least one person?

4. A student is taking a multiple choice exam in which each question has $m$ possible answers, exactly one of which is correct. If the student knows the answer he selects the correct answer. Otherwise he makes a guess at random from the possible answers. Suppose that the probability that he knows the correct answer is 0.7

   (a) What is the probability that on a given question the student gets the correct answer?

   (b) If the student gets the correct answer to a question, what is the probability that he knows the answer?

5. (Polya's Urn) An urn initially contains 5 white and 7 black balls. Each time a ball is selected its colour is noted and it is replaced in the urn along with 2 other balls of the same colour.

   (a) Compute the probability that the first 2 balls selected are white and the next two black.

   (b) Compute the probability that, of the first 4 balls selected, exactly 2 are black.

   (c) Compute the conditional probability that the first ball was black, given that the second ball drawn was white.

6. In a bolt factory machines $A$, $B$ and $C$ manufacture, respectively 25% 35%and 40% of the total. Of their output 5%, 4% and 2% are defective bolts.

   (a) Find the probability that a bolt drawn is defective.

   (b) A bolt is drawn at random from the produce and is found defective. What is the probability that it is manufactured by machine $A$?

7. (a) If $P(A) = 1/2, P(B) = 1/3, P(B|A) = 2/3, P(A \cup B) =$?

   (b) If $P(AB) = 0, P(A) = P(B) = a, P(A|B^c) = P(A^c|B^c)$, then $a =$?

8. Prove or give counterexamples to the following statements:

   (a) If $E$ is independent of $F$ and $E$ is independent of $G$, then $E$ is independent of $F \cup G$.

   (b) If $E$ is independent of $F$ and $E$ is independent of $G$ and $FG = \phi$, then $E$ is independent of $F \cup G$.

9. Show that if $A$, $B$ and $C$ are three events, (assuming the probabilities of some events are nonzero when appropriate so that we do not need to worry about denominator being zero), and $P(C|AB) = P(C|B)$, then $P(A|BC) = P(A|B)$.

10. A and B flip coins in turn with A starting first. The first person who gets head wins. What is the probability that A wins?

11. A company buys tires from two suppliers-1 and 2. Supplier 1 has a record of delivering tires that contain 10% defectives, where as supplier 2 has a defective rate of only 5%. Suppose that 40% if the current supply came from supplier 1. If a tire taken at random from this supply and observed to be defective, what is the probability that it came from supplier 1?

12. An individual tried by a 3-judge panel is declared guilty if at least 2 judges cast votes of guilty. Suppose that when the defendant is in fact guilty, each judge will independently vote guilty with probability 0.6; whereas when the defendant is, in fact, innocent, this probability drops to 0.2. If 50% of defendants are guilty, compute the conditional probability that Judge number 3 votes guilty given that Judges 1 and 2 votes guilty.

Let $J_i$, $i = 1, 2, 3$ denote the event that Judge $i$ casts a guilty vote. Are these events independent? Are they conditionally independent, i.e. are these events independent given a defendant a defendant is guilty?