1. (a) (5)

$$A = \begin{bmatrix} 1 & 3 & 6 \\ 2 & -1 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -2 & 1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 3 & 6 \\ 2 & -1 & 9 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -5 & 6 \\ -18 & 16 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 6 \\ 2 & -1 & 9 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 33 \\ 0 & 7 & 3 \\ 0 & -7 & -3 \end{bmatrix}$$

(b) (5)

$$A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & -2 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix} \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & -2 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 6 \\ 1 & 3 & -7 \\ -1 & -3 & 5 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & -2 \end{bmatrix} \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 4 & 9 & 8 \\ -1 & -7 & -9 \\ 3 & 11 & 13 \end{bmatrix}$$

2. (a) (5)

$$\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} = \det\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix} = -7 \neq 0 \quad \therefore \text{ Yes}$$

(b) (5)

$$\begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix} = \det\begin{pmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{pmatrix}$$

$$= \cancel{0 \cdot \det\begin{pmatrix} 3 & 3 \\ -2 & -2 \end{pmatrix}}^{\,0} - (-2) \cdot \det\begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix} + (-3)\det\begin{pmatrix} 1 & 3 \\ -1 & -2 \end{pmatrix}$$

$$= -1 \neq 0 \quad \therefore \text{ Yes}$$

3. $\begin{cases} 3x - 6y = 11 \\ 2x - 4y = 8 \end{cases}$

(10)

$$\left[\begin{array}{cc|c} 3 & -6 & 11 \\ 2 & -4 & 8 \end{array}\right] \xrightarrow{\frac{1}{3}R_1 \to R_1} \left[\begin{array}{cc|c} 1 & -2 & 11/3 \\ 2 & -4 & 8 \end{array}\right]$$

$$\downarrow R_2 - 2R_1 \to R_2$$

$$\left[\begin{array}{cc|c} 1 & -2 & 11/3 \\ 0 & 0 & 2/3 \end{array}\right]$$

$0 = 2/3$ nonsense

$\therefore$ system has no solution

4. (a) $3x - 4y = -2$

(10) $x + y = 6$

$$x = \frac{\begin{vmatrix} -2 & -4 \\ 6 & 1 \end{vmatrix}}{\begin{vmatrix} 3 & -4 \\ 1 & 1 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 3 & -2 \\ 1 & 6 \end{vmatrix}}{\begin{vmatrix} 3 & -4 \\ 1 & 1 \end{vmatrix}}$$

$$x = \frac{22}{7} \qquad y = \frac{20}{7} \qquad \therefore \left(\frac{22}{7}, \frac{20}{7}\right)$$

(b) $x - 2y - 2z = 3$

(10) $2x - 4y + 4z = 1$

$3x - 3y - 3z = 4$

$$x = \frac{\begin{vmatrix} 3 & -2 & -2 \\ 1 & -4 & 4 \\ 4 & -3 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & -2 \\ 2 & -4 & 4 \\ 3 & -3 & -3 \end{vmatrix}} \qquad y = \frac{\begin{vmatrix} 1 & 3 & -2 \\ 2 & 1 & 4 \\ 3 & 4 & -3 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & -2 \\ 2 & -4 & 4 \\ 3 & -3 & -3 \end{vmatrix}} \qquad z = \frac{\begin{vmatrix} 1 & -2 & 3 \\ 2 & -4 & 1 \\ 3 & -3 & 4 \end{vmatrix}}{\begin{vmatrix} 1 & -2 & -2 \\ 2 & -4 & 4 \\ 3 & -3 & -3 \end{vmatrix}}$$

$$x = \frac{8}{-24} = -\frac{1}{3} \qquad y = -\frac{25}{24} \qquad z = -\frac{15}{24} = -\frac{5}{8}$$

$$\therefore \left(-\frac{1}{3}, -\frac{25}{24}, -\frac{5}{8}\right)$$

5(a)

(10)

$$\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$= \begin{bmatrix} (-1)(a)+(2)(b)+(3)(c) \\ (2)(a)+(1)(b)+(0)(c) \\ (3)(a)+(5)(b)+(-1)(c) \end{bmatrix} = \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix}$$

$\therefore$ LHS = RHS.

(b)

(10)

$$\begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & +\sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \cos^2\theta+\sin^2\theta & 0 & \cos\theta\sin\theta-\cos\theta\sin\theta \\ 0 & 1 & 0 \\ \sin\theta\cos\theta-\sin\theta\cos\theta & 0 & \sin^2\theta+\cos^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

used $\sin^2\theta+\cos^2\theta=1$

LHS = RHS

6.

(a) $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ $\quad B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$

(10)

$$(A+B) = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}$$

$$\therefore (A+B)(A+B) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$2AB = 2\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -6 & 8 \\ -2 & 4 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix}$$

$$\therefore \begin{bmatrix} -2 & 6 \\ -3 & 7 \end{bmatrix} \neq \begin{bmatrix} -4 & 8 \\ -5 & 9 \end{bmatrix}$$

(b) $(A-B) = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}$

(10)

$$(A+B)(A-B) = \begin{bmatrix} 0 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -2 \\ 5 & -5 \end{bmatrix}$$

$$A^2 - B^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 2 & -2 \\ 5 & -5 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 3 & -3 \end{bmatrix}$$

7.

(10)

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1x+d_1 & c_2x+d_2 & c_3x+d_3 \end{vmatrix}$$

$$= (c_1x+d_1)\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - (c_2x+d_2)\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + (c_3x+d_3)\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= c_1x\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} + d_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - c_2x\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} - d_2\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} +$$

$$c_3x\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} + d_3\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

$$= x\left[c_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - c_2\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + c_3x\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\right] +$$

$$\left[d_1\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - d_2\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + d_3\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\right]$$

$$= x\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$$