MA1301 **Name:** ____________________

Semester B 2021-22

HW 1

1. Compute the following integral using method of substitution:

$$(a) \int \frac{e^{1+\frac{1}{x^2}}}{x^3} dx \qquad (b) \int x^{11}\sqrt{1+x^4}dx$$

$$(c) \int \sin 2x\sqrt{\cos x}dx \qquad (d) \int_1^2 xe^{x^2-1}dx$$

$$(e) \int_1^5 \frac{\sin^2(\ln x)}{x}dx \qquad (f) \int \sin^7 x dx$$

$$(g) \int \sqrt{9-16x^2}dx \qquad (h) \int \frac{1}{x^2\sqrt{1-x^2}}dx$$

2. Compute the following integral using integration by parts:

$$(a) \int xe^{-3x}dx \qquad (b) \int_1^e \sqrt{x}\ln x dx$$

$$(c) \int x^2 \sin x dx \qquad (d) \int x\sin^2 x dx$$

$$(e) \int_1^e \left(\frac{\ln x}{x}\right)^2 dx \qquad (f) \int \cos^3 x dx$$

$$(g) \int e^x \sin 3x dx \qquad (h) \int \tan^{-1} x dx$$

3. Compute the following integral using suitable method. You may need to use method of substitution or integration by parts or both.

$$(a) \int e^{2x}\sin(2e^x+1)dx \qquad (b) \int_0^1 \sin(2\sqrt{x})dx$$

$$(c) \int_0^1 \ln(1+x^{\frac{1}{3}})dx \qquad (d) \int \cos(\ln x)dx$$

$$(e) \int \sin(2x)\ln(\sin x)dx \qquad (f) \int (x+1)\ln(x+3)dx$$

$$(g) \int x^2\sqrt{4-x^2}dx \qquad (h) \int x^3\sin(4+x^2)dx$$

4. Show the following inequality without calculating the integral

$$2 \leq \int_{-1}^1 \sqrt{1+x^2}dx \leq 2\sqrt{2}.$$

5. Let $f(x)$ be a differentiable function on $[a,b]$ such that $\int_a^b f(x)dx = 0$ and $f(a) = f(b) = 1$. Find the value of $\int_a^b xf'(x)dx$.

6. Let $f(x)$ be the continuous function, show that for any $a > 0$, we must have

$$\int_0^a x^3 f(x^2)dx - \frac{1}{2}\int_0^{a^2} xf(x)dx = 0.$$

7. Consider the integral

$$I_n = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^n x dx.$$

(a) Show that

$$I_n = \frac{n-1}{n} I_{n-2}, \quad n \geq 2.$$

(b) Using the reduction formula obtained in (a), find the value of

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^5 x dx.$$