PHY1203: General Physics III

Chapter 39
Atomic structure; matter wave; blackbody radiation; uncertainty principle – *Part 1*

Outline

Looking forward at …

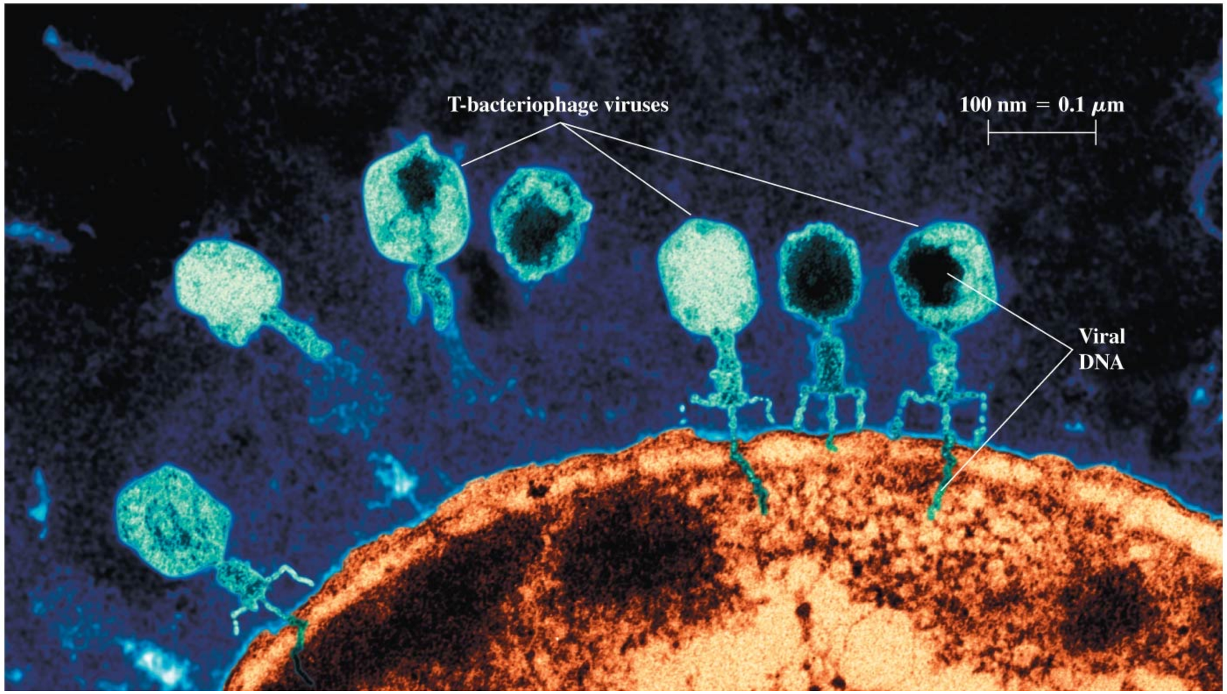
- de Broglie's proposal that electrons and other particles can behave like waves.
- how physicists discovered the atomic nucleus.
- how Bohr's model of electron orbits explained the spectra of hydrogen and hydrogen-like atoms.
- how a laser operates.
- how the idea of electron energy levels, coupled with the photon model of light, explains the spectrum of light emitted by a hot, opaque object.

Introduction

- If light waves can behave like particles, can the particles of matter behave like waves?
- **Yes!**
- Electrons can be made to interfere and diffract just like other kinds of waves.
- We will see that the wave nature of electrons is not merely a laboratory curiosity: It is the fundamental reason why atoms, which according to classical physics should be profoundly unstable, are able to exist.

Introduction

- Viruses (shown in blue) have landed on an *E. coli* bacterium and injected their DNA, converting the bacterium into a virus factory.
- This false-color image was made by using a beam of electrons rather than a light beam.



de Broglie waves

- In 1924 the French physicist Louis de Broglie (pronounced "de broy"), proposed that particles may, in some situations, behave like waves.

"The fundamental idea of [my 1924 thesis] was the following: The fact that, following Einstein's introduction of photons in light waves, one knew that light contains particles which are concentrations of energy incorporated into the wave, suggests that all particles, like the electron, must be transported by a wave into which it is incorporated... ***My essential idea was to extend to all particles the coexistence of waves and particles discovered by Einstein in 1905 in the case of light and photons****."*

Louis de Broglie (1892-1987)

Nobel Prize in Physics 1929

"for his discovery of the wave nature of electrons".

de Broglie waves

- A free particle with rest mass m, moving with non-relativistic speed v, should have a wavelength related to its momentum:

Planck's constant

De Broglie wavelength of a particle
$$\lambda = \frac{h}{p} = \frac{h}{mv}$$
Particle's speed

Particle's momentum Particle's mass

- A particle's frequency is related to its energy in the same way as for a photon:

Planck's constant

Energy of a particle
$$E = hf$$
Frequency

Q39.1

For a photon, the energy E, frequency f, and wavelength λ are related by the equations $E = hf$, $E = hc/\lambda$, and $f = c/\lambda$. (Here h is Planck's constant and c is the speed of light in vacuum.) Which of these equations also applies to *electrons*?

A. $E = hf$

B. $E = hc/\lambda$

C. $f = c/\lambda$

D. two of A, B, and C

E. all three of A, B, and C apply to electrons.

Q39.2

The mass of the proton is 1836 times greater than the mass of the electron. In order for a proton to have the same *momentum* as an electron, how must the de Broglie wavelength λ_p of the proton compare to the de Broglie wavelength λ_e of the electron?

A. λ_p must be equal to $1836\lambda_e$.

B. λ_p must be greater than λ_e, but less than $1836\lambda_e$.

C. λ_p must be equal to λ_e.

D. λ_p must be less than λ_e, but greater than $(1/1836)\lambda_e$.

E. λ_p must be equal to $(1/1836)\lambda_e$.

Q39.3

The mass of the proton is 1836 times greater than the mass of the electron. In order for a proton to have the same *speed* (assumed nonrelativistic) as an electron, how must the de Broglie wavelength λ_p of the proton compare to the de Broglie wavelength λ_e of the electron?

A. λ_p must be equal to $1836\lambda_e$.

B. λ_p must be greater than λ_e, but less than $1836\lambda_e$.

C. λ_p must be equal to λ_e.

D. λ_p must be less than λ_e, but greater than $(1/1836)\lambda_e$.

✔ E. λ_p must be equal to $(1/1836)\lambda_e$.

Davisson and Germer experiment

- de Broglie’s proposal was confirmed experimentally in 1927.

Clinton J. Davisson (1881-1958)

Lester H. Germer (1896-1971)

Nobel Prize in
Physics 1937
Prize share ½

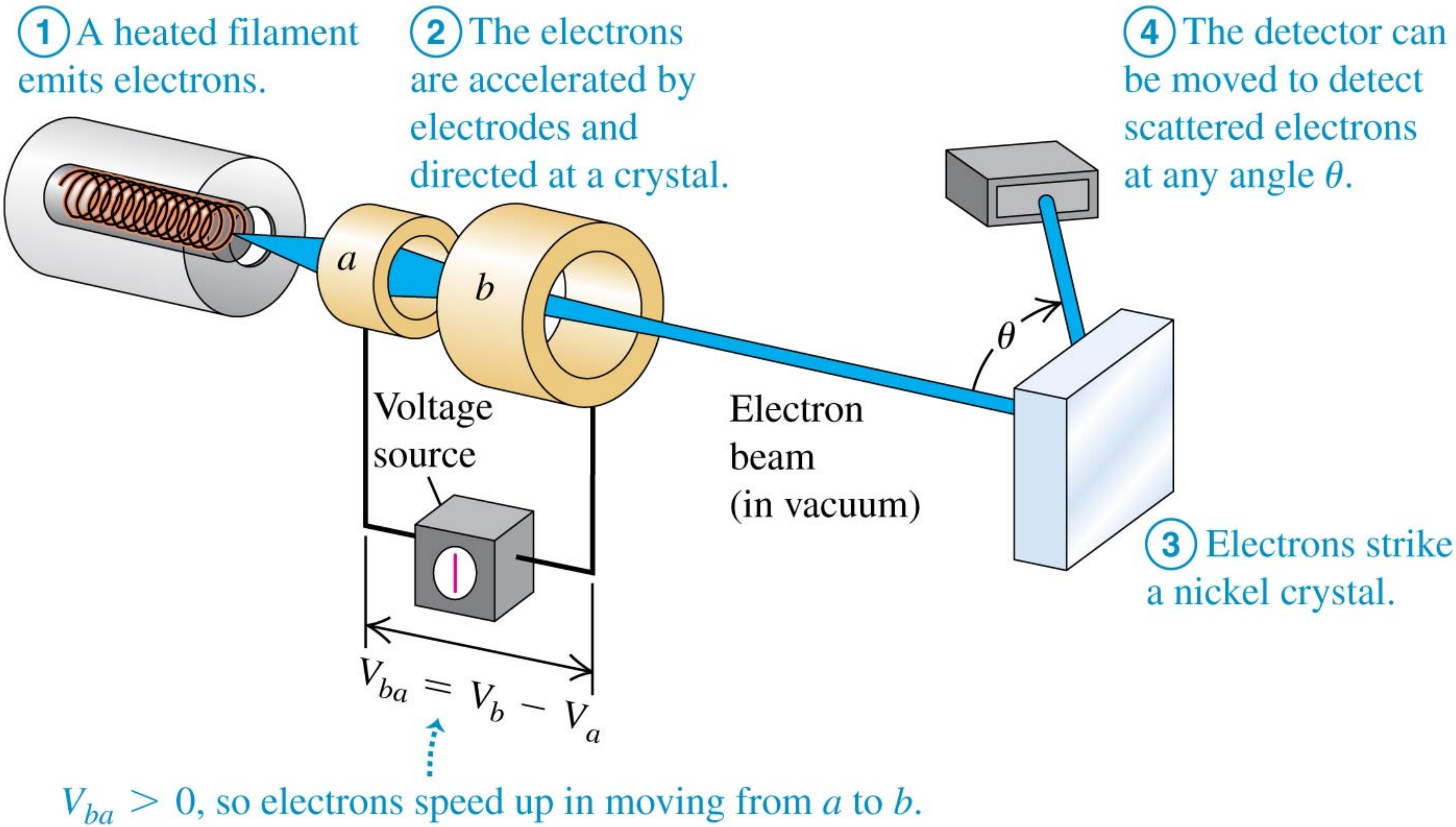
"for their experimental discovery of the diffraction of electrons by crystals".

Elliott Cresson Medal in 1931

"Scattering and Diffraction of Electrons by Crystals".

Davisson and Germer experiment

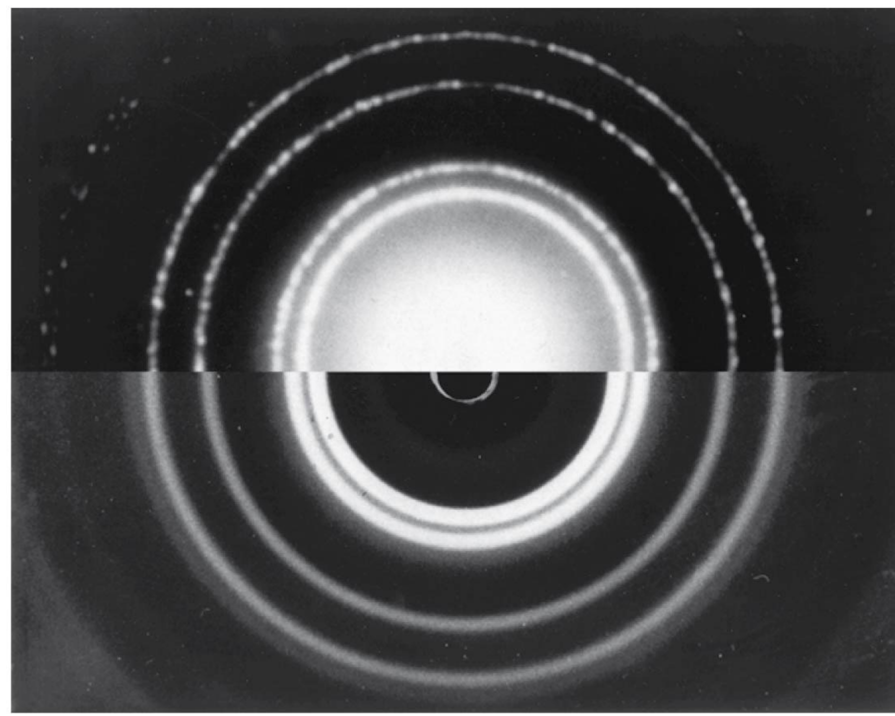
- Shown is an apparatus used to study electron diffraction.



X-ray and electron diffraction

- The upper half of the photo shows the diffraction pattern for 71-pm x rays passing through aluminum foil.
- The lower half, with a different scale, shows the diffraction pattern for 600-eV electrons from aluminum.
- The similarity shows that electrons undergo the same kind of diffraction as x rays.

Top: x-ray diffraction



Bottom: electron diffraction

Davisson and Germer experiment

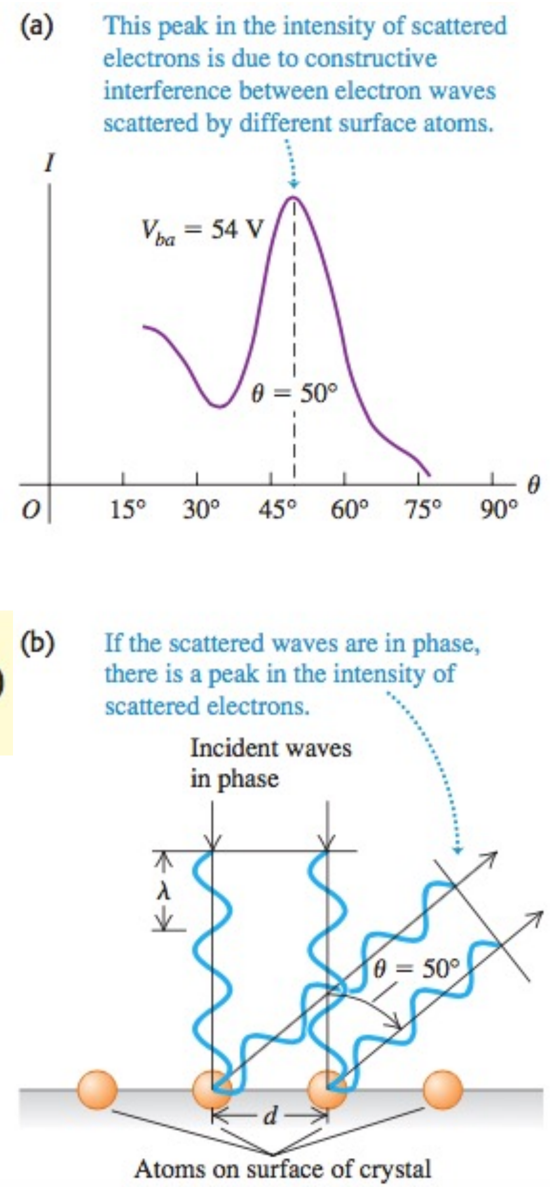
- The momentum of the electrons can be determined as follows: suppose electrons are accelerated from rest at point a to point b through a potential increase V_{ab}, then

$$eV_{ba} = \frac{p^2}{2m} \qquad p = \sqrt{2meV_{ba}}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV_{ba}}} \qquad \text{(de Broglie wavelength of an electron)}$$

- Recall that the maxima of the diffraction pattern are determined as

$$d\sin\theta = m\lambda \qquad (m = 1, 2, 3, \ldots)$$



Q39.4

An electron is accelerated from rest by passing through a voltage V_{ba}. The final wavelength of the electron is λ_1. If the value of V_{ba} is doubled, the final wavelength of the accelerated electron (assumed to be nonrelativistic) changes to

A. $2\lambda_1$.

B. $\sqrt{2}\,\lambda_1$.

C. $\lambda_1/\sqrt{2}$.

D. $\lambda_1/2$.

E. none of the above.

Example 39.1: An electron-diffraction experiment

In an electron-diffraction experiment using an accelerating voltage of 54 V, an intensity maximum occurs for $\theta = 50°$ (see Fig. 39.3a). X-ray diffraction indicates that the atomic spacing in the target is $d = 2.18 \times 10^{-10}$ m $= 0.218$ nm. The electrons have negligible kinetic energy before being accelerated. Find the electron wavelength.

$$\lambda = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{\sqrt{2(9.109 \times 10^{-31}\ \text{kg})(1.602 \times 10^{-19}\ \text{C})(54\ \text{V})}}$$

$$= 1.7 \times 10^{-10}\ \text{m} = 0.17\ \text{nm}$$

Alternatively, using Eq. (39.4) and assuming $m = 1$,

$$\lambda = d \sin\theta = (2.18 \times 10^{-10}\ \text{m}) \sin 50° = 1.7 \times 10^{-10}\ \text{m}$$

EVALUATE: The two numbers agree within the accuracy of the experimental results, which gives us an excellent check on our calculations. Note that this electron wavelength is less than the spacing between the atoms.

Example 39.2: Energy of a thermal neutron

Find the speed and kinetic energy of a neutron ($m = 1.675 \times 10^{-27}$ kg) with de Broglie wavelength $\lambda = 0.200$ nm, a typical interatomic spacing in crystals. Compare this energy with the average translational kinetic energy of an ideal-gas molecule at room temperature ($T = 20°\text{C} = 293$ K).

EXECUTE: From Eq. (39.1), the neutron speed is

$$v = \frac{h}{\lambda m} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{(0.200 \times 10^{-9}\ \text{m})(1.675 \times 10^{-27}\ \text{kg})}$$
$$= 1.98 \times 10^{3}\ \text{m/s}$$

The neutron kinetic energy is

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.675 \times 10^{-27}\ \text{kg})(1.98 \times 10^{3}\ \text{m/s})^2$$
$$= 3.28 \times 10^{-21}\ \text{J} = 0.0205\ \text{eV}$$

Continued

From Eq. (18.16), the average translational kinetic energy of an ideal-gas molecule at $T = 293$ K is

$$\tfrac{1}{2}m(v^2)_{\text{av}} = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(293\ \text{K})$$
$$= 6.07 \times 10^{-21}\ \text{J} = 0.0379\ \text{eV}$$

The two energies are comparable in magnitude, which is why a neutron with kinetic energy in this range is called a *thermal neutron.*

Diffraction of thermal neutrons is used to study crystal and molecular structure in the same way as x-ray diffraction. Neutron diffraction has proved to be especially useful in the study of large organic molecules.

EVALUATE: Note that the calculated neutron speed is much less than the speed of light. This justifies our use of the nonrelativistic form of Eq. (39.1).

De Broglie Waves and the Macroscopic World

- All matters have wave aspects, but why we don’t see ourselves diffracting when we are walking through a door (a single slit)?
- We don’t see it on human scales because the Planck constant h is such a small number. The diffraction does happen, but since the scale are so small for any macroscopic body we see, the wave aspects of them can be safely ignored.

De Broglie Waves and the Macroscopic World

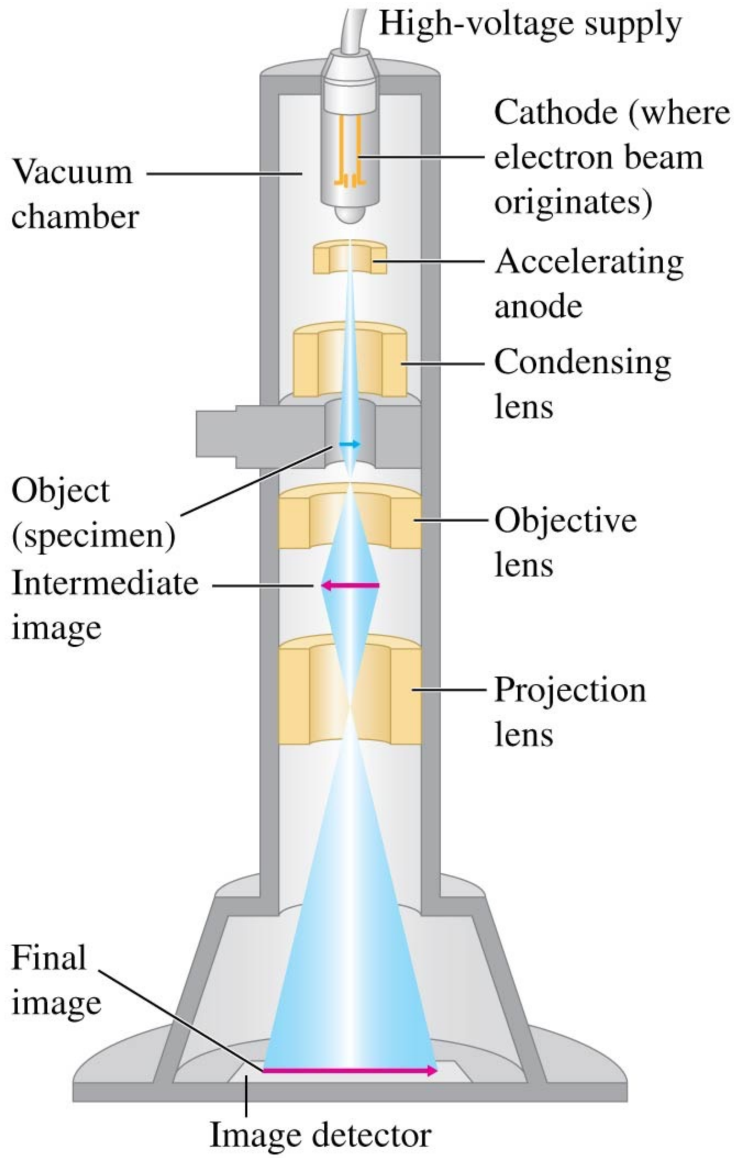
- For instance, what is the wavelength of a falling grain of sand?
- If the grain's mass is $5*10^{-10}$ kg and its diameter is 0.07mm = $7*10^{-5}$ m, it will fall in air with a terminal speed of about 0.4 m/s. The magnitude of its momentum is then
  $p = mv = 15*10^{-10}$kg*0.4 m/s= $2*10^{-10}$kg*m/s.
  The de Broglie wavelength of this falling sand grain is then

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \mathrm{J \cdot s}}{2 \times 10^{-10}\ \mathrm{kg \cdot m/s}} = 3 \times 10^{-24}\ \mathrm{m}$$

- Not only is this wavelength far smaller than the diameter of the sand grain, but it's also far smaller than the size of a typical atom (about 10^{-10}m). A more massive, faster-moving object would have an even larger momentum and an even smaller de Broglie wavelength. The effects of such tiny wavelengths are so small that they are never noticed in daily life.

Electron microscopy

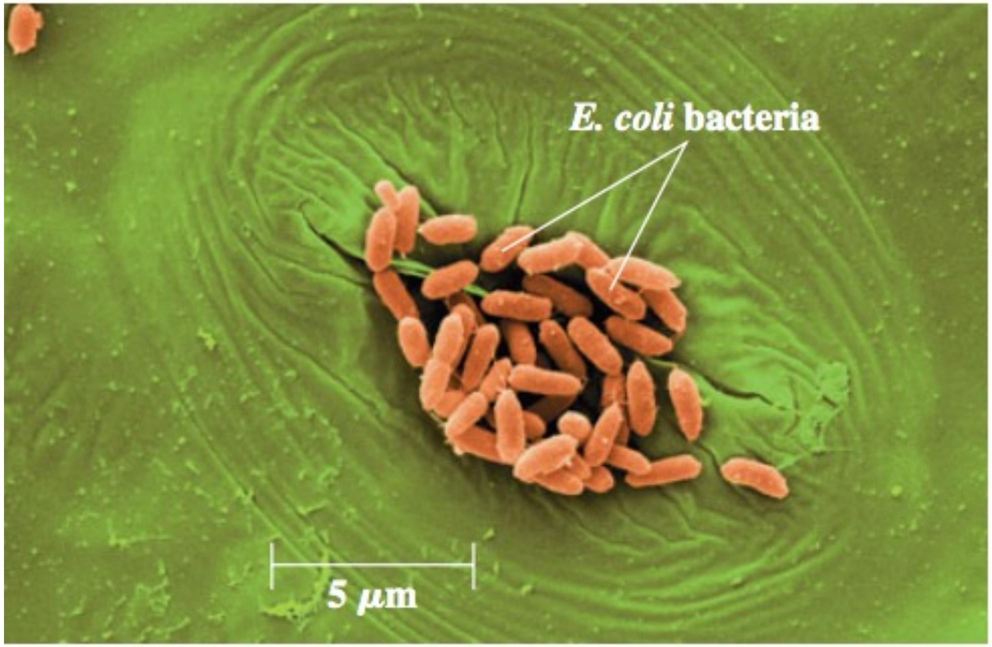
- The wave aspect of electrons means that they can be used to form images, just as light waves can.
- This is the basic idea of the transmission electron microscope (TEM), shown.
- The "lenses" are actually coils that use magnetic fields to focus the electrons.



TEM

E. coli as seen by SEM

39.6 This scanning electron microscope image shows *Escherichia coli* bacteria crowded into a stoma, or respiration opening, on the surface of a lettuce leaf. (False color has been added.) If not washed off before the lettuce is eaten, these bacteria can be a health hazard. The *transmission* electron micrograph that opens this chapter shows a greatly magnified view of the surface of an *E. coli* bacterium.



Example 39.3: An electron microscope

In an electron microscope, the nonrelativistic electron beam is formed by a setup similar to the electron gun used in the Davisson–Germer experiment (see Fig. 39.2). The electrons have negligible kinetic energy before they are accelerated. What accelerating voltage is needed to produce electrons with wavelength 10 pm = 0.010 nm (roughly 50,000 times smaller than typical visible-light wavelengths)?

IDENTIFY, SET UP, and EXECUTE: We can use the same concepts we used to understand the Davisson–Germer experiment. The accelerating voltage is the quantity V_{ba} in Eq. (39.3). Rewrite this equation to solve for V_{ba}:

$$V_{ba} = \frac{h^2}{2me\lambda^2}$$

$$= \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})^2}{2(9.109 \times 10^{-31}\ \mathrm{kg})(1.602 \times 10^{-19}\ \mathrm{C})(10 \times 10^{-12}\ \mathrm{m})^2}$$

$$= 1.5 \times 10^4\ \mathrm{V} = 15{,}000\ \mathrm{V}$$

EVALUATE: It is easy to attain 15-kV accelerating voltages from 120-V or 240-V line voltage using a step-up transformer (Section 31.6) and a rectifier (Section 31.1). The accelerated electrons have kinetic energy 15 keV; since the electron rest energy is 0.511 MeV = 511 keV, these electrons are indeed nonrelativistic.

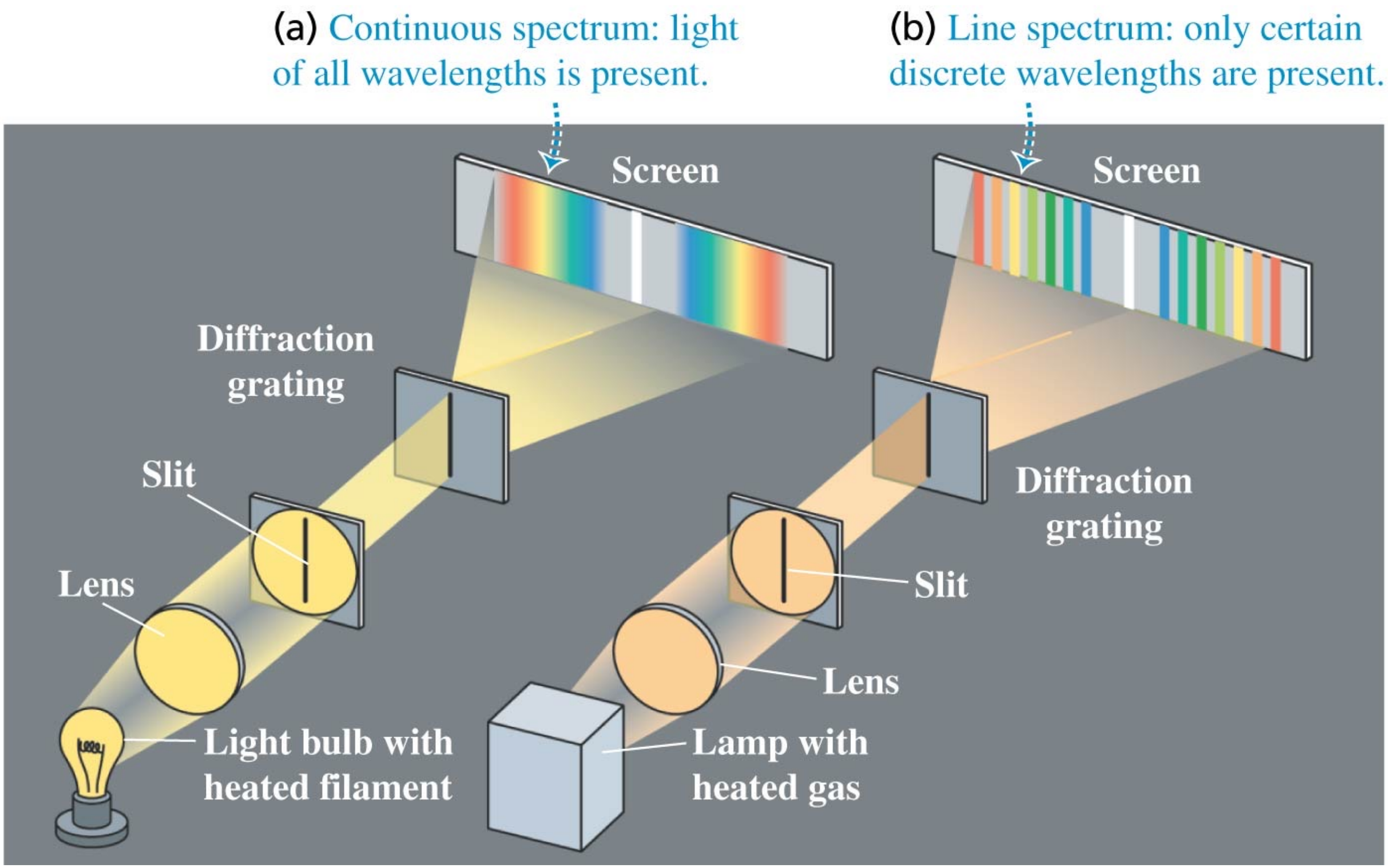
The Nuclear Atom and Atomic Spectra

- Every neutral atom contains at least one electron. How does the wave aspect of electrons affect atomic structure?
- As we will see, it is crucial for understanding not only the structure of atoms but also how they interact with light.
- Historically, the quest to understand the nature of the atom was intimately linked with both the idea that electrons have wave characteristics and the notion that light has particle characteristics.
- Let us start with what was known about atoms—as well as what remained mysterious— by the first decade of the twentieth century.

Line spectra

- Heated materials emit light, and that different materials emit different kinds of light.
- The coils of a toaster glow red when in operation, the flame of a match has a characteristic yellow color, and the flame from a gas range is a distinct blue.
- To analyze these different types of light, we can use a prism or a diffraction grating to separate the various wavelengths in a beam of light into a spectrum.
- If the light source is a hot solid (such as the filament of an incandescent light bulb) or liquid, the spectrum is *continuous;* light of all wavelengths is present.

Atomic line spectra



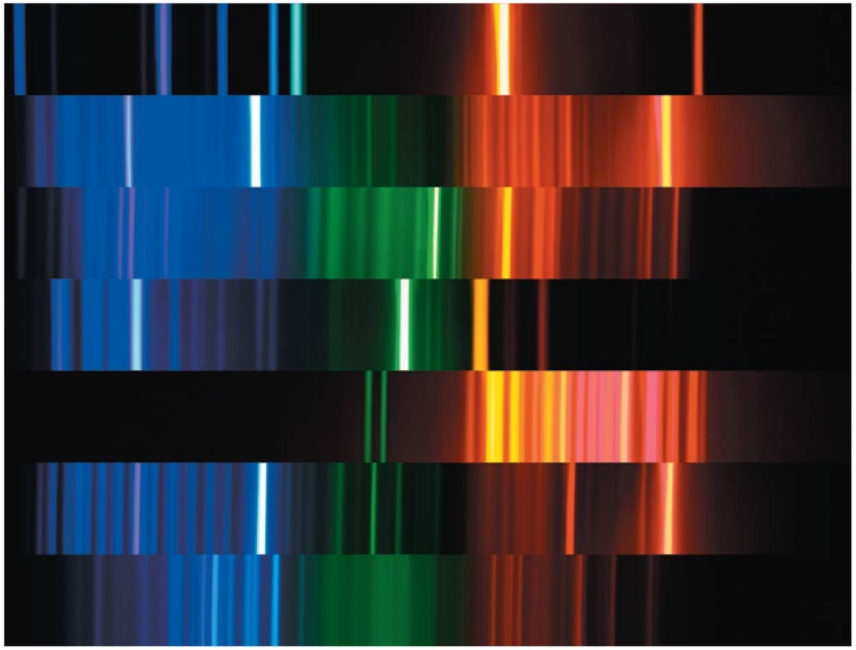
- The light emitted by atoms in a sample of heated gas includes only certain discrete wavelengths. Nineteenth-century physics does not explain this.

Atomic line spectra (emission)

- If the source is a heated *gas*, such as the neon in an advertising sign or the sodium vapor formed when table salt is thrown into a campfire, the spectrum includes only a few colors in the form of isolated sharp parallel lines.
- A spectrum of this sort is called an **emission line spectrum,** and the lines are called **spectral lines.** Each spectral line corresponds to a definite wavelength and frequency.

Atomic line spectra (emission)

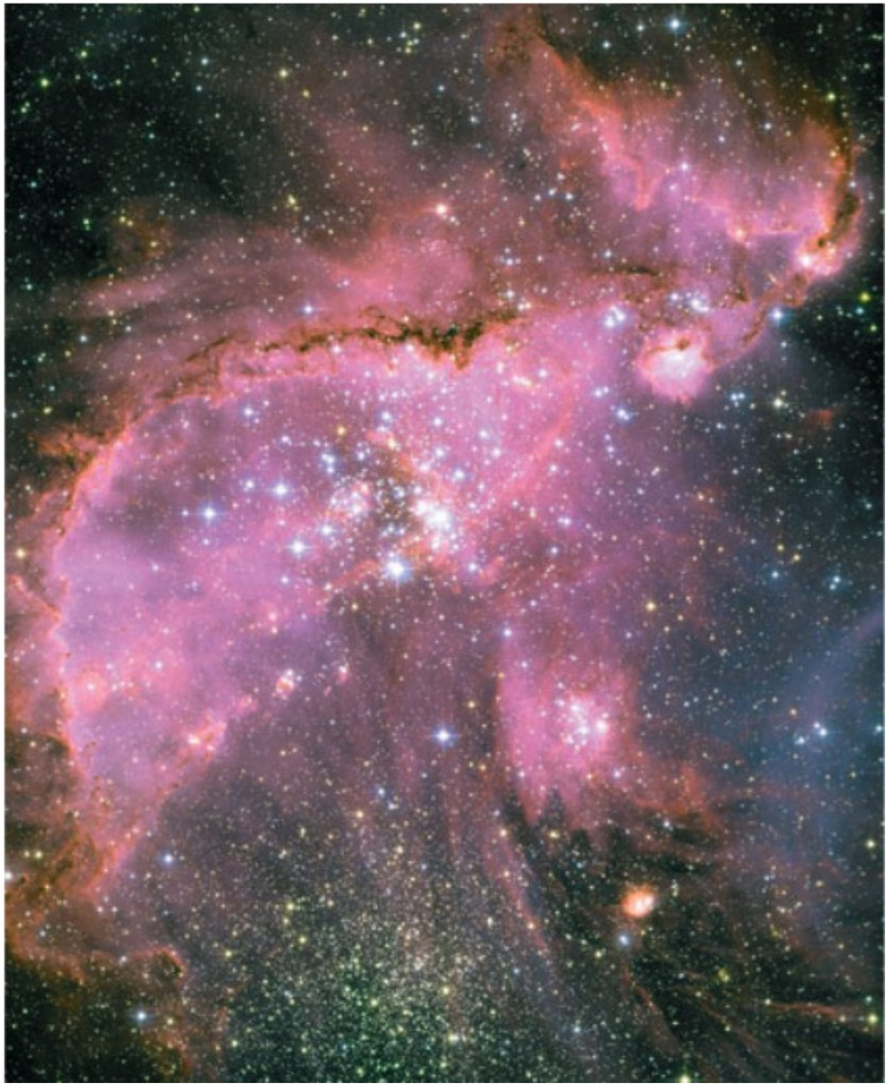
- Shown are the emission line spectra of several kinds of atoms and molecules.
- No two are alike.
- Note that the spectrum of water vapor (H_2O) is similar to that of hydrogen (H_2), but there are important differences that make it straightforward to distinguish these two spectra.



Atomic line spectra (emission)

Application Using Spectra to Analyze an Interstellar Gas Cloud

The light from this glowing gas cloud—located in the Small Magellanic Cloud, a small satellite galaxy of the Milky Way some 200,000 light-years (1.9×10^{18} km) from earth—has an emission line spectrum. Despite its immense distance, astronomers can tell that this cloud is composed mostly of hydrogen because its spectrum is dominated by red light at a wavelength of 656.3 nm, a wavelength emitted by hydrogen and no other element.

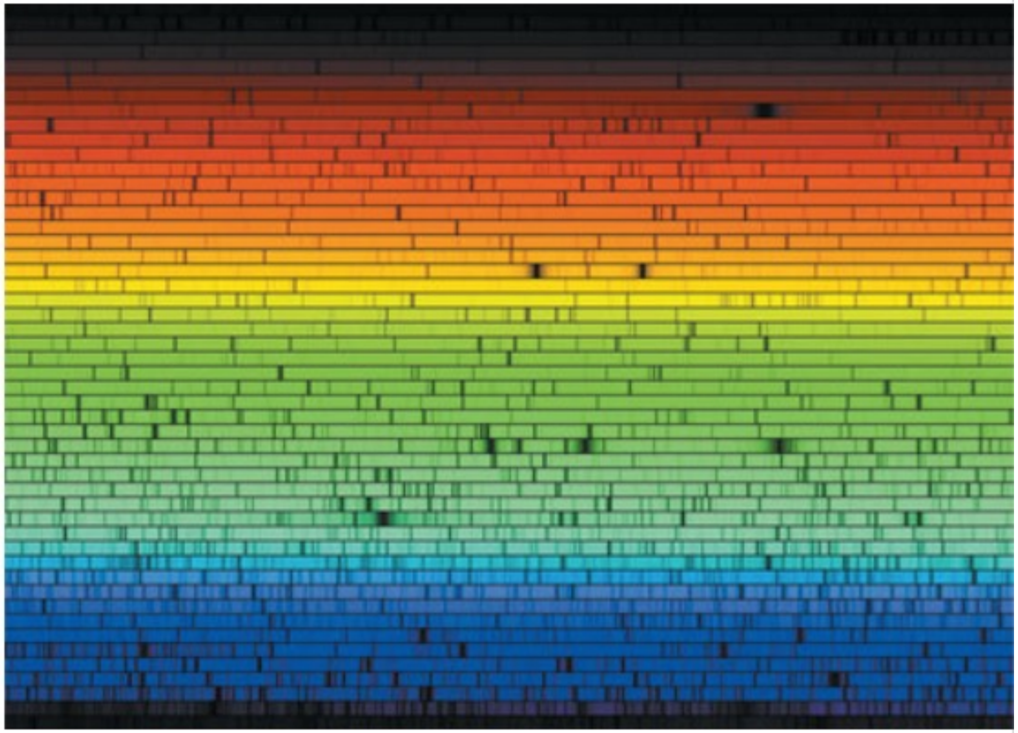


Atomic line spectra (absorption)

- While a *heated* gas selectively *emits* only certain wavelengths, a *cool* gas selectively *absorbs* certain wavelengths.
- If we pass white (continuous-spectrum) light through a gas and look at the *transmitted* light with a spectrometer, we find a series of dark lines corresponding to the wavelengths that have been absorbed.
- This is called an **absorption line spectrum.**
- What's more, a given kind of atom or molecule absorbs the *same* characteristic set of wavelengths when it's cool as it emits when heated. Hence scientists can use absorption line spectra to identify substances in the same manner that they use emission line spectra.

Atomic line spectra (absorption)

39.9 The absorption line spectrum of the sun. (The spectrum "lines" read from left to right and from top to bottom, like text on a page.) The spectrum is produced by the sun's relatively cool atmosphere, which absorbs photons from deeper, hotter layers. The absorption lines thus indicate what kinds of atoms are present in the solar atmosphere.



Why discrete spectra?

- The emission line spectra and absorption line spectra of atoms presented a question to scientists: *Why* does a given kind of atom emit and absorb only certain very specific wavelengths?
- To answer this question, we need to have a better idea of what the inside of an atom is like.
- We know that atoms are much smaller than the wavelengths of visible light, so there is no hope of actually *seeing* an atom using that light. But we can still describe how the mass and electric charge are distributed throughout the volume of the atom.

Where we were in 1910

- In 1897 the English physicist J. J. Thomson (Nobel Prize 1906) had discovered the electron and measured its charge-to-mass ratio e/m.

Sir J. J. Thomson (1856-1940)

Nobel Prize in Physics 1906

"in recognition of the great merits of his theoretical and experimental investigations on the conduction of electricity by gases".

Where we were in 1910

- By 1909, the American physicist Robert Millikan (Nobel Prize 1923) had made the first measurements of the electron charge -*e*.

Robert A. Millikan (1868-1953)

Nobel Prize in Physics 1923

"for his work on the elementary charge of electricity and on the photoelectric effect".

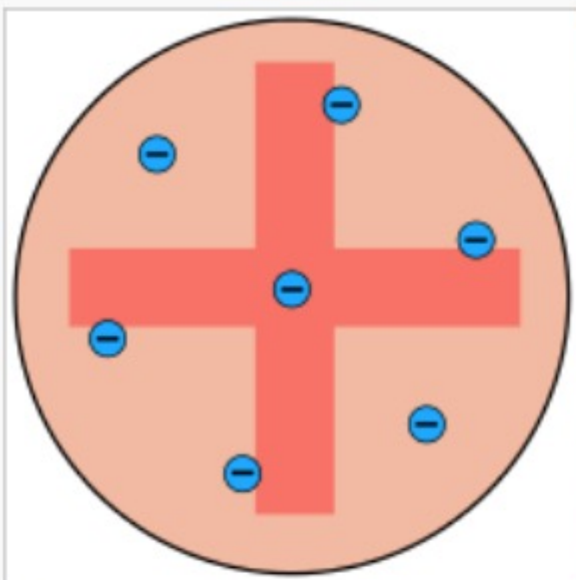
Where we were in 1910

- These and other experiments showed that almost all the mass of an atom had to be associated with the *positive* charge, not with the electrons.
- It was also known that the overall size of atoms is of the order of 10^{-10} m and that all atoms except hydrogen contain more than one electron.

Thomson model of an atom

- In 1910 the best available model of atomic structure was one developed by Thomson (Plum pudding model).

- He envisioned the atom as a sphere of some as yet unidentified positively charged substance, within which the electrons were embedded like raisins in cake.



The Plum Pudding model

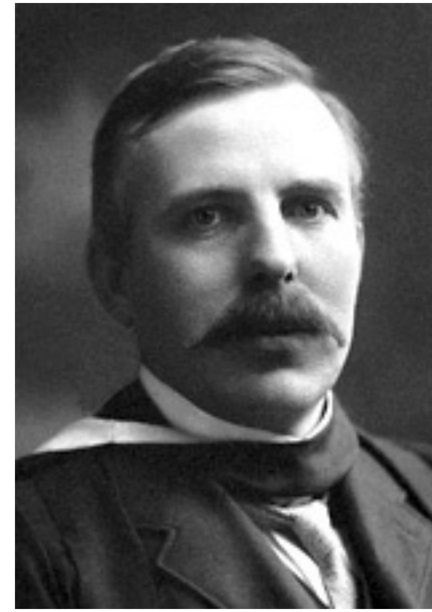
A Christmas pudding

Thomson model of an atom

- The Thomson model offered an explanation for line spectra.
- If the atom collided with another atom, as in a heated gas, each electron would oscillate around its equilibrium position with a characteristic frequency and emit electromagnetic radiation with that frequency.
- If the atom were illuminated with light of many frequencies, each electron would selectively absorb only light whose frequency matched the electron's natural oscillation frequency.
- Can this model be tested experimentally?

The Rutherford scattering experiment (1911)

- The first experiments designed to test Thomson's model by probing the interior structure of the atom were carried out in 1910–1911 by Ernest Rutherford and his students.
- These experiments consisted of shooting a beam of charged particles at thin foils of various elements and observing how the foil deflected the particles.

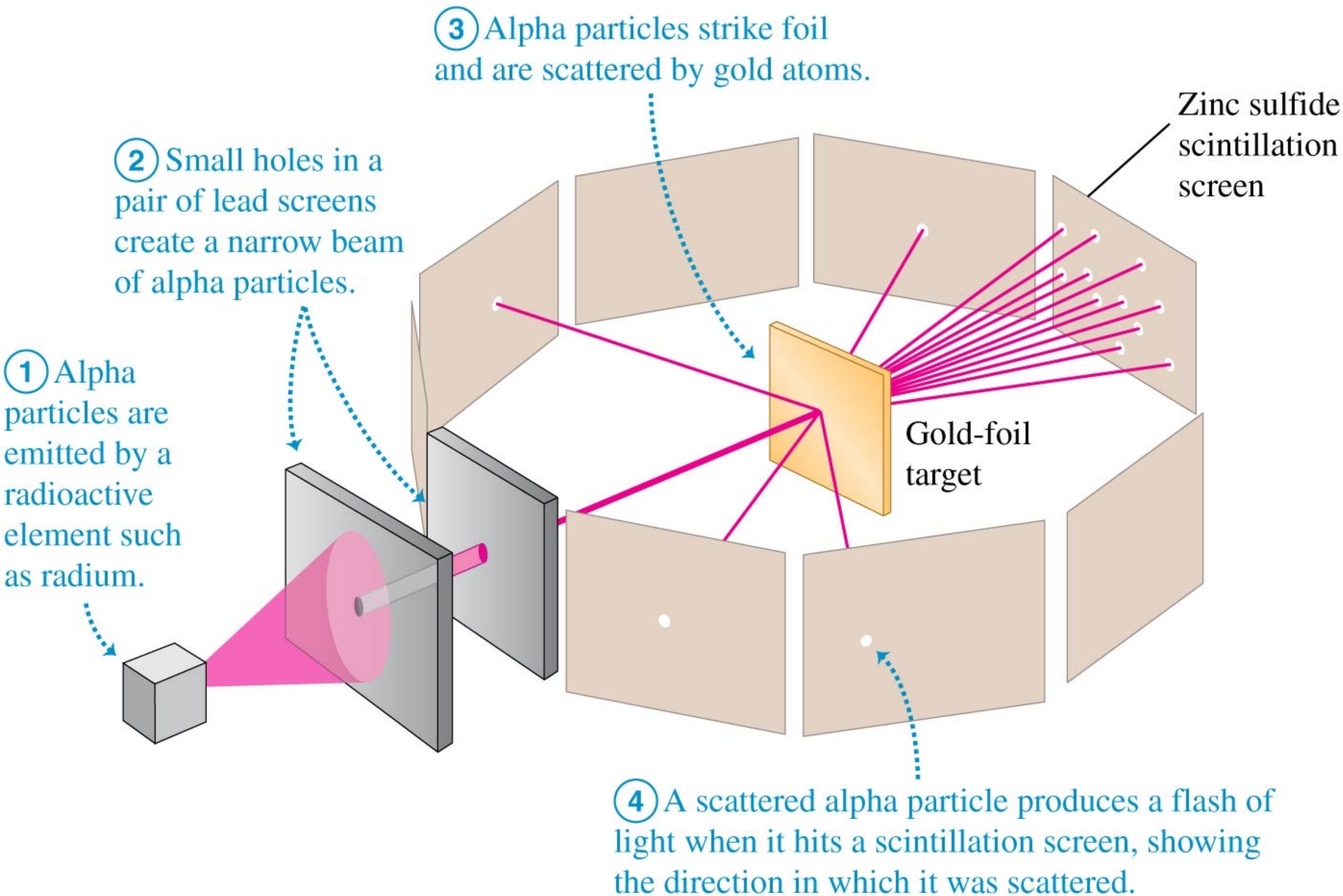


Ernest Rutherford (1871-1937)

Nobel Prize in Chemistry 1908

"for his investigations into the disintegration of the elements, and the chemistry of radioactive substances".

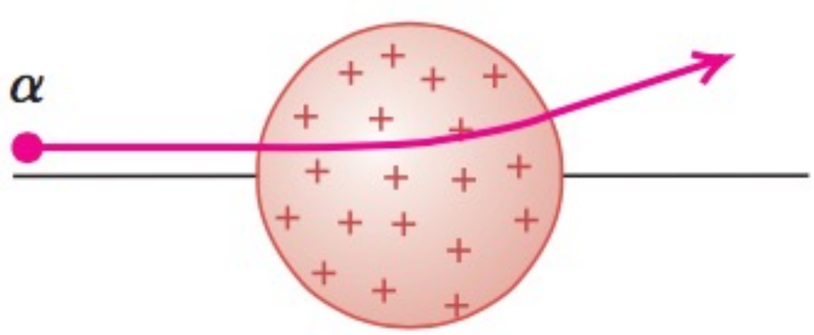
The Rutherford scattering experiment (1911)



Failure of the Thompson model

- In the Thomson model, the positive charge and the negative electrons are distributed through the whole atom. Hence the electric field inside the atom should be quite small, and the electric force on an alpha particle that enters the atom should be quite weak.
- The maximum deflection to be expected is then only a few degrees.

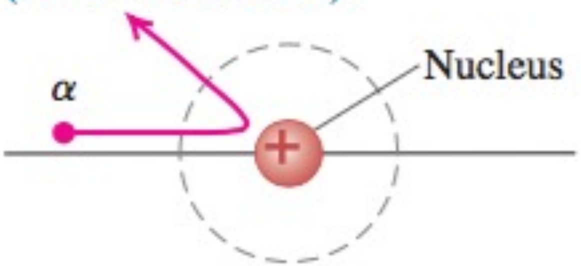
(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.



Failure of the Thompson model

- The results of the Rutherford experiments were very different. Some alpha particles were scattered by nearly 180°—that is, almost straight backward.
- Rutherford later wrote:
  It was quite the most incredible event that ever happened to me in my life. It was almost as incredible as if you had fired a 15-inch shell at a piece of tissue paper and it came back and hit you.

(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).

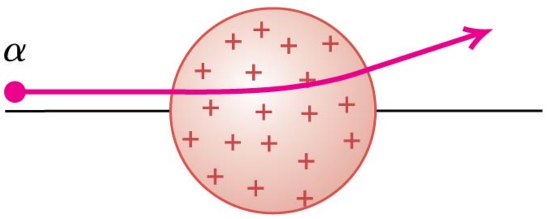


The nuclear atom

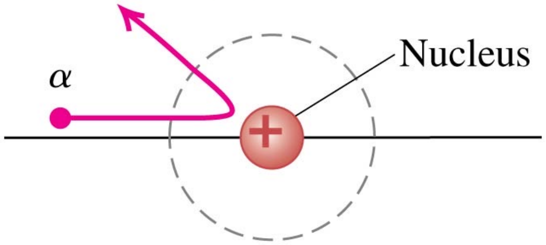
- The results that some alpha particles were scattered by large angles lead Rutherford to conclude that the atom's positive charge is concentrated in a **nucleus** at its center.

- Rutherford's experiments established that the atom does have a nucleus—a very small, very dense structure, no larger than 10^{-14}m in diameter. The nucleus occupies only about 10^{-12} of the total volume of the atom or less, but it contains *all* the positive charge and at least 99.95% of the total mass of the atom.

(a) Thomson's model of the atom: An alpha particle is scattered through only a small angle.

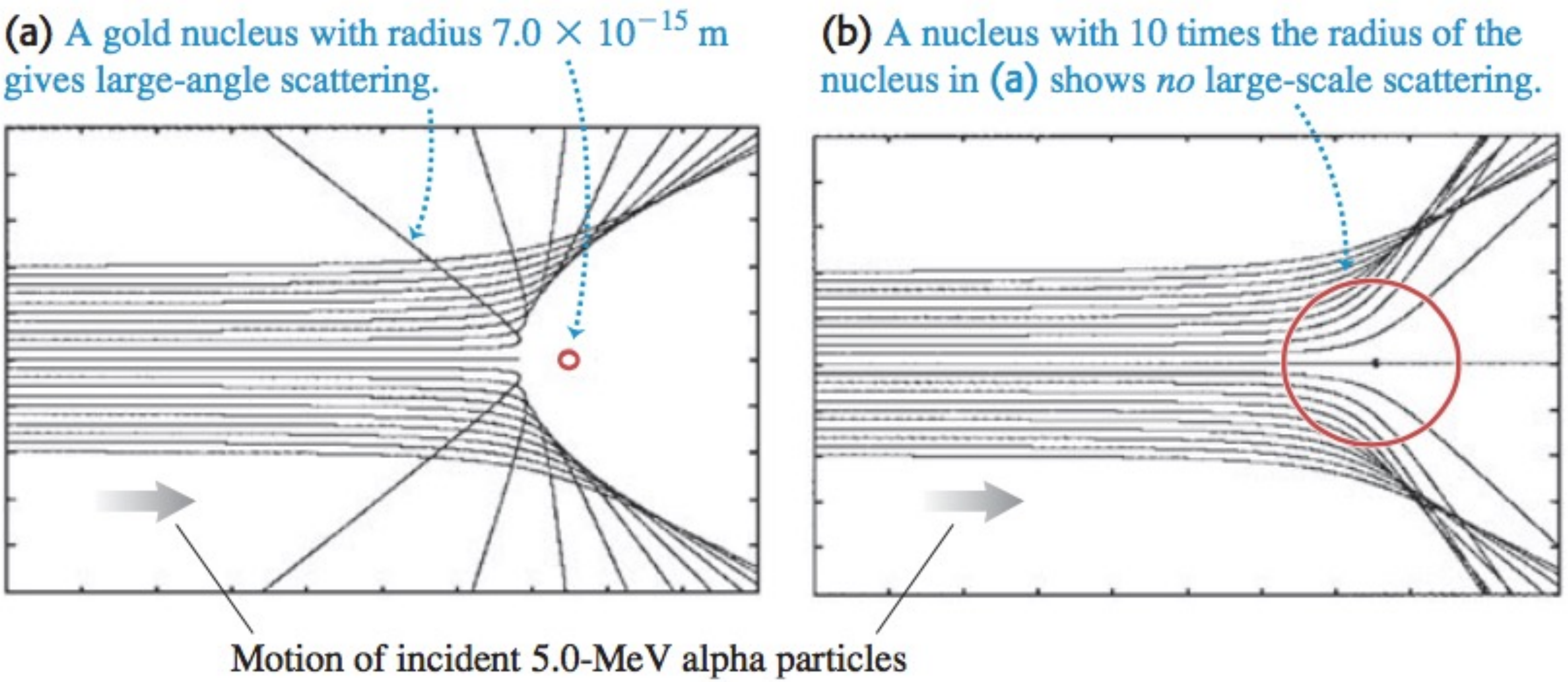


(b) Rutherford's model of the atom: An alpha particle can be scattered through a large angle by the compact, positively charged nucleus (not drawn to scale).



Computer simulation of Rutherford's experiment

39.13 Computer simulation of scattering of 5.0-MeV alpha particles from a gold nucleus. Each curve shows a possible alpha-particle trajectory. (a) The scattering curves match Rutherford's experimental data if a radius of 7.0×10^{-15} m is assumed for a gold nucleus. (b) A model with a much larger radius for the gold nucleus does not match the data.



Example 39.4: A Rutherford experiment

An alpha particle (charge $2e$) is aimed directly at a gold nucleus (charge $79e$). What minimum initial kinetic energy must the alpha particle have to approach within 5.0×10^{-14} m of the center of the gold nucleus before reversing direction? Assume that the gold nucleus, which has about 50 times the mass of an alpha particle, remains at rest.

Continued

EXECUTE: At point 1 the separation r of the alpha particle and gold nucleus is effectively infinite, so from Eq. (23.9) $U_1 = 0$. At point 2 the potential energy is

$$U_2 = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

$$= (9.0 \times 10^9\ \mathrm{N \cdot m^2/C^2})\frac{(2)(79)(1.60 \times 10^{-19}\ \mathrm{C})^2}{5.0 \times 10^{-14}\ \mathrm{m}}$$

$$= 7.3 \times 10^{-13}\ \mathrm{J} = 4.6 \times 10^6\ \mathrm{eV} = 4.6\ \mathrm{MeV}$$

By energy conservation $K_1 + U_1 = K_2 + U_2$, so $K_1 = K_2 + U_2 - U_1 = 0 + 4.6\ \mathrm{MeV} - 0 = 4.6\ \mathrm{MeV}$. Thus, to approach within 5.0×10^{-14} m, the alpha particle must have initial kinetic energy $K_1 = 4.6$ MeV.

EVALUATE: Alpha particles emitted from naturally occurring radioactive elements typically have energies in the range 4 to 6 MeV. For example, the common isotope of radium, ^{226}Ra, emits an alpha particle with energy 4.78 MeV.

Was it valid to assume that the gold nucleus remains at rest? To find out, note that when the alpha particle stops momentarily, all of its initial momentum has been transferred to the gold nucleus. An alpha particle has a mass $m_\alpha = 6.64 \times 10^{-27}$ kg; if its initial kinetic energy $K_1 = \frac{1}{2}mv_1^2$ is 7.3×10^{-13} J, you can show that its initial speed is $v_1 = 1.5 \times 10^7$ m/s and its initial momentum is $p_1 = m_\alpha v_1 = 9.8 \times 10^{-20}\ \mathrm{kg \cdot m/s}$. A gold nucleus (mass $m_{\mathrm{Au}} = 3.27 \times 10^{-25}$ kg) with this much momentum has a much slower speed $v_{\mathrm{Au}} = 3.0 \times 10^5$ m/s and kinetic energy $K_{\mathrm{Au}} = \frac{1}{2}mv_{\mathrm{Au}}^2 = 1.5 \times 10^{-14}\ \mathrm{J} = 0.092$ MeV. This *recoil kinetic energy* of the gold nucleus is only 2% of the total energy in this situation, so we are justified in neglecting it.

The failure of classical physics

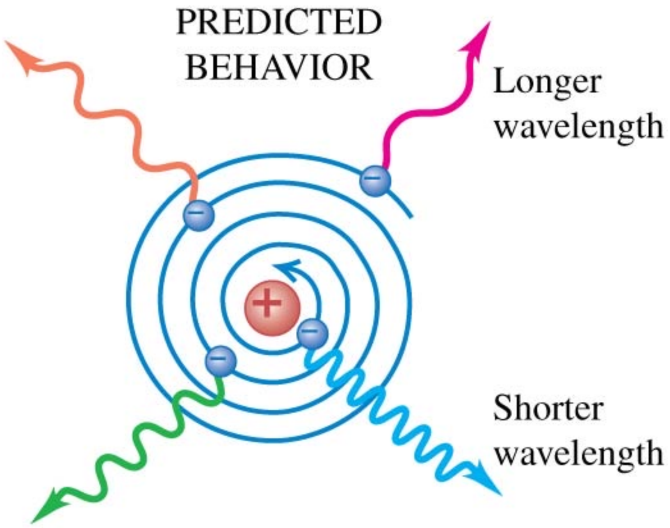
- Rutherford's discovery of the atomic nucleus raised a serious question:
  What prevented the negatively charged electrons from falling into the positively charged nucleus due to the strong electrostatic attraction?

- Rutherford suggested that perhaps the electrons *revolve* in orbits about the nucleus, just as the planets revolve around the sun.

The failure of classical physics

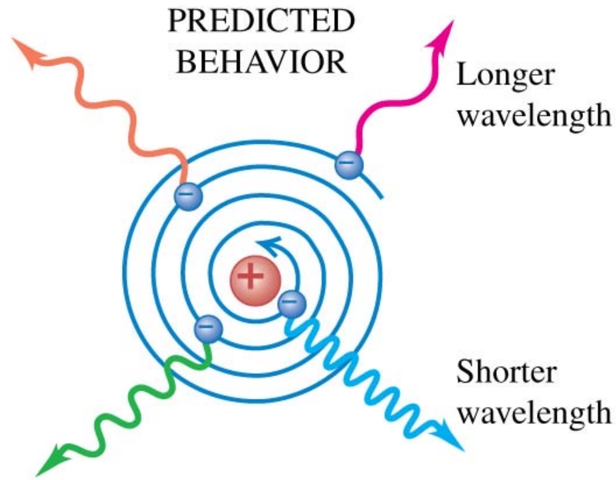
BUT, ACCORDING TO CLASSICAL PHYSICS:

- An orbiting electron is accelerating, so it should radiate electromagnetic waves.



- The electron's angular speed would increase as its orbit shrank, so the frequency of the radiated waves should increase.
- The waves would carry away energy, so the electron should lose energy and spiral inward.
- Thus, classical physics says that atoms should collapse within a fraction of a second and should emit light with a continuous spectrum as they do so (not the line spectrum observed).

The failure of classical physics



- Thus Rutherford's model of electrons orbiting the nucleus, which is based on Newtonian mechanics and classical electromagnetic theory, makes three entirely *wrong* predictions about atoms:
  - They should emit light continuously,
  - they should be unstable, and
  - the light they emit should have a continuous spectrum.
- Clearly a radical reappraisal of physics on the scale of the atom was needed.
- Next we will see the bold idea that led to an new understanding of the atom, and see how this idea meshes with de Broglie's no less bold notion that electrons have wave attributes.

The Bohr model of hydrogen

- Niels Bohr (1885–1962) postulated that each atom has a set of possible **energy levels.**
- An atom can have an amount of internal energy equal to any one of these levels, but it *cannot* have an energy *intermediate* between two levels. All isolated atoms of a given element have the same set of energy levels, but atoms of different elements have different sets.

Niels H. D. Bohr (1885-1962)

Nobel Prize in Physics 1922

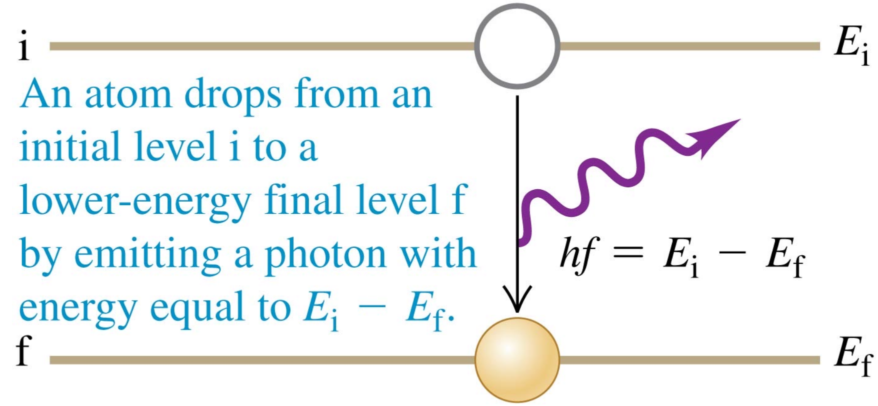
"for his services in the investigation of the structure of atoms and of the radiation emanating from them".

The Bohr model of hydrogen

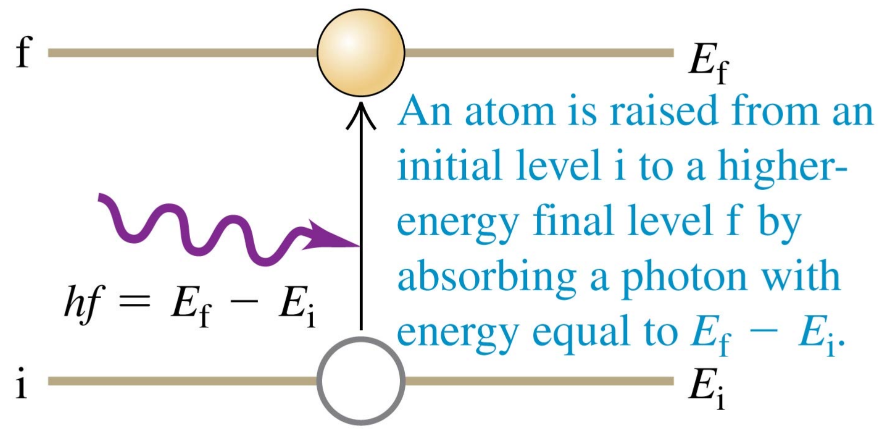
- For example, each energy level of a hydrogen atom corresponds to a specific stable circular orbit of the electron around the nucleus.
- In the Bohr model, an atom radiates energy only when an electron makes a transition from an orbit of energy E_i to a different orbit with lower energy E_f , emitting a photon of energy $hf = E_i - E_f$ in the process.
- Bohr won the 1922 Nobel Prize in physics for these ideas.

Atomic energy levels

- When an atom makes a transition from one energy level to a lower level, it **emits** a photon whose energy equals that lost by the atom.

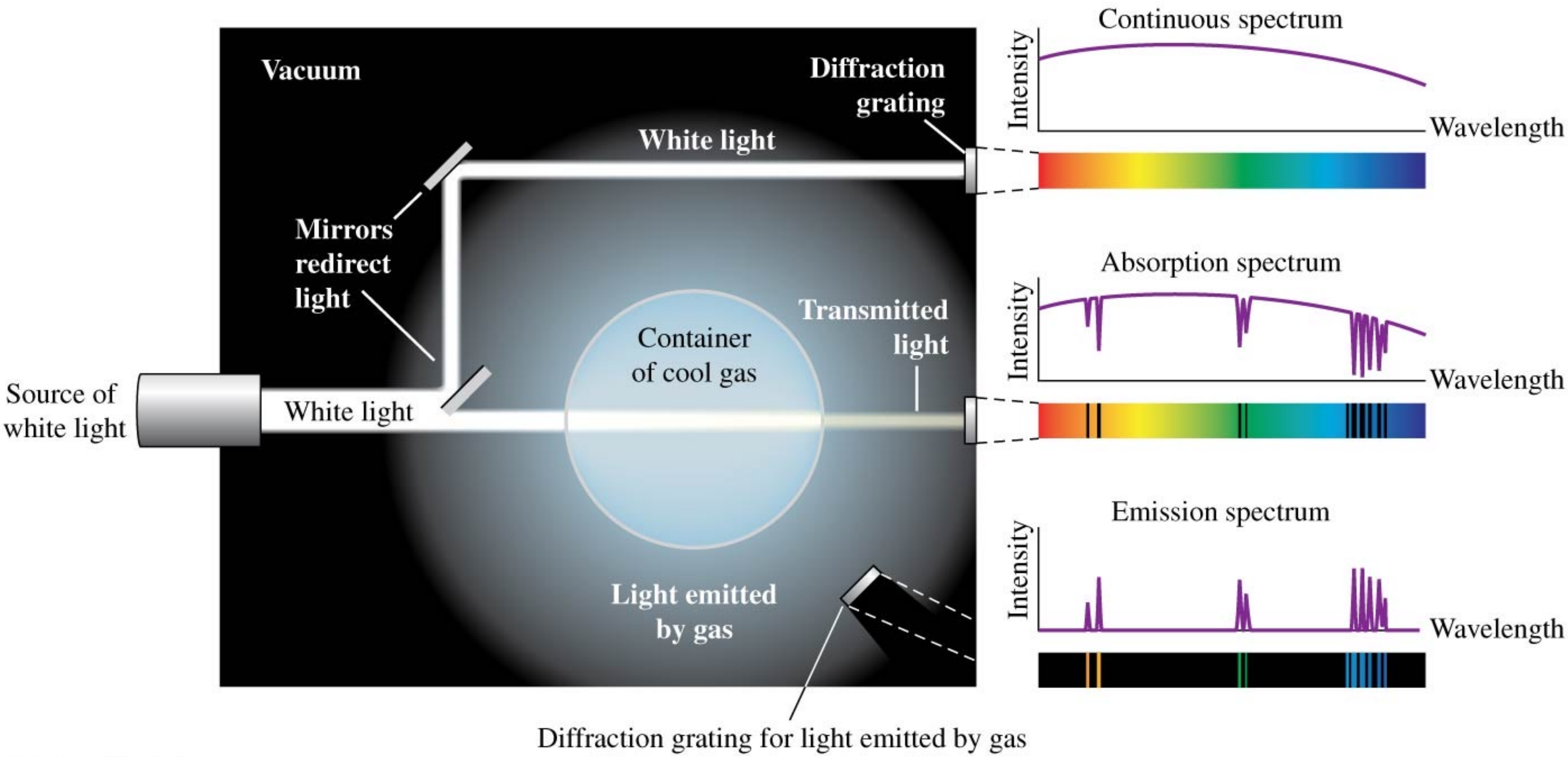


- An atom can also **absorb** a photon, provided the photon energy equals the difference between two energy levels.



Atomic energy levels

- A cool gas that's illuminated by white light to make an absorption line spectrum also produces an emission line spectrum when viewed from the side.

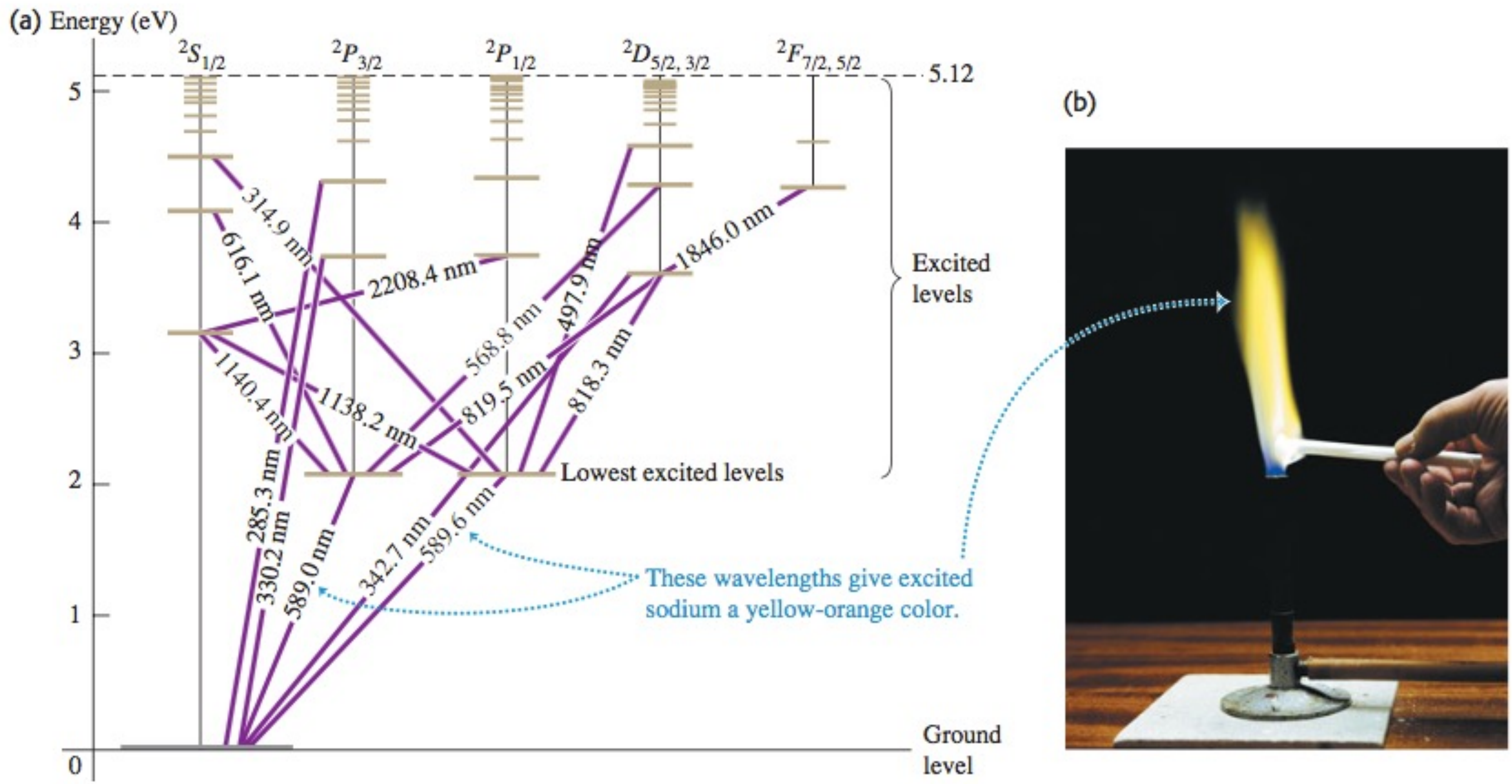


Ground level and excited level

- The observation that atoms are stable means that each atom has a *lowest* energy level, called the **ground level.**
- Levels with energies greater than the ground level are called **excited levels.**
- An atom in an excited level, called an *excited atom*, can make a transition into the ground level by emitting a photon.
- But since there are no levels below the ground level, an atom in the ground level cannot lose energy and so cannot emit a photon.

Energy levels of Na

39.19 (a) Energy levels of the sodium atom relative to the ground level. Numbers on the lines between levels are wavelengths of the light emitted or absorbed during transitions between those levels. The column labels, such as $^2S_{1/2}$, refer to some quantum states of the atom. (b) When a sodium compound is placed in a flame, sodium atoms are excited into the lowest excited levels. As they drop back to the ground level, the atoms emit photons of yellow-orange light with wavelengths 589.0 and 589.6 nm.



Q39.5

A certain atom has two energy levels whose energies differ by 2.5 eV. In order for a photon to excite the atom from the lower energy level to the upper energy level, the energy of the photon

A. can have any value greater than 2.5 eV.

B. can have any value greater than or equal to 2.5 eV.

C. must be exactly 2.5 eV.

D. can have any value less than or equal to 2.5 eV.

E. can have any value less than 2.5 eV.

Example 39.5 Emission and absorption spectra

A hypothetical atom (Fig. 39.20a) has energy levels at 0.00 eV (the ground level), 1.00 eV, and 3.00 eV. (a) What are the frequencies and wavelengths of the spectral lines this atom can emit when excited? (b) What wavelengths can this atom absorb if it is in its ground level?

EXECUTE: (a) The possible energies of emitted photons are 1.00 eV, 2.00 eV, and 3.00 eV. For 1.00 eV, Eq. (39.2) gives

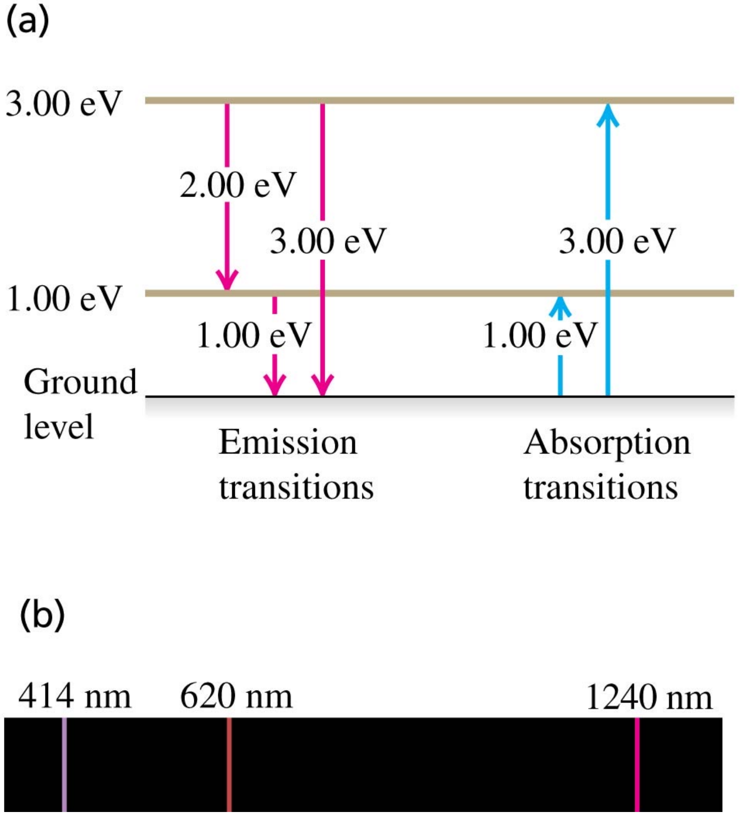
$$f = \frac{E}{h} = \frac{1.00\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV} \cdot \text{s}} = 2.42 \times 10^{14}\ \text{Hz}$$

For 2.00 eV and 3.00 eV, $f = 4.84 \times 10^{14}$ Hz and 7.25×10^{14} Hz, respectively. For 1.00-eV photons,

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\ \text{m/s}}{2.42 \times 10^{14}\ \text{Hz}} = 1.24 \times 10^{-6}\ \text{m} = 1240\ \text{nm}$$

This is in the infrared region of the spectrum (Fig. 39.20b). For 2.00 eV and 3.00 eV, the wavelengths are 620 nm (red) and 414 nm (violet), respectively.

(b) From the ground level, only a 1.00-eV or a 3.00-eV photon can be absorbed (Fig. 39.20a); a 2.00-eV photon cannot be absorbed because the atom has no energy level 2.00 eV above the ground level. Passing light from a hot solid through a gas of these hypothetical atoms (almost all of which would be in the ground state if the gas were cool) would yield a continuous spectrum with dark absorption lines at 1240 nm and 414 nm.

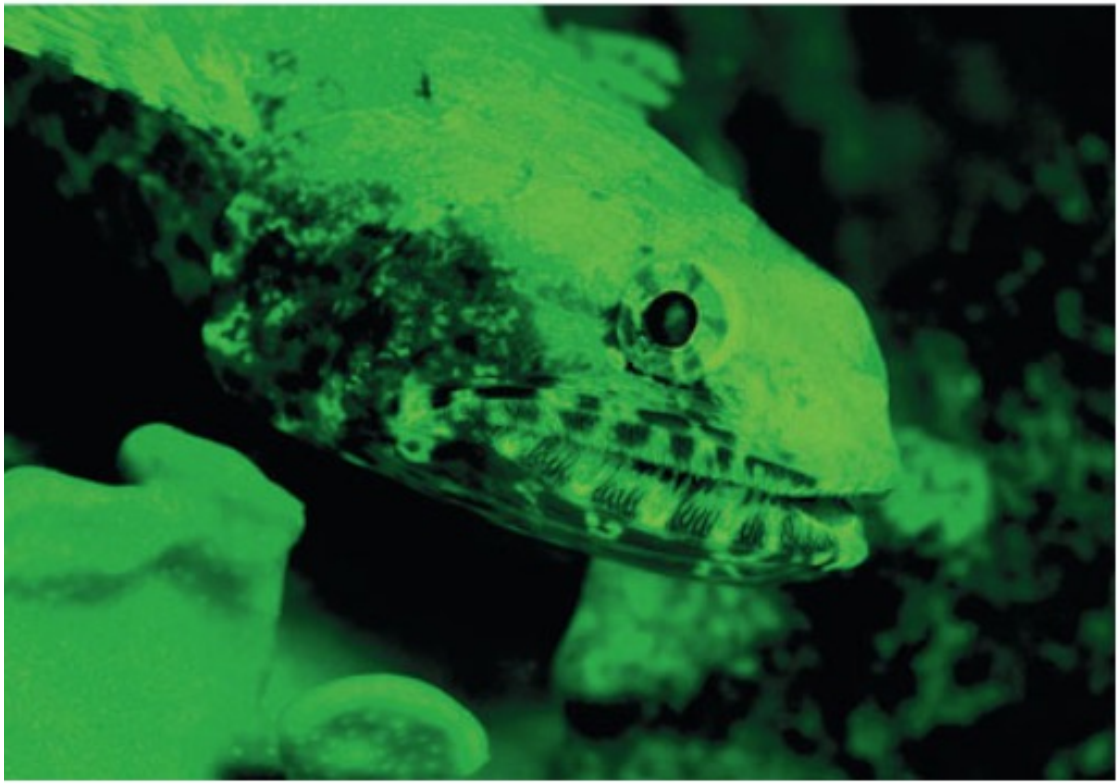


Fluorescence

- Take the hypothetical atom in the last slide as an example: a 3eV photon makes it reach the 2nd excited level from the ground level, but it can transit back to the ground level in two steps: 2eV and then 1eV.
- Thus this gas will emit longer-wavelength radiation than it absorbs, a phenomenon called *fluorescence.*
- For example, the electric discharge in a fluorescent lamp causes the mercury vapor in the tube to emit ultraviolet radiation. This radiation is absorbed by the atoms of the coating on the inside of the tube. The coating atoms then re-emit light in the longer-wavelength, visible portion of the spectrum.
- Fluorescent lamps are more efficient than incandescent lamps in converting electrical energy to visible light because they do not waste as much energy producing (invisible) infrared photons.

Fluorescence

Application **Fish Fluorescence**
When illuminated by blue light, this tropical lizardfish (family *Synodontidae*) fluoresces and emits longer-wavelength green light. The fluorescence may be a sexual signal or a way for the fish to camouflage itself among coral (which also have a green fluorescence).



Are the energy levels real?

- In 1914, the German physicists James Franck and Gustav Hertz found direct experimental evidence for the existence of atomic energy levels.

James Franck (1882-1964)

Nobel Prize in Physics 1925 Prize share ½

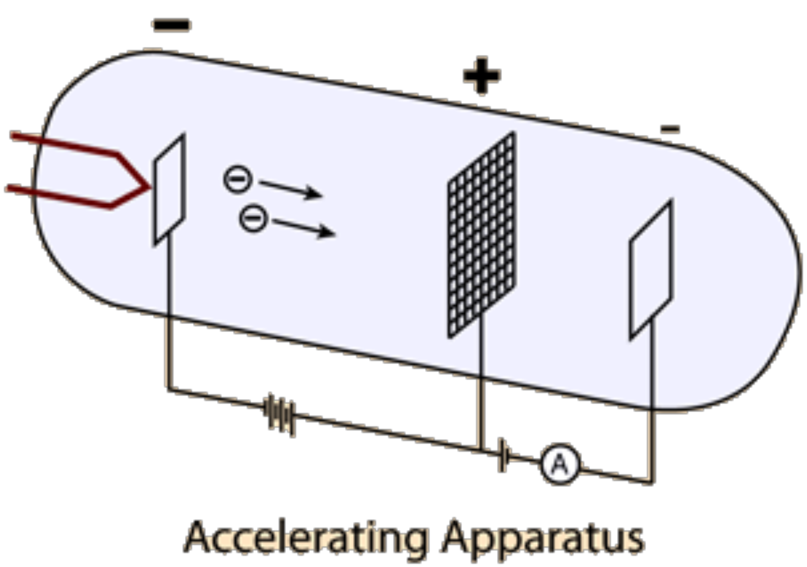
Gustav L. Hertz (1887-1975)

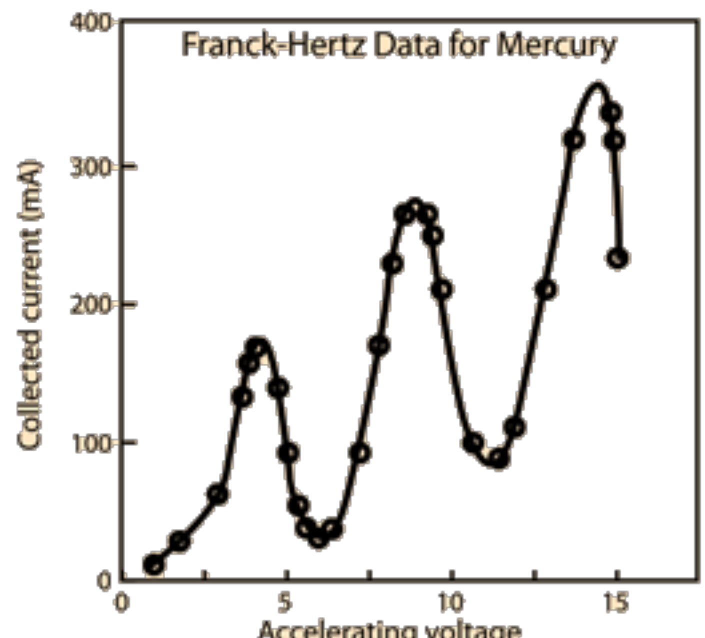
Nobel Prize in Physics 1925 Prize share ½

"for their discovery of the laws governing the impact of an electron upon an atom"

The Franck-Hertz experiment

- Franck and Hertz studied the motion of electrons through mercury vapor under the action of an electric field.
- When the accelerating voltage reaches 4.9 V, the current sharply drops, indicating the sharp onset of a new phenomenon which takes enough energy away from the electrons that they cannot reach the collector.





The Franck-Hertz experiment

- They also found that when the electron kinetic energy was 4.9 eV or greater, the vapor emitted ultraviolet light of wavelength 250 nm.
- Suppose mercury atoms have an excited energy level 4.9 eV above the ground level. An atom can be raised to this level by collision with an electron; it later decays back to the ground level by emitting a photon, the wavelength of which should be

$$\lambda = \frac{hc}{E} = \frac{(4.136 \times 10^{-15}\ \mathrm{eV \cdot s})(3.00 \times 10^{8}\ \mathrm{m/s})}{4.9\ \mathrm{eV}}$$
$$= 2.5 \times 10^{-7}\ \mathrm{m} = 250\ \mathrm{nm}$$

- This is equal to the wavelength that Franck and Hertz measured, which demonstrates that this energy level actually exists in the mercury atom.
- Similar experiments with other atoms yield the same kind of evidence for atomic energy levels.

The Bohr model of hydrogen

- Bohr found that the magnitude of the electron's angular momentum is quantized; that is, this magnitude must be an integral multiple of $h/2\pi$.
- Let's number the orbits by the **principal quantum number** n, where $n = 1, 2, 3, \ldots$, and call the radius of orbit n r_n and the speed of the electron in that orbit v_n.
- The magnitude of the angular momentum of an electron of mass m in such an orbit is:

Quantization of angular momentum:

Orbital angular momentum

Principal quantum number ($n = 1, 2, 3, \ldots$)

$$L_n = m v_n r_n = n\frac{h}{2\pi}$$

Planck's constant

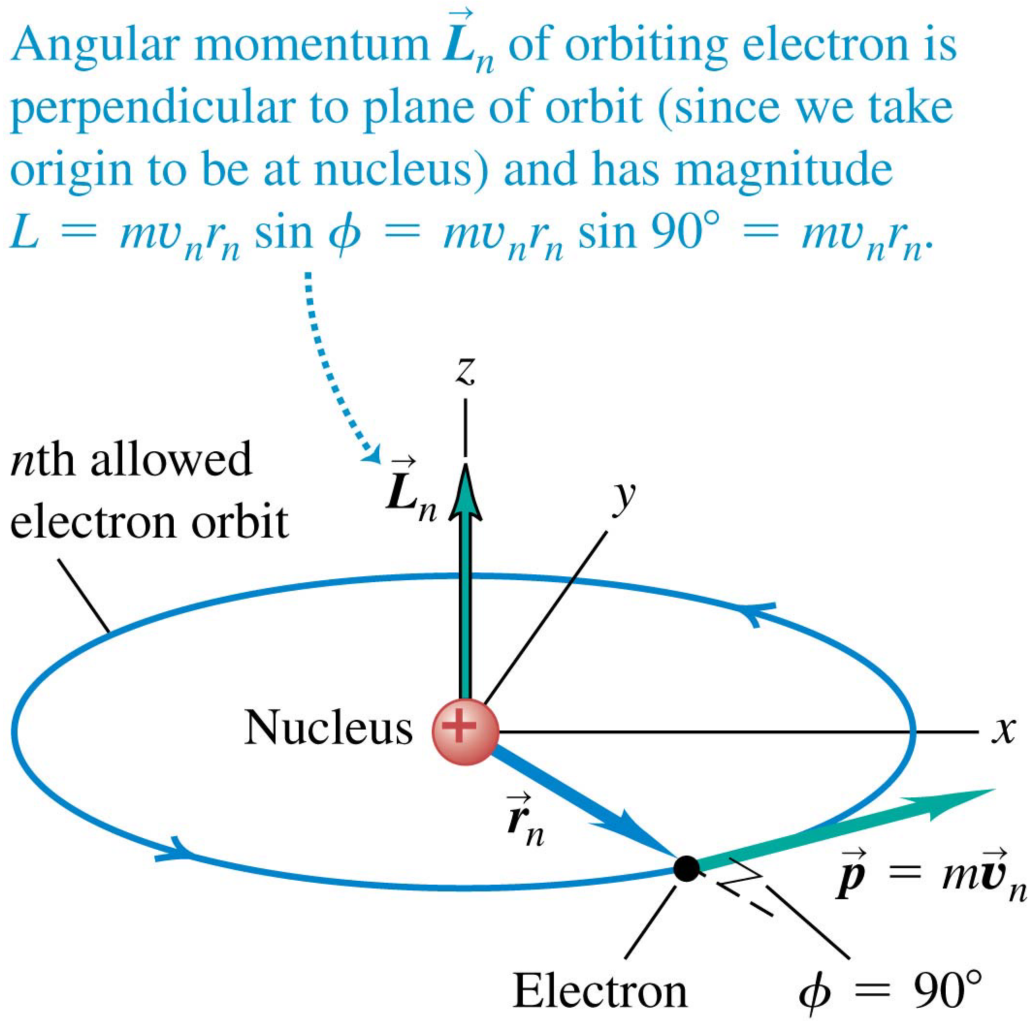
Electron mass

Electron speed

Electron orbital radius

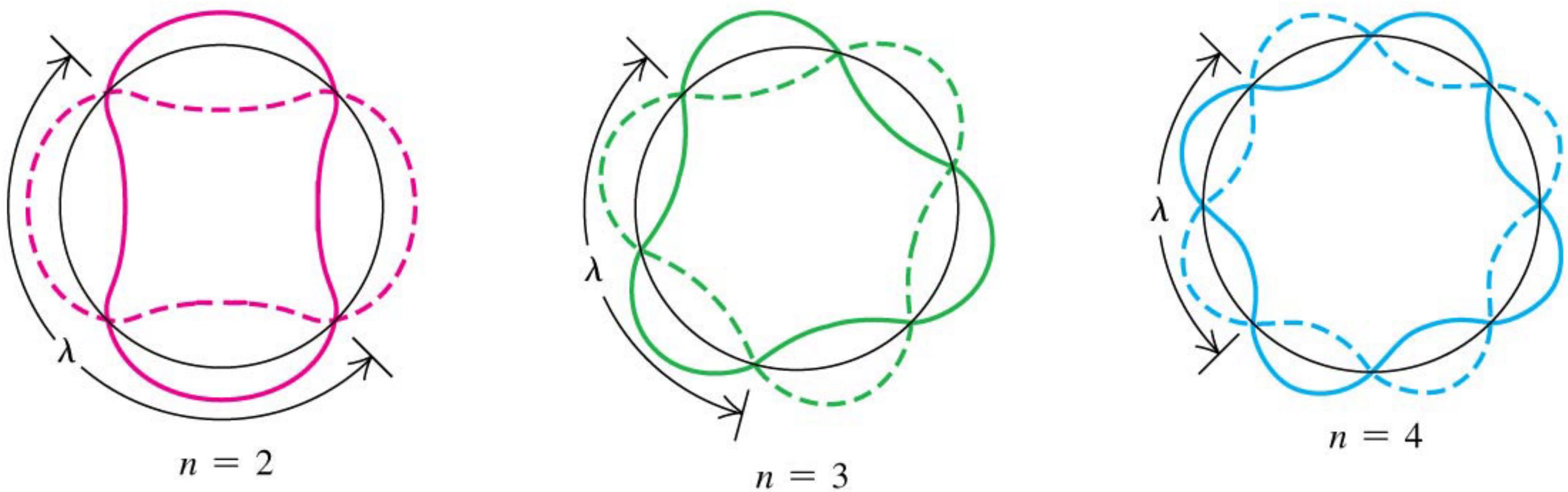
The Bohr model of hydrogen

- Shown is the angular momentum of an electron in a circular orbit around an atomic nucleus.



The Bohr model of hydrogen

- We can also appreciate Bohr's finding in a different way.
- A standing wave on a string transmits no energy, and electrons in Bohr's orbits radiate no energy.
- For the wave to "come out even" and join onto itself smoothly, the circumference of this circle must include some whole number of wavelengths.



The Bohr model of hydrogen

- Hence for an orbit with radius r_n and circumference $2\pi r_n$, we must have $2\pi r_n = n\lambda_n$, where λ_n is the wavelength and $n = 1, 2, 3, \ldots$
- According to the de Broglie relationship, the wavelength of a particle with rest mass m moving with nonrelativistic speed v_n is $\lambda_n = h/mv_n$. Combining $2\pi r_n = n\lambda_n$ and $\lambda_n = h/mv_n$, we find $2\pi r_n = nh/mv_n$ or

$$mv_n r_n = n\frac{h}{2\pi}$$

- same as Bohr's result. Thus a wave picture of the electron leads naturally to the quantization of the electron's angular momentum.

The Bohr model of hydrogen

- Solving v_n , r_n from $mv_n r_n = n\dfrac{h}{2\pi}$ and $\dfrac{1}{4\pi\epsilon_0}\dfrac{e^2}{r_n{}^2} = \dfrac{mv_n{}^2}{r_n}$

(Coulomb's force providing the centripetal force) we have

$$r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2} \qquad \text{(orbital radii in the Bohr model)}$$

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh} \qquad \text{(orbital speeds in the Bohr model)}$$

- Smallest possible r : n=1

$$a_0 = \epsilon_0 \frac{h^2}{\pi m e^2} \qquad \text{(Bohr radius)}$$

$$r_n = n^2 a_0$$

- Permitted orbits have radii a_0, $4a_0$, $9a_0$, and so on.

The Bohr model of hydrogen

- The orbital speed of the electron in Bohr's model of a hydrogen atom is:

Orbital speed in nth orbit in the Bohr model

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh}$$

Magnitude of electron charge
Planck's constant
Electric constant
Principal quantum number ($n = 1, 2, 3, \ldots$)

- The radius of this orbit is:

Radius of nth orbit in the Bohr model

$$r_n = n^2 a_0$$

Bohr radius
Principal quantum number ($n = 1, 2, 3, \ldots$)

where the Bohr radius is $a_0 = 5.29 \times 10^{-11}$ m.

- Total energies in the Bohr model:

$$K_n = \frac{1}{2}mv_n^2 = \frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}$$ (kinetic energies in the Bohr model)

$$U_n = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n} = -\frac{1}{\epsilon_0^2}\frac{me^4}{4n^2h^2}$$ (potential energies in the Bohr model)

$$E_n = K_n + U_n = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}$$ (total energies in the Bohr model)

Q39.7

Complete the sentence: "In the Bohr model of the hydrogen atom, an electron in the $n = 2$ orbit has a __________ total energy and a __________ kinetic energy than an electron in the $n = 1$ orbit."

A. higher, higher

B. lower, higher

C. higher, lower

D. lower, lower

E. none of the above

The Bohr model of hydrogen

- The Bohr model predicts the observable energy levels of the hydrogen atom, which give rise to the hydrogen spectrum, below.

Planck's constant · Speed of light in vacuum · Electron mass · Magnitude of electron charge

Total energy for nth orbit in the Bohr model

$$E_n = -\frac{hcR}{n^2}, \quad \text{where} \quad R = \frac{me^4}{8\epsilon_0^2 h^3 c}$$

Principal quantum number ($n = 1, 2, 3, \ldots$) · Rydberg constant · Electric constant