<u>EE1001 Test 1 (Q&A)</u>
(100 marks, 75 mins in total, KF Tsang)

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EE1001 Test 1, KF Tsang, 16 October 2021, 9:15am -10:30am, 1hr 15min, LT 5, LT 6, online

There are TEN (10) questions.

Answer ALL questions.

CityU approved calculator is allowed.

Observe CityU examination/test guidelines.

ALL students please scan your answer and upload onto CANVAS on or before 10:45am. THREE trials are allowed.

<u>Q1 (10 marks, 2 marks each)</u>

Determine the following statements whether they are true or false.

(i) A valid argument can go from false premises to a true conclusion.
(ii) An unsound argument always has at least one false premise.
(iii) "$a \leftrightarrow b$" is logically equivalent to "$\sim b \leftrightarrow \sim a$".
(iv) "All human beings are not animals" is an unsound argument.
(v) Given $A = \{a_1, a_2, a_3\}$. $\forall x \in A, P(x)$ is logically equivalent to $P(a_1) \wedge P(a_2) \wedge P(a_3)$.

<u>Q2 (13 marks)</u>

Without using a truth table, determine whether $(p \vee q) \wedge (p \rightarrow r) \wedge \sim r \rightarrow q$ is a tautology or a contradiction, or neither. State the reason for each step.

<u>Q3 (15 marks)</u>

There are <u>THREE (3)</u> suspects who are thought to be guilty of a crime. Their statements are as follows:

Suspect A: "If B is guilty, then C and I are innocent."
Suspect B: "I'm innocent, and at least one of the others is guilty."
Suspect C: "None of us are guilty."

Let a, b, and c be
a = "Suspect A is innocent"

b = "Suspect B is innocent"
c = "Suspect C is innocent"

Q3(i) Formulate the statements of the Suspects A, B, and C. (6 marks)

Q3(ii) Given the innocent told the truth and the guilty lied, use truth table to determine whether the Suspects A, B, and C are innocent or guilty. (9 marks)
(Hint: There are **TWO (2)** possible situations)

Q4 (12 marks)

(i) In an arithmetic sequence, the sum of its first nine terms is 2340, and the sum of the next eight terms is 6500. Determine the common difference (d) and the first term (a_1). (6 marks)

(ii) Given a geometric sequence with the first term b and the common ratio r, its sum to infinity is 4. Another geometric sequence has the first term $3b$, common ratio r^2 and sum to infinity 8. Determine the values of b and r. (6 marks)

Q5 (10 marks)

Use 1,2,3 to form a number with a length of 4 (e.g., 1231). Find the number of permutations or combinations under each condition.
(i) These three numbers are reused with no limitations. (2 marks)
(ii) One of the elements (i.e., '1', or '2', or '3') is used TWICE to form the number. (4 marks)
(iii) One of the elements (i.e., '1', or '2', or '3') is used TWICE to form the number, meanwhile the element cannot be adjacent. (e.g., there are two "1" used, then the number cannot be 11xy, x11y, xy11, etc.) (4 marks)

Q6 (12 marks)

There is a group of 100 passengers who plan to visit Hong Kong. It is known that THREE (3) of them are infected by the Cov-19. Given the limited human resources, a custom agent can randomly select 3 passengers to perform medical check. You can assume that the medical check result of the selected 3 passengers is 100% correct.

(i) Find the number of the combinations of the selected 3 customers; (2 marks)
(ii) Find the probability of detecting exactly 1 infected customer; (4 marks)
(iii) Find the probability of detecting at least 1 infected customer; (4 marks)
(iv) It is known that Cov-19 has a high probability to infect others and thus a 1% rule has been established. If the infection rate of passengers is less than 1%, the whole team will be allowed to entry into Hong Kong, otherwise the whole team should undergo quarantine. Comment on the decision to be made. (2 marks)

Q7 (10 marks)

Find the coefficient of the term x^3y^3;

(i) $(x+y^2/x)(x+y)^5$ (5 Marks)

(ii) $(x+y)(2x-y)^5$ (5 Marks)

Q8 (6 marks)

Find *a* if the coefficients of x^2 and x^3 in the expansion of $(2+ax)^8$ are equal ($a \neq 0$).

Q9 (6 marks)

In the expansion of $(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{2x^2})^n$.

(i) n = 8, determine if there exists a term that is independent of x (constant term). (3 marks)

(ii) n = 10, determine if there exists a term that is independent of x (constant term). Please give a detailed explanation. (3 marks)

Q10 (6 marks)

In a training camp, find the minimum number of students in the camp such that three of them are born in the same month.

-- END --

ANSWER

Q1 (10 marks, 2 marks each)

Determine the following statements whether they are true or false.

(i) A valid argument can go from false premises to a true conclusion.

(ii) An unsound argument always has at least one false premise.

(iii) "$a \leftrightarrow b$" is logically equivalent to "$\sim b \leftrightarrow \sim a$".

(iv) "All human beings are not animals" is an unsound argument.

(v) Given $A = \{a_1, a_2, a_3\}$. $\forall x \in A$, $P(x)$ is logically equivalent to $P(a_1) \wedge P(a_2) \wedge P(a_3)$.

Solution of Q1 (10 marks)

(i) True

(ii) False (An invalid and unsound argument may have all true premises with a false conclusion)

(iii) True (can be verified using truth table)

(iv) False (it has only one statement and thus it is not an argument)

(v) True

Q2 (13 marks)

Without using a truth table, determine whether $(p \vee q) \wedge (p \rightarrow r) \wedge \sim r \rightarrow q$ is a tautology or a contradiction, or neither. State the reason for each step. (13 marks)

Solution of Q2 (13 marks)

$(p \vee q) \wedge (p \rightarrow r) \wedge \sim r \rightarrow q$	
$\equiv (p \vee q) \wedge (\sim p \vee r) \wedge \sim r \rightarrow q$	(Definition of $\rightarrow$)
$\equiv (p \vee q) \wedge \sim r \wedge (\sim p \vee r) \rightarrow q$	(Commutative laws)
$\equiv (p \vee q) \wedge [(\sim r \wedge \sim p) \vee (\sim r \wedge r)] \rightarrow q$	(Distributive laws)
$\equiv (p \vee q) \wedge [(\sim r \wedge \sim p) \vee \mathbf{c}] \rightarrow q$	(Negation laws)
$\equiv (p \vee q) \wedge \sim r \wedge \sim p \rightarrow q$	(Identity laws)
$\equiv \sim p \wedge (p \vee q) \wedge \sim r \rightarrow q$	(Commutative laws)
$\equiv [(\sim p \wedge p) \vee (\sim p \wedge q)] \wedge \sim r \rightarrow q$	(Distributive laws)
$\equiv [\mathbf{c} \vee (\sim p \wedge q)] \wedge \sim r \rightarrow q$	(Negation laws)
$\equiv \sim p \wedge q \wedge \sim r \rightarrow q$	(Identity laws)
$\equiv \sim(\sim p \wedge q \wedge \sim r) \vee q$	(Definition of $\rightarrow$)
$\equiv \sim p \vee \sim q \vee r \vee q$	(De Morgan's laws)
$\equiv \sim p \vee r \vee \mathbf{t}$	(Negation laws)
$\equiv \mathbf{t}$	(Universal bound laws)

It's a tautology.

Q3 (15 marks)
There are <u>THREE (3)</u> suspects who are thought to be guilty of a crime. Their statements are as follows:
Suspect *A*: "If *B* is guilty, then *C* and I are innocent."
Suspect *B*: "I'm innocent, and at least one of the others is guilty."
Suspect *C*: "None of us are guilty."

Let *a*, *b*, and *c* be
a = "Suspect *A* is innocent"
b = "Suspect *B* is innocent"
c = "Suspect *C* is innocent"

Q3(i) Formulate the statements of the Suspects *A*, *B*, and *C*. (6 marks)

Q3(ii) Given the innocent told the truth and the guilty lied. Using truth table to determine whether the Suspects *A*, *B*, and *C* are innocent or guilty. (9 marks)
(Hint: There are **<u>TWO (2)</u>** possible situations)

<u>Solution of Q3 (15 marks)</u>
Q3(i)
<u>The statement of the suspect:</u>
"$\sim b \rightarrow a \wedge c$" = *A*'s statement = "If *B* is guilty, then *C* and I are innocent."
"$b \wedge (\sim a \vee \sim c)$" = *B*'s statement = "I'm innocent, and at least one of the others is guilty."
"$a \wedge b \wedge c$" or "$\sim(\sim a \vee \sim b \vee \sim c)$" = *C*'s statement = "None of us are guilty."
(6 marks)

Q3(ii)
The situation is satisfied only when the truth values of the suspects' statuses and the truth values of the suspects' statements are the same, i.e.,
$[a \leftrightarrow (\sim b \rightarrow a \wedge c)] \wedge [b \leftrightarrow (b \wedge (\sim a \vee \sim c))] \wedge [c \leftrightarrow (a \wedge b \wedge c)]$ must be true

a	b	c	$\sim b \rightarrow a \wedge c$	$b \wedge (\sim a \vee \sim c)$	$a \wedge b \wedge c$	$[a \leftrightarrow (\sim b \rightarrow a \wedge c)] \wedge [b \leftrightarrow (b \wedge (\sim a \vee \sim c))] \wedge [c \leftrightarrow (a \wedge b \wedge c)]$
T	T	T	T	F	T	F
T	**T**	**F**	**T**	**T**	**F**	**T**
T	F	T	T	F	F	F
T	F	F	F	F	F	F
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	F	F	F	F
F	**F**	**F**	**F**	**F**	**F**	**T**

Therefore, there are two possible situations:
1. Suspects A and B are innocent, and Suspect C is guilty.
2. All Suspects A, B, and C are guilty.

(9 marks)

Q4 (12 marks)

(i) In an arithmetic sequence, the sum of its first nine terms is 2340, and the sum of the next eight terms is 6500. Determine the common difference (d) and the first term (a_1).

(6 marks)

(ii) Given a geometric sequence with the first term b and the common ratio r, its sum to infinity is 4. Another geometric sequence has the first term $3b$, common ratio r^2 and sum to infinity 8. Determine the values of b and r.

(6 marks)

Solution of Q4 (12 marks)

Q4(i)

The sum of the first nine terms is 2340:

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$

$$S_9 = \frac{9}{2}\left[2a_1 + (9-1)d\right]$$

$$9a_1 + 36d = 2340$$

$$a_1 = 260 - 4d \quad \text{- (1)}$$

The sum of the next eight terms is 6500

$$S_8 = \frac{8}{2}\left[2a_{10} + (8-1)d\right]$$

$$4\left[2(a_1 + (10-1)d) + (8-1)d\right] = 6500$$

$$8a_1 + 100d = 6500$$

$$a_1 = 812.5 - 12.5d \quad \text{- (2)}$$

(1) = (2)

$260 - 4d = 812.5 - 12.5d$

Therefore,

$d = 65$ and $a_1 = 0$,

(6 marks)

Q4(ii)

For the 1st sequence,

$$S_1 = \frac{b}{1-r}$$

$b = 4(1 - r)$ – (1)

For the 2nd sequence,

$$S_2 = \frac{3b}{1-r^2}$$

$8(1 - r^2) = 3b$ – (2)

Substituting (1) into (2),
$8(1 - r^2) = 3\times4(1 - r)$
$8r^2 - 12r + 4 = 0$
$r = 0.5$ or 1 (rejected, since $|r| < 1$)

Substituting $r = 0.5$ into (1), $b = 2$

Therefore, $r = 0.5$ and $b = 2$.

(6 marks)

Q5 (10 marks)

Use 1,2,3 to form a number with a length of 4 (e.g., 1231). Find the number of permutations or combinations under each condition.

(i) These three numbers are reused with no limitations. (2 marks)

(ii) One of the elements (i.e., '1', or '2', or '3') is used TWICE to form the number. (4 marks)

(iii) One of the elements (i.e., '1', or '2', or '3') is used TWICE to form the number, meanwhile the element cannot be adjacent. (e.g., there are two "1" used, then the number cannot be 11xy, x11y, xy11, etc.) (4 marks)

Solution of Q5 (10 marks)

(i) $3^4=81$; (2 MARK)

(ii)

Select the number to use twice, which is ${}_3C_1$;

For each case(1231,1232,1233), note there are two identical numbers, thus the number of permutation is: ${}_4P_4/{}_2P_2$;

Therefore, the total number that satisfies the condition is:

${}_3C_1*{}_4P_4/{}_2P_2=36$ (4 MARKS)

(iii)

Select the number to use twice, which is ${}_3C_1$;

For each case (1231,1232,1233), we can firstly permutate the other two numbers, which is ${}_2P_2$;

Then, we can insert the repeated two numbers into the two of the three formed intervals, which is ${}_3C_2$;

Therefore, the total number that satisfies the condition is:

${}_3C_1*{}_2P_2*{}_3C_2=18$; (4 MARKS)

Q6 (12 marks)

There is a group of 100 passengers who plan to visit Hong Kong. It is known that THREE (3) of them are infected by the Cov-19. Given the limited human resources, a custom agent can randomly select 3 passengers to perform medical check. You can assume that the medical check result of the selected 3 passengers is 100% correct.

(i) Find the number of the combinations of the selected 3 customers; (2 marks)

(ii) Find the probability of detecting exactly 1 infected customer; (4 marks)

(iii) Find the probability of detecting at least 1 infected customer; (4 marks)

(iv) It is known that Cov-19 has a high probability to infect others and thus a 1% rule has been established. If the infection rate of passengers is less than 1%, the whole team will be allowed to entry into Hong Kong, otherwise the whole team should undergo quarantine. Comment on the decision to be made. (2 marks)

Solution of Q6 (12 marks)

(i) ${}_{100}C_3$=161700; (2 MARKS)

(ii) The number of ways that exactly 1 infected passenger in the 3 picked passengers is：
${}_3C_1$*${}_{97}C_2$ = 3*4656 = 13968;
The probability is 13968/161700 = 8.6% (4 MARKS)

(iii)For the case of:
1 infected passenger in the 3 picked passengers is：${}_3C_1$*${}_{97}C_2$ = 3*4753 = 13968
2 infected passenger in the 3 picked passengers is：${}_3C_2$*${}_{97}C_1$ = 3*97 = 291
3 infected passenger in the 3 picked passengers is：${}_3C_3$*${}_{97}C_0$ = 1*1 = 1
Therefore, to total combinations is 13968+291+1= 14260
The probability is 14260/161700=8.8% (4 MARKS)

(iv)Since the Cov-19 has a high probability to infect others, it is necessary to ensure the detection rate of the passenger group. Based on (iii), the infected rate is 8.8% which is far too high a figure. Hence the passenger team is NOT allowed to enter without quarantine. (2 MARKS)

Q7 (10 marks)

Find the coefficient of the term x^3y^3;

(i) $(x+y^2/x)(x+y)^5$ (5 Marks)

(ii) $(x+y)(2x-y)^5$ (5 Marks)

Solution of Q7 (10 marks)

(i) The *(r+1)-th* term in $(x+y)^5$ is ${}_5C_r\, x^r(y)^{5-r}$.
The term x^3y^3 is formed by two pasts as: $x * x^2y^3 + y^2/x * x^4y$
With the ${}_5C_r\, x^r(y)^{5-r}$, it is computed that the coefficient of x^2y^3 is ${}_5C_2$ (r=2);
With the ${}_5C_r\, x^r(y)^{5-r}$, it is computed that the coefficient of x^4y is ${}_5C_4$ (r=4);
Hence, the coefficient is ${}_5C_2 + {}_5C_4$ =15 (5 Marks)

(ii) The *(r+1)-th* term in $(2x-y)^5$ is ${}_5C_r\, 2^r(-1)^{5-r} x^r(y)^{5-r}$.
The term x^3y^3 is formed by two pasts as: $x * x^2y^3 + y * x^3y^2$
With the ${}_5C_r\, 2^r(-1)^{5-r} x^r(y)^{5-r}$, it is computed that the coefficient of x^2y^3 is ${}_5C_2\, 2^2(-1)^3$ (r=2);

With the ${}_5C_r\, 2^r(-1)^{5-r}\, x^r(y)^{5-r}$, it is computed that the coefficient of x^3y^2 is ${}_5C_3\, 2^3(-1)^2$ (r=3);

Hence, the coefficient is ${}_5C_2\, 2^2(-1)^3 + {}_5C_3\, 2^3(-1)^2 = 40$ (5 Marks)

Q8 (6 marks)

Find a if the coefficients of x^2 and x^3 in the expansion of $(2+ax)^8$ are equal ($a \neq 0$).

Solution of Q8 (6 marks)

The term with x^2 is ${}_8C_6\, 2^6(ax)^2$, the coefficient is ${}_8C_6\, 2^6a^2$; (2 MARKS)

The term with x^3 is ${}_8C_5\, 2^5(ax)^3$, the coefficient is ${}_8C_5\, 2^5a^3$; (2 MARKS)

By ${}_8C_6\, 2^6a^2 = {}_8C_5\, 2^5a^3$, it is computed a=1. (2 MARKS)

Q9 (6 marks)

In the expansion of $(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{2x^2})^n$.

(i) n = 8, determine if there exists a term that is independent of x (constant term). (3 marks)

(ii) n = 10, determine if there exists a term that is independent of x (constant term). (3 marks)

Solution of Q9 (6 marks)

(i) The $(r+1)$-*th* term is written as ${}_8C_r\, (\frac{\sqrt{x}}{\sqrt{3}})^r(\frac{\sqrt{3}}{2x^2})^{8-r} = {}_8C_r\, (\frac{1}{\sqrt{3}})^r(\frac{\sqrt{3}}{2})^{8-r}x^{r/2-2(8-r)}$,

The make the term independent of x, r/2-2(8-r)=0,

r = 32/5.

Noted that r should be an integer, therefore, there is no term independent from x. (3 MARKS)

(ii) The $(r+1)$-*th* term is written as ${}_{10}C_r\, (\frac{\sqrt{x}}{\sqrt{3}})^r(\frac{\sqrt{3}}{2x^2})^{10-r} = {}_{10}C_r\, (\frac{1}{\sqrt{3}})^r(\frac{\sqrt{3}}{2})^{10-r}x^{r/2-2(10-r)}$,

The make the term independent of x, r/2-2(10-r)=0,

r = 8.

Since r is an integer, therefore, there is a term independent from x. (3 MARKS)

Q10 (6 marks)

In a training camp, find the minimum number of students in the camp such that three of them are born in the same month.

Solution of Q10 (6 marks): **Number of month n =12**

According to the given condition, K+1 = 3, K = 2,

M = kn +1 = 2*12 + 1 = 25. **(6 MARKS)**

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