

MA1301 Semester B 2021-22
Midterm Exam (C01 & CA1)

MA1301**Name:** _____**Semester B 2021-22****Midterm Exam (C01 & CA1)****14/03/2022**

This exam contains 3 pages (including this page) and 4 questions. Total of points is 100.

Grade Table (for instructor use only)

Question	Points	Score
1	15	
2	50	
3	15	
4	20	
Total:	100	

1. (15 points) (i) [5pts] State the definition of $\int_a^b f(x)dx$ (taking equal sub-intervals and the sample point to be the right endpoint).
(ii) [5pts] Show that

$$\int_0^1 \frac{x}{\sqrt{x^2+1}} dx = \lim_{n \rightarrow +\infty} \left[\frac{1}{\sqrt{1^2 \times n^2 + n^4}} + \frac{2}{\sqrt{2^2 \times n^2 + n^4}} + \frac{3}{\sqrt{3^2 \times n^2 + n^4}} + \cdots + \frac{n}{\sqrt{n^2 \times n^2 + n^4}} \right].$$

- (iii) [5pts] Evaluate the limit in (ii).

2. (50 points) Evaluate the following integrals: (i) [15pts]

$$\int e^{-x} \cos(3x) dx$$

- (ii) [10pts]

$$\int \frac{3x}{\sqrt{4x^2+1}} dx.$$

- (iii) [15pts]

$$\int \frac{x^2 - 5x - 5}{(x-2)(x^2+2x+3)} dx$$

- (iv) [10pts] Determine whether the following improper integral is convergent or divergent. If it converges, please find the value.

$$\int_0^{+\infty} \frac{x+1}{(9+x^2)^{\frac{3}{2}}} dx.$$

3. (15 points) Find the surface area of the surface generated by rotating the region in the first quadrant bounded by the curve $y^2 = 4 - x$ and x -axis about x -axis for one complete revolution.
4. (20 points) (i)[10pts] Let $f(x)$ be a continuously differentiable function on $[a, b]$ such that $\int_a^b f(x)dx=0$ and $f(a) = f(b) = 1$. Find the value of

$$\int_a^b x f'(x) dx$$

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(ii)[10pts] Let $f(x)$ be twice continuously differentiable function on $[0, 1]$ such that $f(0) = f(1) = 1$ and $\int_0^1 f(x)dx = 1$. Using integration by parts, find the value of

$$\int_0^1 x(1-x)f''(x)dx.$$