EE1001 Foundations of Digital Techniques **Semester A, 2021/2022**

Test 2 (**75 min.**) Name: _____ Student ID: _____

1. (10 marks) Prove that for all real numbers x, if $(x+1)^2$ is irrational, then $\frac{x}{2}$ is irrational.

2. (10 marks) Prove by mathematical induction that $4^n > n^2$ for any integers $n \geq 1$.

3. (10 marks) Convert the repeating decimal $0.\overline{45}$ into a fraction of the form $\frac{a}{b}$. Show your steps.

4. (10 marks) Find the 3rd roots of $\left(\overline{4-4\sqrt{3}i}\right)^2$ in polar form.

5. (10 marks) Let $A = (21)_{10}$ and $B = (-17)_{10}$. Convert them into two's complement format in a 6-bit system and compute $A + B$ using two's complement arithmetic. Explain clearly how you obtain the answer.

6. (10 marks) Consider the following statement: For all sets A and B, $(A - B) \cup (A \cap B) = A$. Use algebraic rules to prove it. State the name of the algebraic rule you used in each step. Do ***not*** skip any steps.

7. (10 marks) Perform the addition of the following 2 numbers $(A + B)$ in IEEE 754 floating point 32-bit format and convert the result to decimal number.

	Sign	Exponent	Fraction
A:	0	1000 0011	0011 0000 0000 0000 0000 000
B:	1	1000 0001	1100 0000 0000 0000 0000 001

8. (10 marks) Let $A = \{1, 4, 5\}$, $B = \{2, 3, 4\}$, $C = \{2, 3, 5\}$, $D = \{1, 2, 3, 4, 5\}$.

   a) Is $\{A, C\}$ a partition of D? Explain your answer.

   b) Find $A \times (B \cap C)$.

9. (10 marks) Let $X = \{1, 2, 3, 4\}$ and $Y = \{5, 6, 7, 8. 9\}$. Define $f: X \to Y$ by specifying that

$$f(1) = 5, f(2) = 7, f(3) = 7, f(4) = 8.$$

a) What is the range of f?

b) Is f injective? Is f surjective? Explain your answers.

10. (10 marks) Let $f(x) = x^2 + x - 1$ and $g(x) = 3x + 1$. Determine $f \circ g$ and $g \circ f$ and simplify their expressions.

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