Review Chapter 12

- Pressure definition: $$p = \frac{F_\perp}{A}$$

Force perpendicular to surface

Surface Area

- Pressure inside a fluid

Pressure at depth h in a **fluid of uniform density**

$$p = p_0 + \rho g h$$

Uniform density of fluid

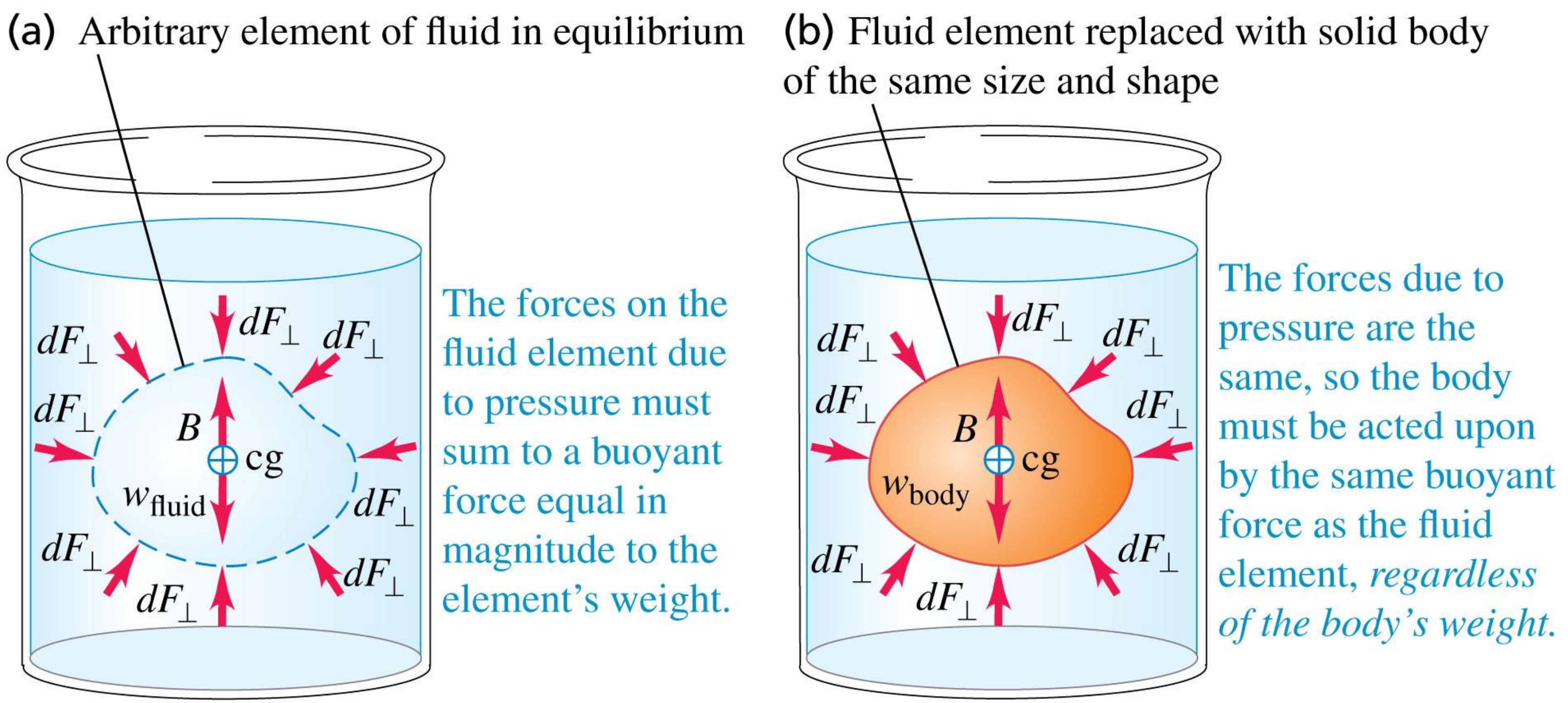
Depth below surface

Pressure at surface of fluid

Acceleration due to gravity ($g > 0$)

Buoyancy

- Buoyancy force B is the sum of $dF_\perp$
- **Archimedes' principle**: Buoyancy force is upward, and is equal to the weight of the fluid displaced.



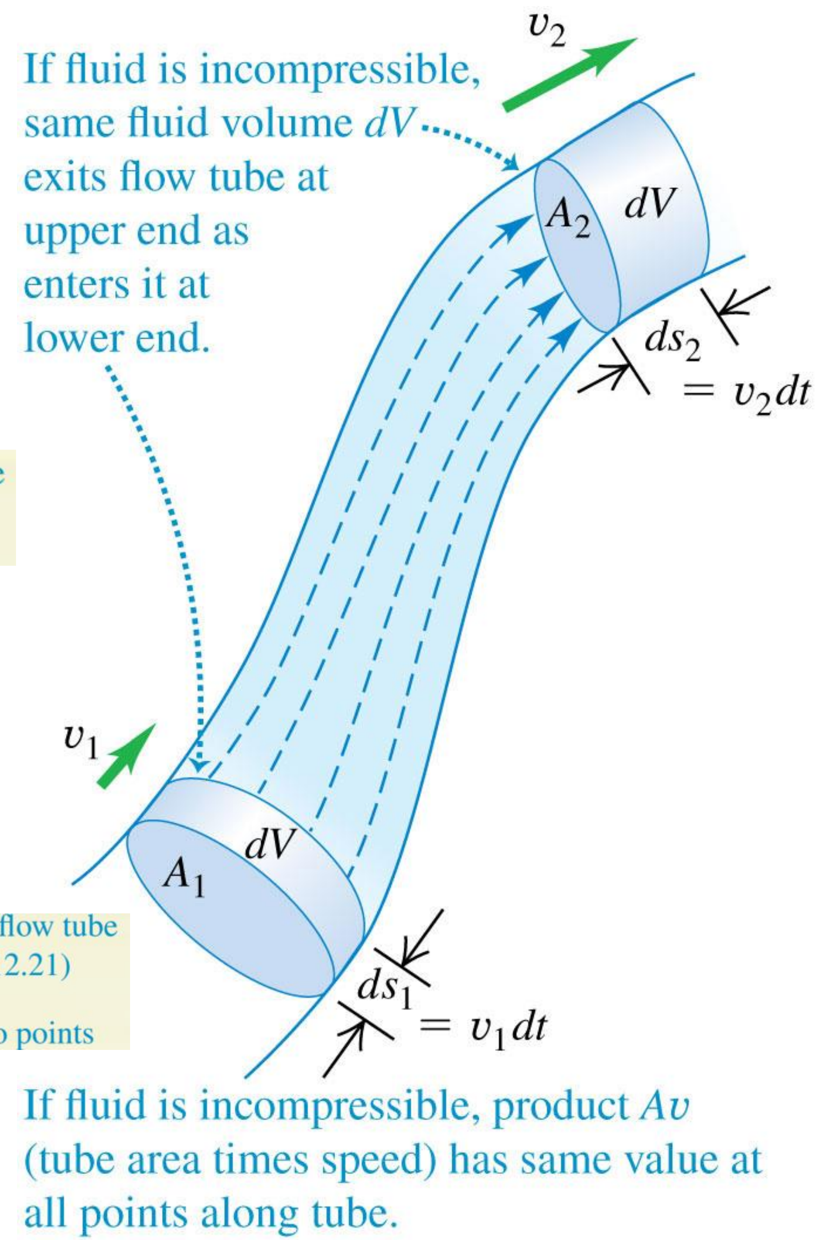
Fluid flow: The continuity equation (matter conservation)

- During time dt, fluid with volume of dV flows through the tube
- The **volume flow rate** is

Volume flow rate of a fluid $\dfrac{dV}{dt} = Av$ Cross-sectional area of flow tube; Speed of flow

- Volume entering must be equal to volume exiting

Continuity equation for an **incompressible fluid** $A_1 v_1 = A_2 v_2$ Cross-sectional area of flow tube at two points (see Fig. 12.21); Speed of flow at the two points



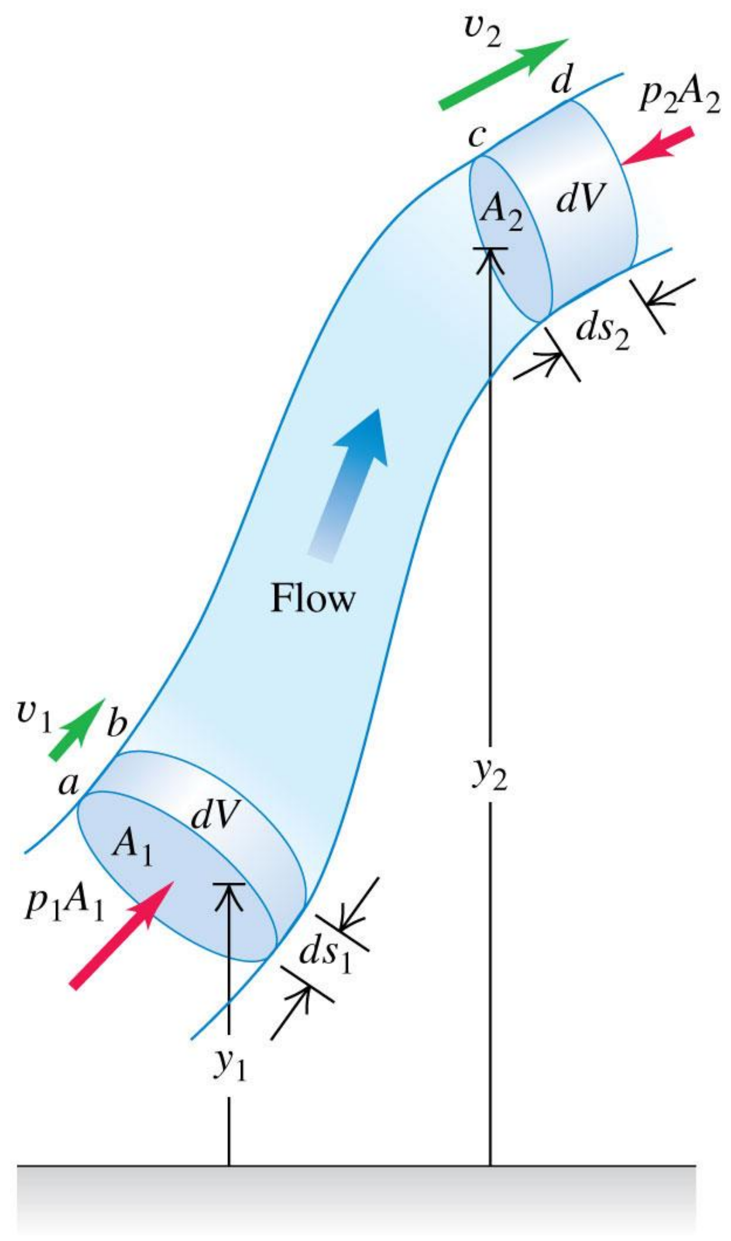
Bernoulli's equation (energy conservation)

- During time dt, work done to the fluid is $w_1 = Fds_1 = p_1A_1ds_1 = p_1dV$
- The fluid does work $w_2 = p_2dV$
- Gravitational potential energy changes by $\Delta E_g = mg(y_2 - y_1) = \rho dVg(y_2 - y_1)$
- Kinetic energy changes by $\Delta E_k = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \frac{1}{2}\rho dV(v_2^2 - v_1^2)$
- Energy is conserved, so $w_1 - w_2 = \Delta E_g + \Delta E_k$
- Simplify: $p_1 + \rho g y_1 + \frac{1}{2}\rho v_1 = p_2 + \rho g y_2 + \frac{1}{2}\rho v_2$, or

Pressure Fluid density Value is **same** at all points in flow tube.

$$p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$$

Acceleration due to gravity Elevation Flow speed

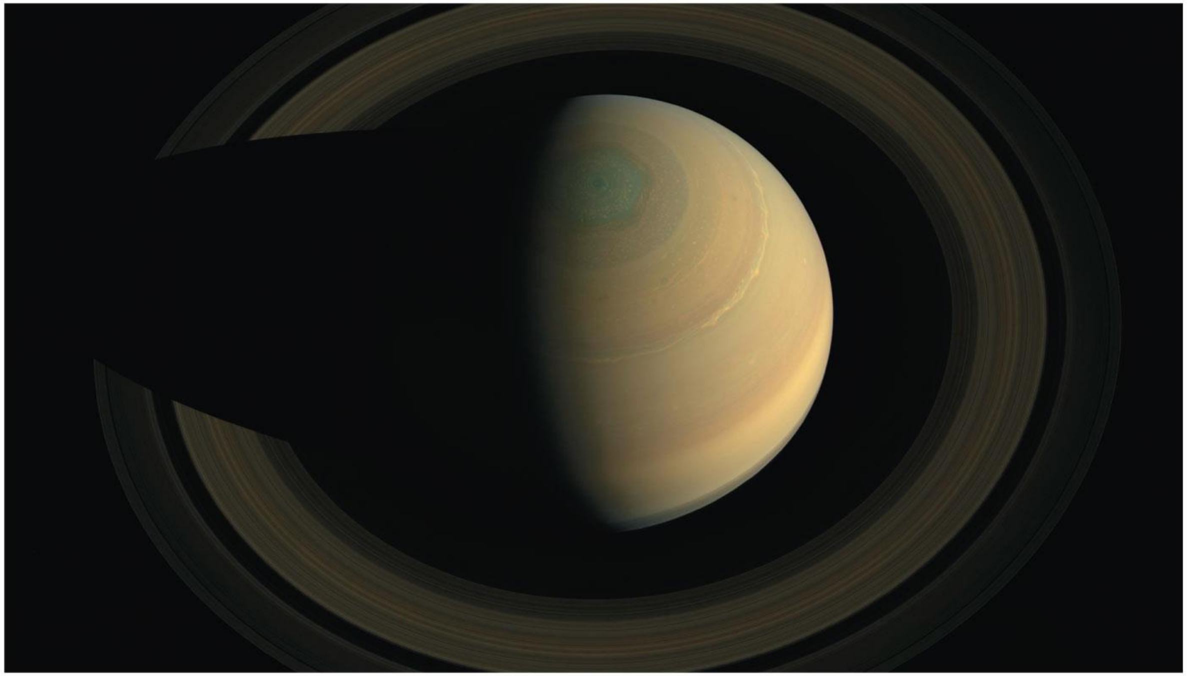


Chapter 13

Gravitation

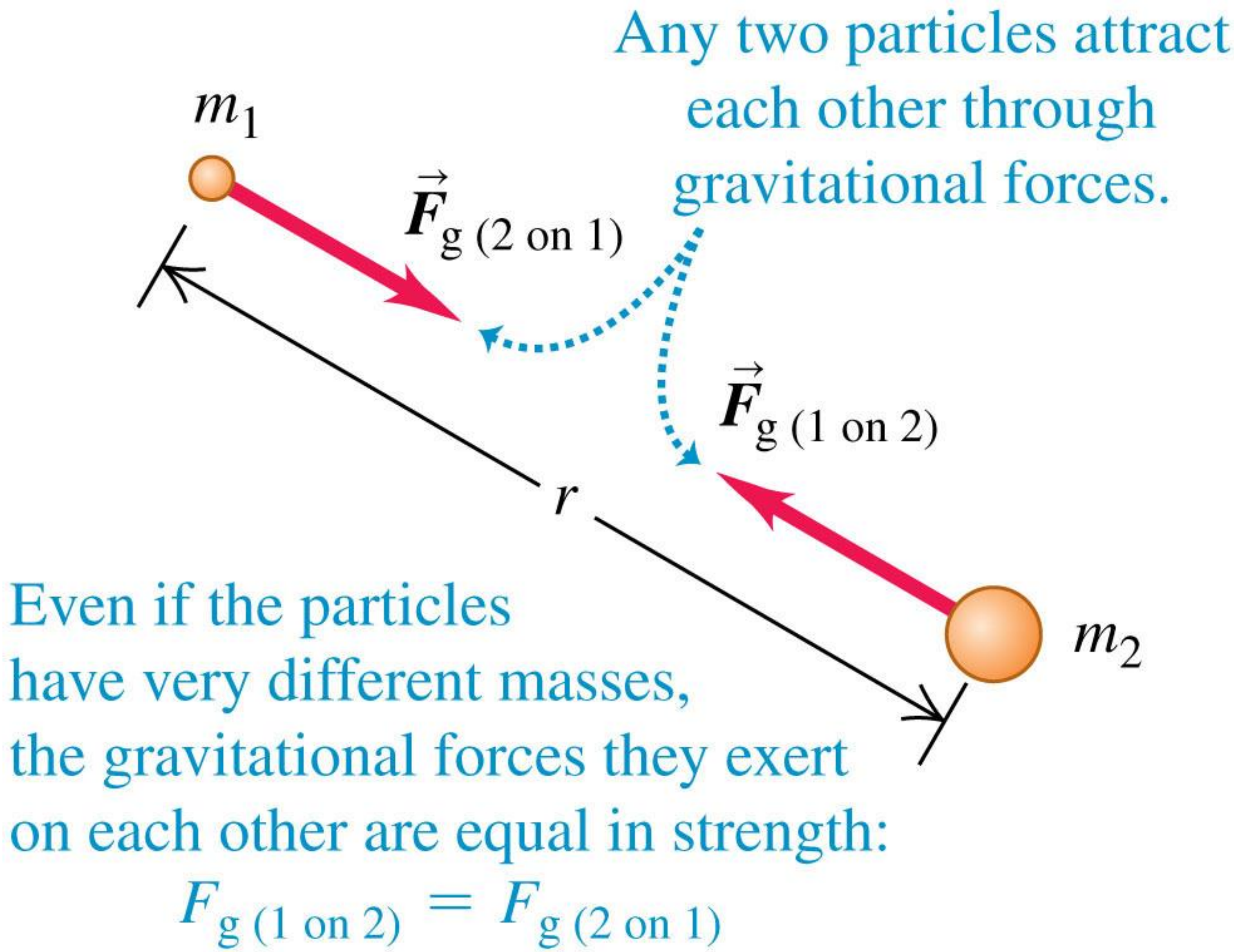
Introduction

- What can we say about the motion of the particles that make up Saturn's rings?



- Why doesn't the moon fall to earth, or the earth into the sun?
- By studying gravitation and celestial mechanics, we will be able to answer these and other questions.

Newton's law of gravitation



Newton's law of gravitation

- *Law of gravitation*: Every particle of matter attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them:

Gravitational constant (same for any two particles)

Newton's **law of gravitation:** Magnitude of attractive gravitational force between any two particles

$$F_g = \frac{Gm_1m_2}{r^2}$$

Masses of particles

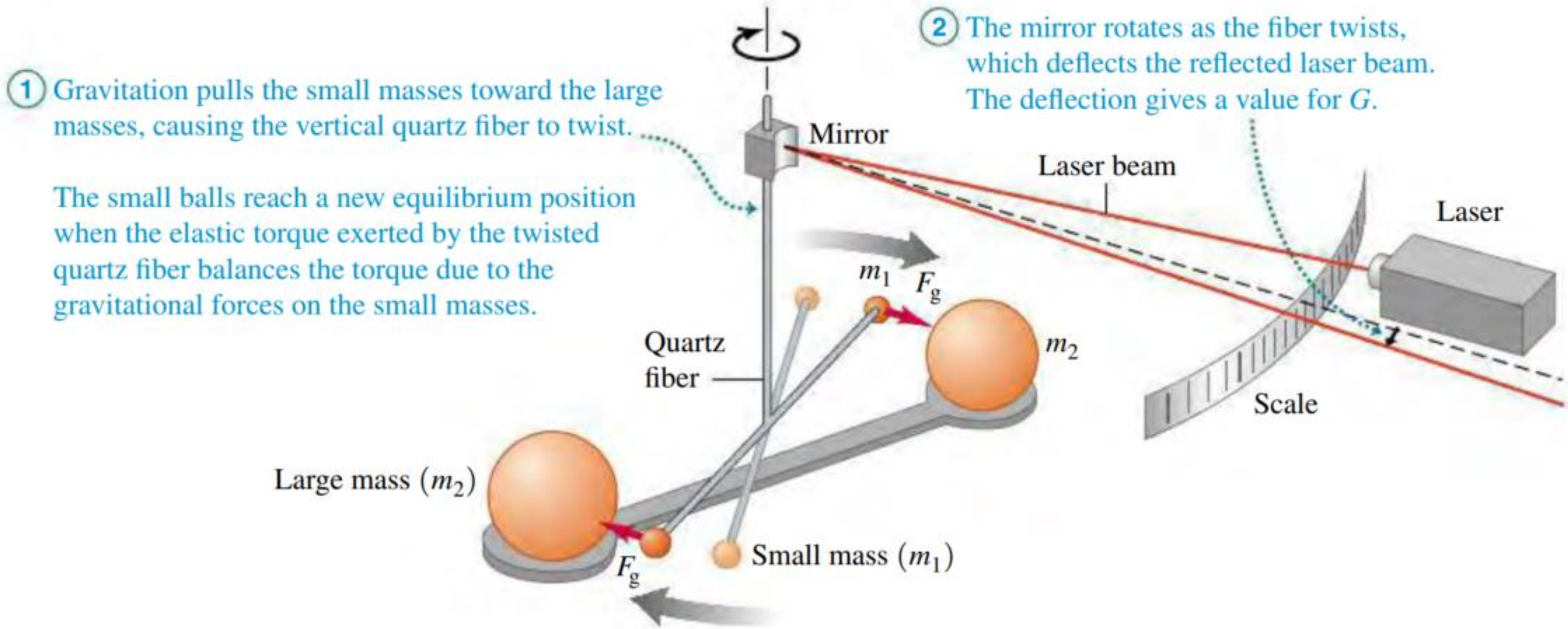
Distance between particles

- The gravitational constant G is a fundamental physical constant that has the same value for any two particles.

- $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.

Measuring G

- G is tiny on laboratory length scale. Here is how to measure it.

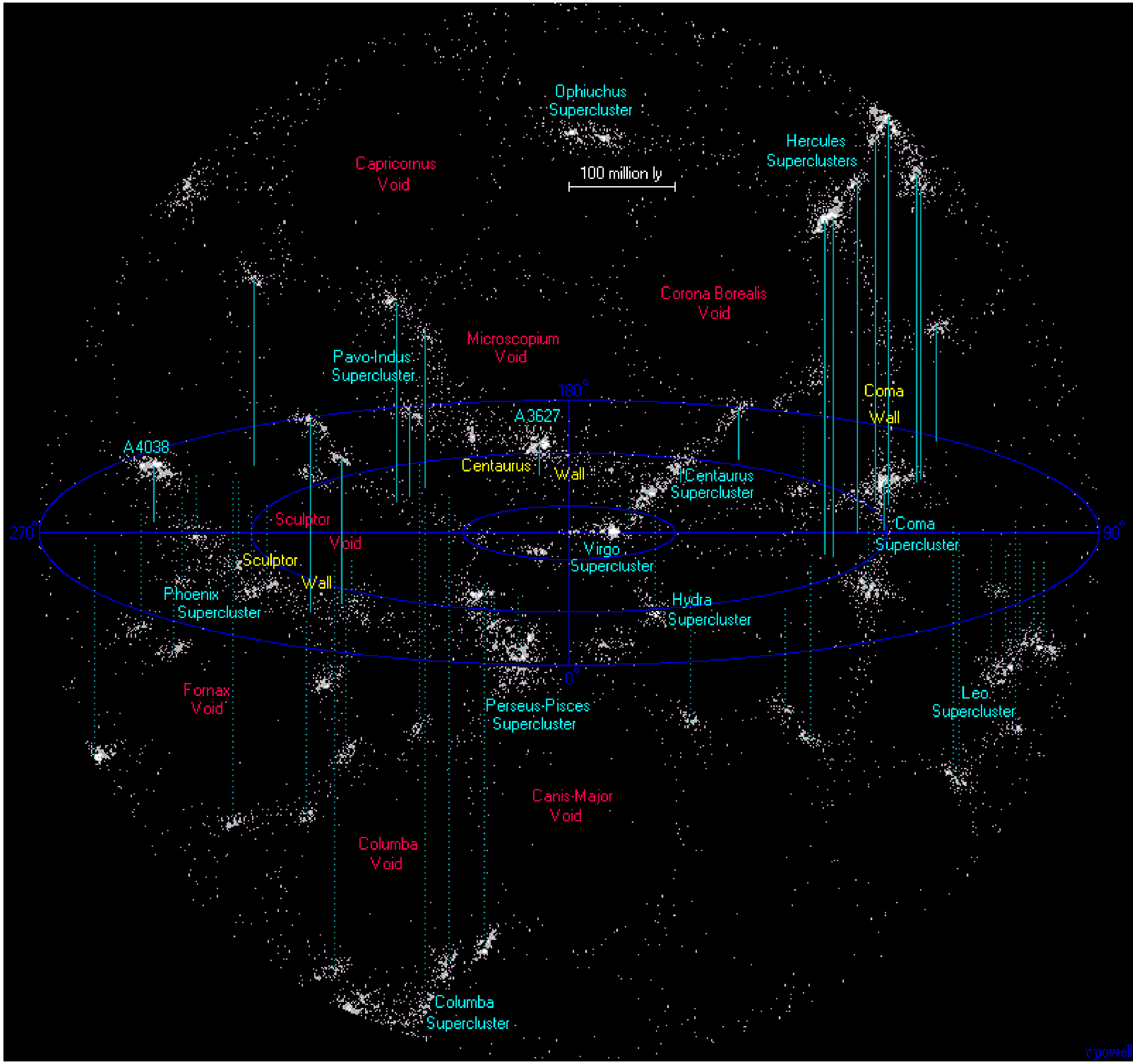


- Gravitation effects are important on a larger scale

- $F_g = \frac{Gm_1 m_2}{r^2}$, while $m_{1,2} \propto r^3$, so $F_g \propto r^4$

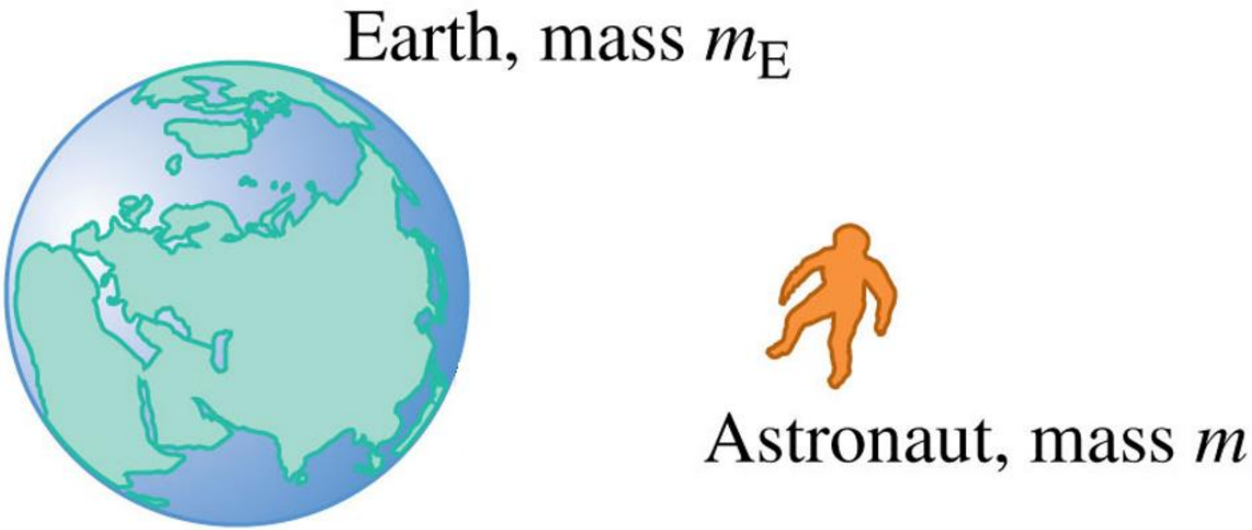
Gravitation is important on larger length scale

- Gravitation makes the universe foam-like with filaments and voids.



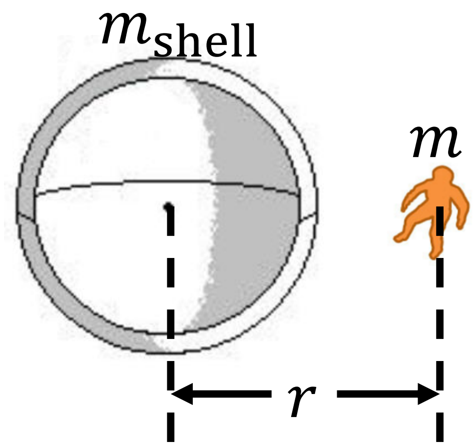
Spherical mass distributions

- Consider the gravitational force between the earth and an astronaut.
- If they are not far away, earth cannot be considered a point mass.
- Should we divide the earth into volume elements and do an integral?
- Fortunately Newton has done this for us.

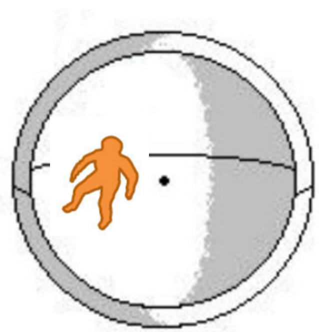


Spherical mass distributions

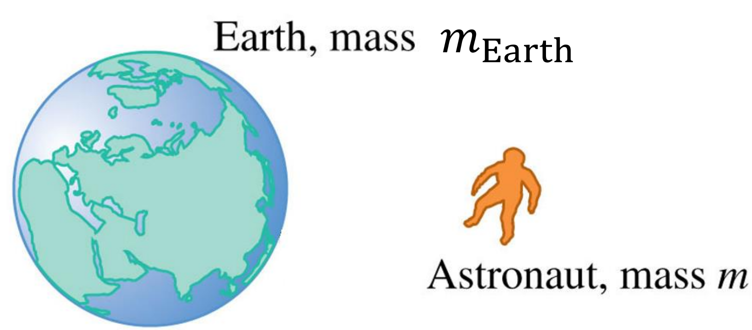
- Outside a spherical shell of mass, the gravitational force of the shell acts as if all the mass of the shell is concentrated at its center:
  - $F = G m_{\text{shell}} m / r^2$



- Inside a spherical shell of mass, the total gravitational force is zero.



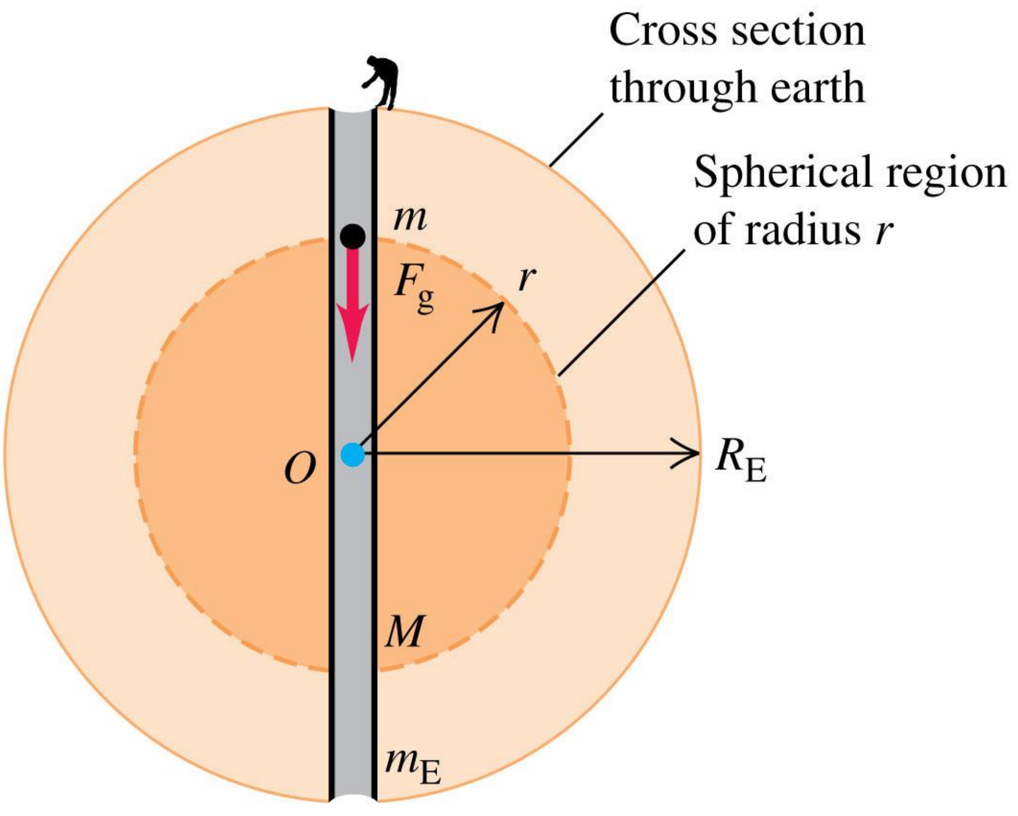
- The earth can be divided into spherical shells. In total,
  - $F = G m_{\text{Earth}} m / r^2$



Inside earth

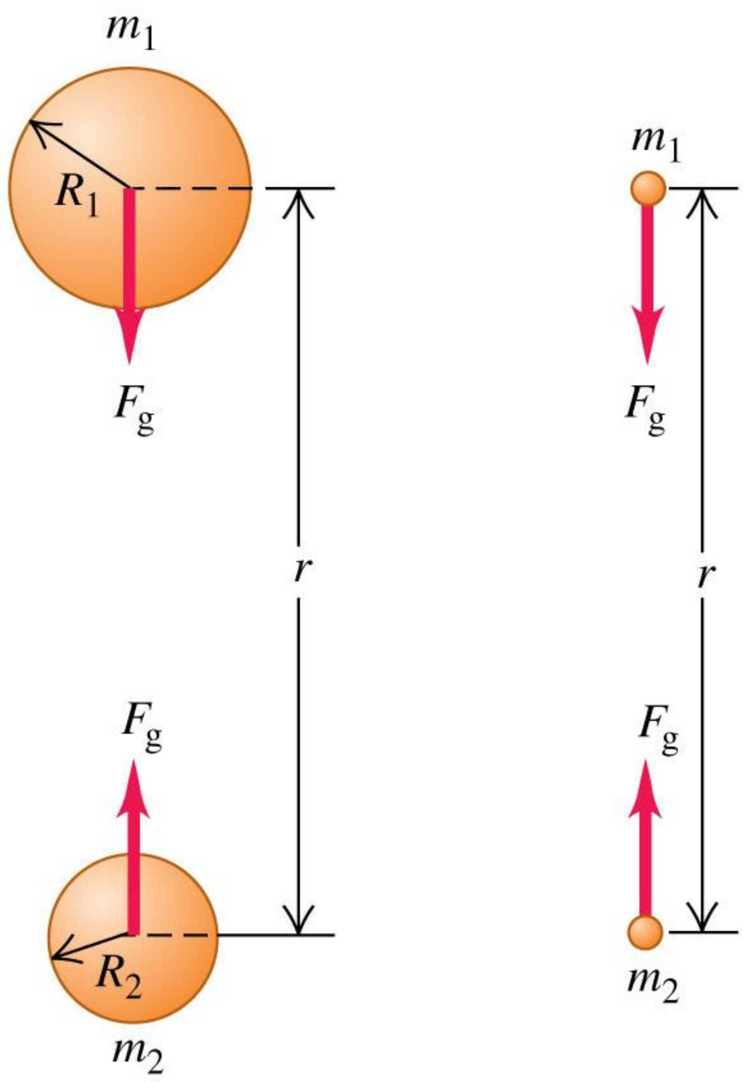
- For mass m inside the earth, at distance r to the center, only the mass inside r exerts a net gravitational force.

- $F = \frac{GMm}{r^2} = \frac{Gm}{r^2}\left(m_E \frac{r^3}{R_E^3}\right) = \frac{Gm_E m}{R_E^3} \cdot r$



Gravitation between two spheres

- The gravitational force between two spheres is the same as if each sphere is replaced by a point mass at its center.



Weight

- The **weight** of a body is the total gravitational force exerted on it by all other bodies in the universe.

- At the surface of the earth, we can neglect all other gravitational forces, so a body's weight is:

Gravitational constant

Mass of the earth

Weight of a body at the earth's surface ...

$$w = F_g = \frac{Gm_E m}{R_E^2}$$

Mass of body

... equals gravitational force the earth exerts on body.

Radius of the earth

- The acceleration due to gravity at the earth's surface is:

Gravitational constant

Mass of the earth

Acceleration due to gravity at the earth's surface

$$g = \frac{Gm_E}{R_E^2}$$

Radius of the earth

Walking and running on the moon

- You automatically transition from a walk to a run when the vertical force the ground exerts on you exceeds your weight.
- This transition from walking to running happens at much lower speeds on the moon, where objects weigh only 17% as much as on earth.
- Hence, the Apollo astronauts found themselves running even when moving relatively slowly during their moon "walks."

Example: weight on Venus

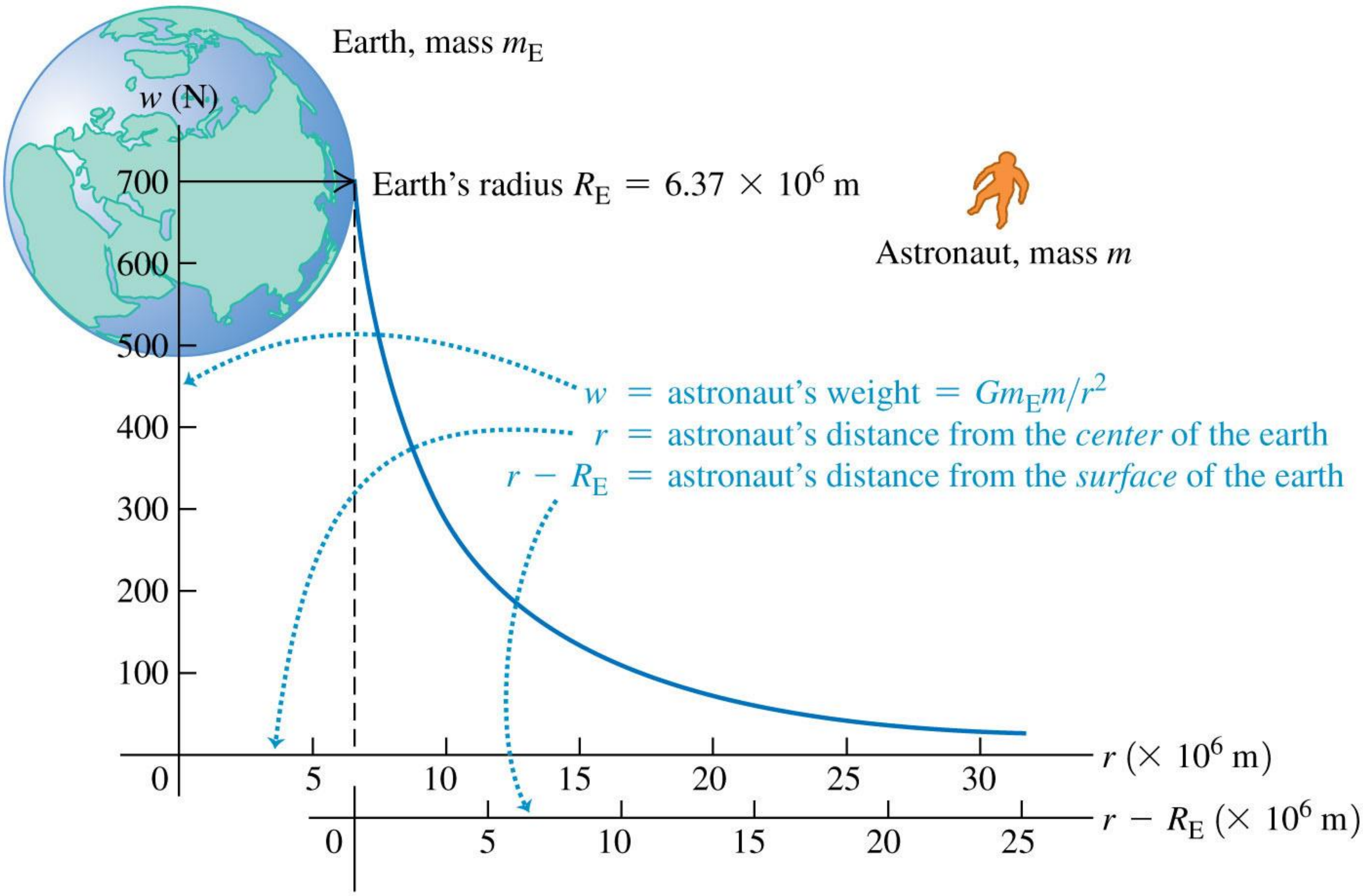
- The mass of Venus is 81.5% that of the earth, and its radius is 94.9% that of the earth. If a rock weighs 75.0 N on earth, what would it weigh at the surface of Venus?

- Answer:

- $g_{\text{Venus}} = \frac{Gm_{\text{Venus}}}{R_{\text{Venus}}^2} = \frac{G(0.815 * m_{\text{Earth}})}{(0.949R_{\text{Earth}})^2} = 0.905\frac{Gm_{\text{Earth}}}{R_{\text{Earth}}^2} = 0.905g_{\text{Earth}}$

- $w_{\text{Venus}} = \frac{g_{\text{Venus}}}{g_{\text{Earth}}} w_{\text{Earth}} = 0.905 * 75.0N = 67.9N$

Weight decreases with altitude



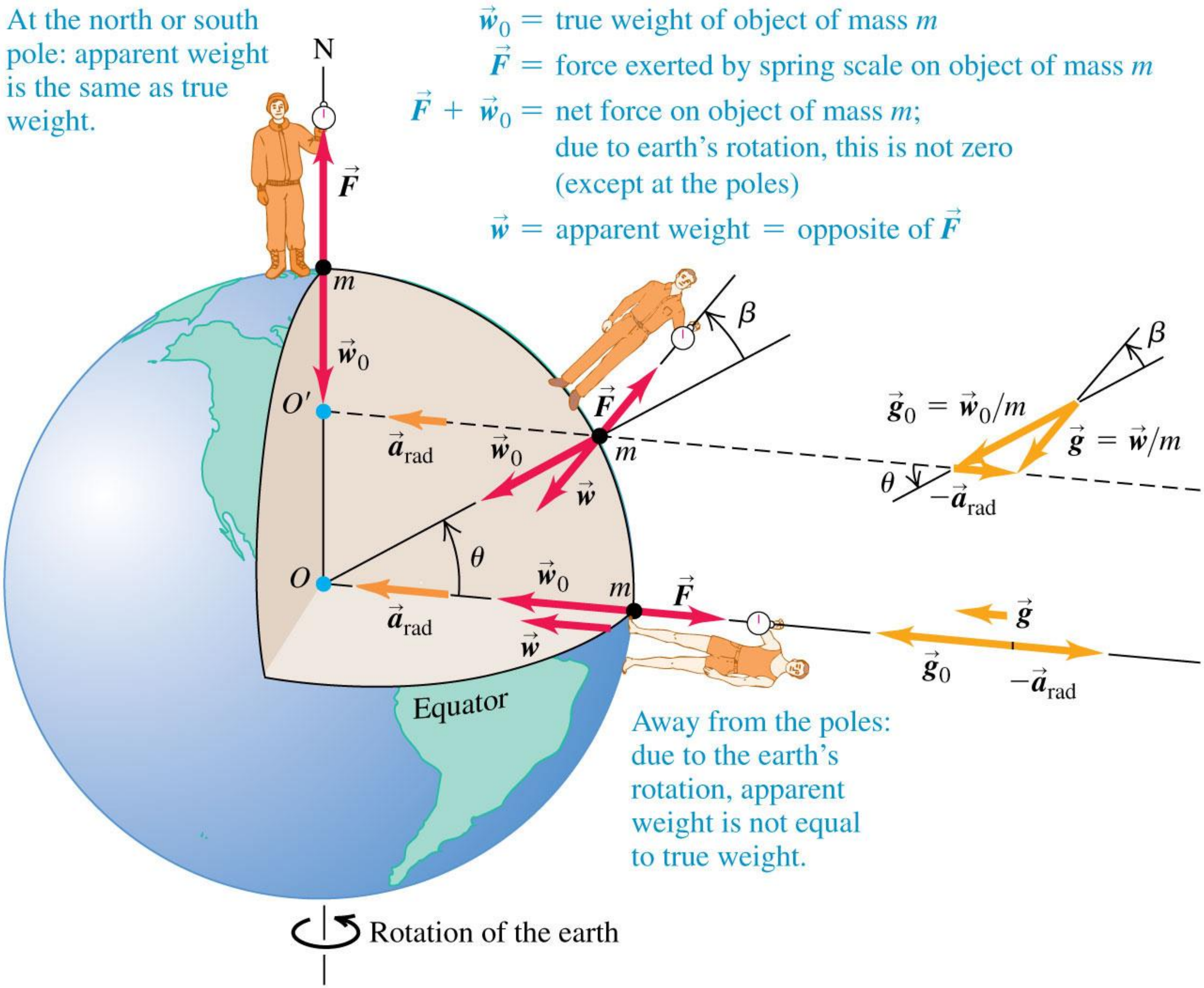
Weight on an airplane

13.7 In an airliner at high altitude, you are farther from the center of the earth than when on the ground and hence weigh slightly less. Can you show that at an altitude of 10 km above the surface, you weigh 0.3% less than you do on the ground?

- $g_{\text{ground}} = \dfrac{Gm_E}{R_E^2}$
- $g_{\text{air}} = \dfrac{Gm_E}{(R_E+10\text{km})^2}$
- $\dfrac{w_{\text{air}}}{w_{\text{ground}}} = \dfrac{g_{\text{air}}}{g_{\text{ground}}}$

$$= \left(\frac{R_E}{R_E+10\text{km}}\right)^2$$
$$= 0.9968\ldots$$
$$\approx 1 - 0.3\%$$

Apparent weight and the earth's rotation

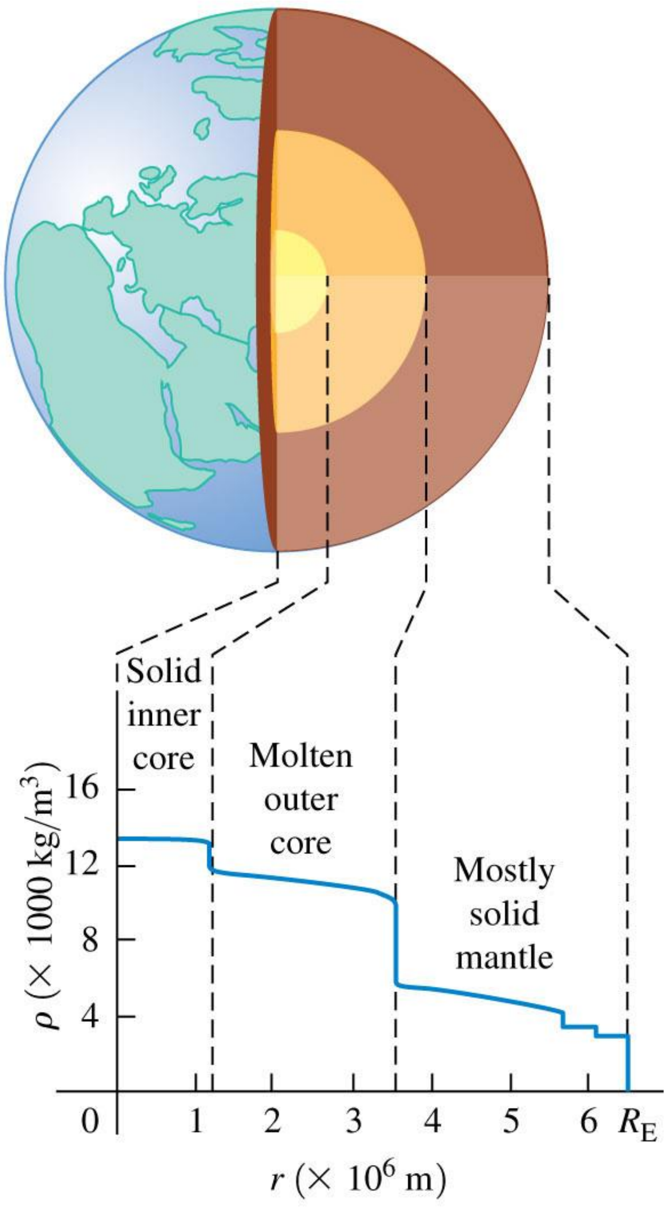


Variations of *g* with latitude and elevation

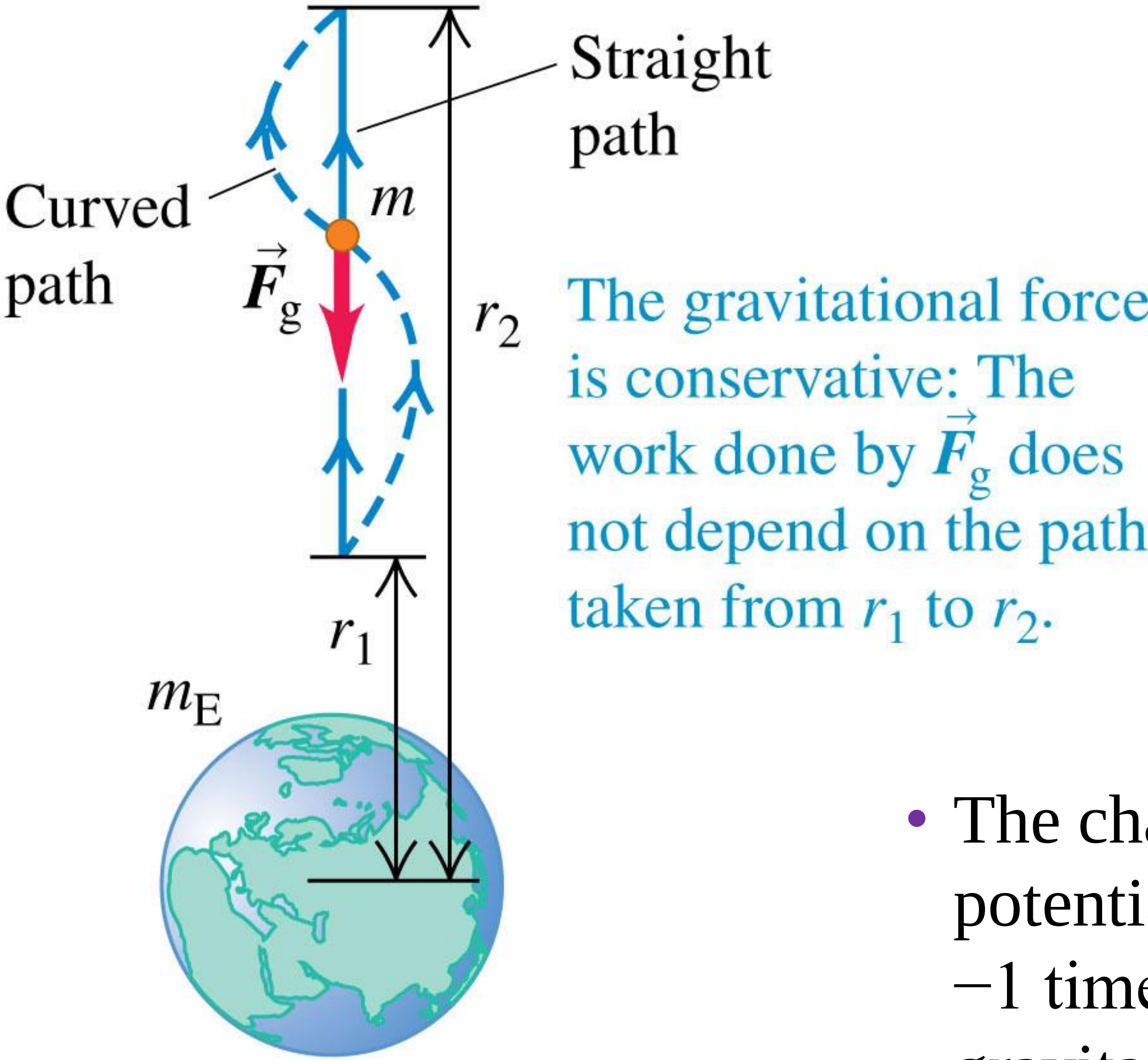
Station	North Latitude	Elevation (m)	g (m/s^2)
Canal Zone	09°	0	9.78243
Jamaica	18°	0	9.78591
Bermuda	32°	0	9.79806
Denver, CO	40°	1638	9.79609
Pittsburgh, PA	40.5°	235	9.80118
Cambridge, MA	42°	0	9.80398
Greenland	70°	0	9.82534

Interior of the earth

- The earth is approximately spherically symmetric, but it is *not* uniform throughout its volume.
- The density ρ decreases with increasing distance r from the center.



Gravitational potential energy



- The change in gravitational potential energy is defined as −1 times the work done by the gravitational force as the body moves from r_1 to r_2.

Gravitational potential energy

- We define the gravitational potential energy U so that $W_{\text{grav}} = U_1 - U_2$:

Gravitational constant · Mass of the earth

Gravitational potential energy (general expression)

$$U = -\frac{Gm_E m}{r}$$

Mass of body

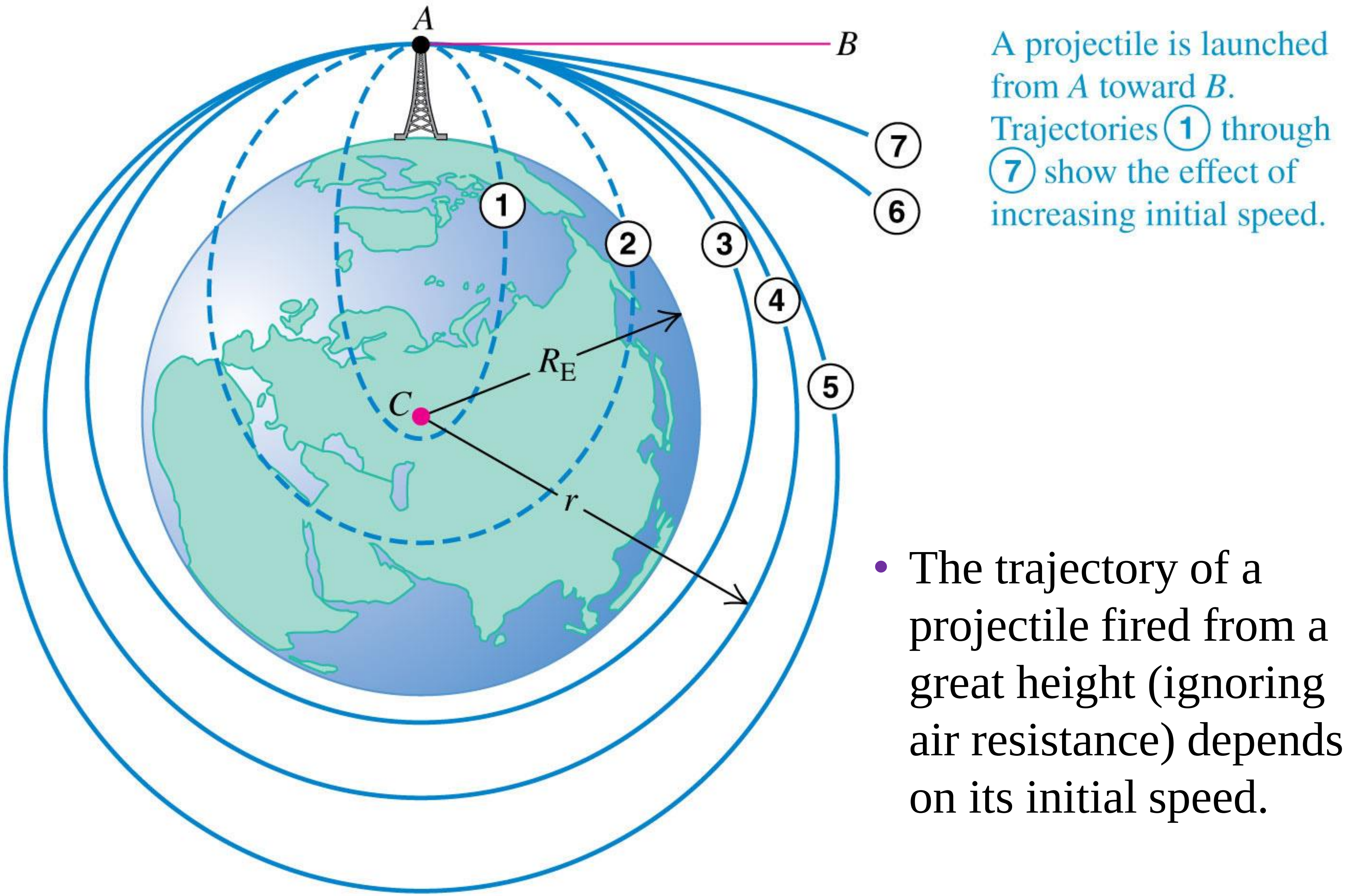
Distance of body from the earth's center

- If the earth's gravitational force on a body is the only force that does work, then the total mechanical energy of the system of the earth and body is constant, or **conserved**.

Example: Escape speed

- In Jules Verne's 1865 story with this title, three men went to the moon in a shell fired from a giant cannon sunk in the earth in Florida. Find the minimum muzzle speed that would allow a shell to escape from the earth completely (the *escape speed*). The earth's radius and mass are $R_E = 6.37 \times 10^6 m$ and $m_E = 5.97 \times 10^{24} kg$
- Answer: When the shell is fired, let its speed be v_1, kinetic energy is $K_1 = \frac{1}{2}mv_1^2$, potential energy $U_1 = -Gm_E m/R_E$
- When the shell escape, $K_2 \geq 0$, $U_2 = -Gm_E m/\infty = 0$
- Energy conservation: $K_1 + U_1 = K_2 + U_2$
- $\frac{1}{2}mv_1^2 = \mathrm{K}_1 = \mathrm{K}_2 + \mathrm{U}_2 - \mathrm{K}_1 \geq Gm_E m/R_E$
- $v_1 \geq \sqrt{2Gm_E/R_E} = 1.12 \times 10^4 m/s$

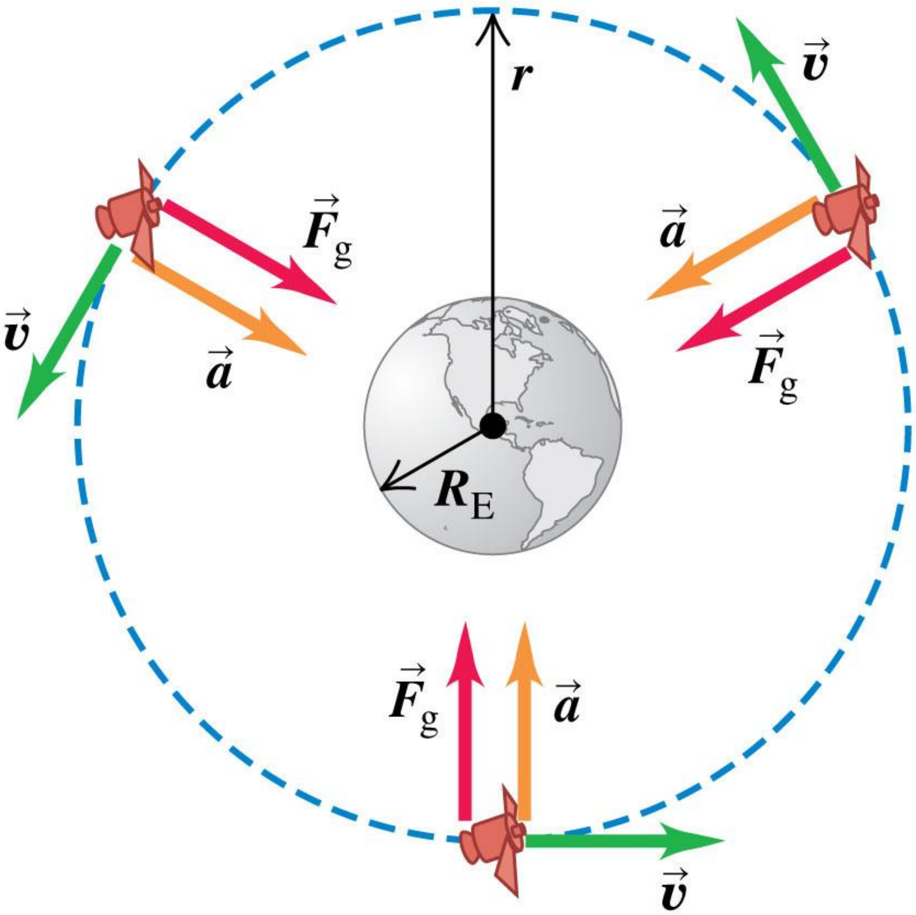
The motion of satellites



- The trajectory of a projectile fired from a great height (ignoring air resistance) depends on its initial speed.

Circular satellite orbits

- For a circular orbit, the speed of a satellite is just right to keep its distance from the center of the earth constant.
- The force $\vec{F}_g$ due to the earth's gravitational attraction provides the centripetal acceleration that keeps a satellite in orbit.



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Circular satellite orbits

- $\frac{Gm_E m}{r^2} = \frac{mv^2}{r}$

Gravitational constant

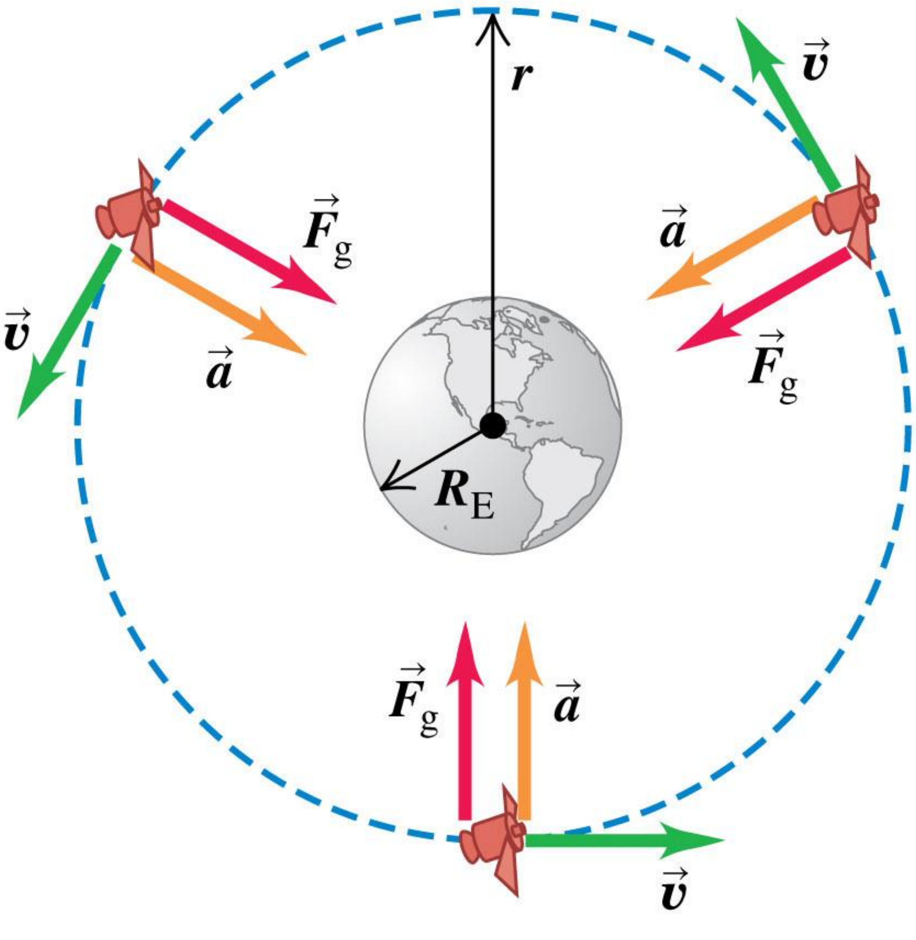
Speed of satellite in a circular orbit around the earth $v = \sqrt{\frac{Gm_E}{r}}$ Mass of the earth

Radius of orbit

- It is smaller than the escape speed.

- The period (the time for one revolution) is

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{\frac{3}{2}}}{\sqrt{Gm_E}}$$



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Circular satellite orbits

- A satellite is constantly falling *around* the earth.
- Astronauts inside the satellite in orbit are in a state of *apparent weightlessness* because they are falling with the satellite.

Example: Galaxy's Core

- Astronomers have observed a small, massive object at the center of our Milky Way galaxy. A ring of material orbits this massive object; the ring has a diameter of about 15 light-years and an orbital speed of about 200 km/s. Determine the mass of the object at the center of the Milky Way galaxy.

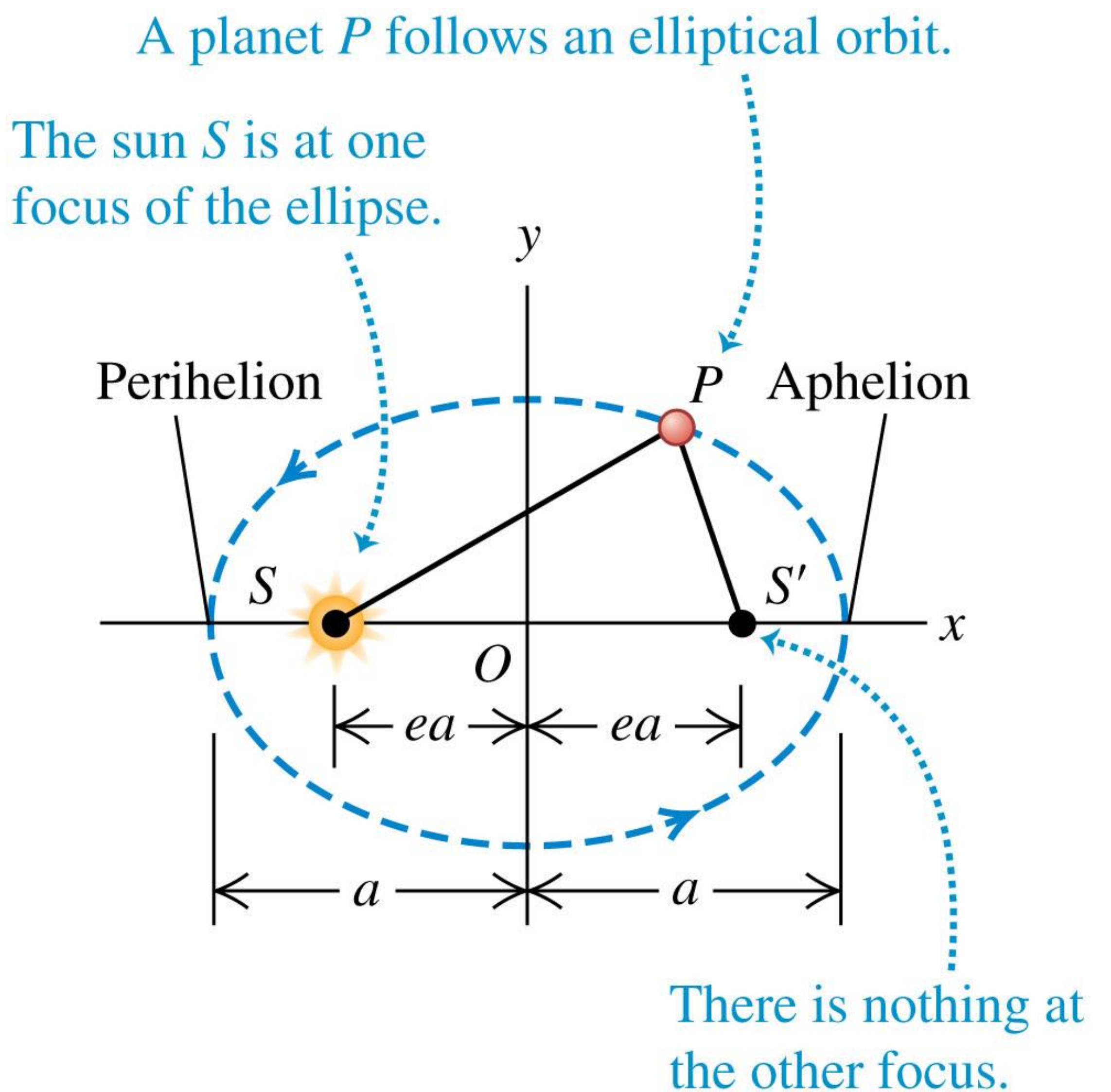
- $r = \frac{D}{2} = \frac{15*365*24*60*60*3.0\times10^{8}}{2} = 7.1 \times 10^{16}\text{m}$

- $v = \sqrt{Gm/r} \Rightarrow m = rv^{2}/G = 4.3 \times 10^{37}\text{kg}$

Elliptical orbits: Kepler's first law

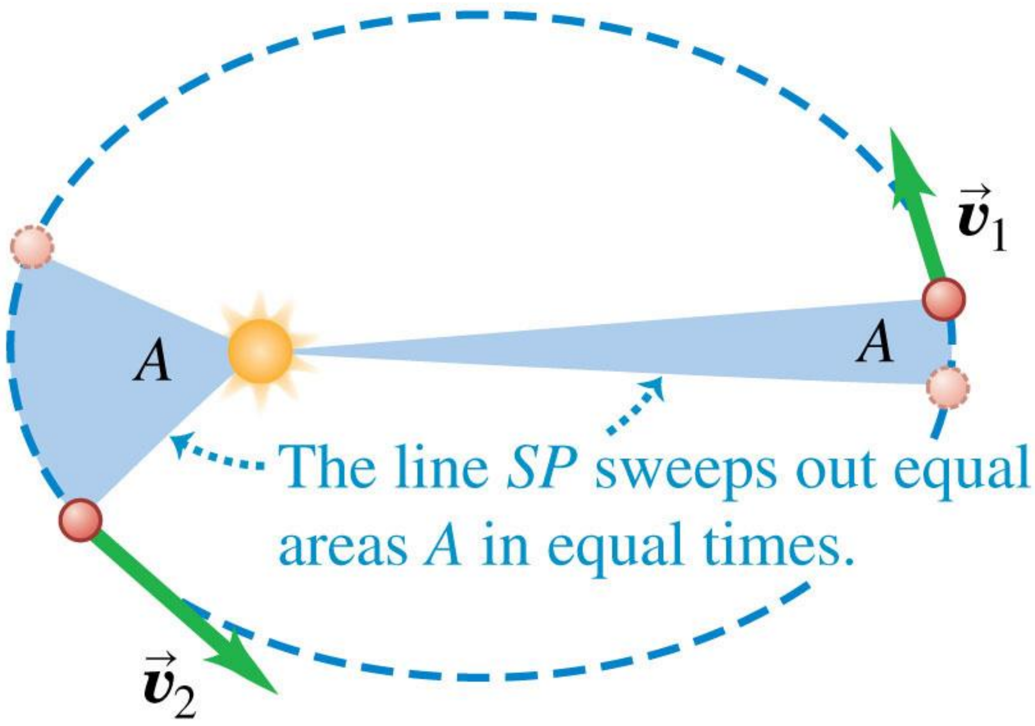
- Each planet moves in an elliptical orbit with the sun at one focus of the ellipse.

- An ellipse is defined such that for all points on it, $PS + PS'$ is a constant.



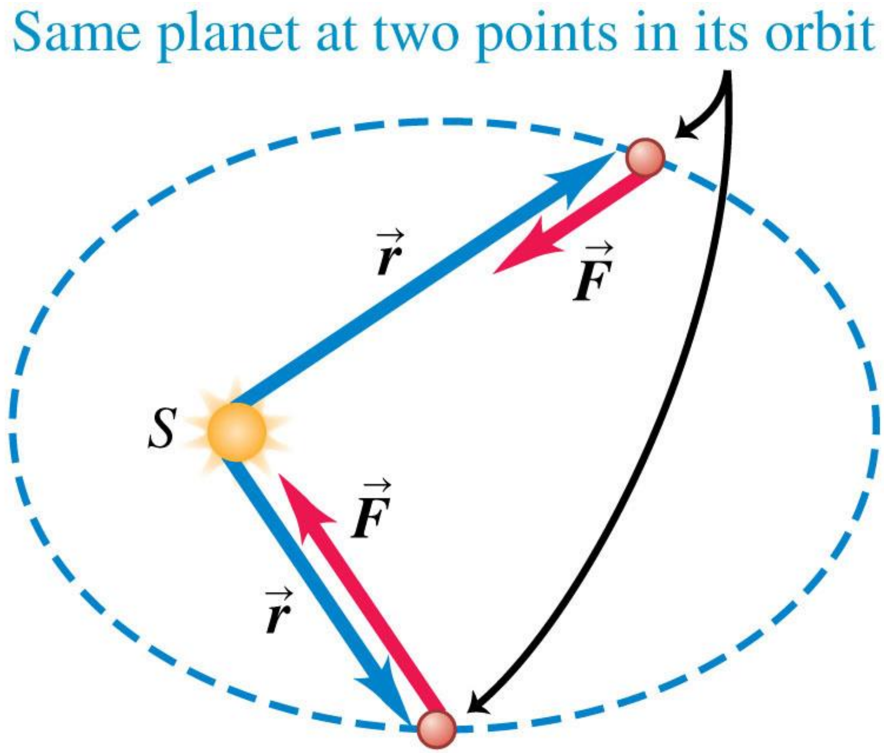
Kepler's second law

- A line from the sun to a given planet sweeps out equal areas in equal times.



Kepler's second law

- Because the gravitational force that the sun exerts on a planet produces zero torque around the sun, the planet's angular momentum around the sun remains constant.



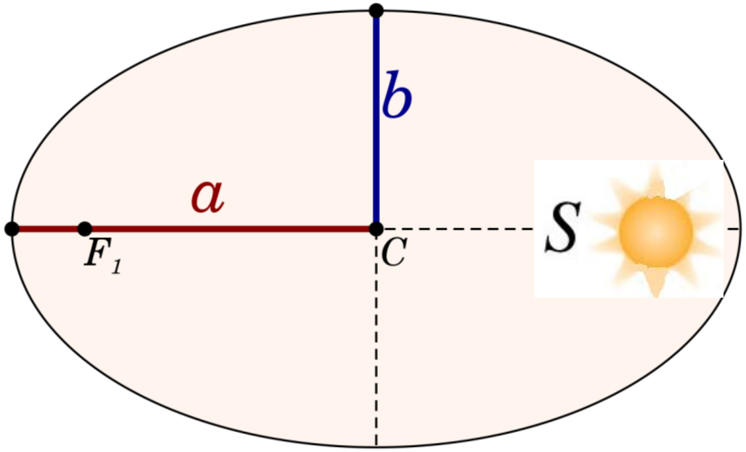
- Gravitational force $\vec{F}$ on planet has different magnitudes at different points but is always opposite to vector $\vec{r}$ from sun S to planet.
- Hence $\vec{F}$ produces zero torque around sun.

Kepler's third law

- The periods of the planets are proportional to the three-halves powers of the semi-major axis lengths of their orbits.

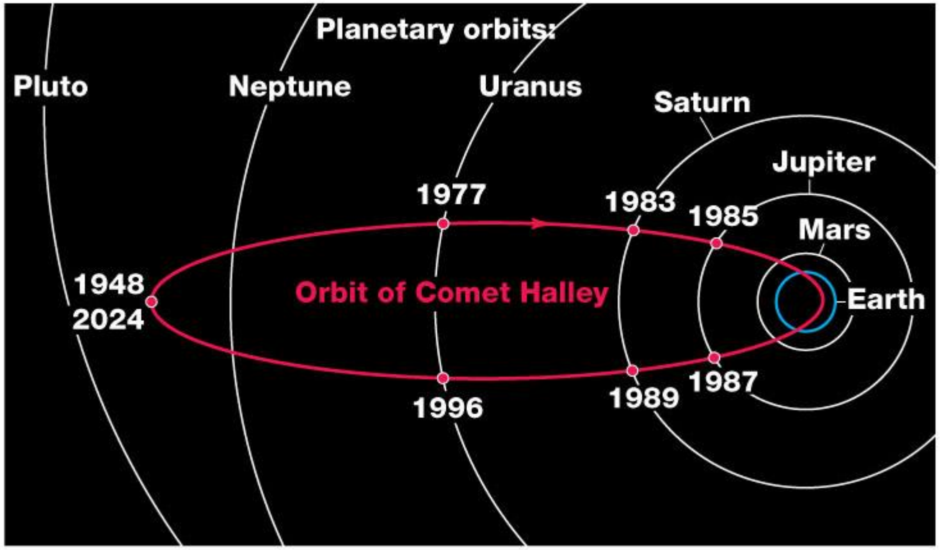
$$T = \frac{2\pi a^{3/2}}{\sqrt{G m_S}} \qquad \text{(elliptical orbit around the sun)}$$

- Note that the period does not depend on the shorter axis b.
- An asteroid in an elongated elliptical orbit with semi-major axis a will have the same orbital period as a planet in a circular orbit of radius a.



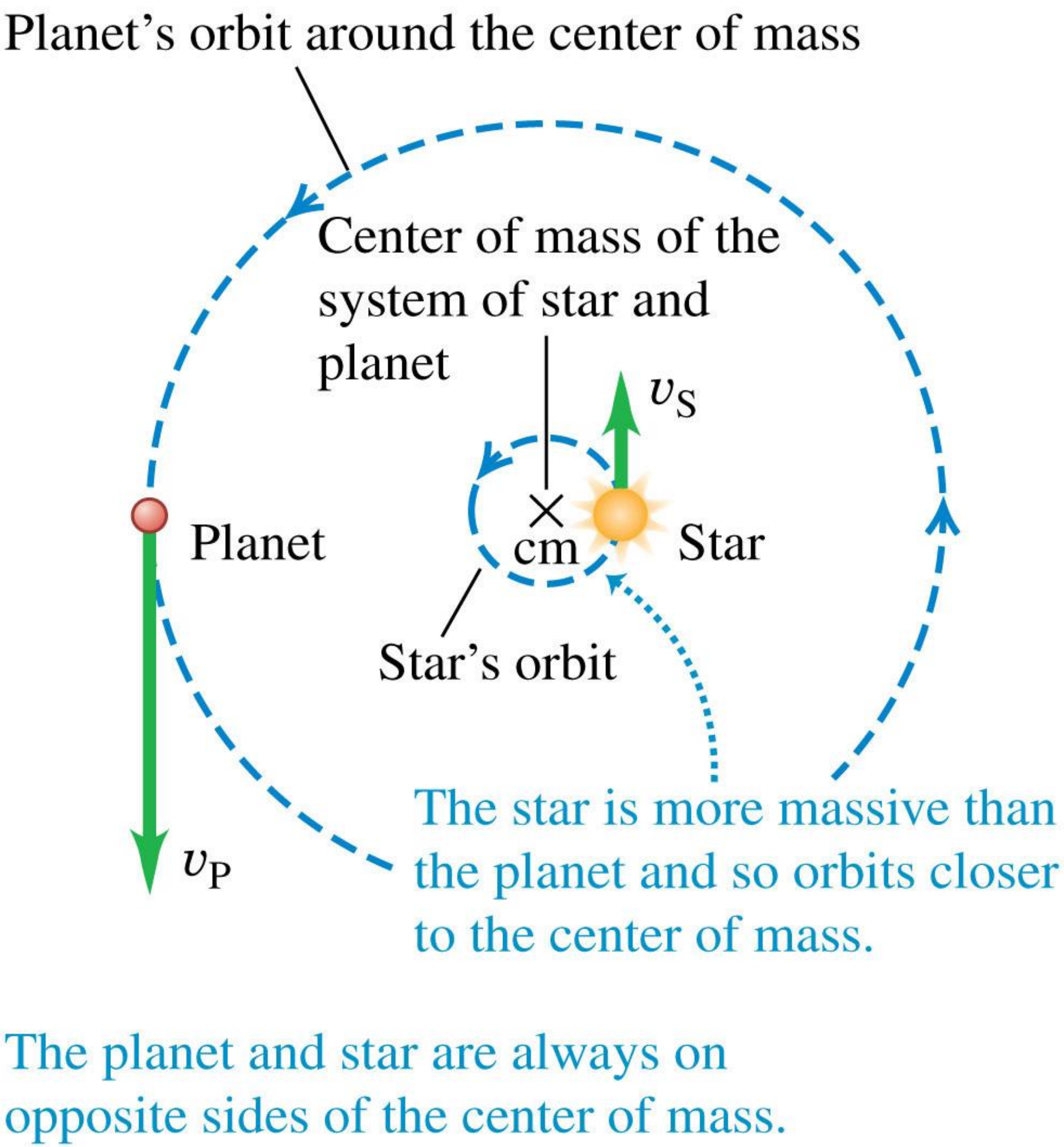
Comet Halley

- At the heart of Comet Halley is an icy body, called the **nucleus**, that is about 10 km across.
- When the comet's orbit carries it close to the sun, the heat of sunlight causes the nucleus to partially evaporate.
- The evaporated material forms the tail, which can be tens of millions of kilometers long.



Planetary motions and the center of mass

- We have assumed that as a planet or comet orbits the sun, the sun remains absolutely stationary.
- In fact, *both* the sun and the planet orbit around their common center of mass.



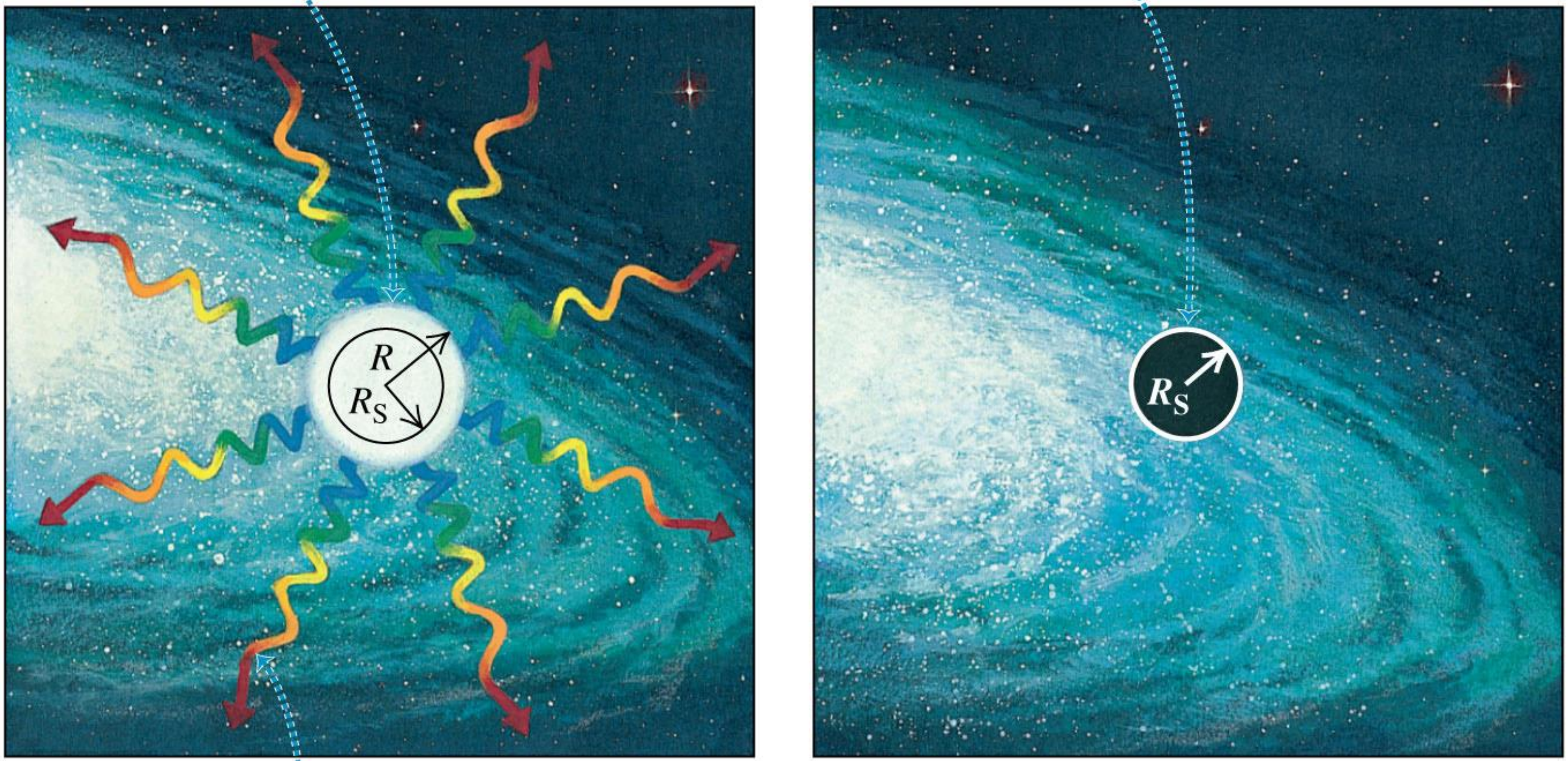
Black holes

- If a spherical nonrotating body has radius less than the **Schwarzschild radius**, nothing can escape from it.
- Escape speed $v = \sqrt{2GM/R} \geq c \implies R \leq 2GM/c^2$

Speed of light in vacuum

- Such a body is a **black hole**.
- The surface of the sphere with radius R_S surrounding a black hole is called the **event horizon**.
- Since light can't escape from within that sphere, we can't see events occurring inside.

Black holes



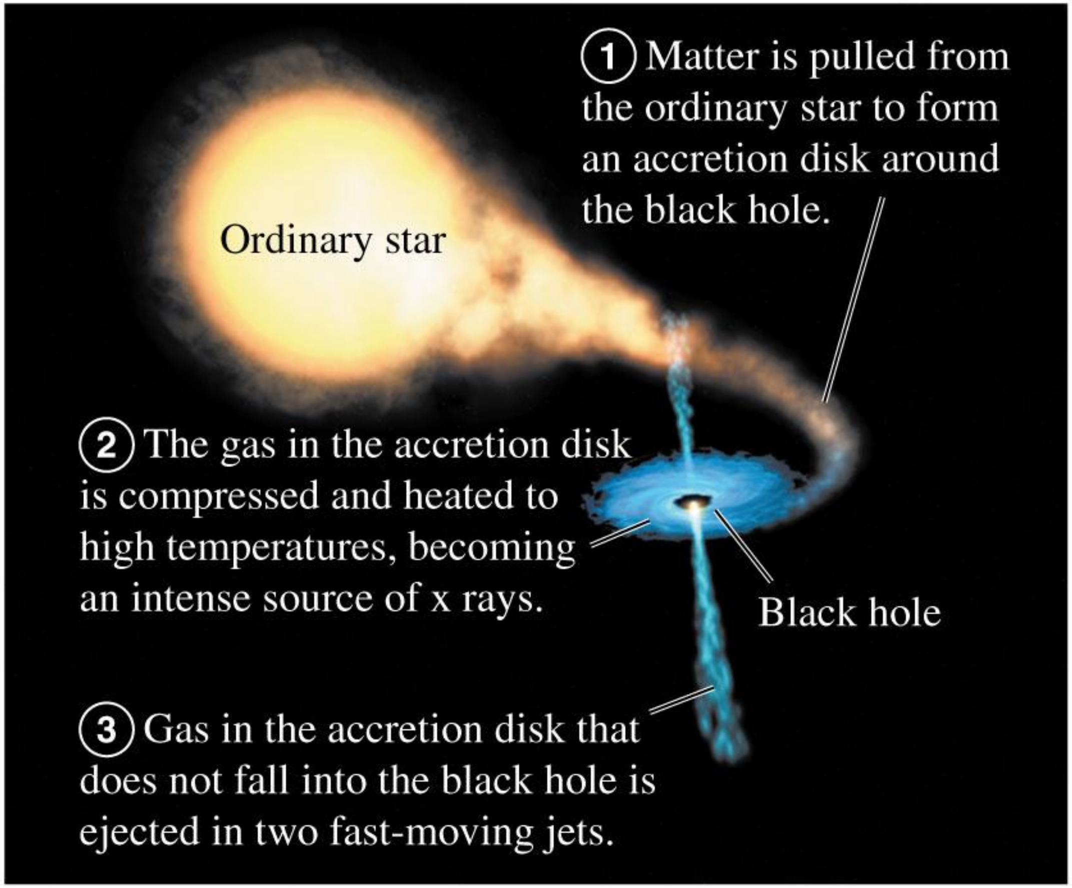
(a) When the radius R of a body is greater than the Schwarzschild radius R_S, light can escape from the surface of the body.

Gravity acting on the escaping light "red shifts" it to longer wavelengths.

(b) If all the mass of the body lies inside radius R_S, the body is a black hole: No light can escape from it.

Detecting black holes

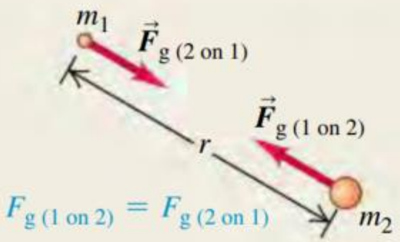
- We can detect black holes by looking for x rays emitted from their *accretion disks*.



Chapter summary

Newton's law of gravitation: *Any* two particles with masses m_1 and m_2, a distance r apart, attract each other with forces inversely proportional to r^2. These forces form an action–reaction pair and obey Newton's third law. When two or more bodies exert gravitational forces on a particular body, the total gravitational force on that individual body is the vector sum of the forces exerted by the other bodies. The gravitational interaction between spherical mass distributions, such as planets or stars, is the same as if all the mass of each distribution were concentrated at the center. (See Examples 13.1–13.3 and 13.10.)

$$F_g = \frac{Gm_1m_2}{r^2}$$



Gravitational force, weight, and gravitational potential energy: The weight w of a body is the total gravitational force exerted on it by all other bodies in the universe. Near the surface of the earth (mass m_E and radius R_E), the weight is essentially equal to the gravitational force of the earth alone. The gravitational potential energy U of two masses m and m_E separated by a distance r is inversely proportional to r. The potential energy is never positive; it is zero only when the two bodies are infinitely far apart. (See Examples 13.4 and 13.5.)

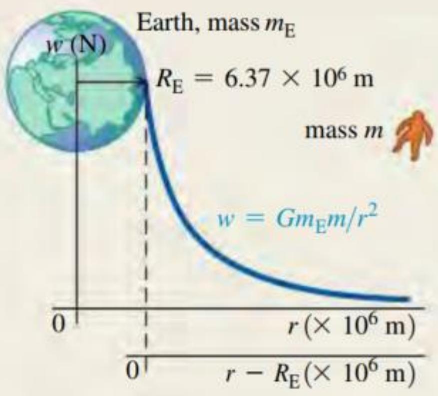
$$w = F_g = \frac{Gm_Em}{R_E^2}$$

(weight at earth's surface)

$$g = \frac{Gm_E}{R_E^2}$$

(acceleration due to gravity at earth's surface)

$$U = -\frac{Gm_Em}{r}$$



Chapter summary

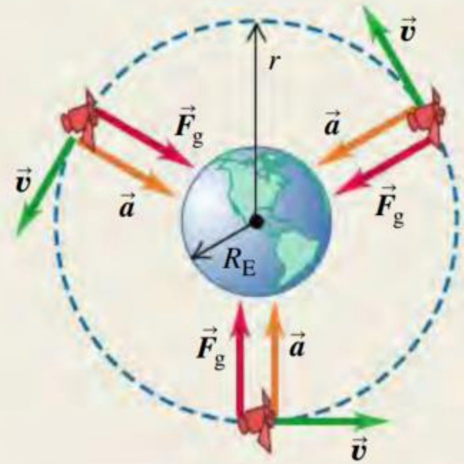
Orbits: When a satellite moves in a circular orbit, the centripetal acceleration is provided by the gravitational attraction of the earth. Kepler's three laws describe the more general case: an elliptical orbit of a planet around the sun or a satellite around a planet. (See Examples 13.6–13.9.)

$$v = \sqrt{\frac{Gm_E}{r}}$$

(speed in circular orbit)

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{Gm_E}} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

(period in circular orbit)



Black holes: If a nonrotating spherical mass distribution with total mass M has a radius less than its Schwarzschild radius R_S, it is called a black hole. The gravitational interaction prevents anything, including light, from escaping from within a sphere with radius R_S. (See Example 13.11.)

$$R_S = \frac{2GM}{c^2}$$

(Schwarzschild radius)

If all of the body is inside its Schwarzschild radius $R_S = 2GM/c^2$, the body is a black hole.