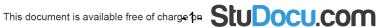
### CITY UNIVERSITY OF HONG KONG

### **Department of Management Sciences**

Module code & title : **CB2200 Business Statistics** Session Semester B, 2015-2016 Time allowed Two hours This paper has EIGHT pages. (Including this page.) *Instructions to students:* 1. This paper consists of FOUR questions. 2. Attempt **ALL FOUR** questions. 3. Start each question on a new page. 4. This paper is not to be taken away. 5. Give your answers to the nearest 4 decimal places, if necessary. Materials, aids and instruments permitted to be used during examination: University approved calculators.

Not to be taken away

Special materials other than standard materials (e.g., answer book or



supplementary sheets) to be supplied to students:

Nil

#### Question 1 (25 marks)

A restaurant offers pizza delivery service to a campus dormitory. Delivery times follow a normal distribution with mean 20 minutes and standard deviation 5 minutes.

a. For a single delivery, state in which of the following ranges (expressed in minutes) delivery time is most likely to lie. Briefly explain your choice without doing any calculation.

18-20 19-21 20-22 21-23 (3 marks)

- b. The restaurant does not charge for the pizza if delivery takes more than 30 minutes. What is the probability of getting a free pizza from a single order? (3 marks)
- c. During examination period, a student plans to order pizza five consecutive evenings. Assume that
  these delivery times are independent of each other. What is the probability that the student will
  get at least one free pizza? (4 marks)
- d. Find the range of times symmetrically distributed around the population mean that includes 60% of all deliveries from this service. (5 marks)
- e. The restaurant selected a random sample of 26 deliveries, what is the probability that the sample mean delivery time is between 18 and 23 minutes? (5 marks)
- f. Suppose the restaurant plans to expand its delivery network, and offers service to commercial areas as well. A random sample of delivery times is required to estimate the new population mean of delivery time. How large a sample is needed to ensure the probability that the sample mean is larger than the new population mean by more than 2 minutes is less than 0.05? Assumed the population standard deviation is 10 minutes.

(5 marks)

#### Question 2 (25 marks)

Many food products contain small quantities of substances that would give an undesirable taste or smell if they were present in large amounts. An example is the "off-odors" caused by sulfur compounds in wine. Oenologists (wine experts) have determined the odor threshold, the lowest concentration of a compound that the human nose can detect. For example, in the oenology literature, the odor threshold for dimethyl sulfide (DMS) is given as 25 micrograms per liter of wine (μg/l). Here are the DMS odor thresholds for 10 randomly selected oenologists:

31	31	47	36	23	34	32	30	20	24

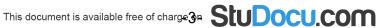
Assume that the standard deviation of the odor threshold is known to be 7µg/l.

a. Sketch a box plot and comment on the shape of this sample data set.

(7 marks)

- b. In order to say the sampling distribution of sample means is Normal, what assumption is required? Why? Should Z or t distribution be used for conducting inferential analysis? (3 marks)
- c. Based on your answer in part (b), give a 95% confidence interval for the mean DMS odor threshold among all oenologists. (5 marks)
- d. Are you convinced that the mean odor threshold for oenologists is higher than the published threshold, 25µg/l? At 5% level of significance, carry out a hypothesis test using critical value approach to justify your answer. (8 marks)
- e. Compute the p-value for part (d).

(2 marks)



## Question 3 (25 marks)

A health club randomly selected 500 of its members and revealed that 34% of them are overweighted. 76% of the selected members were male. 35% of the selected males were overweighted.

- a. What is the probability that a randomly selected member is female and overweighted? (3 marks)
- b. Suppose a randomly selected member is overweighted, what is the chance that the member is male? (3 marks)
- c. Are "overweighted" and "gender" independent? Why or why not? (4 marks)
- d. Construct an 85% confidence interval estimate for the population proportion of overweighted members. (7 marks)
- e. Using the confidence interval constructed in part (d), what conclusion will be drawn for a hypothesis test,  $H_0$ :  $\pi=0.3$  against  $H_1$ :  $\pi\neq0.3$ , at 15% level of significance? Explain. (4 marks)
- f. If you want to be 90% confidence of estimating the proportion of overweighted members in part (d) to within  $\pm 2\%$ , what sample size is needed? (4 marks)

#### Question 4 (25 marks)

The City Office Equipment Corporation sells an imported copier on a franchised basis and performs maintenance service on this copier. The service manager randomly selected 45 recent requests on performing maintenance service. The information being collected include the total time in minutes (Y) spent by the service person, and the number of copiers serviced (X). The service manager carried out a simple linear regression analysis on the data, and part of the Excel output is given as below:

Regression Statistics	
R Square	0.9575

	Coefficients	Standard Error	t Stat	P-value
Intercept	-0.5802	2.8039	???	???
Χ	15.0352	0.4831	???	???

- a. State the estimated simple linear regression equation. Interpret the meaning of the estimated slope coefficient. (4 marks)
- b. Determine the correlation coefficient. Describe the relationship between Y and X. (4 marks)
- c. Why correlation coefficient is preferred over covariance? (3 marks)
- (2 marks) d. Interpret the meaning of R-Square.
- e. At 5% level of significance, is there any positive relationship between Y and X? (8 marks)
- f. Predict the total time needed for maintaining
  - (i) 1 copier, and
  - (ii) 5 copiers. (2 marks)
- g. Given that the observed X values ranged between 2 to 8, of the two predictions obtained for (i) and (ii) in part (f), which is more justifiable? Explain why. (2 marks)



# **Formulae Sheet**

1. 
$$\mu = \frac{1}{N} \sum X_i \quad ; \qquad \qquad \sigma^2 = \frac{1}{N} \sum (X_i - \mu)^2$$
$$\overline{X} = \frac{1}{n} \sum X_i \quad ; \qquad \qquad s^2 = \frac{1}{n-1} \sum (X_i - \overline{X})^2$$

2. 
$$\mu = E(X) = \sum X_i P(X_i)$$
;  $\sigma^2 = Var(X) = \sum (X_i - \mu)^2 P(X_i)$ 

3. If 
$$X \sim Bin(n, p)$$
, then (a)  $P(X = k) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$ 

(b) 
$$\mu = E(X) = np$$
;  $Var(X) = np(1-p)$ 

4. 
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
;

$$P(A \text{ and } B) = P(B \mid A)P(A)$$
; 
$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$

5. If 
$$X \sim N(\mu, \sigma^2)$$
, then  $Z = \frac{X - \mu}{\sigma} \sim N(0, 1^2)$ 

6. If 
$$\overline{X} \sim N(\mu, (\frac{\sigma}{\sqrt{n}})^2)$$
, then  $Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \sim N(0, 1^2)$ 

7. 
$$\overline{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$
;  $\overline{X} \pm t_{\frac{\alpha}{2},n-1} \frac{s}{\sqrt{n}}$ ;  $e = Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$ 

8. 
$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$
;  $t = \frac{\overline{X} - \mu}{\frac{s}{\sqrt{n}}}$ 

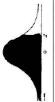
9. If 
$$p \sim N(\pi, \sqrt{\frac{\pi(1-\pi)}{n}}^2)$$
, then  $Z = \frac{p-\pi}{\sqrt{\frac{\pi(1-\pi)}{n}}} \sim N(0,1^2)$ ;

$$p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p(1-p)}{n}}$$
 ;  $e = Z_{\frac{\alpha}{2}} \sqrt{\frac{\pi(1-\pi)}{n}}$ 

10. 
$$s_{xy} = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{n-1}$$
;  $r_{xy} = \frac{S_{XY}}{S_X S_Y}$ ;

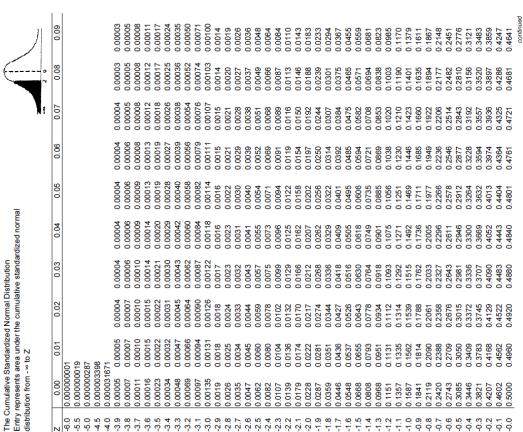
For 
$$Y = \beta_0 + \beta_1 X + \varepsilon$$
,  $b_1 = \frac{\sum (X_i - \overline{X})(Y_i - \overline{Y})}{\sum (X_i - \overline{X})^2}$ ;  $b_0 = \overline{Y} - b_1 \overline{X}$ 

## The Cumulative Standardized Normal Distribution



imulative Standardized Normal Distribution (Coni
Entry represents area under the cumulative standardized normal
distribution from -∞ to Z





Critical Values of t

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	0.7407	1.5332	2.1318	2.7764	3.7469	4.6041
	0.7267	1.4759	2.0150	2.5706	3.3649	4.0322
	0.7176	1.4398	1.9432	2.4469	3.1427	3.7074
	0.7111	1,4149	1.8946		2.9980	3.4995
	0.7064	1.3968	1.8595	2.3060	2.8965	3.3554
	0.7027	1.3830	1.8331		2.8214	3.2498
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7 5	0.6955	1.3562	1.7823	2.1788	2.6810	3.0545
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18	0.6884	1.3304	1.7341	2.1009	2.5524	2.8784
19	0.6876	1.3277	1.7291	2.0930	2.5395	2.8609
20	0.6870	1.3253	1.7247	2.0860	2.5280	2.8453
21	0.6864	1.3232	1.7207	2.0796	2.5177	2.8314
22	0.6858	1.3212	1.7171	2.0739	2.5083	2.8188
27	0.6833	1.3195	1.7139	2.0687	2,4999	2.8073
25	0.0840	1 3163	1 7081	2.0033	2 4851	2 7874
26	0.6840	13150	1 7056	2.0555	24786	2 7787
27	0.6837	1.3137	1.7033	2.0518	2.4727	2.7707
28	0.6834	1.3125	1.7011	2.0484	2.4671	2.7633
29	0.6830	1.3114	1.6991	2.0452	2.4620	2.7564
30	0.6828	1,3104	1.6973	2.0423	2.4573	2.7500
31	0.6825	1.3095	1.6955	2.0395	2.4528	2.7440
32	0.6822	1.3086	1.6939	2.0369	2.4487	2.7385
33	0.6820	1.3077	1.6924	2.0345	2.4448	2.7333
34	0.6818	1.3070	1.6909	2.0322	2.4411	2.7284
35	0.6816	1.3062	1.6896	2.0301	2.4377	2.7238
36	0.6814	1,3055	1.6883	2.0281	2.4345	2.7195
20	0.0012	1.5049	1.00/1	2.0202	2.4314	2.7134
30	0.6810	13036	1,6849	2.0244	2.4200	27070
40	0.6807	1 3031	1 6839		2 4233	2 7045
41	0.6805	1.3025	1.6829	2.0195	2.4208	2.7012
42	0.6804	1.3020	1.6820	2.0181	2.4185	2.6981
43	0.6802	1.3016	1.6811	2.0167	2.4163	2.6951
44	0.6801	1,3011	1.6802	2.0154	2.4141	2.6923
45	0.6800	1.3006	1.6794	2.0141	2.4121	2.6896
46	0.6799	1.3002	1.6787	2.0129	2.4102	2.6870
47	0.6797	1.2998	1.6779	2.0117	2.4083	2.6846
48	0.6796	1.2994	1.6/72	20106		· / MM /.

continued