Test 1 Questions and Answers

1. A box contains seven marbles — one red, two green, and four blue. Consider an experiment that consists of taking one marble from the box, then replacing it in the box and drawing a second marble from the box. What is the probability that the first marble is red and the second marble is green?

1/7X2/7 = 0.04082

2. A box contains seven marbles — one red, two green, and four blue. Consider an experiment that consists of taking one marble from the box, and then drawing a second marble from the box without replacing it in the box. What is the probability that the first marble is red and the second marble is green?

1/7X2/6 = 0.04762

3. A group of 7 boys and 12 girls is lined up in random order— assuming each permutation to be equally likely. What is the probability that the person in the 4th position is a particular boy or a particular girl?

2/(7+12)= 0.1053

4. A group of 8 boys and 12 girls is lined up in random order— assuming each permutation to be equally likely. What is the probability that the person in the 3rd position is a boy and a particular girl is in the 4th position?

8/(8+12)x1/(7+12)= 0.02105

5. Peter is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 7 days out of 1000 days. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 80% of the time. When it doesn't rain, he correctly forecasts no rain 80% of the time. What is the probability that it will not rain on the day of Peter's wedding?

5. *Solution:* The sample space is defined by two mutually-exclusive events - it rains, or it does not rain. Additionally, a third event occurs when the weatherman predicts rain. Notation for these events appears below.

- Event A_1. It rains on Peter's wedding.
- Event A_2. It does not rain on Peter's wedding.
- Event B. The weatherman predicts rain.

In terms of probabilities, we know the following:

- $P(A_1) = 7/1000 =0.007$

- $P(A_2) = 993/1000 = 0.993$
- $P(B \mid A_1) = 0.8$
- $P(B \mid A_2) = 0.2$

We want to know $P(A_1 \mid B)$, the probability it will rain on the day of Peter's wedding, given a forecast for rain by the weatherman. The answer can be determined from Bayes' theorem, as shown below.

$P(A_1 \mid B) = [P(A_1) P(B \mid A_1)]/[P(A_1) P(B \mid A_1) + P(A_2) P(B \mid A_2)]$

$= [0.007 \times 0.8]/[0.007 \times 0.8 + 0.993 \times 0.2]$

$= 0.0274$

Therefore, $P(A_2 \mid B) = 1 - P(A_1 \mid B) = 1 - 0.0274 = 0.9726$

6. John was tested positive for COVID-19. We assume that a person gets COVID-19 with 10% of the chance. When a person actually gets COVID-19, the test incorrectly predicts negative 20% of the time. When a person doesn't gets COVID-19, the test incorrectly predicts positive 20% of the time. What is the probability that John gets COVID-19?

6. *Solution:* The sample space is defined by two mutually-exclusive events – a person gets COVID-19, or a person does not get COVID-19. Additionally, a third event occurs when the test predicts positive for COVID-19. Notation for these events appears below.

- Event A_1. A person gets COVID-19.
- Event A_2. A person does not get COVID-19.
- Event B. The test predicts positive for COVID-19.

In terms of probabilities, we know the following:

- $P(A_1) = 0.1$
- $P(A_2) = 0.9$
- $P(B \mid A_1) = 0.8$
- $P(B \mid A_2) = 0.2$

We want to know $P(A_1 \mid B)$, the probability that John gets COVID-19, given that he was tested positive for COVID-19. The answer can be determined from Bayes' theorem, as shown below.

$P(A_1 \mid B) = [P(A_1) P(B \mid A_1)]/[P(A_1) P(B \mid A_1) + P(A_2) P(B \mid A_2)]$

$= [0.1 \times 0.8]/[0.1 \times 0.8 + 0.9 \times 0.2]$

$= 0.3077$

7. Earthquakes in a certain city occur according to a Poisson process with a rate of 2 per 48 months. Find the probability that one or two earthquakes will occur during a randomly-chosen future 10-year period.

7. Solution:

μ = 2X10/4 = 5

P(x=1; μ=5) + P(x=2; μ=5) = ($e^{-\mu}$) (μ^1) / 1! + ($e^{-\mu}$) (μ^2) / 2! = (5+25/2) e^{-5} = 0.1179

8. Suppose scores on an IQ test are normally distributed, with a population mean of 100 and a population standard deviation of 15. Suppose 20 people are randomly selected and tested. If the standard deviation in the sample group is 15, What is the probability that the average test score in the sample group will be at least 95?

8. Solution:

$$z = [\ \bar{x} - \mu\ ] / [\ \delta / \text{sqrt}(\ n\ )\ ]$$
$$= (\ 95 - 100\ ) / [\ 15/ \text{sqrt}(20)\ ]$$
$$= -1.4907$$

where $\bar{x}$ is the sample mean, μ is the population mean, δ is the standard deviation of the population, and n is the sample size.

Now, use the standard normal calculator. We obtain the cumulative probability: 0.068. Hence, there is a 100%-6.8% = 93.2% chance that the sample average will be at least 95.

9. The population standard deviation of test scores on a certain achievement test is 12. If a random sample of 9 students had a sample mean score of 80 and a sample standard deviation of test scores 15, find a 90 percent confidence interval estimate for the average score of all students.

9. Answer: Use the **Normal Distribution Calculator** to find the critical z-score. 80 ± 1.645(12)/3 = 80 +/- 6.58

10. If a random sample of 9 students on a certain achievement test had a sample mean score of 80 and a sample standard deviation of test scores 13, find a 99 percent confidence interval estimate for the average score of all students.

10. Answer: Use the **t Distribution Calculator** to find the critical t statistic. 80±3.355 (13)/3 = 80 +/- 14.5383