BME2102：

Introduction to Biomechanics

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http://www.cityu.edu.hk/bme/lixidong/

Labs

- Lab 1： Tensile and 3-Point Bending Test: Lab. Sheet No.: BME2818
- Lab 2： 3D Printing of Biomedical Structures: Lab. Sheet No.: BME2820
- Lab 3： Electrophoresis: Lab. Sheet No.: BME2819
- Report (20% to the final grade)
  - Everyone should write a report (Not a team report. Teammates can only share the original data—not the processed ones or any other parts.)
  - For face-to-face sessions: one week after your session (extended for one week for the 1st session)
  - For virtual sessions: one week after your session (extended for one week for the 1st session)—the same as the face-to-face session you have registered
  - Deadlines setup with the submission folders in the Canvas for the reports do not mean your individual deadline but the deadline for the Lab held latest. Very important!!!
  - Late submissions: 1 Day -10%, 2 Days -25%, 3 Days -45%, 5 Days -75 %, 1 Week or longer -100%

Schedules, Teams, Labs, and TAs

Week		Date	Time	Report Due 11:59pm	YU Zejie	LIAO Junchen	CAO Hui	WANG Shuideng
5	T	28-Sep	09:00-11:50	12-Oct	L01A: Lab1	L01C: Lab2	L01B: Lab3	
	W	29-Sep	**14:00-16:50**	13-Oct	L03A: Lab1			L03B: Lab3
	R	30-Sep	09:00-11:50	14-Oct	L04A: Lab1			L04B: Lab3
6	T	5-Oct	09:00-11:50	19-Oct				L02A: Lab1
	R	7-Oct	09:00-11:50	21-Oct	L05A: Lab1	L05C: Lab2	L05B: Lab3	
8	T	19-Oct	09:00-11:50	26-Oct	L01B: Lab1	L01A: Lab2	L01C: Lab3	
	W	20-Oct	**14:00-16:50**	27-Oct		L03A: Lab2		L03B: Lab1
	R	21-Oct	09:00-11:50	28-Oct		L04A: Lab2		L04B: Lab1
9	T	26-Oct	09:00-11:50	2-Nov				L02A: Lab2
	R	28-Oct	09:00-11:50	4-Nov	L05B: Lab1	L05A: Lab2	L05C: Lab3	
11	T	9-Nov	09:00-11:50	16-Nov	L01C: Lab1	L01B: Lab2	L01A: Lab3	
	W	10-Nov	**14:00-16:50**	17-Nov			L03A: Lab3	L03B: Lab2
	R	11-Nov	09:00-11:50	18-Nov			L04A: Lab3	L04B: Lab2
12	T	16-Nov	09:00-11:50	23-Nov				L02A: Lab3
	R	18-Nov	09:00-11:50	25-Nov	L05C: Lab1	L05B: Lab2	L05A: Lab3	

Teams

Name	Lab Session	Team
AU Shun Yan Serene	L01	A
BABIC Marko	L01	A
CHENG Chit Yuen	L01	A
GUO Guihuan	L01	A
HUNG Chi Sing	L01	A
LAM Ka Long	L01	B
PAN Xinyu	L01	B
PANG Yan Yee	L01	B
PRIJADI Shannon Eugenia	L01	B
QU Qingao	L01	B
SIRIPAKDEECHAIKUL Apinant	L01	C
WANG Jiachen	L01	C
WONG Ka Hei	L01	C
ZHAO Zirui	L01	C
KO Yu Fei	L01	C
LI Zongze	L02	A
SHIM Yoonsue	L02	A
SIASAKUN Anchalee	L02	A
TSE Ting Kit	L02	A
YANG Shing Chun	L02	A
ZHANG Boyuan	L02	A

Name	Lab Session	Team
CHAN Hiu Lam	L03	A
CHONG Kin Tung	L03	A
CHU Hon Man Herman	L03	A
LAU Chun Yin	L03	A
LAW Long Hin Marvin	L03	A
LEE Ho Yin	L03	B
SIU Wai Hin	L03	B
SMAT Aidana	L03	B
WU Chiu Yan	L03	B
XING Jiazhen	L03	B
CHONG Hiu Kwan	L04	A
KHAN Abdul Raffay	L04	A
LAM Cheuk Fung	L04	A
LEUNG Shu Wah	L04	A
LI Tsz Hei	L04	B
LO Wai Ping	L04	B
SHIMA IYACU Nys Marlaine	L04	B
WONG Ching Yu	L04	B

Name	Lab Session	Team
CHENG Hoi Man	L05	A
HE Qinshu	L05	A
HO Yan Tung	L05	A
IP Chun Wang	L05	A
KONG Kin Man	L05	A
LAI Sin Yiu	L05	B
LEI Wang Kwo	L05	B
LEUNG Chun Hang	L05	B
LEUNG Hin Wai	L05	B
LUO Tsz Ki	L05	B
NG Chun Wai	L05	C
TSE Chin Wang	L05	C
WAN Ka Yu	L05	C
WONG Yuen Ching	L05	C
YU Cheuk Lam	L05	C

Lab Instructors

- Updates: AIMS and/or Canvas
- Labs (Students in HK must attend on campus)
  - Schedule:
    - L01 : wk 5, 8, 11 T (Y1501, B1667)
    - L02 : wk 6, 9, 12 T (Y1501, B1667)
    - L03 : wk 5, 8, 11 W (Y1501, B1667)
    - L04 : wk 5, 8, 11 R (Y1501, B1667)
    - L05 : wk 6, 9, 12 R (Y1501, B1667)
  - Y1501, Yeung Kin Man Academic Building
- TAs (Lab tutoring and report grading) – For questions about the lab and your report grades, please contact your TA.
  - **YU Zejie**: zejieyu2-c@my.cityu.edu.hk
  - **LIAO Junchen**: junchliao2-c@my.cityu.edu.hk
  - **CAO Hui**: huicao8-c@my.cityu.edu.hk
  - **WANG Shuideng**: sdwang8-c@my.cityu.edu.hk

Schedule

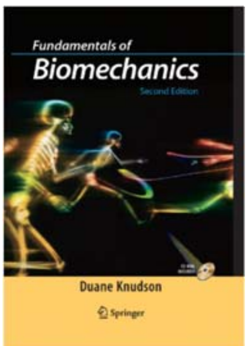
			15331	15332	15333	15334	15335	15336	15337
			C01	**T01**	**L01**	**L02**	**L03**	**L04**	**L05**
			17:00 - 18:50	10:00 - 11:50	09:00 - 11:50	09:00 - 11:50	14:00-16:50	09:00 - 11:50	09:00 - 11:50
			YEUNG	YEUNG	YEUNG	YEUNG	YEUNG	YEUNG	YEUNG
			LT-18	LT-18	Y1501	Y1501	Y1501	Y1501	Y1501
Week	Date	Instructor	DONG Lixin	DONG Lixin					
1	3/9	F	Lect 1						
2	10/9	F	Lect 2						
3	16/9	R							
	17/9	F	Lect 3						
4	23/9	R							
	24/9	F	Lect 4						
5	28/9	T			Lab				
	29/9	W					Lab		
	30/9	R						Lab	
	1/10	F	National Day						
6	5/10	T				Lab			
	7/10	R							Lab
	8/10	F	Lect 5	Tutorial 1					
7	15/10	F	In-class Test						
8	19/10	T			Lab				
	20/10	W					Lab		
	21/10	R						Lab	
	22/10	F	Lect 6						
9	26/10	T				Lab			
	28/10	R							Lab
	29/10	F	Lect 7						
10	4/11	R							
	5/11	F	Lect 8						
11	9/11	T			Lab				
	10/11	W					Lab		
	11/11	R						Lab	
	12/11	F	Lect 9						
12	16/11	T				Lab			
	18/11	R							Lab
	19/11	T	Lect 10	Tutorial 2					
13	26/11	F	Lect 11						

In-class Test

- 17:00-18:50, Oct. 15, F
- On Campus (Zoom will only be available to students not in HK)
- Closed book and notes but formula sheet will be provided (Posted into the Canvas)
- 4-5 questions (Sample exam posted into the Canvas)
- Covers everything we have learned and will learn by the end of class of today (Oct. 8):
  - I. Introduction
  - II. Rigid-body Mechanics: Linear Motion and Newton's Laws
  - III. Angular Motion and Euler's Laws
  - IV. Mechanics of Biomaterials
  - HW1-3 (HW1 and 2 solutions posted. HW3 assignment posted, due on next Tu (Oct. 12). Solutions will be posted by next Wed.)
  - Labs: Not covered

In-class Test

- Review
  - Lecture Notes:
    - Lects 1-3
    - Lects 5-6
  - Textbooks



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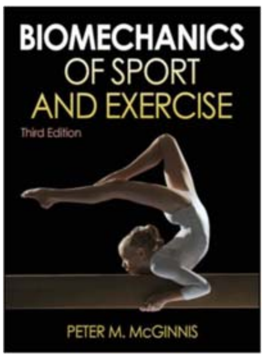

and

Review

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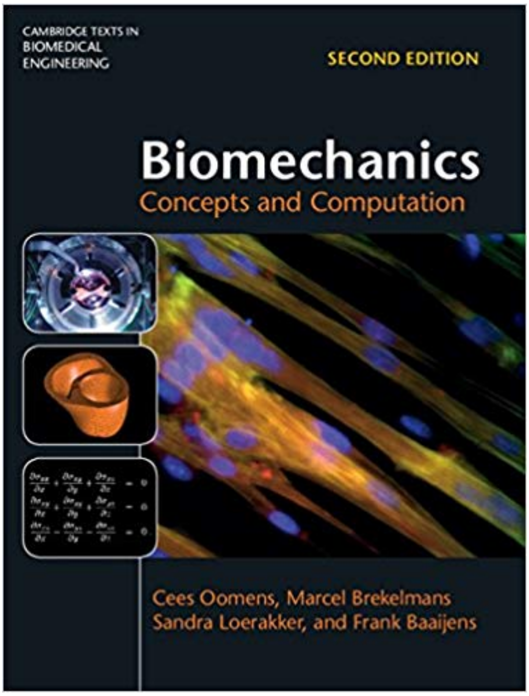

Review

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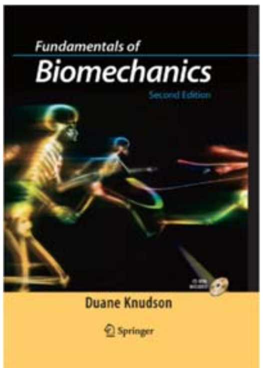


Review

CHAPTER 4
MECHANICS OF THE MUSCULOSKELETAL SYSTEM

PART III
MECHANICAL BASES

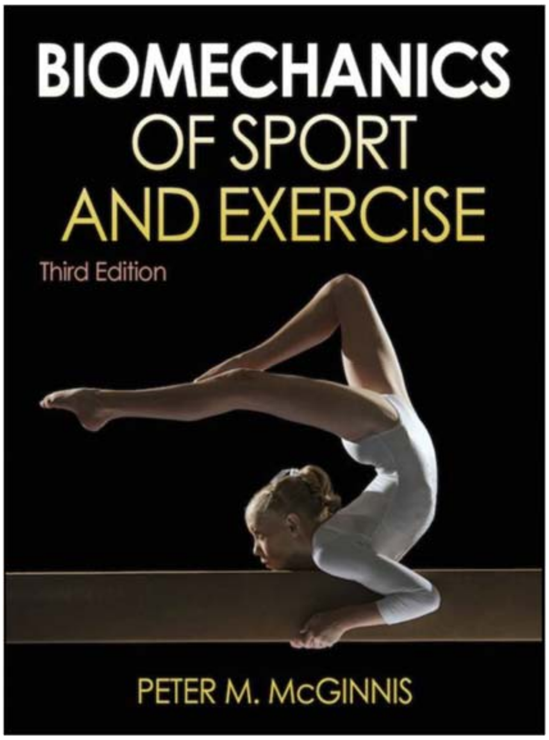
CHAPTER 5
LINEAR AND ANGULAR KINEMATICS

CHAPTER 6
LINEAR KINETICS

Review

CHAPTER 7
ANGULAR KINETICS

Review



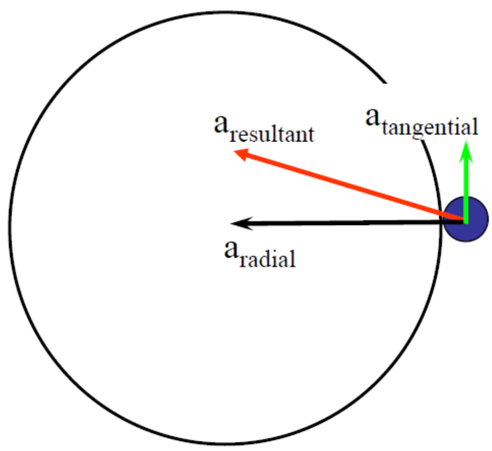
contents

Review

III. Angular Motion and Euler's Laws

Angular Motion

- Angular motion occurs when all points on an object move in a circular path around the same fixed axis
  - Axis can be inside the object (CoG)
  - Or outside the body



Angular Position

- Orientation of a line with another line or a plane of reference
- Greek letter θ
  - Absolute Angular Position
    - If the angle of interest is being compared to a plane of reference (fixed & immovable)
  - Relative Angular Position
    - If the angle of interest is being compared to another line capable of moving

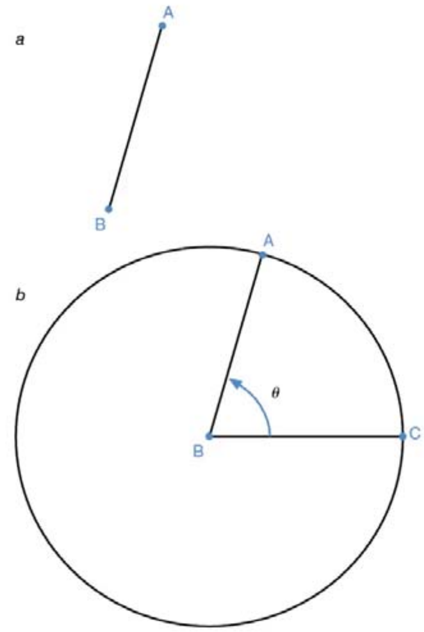


Figure 6.2 A circle is used in describing the angle of a line *(a)* if the center of the circle coincides with the inter-

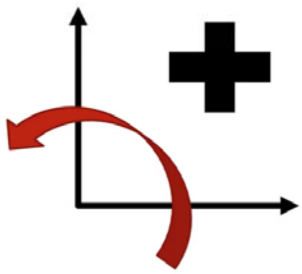
Angular Displacement

- Similar to linear displacement
- Change in absolute angular position between final and initial positions of a rotating line
- Measured in Degrees or Radians
  - 57.3° in 1 Radian

$$\Delta\theta = \theta_f - \theta_i$$

Determining Direction: Positive vs Negative

Mechanically
- Based on observation view
- Typically used for objects & non-anatomical relation movements



Anatomically
- Based on Joint Actions
- Increasing Joint Angle = Positive (+)
- Decreasing Joint Angle = Negative (-)

Joint Actions Reference

Increasing Angles (+):
- Extension
- Plantar Flexion
- Abduction
- Eversion
- Radial Deviation
- External Rotation
- Pronation

Decreasing Angles (-):
- Flexion
- Dorsiflexion
- Adduction
- Inversion
- Ulnar Deviation
- Internal Rotation
- Supination

Angular Displacement Example

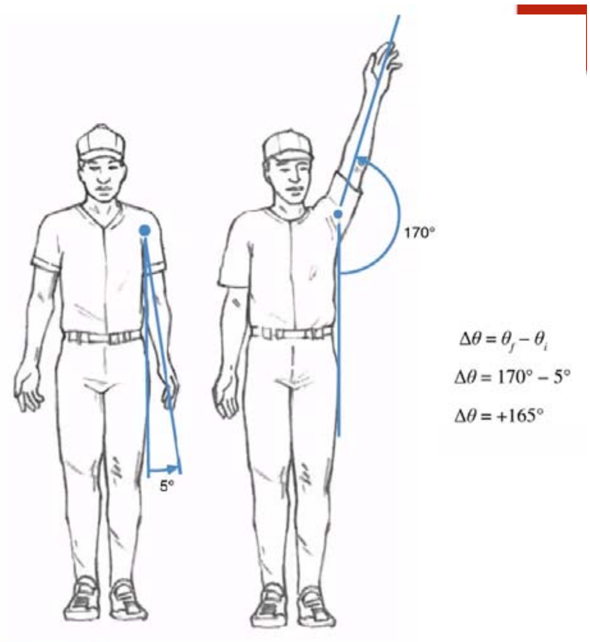
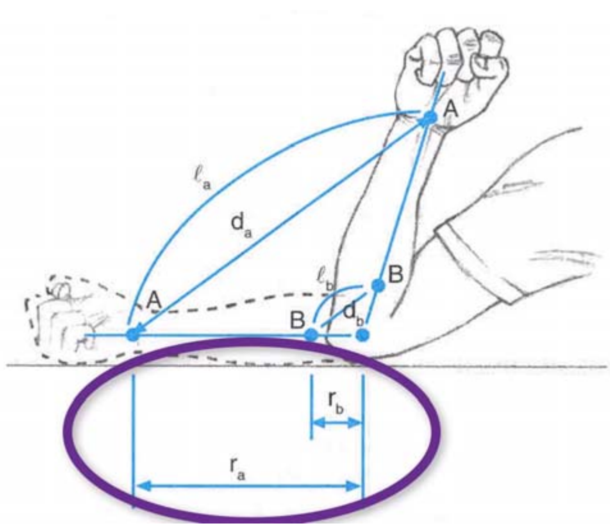


Figure 6.5 Angular displacement of a pitcher's arm at the shoulder joint around the anteroposterior axis.

Linear & Angular Displacement Relationship

- Linear motion of a point is dictated by the distance from the axis of rotation (radius)



Linear & Angular Displacement Relationship

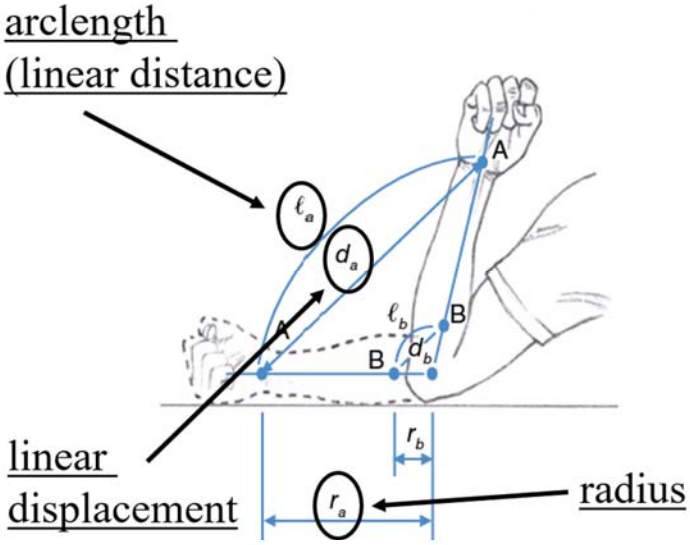
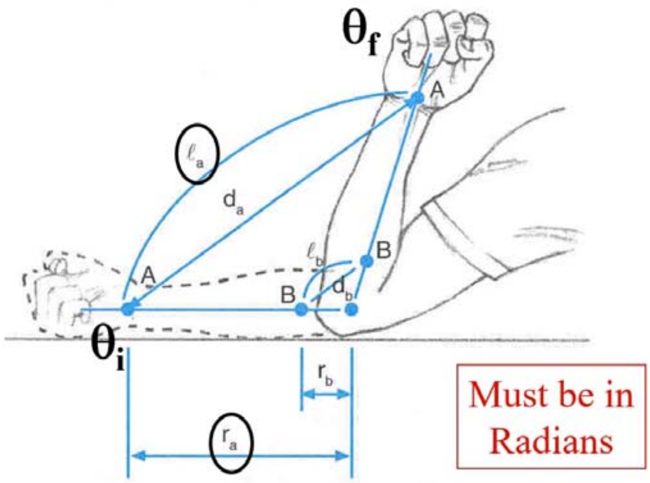


Figure 6.6 The distance that the hand or wrist (A) moves (ℓ_a or d_a) when your elbow (B) flexes is greater than the distance that the insertion point of the biceps moves (ℓ_b or d_b). The ratio of these distances to each other is the same as the ratio of the radius r_a to the radius r_b.

Calculating Arclength (Linear Distance in Angular Motion)



$$\ell_a = r_a\,(\Delta\theta)$$

Angular Velocity

- Defined as the rate of angular displacement
- Think: “How quickly is something changing its angle?”
- Represented with Greek letter omega (ω)
- Measured in Degrees/Second (°/s) or Radians/Second (Rad/s) or Rotations per Minute (rpm)

$$\omega = \Delta\theta/\Delta t \quad = \quad (\theta_f - \theta_i)/\Delta t$$

$1\ rpm = \frac{2\pi}{60s} \qquad rad/s = \frac{360}{60}\ °/s$

Angular Velocity Example

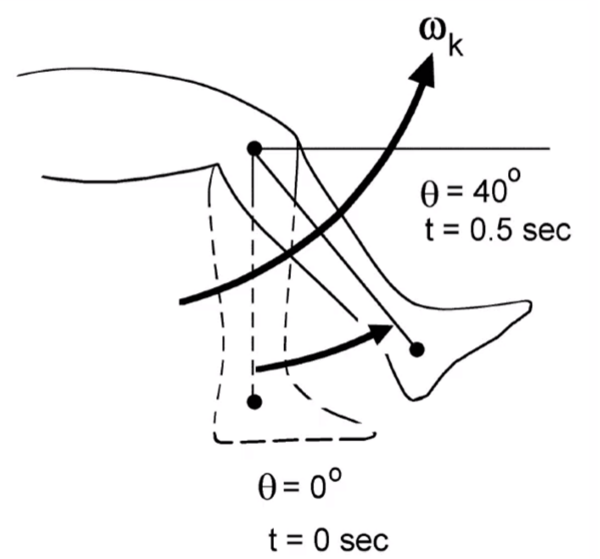


Figure 5.11. The average angular velocity of the first half of a knee extension exercise can be calculated from the change in angular displacement divided by the change in time.

$\omega = \Delta\theta/\Delta t$
$\omega = (40 - 0)/(0.5 - 0)$
$\omega = 40/0.5$
$\omega = 80°/s$

Angular & Linear Velocity

- Individual Points on an object displace difference distances while under angular motion.
- For any two points
  - Angular Velocity will be the same: $\omega_a = \omega_b$
  - Linear Velocity will be different: $V_a \neq V_b$

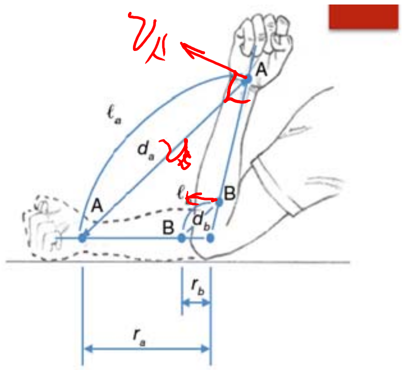


Figure 6.6 The distance that the hand or wrist (A) moves (ℓ_a or d_a) when your elbow (B) flexes is greater than the distance that the insertion point of the biceps moves (ℓ_b or d_b). The ratio of these distances to each other is the same as the ratio of the radius r_a to the radius r_b.

Angular & Linear Velocity

- Linear Velocity of a point in angular motion is called Tangential Velocity: $V_t = \omega r$

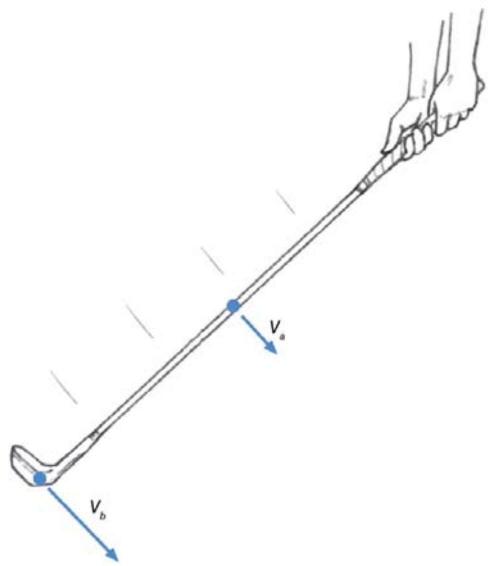
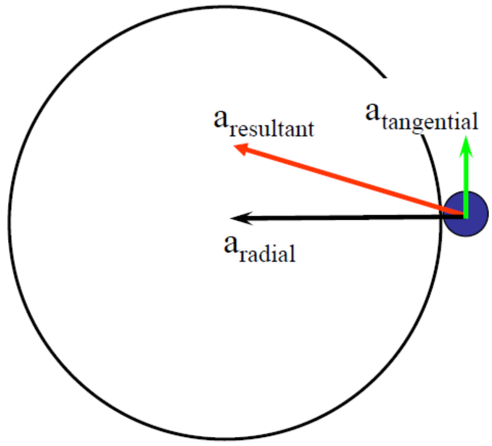


Figure 6.8 The linear velocity of the club head (v_b) is faster than the linear velocity of a point on the shaft (v_a) because the club head is farther from the axis of rotation.

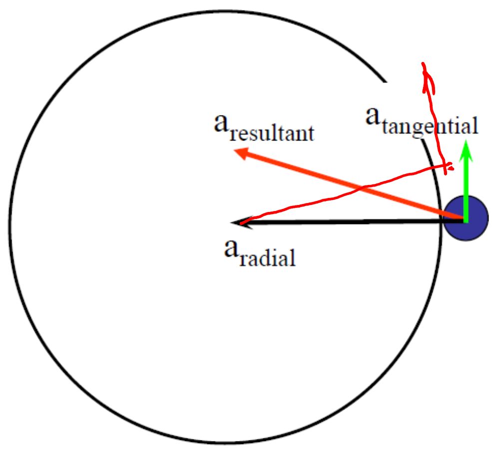
Radial Forces

- Remember: objects must be forced to follow a curved path
- Two forces play a role in radial acceleration (action-reaction pair)
  - Centripetal force
    - “center seeking” force
      - force that causes radial acceleration
      - directed in toward center of rotation (along radius)
  - Centrifugal force
    - “center fleeing” force
      - reaction force to centripetal force
      - directed out away from center of rotation (along radius)



Tangential Acceleration

- Tangential acceleration (a_T) - the linear acceleration that serves to describe the rate of change in magnitude of tangential velocity.
- $a_T = (v_{Tf} – v_{Ti})/t$
- Although a_T may appear to be a new term, it is simply the change in linear or tangential velocity of the point of interest.



Radial Acceleration

- Radial acceleration (a_R) - the linear acceleration that serves to describe the change in direction of an object following a curved path.
  - Radial acceleration is a linear quantity
  - It is always directed inward, toward the center of a curved path.
  - $a_R = v_T^2/r = (\omega r)^2/r = \omega^2 r$
    - for a given r, higher v_T is related to a higher a_R; which means a higher force is needed to produce a_R (i.e., to maintain curved path).
    - for a given r, higher ω is also related to a higher a_R; which means a higher force is needed to produce a_R (i.e., to maintain curved path).
    - for a given v_T, lower r (i.e., a tighter "turning radius") results in a higher a_R (and the need for a greater force to maintain a curved path)

Angular Acceleration

- Defined as: the rate of change in Angular Velocity
- Think: "How quickly is the angular speed changing?"
- Represented with Greek letter alpha (α)
- Measured in Degrees/Second/Second (°/s²) or Radians/Second/Second (Rad/s²)
  - $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{(\omega_f - \omega_i)}{\Delta t}$

Torque/Moment

- Linear Kinematics
  - Force acting THROUGH axis of rotation causes LINEAR motion
- Angular Kinetics
  - Force acting OUTSIDE of axis of rotation causes ANGULAR motion
  - Termed: Torque
  - Torque is the turning effect of a force
  - Torque is the CAUSE of angular motion

Determining Torque (T)/Moment (M)

- Determined by the combination of two factors
  - The amounts of FORCE applied
  - The size of the MOMENT ARM
    - Moment Arm: The perpendicular distance from the line of the force to the axis of rotation

$$T = F \times r$$

where

T = torque (or moment of force),

F = force, and

r = moment arm (or perpendicular distance).

Torque

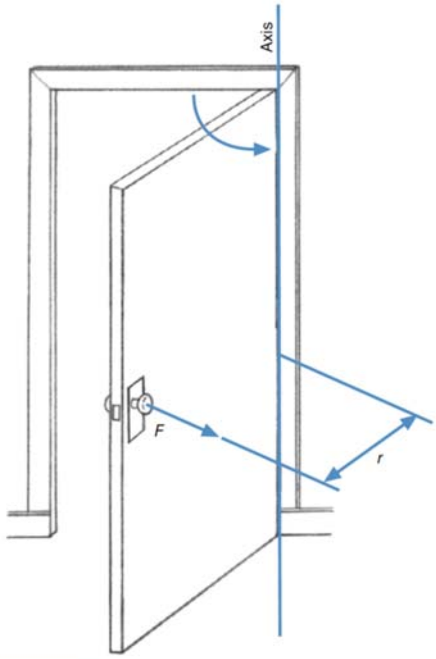


Figure 5.3 The torque created by the pulling force on the doorknob causes the door to swing open.

Torque

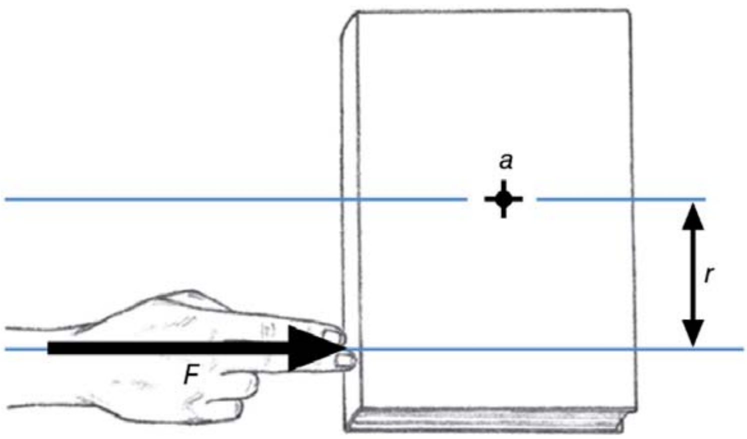


Figure 5.2 The moment arm *(r)* of a force *(F)* is the perpendicular distance between the line of action of the force and a parallel line passing through the axis of rotation *(a)*.

Torque

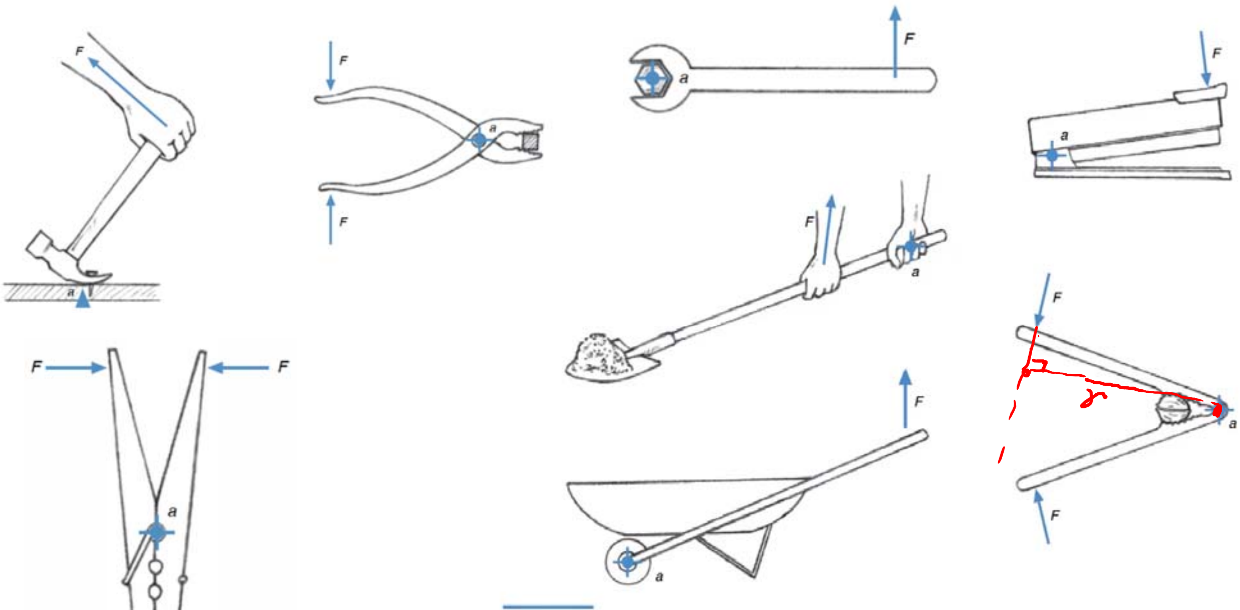


Figure 5.4 Common tools that you exert forces *(F)* on, thus creating torques around the axes *(a)*.

Torque

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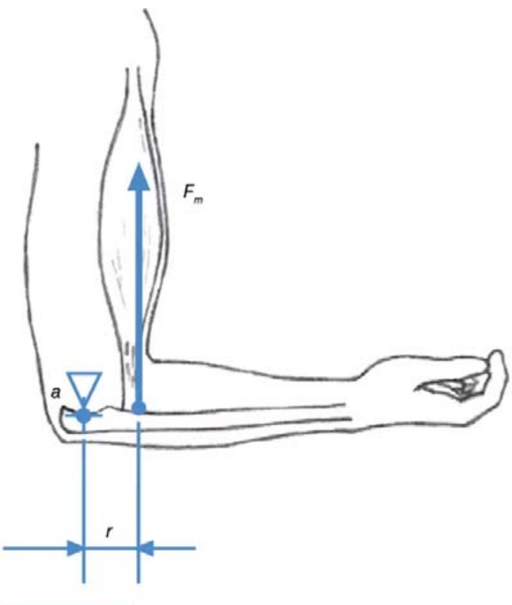


Figure 5.6 The biceps brachii exerts a torque around the axis of the elbow joint by producing a force *(F_m)* with a moment arm *(r)* around the joint.

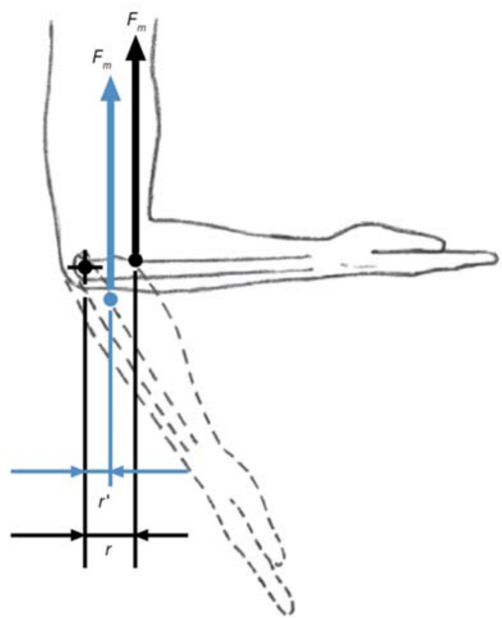
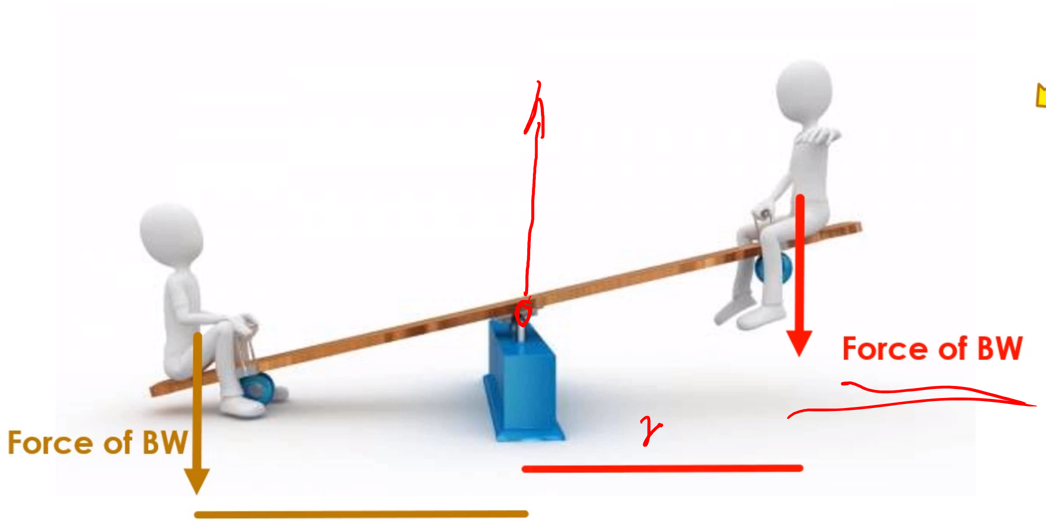


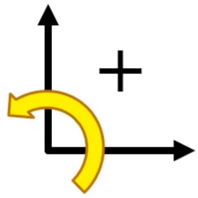
Figure 5.7 The moment arm of the biceps brachii muscle decreases from *r* to *r'* as the elbow extends from 90°.

Forces and Torques in Equilibrium

$\sum F = 0$ $\sum F$ = net external force and

$\sum T = 0$ $\sum T$ = net torque.





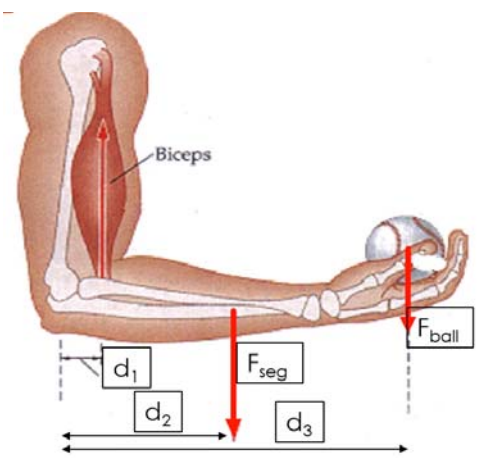
Seesaw

Net Torque

- Torque acting around a single axis of rotation can be summed to determine the effects of the torques
- Similar concept to Net Force

$\sum T = T1 + T2 + T3$

$\sum T = (F_{bicep} * d_1) + (F_{seg} * d_2) + (F_{ball} * d_3)$



Net Torque

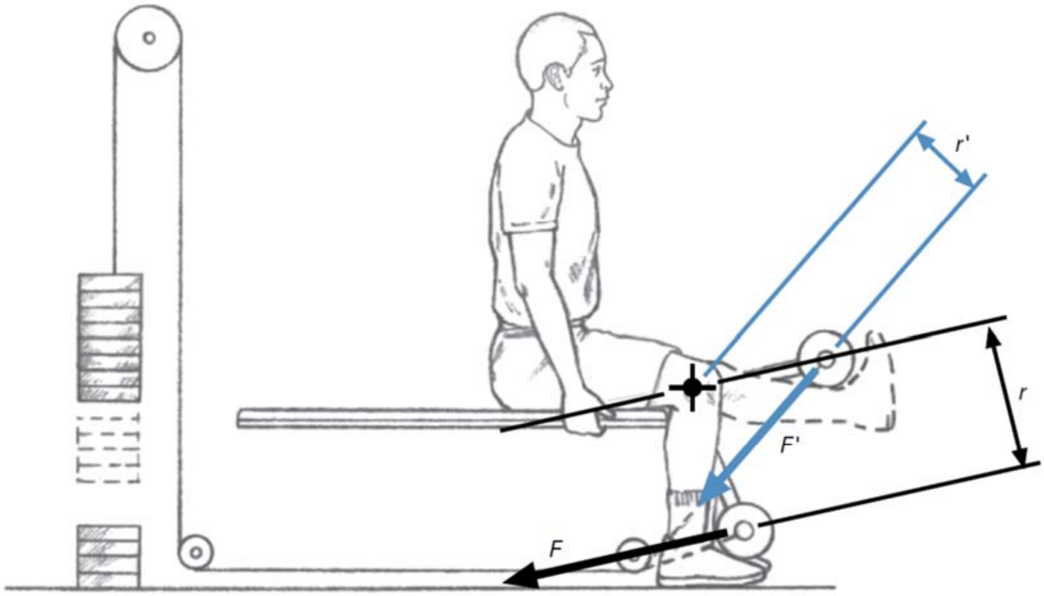
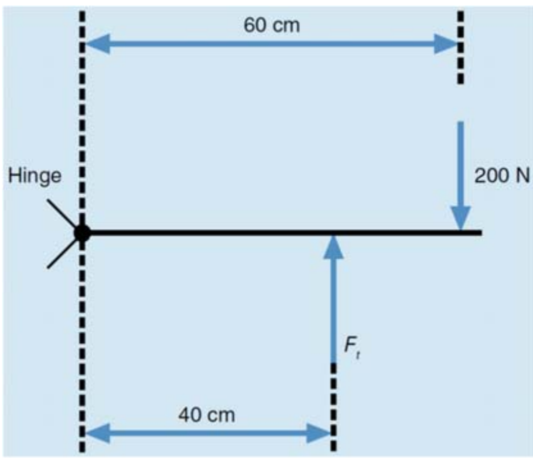


Figure 5.8 A leg extension machine. The torque varies with position due to the change in the size of its moment arm (r).

Example

- Jeff is pushing on a door with a horizontal force of 200 N. The moment arm of this force around the hinges of the door is 60 cm. Ted is pushing in the opposite direction on the other side of the door. The moment arm of his pushing force is 40 cm. How large is the force that Ted pushes with if the door is in static equilibrium?



Mathematical Determination of the Center-of-Gravity (CoG) Location

- The center of gravity is the point at which the entire mass or weight of the body may be considered to be concentrated.

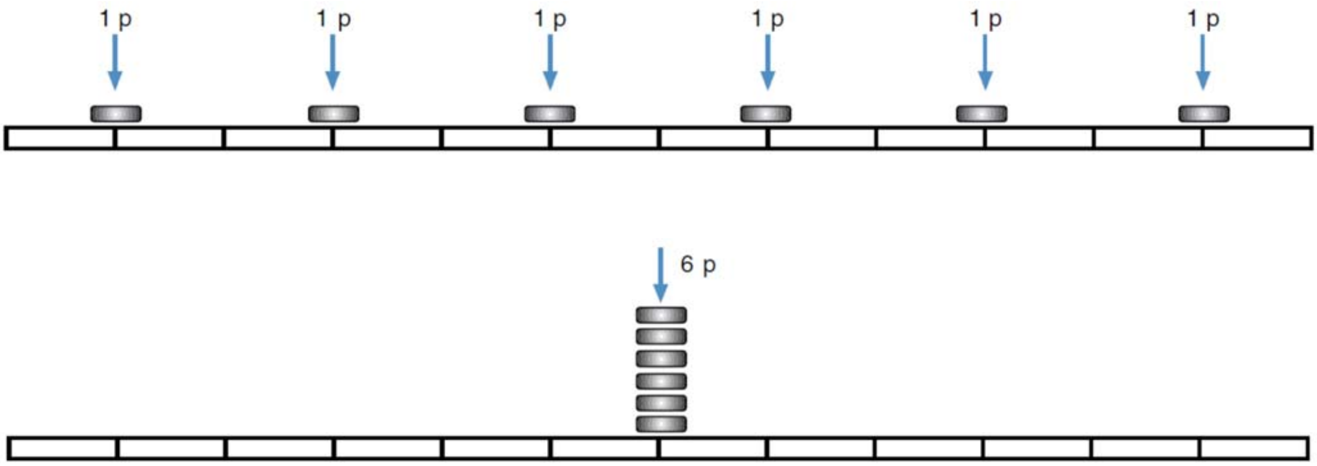


Figure 5.12 A ruler with six pennies on it, one placed every 2 in., feels the same and is equivalent to a ruler with six pennies stacked at the center of the ruler.

Mathematical Determination of the Center-of-Gravity (CoG) Location

$$\sum T = \sum (W \times r) = \left(\sum W\right) \times r_{cg}$$

$$\sum (W \times r) = (1\ \text{p})(1\ \text{in.}) + (1\ \text{p})(3\ \text{in.}) + (1\ \text{p})(5\ \text{in.}) + (1\ \text{p})(7\ \text{in.}) + (1\ \text{p})(9\ \text{in.}) + (1\ \text{p})(11\ \text{in.}) = 36\ \text{p·in.}$$

$$= \left(\sum \text{W}\right) \times r_{cg} \qquad \frac{36\ \text{p·in.}}{6\ \text{p}} = r_{cg} \qquad r_{cg} = 6\ \text{in.}$$

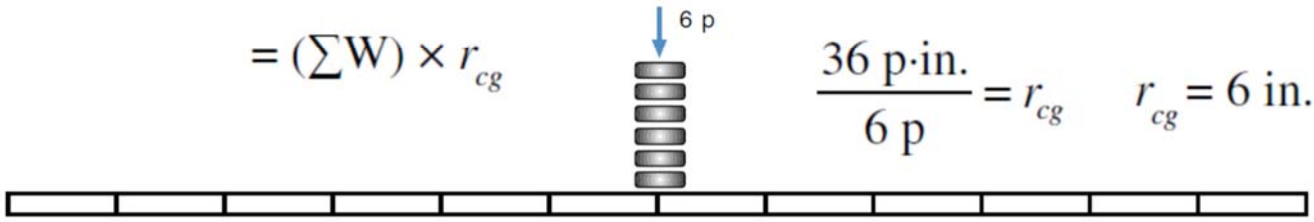
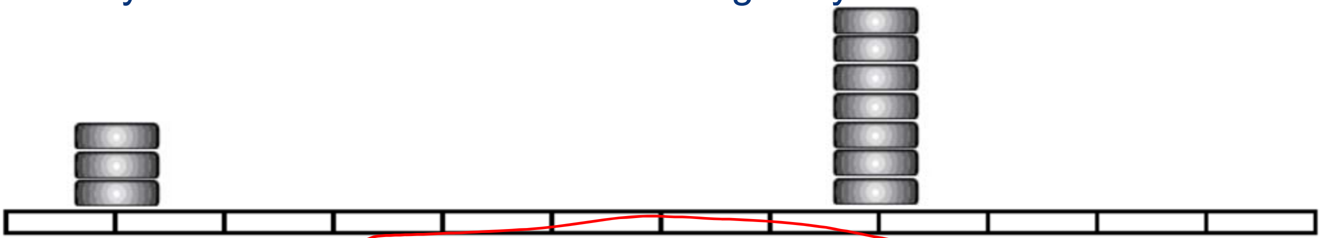


Figure 5.12 A ruler with six pennies on it, one placed every 2 in., feels the same and is equivalent to a ruler with six pennies stacked at the center of the ruler.

Example

- Place three pennies on the ruler at the 1 in. (2.5 cm) mark and seven pennies on the ruler at the 8 in. (20 cm) mark. Can you determine where its center of gravity is?



$$\sum(W_i \times r_i) = (\sum W) \times r_{cg}$$

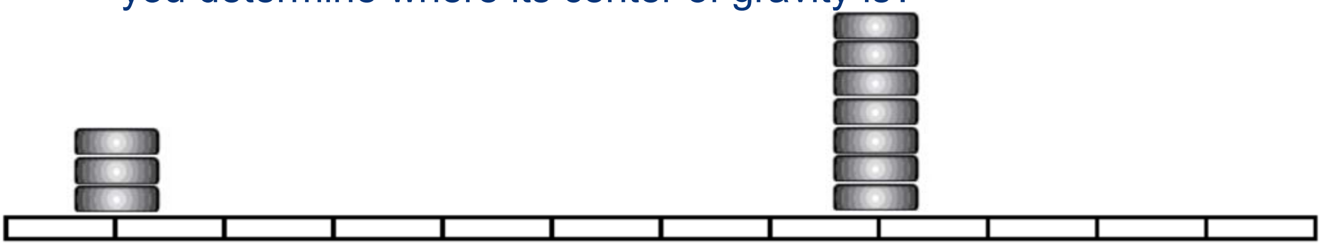
$$\sum(W \times r) = (3\text{ p})(1\text{ in.}) + (7\text{ p})(8\text{ in.}) = (3\text{ p} + 7\text{ p}) \times r_{cg} = (\sum W) \times r_{cg}$$

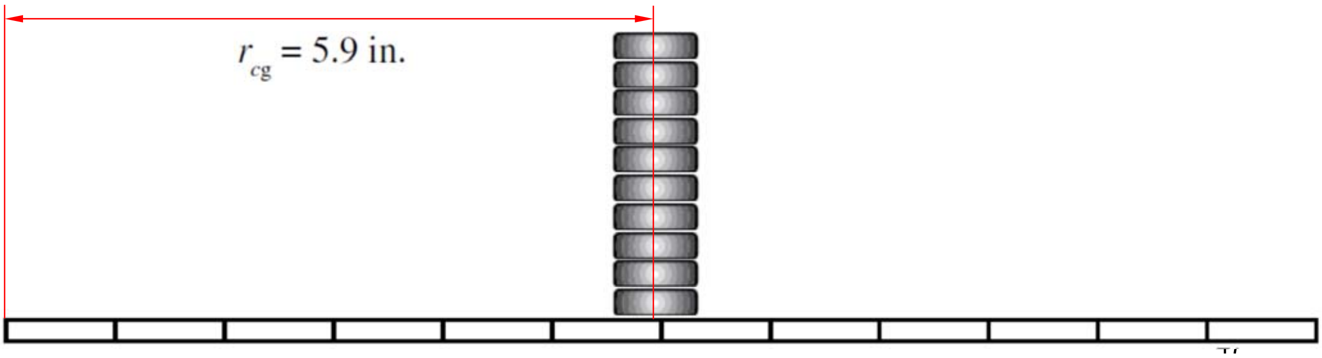
$$3\text{ p}\cdot\text{in.} + 56\text{ p}\cdot\text{in.} = (10\text{ p}) \times r_{cg}$$

$$\frac{59\text{ p}\cdot\text{in.}}{10\text{ p}} = r_{cg} \qquad r_{cg} = 5.9\text{ in.}$$

Example

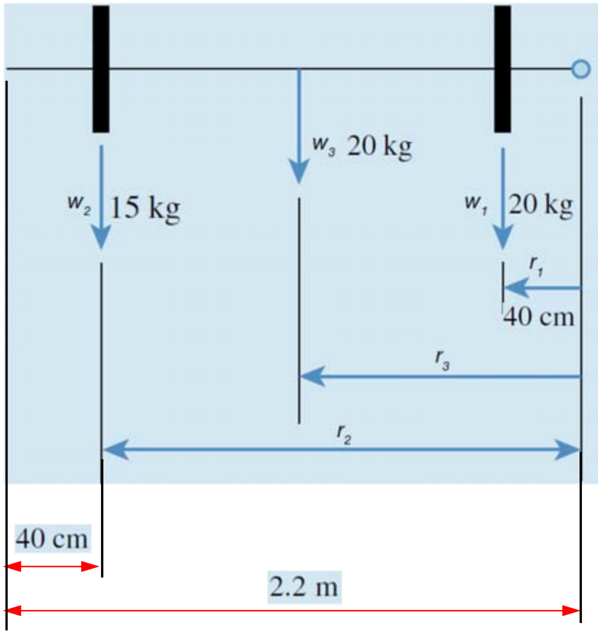
- Place three pennies on the ruler at the 1 in. (2.5 cm) mark and seven pennies on the ruler at the 8 in. (20 cm) mark. Can you determine where its center of gravity is?



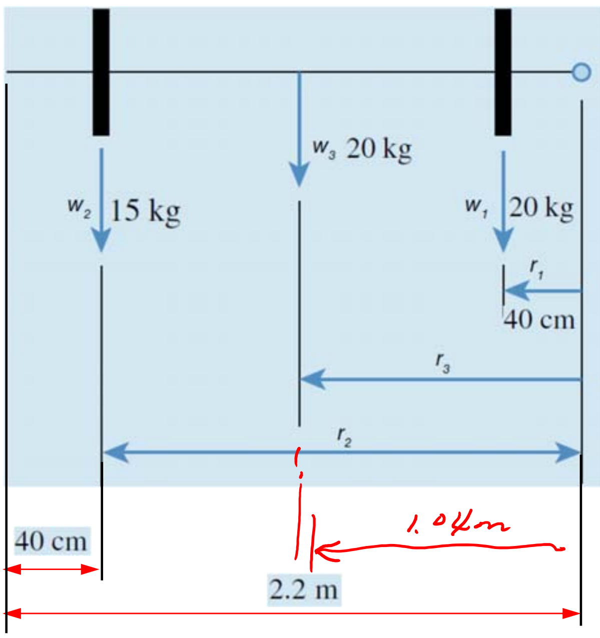


Example

- A weightlifter has mistakenly placed a 20 kg plate on one end of the barbell and a 15 kg plate on the other end of the barbell. The barbell is 2.2 m long and has a mass of 20 kg without the plates on it. The 20 kg plate is located 40 cm from the right end of the barbell, and the 15 kg plate is located 40 cm from the left end of the barbell. Where is the center of gravity of the barbell with the weight plates on it?



Example



$$\sum T = W_1 r_1 + W_2 r_2 + W_3 r_3$$

$$= (m_1 g) r_1 + (m_2 g) r_2 + (m_2 g) r_3$$

$$= g\,(m_1 r_1 + m_2 r_2 + m_2 r_3)$$

$$= g\,[(20\text{ kg})(0.4\text{ m})$$

$$+ (15\text{ kg})(1.8\text{ m}) + (20\text{ kg})(1.1\text{ m})]$$

$$= g\,(57\text{ kg·m})$$

$$= W_{total}\, r_{cg} = (m_{total} g) r_{cg}$$

$$g\,(55\text{ kg})\, r_{cg} = g\,(57\text{ kg·m})$$

$$r_{cg} = (57\text{ kg·m})/55\text{ kg}$$

$$r_{cg} = 1.04\text{ m}$$

The center of gravity is 1.04 m from the right end of the barbell.

Moment

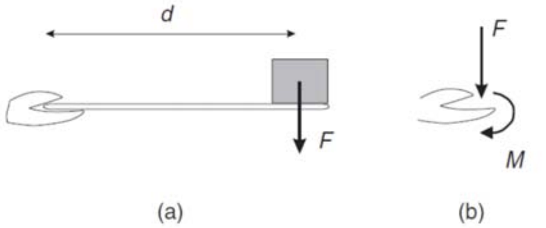
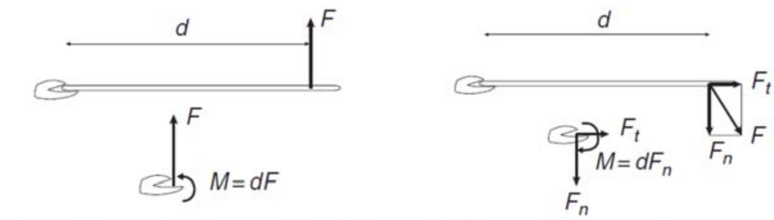


Figure 2.14

(a) Weight of an object on a tray (b) Loading on the hand.

$$M = dF$$



(a) Moment due to reversed force F (b) Moment due to oriented force F

Figure 2.15

Moment due to various forces F.

$$M = dF_n$$

Moment Vector

- A point in space may be identified by its position vector $\vec{x}$, see for instance the three-dimensional example in Fig. 2.16, where O denotes the location of the origin of the Cartesian vector basis $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$.
- Assume that a force $\vec{F}$ is applied to a point Q with location $\vec{x}_Q$. The moment vector is defined with respect to a point in space, say P having location $\vec{x}_P$. The moment exerted by the force $\vec{F}$ with respect to point P is defined as

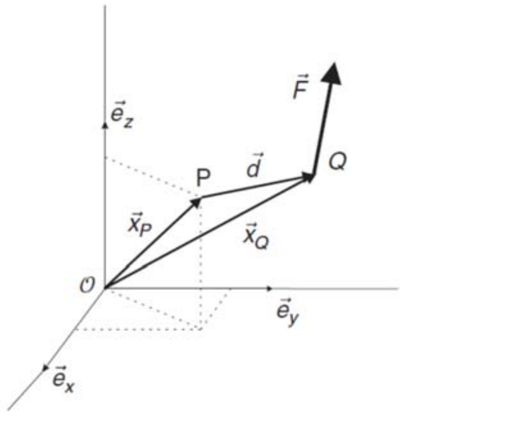


Figure 2.16

A point in space identified by its position vector $\vec{x}$.

$$\vec{M} = (\vec{x}_Q - \vec{x}_P) \times \vec{F} = \vec{d} \times \vec{F}$$

Example

Let the origin of the Cartesian coordinate system be the point with respect to which the moment vector is computed, i.e.

$$\vec{x}_{\mathrm{P}} = \vec{0}.$$

The point of application of the force vector $\vec{F}$, is denoted by:

$$\vec{x}_{\mathrm{Q}} = 2\vec{e}_x + \vec{e}_y,$$

which means that this point is located in the xy-plane. The force vector is also located in this plane:

$$\vec{F} = 5\vec{e}_y.$$

The moment of the force $\vec{F}$ with respect to the point P follows from

$$\vec{M} = (\vec{x}_{\mathrm{Q}} - \vec{x}_{\mathrm{P}}) \times \vec{F} = (2\vec{e}_x + \vec{e}_y) \times 5\vec{e}_y = 10\underbrace{\vec{e}_x \times \vec{e}_y}_{\vec{e}_z} + 5\underbrace{\vec{e}_y \times \vec{e}_y}_{\vec{0}} = 10\vec{e}_z.$$

Moment Vector (2D Case)

$$\vec{M} = (\vec{x}_Q - \vec{x}_P) \times \vec{F} = \vec{d} \times \vec{F}$$

$$\underset{\sim}{d} = \begin{bmatrix} d_x \\ d_y \\ d_z \end{bmatrix}, \quad \underset{\sim}{F} = \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

$$\underset{\sim}{M} = \begin{bmatrix} d_y F_z - d_z F_y \\ d_z F_x - d_x F_z \\ d_x F_y - d_y F_x \end{bmatrix}$$

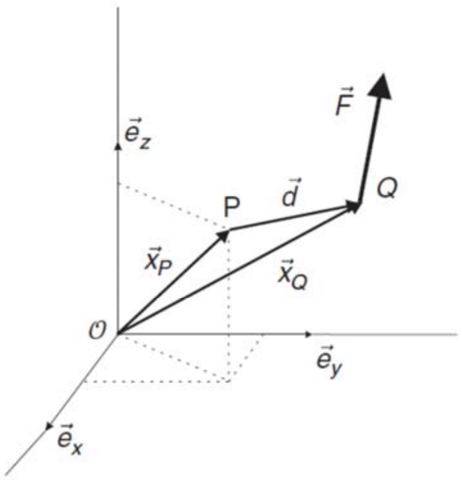


Figure 2.16

A point in space identified by its position vector $\vec{x}$.

Moment Vector (2D Case)

$$\vec{M} = (\vec{x}_Q - \vec{x}_P) \times \vec{F} = \vec{d} \times \vec{F}$$

$$\underset{\sim}{d} = \begin{bmatrix} d_x \\ d_y \\ 0 \end{bmatrix}, \quad \underset{\sim}{F} = \begin{bmatrix} F_x \\ F_y \\ 0 \end{bmatrix}$$

$$\vec{M} = (d_x F_y - d_y F_x)\vec{e}_z$$

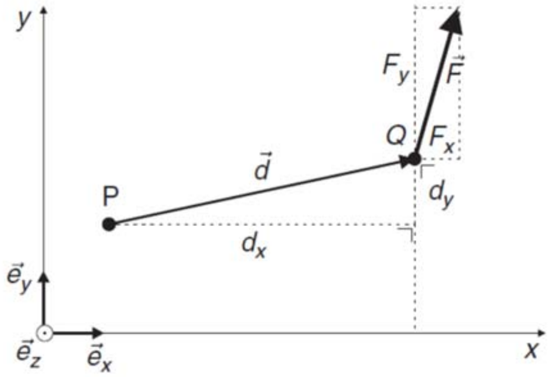


Figure 2.17

The moment of a force acting at point Q with respect to point P.

Static Equilibrium Conditions

- Suppose that *n* forces $\vec{F}_i$ (*i* = 1, 2, ..., *n*) are applied to the body. Each of these forces will have a moment M_i with respect to an arbitrary point *P*. There may be a number of additional moments $\vec{M}_j$ (*j* = 1, 2, ..., *m*) applied to the body. Static equilibrium then requires that

$$\sum_{i=1}^{n} \vec{F}_i = \vec{0}$$

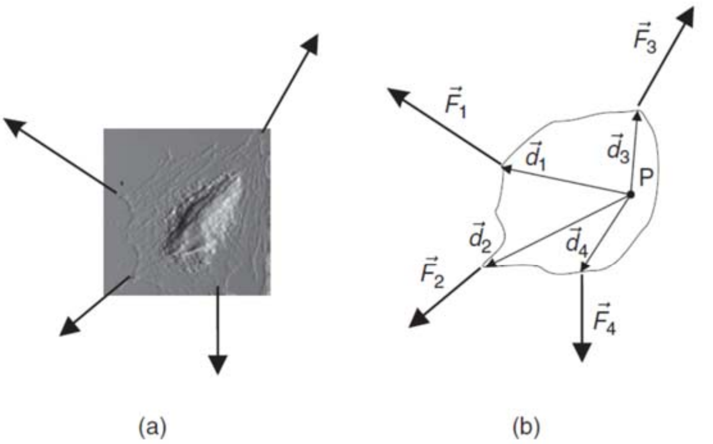
$$\sum_{i=1}^{n} \vec{M}_i + \sum_{j=1}^{m} \vec{M}_j = \vec{0}.$$

(a) Static equilibrium (b) **No** static equilibrium (c) Static equilibrium

Figure 3.1

Examples of satisfaction and violation of static equilibrium.

Example



$$\sum_{i=1}^{4} \vec{F}_i = \vec{0}$$

$$\sum_{i=1}^{4} (\vec{d}_i \times \vec{F}_i) = \vec{0}.$$

Figure 3.2

An image and a model of a cell.

Example

Consider the beam construction, sketched in Fig. 3.8(a), loaded by a force P. The beam is clamped at point A and we want to determine the reaction loads at point A.

First of all a coordinate system is introduced and a free body diagram of the loaded beam construction is drawn, as in Fig. 3.8(b). The applied load is represented by the vector:

$$\vec{P} = -P\vec{e}_z.$$

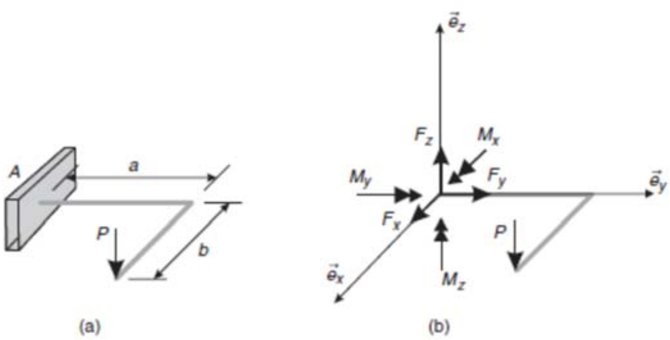


Figure 3.8

A beam construction loaded by a force P and the free body diagram.

Example

The reaction force vector on the beam construction at point A is denoted by $\vec{F}$ and is decomposed according to:

$$\vec{F} = F_x\vec{e}_x + F_y\vec{e}_y + F_z\vec{e}_z,$$

while the reaction moment vector at point A is written as

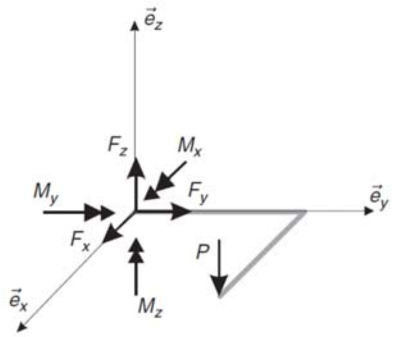
$$\vec{M} = M_x\vec{e}_x + M_y\vec{e}_y + M_z\vec{e}_z.$$

The requirement that the sum of all forces is equal to zero implies that

$$\vec{F} + \vec{P} = \vec{0},$$

and consequently

$$F_x = 0, \qquad F_y = 0, \qquad F_z - P = 0.$$



Example

The requirement that the sum of all moments with respect to A equals zero leads to:

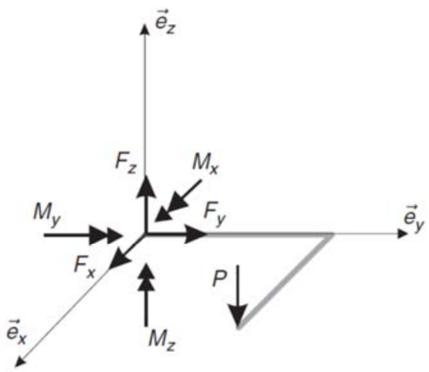
$$\vec{M} + \vec{d} \times \vec{P} = \vec{0},$$

where the distance vector $\vec{d}$ is given by

$$\vec{d} = b\vec{e}_x + a\vec{e}_y,$$

hence

$$\vec{d} \times \vec{P} = (b\vec{e}_x + a\vec{e}_y) \times (-P\vec{e}_z)$$
$$= bP\vec{e}_y - aP\vec{e}_x.$$



Consequently

$$M_x - aP = 0, \qquad M_y + bP = 0, \qquad M_z = 0.$$

Newton's 1st Law

- Law of Inertia
- Linearly
  - Inertia is an object's resistance to change in its state of motion
- Angularly
  - The Moment of Inertia is an object's resistance to a change in its angular state of motion.
  - The moment of inertia of such an object about an axis through its center of gravity can be defined mathematically as follows:

$$I_a = \Sigma m_i r_i^2$$

where

I_a = moment of inertia about axis a through the center of gravity,

Σ = summation symbol,

m_i = mass of particle i, and

r_i = radius (distance) from particle i to axis of rotation through the center of gravity.

Moment of Inertia

$$I_a = mk_a^2$$

where

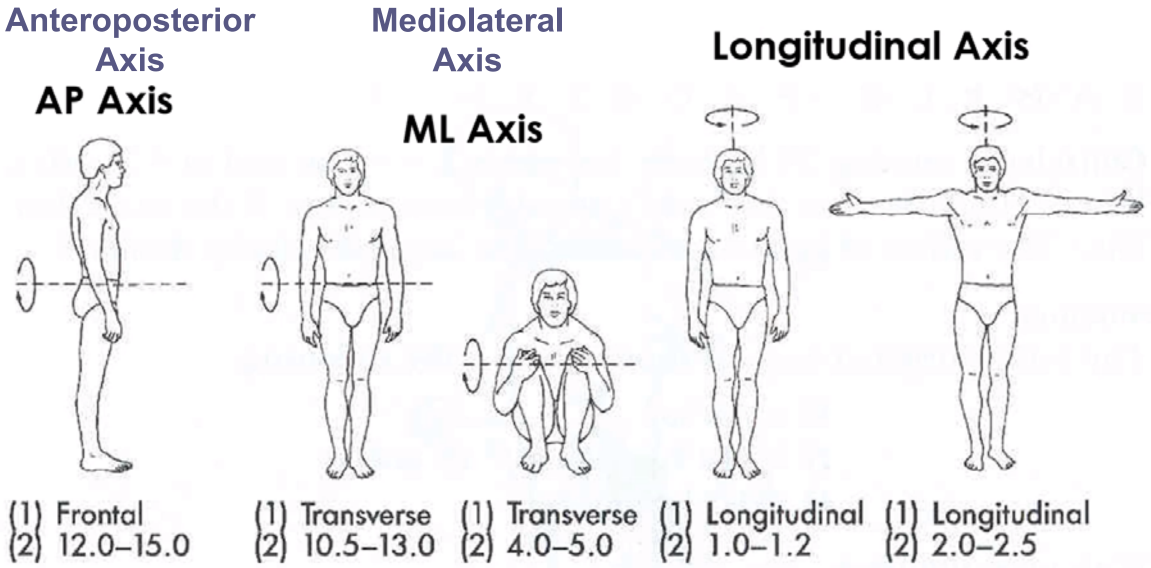
I_a = moment of inertia about axis a through the center of gravity,

m = mass of the object, and

k_a = radius of gyration about axis a through the center of gravity.

- The radius of gyration is a length measurement that represents how far from the axis of rotation all of the object's mass must be concentrated to create the same resistance to change in angular motion as the object had in its original shape.

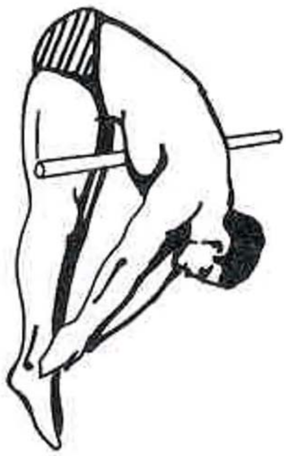
Moment of Inertia – Whole Body



Sports Examples

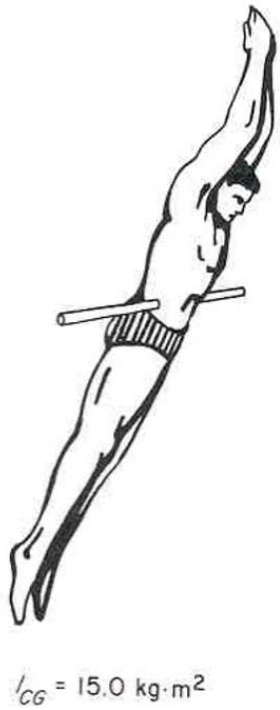
- Key point: CoM is the axis of rotation

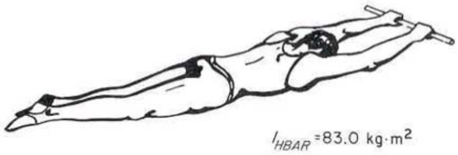
$I_{CG} = 3.5\ kg \cdot m^2$



$I_{CG} = 6.5\ kg \cdot m^2$

More Examples





Axis of rotation is outside of body

Angular Momentum (H)

- Newton's 1st Law
  - Angular motion will maintain the state of motion unless a net external torque is exerted on it—constant angular motion
  - The quantity of angular motion is "Angular Momentum" (H)

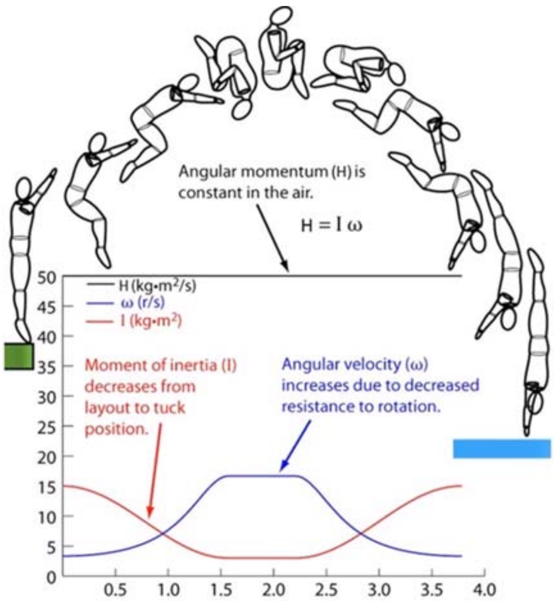
$$H = \underbrace{I}_{\text{Moment of inertia}} \ \underbrace{\omega}_{\text{Angular velocity}}$$

Unit: kg m²/s

Conservation of Angular Momentum

- Unless acted upon by an external torque, angular momentum conserved.

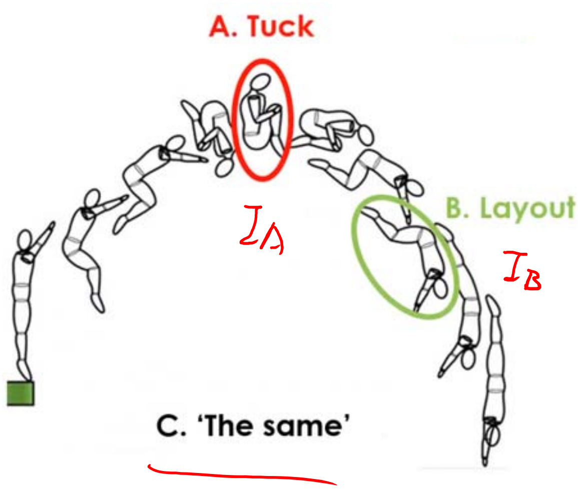
$$H = I\,\omega$$



$I_1\omega_1 = I_2\omega_2$

Conservation of Angular Momentum

- Angular Momentum: $H = I\omega$
- Moment of Inertia: $I = mk^2$
- Example:
  - Which position has greater H_{trans}? C
  - Which position has greater I_{trans}? B
  - Which position has greater ω_{trans}? A



Conservation of Angular Momentum

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$H = I\omega$

$I = mk^2$

Example/Assignment:

Gymnast (m = 50 kg)

k = 0.481 m

ω= 1 rad/s

H_{trans} = ?

H_{trans} = ? $11.52\ kg\,m^2/s$

k = 0.232 m

ω = ?

$\omega = \dfrac{H}{mk^2}$

$= \dfrac{11.52\ kg\,m^2/s}{50kg \cdot 0.232m}$

$= 4.28\ rad/s$

$H = I\omega = mk^2\omega$

$= 50kg \cdot (0.481m)^2 \cdot 1 rad/s$

$= 11.52\ kg \cdot m^2/s$

H_1

Angular – Impulse Momentum

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- If external torques are present, then Angular Momentum is not conserved
  - But the change is predictable!

Torque applied:
Δ Angular momentum

Angular – Impulse Momentum

$$\underbrace{T\,\Delta t}_{\text{Angular Impulse}} = \overbrace{H_f - H_i}^{\text{Angular Momentum}}$$

$$(I\omega_f - I\omega_i)$$

$$I(\omega_f - \omega_i)$$

$$T = I(\omega_f - \omega_i)/\Delta t$$

$$T = I\alpha$$

Newton's 2nd law

Newton's 2nd Law (Angular Version 1)

- A net external torque produces an angular acceleration directly proportional to the net torque

$$\sum T = I\alpha$$

Newton's 2nd Law (Angular Version 2)

- A net external torque exerted on a body is directly proportional to the rate of change in angular momentum (angular motion)

$$\sum T = (I\omega_f - I\omega_i)/\Delta t$$

Comparison of Linear and Angular Kinetic Quantities

Quantity	Symbol and equation for definition	SI units
Linear		
Inertia (mass)	m	kg
Force	F	N
Linear momentum	$L = mv$	kg·m/s
Impulse	$\Sigma\bar{F}\Delta t$	N·s
Angular		
Moment of inertia	$I = \Sigma mr^2 = mk^2$	kg·m²
Torque of moment of force	$T = F \times r$	Nm
Angular momentum	$H = I\omega$	kg·m²/s
Angular impulse	$\Sigma\bar{T}\Delta t$	Nm·s

Summary

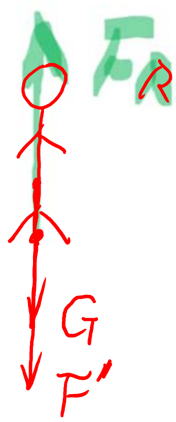
- Angular Momentum is the combination of the moment of inertia and the angular velocity
- Angular Momentum is conserved unless acted upon by a net external torque (Newton's 1st Law)
- If a net external torque acts on an object a proportional angular acceleration is produced (Newton's 2nd Law)
- A net external torque acting on an object will result in a directly proportional rate of change in angular momentum

Tutorial 3

- A 75 kg jumper lands stiff-legged on the floor and changes his velocity from –4.5 m/s to zero in 0.15 seconds. Compute the average ground reaction force under his feet during this time interval. If he increased the impact time to 0.2 s, what happens to the ground reaction force? (Caution: *net force* and *ground reaction force* is not the same thing! Be careful here!).

$F_R = G + F$

$F = m \frac{\Delta v}{t}$



$$\boxed{F \Delta t = m \Delta v}$$

$$F = m \frac{\Delta v}{\Delta t} = ma$$

Tutorial 3

- A football player pushed a 60 kg blocking sled with a constant horizontal force of 400 N. The coefficient of kinetic friction between the sled and ground is 0.5. How much horizontal force opposes the forward motion of the sled? What is the sled's horizontal acceleration? (assume that the playing surface is level).

Tutorial 3

- Two ice skaters start out motionless in the center of the ice rink. They then push each other apart. The man (mass = 80 kg) moves to the right with a speed of 2.5 m/s. The woman moves to the left at some unknown speed.
  - a) If her mass is 58 kg, calculate that speed. (assume frictionless ice)
  - b) What has happened to the total momentum of the system (woman + man) during the push-off? Why?

Tutorial 3

- A child tries to swing an adult size baseball bat without choking up on it. She can manage only 200 deg/s of angular velocity with a radius of 80 cm. But then she chokes up on the bat thereby reducing the radius of rotation by 10 cm. She can now generate an angular velocity of 300 deg/s. Compute the linear velocity of the endpoint of the bat in each case in m/s (watch your units!).

Tutorial 3

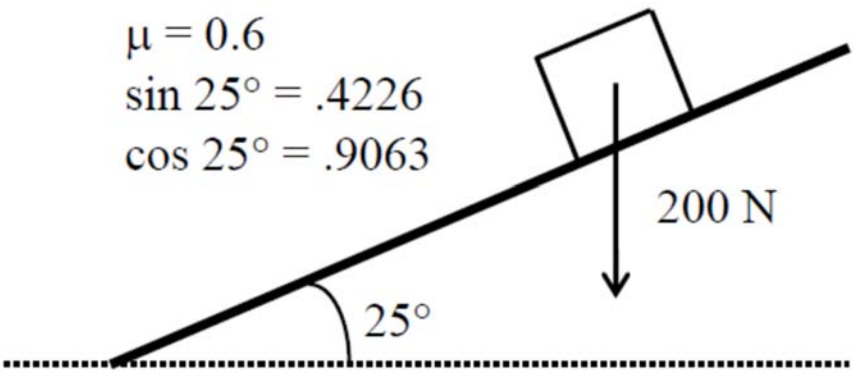
- A golfer accelerates the club from the top of the backswing until impact with the ball with an average angular acceleration of 30 rad/s^2 for a period of 0.5 s. The radius of rotation is 1.1 m. Compute the angular velocity of the club at impact, the linear velocity of the clubhead at impact, and the radial acceleration of the clubhead at impact.

Tutorial 3

- A champion hammer thrower rotates at a rate of 3.2 rev/s just prior to releasing the hammer.
  - a) If the hammer is located 1.6 m away from the axis of rotation, what is the radial acceleration experienced by the athlete?
  - b) How much tension (i.e. force) is needed to produce this radial acceleration if the mass of the hammer is 7.3 kg?

Tutorial 3

- The 200 N box in the figure to the right will
  - a) require an additional force of at least 24.3 N to initiate sliding
  - b) have a friction force of 84.5 N opposing sliding
  - c) have a limiting value of friction equal to 120 N
  - d) both a and b

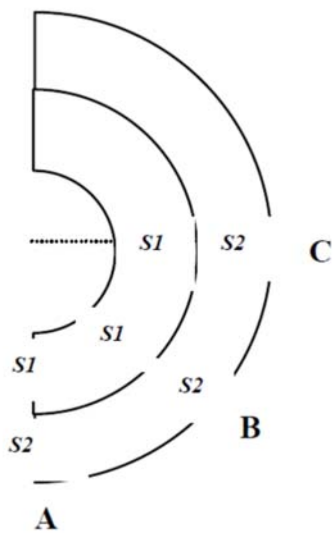


Tutorial 3

- A 65 kg gymnast begins to prepare for his dismount from the high bar by increasing his angular velocity by a factor of 3. By what factor does the centripetal force change? (you may assume that *r* does not change)
  - a) increases by a factor of 3
  - b) increases by a factor of 6
  - c) increases by a factor of 9
  - d) increases by a factor of 12

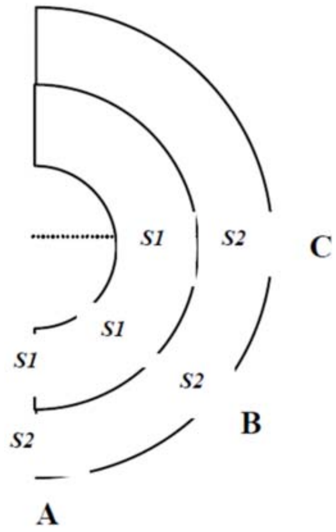
Tutorial 4

- Two speed skaters (*S1* and *S2*) enter the final curve (point A) with exactly the same velocity (say, 20 m/s). At this instant they are tied. Throughout the first half of the curve (points A-C), it appears that the athlete in the outside lane (*S2*) remains tied with the athlete in the inside lane (*S1*). Assume that the athlete in the inside lane maintains a constant speed. Using relative terms like “constant”, “zero”, “greater than”, “less than”, “stays the same”, etc. answer the following questions.
  - a) Briefly discuss the differences between the linear distances traveled by the athletes between points A and C.



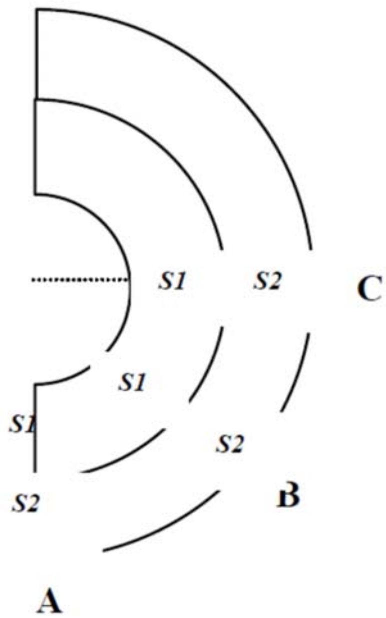
Tutorial 4

– b) Briefly discuss the differences between the tangential velocities of the athletes at points A and C. On the figure to the right, draw the tangential velocity vectors (arrows) for each athlete at points A and C. Indicate relative differences in magnitude by the relative lengths of the vectors (arrows). Be sure to orient your vector in the correct direction.



Tutorial 4

- c) Briefly discuss the differences between the tangential accelerations of athletes between points A and C. On the figure to the right, draw the tangential acceleration vectors (arrows) for each athlete at point B. Indicate relative differences in magnitude by the relative lengths of the vectors (arrows). Be sure to orient your vector in the correct direction.



Tutorial 4

- d) Briefly discuss the differences between the radial accelerations of the athletes between points A and C. On the figure to the right, draw the radial acceleration vectors (arrows) for each athlete at point B. Indicate relative differences in magnitude by the relative lengths of the vectors (arrows). Be sure to orient your vector in the correct direction.