MA1300 Self Practice # 9

1. (P205, #34, 39, 40) Find the critical numbers of the function.

$$g(t) = |3t - 4|, \qquad F(x) = x^{4/5}(x-4)^2, \qquad g(\theta) = 4\theta - \tan\theta.$$

2. (P205, #47, 54, 56) Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = 2x^3 - 3x^2 - 12x + 1, \quad [-2, 3];$$
$$f(t) = \sqrt[3]{t}(8-t), \quad [0, 8];$$
$$f(t) = t + \cot(t/2), \quad [\pi/4, 7\pi/4].$$

3. (P205, #57) If a and b are positive numbers, find the maximum value of $f(x) = x^a(1-x)^b$, $0 \le x \le 1$.

4. (P205, #63) Between 0°C and 30°C, the volume V (in cubic centimeters) of 1 kg of water at a temperature T is given approximately by the formula

$$V = 999.87 - 0.06426T + 0.0085043T^2 - 0.0000679T^3.$$

Find the temperature at which water has its maximum density.

5. (P206, #69) Prove that the function

$$f(x) = x^{101} + x^{51} + x + 1$$

has neither a local maximum nor a local minimum.

6. (P206, #72) A cubic function is a polynomial of degree 3; that is, it has the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

a Show that a cubic function can have two, one or no critical number(s). Give examples and sketches to illustrate the three possibilities.

b How many local extreme values can a cubic function have?

7. (P212, #5) Let $f(x) = 1 - x^{2/3}$. Show that $f(-1) = f(1)$ but there is no number c in $(-1, 1)$ such that $f'(c) = 0$. Why does this not contradict Rolle's Theorem?

8. (P213, #18) Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root.

9. (P213, #19) Show that the equation $x^3 - 15x + c = 0$ has at most one root in the interval $[-2, 2]$.

10. (P213, #21)

a Show that a polynomial of degree 3 has at most three real roots.

b Show that a polynomial of degree n has at most n real roots.

11. (P213, #23) If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

12. (P213, #27) Show that $\sqrt{1+x} < 1 + \frac{1}{2}x$ if $x > 0$.

13. (P213, #34) A number a is called a **fixed point** of a function f if $f(a) = a$. Prove that if $f'(x) \neq 1$ for all real numbers x, then f has at most one fixed point.

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