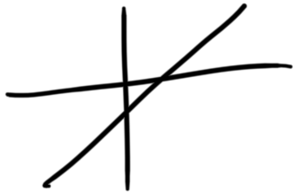
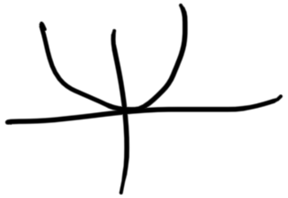
LINEAR AND NON-LINEAR EQUATION

ie.

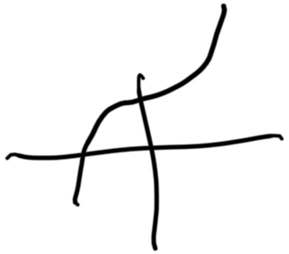
Linear function - $\boxed{y} = m\boxed{x} + c$



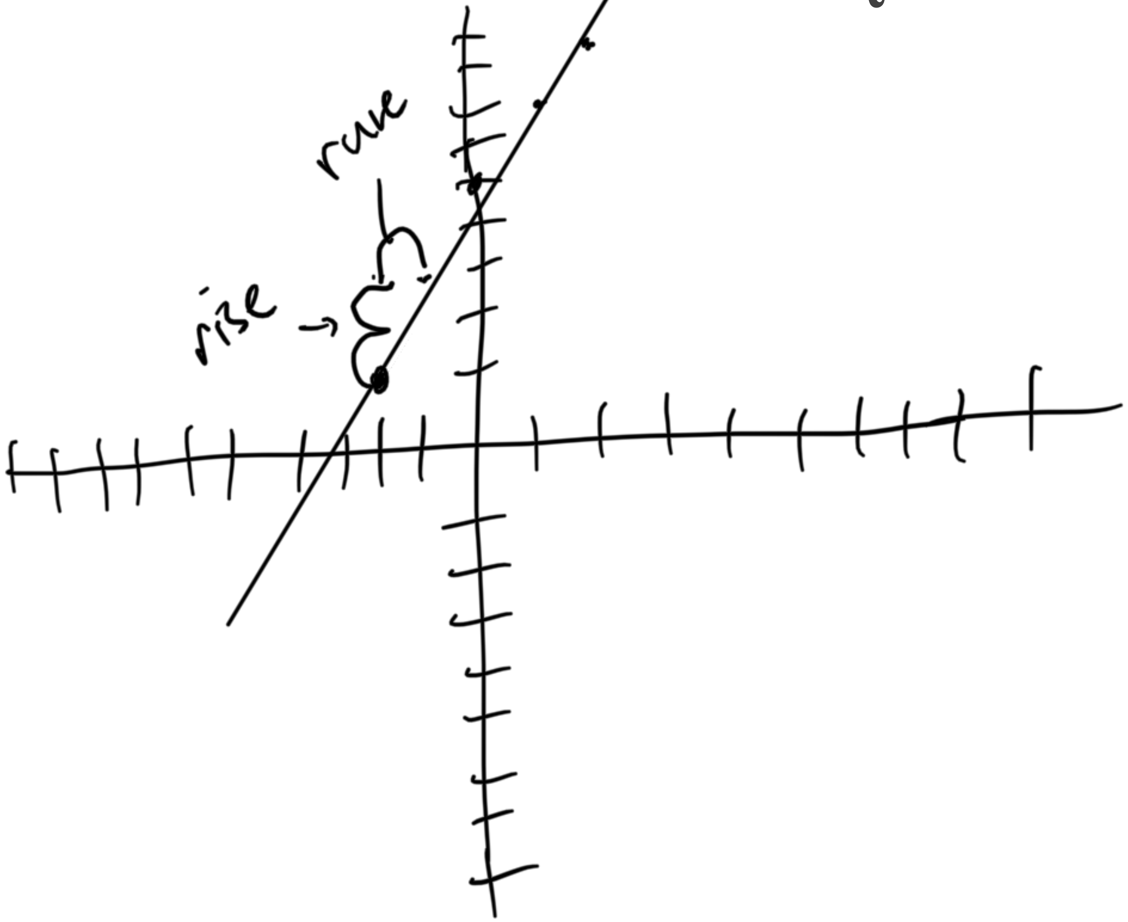
non-linear function ie. $y = x^2 + 4$ degree of 2.



ie. $y = x^3 + 4$ degree of 3



GRAPHING LINEAR EQUATION



$y = \boxed{2}x + 5$

Method 1

x	y
-2	$2(-2)+5 = 1$
-1	$2(-1)+5 = 3$
0	$2(0)+5 = 5$
1	$2(1)+5 = 7$
2	$2(2)+5 = 9$

Method 2

$\frac{\text{rise}}{\text{run}}$

$\boxed{\frac{2}{1} = m}$

y intercept when $x = 0$

$y = 2(0)+5$

$y = 5$

TO DETERMINE THE EQUATION OF A STRAIGHT LINE

- Given the values of m and c
- Given the gradient and a point on the line
- Given two points on the line

GIVEN THE VALUES OF *M* AND *C*

$y = mx + c$

Eg. State the equation of a line whose gradient is 3 and whose y intercept is 2.

Ans

$m = 3 \quad y = 2$

$(0, 2)$

y intercept

$y = mx + c$

$y = 3x + 2$

GIVEN THE GRADIENT AND A POINT ON THE LINE

Ex. Given the line passing thru pt (3,5) w/ gradient of 2

x, y

m = 2

$y = mx + c$

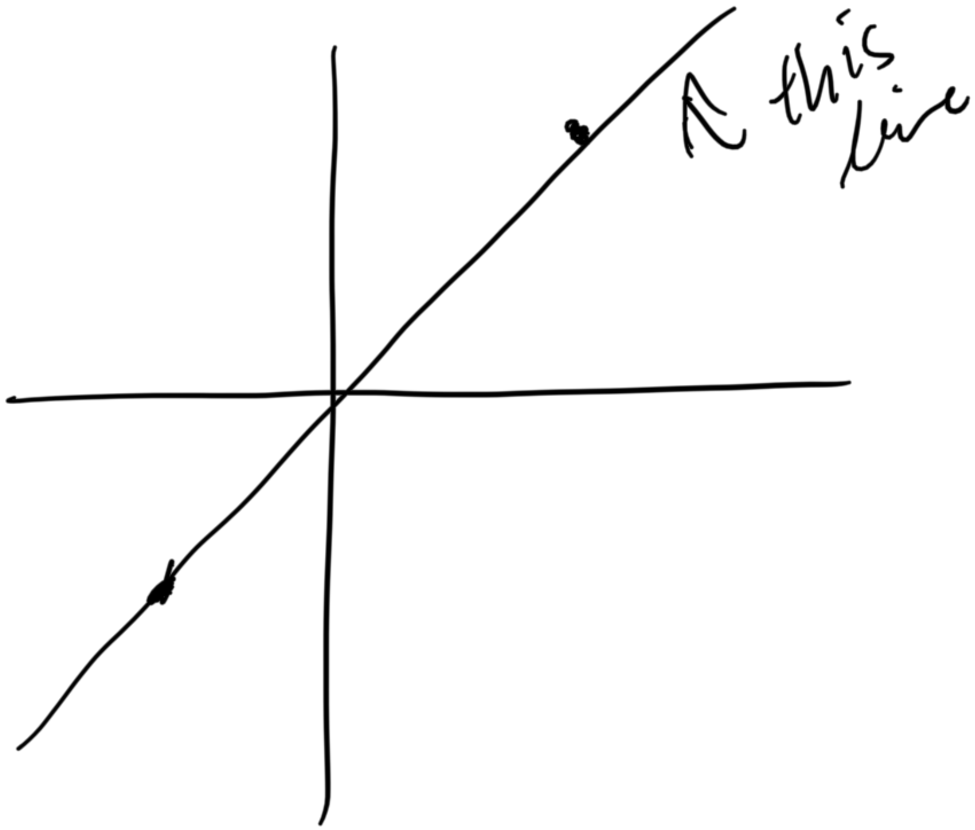
$5 = 2(3) + c$

$5 = 6 + c$

$-1 = c$

$\therefore\ y = 2x - 1$

GIVEN TWO POINTS ON THE LINE



Ex 1

Determine equation of the line

① that passes through the points $(2,1)$ and $(4,7)$

$x_1\ y_1 \qquad x_2\ y_2$

$$m = \frac{7-1}{4-2} = \frac{6}{2} = 3$$

$$y-1 = 3(x-2)$$

$$y-1 = 3x-6$$

$$\therefore\ y = 3x-5$$

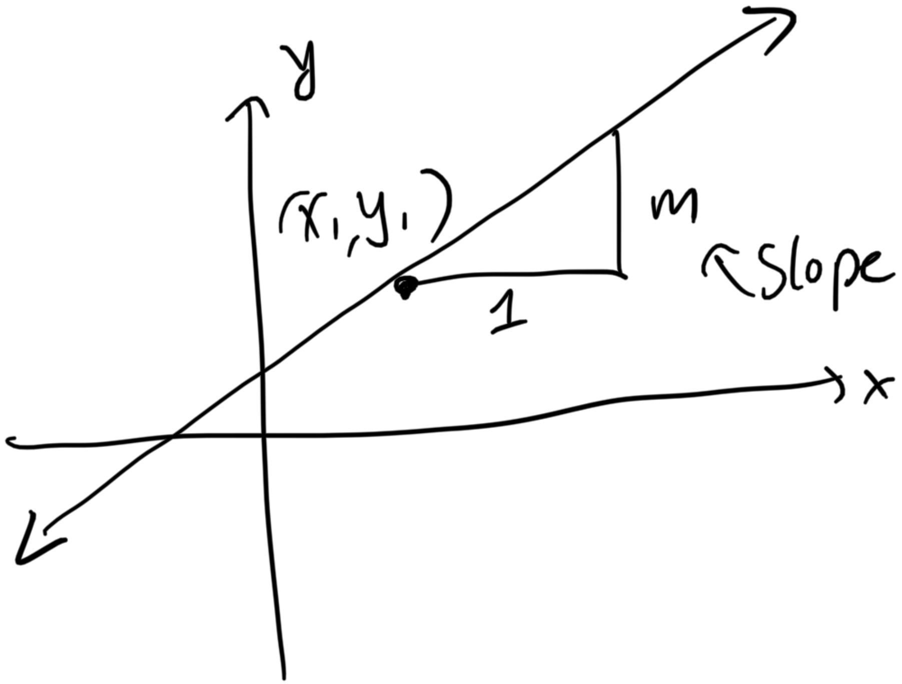
② Determine the equation of the straight line that passes through the origin and which is perpendicular to the equation $y = 2x$. ← slope $= -\frac{1}{2}$

$$m = \frac{y-0}{x-0} = -\frac{1}{2} \qquad 2y = -x \qquad y = -\frac{1}{2}x$$

STRAIGHT LINE IN VARIOUS FORMS

- Point-Slope
- Slope-Intercept
- Line through two points (intercept-intercept form)

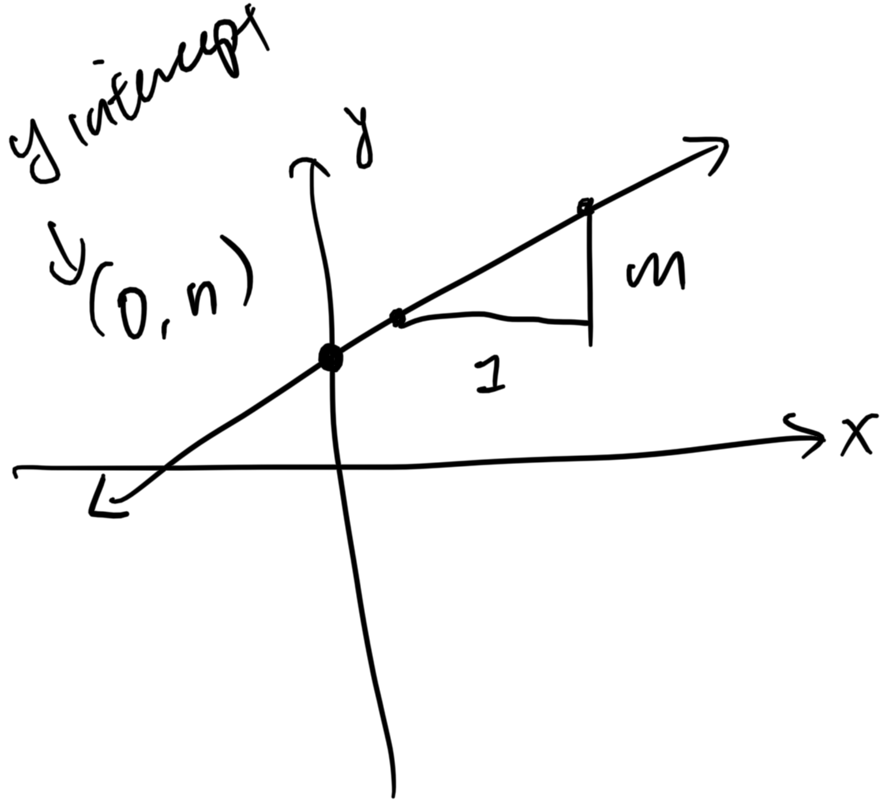
POINT-SLOPE



$$m = \frac{y - y_1}{x - x_1}$$

$$y - y_1 = m(x - x_1)$$

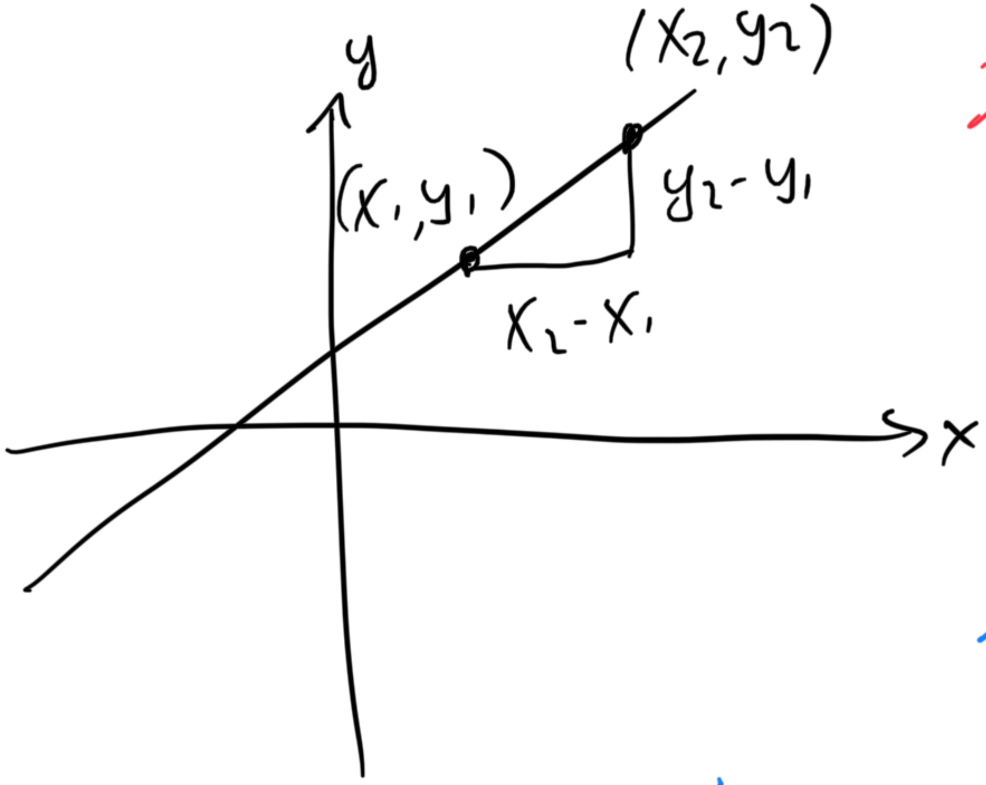
SLOPE-INTERCEPT



$$m = \frac{y-n}{x-0}$$

$$y = mx + n$$

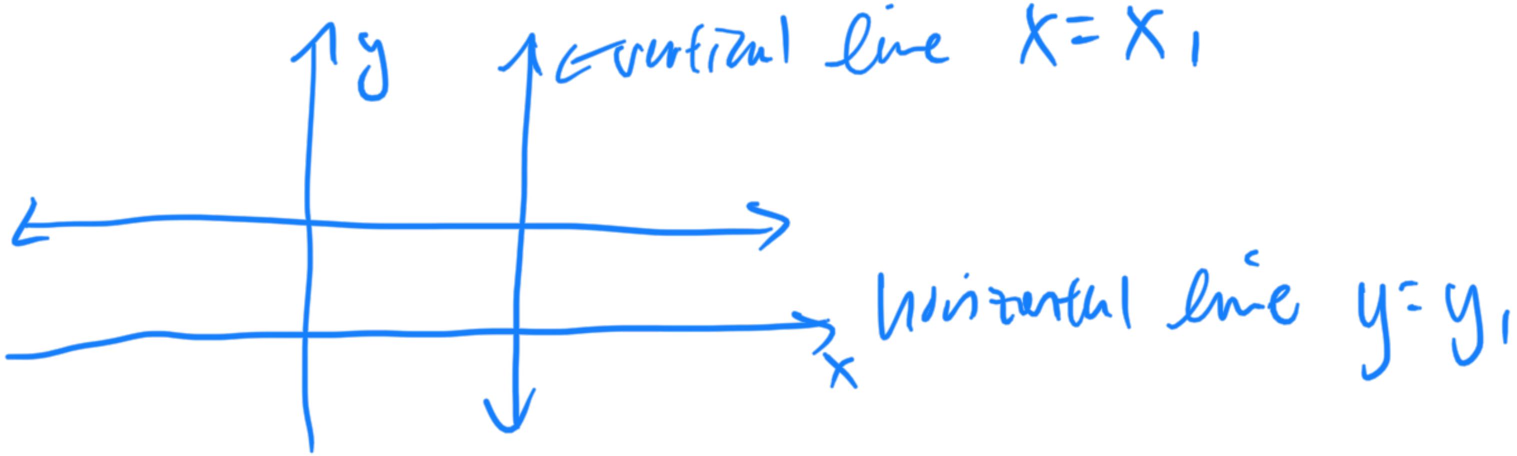
LINE THROUGH TWO POINTS (INTERCEPT-INTERCEPT)



$$\boxed{y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)}$$

$$\underbrace{\frac{y_2 - y_1}{x_2 - x_1}}_{\text{slope}}$$

what happens if we have $x_1 = x_2$



EXAMPLE

Find the expression of straight line which passes thru $(4,0)$ and $(0,-3)$

x y x y

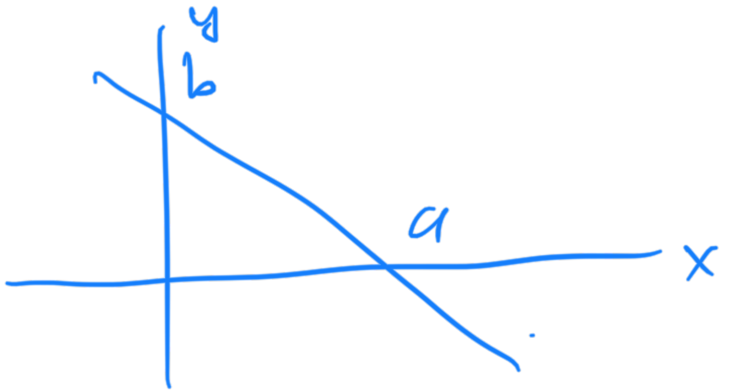
using intercept - intercept form

$$\boxed{y - 0 = \frac{-3-0}{0-4}(x-4)}$$

$$y = \frac{3}{4}(x-4)$$

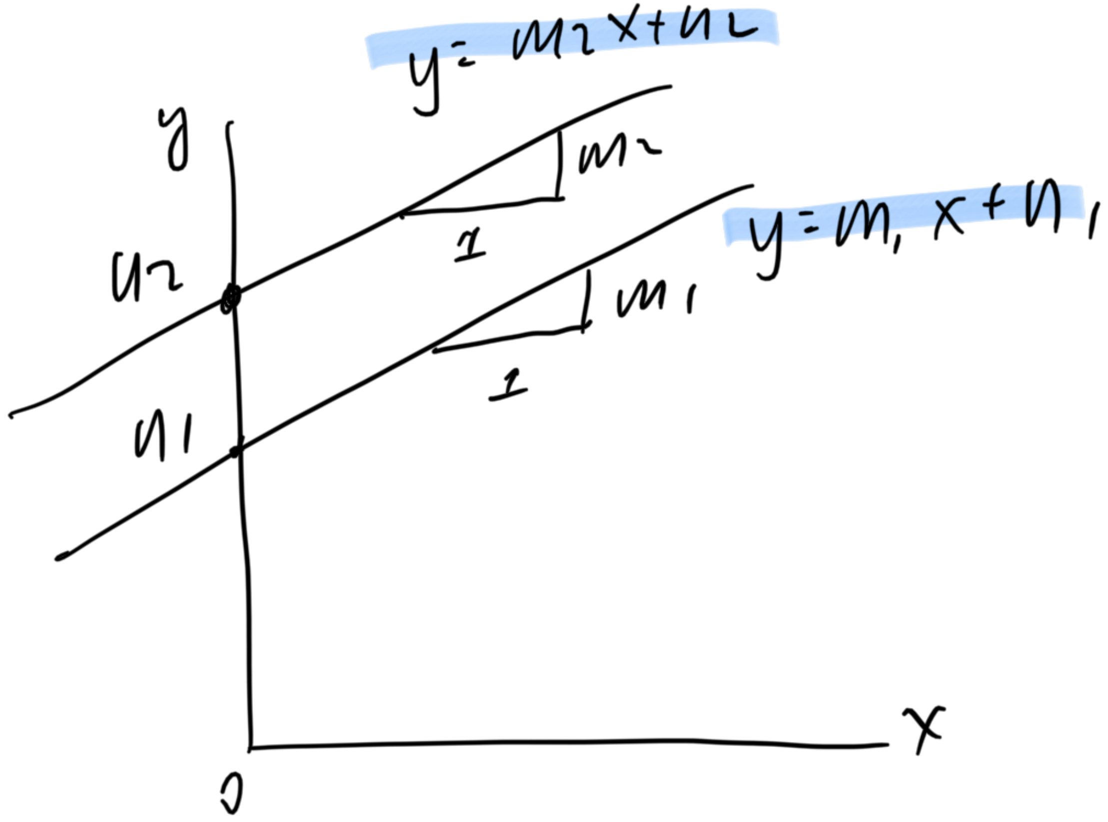
$$3x - 4y = 12$$

(Remark)



$$\frac{x}{a} + \frac{y}{b} = 1$$

PARALLEL STRAIGHT LINES



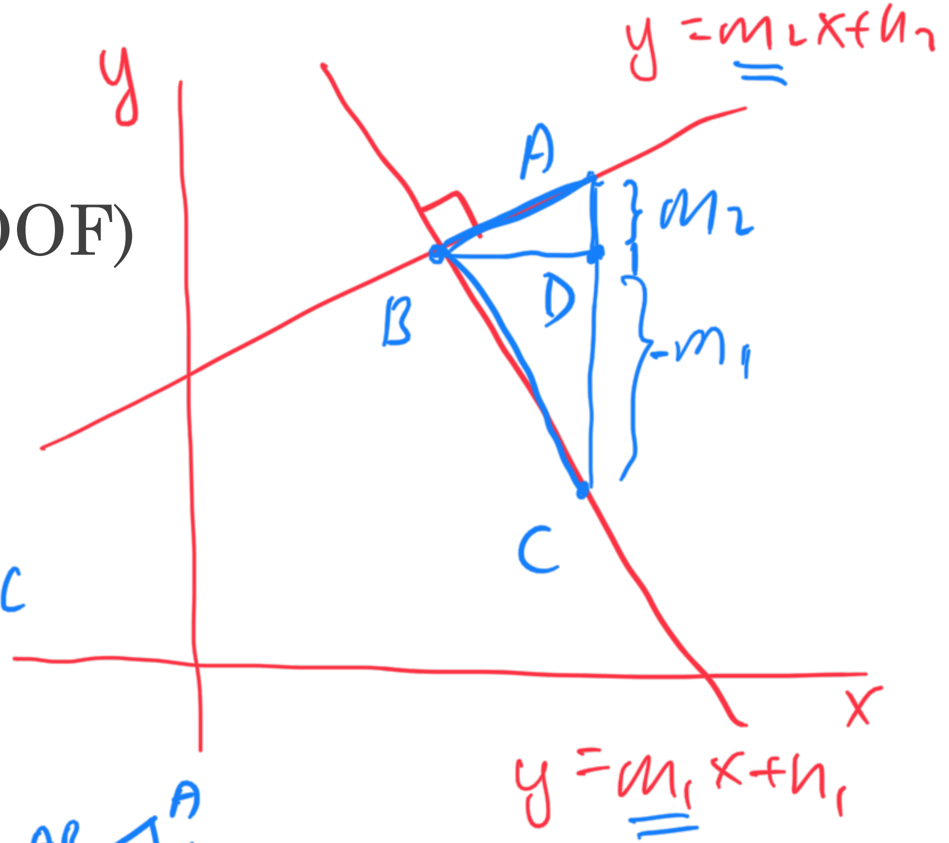
note: $m_1 = m_2$

remark
doesn't matter
if $n_1 \neq n_2$
as long as $m_1 = m_2$
then parallel

PERPENDICULAR STRAIGHT LINES (PROOF)

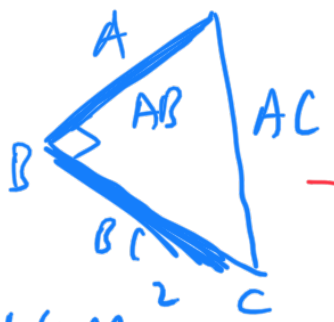
$y = m_1 x + n_1 \qquad y = m_2 x + n_2$

$m_1 m_2 = -1$

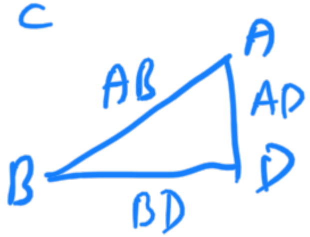


Proof:

$\Delta ABC: \; AB^2 + BC^2 = AC^2$



$\Delta ABD: \; AB^2 = BD^2 + AD^2 = 1 + m_2^2$



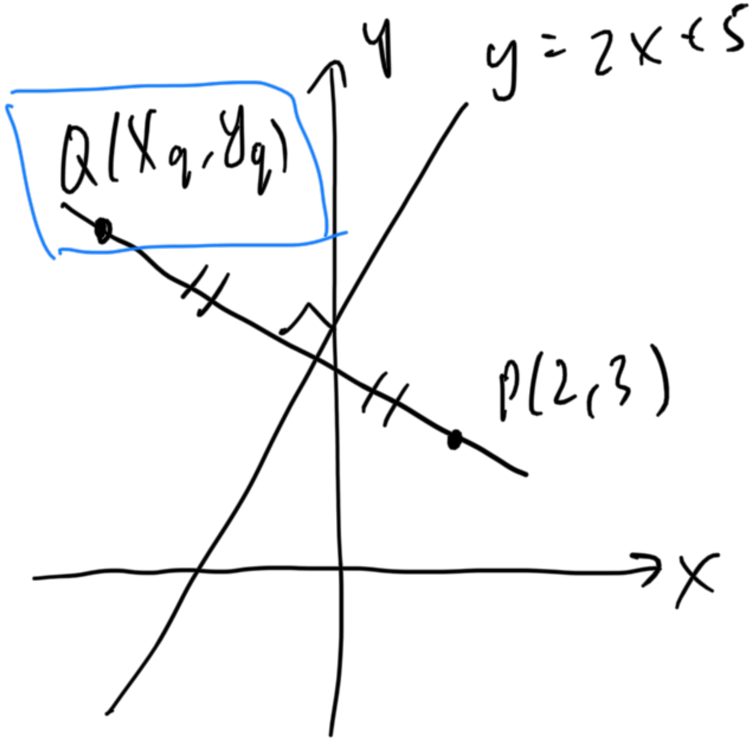
$\Delta BCD: \; BC^2 = 1 + (-m_1)^2$

$1 + m_2^2 + 1 + (-m_1)^2 = (m_2 - m_1)^2$

$\therefore m_1 m_2 = -1$

EXAMPLE Find the reflected image of point (2,3) about the mirror line $y=2x+5$

$$\boxed{y-3 = -\frac{1}{2}(x-2)}$$



PQ $2\cdot\left(\dfrac{y_q-3}{x_q-2}\right) = -1$

Middle point

$$\frac{3+y_q}{2} = 2\cdot\left(\frac{2+x_q}{2}\right)+5$$

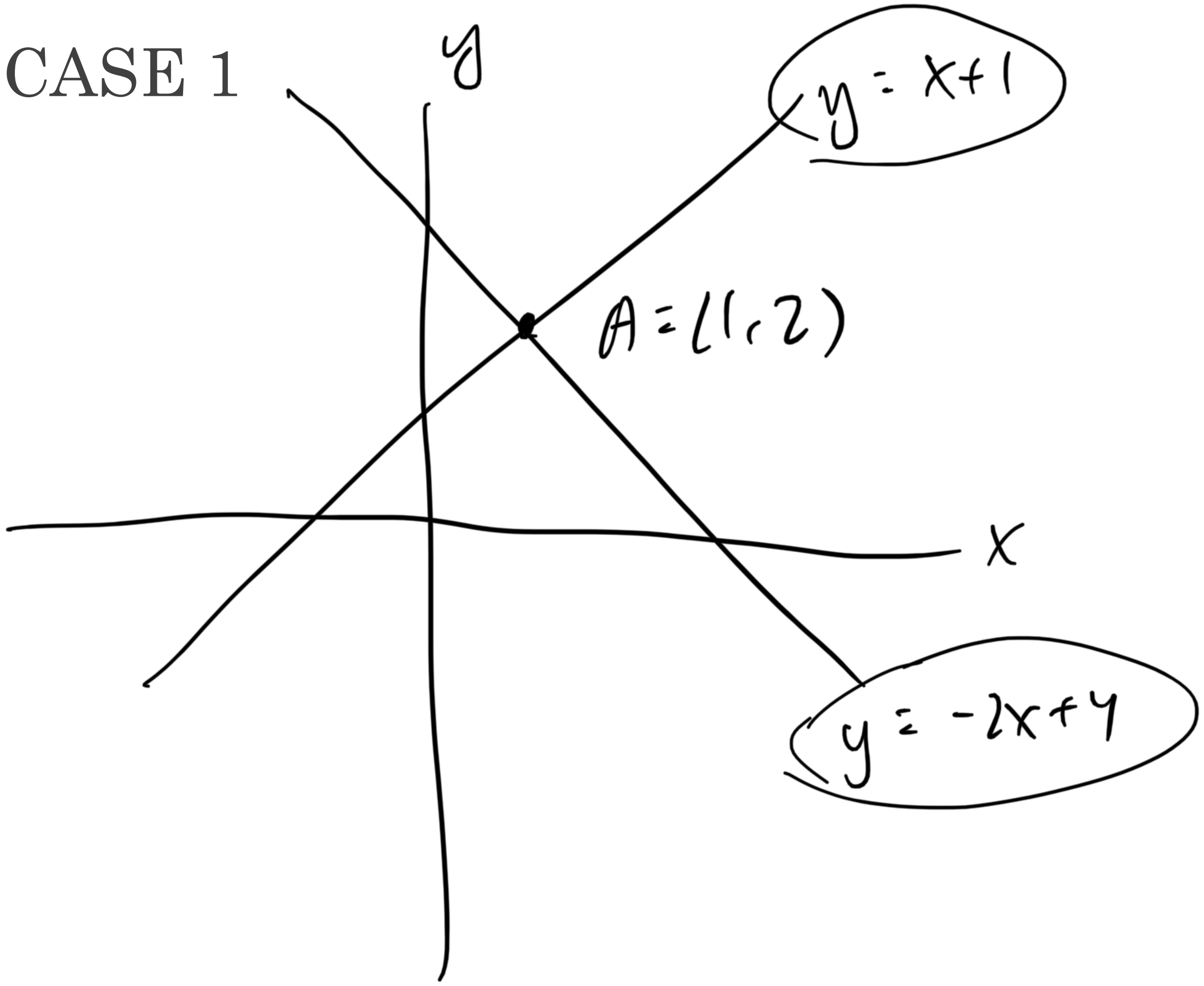
$x_q = -\frac{14}{5}$ $y_q = \frac{27}{5}$ $Q = \left(-\frac{14}{5}, \frac{27}{5}\right)$

$$y-\frac{27}{5} = -\frac{1}{2}\left(x+\frac{14}{5}\right)$$

INTERSECTION OF TWO LINES

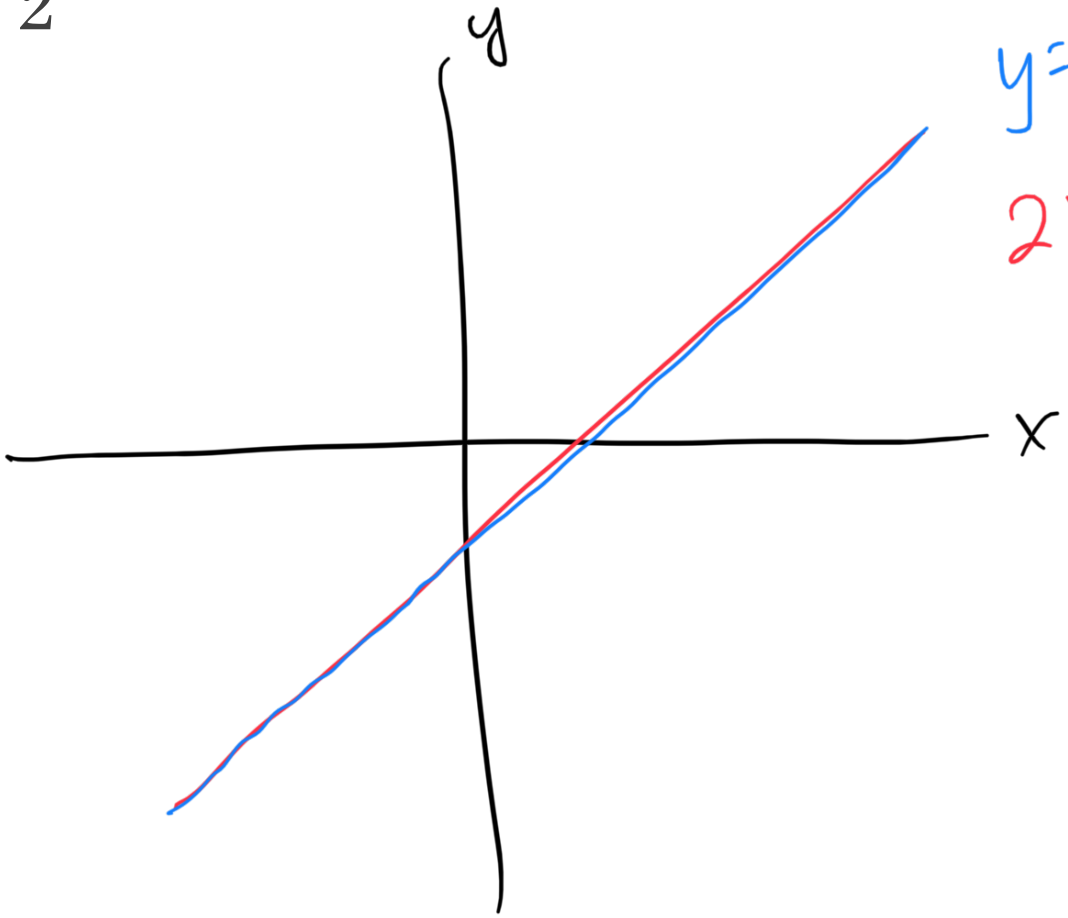
- 3 cases…

CASE 1



1 ans

CASE 2

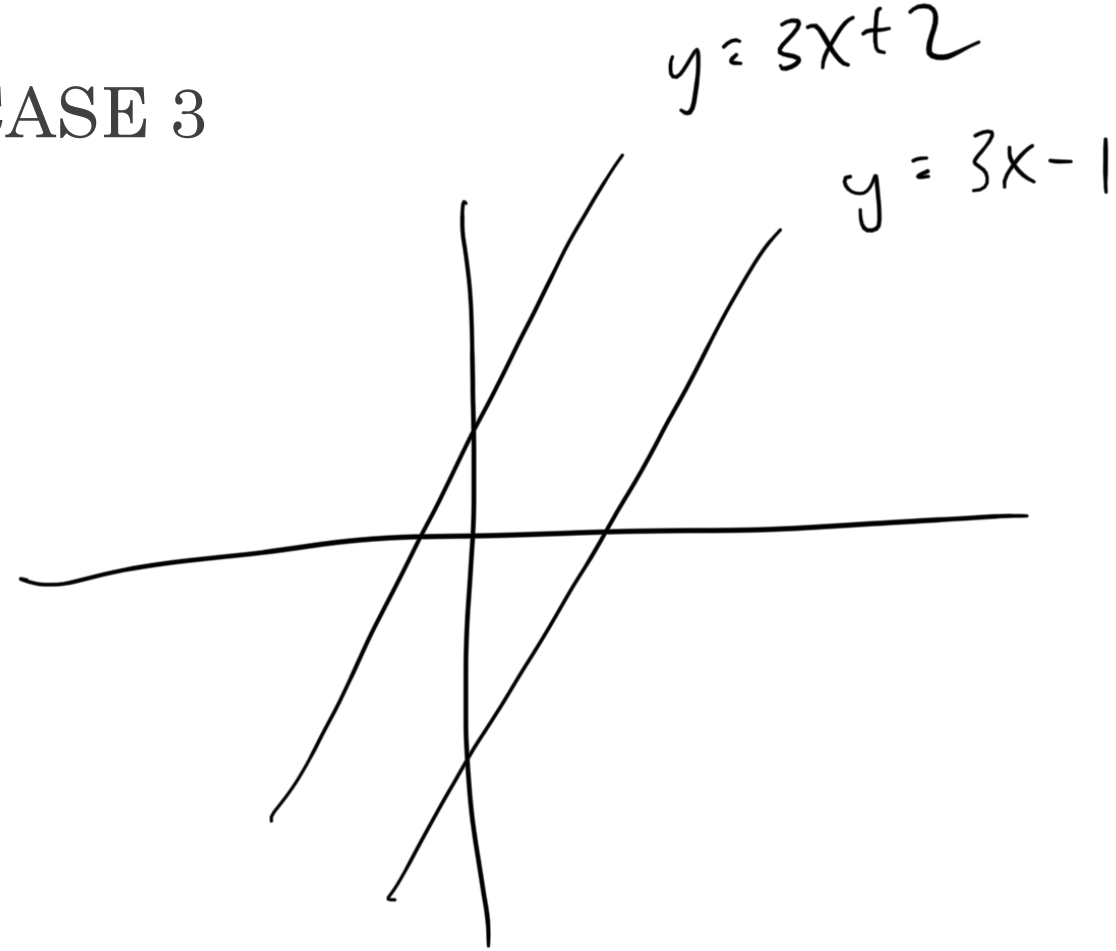


$y = x - 1$

$2x - 2y - 2 = 0$

∞ amt of answers.

CASE 3



no intersections

EXAMPLE

(a) $y = x + 1$ and $y = 1 - x$

$x + 1 = 1 - x$

$2x = 0$

$x = 0$

$y = 0 + 1$

$y = 1$

(b) $y = 3$ and $y = 2x + 1$

$3 = 2x + 1$

$2 = 2x$

$1 = x$

$(1, 3)$

(c) $x + y = 2$ and $y = x$

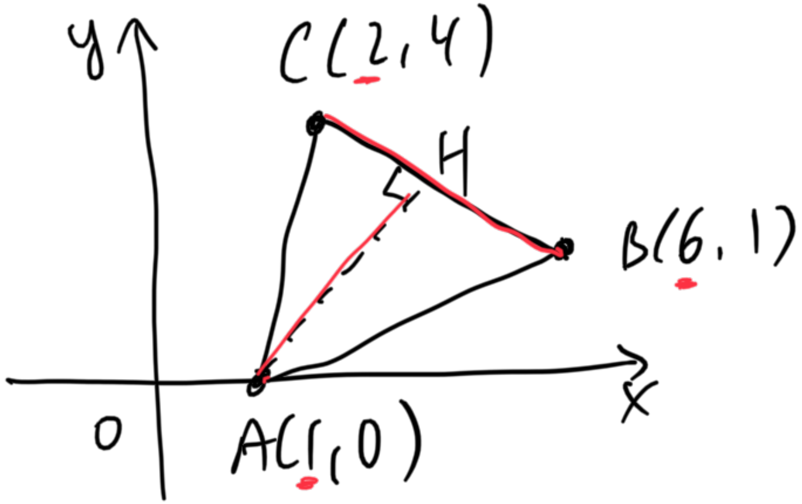
$y + y = 2$

$2y = 2$

$y = 1$

$(1, 1)$

PUTTING IT ALL TOGETHER QUESTION



① Shift the triangle 1 unit leftward and find coordinates of new positions (A, B, C)

② Find A'H after transformation

③ Find area of triangle $= \frac{\text{Base} \times \text{Height}}{2}$

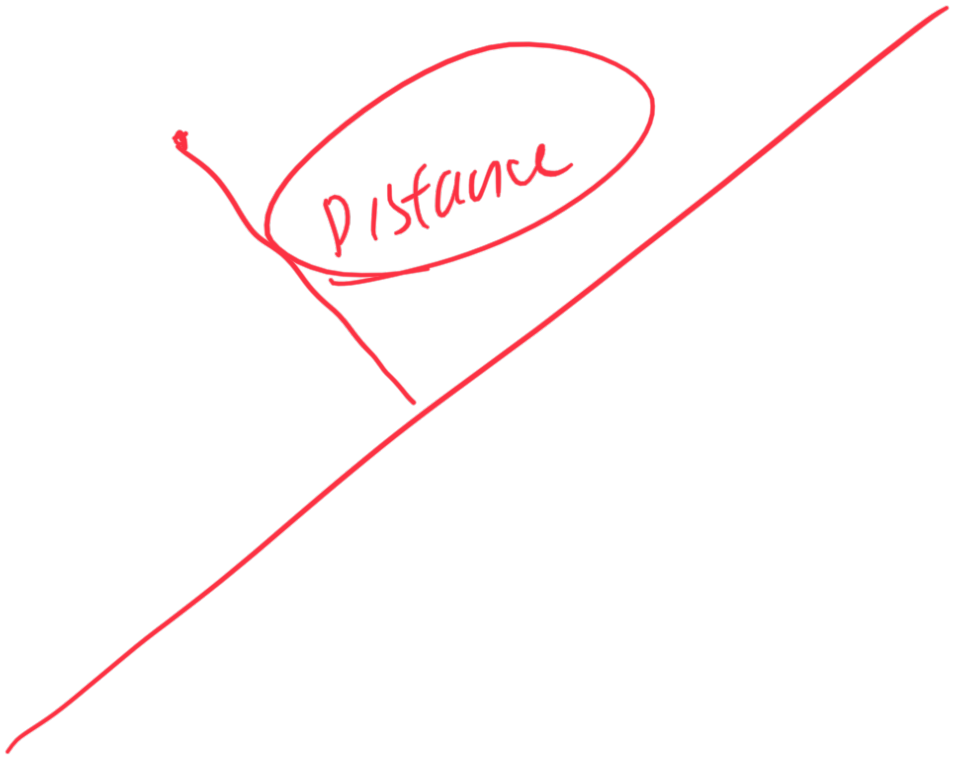
① $A'(0,0)$ $B'(5,1)$ $C'(1,4)$

② $y-4 = \frac{1-4}{5-1}(x-1) \Rightarrow y-4 = -\frac{3}{4}(x-1) \therefore 3x+4y-19=0 \Leftarrow \overline{CB}$

length A'H $d = \frac{|-19|}{\sqrt{3^2+4^2}} = \frac{19}{5} \leftarrow$ Height

③ $\sqrt{(5-1)^2+(1-4)^2}=5$ base $\therefore \frac{1}{2}\cdot\frac{19}{5}\cdot 5=\frac{19}{2}=9.5$

THE DISTANCE FROM A POINT TO A LINE



- Class activity…

EXAMPLE (PART A)

A robot is moving along the line $20x+30y=600$. A homing beacon sits at the point $(35,40)$

(i) Where on this line the robot hears the loudest ping?

Shortest $\perp$ distance

$y=\frac{-20x}{30}+\frac{600}{30}$ $\quad m=-\frac{2}{3}$ $\therefore m_{\perp}=\frac{3}{2}$

Use substitution

$y-40=\frac{3}{2}(x-35)$ or $y=\frac{3}{2}(x-35)+40$ ← $\perp$ line

$\hookrightarrow 20x+30\left(\frac{3}{2}(x-35)+40\right)=600$

$x=15$

$20(15)+30y=600$ $\therefore y=10$

$15,10$

EXAMPLE (PART B)

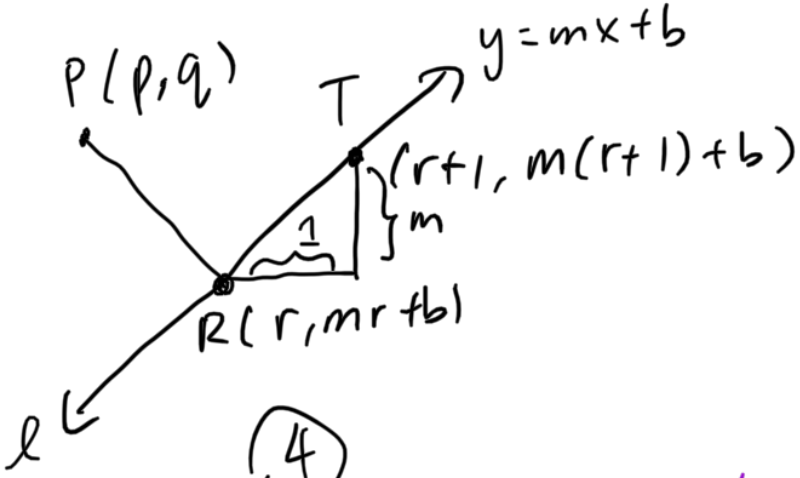
At this point, how far will the robot be from the beacon?

$$\text{distance} = \sqrt{(35-15)^2 + (40-10)^2}$$

$$= \sqrt{(20)^2 + (30)^2}$$

$$= \sqrt{1300} \text{ or } 10\sqrt{13}$$

THE CONSTRUCTION OF A FORMULA FOR THE DISTANCE TO A POINT FROM THE LINE



1 $\overline{PR}$ must be $\perp$ to ℓ

ΔPTR right triangle w/ the right angle at vertex R.

$\overline{PR} \perp RT$

2 $\overline{PR} \perp RT$

3 $(p-r)(1)+(q-(mr+b))m=0$

$p-r+qm-m^2r-bm=0$

$-r-mr^2=-p-qm+bm$

$r(1+m^2)=p+qm-bm$

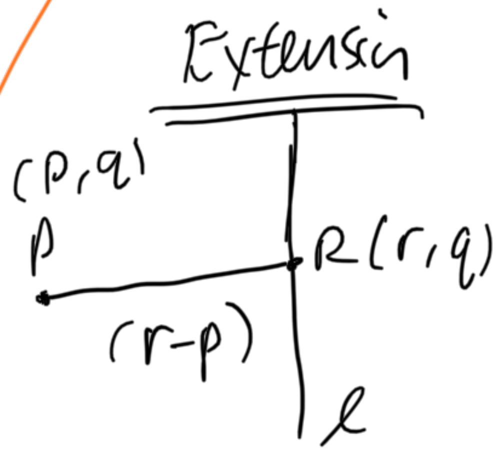
$r=\dfrac{p+qm-bm}{1+m^2}$

4 $R\left(\dfrac{p+qm-bm}{1+m^2},\ m\left(\dfrac{p+qm-bm}{1+m^2}\right)+b\right)$

and

$P(p,q)$

5 $d=\sqrt{(x_2-x_1)^2-(y_2-y_1)^2}$



$$d = \sqrt{\left(\frac{p+qm-bm}{1+m^2} - p\right)^2 + \left(m\left(\frac{p+qm-bm}{1+m^2}\right)+b-q\right)^2}$$

EXAMPLE (PART A)

(a) $P(0,0)$ and the line $y=10$

$p=0 \quad q=0 \quad m=0 \quad b=10$

$$d = \sqrt{\left(\frac{0+0+10(0)}{1+0} - 0\right)^2 + \left(0\left(\frac{0+0-10(0)}{1+0^2}\right)+10-0\right)^2}$$

$$d = \sqrt{0+10^2} = 10$$

EXAMPLE (PART B)

$P(0,0) \quad y = x + 10$

$p = 0 \quad q = 0 \quad m = 1 \quad b = 10$

$$d = \sqrt{\left(\frac{0+0(1)-10(1)}{1+1^2} - 0\right)^2 + \left(1\left(\frac{0+(0)(1)-10(1)}{1+1^2}\right)+10-0\right)^2}$$

$$d = \sqrt{5^2+5^2} = \sqrt{50} = 5\sqrt{2}$$

Ex. $P(0,0)$ $y = x - 6$

$p = 0$ $q = 0$ $m = 1$ $b = -6$

$$d = \sqrt{\left(\frac{0 + 0(1) - (-6)(1)}{1+1^2} - 0\right)^2 + \left(1\left(\frac{0 + 0(1) - (-6)(1)}{1+1^2}\right) + (-6) - 0\right)^2}$$

$$d = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$$

DEFINITION AND PROPERTIES OF BETWEENNESS

- Definition 3.15 (Betweenness) For any three points A,B and C, we say that B is between A and C, and we write $A - B - C$, iff A, B, and C are distinct, collinear points, and $AB + BC = AC$.
- Theorem 3.16 If $\underbrace{A - B - C \text{ then } C - B - A}_{\text{using definition}}$, and neither $A - C - B$ nor $B - A - C$.

$A - B - C$

dash

PROOF

let's proof $A-C-B$ is not true

Assume $A-C-B$ then $AC+CB=AB$

Since $A-B-C$, $AB+BC=AC$

so $AC+CB+BC=AB+BC=AC$ ie. $2BC=0$

then $B=C$ which is contradicts

PARAMETRIC EQUATION OF A LINE 2D. or $\mathbb{R}^2$

$x = x(t)$ t: an extra parameter

$y = y(t)$ x and y are both given as functions of a third variable $t \in \mathbb{R}$.

As t varies, the point $(x(t), y(t))$ varies and traces out a curve. A parameter t is usually time but not necessarily

(t denotes time as $x(t), y(t)$ represents the position of a particle at time t.)

EXAMPLE

We want parametric equation of a line through 2 points (x_1, y_1) and (x_2, y_2), how to show this?

preferred.

$$\begin{cases} x = x_1 + t(x_2 - x_1) \\ y = y_1 + t(y_2 - y_1) \end{cases} \longrightarrow$$

$$y = mx + c$$

$$\boxed{y - y_1 = m(x - x_1) \;??}$$

or

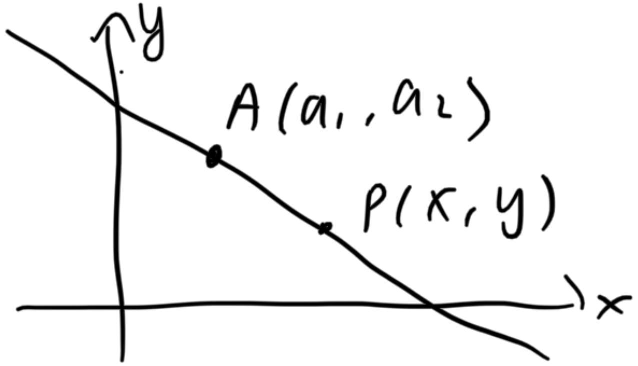
$$x = (1-t)x_1 + tx_2$$

$$y = (1-t)y_1 + ty_2$$

VECTOR EQUATION OF A LINE AND DIRECTION OF A VECTOR

$$\boxed{\vec{r} = \vec{a} + t\vec{b}}, \quad t \in \mathbb{R}$$

$$\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + t\begin{pmatrix} b_1 \\ b_2 \end{pmatrix}}$$



P(x,y) is any point on the line. A(a_1, a_2) is a fixed point

$\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$ direction of the vector of the line which enables us to calculate for the slope.

$$\therefore \boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_1 + tb_1 \\ a_2 + tb_2 \end{pmatrix}}$$ Parametric equation of the line where $t \in \mathbb{R}$

t : parameter

In Relation to Cartesian

$$x = a_1 + tb_1 \qquad y = a_2 + tb_2$$

step 1
isolate for
t

$$\frac{x - a_1}{b_1} = t \qquad \frac{y - a_2}{b_2} = t$$

equate.

step 2
equate t

$$\frac{x - a_1}{b_1} = \frac{y - a_2}{b_2}$$

$$\frac{b_2(x - a_1)}{\cancel{b_1 b_2}} = \frac{b_1(y - a_2)}{\cancel{b_1 b_2}}$$

step 3
get rid
of
denominator

$$xb_2 - a_1b_2 = yb_1 - a_2b_1$$

cartesian of the line

EXAMPLE

Find the vector equation, the parametric equation and cartesian equation of line passing through the point $A(1,5)$ with direction $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

Cartesian

$2x - 3y = 13$

vector

$\vec{a} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ $\vec{b} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\vec{r} = \vec{a} + t\vec{b}$ $\boxed{\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 2 \end{pmatrix}, \ t \in \mathbb{R}}$

parametric

$$\boxed{\left.\begin{aligned} x &= 1 + 3t \\ y &= 5 + 2t \end{aligned}\right\} t \in \mathbb{R}.}$$

cartesian

$x = 1 + 3t$ $y = 5 + 2t$

$t = \frac{x-1}{3}$ $t = \frac{y-5}{2}$

$\frac{x-1}{3} = \frac{y-5}{2} \ldots$

$\boxed{2x - 3y + 13 = 0}$

$\vec{a} \cdot \vec{b} = (a_1 b_1) + (a_2 b_2) = |\vec{a}||\vec{b}| \cos\theta$ $\quad \therefore \cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

THE SCALAR PRODUCT OF TWO VECTORS

Remark

let $\vec{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\vec{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\vec{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

(special unit vector in positive direction)

lenght $\vec{a}$: $\sqrt{3^2 + (-4)^2} = 5 = |\vec{a}|$ $\qquad \vec{a} = 3\vec{i} - 4\vec{j}$

length $\vec{b}$: $\sqrt{1^2 + 2^2} = \sqrt{5} = |\vec{b}|$ $\qquad \vec{b} = \vec{i} + 2\vec{j}$

Def: Given 2 vectors $\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$

$\vec{a} = a_1\vec{i} + a_2\vec{j}$ $\qquad \vec{b} = b_1\vec{i} + b_2\vec{j}$

θ acute then $\cos\theta > 0$ $\quad \vec{a} \cdot \vec{b} > 0$

θ obtuse then $\cos\theta < 0$ $\quad \vec{a} \cdot \vec{b} < 0$

$\vec{a} \cdot \vec{b} = 0$ perpendicular or orthogonal

EXAMPLE

Find $t \in \mathbb{R}$ such that the vectors $\vec{a} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$ and $\vec{b} = \begin{pmatrix} 2 \\ t \end{pmatrix}$ are orthogonal.

$$\vec{a} \cdot \vec{b} = 0$$

$$(-1)(2) + (5)(t) = 0$$

$$-2 + 5t = 0$$

$$5t = 2$$

$$t = 2/5$$

EXAMPLE

Q7

Find the measure of the angle between 2 lines

$l_1 = 2x + y = 5$

$l_2 = 3x - 2y = 8$

Ans

$l_1 = y = 5 - 2x$

$m = -2$

$\vec{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$l_2 \; y = \frac{3}{2}x - 4$

$m = \frac{3}{2}$

$\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{(1)(2) + (-2)(3)}{\sqrt{1+4}\,\sqrt{4+9}} = -\frac{4}{\sqrt{5}\,\sqrt{13}} = -\frac{4}{\sqrt{65}}$$

$$\cos\theta = -\frac{4}{\sqrt{65}}$$

$$\theta = \left(\cos^{-1}\left(-\frac{4}{\sqrt{65}}\right)\right)$$

$$= 119.7^\circ$$

$\therefore 60.3^\circ$

Acute angle

EXAMPLE

Find the angle between the lines given by parametric equation.

$\ell_1 : x = 2-3t \ , \ y = -1+t \qquad t \in \mathbb{R}$

$\ell_2 : x = 1+2s \ , \ y = -4+3s \qquad s \in \mathbb{R}.$

$\vec{b}_1 = \begin{pmatrix} -3 \\ 1 \end{pmatrix} \qquad b_2 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

$\cos\theta = \dfrac{-6+3}{\sqrt{10}\,\sqrt{13}} \qquad \theta = 74.7^\circ$