GE2256: Game Theory Applications to Business
Lecture 1

Department of Economics and Finance
City University of Hong Kong
Sem A, 2021-2022

Welcome!!

Introduction

- *GAME THEORY* aims to help us understand situations in which decision-makers interact.
- As opposed to individual decision making in isolation, Game Theory is for situations where one's payoff/well being is affected by the actions of others: Outcomes and payoffs are dependent not only on my actions but also on actions by others.
- Analysis is done through the development of simple mathematical *models*.
- **Game Theory** is also known as **Theory of Strategic Interaction**.

The Road Ahead...

平衡概念,

First-half:

- One-shot simultaneous-move games; equilibrium concept (Nash equilibrium)
- Pure strategy equilibrium; Mixed strategy equilibrium
- Dominant strategy games; rationalizability

占主导地位的

The Road Ahead...

Second-half:

- Sequential-move games (including centipede game, ultimatum game, dictator game)
- Games with continuous action space: Cournot, Stackelberg
- Simultaneous-move games with incomplete information: Bayes-Nash equilibrium
- Repeated Games & reputation

Class Details: CANVAS

- Please read the syllabus (available on Canvas).
- I will use the Canvas site to distribute lecture slides every week.
- Assignments and other communication will also be done through Canvas.
- So, please make sure to regularly visit the Canvas page for GE2256.

Class Details: Structure of the Lectures

- I will present the material (lecture slides) in class for the first 45-50 mins.
- Then we will take a break for 10-15 mins.
- I will present again for 45-50 mins.
- We will take a break of 10-15 mins.
- I will present again to finish off the material for the day.

Class Details: Prerequisites, Textbook

- No specific prerequisites required.
- Textbook on which the lectures are based: Dixit, A., Skeath, S. and Reiley, D, 2015: **Games of Strategy, 4th edition** (Publisher: W. W. Norton).
- Additional reference: Watson, J, 2013: **Strategy: An Introduction to Game Theory, 3rd edition** (Publisher: W. W. Norton).

Class Details: Lecture Notes

- NOTE: The lecture notes are self-contained. What this means is that **you should be able to solve the problems in assignments, mid-term exam and final exam if you go through the lecture notes only.**
- **However, you are strongly advised to go through the textbook for enhanced learning experience.**

Class Details: Doubt Clearance

- E-mail: nilanroy@cityu.edu.hk
- Please e-mail me (via canvas or directly) whenever you have doubts about concepts or any issue related to the course.

Class Details: Marks Breakdown

- Homework assignments: 10%
- Group project: 30%
- Mid-term exam: 10%
- Final exam: 40%
- In-class exercise/experiment participation: 10%
- ***Students are required to pass both coursework (mid-term exam + homework assignments + in-class participation + group project) and examination (final examination) components separately in order to pass the course.***

Class Details: HW assignments

- There will be three homework assignments in total.
- Please note that you need to submit your assignment online by the deadline specified. No late submission is allowed.
- Homework assignments will not be graded but submission and completion of all questions is mandatory in order to get full grades.
- The solutions to homework assignments will be posted on canvas within one week of the deadline for submission.

Class Details: Group Project

- At the end of the term, you have to submit a group project.
- The project must be on “an application of a theoretical concept that you encounter in this course to address some real-word problem”.
- More details and operational aspects will be announced later (in late September).

Class Details: In-class exercise participation

- There will be few (about 3-4) simple in-class exercises/activities.
- You need to simply participate in these exercises and you will get full grade. You need to then submit it online.
- No right or wrong answer for this.
- If you are absent and there is an in-class exercise, you lose the points.

Class Details: Exams

- Mid-term exam (a MCQ quiz) will be conducted within class hours on OCT 21.
- Final exam (written type) will be conducted in the month of DEC. Exact date, time and venue to be decided by the university.

Decisions versus Games

- In an individual decision-making task, there is no interdependency among different individuals. There is no strategic interaction.
- The outcomes are determined by the action choice of the decision maker and not affected by the decisions of other individuals.
- Example: A decision-maker choosing how much money to allocate among risk-free bonds and risky stocks.

债券, 股票

Decisions versus Games

相互依存的

- In contrast to “decisions”, there is interdependency among different individuals in a “game”.
- The outcomes are determined by the action choices of different individuals.
- A Motivating Example: Consider that you are about to cross a road and you see an approaching car. There is no traffic signal.
- You can either **cross the road** or **wait for the car to pass**.
- The driver of the car can press the accelerator and **speed up** or press the **brake** and let you cross.

Decisions versus Games

- Different outcomes might be realized depending on the choices made by you and the driver of the car.
- Four outcomes are possible in this case.
- In one of the outcomes, collision is set to take place when you choose to cross the road and the driver of the car speeds up.
- There is interdependency among you and the car driver (the two individuals).
- This is in contrast to the individual decision making task.

Individual Decision Making: LOTTERY CHOICE EXPERIMENT (LCE)

- You have to choose between option A and option B.

  OPTION "A": \$2 with probability p and \$1.6 with probability $1 - p$.

  OPTION "B": \$3.85 with probability p and \$0.1 with probability $1 - p$.

- You have to make 10 decisions as the probability p is varied. Refer to the LCE sheet.
- For each decision, which option will you choose?

Individual Decision Making: LOTTERY CHOICE EXPERIMENT (LCE)

- For each decision (value of p), you can calculate the expected payoff from option A and option B.
- Expected payoff from option "A": $p(2) + (1-p)(1.6)$=$1.6 + (0.4)p$.
- Expected payoff from option "B": $p(3.85) + (1-p)(0.1)$=$0.1 + (3.75)p$.
- If you are an expected payoff maximizer (also known as risk-neutral individual), then you will choose option A over B if and only if the expected payoff from A is higher than expected payoff from B:

$$1.6 + (0.4)p > 0.1 + (3.75)p \implies p < 0.45$$

Individual Decision Making: LOTTERY CHOICE EXPERIMENT (LCE)

The expected payoffs for different values of p are listed below. '✓' denotes the preferred lottery for a expected payoff maximizer:

p	A	B
0.1	1.64 ✓	0.48
0.2	1.68 ✓	0.85
0.3	1.72 ✓	1.23
0.4	1.76 ✓	1.6
0.5	1.8	1.98 ✓
0.6	1.84	2.35 ✓
0.7	1.88	2.73 ✓
0.8	1.92	3.1 ✓
0.9	1.96	3.48 ✓
1.0	2	3.85 ✓

Individual Decision Making: LOTTERY CHOICE EXPERIMENT (LCE)

- Observe that for the decision 10 with $p = 1$, an individual should choose lottery B as it surely gives you a higher payoff of 3.85 as compared to A which gives 2.
- Different individuals have different tolerance levels of risk and will have different preferences for decisions 1-9.
- If you are “very” risk-averse, you might choose lottery A over B for all decisions 1-9.
- Similarly, if you are an extreme risk-seeker, you might choose lottery B over A for all decisions 1-9.
- Typically, it has been seen that most of us are risk-averse with the degree of aversion differing across individuals.

A Strategic Game

- A strategic game is a model of interacting decision-makers.
- We refer to the decision-makers as *players*.
- Each player has a set of possible *actions* or *strategies*.
- The model captures interaction between the players by allowing each player to be affected by the actions of *all* players, not only her own action.

A Strategic Game

A strategic game consists of

- a set of **players**
- for each player, a set of **actions**
- for each player, **preferences** over the set of action profiles (usually known as outcomes). Typically, we will work with **payoff functions** assigned to each player.

A Strategic Game

- A wide range of situations may be modeled as strategic games.
- For example, the players may be firms, the actions prices, and the preferences a reflection of the firms' profits (ECONOMICS/BUSINESS!).
- Or the players may be candidates for political office, the actions campaign expenditures, and the preferences a reflection of the candidates' probabilities of winning (POLITICAL SCIENCE!).
- Or the players may be animals fighting over some prey, the actions concession times, and the preferences a reflection of whether an animal wins or loses (BIOLOGY!).
- The applications of game theory ranges from business to political science to biology to sociology to computer science. Game theory has enormous applications over a spectrum of fields.

Rationality

- We will assume that players are rational. The definition of rationality is that a player selects the strategy that he most prefers.
- In other words, players seek to maximize their expected payoff, given their beliefs about the strategies of the other players.
- Note that assuming that players are rational does not mean that they are selfish or seek to maximize their own monetary gains. A "payoff" is not necessarily the same as a monetary gain: it might also incorporate other regarding preferences, preferences for fairness and other aspects.
- All that rationality implies is that given own beliefs and own payoff functions, players maximize their expected payoff.

Common Knowledge

- In modeling a strategic situation as a game, we assume that the players share a common understanding of the game being played (the number of players, set of actions available to each player, payoff functions of each player and so on).
- That is, there is *common knowledge* of the game being played.
- A fact "F" is common knowledge if each player knows F, each player knows that the other player knows F, each player knows that the other player knows that each player knows F, and so on (I know that you know that I know that you know that........).

Simultaneous Games

同时的

- We will start our analysis by introducing **simultaneous-move** games.
- Each player chooses her action once and for all (**one-shot**), and the players choose their actions “simultaneously” in the sense that no player is informed, when she chooses her action, of the action chosen by any other player.
- We will start our discussion with the introduction of two player, two action (2x2) games. Two player games are also known as bilateral games. *It should be noted that a game can have any arbitrary number of players and any number of actions (finite or infinite action space); More on this later..*

Simultaneous Games

Consider a two-player game. It is typically summarized using a payoff matrix as follows:

	$Left(L)$	$Right(R)$
$Up(U)$	$u_1(U,L), u_2(U,L)$	$u_1(U,R), u_2(U,R)$
$Down(D)$	$u_1(D,L), u_2(D,L)$	$u_1(D,R), u_2(D,R)$

- There are two players: player 1 is choosing a row (row player) and player 2 is choosing a column (column player).
- There are two actions available for each player: player 1 can choose either "Up" or "Down"; player 2 can choose between "Left" or "Right".

Simultaneous Games

	$Left(L)$	$Right(R)$
$Up(U)$	$u_1(U,L), u_2(U,L)$	$u_1(U,R), u_2(U,R)$
$Down(D)$	$u_1(D,L), u_2(D,L)$	$u_1(D,R), u_2(D,R)$

- Player 1's payoff function is $u_1(.)$ and player 2's payoff function is $u_2(.)$.
- These payoff functions are defined over the outcomes. There are four possible outcomes of the play of the one-shot simultaneous game: (U,L), (U,R), (D,L), (D,R). The first entry is for player 1's choice and second entry is for player 2's choice of action.

Coordination Game

	Opera	*Football*
Opera	$2, 1$	$0, 0$
Football	$0, 0$	$1, 2$

- Imagine a couple (husband and wife) that agreed to meet this evening, but cannot recall if they will be attending the opera or a football match (perhaps they did not finalize the meeting place!).
- No communication is allowed between the players. Assume that one of their mobile phones is not working or the battery is down so that they cannot call/message each other.

Coordination Game

	Opera	*Football*
Opera	$2, 1$	$0, 0$
Football	$0, 0$	$1, 2$

- The husband would like to go to the football game whereas the wife would like to go to the opera. Both would prefer to go to the same place ("togetherness") than to spend the evening alone.
- Wife is choosing a row (row player) and husband is choosing a column (column player).
- This two person coordination game is also known as the "Battle of the Sexes" game.

Matching Pennies Game

- Two people choose, simultaneously, whether to show the head or the tail of a coin.
- If they show the same side, person 2 pays person 1 a dollar.
- If they show different sides, person 1 pays person 2 a dollar.

	Head	*Tail*
Head	$1, -1$	$-1, 1$
Tail	$-1, 1$	$1, -1$

Person 1 is the row player and person 2 is the column player.

Matching Pennies Game

	Head	*Tail*
Head	$1, -1$	$-1, 1$
Tail	$-1, 1$	$1, -1$

- In this game, the players' interests are diametrically opposed: such a game is called "strictly competitive".
- Player 1 wants to take the same action as the other player (match the other player's action), whereas player 2 wants to take the opposite action.
- This game is also an example of a "zero-sum game" where one player's gain is exactly equal to the other player's loss under each possible outcome realization.

Stag Hunt Game

	Stag	*Hare*
Stag	$2, 2$	$0, 1$
Hare	$1, 0$	$1, 1$

- Two individuals go out on a hunt. Each can individually choose to hunt a stag (a male deer) or hunt a hare.
- Each player must choose an action without knowing the choice of the other.
- If an individual hunts a stag, he must have the cooperation of his partner in order to succeed: you cannot hunt a stag alone.

Stag Hunt Game

	Stag	*Hare*
Stag	2, 2	0, 1
Hare	1, 0	1, 1

- An individual can get a hare by himself, but a hare is worth less than a stag.
- This is a game of coordination. Each player wants to coordinate on hunting the same animal.

Game of Chicken/ Hawk-Dove Game

	Turn	*Straight*
Turn	$0, 0$	$-1, +1$
Straight	$+1, -1$	$-10, -10$

- Two drivers drive towards each other on a collision course: one must turn, or both may die in the crash (or incur severe injuries).
- If one driver turns and the other does not, the one who turned will be called a “chicken” meaning a coward.
- This game is also known as the “snowdrift” game. This is an influential model of conflict for two players.

Constant-sum Game

	L	R
U	$5, 3$	$4, 4$
D	$0, 8$	$2, 6$

- The above game is an example of a strategic situation where the sum of the players' payoffs add up to a constant for each outcome. Such a game is known as a constant-sum game.
- In this example, the constant is 8.

References

Although the lecture slide is self-contained, you might want to refer to the following sections of the textbook (GAMES OF STRATEGY by Dixit, Skeath, Reiley):

2.1 Decisions versus games

2.2 Classifying games: subsections A and B

2.3 Some terminology and background assumptions: subsections A and B

2.4 The uses of game theory

4.1 Depicting simultaneous-move games with discrete strategies