# Lecture 1

# Vector
# and
# Fundamental Forces in Nature

PHY 1202

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香港城市大學
City University of Hong Kong

# Lecture Outline

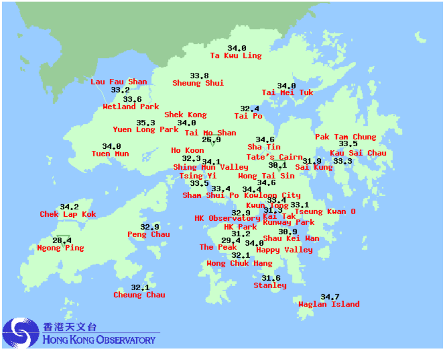
- **Chapter 3,** D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Review of Vectors
  - Vector in Cartesian Coordinate
  - Position and unit vectors
  - Vector algebra
  - Vector product
  - Vectors in Cylindrical and Spherical coordinates.
- Fundamental interactions in Nature.
  - Forces in Nature.

# 3.2 Vectors and Scalars

## Air Temperature Distribution

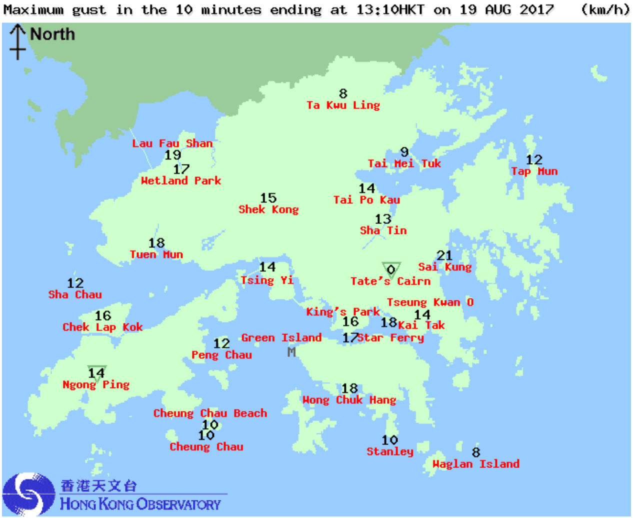
Air temperature at 13:10 HKT on 19 AUG 2017 (°C)




## Average Wind Speed and Wind Direction Distribution

Mean wind in the 10 minutes ending at 13:10HKT on 19 AUG 2017

# Maximum Wind Gust in 10 Minutes



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# 3.2 Vectors and Scalars

Arrows are used to represent vectors.

- ➢ The length of the arrow signifies magnitude
- ➢ The head of the arrow signifies direction

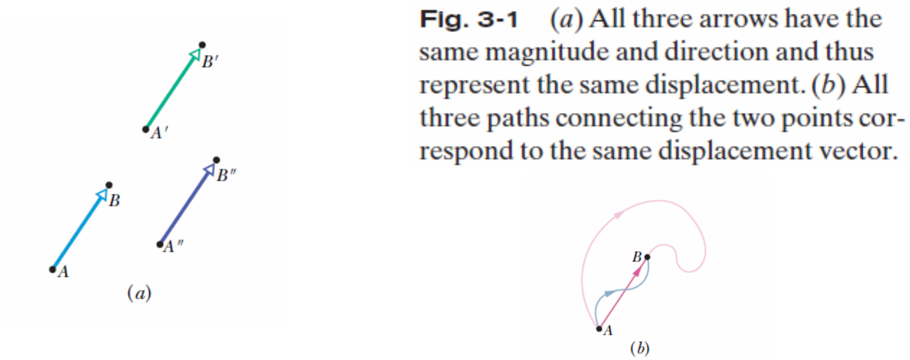



**Fig. 3-1** (*a*) All three arrows have the same magnitude and direction and thus represent the same displacement. (*b*) All three paths connecting the two points correspond to the same displacement vector.

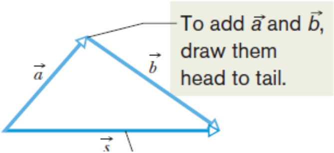
Sometimes the vectors are represented by bold lettering, such as vector **a**. Sometimes they are represented with arrows on the top, such as $\vec{a}$

# 3.3 Adding vectors geometrically

Vector **a** and vector **b** can be added geometrically to yield the resultant vector sum, **s**.

$$\vec{s} = \vec{a} + \vec{b},$$

Place the second vector, **b**, with its tail touching the head of the first vector, **a**. The vector sum, **s**, is the vector joining the tail of **a** to the head of **b**.



# 3.3 Adding vectors geometrically

Some rules:

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad \text{(commutative law).}$$

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \qquad \text{(associative law).}$$

$$\vec{b} + (-\vec{b}) = 0.$$

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) \qquad \text{(vector subtraction)}$$



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# 3.3 Adding vectors…sample problem

In an orienteering class, you have the goal of moving as far (straight-line distance) from base camp as possible by making three straight-line moves. You may use the following displacements in any order: (a) $\vec{a}$, 2.0 km due east (directly toward the east); (b) $\vec{b}$, 2.0 km 30° north of east (at an angle of 30° toward the north from due east); (c) $\vec{c}$, 1.0 km due west. Alternatively, you may substitute either $-\vec{b}$ for $\vec{b}$ or $-\vec{c}$ for $\vec{c}$. What is the greatest distance you can be from base camp at the end of the third displacement?

***Reasoning:*** Using a convenient scale, we draw vectors $\vec{a}$, $\vec{b}$, $\vec{c}$, $-\vec{b}$, and $-\vec{c}$ as in Fig. 3-7*a*. We then mentally slide the vectors over the page, connecting three of them at a time in head-to-tail arrangements to find their vector sum $\vec{d}$. The tail of the first vector represents base camp. The head of the third vector represents the point at which you stop. The vector sum $\vec{d}$ extends from the tail of the first vector to the head of the third vector. Its magnitude $d$ is your distance from base camp.

We find that distance $d$ is greatest for a head-to-tail arrangement of vectors $\vec{a}$, $\vec{b}$, and $-\vec{c}$. They can be in any order, because their vector sum is the same for any order.

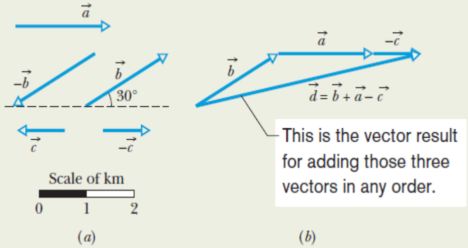


**Fig. 3-7** (*a*) Displacement vectors; three are to be used. (*b*) Your distance from base camp is greatest if you undergo displacements $\vec{a}$, $\vec{b}$, and $-\vec{c}$, in any order.

The order shown in Fig. 3-7*b* is for the vector sum

$$\vec{d} = \vec{b} + \vec{a} + (-\vec{c}).$$

Using the scale given in Fig. 3-7*a*, we measure the length $d$ of this vector sum, finding

$$d = 4.8 \text{ m}. \qquad \text{(Answer)}$$

# 3.4 Components of vectors



The component of a vector along an axis is the projection of the vector onto that axis.

The process of finding the components of a vector is called resolution of the vector.

In 3-dimensions, there are three components of a vector along pre-defined x-, y-, and z-axes.

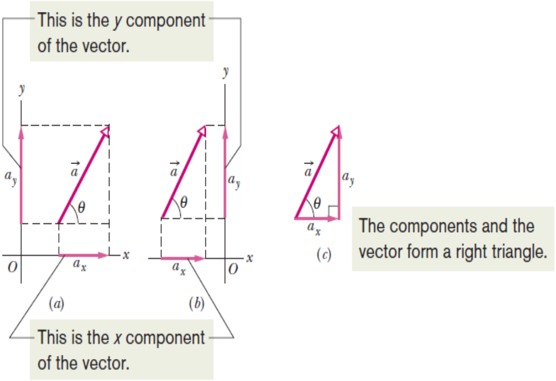


**Fig. 3-8** (a) The components $a_x$ and $a_y$ of vector $\vec{a}$. (b) The components are unchanged if the vector is shifted, as long as the magnitude and orientation are maintained. (c) The components form the legs of a right triangle whose hypotenuse is the magnitude of the vector.

**We find the components of a vector by using the right triangle rules.**

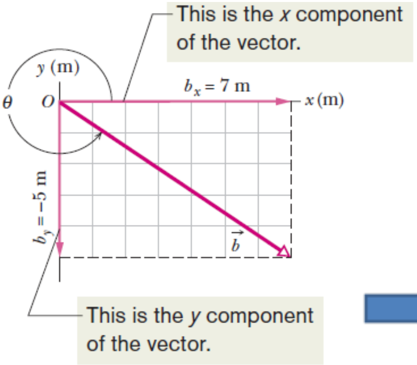


**Fig. 3-9** The component of $\vec{b}$ on the $x$ axis is positive, and that on the $y$ axis is negative.

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x}$$

# Example, vectors:



A small airplane leaves an airport on an overcast day and is later sighted 215 km away, in a direction making an angle of 22° east of due north. How far east and north is the airplane from the airport when sighted?

**KEY IDEA**

We are given the magnitude (215 km) and the angle (22° east of due north) of a vector and need to find the components of the vector.

***Calculations:*** We draw an *xy* coordinate system with the positive direction of *x* due east and that of *y* due north (Fig. 3-10). For convenience, the origin is placed at the airport. The airplane's displacement $\vec{d}$ points from the origin to where the airplane is sighted.

To find the components of $\vec{d}$, we use Eq. 3-5 with $\theta = 68° (= 90° - 22°)$:

$$d_x = d\cos\theta = (215\text{ km})(\cos 68°) = 81\text{ km} \qquad \text{(Answer)}$$

$$d_y = d\sin\theta = (215\text{ km})(\sin 68°) = 199\text{ km} \approx 2.0\times10^2\text{ km}. \qquad \text{(Answer)}$$

Thus, the airplane is 81 km east and $2.0 \times 10^2$ km north of the airport.

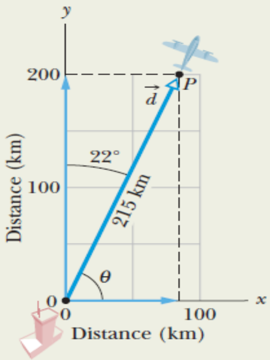


**Fig. 3-10** A plane takes off from an airport at the origin and is later sighted at *P*.



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# 3.4: Problem Solving Check-points

# 3.5: Unit vectors

A unit vector is a vector of unit magnitude, pointing in a particular direction.

Unit vectors pointing in the x-, y-, and z-axes are usually designated by

respectively $\hat{i}, \hat{j}, \hat{k}$

Therefore vector, $\vec{a}$,
with components ax and ay in the x- and y-directions,
can be written in terms of the following vector sum:

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

The unit vectors point along axes.

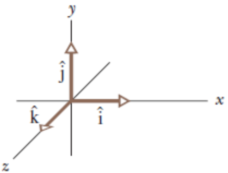


**Fig. 3-13** Unit vectors $\hat{i}$, $\hat{j}$, and $\hat{k}$ define the directions of a right-handed coordinate system.

$a_x$ is the magnitud

$a_y$ is the magnitud

2 dimension:

# 3.6: Vector Algebra - Adding vectors by components

If $\vec{r} = \vec{a} + \vec{b}$,

then
$$r_x = a_x + b_x$$
$$r_y = a_y + b_y$$
$$r_z = a_z + b_z.$$

Therefore, two vectors must be equal if their corresponding components are equal.

The procedure of adding vectors also applies to vector subtraction.

Therefore, $\vec{d} = \vec{a} - \vec{b}$ $\Rightarrow$ $d_x = a_x - b_x, \quad d_y = a_y - b_y, \quad \text{and} \quad d_z = a_z - b_z,$

where $\vec{d} = d_x\hat{\mathrm{i}} + d_y\hat{\mathrm{j}} + d_z\hat{\mathrm{k}}.$



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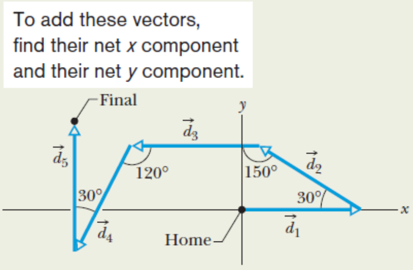
# Example

The desert ant *Cataglyphis fortis* lives in the plains of the Sahara desert. When one of the ants forages for food, it travels from its home nest along a haphazard search path, over flat, featureless sand that contains no landmarks. Yet, when the ant decides to return home, it turns and then runs directly home. According to experiments, the ant keeps track of its movements along a mental coordinate system. When it wants to return to its home nest, it effectively sums its displacements along the axes of the system to calculate a vector that points directly home. As an example of the calculation, let's consider an ant making five runs of 6.0 cm each on an *xy* coordinate system, in the directions shown in Fig. 3-16*a*, starting from home. At the end of the fifth run, what are the magnitude and angle of the ant's net displacement vector $\vec{d}_{net}$, and what are those of the homeward vector $\vec{d}_{home}$ that extends from the ant's final position back to home? In a real situation, such vector calculations might involve thousands of such runs.



(*a*)

# Example, vector addition:

The desert ant *Cataglyphis fortis* lives in the plains of the Sahara desert. When one of the ants forages for food, it travels from its home nest along a haphazard search path, over flat, featureless sand that contains no landmarks. Yet, when the ant decides to return home, it turns and then runs directly home. According to experiments, the ant keeps track of its movements along a mental coordinate system. When it wants to return to its home nest, it effectively sums its displacements along the axes of the system to calculate a vector that points directly home. As an example of the calculation, let's consider an ant making five runs of 6.0 cm each on an $xy$ coordinate system, in the directions shown in Fig. 3-16*a*, starting from home. At the end of the fifth run, what are the magnitude and angle of the ant's net displacement vector $\vec{d}_{\text{net}}$, and what are those of the homeward vector $\vec{d}_{\text{home}}$ that extends from the ant's final position back to home? In a real situation, such vector calculations might involve thousands of such runs.

**KEY IDEAS**

(1) To find the net displacement $\vec{d}_{\text{net}}$, we need to sum the five individual displacement vectors:

$$\vec{d}_{\text{net}} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 + \vec{d}_4 + \vec{d}_5.$$

(2) We evaluate this sum for the $x$ components alone,

$$d_{\text{net},x} = d_{1x} + d_{2x} + d_{3x} + d_{4x} + d_{5x},$$

and for the $y$ components alone,

$$d_{\text{net},y} = d_{1y} + d_{2y} + d_{3y} + d_{4y} + d_{5y}.$$

(3) We construct $\vec{d}_{\text{net}}$ from its $x$ and $y$ components.

**Calculations:**

$$d_{1x} = (6.0\text{ cm})\cos 0° = +6.0\text{ cm}$$
$$d_{2x} = (6.0\text{ cm})\cos 150° = -5.2\text{ cm}$$
$$d_{3x} = (6.0\text{ cm})\cos 180° = -6.0\text{ cm}$$
$$d_{4x} = (6.0\text{ cm})\cos(-120°) = -3.0\text{ cm}$$
$$d_{5x} = (6.0\text{ cm})\cos 90° = 0.$$

$$\begin{aligned} d_{\text{net},x} &= +6.0\text{ cm} + (-5.2\text{ cm}) + (-6.0\text{ cm}) \\ &\quad + (-3.0\text{ cm}) + 0 \\ &= -8.2\text{ cm}. \end{aligned}$$

$$d_{\text{net},y} = +3.8\text{ cm}.$$

**TABLE 3-1**

| Run | $d_x$ (cm) | $d_y$ (cm) |
|---|---|---|
| 1 | +6.0 | 0 |
| 2 | −5.2 | +3.0 |
| 3 | −6.0 | 0 |
| 4 | −3.0 | −5.2 |
| 5 | 0 | +6.0 |
| net | −8.2 | +3.8 |

Vector $\vec{d}_{\text{net}}$ and its $x$ and $y$ components are shown in Fig. 3-16*b*. To find the magnitude and angle of $\vec{d}_{\text{net}}$ from its components, we use Eq. 3-6. The magnitude is

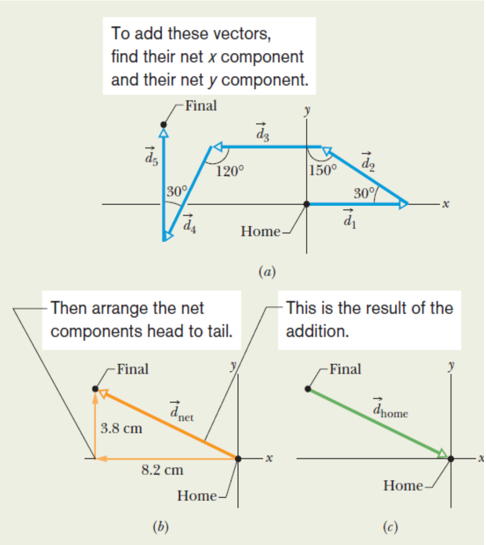
$$\begin{aligned} d_{\text{net}} &= \sqrt{d_{\text{net},x}^2 + d_{\text{net},y}^2} \\ &= \sqrt{(-8.2\text{ cm})^2 + (3.8\text{ cm})^2} = 9.0\text{ cm}. \end{aligned}$$

To find the angle (measured from the positive direction of $x$), we take an inverse tangent:

$$\begin{aligned} \theta &= \tan^{-1}\left(\frac{d_{\text{net},y}}{d_{\text{net},x}}\right) \\ &= \tan^{-1}\left(\frac{3.8\text{ cm}}{-8.2\text{ cm}}\right) = -24.86°. \end{aligned}$$

# Example, vector addition:

## Note:




*Caution:* Taking an inverse tangent on a calculator may not give the correct answer. The answer −24.86° indicates that the direction of $\vec{d}_{net}$ is in the fourth quadrant of our *xy* coordinate system. However, when we construct the vector from its components (Fig. 3-16*b*), we see that the direction of $\vec{d}_{net}$ is in the second quadrant. Thus, we must "fix" the calculator's answer by adding 180°:

$$\theta = -24.86^\circ + 180^\circ = 155.14^\circ \approx 155^\circ.$$

Thus, the ant's displacement $\vec{d}_{net}$ has magnitude and angle

$$d_{net} = 9.0 \text{ cm at } 155^\circ. \qquad \text{(Answer)}$$

Vector $\vec{d}_{home}$ directed from the ant to its home has the same magnitude as $\vec{d}_{net}$ but the opposite direction (Fig. 3-16*c*). We already have the angle ($-24.86^\circ \approx -25^\circ$) for the direction opposite $\vec{d}_{net}$. Thus, $\vec{d}_{home}$ has magnitude and angle

$$d_{home} = 9.0 \text{ cm at } -25^\circ. \qquad \text{(Answer)}$$

A desert ant traveling more than 500 m from its home will actually make thousands of individual runs. Yet, it somehow knows how to calculate $\vec{d}_{home}$ (without studying this chapter).

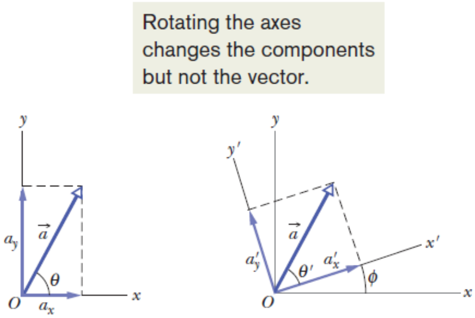
## Freedom of choosing a coordinate system

Relations among vectors do not depend on the origin or the orientation of the axes.

Relations in physics are also independent of the choice of the coordinate system.

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{a_x'^2 + a_y'^2}$$

$$\theta = \theta' + \phi.$$



# 3.8: Multiplying vectors

## *A. Multiplying a vector by a scalar*

Multiplying a vector by a scalar changes the magnitude but not the direction:

$$\vec{a} \times s = s\vec{a}$$

## B. Multiplying a vector by a vector: Scalar (Dot) Product

The scalar product between two vectors is written as:

$$\vec{a}\cdot\vec{b}$$

It is defined as:

$$\vec{a}\cdot\vec{b} = ab\cos\phi,$$

Here, a and b are the magnitudes of vectors a and b respectively, and f is the angle between the two vectors.
The right hand side is a scalar quantity.

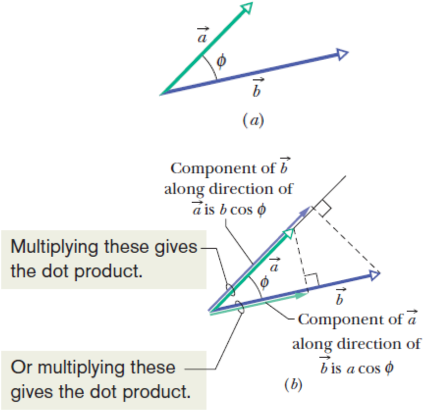



**Fig. 3-18** (*a*) Two vectors $\vec{a}$ and $\vec{b}$, with an angle $\phi$ between them. (*b*) Each vector has a component along the direction of the other vector.

# Dot product in terms of components

$$\hat{\imath}\cdot\hat{\imath} = \hat{\jmath}\cdot\hat{\jmath} = \hat{k}\cdot\hat{k} = (1)(1)\cos 0° = 1$$
$$\hat{\imath}\cdot\hat{\jmath} = \hat{\imath}\cdot\hat{k} = \hat{\jmath}\cdot\hat{k} = (1)(1)\cos 90° = 0$$

$$\begin{aligned}\vec{A}\cdot\vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})\cdot(B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\ &= A_x\hat{\imath}\cdot B_x\hat{\imath} + A_x\hat{\imath}\cdot B_y\hat{\jmath} + A_x\hat{\imath}\cdot B_z\hat{k} \\ &\quad + A_y\hat{\jmath}\cdot B_x\hat{\imath} + A_y\hat{\jmath}\cdot B_y\hat{\jmath} + A_y\hat{\jmath}\cdot B_z\hat{k} \\ &\quad + A_z\hat{k}\cdot B_x\hat{\imath} + A_z\hat{k}\cdot B_y\hat{\jmath} + A_z\hat{k}\cdot B_z\hat{k} \\ &= A_xB_x\hat{\imath}\cdot\hat{\imath} + \underline{A_xB_y\hat{\imath}\cdot\hat{\jmath}} + \underline{A_xB_z\hat{\imath}\cdot\hat{k}} \\ &\quad + \underline{A_yB_x\hat{\jmath}\cdot\hat{\imath}} + A_yB_y\hat{\jmath}\cdot\hat{\jmath} + \underline{A_yB_z\hat{\jmath}\cdot\hat{k}} \\ &\quad + \underline{A_zB_x\hat{k}\cdot\hat{\imath}} + \underline{A_zB_y\hat{k}\cdot\hat{\jmath}} + A_zB_z\hat{k}\cdot\hat{k}\end{aligned}$$

Terms underlined red are zero

$$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$$ (scalar (dot) product in terms of components)

You only need to know this expression for examination, no need to understand the derivation for examination

# C. Multiplying a vector with a vector: Vector (Cross) Product

The vector product between two vectors a and b can be written as:

$$\vec{a} \times \vec{b}.$$

The result is a new vector c, which is:

$$c = ab \sin \phi, \qquad \text{Eq. 3-27}$$

Here a and b are the magnitudes of vectors **a** and **b** respectively, and f is the smaller of the two angles between **a** and **b** vectors.

The right-hand rule allows us to find the direction of vector c.

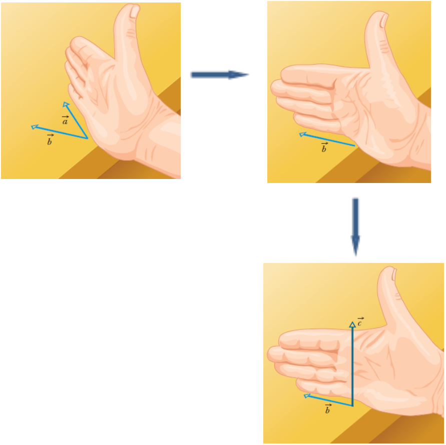
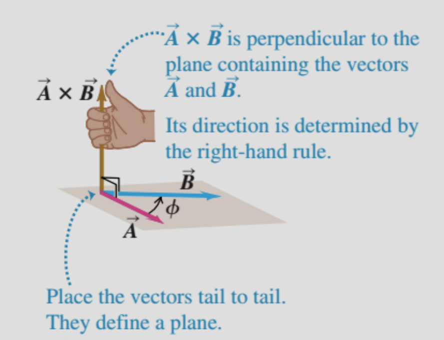



**Fig. 3-19** Illustration of the right-hand rule for vector products. (*a*) Sweep vector $\vec{a}$ into vector $\vec{b}$ with the fingers of your right hand. Your outstretched thumb shows the direction of vector $\vec{c} = \vec{a} \times \vec{b}$.

# Direction of cross product(vector product)

•

- $\vec{C} = \vec{A} \times \vec{B}$
- Directio … ndicu perpend … an cc



## 3.8: Multiplying vectors; vector product in unit-vector notation:

$$\vec{a}\times\vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}}) \times (b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$$

$$= (a_yb_z - b_ya_z)\hat{\mathrm{i}} + (a_zb_x - b_za_x)\hat{\mathrm{j}} + (a_xb_y - b_xa_y)\hat{\mathrm{k}}.$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_yb_z - b_ya_z)\hat{i} + (a_zb_x - b_za_x)\hat{j} + (a_xb_y - b_xa_y)\hat{k}$$

Evaluate this determinant

Note that: $a_x\hat{\mathrm{i}} \times b_x\hat{\mathrm{i}} = a_xb_x(\hat{\mathrm{i}} \times \hat{\mathrm{i}}) = 0,$

And, $a_x\hat{\mathrm{i}} \times b_y\hat{\mathrm{j}} = a_xb_y(\hat{\mathrm{i}} \times \hat{\mathrm{j}}) = a_xb_y\hat{\mathrm{k}}$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \vec{i}\cdot\begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} - \vec{j}\cdot\begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} + \vec{i}\cdot\begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix}$$

$$\vec{a}\times\vec{b} = (a_yb_z - a_zb_y)\cdot\vec{i} + (a_xb_z - a_zb_x)\cdot\vec{j} + (a_xb_y - a_yb_x)\cdot\vec{k}.$$



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# Derivation of the cross product expression this derivation is not needed for examination

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\begin{aligned}
\hat{\imath} \times \hat{\jmath} &= -\hat{\jmath} \times \hat{\imath} = \hat{k} \\
\hat{\jmath} \times \hat{k} &= -\hat{k} \times \hat{\jmath} = \hat{\imath} \\
\hat{k} \times \hat{\imath} &= -\hat{\imath} \times \hat{k} = \hat{\jmath}
\end{aligned}$$

$$\begin{aligned}
\vec{A} \times \vec{B} &= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) \times (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}) \\
&= A_x\hat{\imath} \times B_x\hat{\imath} + A_x\hat{\imath} \times B_y\hat{\jmath} + A_x\hat{\imath} \times B_z\hat{k} \\
&\quad + A_y\hat{\jmath} \times B_x\hat{\imath} + A_y\hat{\jmath} \times B_y\hat{\jmath} + A_y\hat{\jmath} \times B_z\hat{k} \\
&\quad + A_z\hat{k} \times B_x\hat{\imath} + A_z\hat{k} \times B_y\hat{\jmath} + A_z\hat{k} \times B_z\hat{k}
\end{aligned}$$

$$\vec{A} \times \vec{B} = (A_yB_z - A_zB_y)\,\hat{\imath} + (A_zB_x - A_xB_z)\hat{\jmath} + (A_xB_y - A_yB_x)\hat{k}$$

# Sample problem, vector product

In Fig. 3-20, vector $\vec{a}$ lies in the $xy$ plane, has a magnitude of 18 units and points in a direction 250° from the positive direction of the $x$ axis. Also, vector $\vec{b}$ has a magnitude of 12 units and points in the positive direction of the $z$ axis. What is the vector product $\vec{c} = \vec{a} \times \vec{b}$?

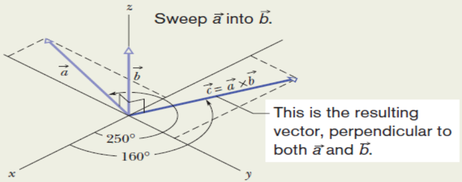



**Fig. 3-20** Vector $\vec{c}$ (in the $xy$ plane) is the vector (or cross) product of vectors $\vec{a}$ and $\vec{b}$.

**KEY IDEA**

When we have two vectors in magnitude-angle notation, we find the magnitude of their cross product with Eq. 3-27 and the direction of their cross product with the right-hand rule of Fig. 3-19.

***Calculations:*** For the magnitude we write

$$c = ab \sin \phi = (18)(12)(\sin 90°) = 216. \qquad \text{(Answer)}$$

To determine the direction in Fig. 3-20, imagine placing the fingers of your right hand around a line perpendicular to the plane of $\vec{a}$ and $\vec{b}$ (the line on which $\vec{c}$ is shown) such that your fingers sweep $\vec{a}$ into $\vec{b}$. Your outstretched thumb then

gives the direction of $\vec{c}$. Thus, as shown in the figure, $\vec{c}$ lies in the $xy$ plane. Because its direction is perpendicular to the direction of $\vec{a}$ (a cross product always gives a perpendicular vector), it is at an angle of

$$250° - 90° = 160° \qquad \text{(Answer)}$$

from the positive direction of the $x$ axis.

# Sample problem, vector product, unit vector notation

If $\vec{a} = 3\hat{i} - 4\hat{j}$ and $\vec{b} = -2\hat{i} + 3\hat{k}$, what is $\vec{c} = \vec{a} \times \vec{b}$?

**KEY IDEA**

When two vectors are in unit-vector notation, we can find their cross product by using the distributive law.

***Calculations:*** Here we write

$$\begin{aligned}\vec{c} &= (3\hat{i} - 4\hat{j}) \times (-2\hat{i} + 3\hat{k})\\ &= 3\hat{i} \times (-2\hat{i}) + 3\hat{i} \times 3\hat{k} + (-4\hat{j}) \times (-2\hat{i})\\ &\quad + (-4\hat{j}) \times 3\hat{k}.\end{aligned}$$

We next evaluate each term with Eq. 3-27, finding the direction with the right-hand rule. For the first term here, the angle $\phi$ between the two vectors being crossed is 0. For the other terms, $\phi$ is 90°. We find

$$\begin{aligned}\vec{c} &= -6(0) + 9(-\hat{j}) + 8(-\hat{k}) - 12\hat{i}\\ &= -12\hat{i} - 9\hat{j} - 8\hat{k}. \qquad \text{(Answer)}\end{aligned}$$

This vector $\vec{c}$ is perpendicular to both $\vec{a}$ and $\vec{b}$, a fact you can check by showing that $\vec{c}\cdot\vec{a} = 0$ and $\vec{c}\cdot\vec{b} = 0$; that is, there is no component of $\vec{c}$ along the direction of either $\vec{a}$ or $\vec{b}$.

# Vectors in Other Coordinate Systems

- It is often useful to express the position vector in coordinate systems other than Cartesian to handle geometries consisting certain symmetries.

- For example, problems concerning a long pipe or straight wire are best expressed in the cylindrical coordinate, while problems involving a point source or spheres are best solved in the spherical coordinate system.

# Cylindrical Coordinates

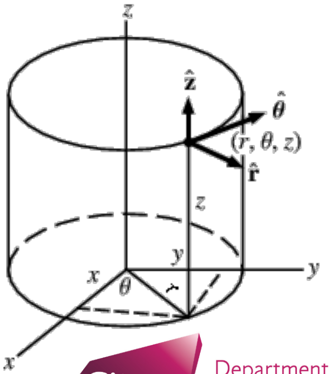
- Cylindrical coordinates are just the generalization of the two-dimensional polar coordinate ($r$, q ) to three-dimension by adding the $z$-axis.
- The coordinates are ($r$, q , $z$) corresponds to the (radial, azimuthal, vertical) axes.

- From Cylindrical to Cartesian

$$x = r\cos(\theta)$$
$$y = r\sin(\theta)$$
$$z = z$$

- From Cartesian to Cylindrical

$$r^2 = x^2 + y^2$$
$$\tan(\theta) = \frac{y}{x}$$
$$z = z$$

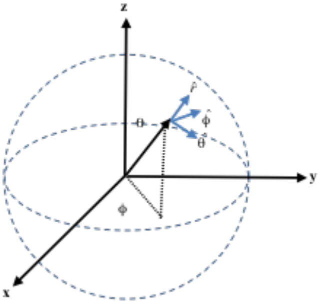


# Spherical Coordinates

- Spherical coordinates or spherical polar coordinates ($r$, q , f) is ideal for describing position on a sphere.
- The coordinates are ($r$, q , f) corresponds to the (radial, polar, azimuthal) axes.

- From Spherical to Cartesian

$$x = r\sin(\theta)\cos(\phi)$$
$$y = r\sin(\theta)\sin(\phi)$$
$$z = r\cos(\theta)$$




- From Cartesian to Spherical

$$r = \sqrt{x^2 + y^2 + z^2}$$
$$\theta = \cos^{-1}\left(\frac{z}{r}\right)$$
$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

# The Fundamental Forces of Nature

- The forces of nature are responsible for providing the energy we used from the energy and matter relationship.

- There are only four fundamental forces of nature that hold the whole world together.

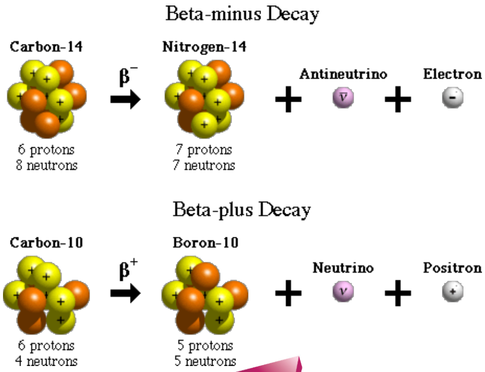
- The fundamental forces are:
  - Strong Nuclear Force
  - Weak Nuclear Force
  - Gravitational Force
  - Electromagnetic Force.

# The Strong Nuclear Force

- This force binds the protons and neutrons together within the nucleus.
- It is the strongest of the four forces
- It acts at extremely short range within the nucleus that keeps the protons from repelling each other.
- If disrupted, the energy released is tremendous (example: Sun, nuclear bombs)

# The Weak Nuclear Force

- Also acts at extremely small distances within the nucleus.
- It is responsible for radiative decay of elements called beta b-decay
- Weak force arises when a neutron is changed into a proton or a proton changes into a neutron. This also changes the identify of the element!
- The weak force (interaction) is responsible for radiative decay that plays an essential role in nuclear fission (the splitting of an atom).
- Energy is released in the form of electrons can be used to do work.



# Gravitational Force

- It is the weakest of the four fundamental forces
- It is an attractive force between all mass in the universe.
- It has infinite range that acts across the entire universe.
- The ‘particle’ that carries the force is called a graviton, however it has never been found.

# Electromagnetic Force

- It is the force between two charged bodies.
- It is an attractive force between positive and negative charges and is a repulsive force between like charges particles.
- This is the force that holds the atoms and all objects including you and me together.
- It also has infinite range.
- The 'messenger' particle that carries this force is called a photon.
- Electricity, magnetism, wireless, fiber optics, light, AP 1202, AP 2191, AP 3205, AP 4254, …. are all examples of this force.

# Calculus

- In physics we need to know some calculus for calculating some physical quantities
- We need to know how to do differentiation and integration
- In this course, we use a little bit calculus to understand the course content. They are not required in examination

# Differentiation (finding rate of change)

- Differentiation helps us to find the instantaneous rate of change of a variable with respect to another variable, such as growth rate
- For example, y changes with x. y is function of x. the rate of change of y with respect to x represents how fast y changes with x.
- Instantaneous means the rate of change at a particular value of x

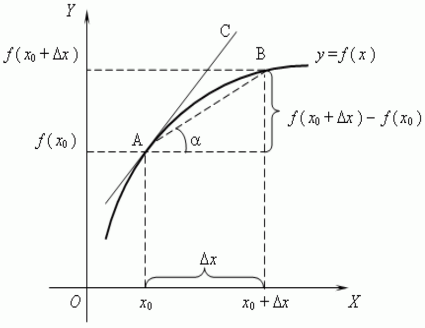



Fig. 1



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# Average and instantaneous rate of change

- X is changed from x to x0+Δx and y is change from f(x0) to f(x0+Δx).
- Δy/ Δx= (f(x0+Δx)- f(x0))/ Δx =Δf/Δx is the average rate of change for interval Δx
- The rate of change at point A is the instantaneous rate of change.
- This is obtained by putting Δx very small.

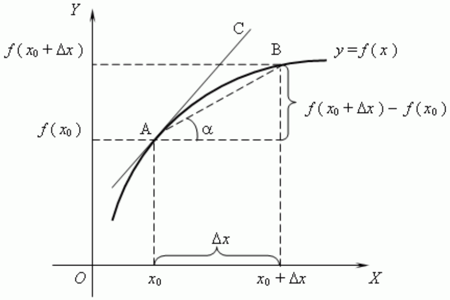


Fig. 1



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# Simple calculus

- Consider a function of
- The derivative (differe
  $\frac{df}{dx} = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$
  to x is defined as $\lim_{\Delta x \to 0}$
- $\Delta f = f(x_0+\Delta x) - f(x_0)$
- i.e., divide Δf by Δx an
  very small value (the r

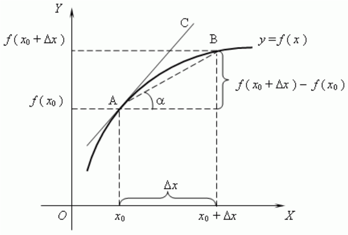


Fig. 1

- If f is distance, then the derivative with respect to time is the speed



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# Derivative of simple functions

$$f(x) = c,\ \frac{df}{dx} = 0 \quad ; \qquad f(x) = ax\,,\ \frac{df}{dx} = a$$

$$f(x) = x,\ \frac{df}{dx} = 1\ ; \quad f(x) = x^2\,,\ \frac{df}{dx} = 2x$$

$$f(x) = x^n\,,\ \frac{df}{dx} = nx^{n-1}\,; \quad f(x) = \sin x,\ \frac{df}{dx} = \cos x$$

$$f(x) = \cos x \quad ,\ \frac{df}{dx} = -\sin x$$

$$f(x) = cy(x)\,,\ \frac{df}{dx} = c\frac{dy}{dx}$$

*c is constant*



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# Sum rule and chain rule of differentiation

- Derivative of sum of two functions is the sum of the derivatives of the two functions
- For the exercise, you need the following formula and the rule given above.
- In examination, differentiation formula are given. What you need is understand the concepts. Know the calculation in the exercise and you should be able to do the calculation in examination

$$\frac{d(f(x) + g(}{dx}$$

$$f(x) =$$

$$\frac{df}{dx} = \frac{d(2x^2)}{dx} +$$



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# Example

$$\frac{d}{dx}(3x^2 - 2x + 1)$$

$$= 3\frac{d}{dx}(x^2) - 2\frac{d}{dx}(x) + \frac{d}{dx}(1)$$

$$= 3(2x) - 2(1) + (0)$$

$$= 6x - 2$$

$$f(x) = \sin($$

$$df \quad d$$



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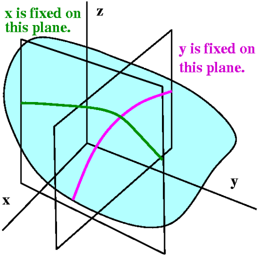
# Partial differentiation (partial derivative)

- The function is a function of two variables, x, y $z = f(x, y)$
- Keep one variable x or y constant and differentiate with respect to the other variable y or x.

$$\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x} = \frac{df}{dx} \quad \text{assuming y a constant}$$

$$\frac{\partial z}{\partial y} = \frac{\partial f}{\partial y} = \frac{df}{dy} \quad \text{assuming x a constant}$$



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# Integration

- Integration is reverse of diffe
- f is the differentiation (deriv
- then g is the integration of f
- $g = \int f \, dx$; g is called the ir
  to x



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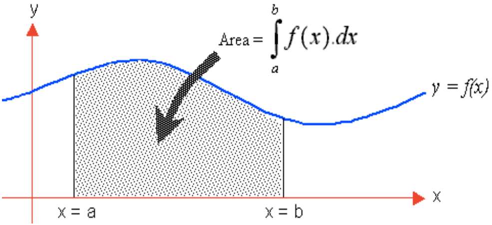
# Definite integral

Definite integral of f with re




- Integration of a curve from point a to point b represents the area under the curve



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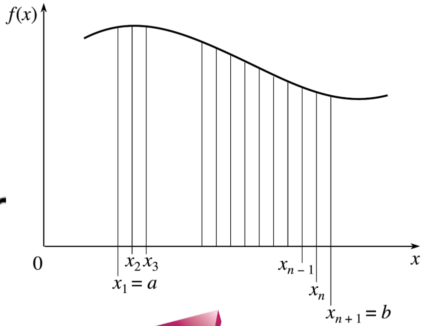
# Proof

Cut the area under the curve int

$x_{i+1}$ is a small rectangle.

The total area is approximately t

$$Area = \lim_{n\to\infty}\sum_{i=1}^{n} f\left(x_i\right)\Delta x = \int_a^b f(x)\,dx.$$




$\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$ m

the summation, when

larger (go to infinity) a

- 
  - $f(x_i) = \frac{dg}{dx} \cong \frac{g(x_i+\Delta x)-g(x_i)}{\Delta x}$
  - $f(x_i)\Delta x = g(x_i + \Delta x) - g(x$
  - $\sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n}(g(x_i +$

There are terr
$- g(x_i)$, when



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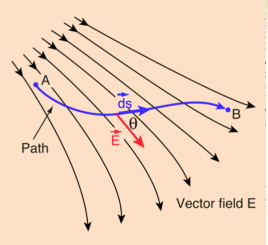
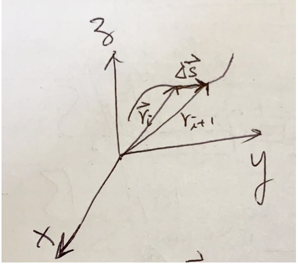
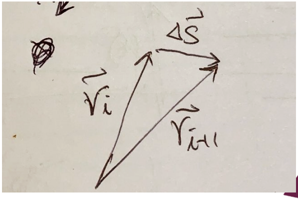
# Line integral of vector function (vector field)

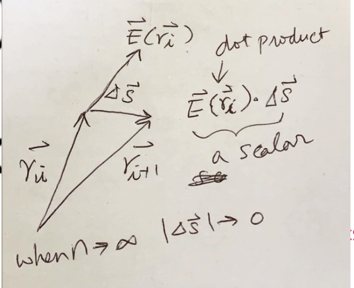
In a two dimension a
space, there is a vec
function) $\vec{E}$. Vector f
position there is a ve
function of position $\vec{r}$



- The line integral of a vecto
value of the sum of the ter
larger and larger, i.e. $n \to$ c
$\int_{path\ A\ to\ B} \vec{E}(\vec{r})$
- The general calc

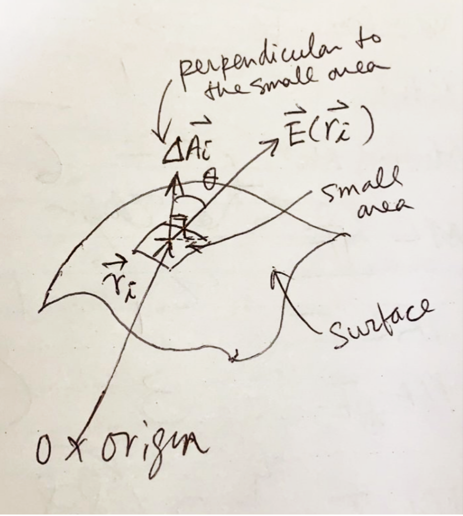


# A simple case of line integral of a vector function

- 
  - The path is a straight line be
  
    function is a constant vect
  - The line integral $\int_A^B \vec{E} \cdot d\vec{s} =$
  
    $(\vec{r}_B - \vec{r}_A)/n$
  - $\sum_{i=1}^{n} \vec{E} \cdot (\vec{r}_B - \vec{r}_A)/n = \vec{E} \cdot ($

# Surface integral of a vector function (vector field)

- Consider the
  (vector field)
- Consider a su
- You divide th
  many small a
- Each small ar



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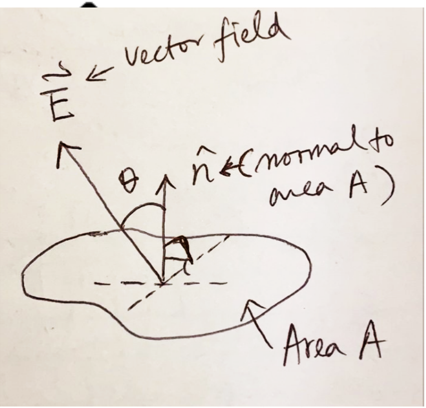
# Surface integral of vector function

- Definition: $\int_{surface} \vec{E} \cdot d\vec{A} =$
- $= \lim_{n\to\infty} \sum_{i=1}^{n} E\,(\vec{r}_i)\Delta \mathrm{A}_{\mathrm{i}} \cos\theta,$
- For each small area, we calcul
  then we add all the n terms to

# Simple example of surface integral

- 
  - Consider the simple case, w
  - constant $\vec{E}$ and the surface i
  - normal directio
  - The surface int
  - $\vec{\mathrm{E}} \cdot \vec{A} = \vec{E} \cdot \hat{n}A$



# Integration

- What you expect to know and understand for integration:
- The meaning of integration:
- Reverse of differentiation
- Area under a curve
- Line and surface integral



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