Chapter 4

Work, Kinetic Energy, Potential Energy

Topics for Part 1

- Definition of work done by a force?
- the definition of kinetic energy
- relation between work and kinetic energy
- Work done by a varying force

Reason for learning work and energy

- The simple methods of chapter 3 using Newton's laws are inadequate when the forces are not constant (a is not constant).
- Use concepts of *work*, *energy*, and the *conservation of energy* to help us solve problems without involvement of acceleration.
- Work and energy are used to understand many phenomena of heat. Fundamental concepts of heat physics (thermodynamics)
- Human activities require Work and Energy (fuel, electricity, food)



What is Work ?

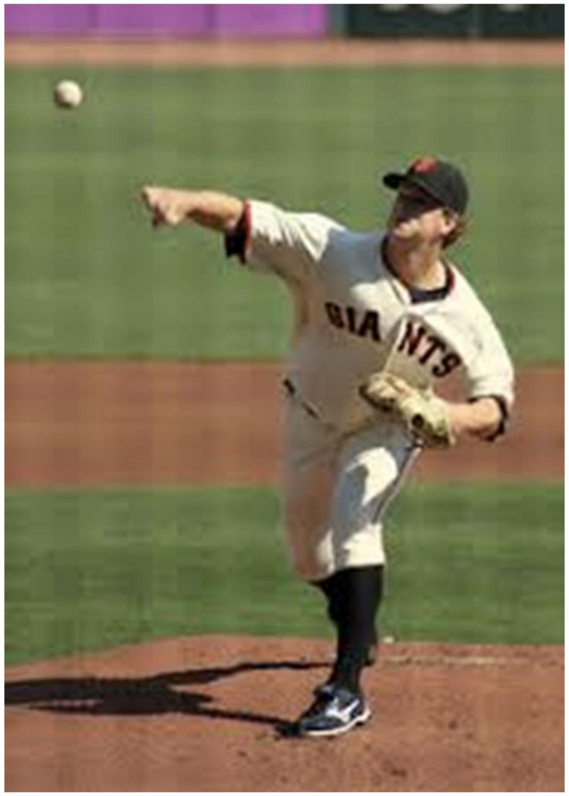
- When we use force to move an object with mass, we need to work hard and afterward we feel that we have done and output something because we are tired

- We need to quantify this something we output or spent in moving an object with a force

- This is the origin of the concept of work and it is related to force

What is energy?

- Player throws the ball, he has "done something" on the ball
- The ball moves fast, and acquires something really powerful -- it can smash a glass of window.



- The thing that the ball acquires is known as energy or more specifically, kinetic energy
- The player also has to work hard for the ball to have energy

Work → Energy

- Consider the following kinematic relation:

$$2a_x \Delta x = v_x^2 - v_{x0}^2$$

An object accelerated with a_x for a distance of Δx will acquire a velocity change from v_{x0} to v_x.

- But acceleration a_x is produced by a force F_x (Nt's 2nd law):

$$F_x = ma_x$$

- Multiply first equation by m and it becomes:

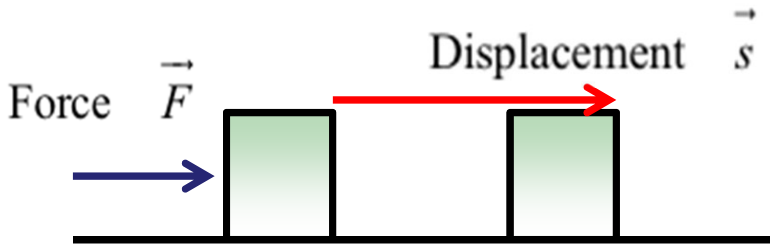
$$\underbrace{\boxed{F_x \Delta x}}_{\text{Work by a force}} = \underbrace{\boxed{\frac{1}{2}mv_x^2 - \frac{1}{2}mv_{x0}^2}}_{\text{Energy acquired by an object}}$$

Work done by a force

$$\boxed{F_x \Delta x} = \frac{1}{2} m v_x^2 - \frac{1}{2} m v_{x0}^2$$

$\boxed{\text{Work done} = \text{Force} \times \text{distance} = Fs}$

- Larger force = more energy = larger work done
- Longer distance = more energy = larger work done



People push the car with a force over a distance do work on the car

2-Dimensional Work done by a force

$$F_x \Delta x = \frac{1}{2} m v_x^2 - \frac{1}{2} m v_{x0}^2$$

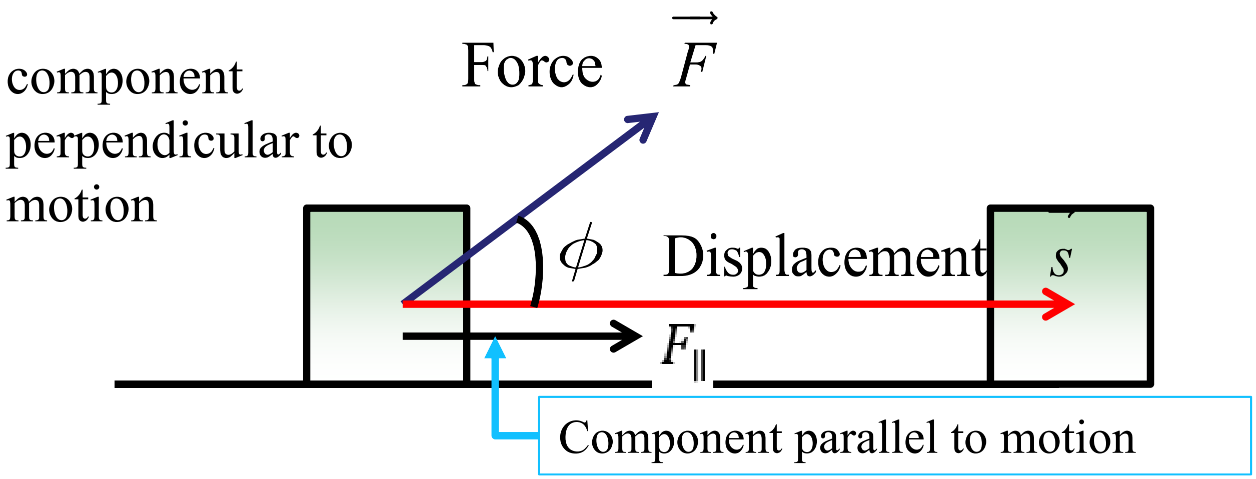
$$F_y \Delta y = \frac{1}{2} m v_y^2 - \frac{1}{2} m v_{y0}^2$$

Add the two:

$$\boxed{F_x \Delta x + F_y \Delta y} = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 - \frac{1}{2} m v_{x0}^2 - \frac{1}{2} m v_{y0}^2$$

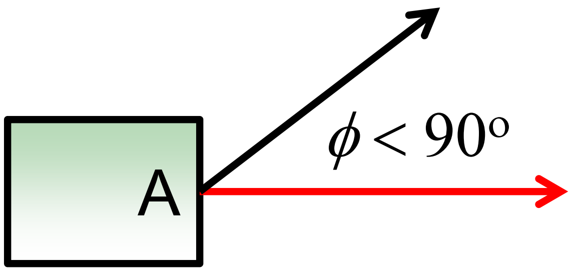
$$\Delta \vec{s} = \Delta x \hat{\imath} + \Delta y \hat{\jmath}$$

$$F_x \Delta x + F_y \Delta y = \boxed{\vec{F} \cdot \Delta \vec{s} = \text{Work} = F s \cos\phi} = F_{\parallel} \Delta s$$

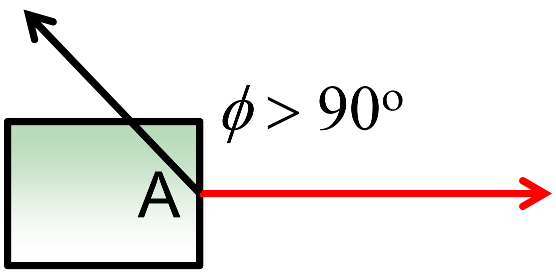


Positive, negative, and zero work

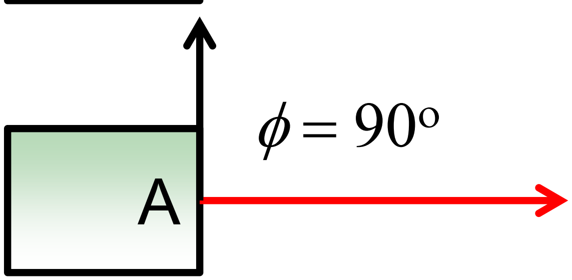
- The work done by a force can be positive, negative, or zero
- It depends on the angle between the force and the displacement $(\cos\phi)$.



Angle between force and displacement < 90 degree, positive work $\cos\phi > 0$
Energy transferred by force to the object A moved by the force, you give A some energy

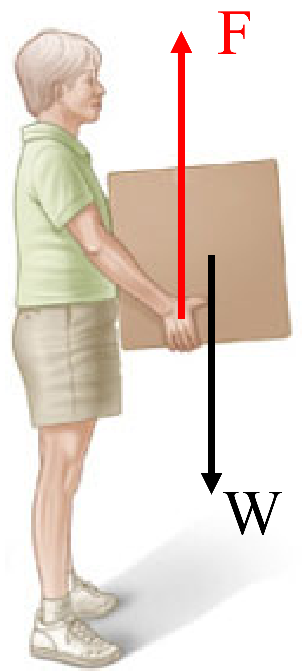


Angle between force and displacement > 90 degree, negative work $\cos\phi < 0$
Energy transferred from the moving object A to you



Angle between force and displacement = 90 degree, zero work $\cos\phi = 0$
No Energy transferred to and from the moving object A

Zero work, but why I feel tired by just holding



- A person is holding a heavy box but not lifting so he does zero work. But after a while of holding, his arm will feel tired. He may sweat if holding very long. Why?

- What happens is that he needs to produce a force to hold the heavy box by his arm to counter gravity.

- To produce a force , chemical reactions have to occur in his body. Chemical energy is consumed but is converted to heat energy (later chapter).

Kinetic energy (KE): definition

$$F_x \Delta x + F_y \Delta y = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 - \frac{1}{2}mv_{x0}^2 - \frac{1}{2}mv_{y0}^2 \quad = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

- The *kinetic energy (KE)* of a particle with mass m and speed v: $KE = 1/2\ mv^2$.

- **When an object (m) moves (*v*), it has K.E.** $= \frac{1}{2}mv^2$

Work and Kinetic energy (KE):

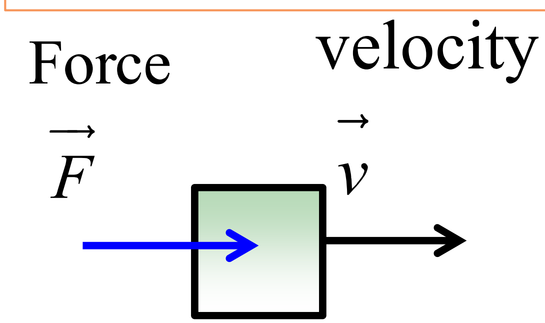
$$\boxed{F_x\Delta x + F_y\Delta y} = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 - \frac{1}{2}mv_{x0}^2 - \frac{1}{2}mv_{y0}^2 \quad = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

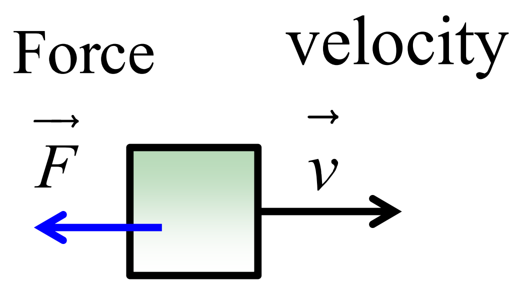
$$F_{\parallel}\Delta s \quad = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

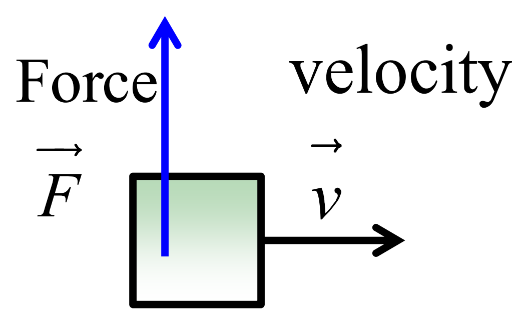
Force and velocity in the same direction, speed is increased, kinetic energy is increased. (positive work)

Force and velocity in the opposite direction, speed is decreased, kinetic energy is decreased (negative work)

Force and velocity are perpendicular, speed is not changed, kinetic energy is not changed (zero work)







The work-energy theorem, relation between work and K.E.

- **The *work-energy theorem*:** The work done by the net force on a particle equals the change in the particle's kinetic energy. **Work is converted into kinetic energy**
- Mathematically, the work-energy theorem is

$$\boxed{W_{\text{tot}} = K_{final} - K_{initial} = \Delta K.}$$

→ equivalent to 2nd law

Proof (for constant force):

$$F_x \Delta x = \frac{1}{2} m v_x^2 - \frac{1}{2} m v_{x0}^2 \qquad F_y \Delta y = \frac{1}{2} m v_y^2 - \frac{1}{2} m v_{y0}^2$$

$$\Rightarrow F_x \Delta x + F_y \Delta y = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 - \frac{1}{2} m v_{x0}^2 - \frac{1}{2} m v_{y0}^2$$

$$\Rightarrow \vec{F} \cdot \Delta \vec{s} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

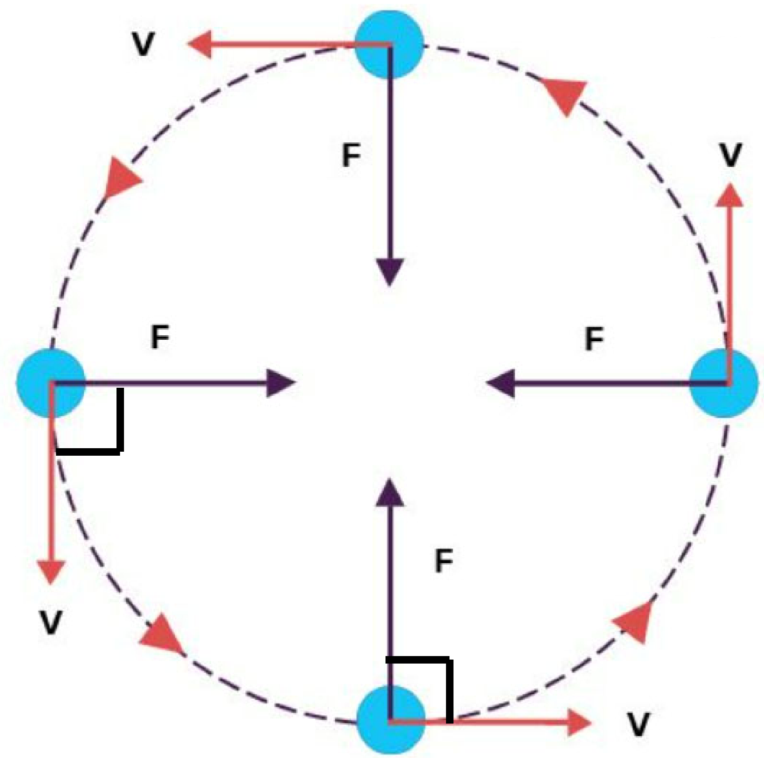
Work-energy in uniform circular motion

$$\vec{F} \cdot \Delta\vec{s} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\vec{F} \cdot \Delta\vec{s} = F\Delta s \cos 90^o = 0$$

$\rightarrow$ No K.E. change

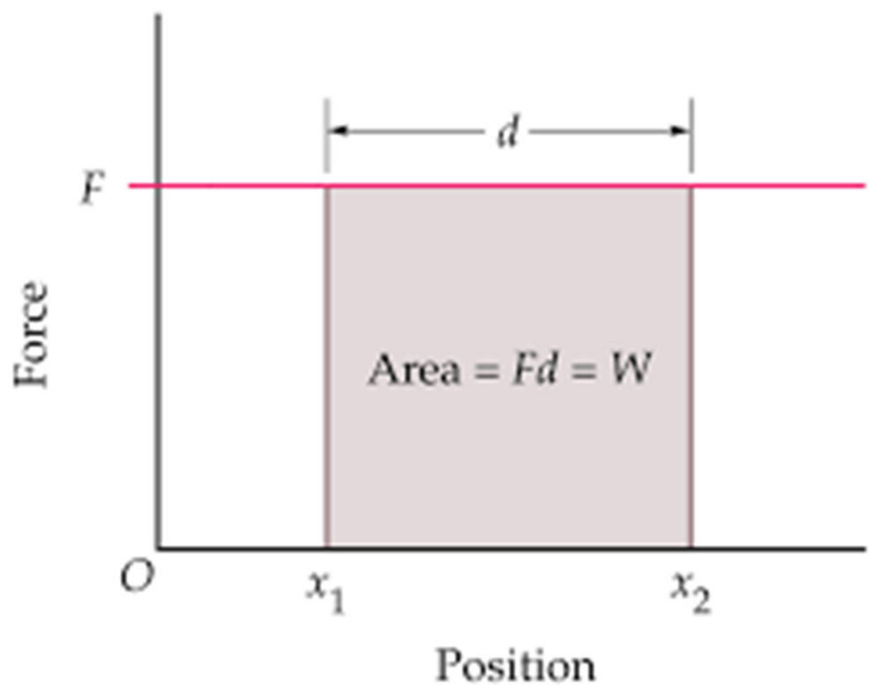
$$v_f = v_i = \text{v}$$



Calculate work done by a constant force graphically

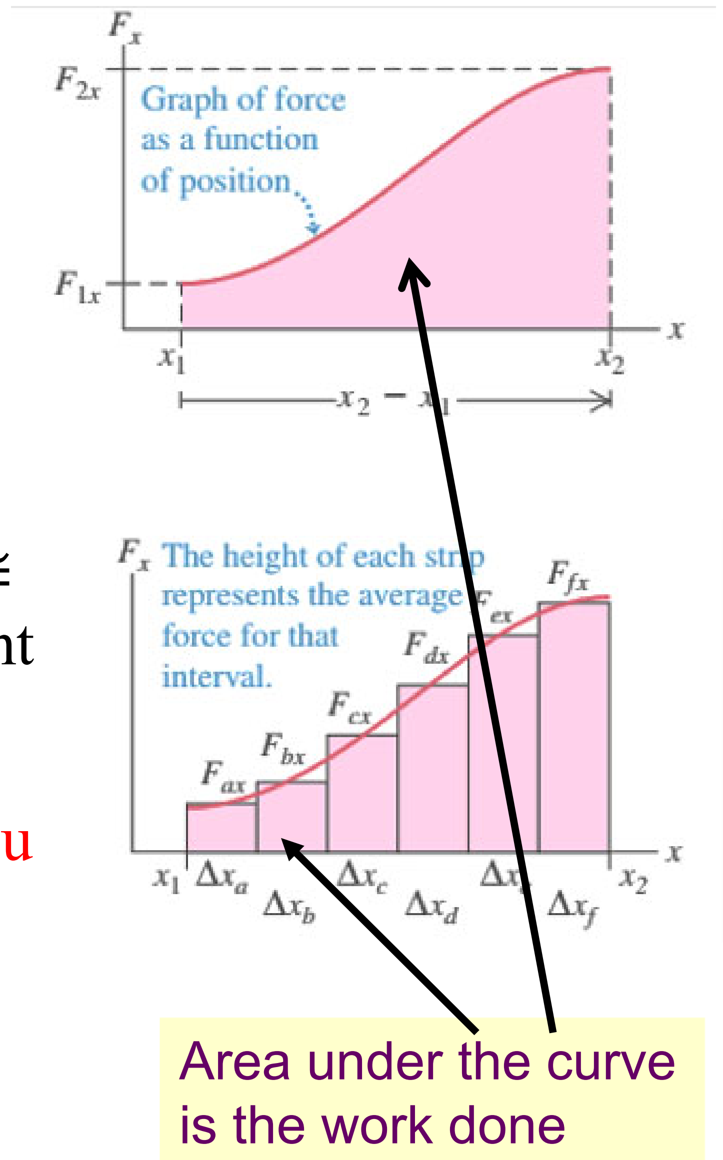
Plot force vs position

The work done by a force F in a displacement of d is $W = Fd$, which equals to the area under the curve between x_2 and x_1. $d = x_2 - x_1$.



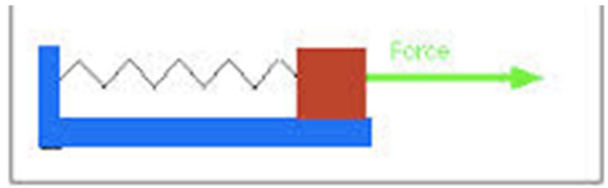
Work done by a changing force

- For a changing force, its work = the area under the curve for Force vs position between the initial and final position x_1 and x_2
- We approximate the area under the curve by total area of many small rectangles.
- The area of a small rectangle $\cong F\Delta x \cong$ the work done in the small displacement Δx
- the total areas of the rectangles give you **approximately** the areas under the curve and this also equals the work for the displacement from x_1 to x_2



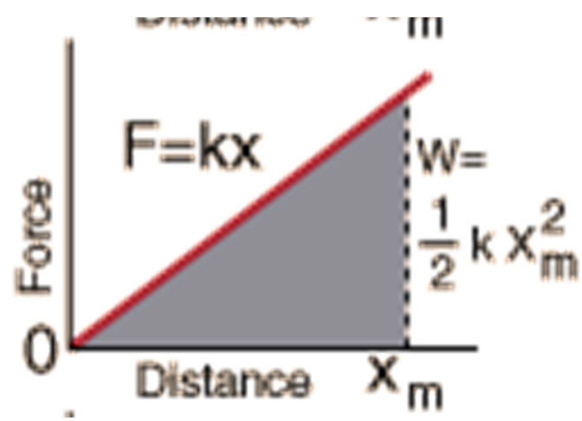
Work needed to stretch a spring

- The force required to hold a string stretched by a distance x is proportional to x: $F_x = kx$.



- k is the *force constant* (or *spring constant*) of the spring.

- The area under the graph represents the work done on the spring to stretch it a distance X_m:



- $area\ of\ triangle = base \times \frac{height}{2}$ → $W = kX_m^2/2$ → Elastic energy of the spring

Lecture 4 part 2

Potential Energy and Energy Conservation

Topics in Part 2

- gravitational potential energy in vertical motion
- elastic potential energy stored in a spring
- To solve problems using conservation of mechanical energy

Projectile revisited

$$v_y^2 - v_{y0}^2 = -2g\Delta H$$

$$v^2 = v_x^2 + v_y^2 \qquad v_x = v_{x0}$$

$$v_x^2 - v_{x0}^2 = 0$$

$$v^2 - v_0^2 = -2g\Delta H \qquad \Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = -mg(H - H_0)$$

- kinetic energy change is related to the height change of the object

or $\boxed{\frac{1}{2}mv^2} + \boxed{mgH} = \frac{1}{2}mv_0^2 + mgH_0 =$ true for all

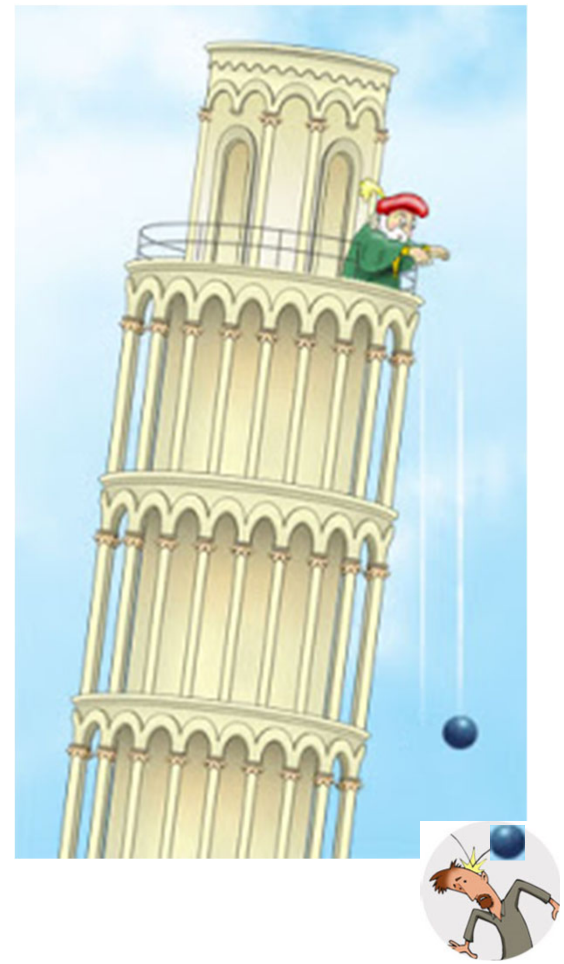
- This seems like some kind of conservation that is true for all height

kinetic energy (KE) | Something related to height

Objects in high rise building are dangerous!

- An object at high altitude can fall and hit someone causing real damage

- Altitude stores some energy



Potential energy

- A falling object increases in speed, gaining kinetic energy
- Where does the energy come from?

$$KE + mgH = \text{constant}$$

- Energy is stored in height and then converted into KE as H drops.
- Energy is stored in height is the potential energy (PE) due to gravity

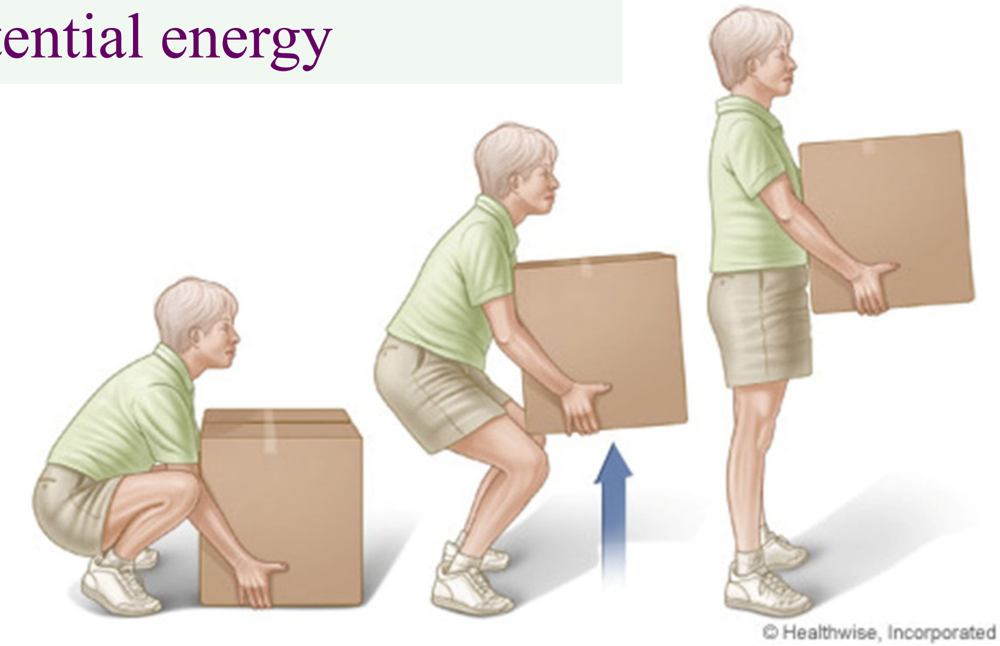
$$PE = mgH$$

Energy transfer related to gravity in lifting an object

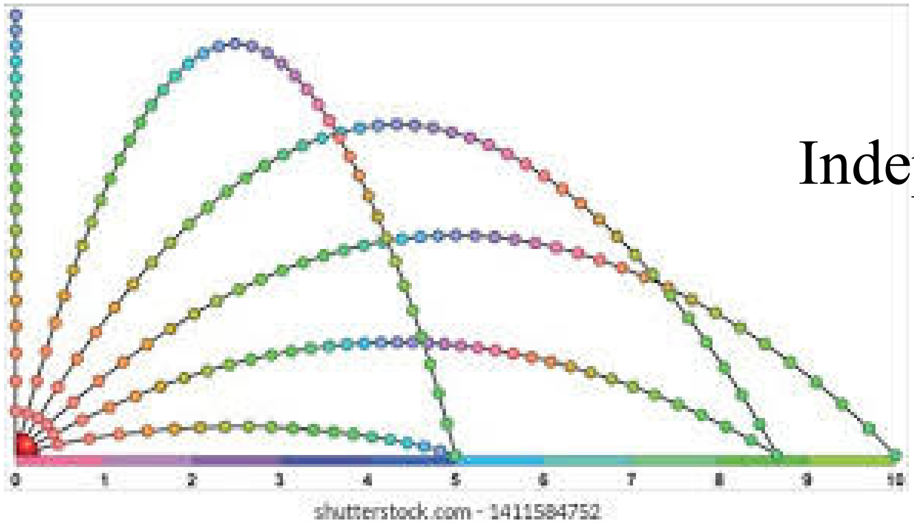
- You lift an object,
- You use force to move the object by a distance
- you do work, the box has no KE gain

- Where does the work go?
- Work is converted into potential energy

The energy possessed by an object which is related to the position of the object is the potential energy



Conservation of energy in a projectile



Independent of initial angle

$$\boxed{\frac{1}{2}mv^2} + \boxed{mgH} = \frac{1}{2}mv_0^2 + mgH_0$$

$$KE + PE = KE_0 + PE_0 = \text{constant}$$

Conservation of mechanical energy

- When an object falls from height y_2 to y_1, the object's kinetic energy is increased. The increase in kinetic energy equals to the decrease in the gravitational potential energy of the object

$$\text{Decrease in } PE = \text{increase in } KE$$
$$PE(\text{initial}) - PE(\text{final}) = KE(\text{final}) - KE(\text{initial})$$
$$PE(\text{initial}) + KE(\text{initial}) = PE(\text{final}) + KE(\text{final}) = \text{mechanical energy}$$

- So when the object's gravitational potential energy is added to the object's kinetic energy. This quantity is unchanged, i.e. it is conserved. This quantity is called mechanical energy.

- Definition: mechanical energy = potential energy + kinetic energy

When an object is only influenced by the gravitation force (there is **no other forces, or other forces do not do work**), the mechanical energy is conserved.

Derivation from work-energy theorem

- *Gravitational potential energy* is caused by gravitational force (weight) on the object
- It is related to the work done by the gravitational force

Proof: Work done by gravity $W_G = (-mg)\Delta y = -\Delta PE$

opposite to height increase

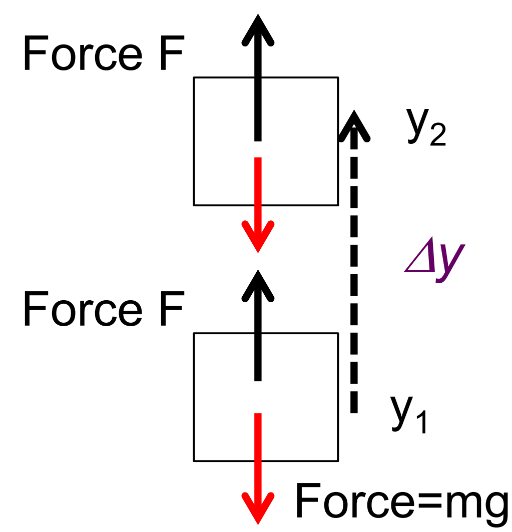
Using work-energy theorem:

$$W_G = \Delta KE \quad \text{or} \quad -\Delta PE = \Delta KE$$

$$\rightarrow \quad \Delta PE + \Delta KE = 0 \quad \text{or} \quad \Delta ME = 0$$

$$\rightarrow \quad ME_f - ME_i = 0 \text{ or } ME_f = ME_i$$

$\rightarrow$ Conservation of Mechanical Energy

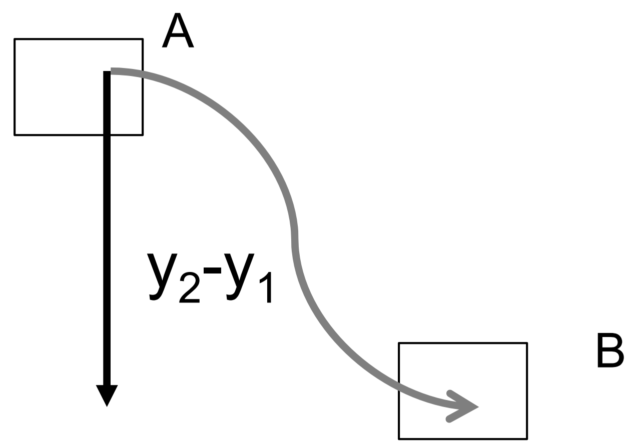


PE only depend on vertical coordinate y

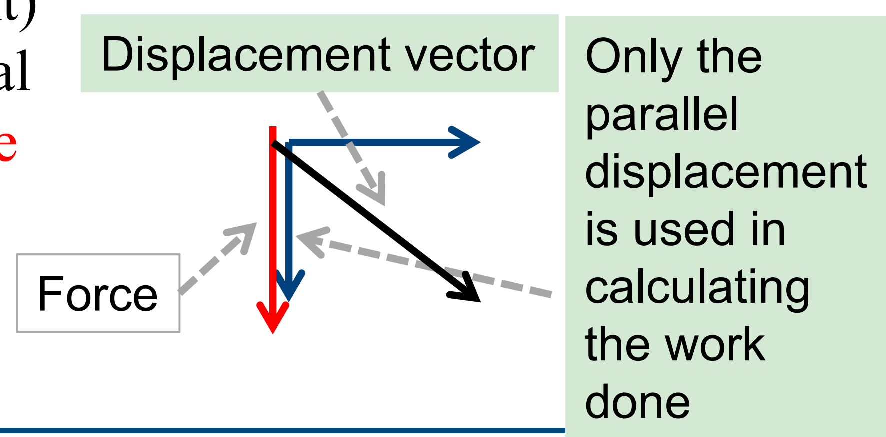
- We can use the same expression for gravitational potential energy whether the body' s falling path is curved or straight. It only depends on *y* not on *x*
- When work done by a force is calculated, only displacement parallel to the force is considered

$$\text{Work} = Fs\cos\phi = F\Delta s_{\parallel}$$

- Only vertical distance (y displacement) is needed in the expression of potential energy, because the gravitational force is vertical.
- Potential Energy depends on height and mass. $P.E. = mgy$



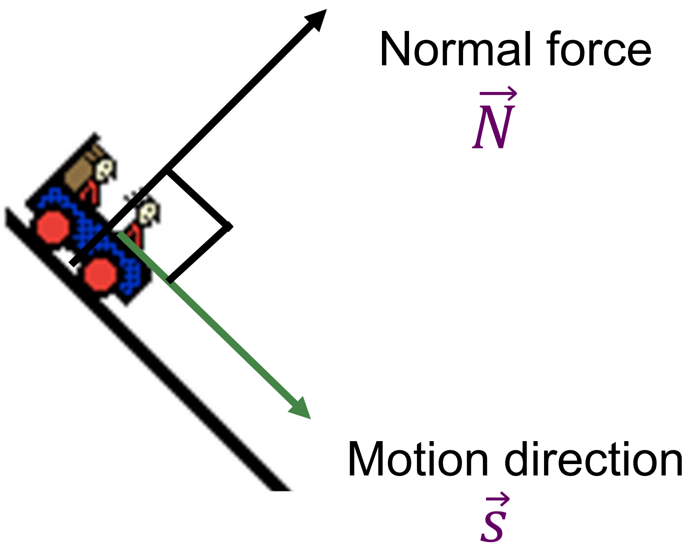
The potential energy difference of point A and B depends on the difference of the vertical distance of the two points y_2-y_1



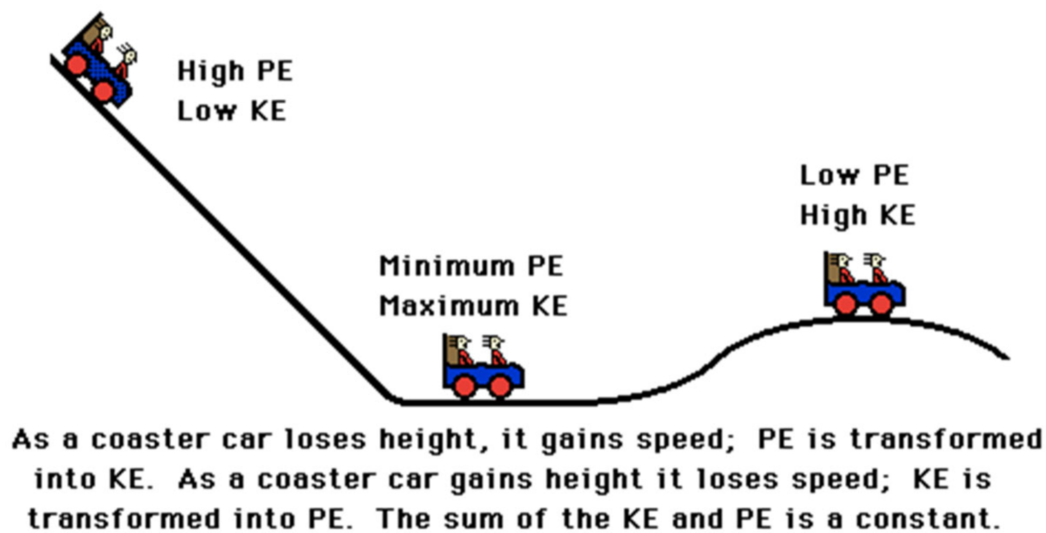
Conservation of mechanical energy in a roller coaster

- The contact force from the track on the roller coaster **is always perpendicular to the motion, as there is no friction. It is a normal force**, normal force has no work: $W_N = \vec{N} \cdot \vec{s} = 0$. The only force can have work is the gravitational force.

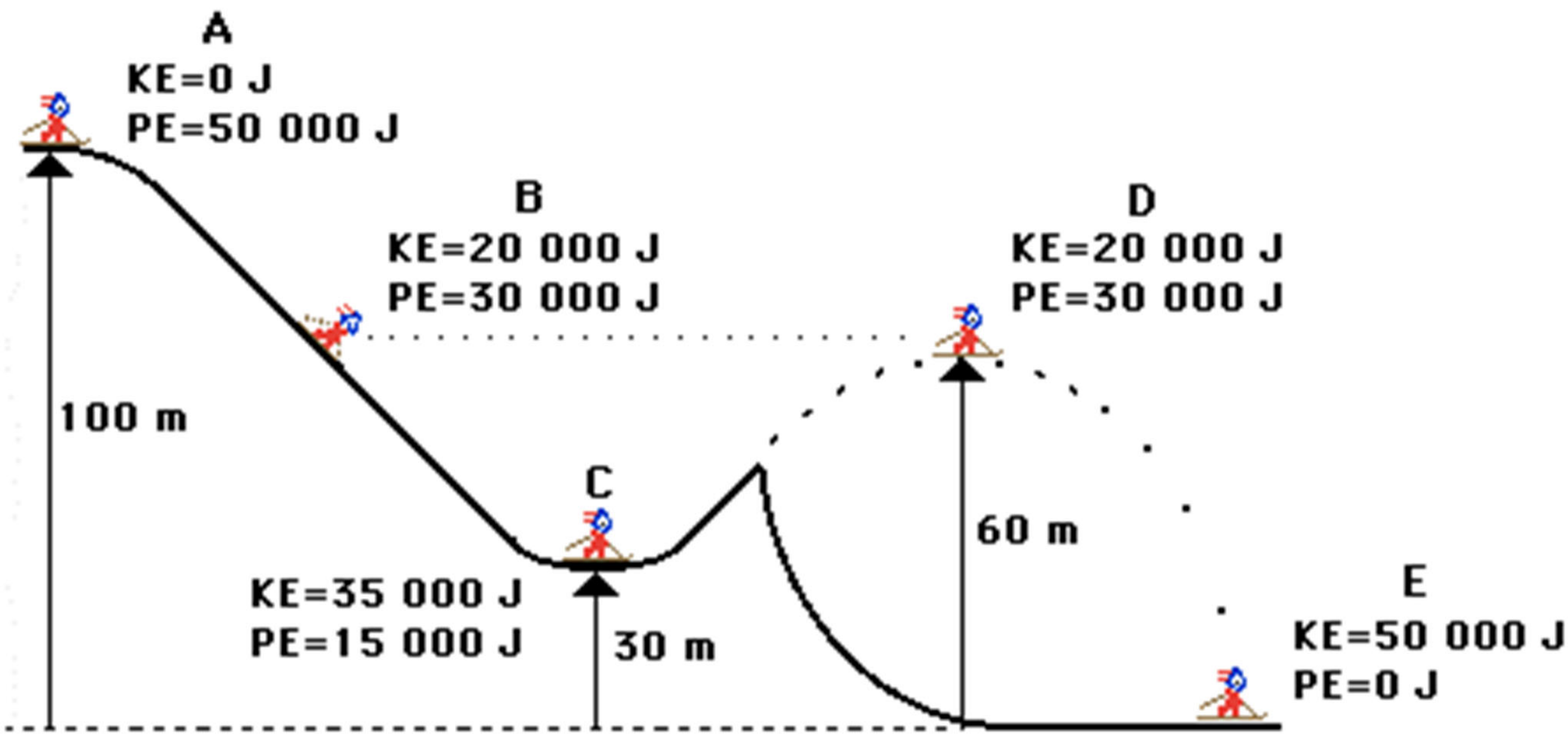
- When only the force of gravity does work on a system (there is no friction), the total mechanical energy of the roller coaster is conserved.



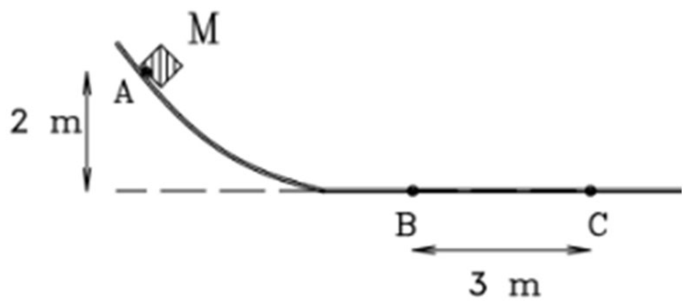
$$ME = KE + PE = \text{constant}$$

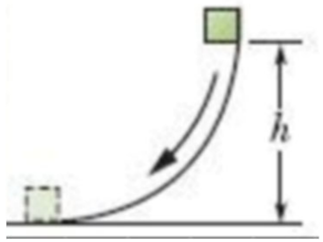


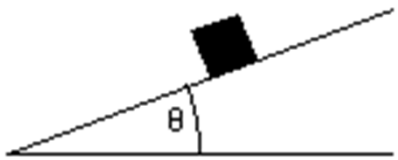
Mechanical energy is a constant=50000J



An object slide down a surface







Same as roller coaster for other smooth surfaces

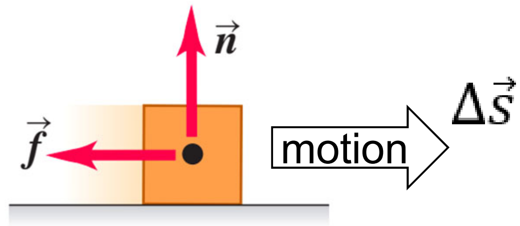
When there is no friction and the force from the surface on the object is always normal to the surface and thus normal to the path.

Only force has work done on the object is the gravitational force.
→ Mechanical Energy = K.E.+P.E. is conserved

What if there is friction?

- Friction is always opposite to motion

→ Work done by friction is negative

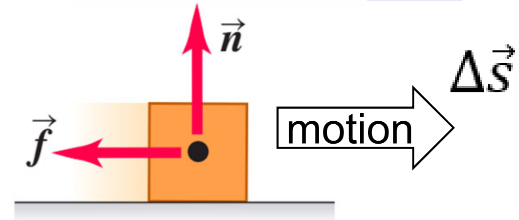


- The effect of friction mostly reduce the speed of an object

→ kinetic energy loss

All kind energy loss including PE??

Work energy theorem with friction



- Check with Work-energy theorem:

$$W_f + W_G = \Delta KE \quad \text{or} \quad W_f - \Delta PE = \Delta KE \qquad (W_G = -\Delta PE)$$

$$\rightarrow \qquad W_f = \Delta PE + \Delta KE = \Delta ME = ME_{final} - ME_{initial}$$

- Note W_f is negative → Energy is lost due to friction (W_f = - E_{lost})

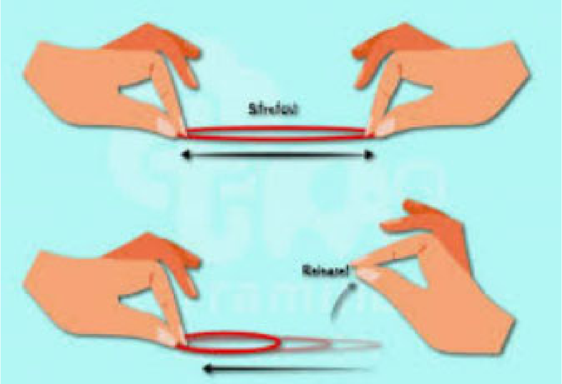
Initial mechanical energy

= final mechanical energy + lost energy due to friction(positive value)

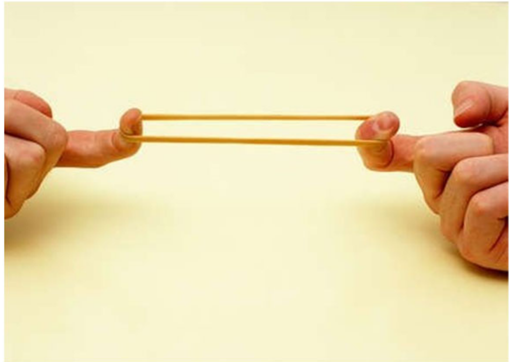
$$\boxed{\text{Work-Energy theorem with friction:} \quad W_f = ME_{final} - ME_{initial}}$$

More potential energy: Elastic potential energy

- There is another kind of potential energy, which is elastic potential energy

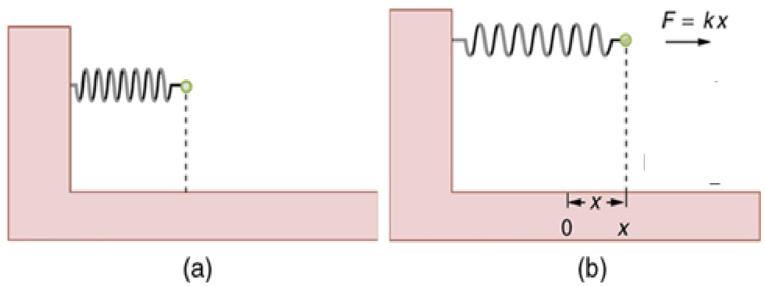


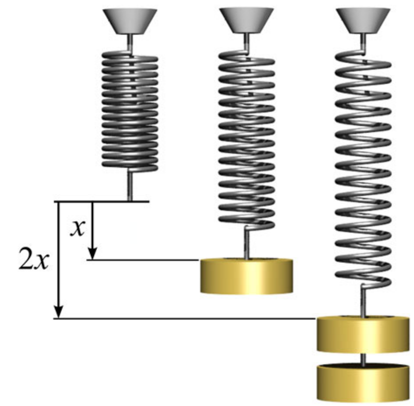
- A body is *elastic* if it returns to its original shape after being deformed.
- When an elastic object is stretched or compressed, you do work on the object and energy is stored in the object as elastic potential energy



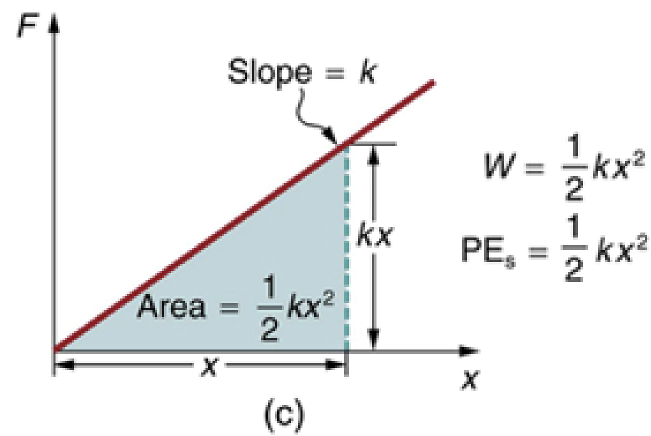
Energy stored in stretching a spring

- Elastic force required to stretch a spring a distance x is proportional to x: $F_x = kx$, *where* k is the *force constant* (or *spring constant*) of the spring.





- The area under the graph represents the work done on the spring to stretch it a distance x: $W = 1/2\ kx^2$.

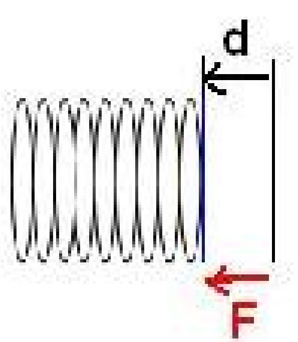


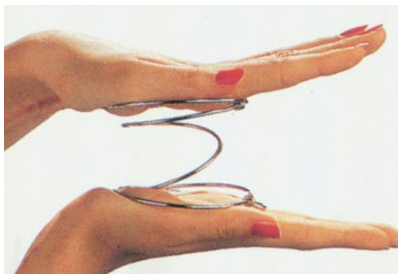
- The work is stored in the spring as elastic potential energy:

Elastic potential energy= $1/2\ kx^2$

Energy stored when a spring is compressed

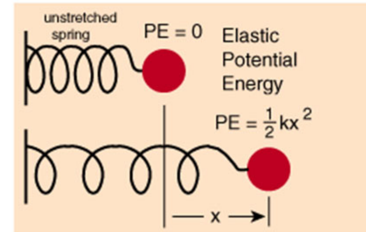
- When you compress a spring, your force and displacement are in the same direction, so you do positive work on the spring, the energy you used is stored as elastic energy in the spring. Elastic energy of the spring is increased as a result.
- Force used to compress = kd, d = compressed distance
- work required to compress a spring by d is $\frac{1}{2}kd^2 =$ elastic energy stored in the spring

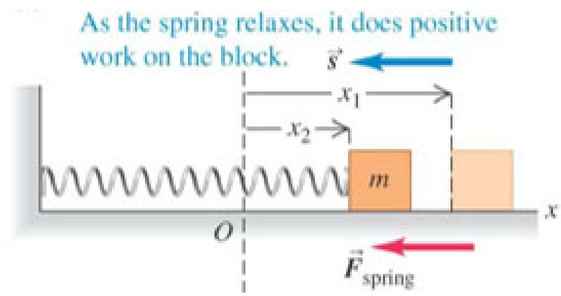




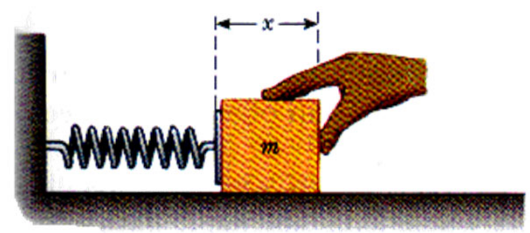
When you release the stretched or compressed spring

- When you release a stretched spring, the spring exert a force on an object and move the object back; so, it does positive work on the object and increases its kinetic energy



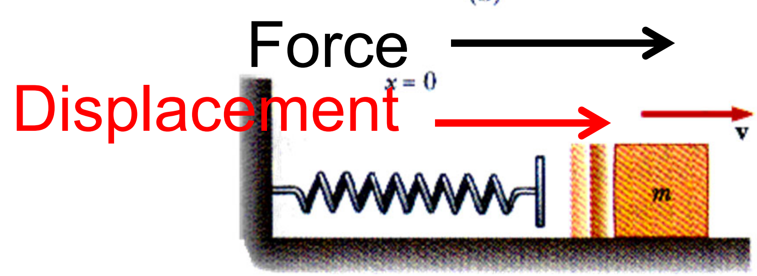


- Similarly, a compressed spring pushes out the object and increases the kinetic energy of the object.



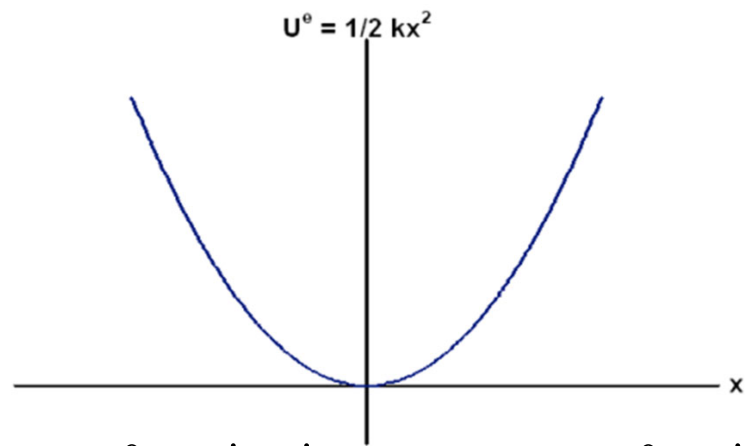
(b)

- The elastic energy stored in a stretched or compressed spring is converted into kinetic energy of the object.



Elastic potential energy in a spring (graphical representation)

- The elastic potential energy stored in an ideal spring is
  $U_{el} = 1/2\ kx^2$.
- The figure at the right shows a graph of the elastic potential energy for an ideal spring.
- It is a parabola > 0

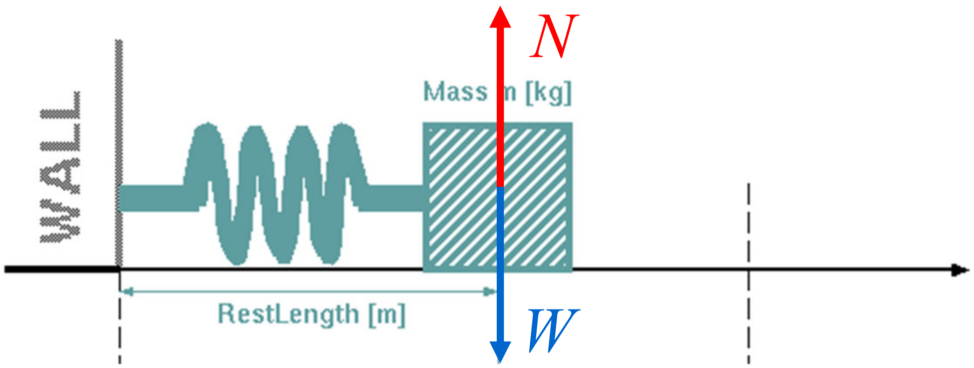


$x < 0$, spring is compressed

$x > 0$ spring is stretched

Spring mass system

- A mass is attached to a spring. The mass is free to move and it can compress and elongate the spring by its motion.



- The mass has kinetic energy due to motion. The spring has elastic energy due to compression or elongation

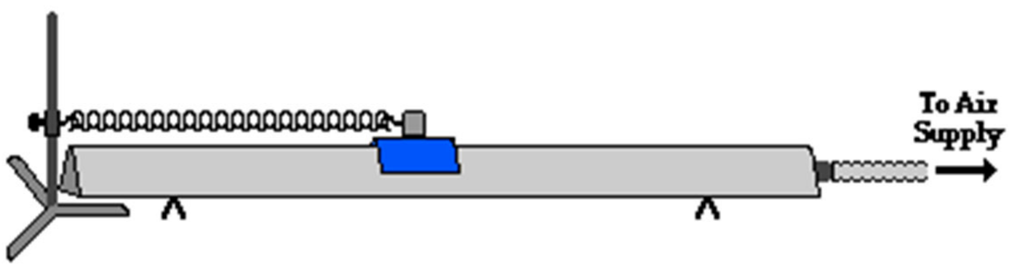
- The elastic energy can be converted in kinetic energy or vice versa

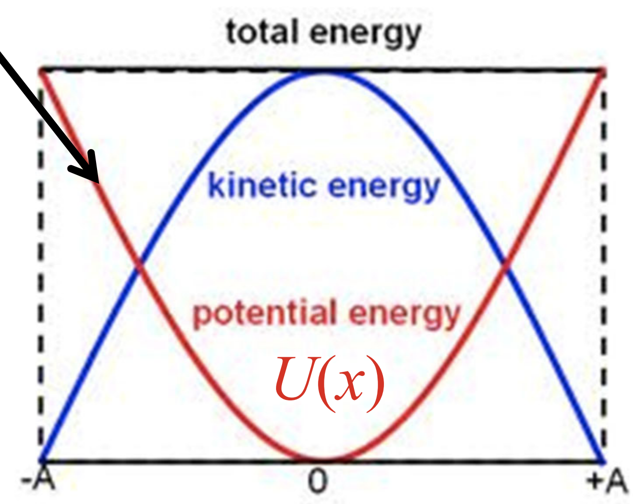
- i.e. the elastic potential energy + the kinetic energy is a constant for the spring mass system, if there is no other external force.

- The gravitational force is cancelled by the normal force. So they are not considered

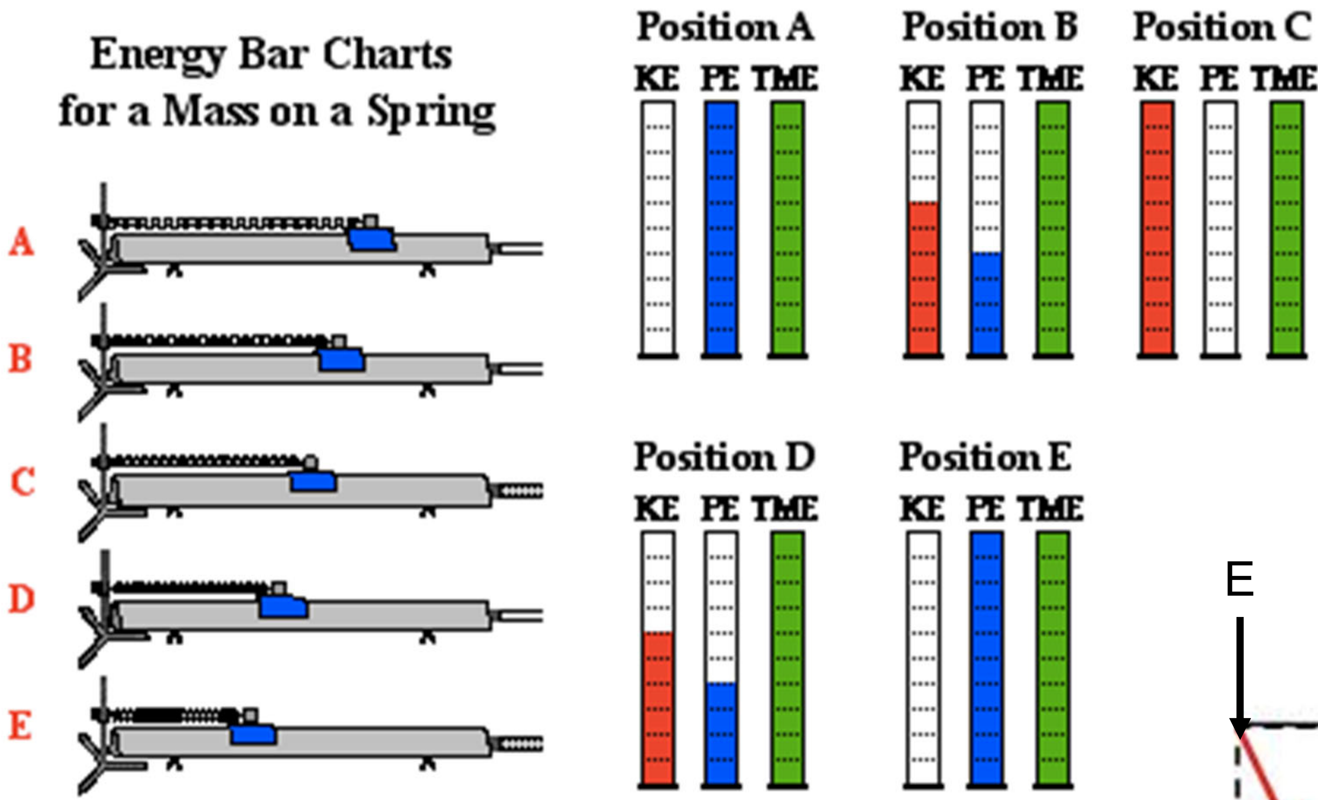
Energy diagram of spring mass system

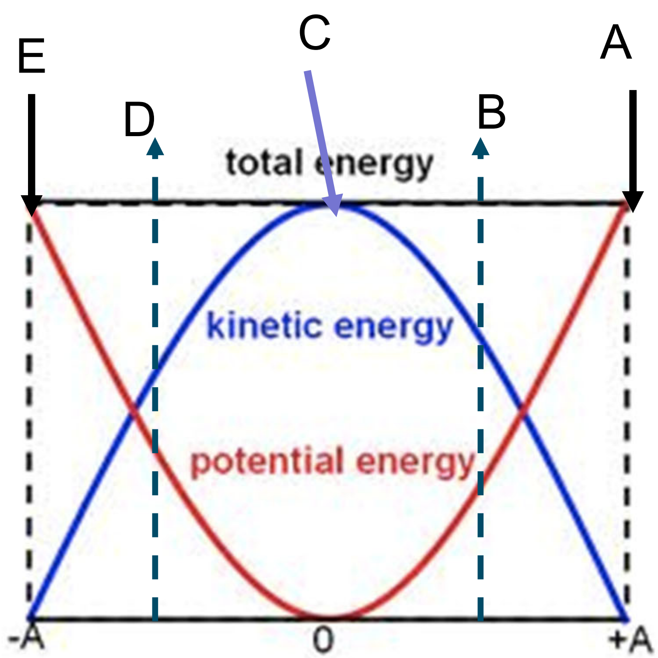
- The *energy diagram* is the graph that shows the potential-energy function $PE = U(x)$, kinetic energy KE and the total mechanical energy $E=KE+PE$ *of the mass spring system in an air track*
- In the graph the kinetic energy equals E-$U(x)$





Energy changes in a mass spring system

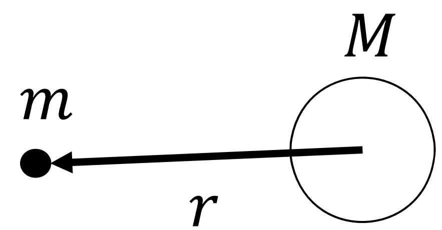




More potential energy: other potential energy

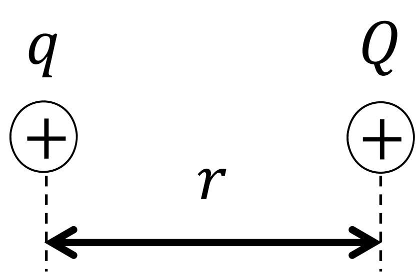
- Newtonian Gravitational potential outside earth

$$PE_G = -G\frac{Mm}{r}$$



- Coulomb electric potential of electric charges

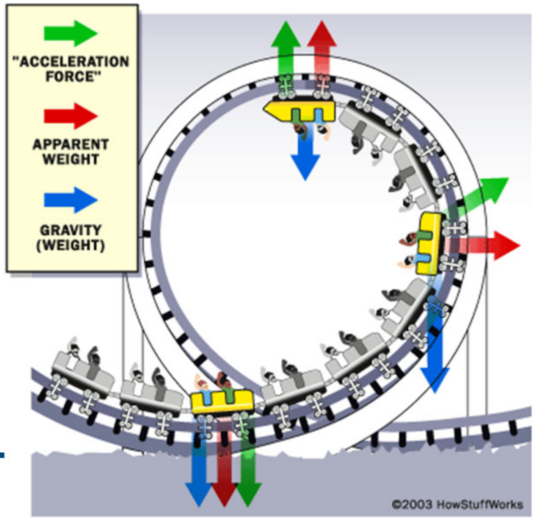
$$PE_e = k\frac{Qq}{r}$$



Application of conservation of mechanical energy

- We can use the conservation of mechanical energy to solve problems.
- For example, we want to find the velocity of a falling object; we can first find its kinetic energy, which equals the change in potentials energy.
- From the kinetic energy, we can find the velocity easily.
- This can avoid using the force. This is convenient if the force considered in not a constant. For example the force experienced in a roller-coaster

The resultant of other forces (not include gravitational force) are normal to the motion in the roller coaster in this case and can be ignored in the energy consideration

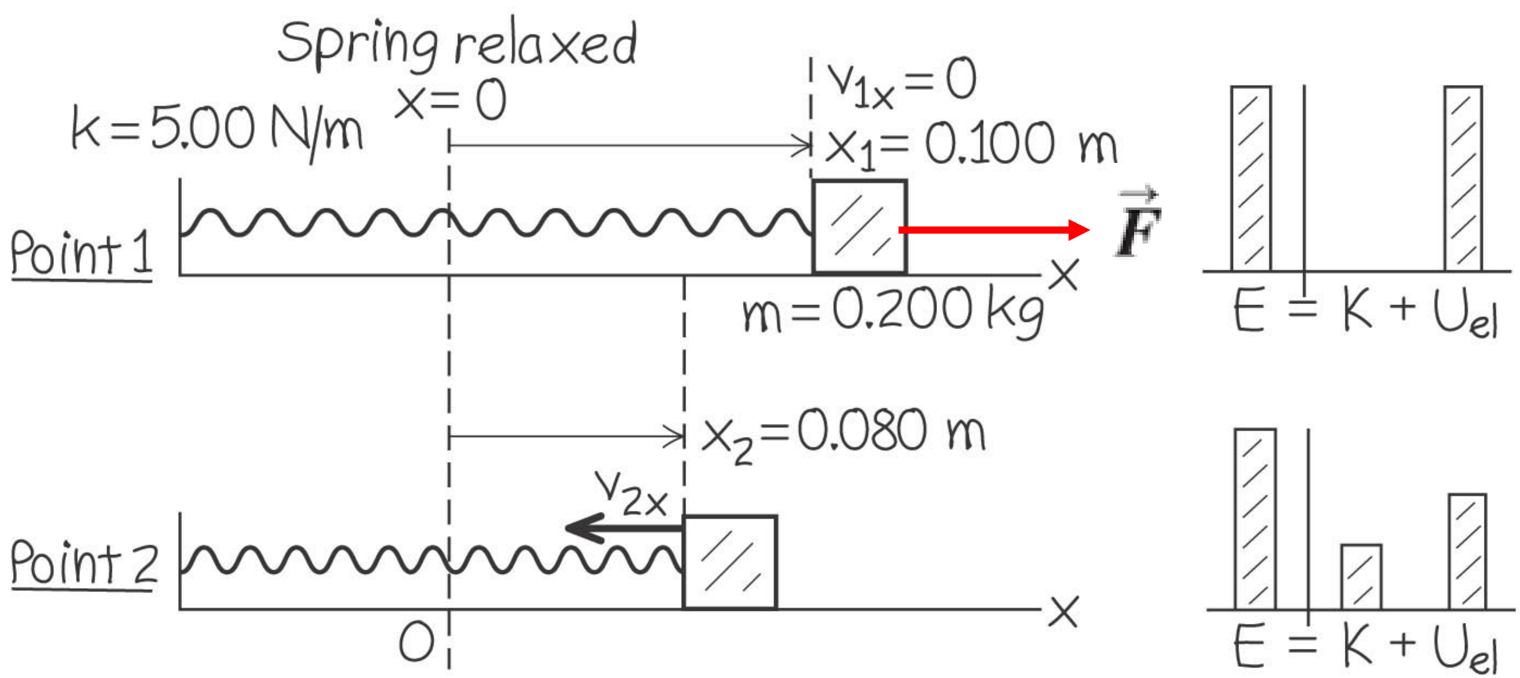


Examples

Motion with elastic potential energy

Example 7.8 **Motion with elastic potential energy and work done by other forces**

Suppose the glider in Example 7.7 is initially at rest at $x = 0$, with the spring unstretched. You then push on the glider with a constant force $\vec{F}$ (magnitude 0.610 N) in the $+x$-direction. What is the glider's velocity when it has moved to $x = 0.100$ m?



SOLUTION

IDENTIFY and SET UP: Although the force $\vec{F}$ you apply is constant, the spring force isn't, so the acceleration of the glider won't be constant. Total mechanical energy is not conserved because of

the work done by the force $\vec{F}$, so we must use the generalized energy relationship given by Eq. (7.13). As in Example 7.7, we ignore gravitational potential energy because the glider's height doesn't change. Hence we again have $U = U_{el} = \frac{1}{2}kx^2$. This time, we let point 1 be at $x_1 = 0$, where the velocity is $v_{1x} = 0$, and let point 2 be at $x = 0.100$ m. The glider's displacement is then $\Delta x = x_2 - x_1 = 0.100$ m. Our target variable is v_{2x}, the velocity at point 2.

EXECUTE: The force $\vec{F}$ is constant and in the same direction as the displacement, so the work done by this force is $F\Delta x$. Then the energy quantities are

$$K_1 = 0$$

$$U_1 = \tfrac{1}{2}kx_1^2 = 0$$

$$K_2 = \tfrac{1}{2}mv_{2x}^2$$

$$U_2 = \tfrac{1}{2}kx_2^2 = \tfrac{1}{2}(5.00\ \text{N/m})(0.100\ \text{m})^2 = 0.0250\ \text{J}$$

$$W_{\text{other}} = F\Delta x = (0.610\ \text{N})(0.100\ \text{m}) = 0.0610\ \text{J}$$

→

The initial total mechanical energy is zero; the work done by $\vec{F}$ increases the total mechanical energy to 0.0610 J, of which $U_2 = 0.0250$ J is elastic potential energy. The remainder is kinetic energy. From Eq. (7.13),

$$K_1 + U_1 + W_{\text{other}} = K_2 + U_2$$

$$K_2 = K_1 + U_1 + W_{\text{other}} - U_2$$

$$= 0 + 0 + 0.0610 \text{ J} - 0.0250 \text{ J} = 0.0360 \text{ J}$$

$$v_{2x} = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.0360 \text{ J})}{0.200 \text{ kg}}} = 0.60 \text{ m/s}$$

We choose the positive square root because the glider is moving in the $+x$-direction.

EVALUATE: To test our answer, think what would be different if we disconnected the glider from the spring. Then only $\vec{F}$ would do work, there would be zero elastic potential energy at all times, and Eq. (7.13) would give us

$$K_2 = K_1 + W_{\text{other}} = 0 + 0.0610\ \text{J}$$

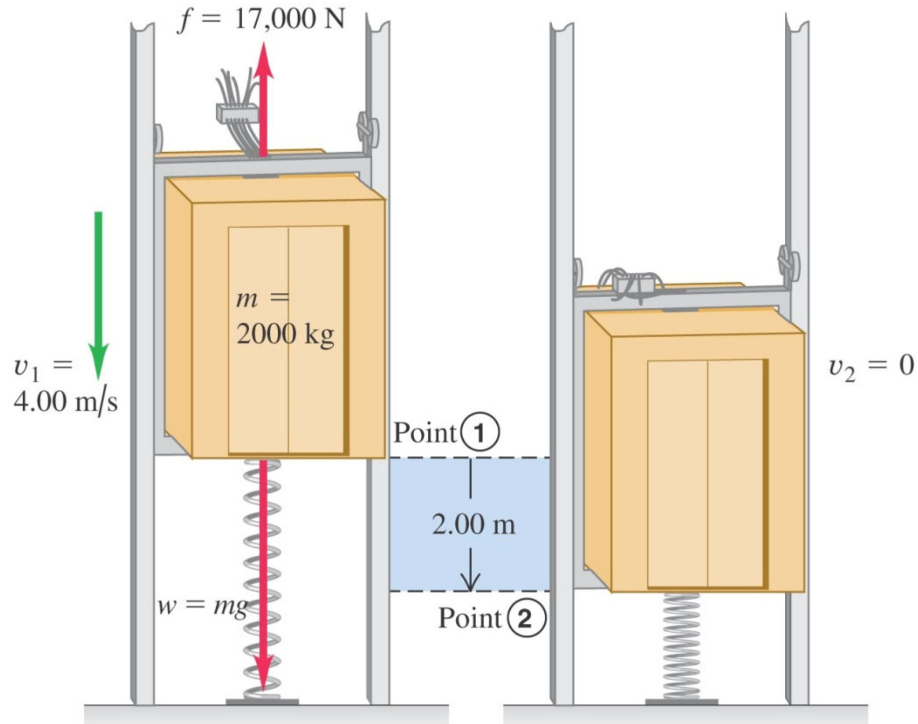
$$v_{2x} = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(0.0610\ \text{J})}{0.200\ \text{kg}}} = 0.78\ \text{m/s}$$

Our answer $v_{2x} = 0.60\ \text{m/s}$ is less than $0.78\ \text{m/s}$ because the spring does negative work on the glider as it stretches (see Fig. 7.13b).

If you stop pushing on the glider when it reaches $x = 0.100$ m, only the spring force does work on it thereafter. Hence for $x > 0.100$ m, the total mechanical energy $E = K + U = 0.0610$ J is constant. As the spring continues to stretch, the glider slows down and the kinetic energy K decreases as the potential energy increases. The glider comes to rest at some point $x = x_3$, at which the kinetic energy is zero and the potential energy $U = U_{el} = \frac{1}{2}kx_3^2$ equals the total mechanical energy 0.0610 J. Can you show that $x_3 = 0.156$ m? (It moves an additional 0.056 m after you stop pushing.) If there is no friction, will the glider remain at rest?

Example 7.9 Motion with gravitational, elastic, and friction forces

A 2000-kg (19,600-N) elevator with broken cables in a test rig is falling at 4.00 m/s when it contacts a cushioning spring at the bottom of the shaft. The spring is intended to stop the elevator, compressing 2.00 m as it does so (Fig. 7.17). During the motion a safety clamp applies a constant 17,000-N frictional force to the elevator. What is the necessary force constant k for the spring?



SOLUTION

IDENTIFY and SET UP: We'll use the energy approach to determine k, which appears in the expression for elastic potential energy. This problem involves *both* gravitational and elastic potential energy. Total mechanical energy is not conserved because the friction force does negative work W_{other} on the elevator. We'll therefore use the most general form of the energy relationship, Eq. (7.13). We take point 1 as the position of the bottom of the elevator when it contacts the spring, and point 2 as its position when it stops. We choose the origin to be at point 1, so $y_1 = 0$ and $y_2 = -2.00$ m. With this choice the coordinate of the upper end of the spring after contact is the same as the coordinate of the elevator, so the elastic potential energy at any point between points 1 and 2 is $U_{el} = \frac{1}{2}ky^2$. The gravitational potential energy is $U_{grav} = mgy$ as usual. We know the initial and final speeds of the elevator and the magnitude of the friction force, so the only unknown is the force constant k (our target variable).

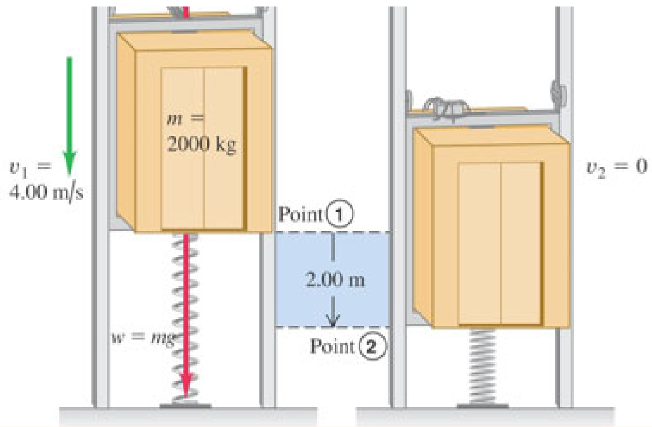
$$K_1 + U_{grav,1} + U_{el,1} + W_{other} = K_2 + U_{grav,2} + U_{el,2} \quad \text{(valid in general)} \qquad (7.13)$$

EXECUTE: The elevator's initial speed is $v_1 = 4.00\ \text{m/s}$, so its initial kinetic energy is

$$K_1 = \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}(2000\ \text{kg})(4.00\ \text{m/s})^2 = 16{,}000\ \text{J}$$

The elevator stops at point 2, so $K_2 = 0$. At point 1 the potential energy $U_1 = U_{\text{grav}} + U_{\text{el}}$ is zero; U_{grav} is zero because $y_1 = 0$, and $U_{\text{el}} = 0$ because the spring is uncompressed. At point 2 there is both gravitational and elastic potential energy, so

$$U_2 = mgy_2 + \tfrac{1}{2}ky_2^2$$



$$K_1 + U_{grav,1} + U_{el,1} + W_{other} = K_2 + U_{grav,2} + U_{el,2} \quad \text{(valid in general)} \tag{7.13}$$

The gravitational potential energy at point 2 is

$$mgy_2 = (2000\ \text{kg})(9.80\ \text{m/s}^2)(-2.00\ \text{m}) = -39{,}200\ \text{J}$$

The "other" force is the constant 17,000-N friction force. It acts opposite to the 2.00-m displacement, so

$$W_{other} = -(17{,}000\ \text{N})(2.00\ \text{m}) = -34{,}000\ \text{J}$$

We put these terms into Eq. (7.14), $K_1 + U_1 + W_{other} = K_2 + U_2$:

$$K_1 + 0 + W_{other} = 0 + (mgy_2 + \tfrac{1}{2}ky_2^2)$$

$$\begin{aligned} k &= \frac{2(K_1 + W_{other} - mgy_2)}{y_2^2} \\ &= \frac{2[16{,}000\ \text{J} + (-34{,}000\ \text{J}) - (-39{,}200\ \text{J})]}{(-2.00\ \text{m})^2} \\ &= 1.06 \times 10^4\ \text{N/m} \end{aligned}$$

This is about one-tenth the force constant of a spring in an automobile suspension.

EVALUATE: There might seem to be a paradox here. The elastic potential energy at point 2 is

$$\tfrac{1}{2}ky_2^2 = \tfrac{1}{2}(1.06 \times 10^4\,\text{N/m})(-2.00\,\text{m})^2 = 21{,}200\,\text{J}$$

This is *more* than the total mechanical energy at point 1:

$$E_1 = K_1 + U_1 = 16{,}000\,\text{J} + 0 = 16{,}000\,\text{J}$$

But the friction force *decreased* the mechanical energy of the system by 34,000 J between points 1 and 2. Did energy appear from nowhere? No. At point 2, which is below the origin, there is also *negative* gravitational potential energy $mgy_2 = -39{,}200\ \mathrm{J}$. The total mechanical energy at point 2 is therefore not 21,200 J but rather

$$\begin{aligned} E_2 &= K_2 + U_2 = 0 + \tfrac{1}{2}ky_2^2 + mgy_2 \\ &= 0 + 21{,}200\ \mathrm{J} + (-39{,}200\ \mathrm{J}) = -18{,}000\ \mathrm{J} \end{aligned}$$

This is just the initial mechanical energy of 16,000 J minus 34,000 J lost to friction.

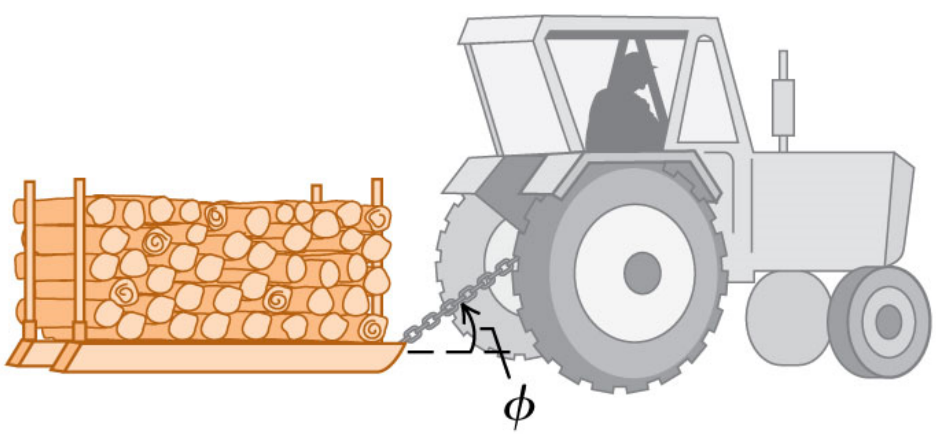
Will the elevator stay at the bottom of the shaft? At point 2 the compressed spring exerts an upward force of magnitude $F_{\text{spring}} = (1.06 \times 10^4\ \text{N/m})(2.00\ \text{m}) = 21{,}200\ \text{N}$, while the downward force of gravity is only $w = mg = (2000\ \text{kg})(9.80\ \text{m/s}^2) = 19{,}600\ \text{N}$. If there were no friction, there would be a net upward force of $21{,}200\ \text{N} - 19{,}600\ \text{N} = 1600\ \text{N}$, and the elevator would rebound. But the safety clamp can exert a kinetic friction force of 17,000 N, and it can presumably exert a maximum static friction force greater than that. Hence the clamp will keep the elevator from rebounding.

Work done by several forces

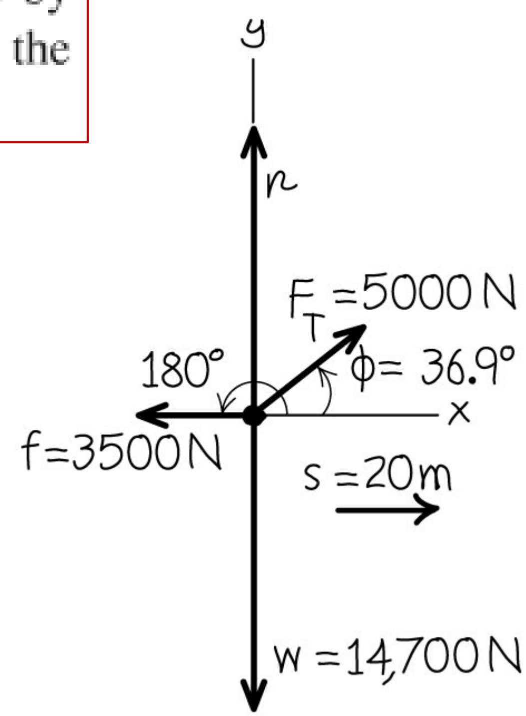
Example 6.2 Work done by several forces

A farmer hitches her tractor to a sled loaded with firewood and pulls it a distance of 20 m along level ground (Fig. 6.7a). The total weight of sled and load is 14,700 N. The tractor exerts a constant 5000-N force at an angle of 36.9° above the horizontal. A 3500-N friction force opposes the sled's motion. Find the work done by each force acting on the sled and the total work done by all the forces.

(a)



Free-body diagram for sled



SOLUTION

IDENTIFY AND SET UP: Each force is constant and the sled's displacement is along a straight line, so we can calculate the work using the ideas of this section. We'll find the total work in two ways: (1) by adding the work done on the sled by each force and (2) by finding the work done by the net force on the sled. We first draw a free-body diagram showing all of the forces acting on the sled, and we choose a coordinate system (Fig. 6.7b). For each force—weight, normal force, force of the tractor, and friction force—we know the angle between the displacement (in the positive x-direction) and the force. Hence we can use Eq. (6.2) to calculate the work each force does.

As in Chapter 5, we'll find the net force by adding the components of the four forces. Newton's second law tells us that because the sled's motion is purely horizontal, the net force can have only a horizontal component.

EXECUTE: (1) The work W_w done by the weight is zero because its direction is perpendicular to the displacement (compare Fig. 6.4c). For the same reason, the work W_n done by the normal force is also zero. (Note that we don't need to calculate the magnitude n to conclude this.) So $W_w = W_n = 0$.

That leaves the work W_T done by the force F_T exerted by the tractor and the work W_f done by the friction force f. From Eq. (6.2),

$$W_T = F_T s \cos\phi = (5000\ \text{N})(20\ \text{m})(0.800) = 80{,}000\ \text{N}\cdot\text{m}$$
$$= 80\ \text{kJ}$$

The friction force $\vec{f}$ is opposite to the displacement, so for this force $\phi = 180°$ and $\cos\phi = -1$. Again from Eq. (6.2),

$$W_f = fs\cos 180° = (3500\ \text{N})(20\ \text{m})(-1) = -70{,}000\ \text{N}\cdot\text{m}$$
$$= -70\ \text{kJ}$$

The total work W_{tot} done on the sled by all forces is the *algebraic* sum of the work done by the individual forces:

$$W_{tot} = W_w + W_n + W_T + W_f = 0 + 0 + 80\ \text{kJ} + (-70\ \text{kJ})$$
$$= 10\ \text{kJ}$$

(2) In the second approach, we first find the *vector* sum of all the forces (the net force) and then use it to compute the total work. The vector sum is best found by using components. From Fig. 6.7b,

$$\sum F_x = F_{\mathrm{T}} \cos\phi + (-f) = (5000\ \mathrm{N})\cos 36.9^\circ - 3500\ \mathrm{N}$$
$$= 500\ \mathrm{N}$$

$$\sum F_y = F_{\mathrm{T}} \sin\phi + n + (-w)$$
$$= (5000\ \mathrm{N})\sin 36.9^\circ + n - 14{,}700\ \mathrm{N}$$

We don't need the second equation; we know that the y-component of force is perpendicular to the displacement, so it does no work. Besides, there is no y-component of acceleration, so $\sum F_y$ must be zero anyway. The total work is therefore the work done by the total x-component:

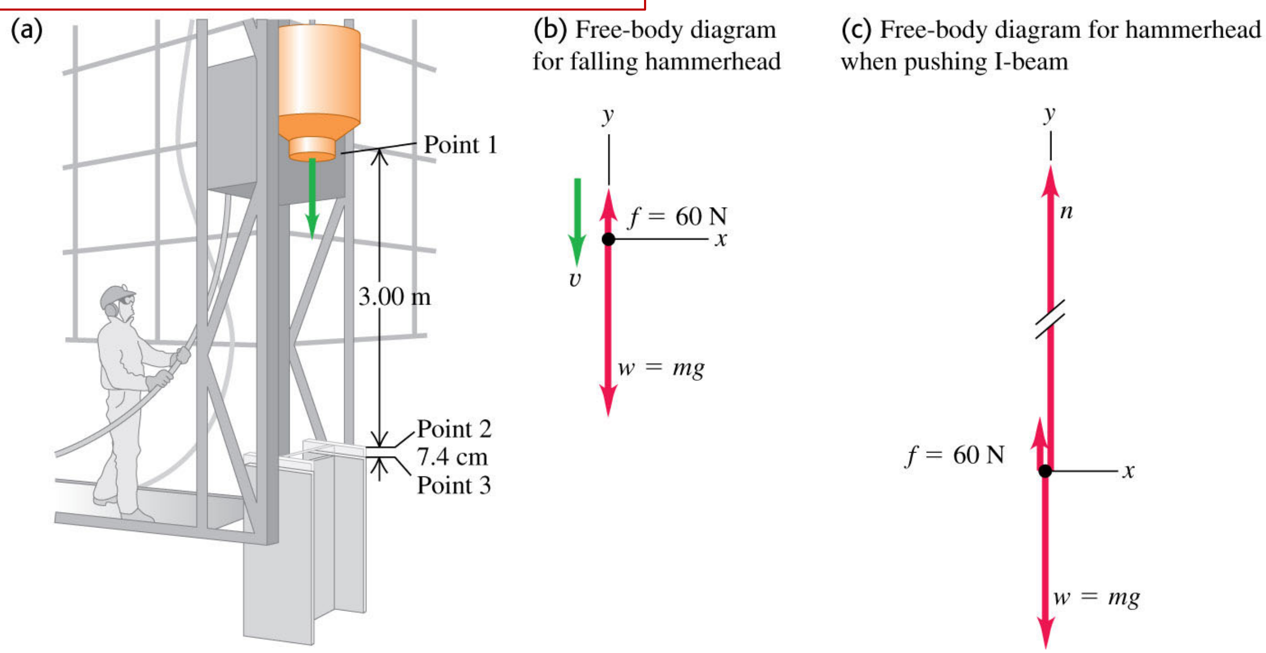
$$W_{\mathrm{tot}} = \left(\sum \vec{F}\right) \cdot \vec{s} = \left(\sum F_x\right) s = (500\ \mathrm{N})(20\ \mathrm{m}) = 10{,}000\ \mathrm{J}$$
$$= 10\ \mathrm{kJ}$$

EVALUATE: We get the same result for W_{tot} with either method, as we should. Note also that the net force in the x-direction is *not* zero, and so the sled must accelerate as it moves. In Section 6.2 we'll return to this example and see how to use the concept of work to explore the sled's changes of speed.

Forces on a hammerhead

Example 6.4 Forces on a hammerhead

The 200-kg steel hammerhead of a pile driver is lifted 3.00 m above the top of a vertical I-beam being driven into the ground (Fig. 6.12a). The hammerhead is then dropped, driving the I-beam 7.4 cm deeper into the ground. The vertical guide rails exert a constant 60-N friction force on the hammerhead. Use the work–energy theorem to find (a) the speed of the hammerhead just as it hits the I-beam and (b) the average force the hammerhead exerts on the I-beam. Ignore the effects of the air.



Example 6.4

SOLUTION

IDENTIFY: We'll use the work–energy theorem to relate the hammerhead's speed at different locations and the forces acting on it. There are *three* locations of interest: point 1, where the hammerhead starts from rest; point 2, where it first contacts the I-beam; and point 3, where the hammerhead and I-beam come to a halt (Fig. 6.12a). The two target variables are the hammerhead's speed at point 2 and the average force the hammerhead exerts between points 2 and 3. Hence we'll apply the work–energy theorem twice: once for the motion from 1 to 2, and once for the motion from 2 to 3.

Example 6.4

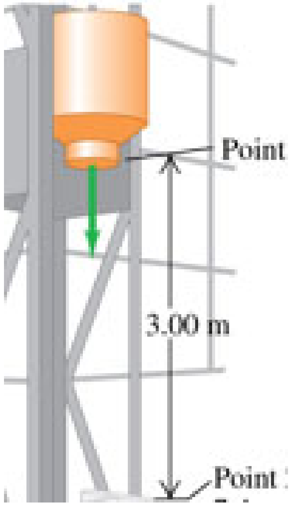
SET UP: Figure 6.12b shows the vertical forces on the hammerhead as it falls from point 1 to point 2. (We can ignore any horizontal forces that may be present because they do no work as the hammerhead moves vertically.) For this part of the motion, our target variable is the hammerhead's final speed v_2.

Figure 6.12c shows the vertical forces on the hammerhead during the motion from point 2 to point 3. In addition to the forces shown in Fig. 6.12b, the I-beam exerts an upward normal force of magnitude n on the hammerhead. This force actually varies as the hammerhead comes to a halt, but for simplicity we'll treat n as a constant. Hence n represents the *average* value of this upward force during the motion. Our target variable for this part of the motion is the force that the *hammerhead* exerts on the I-beam; it is the reaction force to the normal force exerted by the I-beam, so by Newton's third law its magnitude is also n.

Example 6.4

EXECUTE: (a) From point 1 to point 2, the vertical forces are the downward weight $w = mg = (200\ \text{kg})(9.8\ \text{m/s}^2) = 1960\ \text{N}$ and the upward friction force $f = 60\ \text{N}$. Thus the net downward force is $w - f = 1900\ \text{N}$. The displacement of the hammerhead from point 1 to point 2 is downward and equal to $s_{12} = 3.00\ \text{m}$. The total work done on the hammerhead between point 1 and point 2 is then

$$W_{\text{tot}} = (w - f)s_{12} = (1900\ \text{N})(3.00\ \text{m}) = 5700\ \text{J}$$



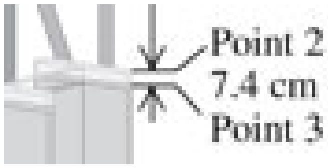
At point 1 the hammerhead is at rest, so its initial kinetic energy K_1 is zero. Hence the kinetic energy K_2 at point 2 equals the total work done on the hammerhead between points 1 and 2:

$$W_{\text{tot}} = K_2 - K_1 = K_2 - 0 = \tfrac{1}{2}mv_2^2 - 0$$

$$v_2 = \sqrt{\frac{2W_{\text{tot}}}{m}} = \sqrt{\frac{2(5700\ \text{J})}{200\ \text{kg}}} = 7.55\ \text{m/s}$$

This is the hammerhead's speed at point 2, just as it hits the I-beam.

(b) As the hammerhead moves downward from point 2 to point 3, its displacement is $s_{23} = 7.4 \text{ cm} = 0.074 \text{ m}$ and the net downward force acting on it is $w - f - n$ (Fig. 6.12c). The total work done on the hammerhead during this displacement is



$$W_{\text{tot}} = (w - f - n)s_{23}$$

The initial kinetic energy for this part of the motion is K_2, which from part (a) equals 5700 J. The final kinetic energy is $K_3 = 0$ (the hammerhead ends at rest). From the work–energy theorem,

$$W_{\text{tot}} = (w - f - n)s_{23} = K_3 - K_2$$

$$n = w - f - \frac{K_3 - K_2}{s_{23}}$$

$$= 1960 \text{ N} - 60 \text{ N} - \frac{0 \text{ J} - 5700 \text{ J}}{0.074 \text{ m}} = 79{,}000 \text{ N}$$

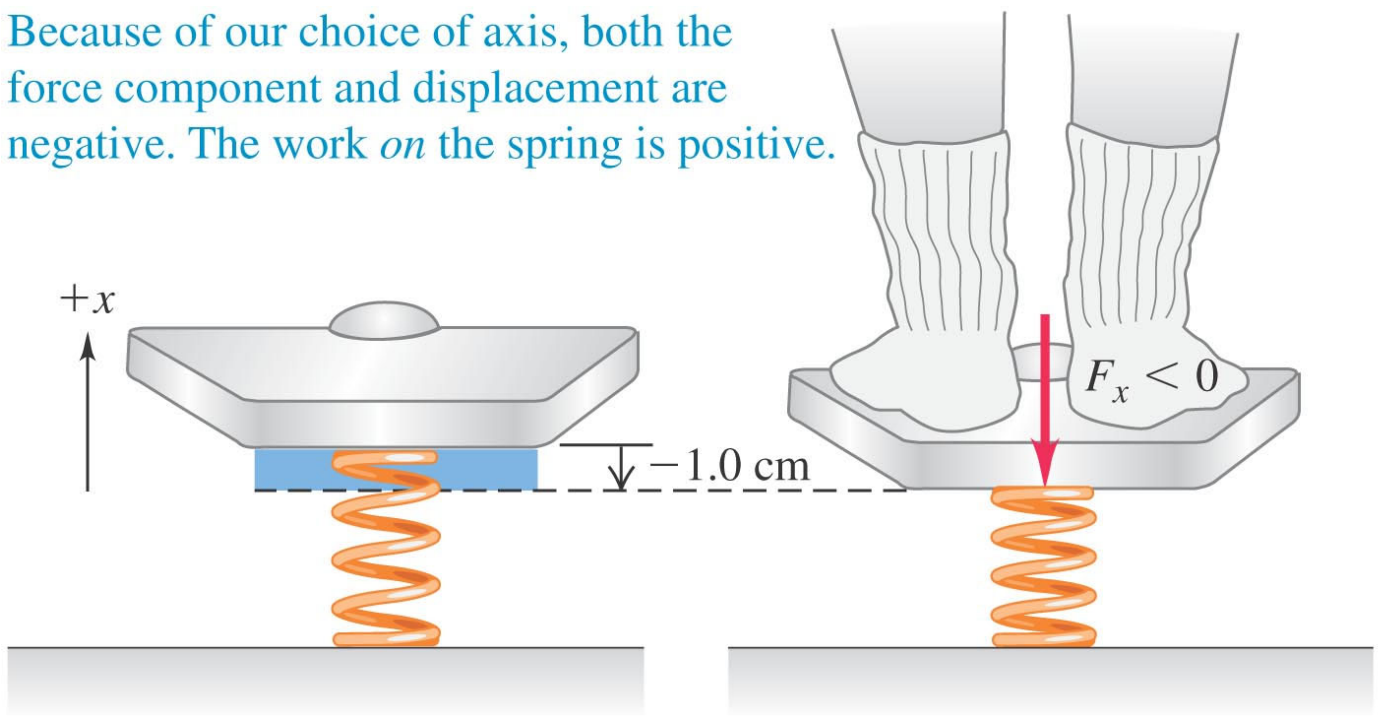
The downward force that the hammerhead exerts on the I-beam has this same magnitude, 79,000 N (about 9 tons)—more than 40 times the weight of the hammerhead.

Example 6.4

EVALUATE: The net change in the hammerhead's kinetic energy from point 1 to point 3 is zero; a relatively small net force does positive work over a large distance, and then a much larger net force does negative work over a much smaller distance. The same thing happens if you speed up your car gradually and then drive it into a brick wall. The very large force needed to reduce the kinetic energy to zero over a short distance is what does the damage to your car—and possibly to you.

Work done on a spring scale

- Example 6.6. A woman 600N steps on a bathroom scale that contains a stiff spring (see figure below). In equilibrium, the spring is compressed 1.0cm under her weight. Find the force constant of the spring and the total work done on it during the compression.



SOLUTION

IDENTIFY and SET UP: In equilibrium the upward force exerted by the spring balances the downward force of the woman's weight. We'll use this principle and Eq. (6.8) to determine the force constant k, and we'll use Eq. (6.10) to calculate the work W that the woman does on the spring to compress it. We take positive values of x to correspond to elongation (upward in Fig. 6.21), so that the displacement of the end of the spring (x) and the x-component of the force that the woman exerts on it (F_x) are both negative. The applied force and the displacement are in the same direction, so the work done on the spring will be positive.

EXECUTE: The top of the spring is displaced by $x = -1.0 \text{ cm} = -0.010 \text{ m}$, and the woman exerts a force $F_x = -600 \text{ N}$ on the spring. From Eq. (6.8) the force constant is then

$$k = \frac{F_x}{x} = \frac{-600 \text{ N}}{-0.010 \text{ m}} = 6.0 \times 10^4 \text{ N/m}$$

Then, using $x_1 = 0$ and $x_2 = -0.010$ m in Eq. (6.10), we have

$$W = \tfrac{1}{2}kx_2^2 - \tfrac{1}{2}kx_1^2$$

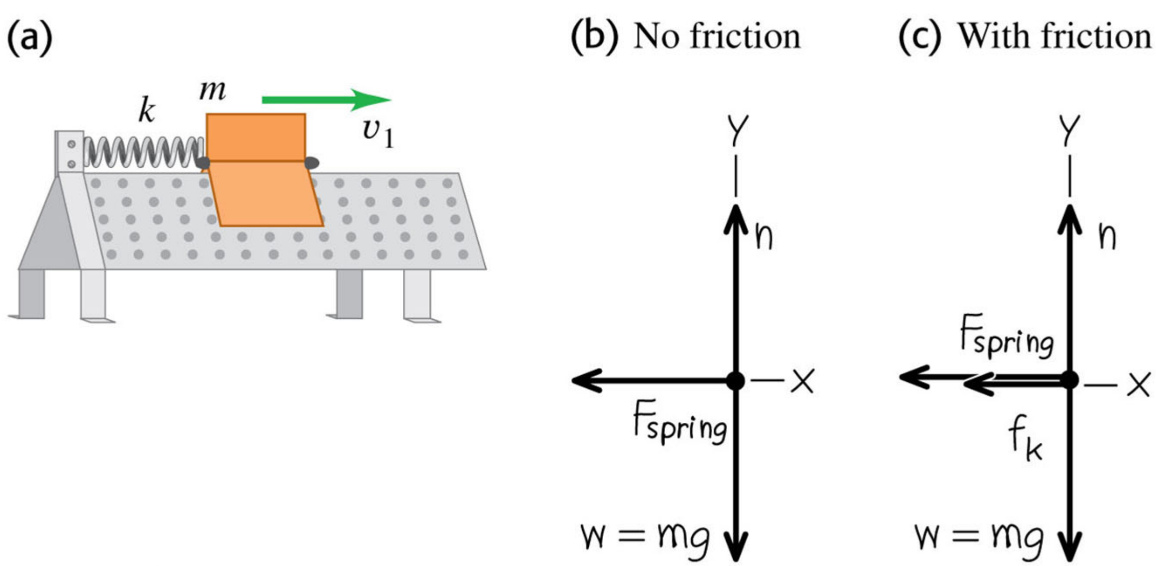
$$= \tfrac{1}{2}(6.0 \times 10^4 \text{ N/m})(-0.010 \text{ m})^2 - 0 = 3.0 \text{ J}$$

EVALUATE: The work done is positive, as expected. Our arbitrary choice of the positive direction has no effect on the answer for W. You can test this by taking the positive x-direction to be downward, corresponding to compression. Do you get the same values for k and W as we found here?

Motion with a varying force

- **Example 6.7** Motion with a varying force

- An air-track glider of mass 0.100 kg is attached to the end of a horizontal air track by a spring with force constant 20.0 N/m (Fig. 6.22a). Initially the spring is unstretched and the glider is moving at 1.50 m/s to the right. Find the maximum distance d that the glider moves to the right (a) if the air track is turned on, so that there is no friction, and (b) if the air is turned off, so that there is kinetic friction with coefficient $\mu_k = 0.47$.



SOLUTION

IDENTIFY and SET UP: The force exerted by the spring is not constant, so we *cannot* use the constant-acceleration formulas

work–energy theorem, since the total work done involves the distance moved (our target variable). In Figs. 6.22b and 6.22c we choose the positive x-direction to be to the right (in the direction of the glider's motion). We take $x = 0$ at the glider's initial position (where the spring is unstretched) and $x = d$ (the target variable) at the position where the glider stops. The motion is purely horizontal, so only the horizontal forces do work. Note that Eq. (6.10) gives the work done by the *glider* on the *spring* as it stretches; to use the work–energy theorem we need the work done by the *spring* on the *glider*, which is the negative of Eq. (6.10). We expect the glider to move farther without friction than with friction.

EXECUTE: (a) Equation (6.10) says that as the glider moves from $x_1 = 0$ to $x_2 = d$, it does an amount of work $W = \frac{1}{2}kd^2 - \frac{1}{2}k(0)^2 = \frac{1}{2}kd^2$ on the spring. The amount of work that the *spring* does on the *glider* is the negative of this, $-\frac{1}{2}kd^2$. The spring stretches until the glider comes instantaneously to rest, so the final kinetic energy K_2 is zero. The initial kinetic energy is $\frac{1}{2}mv_1{}^2$, where $v_1 = 1.50$ m/s is the glider's initial speed. From the work–energy theorem,

$$\Rightarrow \quad -\tfrac{1}{2}kd^2 = 0 - \tfrac{1}{2}mv_1{}^2$$

We solve for the distance d the glider moves:

$$\Rightarrow \quad d = v_1\sqrt{\frac{m}{k}} = (1.50\ \text{m/s})\sqrt{\frac{0.100\ \text{kg}}{20.0\ \text{N/m}}}$$
$$= 0.106\ \text{m} = 10.6\ \text{cm}$$

The stretched spring subsequently pulls the glider back to the left, so the glider is at rest only instantaneously.

(b) If the air is turned off, we must include the work done by the kinetic friction force. The normal force n is equal in magnitude to the weight of the glider, since the track is horizontal and there are no other vertical forces. Hence the kinetic friction force has constant magnitude $f_k = \mu_k n = \mu_k mg$. The friction force is directed opposite to the displacement, so the work done by friction is

$$\Rightarrow \quad W_{\text{fric}} = f_k d \cos 180^\circ = -f_k d = -\mu_k mgd$$

The total work is the sum of W_{fric} and the work done by the spring, $-\frac{1}{2}kd^2$. The work–energy theorem then says that

$$\Rightarrow \quad -\mu_k mgd - \tfrac{1}{2}kd^2 = 0 - \tfrac{1}{2}mv_1^2 \qquad \text{or}$$

$$\tfrac{1}{2}kd^2 + \mu_k mgd - \tfrac{1}{2}mv_1^2 = 0$$

This is a quadratic equation for d. The solutions are

$$d = -\frac{\mu_k mg}{k} \pm \sqrt{\left(\frac{\mu_k mg}{k}\right)^2 + \frac{mv_1^2}{k}}$$

We have

$$\frac{\mu_k mg}{k} = \frac{(0.47)(0.100\ \text{kg})(9.80\ \text{m/s}^2)}{20.0\ \text{N/m}} = 0.02303\ \text{m}$$

$$\frac{mv_1^{\ 2}}{k} = \frac{(0.100\ \text{kg})(1.50\ \text{m/s})^2}{20.0\ \text{N/m}} = 0.01125\ \text{m}^2$$

so

$$\begin{aligned} d &= -(0.02303\ \text{m}) \pm \sqrt{(0.02303\ \text{m})^2 + 0.01125\ \text{m}^2} \\ &= 0.086\ \text{m} \quad \text{or} \quad -0.132\ \text{m} \end{aligned}$$

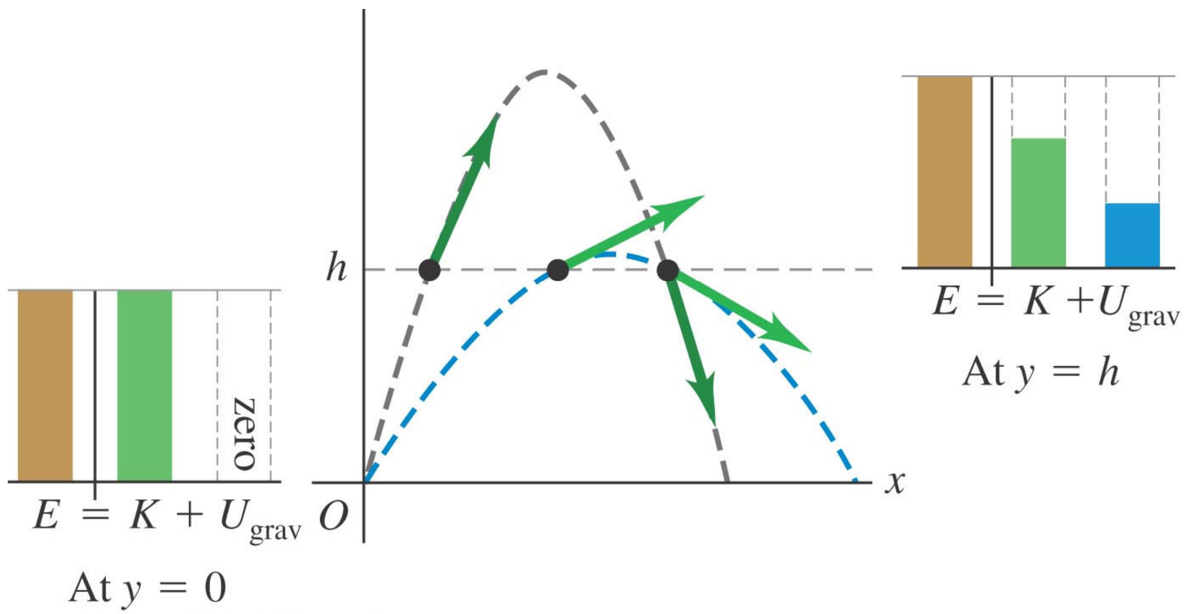
The quantity d is a positive displacement, so only the positive value of d makes sense. Thus with friction the glider moves a distance $d = 0.086\ \text{m} = 8.6\ \text{cm}$.

EVALUATE: Note that if we set $\mu_k = 0$, our algebraic solution for d in part (b) reduces to $d = v_1\sqrt{m/k}$, the zero-friction result from part (a). With friction, the glider goes a shorter distance. Again the glider stops instantaneously, and again the spring force pulls it toward the left; whether it moves or not depends on how great the *static* friction force is. How large would the coefficient of static friction μ_s have to be to keep the glider from springing back to the left?

Energy in projectile motion

- **Conceptual Example 7.3** Energy in projectile motion

- A batter hits two identical baseballs with the same initial speed and from the same initial height but at different initial angles. Prove that both balls have the same speed at any height h if air resistance can be neglected.

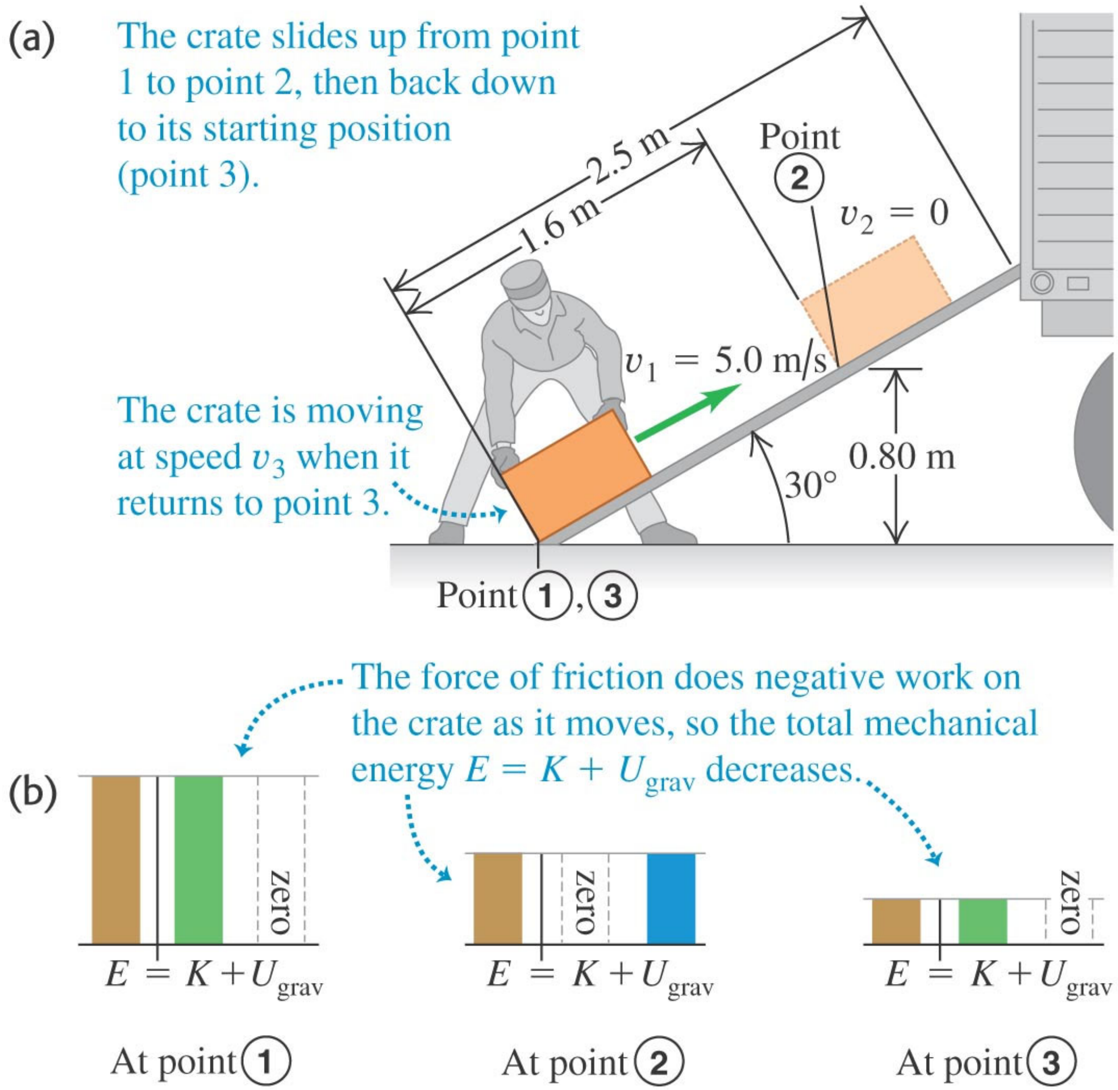


SOLUTION

The only force acting on each ball after it is hit is its weight. Hence the total mechanical energy for each ball is constant. Figure 7.8 shows the trajectories of two balls batted at the same height with the same initial speed, and thus the same total mechanical energy, but with different initial angles. At all points at the same height the potential energy is the same. Thus the kinetic energy at this height must be the same for both balls, and the speeds are the same.

Moving a crate on an inclined plane with friction

- Follow Example 7.6 using Figure 7.11 to the right.
- Notice that mechanical energy was lost due to friction.

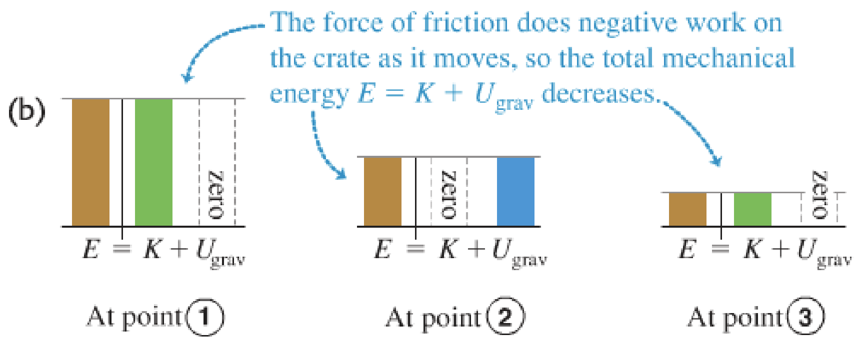


Example 7.6 An inclined plane with friction

We want to slide a 12-kg crate up a 2.5-m-long ramp inclined at 30°. A worker, ignoring friction, calculates that he can do this by giving it an initial speed of 5.0 m/s at the bottom and letting it go. But friction is *not* negligible; the crate slides only 1.6 m up the ramp, stops, and slides back down (Fig. 7.11a). (a) Find the magnitude of the friction force acting on the crate, assuming that it is constant. (b) How fast is the crate moving when it reaches the bottom of the ramp?

SOLUTION

IDENTIFY and SET UP: The friction force does work on the crate as it slides. The first part of the motion is from point 1, at the bottom of the ramp, to point 2, where the crate stops instantaneously ($v_2 = 0$). In the second part of the motion, the crate returns to the bottom of the ramp, which we'll also call point 3 (Fig. 7.11a). We take the positive y-direction upward. We take $y = 0$ (and hence $U_{grav} = 0$) to be at ground level (point 1), so that $y_1 = 0$, $y_2 = (1.6\text{ m})\sin 30° = 0.80\text{ m}$, and $y_3 = 0$. We are given $v_1 = 5.0\text{ m/s}$. In part (a) our target variable is f, the magnitude of the friction force as the crate slides up; as in Example 7.2, we'll find this using the energy approach. In part (b) our target variable is v_3, the crate's speed at the bottom of the ramp. We'll calculate the work done by friction as the crate slides back down, then use the energy approach to find v_3.



EXECUTE: (a) The energy quantities are

$$
\begin{aligned}
K_1 &= \tfrac{1}{2}(12\ \text{kg})(5.0\ \text{m/s})^2 = 150\ \text{J} \\
U_{\text{grav},1} &= 0 \\
K_2 &= 0 \\
U_{\text{grav},2} &= (12\ \text{kg})(9.8\ \text{m/s}^2)(0.80\ \text{m}) = 94\ \text{J} \\
W_{\text{other}} &= -fs
\end{aligned}
$$

Here $s = 1.6$ m. Using Eq. (7.7), we find

$$
\begin{aligned}
K_1 + U_{\text{grav},1} + W_{\text{other}} &= K_2 + U_{\text{grav},2} \\
W_{\text{other}} = -fs &= (K_2 + U_{\text{grav},2}) - (K_1 + U_{\text{grav},1}) \\
&= (0 + 94\ \text{J}) - (150\ \text{J} + 0) = -56\ \text{J} = -fs \\
f = \frac{W_{\text{other}}}{s} &= \frac{56\ \text{J}}{1.6\ \text{m}} = 35\ \text{N}
\end{aligned}
$$

The friction force of 35 N, acting over 1.6 m, causes the mechanical energy of the crate to decrease from 150 J to 94 J (Fig. 7.11b).

(b) As the crate moves from point 2 to point 3, the work done by friction has the same negative value as from point 1 to point 2. (The friction force and the displacement both reverse direction but have the same magnitudes.) The total work done by friction between points 1 and 3 is therefore

$$W_{\text{other}} = W_{\text{fric}} = -2fs = -2(56\text{ J}) = -112\text{ J}$$

From part (a), $K_1 = 150\text{ J}$ and $U_{\text{grav},1} = 0$. Equation (7.7) then gives

$$K_1 + U_{\text{grav},1} + W_{\text{other}} = K_3 + U_{\text{grav},3}$$

$$\begin{aligned} K_3 &= K_1 + U_{\text{grav},1} - U_{\text{grav},3} + W_{\text{other}} \\ &= 150\text{ J} + 0 - 0 + (-112\text{ J}) = 38\text{ J} \end{aligned}$$

The crate returns to the bottom of the ramp with only 38 J of the original 150 J of mechanical energy (Fig. 7.11b). Since $K_3 = \frac{1}{2}mv_3^2$,

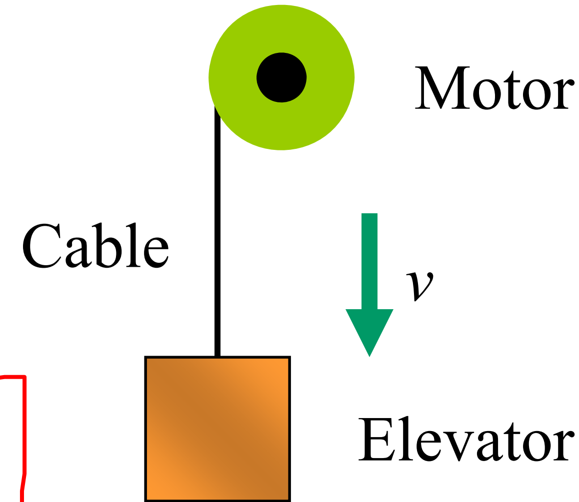
$$v_3 = \sqrt{\frac{2K_3}{m}} = \sqrt{\frac{2(38\text{ J})}{12\text{ kg}}} = 2.5\text{ m/s}$$

Think questions

TQ 4.1

$W = \vec{F}\,\Delta s \cos\phi$

An elevator is being *lowered* at a constant speed by a steel cable attached to an electric motor. Which statement is correct?



$W_c < 0$

A. The cable does positive work on the elevator, and the elevator does positive work on the cable.

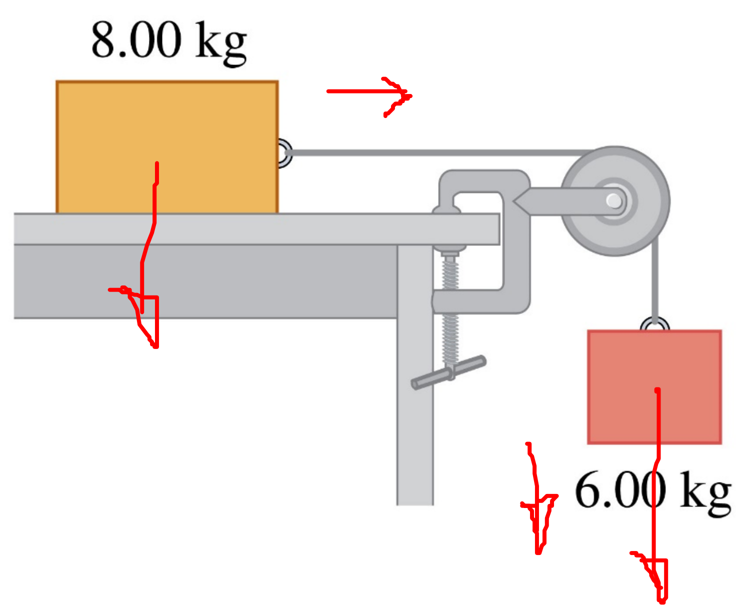
B. The cable does positive work on the elevator, and the elevator does negative work on the cable.

C. The cable does negative work on the elevator, and the elevator does positive work on the cable.

D. The cable does negative work on the elevator, and the elevator does negative work on the cable.

TQ 4.2

A 6.00-kg block and an 8.00-kg block are connected as shown. When released, the 6.00-kg block accelerates downward and the 8.00-kg block accelerates to the right. After each block has moved 2.00 cm, the force of gravity has done



A. more work on the 8.00-kg block than on the 6.00-kg block.

B. the same amount of work on both blocks.

C. less work on the 8.00-kg block than on the 6.00-kg block.

D. not enough information given to decide

TQ 4.3

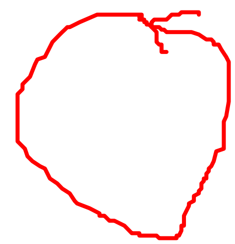
A nonzero net force acts on an object. Which of the following quantities could be *constant?*

A. the object's kinetic energy

B. the object's velocity

C. both of the above

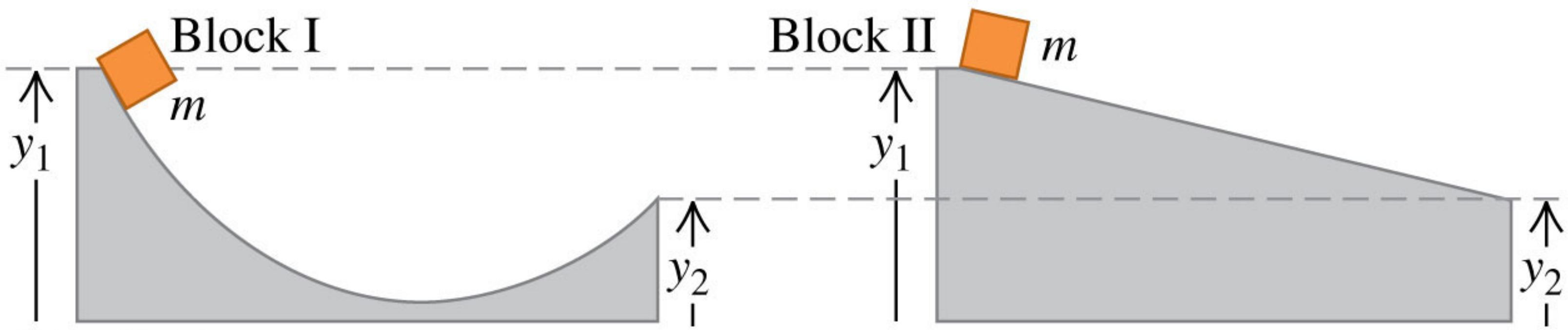
D. none of the above



$\vec{v} \neq$ const

TQ 4.5

The two ramps shown are both frictionless. The heights y_1 and y_2 are the same for each ramp. A block of mass m is released from rest at the left-hand end of each ramp. Which block arrives at the right-hand end with the greater speed?



A. the block on the curved track

B. the block on the straight track

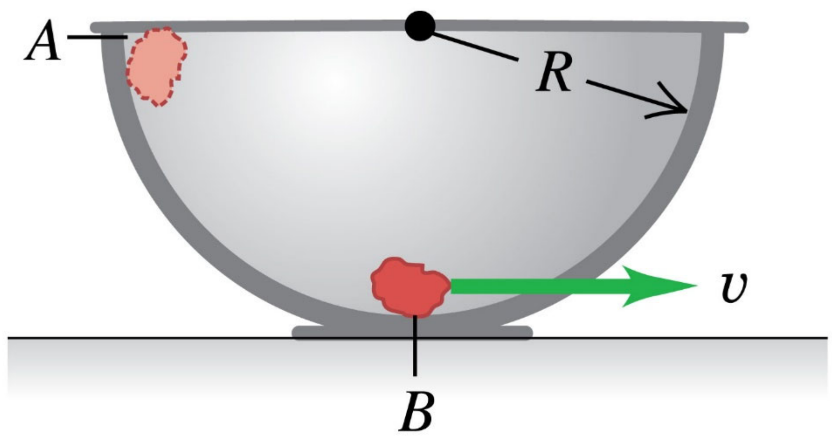
C. Both blocks arrive at the right-hand end with the same speed.

D. The answer depends on the shape of the curved track.

TQ 4.6

As a rock slides from *A* to *B* along the inside of this frictionless hemispherical bowl, mechanical energy is conserved. Why?

(Ignore air resistance.)



A. The bowl is hemispherical.

B. The normal force is balanced by centrifugal force.

C. The normal force is balanced by centripetal force.

D. The normal force acts perpendicular to the bowl's surface.

E. The rock's acceleration is perpendicular to the bowl's surface.