EE1001
Foundations of Digital Techniques

Logic

Assignment 1

Validity and Soundness of Argument
Propositional Logic
Conditionals

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Question 1

- Q1)
- $A = \{4, 14, 66, 70\}$, $\exists\, x \in A$ such that x is an odd number. Determine whether the statement is T or F.

Question 1

- **Q1)**
- **A = {4, 14, 66, 70}, ∃ x ∈ A such that x is an odd number. Determine whether the statement is T or F.**

- Solution Q1)
- Consider A = {4, 14, 66, 70}. Let p: ∃ x ∈ A such that x is an odd number. Here, the statement p uses the quantifier ‘there exists’ (∃). This statement is true if at least one element of set A satisfies the condition ‘x is an odd number’ and is false otherwise. Here, the given statement is false as none of the elements of set A satisfy the condition, ‘x ∈ A such that x is an odd number’.

Question 2

- Q2)
- $A = \{1, 2, 3\}$, $p: \forall x \in A, x < 4$. Determine whether the statement is T or F.
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Question 2

- **Q2)**
- **A = {1, 2, 3}, p: $\forall$ x $\in$ A, x < 4. Determine whether the statement is T or F.**
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- Solution Q2)
- A = {1, 2, 3} Let p: $\forall$ x $\in$ A, x < 4 Here, the statement p uses the quantifier 'for all'($\forall$). This statement is true if and only if each and every element of set A satisfies the condition 'x < 4' and is false otherwise. Here, the given statement is true for all the elements of set A, as 1, 2, 3 satisfy the condition, 'x $\in$ A, x < 4'.

Question 3

- Q3)
- Write the negations of following statements.
- i. $\forall$ n $\in$ N, n + 1 > 2.
- ii. $\forall$ x $\in$ N, x 2 + x is even number.

Question 3

Write the negations of following statements.

i) $\forall n \in \mathbf{N},\ n+1 > 2$

ii) $\forall x \in \mathbf{N},\ x^2 + x$ is an even number

Ans:

i) $\sim(\forall n \in \mathbf{N},\ n+1 > 2)$

$= \exists n \in \mathbf{N},\ n+1 \leq 2$

Ans:

ii) $\sim(\forall x \in \mathbf{N},\ x^2 + x$ is an even number$)$

$= \exists x \in \mathbf{N},\ x^2 + x$ is not an even number

Question 4

1) If **S**uperheroexists, then Superhero is all-**p**owerful) and perfectly-**g**ood.

2). If Superhero is all-powerful, then he would be **a**ble to prevent crimes.

3). If Superhero is perfectly-good, then he would be **w**illing to prevent crimes.

4). If Superhero is able to and willing to prevent crimes, then there would be no crime.

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5) There are **c**rimes.

Conclusion: Superhero does not exist.

Use inference rules and logical equivalence relation to determine the validity of the argument above.

Question 4

1) If **S**uperhero exists, then Superhero is all-**p**owerful) and perfectly-**g**ood.

2). If Superhero is all-powerful, then he would be **a**ble to prevent crimes.

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4). If Superhero is able to and willing to prevent crimes, then there would be no crime.

5) There are **c**rimes.

Conclusion: Superhero does not exist.

Use inference rules and logical equivalence relation to determine the validity of the argument above.

Ans:

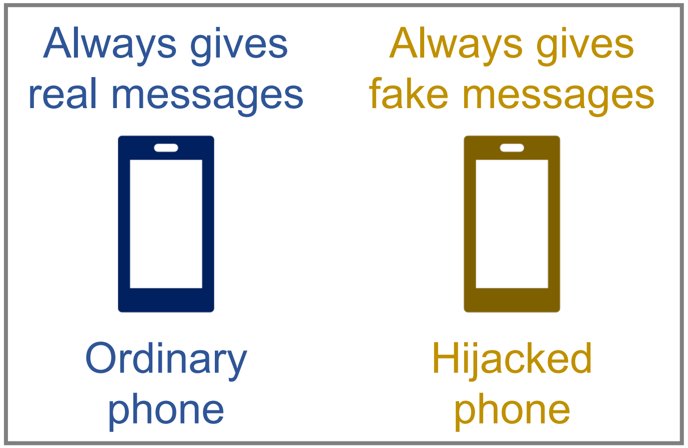
1) $S \rightarrow (p \wedge g)$
2) $p \rightarrow a$
3) $g \rightarrow w$
4) $(a \wedge w) \rightarrow \sim e$
5) e
6) $\sim(a \wedge w)$ (MT 4,5)
7) $\sim a \vee \sim w$ (De Morgan 6)
8) $\sim a \rightarrow \sim p$ (contrapositive 2)
9) $\sim w \rightarrow \sim g$ (contrapositive 3)
10) $(\sim a \rightarrow \sim p) \wedge (\sim w \rightarrow \sim b)$ (C 8,9)
11) $\sim p \vee \sim g$ (CD 10,7)
12) $\sim(p \wedge g)$ (De Morgan 11)
13) $\sim S$ (MT 1,12)

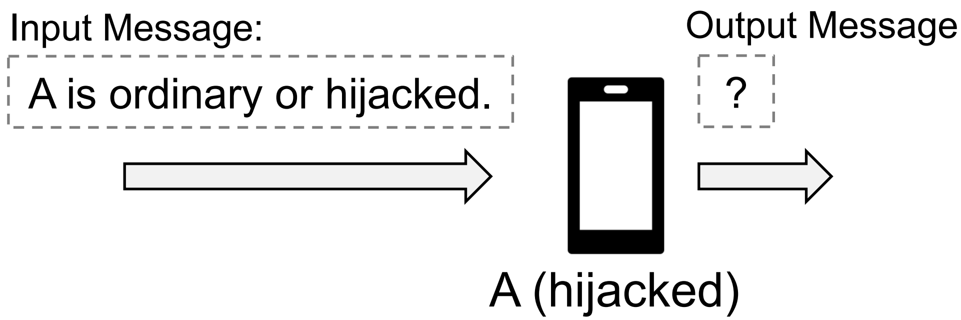
Therefore, the argument is valid

Caution: A valid argument may not be sound.

Question 5

Q5. There are two types of phones:

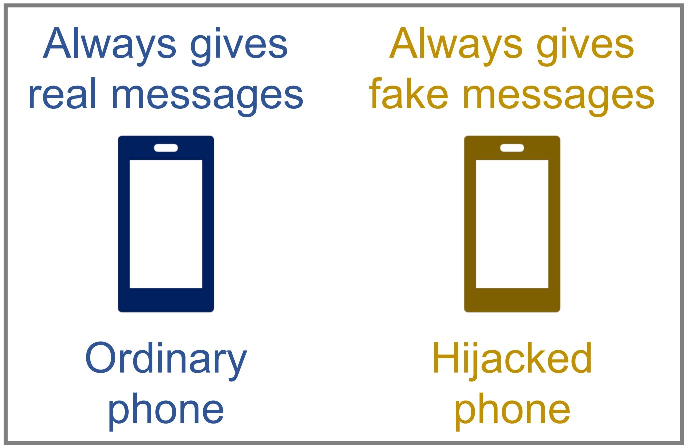


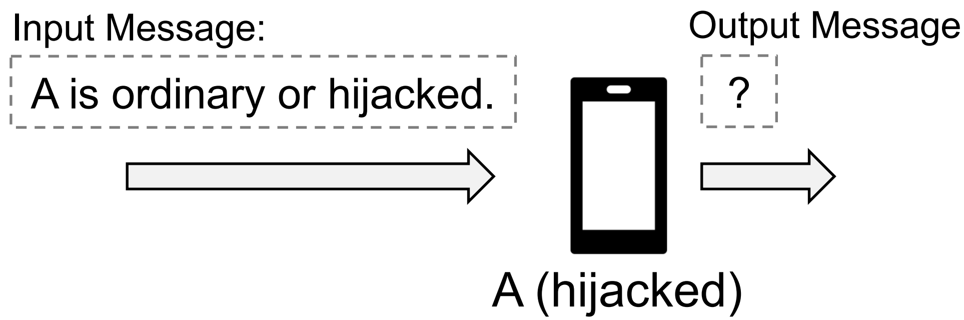


Question:
A is a hijacked phone, and my input message is "A is ordinary or hijacked".
<u>What message will A output</u>?

Question 5

Q5. There are two types of phones:





Question:
A is a hijacked phone, and my input message is "A is ordinary or hijacked".
<u>What message will A output</u>?

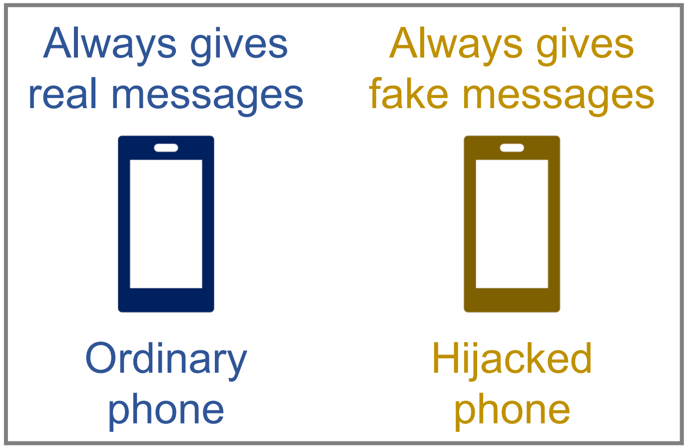
Ans:

Let p = "A is an ordinary phone"
The statement "A is ordinary or hijacked" can be formulated as **"**$p \vee \sim p$**"**
Given A is a hijacked phone, its output message should be the <u>**negation**</u> of the input message, i.e.,

$$\begin{aligned} \text{Output message} &= \sim(p \vee \sim p) \\ &= \sim p \wedge \sim(\sim p) \qquad \text{(De Morgan's laws)} \\ &= \sim p \wedge p \\ &= \text{A is hijacked and ordinary} \end{aligned}$$

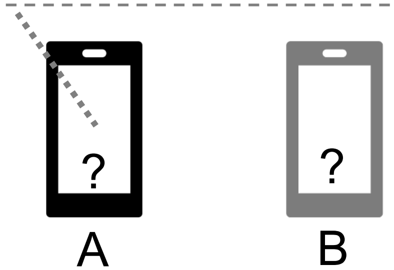
Question 6

Q6. There are two types of phones:



The Message given by A:

We are both ordinary phones.

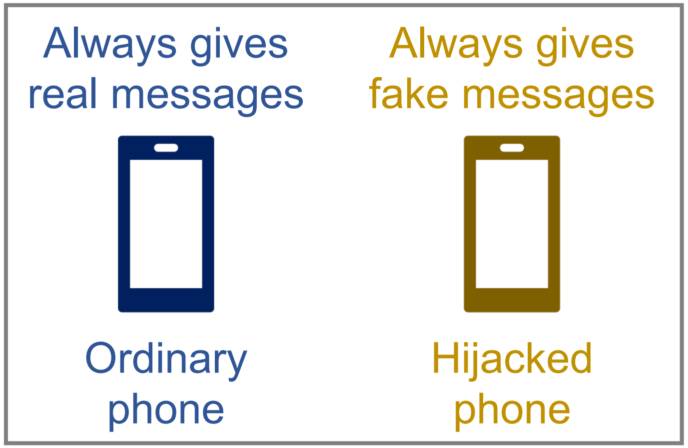


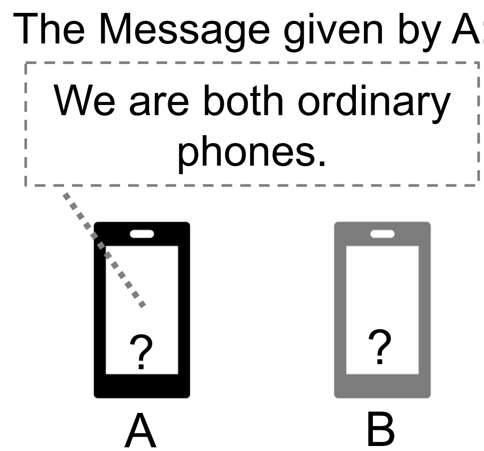
Question:

Are A and B ordinary or hijacked? Use truth table to justify.

Question 6

Q6. There are two types of phones:





Question:
Are A and B ordinary or hijacked? Use truth table to justify.

Ans:
Let p = "A is an ordinary phone", and q = "B is an ordinary phone".
Therefore, the statement "A and B are ordinary" can be formulated as "$p \wedge q$"
The condition is satisfied only when $\boldsymbol{p \leftrightarrow (p \wedge q)}$ **= True**

p	q	$p \wedge q$	$p \leftrightarrow (p \wedge q)$
T	T	T	**T**
T	F	F	F
F	T	F	**T**
F	F	F	**T**

∴ Three possible solutions

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