MA1300 Self Practice # 7

1. (P156, #85)

   a If n is a positive integer, prove that
$$\frac{d}{dx}(\sin^n x \cos nx) = n\sin^{n-1} x \cos(n+1)x.$$

   b Find a formula for the derivative of $y = \cos^n x \cos nx$ that is similar to the on in part **a**.

2. (P156, #88)

   a Write $|x| = \sqrt{x^2}$ and use the Chain Rule to show that
$$\frac{d}{dx}|x| = \frac{x}{|x|}.$$

   b If $f(x) = |\sin x|$, find $f'(x)$ and sketch the graphs of f and f'. Where is f not differentiable?

   c If $g(x) = \sin|x|$, find $g'(x)$ and sketch the graphs of g and g'. Where is g not differentiable?

3. (P156, #89) If $y = f(u)$ and $u = g(x)$, where f and g are twice differentiable functions, show that
$$\frac{d^2y}{dx^2} = \frac{d^2y}{du^2}\left(\frac{du}{dx}\right)^2 + \frac{dy}{du}\frac{d^2u}{dx^2}.$$

4. (P156, #90) If $y = f(u)$ and $u = g(x)$, where f and g possess third derivatives, find a formula for d^3y/dx^3 similar to the one given in Exercise 3.

5. (P161, #2, 4) For each of the following implicit function relations
$$4x^2 + 9y^2 = 36, \qquad \cos x + \sqrt{y} = 5,$$

   a Find y' by implicit differentiation.

   b Solve the equation explicitly for y and differentiate to get y' in terms of x.

   c Check that your solutions to parts **a** and **b** are consistent by substituting the expression for y into your solution for part **a**.

6. (P161, #12, 15) Find dy/dx by implicit differentiation.
$$1 + x = \sin(xy^2), \qquad \tan\frac{x}{y} = x + y.$$

7. (P161, #21) If $f(x) + x^2[f(x)]^3 = 10$ and $f(1) = 2$, find $f'(1)$.

8. (P161, #22) If $g(x) + x\sin g(x) = x^2$, find $g'(0)$.

9. (P162, #33)

a The curve with equation $y^2 = 5x^4 - x^2$ is called a **kampyle of Eudoxus**. Find an equation of the tangent line to this curve at the point $(1, 2)$.

b Illustrate part **a** by graphing the curve and the tangent line on a common screen. (If your graphing device will graph implicitly defined curves, then use that capability. If not, you can still graph this curve by graphing its upper and lower halves separately.)

10. (P162, #34)

a The curve with equation $y^2 = x^3 + 3x^2$ is called the **Tschirnhausen cubic**. Find an equation of the tangent line to this curve at the point $(1, -2)$.

b At what points does this curve have horizontal tangents?

c Illustrate parts **a** and **b** by graphing the curve and the tangent lines on a common screen.

11. (P162, #44) Show by implicit differentiation that the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at the point (x_0, y_0) is

$$\frac{x_0 x}{a^2} + \frac{y_0 y}{b^2} = 1.$$

12. (P162, #45) Find an equation of the tangent line to the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

at point (x_0, y_0).

13. (P162, #47) Show, using implicit differentiation, that any tangent line at a point P to a circle with center O is perpendicular to the radius OP.