
WEEK 2
AURORA

CHAPTER ZERO REVISION

- Coordinate
- Vector and Scalar
- **Matrices**

MATRICES

- Definition and properties of matrices
- Determinants
- Special types of matrices
- Solving the system of equation

DEFINITION

Matrix : A set of mn numbers (real or complex), arranged in a rectangular formation having m rows and n columns.

capital letter → $\underline{\underline{A}}$ = square bracket →

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

$m \times n$ (m by n matrix)

a_{ij} i^{th} row

j^{th} column

ORDER OF MATRIX

column matrix = column vector

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad C = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

square matrix

2×3 $\quad$ 3×1 $\quad$ 3×3

$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ (row matrix) = row vector

1×3

Remark order of matrix matters $3 \times 1 \neq 1 \times 3$

REPRESENTATION OF MATRIX

- Row Matrix and Column Matrix
- Zero Matrix
- Square matrix
- Square matrix: diagonal matrix
- Scalar Matrix
- Identity Matrix or Unit matrix
- Equal Matrix
- Negative of a Matrix

ROW MATRIX AND COLUMN MATRIX

column matrix = column vector

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

2×3 $\qquad$ 3×1

ZERO

ie.

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A + O = O + A = A$$

SQUARE MATRIX

$A_{m \times n} \therefore m = n \quad A_n$ or $n \times n$

ie. $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \quad \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

SQUARE MATRIX: DIAGONAL MATRIX

ie. $\begin{bmatrix} 1 & 3 & -1 \\ 5 & 2 & 3 \\ 6 & 4 & 0 \end{bmatrix}$

a_{11}, a_{22}, a_{33}

$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

All elements are zero except for the principal diagonal. however, it is possible to have some zero values as principal diagonal.

SCALAR MATRIX

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \text{ and } \begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix}$$

principal diagonal elements are the same.

IDENTITY MATRIX OR UNIT MATRIX

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AI = IA = A$$

EQUAL MATRIX

$A = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} \frac{4}{2} & 2-1 \\ \sqrt{9} & 0 \end{bmatrix}$

Note: # are different $B = \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix}$ $\therefore$ $A = B$

Ex. $\begin{bmatrix} x+3 & 2y+x \\ z-1 & 4a-b \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2a \end{bmatrix}$ Find x, y, z

$x+3=0$ $z-1=3$

$2y+x=-7$ $4a-b=2a$

$2y-3=-7$ $2y=-4$ $y=-2$

$\begin{cases} x=-3 \\ z=4 \\ y=-2 \end{cases}$

NEGATIVE OF A MATRIX

ie. $A_{3\times 2} = \begin{bmatrix} 3 & -1 \\ 2 & -2 \\ -4 & 5 \end{bmatrix}$ $-A_{3\times 2} = \begin{bmatrix} -3 & 1 \\ -2 & 2 \\ 4 & -5 \end{bmatrix}$

$A + (-A) = (-A) + A = 0$

Remark: $-A$ is the additive inverse of A

$\therefore B_{m\times n} + (-A_{m\times n}) = B_{m\times n} - A_{m\times n}$

OPERATIONS ON MATRICES

- Multiplication of a Matrix by a Scalar
- Addition and subtraction of Matrices
- Product of Matrices

MULTIPLICATION OF A MATRIX BY A SCALAR

ie. $A = \begin{bmatrix} 1 & 4 \\ 2 & -5 \\ 3 & 6 \end{bmatrix}$ $kA = \begin{bmatrix} k & 4k \\ 2k & -5k \\ 3k & 6k \end{bmatrix}$ where k is a scalar.

ie. $3A = \begin{bmatrix} 3 & 12 \\ 6 & -15 \\ 9 & 18 \end{bmatrix}$

ADDITION AND SUBTRACTION OF MATRICES

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 3 & 0 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3+1 & 1+0 & 2+2 \\ 2+-1 & 1+3 & 4+0 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 4 \\ 1 & 4 & 4 \end{bmatrix} \quad A+B=B+A$$

$$A-B = \begin{bmatrix} 3-1 & 1-0 & 2-2 \\ 2-(-1) & 1-3 & 4-0 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 3 & -2 & 4 \end{bmatrix}$$

Remarks: $(A+B)+C = A+(B+C)$

PRODUCT OF MATRICES

$$A_{2\times 2} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \qquad B_{2\times 2} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{bmatrix}$$

Remark ① $AB \neq BA$ (not always true)

② $AA = A^2$

$AA^2 = A^3$

$A^2A^2 = A^4$

EXAMPLE

Ex. $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}_{2\times 3}$ $B = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}_{3\times 2}$

$$AB = \begin{bmatrix} 3+2+6 & -3+1+2 \\ 1+0+3 & -1+0+1 \end{bmatrix} = \begin{bmatrix} 11 & 0 \\ 4 & 0 \end{bmatrix}$$

REMARK

Remark. If A, B, C to be in the order of $(m\times p)$, $(p\times q)$, and $(q\times n)$

$$\begin{cases} (AB)C = A(BC) \\ C(A+B) = CA+CB \\ (A+B)C = AC+BC \end{cases}$$

* $AI = IA = A$

* $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

DETERMINANTS

- Definition
- Minor and Cofactor of Element
- Properties of the Determinant

DEFINITION

A determinant of a matrix is a scalar number. It is only defined for square matrices

$|A|$ or $\det A$

ie. Determinant of $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

notation

$$|A| = \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

$$= a_{11}a_{22} - a_{12}a_{21}$$

"2x2"

ie. $A = \begin{bmatrix} 3 & 1 \\ -2 & 3 \end{bmatrix}$ Find determinant

Ans:

$$|A| = \begin{vmatrix} 3 & 1 \\ -2 & 3 \end{vmatrix} = (3)(3) - (-2)(1) = 9 + 2 = 11$$

ie. $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$$\det A = |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{31}a_{23}) + a_{13}(a_{21}a_{32} - a_{31}a_{22})$$

$$= \#$$

Ex. $|A| = \begin{vmatrix} 3 & 2 & 1 \\ 0 & 1 & -2 \\ 1 & 3 & 4 \end{vmatrix} = 3\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} - 2\begin{vmatrix} 0 & -2 \\ 1 & 4 \end{vmatrix} + 1\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix}$

$= 3(4+6) - 2(0+2) + 1(0-1)$

$= 30 - 4 - 1 = 25$ ✓

method 2

$|A| = 3\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} - 0 \cancel{\begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix}}^{0} + 1\begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix}$

$= 3(4+6) + 1(-4-1)$

$= 30 - 5 = 25$ ✓

PROPERTIES OF THE DETERMINANT (TOTAL 9)

1. Interchange rows and column is possible. As a result, determinant value doesn't change.

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \rightarrow |B| = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$|A| = |B|$$

2. If two rows or two columns of a determinant are interchanged, the sign of the determinant is changed but its absolute value is unchanged.

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad |B| = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$|B| = -a_1(b_2c_3 - b_3c_2) + b_1(a_2c_3 - a_3c_2) - c_1(a_2b_5 - a_3b_2)$$

$$= -(a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2))$$

$$\therefore \quad |B| = -\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \text{or} \quad |B| = -|A|$$

3. If every element of a row or column of a determinant is zero, the value of the determinant is zero.

$$|A| = \begin{vmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$= 0(b_2c_3 - b_3c_2) - 0(a_2c_3 - a_3c_2) + 0(a_2b_3 - a_3b_2)$$

$$|A| = 0$$

4.

4. If 2 rows or columns of a determinant are identical, the value of the determinant is zero.

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1(b_1c_3 - b_3c_1) - b_1(a_1c_3 - a_3c_1) + c_1(a_1b_3 - a_3b_1)$$

$$= 0$$

5.

If every element of a row or column of a determinant is multiplied by the same constant k, the value of the determinant is multiplied by that constant

Ex.

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \qquad |B| = \begin{vmatrix} ka_1 & kb_2 & kc_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$|B| = ka_1(b_2c_3 - b_3c_3) - kb_1(a_2c_3 - a_3c_2) + kc_1(a_2b_3 - a_3b_2)$$

$$= k[\underline{a_1(b_2c_3 - b_3c_3) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)}]$$

$$|B| = k|A|$$

6.

The value of a determinant is not changed if each element of any row or of any column is added to or subtracted from a constant multiple of the corresponding element of another row or column

i.e. $|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

$$|B| = \begin{vmatrix} a_1 + ka_2 & b_1 + kb_2 & c_1 + kc_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$|B| = (a_1 + ka_2)(b_2c_3 - b_3c_2) - (b_1 + kb_2)(a_2c_3 - a_3c_2) + (c_1 + kc_2)(a_2b_3 - a_3b_2)$$

$$|B| = [a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)]$$

$$|B| = [ka_2(b_2c_3 - b_3c_2) - kb_2(a_2c_3 - a_3c_2) + kc_2(a_2b_3 - a_3b_2)]$$

$$|B| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + k \underbrace{\begin{vmatrix} a_2 & b_2 & c_2 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}}_{0}$$

using property 4.

$|B| = |A|$

7.

Diagonal matrix : the determinant of DM is equal to the product of its diagonal elements.

DM

$$|A| : \begin{vmatrix} 2 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 2(-15-0)-(0-0)+0(0-0)$$

$$= -30$$

Remark $2 \times -5 \times 3 = -30$

8.

The determinant of the product of 2 matrices is equal to the product of determinants of 2 matrices

$$|AB| = |A||B|$$

ie. $A = \begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

$|AB| = |A| \; |B|$

$\begin{vmatrix} 1+4 & 0+2 \\ 3+0 & 0+0 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} \; \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$

$\begin{vmatrix} 5 & 2 \\ 3 & 0 \end{vmatrix} = 0-6=-6 \quad 1-0=1$

$0-6=-6 = (-6)(1)$

$-6 = -6$

9. The determinant in which each element in any row or column consists of 2 terms, then the determinant can be expressed as the sum of 2 other determinants

$$\begin{vmatrix} a_1+\alpha_1 & b_1 & c_1 \\ a_2+\alpha_2 & b_2 & c_2 \\ a_3+\alpha_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} + \begin{vmatrix} \alpha_1 & b_1 & c_1 \\ \alpha_2 & b_2 & c_2 \\ \alpha_3 & b_3 & c_3 \end{vmatrix}$$

Expand at home! :)

TYPES OF SPECIAL MATRICES

- Transpose
- Symmetric
- Skew
- Singular and Non-singular Matrices
- Adjoint of a Matrix
- Inverse

TRANSPOSE

ie.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \qquad A^t = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

rows and
columns
are interchangable

SYMMETRIC

i.e. $A = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix}$

$A^t = \begin{bmatrix} a & b & c \\ b & d & e \\ c & e & f \end{bmatrix} = A$

If $A^t = A$
then
symmetric

SKEW

ie.

$$B = \begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix} \qquad B^t = \begin{bmatrix} 0 & 4 & -1 \\ -4 & 0 & 3 \\ 1 & -3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -4 & 1 \\ 4 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}$$

$$B^t = -B$$

SINGULAR AND NON-SINGULAR MATRICES

A square matrix A is called singular if $|A| = 0$
nonsingular if $|A| \neq 0$

ie. $A = \begin{bmatrix} 3 & 2 \\ 9 & 6 \end{bmatrix}$ then $|A| = 0$ $\therefore$ singular

ie. $A = \begin{bmatrix} 3 & 1 & 6 \\ -1 & 3 & 2 \\ 1 & 0 & 0 \end{bmatrix} = 1\begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} \neq 0$ non singular

Ex. find k if $A = \begin{bmatrix} k-2 & 1 \\ 5 & k+2 \end{bmatrix}$ is singular

Ans $\begin{vmatrix} k-2 & 1 \\ 5 & k+2 \end{vmatrix} = 0$

$(k-2)(k+2) - 5 = 0$

$k^2 - 4 - 5 = 0$

$k^2 - 9 = 0$

$(k+3)(k-3) = 0$

$k = \pm 3$

INVERSE

ie. $A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$ show that $AB = BA = I$

$$AB = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\therefore \quad AB = BA = I$

$\therefore \quad B = \underset{\substack{\uparrow \\ \text{Inverse}}}{A^{-1}} = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$

CRAMER'S RULE (SOLVING SYSTEM OF EQUATIONS)

2x2

$$a_1x + b_1y = c_1 \quad \cdots ①$$

$$a_2x + b_2y = c_2 \quad \cdots ②$$

$$x(a_1b_2 - a_2b_1) = b_2c_1 - b_1c_2$$

$$y(a_2b_1 - a_1b_2) = a_2c_1 - a_1c_2$$

$$x = \frac{b_2c_1 - b_1c_2}{a_1b_2 - a_2b_1}$$

$$y = \frac{a_2c_1 - a_1c_2}{a_2b_1 - a_1b_2}$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

$$|A| = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$$ determinat of the coefficient of x and y

$$|A_x| = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}$$

$$\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = |A_y|$$

$$\therefore \quad x = \frac{|A_x|}{|A|} \qquad y = \frac{|A_y|}{|A|}$$

Now $\underline{\underline{3 \times 3}}$.

$$a_1 x + b_1 y + c_1 z = d_1$$
$$a_2 x + b_2 y + c_2 z = d_2$$
$$a_3 x + b_3 y + c_3 z = d_3$$

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}, \quad |A| \neq 0$$

$$x = \frac{\begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}}{|A|} \qquad y = \frac{\begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}}{|A|} \qquad z = \frac{\begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}}{|A|}$$

$$x = \frac{|A_x|}{|A|} \qquad y = \frac{|A_y|}{|A|} \qquad z = \frac{|A_z|}{|A|}$$

EXAMPLE

$$-4x + 2y - 9z = 2$$
$$3x + 4y + z = 5$$
$$x - 3y + 2z = 8$$

step 1

$$|A| = \begin{vmatrix} -4 & 2 & -9 \\ 3 & 4 & 1 \\ 1 & -3 & 2 \end{vmatrix} = -4(8+3) - 2(6-1) - 9(-9-4)$$

$$= -44 - 10 + 117$$

$$= 63$$

$$|A_x| = \begin{vmatrix} 2 & 2 & -9 \\ 5 & 4 & 1 \\ 8 & -3 & 2 \end{vmatrix} = 2(11) - 2(2) - 9(-47)$$

$$= 22 - 4 + 423$$

$$= 441$$

$$|A_y| = \begin{vmatrix} -4 & 2 & -9 \\ 3 & 5 & 1 \\ 1 & 8 & 2 \end{vmatrix} = -4(2) - 2(5) - 9(19)$$

$$= -8 - 10 - 171 = -189$$

$$(A_z) = \begin{vmatrix} -4 & 2 & 2 \\ 3 & 4 & 5 \\ 1 & -3 & 8 \end{vmatrix} = -4(47) - 21(9) + 2(-13)$$

$$= -188 - 38 - 26$$

$$= -252$$

$$x = \frac{|A_x|}{|A|} = \frac{441}{63} = 7$$

$$y = \frac{|A_y|}{|A|} = \frac{-189}{63} = -3$$

$$z = \frac{|A_z|}{|A|} = \frac{-252}{63} = -4$$

GAUSSIAN ELIMINATION 2×2.

Left method

Ex. $x - 2y = 1$

$3x + 2y = 11$

$\cancel{3x} - 6y = 3$

$(-)\ \cancel{3x} + 2y = 11$

$-8y = -8$

$8y = 8$

$y = 1$ ✓

$x - 2(1) = 1$

$x = 3$ ✓

$$\left[\begin{array}{cc|c} 1 & -2 & 1 \\ 3 & 2 & 11 \end{array}\right] \xrightarrow{R_2 - 3R_1} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 8 & 8 \end{array}\right] \xrightarrow{\frac{1}{8}R_2} \left[\begin{array}{cc|c} 1 & -2 & 1 \\ 0 & 1 & 1 \end{array}\right] \xrightarrow{R_1 + 2R_2} \left[\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right]$$

$\therefore\ x = 3$ ✓

$y = 1$ ✓