Chapter 4 Transients

1. First-order *RC* or *RL* circuits.
2. Transient response and steady-state response.
3. Transient response of a first-order circuit with respect to its time constant.

What are Transients?

The time-varying currents and voltages resulting from the sudden application of sources, usually due to switching, are called **transients**. By writing circuit equations, we obtain integrodifferential equations.

First-order RC circuits

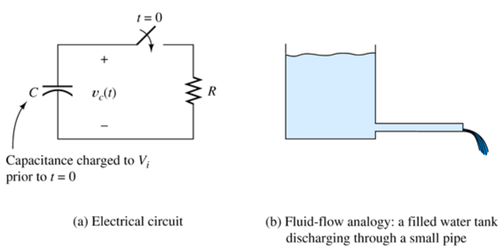


Figure 4.1 A capacitance discharging through a resistance and its fluid-flow analogy. The capacitor is charged to V_i prior to $t = 0$ (by circuitry that is not shown). At $t = 0$, the switch closes and the capacitor discharges through the resistor.

First-order RC circuits

What is the voltage across the capacitor at t=0+?

How much energy is stored in the capacitor at t=0+?

What is the current in the capacitor at t=0+?

Discharge of a Capacitance through a Resistance

$$Ri(t) + v_C(t) = RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

Initial Conditions?
The voltage of the capacitor cannot change instantaneously when the switch closes.

$$v_C(0+) = v_C(0-)$$

Solution to first-order differential equation

$$Ri(t) + v_C(t) = RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

$$RCKse^{st} + Ke^{st} = 0$$

$$v_C(0+) = V_i$$

$$v_C(t) = V_i e^{-t/RC}$$

Typical response of discharge

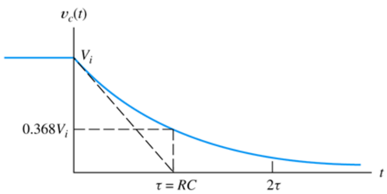


Figure 4.2 Voltage versus time for the circuit of Figure 4.1a. When the switch is closed, the voltage across the capacitor decays exponentially to zero. At one time constant, the voltage is equal to 36.8% of its initial value.

Time constant

The time interval $\tau = RC$ is called the *time constant* of the circuit.

Questions

What is the voltage across the capacitor at t →∞?

How much energy is stored in the capacitor at t →∞?

What is the current in the capacitor at t →∞ ?

Where did the energy go?

Capacitor charging

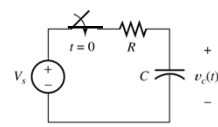


Figure 4.3 Capacitance charging through a resistance. The switch closes at $t = 0$ connecting the dc source V_s to the circuit.

$$RC\frac{dv_C(t)}{dt} + v_C(t) = V_s$$

$$v_C(t) = V_s - V_s e^{-t/\tau}$$

Solution with external source

$$v_C(t) = K_1 + K_2 e^{st}$$

$$(1 + RCs)K_2 e^{st} + K_1 = V_s$$

$$v_C(0) = V_s + K_2 e^0 = 0$$

$$v_C(t) = V_s - V_s e^{-t/\tau}$$

Typical response of charge

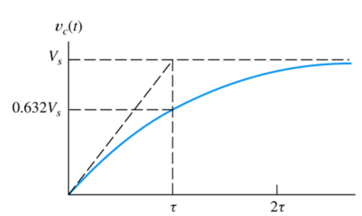


Figure 4.4 The charging transient for the RC circuit of Figure 4.3.

RC or RL Circuit Transients

Procedure:

1. Apply Kirchhoff's current and voltage laws to write the differential circuit equation.
2. Assume a solution of the form $K_1 + K_2e^{st}$.
3. Substitute the solution into the differential equation to determine the values of K_1 and s .
4. Use the initial conditions to determine the value of K_2.
5. Write the final solution.

First-order RL circuits

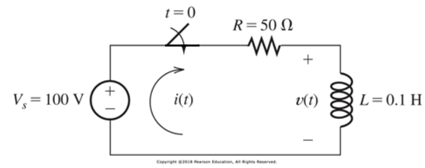


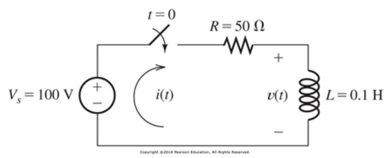
Figure 4.8 The circuit analyzed in Example 4.4.

What is the voltage across the inductor at t=0+?

How much energy is stored in the inductor at t=0+?

What is the current in the inductor at t=0+?

RL Transient Analysis: Example 4.4



Initial Conditions?

The current of the inductor cannot change instantaneously when the switch closes.

$$i_L(0+) = i_L(0-)$$

RL Transient Analysis

$$Ri(t) + L\frac{di(t)}{dt} = V_s \qquad i(t) = K_1 + K_2e^{st}$$

$$RK_1 + (RK_2 + sLK_2)e^{st} = V_s$$

$$i(0+) = K_1 + K_2e^0 = 0$$

$$i(t) = 2 - 2e^{-t/\tau} \qquad \tau = \frac{L}{R}$$

Typical responses

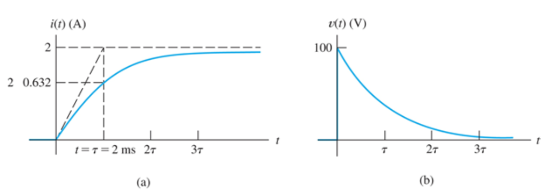


Figure 4.9 Current and voltage versus time for the circuit of Figure 4.8.

Example 4.5

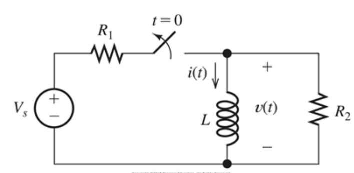


Figure 4.10 The circuit analyzed in Example 4.5.

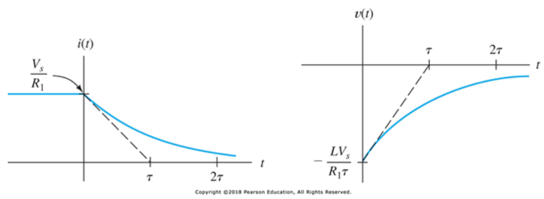


Figure 4.11 The current and voltage for the circuit of Figure 4.10.

Exercise 4.6

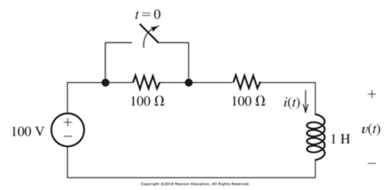


Figure 4.13 The circuit for Exercise 4.6.

RC or RL **Transient Analysis**: **Easy way**

1. Combine any networks of inductors (capacitors) into a single equivalent.
2. Use DC analysis to solve for the current (voltage) flowing the inductor (across the capacitor) at t=0-.
3. Use DC analysis and property of inductor (capacitor) to find the initial value of the variables of interest at t=0+.
4. Use DC analysis to find the steady state value of the variables of interest as t →∞.
5. Find the equivalent resistance seen by the inductor (capacitor) for t>0.
6. Solve for the time constant τ and plug into the general equation.

RC or RL Transient Analysis Easy way

$$i(t) = i(\infty) + [i(0+) - i(\infty)]e^{-t/\tau}$$
$$v(t) = v(\infty) + [v(0+) - v(\infty)]e^{-t/\tau}$$

$$\tau = RC \qquad \tau = \frac{L}{R}$$

Exercise 4.6 -revisited

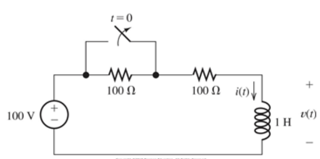


Figure 4.13 The circuit for Exercise 4.6.