MA1300 Self Practice # 6

1. (P138, #81) Find an equation of the normal line to the parabola $y = x^2 - 5x + 4$ that is parallel to the line $x - 3y = 5$.

2. (P138, #82) Where does the normal line to the parabola $y = x - x^2$ at the point $(1, 0)$ intersect the parabola a second time? Illustrate with a sketch.

3. (P139, #85)

a Use the Product Rule twice to prove that if f, g, and h are differentiable, then $(fgh)' = f'gh + fg'h + fgh'$.

b Taking $f = g = h$ in part **a**, show that

$$\frac{d}{dx}[f(x)]^3 = 3[f(x)]^2 f'(x).$$

c Use part **b** to differentiate $y = (x^4 + 3x^3 + 17x + 82)^3$

4. (P139, #95)

a For what values of x is the function $f(x) = |x^2 - 9|$ differentiable? Find a formula for f'.

b Sketch the graphs of f and f'.

5. (P139, #98)

a If $F(x) = f(x)g(x)$, where f and g have derivatives of all orders, show that $F'' = f''g + 2f'g' + fg''$.

b Find similar formulas for F''' and $F^{(4)}$.

c Guess a formula for $F^{(n)}$.

6. (P146, #8) Differentiate

$$y = u(a\cos u + b\cot u).$$

7. (P147, #31)

a Use the Quotient Rule to differentiate the function

$$f(x) = \frac{\tan x - 1}{\sec x}.$$

b Simplify the expression for $f(x)$ by writing it in terms of $\sin x$ and $\cos x$, and then find $f'(x)$.

c Show that your answers to parts **a** and **b** are equivalent.

8. (P147, #32) Suppose $f(\pi/3) = 4$ and $f'(\pi/3) = -2$, and let

$$g(x) = f(x)\sin x,$$

and

$$h(x) = \frac{\cos x}{f(x)}.$$

Find **(a)** $g'(\pi/3)$, and **(b)** $h'(\pi/3)$.

9. (P147, #36) An elastic band is hung on a hook and a mass is hung on the lower end of the band. When the mass is pulled downward and then released, it vibrates vertically. The equation of motion is $s = 2\cos t + 3\sin t$, $t \geq 0$, where s is measured in centimeters and t in seconds. (Take the positive direction to be downward.)

a Find the velocity and acceleration at time t.

b Graph the velocity and acceleration functions.

c When does the mass pass through the equilibrium position for the first time?

d How far from its equilibrium position does the mass travel?

e When is the speed the greatest?

10. (P147, #38) An Object with mass m is dragged along a horizontal plane by a force acting along a rope attached to the object. If the rope makes an angle θ with the plane, then the magnitude of the force is

$$F = \frac{\mu mg}{\mu\sin\theta + \cos\theta},$$

where μ is a constant called the *coefficient of friction*.

a Find the rate of change of F with respect to θ.

b When is this rate of change equal to 0?

c If $m = 20$ kg, $g = 9.8$ m/s^2, and $\mu = 0.6$, draw the graph of F as a function of θ and use it to locate the value of θ for which $dF/d\theta = 0$. Is the value consistent with your answer to part **b**?

11. (P154, #12, 13, 37, 40, 45) Find the derivative of the function.

$$f(t) = \sqrt[3]{1+\tan t}, \qquad y = \cos(a^3 + x^3), \qquad y = \cot^2(\sin\theta),$$

$$y = \sin(\sin(\sin x)), \qquad y = \cos\sqrt{\sin(\tan \pi x)}.$$

12. (P155, #68) Suppose f is differentiable on $\mathbb{R}$ and α is a real number. Let $F(x) = f(x^\alpha)$ and $G(x) = [f(x)]^\alpha$. Find expressions for **(a)** $F'(x)$ and **(b)** $G'(x)$.

13. (P155, #71) If $F(x) = f(3f(4f(x)))$, where $f(0) = 0$ and $f'(0) = 2$, find $F'(0)$.

14. (P155, #76) If the equation of motion of a particle is given by $s = A\cos(\omega t + \delta)$, the particle is said to undergo *simple harmonic motion*.

a Find the velocity of the particle at time t.

b When is the velocity 0?