BME2102：Introduction to Biomechanics

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- Assignments: drop box for posting your answers in a single Word/PDF file
- Files
  - Homework: HW1 due 23:59 next Monday; HW2 posted, due 23:59, 27/09, Monday
  - Lecture Notes: preview and final versions (Lect 2 both versions posted, Lect 3 preview version posted)
- Labs: Lab instructions (Labs 1-3: All materials posted) Preview them starting from the ppt file there.
- Panopto Recordings: Videos of lectures (Lect 2 posted)
- Zoom: Links to tutorials
- Office hours (updated):
  - 3:30-4:30 Monday and Friday

Review

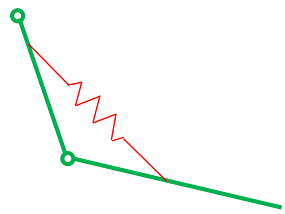
- Free Body Diagrams allow us to view significant forces in an impactful way
- Forces are the push and pull interactions between two objects
- Internal forces act within the body
  - Can move parts of the body but not the whole body
- External forces act between two objects
- Mass = Inertia; Resistance to a change in motion
- Weight = The acceleration due to gravity's effect on mass
- RoM is the quantity of motion during a movement
- The purpose of the movement dictates the needed RoM
- Lower Effort and Higher Accuracy = Lower RoM
- Higher Effort and Lower Accuracy = Higher RoM

Review

- Balance is the ability to control body position
- A tradeoff between Stability and Mobility
- Manipulated by Base of Support and the CoM position relative to it
- Bigger Base of Support = More Stable
- Stability increases the closer CoM is to the center of Base of Support

Review

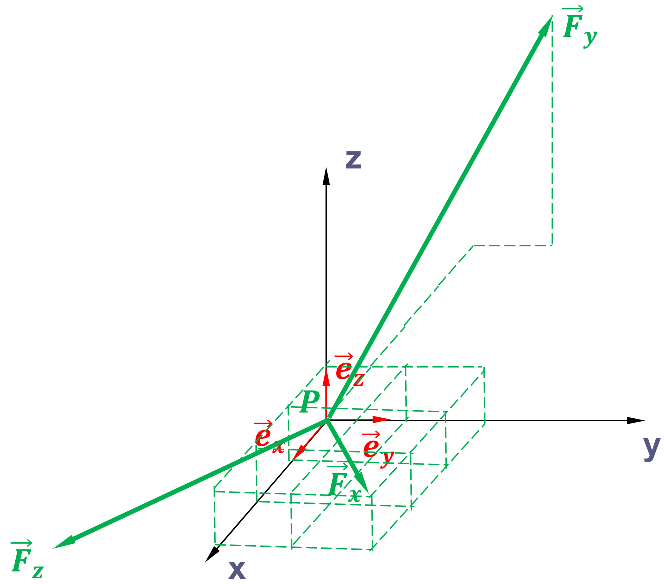
- Agonist Muscles are your active muscles / Antagonist muscles are their counterparts
- Your Agonist will be Active throughout a movement / Your antagonist will be passive
- Concentric Muscle action is an active shortening of the muscle, moving against the line of the force
- Eccentric Muscle action is an active lengthening of the muscle, moving with the line of the force
- Isometric Muscle action is muscle activation with no change in muscle length



Tutorial 1

- 9. Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be an orthonormal vector basis. The force vectors $\vec{F}_x = 3\vec{e}_x + 2\vec{e}_y + \vec{e}_z$ and $\vec{F}_y = -4\vec{e}_x + \vec{e}_y + 4\vec{e}_z$ act on point P. Calculate a vector $\vec{F}_z$ acting on P in such a way that the sum of all force vectors is the zero vector.

$$\vec{F}_x = 3\vec{e}_x + 2\vec{e}_y + 1\vec{e}_z$$
$$\vec{F}_y = -4\vec{e}_x + 1\vec{e}_y + 4\vec{e}_z$$
$$\vec{F}_z = a\vec{e}_x + b\vec{e}_y + c\vec{e}_z$$
$$\vec{F}_x + \vec{F}_y + \vec{F}_z = 0\vec{e}_x + 0\vec{e}_y + 0\vec{e}_z$$
$$\vec{F}_z = ?\vec{e}_x + ?\vec{e}_y + ?\vec{e}_z$$



Tutorial 1

- 9. Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be an orthonormal vector basis. The force vectors $\vec{F}_x = 3\vec{e}_x + 2\vec{e}_y + \vec{e}_z$ and $\vec{F}_y = -4\vec{e}_x + \vec{e}_y + 4\vec{e}_z$ act on point P. Calculate a vector $\vec{F}_z$ acting on P in such a way that the sum of all force vectors is the zero vector.

$\vec{F}_x = 3\vec{e}_x + 2\vec{e}_y + 1\vec{e}_z$

$\vec{F}_y = -4\vec{e}_x + 1\vec{e}_y + 4\vec{e}_z$

$\vec{F}_z = a\vec{e}_x + b\vec{e}_y + c\vec{e}_z$

$\vec{F}_x + \vec{F}_y + \vec{F}_z$

$= (3\vec{e}_x + 2\vec{e}_y + 1\vec{e}_z) + (-4\vec{e}_x + 1\vec{e}_y + 4\vec{e}_z) +$
$(a\vec{e}_x + b\vec{e}_y + c\vec{e}_z)$

$= (3 - 4 + a)\vec{e}_x + (2 + 1 + b)\vec{e}_y + (1 + 4 + c)\vec{e}_z$

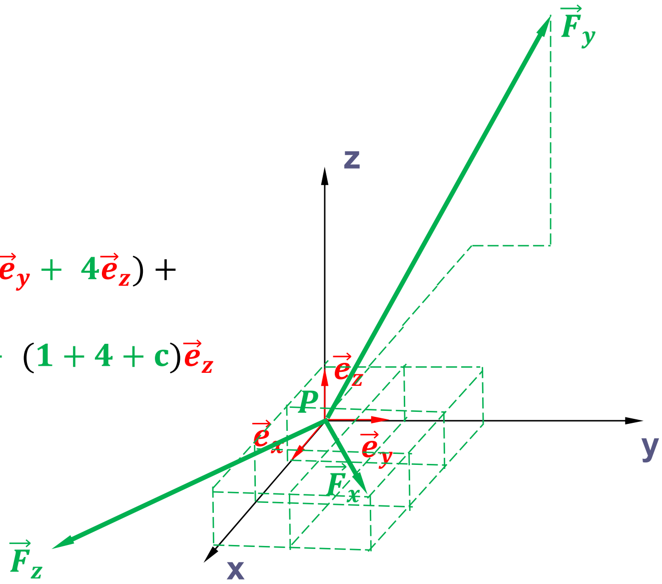
$= 0\vec{e}_x + 0\vec{e}_y + 0\vec{e}_z$

$3 - 4 + a = 0 \therefore a = 1$

$2 + 1 + b = 0 \therefore b = -3$

$1 + 4 + c = 0 \therefore c = -5$

$\vec{F}_z = 1\vec{e}_x - 3\vec{e}_y - 5\vec{e}_z$



HW 1

- 1. Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be a right-handed and orthonormal vector basis. The following vectors are given: $\vec{a} = 4\vec{e}_z$, $\vec{b} = -3\vec{e}_y + 4\vec{e}_z$ and $\vec{c} = \vec{e}_x + 2\,\vec{e}_z$.
  - (a) Write the vectors in column notation.
  - (b) Determine $\vec{a} + \vec{b}$ and $3(\vec{a} + \vec{b} + \vec{c})$.
  - (c) Determine $\vec{a} \cdot \vec{b}$, $\vec{b} \cdot \vec{a}$, $\vec{a} \times \vec{b}$ and $\vec{b} \times \vec{a}$.
  - (d) Determine $|\vec{a}|$, $|\vec{b}|$, $|\vec{a} \times \vec{b}|$ and $|\vec{b} \times \vec{a}|$.
  - (e) Determine the smallest angle between $\vec{a}$ and $\vec{b}$.
  - (f) Determine a unit normal vector on the plane defined by $\vec{a}$ and $\vec{b}$.
  - (g) Determine $\vec{a} \times \vec{b} \cdot \vec{c}$ and $\vec{a} \times \vec{c} \cdot \vec{b}$.
  - (h) Determine $\vec{a}\vec{b} \cdot \vec{c}$, $(\vec{a}\vec{b})^{\mathrm{T}} \cdot \vec{c}$ and $\vec{b}\vec{a} \cdot \vec{c}$.
  - (i) Do the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a suitable vector basis? If the answer is yes, do they form an orthogonal basis? If the answer is yes, do they form an orthonormal basis?

Dyadic or Tensor Product of Two Vectors (Hint to HW1 Q1e)

- The dyadic or tensor product of two vectors $\vec{a}$ and $\vec{b}$ defines a linear transformation operator called a dyad $\vec{a}\vec{b}$. Application of a dyad $\vec{a}\vec{b}$ to a vector $\vec{p}$ yields a vector into the direction of $\vec{a}$, where $\vec{a}$ is multiplied by the inner product of $\vec{b}$ and $\vec{p}$:

$$\vec{a}\vec{b}\cdot\vec{p} = \vec{a}\ (\vec{b}\cdot\vec{p}).$$

- So, application of a dyad to a vector transforms this vector into another vector.

Dyadic or Tensor Product of Two Vectors

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- $\vec{a}\vec{b}\cdot\vec{p} = \vec{a}\ (\vec{b}\cdot\vec{p})$

This transformation is linear, as can be seen from

$$\vec{a}\vec{b}\cdot(\alpha\vec{p}+\beta\vec{q}) = \vec{a}\vec{b}\cdot\alpha\vec{p}+\vec{a}\vec{b}\cdot\beta\vec{q} = \alpha\vec{a}\vec{b}\cdot\vec{p}+\beta\vec{a}\vec{b}\cdot\vec{q}. \tag{1.14}$$

The transpose of a dyad $(\vec{a}\vec{b})^{\mathrm{T}}$ is defined by

$$(\vec{a}\vec{b})^{\mathrm{T}}\cdot\vec{p} = \vec{b}\vec{a}\cdot\vec{p}, \tag{1.15}$$

or simply

$$(\vec{a}\vec{b})^{\mathrm{T}} = \vec{b}\vec{a}. \tag{1.16}$$

An operator $\boldsymbol{A}$ that transforms a vector $\vec{a}$ into another vector $\vec{b}$ according to

$$\vec{b} = \boldsymbol{A}\cdot\vec{a}, \tag{1.17}$$

is called a second-order tensor $\boldsymbol{A}$. This implies that the dyadic product of two vectors is a second-order tensor.

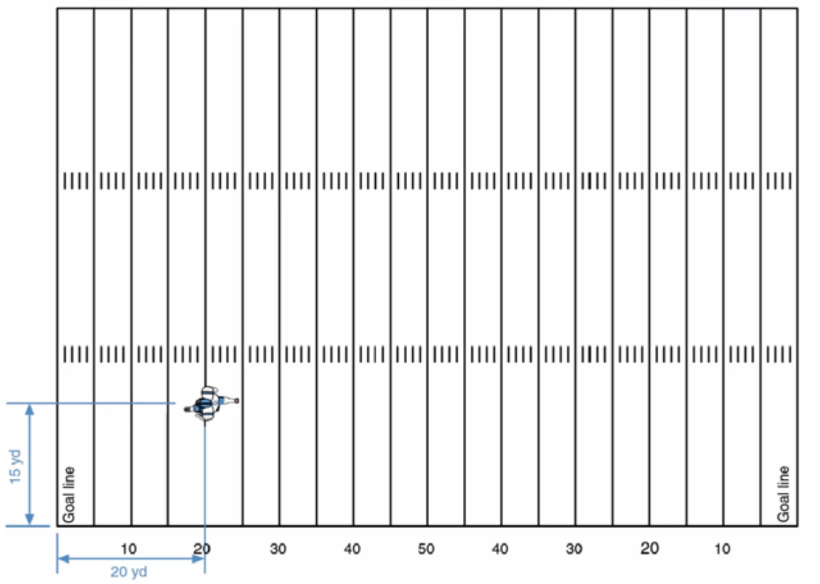
II. Rigid-body Mechanics: Linear Motion and Newton's Laws

Quantifying Linear Motion

- Primary Variables
  - Position
  - Displacement (Distance)
  - Velocity (Speed)
  - Acceleration (Slowing Down/Speeding Up)
  - Force

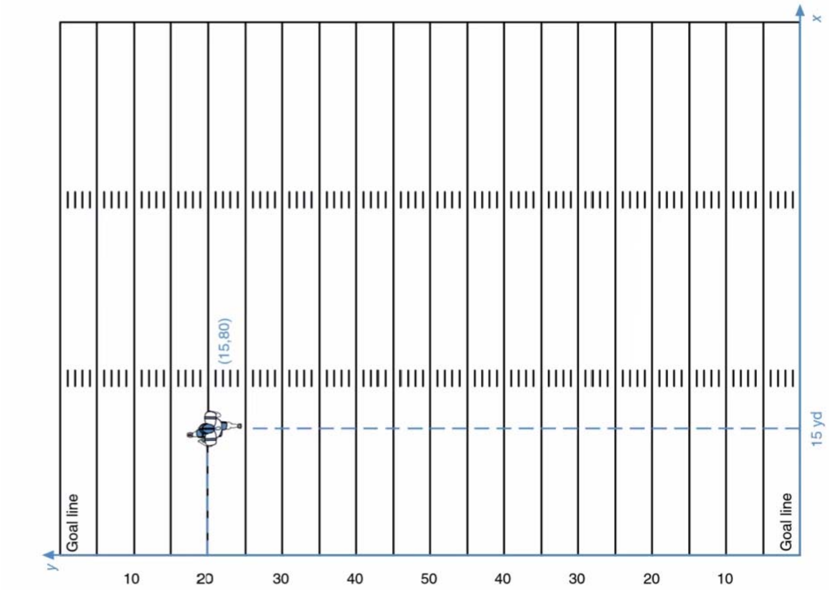
Position

- Defined as "objects location in space"
- Movement = change in position
- "Where is the object at the beginning and the end" of a movement of interest



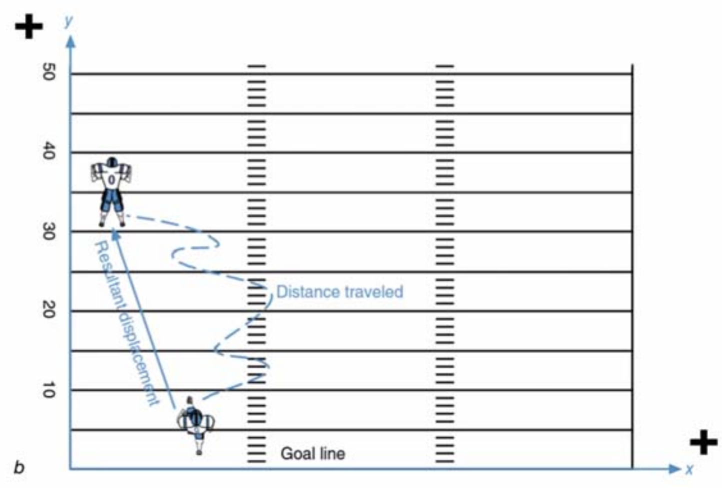
Position

- Cartesian Coordinate System
- Position in reference to a fixed point termed "the origin" $(0_x, 0_y)$



Distance vs Displacement

- Distance Traveled
  - A measurement of the length of the path traveled from initial position to final position
  - Scalar Measurement
- Displacement
  - Straight line distance in a specific direction from the initial position to the final position
  - Vector Measure
  - Represented with “d”
  - Units = meters



$$d_y = \Delta y = y_f - y_i$$

$$d_x = \Delta x = x_f - x_i$$

Acceleration

- Defined as the rate of change in velocity
- Acceleration occurs anytime an object slows down or speeds ups, starts or stops, or changes direction
- Vector (since velocity is a vector)
- Represented with “a”
- Units
  - Meters per second per second
  - m/s/s
  - m/s^2

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \quad [m/s^2] \qquad a = \frac{dv}{dt} \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

Acceleration

− Direction +	*v* (Direction of motion)	Change in motion (Speeding up +; slowing down −)	*a* (Direction of acceleration)
Speeding up — *v* → *a* →	+	+	+
Not changing — *v* → $a = 0$	+	0 (Constant velocity)	0
Slowing down — *v* → *a* ←	+	−	−
Speeding up — *a* ← *v* ←	−	+	−
Not changing — $a = 0$ *v* ←	−	0 (Constant velocity)	0
Slowing down — *a* → *v* ←	−	−	+

Figure 2.5 The direction of motion and direction of acceleration are the same when the object is speeding up, but opposite to each other when the object is slowing down.

Sir Isaac Newton

- One of the most influential scientist of all time
- An apple may have fallen on his head
- Set the foundations for mechanics in the late 1600s
- Developed the laws of motion and gravitation
- Among many other nerdy theories and discoveries

Newton's Law of Universal Gravitation

- This law was purportedly inspired by the fall of an apple on his family's farm in Lincolnshire while he was residing there during the **plague** years.
- All objects attract each other with a gravitational force that is inversely proportional to the square of the distance between the objects.
- This force of gravity was proportional to the mass of each of the two bodies being attracted to each other.

$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$

Figure 3.8 The alleged inspiration for Newton's law of universal gravitation.

Newton's Law of Universal Gravitation

$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$

where *F* is the force of gravity, G is the universal constant of gravitation, m_1 and m_2 are the masses of the two objects involved, and *r* is the distance between the centers of mass of the two objects.

$$g = G\left(\frac{m_2}{r^2}\right)$$

$$F = mg$$

or

$$W = mg$$

where *W* is the force of the earth's gravity acting on the object, or the weight of the object; *m* is the mass of the object; and g is the acceleration of the object caused by the earth's gravitational force.

Gravitation Near Earth's Surface

$$F = G\left(\frac{m_1 m_2}{r^2}\right)$$

$$g = G\left(\frac{m_2}{r^2}\right)$$

$$F = mg$$

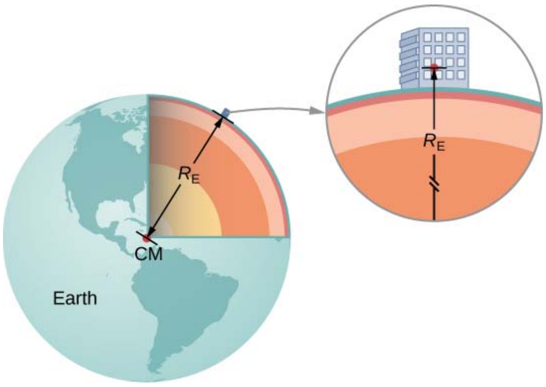
or

$$W = mg$$

$G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$

$m_2 = M_E = 5.96 \times 10^{24}\ kg$

$r = R_E = 6.37 \times 10^{6}\ m$



$g = GM_E / R_E^2$
$= 6.67\times10^{-11}\ N\cdot m^2/kg^2 \times 5.96\times10^{24}\ kg / (6.37\times10^{6}\ m)^2$
$= 9.8\ m/s^2$

Newton's First Law of Motion —Law of Inertia

- A body will maintain its state of rest or linear motion unless a NET External force acts on it
- Change in Motion requires a net external force
  - Change in Motion = Change in Velocity
  - Change in Velocity = Acceleration
  - No Net External Force = No Acceleration

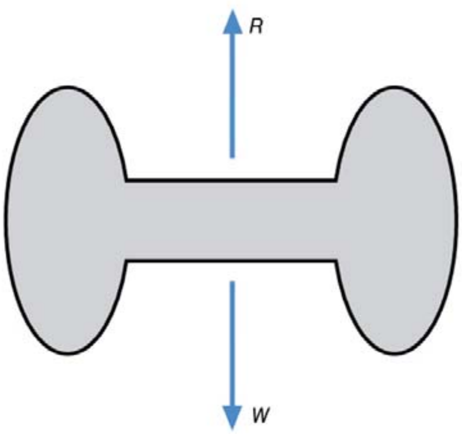


Figure 3.1 Free-body diagram of a dumbbell when held still in the hand. According to Newton's first law, this diagram is also accurate for a dumbbell moving at a constant velocity in a straight line.

Force vectors

- Imagine pulling on a thin wire that is attached to a wall. The pulling force exerted on the point of application is a vector with a physical meaning, it has
  - a **length**: the magnitude of the pulling force
  - an **orientation** in space: the direction of the wire
  - a **line-of-action**, which is the line through the force vector.
- Force vector: $\vec{F}$
  - The 'shaft' of the arrow indicates the orientation in space of the force vector.
  - The point of application of the force vector is denoted by the point *P*.

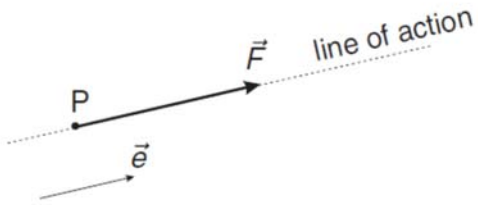


Figure 2.1

The force vector $\vec{F}$ and unit vector $\vec{e}$.

Force vectors

- The magnitude of the force vector is denoted by $|\vec{F}|$. If $\vec{e}$ denotes a unit vector the force vector may be written as

$$\vec{F} = F\vec{e}.$$

- The absolute value $|\vec{F}|$ of the number F is equal to the magnitude of force vector:

$$|F| = |\vec{F}|$$

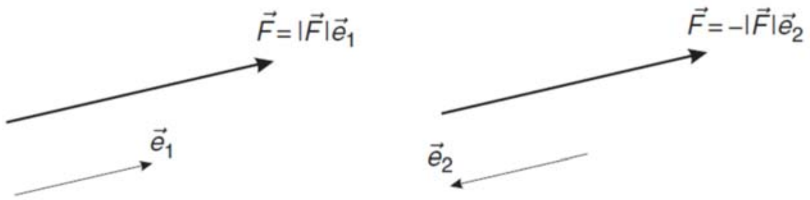
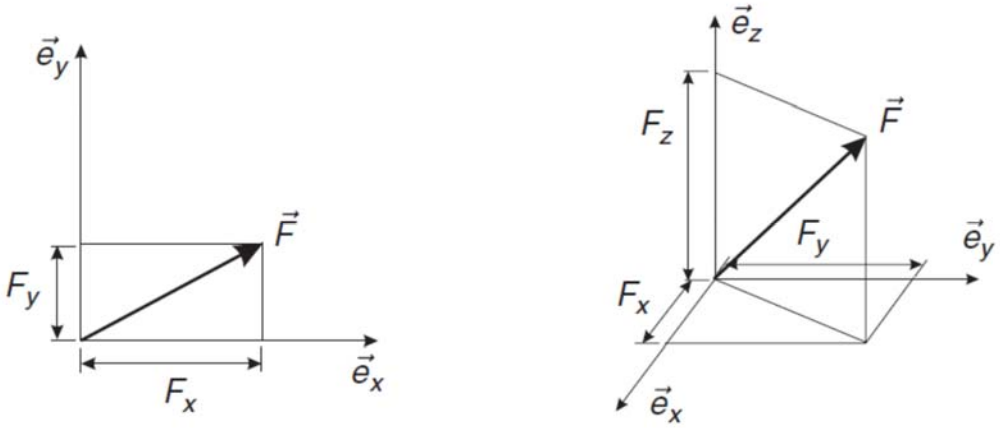


Figure 2.2

Force vector $\vec{F}$ written with respect to $\vec{e}_1$ and written with respect to $\vec{e}_2$.

Force vectors



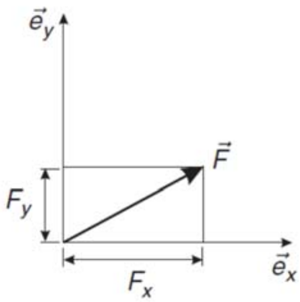
(a) 2D Cartesian vector basis:
$\vec{F}=F_x\,\vec{e}_x+F_y\,\vec{e}_y$

(b) 3D Cartesian vector basis:
$\vec{F}=F_x\,\vec{e}_x+F_y\,\vec{e}_y+F_z\,\vec{e}_z$

Figure 2.8

Decomposition of $\vec{F}$ in a two- or three-dimensional Cartesian basis.

Force vectors



(a) 2D Cartesian vector basis:
$\vec{F}=F_x\,\vec{e}_x+F_y\,\vec{e}_y$

Clearly, vector addition is straightforward, for example if

$$\vec{F}_1 = 2\vec{e}_x + 5\vec{e}_y, \qquad \vec{F}_2 = -\vec{e}_x + 3\vec{e}_y,$$

then

$$\vec{F}_1 + \vec{F}_2 = \vec{e}_x + 8\vec{e}_y, \qquad \vec{F}_1 + 3\vec{F}_2 = -\vec{e}_x + 14\vec{e}_y.$$

Example

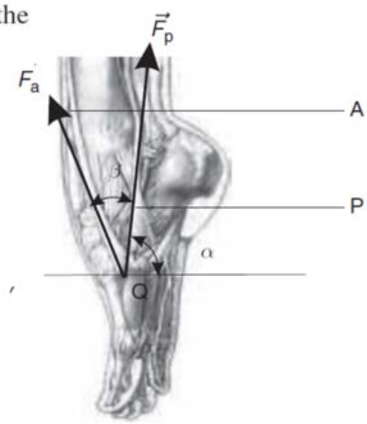
In the foot, the tendons of the tibialis anterior and the tibialis posterior may be identified, see Fig. 2.9. Let the magnitude of the force vectors be given by:

$$F_a = |\vec{F}_a| = 50\,[\mathrm{N}], \qquad F_p = |\vec{F}_p| = 60[\mathrm{N}],$$

while the angles α and β are specified by:

$$\alpha = \frac{5\pi}{11}, \qquad \beta = \frac{\pi}{6}.$$

What is the net force acting on the attachment point Q of the two muscles on the foot?



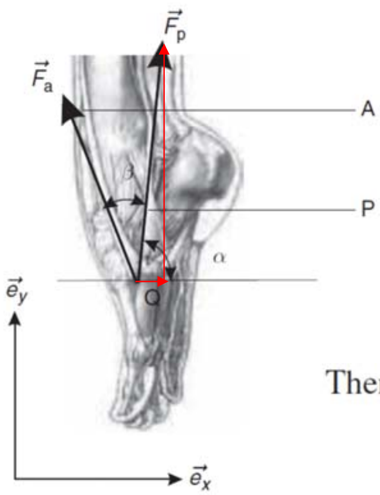
Example

First, the force vectors $\vec{F}_a$ and $\vec{F}_p$ are written with respect to the Cartesian coordinate system. Clearly:

$$\vec{F}_a = F_a\left[\cos(\alpha+\beta)\,\vec{e}_x + \sin(\alpha+\beta)\,\vec{e}_y\right] \approx -18.6\vec{e}_x + 46.4\vec{e}_y \quad [\mathrm{N}],$$

and

$$\vec{F}_p = F_p\left[\cos(\alpha)\,\vec{e}_x + \sin(\alpha)\,\vec{e}_y\right] \approx 8.5\vec{e}_x + 59.4\vec{e}_y \quad [\mathrm{N}].$$



Therefore, the net force due to $\vec{F}_a$ and $\vec{F}_p$ acting on point Q equals

$$\vec{F} = \vec{F}_a + \vec{F}_p = -10.1\vec{e}_x + 105.8\vec{e}_y \quad [\mathrm{N}].$$

Figure 2.9

Forces of the tendons of the tibialis anterior $\vec{F}_a$ and posterior $\vec{F}_p$, respectively.

Newton's Second Law of Motion —Law of Acceleration

- If a nonzero net force acts on a body, the body will experience on acceleration proportional to Net External force applied
  - $F = ma$
- Force and Acceleration = Cause and Effect
- The larger the force, the greater the acceleration
- The smaller the force, the smaller the acceleration

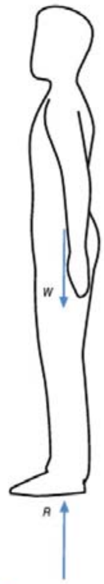


Figure 3.6 Free-body diagram of a person standing in an elevator.

If:

R>W : a = +

R=W : a = 0

R<W : a = -

Newton's Second Law of Motion —Vector Form

- Let the position of a material particle in space be given by the vector $\vec{x}$. If the particle moves in 3D space, this vector will be a function of the time *t*, i.e.

$$\vec{x} = \vec{x}(t)$$

- The velocity $\vec{v}$ of the particle is given by

$$\vec{v}(t) = \frac{d\vec{x}}{dt}$$

- and the acceleration $\vec{a}$ follows from

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = \frac{d^2\vec{x}}{dt^2}$$

- Newton's second law in vector form may be formulated as

$$\vec{F} = m\vec{a}$$

Example

Let the position of a particle with mass m for $t \geq 0$ be given by

$$\vec{x}(t) = \left(1 + \left(\frac{t}{\tau}\right)^2\right)\vec{x}_0$$

where $\vec{x}_0$ denotes the position of the particle at $t = 0$ and τ is a constant, characteristic time. What is the force on this particle? The velocity of this particle is obtained from

$$\vec{v} = \frac{d\vec{x}}{dt} = \frac{d}{dt}\left((1+(t/\tau)^2)\vec{x}_0\right) = \frac{d(1+(t/\tau)^2)}{dt}\vec{x}_0 = (2t/\tau^2)\vec{x}_0,$$

while the acceleration follows from

$$\vec{a} = \frac{d\vec{v}}{dt} = (2/\tau^2)\vec{x}_0.$$

Example

Let the position of a particle with mass m for $t \geq 0$ be given by

$$\vec{x}(t) = \left(1 + \left(\frac{t}{\tau}\right)^2\right)\vec{x}_0$$

where $\vec{x}_0$ denotes the position of the particle at t=0 and τ is a constant, characteristic time. What is the force on this particle?

The force on this particle equals

$$\vec{F} = (2m/\tau^2)\vec{x}_0$$

Uniform Accelerated Motion

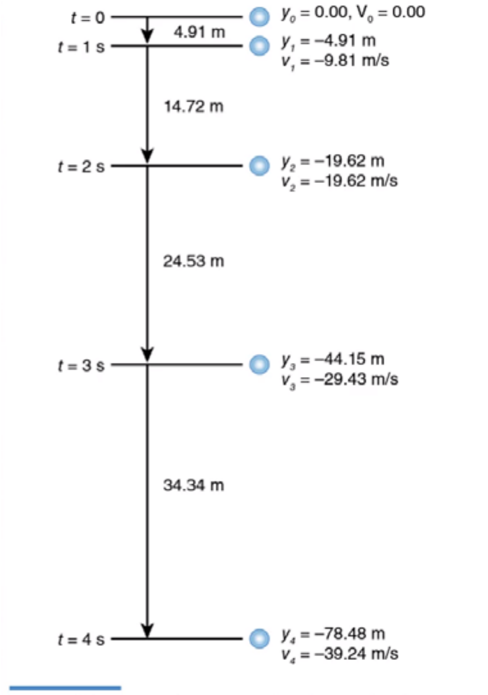


Figure 2.6 Vertical position of a dropped ball at each 1 s interval.

Uniform Accelerated Motion Equations

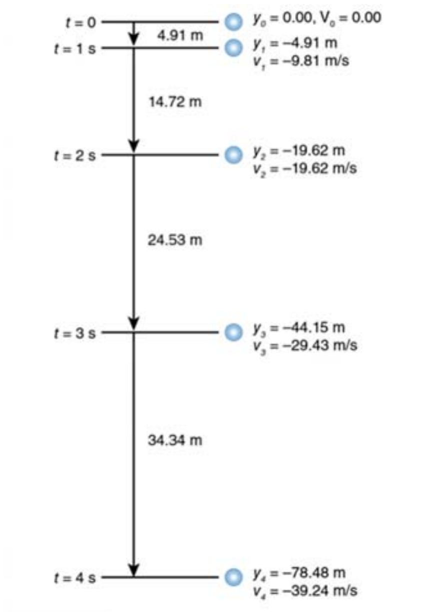


Figure 2.6 Vertical position of a dropped ball at each 1 s interval.

KEY POINT:
- Gravity is the only force
- a_y is acceleration due to gravity
- a_y = -9.81 m/s^2
- a_x = 0 m/s^2

$$a = \frac{v_f - v_i}{t}$$

$$v_f = v_i + at$$

$$(v_f)^2 = (v_i + at)^2$$

$$(v_f)^2 = (v_i)^2 + 2av_it + a^2\,t^2$$
$$= (v_i)^2 + 2a\left(v_it + \frac{1}{2}at^2\right)$$

$$d = v_it + \frac{1}{2}at^2$$

$$(v_f)^2 = (v_i)^2 + 2ad$$

Uniform Accelerated Motion

Occurs when acceleration is constant

Projectile motion

- Constantly & Uniformly accelerated due to gravity
- Gravity is the only force in projectile motion
- Acts downward in the vertical direction
- Vertical Acceleration is constantly a -9.81 m/s^2
- Horizontal Acceleration is constantly 0 m/s^2

Perfect Parabolic Path

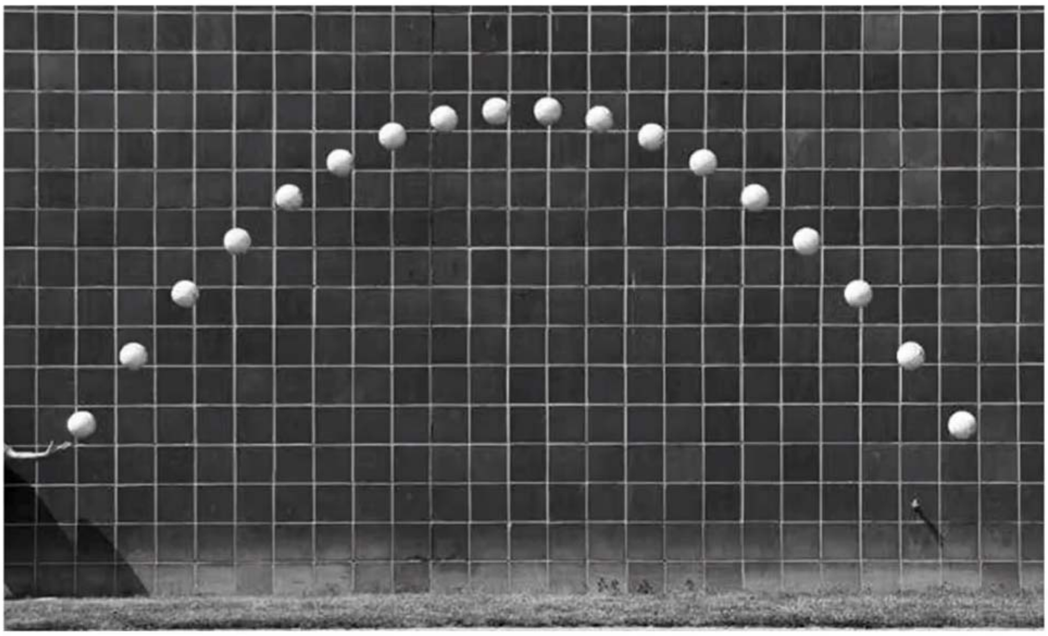
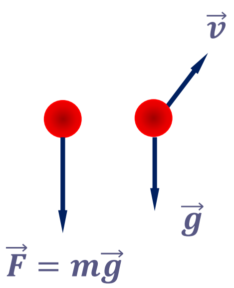


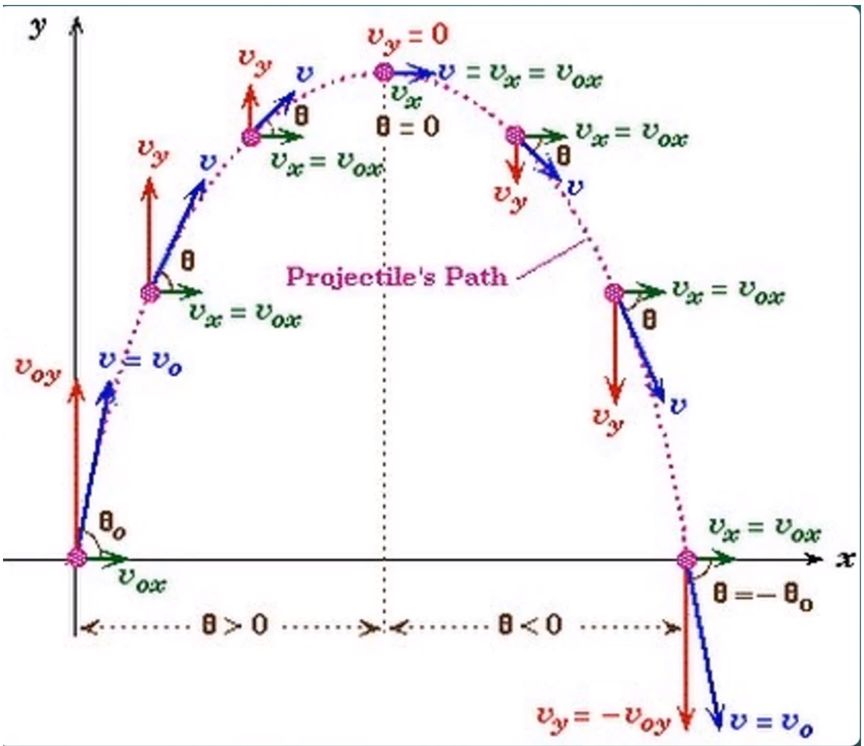
Figure 2.9 Stroboscopic photos of a ball in flight taken at equally spaced time intervals. Note the parabolic trajectory.

Projectile Motion

- A projectile is (almost) any object in the air (on earth)
  - An object that only has one force acting on it, that force being gravity
  - If this object is on the ground there is a GRF

Perfect Parabolic Path

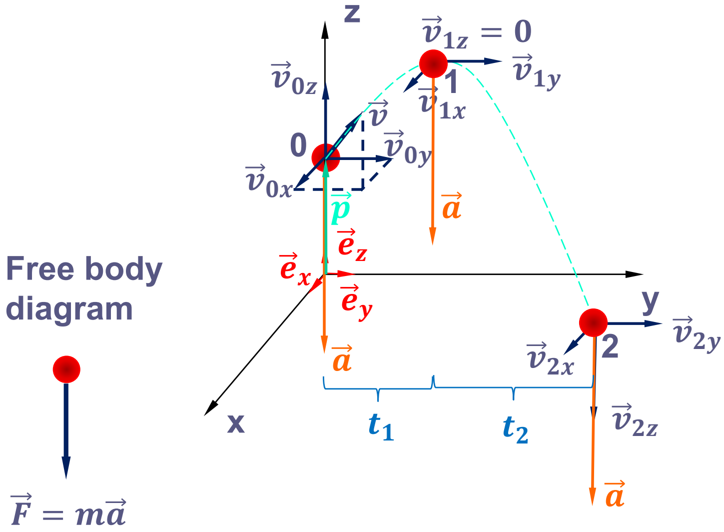




HW1 Q5: Hints

5. Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be a right-handed Cartesian basis. If a shot is put with a velocity of $\vec{v} = 2\vec{e}_x + 4\vec{e}_y + 5\vec{e}_z$ m/s and an acceleration of $\vec{a} = -9.81\vec{e}_z$ m/s^2, and released from a position of $\vec{p} = 10\vec{e}_z$ m and later lands on the ground (height = 0 m).

(a) What was the total time of flight?



$$v_{1z} = v_{0z} + a_z t_1 = 0$$

$$v_{2z} = v_{1z} + a_z t_2 = a_z t_2$$

$$d_{1z} = v_{0z} t_1 + \frac{1}{2} a_z {t_1}^2$$

$$d_{2z} = d_{1z} + p$$

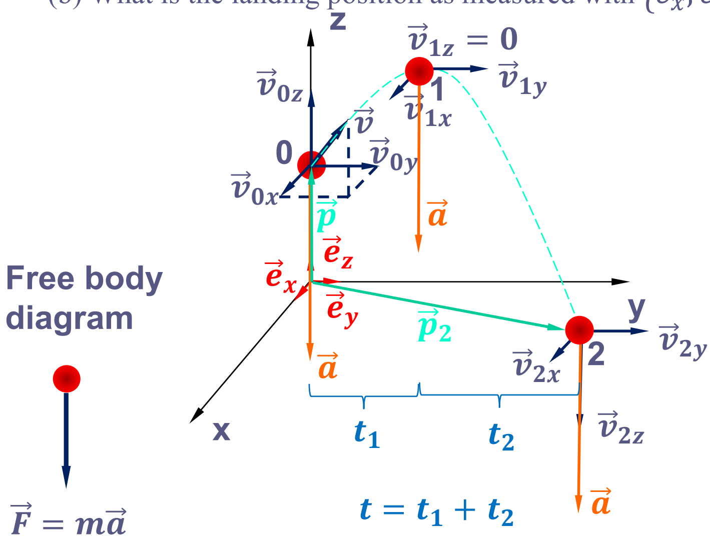
$$d_{2z} = v_{1z} t_2 + \frac{1}{2} a_z {t_2}^2 = \frac{1}{2} a_z {t_2}^2$$

$$t = t_1 + t_2$$

HW1 Q5: Hints

4. Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be a right-handed Cartesian basis. If a shot is put with a velocity of $\vec{v} = 2\vec{e}_x + 4\vec{e}_y + 5\vec{e}_z$ m/s and an acceleration of $\vec{a} = -9.81\vec{e}_z$ m/s^2, and released from a position of $\vec{p} = 10\vec{e}_z$ m and later lands on the ground (height = 0 m).

(b) What is the landing position as measured with $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$?



$$x_2 = v_{0x} t = ?\, t$$

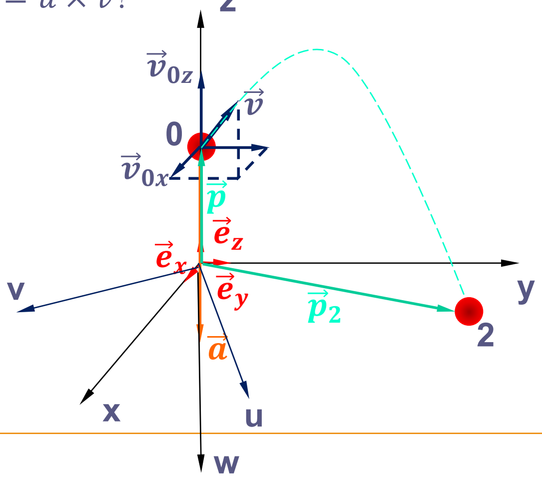
$$y_2 = v_{0y} t = ?\, t$$

$$z_2 = 0$$

$$\vec{p}_2 = x_2\vec{e}_x + y_2\vec{e}_y + z_2\vec{e}_z$$

HW1 Q5: Hints

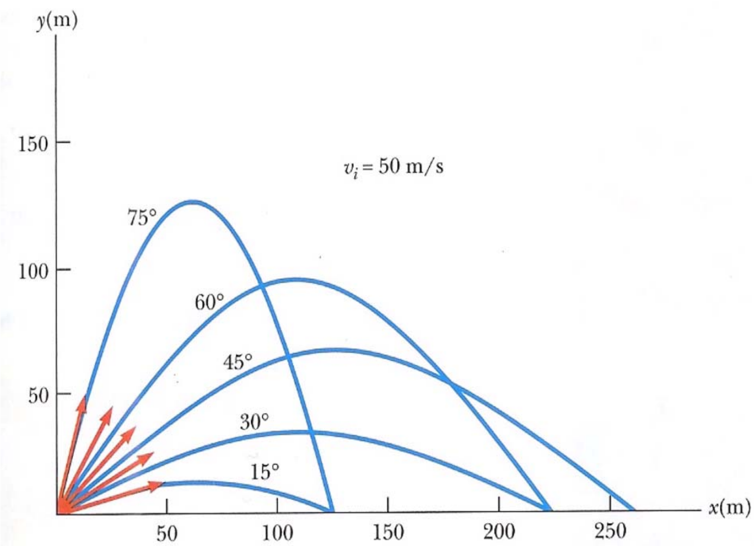
4. Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be a right-handed Cartesian basis. If a shot is put with a velocity of $\vec{v} = 2\vec{e}_x + 4\vec{e}_y + 5\vec{e}_z$ m/s and an acceleration of $\vec{a} = -9.81\vec{e}_z$ m/s^2, and released from a position of $\vec{p} = 10\vec{e}_z$ m and later lands on the ground (height = 0 m).
   (a) What was the total time of flight?
   (b) What is the landing position as measured with $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$?
   (c) What is the landing position as measured with $\vec{u} = 4\vec{e}_x + 3\vec{e}_y$, $\vec{v} = 3\vec{e}_x - 4\vec{e}_y$ and $\vec{w} = \vec{u} \times \vec{v}$?



$$\vec{p}_2 = x_2\vec{e}_x + y_2\vec{e}_y + z_2\vec{e}_z$$
$$\vec{p}_2 = U\vec{u} + V\vec{v} + W\vec{w}$$

Optimal Projection Principle

- In most sporting applications involving projectile motion there is an ideal angle of release range for best performance
- Optimal Release/Takeoff Angle depends on the goal of performance
- Assuming Air Resistance is ignored (Horizontal and Vertical)



Air Resistance

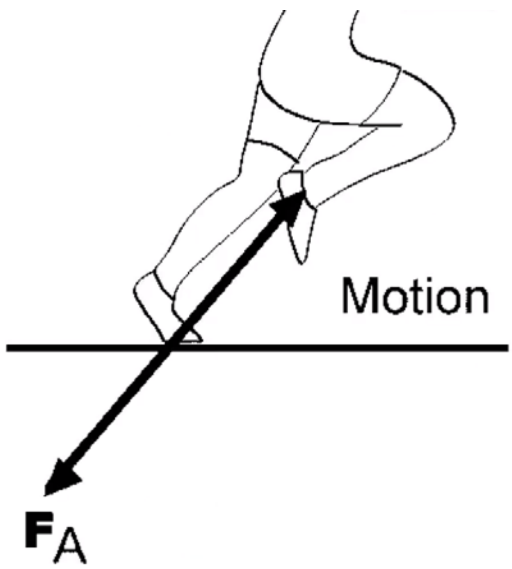
Air Resistance is (mostly) ignored while quantifying projectile motion

Air Resistance is negligible when:

- The object's weight is large
- There is a small cross sectional area perpendicular to the movement direction
- The object has streamlined shape
- The velocity of movement is relatively low

Newton's Third Law of Motion —Law of Action-Reaction

- For every action (force), there is an equal and opposite reaction (another force)
- Force exerted on objects with larger inertia will often create motion in the opposite direction



Ground Reaction Force (GRF)

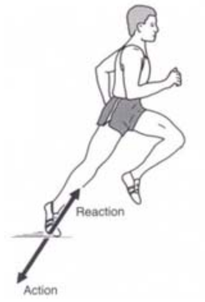
- Consequence of Newton's 3rd Law: Action-Reaction
- GRF is the force the ground exerts on an object in contact with it
- Accelerates the body
  - Produces movement
  - Absorb impact

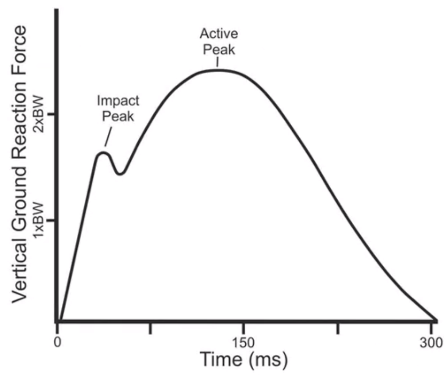
Summary

- Isaac Newton developed universal laws to describe and understand motion
- 1^{st} Law: Inertia — an object won't change its motion unless acted on by a net external force
- 2^{nd} Law: Acceleration — Acceleration is proportional to the net external force
- 3^{rd} Law: Action-Reaction — For every force there is an equal and opposite force

Implications

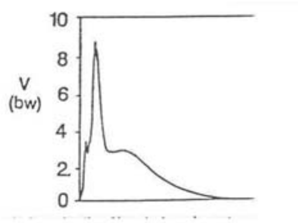
- Running GRF
  - Peak vertical force about 2-3 times BW (Body Weight) each step
  - Consider a 5K run
    - ➢ Assume SL = 1m
    - ➢ 5000 steps required





Another Implication

- Landing GRF from a max. jump
  - Peak vertical force can range from 7 to 10 times of BW
  - A middle block in an elite men's VB game
    - ➢ Block jump = 12 times
    - ➢ Attack jump =8 times
    - ➢ Jump serve = 4 times
    - ➢ Total = 24 times (winning team more)
  - Injury implication
    - ➢ Average BW of middle blockers = 211 lb
    - ➢ Each peak landing force is about 1477 to 2110 lb
    - ➢ Landing on one leg or two legs



Impulse and Momentum

- Newton's 2nd Law
  - F = ma
  - Gives us a view of the effects of a force at an instant in time
  - Often more concerned with impacts of a force over a duration of time
- Impulse and Momentum

$$\sum F = ma$$

$$\sum F = m\,\frac{\Delta v}{\Delta t}$$

$$\underbrace{\sum F \times \Delta t}_{\substack{\textbf{Impulse} \\ \textbf{J (N·s)}}} = \underbrace{m \times \Delta v}_{\substack{\textbf{Momentum} \\ \textbf{p (Kg·m/s)}}}$$

Impulse and Momentum

Impulse:

A force applied to an object for a duration of time

= F x t

Momentum

A quantification of an objects movement taking into account both mass and changing velocity

= m x Δv

Manipulating Momentum

- We apply an impulse to manipulate momentum
  - Momentum is m x Δv (m is constant, Δv is our outcome)
  - We can manipulate F or T

- Momentum can be maximized or minimized
  - (Unless you're Von Miller – can't be stopped)
- Depending on the goal of the activity
  - Ex: Jumping vs Landing
  - Ex: Throwing vs Catching

Manipulating Momentum

Goal: Maximize Momentum (increase velocity)

- Object is starting from a set velocity (0 potentially) Impulse & Momentum can vary
- Apply a similar force for a longer time
- Apply a greater force over a similar time
- Apply a greater force over a longer time

Goal: Minimize Momentum (decrease velocity)

- Object begins with a specific velocity and decreases to a specific velocity – Impulse Magnitude is already decided
- Increase time & Decrease force
- Decrease time & Increase force

Friction

- Friction is a contact force (reaction force)
- Key Properties
  - Resists Rolling & Sliding
  - Always acts opposite of relative motion
  - Broken into two components (factors): vertical (perpendicular) & horizontal (parallel)

Two Components of Friction

- $F = \mu \times R$

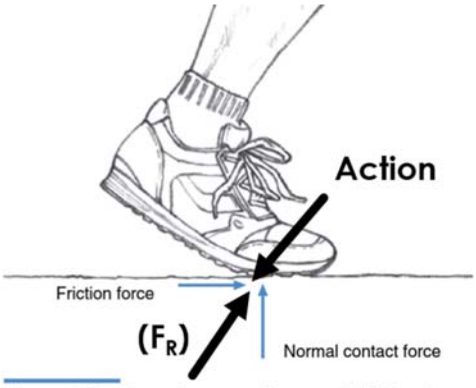
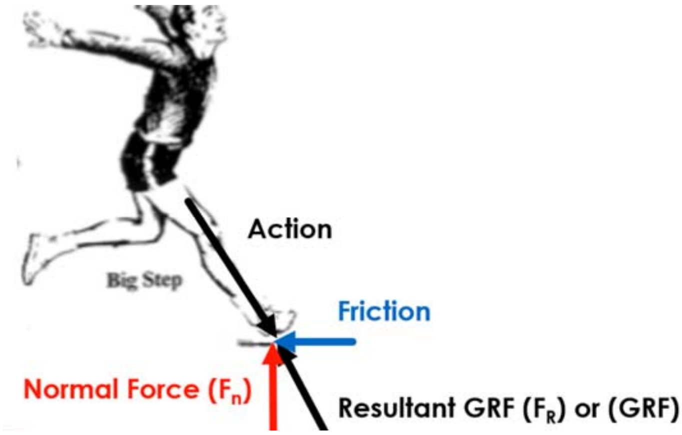


Figure 1.3 Normal contact force and friction force acting on a runner's foot during push-off.



Two Components of Friction

- $F = \mu \times R$

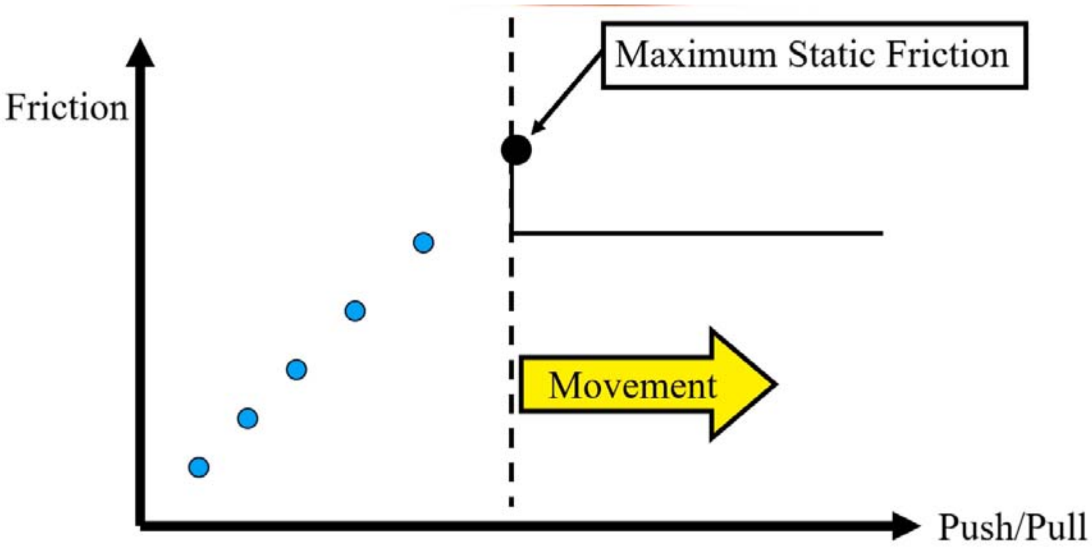
The Nature of the Surface (μ)

- "How rough/sticky is the surface"
- Think Ice vs Grass
- Horizontal Component of Friction
- Always Parallel with Surface

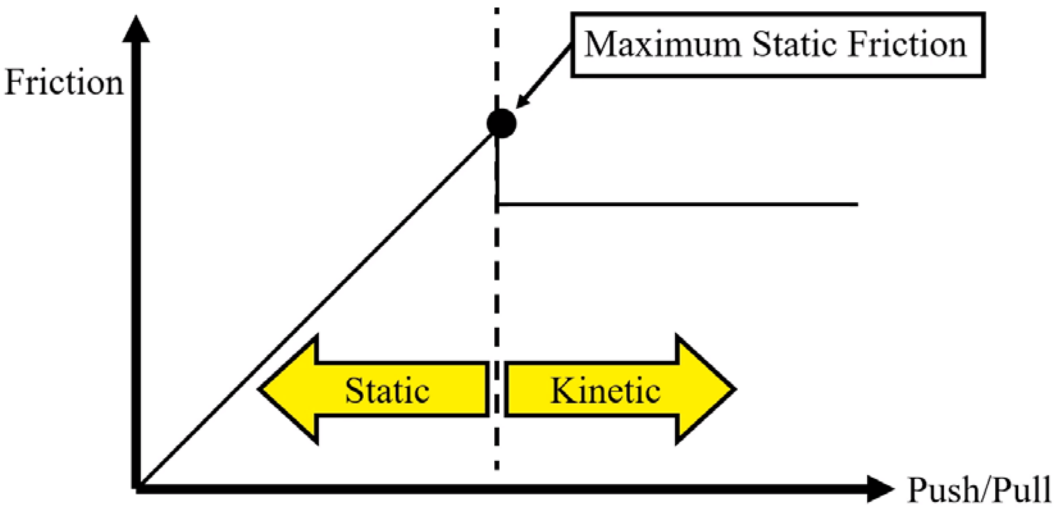
The Normal Force (R)

- Force holding the two surfaces together
- Determined by weight (More weight = More Normal Force)
- Vertical Component of Friction
- Always Perpendicular to surface

Relationship of Friction & Force (Push or Pull)

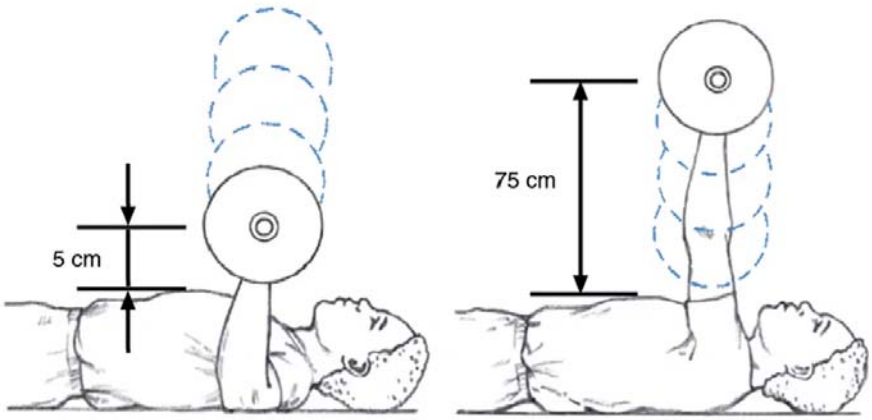


Friction

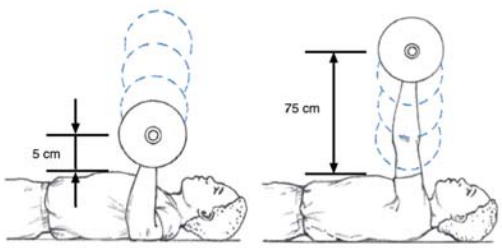


Work-Energy Relationship

- When force is applied to an object and results in movement, Mechanical Work has occurred
- Work = U = Fd (J or joules)



Mechanical Work

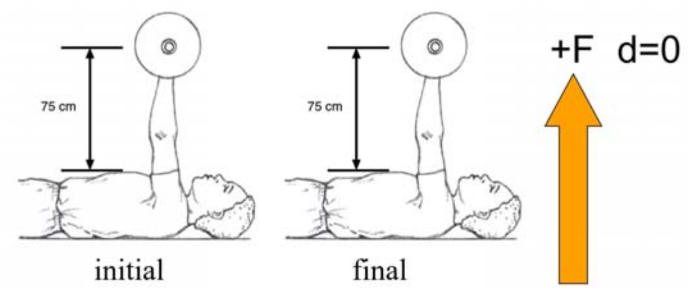


initial final

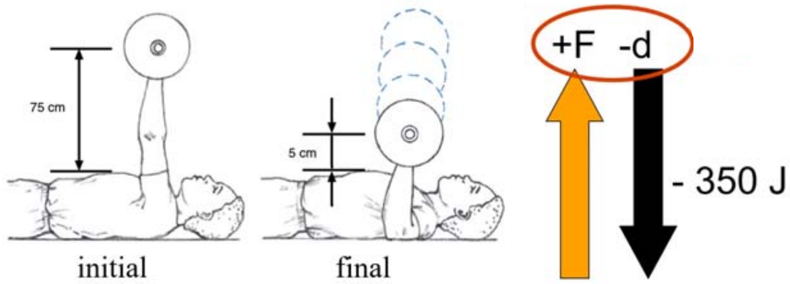
U = F d =
500 N (0.7 m) = 350 J

- Work is Positive (+)
- "Done on the object by the person"
- Muscle Action: Concentric

Mechanical Work



Work is Zero (0)
Muscle Action: Isometric



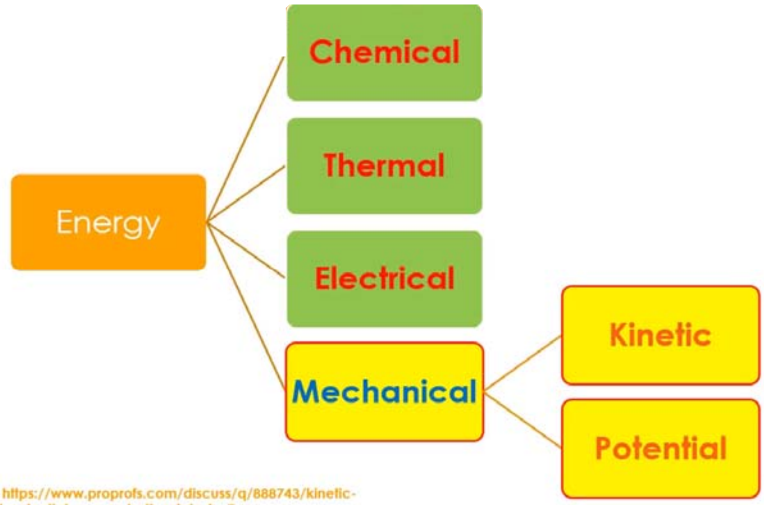
Work is Negative (-)
Done on the person by the object
Muscle Action: Eccentric

Mechanical Work & Muscle Actions

- Positive (concentric muscle actions)
  - lifting, uphill locomotion, pushing
- Negative (eccentric muscle actions)
  - lowering, downhill locomotion, being pushed
- Zero (isometric muscle actions)
  - static

Energy

- Capacity to do work



IRF: https://www.proprofs.com/discuss/q/888743/kinetic-and-potential-energy-both-relate-to-7va

Kinetic Energy (KE)

- Energy of Motion
- Energy of an object due to its motion

- $KE = \frac{1}{2} \cdot m \cdot v^2$ (J or joules)

- Ex: A 60 kg long distance runner, $v = 5$ m/s

  $KE = \frac{1}{2} \cdot m \cdot v^2$

  $= \frac{1}{2} \cdot 60 \cdot 5^2$

  $= 750$ (J)

(Gravitational) Potential Energy (PE)

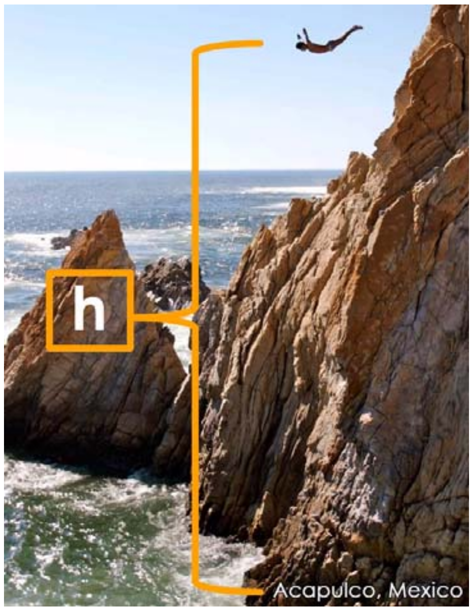
- Potential for Motion (Potential to fall)
- Energy due to an object's position relative to the earth
- PE = m·g·h (J or joules)

- Ex: A 60 kg diver at a diving board's edge (10m)

  PE = m·g·h

  = 60· (-9.81) ·(-10)

  = 5886 (J)

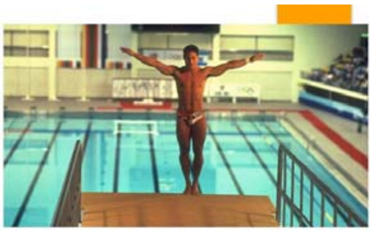


Conservation of Mechanical Energy

- The total mechanical energy remains constant if gravity is the only external force
- Energy cannot be destroyed – Transferred between one form to another

- Ex:
  - A 60 kg diver at a diving board's edge (10 m)
    - PE = 5886 (J)
    - KE = 0 (J)
    - Total Mechanical Energy = 5886 (J)
  - What is the total Mechanical Energy in the mid fall from diving board?

Conservation of Mechanical Energy



d = -5 m

d = -5 m

v = -9.9045 m/s

$KE = \frac{1}{2} \cdot m \cdot v^2$

$KE = (0.5) \cdot 60 \cdot (-9.9045)^2$

$\boxed{KE = 2942.9736\ J}$

$PE = m \cdot a_g \cdot h$

$PE = 60 \cdot (-9.81) \cdot (-5)$

$\boxed{PE = 2943\ J}$

Total Mechanical Energy = 5886J

Conservation of Mechanical Energy

$V_f = -14.00714104$ m/s

PE = 0 J

KE = 5886.00003 J

Total Mechanical Energy = 5886J

Conservation of Mechanical Energy

The falling diver's energy

PE decreasing, KE increasing

As you jump up

KE decreasing, PE increasing

TOTAL mechanical energy remains **CONSTANT**

Work – Energy Principle

- Energy is the capacity to do work
- The Work done by the body is = to the Change in Energy of the body

  - Work = Δ Energy
  - Work = $\Delta KE + \Delta PE$
  - Work = Fd = Δ(Energy)

No Knowledge of force required

Power

- In mechanics, **power** is the rate of doing work, or how much work is done in a specific amount of time.

$$P = \frac{U}{\Delta t}$$

where

P = power,

U = work done, and

Δt = time taken to do the work.

- The SI units for power are **watts** (abbreviated with the letter W), named after the Scottish inventor James Watt; 1 W equals 1 J/s.

Power

$$P = \frac{U}{\Delta t}$$

$$P = \frac{\bar{F}(d)}{\Delta t} = \bar{F}\left(\frac{d}{\Delta t}\right)$$

$$P = \bar{F}\bar{v}$$

Momentum and Energy

- Momentum
  - Vector
  - Direction
  - State of motion

Momentum = m · Δ v

- Energy
  - Scalar
  - Work
  - Potential for future interaction

KE = 1/2 · m · v^2

Momentum and Energy

- Momentum
  - Vector
  - Direction
  - State of motion

Momentum = m · Δ v

- Energy
  - Scalar
  - Work
  - Potential for future interaction

KE = 1/2 · m · v^2

Conservation of Momentum

- Momentum is constant if the net external force is zero.
  - L = linear momentum = Constant, if $\Sigma F = 0$
- Conservation of momentum applies to the components of momentum, so the above equation can be represented by equations for the three dimensions (vertical, horizontal—forward and backward, and horizontal—side to side).

$$L_x = \text{constant} \qquad \text{if } \Sigma F_x = 0$$

$$L_y = \text{constant} \qquad \text{if } \Sigma F_y = 0$$

$$L_z = \text{constant} \qquad \text{if } \Sigma F_z = 0$$

Conservation of Momentum

- Momentum is constant if the net external force is zero.
  - L = linear momentum = Constant, if $\Sigma F = 0$
- In the case of a multi-object system, if one object's velocity increases, another object's velocity decreases to keep the total momentum of the system constant.

$$L_i = \Sigma(mu) = m_1u_1 + m_2u_2 + m_3u_3 + \ldots$$

$$= m_1v_1 + m_2v_2 + m_3v_3 + \ldots$$

$$= \Sigma(mv) = L_f = \text{constant}$$

where

L_i = initial linear momentum,

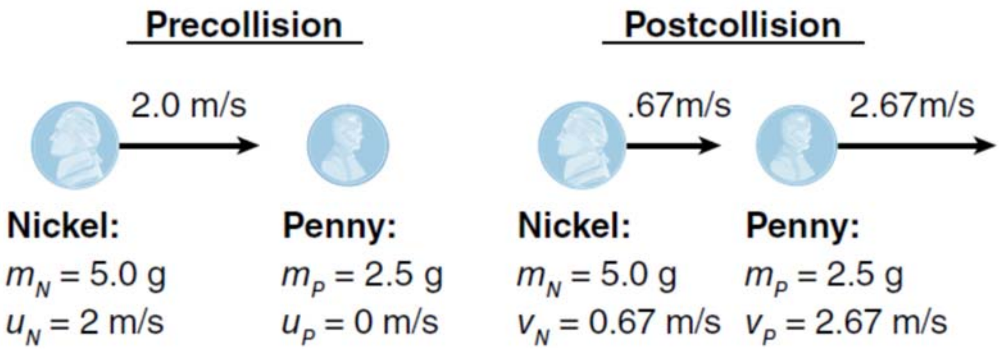
L_f = final linear momentum,

m = mass of part of the system,

u = initial velocity, and

v = final velocity.

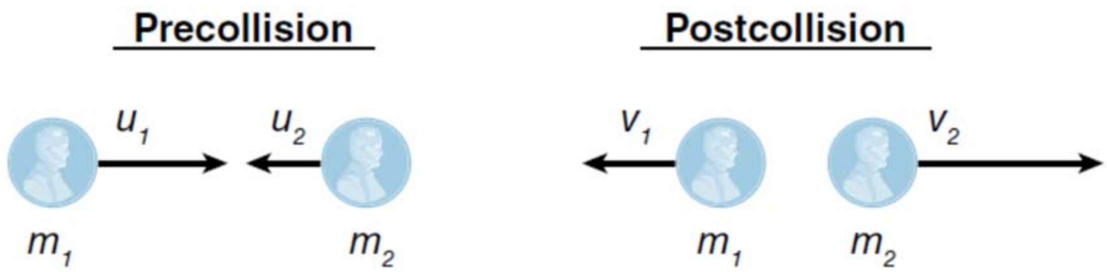
Elastic Collisions



$$m_P u_P + m_N m_N = m_P v_P + m_N v_N$$
$$(5\text{ g})(2\text{ m/s}) + (2.5\text{ g})(0\text{ m/s}) = (2.5\text{ g})(2.67\text{ m/s}) + (5.0\text{ g})\ 0.67\text{ m/s}$$
$$10\text{ g·m/s} = 10\text{ g·m/s}$$

Figure 3.2 Perfectly elastic collision of a moving nickel with a stationary penny.

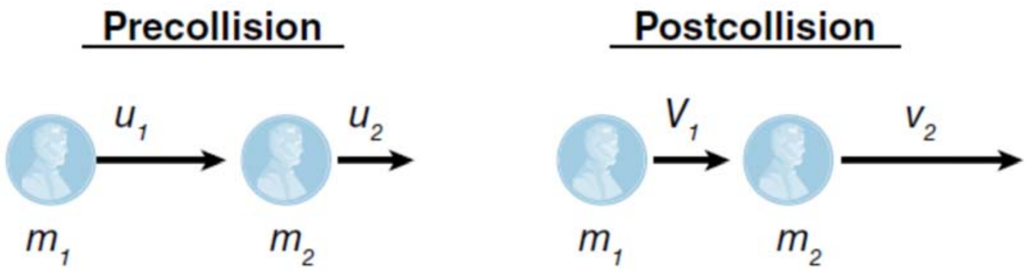
Elastic Collisions



$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1u_1 = m_2v_2$$
$$m_2u_2 = m_1v_1$$

Figure 3.3 Perfectly elastic head-on collision of two pennies moving in opposite directions.

Elastic Collisions



$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$m_1u_1 = m_2v_2$$
$$m_2u_2 = m_1v_1$$

Figure 3.4 Perfectly elastic overtaking collision of two pennies moving in the same direction.

Inelastic Collisions

In a perfectly inelastic collision,

$$v_1 = v_2 = v = \text{final velocity.}$$

Therefore,

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v.$$

Coefficient of Restitution

- The **coefficient of restitution** is defined as the absolute value of the ratio of the velocity of separation to the velocity of approach. The velocity of separation is the difference between the velocities of the two colliding objects just after the collision.

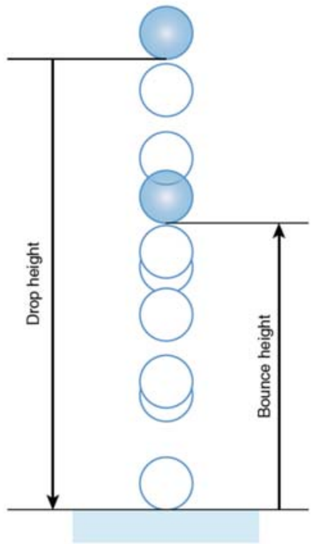
$$e = \left|\frac{v_1 - v_2}{u_1 - u_2}\right| = \left|\frac{v_2 - v_1}{u_1 - u_2}\right|$$

where

e = coefficient of restitution,

v_1, v_2 = postimpact velocities of objects one and two, and

u_1, u_2 = preimpact velocities of objects one and two.



$$\text{Coefficient of restitution} = e = \sqrt{\frac{\text{bounce height}}{\text{drop height}}}$$

Figure 3.5 Determination of the coefficient of restitution from the drop and bounce heights.

Example

- A golf ball is struck by a golf club. The mass of the ball is 46 g, and the mass of the club head is 210 g. The club head's velocity immediately before impact is 50 m/s. If the coefficient of restitution between the club head and the ball is 0.80, how fast is the ball moving immediately after impact?

Solution:

Step 1: List the known quantities.

$m_{ball} = 46$ g

$m_{club} = 210$ g

$u_{ball} = 0$ m/s

$u_{club} = 50$ m/s

$e = 0.80$

Step 2: Identify the variable to solve for.

$v_{ball} = ?$

Example

Step 3: Search for equations with the known and unknown variables in them.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$m_{ball} u_{ball} + m_{club} u_{club} = m_{ball} v_{ball} + m_{club} v_{club}$$

$$e = \left| \frac{v_1 - v_2}{u_1 - u_2} \right| = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_{\mathrm{club}} - v_{ball}}{u_{ball} - u_{\mathrm{club}}}$$

Example

Step 4: We have two unknown variables, v_{club} and v_{ball}, which represent the postimpact velocities of the club and the ball. We also have two equations to use. If the number of independent equations is equal to the number of unknown variables, the unknown variables can be computed. We need to solve one of the equations for one of the unknown variables in terms of the other. Let's use the coefficient of restitution equation and solve for the postimpact velocity of the club. We want to manipulate the equation so that the postimpact velocity of the club, v_{club}, is on one side of the equation by itself.

$$e = \frac{v_{\mathrm{club}} - v_{ball}}{u_{ball} - u_{\mathrm{club}}}$$

$$e\,(u_{ball} - u_{club}) = v_{club} - v_{ball}$$

$$v_{club} = e\,(u_{ball} - u_{club}) + v_{ball}$$

Example

Step 5: Now let's substitute this expression for the postimpact velocity of the club into the conservation of momentum equation.

$$m_{ball}u_{ball} + m_{club}u_{club} = m_{ball}v_{ball} + m_{club}v_{club}$$

$$m_{ball}u_{ball} + m_{club}u_{club} = m_{ball}v_{ball} + m_{club}$$
$$(e\,(u_{ball} - u_{club}) + v_{ball})$$

Example

Step 6: Substitute known values and solve for the postimpact velocity of the ball.

$$(46\text{ g})(0) + (210\text{ g})(50\text{ m/s}) = (46\text{ g})v_{ball} +$$
$$(210\text{ g}) \times [0.80\,(0 - 50\text{ m/s}) + v_{ball}]$$

$$(210\text{ g})(50\text{ m/s}) = v_{ball}\,(46\text{ g} + 210\text{ g}) - (210$$
$$\text{g})(0.8)(50\text{ m/s})$$

$$(210\text{ g})(50\text{ m/s}) + (210\text{ g})(0.8)(50\text{ m/s}) = v_{ball}$$
$$(256\text{ g})$$

$$v_{ball} = \frac{(210\text{ g})(90\text{ m/s})}{256\text{ g}}$$

$$v_{ball} = 74\text{ m/s}$$

Step 7: Common sense check.
This velocity is over 150 mi/h, but that seems about right when you think about how fast a golf ball rockets off the tee.