

# Lecture 2

## Electric Charge



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# Lecture Outline

- **Chapter 21**, D Halliday, R Resnick, and J Walker, “Fundamentals of Physics” 9th Edition, Wiley (2005).
- Electric Charge
  - Conductors and Insulators
  - Coulomb’s Law
  - Quantization of Charge
  - Charge Conservation

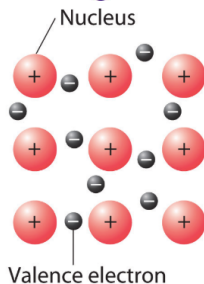
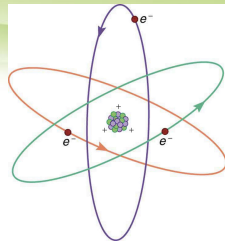
# Static electricity

- Rub a comb against your hair, the comb can attract small piece of paper
- Take off a cloth in dry weather
- There is spark
- Cloth and comb are charged as a result of the action



# Explanation

- Matter consists of positive and negative charge
- Positive, nucleus, cannot move
- Negative electrons, can move
- Rub the comb, negative charge go to the comb
- Negative and positive charge attracts each other
- Same charge repels



# Charged comb attracts paper

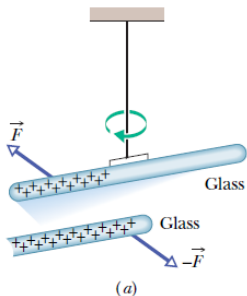
- Negative charge in the comb **repels negative** charge in the paper and **attracts the positive** charge in the paper (**induced charge in paper**)
- The **negative charge is moved to a farther distance** from the comb



# Static electricity-Summary

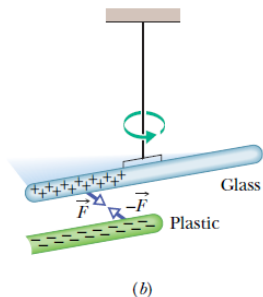
- Matters have positive and negative charges
- You can make objects positively charged or negative charged with appropriate actions.
- Like charges repel and unlike charges attract

## 21.2 Electric Charge:



**Fig. 21-2** (a) Two charged rods of the same sign repel each other. (b) Two charged rods of opposite signs attract each other. Plus signs indicate a positive net charge, and minus signs indicate a negative net charge.

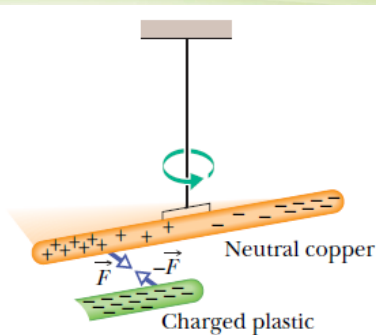
Electric charge has two different signs.



Charges with the same electrical sign repel each other, and charges with the opposite electrical signs attract each other.

Charges exert forces on each other

## 21.3 Conductors and Insulators



**Fig. 21-4** A neutral copper rod is electrically isolated from its surroundings by being suspended on a nonconducting thread. Either end of the copper rod will be attracted by a charged rod. Here, conduction electrons in the copper rod are repelled to the far end of that rod by the negative charge on the plastic rod. Then that negative charge attracts the remaining positive charge on the near end of the copper rod, rotating the copper rod to bring that near end closer to the plastic rod.

The properties of conductors and insulators are due to the structure and electrical nature of atoms.

Atoms consist of positively charged **protons**, negatively charged **electrons**, and electrically neutral **neutrons**. The protons and neutrons are packed tightly together in a central nucleus.

When atoms of a conductor come together to form the solid, some of their outermost (and so most loosely held) electrons become free to wander about within the solid, leaving behind positively charged atoms (*positive ions*). We call the mobile electrons **conduction or free electrons**.

There are few (if any) free electrons in a nonconductor.



## 21.3 Conductors and Insulators

**Conductors** are materials through which charge can move freely; examples include *metals (such as copper in common lamp wire), the human body, and tap water.*

**Nonconductors**—also called **insulators**—are materials through which charge cannot move freely; examples include *rubber, plastic, glass, and chemically pure water.*

**Semiconductors** are materials that are intermediate between conductors and insulators; examples include *silicon and germanium in computer chips.*

**Superconductors** are materials that are perfect conductors, allowing charge to move without any hindrance.

## 21.4 Coulomb's Law

The equation giving the **electrostatic force** for charged particles is called **Coulomb's law**:

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r} \quad (\text{Coulomb's law}),$$

The SI unit of charge is the **coulomb**.

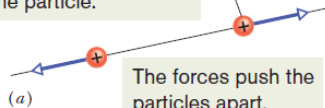
The **electrostatic constant** is

$$k = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2.$$

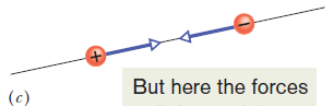
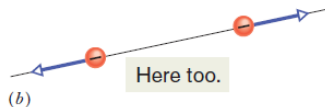
The quantity  $\epsilon_0$  is called the **permittivity constant**

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2.$$

Always draw the force vector with the tail on the particle.



The forces push the particles apart.



**Fig. 21-6** Two charged particles repel each other if they have the same sign of charge, either (a) both positive or (b) both negative. (c) They attract each other if they have opposite signs of charge.

The force between two point charges is directed along the line joining the charges. The unit vector denoting the direction from the first charge to the second is  $\hat{r}_{12}$ .

$$i = \frac{dq}{dt} \quad (\text{electric current}),$$

The unit of electric current is the ampere (A). The unit of electric charge is the coulomb (C) is defined as  $1 \text{ C} = (1 \text{ A})(1 \text{ s})$ .

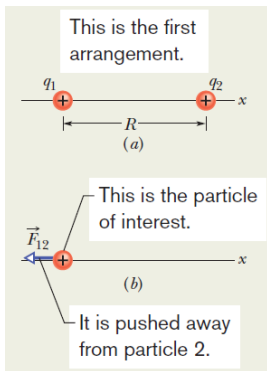
## 21.4 Coulomb's Law

If there are  $n$  charged particles, they interact independently in pairs, and the force on any one of them, say particle 1, is given by the vector sum

$$\vec{F}_{1,\text{net}} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} + \cdots + \vec{F}_{1n}$$

## Example, The net force due to two other particles:

(a) Figure 21-8a shows two positively charged particles fixed in place on an  $x$  axis. The charges are  $q_1 = 1.60 \times 10^{-19} \text{ C}$  and  $q_2 = 3.20 \times 10^{-19} \text{ C}$ , and the particle separation is  $R = 0.0200 \text{ m}$ . What are the magnitude and direction of the electrostatic force  $\vec{F}_{12}$  on particle 1 from particle 2?



**Fig. 21-8** (a) Two charged particles of charges  $q_1$  and  $q_2$  are fixed in place on an  $x$  axis. (b) The free-body diagram for particle 1, showing the electrostatic force on it from particle 2.

$$\begin{aligned} F_{12} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_2|}{R^2} \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \\ &\quad \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(0.0200 \text{ m})^2} \\ &= 1.15 \times 10^{-24} \text{ N}. \end{aligned}$$

Thus, force  $\vec{F}_{12}$  has the following magnitude and direction (relative to the positive direction of the  $x$  axis):

$$1.15 \times 10^{-24} \text{ N} \quad \text{and} \quad 180^\circ. \quad (\text{Answer})$$

We can also write  $\vec{F}_{12}$  in unit-vector notation as

$$\vec{F}_{12} = -(1.15 \times 10^{-24} \text{ N})\hat{i}. \quad (\text{Answer})$$

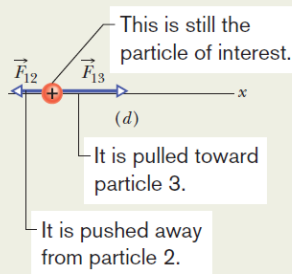
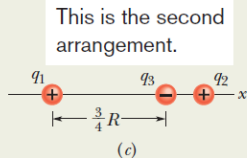
# Example, The net force due to two other particles, cont.:

(b) Figure 21-8c is identical to Fig. 21-8a except that particle 3 now lies on the  $x$  axis between particles 1 and 2. Particle 3 has charge  $q_3 = -3.20 \times 10^{-19} \text{ C}$  and is at a distance  $\frac{3}{4}R$  from particle 1. What is the net electrostatic force  $\vec{F}_{1,\text{net}}$  on particle 1 due to particles 2 and 3?

$$\begin{aligned} F_{13} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_3|}{(\frac{3}{4}R)^2} \\ &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \\ &\quad \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(\frac{3}{4})^2(0.0200 \text{ m})^2} \\ &= 2.05 \times 10^{-24} \text{ N}. \end{aligned}$$

We can also write  $\vec{F}_{13}$  in unit-vector notation:

$$\vec{F}_{13} = (2.05 \times 10^{-24} \text{ N})\hat{i}.$$



The net force  $\vec{F}_{1,\text{net}}$  on particle 1 is the vector sum of  $\vec{F}_{12}$  and  $\vec{F}_{13}$ ; that is, from Eq. 21-7, we can write the net force  $\vec{F}_{1,\text{net}}$  on particle 1 in unit-vector notation as

$$\begin{aligned} \vec{F}_{1,\text{net}} &= \vec{F}_{12} + \vec{F}_{13} \\ &= -(1.15 \times 10^{-24} \text{ N})\hat{i} + (2.05 \times 10^{-24} \text{ N})\hat{i} \\ &= (9.00 \times 10^{-25} \text{ N})\hat{i}. \end{aligned} \quad \text{(Answer)}$$

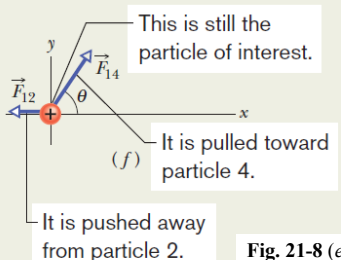
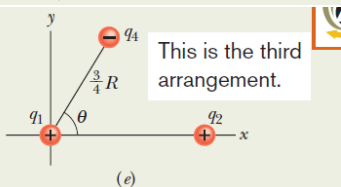
Thus,  $\vec{F}_{1,\text{net}}$  has the following magnitude and direction (relative to the positive direction of the  $x$  axis):

$$9.00 \times 10^{-25} \text{ N} \quad \text{and} \quad 0^\circ. \quad \text{(Answer)}$$

**Fig. 21-8** (c) Particle 3 included. (d) Free-body diagram for particle 1.

## Example, The net force due to two other particles, cont.:

(c) Figure 21-8e is identical to Fig. 21-8a except that particle 4 is now included. It has charge  $q_4 = -3.20 \times 10^{-19} \text{ C}$ , is at a distance  $\frac{3}{4}R$  from particle 1, and lies on a line that makes an angle  $\theta = 60^\circ$  with the  $x$  axis. What is the net electrostatic force  $\vec{F}_{1,\text{net}}$  on particle 1 due to particles 2 and 4?



**Fig. 21-8** (e) Particle 4 included. (f) Freebody diagram for particle 1.

$$\begin{aligned} F_{14} &= \frac{1}{4\pi\epsilon_0} \frac{|q_1||q_4|}{(\frac{3}{4}R)^2} \\ &= (8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2) \\ &\quad \times \frac{(1.60 \times 10^{-19} \text{ C})(3.20 \times 10^{-19} \text{ C})}{(\frac{3}{4})^2(0.0200 \text{ m})^2} \\ &= 2.05 \times 10^{-24} \text{ N}. \end{aligned}$$

$$\begin{aligned} F_{1,\text{net},x} &= F_{12,x} + F_{14,x} = F_{12} + F_{14} \cos 60^\circ \\ &= -1.15 \times 10^{-24} \text{ N} + (2.05 \times 10^{-24} \text{ N})(\cos 60^\circ) \\ &= -1.25 \times 10^{-25} \text{ N}. \end{aligned}$$

The sum of the  $y$  components gives us

$$\begin{aligned} F_{1,\text{net},y} &= F_{12,y} + F_{14,y} = 0 + F_{14} \sin 60^\circ \\ &= (2.05 \times 10^{-24} \text{ N})(\sin 60^\circ) \\ &= 1.78 \times 10^{-24} \text{ N}. \end{aligned}$$

The net force  $\vec{F}_{1,\text{net}}$  has the magnitude

$$F_{1,\text{net}} = \sqrt{F_{1,\text{net},x}^2 + F_{1,\text{net},y}^2} = 1.78 \times 10^{-24} \text{ N}. \quad (\text{Answer})$$

To find the direction of  $\vec{F}_{1,\text{net}}$ , we take

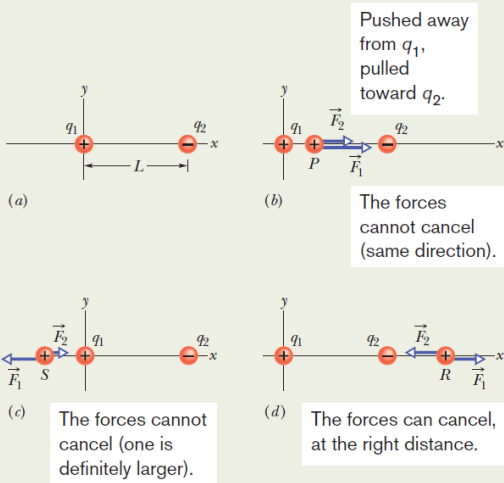
$$\theta = \tan^{-1} \frac{F_{1,\text{net},y}}{F_{1,\text{net},x}} = -86.0^\circ.$$

However, this is an unreasonable result because  $\vec{F}_{1,\text{net}}$  must have a direction between the directions of  $\vec{F}_{12}$  and  $\vec{F}_{14}$ . To correct  $\theta$ , we add  $180^\circ$ , obtaining

$$-86.0^\circ + 180^\circ = 94.0^\circ. \quad (\text{Answer})$$

## Example, Equilibrium of two forces

Figure 21-9a shows two particles fixed in place: a particle of charge  $q_1 = +8q$  at the origin and a particle of charge  $q_2 = -2q$  at  $x = L$ . At what point (other than infinitely far away) can a proton be placed so that it is in *equilibrium* (the net force on it is zero)? Is that equilibrium *stable* or *unstable*? (That is, if the proton is displaced, do the forces drive it back to the point of equilibrium or drive it farther away?)



$$\frac{1}{4\pi\epsilon_0} \frac{8qq_p}{x^2} = \frac{1}{4\pi\epsilon_0} \frac{2qq_p}{(x-L)^2}.$$

$$\left( \frac{x-L}{x} \right)^2 = \frac{1}{4}.$$

$$\frac{x-L}{x} = \frac{1}{2},$$

$$x = 2L.$$

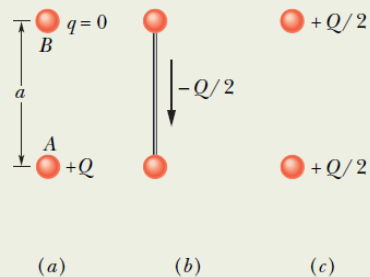
The equilibrium at  $x=2L$  is unstable; that is, if the proton is displaced leftward from point  $R$ , then  $F_1$  and  $F_2$  both increase but  $F_2$  increases more (because  $q_2$  is closer than  $q_1$ ), and a net force will drive the proton farther leftward. If the proton is displaced rightward, both  $F_1$  and  $F_2$  decrease but  $F_2$  decreases more, and a net force will then drive the proton farther rightward. In a stable equilibrium, if the proton is displaced slightly, it returns to the equilibrium position.



## Example, Charge Sharing:

In Fig. 21-10a, two identical, electrically isolated conducting spheres  $A$  and  $B$  are separated by a (center-to-center) distance  $a$  that is large compared to the spheres. Sphere  $A$  has a positive charge of  $+Q$ , and sphere  $B$  is electrically neutral. Initially, there is no electrostatic force between the spheres. (Assume that there is no induced charge on the spheres because of their large separation.)

(a) Suppose the spheres are connected for a moment by a conducting wire. The wire is thin enough so that any net charge on it is negligible. What is the electrostatic force between the spheres after the wire is removed?



**Fig. 21-10** Two small conducting spheres  $A$  and  $B$ . (a) To start, sphere  $A$  is charged positively. (b) Negative charge is transferred from  $B$  to  $A$  through a connecting wire. (c) Both spheres are then charged positively.

- (1) Since the spheres are identical, connecting them means that they end up with identical charges (same sign and same amount).
- (2) The initial sum of the charges (including the signs of the charges) must equal the final sum of the charges.

**Reasoning:** When the spheres are wired together, the (negative) conduction electrons on  $B$  move away from one another (along the wire to positively charged  $A$ — Fig. 21-10b.)

As  $B$  loses negative charge, it becomes positively charged, and as  $A$  gains negative charge, it becomes less positively charged. The transfer of charge stops when the charge on  $B$  has increased to  $Q/2$  and the charge on  $A$  has decreased to  $Q/2$ , which occurs when  $Q/2$  has shifted from  $B$  to  $A$ .

The spheres, now positively charged, repel each other.

$$F = \frac{1}{4\pi\epsilon_0} \frac{(Q/2)(Q/2)}{a^2} = \frac{1}{16\pi\epsilon_0} \left( \frac{Q}{a} \right)^2.$$

## 21.5 Charge is Quantized

Since the days of Benjamin Franklin, our understanding of the nature of electricity has changed from being a type of ‘continuous fluid’ to a collection of smaller charged particles. The total charge was found to always be a multiple of a certain **elementary charge**, “e”:

$$q = ne, \quad n = \pm 1, \pm 2, \pm 3, \dots,$$

The value of this elementary charge is one of the fundamental constants of nature, and it is the magnitude of the charge of both the proton and the electron. The value of “e” is:

$$e = 1.602 \times 10^{-19} \text{ C.}$$

(Millikan’s oil drop experiment in 1909)

**Table 21-1**

**The Charges of Three Particles**

Particle	Symbol	Charge
Electron	$e$ or $e^-$	$-e$
Proton	$p$	$+e$
Neutron	$n$	$0$

# Quarks have charge of  $\pm \frac{1}{3} e$  or  $\pm \frac{2}{3} e$ , but they never appear individually (color confinement).



Many descriptions of electric charge use terms that might lead you to the conclusion that charge is a substance. Phrases like:

“Charge on a sphere”

“Charge transferred”

“Charge carried on the electron”

However, **charge is a *property* of particles**, one of many properties, such as mass.

## Example, Mutual Electric Repulsion in a Nucleus:

The nucleus in an iron atom has a radius of about  $4.0 \times 10^{-15} \text{ m}$  and contains 26 protons.

(a) What is the magnitude of the repulsive electrostatic force between two of the protons that are separated by  $4.0 \times 10^{-15} \text{ m}$ ?

### KEY IDEA

The protons can be treated as charged particles, so the magnitude of the electrostatic force on one from the other is given by Coulomb's law.

**Calculation:** Table 21-1 tells us that the charge of a proton is  $+e$ . Thus, Eq. 21-4 gives us

$$\begin{aligned} F &= \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} \\ &= \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.602 \times 10^{-19} \text{ C})^2}{(4.0 \times 10^{-15} \text{ m})^2} \\ &= 14 \text{ N}. \end{aligned} \quad (\text{Answer})$$

**No explosion:** This is a small force to be acting on a macroscopic object like a cantaloupe, but an enormous force to be

acting on a proton. Such forces should explode the nucleus of any element but hydrogen (which has only one proton in its nucleus). However, they don't, not even in nuclei with a great many protons. Therefore, there must be some enormous attractive force to counter this enormous repulsive electrostatic force.

(b) What is the magnitude of the gravitational force between those same two protons?

### KEY IDEA

Because the protons are particles, the magnitude of the gravitational force on one from the other is given by Newton's equation for the gravitational force (Eq. 21-2).

**Calculation:** With  $m_p (= 1.67 \times 10^{-27} \text{ kg})$  representing the mass of a proton, Eq. 21-2 gives us

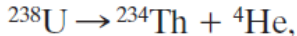
$$\begin{aligned} F &= G \frac{m_p^2}{r^2} \\ &= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.67 \times 10^{-27} \text{ kg})^2}{(4.0 \times 10^{-15} \text{ m})^2} \\ &= 1.2 \times 10^{-35} \text{ N}. \end{aligned} \quad (\text{Answer})$$

## 21.6 Charge is Conserved

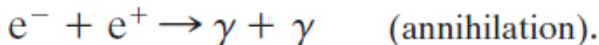
The hypothesis of **conservation of charge** has stood up under close examination, both for large-scale charged bodies and for atoms, nuclei, and elementary particles.

Example 1: Radioactive decay of nuclei, in which a nucleus transforms into (becomes) a different type of nucleus.

A uranium-238 nucleus ( $^{238}\text{U}$ ) transforms into a thorium-234 nucleus ( $^{234}\text{Th}$ ) by emitting an *alpha particle*. An alpha particle has the same makeup as a helium-4 nucleus, it has the symbol  $^4\text{He}$ . Here the net charge is  $92 = 90 + 2$ .



Example 2: An electron  $e^-$  (charge  $-e$ ) and its antiparticle, the positron  $e^+$  (charge  $+e$ ), undergo an annihilation process, transforming into two gamma rays (high-energy light). Here the net charge is  $0 = 0$ .



Example 3: A gamma ray (in a certain environment) transforms into an electron and a positron. Here the net charge is again  $0 = 0$ .

