# CITY UNIVERSITY OF HONG KONG

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| Course code & title | : | GE2256/Applications of Game Theory to Business |
| Session | : | Semester A 2020/21 |
| Time allowed | : | Two hours |

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This paper has THREE pages (excluding this cover page).

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1. This paper consists of 6 questions. Answer ALL questions.

2. Total mark is 100. Different questions carry different marks.

3. You must show all your work with proper explanation in order to get full credit for a correct answer.

4. This is a closed-book, closed-notes, closed-assignments examination.

5. The method of submission for your answers is online. You must write out your answers on sheets of papers, then take photos of the sheets, and finally upload them through canvas.

6. Useful hotlines during the Exam:

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| Course Leader | Dr. Nilanjan Roy | 6778-5890 |
| e-learning Team | 3442-6727 | |
| Computer Services Centre (CSC) | 3442-6488 | |
| Departmental hotline | 3442-2688 | |

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**Academic Honesty** *(*Students must reaffirm the honesty pledge* ***by writing*** *"I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties" onto the first examination answer sheet.)*

I pledge that the answers in this examination are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- I will not plagiarize (copy without citation) from any source;
- I will not communicate or attempt to communicate with any other person during the examination; neither will I give or attempt to give assistance to another student taking the examination; and
- I will use only approved devices (e.g., calculators) and/or approved device models.
- I understand that any act of academic dishonesty can lead to disciplinary action.

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*This is a **closed-book** examination.*

*Students are allowed to use the following materials/aids:*

*Calculator*

*Materials/aids other than those stated above are not permitted. Students will be subject to disciplinary action if any unauthorized materials or aids are found on them.*

**Question 1**: *(15 points)* Two players (player 1 and player 2) are bargaining over how to split one dollar. Both of them simultaneously name shares they would like to have, $s_1$ and $s_2$, where $0 \leq s_1 \leq 1$ and $0 \leq s_2 \leq 1$. If $s_1 + s_2 \leq 1$, then the players receive the shares they named; if $s_1 + s_2 > 1$, then both players receive zero. What are the pure strategy Nash equilibria of this game?

**Question 2**: Consider the following game:

| | | q | | 1-q | |
|---|---|---|---|---|---|
| | | W | ~~X~~ | Y | ~~Z~~ |
| p | A | 3,3 | 2,1 | 0,2 | 2,1 |
| | ~~B~~ | ~~1,1~~ | ~~1,2~~ | ~~1,0~~ | ~~1,4~~ |
| 1-p | C | 0,0 | 1,0 | 3,2 | 1,1 |
| | ~~D~~ | ~~0,0~~ | ~~0,5~~ | ~~0,2~~ | ~~3,1~~ |

$3q = 3(1-q)$

$q = \frac{1}{2}$

$3p = \cancel{2p+1}\ 2$

$p = \frac{2}{3}$

(a) (*12 points*) Find out the rationalizable strategies for each player.

(b) (*8 points*) What are the pure strategy and mixed strategy Nash equilibrium/equilibria of this game?

**Question 3**: Three players move sequentially, one by one, such that player 1 moves first, then after observing player 1's choice, player 2 moves and finally, after observing player 2's choice, player 3 moves. Each player can choose whether to contribute (C) or not contribute (N) towards the construction of a public good. If there are a total of 2 or 3 contributors, then the benefit of the public good to each person is 4. If there are a total of 0 or 1 contributors, then the benefit of the public good to each person is 2. Contributing costs 1, and if a player does not contribute, the cost is 0. The payoff to a player is benefit minus cost.

(a) (*6 points*) Draw the extensive form (or game tree) for this game. How many proper subgames are there? 7

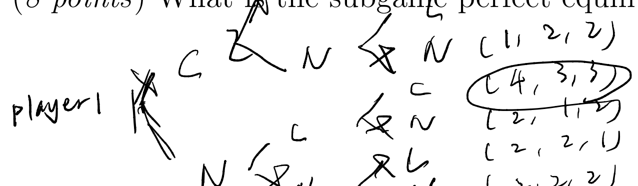
(3, 3, 3) C, N    CC, CN, NC, NN,

(b) (*6 points*) Write down all possible strategies of players 1 and 2. How many strategies does player 3 have?

N (3, 3, 4)    8

(3, 4, 3)

(c) (*8 points*) What is the subgame perfect equilibrium of this game?



**Question 4**: *(15 points)* Consider the Stackelberg game of sequential quantity choice where firm 1 moves first and firm 2 is the follower. After firm 1 sets a quantity ($q_1$), firm 2 chooses it's quantity ($q_2$) after observing firm 1's choice. The demand function in the market is given by:

$u_1 = (800 - 2q_1 - 2q_2)q_1$

$$p = 1000 - 2q_1 - 2q_2$$

$u_2 = (800 - 2q_1 - 2q_2)q_2$

Each firm incurs a cost of 200 for each unit of output it produces. Find the subgame perfect equilibrium of the game.

FOC: $-2q_2 + 800 - 2q_1 - 2q_2 = 0$ $\quad q_2 = \frac{\cancel{8}\,400 - q_1}{2}$

**Question 5** *(15 points)* Two people are involved in a dispute. Person 1 can be of only one type. Person 1 does not know whether person 2 is strong or weak; she assigns probability $\alpha$ to person 2's being strong. Person 2 is fully informed (about his own type). Each person can either fight or yield. Each person's payoff function assigns a payoff of 0 if she yields (regardless of the other person's action) and a payoff of 1 if she fights and her opponent yields; if both people fight, then their payoffs are (-1,1) if person 2 is strong and (1,-1) if person 2 is weak. Formulate this situation as a Bayesian game and find its Bayes-Nash equilibria if $\alpha < 1/2$ and if $\alpha > 1/2$.

$u_1$: $\cancel{800}$ $\cancel{400} - q_1$ $\quad 800 - 2q_1 - 400 + q_1$ $\quad q_1 = 200$

$(400 - q_1)q_1$ $\quad -2q_1 + 400 = 0$ $\quad q_2 = 100$

**Question 6**: There is a market with two firms who are contemplating acting collusively. If they cut production so that each is making half the monopoly quantity, they each get $\pi_m = 2$. If they act as competitors, they each get $\pi_c = 1$. If one player chooses the monopoly quantity, the other player can take advantage of the situation and increase output, getting a larger share of the market for himself at the expense of his partner, who gets 0 if this occurs; call this value $\pi_d$. Here is the payoff matrix for the game with firm A as the row player and firm B as the column player:

| | Collude | Compete |
|---|---|---|
| Collude | 2, 2 | 0, $\pi_d$ |
| Compete | $\pi_d$, 0 | 1,1 |

$\pi_d > 2$

(a) (*3 points*) For what values of $\pi_d$ does this game have the form of a prisoner's dilemma, i.e., each player has a strictly dominant strategy to play competitively? Assume this inequality holds for the rest of the problem.

(b) (*12 points*) Assume the game is repeated infinitely with discount factor $\delta$. Use a "trigger strategy" (similar to the one discussed in class) to find the condition on $\delta$ such that the play of (Collude, Collude) every period can be supported as a subgame-perfect equilibrium of the infinitely repeated game.

*******END OF THE EXAMINATION PAPER*******