MA1300 Self Practice # 11

1. (P221, #11, 14) For the following two functions:

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the local maximum and minimum values of f.

(c) Find the intervals of concavity and the inflection points.

$$f(x) = x^4 - 2x^2 + 3,$$
$$f(x) = \cos^2 x - 2\sin x, \quad 0 \le x \le 2\pi.$$

2. (P221, #26) Suppose $f(3) = 2$, $f'(3) = 1/2$, and $f'(x) > 0$, and $f''(x) < 0$ for all x.

(a) Sketch a possible graph for f.

(b) How many solutions does the equation $f(x) = 0$ have? Why?

(c) Is it possible that $f'(2) = 1/3$? Why?

3. (P222, #53) Find a cubic function $f(x) = ax^3 + bx^2 + cx + d$ that has a local maximum value of 3 at -2 and a local minimum value of 0 at 1.

4. (P223, #59)

(a) If f and g are positive, increasing, concave upward functions on I, show that the product function fg is concave upward on I.

(b) Show that part (a) remains true if f and g are both decreasing.

(c) Suppose f is increasing and g is decreasing. Show, by giving three examples, that fg may be concave upward, concave downward, or linear. Why doesn't the argument in parts (a) and (b) work in this case?

5. (P223, #61) Show that $\tan x > x$ for $0 < x < \pi/2$. [Hint: Show that $f(x) = \tan x - x$ is increasing on $(0, \pi/2)$.]

6. (P223, #62) Prove that, for all $x > 1$,

$$2\sqrt{x} > 3 - \frac{1}{x}.$$

7. (P235, #13, 19) Find the limit.

$$\lim_{t\to\infty} \frac{\sqrt{t} + t^2}{2t - t^2}, \qquad \lim_{x\to\infty} \left(\sqrt{9x^2 + x} - 3x\right).$$

8. (P235, #45) Find the horizontal asymptotes of the curve and use them, together with concavity and intervals of increase and decrease, to sketch the curve.

$$y = \frac{1-x}{1+x}.$$

9. (P243, #27) Use the guidelines of Section 3.5 (A∼H) to sketch the curve.

$$y = \frac{\sqrt{1-x^2}}{x}.$$

10. (P243, #41) In the theory of relativity, the mass of a particle is

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}},$$

where m_0 is the rest mass of the particle, m is the mass when the particle moves with speed v relative to the observer, and c is the speed of light. Sketch the graph of m as a function of v.

11. (P243, #44) Coulomb's Law states that the force of attraction between two charged particles is directly proportional to the product of the charges and inversely proportional to the square of the distance between them. The figure 1 shows particles with charge 1 located at positions 0 and 2 on a coordinate line and a particle with charge -1 at a position x between them. It follows from Coulomb's Law that the net force acting on the middle particle is

$$F(x) = -\frac{k}{x^2} + \frac{k}{(x-2)^2}, \qquad 0 < x < 2,$$

where k is a positive constant. Sketch the graph of the net force function. What does the graph say about the force?

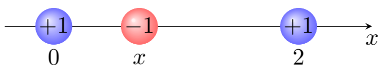

Figure 1: The positions of the charged particles.

12. (P257, #9) A model used for the yield Y of an agricultural crop as a function of the nitrogen level N in the soil (measured in appropriate units) is

$$Y = \frac{kN}{1+N^2},$$

where k is a positive constant. What nitrogen level gives the best yield?

13. (P257, #10) The rate (in mg carbon/m^3/h) at which photosynthesis takes place for a species of phytoplankton is modeled by the function

$$P = \frac{100I}{I^2+I+4},$$

where I is the light intensity (measured in thousands of foot-candles). For what light intensity is P a maximum?

14. (P258, #43) If a resistor of R ohms is connected across a battery of E volts with internal resistance r ohms, then the power (in watts) in the external resistor is

$$P = \frac{E^2 R}{(R + r)^2}.$$

If E and r are fixed but R varies, what is the maximum value of the power?

15. (P260, #67) Let v_1 be the velocity of light in air and v_2 the velocity of light in water. According to Fermat's Principle, a ray of light will travel from a point A in the air to a point B in the water by a path ACB that minimizes the time taken. Show that

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{v_1}{v_2},$$

where θ_1 (the angle of incidence) and θ_2 (the angle of refraction) are as shown in Figure 2. This equation is known as Snell's Law.

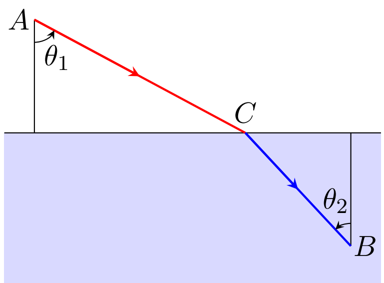

Figure 2: Figure of Problem 15.

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