

EE1001 Test 1, KF Tsang, 16 October 2021, 9:15am -10:30am, 1hr 15min, LT 5, LT 6, online

There are TEN (10) questions.

Answer ALL questions.

CityU approved calculator is allowed.

Observe CityU examination/test guidelines.

ALL students please scan your answer and upload onto CANVAS on or before 10:45am. THREE trials are allowed.

**Q1 (10 marks, 2 marks each)**

Determine the following statements whether they are true or false.

- (i) A valid argument can go from false premises to a true conclusion.
- (ii) An unsound argument always has at least one false premise.
- (iii) " $a \leftrightarrow b$ " is logically equivalent to " $\sim b \leftrightarrow \sim a$ ".
- (iv) "All human beings are not animals" is an unsound argument.
- (v) Given  $A = \{a_1, a_2, a_3\}$ .  $\forall x \in A, P(x)$  is logically equivalent to  $P(a_1) \wedge P(a_2) \wedge P(a_3)$ .

**Q2 (13 marks)**

Without using a truth table, determine whether  $(p \vee q) \wedge (p \rightarrow r) \wedge \sim r \rightarrow q$  is a tautology or a contradiction, or neither. State the reason for each step.

**Q3 (15 marks)**

There are THREE (3) suspects who are thought to be guilty of a crime. Their statements are as follows:

- Suspect A: "If  $B$  is guilty, then  $C$  and I are innocent."  
Suspect B: "I'm innocent, and at least one of the others is guilty."  
Suspect C: "None of us are guilty."

Let  $a$ ,  $b$ , and  $c$  be

$a$  = "Suspect A is innocent"

$b$  = "Suspect B is innocent"

$c$  = "Suspect C is innocent"

Q3(i) Formulate the statements of the Suspects  $A$ ,  $B$ , and  $C$ . (6 marks)

Q3(ii) Given the innocent told the truth and the guilty lied, use truth table to determine whether the Suspects  $A$ ,  $B$ , and  $C$  are innocent or guilty. (9 marks)

(Hint: There are **TWO (2)** possible situations)

**Q4 (12 marks)**

(i) In an arithmetic sequence, the sum of its first nine terms is 2340, and the sum of the next eight terms is 6500. Determine the common difference ( $d$ ) and the first term ( $a_1$ ).

(6 marks)

(ii) Given a geometric sequence with the first term  $b$  and the common ratio  $r$ , its sum to infinity is 4. Another geometric sequence has the first term  $3b$ , common ratio  $r^2$  and sum to infinity 8. Determine the values of  $b$  and  $r$ .

(6 marks)

**Q5 (10 marks)**

Use 1,2,3 to form a number with a length of 4 (e.g., 1231). Find the number of permutations or combinations under each condition.

(i) These three numbers are reused with no limitations. (2 marks)

(ii) One of the elements (i.e., '1', or '2', or '3') is used TWICE to form the number. (4 marks)

(iii) One of the elements (i.e., '1', or '2', or '3') is used TWICE to form the number, meanwhile the element cannot be adjacent. (e.g., there are two "1" used, then the number cannot be 11xy, x11y, xy11, etc.) (4 marks)

**Q6 (12 marks)**

There is a group of 100 passengers who plan to visit Hong Kong. It is known that THREE (3) of them are infected by the Cov-19. Given the limited human resources, a custom agent can randomly select 3 passengers to perform medical check. You can assume that the medical check result of the selected 3 passengers is 100% correct.

(i) Find the number of the combinations of the selected 3 customers; (2 marks)

(ii) Find the probability of detecting exactly 1 infected customer; (4 marks)

(iii) Find the probability of detecting at least 1 infected customer; (4 marks)

(iv) It is known that Cov-19 has a high probability to infect others and thus a 1% rule has been established. If the infection rate of passengers is less than 1%, the whole team will be allowed to entry into Hong Kong, otherwise the whole team should undergo quarantine. Comment on the decision to be made. (2 marks)

**Q7 (10 marks)**

Find the coefficient of the term  $x^3y^3$ ;

(i)  $(x+y^2/x)(x+y)^5$  (5 Marks)

(ii)  $(x+y)(2x-y)^5$  (5 Marks)

**Q8 (6 marks)**

Find  $a$  if the coefficients of  $x^2$  and  $x^3$  in the expansion of  $(2+ax)^8$  are equal ( $a \neq 0$ ).

**Q9 (6 marks)**

In the expansion of  $(\frac{\sqrt{x}}{\sqrt{3}} + \frac{\sqrt{3}}{2x^2})^n$ .

(i)  $n = 8$ , determine if there exists a term that is independent of  $x$  (constant term). (3 marks)

(ii)  $n = 10$ , determine if there exists a term that is independent of  $x$  (constant term).  
Please give a detailed explanation. (3 marks)

**Q10 (6 marks)**

In a training camp, find the minimum number of students in the camp such that three of them are born in the same month.

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