MA1300 Self Practice # 4

1. (P92, #46) Find the values of a and b that make f continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3. \end{cases}$$

2. (P92, #51, 53) Use the Intermediate Value Theorem to show that there is a root of the given equation in the specified interval.

(a) $x^4 + x - 3 = 0, \qquad (1, 2)$

(b) $\cos x = x, \qquad (0, 1)$

3. (P93, #65) Is there a number that is exactly 1 more than its cube?

4. (P93, #66) If a and b are positive numbers, prove that the equation

$$\frac{a}{x^3 + 2x^2 - 1} + \frac{b}{x^3 + x - 2} = 0$$

has at least one solution in the interval $(-1, 1)$.

5. (P96, #26, 28, 32, 34, 37) Find the limit.

$$\lim_{x\to -3} \frac{x^2-9}{x^2+2x-3} \qquad \lim_{x\to 1^+} \frac{x^2-9}{x^2+2x-3} \qquad \lim_{v\to 4^+} \frac{4-v}{|4-v|}$$

$$\lim_{x\to 3} \frac{\sqrt{x+6}-x}{x^3-3x^2} \qquad \lim_{x\to 0} \frac{1-\sqrt{1-x^2}}{x}$$

6. (P96, #40) Prove that

$$\lim_{x\to 0} x^2 \cos\frac{1}{x^2} = 0.$$

7. (P96, #45) Let

$$f(x) = \begin{cases} \sqrt{-x}, & \text{if } x < 0, \\ 3 - x, & \text{if } 0 \le x \le 3, \\ (x-3)^2, & \text{if } x > 3. \end{cases}$$

(a) Evaluate each limit, if it exists.

(i) $\lim\limits_{x\to 0^+} f(x)$ (ii) $\lim\limits_{x\to 0^-} f(x)$ (iii) $\lim\limits_{x\to 0} f(x)$

(iv) $\lim\limits_{x\to 3^-} f(x)$ (v) $\lim\limits_{x\to 3^+} f(x)$ (vi) $\lim\limits_{x\to 3} f(x)$

(b) Where is f discontinuous?

(c) Sketch the graph of f.

8. (P96, #46) Let

$$g(x) = \begin{cases} 2x - x^2 & \text{if } 0 \leq x \leq 2 \\ 2 - x & \text{if } 2 < x \leq 3 \\ x - 4 & \text{if } 3 < x < 4 \\ \pi & \text{if } x \geq 4 \end{cases}$$

(a) For each of the numbers 2, 3, and 4, discover whether g is continuous from the left, continuous from the right, or continuous at the number.

(b) Sketch the graph of g.