BME2102 Introduction of Biomechanics 2020/21B

In-Class Test

17:00-18:50, Oct. 15, 2021

Closed book and notes. Formula sheet provided.

Student Name (Print)								
Student ID	1st	2nd	3rd	4th digit: i_4	5th digit: i_5	6th digit: i_6	7th digit: i_7	8th digit: i_8

Problem	1	2	3	4	Total
Points	25	25	25	25	100
Score					

- *Justify your answers and show the details of your calculations.*
- *Box or circle your answers.*
- *Please note for each problem there is a variable* ***i*** *associating to the 4th-8th digit of your Student ID, respectively.*
- *If you are approved to take the exam via the Zoom, your answer script should be posted by 18:50 to the Canvas (**Assignments: In-class Test**). Leave a back-up copy of the answer sheet for your own record. Videotape yourself should any technical issues prevent you from submitting your answers.*

- ***University Policy on Academic Honesty**: Students should be reminded about the importance of academic honesty, and about the honesty pledge they made when joining CityU. Require students to reaffirm their academic honesty pledge with each online assessment task. An example is provided below:*
  - *"I pledge that the answers in this exam/quiz are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,*
    - *I will not plagiarize (copy without citation) from any source;*
    - *I will not communicate or attempt to communicate with any other person during the exam/quiz; neither will I give or attempt to give assistance to another student taking the exam/quiz; and*
    - *I will use only approved devices (e.g., calculators) and/or approved device models.*
  - *I understand that any act of academic dishonesty can lead to disciplinary action."*
  - *Write '**I affirm the stated academic honesty pledge**.' below and **sign** your name.*

Sign here: ______________________________

1. Let $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$ be a right-handed Cartesian basis. If a shot is put with a velocity of $\vec{v} = 30\vec{e}_x + 4\vec{e}_y + 5.i\vec{e}_z$ m/s (where, $\boldsymbol{i} = |\boldsymbol{i_4} - \boldsymbol{i_8}|$, **i.e. the difference between the 4th and 8th digits of your Student ID**) and an acceleration of $\vec{a} = \vec{e}_y - 9.81\vec{e}_z$ m/s^2, and released from a position of $\vec{p} = 2\vec{e}_z$ m. (**25 points**)
   a. If the shot is later lands on the ground (height = 0 m), what was the total time of flight? What is the landing position as measured with $\{\vec{e}_x, \vec{e}_y, \vec{e}_z\}$?
   b. If an infinitely large target plate is placed in the mid-way of landing and perpendicular to $\vec{e}_x$, what is the location of the shot as measured with $\vec{u} = 4\vec{e}_x + 3\vec{e}_y$, $\vec{v} = 3\vec{e}_x - 4\vec{e}_y$ and $\vec{w} = \vec{u} \times \vec{v}$?

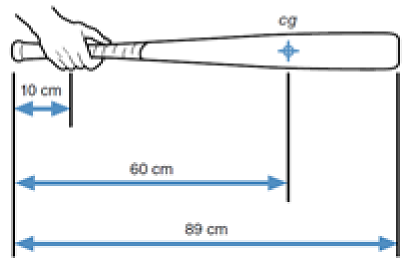
$\boldsymbol{i}$ =

2. A baseball batter sets up in the batting box and holds his bat still over the plate for a moment. The batter holds the 1 kg bat in a horizontal position with only one hand. The hand is 10 cm from the knob end of the bat. The bat has a moment of inertia around a transverse axis through its center of gravity of 500.i kg·cm^2 (where, $i = |i_5 - i_8|$, **i.e. the difference between the 5th and 8th digits of your Student ID**). The bat is 89 cm long, and its center of gravity is 60 cm from the knob end of the bat. (**25 points**)

a. How large is the vertical force exerted by the batter's hand on the bat?

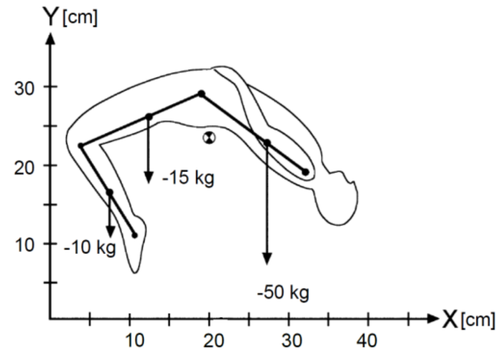
b. How large is the torque exerted by the batter's hand on the bat?

c. What is the moment of inertia of the bat about an axis through the handle where the hand holds the bat?



i =

3. In the segmental method, the body is mathematically broken up into segments. Figure 1 depicts the center of gravity (CoG) of a high jumper clearing the bar using a three-segment biomechanical model. This simple model (head + arms + trunk, thighs, legs + feet) illustrates the segmental method of calculating the CoG of a linked biomechanical system. (**25 points**)
   a. Given CoGs of the segments as thighs (12cm, 26cm), shank/feet (8cm, 16cm), and head + arms + trunk (28cm, 23cm), calculate the horizontal position of the whole-body CoG of the high jumper using the segmental method and a three-segment model of the body.
   b. If the whole-body CoG is coincident to the bar and the bar is at a height of 2.i m (where, $i = |i_6 - i_8|$, **i.e. the difference between the 6th and 8th digits of your Student ID**) from the ground. Suppose at that position the high jumper has a velocity of 1 m/s in horizontal direction and 0 m/s in vertical direction, what are the kinetic and potential energy at that position? Determine the landing position and kinetic and potential energy there.



i =

4. A neuro-probe is a needle-like probe used to collect signals from a brain. Suppose the neuro-probe is a flexible one with a Young's modulus of 3 GPa, and a rectangular cross-sectional area of 20 μm x 3 μm with a length of 2.***i*** mm (where, ***i*** = **$|i_7 - i_8|$, i.e., the difference between the 7th and 8th digits of your Student ID**). (**25 points**)

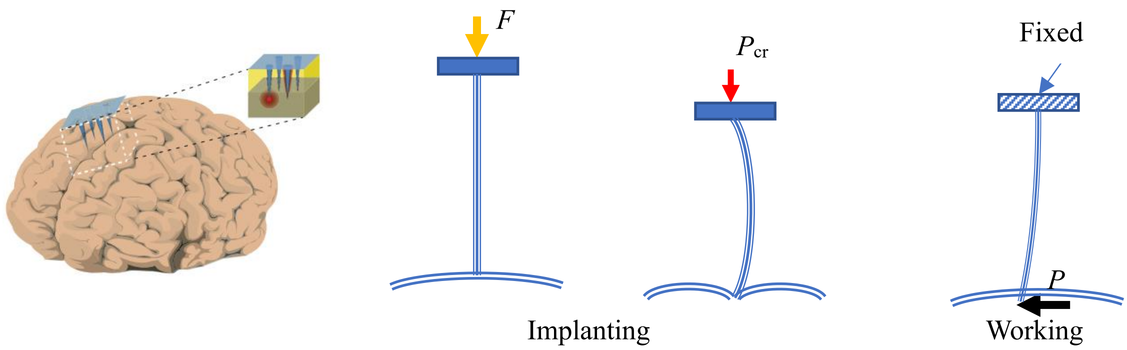
   a. When it punctures against the cerebral cortex, at the beginning, ignore the deformation of the cerebral cortex before the compression force reaches to a certain value. What are the stress and strain of the neuro-probe when the compression force is 100 μN?

   b. The neuro-probe can be regarded as a slender beam, and it will buckle as the force is larger than a critical value, P_{cr}, which is given by Euler's formula:

$$P_{\text{cr}} = \frac{\pi^2 E I}{L^2}$$

   Applying this equation, find the critical buckling force.

   c. The upper end of the neuro-probe will be fixed on the skull, while the lower tip is implanted in the brain. The micromotion of the brain will cause the probe to bend, what is the bending force P caused by the micromotion of the brain of a 10 μm?



i =

Quick-Reference Equations

MATHEMATICAL FORMULAS

Pythagorean theorem

$$A^2 + B^2 = C^2 \quad \textbf{(1.5)}$$

Trigonometric functions

$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}} \quad \textbf{(1.6)}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}} \quad \textbf{(1.7)}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}} \quad \textbf{(1.8)}$$

$$\theta = \arcsin\left(\frac{\text{opposite side}}{\text{hypotenuse}}\right) \quad \textbf{(1.9)}$$

$$\theta = \arccos\left(\frac{\text{adjacent side}}{\text{hypotenuse}}\right) \quad \textbf{(1.10)}$$

$$\theta = \arctan\left(\frac{\text{opposite side}}{\text{adjacent side}}\right) \quad \textbf{(1.11)}$$

LINEAR KINEMATICS

Average speed

$$\bar{s} = \frac{\ell}{\Delta t} \quad \textbf{(2.5)}$$

Average velocity

$$\bar{v} = \frac{d}{\Delta t} \quad \textbf{(2.6)}$$

Average acceleration

$$\bar{a} = \frac{v_f - v_i}{\Delta t} \quad \textbf{(2.9)}$$

PROJECTILE EQUATIONS

Vertical motion (y)

Vertical position:

$$y_f = y_i + v_i\Delta t + \frac{1}{2}g(\Delta t)^2 \quad \textbf{(2.14)}$$

$$y_f = \frac{1}{2}g(\Delta t)^2 \quad \text{if } y_i = 0 \text{ and } v_i = 0 \quad \textbf{(2.16)}$$

Vertical velocity:

$$v_f = v_i + g\Delta t \quad \textbf{(2.11)}$$

$$v^2 = v^2 + 2g\Delta y \quad \textbf{(2.15)}$$

$$v_{peak} = 0 \quad \textbf{(2.19)}$$

$$v_f = g\Delta t \quad \text{if yi} = 0 \text{ and vi} = 0 \quad \textbf{(2.17)}$$

$$v^2 = 2g\Delta y \quad \text{if } v_i = 0 \quad \textbf{(2.18)}$$

Vertical acceleration:

$$a = g = -9.81 \text{ m/s}^2 \quad \textbf{(2.10)}$$

Horizontal motion (x)

Horizontal position:

$$x = v\Delta t \quad \textbf{(2.26)}$$

Horizontal velocity:

$$v = v_f = v_i = \text{constant} \quad \textbf{(2.22)}$$

Horizontal acceleration:

$$a = 0 \quad \textbf{(2.23)}$$

Other equations governing projectile motion

Time of flight:

$$\Delta t_{up} = \Delta t_{down} \quad \text{if } y_f = y_i \quad \textbf{(2.20)}$$

$$\Delta t_{flight} = 2\Delta t_{up} \quad \text{if } y_f = y_i \quad \textbf{(2.21)}$$

Parabolic equation:

$$y_f = y_i + v_{y_i}\left(\frac{x}{v_x}\right) + \frac{1}{2}g\left(\frac{x}{v_x}\right)^2 \quad \textbf{(2.27)}$$

LINEAR KINETICS

Weight

$$W = mg \quad \textbf{(1.2)}$$

Static and dynamic friction

$$F_s = \mu_s R \quad \textbf{(1.3)}$$

$$F_d = \mu_d R \quad \textbf{(1.4)}$$

Static equilibrium

$$\Sigma F = 0 \quad \textbf{(1.12)}$$

$$\Sigma F_x = 0 \quad \textbf{(1.13)}$$

$$\Sigma F_y = 0 \quad \textbf{(1.14)}$$

Newton's 1st law: law of inertia

$$v = \text{constant if } \Sigma F = 0 \quad \textbf{(3.1a)}$$

or

$$\Sigma F = 0 \quad \text{if } v = \text{constant} \quad \textbf{(3.1b)}$$

Linear momentum

$$L = mv \quad \textbf{(3.6)}$$

Conservation of momentum

$$L = \text{constant if } \Sigma F = 0 \quad \textbf{(3.7)}$$

$$L_x = \text{constant if } \Sigma F_x = 0 \quad \textbf{(3.8)}$$

$$L_y = \text{constant if } \Sigma F_y = 0 \quad \textbf{(3.9)}$$

$$L_i = \Sigma(mu) = m_1u_1 + m_2u_2 + m_3u_3 + \ldots = m_1v_1 + m_2v_2 + m_3v_3 + \ldots = \Sigma(mv) = L_f = \text{constant if } \Sigma F = 0 \quad \textbf{(3.11)}$$

Perfectly elastic collision of two objects

$$v_1 = \frac{2m_2u_2 + (m_1 - m_2)u_1}{m_1 + m_2} \quad \textbf{(3.17)}$$

Perfectly inelastic collision of two objects

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v \quad \textbf{(3.19)}$$

Coefficient of restitution

$$e = \left|\frac{v_1 - v_2}{u_1 - u_2}\right| = \left|\frac{v_2 - v_1}{u_1 - u_2}\right| \quad \textbf{(3.20)}$$

Newton's 2nd law: law of acceleration

$$\Sigma F = ma \quad \textbf{(3.22)}$$

$$\Sigma F_x = ma_x \quad \textbf{(3.23)}$$

$$\Sigma F_y = ma_y \quad \textbf{(3.24)}$$

Impulse–momentum equation

$$\Sigma\bar{F}\ \Delta t = m(v_f - v_i) \quad \textbf{(3.29)}$$

Universal law of gravitation: gravitational force

$$F = G\left(\frac{m_1m_2}{r^2}\right) \quad \textbf{(3.30)}$$

WORK, POWER, AND ENERGY

Work

$$U = \bar{F}(d) \quad \textbf{(4.2)}$$

Kinetic energy

$$KE = \frac{1}{2}mv^2 \quad \textbf{(4.4)}$$

Gravitational potential energy

$$PE = Wh \quad \textbf{(4.5)}$$

Strain energy

$$SE = \frac{1}{2}k\ \Delta x^2 \quad \textbf{(4.7)}$$

Work–energy principle

$$U = \Delta E \quad \textbf{(4.8)}$$

Power

$$P = \frac{U}{\Delta t} \quad \textbf{(4.12)}$$

$$P = \bar{F}\bar{v} \quad \textbf{(4.13)}$$

ANGULAR KINEMATICS

Angular position measured in radians

$$\theta = \frac{\text{arc length}}{r} = \frac{\ell}{r} \quad \textbf{(6.1)}$$

Angular displacement and arc length

$$\ell = \Delta\theta r \quad \textbf{(6.4)}$$

Average angular velocity

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{\theta_f - \theta_i}{\Delta t} \quad \textbf{(6.6)}$$

Angular velocity and linear velocity

$$v_T = \omega r \quad \textbf{(6.8)}$$

Average angular acceleration

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} \quad \textbf{(6.9)}$$

Tangential acceleration

$$a_T = \alpha r \quad \textbf{(6.10)}$$

Centripetal acceleration

$$a_r = \frac{v_T^2}{r} \quad \textbf{(6.11)}$$

$$a_r = \omega^2 r \quad \textbf{(6.12)}$$

ANGULAR KINETICS

Torque

$$T = F \times r \quad \textbf{(5.1)}$$

Static equilibrium

$$\Sigma T = 0 \quad \textbf{(5.2)}$$

Center of gravity

$$\Sigma(W \times r) = (\Sigma W) \times r_{cg} \quad \textbf{(5.3)}$$

Moment of inertia

$$I_a = \Sigma m_i r_i^2 \quad \textbf{(7.1)}$$

$$I_a = m k_a^2 \quad \textbf{(7.2)}$$

Moment of inertia: parallel axis theorem

$$I_b = I_{cg} + mr^2 \quad \textbf{(7.3)}$$

Angular momentum

$$H_a = I_a \omega_a \quad \textbf{(7.4)}$$

Angular momentum of the human body

$$H_a = \Sigma(I_i\,\omega_i + m_i r^2_{i/cg}\,\omega_{i/cg}) \quad \textbf{(7.5)}$$

Conservation of angular momentum

$$H_i = I_i\omega_i = I_f\omega_f = H_f = \text{constant} \ \text{ if } \Sigma T = 0 \quad \textbf{(7.7)}$$

Angular version of Newton's 2nd law

$$\Sigma T_a = I_a \alpha_a \quad \textbf{(7.9)}$$

$$\Sigma\overline{T}_a = \frac{\Delta H_a}{\Delta t} = \frac{\left(H_f - H_i\right)}{\Delta t} \quad \textbf{(7.10)}$$

Angular impulse-momentum

$$\Sigma\overline{T}_a \Delta t = (H_f - H_i)_a \quad \textbf{(7.11)}$$

FLUID MECHANICS

Pressure

$$P = \frac{F}{A}$$

Density

$$\rho = \frac{m}{V} \quad \textbf{(8.3)}$$

Drag force

$$F_D = \frac{1}{2} C_D \rho A v^2 \quad \textbf{(8.5)}$$

Lift force

$$F_L = \frac{1}{2} C_L \rho A v^2 \quad \textbf{(8.6)}$$

MECHANICS OF MATERIALS

Stress

$$\sigma = \frac{F}{A} \quad \textbf{(9.1)}$$

Shear stress

$$\tau = \frac{F}{A} \quad \textbf{(9.2)}$$

Strain

$$\varepsilon = \frac{\ell - \ell_o}{\ell_o} \quad \textbf{(9.4)}$$

Elastic modulus

$$E = \frac{\Delta\sigma}{\Delta\varepsilon} \quad \textbf{(9.5)}$$

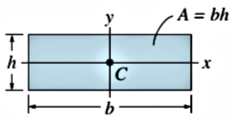
Abbreviations for Variables and Subscripts Used in Equations

Variables

a = instantaneous linear acceleration
$\bar{a}$ = average linear acceleration
A = area
C_D = coefficient of drag
C_L = coefficient of lift
d = displacement
e = coefficient of restitution
E = energy
E = elastic modulus or Young's modulus
F = force
$\bar{F}$ = average force
F_d = dynamic friction force
F_s = static friction force
ΣF = net force = sum of forces
g = acceleration due to gravity
G = gravitational constant
h = height
H = angular momentum
I = moment of inertia
k = radius of gyration
k = stiffness or spring constant
KE = kinetic energy
ℓ = distance traveled or length
L = linear momentum
m = mass
P = power
P = pressure
P = force
PE = gravitational potential energy
r = radius
r = moment arm
R = normal contact force
s = instantaneous linear speed
$\bar{s}$ = average linear speed
t = time
T = torque
u = pre-impact velocity
U = work done
v = instantaneous linear velocity
v = post-impact velocity
$\bar{v}$ = average linear velocity
V = volume
W = weight
x = horizontal position
y = vertical position
α = instantaneous angular acceleration
$\bar{\alpha}$ = average angular acceleration
Δ = change in … = final – initial
ε = strain
μ = coefficient of friction
ρ = density
σ = stress
Σ = sum of …
τ = shear stress
θ = angular position
ω = instantaneous angular velocity
$\bar{\omega}$ = average angular velocity

Subscripts

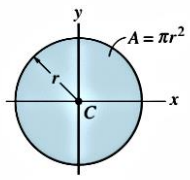
a = axis
b = axis
d = dynamic
cg = center of gravity
D = drag
f = final or ending
i = initial or starting
i = one of a number of parts
L = lift
o = original or undeformed
r = radial
s = static
T = tangential
x = horizontal
y = vertical



Rectangular area

$I_x = \frac{1}{12} bh^3$

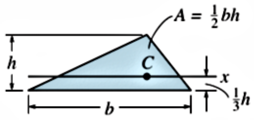
$I_y = \frac{1}{12} hb^3$



Circular area

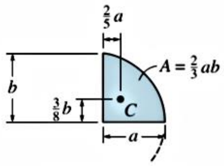
$I_x = \frac{1}{4} \pi r^4$

$I_y = \frac{1}{4} \pi r^4$

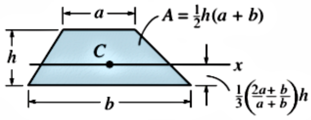


Triangular area

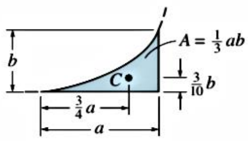
$I_x = \frac{1}{36} bh^3$



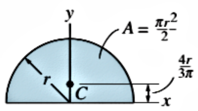
Semiparabolic area



Trapezoidal area



Exparabolic area

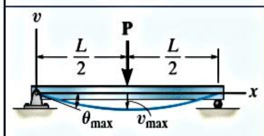
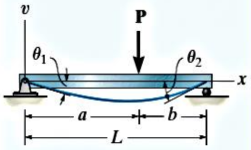
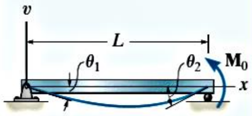
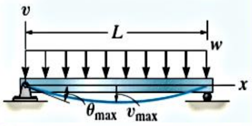


Semicircular area

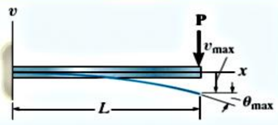
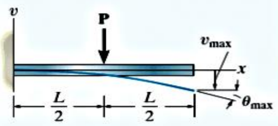
$I_x = \frac{1}{8} \pi r^4$

$I_y = \frac{1}{8} \pi r^4$

Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{16EI}$	$v_{max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI}(3L^2 - 4x^2)$ $0 \le x \le L/2$
	$\theta_1 = \frac{-Pab(L + b)}{6EIL}$ $\theta_2 = \frac{Pab(L + a)}{6EIL}$	$v\Big\rvert_{x=a} = \frac{-Pba}{6EIL}(L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL}(L^2 - b^2 - x^2)$ $0 \le x \le a$
	$\theta_1 = \frac{-M_0 L}{6EI}$ $\theta_2 = \frac{M_0 L}{3EI}$	$v_{max} = \frac{-M_0 L^2}{9\sqrt{3}\,EI}$ at $x = 0.5774L$	$v = \frac{-M_0 x}{6EIL}(L^2 - x^2)$
	$\theta_{max} = \frac{-wL^3}{24EI}$	$v_{max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI}(x^3 - 2Lx^2 + L^3)$

Cantilevered Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{max} = \frac{-PL^2}{2EI}$	$v_{max} = \frac{-PL^3}{3EI}$	$v = \frac{-Px^2}{6EI}(3L - x)$
	$\theta_{max} = \frac{-PL^2}{8EI}$	$v_{max} = \frac{-5PL^3}{48EI}$	$v = \frac{-Px^2}{12EI}(3L - 2x) \quad 0 \le x \le L/2$ $v = \frac{-PL^2}{48EI}(6x - L) \quad L/2 \le x \le L$