MA1300 Self Practice # 5

1. (P111, #7) Find an equation of the tangent line to the curve at the given point.

$$y = \sqrt{x}, \qquad (1, 1).$$

2. (P111, #14) If a rock is thrown upward on the planet Mars with an velocity of 10 m/s, its height (in meters) after t seconds is given by $H = 10t - 1.86t^2$.

a Find the velocity of the rock after one second.

b Find the velocity of the rock when $t = a$.

c When will the rock hit the surface?

d With what velocity will the rock hit the surface?

3. (P111, #15) The displacement (in meters) of a particle moving in a straight line is given by the equation of motion $s = 1/t^2$, where t is measured in seconds. Find the velocity of the particle at times $t = a$, $t = 1$, $t = 2$, and $t = 3$.

4. (P111, #18) Find an equation of the tangent line to the graph of $y = g(x)$ at $x = 5$ if $g(5) = -3$ and $g'(5) = 4$.

5. (P111, #20) If the tangent line to $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$, find $f(4)$ and $f'(4)$.

6. (P113, #46) If a cylindrical tank holds $100,000$ liters of water, which can be drained from the bottom of the tank in an hour, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V(t) = 100,000\left(1 - \frac{t}{60}\right)^2 \qquad 0 \le t \le 60.$$

Find the rate at which the water is flowing out of the tank (the instantaneous rate of change of V with respect to t) as a function of t. What are its units? For times $t = 0$, 10, 20, 30, 40, 50 and 60 min, find the flow rate and the amount of water remaining in the tank. Summarize your findings in a sentence or two. At what time is the flow rate the greatest? The least?

7. (P125, #52) Where is the greatest integer function $f(x) = [\![x]\!]$ not differentiable? Find a formula for f' and sketch its graph.

8. (P126, #54) The **left-hand** and **right-hand derivatives** of f at a are defined by

$$f'_-(a) = \lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h},$$

and

$$f'_+(a) = \lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h},$$

if these limits exist. Then $f'(a)$ exists if and only if these one-sided derivatives exist and are equal.

a Find $f'_-(4)$ and $f'_+(4)$ for the function

$$f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ 5 - x & \text{if } 0 < x < 4 \\ \frac{1}{5-x} & \text{if } x \ge 4. \end{cases}$$

b Sketch the graph of f.

c Where is f discontinuous?

d Where is f not differentiable?

9. (P126, #55) Recall that a function f is called *even* if $f(-x) = f(x)$ for all x in its domain and *odd* if $f(-x) = -f(x)$ for all such x. Prove each of the following.

a The derivative of an even function is an odd function.

b The derivative of an odd function is an even function.

10. (P136, #21, 22) Differentiate the function.

$$u = \sqrt[5]{t} + 4\sqrt{t^5}, \qquad v = \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2.$$

11. (P136, #27, P137, #31, 43, 44) Differentiate.

$$F(y) = \left(\frac{1}{y^2} - \frac{3}{y^4}\right)(y + 5y^3),$$

$$y = \frac{x^3}{1 - x^2}, \qquad f(x) = \frac{x}{x + \frac{c}{x}}, \qquad f(x) = \frac{ax + b}{cx + d}.$$

12. (P137, #45) The general polynomial of degree n has the form

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0,$$

where $a_n \neq 0$. Find the derivative of P.

13. (P137, #62) Find the first and second derivatives of the function

$$f(x) = \frac{1}{3 - x}.$$

14. (P137, #63) The equation of motion of a particle is $s = t^3 - 3t$, where s is in meters and t is in seconds. Find

a the velocity and acceleration as functions of t.

b the acceleration after $2s$, and

c the acceleration when the velocity is 0.

15. (P138, #69) If $f(x) = \sqrt{x}g(x)$, where $g(4) = 8$, and $g'(4) = 7$, find $f'(4)$.

16. (P138, #70) If $h(2) = 4$ and $h'(2) = -3$, find

$$\frac{d}{dx}\left(\frac{h(x)}{x}\right)\bigg|_{x=2}.$$

17. (P138, #80) Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - 2y = 2$.