Problem 1

(a) $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (\vec{\imath} + 2\vec{\jmath}) - (\vec{\jmath} - \vec{k}) = \vec{\imath} + \vec{\jmath} + \vec{k}.$

$$\overrightarrow{AX} = \underbrace{\frac{2}{3}\left|\overrightarrow{AB}\right|}_{magnitude} \times \underbrace{\left(\widehat{AB}\right)}_{direction} = \frac{2}{3}\left|\overrightarrow{AB}\right| \times \frac{\overrightarrow{AB}}{\left|\overrightarrow{AB}\right|} = \frac{2}{3}\left(\vec{\imath} + \vec{\jmath} + \vec{k}\right) = \frac{2}{3}\vec{\imath} + \frac{2}{3}\vec{\jmath} + \frac{2}{3}\vec{k}.$$

(b) Using the fact that $\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA}$, we have

$$\overrightarrow{OX} = \overrightarrow{AX} - \overrightarrow{OA} = \left(\frac{2}{3}\vec{\imath} + \frac{2}{3}\vec{\jmath} + \frac{2}{3}\vec{k}\right) - \left(\vec{\jmath} - \vec{k}\right) = \frac{2}{3}\vec{\imath} - \frac{1}{3}\vec{\jmath} + \frac{5}{3}\vec{k}.$$

Problem 2

(Method 1)

Let $\theta = \angle BAC$, then

$$\cos\theta = \frac{\overrightarrow{AB} \cdot \overrightarrow{AC}}{\left|\overrightarrow{AB}\right|\left|\overrightarrow{AC}\right|} = \frac{2}{(4)(4)} = \frac{1}{8}.$$

Using cosine law, we have

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\theta = 4^2 + 4^2 - 2(4)(4)\left(\frac{1}{8}\right) = 28 \Rightarrow BC = \sqrt{28}.$$

(Method 2)

Using the fact that $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC} = \overrightarrow{AC} - \overrightarrow{AB}$ and $|\vec{a}|^2 = \vec{a} \cdot \vec{a}$, we get

$$BC = \left|\overrightarrow{BC}\right| = \sqrt{\overrightarrow{BC} \cdot \overrightarrow{BC}} = \sqrt{\left(\overrightarrow{AC} - \overrightarrow{AB}\right) \cdot \left(\overrightarrow{AC} - \overrightarrow{AB}\right)}$$

$$= \sqrt{\left(\overrightarrow{AC} \cdot \overrightarrow{AC}\right) - 2\left(\overrightarrow{AC} \cdot \overrightarrow{AB}\right) + \left(\overrightarrow{AB} \cdot \overrightarrow{AB}\right)} = \sqrt{\left|\overrightarrow{AC}\right|^2 - 2\left(\overrightarrow{AB} \cdot \overrightarrow{AC}\right) + \left|\overrightarrow{AB}\right|^2}$$

$$= \sqrt{4^2 - 2(2) + 4^2} = \sqrt{28}.$$

Problem 3

(a) $$\vec{a} \times \vec{b} = (\vec{\imath} + 3\vec{\jmath}) \times \left(-2\vec{\jmath} + 5\vec{k}\right) = -2(\vec{\imath} \times \vec{\jmath}) + 5\left(\vec{\imath} \times \vec{k}\right) - 6(\vec{\jmath} \times \vec{\jmath}) + 15\left(\vec{\jmath} \times \vec{k}\right)$$
$$= -2\vec{k} + 5(-\vec{\jmath}) - 6\left(\vec{0}\right) + 15\vec{\imath} = 15\vec{\imath} - 5\vec{\jmath} - 2\vec{k}.$$

(b) $$\vec{a} \times \vec{b} = \left(\vec{\imath} + \vec{\jmath} - 2\vec{k}\right) \times \left(-3\vec{\imath} + 2\vec{\jmath} + 5\vec{k}\right)$$
$$= -3(\vec{\imath} \times \vec{\imath}) + 2(\vec{\imath} \times \vec{\jmath}) + 5\left(\vec{\imath} \times \vec{k}\right) - 3(\vec{\jmath} \times \vec{\imath}) + 2(\vec{\jmath} \times \vec{\jmath}) + 5\left(\vec{\jmath} \times \vec{k}\right) + 6\left(\vec{k} \times \vec{\imath}\right)$$
$$- 4\left(\vec{k} \times \vec{\jmath}\right) - 10\left(\vec{k} \times \vec{k}\right)$$
$$= -3\left(\vec{0}\right) + 2\vec{k} + 5(-\vec{\jmath}) - 3\left(-\vec{k}\right) + 2\left(\vec{0}\right) + 5\vec{\imath} + 6\vec{\jmath} - 4(-\vec{\imath}) - 10\left(\vec{0}\right) = 9\vec{\imath} + \vec{\jmath} + 5\vec{k}.$$

(c) $$\vec{a} \times \vec{b} = \left(-3\vec{\imath} + \vec{\jmath} + 3\vec{k}\right) \times \left(6\vec{\jmath} + \vec{k}\right)$$
$$= -18(\vec{\imath} \times \vec{\jmath}) - 3\left(\vec{\imath} \times \vec{k}\right) + 6(\vec{\jmath} \times \vec{\jmath}) + \left(\vec{\jmath} \times \vec{k}\right) + 18\left(\vec{k} \times \vec{\jmath}\right) + 3\left(\vec{k} \times \vec{k}\right)$$
$$= -18\vec{k} - 3(-\vec{\jmath}) + 6\left(\vec{0}\right) + \vec{\imath} + 18(-\vec{\imath}) + 3\left(\vec{0}\right) = -17\vec{\imath} + 3\vec{\jmath} - 18\vec{k}.$$

(d) $$\vec{a} \times \vec{b} = \left(\vec{\jmath} + \vec{k}\right) \times \left(3\vec{\imath} - \vec{\jmath} + 2\vec{k}\right)$$
$$= 3(\vec{\jmath} \times \vec{\imath}) - (\vec{\jmath} \times \vec{\jmath}) + 2\left(\vec{\jmath} \times \vec{k}\right) + 3\left(\vec{k} \times \vec{\imath}\right) - \left(\vec{k} \times \vec{\jmath}\right) + 2\left(\vec{k} \times \vec{k}\right)$$
$$= 3\left(-\vec{k}\right) - \left(\vec{0}\right) + 2\vec{\imath} + 3\vec{\jmath} - (-\vec{\imath}) + 2\left(\vec{0}\right) = 3\vec{\imath} + 3\vec{\jmath} + 3\vec{k}.$$

Problem 4

(a) We first note that $\begin{cases} \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 2\vec{\imath} - 3\vec{\jmath} - 2\vec{k} \\ \overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = -3\vec{\imath} - 2\vec{\jmath} + \vec{k} \end{cases}$. According to the definition of vector product, the vector $\overrightarrow{AB} \times \overrightarrow{AC}$ is perpendicular to both $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Thus the required vector is found to be

$$\overrightarrow{AB} \times \overrightarrow{AC} = (2\vec{\imath} - 3\vec{\jmath} - 2\vec{k}) \times (-3\vec{\imath} - 2\vec{\jmath} + \vec{k})$$

$$= -6(\vec{\imath} \times \vec{\imath}) - 4(\vec{\imath} \times \vec{\jmath}) + 2(\vec{\imath} \times \vec{k}) + 9(\vec{\jmath} \times \vec{\imath}) + 6(\vec{\jmath} \times \vec{\jmath}) - 3(\vec{\jmath} \times \vec{k}) + 6(\vec{k} \times \vec{\imath}) + 4(\vec{k} \times \vec{\jmath}) - 2(\vec{k} \times \vec{k})$$
$$= -6(\vec{0}) - 4(\vec{k}) + 2(-\vec{\jmath}) + 9(-\vec{k}) + 6(\vec{0}) - 3(\vec{\imath}) + 6(\vec{\jmath}) + 4(-\vec{\imath}) - 2(\vec{0})$$
$$= -7\vec{\imath} + 4\vec{\jmath} - 13\vec{k}.$$

(b) Note that $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = -5\vec{\imath} + \vec{\jmath} + 3\vec{k}$, the required vector is given by

$$\vec{a} = |\overrightarrow{BC}| \times (\widehat{AB \times AC}) = \sqrt{35} \times \frac{\overrightarrow{AB} \times \overrightarrow{AC}}{|\overrightarrow{AB} \times \overrightarrow{AC}|} = \sqrt{35} \times \frac{-7\vec{\imath} + 4\vec{\jmath} - 13\vec{k}}{\sqrt{234}}$$
$$= -\frac{7\sqrt{35}}{\sqrt{234}}\vec{\imath} + \frac{4\sqrt{35}}{\sqrt{234}}\vec{\jmath} - \frac{13\sqrt{35}}{\sqrt{234}}\vec{k}.$$

(c) For any point $X = (x, y, z)$ in the plane, the vector $\overrightarrow{AX}$ lies on the same plane and its perpendicular to the vector $\overrightarrow{AB} \times \overrightarrow{AC}$. Note that $\overrightarrow{AX} = \overrightarrow{OX} - \overrightarrow{OA} = (x-1)\vec{\imath} + (y-2)\vec{\jmath} + z\vec{k}$, then we have

$$\overrightarrow{AX} \cdot (\overrightarrow{AB} \times \overrightarrow{AC}) = 0$$
$$\Rightarrow -7(x-1) + 4(y-2) - 13(z) = 0$$
$$\Rightarrow 7x - 4y + 13z = -1.$$

Thus the equation of plane is $7x - 4y + 13z = -1$.

Problem 5

(a) Since

$$|\vec{a} \times \vec{b}| = |(\vec{\imath} - 2\vec{\jmath}) \times (2\vec{\imath} + \vec{\jmath})| = |5\vec{k}| = 5 \neq 0,$$

thus $\vec{a}$ and $\vec{b}$ are not collinear and these two vectors are linearly independent.

(b) Note that

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (-17\vec{\imath} + 9\vec{\jmath} - 11\vec{k}) = (\vec{\imath} - 2\vec{\jmath} + 3\vec{k}) \cdot (-17\vec{\imath} + 9\vec{\jmath} - 11\vec{k})$$
$$= (1)(-17) - 2(9) + (3)(-11) = -68 \neq 0.$$

The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are not coplanar, these three vectors are linearly independent.

(c) Note that

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{a} \cdot (-4\vec{\imath} - 8\vec{\jmath} - 4\vec{k}) = (\vec{\imath} + 2\vec{\jmath} - 5\vec{k}) \cdot (-4\vec{\imath} - 8\vec{\jmath} - 4\vec{k})$$
$$= (1)(-4) + 2(-8) + (-5)(-4) = 0.$$

The vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are coplanar, these three vectors are linearly dependent.

Problem 6

$$|z_1 + z_2|^2 - |z_1 - z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)} - (z_1 - z_2)\overline{(z_1 - z_2)}$$
$$= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) - (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$
$$= z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 + z_2\bar{z}_2 - z_1\bar{z}_1 + z_1\bar{z}_2 + z_2\bar{z}_1 - z_2\bar{z}_2 = 2(z_1\bar{z}_2 + z_2\bar{z}_1)$$
$$= 2(z_1\bar{z}_2 + \overline{z_1\bar{z}_2}) = 4Re(z_1\bar{z}_2).$$

Remark: The last equality follows from the fact that $z + \bar{z} = 2Re(z)$.

Problem 7

(a)
$$z^6 = -3+\sqrt{3}i \Rightarrow z = \sqrt[6]{-3+\sqrt{3}i} = \sqrt[6]{\sqrt{12}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)}$$
$$= 12^{\frac{1}{12}}\left(\cos\frac{2k\pi+\frac{5\pi}{6}}{6} + i\sin\frac{2k\pi+\frac{5\pi}{6}}{6}\right)$$
$$= 12^{\frac{1}{12}}\left(\cos\left(\frac{k\pi}{3}+\frac{5\pi}{36}\right) + i\sin\left(\frac{k\pi}{3}+\frac{5\pi}{36}\right)\right),\ \ for\ \ k = 0,1,2,\dots,5.$$

(b)
$$(1-z)^7 + (1+z)^7 = 0 \Rightarrow \left(\frac{1-z}{1+z}\right)^7 = -1 \Rightarrow \frac{1-z}{1+z} = (\cos\pi + i\sin\pi)^{\frac{1}{7}}$$
$$\Rightarrow \frac{1-z}{1+z} = \underbrace{\cos\frac{2k\pi+\pi}{7} + i\sin\frac{2k\pi+\pi}{7}}_{\omega_k},\ \ k = 0,1,2,\dots,6.$$
$$\Rightarrow z = \frac{1-\omega_k}{1+\omega_k}$$

(c)
$$z^{10} - 5z^5 - 6 = 0 \Rightarrow (z^5-6)(z^5+1) = 0$$
$$\Rightarrow z^5 = 6\ \ or\ \ z^5 = -1$$
$$\Rightarrow z = \sqrt[5]{6(\cos 0 + i\sin 0)} = 6^{\frac{1}{5}}\left(\cos\frac{2k\pi}{5} + i\sin\frac{2k\pi}{5}\right)\ \ or$$
$$z = \sqrt[5]{(\cos\pi + i\sin\pi)} = \cos\frac{2k\pi+\pi}{5} + i\sin\frac{2k\pi+\pi}{5}$$
where $k = 0,1,2,3,4$.

Problem 8

(a) We first note that
$$(\cos\theta + i\sin\theta)^5 = \cos 5\theta + i\sin 5\theta \dots (1)$$
On the other hand, one cause Binomial theorem and obtain
$$(\cos\theta + i\sin\theta)^5$$
$$= \cos^5\theta + 5\cos^4\theta\sin\theta\, i + 10\cos^3\theta\sin^2\theta\, i^2 + 10\cos^2\theta\sin^3\theta\, i^3 + 5\cos\theta\sin^4\theta\, i^4 + \sin^5\theta\, i^5$$
$$= (\cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta) + i(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta) \dots (2).$$
By comparing the *real part* between the equations (1) and (2), we get
$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$
$$= \cos^5\theta - 10\cos^3\theta\,(1-\cos^2\theta) + 5\cos\theta\,(1-\cos^2\theta)^2$$
$$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta.$$

(b) We shall consider the expression $(\cos\theta + i\sin\theta)^3$. We first note that
$$(\cos\theta + i\sin\theta)^3 = \cos 3\theta + i\sin 3\theta \dots (1)$$
On the other hand, one cause Binomial theorem and obtain
$$(\cos\theta + i\sin\theta)^3$$
$$= \cos^3\theta + 3\cos^2\theta\sin\theta\, i + 3\cos\theta\sin^2\theta\, i^2 + \sin^3\theta\, i^3$$
$$= (\cos^3\theta - 3\cos\theta\sin^2\theta) + i(3\cos^2\theta\sin\theta - \sin^3\theta) \dots (2).$$
By comparing the *imaginary part* between the equation (1) and (2), we obtain
$$\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta = 3(1-\sin^2\theta)\sin\theta - \sin^3\theta = 3\sin\theta - 4\sin^3\theta.$$

Problem 9

(a)

$$\left(\begin{array}{ccc|c} 1 & -1 & 3 & 15 \\ -3 & 2 & 1 & 4 \\ 2 & -3 & 2 & 9 \end{array}\right) \overset{\substack{R_2+3R_1 \\ R_3-2R_1}}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & 3 & 15 \\ 0 & -1 & 10 & 49 \\ 0 & -1 & -4 & -21 \end{array}\right) \overset{R_3-R_2}{\sim} \left(\begin{array}{ccc|c} 1 & -1 & 3 & 15 \\ 0 & -1 & 10 & 49 \\ 0 & 0 & -14 & -70 \end{array}\right).$$

Since there is no column with no pivot, the system has unique solution.

The system can be expressed as $\begin{cases} x - y + 3z = 15 \\ -y + 10z = 49 \\ -14z = -70 \end{cases}$. Solving the equations backwards, we obtain $z = 5,\ y = 1,\ x = 1$.

(b)

$$\left(\begin{array}{ccc|c} 2 & 1 & -3 & 12 \\ 4 & 0 & 1 & 5 \\ 3 & -1 & 2 & 1 \end{array}\right) \overset{R_1 \div 2}{\sim} \left(\begin{array}{ccc|c} 1 & 1/2 & -3/2 & 6 \\ 4 & 0 & 1 & 5 \\ 3 & -1 & 2 & 1 \end{array}\right) \overset{\substack{R_2-4R_1 \\ R_3-3R_2}}{\sim} \left(\begin{array}{ccc|c} 1 & 1/2 & -3/2 & 6 \\ 0 & -2 & 7 & -19 \\ 0 & -5/2 & 13/2 & -17 \end{array}\right)$$

$$\overset{R_2 \div (-2)}{\sim} \left(\begin{array}{ccc|c} 1 & 1/2 & -3/2 & 6 \\ 0 & 1 & -7/2 & 19/2 \\ 0 & -5/2 & 13/2 & -17 \end{array}\right) \overset{R_2+\frac{5}{2}R_1}{\sim} \left(\begin{array}{ccc|c} 1 & 1/2 & -3/2 & 6 \\ 0 & 1 & -7/2 & 19/2 \\ 0 & 0 & 61/4 & -163/4 \end{array}\right).$$

Since there is no column with no pivot, the system has unique solution.

The system can be expressed as $\begin{cases} x + \frac{1}{2}y - \frac{3}{2}z = 6 \\ y - \frac{7}{2}z = \frac{19}{2} \\ \frac{61}{4}z = -\frac{163}{4} \end{cases}$. Solving the equations backwards, we obtain $z = -3,\ y = -1,\ x = 2$.

Problem 10

$$\left(\begin{array}{ccc|c} 2 & 1 & -b & 3 \\ 0 & a & -1 & 2 \\ -2 & 5 & 0 & 1 \end{array}\right) \overset{R_3+R_1}{\sim} \left(\begin{array}{ccc|c} 2 & 1 & -b & 3 \\ 0 & a & -1 & 2 \\ 0 & 6 & -b & 4 \end{array}\right) \overset{R_3 \leftrightarrow R_2}{\sim} \left(\begin{array}{ccc|c} 2 & 1 & -b & 3 \\ 0 & 6 & -b & 4 \\ 0 & a & -1 & 2 \end{array}\right)$$

$$\overset{R_2 \div 6}{\sim} \left(\begin{array}{ccc|c} 2 & 1 & -b & 3 \\ 0 & 1 & -b/6 & 2/3 \\ 0 & a & -1 & 2 \end{array}\right) \overset{R_3-aR_2}{\sim} \left(\begin{array}{ccc|c} 2 & 1 & -b & 3 \\ 0 & 1 & -b/6 & 2/3 \\ 0 & 0 & ab/6-1 & 2-2a/3 \end{array}\right)$$

(a) The system has unique solution if there is no column with no pivot. This happens when $\frac{ab}{6} - 1 \neq 0 \Rightarrow ab \neq 6$.

(b) The system has infinitely many solutions if there is column with no pivot and there is no row with $(0,0,0|b), b \neq 0$. This happens when $\begin{cases} \frac{ab}{6} - 1 = 0 \\ 2 - \frac{2a}{3} = 0 \end{cases} \Rightarrow a = 3, b = 2$.

(c) The system has no solution if there is a row with $(0,0,0|b), b \neq 0$. This happens when $\begin{cases} \frac{ab}{6} - 1 = 0 \\ 2 - \frac{2a}{3} \neq 0 \end{cases} \Rightarrow ab = 6,\ a \neq 3$.

Problem 11

(a)

$$\left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ -3 & 4 & 0 & 1 \end{array}\right) \overset{R_2+3R_1}{\approx} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 10 & 3 & 1 \end{array}\right) \overset{R_2\div 10}{\approx} \left(\begin{array}{cc|cc} 1 & 2 & 1 & 0 \\ 0 & 1 & 3/10 & 1/10 \end{array}\right)$$

$$\overset{R_1-2R_2}{\approx} \left(\begin{array}{cc|cc} 1 & 0 & 4/10 & -2/10 \\ 0 & 1 & 3/10 & 1/10 \end{array}\right)$$

Thus we conclude that $\begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{4}{10} & -\frac{2}{10} \\ \frac{3}{10} & \frac{1}{10} \end{pmatrix}$.

(b)

$$\left(\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ -6 & 4 & 0 & 1 \end{array}\right) \overset{R_2+2R_1}{\approx} \left(\begin{array}{cc|cc} 3 & 1 & 1 & 0 \\ 0 & 6 & 2 & 1 \end{array}\right) \overset{\substack{R_1\div 3 \\ R_2\div 6}}{\approx} \left(\begin{array}{cc|cc} 1 & 1/3 & 1/3 & 0 \\ 0 & 1 & 1/3 & 1/6 \end{array}\right)$$

$$\overset{R_1-\frac{1}{3}R_2}{\approx} \left(\begin{array}{cc|cc} 1 & 0 & 2/9 & -1/18 \\ 0 & 1 & 1/3 & 1/6 \end{array}\right)$$

Thus we deduce that $\begin{pmatrix} 3 & 1 \\ -6 & 4 \end{pmatrix}^{-1} = \begin{pmatrix} \frac{2}{9} & -\frac{1}{18} \\ \frac{1}{3} & \frac{1}{6} \end{pmatrix}$.

(c)

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 4 & 2 & 1 & 0 & 0 & 1 \end{array}\right) \overset{\substack{R_2-3R_1 \\ R_3-4R_1}}{\approx} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -3 & -3 & 1 & 0 \\ 0 & -2 & -7 & -4 & 0 & 1 \end{array}\right)$$

$$\overset{R_3-2R_2}{\approx} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & -1 & -3 & -3 & 1 & 0 \\ 0 & 0 & -1 & 2 & -2 & 1 \end{array}\right) \overset{\substack{R_2\div(-1) \\ R_3\div(-1)}}{\approx} \left(\begin{array}{ccc|ccc} 1 & 1 & 2 & 1 & 0 & 0 \\ 0 & 1 & 3 & 3 & -1 & 0 \\ 0 & 0 & 1 & -2 & 2 & -1 \end{array}\right)$$

$$\overset{\substack{R_1-2R_3 \\ R_2-3R_3}}{\approx} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 5 & -4 & 2 \\ 0 & 1 & 0 & 9 & -7 & 3 \\ 0 & 0 & 1 & -2 & 2 & -1 \end{array}\right) \overset{R_1-R_2}{\approx} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & 3 & -1 \\ 0 & 1 & 0 & 9 & -7 & 3 \\ 0 & 0 & 1 & -2 & 2 & -1 \end{array}\right)$$

Thus we conclude that $\begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 3 \\ 4 & 2 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} -4 & 3 & -1 \\ 9 & -7 & 3 \\ -2 & 2 & -1 \end{pmatrix}$

(d)

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 5 & 5 & 1 & 0 & 0 & 1 \end{array}\right) \overset{R_3-5R_1}{\approx} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 3 & 0 & 1 & 0 \\ 0 & 0 & -4 & -5 & 0 & 1 \end{array}\right)$$

$$\overset{\substack{R_2\div 2 \\ R_3\div(-4)}}{\approx} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 3/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1 & 5/4 & 0 & -1/4 \end{array}\right) \overset{\substack{R_1-R_3 \\ R_2-\frac{3}{2}R_3}}{\approx} \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & -1/4 & 0 & 1/4 \\ 0 & 1 & 0 & -15/8 & 1/2 & 3/8 \\ 0 & 0 & 1 & 5/4 & 0 & -1/4 \end{array}\right)$$

$$\overset{R_1-R_2}{\approx} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 13/8 & -1/2 & -1/8 \\ 0 & 1 & 0 & -15/8 & 1/2 & 3/8 \\ 0 & 0 & 1 & 5/4 & 0 & -1/4 \end{array}\right).$$

Thus we conclude that $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 13/8 & -1/2 & -1/8 \\ -15/8 & 1/2 & 3/8 \\ 5/4 & 0 & -1/4 \end{pmatrix}$.