Tutorial 8 (Chapter 8 and 9)

1. The shopping times of $n = 64$ randomly selected customers at a local supermarket were recorded. The average and variance of the 64 shopping times were 33 minutes and 256 minutes^2, respectively. Estimate μ, the true average shopping time per customer, with a confidence coefficient of $1 - \alpha = 0.9$. (Use large sample CI.)

   Solution $\bar{y} \pm z_{0.05}\frac{s}{\sqrt{n}} = 33 \pm 1.645\frac{16}{8} = (29.71, 36.29)$.

2. Is America's romance with movies on the wane? In a Poll of $n = 800$ randomly chosen adults, 45% indicated that movies were getting better whereas 43% indicated that movies were getting worse. Find a 98% confidence interval for p, the overall proportion of adults who say that movies are getting better. Does the interval include the value $p = 0.5$? Do you think that a majority of adults say that movies are getting better?

   Solution With $z_{0.01} = 2.326$, the 98% CI is $0.45 \pm 2.326\sqrt{\frac{0.45(0.55)}{800}}$ or 0.45 ± 0.041. Since the value 0.5 is not contained in the interval, there is not compelling evidence that a majority of adults feel that movies are getting better.

3. The reaction of an individual to a stimulus in a psychological experiment may take one of two forms, A or B. If an experimenter wishes to estimate the probability p that a person will react in manner A, how many people must be included in the experiment? Assume that the experimenter will be satisfied if the error of estimation is less than 0.04 with probability equal to 0.9 (in other words, the 0.9 CI is required to have length less than 0.08). Assume also that he expects p to lie somewhere in the neighborhood of 0.6.

   Solution The half length of the CI is $1.645\sqrt{\frac{p(1-p)}{n}}$ and thus we require that $1.645\sqrt{\frac{0.6(0.4)}{n}} \leq 0.04$. This implies $n \geq 406$.

4. A manufacturer of gunpowder has developed a new powder, which was tested in eight shells. The resulting muzzle velocities, in feet per second, were as follows: 3005 2925 2935 2965 2995 3005 2937 2905. Find a 95% confidence interval for the true average velocity μ for shells of this type. Assume that muzzle velocities are approximately normally distributed (small sample CI).

   Solution $\bar{Y} \pm t_{7,0.025}\frac{S}{\sqrt{n}} = 2959 \pm 2.365\frac{39.1}{\sqrt{8}} = 2959 \pm 32.7$.

5. Suppose that a political candidate, Jones, claims that he will gain more than 50% of the votes in a city election and thereby emerge as the winner. $n = 15$ voters were sampled. We wish to test $H_0 : p = 0.5$ against the alternative, $H_a : p < 0.5$. The test statistic is Y, the number of sampled voters favoring Jones. Calculate α if we select $RR = \{y \leq 2\}$ as the rejection region. Suppose that he will actually receive 30% of the votes ($p = 0.3$). What is the probability β that the sample will erroneously lead us to conclude that H_0 is true and that Jones is going to win?

   Solution $\alpha = P(\text{ type I error }) = P(Y \leq 2 \text{ when } p = 0.5)$. Thus $\alpha = \sum_{i=0}^{2}\binom{15}{i}0.5^i 0.5^{15-i} = 0.04$. $\beta = P(Y > 2 \text{ when } p = 0.3) = \sum_{i=3}^{15}\binom{15}{i}0.3^i 0.7^{15-i} = 0.873$.

6. A vice president in charge of sales for a large corporation claims that salespeople are averaging no more than 15 sales contacts per week. As a check on his claim, $n = 36$ salespeople are selected at random, and the number of contacts made by each is recorded for a single randomly selected week. The mean and variance of the 36 measurements were 17 and 9, respectively. Does the evidence *contradict* the vice president's claim? Use a test with level $\alpha = 0.05$ by calculating the rejection region (use large sample test).

   Solution Test is $H_0 : \mu = 15$, $H_a : \mu > 15$. The test statistic is $Z = \frac{\bar{Y} - \mu_0}{s/\sqrt{n}}$ with observed value $\frac{17-15}{3/\sqrt{36}} = 4$. The RR is $Z > z_{0.05} = 1.645$. Thus, at the $\alpha = 0.05$ level of significance, the evidence is sufficient to indicate that the vice president's claim is incorrect and that the average number of sales contacts per week exceeds 15.

7. An example above gives muzzle velocities of eight shells tested with a new gunpowder, along with the sample mean and sample standard deviation, $\bar{y} = 2959$ and $s = 39.1$. The manufacturer claims that the new gunpowder produces an average velocity of not less than 3000 feet per second. Do the sample data provide sufficient evidence to contradict the manufacturer's claim at the 0.025 level of significance? (Use rejection rejection and small sample test.)

   Solution We want to test $H_0 : \mu = 3000$ versus the alternative, $H_a : \mu < 3000$. The rejection region is given by $t < t_{7,0.025} = -2.365$. The observed value of the test statistic is $t = \frac{\bar{y}-\mu_0}{s/\sqrt{n}} = \frac{2959-3000}{39.1/\sqrt{8}} = -2.966$. We conclude that sufficient evidence exists to contradict the manufacturer's claim and that the true mean muzzle velocity is less than 3000 feet per second at the 0.025 level of significance.

8. What is the p-value associated with the statistical test in the previous example?

   Solution p-value $= P(T < -2.966) = 0.01046$. (If we use statistical tables we probably will only get $0.01 \leq p - value \leq 0.025$.)