MA1300 Self Practice # 13

1. (P724, #25, 30) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$\text{(a)}.\ a_n = \frac{3+5n^2}{n+n^2},$$
$$\text{(b)}.\ a_n = \sqrt{\frac{n+1}{9n+1}}.$$

2. (P725, #80) A sequence $\{a_n\}$ is given by $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2+a_n}$.

(a) By mathematical induction, show that $\{a_n\}$ is increasing and bounded above by 3. Apply the Monotonic Sequence Theorem to show that $\lim_{n\to\infty} a_n$ exists.

(b) Find $\lim_{n\to\infty} a_n$.

3. (P725, #82) Show that the sequence defined by

$$a_1 = 2, \qquad a_{n+1} = \frac{1}{3-a_n}$$

satisfies $0 < a_n \le 2$ and is decreasing. Deduce that the sequence is convergent and find its limit.

4. (P735, #23, 30, 40) Determine whether the series is convergent or divergent. If it is convergent, find its sum.

$$\text{(a)}.\ \sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{4^n},$$
$$\text{(b)}.\ \sum_{k=1}^{\infty} \frac{k(k+2)}{(k+3)^2},$$
$$\text{(c)}.\ \sum_{n=1}^{\infty} \left(\frac{3}{5^n} + \frac{2}{n}\right).$$

5. (P736, #64) We have seen that the harmonic series is a divergent series whose terms approach 0. Show that

$$\sum_{n=1}^{\infty} \ln\left(1+\frac{1}{n}\right)$$

is another series with this property.