Chapter 3

Newton's Laws of Motion

and Applications

Topics for chapter 3 Part 1

- The cause of motion: force
- The Newton's Laws of Motion
- Learn different types of forces
- Finding and analyzing the forces

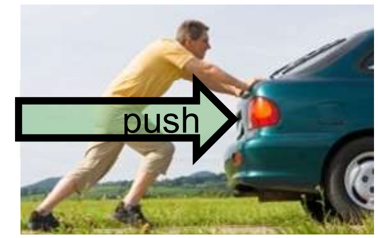
What causes motion?

- We studied motion in chapter 2
- What *causes* motion?
- Answer: Force causes change of motion, and thus causes motion
- They are Newton's laws of motion, which describe the relation between force and motion.
- Discovered by Isaac Newton in 1600s
- Newton formulated three laws from huge amounts of *experimental evidence.*

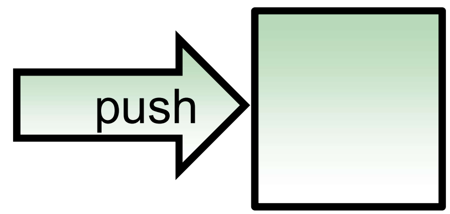
This man pushes car with a force

Characteristics of force

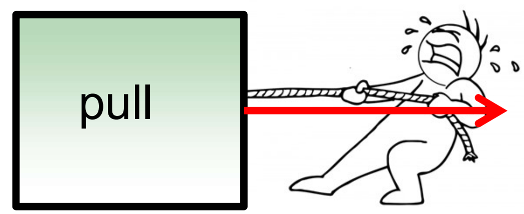
- Force causes change in the motion of an object (e.g. from not moving to moving)



- A force is an interaction between two objects, it involves two objects



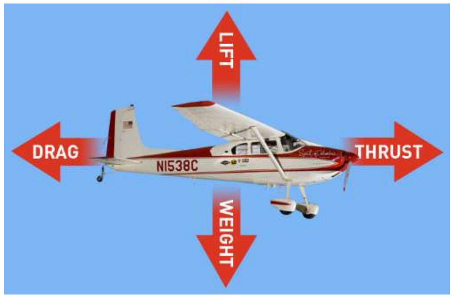
- It is a vector quantity, (has direction and magnitude)
- Examples of force: a push and a pull are forces



.

Newton's First Law

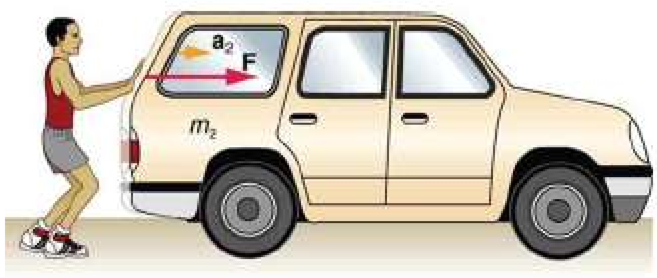
- Establish a qualitative relation between force and motion
- Simple version : *"Not under any force, a stationary object remains at rest, and a moving object stays in uniform motion." Uniform motion means constant velocity*
- Proper version "If there is no force or no net force on a body, the body will remain its state of motion: it moves with constant velocity or remain at rest"
- *No net force=all forces added up to zero= forces cancel each other*
- Mathematical version: $\sum \vec{F} = 0 \Longrightarrow \vec{v} = constant \; or \; \vec{a} = 0$



All the forces on the airplane cancel, so it moves with a constant velocity

Newton's Second Law: change of the state of motion

- If there is a non-zero net force on an object, it causes the object to accelerate (change in motion).
- Example, If you push a car with a force F, the car move faster (it has acceleration).



what is the mathematical relation between force and acceleration (quantitative)?
This is given in the second law

Relation between Mass, Force and acceleration

- From many experimental studies, 2nd law establishes the mathematical relation between force and acceleration
- The acceleration of an object is directly proportional to the net force (the resultant force) on the object. $\vec{a} \propto \sum \vec{F}$
- The object's acceleration is inversely proportional to the object's mass if the net force remains fixed. $\vec{a} \propto \frac{1}{m}$
- Combining the two relations above, we obtain $\vec{a} = \sum \vec{F}/m$

  or Newton's 2nd *Law*: $\boxed{\sum \vec{F} = m\vec{a}}$

- The SI unit for force is the newton (N) ; 1 N = 1 kg·m/s^2

What is mass?

- A physical quantity of an object tells you the amount of matter in the object.
- Mass is proportional to the amount of matter.
  More matter means larger in mass. (*the mass of 2 litres of coca cola is 2 times the mass of 1 litre*)
- 2^{nd} law → larger mass means larger resistance to change of motion, which means smaller acceleration under the same force
- In physics, mass is the characteristic of a body that resists the change of motion, a tendency to stay unchanged.

The amount of gold atoms in 1kg gold bar is double the number of gold atoms in 0.5kg bar

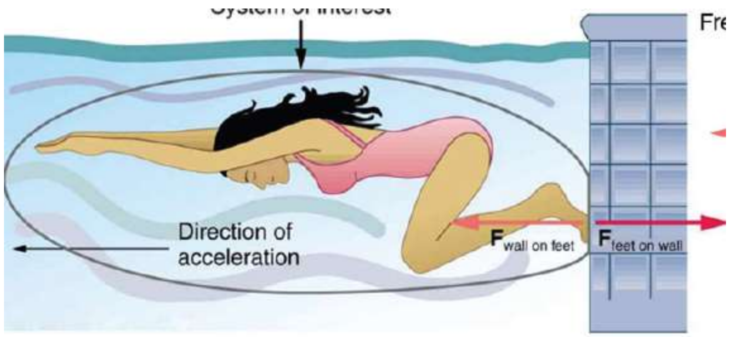
SI unit for mass is kilogram = kg

Newton's Third Law

- This law is about the relationship of the forces between two objects, which exert forces on each other.

- If you exert a force (action) on an object, the object always exerts a force (the "reaction") back upon you.

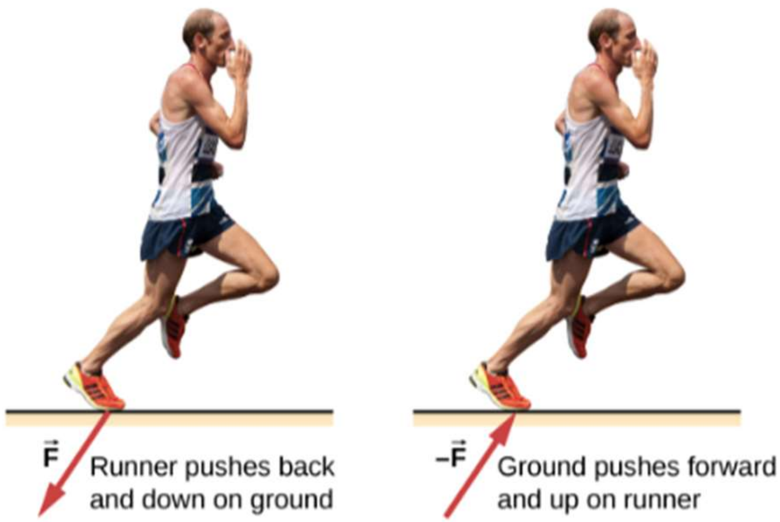
- It is called "the action-reaction pair."
  (see figure)

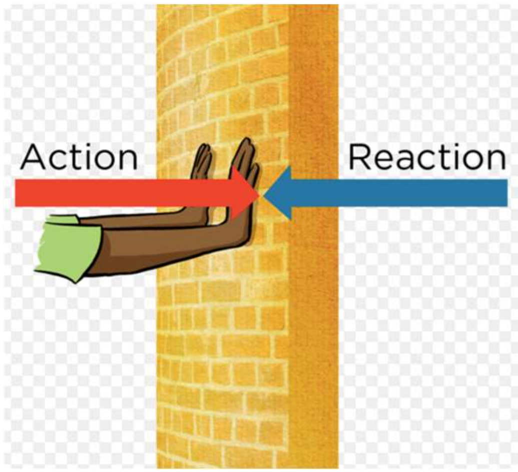


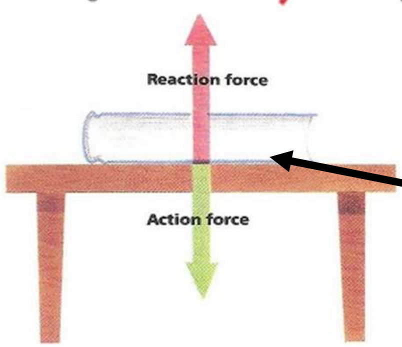
The swimmer is accelerated by the force from the wall on the swimmer. In the meantime, she also exerts a force on the wall

- A force and its reaction force have the *same magnitude but opposite directions*. These forces act on *different bodies*.

Example of action and reaction pair



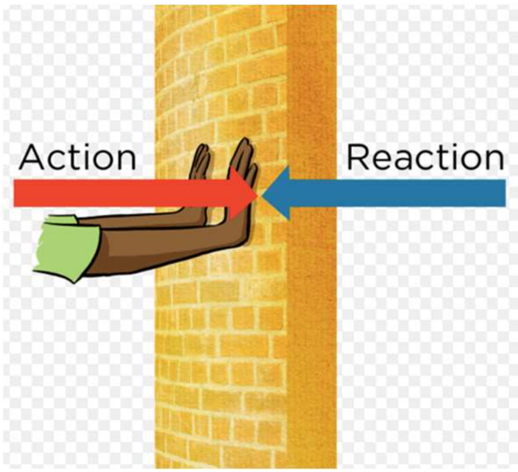




The weight of the book exerts a force on the table, the table give the book a normal force

Will action and reaction forces cancel each other?

$\vec{F}_{H\ on\ W}$

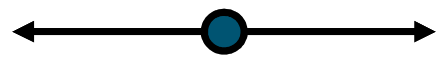


$\vec{F}_{W\ on\ H}$

Third law:

$$\vec{F}_{H \text{ on } W} = -\vec{F}_{W \text{ on } H}$$

$\vec{F}_{W\ on\ H}$



$\vec{F}_{H\ on\ W}$

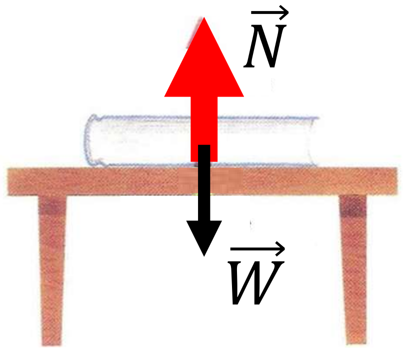
cancel each other?

$\vec{F}_{W\ on\ H}$ is the force on the hand (H)
but $\vec{F}_{H\ on\ W}$ is the force on the wall (W)

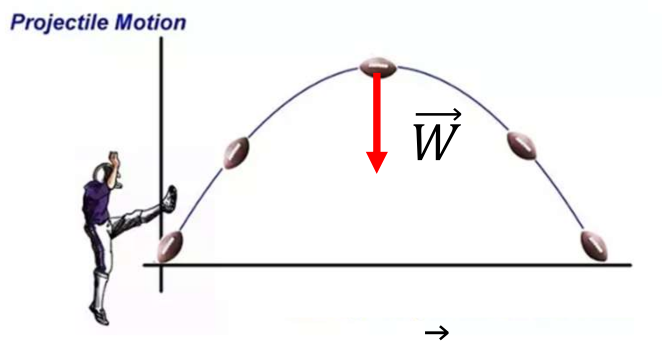
The two forces are on different objects, so they cannot cancel each other.

What the reaction force of weight (the gravity force on an object)?

Is the reaction force for weight $\vec{W}$ the normal force $\vec{N}$?

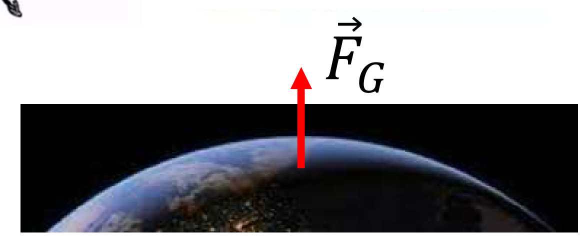


What if there is no the normal force $\vec{N}$ in the example of projectile motion?



reaction force of $\vec{W}$? Who gives $\vec{W}$?

Actually, there is a gravitational pulling force $\vec{F}_G$ from the football on earth! That is the reaction force of $\vec{W}$.



The use of Newton's Laws

- Newton's law tell you relation between force and acceleration
- If you find the net force, you can find acceleration and then find the motion (position vs time from acceleration)
- To find the net force, we add all the forces on the object
- So, we need to know how to find and identify the different types of forces in real life

Common forces found in real life 1

Non-contacting Forces

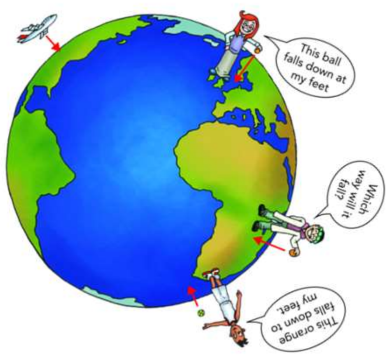
- *Weight*: The pull force of gravity on an object. The force is gravitational attraction from earth. This is a long-range force. No contact with the earth is needed.

$$\text{weight } \vec{W} = m\vec{g}$$

m *is mass of the object*
$\vec{g}$ *is gravitational acceleration*
(g = 9.8 m/s^2)

$\vec{W} = m\vec{g}$

Weight is a force on an object due to gravitational force



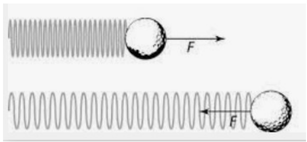
- *Electromagnetic Forces (later in PHY1202)*

Common forces found in real life 2

Contacting Forces

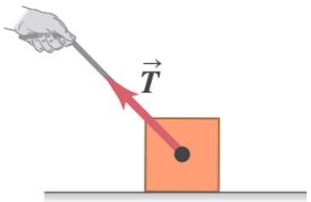
- *Spring force*: A restoring force.

Deformed spring will exert a force to restore the original



- *Tension force*: A pulling force exerted on an object by a rope or cord. This is a contact force.

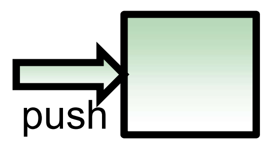
Tension in the rope cause a force on the box



.

Common Forces found in real life 3

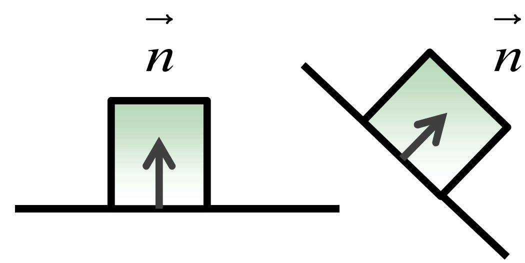
Push: You can exert a force on an object by pushing it. This is also a contact force. Push is exerted by another object by contact.



Common Forces found in real life 4

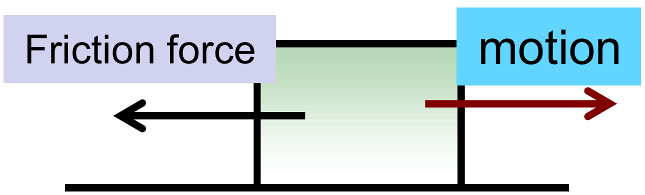
Surface Contacting Forces

- The *normal force*: When an object pushes against a surface, the surface pushes back on the object. There is force from the surface to the object which is perpendicular to the surface. This is a contact force.



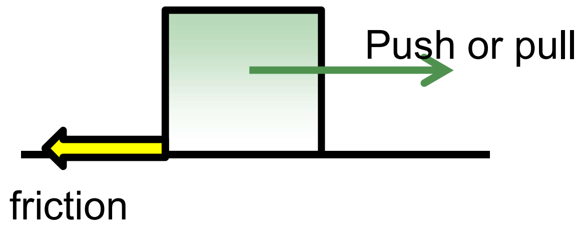
Normal force by the surface on the object, equal and opposite to the force from object on the surface, action-reaction pair

- *Friction force*: This force occurs between the rough surface and an object, when a surface resists sliding of an object. It is parallel to the surface. Friction is a contact force. (smooth surface can have negligible friction (zero friction))



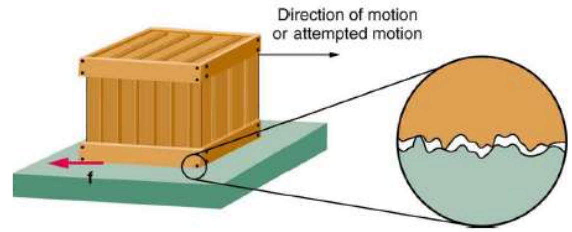
Frictional forces

- When two surfaces want to move relative to each other, a friction force exists between the two surfaces resisting this motion or tendency of motion



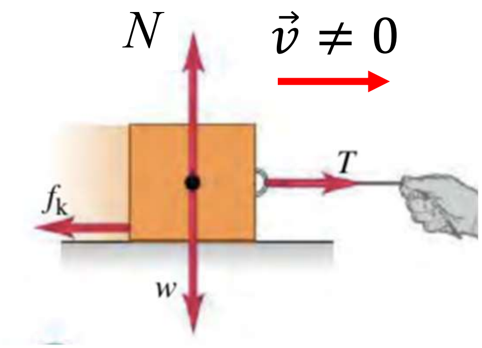
- *It* is parallel to the surfaces.

- Friction between two surfaces arises from interactions between the surfaces due to roughness of the surfaces – the teeth from the roughness try to block and prevent the relative motion.



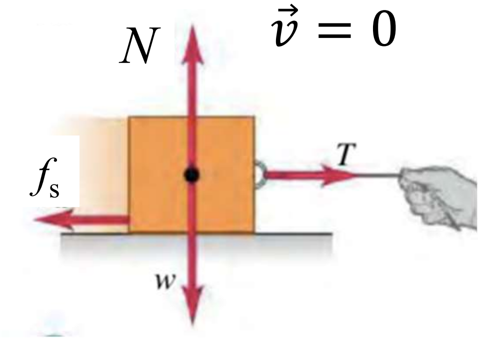
Two types of friction: kinetic and static friction

- *The friction is called Kinetic friction* when there is relative motion between the two surfaces



- The *kinetic friction force* is proportional to the normal force between the surface, the proportional constant is called coefficient of kinetic friction, $f_k = \mu_k N$. (N is the normal force)

- *Friction* exists even when there is no relative motion but a tendency of relative motion. It is called *static friction.*



- The *static friction force* can vary between zero and its maximum value: $f_s \leq f_{s,M} = \mu_s N$. ($N$ is the normal force)

Static friction

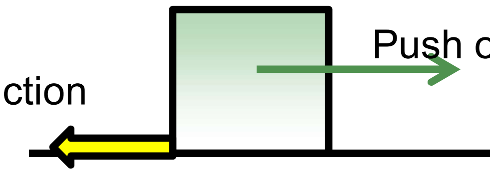
In the beginning, there is a gridlock,  the object does not move and the static friction always balances the force applied to move the object

$$f_{\text{static friction}} = F_{\text{applied force}}$$

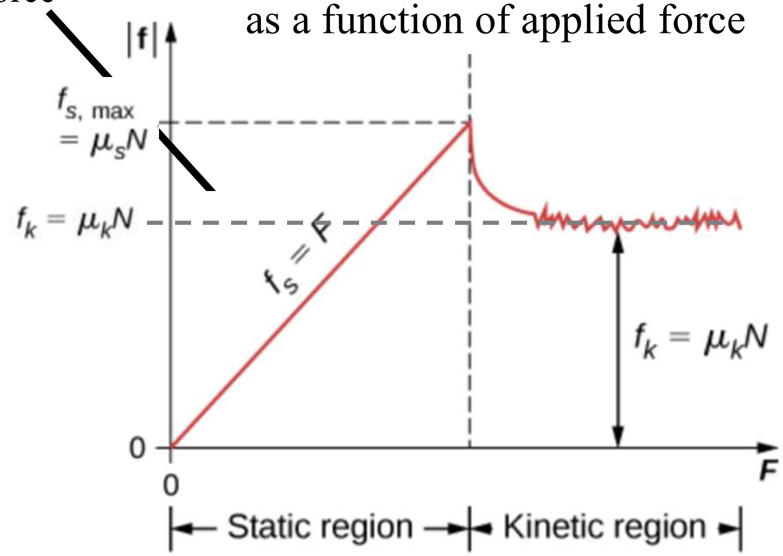
until the gridlock is broken and the maximum static friction = $\mu_s N$ is reached.

Once the gridlock is broken and the object is in motion, the motion makes it easier to break further and move forward. So, we have

$$f_k < f_{s,M} \quad or \quad \mu_k < \mu_s$$



Magnitude of frictional force $|f|$ as a function of applied force



Some coefficients of friction

- Give you a feeling about the values of the coefficients of different materials.
- No need to memorize

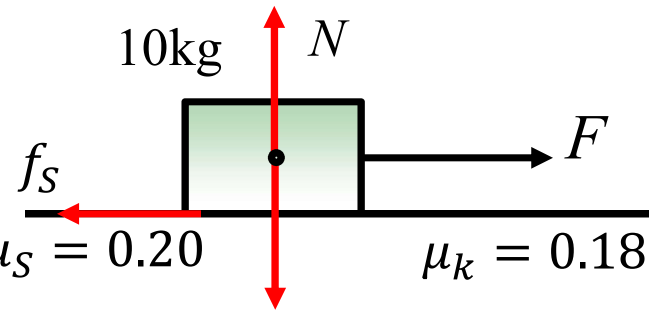
- This is why all tires are made of rubbers

Table 5.1 Coefficients of Static and Kinetic Friction

System	Static friction μ_s	Kinetic friction μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

Static vs kinetic friction

1. What is the friction force if $F = 10N$? Will the crate move?



$N = W = 98N \quad \rightarrow f_S^M = \mu_S N = 20N$

$F = 10N < f_S^M \quad \rightarrow$ No motion, $f_S = F = 10N$

2. What is the minimum F_m to make it move? $F_m = f_S^M = 20N$

3. Once moving, what is the acceleration if the force is kept at F_m?

$F_m - f_k = ma$

But $f_k = \mu_k N = 18N \quad \rightarrow a = \frac{F_m - f_k}{m} = \frac{20-18}{10} = 0.2\text{m/s}^2$

Application of Newton's Law

Newton's 2nd law is about forces and acceleration: $\vec{F}_{net} = m\vec{a}$.

- Usually, there is not just one force
  → force analysis

- Draw a free body (force) diagram of all the forces acting on each object.

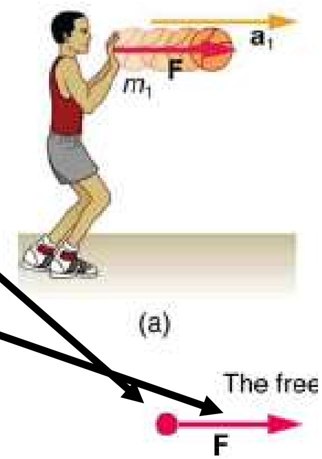
- Below we learn how to draw the free body (force) diagram

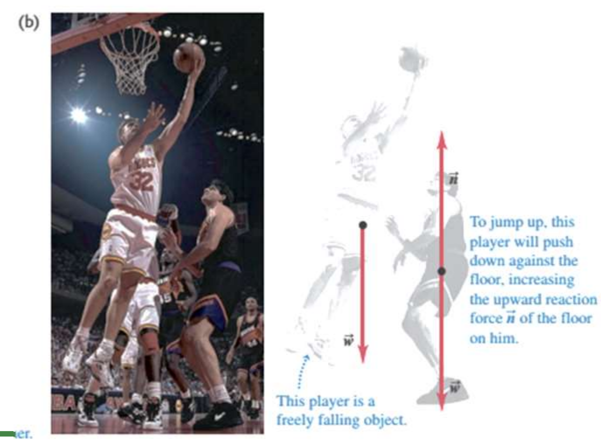
Drawing force vectors in force diagram

- Isolate the object (ball)

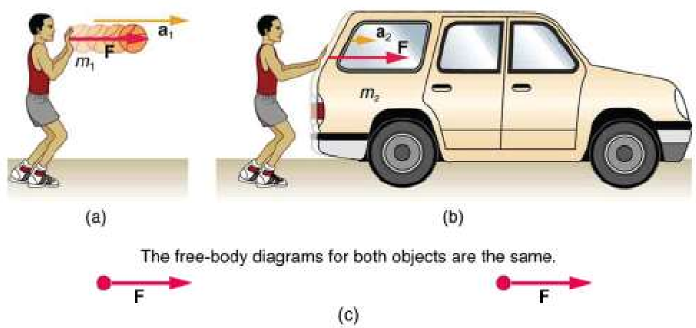
- Use a vector arrow to indicate the magnitude and direction of the forces in a free body diagram or force diagram.

- Draw all the forces in the problem. Then you have the force diagram which helps you to solve the problem

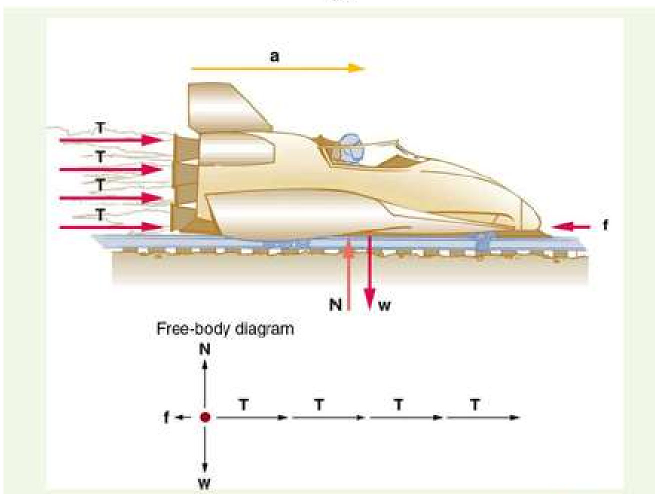




Example of free body (force)diagram



Man push a car



The jet engine pushes the gas out. The outgoing gas push back on the engine. For engine, it experience a thrust from the gas

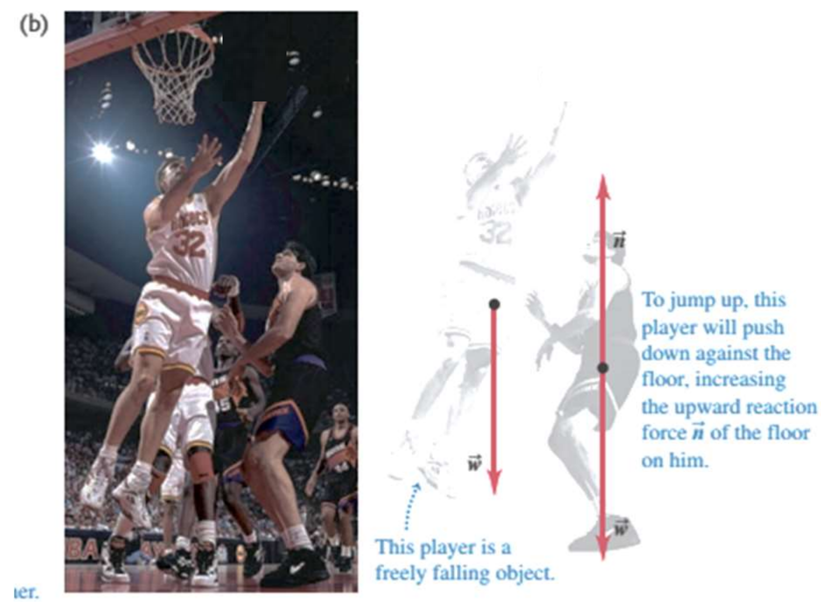
Finding all forces

- For non-contact forces, it is straightforward:

Gravity $\vec{W} = m\vec{g}$

- For contact forces, a rule of thumb is to look for

"contacts"

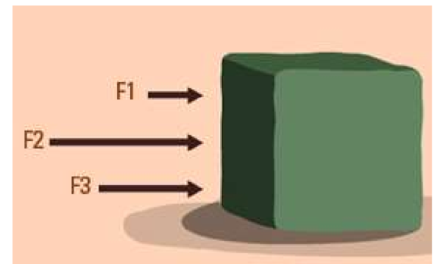


.

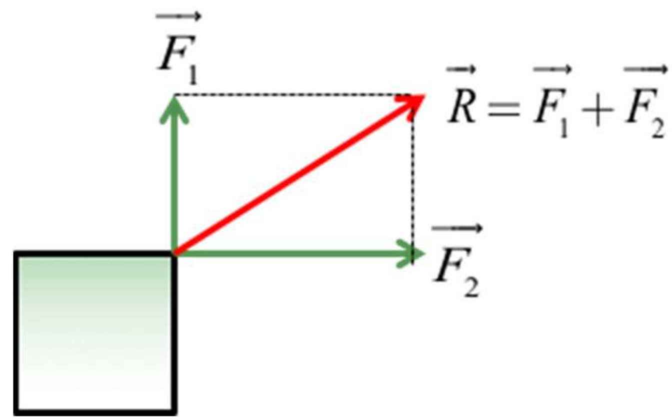
Adding forces together

Newton's 2nd law is about net force: $\vec{F}_{net} = m\vec{a}$.

When we have several forces acting on an object, what is the total effect of these forces on an object?



- The effect of the resultant force of all the forces on the object is the same as the total effect of all the forces on the object.
- The sum of the forces is the resultant force.
- $\vec{R}$ has the same effect of $\overrightarrow{F_1}$ and $\overrightarrow{F_2}$

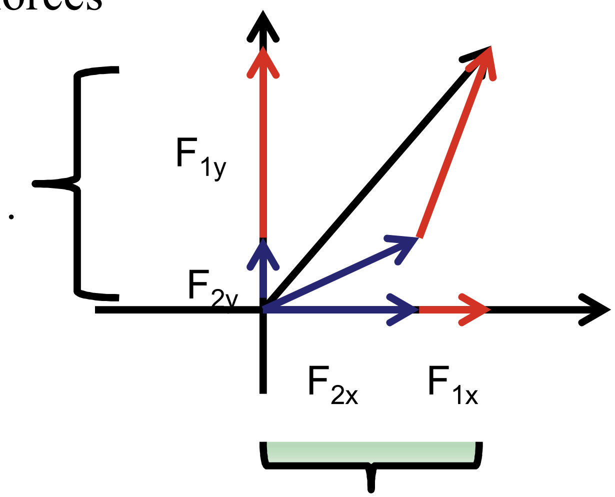


Decompose forces in a convenient coordinate system

- *After you draw the force diagram, choose a convenient coordinate system and find the components of all the forces along the x and y direction*
- *Then find the components of the resultant* force or the *net forces by adding* the components of all the forces

The y component of the resultant is the sum of the y components of all the forces

$$R_y = F_{1y} + F_{2y} + F_{3y} + \dots .$$



The x component of the resultant is the sum of the x components of all the forces

$$R_x = F_{1x} + F_{2x} + F_{3x} + \dots ,$$

.

Adding the forces using the components: more than two forces

- Find components of each force and add the components. Expression for more than two forces
- $\vec{R} = \overrightarrow{F_1} + \overrightarrow{F_2} + \overrightarrow{F_3} + \cdots = \sum \overrightarrow{F_i}$
- $R_x = F_{1x} + F_{2x} + F_{3x} + \ldots ,$ $\quad R_x = ma_x$
- $R_y = F_{1y} + F_{2y} + F_{3y} + \ldots .$ $\quad R_y = ma_y$

Consideration in choosing the convenient coordinate systems

- $F_{1x} + F_{2x} + F_{3x} + \ldots = ma_x$
- $F_{1y} + F_{2y} + F_{3y} + \ldots = ma_y$

Too many quantities and unknowns

- We can make some components zero by choosing a convenient coordinate system

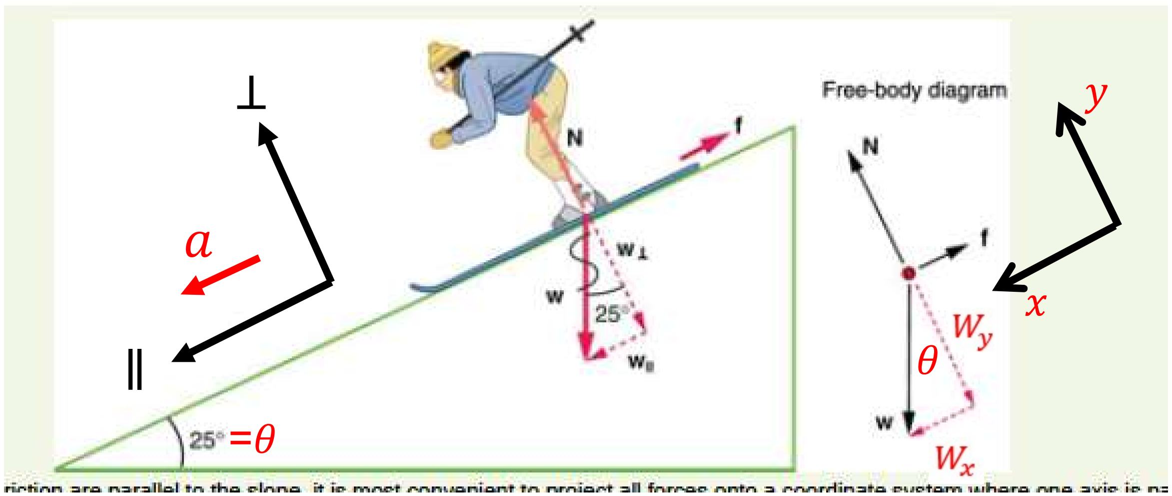
$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

choose $\theta = 0$ or 90^o

- We choose a coordinate system with one axis, say x, as the direction of motion. Then $a_y = 0$ → one less unknown

Another free body (force) diagram



$N_x = 0$ $\quad f_x = -f \quad W_x = W\sin\theta$

$N_y = N \quad f_y = 0 \quad W_y = -W\cos\theta$

$W = mg$

$x\text{: } mg\sin\theta - f = ma$

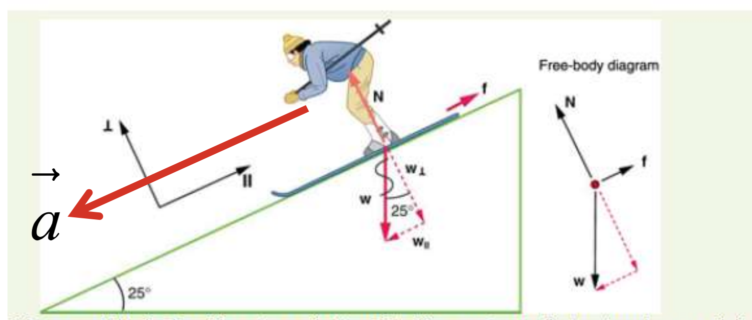
$y\text{: } N - mg\cos\theta = 0$

$f = \mu N$

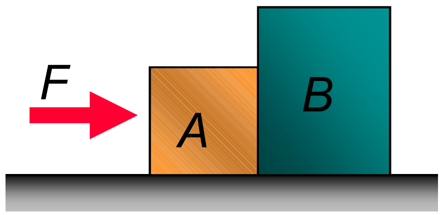
$$\boxed{a = g(\sin\theta - \mu\cos\theta)}$$

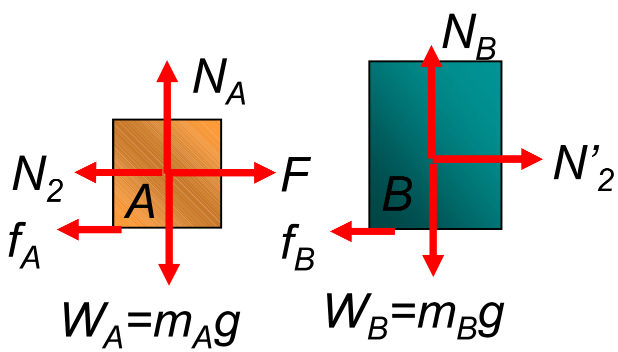
Two common free-body diagram errors

- The normal force must be perpendicular to the surface.
- Don’t mix up force and acceleration. There is no “$m\vec{a}$ force.” $\vec{a}$ is a vector showing the acceleration to remind us of the consequence of the net force



Application of Newton’s Law to a system of multiple objects





- Isolate the objects into individual one
- Then, draw a free body (force) diagram of all the forces acting on each object.
- Use the third law for action and reaction

$$\boxed{N_2 = N'_2}$$

- Write Newton’s equation for each object

Ay: $N_A - W_A = 0$

By: $N_B - W_B = 0$

Ax: $F - f_A - \boxed{N_2} = m_A a$

Bx: $\boxed{N'_2} - f_B = m_B a$

$f_A = \mu N_A \quad f_B = \mu N_B$

$$\boxed{a = F/(m_A + m_B) - \mu g}$$

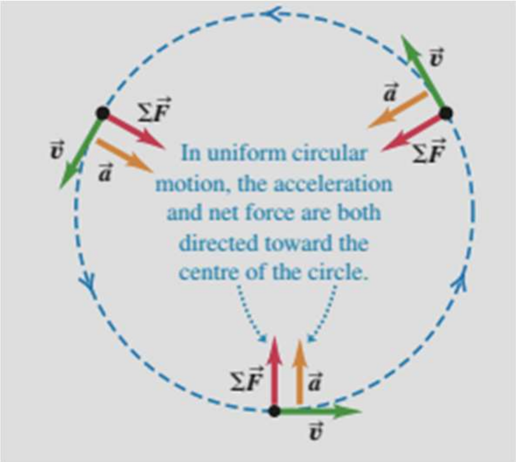
Summary of Steps for applying Newton's Law

- Find and draw the forces of the free body (force) diagram (looking for contacts)
- Set up coordinates according to convenience
- Decompose all the forces into components and find the resultant force (vector sum) of all the forces in terms of the components.
- A. If it is a static problem (the object is not moving, or with constant velocity), the resultant force should be zero, according to Newton's 1st Law
- B. If it is a dynamic problem (the object is moving), the resultant force equals mass time acceleration according to Newton's 2nd Law.
- Apply one of the conditions, A & B, to your problem to get some equations
- Solve the equations to find your required answers (unknown).

Dynamics of circular motion

Dynamics of circular motion

- In uniform circular motion, the object's acceleration (centripetal) is directed toward the center of the circle.

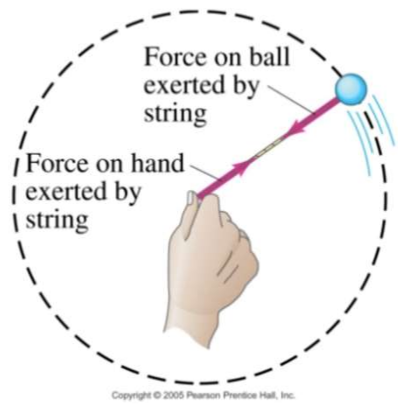


- By Newton's 2^{nd} law, a net force is required for this acceleration and the net force points towards the center of the circle

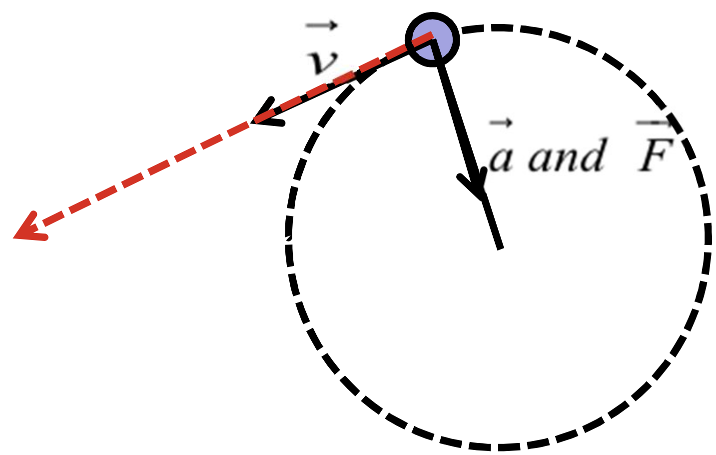
- Here, the force is the cause not the result.

What if the string breaks?

- The string provides a force, which causes the acceleration to keep the ball in circular motion



- If the string breaks, no net force acts on the ball, so it obeys Newton's first law and moves in a straight line (red dashed line with arrow).



Don’t use “centrifugal force”

- When you are in a rotating merry go-round, you think you experience a “centrifugal force”.
- If you do not hold the hand of your friend you will be thrown out from the merry go-round. It seems like a force that throws you out.
- In fact, there is no such a force called “centrifugal force”. It is “fictious”

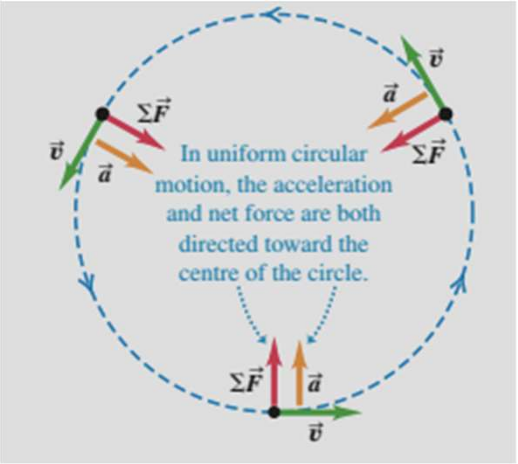
- If you do not hold tight, there is no centripetal force to keep you in the circular motion, then you appear to be thrown out from the merry-go-round.
- Actually, without the centripetal force, you keep your linear uniform motion, which is away from the merry-go-around

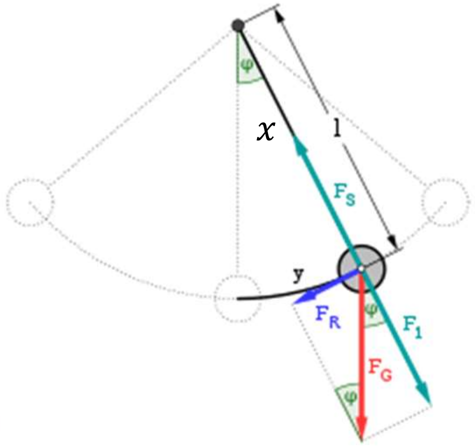
Working on circular motion

- The net force on the object is $F_{\text{net}} = m\, a_c$ with centripetal acceleration $a_c = v^2/R$
- To find F_{net}, we again need to do a free body diagram.
- We can use center direction as one of the axes (x-axis is along $\vec{a}_c$)

$$F_s - F_1 = ma_c = m\frac{v^2}{l}$$

$$F_s = m\frac{v^2}{l} + mg\cos\varphi$$





Chapter 3 part 2

Examples of applying Newton's Laws

Goals for part 2

- This part contains some examples in which the Newton's laws are applied
- They cover from easy to hard problems on one or many objects
- We work on circular motion

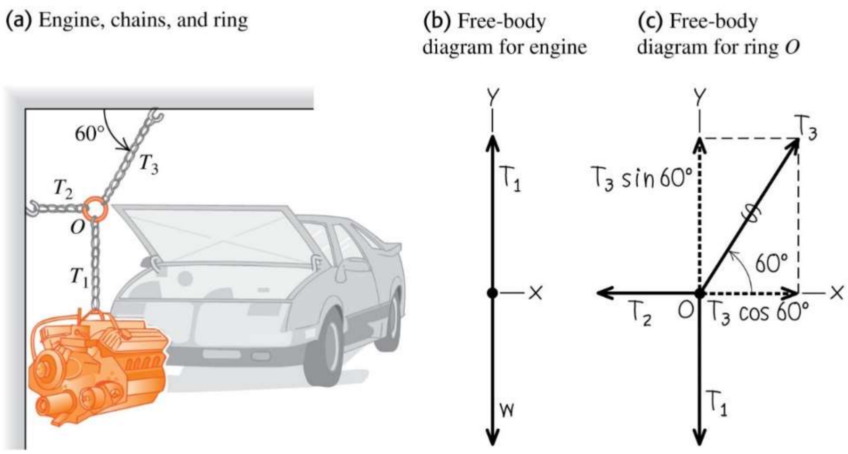
Summary of steps for problem solving

- Draw a free body diagram (force diagram)
- Add all the forces, best approach is to resolve the force into components and add all the components and find the resultant force
- Apply Newton's Law (first law for uniform motion, static equilibrium), (second law for constant acceleration) to the resultant force.
- This will give you some equations. Solve the equations for the unknowns.
- These steps are followed more or less in the examples below

Two-dimensional equilibrium

Example 5.3 Two-dimensional equilibrium

In Fig. 5.3a, a car engine with weight w hangs from a chain that is linked at ring O to two other chains, one fastened to the ceiling and the other to the wall. Find expressions for the tension in each of the three chains in terms of w. The weights of the ring and chains are negligible compared with the weight of the engine.



SOLUTION

IDENTIFY and SET UP: The target variables are the tension magnitudes T_1, T_2, and T_3 in the three chains (Fig. 5.3a). All the bodies are in equilibrium, so we'll use Newton's first law. We need three independent equations, one for each target variable. However, applying Newton's first law to just one body gives us only *two* equations, as in Eqs. (5.2). So we'll have to consider more than one body in equilibrium. We'll look at the engine (which is acted on by T_1) and the ring (which is acted on by all three chains and so is acted on by all three tensions).

Figures 5.3b and 5.3c show our free-body diagrams and choice of coordinate axes. There are two forces that act on the engine: its weight w and the upward force T_1 exerted by the vertical chain.

Three forces act on the ring: the tensions from the vertical chain (T_1), the horizontal chain (T_2), and the slanted chain (T_3). Because the vertical chain has negligible weight, it exerts forces of the same magnitude T_1 at both of its ends (see Example 5.1). (If the weight of this chain were not negligible, these two forces would have different magnitudes like the rope in Example 5.2.) The weight of the ring is also negligible, which is why it isn't included in Fig. 5.3c.

EXECUTE: The forces acting on the engine are along the y-axis only, so Newton's first law says

$$\text{Engine:} \quad \sum F_y = T_1 + (-w) = 0 \quad \text{and} \quad T_1 = w$$

The horizontal and slanted chains don't exert forces on the engine itself because they are not attached to it. These forces do appear when we apply Newton's first law to the ring, however. In the free-body diagram for the ring (Fig. 5.3c), remember that T_1, T_2, and T_3 are the *magnitudes* of the forces. We resolve the force with magnitude T_3 into its x- and y-components. The ring is in equilibrium, so using Newton's first law we can write (separate) equations stating that the x- and y-components of the net force on the ring are zero:

$$\text{Ring:} \quad \sum F_x = T_3 \cos 60^\circ + (-T_2) = 0$$
$$\text{Ring:} \quad \sum F_y = T_3 \sin 60^\circ + (-T_1) = 0$$

Because $T_1 = w$ (from the engine equation), we can rewrite the second ring equation as

$$T_3 = \frac{T_1}{\sin 60^\circ} = \frac{w}{\sin 60^\circ} = 1.2w$$

We can now use this result in the first ring equation:

$$T_2 = T_3 \cos 60^\circ = w\frac{\cos 60^\circ}{\sin 60^\circ} = 0.58w$$

EVALUATE: The chain attached to the ceiling exerts a force on the ring with a *vertical* component equal to T_1, which in turn is equal to w. But this force also has a horizontal component, so its magnitude T_3 is somewhat larger than w. This chain is under the greatest tension and is the one most susceptible to breaking.

To get enough equations to solve this problem, we had to consider not only the forces on the engine but also the forces acting on a second body (the ring connecting the chains). Situations like this are fairly common in equilibrium problems, so keep this technique in mind.

Two-body Problem

EXAMPLE 5.12 TWO BODIES WITH THE SAME MAGNITUDE OF ACCELERATION

SOLUTION

Figure 5.15a shows an air-track glider with mass m_1 moving on a level, frictionless air track in the physics lab. The glider is connected to a lab weight with mass m_2 by a light, flexible, non-stretching string that passes over a stationary, frictionless pulley. Find the acceleration of each body and the tension in the string.

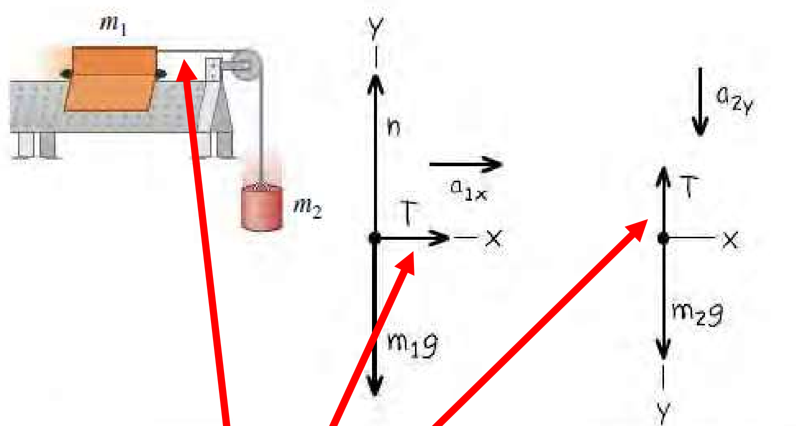
SOLUTION

IDENTIFY and SET UP: The glider and weight are accelerating, so again we must use Newton's second law. Our three target variables are the tension T in the string and the accelerations of the two bodies.

The two bodies move in different directions—one horizontal, one vertical—so we can't consider them to be a single unit as we did the bodies in Example 5.11. Figures 5.15b and 5.15c show our free-body diagrams and coordinate systems. It's convenient to have both bodies accelerate in the positive axis directions, so we chose the positive y-direction for the lab weight to be downward.

We consider the string to be massless and to slide over the pulley without friction, so the tension T in the string is the same throughout and it applies a force of the same magnitude T to each body. (You may want to review Conceptual Example 4.10, in which we discussed the tension force exerted by a massless rope.) The weights are m_1g and m_2g.

5.15 Our sketches for this problem.



Continued

Tension in a massless string is same everywhere

While the *directions* of the two accelerations are different, their *magnitudes* are the same. (That's because the string doesn't stretch, so the two bodies must move equal distances in equal times and their speeds at any instant must be equal. When the speeds change, they change at the same rate, so the accelerations of the two bodies must have the same magnitude a.) We can express this relationship as $a_{1x} = a_{2y} = a$, which means that we have only *two* target variables: a and the tension T.

What results do we expect? If $m_1 = 0$ (or, approximately, for m_1 much less than m_2) the lab weight will fall freely with acceleration g, and the tension in the string will be zero. For $m_2 = 0$ (or, approximately, for m_2 much less than m_1) we expect zero acceleration and zero tension.

EXECUTE: Newton's second law gives

$$\text{Glider:}\quad \sum F_x = T = m_1 a_{1x} = m_1 a$$

$$\text{Glider:}\quad \sum F_y = n + (-m_1 g) = m_1 a_{1y} = 0$$

$$\text{Lab weight:}\quad \sum F_y = m_2 g + (-T) = m_2 a_{2y} = m_2 a$$

(There are no forces on the lab weight in the x-direction.) In these equations we've used $a_{1y} = 0$ (the glider doesn't accelerate vertically) and $a_{1x} = a_{2y} = a$.

The x-equation for the glider and the equation for the lab weight give us two simultaneous equations for T and a:

$$\text{Glider:}\quad T = m_1 a$$

$$\text{Lab weight:}\quad m_2 g - T = m_2 a$$

We add the two equations to eliminate T, giving

$$m_2 g = m_1 a + m_2 a = (m_1 + m_2)a$$

and so the magnitude of each body's acceleration is

$$a = \frac{m_2}{m_1 + m_2} g$$

Substituting this back into the glider equation $T = m_1 a$, we get

$$T = \frac{m_1 m_2}{m_1 + m_2} g$$

EVALUATE: The acceleration is in general less than g, as you might expect; the string tension keeps the lab weight from falling freely. The tension T is *not* equal to the weight $m_2 g$ of the lab weight, but is *less* by a factor of $m_1/(m_1 + m_2)$. If T *were* equal to $m_2 g$, then the lab weight would be in equilibrium, and it isn't.

As predicted, the acceleration is equal to g for $m_1 = 0$ and equal to zero for $m_2 = 0$, and $T = 0$ for either $m_1 = 0$ or $m_2 = 0$.

CAUTION **Tension and weight may not be equal** It's a common mistake to assume that if an object is attached to a vertical string, the string tension must be equal to the object's weight. That was the case in Example 5.5, where the acceleration was zero, but it's not the case in this example! The only safe approach is *always* to treat the tension as a variable, as we did here.

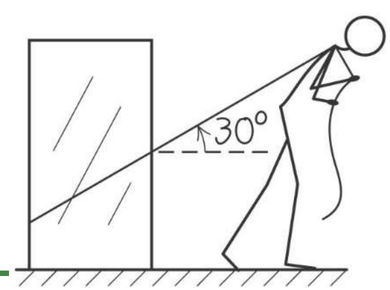
Pulling a crate at an angle

- The angle of the pull affects the normal force, which in turn affects the friction force.

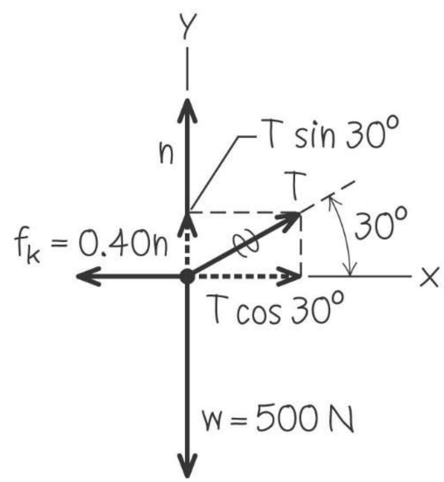
Example 5.15 Minimizing kinetic friction

In Example 5.13, suppose you move the crate by pulling upward on the rope at an angle of 30° above the horizontal. How hard must you pull to keep it moving with constant velocity? Assume that $\mu_k = 0.40$.

(a) Pulling a crate at an angle



(b) Free-body diagram for moving crate



EXECUTE: From the equilibrium conditions and the equation $f_k = \mu_k n$, we have

$$\sum F_x = T\cos 30^\circ + (-f_k) = 0 \quad \text{so} \quad T\cos 30^\circ = \mu_k n$$
$$\sum F_y = T\sin 30^\circ + n + (-w) = 0 \quad \text{so} \quad n = w - T\sin 30^\circ$$

These are two equations for the two unknown quantities T and n. One way to find T is to substitute the expression for n in the second equation into the first equation and then solve the resulting equation for T:

$$T\cos 30^\circ = \mu_k(w - T\sin 30^\circ)$$
$$T = \frac{\mu_k w}{\cos 30^\circ + \mu_k \sin 30^\circ} = 188\ \text{N}$$

We can substitute this result into either of the original equations to obtain n. If we use the second equation, we get

$$n = w - T\sin 30^\circ = (500\ \text{N}) - (188\ \text{N})\sin 30^\circ = 406\ \text{N}$$

$n \neq w = mg!$

SOLUTION

IDENTIFY and SET UP: The crate is in equilibrium because its velocity is constant, so we again apply Newton's first law. Since the crate is in motion, the floor exerts a *kinetic* friction force. The target variable is the magnitude T of the tension force.

Figure 5.21 shows our sketch and free-body diagram. The kinetic friction force f_k is still equal to $\mu_k n$, but now the normal

force n is *not* equal in magnitude to the crate's weight. The force exerted by the rope has a vertical component that tends to lift the crate off the floor; this *reduces* n and so reduces f_k.

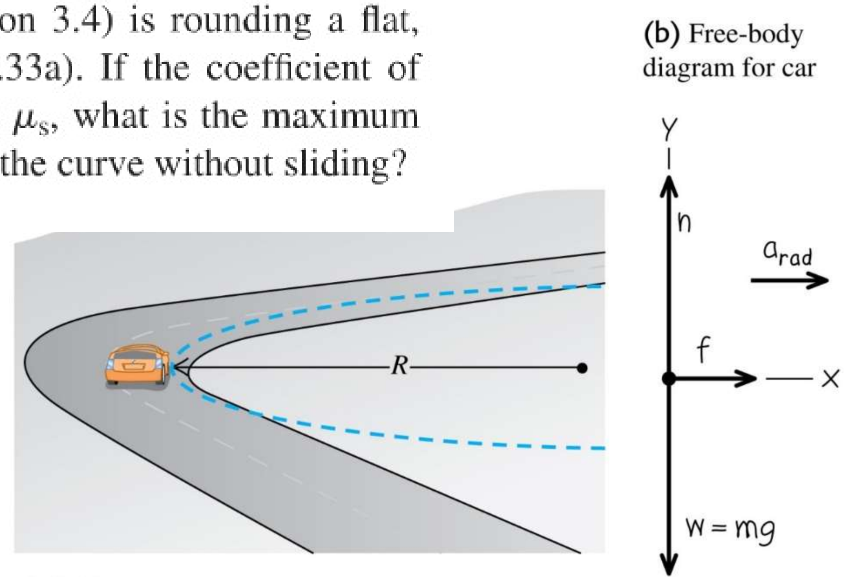
EVALUATE: As expected, the normal force is less than the 500-N weight of the box. It turns out that the tension required to keep the crate moving at constant speed is a little less than the 200-N force needed when you pulled horizontally in Example 5.13. Can you find an angle where the required pull is *minimum*? (See Challenge Problem 5.121.)

A car rounds a flat curve

- A car rounds a flat unbanked curve. What is its maximum speed?

Example 5.21 **Rounding a flat curve**

The sports car in Example 3.11 (Section 3.4) is rounding a flat, unbanked curve with radius R (Fig. 5.33a). If the coefficient of static friction between tires and road is μ_s, what is the maximum speed v_{max} at which the driver can take the curve without sliding?



SOLUTION

IDENTIFY and SET UP: The car's acceleration as it rounds the curve has magnitude $a_{rad} = v^2/R$. Hence the maximum speed v_{max} (our target variable) corresponds to the maximum acceleration a_{rad} and to the maximum horizontal force on the car toward the center of its circular path. The only horizontal force acting on the car is the friction force exerted by the road. So to solve this problem we'll need Newton's second law, the equations of uniform circular motion, and our knowledge of the friction force from Section 5.3.

The free-body diagram in Fig. 5.33b includes the car's weight $w = mg$ and the two forces exerted by the road: the normal force n and the horizontal friction force f. The friction force must point toward the center of the circular path in order to cause the radial acceleration. The car doesn't slide toward or away from the center of the circle, so the friction force is *static* friction, with a maximum magnitude $f_{max} = \mu_s n$ [see Eq. (5.6)].

EXECUTE: The acceleration toward the center of the circular path is $a_{rad} = v^2/R$. There is no vertical acceleration. Thus we have

$$\sum F_x = f = ma_{rad} = m\frac{v^2}{R}$$

$$\sum F_y = n + (-mg) = 0$$

The second equation shows that $n = mg$. The first equation shows that the friction force *needed* to keep the car moving in its circular path increases with the car's speed. But the maximum friction force *available* is $f_{max} = \mu_s n = \mu_s mg$, and this determines the car's maximum speed. Substituting $\mu_s mg$ for f and v_{max} for v in the first equation, we find

$$\mu_s mg = m\frac{v_{max}^2}{R} \qquad \text{so} \qquad v_{max} = \sqrt{\mu_s gR}$$

As an example, if $\mu_s = 0.96$ and $R = 230$ m, we have

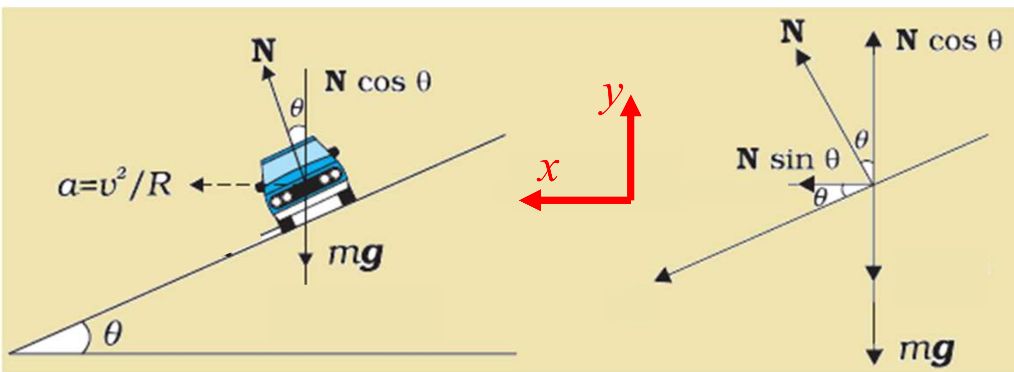
$$v_{max} = \sqrt{(0.96)(9.8\ \text{m/s}^2)(230\ \text{m})} = 47\ \text{m/s}$$

or about 170 km/h (100 mi/h). This is the maximum speed for this radius.

EVALUATE: If the car's speed is slower than $v_{\max} = \sqrt{\mu_s g R}$, the required friction force is less than the maximum value $f_{\max} = \mu_s m g$, and the car can easily make the curve. If we try to take the curve going *faster* than $v_{\max}$, we will skid. We could still go in a circle without skidding at this higher speed, but the radius would have to be larger.

The maximum centripetal acceleration (called the "lateral acceleration" in Example 3.11) is equal to $\mu_s g$. That's why it's best to take curves at less than the posted speed limit if the road is wet or icy, either of which can reduce the value of μ_s and hence $\mu_s g$.

Banking of a curved track – no need for friction



Use acceleration direction as x-axis: $\boxed{a_y = 0}$

$$\boxed{N\cos\theta = mg}$$

$$N\sin\theta = \frac{mv^2}{r} = ma_c$$

By dividing the equations we get $\tan\theta = \dfrac{v^2}{rg}$

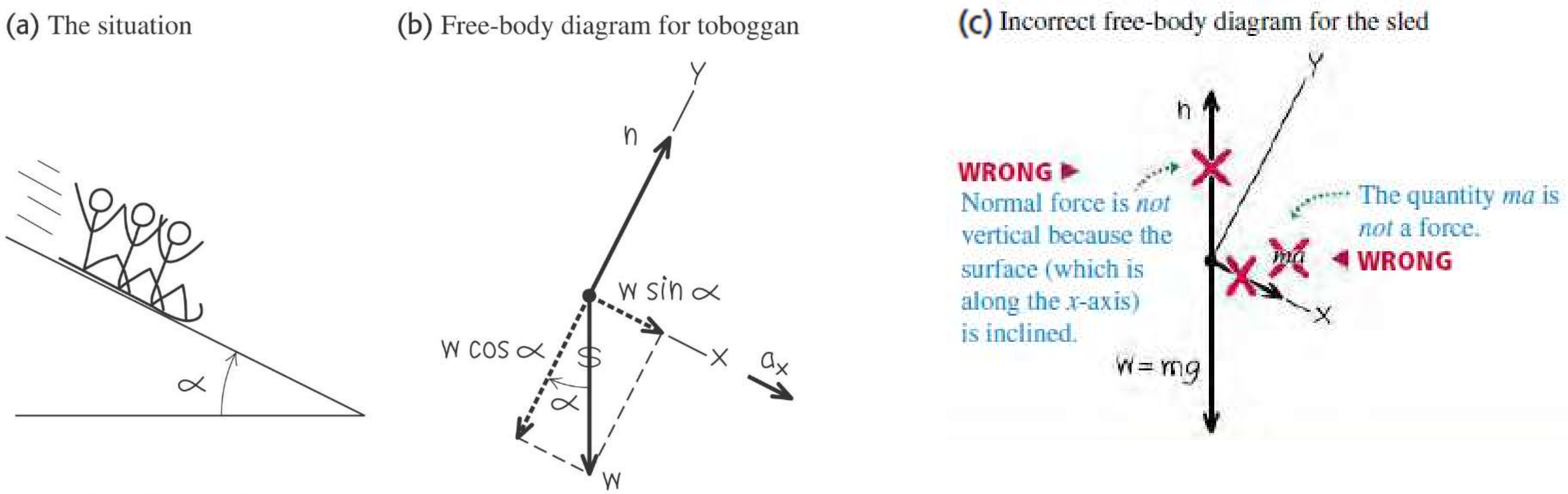
$$v = \sqrt{rg\tan\theta}$$

For a race track of R = 300 m and v = 100 mi/hr = 45m/s, $\theta = 45^{o}$

Acceleration down a hill

- **Example 5.10 Acceleration down a hill**

A toboggan loaded with students (total weight w) slides down a snow-covered slope. The hill slopes at a constant angle α, and the toboggan is so well waxed that there is virtually no friction. What is its acceleration?



SOLUTION

IDENTIFY and SET UP: Our target variable is the acceleration, which we'll find using Newton's second law. There is no friction, so only two forces act on the toboggan: its weight w and the normal force n exerted by the hill.

Figure 5.12 shows our sketch and free-body diagram. As in Example 5.4, the surface is inclined, so the normal force is not vertical and is not equal in magnitude to the weight. Hence we must use both components of $\sum\vec{F} = m\vec{a}$ in Eqs. (5.4). We take axes parallel

and perpendicular to the surface of the hill, so that the acceleration (which is parallel to the hill) is along the positive x-direction.

EXECUTE: The normal force has only a y-component, but the weight has both x- and y-components: $w_x = w \sin\alpha$ and $w_y = -w \cos\alpha$. (In Example 5.4 we had $w_x = -w \sin\alpha$. The difference is that the positive x-axis was uphill in Example 5.4 but is downhill in Fig. 5.12b.) The wiggly line in Fig. 5.12b reminds us that we have resolved the weight into its components. The acceleration is purely in the $+x$-direction, so $a_y = 0$. Newton's second law in component form then tells us that

$$\sum F_x = w \sin\alpha = ma_x$$
$$\sum F_y = n - w \cos\alpha = ma_y = 0$$

Since $w = mg$, the x-component equation tells us that $mg \sin\alpha = ma_x$, or

$$a_x = g \sin\alpha$$

Note that we didn't need the y-component equation to find the acceleration. That's part of the beauty of choosing the x-axis to lie along the acceleration direction! The y-equation tells us the magnitude of the normal force exerted by the hill on the toboggan:

$$n = w \cos\alpha = mg \cos\alpha$$

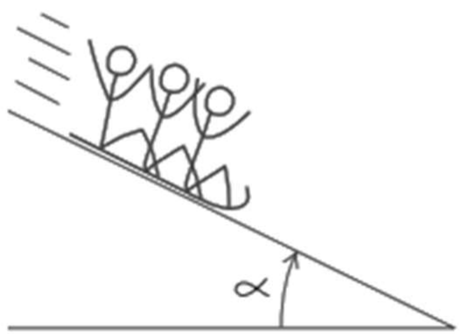
EVALUATE: Notice that the normal force n is not equal to the toboggan's weight (compare Example 5.4). Notice also that the mass m does not appear in our result for the acceleration. That's because the downhill force on the toboggan (a component of the weight) is proportional to m, so the mass cancels out when we use $\sum F_x = ma_x$ to calculate a_x. Hence *any* toboggan, regardless of its mass, slides down a frictionless hill with acceleration $g \sin \alpha$.

If the plane is horizontal, $\alpha = 0$ and $a_x = 0$ (the toboggan does not accelerate); if the plane is vertical, $\alpha = 90°$ and $a_x = g$ (the toboggan is in free fall).

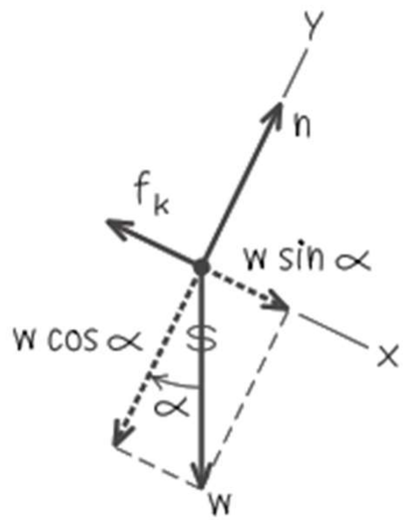
Example 5.16 Toboggan ride with friction I

Let's go back to the toboggan we studied in Example 5.10 (Section 5.2). The wax has worn off and there is now a nonzero coefficient of kinetic friction μ_k. The slope has just the right angle to make the toboggan slide with constant speed. Derive an expression for the slope angle in terms of w and μ_k.

(a) The situation



(b) Free-body diagram for toboggan



EXECUTE: The equilibrium conditions are

$$\sum F_x = w\sin\alpha + (-f_k) = w\sin\alpha - \mu_k n = 0$$
$$\sum F_y = n + (-w\cos\alpha) = 0$$

(We used the relationship $f_k = \mu_k n$ in the equation for the x-components.) Rearranging, we get

$$\mu_k n = w\sin\alpha \qquad \text{and} \qquad n = w\cos\alpha$$

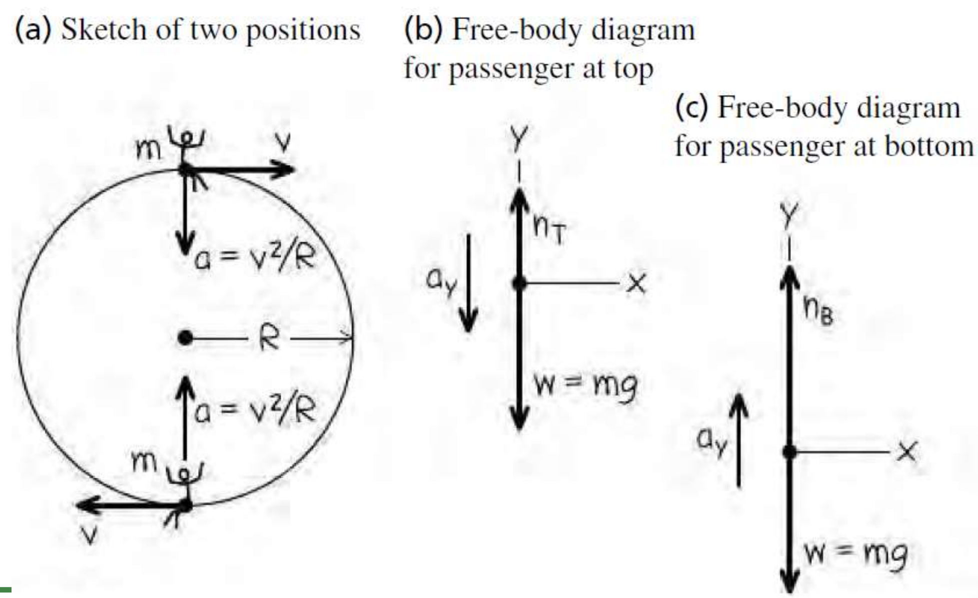
Just as in Example 5.10, the normal force n is *not* equal to the weight w. When we divide the first of these equations by the second, we find

$$\mu_k = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha \qquad \text{so} \qquad \alpha = \arctan\mu_k$$

EXAMPLE 5.23 UNIFORM CIRCULAR MOTION IN A VERTICAL CIRCLE

A passenger on a carnival Ferris wheel moves in a vertical circle of radius R with constant speed v. The seat remains upright during the motion. Find expressions for the force the seat exerts on the passenger when at the top of the circle and when at the bottom.

5.36 Our sketches for this problem.



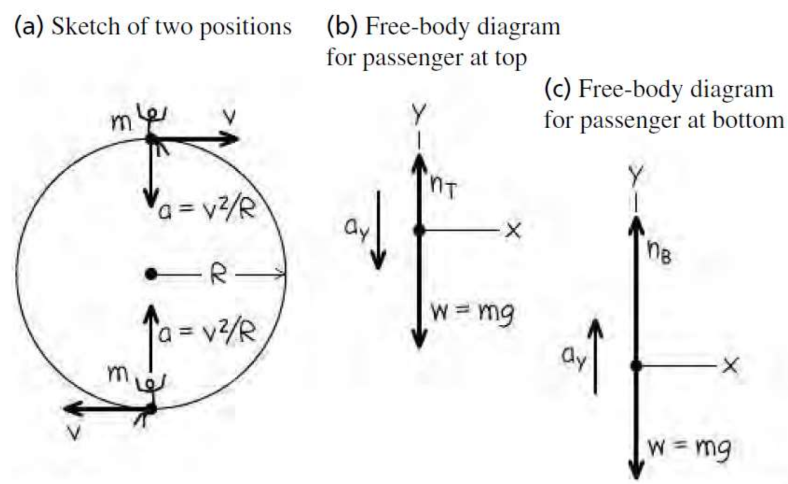
EXAMPLE 5.23 UNIFORM CIRCULAR MOTION IN A VERTICAL CIRCLE

SOLUTION

IDENTIFY and SET UP: The target variables are n_T, the upward normal force the seat applies to the passenger at the top of the circle, and n_B, the normal force at the bottom. We'll find these by using Newton's second law and the uniform circular motion equations.

Figure 5.36a shows the passenger's velocity and acceleration at the two positions. The acceleration always points toward the center of the circle—downward at the top of the circle and upward at the bottom of the circle. At each position the only forces acting are vertical: the upward normal force and the downward force of gravity. Hence we need only the vertical component of Newton's second law. Figures 5.36b and 5.36c show free-body diagrams for the two positions. We take the positive y-direction as upward in both cases (that is, *opposite* the direction of the acceleration at the top of the circle).

5.36 Our sketches for this problem.



EXAMPLE 5.23 UNIFORM CIRCULAR MOTION IN A VERTICAL CIRCLE

EXECUTE: At the top the acceleration has magnitude v^2/R, but its vertical component is negative because its direction is downward.

Hence $a_y = -v^2/R$ and Newton's second law tells us that

$$\text{Top:} \qquad \sum F_y = n_T + (-mg) = -m\frac{v^2}{R} \qquad \text{or}$$

$$n_T = mg\left(1 - \frac{v^2}{gR}\right)$$

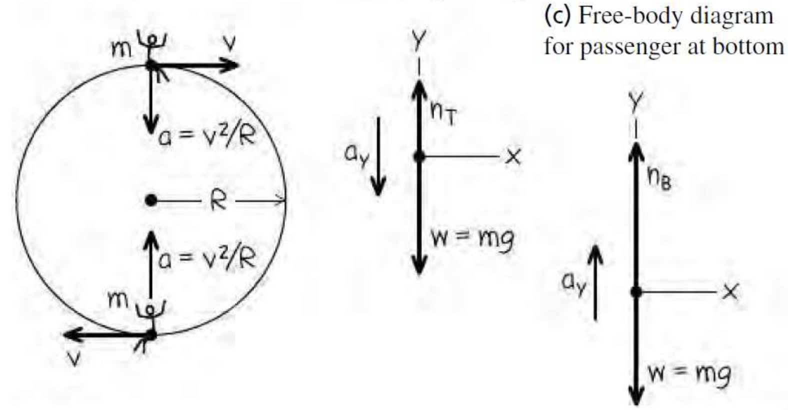
At the bottom the acceleration is upward, so $a_y = +v^2/R$ and Newton's second law says

$$\text{Bottom:} \qquad \sum F_y = n_B + (-mg) = +m\frac{v^2}{R} \qquad \text{or}$$

$$n_B = mg\left(1 + \frac{v^2}{gR}\right)$$

5.36 Our sketches for this problem.

(a) Sketch of two positions (b) Free-body diagram for passenger at top (c) Free-body diagram for passenger at bottom



EXAMPLE 5.23 UNIFORM CIRCULAR MOTION IN A VERTICAL CIRCLE

$$n_T = mg\left(1 - \frac{v^2}{gR}\right)$$

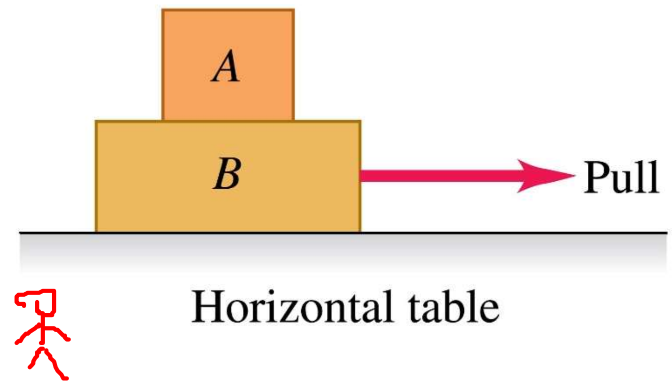
EVALUATE: Our result for n_T tells us that at the top of the Ferris wheel, the upward force the seat applies to the passenger is *smaller* in magnitude than the passenger's weight $w = mg$. If the ride goes fast enough that $g - v^2/R$ becomes zero, the seat applies *no* force, and the passenger is about to become airborne. If v becomes still larger, n_T becomes negative; this means that a *downward* force (such as from a seat belt) is needed to keep the passenger in the seat. By contrast, the normal force n_B at the bottom is always *greater* than the passenger's weight. You feel the seat pushing up on you more firmly than when you are at rest. You can see that n_T and n_B are the values of the passenger's *apparent weight* at the top and bottom of the circle (see Section 5.2).

Thinking questions

TQ3.1

A person pulls horizontally on block *B*, causing both blocks to move horizontally as a unit. There is friction between block *B* and the horizontal table.

If the two blocks are moving to the right at constant velocity,



A. the horizontal force that *B* exerts on *A* points to the left.

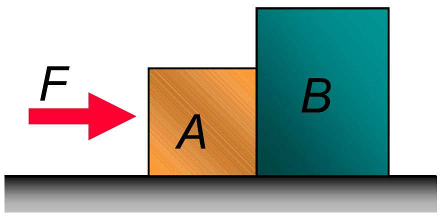
B. the horizontal force that *B* exerts on *A* points to the right.

C. *B* exerts no horizontal force on *A*.

D. not enough information given to decide

TQ 3.2

A lightweight crate (*A*) and a heavy crate (*B*) are side-by-side on a horizontal floor. You apply a horizontal force *F* to crate *A*. There is friction between the crates and the floor.



If the two crates are *accelerating to the right*,

A. crate *A* exerts more force on crate *B* than *B* exerts on *A*

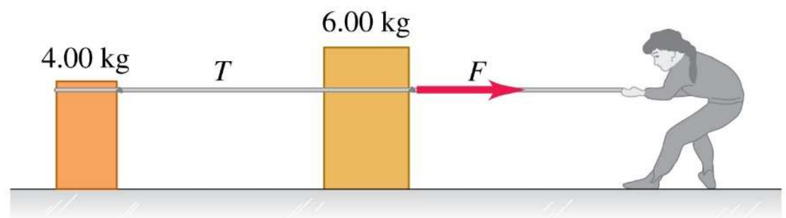
B. crate *A* exerts less force on crate *B* than *B* exerts on *A*

C. crate *A* exerts as much force on crate *B* as *B* exerts on *A*

D. Answer depends on the details of the friction force

TQ3.3

A woman pulls on a 6.00-kg crate, which in turn is connected to a 4.00-kg crate by a light rope. The light rope remains taut.



Compared to the 6.00-kg crate, the lighter 4.00-kg crate

A. is subjected to the same net force and has the same acceleration.

B. is subjected to a smaller net force and has the same acceleration.

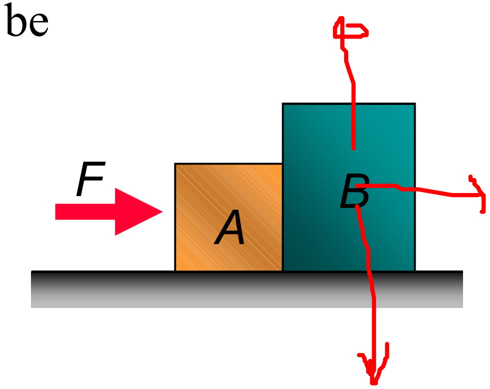
C. is subjected to the same net force and has a smaller acceleration.

D. is subjected to a smaller net force and has a smaller acceleration.

E. none of the above

TQ3.4

A lightweight crate (*A*) and a heavy crate (*B*) are side by side on a frictionless horizontal surface. You are applying a horizontal force *F* to crate *A*. Which of the following forces *should* be included in a free-body diagram for crate *B?*



A. the weight of crate *B*

B. the force of crate *B* on crate *A*

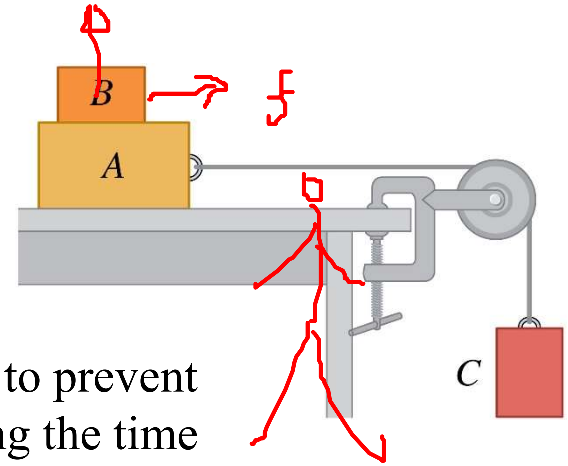
C. the force *F* that you exert

D. the acceleration of crate *B*

E. more than one of the above

TQ3.5

Blocks *A* and *C* are connected by a string as shown. When released, block *A* accelerates to the right and block *C* accelerates downward.



There is friction between blocks *A* and *B*, but not enough to prevent block *B* from slipping. If you stood next to the table during the time that block *B* is slipping on top of block *A*, you would see

A. block *B* accelerating to the right.

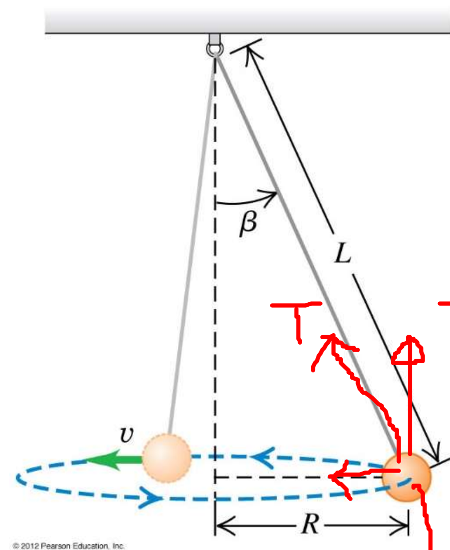
B. block *B* accelerating to the left.

C. block *B* moving at constant speed to the right.

D. block *B* moving at constant speed to the left.

TQ3.6

A pendulum bob of mass m is attached to the ceiling by a thin wire of length L. The bob moves at constant speed in a horizontal circle of radius R, with the wire making a constant angle β with the vertical. The tension in the wire



A. is greater than mg.

B. is equal to mg.

C. is less than mg.

D. is any of the above, depending on the bob's speed v.

T

$T \cos = W$

$ma_c = T \sin$

W

$T = W / \cos$