



WEEK 12





CHAPTER FIVE

- The Quadric Surfaces



OUTLINE

- Sphere, ellipsoid, cone
- Elliptic paraboloid
- Hyperboloid of one sheet
- Hyperboloid of two sheets
- Many more...



FYI

- <https://www.pinterest.com/pin/544161567467205578/>
- <https://www.re-thinkingthefuture.com/materials-construction/a2294-what-are-hyperbolic-paraboloid-shells/>

A HISTORY LESSON...

- Rene' Decarte (1596-1650)
- <https://plato.stanford.edu/entries/descartes/>
- First laid the foundation of algebraic geometry by combining two main branches of mathematics: algebra and geometry.
- He made an astonishing observation of his time that the position of a tiny object on a wall can be located by drawing two perpendiculars from the object on the respective axes defining the wall
- Later his idea was used to study the shape of curves in the xy-plane defined by algebraic funtions. The resultant curve is a graph of the function $y=f(x)$.



STRAIGHT LINE



ALL OTHER SHAPES



2D TO 3D



QUADRIC SURFACE

- The quadric surface is a graph in space of a second degree equation in x , y and z .



STEPS TO SKETCHING EXAMPLE

- We study these surfaces with a simple observation that a surface is generated by infinite family of curves that lie on it. Normally a surface can be drawn by tracing out a few curves and joining them together to form it. To do this we need to formally define a procedure which is known as sketching the traces.
- Warm up: Suppose we have a sphere which is intersected from a plane right in the middle. What geometrical object you get on the plane?



TRACE

- A trace of a surface S is a geometrical curve which one obtain on a plane that intersects the surface.



EXAMPLE OF A BASIC FUNCTION



EXAMPLE: STRAIGHT LINE



EXAMPLE: CIRCLE



EXAMPLE: ELLIPSOID



EXAMPLE: SPHERE



EXAMPLE: ELLIPTIC CONE



EXAMPLE HYPERBOLOID OF ONE SHEET



EXAMPLE: HYPERBOLOID OF TWO SHEETS



EXAMPLE: ELLIPTIC PARABOLOID

FYI

- The Clebsch Surface (1871)
- <https://blogs.ams.org/visualinsight/2016/03/01/clebsch-surface/>
- Penrose Twistor (1969)
- <https://www.youtube.com/watch?v=j16eVLDt2HI>
- Klein Bottle (1869)
- <https://www.youtube.com/watch?v=dj3HqRtC-T8>



RECALL FROM PREVIOUS WEEKS...

Find an equation in cylindrical coordinates for the ellipsoid: $4x^2+4y^2+z^2=1$



RECALL FROM PREVIOUS WEEKS...

Find an equation in spherical coordinates for the hyperboloid of two sheets: $x^2 - y^2 - z^2 = 1$



SUMMARY

- VERY Useful website
- <https://www.geogebra.org/3d?lang=en>