EE 1001 Assignment 2
Please submit Assignment #2 on or before 10 October 2021 Sunday, 23:59

EE 1001 Assignment 4

Q1. Write the first five terms of the sequences with nth term $a_n = (-1)^{n-1} 5^{n+1}$

Ans:

Substituting $n = 1, 2, 3, 4, 5$, we obtain

$$a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$$
$$a_2 = (-1)^{2-1} 5^{2+1} = -5^3 = -125$$
$$a_3 = (-1)^{3-1} 5^{3+1} = 5^4 = 625$$
$$a_4 = (-1)^{4-1} 5^{4+1} = -5^5 = -3125$$
$$a^5 = (-1)^{5-1} 5^{5+1} = 5^6 = 15625$$

Therefore, the required terms are 25, –125, 625, –3125, and 15625.

Q2.

2-1) Find the number of words (with or without meaning) with a length of FIVE (5) that can be formed by using the letters of the word "apple".

2-2) Each student is requested to generate one word (with or without meaning) with a length of FIVE (5), what is the probability that the two students generate exactly the same word?

2-3) Each student is requested to generate one word (with or without meaning) with a length of THREE (3) letters, what is the probability that the two students generate exactly the same word?

Solution:

2-1) The word 'apple' contains 5 letters, and the letters 'p' comes twice. Then, this is to find the distinguishable permutations. The number of words formed = 5!/(2!) = 60

2-2) (a) For each student, he/she has 60 choices, thus, p=1/60*1/60*60=1/3600*60=1/60;

(b) If your assumption is any 5 letters from 26 letters.

(b) (i) 5 letters (non-repeat) from 26 letters (a-z):

For each student, he/she has ${}_{26}P_5$ = 7893600 choices,

Thus, p=1/7893600*1/7893600*7893600=1/7893600;

(b) (ii) 5 letters (may repeat) from 26 letters (a-z):

For each student, he/she has 26^5 = 11881376 choices,

Thus, p=1/11881376*1/11881376*11881376=1/11881376;

2-3)

(a) For the word contains 1 or 0 "p", the number of words are ${}_4P_3$=24;

For the word contains 2 "p", the number of words are : ${}_3C_1$ * 3!/(2!)=9

So there are total 24+9 possible words. Thus, the probability is p=1/33*1/33*33=1/33

(b) (i) 3 letters (non-repeat) from total 26 letters (a-z):
For each student, he/she has ${}_{26}P_3 = 15600$ choices,
Thus, p=1/15600*1/15600*15600=1/15600;

(b) (ii) 3 letters (may repeat) from 26 letters (a-z):
For each student, he/she has $26^3 = 17576$ choices,
Thus, p=1/17576*1/17576*17576=1/17576;

Q3. Given m, n are positive integers, $f(x) = (1+x)^m+(1+x)^n$. It is known that the coefficients of the terms x and x^2 are 7 and 9 respectively. Compute:

(1) The values of m and n; (6 points)

(2) The coefficient of the term x^3; (6 points)

(3) Use the binomial theorem to compute $(1.01)^4$

Solution:

(1) With the binomial theorem, we have: $m+n=7$; ${}_mC_2+{}_nC_2=(m^2+n^2-m-n)/2=9$;
It is computed: m=4, n=3, or m=3, n=4;

(2) No matter m=4, n=3, or m=3, n=4; $f(x) = (1+x)^3+(1+x)^4$;
Thus, the coefficient of the term x3 is ${}_3C_3+{}_4C_3 = 5$;

(3) $(1.01)^4=(1+0.01)^4={}_4C_0(1)^4(0.01)^0 + {}_4C_1(1)^3(0.01)^1 + {}_4C_2(1)^2(0.01)^2 + {}_4C_3(1)^1(0.01)^3 + {}_4C_1(1)^0(0.01)^4$
$= 1+ 0.04+0.0006+0.000004+0.00000001=1.04060401$

Q4. Let A= {2, 3, 5, 6, 7, 9}; B = {3, 6, 9}, and C = {2, 4, 5, 6, 8}. Find each of the following:

(1) A ∪ B
(2) A ∩ B
(3) A ∪ C
(4) A ∩ C
(5) A - B
(6) B - A
(7) B ∪ C
(8) B ∩ C

Solution:

(1) {2, 3, 5, 6, 7, 9}
(2) {3, 6, 9}
(3) {2, 3, 4, 5, 6, 7, 8, 9}
(4) {2, 5, 6}
(5) {2 ,5, 7}
(6) ∅
(7) {2, 3, 4, 5, 6, 8, 9}
(8) {6}

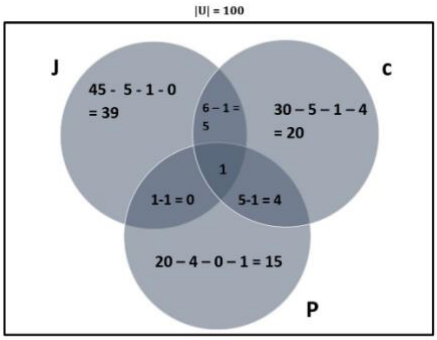
Q5. An AI development company employs 100 computer programmers. Amongst them, 45 are proficient in Java, 30 in C#, 20 in Python, six in C# and Java, one in Java and Python, five in C# and Python, and just one programmer is proficient in all three languages above. Determine the number of computer programmers that are not proficient in any of these three languages.

Solution:

Let U denotes the set of all employed computer programmers and let J, C and P denotes respectively the set of programmers proficient in Java, C# and Python, respectively. Thus:

$$|U| = 100 \quad |J| = 45 \quad |C| = 30 \quad |P| = 20$$
$$|J \cap C| = 6 \quad |J \cap P| = 1 \quad |C \cap P| = 5 \quad |J \cap C \cap P| = 1$$

With Venn diagram, it is easy to obtain:



we need to determine the complement of the set J ∪ C ∪ P.

Calculate |J ∪ C ∪ P| first before determining the complement value:

|J ∪ C ∪ P|= 39 + 5 +20 +4 +15 + 1= 84

Now calculate the complement: |(J ∪ C ∪ P)' | = |U| – |J ∪ C ∪ P| = 100 – 84 = 16

16 programmers are not proficient in any of the three languages.

Q6.

6-1) A drawer contains 12 red and 12 blue socks, all unmatched. A person takes socks out at random in the dark. How many socks must be taken out to ensure that he has at least two blue socks?

6-2) Three students are running for a student government. There are 202 students voting, what is the minimum number of votes required to win the election?

6-3) Three students are running for a student government. There are 202 students voting, what is the minimum number of votes required to ensure the winning of the election?

Solution:

(1) Given 12 red and 12 blue socks so, in order to take out at least 2 blue socks, first we need to take out 12 shocks (which might end up red in worst case) and then take out 2 socks (which would be definitely blue). Thus, we need to take out total 14 socks.

(2) By pigeonhole, there exists a person who has gotten at least $\lceil 202/3 \rceil = 68$ votes. So, someone could win with a 67 − 67 − 68 split.

(3) To ensure the wining, the one need more than 50% vote, which is 202*50% +1=102

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