Review (Final)

Calculus, Spring 2012
Instructor: Dr. Jeffrey Scott

① Polar

② Cartesian plane.

$r>0$ terminal side

$r<0$ terminal side $(-r,\theta)$

$r=0$ $(0,0)$

θ = clockwise

Ex. $(2,\frac{\pi}{3}) \to (x,y)$

$r=2$, $\theta=\frac{\pi}{3}$

$(2,\frac{\pi}{3})$

$\boxed{(1,\sqrt{3})}$

$-\frac{5\pi}{3}$

* Add a multiple of 2π to the angle we don't change.
Add a multiple of π to the angle we change r.

Ex. $(-8,\frac{\pi}{3}) \to (x,y)$ NOT EXAM ON EXAM

$(-8,\frac{\pi}{3})$

$\boxed{(-4,-4\sqrt{3})}$

$x=r\cos\theta$
$y=r\sin\theta$

$\tan\theta=\frac{y}{x}$
$\tan\theta=\frac{-8\sqrt{3}}{-8}$

$x=r\cos\theta$
$x=-8\cos\frac{\pi}{3}$
$x=-8(\frac{1}{2})$
$x=-4$

$y=r\sin\theta$
$y=-8\sin\frac{\pi}{3}$
$y=-8(\frac{\sqrt{3}}{2})$
$y=-4\sqrt{3}$

Ex. $(2,-2)$ w/ $r>0$ and $\theta\le 0<2\pi$

$r=\sqrt{x^2+y^2}$
$r=\sqrt{2^2+(-2)^2}$
$r=\sqrt{8}$

$\tan\theta=\frac{y}{x}$
$\tan\theta=\frac{-2}{2}$
$\theta=2\pi-\frac{\pi}{4}$
$\theta=\frac{7\pi}{4}$

$\therefore (\sqrt{8},\frac{7\pi}{4})$

Ex. $r\cos\theta=\sin\theta \to$ rectangular

$x+y=0$

Cylindrical

$x=r\cos\theta$
$y=r\sin\theta$
$z=z$
$r^2=x^2+y^2$
$\tan\theta=\frac{y}{x}$

NOT EXAM ON EXAM

Ex. Plot $P(2,\frac{2\pi}{3},1) \to$

$x=r\cos\theta=2\cos\frac{2\pi}{3}=-1$
$y=r\sin\theta=2\sin\frac{2\pi}{3}=\sqrt{3}$
$z=1$

Ex. $(1,-3,-7) \to$ polar

$z=-7$
$r=\sqrt{x^2+y^2}=\sqrt{(1)^2+(-3)^2}=\sqrt{10}$
$\tan\theta=\frac{y}{x}=-\frac{3}{1}$

$(\sqrt{10},-\frac{\pi}{3},-7)$

Spherical

$x=r\sin\phi\cos\theta=2\sin\frac{\pi}{3}\cos(\frac{\pi}{3})=\frac{\sqrt{3}}{2}$
$y=r\sin\phi\sin\theta=2\sin\frac{\pi}{3}\sin(\frac{\pi}{3})=\frac{3}{2}$
$z=r\cos\phi=2\cos\frac{\pi}{3}=1$

$\boxed{(\frac{\sqrt{3}}{2},\frac{3}{2},1)}$

$x=r\sin\phi\cos\theta$
$y=r\sin\phi\sin\theta$
$z=r\cos\phi$
$r^2=x^2+y^2+z^2$
$\tan\theta=\frac{y}{x}$
$\cos\phi=\frac{z}{\sqrt{x^2+y^2+z^2}}$

NOT EXAM ON EXAM

$r=\sqrt{x^2+y^2+z^2}$

Ex. $(2,\frac{2\pi}{3},\frac{\pi}{3}) \to$ rectangular

$x=r\sin\phi\cos\theta=2\sin\frac{\pi}{3}\cos(\frac{2\pi}{3})=\frac{\sqrt{3}}{2}(-\frac{1}{2})$
$y=r\sin\phi\sin\theta=2\sin\frac{\pi}{3}\sin\frac{2\pi}{3}=\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{3}}{2}$
$z=r\cos\phi=2\cos\frac{\pi}{3}=1$

$\boxed{(-\frac{\sqrt{3}}{4},\frac{3}{4},1)}$

Ex. $(0,2\frac{\pi}{3},-3) \to$ polar

$r=-3$
$r=\sqrt{x^2+y^2+z^2}=\sqrt{0^2+(\frac{2\pi}{3})^2+(-3)^2}=\frac{\pi}{3}$
$\theta=\frac{2\pi}{3}$

$\cos\phi=\frac{z}{r}=\frac{-3}{\frac{\pi}{3}}=-\frac{9}{\pi}$ $\phi=\frac{2\pi}{3}$

$(\frac{\pi}{3},\frac{2\pi}{3},1)$

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4 different shapes

Function

Ex. find final divisor of $y=-2x^2$

result. $\frac{x^2}{4}=\frac{1}{4}y$
$y=4px$

Step 1. $x^2=4py$

Step 2. $y=-2x^2$
$\frac{y}{-2}=x^2$
$x^2=-\frac{1}{2}y$
$p=-\frac{1}{8}$
$p=-\frac{1}{8}$

Step 3. $\text{Focus}(0,p)$ → $(0,-\frac{1}{8})$
Directrix
$y=-p$
$y=-(-\frac{1}{8})$
$y=\frac{1}{8}$

Ex. vertex $(0,0)$
focus $(2,0)$
→ directrix ?
opp → primary focal divid.

Step 1 axis of symmetry → horizontal
directrix
$y^2=4px$

$p=2$ focus $(p,0)$
$(p,0)$

$|p|=|(4)(2)|=8$

Step 2 $p=2$
$y^2=4(1)x$
$y^2=8x$

Step 3. $x=-p$
$p=2$ $\boxed{x=-2}$

Step 4. $\boxed{\text{opp}}$

4 different shapes

Ex. $y^2+4y+2x+5=0$
↓ complete square
$(y+2)^2-4+2x+5=0$
$(y+2)^2=-2(x+2)$
$(-2,-2)$ vertex
$4p=-8$
$p=-2$
$p<0$ opens to left

Analysis. $(y-k)^2=4p(x-h)$

Vertex $(-2,-2)$
focus $(h+p,k)=(-4,-2)$
directrix $x=h-p=-2-(-2)=0$ $\boxed{x=0}$
axis of symmetry $\boxed{y=-2}$

Conic $x^2+y^2=1$
$(x-1)^2+(y+1)^2=4$ $r=2$
center $(1,-1)$

Ex. find a center of $(1,-2)$ and passes thru $(-1,1)$ on the edge.

$x=-1$ $y=1$ $h=1$ $k=-2$

find step of ex circle

$r=\sqrt{(x-h)^2+(y-k)^2}$
$r=\sqrt{(-1-1)^2+(y-(-2))^2}$
$r=\sqrt{(-2)^2+y^2+4y+4}$
$\boxed{(x-1)^2+(y+1)^2=25}$ Standard form
$x^2+y^2-2x+4y-15=0$ General form

Ex. find standard form of the equation of an ellipse that has major axis of length 6 foci at $(-2,0)$ and $(2,0)$ and center at $(0,0)$

Step 1 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $0<b<a$

Step 2 $2a=6$
$a=3$

Step 3 $\frac{x^2}{3^2}+\frac{y^2}{b^2}=1$
$\frac{x^2}{9}+\frac{y^2}{b^2}=1$

Step 4 $\frac{x^2}{9}+\frac{y^2}{5}=1$

Ex. $4x^2+y^2=36$ identify vertices

Step 1 $\frac{x^2}{9}+\frac{y^2}{36}=1$

$4x^2+y^2=36$
$\frac{x^2}{9}+\frac{y^2}{36}=1$

$a>b \to (0,-6)$ $(0,6)$
$a>b \to (-3,0)$ $(3,0)$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $c^2=a^2-b^2=25$ foci $(0,-5)$ and $(0,5)$

$\frac{x^2}{3^2}+\frac{y^2}{6^2}=1$

Step 2 $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$ major axis version

Hyperbola

Horizontal graph $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

Vertical graph $\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$

Horizontal Asymptotes
$y=k\pm\frac{b}{a}(x-h)$

Vertical Asymptotes
$y=k\pm\frac{a}{b}(x-h)$

Ex. $\frac{(x-2)^2}{4}-\frac{(y+1)^2}{25}=1$

center $(2,-1)$
vertices $(h\pm a,k)$ $(2\pm 2,-1)$
foci $(h\pm\sqrt{a^2+b^2},k)$

$a=2$
$b=5$

Asymptotes: $y=k\pm\frac{b}{a}(x-h)$ $\boxed{\text{Horizontal}}$

Ex. $9y^2-x^2-4x-10=0$

$\frac{x^2}{9}-\frac{y^2}{1}=1$

$a=3$
$b=1$

center $(2,0)$
vertices $(-1,0)$ and $(5,0)$
foci $(-1,0)$ & $(\frac{5}{2},0)$
Asymptotes $y=\pm\frac{1}{3}(x+2)$

$9y^2-x^2-4x-10=0$
$9y^2-(x+2)^2=1$
$\frac{y^2}{\frac{1}{9}}-\frac{(x+2)^2}{1}=1$ $\boxed{\text{Vertical}}$

Ex eccentricity of hyperbola $\frac{5}{3}$ foci $(-1,-2)$ and $(2,-2)$ equation?

$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$

$c=\frac{5}{2}$
$\frac{c}{a}=\frac{5}{3}$

$\frac{(x+1)}{2}$ $\frac{-2+2}{2}$
$(\frac{1}{2},-2)$

$\therefore \frac{(x-\frac{1}{2})^2}{36}-\frac{(y+2)^2}{13}=1$

$\frac{c}{a}=\frac{5}{2}$
$\frac{a}{c}=\frac{2}{5}$
$b^2=c^2-a^2$
$b^2=\frac{25}{4}-\frac{4}{25}$
$=\frac{609}{100}$
$b^2=\frac{609}{100}$ $\boxed{\text{Vertical}}$

Quadric surfaces

1) Describe surface where cylindrical equation is $z=r$

$z^2=r^2$
$z^2=x^2+y^2$
$z=\sqrt{x^2+y^2}$ ← cone axis on z-axis

2) Cylindrical eq. for $x^2+y^2+z^2=1$ $\boxed{\text{ellipsoid}}$

$r^2+z^2=1$
$r^2+z^2=1$
$r^2+z^2=1$

3) Cylindrical for $x^2+y^2+z^2=1$ $\boxed{\text{ellipsoid}}$

$r^2\cos^2\theta+r^2\sin^2\theta+z^2=1$
or
$r^2+z^2\sin^2\theta+z^2=1$

change cylindrical coords to spherical

$z=r\cos\phi$
$x=r\sin\phi$
$y=y$

(r,ϕ) polar coordinate that projects onto xy-plane

$x^2+y^2+z^2=1$
$r^2+z^2=1$

Ex. $x^2-y^2-z^2=1$ $\boxed{\text{hyperboloid}}$

$(r\cos\theta)^2-(r\sin\theta)^2-(r\cos\phi)^2=1$
$r^2\cos^2\theta-r^2\sin^2\theta-r^2\cos^2\phi=1$
$r^2(\cos^2\theta-\sin^2\theta)-r^2\cos^2\phi=1$
$r^2(\cos^2\theta-\sin^2\theta)-\cos^2\phi=1$

Ex. $r^2=2\sin\theta\sin\phi$ → rectangular

$r^2=2(r\sin\theta)\sin\phi$
$x^2+y^2=2y$
or
$x^2+(y-1)^2+z^2=1$ ← sphere w/ radius=1 center $(0,1,0)$

Application

The cables of Thing Ma Bridge are in the shape of a parabola. The pillars supporting the cables are 800 feet apart and the two ends are same the road. At the lowest point of the cable and the road is 50 feet apart and there is 100 feet way between the pillars.
What's the height of the cable 400 feet from the center of the Thing Ma Bridge?

vertex $(0,50)$
pillar $(400,800)$

$(x-h)^2=4p(y-k)$
$(800-0)^2=4p(800-50)$
$p=1333.3$

$\therefore (x-0)^2=4(1333.3)(y-50)$
$y^2=5333.3(y-50)$
At 400 feet
$400^2=5333.3(y-50)$
$y=50.075$ ft.