Homework 1

Due: **Saturday, 25 September**

IMPORTANT NOTE: Please submit your assignment online through canvas, latest by the due date or deadline. You may write your answers in MS Word, or use any other processor (e.g. LaTeX). However, the final file that you will upload on canvas must be a pdf document. It is also fine if you write out your answers on a piece of paper and then scan it or take high quality image using your camera/phone and then create a pdf document to upload on canvas.

Other Notes: *Please note that the questions are based on the material covered in lectures 1-3. There are five questions in this assignment. While your answers will not be graded, you must attempt and write your answers to all the five questions to get full credits for this assignment.*

Question 1: Determine which of the following situations describe strategic interactions (or games) and which describe individual decisions. In each case, indicate what specific features of the situation caused you to classify it as you did.

(a) A group of grocery shoppers in the dairy section, with each shopper choosing a flavor of yogurt to purchase individual decision because there is no interdependency among different shoppers.

(b) A college student considering what type of postgraduate education to pursue individual decision Because there is only one person in the situation,

(c) The *New York Times* and the *Wall Street Journal* choosing the prices for their online subscriptions this year strategic interaction. Because one persons' decision will effect the other person's decision and the outcomes.

Question 2: Each of two players has two possible actions, *Quiet* and *Fink*; each action pair results in the players' receiving amounts of money equal to the numbers corresponding to that action pair in the following game matrix:

	Quiet	*Fink*
Quiet	$2, 2$	$0, 3$
Fink	$3, 0$	$1, 1$

For example, if player 1 chooses *Quiet* and player 2 chooses *Fink*, then player 1 receives nothing, whereas player 2 receives \$3.

The players are not "selfish"; rather the preferences of each player i are represented by the payoff function $m_i(a) + \alpha m_j(a)$, where $m_i(a)$ is the amount of money received by player i when the action profile is a, j is the other player, and α is a given non-negative number. Player 1's payoff to the action pair *(Quiet,Quiet)*, for example, is $2 + 2\alpha$.

(a) Formulate a strategic game that models this situation in the case $\alpha = 1$. Is this game the *Prisoner's Dilemma*? NO.

(b) Find the range of values of α for which the resulting game is the *Prisoner's Dilemma*.

$\begin{cases} 3 > 2 + 2\alpha \\ 1 + \alpha > 3\alpha \end{cases}$ $0 < \alpha < \frac{1}{2}$

Question 3: An employer hires an employee and promises him wage w. The employee can work (W) or shirk (S). Working is associated with cost of effort, e, where $w > e > 0$. If the employee works, the employer obtains revenue r and otherwise $\frac{r}{2}$.
Employer cannot tell whether the employee is working or shirking unless she chooses to run inspection (I) at a cost of $c > 0$. Inspection reveals whether the employee is working or not and in the latter case the wage is withheld. The decisions of the two players (W/S and I/NI) are taken simultaneously. The payoff matrix is as follows:

	I	NI
W	w-e, r-c-w	w-e,r-w
S	0,$r/2$-c	w,$r/2$-w

Employee is choosing a row (row player) and the employer is choosing a column (column player).
Under what condition(s) will this game have a pure strategy Nash equilibrium?
If $c > w$. then $S + NI$ is a pure strategy Nash equilibrium.

Question 4: An airline loses two suitcases belonging to two different travelers. Both suitcases happen to be identical and therefore have equal replacement value. An airline manager tasked to settle the claims of both travelers explains that the airline is liable for a maximum of $100 per suitcase, and in order to determine an honest appraised value of the suitcases the manager separates both travelers so they can't confer, and asks them to write down the amount of their value at no less than $2 and no larger than $100. He also tells them that if both write down the same number, he will treat that number as the true dollar value of both suitcases and reimburse both travelers that amount. However, if one writes down a smaller number than the other, this smaller number will be taken as the true dollar value, and both travelers will receive that amount along with a bonus/malus: $2 extra will be paid to the traveler who wrote down the lower value and a $2 deduction will be taken from the person who wrote down the higher amount. Describe the set of strategies of both players and find all the pure strategy Nash equilibria of the game. Restrict yourself to integer numbers only.

Question 5: Consider a game in which there is a prize worth $30. There are three contestants, Larry, Curly, and Moe. Each can buy a ticket worth $15 or $30 or not buy a ticket at all. They make these choices simultaneously and without any communication. Then, knowing the ticket-purchase decisions, the game organizer awards the prize. If no one has bought a ticket, the prize is not awarded. Otherwise, the prize is awarded to the buyer of the highest-cost ticket if there is only one such player or is split equally between two or three if there are ties among the highest-cost ticket buyers. Show this game in normal form by writing out the payoff matrix, using Larry as the row player, Curly as the column player, and Moe as the table player. Find all pure-strategy Nash equilibria.