ASSIGMENT 3.



1. $r = \sin(2\theta)$

(20) $r = 2\sin\theta\cos\theta$

$r = 2 \cdot \frac{x}{r} \cdot \frac{y}{r}$

$r = \frac{2xy}{r^2}$

$r^3 = 2xy$

$\left(\sqrt{x^2+y^2}\right)^3 = 2xy$ //

or $(x^2+y^2)^{3/2} = 2xy$ // or $x^2+y^2 = (2xy)^{2/3}$ //

accept $(x^2+y^2)^3 = 4(xy)^2$

2. (a) $(2, 2)$

(20) (b) $x = 2 \quad y = 2$

$r^2 = x^2+y^2 = 2^2+2^2 = 4+4 = 8 \quad r = \sqrt{8}$

$\tan\theta = \frac{y}{x} = \frac{2}{2} = 1$

$\theta = \frac{\pi}{4}$ $\therefore$ polar coordinate $P(2,2)$ are $\left(\sqrt{8}, \frac{\pi}{4}\right)$ //

(b) $(1, -\sqrt{3})$

(7) $r = 2 \quad \theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -\frac{\pi}{3}$

$\therefore$ polar coordinate $P(1, -\sqrt{3})$ are $\left(2, -\frac{\pi}{3}\right)$ //

(c) $(-1, \sqrt{3})$

(7) $r = 2 \quad \theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = \frac{2\pi}{3}$

$\therefore$ polar coordinate $P(-1, \sqrt{3})$ are $\left(2, \frac{2\pi}{3}\right)$ //

3. (5 bonus)

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\sin\theta + \theta\cos\theta}{\cos\theta - \theta\sin\theta}$$

$x = \theta\cos\theta$ $\quad$ $y = \theta\sin\theta$ $\quad$ ②

$f = \theta$ $\quad g = \cos\theta$ $\quad$ | $\quad f = \theta$ $\quad g = \sin\theta$

$f' = 1$ $\quad g' = -\sin\theta$ $\quad$ | $\quad f' = 1$ $\quad g' = \cos\theta$

by product rule

when $\theta = \frac{\pi}{2}$ $\quad \frac{dy}{dx} = \frac{\sin\frac{\pi}{2} + \frac{\pi}{2}\cos\frac{\pi}{2}}{\cos\frac{\pi}{2} - \frac{\pi}{2}\sin\frac{\pi}{2}}$

$= -\frac{2}{\pi}$

note when $\theta = \frac{\pi}{2}$

polar coordinates $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$

cartesian is $\left(0, \frac{\pi}{2}\right)$
$\quad x \quad y$

$\therefore$ $y - \frac{\pi}{2} = -\frac{2}{\pi}(x - 0)$

$\therefore$ $y - \frac{\pi}{2} = -\frac{2}{\pi}x$ //

4. $A(5,7)$ $\quad B(-5,-7)$

(20) (a) (10)

center of circle = midpt.

Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

$= \left(\frac{0}{2}, \frac{0}{2}\right)$

$= (0, 0)$

radius $= \frac{\text{Diameter}}{2}$ $\quad \therefore$ or use midpt.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$r = d = \sqrt{(5-0)^2 + (7-0)^2}$

$r = d = \sqrt{25 + 49}$

$r = \sqrt{74}$

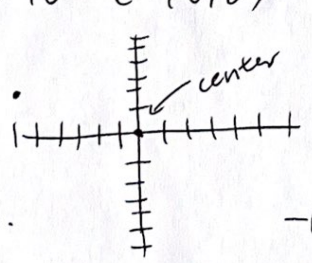
$\therefore$ $(x-0)^2 + (y-0)^2 = (\sqrt{74})^2$

$x^2 + y^2 = 74$ //

(b) $x^2 + y^2 = 40$ $\quad c: (0,0)$

(10)

$(-6, 2)$
$x_1 \; y_1$




Midpoint : $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Midpt $= \left(\frac{-6 + x_2}{2}, \frac{2 + y_2}{2}\right)$

$\frac{-6 + x_2}{2} = 0$ $\quad \frac{2 + y_2}{2} = 0$

$x_2 = 6$ $\quad y_2 = -2$

$\therefore$ $(6, -2)$ //

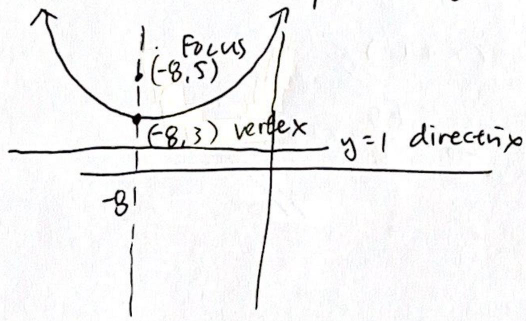
5. (20)

(a) (10) vertex $(h,k) = (-8, 3)$ $\quad 4p = 8 \quad p = 2$

focus $(h, k+p) = (-8, 5)$

directrix $= y = k - p = 1 = y$

axis of symmetry $x = h = -8 = x$



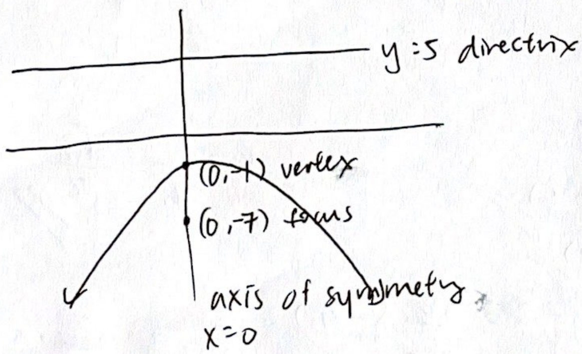

(b) $x^2 = -24(y+1)$ $\quad 4p = -24 \quad p = -6$

(10) vertex $(h,k) = (0, -1)$

focus $(h, k+p) = (0, -7)$

directrix $y = k - p \quad y = 5$

axis of symmetry $x = h = 0$



$a = \sqrt{64} = 8 \quad b = \sqrt{58}$ ④

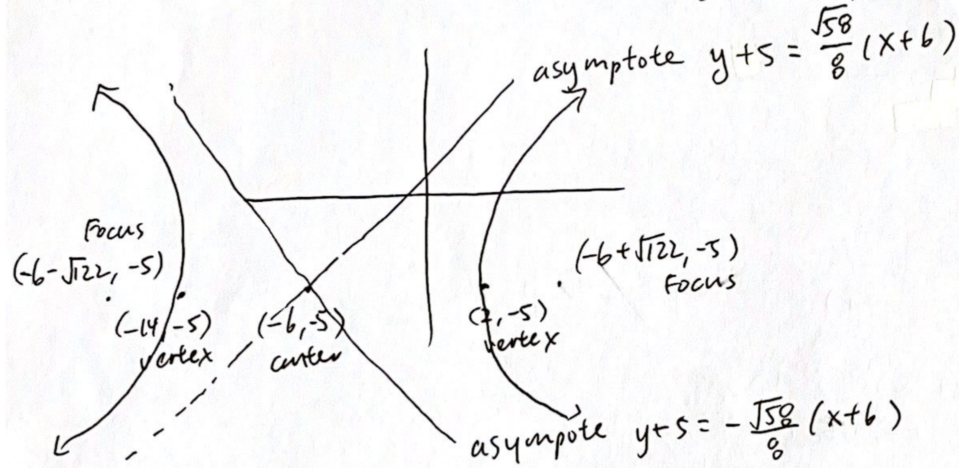
6. (a) center $(-6, -5)$ //

(20) (10) verticies $(2, -5)$ // and $(-14, -5)$ // $(h \pm a, k)$ $\quad c = \sqrt{64 + 58} = \sqrt{122}$

foci $(h \pm c, k) = (-6 \pm \sqrt{122}, -5)$ //

asymptotes $y - k = \pm \frac{b}{a}(x - h)$

$y + 5 = \pm \frac{\sqrt{58}}{8}(x + 6)$ //




(b)

(10) $\frac{(y-3)^2}{8} - \frac{(x-9)^2}{6} = 1$ $\quad c = \sqrt{8+6} = \sqrt{14}$

$a = \sqrt{8} \quad b = \sqrt{6}$

center $(9, 3)$ //

verticies $(9, 3 \pm \sqrt{8})$ //

foci $(9, 3 \pm \sqrt{14})$ //

$e = \frac{c}{b} = \frac{\sqrt{7}}{2}$ //

asymptotes $x - 9 = \pm \frac{\sqrt{6}}{2\sqrt{2}}(y - 3)$ //