Chapter 2

Motion Along a Straight Line

Introduction

An Overview of Mechanics

Mechanics: The study of how bodies react to forces acting on them.

Statics: The study of bodies in equilibrium.

Dynamics:

1. **Kinematics** – concerned with the geometric aspects of motion
2. **Kinetics** – concerned with the forces causing the motion

Introduction

- **Kinematics** is the study of motion.
- *Velocity* and *acceleration* are important physical quantities.
- A typical runner gains speed gradually during the course of a sprinting foot race and then slows down after crossing the finish line.

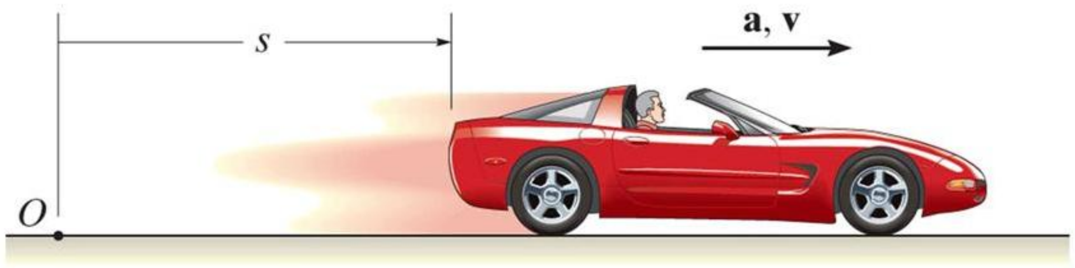
Introduction

The motion of large objects, such as rockets, airplanes, or cars, can often be analyzed as if they were particles.

Why?

If we measure the altitude of this rocket as a function of time, how can we determine its velocity and acceleration?

Introduction



A sports car travels along a straight road.

Can we treat the car as a particle?

If the car accelerates at a constant rate, how can we determine its position and velocity at some instant?

Displacement, time, and average velocity

- A particle moving along the x-axis has a coordinate x.
- The change in the particle's coordinate is $\Delta x = x_2 - x_1$.
- The average x-velocity of the particle is $v_{\text{av-}x} = \Delta x / \Delta t$.

Average x-velocity of a particle in **straight-line motion** during time interval from t_1 to t_2

x-component of the particle's displacement

$$v_{\text{av-}x} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

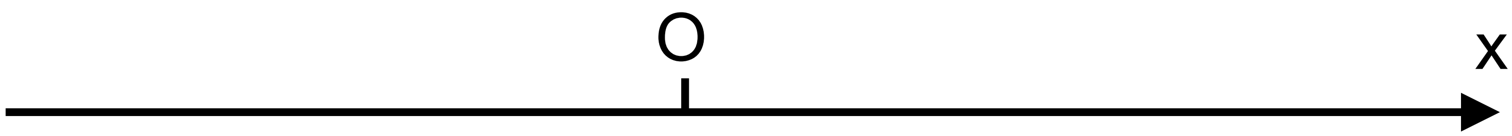
Final x-coordinate minus initial x-coordinate

Time interval

Final time minus initial time

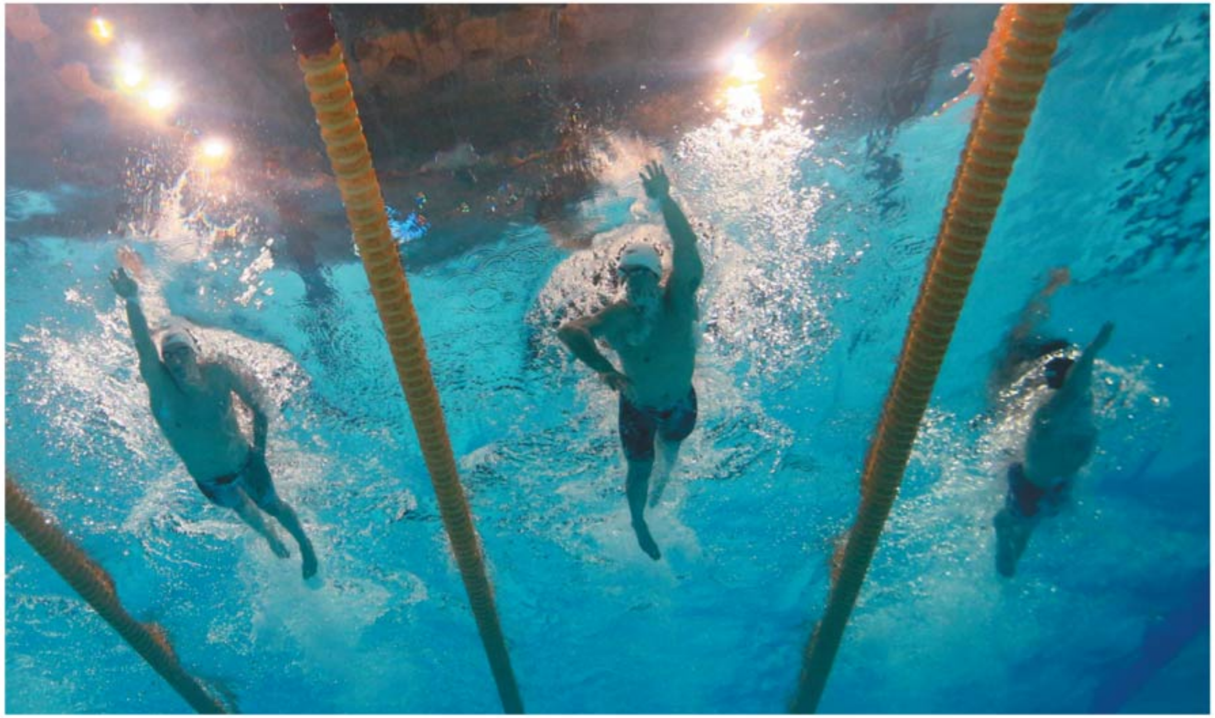
Rules for the sign of *x*-velocity

If x-coordinate is:	. . . x-velocity is:
Positive & increasing (getting more positive)	Positive: Particle is moving in $+x$-direction
Positive & decreasing (getting less positive)	Negative: Particle is moving in $-x$-direction
Negative & increasing (getting less negative)	Positive: Particle is moving in $+x$-direction
Negative & decreasing (getting more negative)	Negative: Particle is moving in $-x$-direction

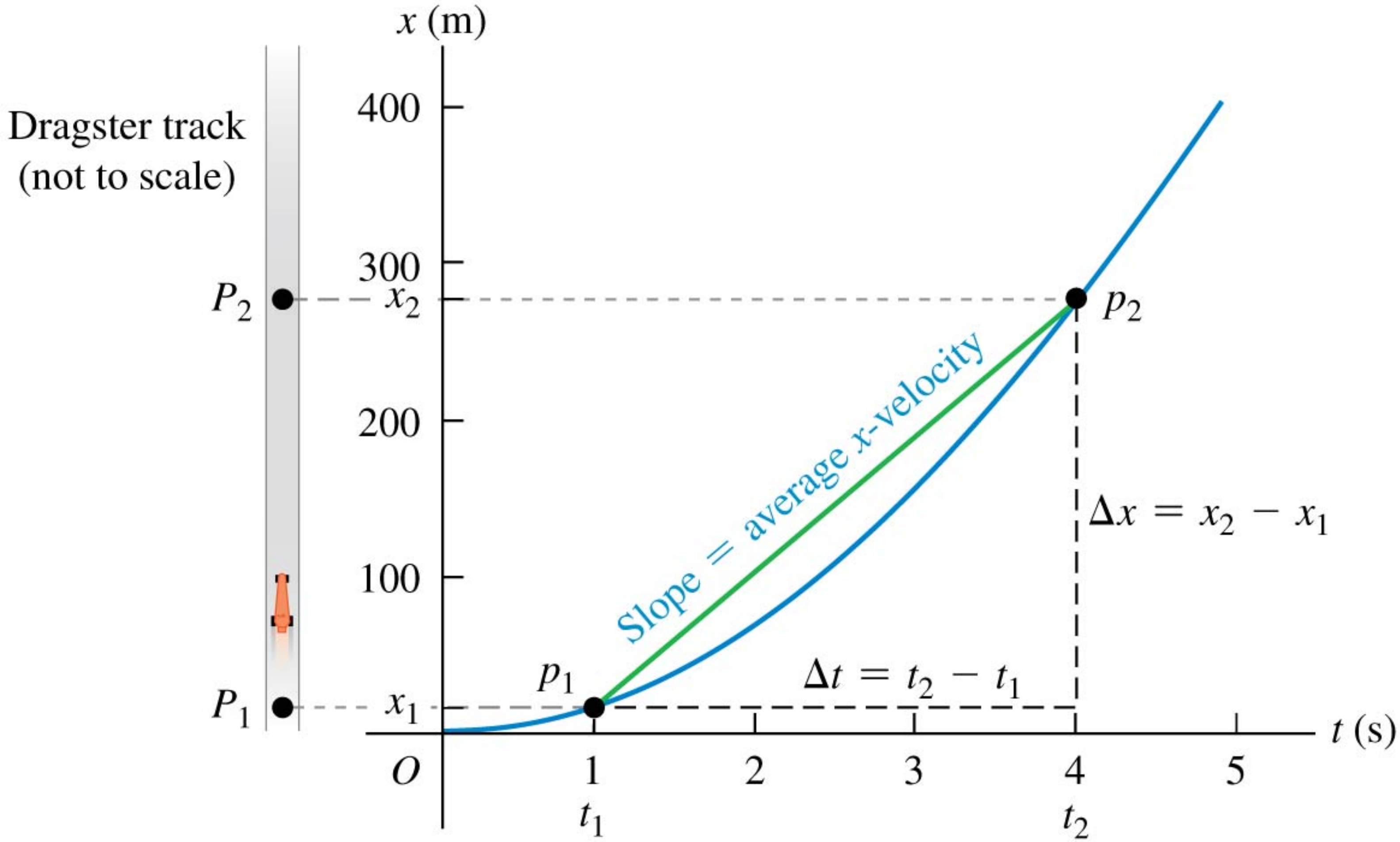


Average velocity

- The winner of a 50-m swimming race is the swimmer whose average velocity has the greatest magnitude.
- That is, the swimmer who traverses a displacement Δx of 50 m in the shortest elapsed time Δt.



A position-time graph



Instantaneous velocity

- The **instantaneous velocity** is the velocity at a specific instant of time or specific point along the path and is given by $v_x = dx/dt$.

- The average speed is *not* the magnitude of the average velocity!

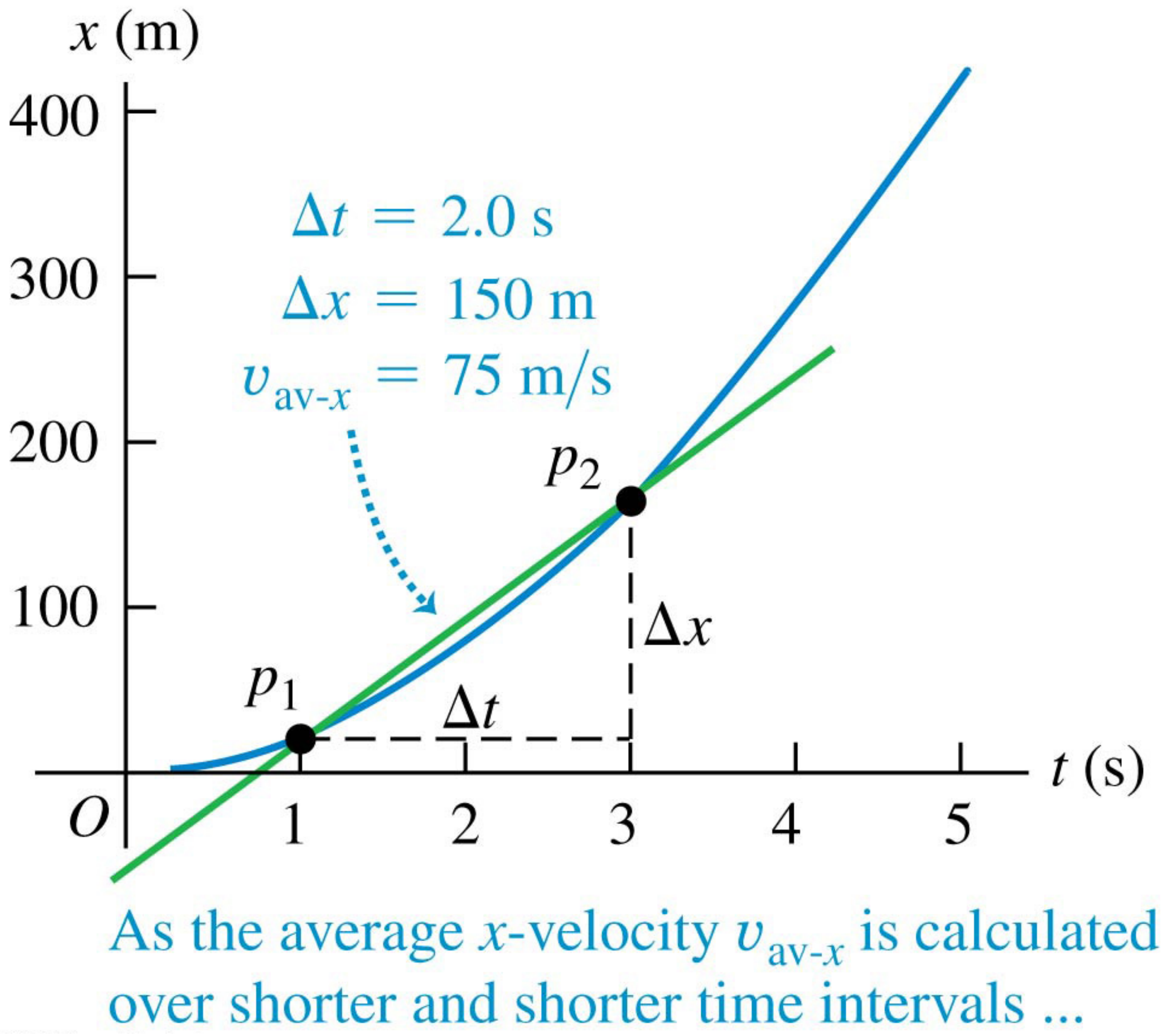
The **instantaneous** ***x*-velocity** of a particle in **straight-line motion** ...

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

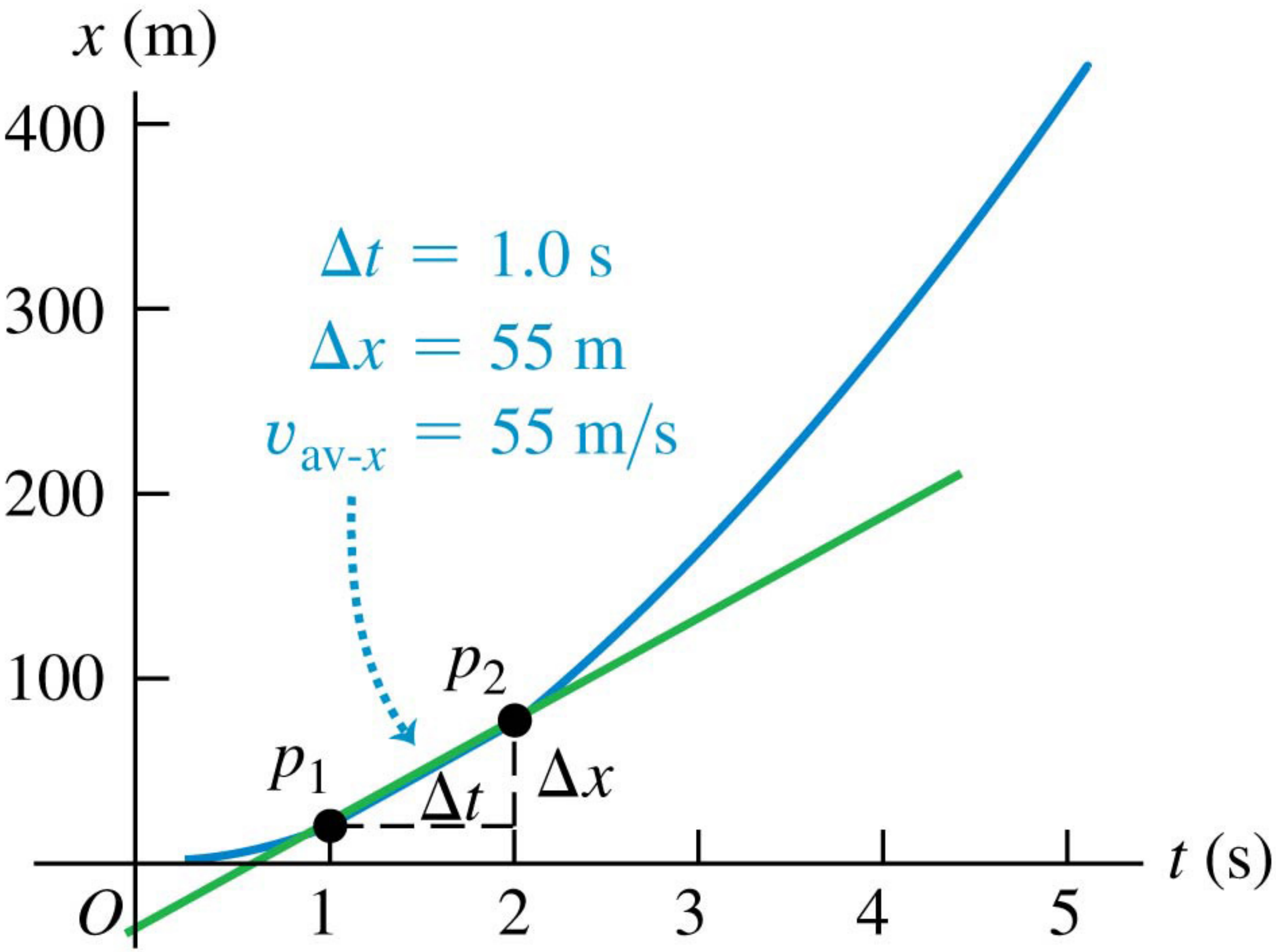
... equals the limit of the particle's average *x*-velocity as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's *x*-coordinate.

Finding velocity on an *x*-*t* graph

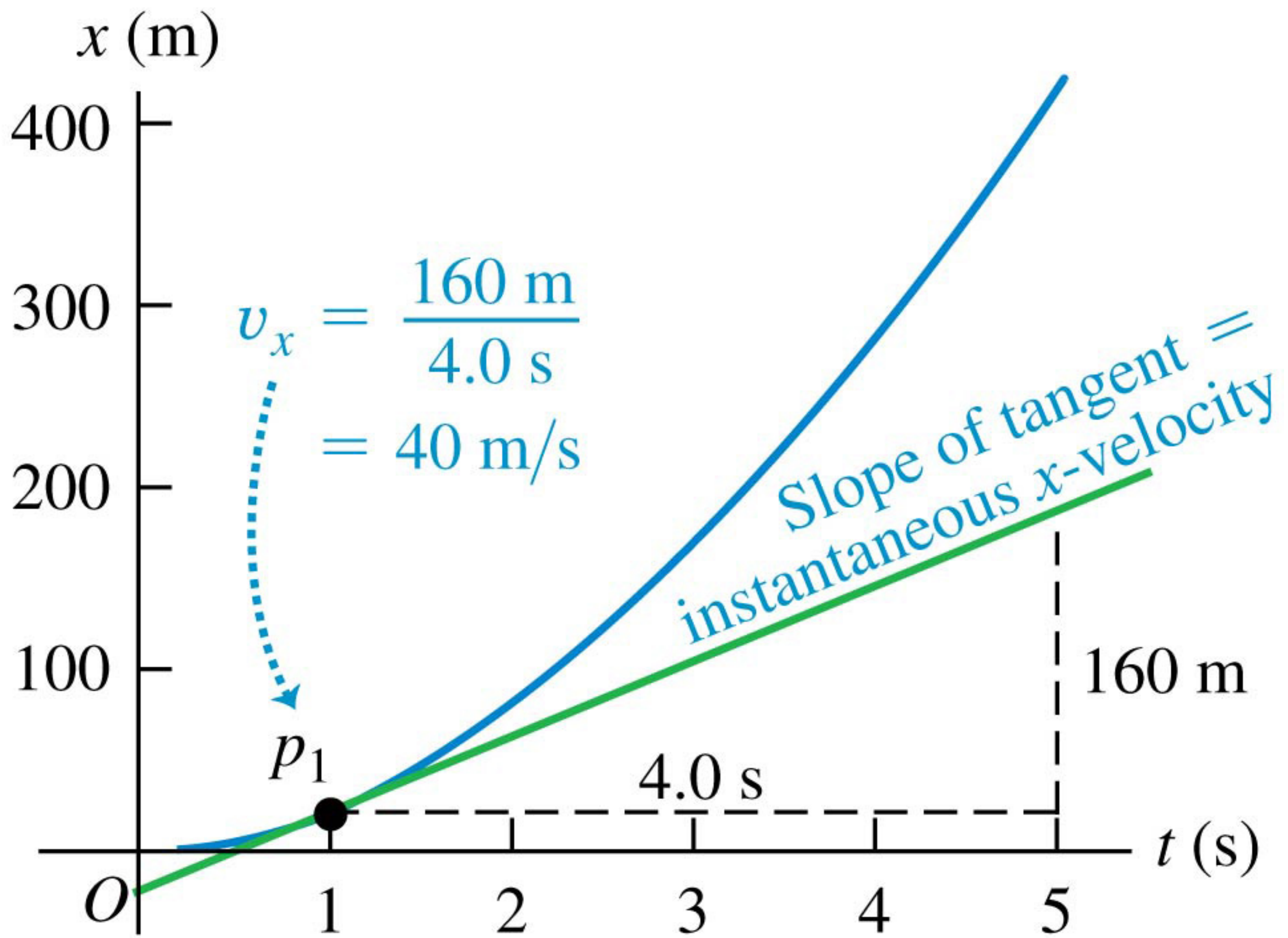


Finding velocity on an *x*-*t* graph



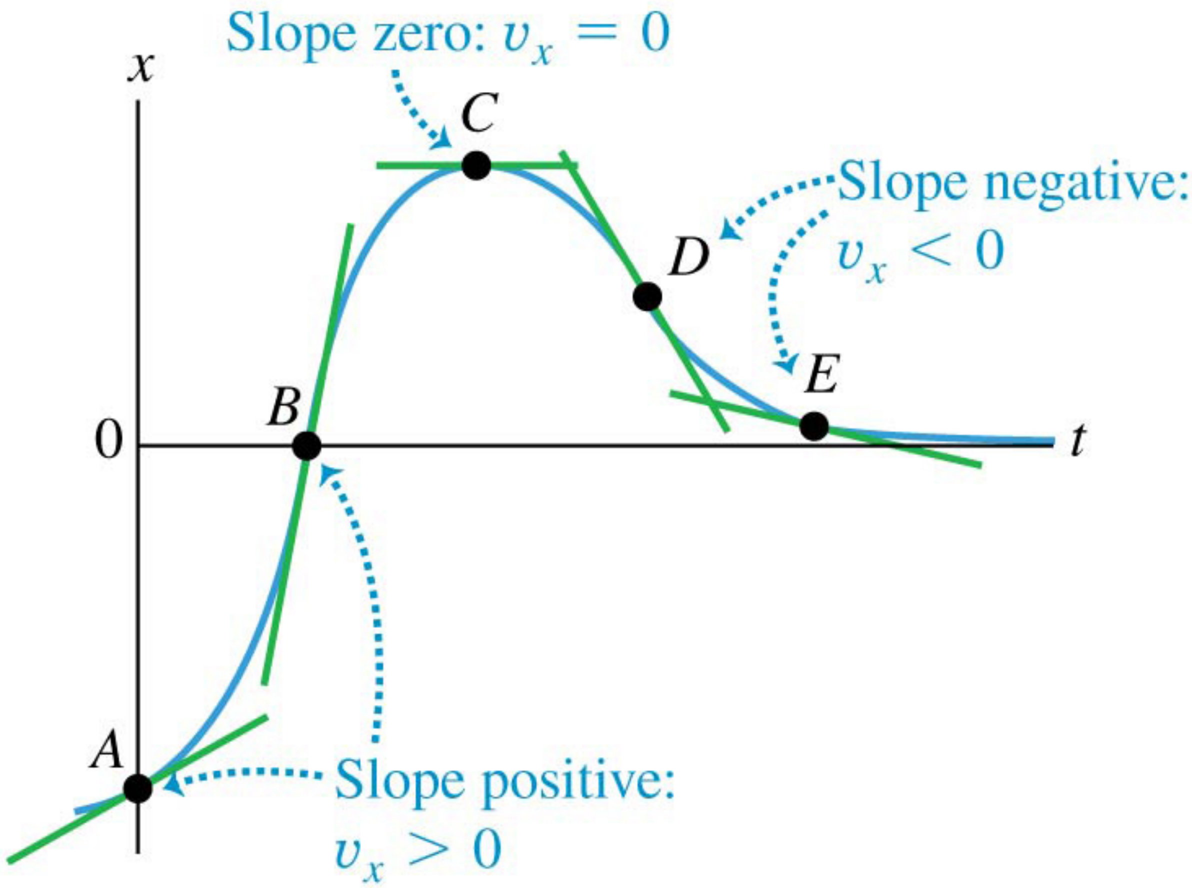
... its value $v_{av\text{-}x} = \Delta x/\Delta t$ approaches the instantaneous x-velocity.

Finding velocity on an *x*-*t* graph



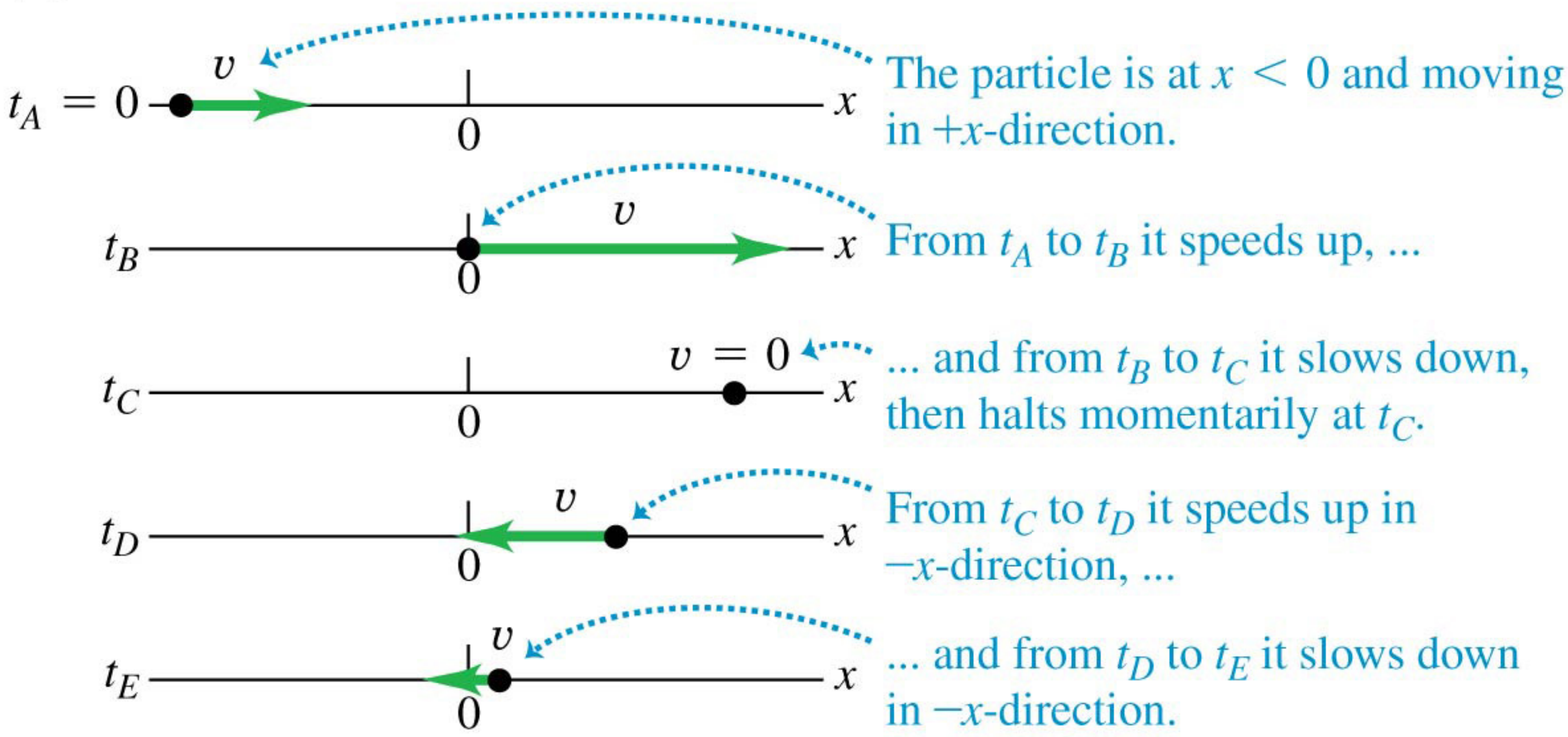
The instantaneous x-velocity v_x at any given point equals the slope of the tangent to the x-t curve at that point.

x-*t* graphs


Slope zero: $v_x = 0$
Slope negative: $v_x < 0$
Slope positive: $v_x > 0$
x
t
0
A
B
C
D
E

Motion diagrams

- Here is a *motion diagram* of the particle in the previous x-t graph.



Average acceleration

- Acceleration describes the rate of change of velocity with time.

- The *average* x-acceleration is $a_{\text{av-}x} = \Delta v_x / \Delta t$.

Average x-acceleration of a particle in **straight-line motion** during time interval from t_1 to t_2

Change in x-component of the particle's velocity

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{v_{2x} - v_{1x}}{t_2 - t_1}$$

Final x-velocity minus initial x-velocity

Time interval

Final time minus initial time

Instantaneous acceleration

- The *instantaneous* acceleration is $a_x = dv_x/dt$.

The **instantaneous** ***x*-acceleration** of a particle in **straight-line motion** ...

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

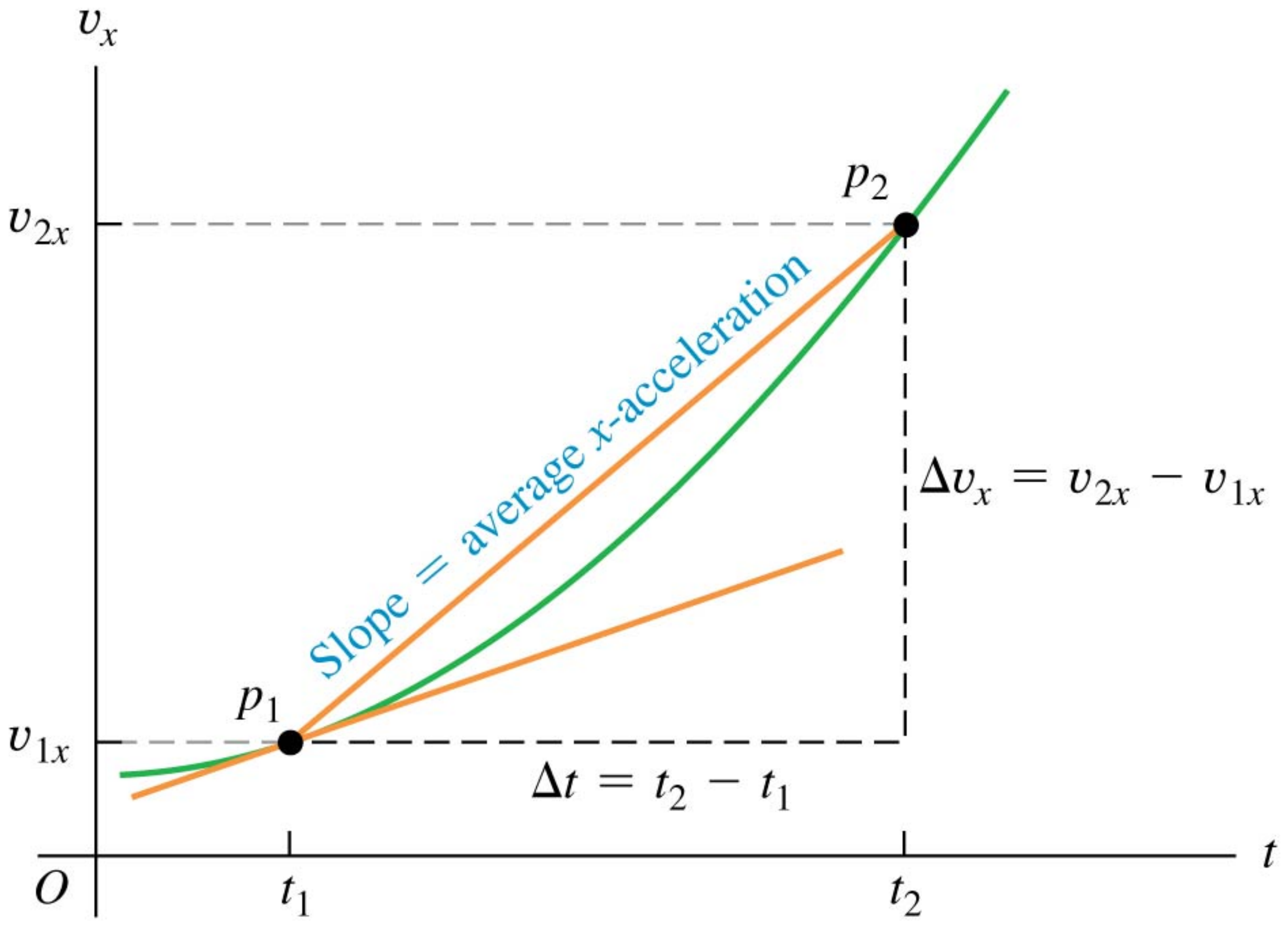
... equals the limit of the particle's average *x*-acceleration as the time interval approaches zero ...

... and equals the instantaneous rate of change of the particle's *x*-velocity.

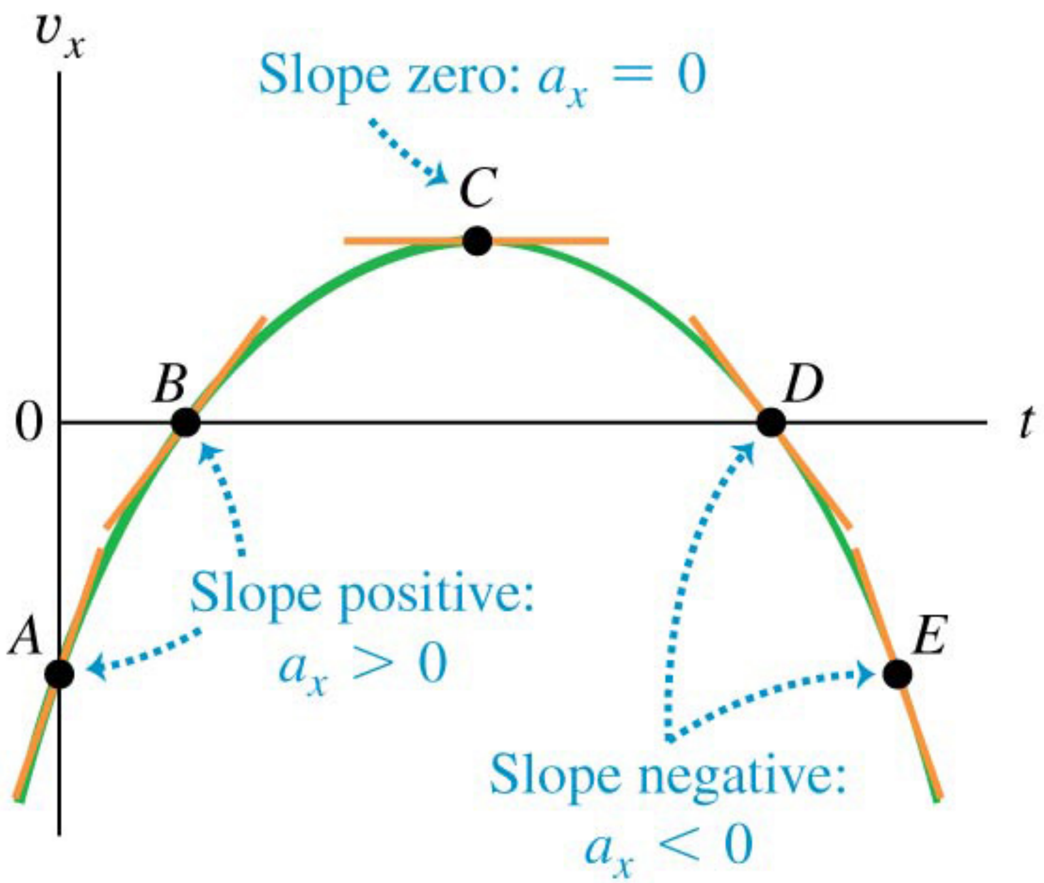
Rules for the sign of *x*-acceleration

If x-velocity is:	. . . x-acceleration is:
Positive & increasing (getting more positive)	Positive: Particle is moving in $+x$-direction & speeding up
Positive & decreasing (getting less positive)	Negative: Particle is moving in $+x$-direction & slowing down
Negative & increasing (getting less negative)	Positive: Particle is moving in $-x$-direction & slowing down
Negative & decreasing (getting more negative)	Negative: Particle is moving in $-x$-direction & speeding up

Finding acceleration on a v_x-t graph



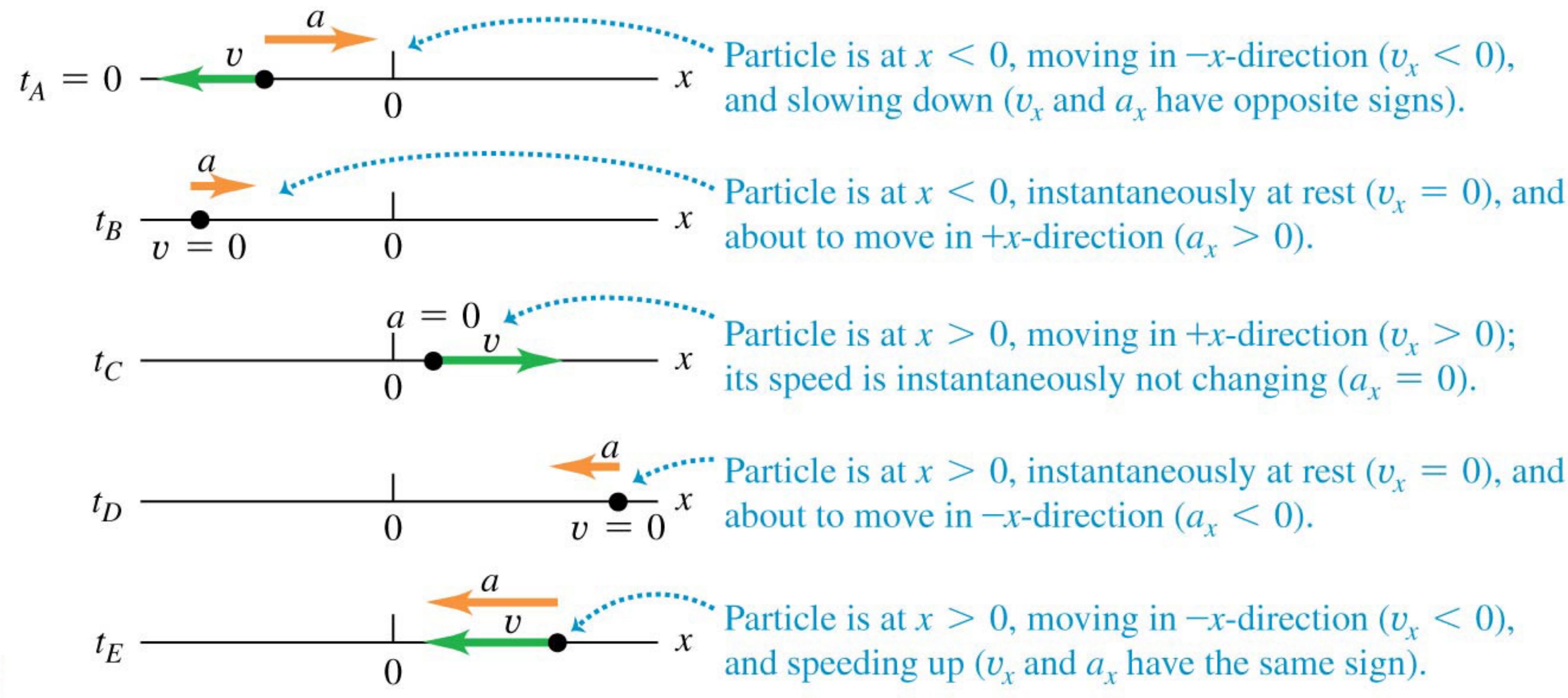
A v_x-t graph



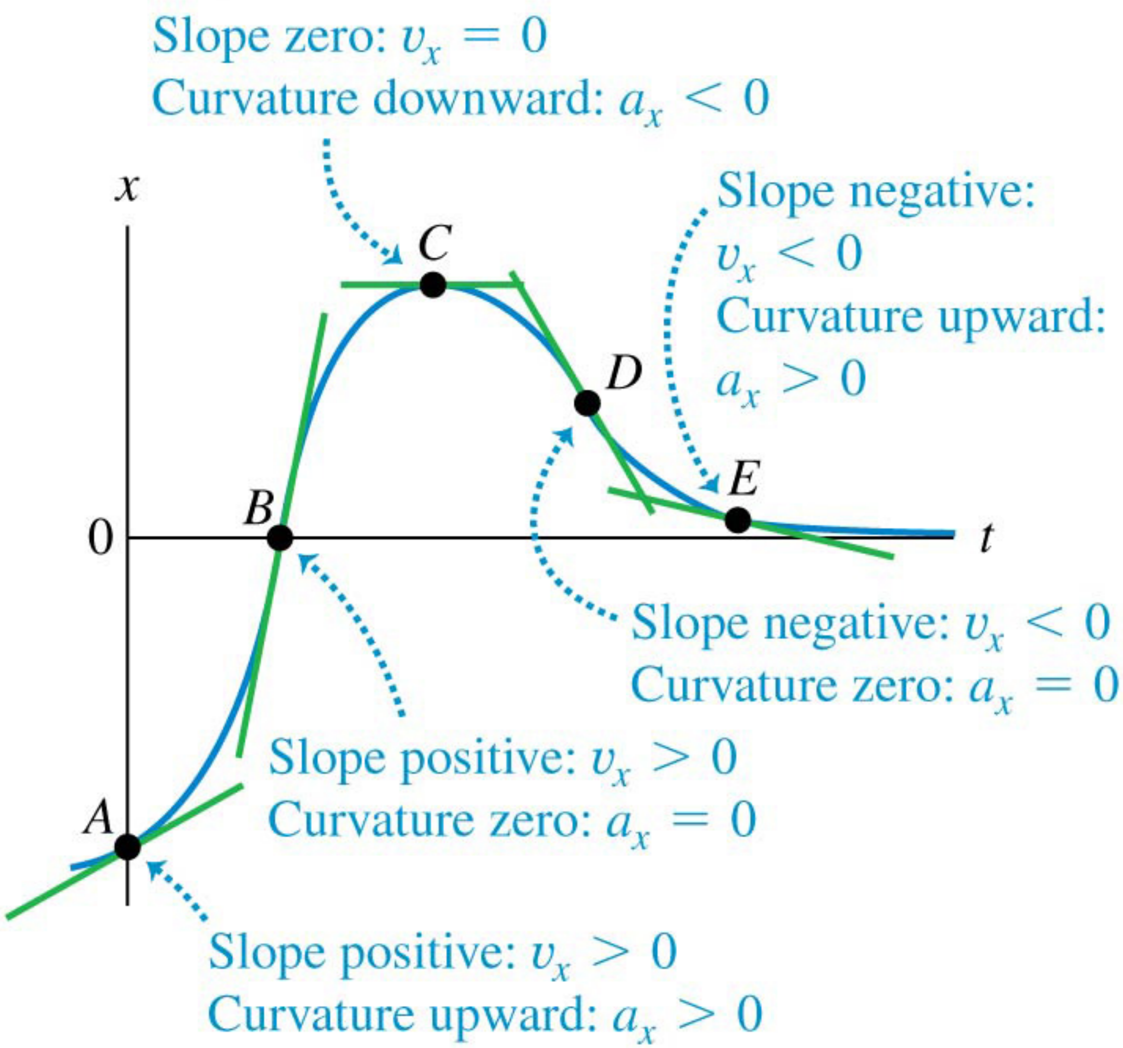
v_x
Slope zero: $a_x = 0$
C
B
D
0
t
A
E
Slope positive: $a_x > 0$
Slope negative: $a_x < 0$

Motion diagrams

- Here is the motion diagram for the particle in the previous v_x-t graph.



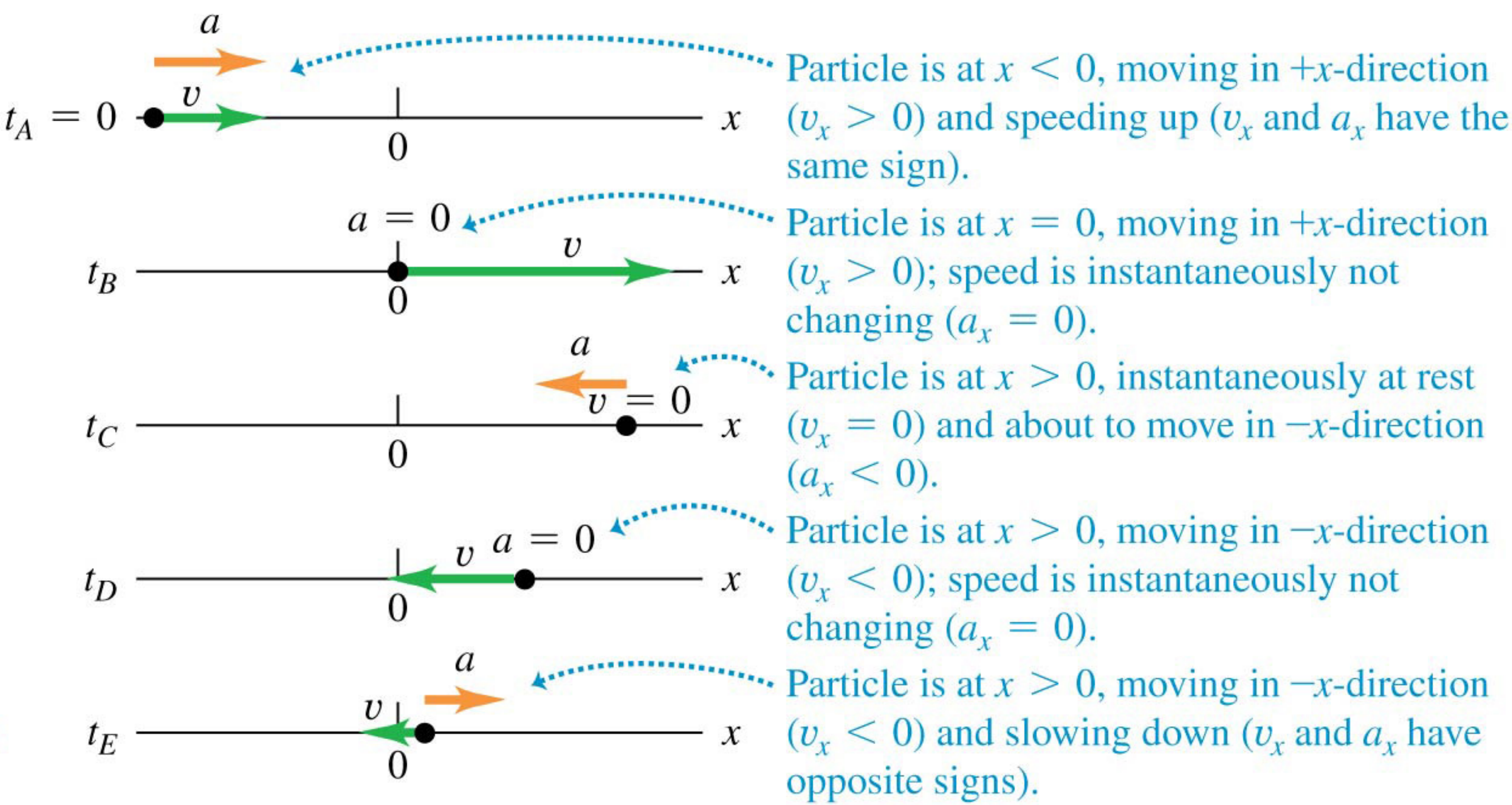
A *x*-*t* graph



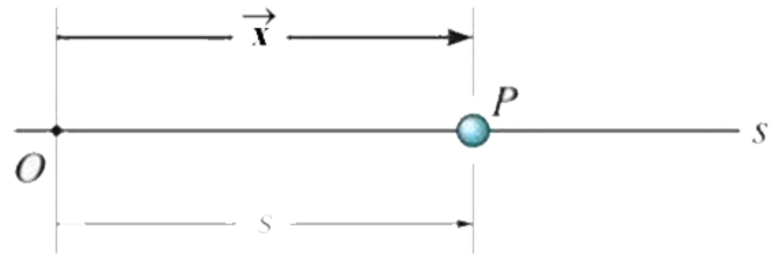
Slope zero: $v_x = 0$
Curvature downward: $a_x < 0$
Slope negative: $v_x < 0$
Curvature upward: $a_x > 0$
x
C
D
E
B
0
t
A
Slope negative: $v_x < 0$
Curvature zero: $a_x = 0$
Slope positive: $v_x > 0$
Curvature zero: $a_x = 0$
Slope positive: $v_x > 0$
Curvature upward: $a_x > 0$

Motion diagrams

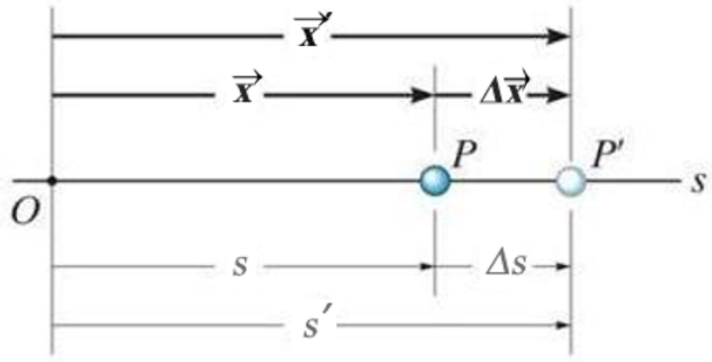
- Here is the motion diagram for the particle in the previous v_x-t graph.



Rectilinear Kinematics: General Formalism



Position



Displacement

A particle travels along a straight-line path defined by the coordinate axis s. The position of the particle at any instant, relative to the origin, O, is defined by the position vector $\vec{x}$, or the scalar s. Scalar s can be positive or negative. Typical units for $\vec{x}$ and s are meters (m) or feet (ft).

The displacement of the particle is defined as its change in position.

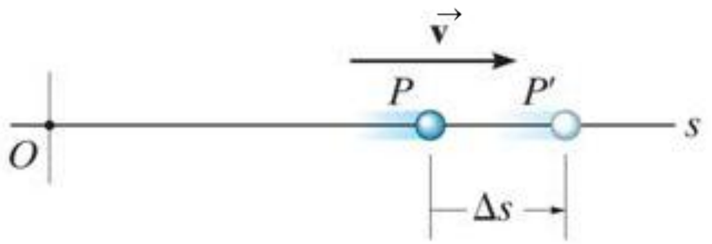
Vector form: $\Delta \vec{x} = \vec{x}' - \vec{x}$

Scalar form: $\Delta s = s' - s$

The total distance traveled by the particle, s_T, is a positive scalar that represents the total length of the path over which the particle travels.

Rectilinear Kinematics: General Formalism

Velocity is a measure of the rate of change in the position of a particle. It is a vector quantity (it has both magnitude and direction). The magnitude of the velocity is called speed, with units of m/s or ft/s.

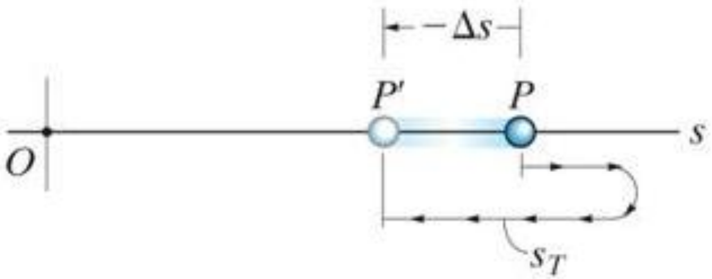


Velocity

The average velocity of a particle during a time interval Δt is

$$\vec{v}_{avg} = \Delta\vec{x} / \Delta t$$

The instantaneous velocity is the time-derivative of position. $\vec{v} = d\vec{x} / dt$



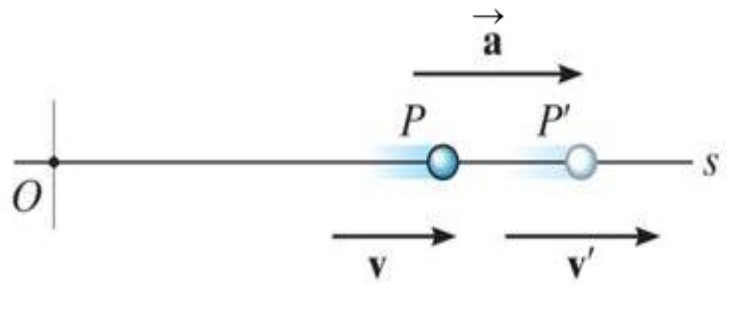
Average velocity and Average speed

Speed is the magnitude of velocity: $v = ds / dt$

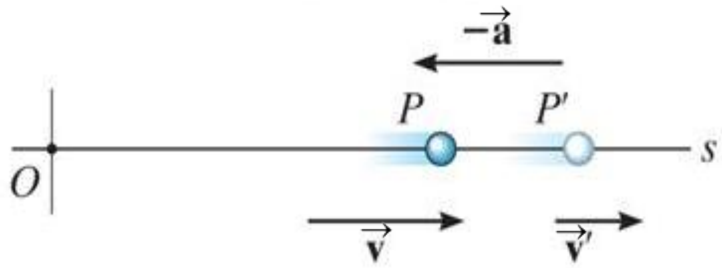
Average speed is the total distance traveled divided by elapsed time: $(v_{sp})_{avg} = s_T / \Delta t$

Rectilinear Kinematics: General Formalism

Acceleration is the rate of change in the velocity of a particle. It is a vector quantity. Typical units are m/s^2.



Acceleration



Deceleration

The instantaneous acceleration is the time derivative of velocity.

Vector form: $\vec{a} = d\,\vec{v}\,/\,dt$

Scalar form: $a = dv\,/\,dt = d^2s\,/\,dt^2$

Acceleration can be positive (speed increasing) or negative (speed decreasing).

The derivative equations for velocity and acceleration can be manipulated to get $a\,ds = v\,dv$

Quiz

1. In dynamics, a particle is assumed to have __________.

   A) both translation and rotational motions

   ✔ B) only a mass

   C) a mass but the size and shape cannot be neglected

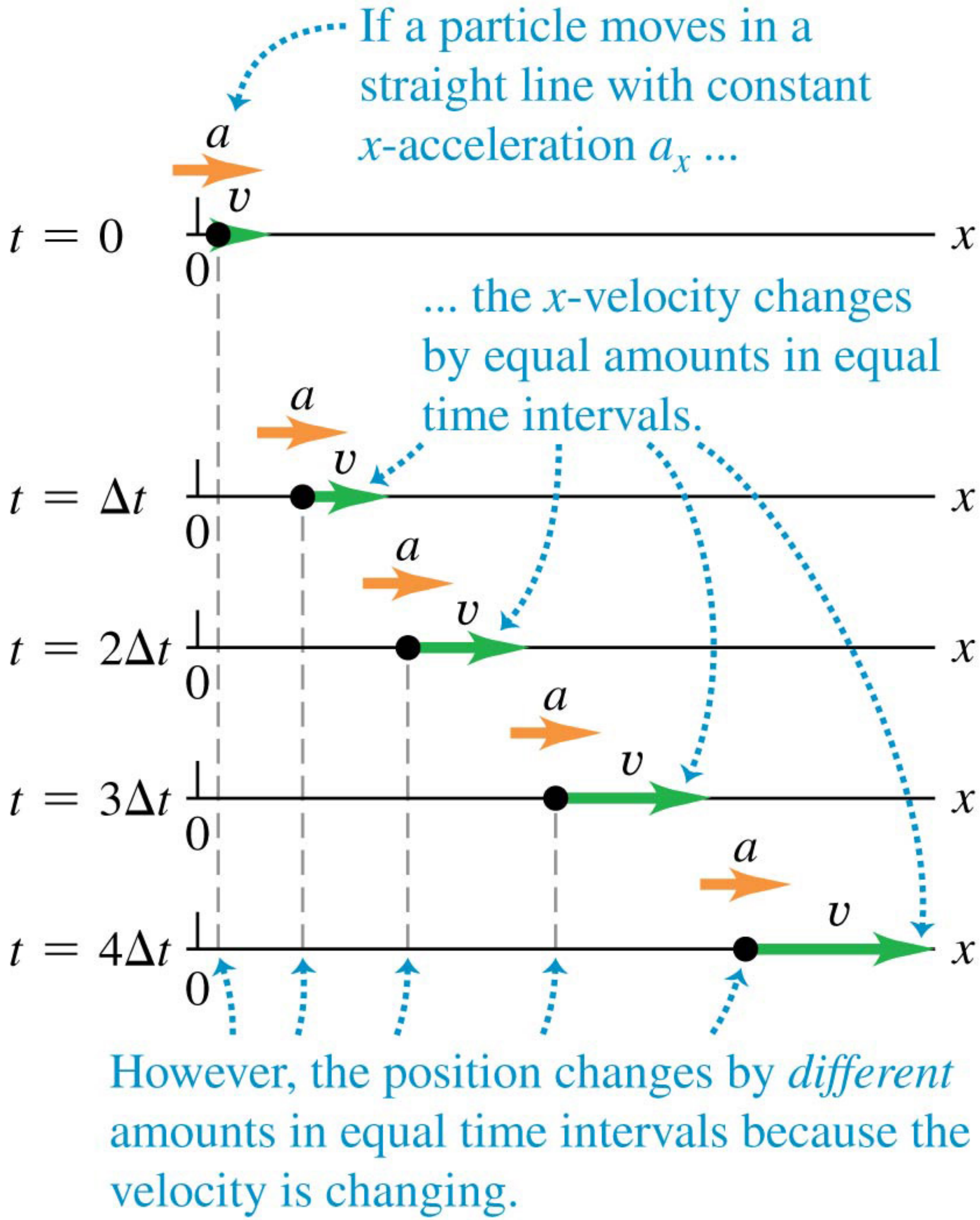
   D) no mass or size or shape, it is just a point

2. The average speed is defined as __________.

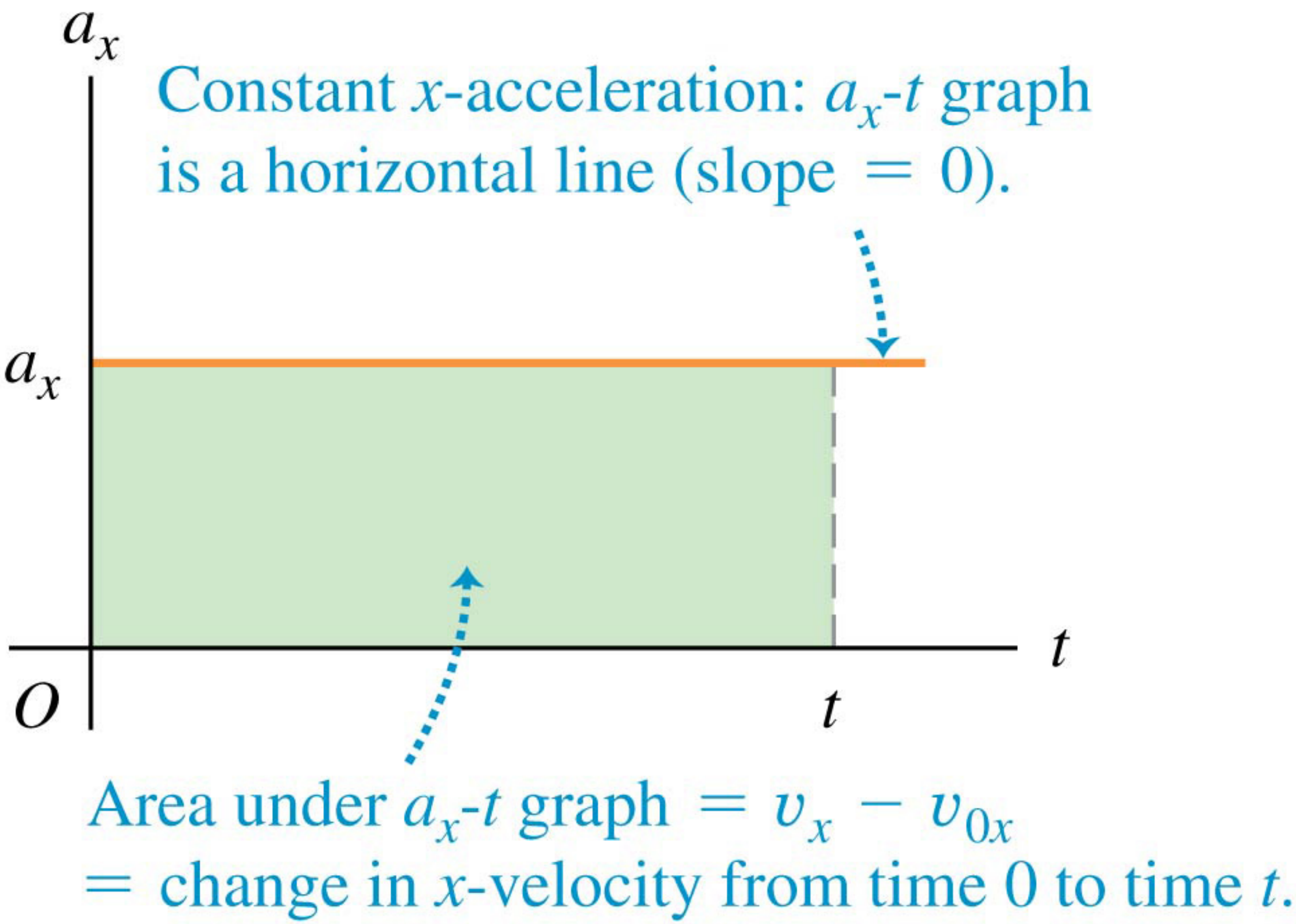
   A) $\Delta x / \Delta t$ B) $\Delta s / \Delta t$

   ✔ C) $s_T / \Delta t$ D) None of the above.

Motion with constant acceleration



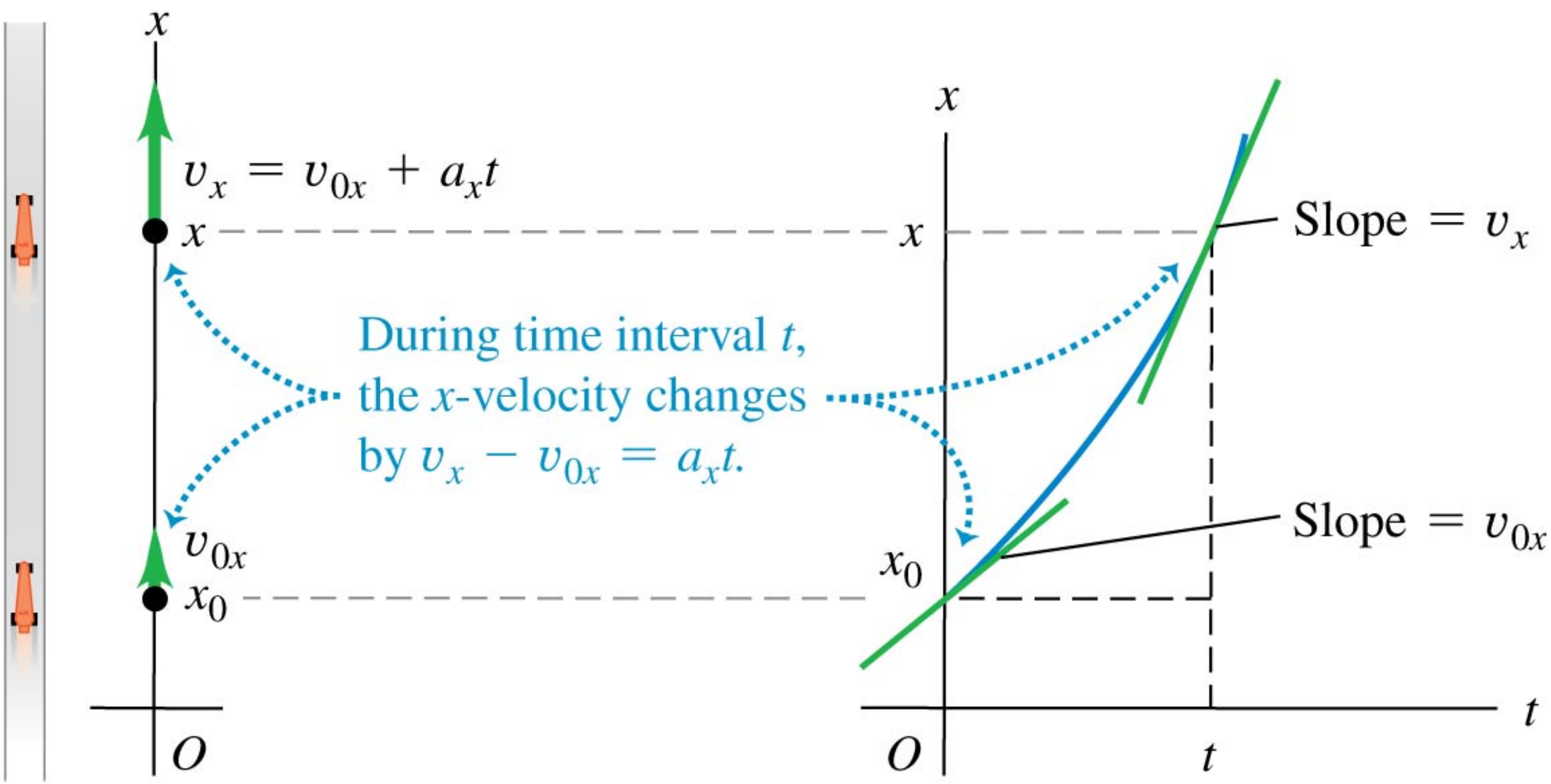
Motion with constant acceleration



A position-time graph

(a) A race car moves in the x-direction with constant acceleration.

(b) The x-t graph



The equations of motion with constant acceleration

- The four equations below apply to any straight-line motion with constant acceleration a_x.

Equation		Includes Quantities			
$v_x = v_{0x} + a_x t$	(2.8)	t		v_x	a_x
$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$	(2.12)	t	x		a_x
$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$	(2.13)		x	v_x	a_x
$x - x_0 = \frac{1}{2}(v_{0x} + v_x)t$	(2.14)	t	x	v_x	

The equations of motion with constant acceleration

- The four equations apply to any straight-line motion with constant acceleration a_x.
- First equation gives the velocity at time t
- Second equation gives the position of the particle at time t

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x \left(x - x_0 \right)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t$$

You should be able to solve most of the problems using these four equations

The equations of motion with constant acceleration

- Third equation gives you the velocity at t in terms of the initial velocity, acceleration and the distance travelled (displacement between t=0 and t).

- Fourth equation gives the distance travelled in terms of initial velocity, final velocity and time spent.

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x\left(x - x_0\right)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2}\right) t$$

Derivations

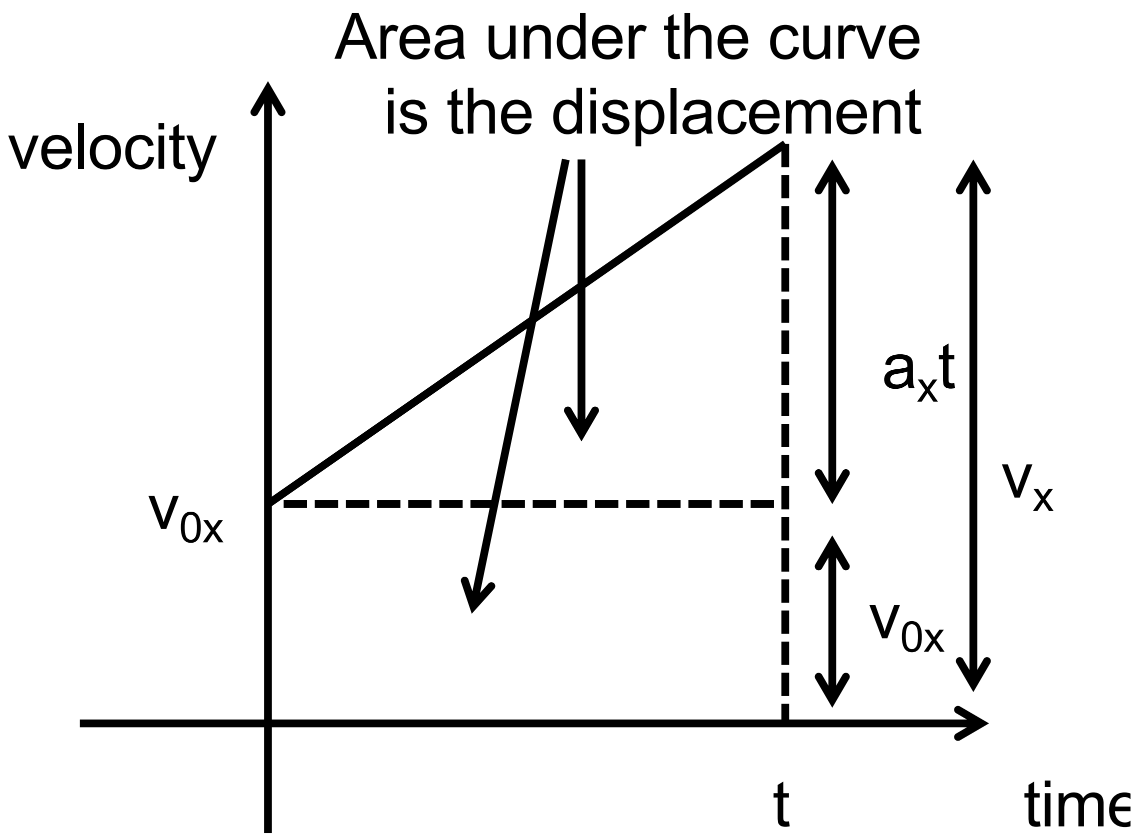
- The 1st ,2nd and 4th equations can be derived using the velocity and time graph.
- 3rd equation derived from 1st and 2nd

$$v_x = v_{0x} + a_x t$$

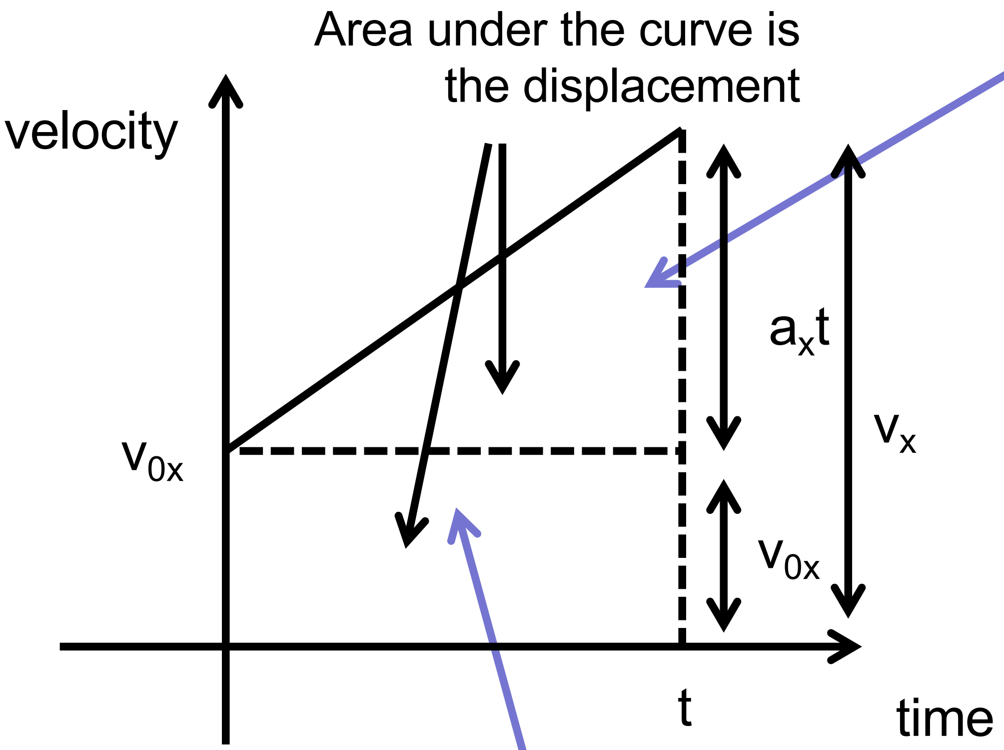
$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x \left(x - x_0 \right)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t$$



Derivations



1st eqt.

$a_x t = v_x - v_{0x}$

$v_x = v_{0x} + a_x t$

3rd eqt.

$$v_x = v_{0x} + a_x t \Rightarrow v_{0x} = v_x - a_x t$$

$$\Rightarrow v_{0x}^2 = v_x^2 - 2a_x v_x t + a_x^2 t^2$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x - x_0 = v_{0x} t + \frac{1}{2} a_x t^2$$

$$\Rightarrow x - x_0 = (v_x - a_x t)t + \frac{1}{2} a_x t^2$$

$$\Rightarrow 2a_x (x - x_0) = 2a_x v_x t - a_x^2 t^2$$

$$\Rightarrow 2a_x (x - x_0) = v_x^2 - v_{0x}^2$$

$$\Rightarrow v_x^2 = v_{0x}^2 + 2a_x \left(x - x_0\right)$$

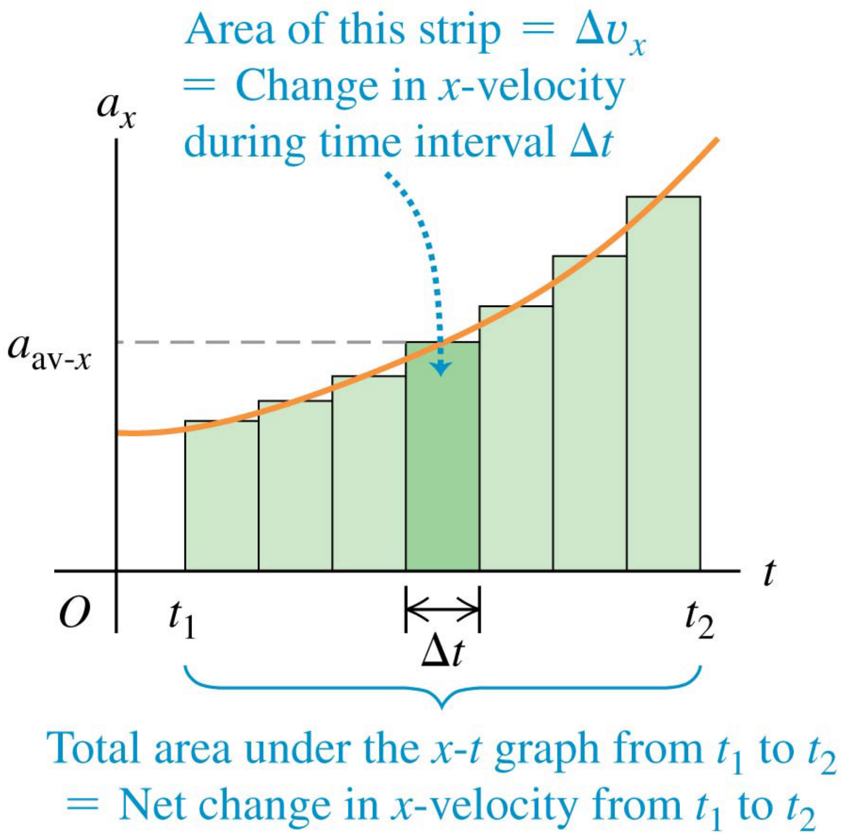
2nd & 4th eqt.

Area $= x - x_0 = (v_x + v_{0x})t/2$

$x - x_0 = (v_{0x} + v_{0x} + a_x t)t/2 = v_{0x} t + a_x t^2/2$

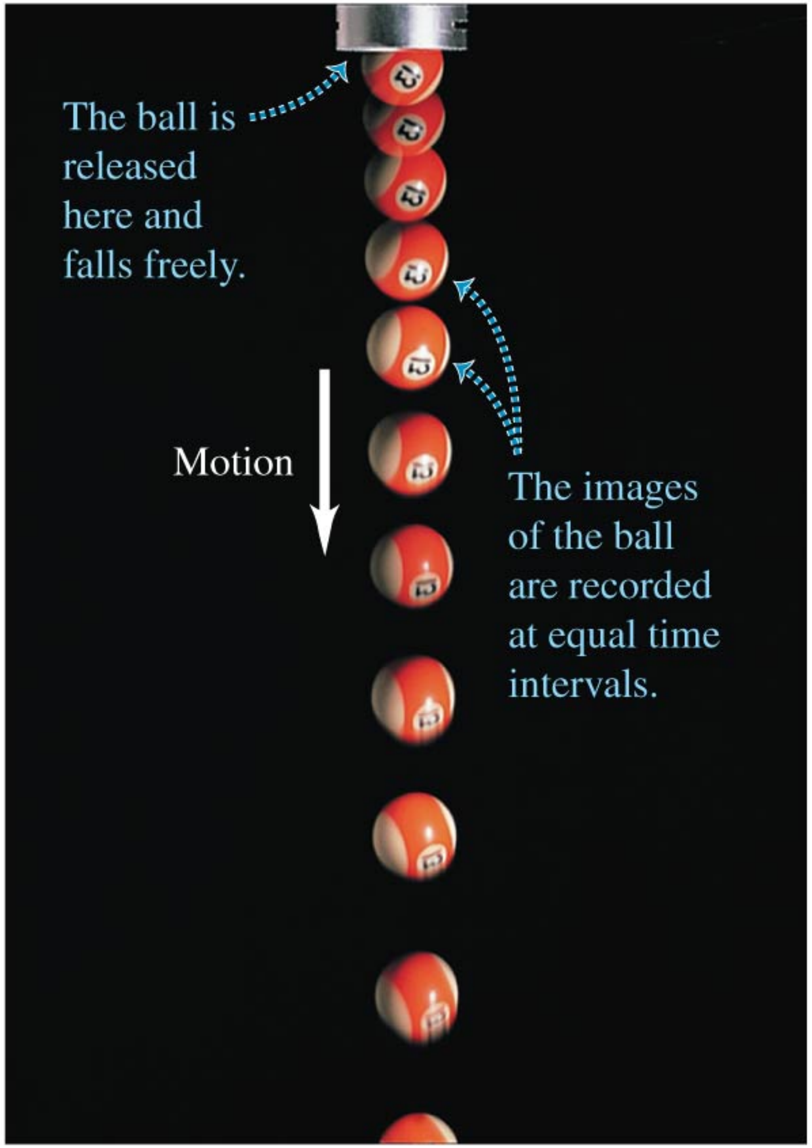
Velocity and position by integration

- The acceleration of a car is not always constant.
- The motion may be integrated over many small time intervals to give $v_x = v_{0x} + \int_0^t a_x dt$ and $x = x_0 + \int_0^t v_x dt$.



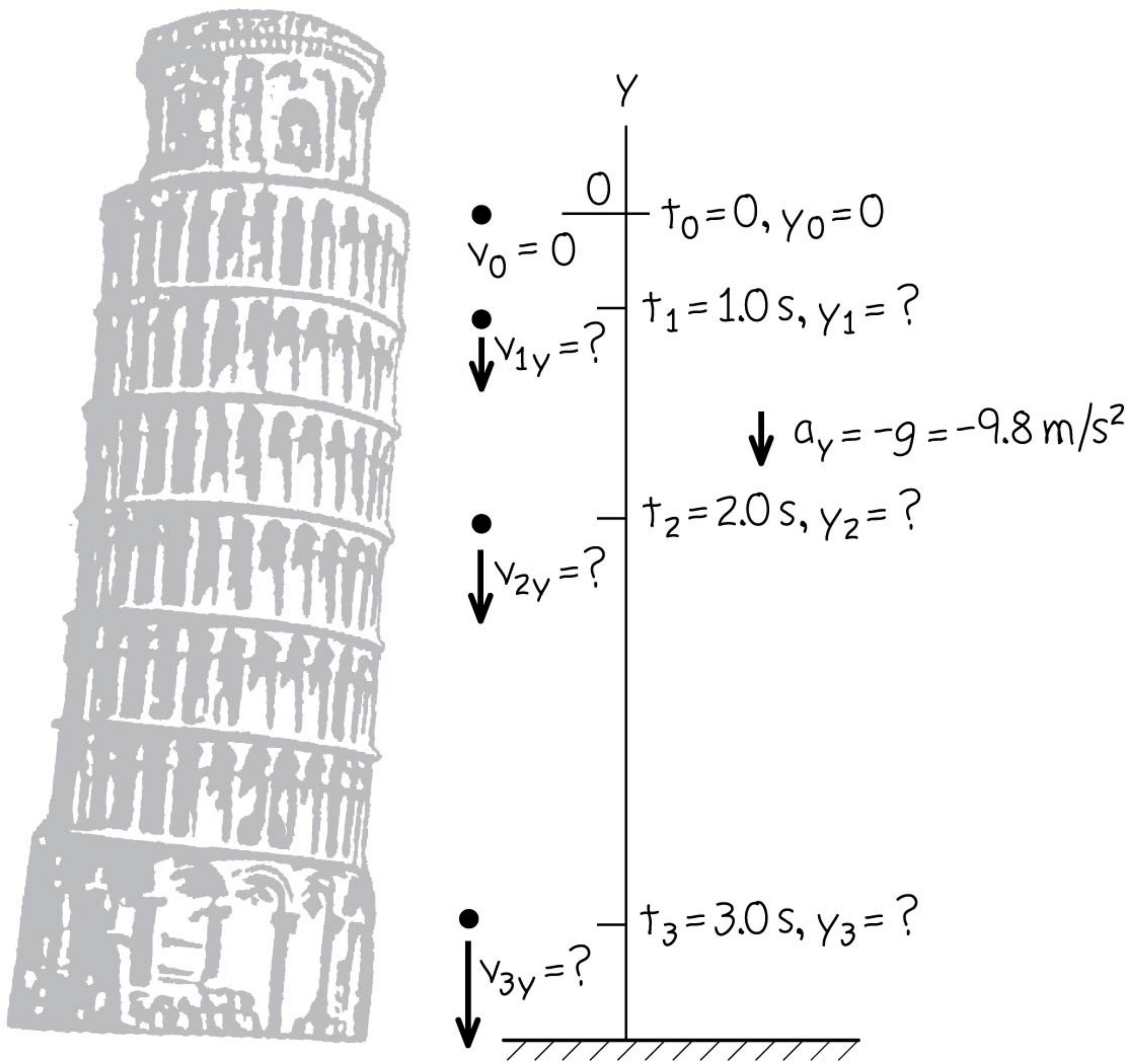
Freely falling bodies

- **Free fall** is the motion of an object under the influence of only gravity.
- In the figure, a strobe light flashes with equal time intervals between flashes.
- The velocity change is the same in each time interval, so the acceleration is constant.



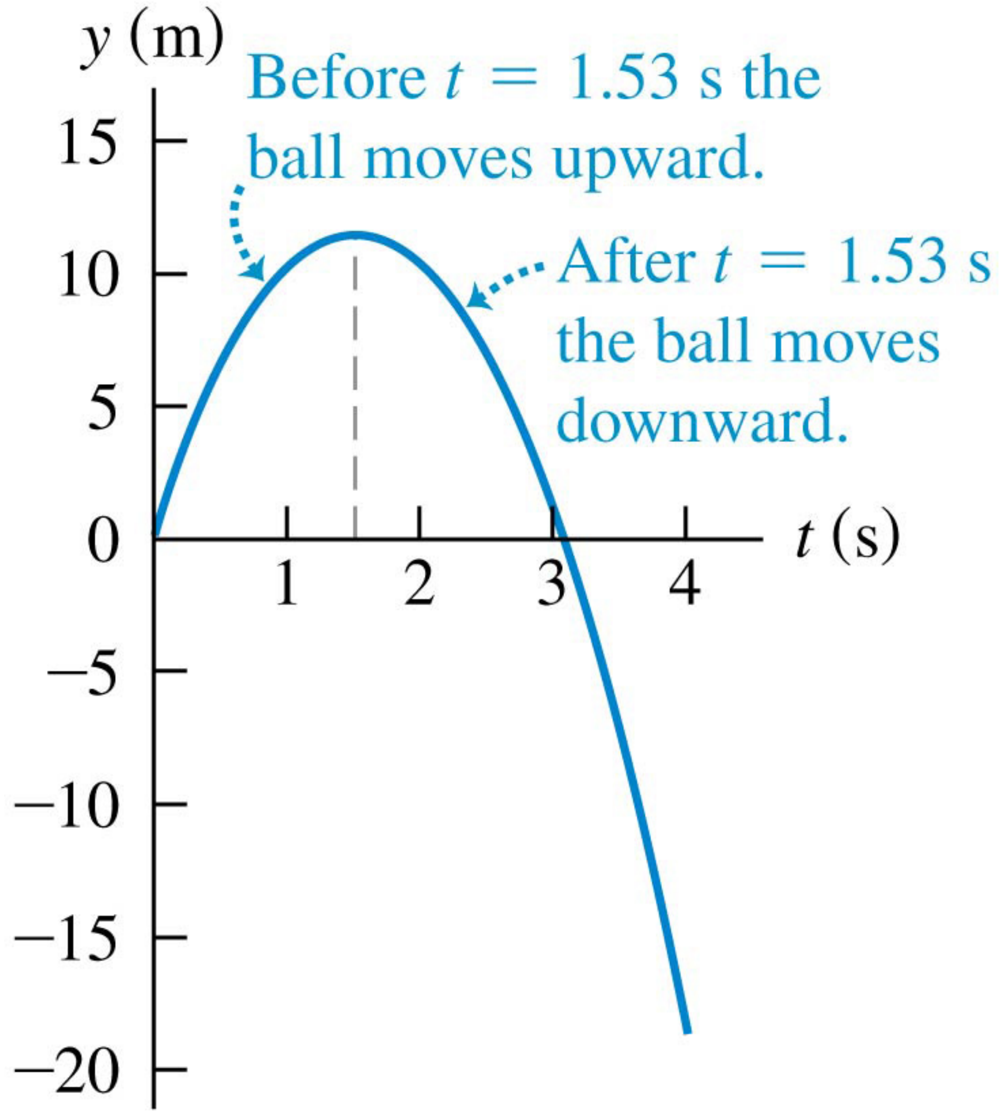
A freely falling coin

- If there is no air resistance, the downward acceleration of any freely falling object is $g = 9.8\ \text{m/s}^2 = 32\ \text{ft/s}^2$.



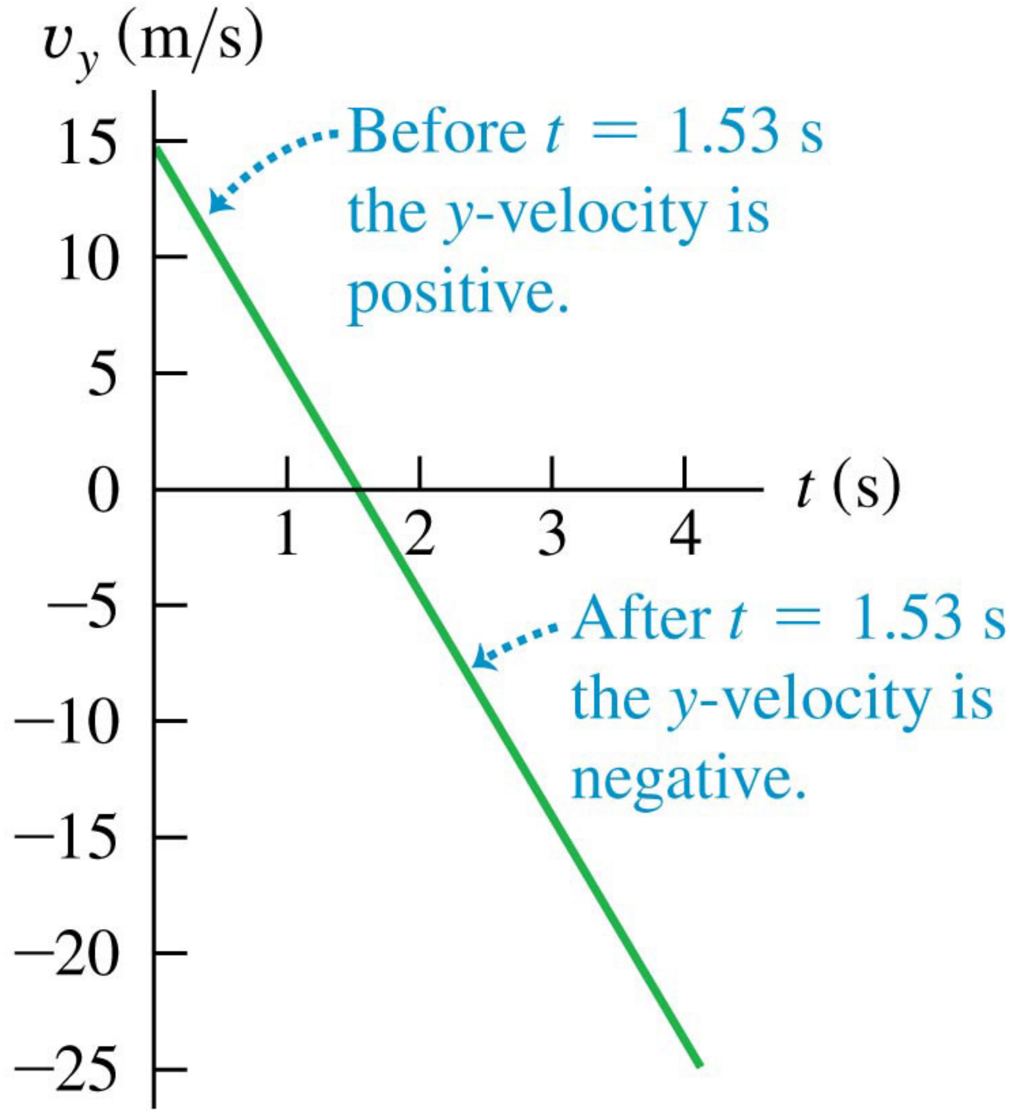
Up-and-down motion in free fall

- Position as a function of time for a ball thrown upward with an initial speed of 15.0 m/s.



Up-and-down motion in free fall

- Velocity as a function of time for a ball thrown upward with an initial speed of 15.0 m/s.
- The vertical velocity, but *not the acceleration*, is zero at the highest point.



Rectilinear kinematics: general formalism

- Differentiate position to get velocity and acceleration.

$$v = dx/dt\,; \qquad a = dv/dt \quad \text{or} \quad a = v\,dv/dx$$

- Integrate acceleration for velocity and position.

Velocity:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a\,dt \quad \text{or} \quad \int_{v_o}^{v} v\,dv = \int_{x_o}^{x} a\,dx$$

Position:

$$\int_{x_o}^{x} dx = \int_{o}^{t} v\,dt$$

- Note that x_o and v_o represent the initial position and velocity of the particle at $t = 0$.

Rectilinear kinematics: constant acceleration

The three kinematic equations can be integrated for the special case when acceleration is constant ($a = a_c$). A common example of constant acceleration is gravity; i.e., a body freely falling toward earth. In this case, $a_c = g = 9.81$ m/s^2 downward. These equations are:

$$\int_{v_o}^{v} dv = \int_{o}^{t} a_c \, dt \qquad \text{yields} \qquad v = v_o + a_c t$$

$$\int_{x_o}^{x} dx = \int_{o}^{t} v \, dt \qquad \text{yields} \qquad x = x_o + v_o t + (1/2)\, a_c t^2$$

$$\int_{v_o}^{v} v \, dv = \int_{x_o}^{x} a_c \, dx \qquad \text{yields} \qquad v^2 = (v_o)^2 + 2a_c(x - x_o)$$

Example 1

Given: A particle travels along a straight line to the right with a velocity of $v = (4\,t - 3\,t^2)$ m/s where t is in seconds. Also, $s = 0$ when $t = 0$.

Find: The position and acceleration of the particle when $t = 4$ s.

Plan: Establish the positive coordinate, s, in the direction the particle is traveling. Since the velocity is given as a function of time, take a derivative of it to calculate the acceleration. Conversely, integrate the velocity function to calculate the position.

Example 1

Solution:

1) Take a derivative of the velocity to determine the acceleration.

$$a = dv / dt \quad = d(4\,t - 3\,t^2) / dt = 4 - 6\,t$$

$$\Rightarrow \quad a = -\,20\ m/s^2 \text{ (or in the } \leftarrow \text{ direction) when } t = 4\ s$$

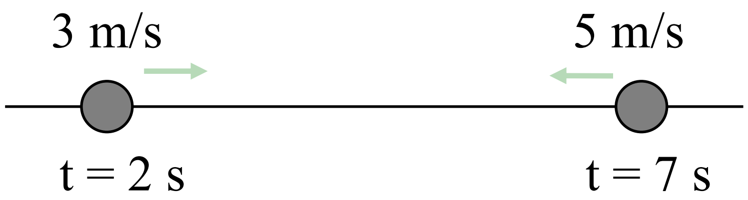
2) Calculate the distance traveled in 4s by integrating the velocity using $x_o = 0$:

$$v = ds / dx \quad \Rightarrow \quad dx = v\,dt \quad \Rightarrow \int_{x_o}^{x} dx = \int_{o}^{t} (4\,t - 3\,t^2)\,dt$$

$$\Rightarrow \quad x - x_o = 2\,t^2 - t^3$$

$$\Rightarrow \quad x - 0 = 2(4)^2 - (4)^3 \quad \Rightarrow \quad x = -\,32\ m\ (\text{ or } \leftarrow)$$

Quiz



1. A particle moves along a horizontal path with its velocity varying with time as shown. The average acceleration of the particle is __________.

   A) $0.4\ m/s^2$ $\rightarrow$ B) $0.4\ m/s^2$ $\leftarrow$

   C) $1.6\ m/s^2$ $\rightarrow$ ✔ D) $1.6\ m/s^2$ $\leftarrow$

2. A particle has an initial velocity of 30 m/s to the left. If it then passes through the same location 5 seconds later with a velocity of 50 m/s to the right, the average velocity of the particle during the 5 s time interval is ________.

   A) 10 m/s $\rightarrow$ B) 40 m/s $\rightarrow$

   C) 16 m/s $\rightarrow$ ✔ D) 0 m/s

Example 2

Given: A particle is moving along a straight line such that its velocity is defined as $v = (-4s^2)$ m/s, where s is in meters.

Find: The velocity and acceleration as functions of time if $s = 2$ m when $t = 0$.

Plan: Since the velocity is given as a function of distance, use the equation $v=ds/dt$.

1) Express the distance in terms of time.

2) Take a derivative of it to calculate the velocity and acceleration.

Example 2

Solution:

1) Since $v = (-4s^2)$

$$v = \frac{ds}{dt} = -4\,s^2 \implies -4\,dt = \frac{ds}{s^2}$$

Determine the distance by integrating using $s_0 = 2$.

$$\int_0^t (-4)dt = \int_2^s s^{-2}ds$$ Notice that s = 2 m when t = 0.

$$-4\,t\Big]_0^t = -\frac{1}{s}\Big]_2^s \Rightarrow -4\,t = -\left(\frac{1}{s} - \frac{1}{2}\right)$$

$$\Rightarrow \quad 4\,t + \frac{1}{2} = \frac{1}{s} \qquad \Rightarrow \qquad s = \frac{2}{8t+1}$$

Example 2

2) Take a derivative of distance to calculate the velocity and acceleration.

$$s = \frac{2}{8t+1} \quad \text{m}$$

$$\Rightarrow \quad v = \frac{ds}{dt} = \frac{2\cdot(-1)\cdot 8}{(8t+1)^2} = -\frac{16}{(8t+1)^2} \quad \text{m/s}$$

$$\Rightarrow \quad a = \frac{dv}{dt} = -\frac{16\cdot(-2)\cdot 8}{(8t+1)^3} = \frac{256}{(8t+1)^3} \qquad \text{m/s}^2$$

Quiz

3. A particle has an initial velocity of 3 m/s to the left at $s_0 = 0$ m. Determine its position when t = 3 s if the acceleration is 2 m/s^2 to the right.

✔ A) 0.0 m B) 6.0 m ←

C) 18.0 m → D) 9.0 m →

4. A particle is moving with an initial velocity of v = 12 m/s and constant acceleration of 3.78 m/s^2 in the same direction as the velocity. Determine the distance the particle has traveled when the velocity reaches 30 m/s.

A) 50 m ✔ B) 100 m

C) 150 m D) 200 m