

# BME2102: Introduction to Biomechanics

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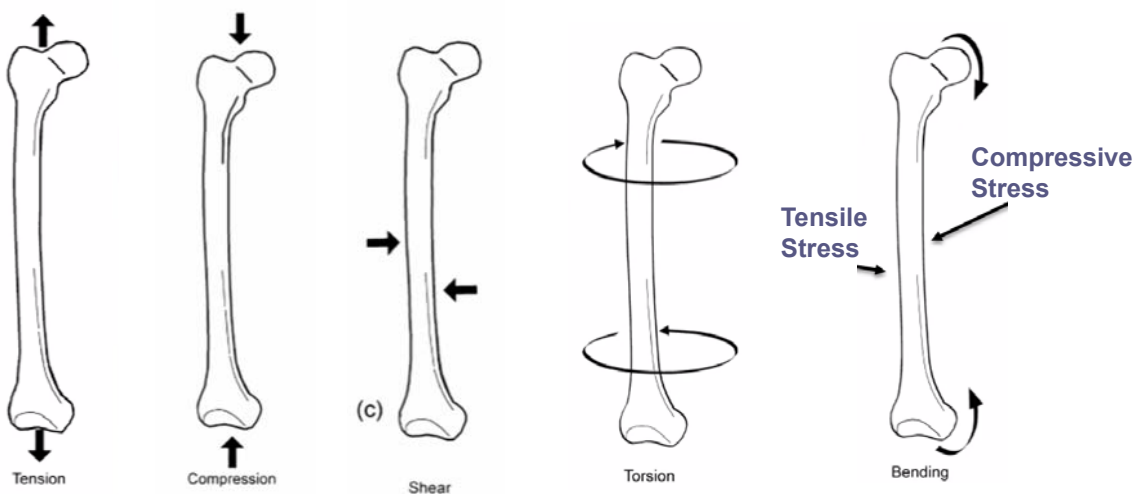
  
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## IV. Mechanics of Biomaterials

- Loading occurs when forces act on materials or tissues
  - Forces acting on musculoskeletal tissues are a good example
- Types of Loading
  - Tension
  - Compression
  - Shear
  - Torsion
  - Bending

## Tension/Tensile, Compression, Shear, Torsion, and Bending



- Stress

- Force has to be normalized – Termed: “Stress”
- Force per square meter

$$\sigma = \frac{F}{A} (\text{N/m}^2)$$

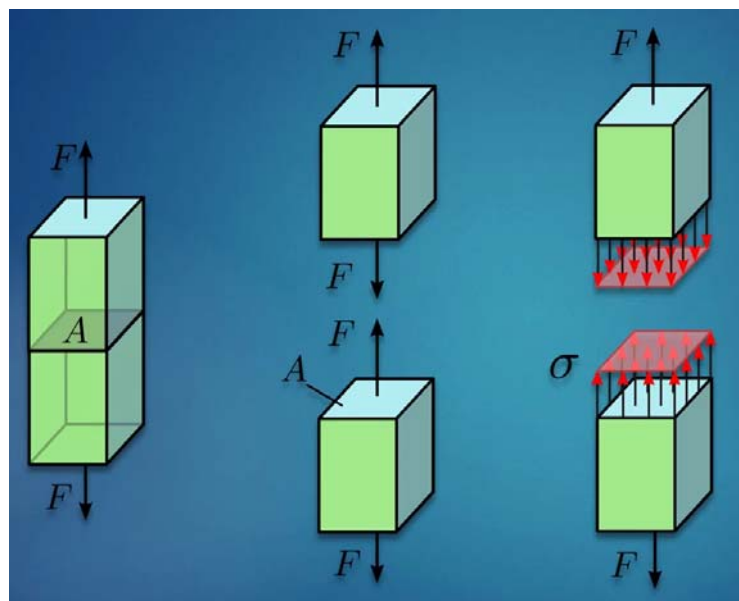
- Where F is internal force and A is internal area (analysis plane)

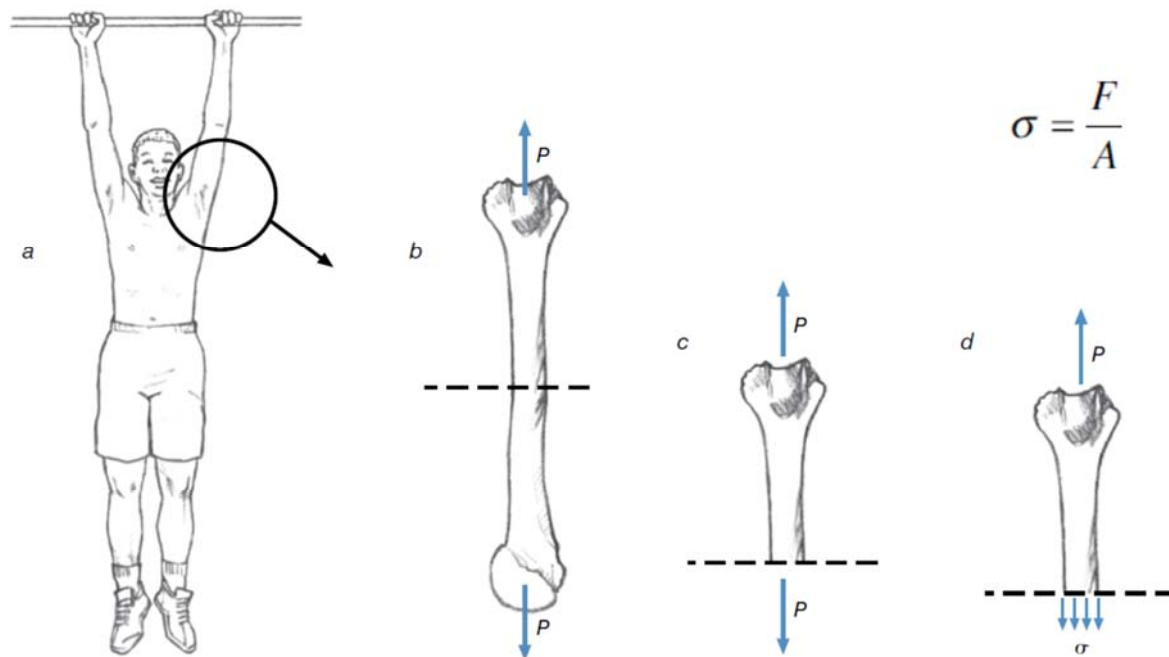


## Normal Stress

- Stress

- The intensity of the distributed force, F  
OR
- Force per 1 square meter ( $\text{N/m}^2 = \text{Pa}$ )





**Figure 9.2** The humerus is loaded axially in tension when you do a chin-up.

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## Sample

- A sample of biological material is loading in a material-testing machine. The cross-sectional area of the specimen is  $1 \text{ cm}^2$ . The machine applies a tensile load to the specimen until it fails. The maximum tensile force applied was  $700,000 \text{ N}$ . What was the maximum stress in the material when it carried this load?

**Step 1:** Identify the known quantities.

$$A = 1 \text{ cm}^2 = 0.0001 \text{ m}^2$$

$$F = 700,000 \text{ N}$$

**Step 2:** Identify the unknown variable to solve for.

$$\sigma = ?$$

**Step 3:** Search for an equation with the known and unknown variables.

$$\sigma = F/A$$

**Step 4:** Substitute the known quantities and solve for the unknown variable.

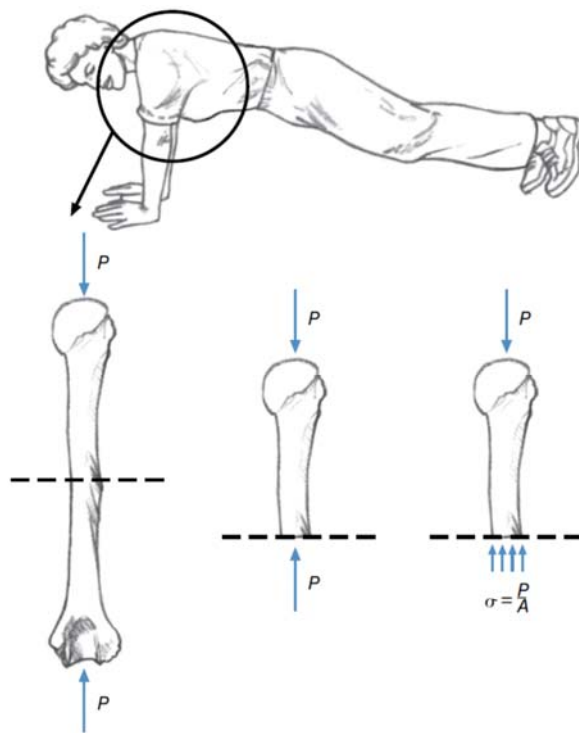
$$\sigma = (700,000 \text{ N}) / (0.0001 \text{ m}^2)$$

$$\sigma = 7,000,000,000 \text{ Pa} = 7 \text{ GPa}$$

(The prefix G before Pa means giga-, or 1 billion. So the stress is 7 billion pascal. Other prefixes used in the SI system are shown in table A.3 in appendix A.)

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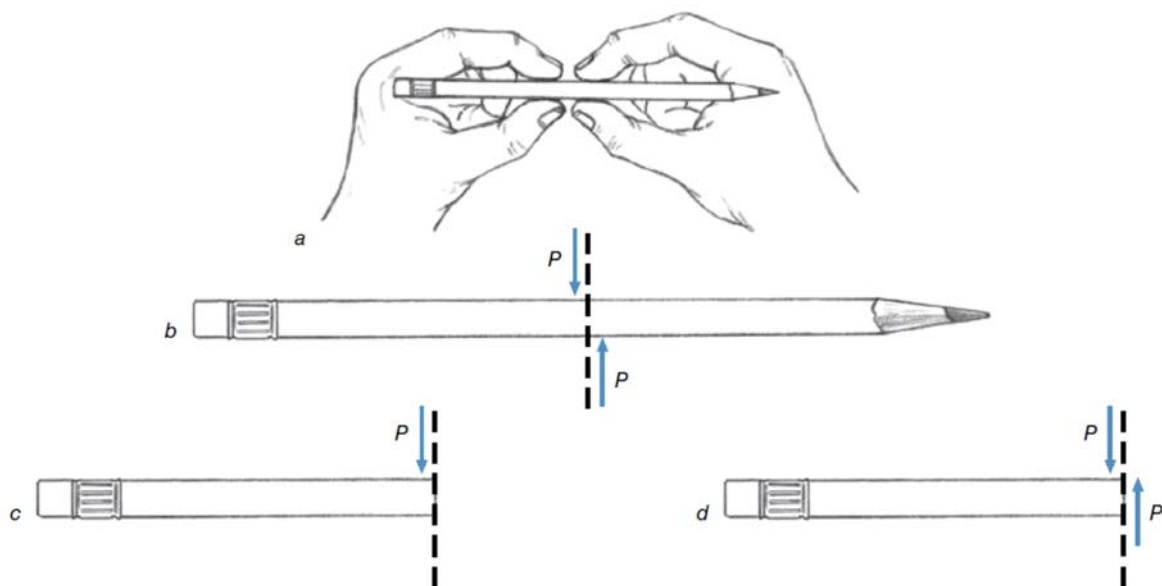
# Compression: Compressive Stress



**Figure 9.3** The humerus is loaded axially in compression when you do a push-up.

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# Shear: Shear Stress

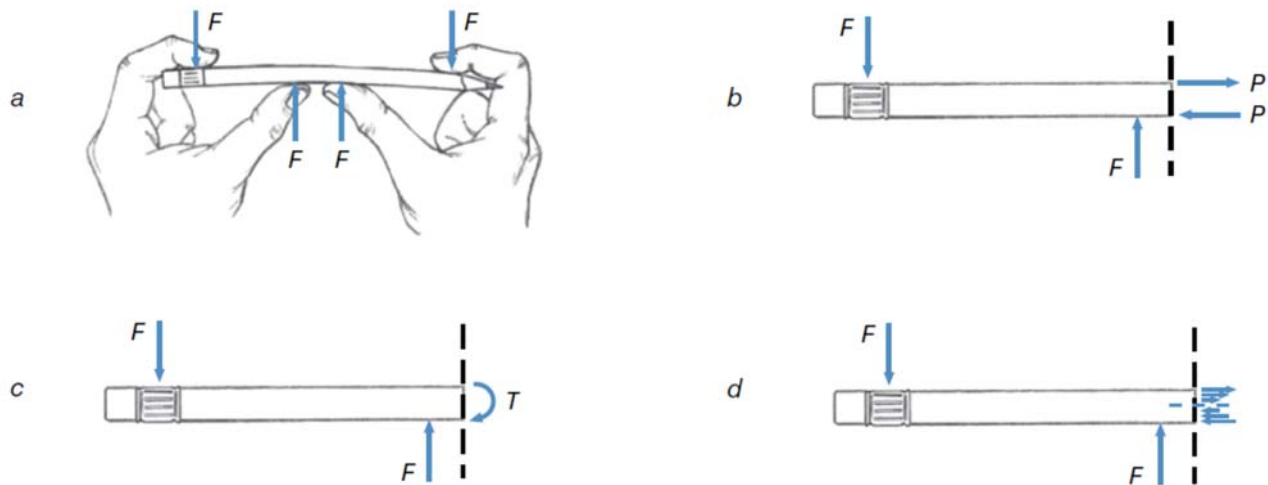


**Figure 9.4** Analysis of a pencil withstanding a shear load.

$$\tau = \frac{F}{A}$$

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# Beading: Tensile and Compressive Stresses

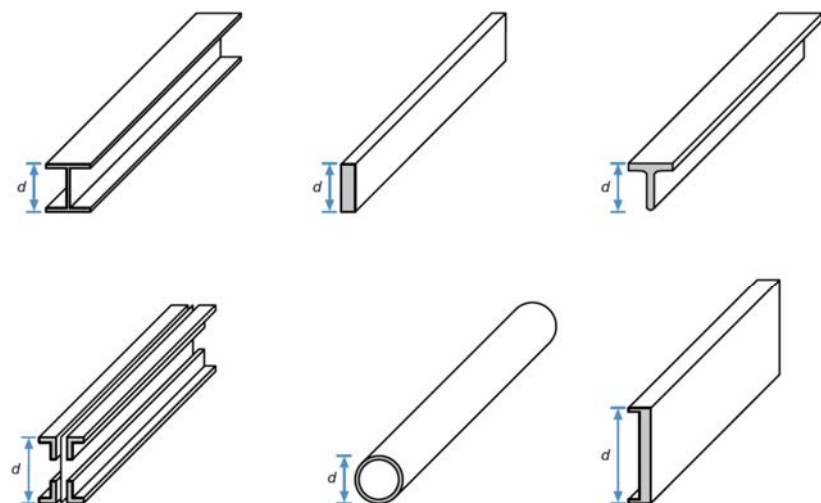


**Figure 9.6** A bending load creates both tensile and compressive stresses.

$$T = F \times r$$

## Cross Sections

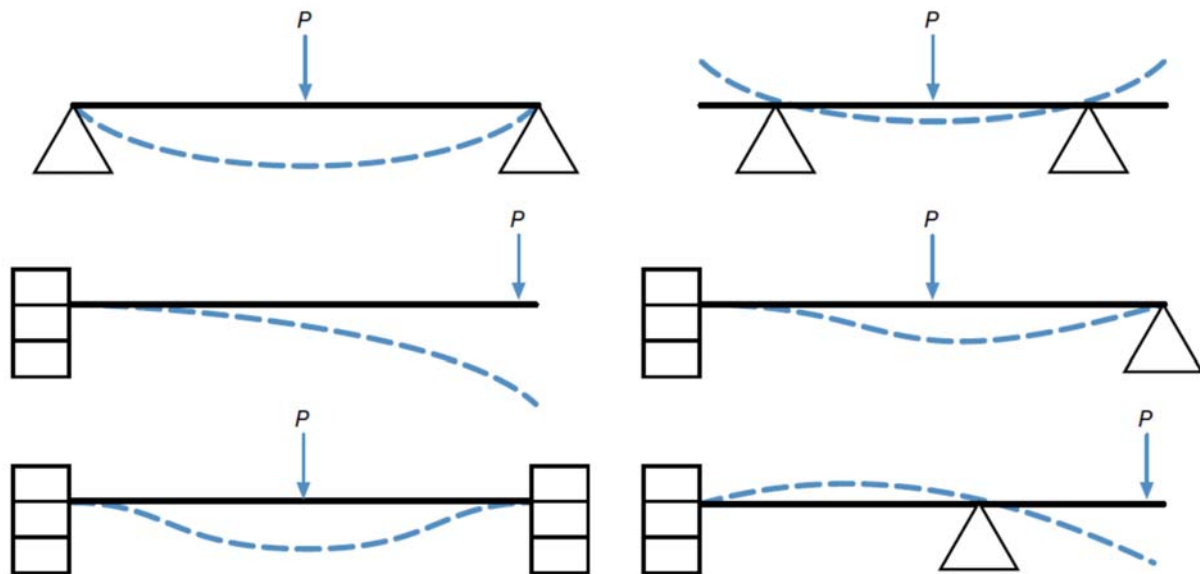
- An object with greater depth (and more cross-sectional area farther from its neutral axis) is able to withstand greater bending loads because it has a larger moment arm.



**Figure 9.7** Cross sections of a variety of beam shapes used in construction. The depth of the beam ( $d$ ) greatly affects its ability to withstand bending loads.



# Beam Support Configurations

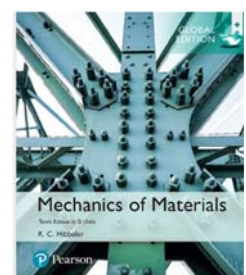


**Figure 9.8** Various beam support configurations and the resulting deflections under a single concentrated load. The triangle supports represent pinned or hinged connections that allow the beam to rotate at the support point. The blocks represent rigid connections that do not allow the beam to rotate.

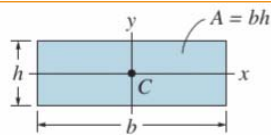
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# Beam Bending

Simply Supported Beam Slopes and Deflections		
Beam	Slope	
	$\theta_{\max} = \frac{-PL^2}{16EI}$	
Deflection	Elastic Curve	
$v_{\max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI} (3L^2 - 4x^2)$ $0 \leq x \leq L/2$	



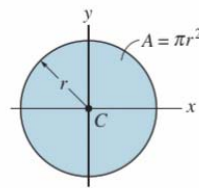
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Rectangular area

$$I_x = \frac{1}{12}bh^3$$

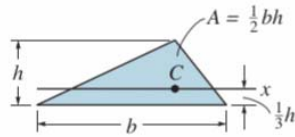
$$I_y = \frac{1}{12}hb^3$$



Circular area

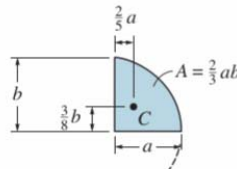
$$I_x = \frac{1}{4}\pi r^4$$

$$I_y = \frac{1}{4}\pi r^4$$

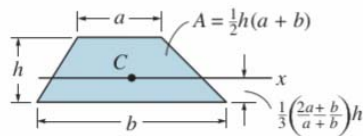


Triangular area

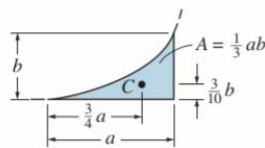
$$I_x = \frac{1}{36}bh^3$$



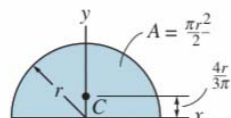
Semiparabolic area



Trapezoidal area



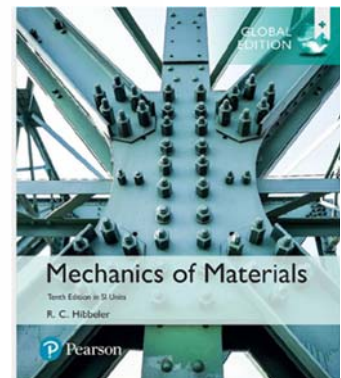
Exparabolic area



Semicircular area

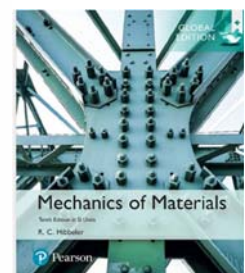
$$I_x = \frac{1}{8}\pi r^4$$

$$I_y = \frac{1}{8}\pi r^4$$

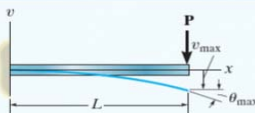
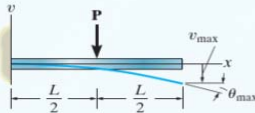
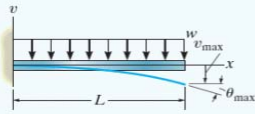
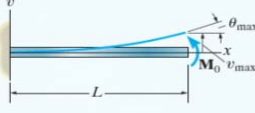
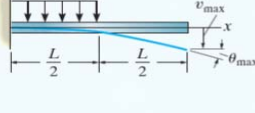
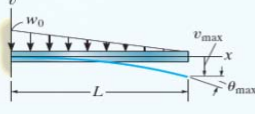


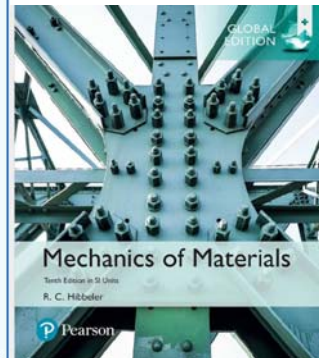
## Simply Supported Beam Slopes and Deflections

Beam	Slope	Deflection	Elastic Curve
	$\theta_{\max} = \frac{-PL^2}{16EI}$	$v_{\max} = \frac{-PL^3}{48EI}$	$v = \frac{-Px}{48EI} (3L^2 - 4x^2)$ $0 \leq x \leq L/2$
	$\theta_1 = \frac{-Pab(L+b)}{6EIL}$ $\theta_2 = \frac{Pab(L+a)}{6EIL}$	$v _{x=a} = \frac{-Pba}{6EIL} (L^2 - b^2 - a^2)$	$v = \frac{-Pbx}{6EIL} (L^2 - b^2 - x^2)$ $0 \leq x \leq a$
	$\theta_1 = \frac{-M_0 L}{6EI}$ $\theta_2 = \frac{M_0 L}{3EI}$	$v_{\max} = \frac{-M_0 L^2}{9\sqrt{3}EI}$ at $x = 0.5774L$	$v = \frac{-M_0 x}{6EIL} (L^2 - x^2)$
	$\theta_{\max} = \frac{-wL^3}{24EI}$	$v_{\max} = \frac{-5wL^4}{384EI}$	$v = \frac{-wx}{24EI} (x^3 - 2Lx^2 + L^3)$
	$\theta_1 = \frac{-3wL^3}{128EI}$ $\theta_2 = \frac{7wL^3}{384EI}$	$v _{x=L/2} = \frac{-5wL^4}{768EI}$ $v_{\max} = -0.006563 \frac{wL^4}{EI}$ at $x = 0.4598L$	$v = \frac{-wx}{384EI} (16x^3 - 24Lx^2 + 9L^3)$ $0 \leq x \leq L/2$ $v = \frac{-wL}{384EI} (8x^3 - 24Lx^2 + 17L^2x - L^3)$ $L/2 \leq x < L$
	$\theta_1 = \frac{-7w_0 L^3}{360EI}$ $\theta_2 = \frac{w_0 L^3}{45EI}$	$v_{\max} = -0.00652 \frac{w_0 L^4}{EI}$ at $x = 0.5193L$	$v = \frac{-w_0 x}{360EIL} (3x^4 - 10L^2x^2 + 7L^4)$

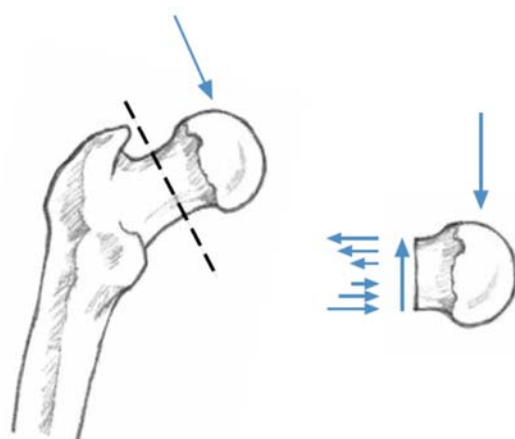




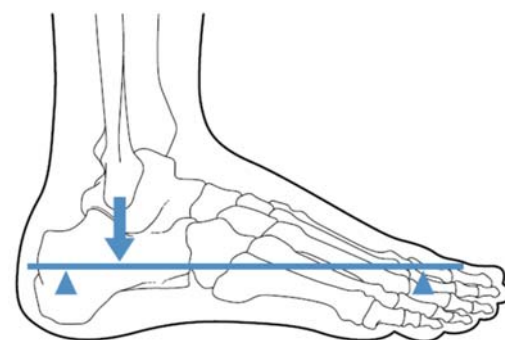
Beam	Slope	Deflection	Elastic Curve
	$\theta_{\max} = \frac{-PL^2}{2EI}$	$v_{\max} = \frac{-PL^3}{3EI}$	$v = \frac{-Px^2}{6EI} (3L - x)$
	$\theta_{\max} = \frac{-PL^2}{8EI}$	$v_{\max} = \frac{-5PL^3}{48EI}$	$v = \frac{-Px^2}{12EI} (3L - 2x) \quad 0 \leq x \leq L/2$ $v = \frac{-PL^2}{48EI} (6x - L) \quad L/2 \leq x \leq L$
	$\theta_{\max} = \frac{-wL^3}{6EI}$	$v_{\max} = \frac{-wL^4}{8EI}$	$v = \frac{-wx^2}{24EI} (x^2 - 4Lx + 6L^2)$
	$\theta_{\max} = \frac{M_0L}{EI}$	$v_{\max} = \frac{M_0L^2}{2EI}$	$v = \frac{M_0x^2}{2EI}$
	$\theta_{\max} = \frac{-wL^3}{48EI}$	$v_{\max} = \frac{-7wL^4}{384EI}$	$v = \frac{-wx^2}{24EI} (x^2 - 2Lx + \frac{3}{2}L^2) \quad 0 \leq x \leq L/2$ $v = \frac{-wL^3}{384EI} (8x - L) \quad L/2 \leq x \leq L$
	$\theta_{\max} = \frac{-w_0L^3}{24EI}$	$v_{\max} = \frac{-w_0L^4}{30EI}$	$v = \frac{-w_0x^2}{120EI} (10L^3 - 10L^2x + 5Lx^2 - x^3)$



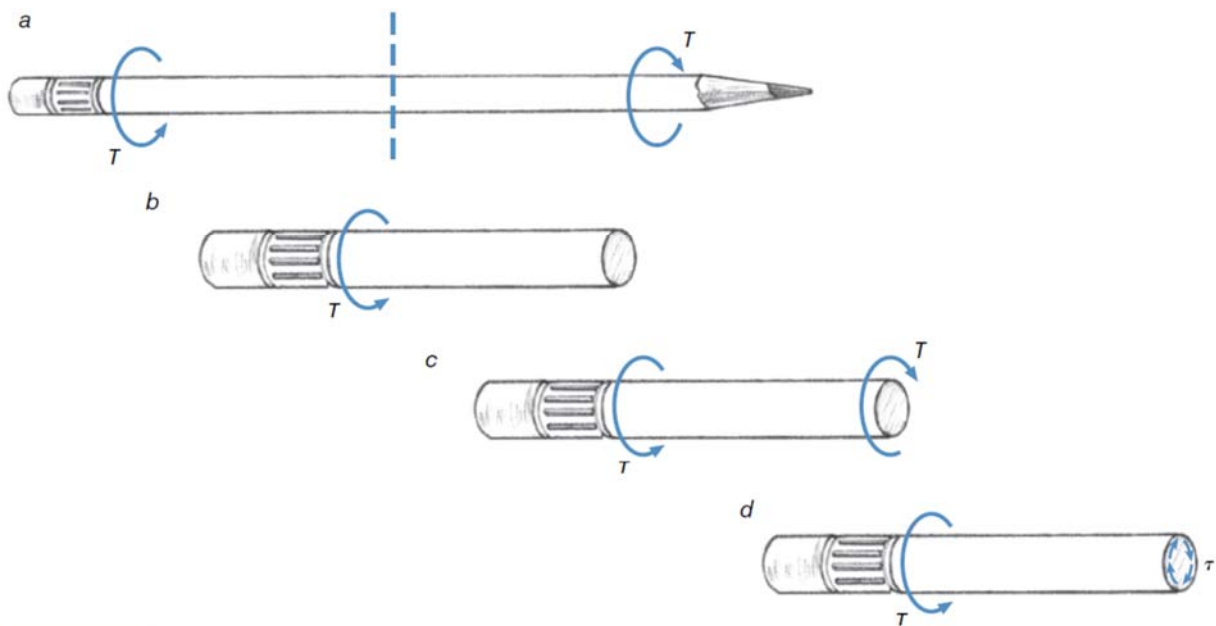
## Examples of Beam Support



**Figure 9.9** The neck of the femur acts as a cantilever beam and develops tensile stress in its upper part and compressive stress in its lower part.

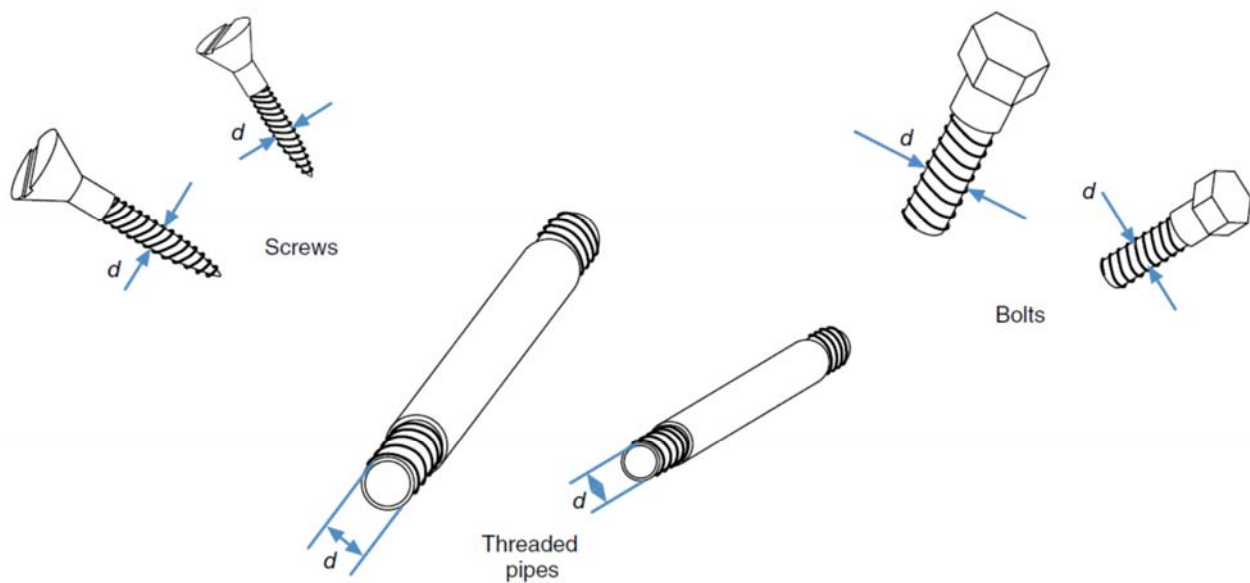


**Figure 9.10** The foot as an example of a simple beam.



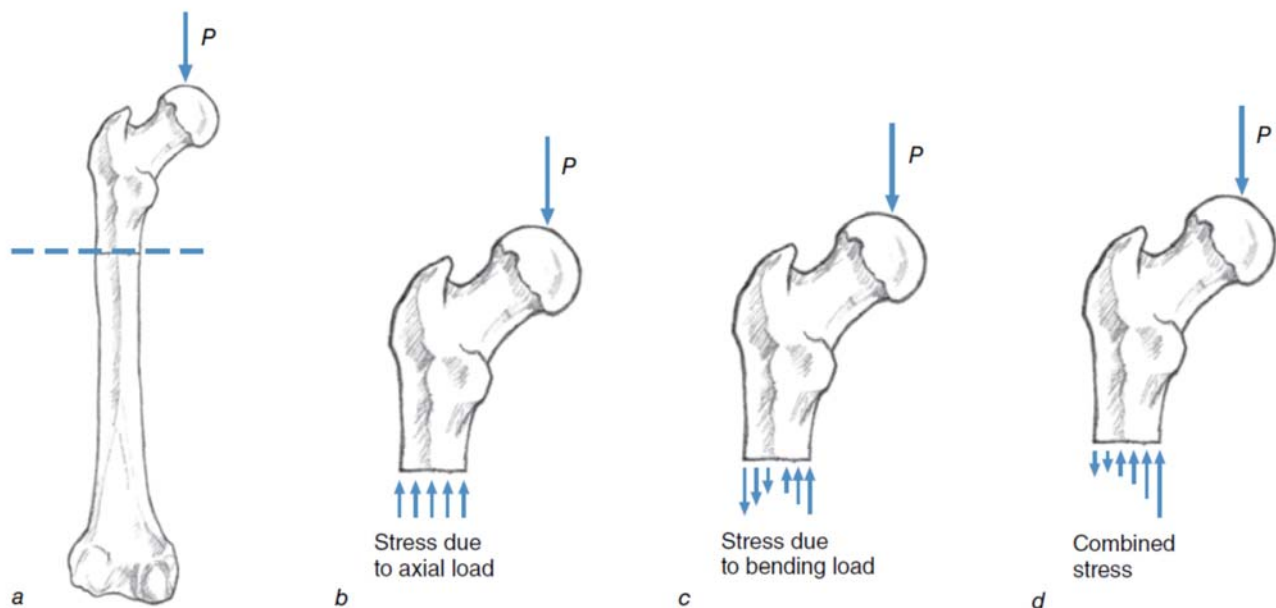
**Figure 9.11** A pencil withstanding a torsional load is subjected to shear stress.

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**Figure 9.12** The diameter ( $d$ ) of an object's cross section greatly affects its ability to withstand torsional loads.

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**Figure 9.13** The combined stresses due to the compressive load and bending load on the femur.

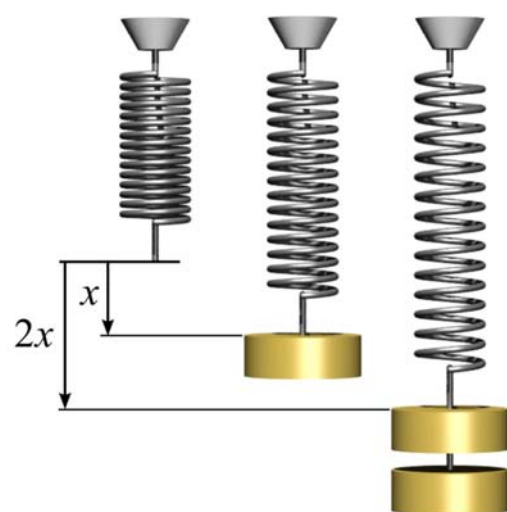
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## Hooke's Law

- Consider a simple helical spring that has one end attached to some fixed object, while the free end is being pulled by a force  $F_s$ . Let  $x$  be the amount by which the free end of the spring was displaced. Hooke's law states that

$$F_s = kx \quad x = \frac{F_s}{k}$$

where  $k$  is a positive real number, characteristic of the spring, AKA spring constant or stiffness.



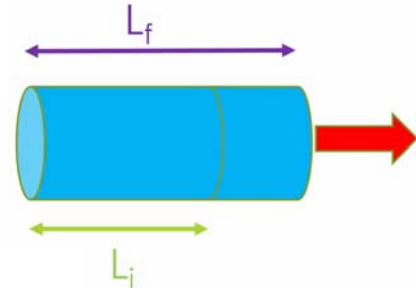
[https://en.wikipedia.org/wiki/Hooke%27s\\_law](https://en.wikipedia.org/wiki/Hooke%27s_law)

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- As forces act on materials (Stress), the material may deform
- The measure of this deformation is termed: Strain
- Measurement of deformation
  - Strain

$$\varepsilon = \frac{L_f - L_i}{L_i} = \frac{\Delta L}{L_i}$$

$$\varepsilon = \frac{L_f - L_i}{L_i} = \frac{\Delta L}{L_i} \times 100 = \% \text{ Strain}$$



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## Sample

- A sample of biological material is loaded into a material-testing machine. The material is 2 cm long in its unloaded state. A 6000 N tensile force is applied to the material, and it stretches to a length of 2.0004 cm as a result of this force. What is the strain in the specimen when it is stretched this much?

**Step 1:** Identify the known quantities.

$$\ell = 2.0004 \text{ cm}$$

$$\ell_o = 2.0 \text{ cm}$$

**Step 2:** Identify the unknown variable to solve for.

$$\varepsilon = ?$$

**Step 3:** Search for an equation with the known and unknown variables.

$$\varepsilon = \frac{\ell - \ell_o}{\ell_o}$$

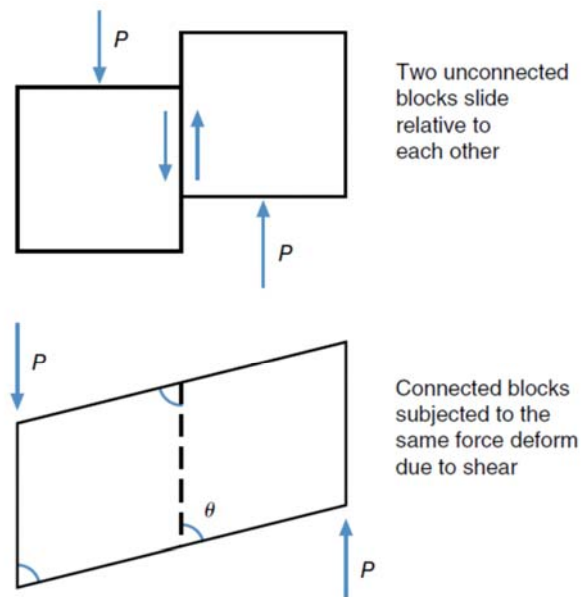
**Step 4:** Substitute the known quantities and solve for the unknown variable.

$$\varepsilon = \frac{\ell - \ell_o}{\ell_o} = \frac{2.0004 - 2.000}{2.000}$$

$$\varepsilon = 0.0002 = 0.02\%$$

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- Shear strain occurs with a change in orientation of adjacent molecules as a result of these molecules slipping past each other.

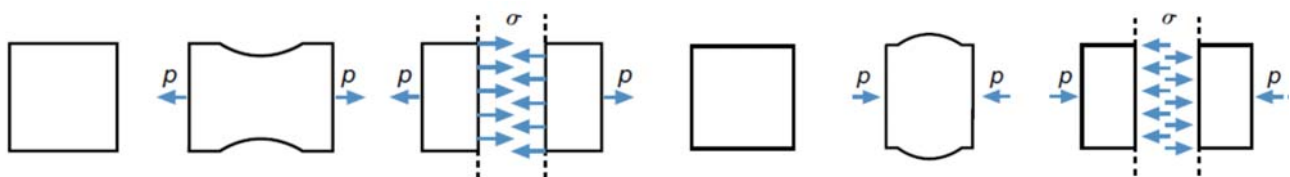


**Figure 9.14** Illustration of deformation caused by shear. The change in the angle ( $\theta$ ) indicates the shear strain.

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## Poisson's Ratio

- Poisson effect



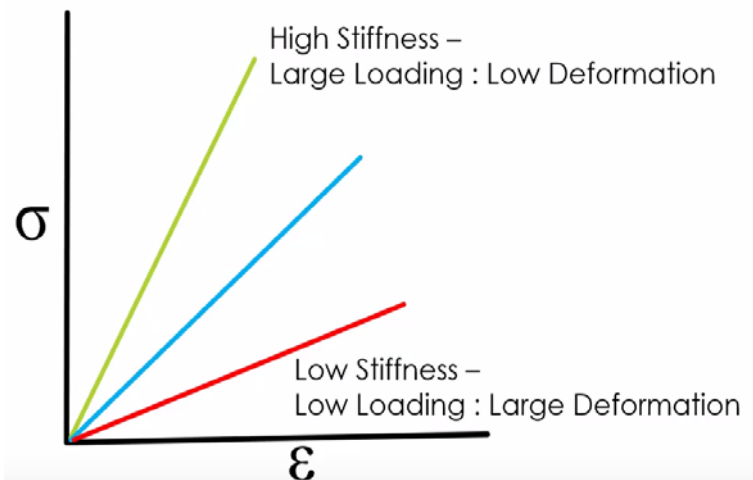
- Poisson's ratio**
  - A specific ratio of strain in the axial direction to strain in the transverse direction exists for each different type of material.
  - Values of Poisson's ratio can be as low as 0.1 and as high as 0.5, but for most materials they are between 0.25 and 0.35.

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- As forces act on a material (Stress) the material may deform (Strain)
- Depending on the “Stiffness” of a material the relationship curve may differ
- Stiffness AKA Young’s Modulus or Elastic Modulus

$$E = \frac{\Delta\sigma}{\Delta\epsilon}$$



$E$  = elastic modulus,

$\Delta\sigma$  = change in stress, and

$\Delta\epsilon$  = change in strain.

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## Sample

- A material is subjected to a tensile load of 80,000 N (80 kN). Its cross-sectional area is 1 cm<sup>2</sup>. The elastic modulus for this material is 70 GPa. What strain results from this tensile load?

**Step 1:** Identify the known quantities.

$$F = 80,000 \text{ N}$$

$$A = 1 \text{ cm}^2 = 0.0001 \text{ m}^2$$

$$E = 70 \text{ GPa}$$

**Step 2:** Identify the unknown variable to solve for.

$$\epsilon = ?$$

**Step 3:** Search for an equation with the known and unknown variables.

$$E = \frac{\sigma}{\epsilon}$$

$$\sigma = \frac{F}{A}$$

**Step 4:** Substitute the known quantities and solve for the unknown variable.

$$\sigma = (80,000 \text{ N}) / (0.0001 \text{ m}^2) = 800,000,000 = 800 \text{ MPa}$$

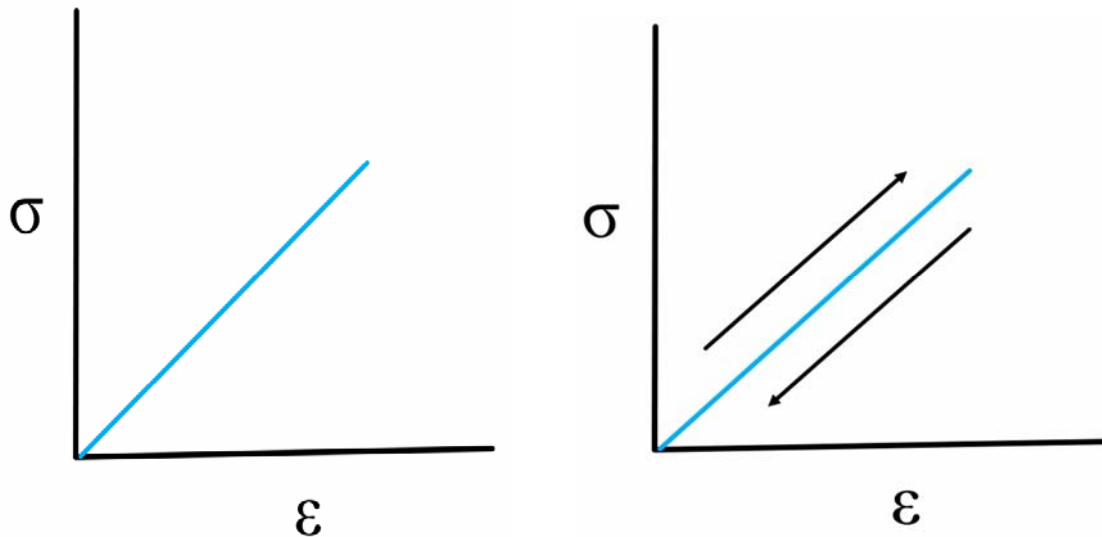
$$E = 70 \text{ GPa} = (800 \text{ MPa}) / \epsilon$$

$$\epsilon = (800 \text{ MPa}) / (70 \text{ GPa}) = (800,000,000 \text{ Pa}) / (70,000,000,000 \text{ Pa})$$

$$\epsilon = 0.0114 = 1.14\%$$

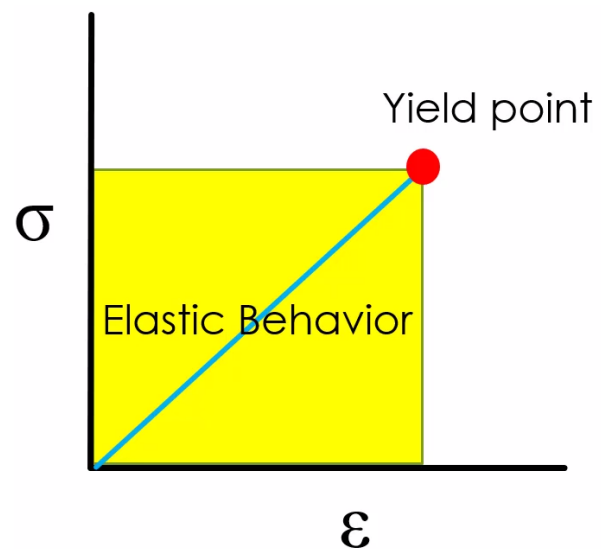
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- Stiffness refers to the “Elastic Region” of a material’s Stress/Strain curve
- Material deforms and then returns to its original form



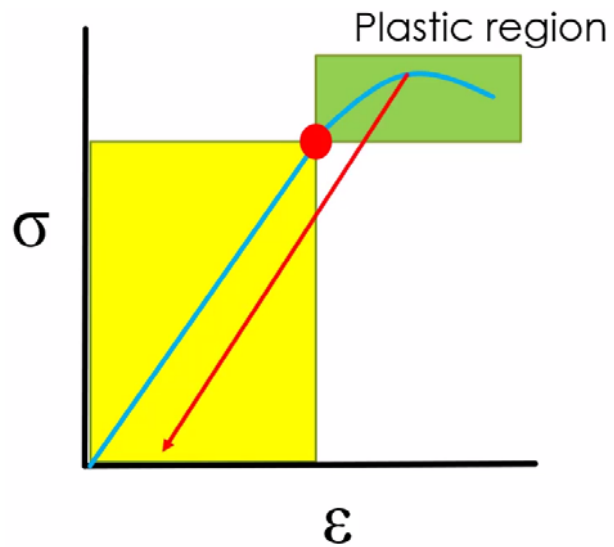
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- Further stress results in permanent deformation
- This point is called the Yield point



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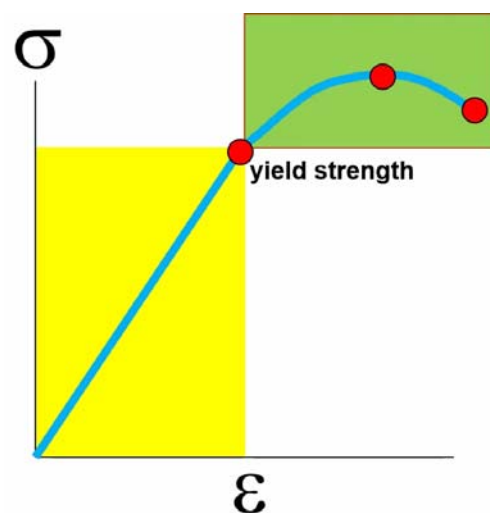
- When stress is removed, material will return to new length.
- It will be permanently deformed when strained beyond the Yield Point
- This area is called the Plastic Region



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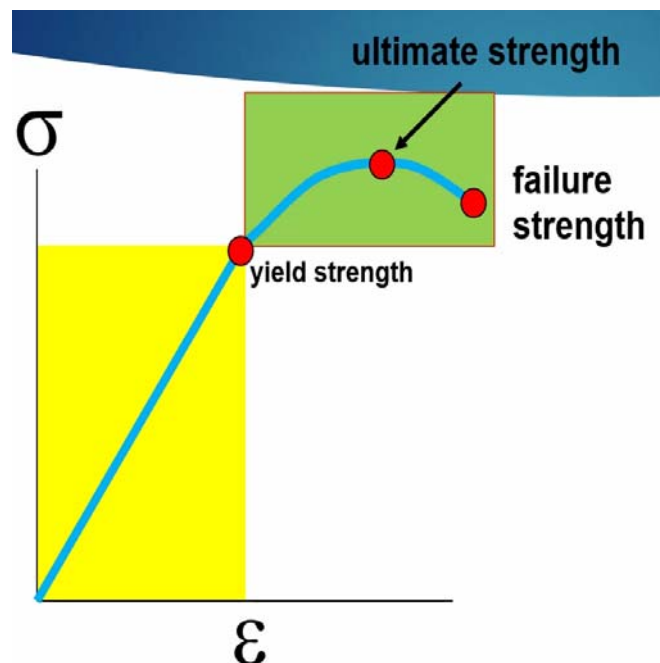
## Mechanical Strength

- Ultimate Strength is the highest amount of stress a material can handle without damaging
- Failure Strength is the point at which the material fails and tears or ruptures



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- Ultimate Strength is the highest amount of stress a material can handle without damaging
- Failure Strength is the point at which the material fails and tears or ruptures



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Figure 4.3. The regions and key variables in a load-deformation graph of an elastic material.

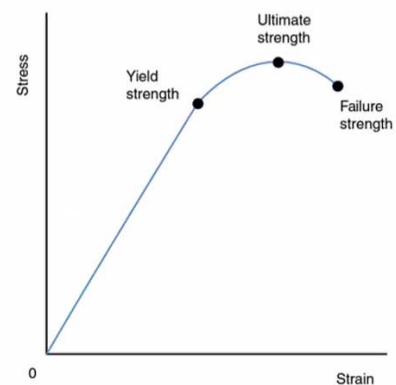
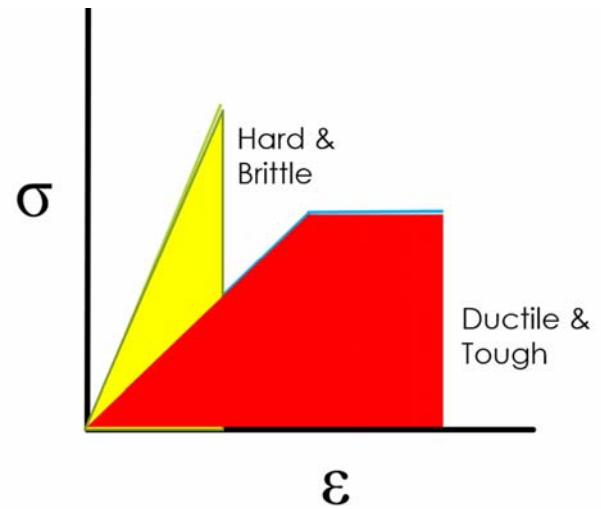


Figure 9.19 Material measures of strength shown on a stress-strain curve.

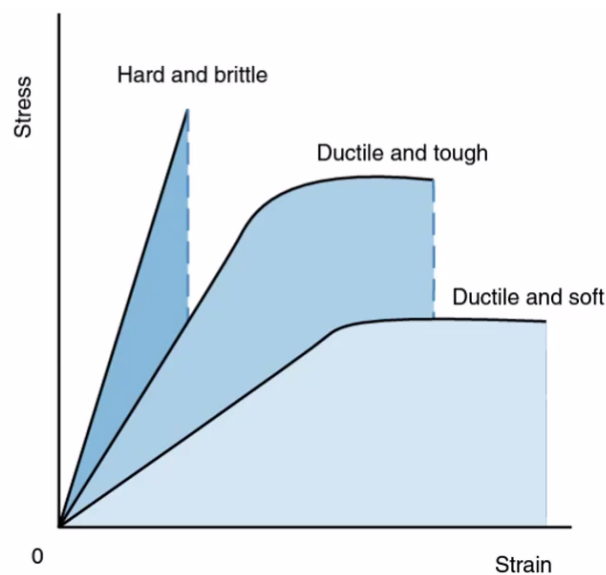
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- Combination of stress & strain
  - Overall strength of the material
  - Ability to absorb energy
    - Area under curve = Energy Absorbed/Released
    - Work – Energy relationship
      - ❖ Work = Energy
      - ❖ Work =  $Fd$
      - ❖ Stress  $\times$  Strain =  $Fd$



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## Material Toughness



**Figure 9.21** The toughness of different materials is indicated by the area under the material's stress-strain curve.

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- Loading occurs when forces act on materials or tissues
- Types of Loading
  - Tension
  - Compression
  - Shear
  - Torsion
  - Bending
- Stress is normalized force of the intensity of the distributed force ( $\text{N/m}^2 = \text{Pa}$ )

$$\sigma = \frac{F}{A} (\text{N/m}^2)$$

$$\tau = \frac{F}{A}$$

- Loading causes deformation

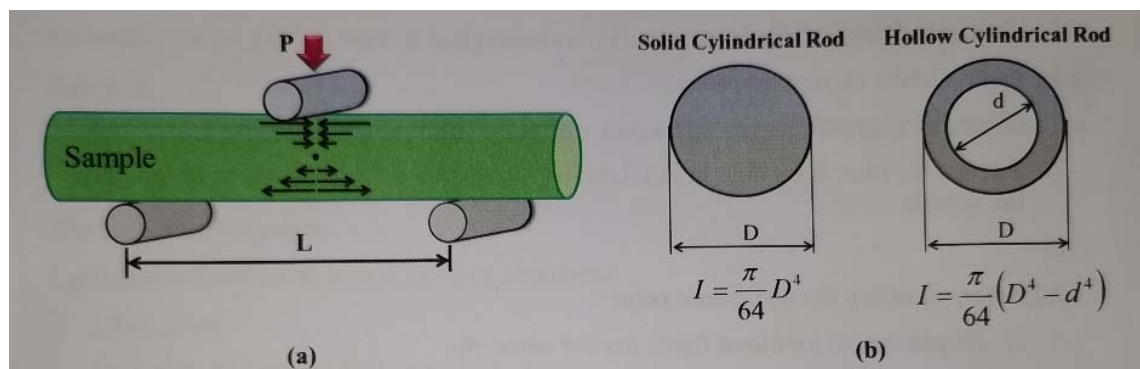


A material-testing machine

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## 3-Point Bending Test

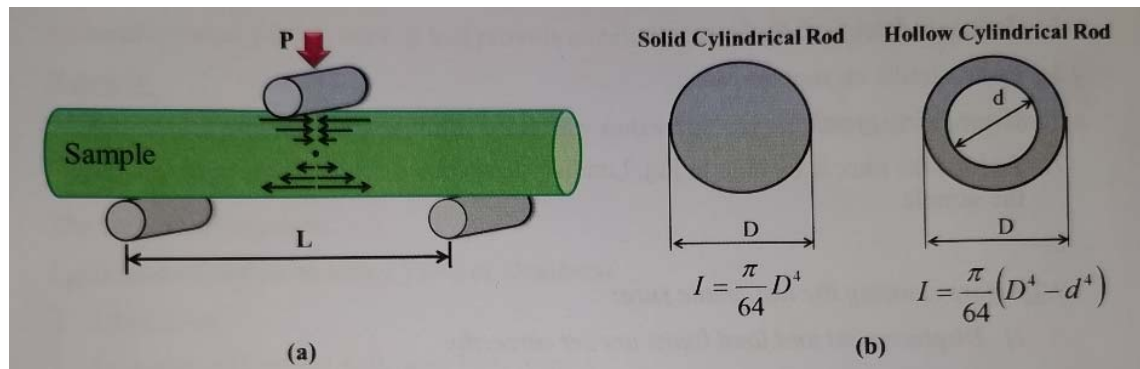
- 3 -point bending test
  - One of the most standardized and frequently adopted methods
  - Usually performed on samples beam shapes with either circular or rectangular cross sections
  - Test sample is placed on two supporting rods and bent by the third rod which applies concentrate load at the mid-section of the sample.



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## 3-Point Bending Test

- The force  $P$  is measured by a loading cell attached at the mechanical tester.
- Deflection of the sample  $\delta$  could be read approximately as the displacement recorded by the software associated with the mechanical tester or measured more accurately with an *in-situ* image recording system.

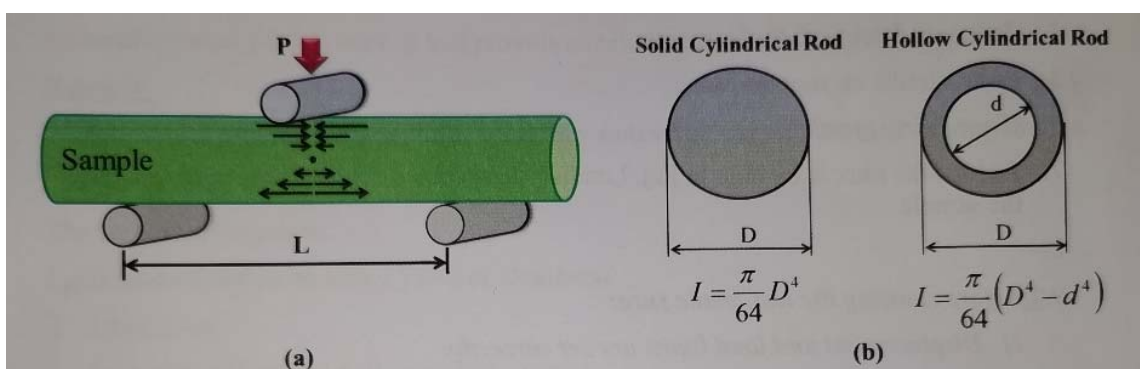


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## 3-Point Bending Test

- In this lab, a Tinius Olsen H50KT system is used to perform 3-point bending tests under displacement control at a constant displacement rate at room temperature in air.
- Flexural stress,  $\sigma_f$ , at the out surface of the center section of the beam can be calculated based on simple beam theory by equation

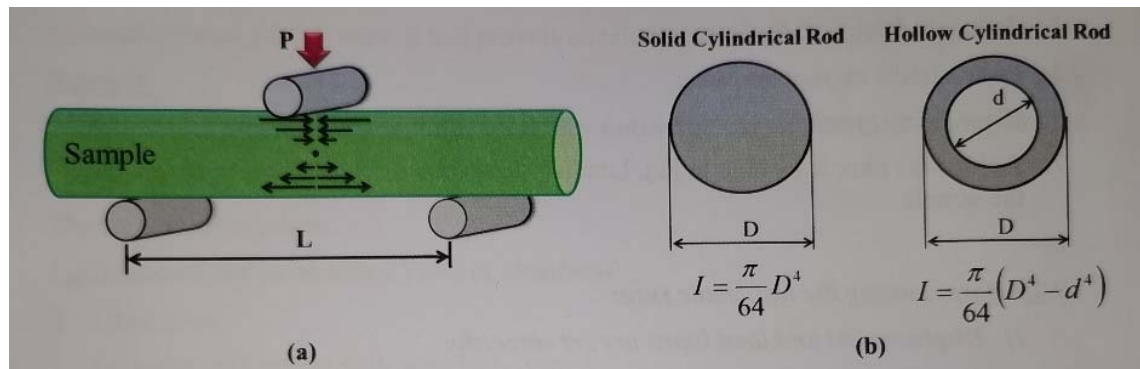
$$\sigma_f = \frac{PLD}{8I}$$



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## 3-Point Bending Test

- Flexural stress  $\sigma_f = \frac{PLD}{8I}$
- where,  $P$  is the value of concentration load,  $L$  is the supporting span,  $D$  is the outer diameter of the cross section of the beam, and  $I$  is the geometry dependent moment of inertia which could be calculated by the equations shown in Figure (b).



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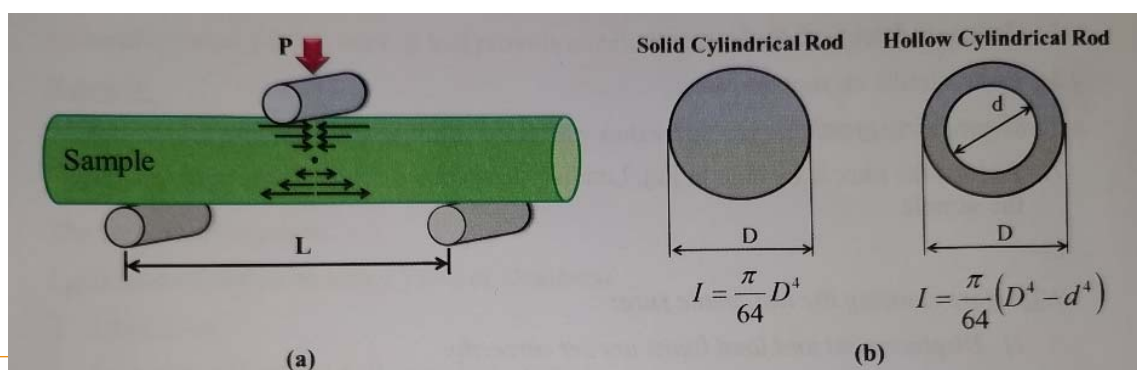
## 3-Point Bending Test

- The elastic flexural deflection  $\delta_{elastic}$  at the center of the beam can be calculated by

$$\delta_{elastic} = \frac{PL^3}{48EI}$$

where,  $E$  is elastic modulus which can be obtained from this equation once  $\delta_{elastic}$  is measured.

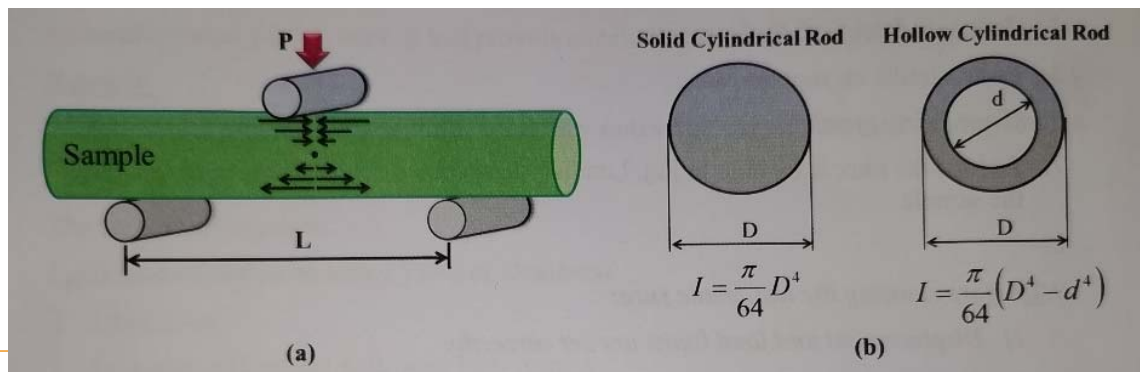
$$E = \frac{PL^3}{48I\delta_{elastic}}$$



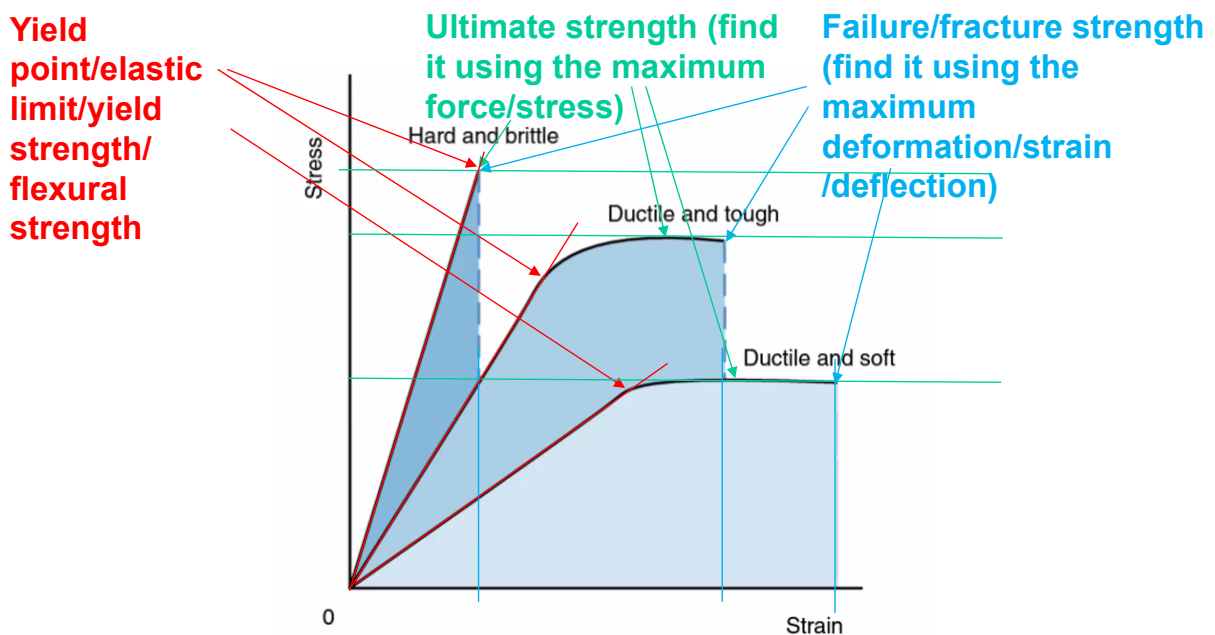
# 3-Point Bending Test

- The flexural strain  $\varepsilon_f$  can then be calculated by

$$\varepsilon_f = \frac{\sigma_f}{E} = \frac{\frac{PLD}{8I}}{\frac{PL^3}{48I\delta_{elastic}}} = \frac{6D\delta_{elastic}}{L^2}$$



# Material Toughness



**Figure 9.21** The toughness of different materials is indicated by the area under the material's stress-strain curve.

- 
- 1. A 1 cm long section of the patellar ligament stretches to 1.001 cm when it is subjected to a tensile force of 10,000 N. What is the strain in this segment of ligament?

- 
- 2. The section of the patellar ligament in the previous question is 50 mm<sup>2</sup> in cross section. What is the stress in this section of ligament as a result of the 10,000 N tensile force?



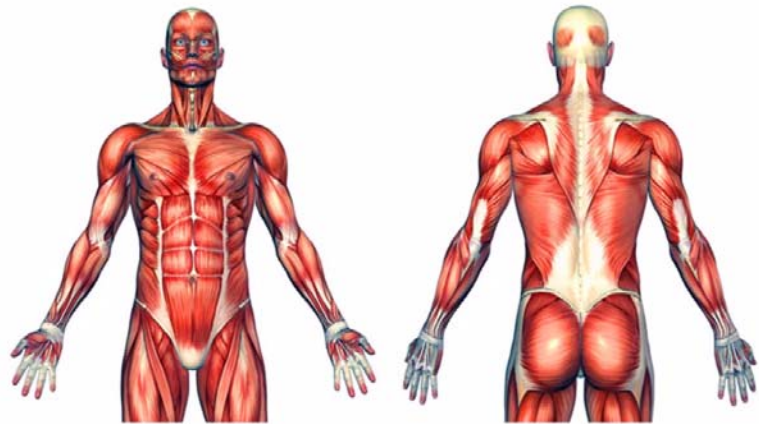
- 3. The modulus of elasticity (for compression) for a section of compact bone in the femur is 12 GPa ( $12 \times 10^9$  Pa). If this bone is subjected to a compression stress of 60 MPa ( $60 \times 10^6$  Pa), what strain results from this compression?

- 5. The Achilles tendon is subjected to a large tension stress that results in a strain of 6%. If the unloaded tendon is 10 cm long, how much does it elongate as a result of this strain?

- 
- 6. The yield strength of a material is 10 MPa. The yield strain for this material is 0.10%. What is the modulus of elasticity for this material?

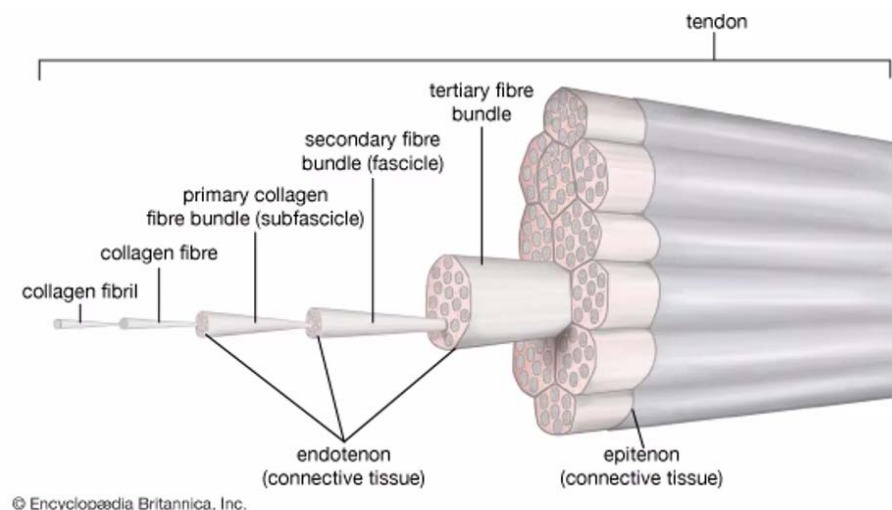
- 
- 7. The modulus of elasticity for a prosthetic material is 20 GPa. A 3 cm long sample of this material is circular in cross section with a radius of 1 cm. This sample is stretched 3.003 cm. What tensile force was applied to the material to create this stretch?

- Bone
- Cartilage
- Tendon
  - Muscle to Bone
- Ligament
  - Bone to Bone



## Connective Tissues

- Cell
- Collagen
- Mineral
- Water
- Elastin

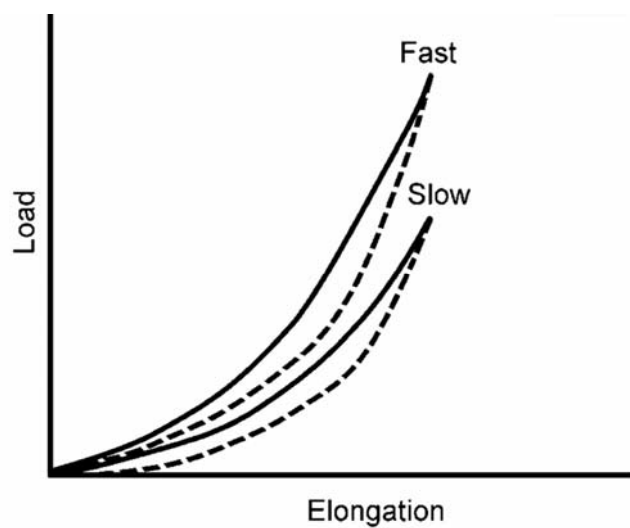


- Bone
  - 30% & 20% (45% mineral)
- Cartilage
  - 20% & 70%
- Tendon & Ligament
  - 25% & 70%

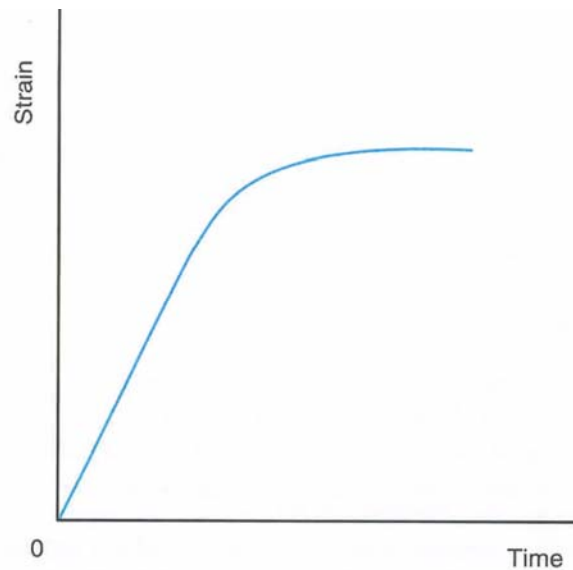


## Viscoelasticity

- Viscoelastic properties occur when the stress and strain on a materials are dependent on how quickly or slowly the load applied



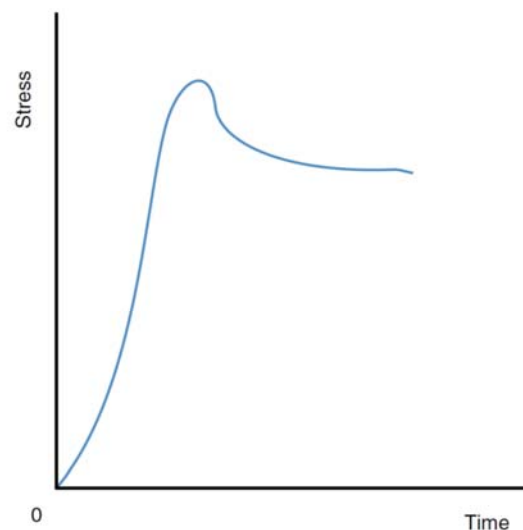
- **Constant compressive stress**
  - Increases strain as water content is squeezed out
  - Strain reaches a maximum
- **Ex**
  - Gravity & Spinal Cord



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# Viscoelastic Stress Relaxation

- **Constant strain results in**
  - Stress increases as water content is squeezed out
  - Stress maxes out
  - Stress then decreases
- **Ex**
  - A long static stretch

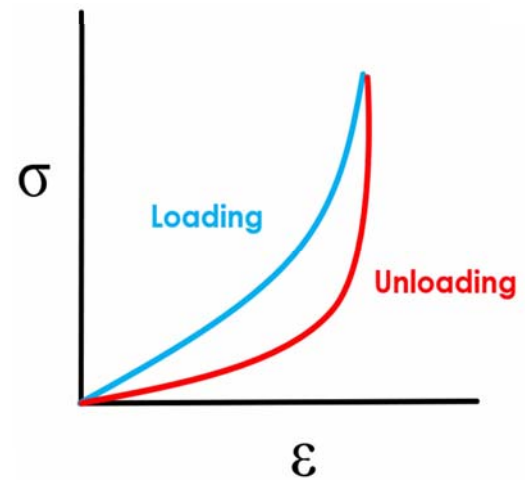


**Figure 9.26** Stress relaxation in articular cartilage under constant compressive strain.

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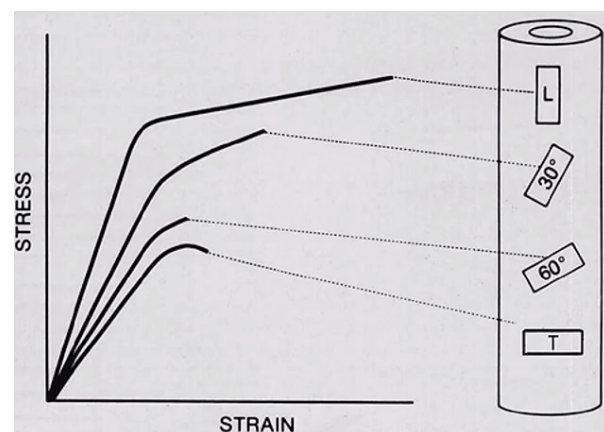
- Hysteresis is an elastic properly describing a material that has a different stress-strain curve when being uploaded compared to loaded
- Area under curved is energy stored/released
- Difference between loaded/unloaded energy lost
  - Energy lost because few materials are perfectly elastic
  - Less area = more elastic or more efficient



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## Isotropic vs Anisotropic Behavior

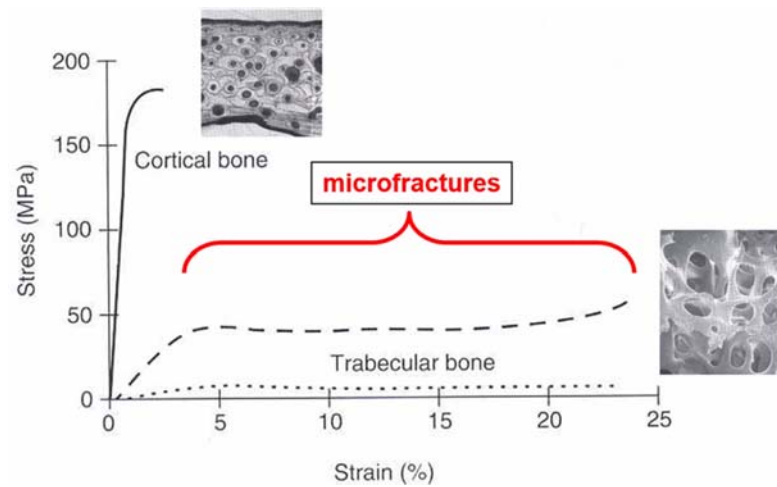
- **Isotropic**
  - A material that has the same mechanical properties regardless of loading direction
- **Anisotropic**
  - A material that has different mechanical properties dependent on how it is loaded



**FIG. 12-8** Anisotropic behavior of cortical bone specimens machined from a human femoral shaft and tested in tension. The orientation of load application—longitudinal (L), tilted 30° with respect to the bone axis, tilted 60°, and transverse (T)—strongly influences both the stiffness and the ultimate strength. (Frankel VH, Nordin M: Basic Biomechanics of the Skeletal System, p. 22. Philadelphia, Lea & Febiger, 1980)

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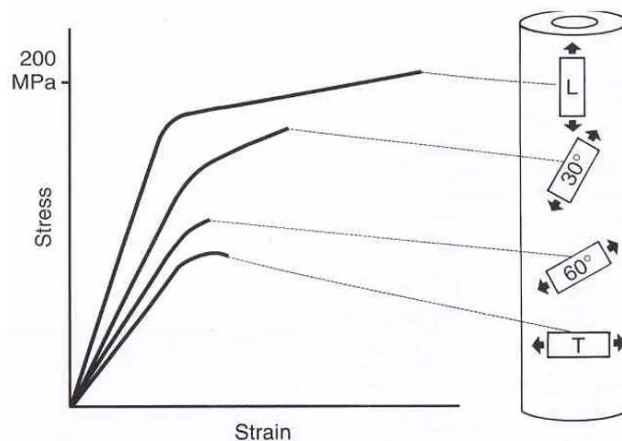
- Tested in Compression



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## Bone Loading

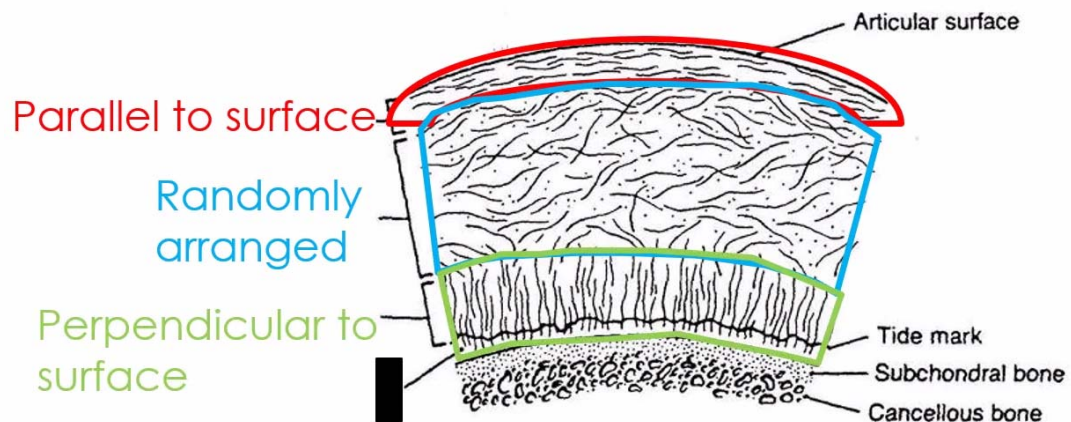
- Ultimate strength of bone



Loading Type	Ultimate Strength
Compression	200 MPa (29,000lb/in <sup>2</sup> )
Tension	125 MPa
Shear	65 MPa (9,425lb/in <sup>2</sup> )

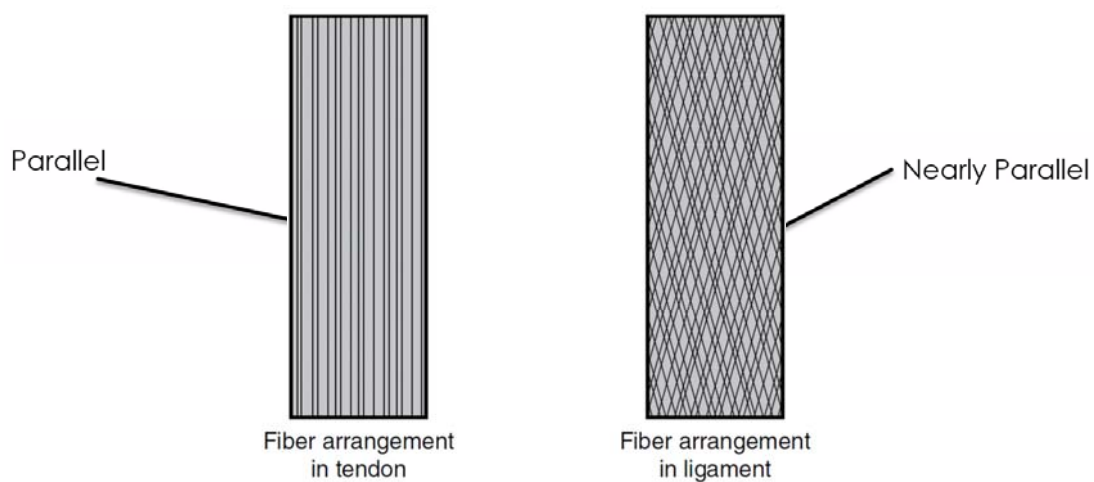
**Hayes, 1986**

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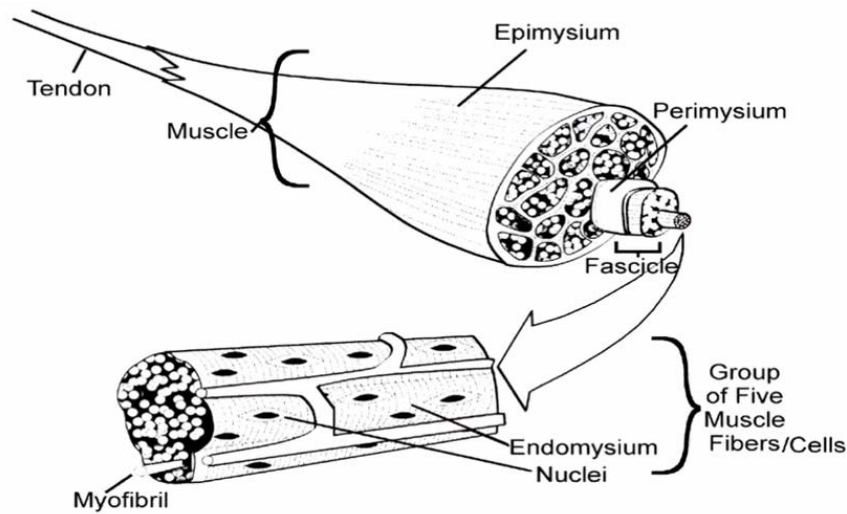
65

# Collagen in Tendon & Ligaments



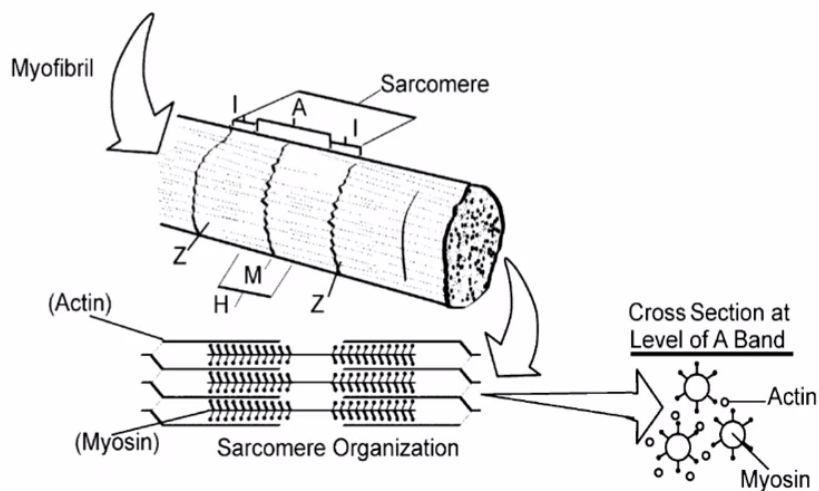
**Figure 9.27** Parallel arrangement of collagen fibers in tendon, and nearly parallel arrangement of collagen fibers in ligament.

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**Figure 3.6.** The macroscopic structure of muscle includes several layers of connective tissue and bundles of muscle fibers called fascicles. Muscle fibers (cells) are multinucleated and composed of many myofibrils.

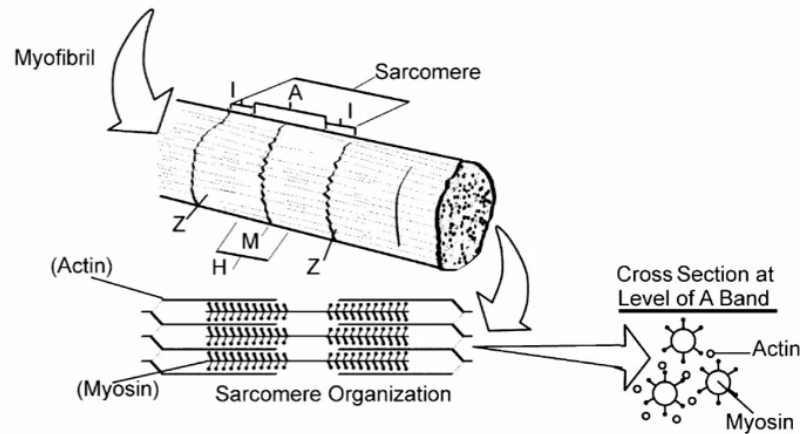
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**Figure 3.8.** The microscopic structure of myofibril components of muscle fibers. Schematics of the sarcomere, as well as of the actin and myosin filaments are illustrated.

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- Myosin will create a crossbridge with Actin and pull forwards the center
- Sarcomere Shortens = Muscle Contraction

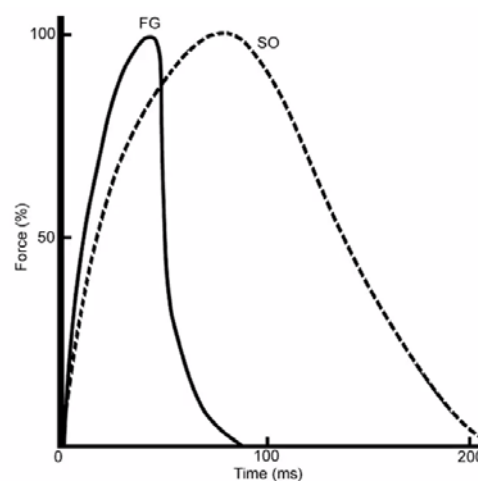


**Figure 3.8.** The microscopic structure of myofibril components of muscle fibers. Schematics of the sarcomere, as well as of the actin and myosin filaments are illustrated.

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## Muscle Types

- Type I
  - Slow Twitch
  - Slow Oxidative
- Type II
  - Fast Twitch
  - Fast Glycolytic

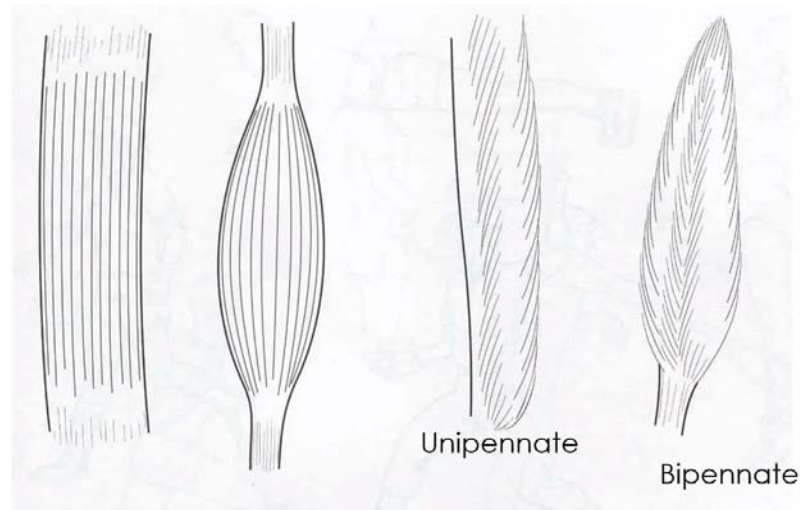


**Figure 4.9.** The twitch response of fast-twitch (FG) and slow-twitch (SO) muscle fibers. Force output is essentially identical for equal cross-sectional areas, but there are dramatic differences in the rise and decay of tension between fiber types that affect the potential speed of movement.

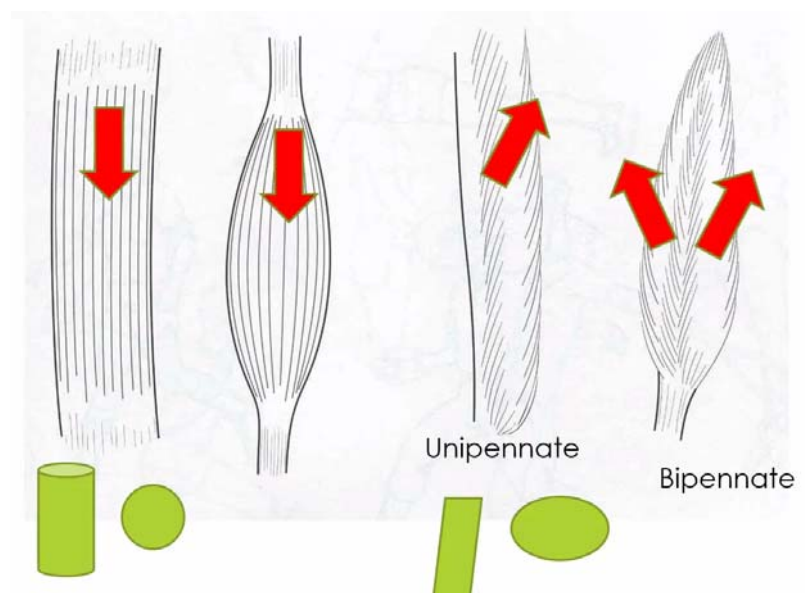
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- **Parallel**
  - Greater ROM
  - Less tension
- **Pennate**
  - Less ROM
  - Greater tension
    - Cross-sectional area



- **Parallel**
  - Greater ROM
  - Less tension
- **Pennate**
  - Less ROM
  - Greater tension
    - Cross-sectional area



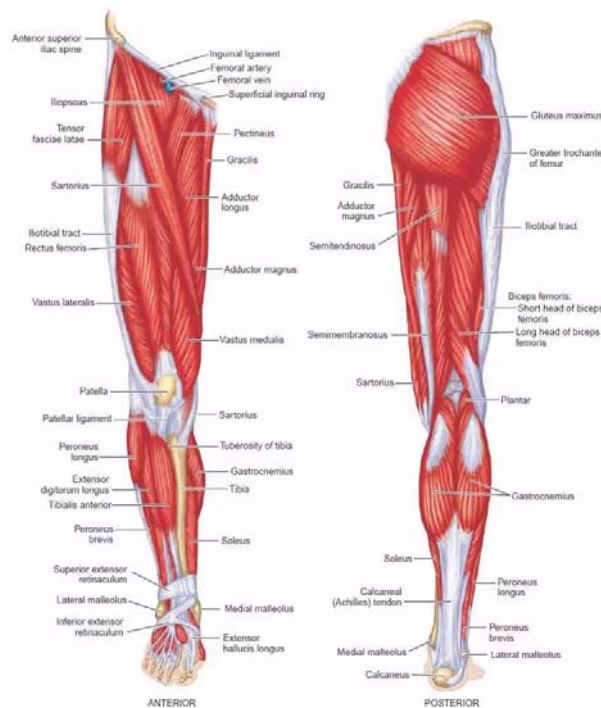


- **Single Joint Muscles**

- Muscles that cross one single joint
- Ex: Vastus Group

- **Biarticular Muscles**

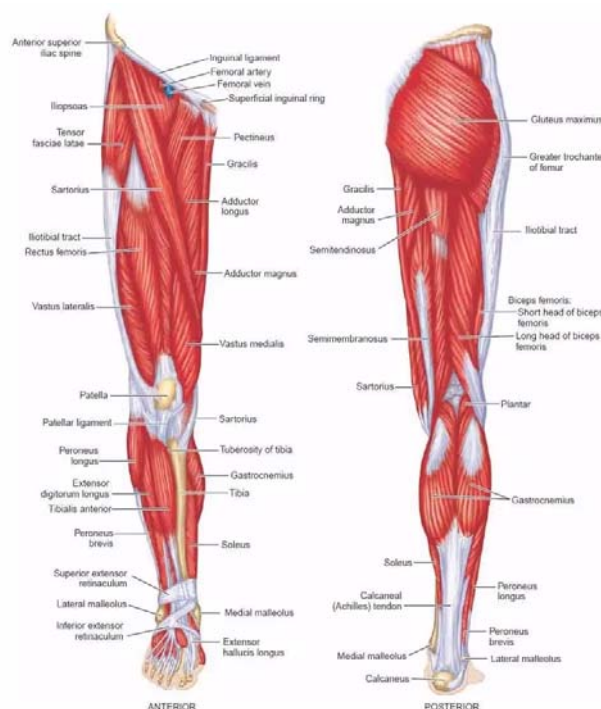
- Muscle that cross/span two joints
- More complex movement
- Ex: Rectus Femoris



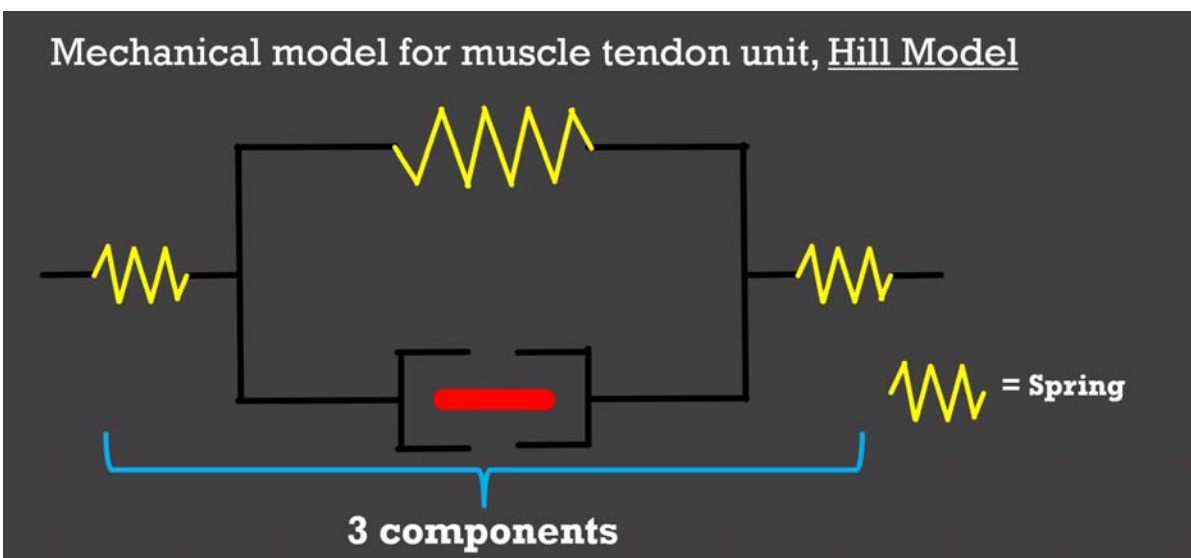
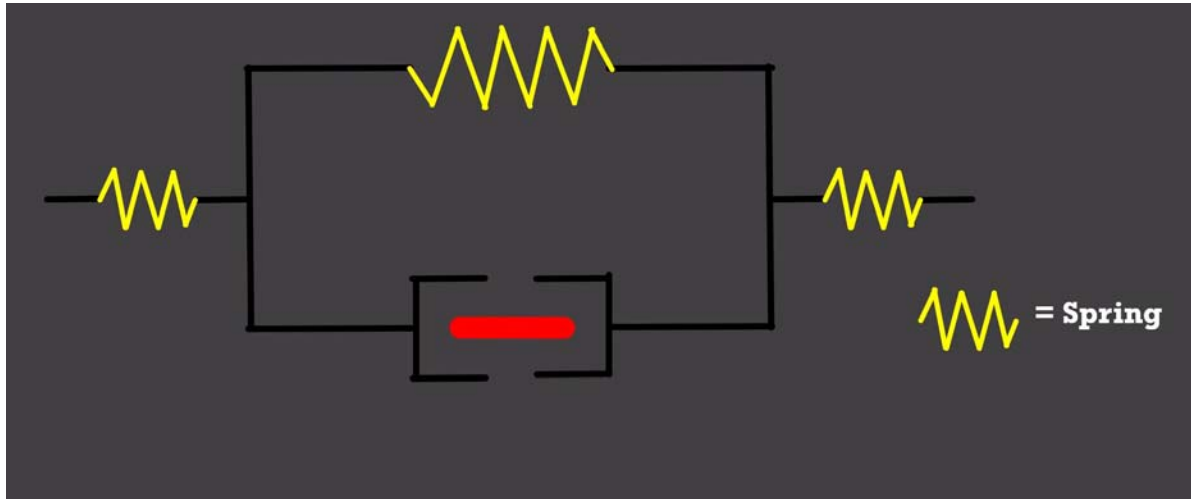
73

- **3 Leg Biarticulate Muscles**

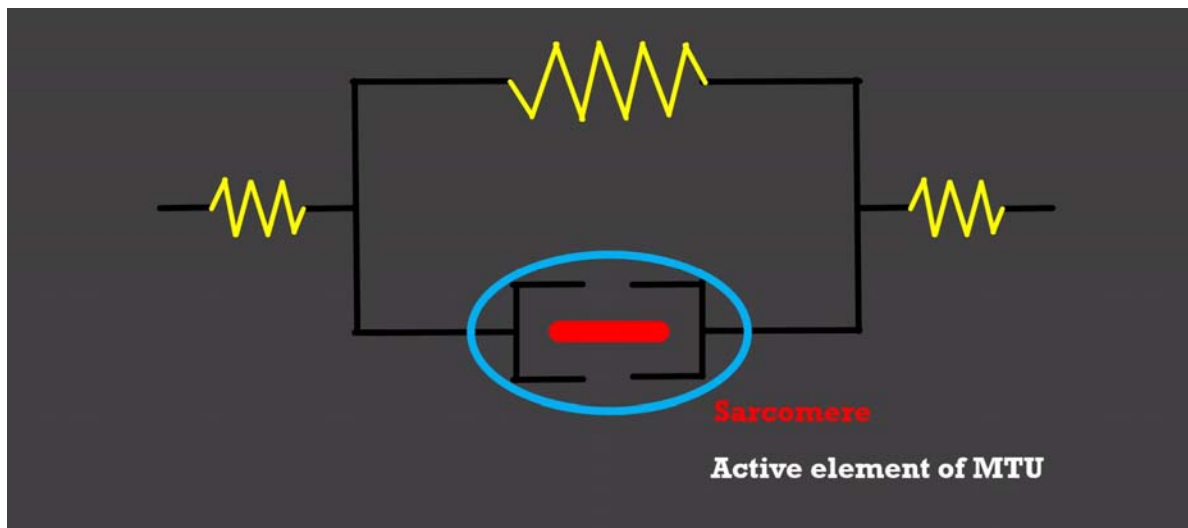
- Hamstrings: Semitendinosus, Semimembranosus, & Biceps Femoris
  - Joints: Hip & Knee
  - Action: Hip Extension & Knee Flexion
- Rectus Femoris
  - Joints: Hip & Knee
  - Action: Hip Flexion & Knee Extension
- Gastrocnemius
  - Joints: Knee & Ankle
  - Action: Knee Flexion & Ankle Plantar Flexion



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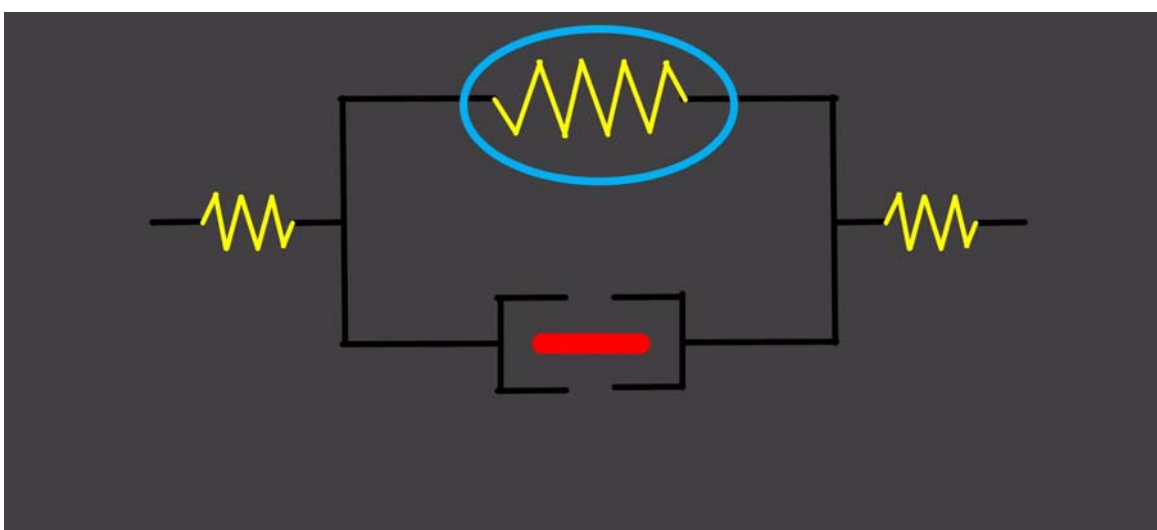


# Contractile Component



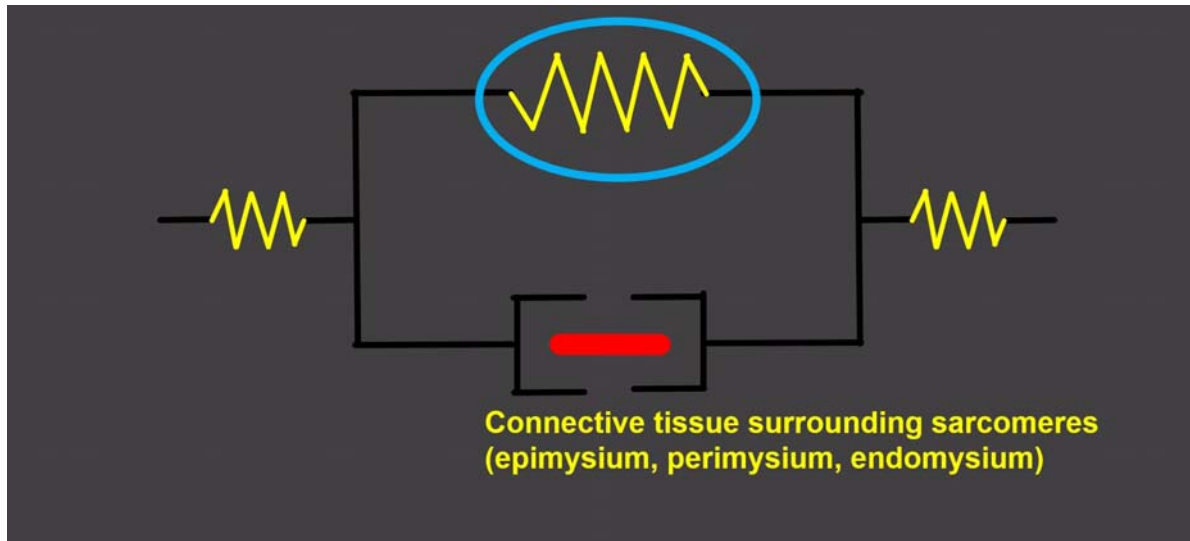
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# Parallel Elastic Component



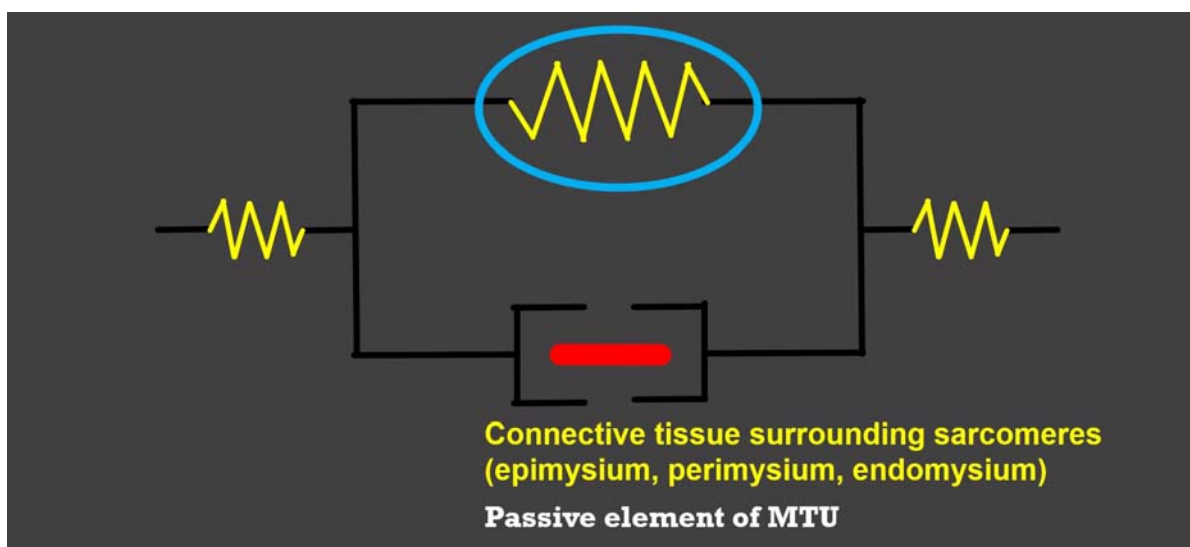
78

# Parallel Elastic Component



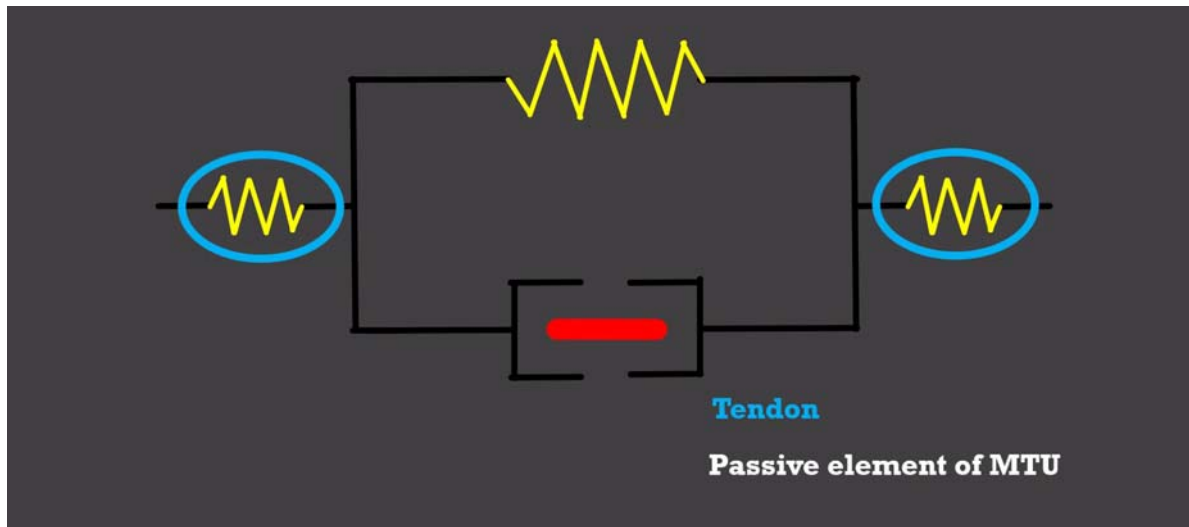
79

# Parallel Elastic Component



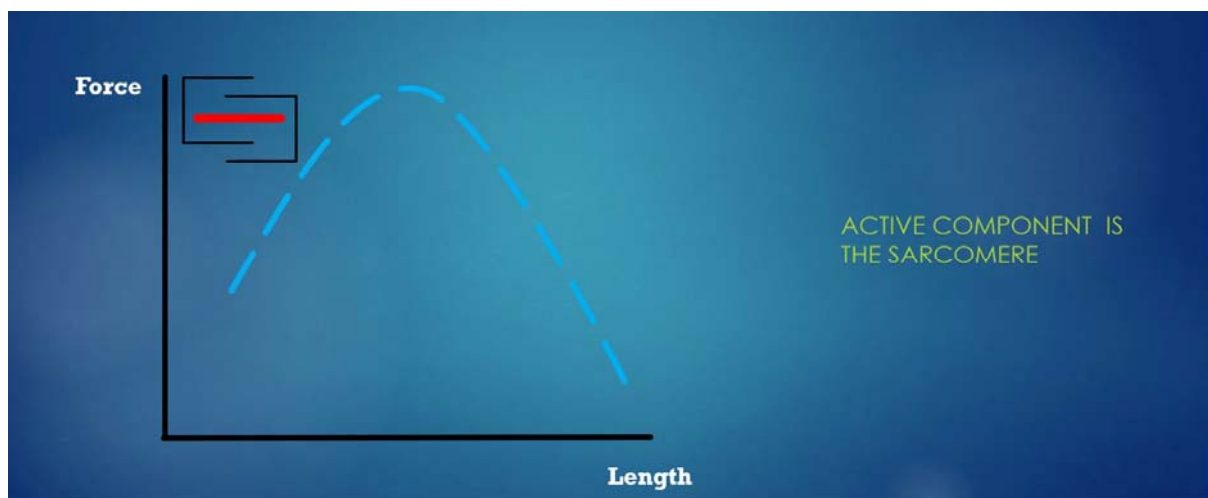
80

# Series Elastic Component



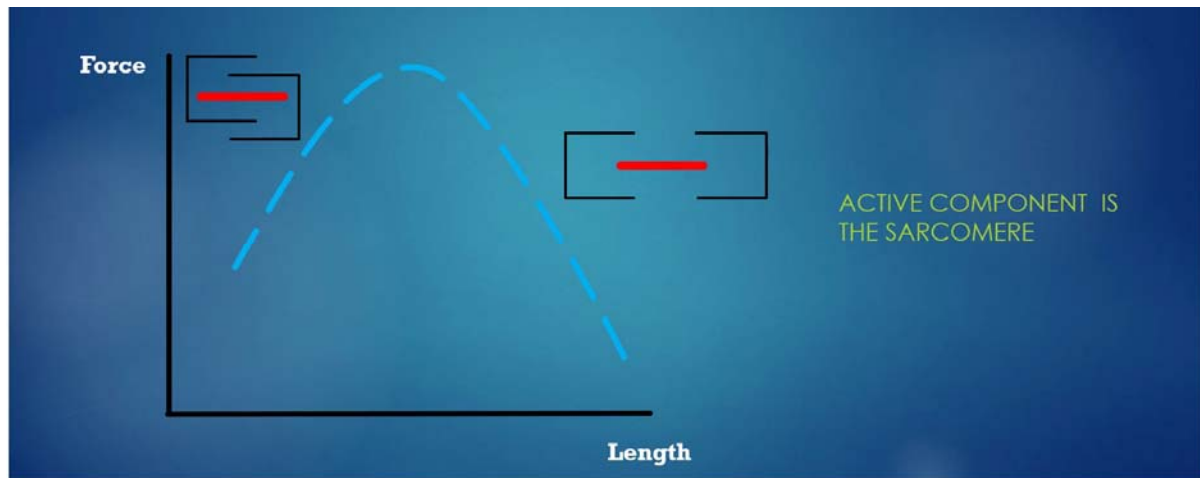
81

# Force-Length Principle



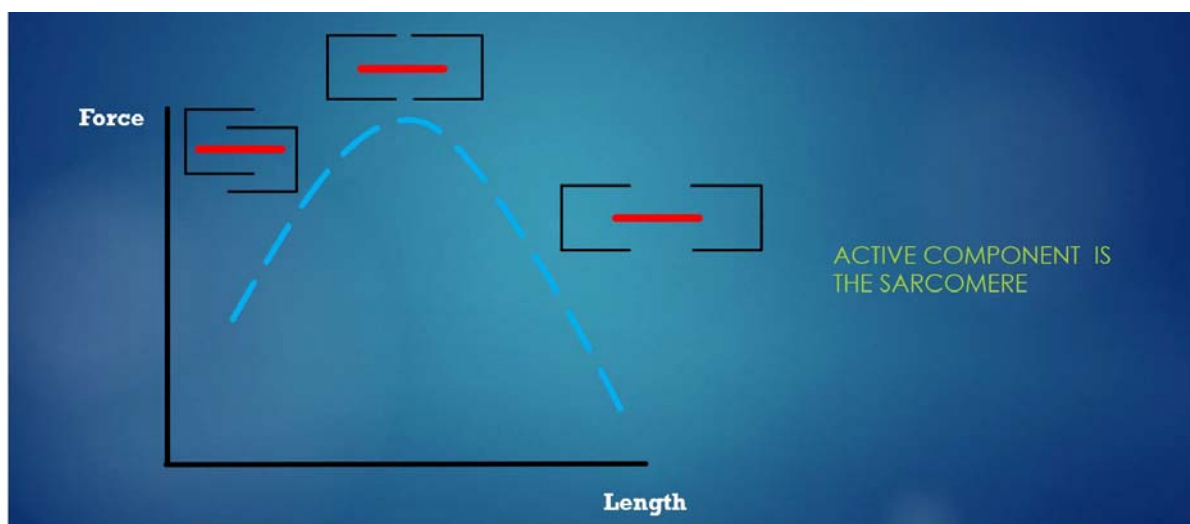
82

# Force-Length Principle



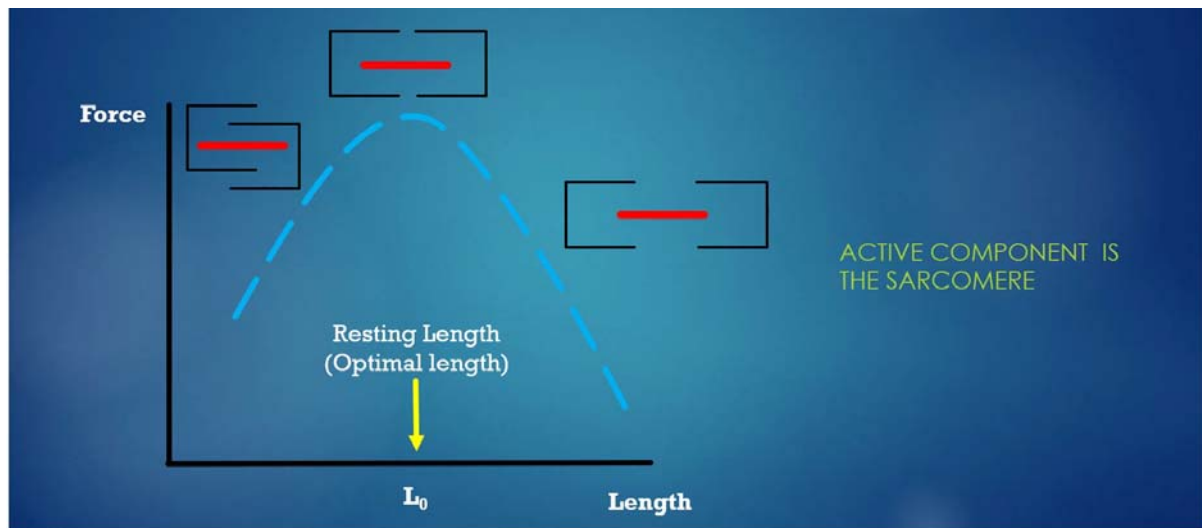
83

# Force-Length Principle



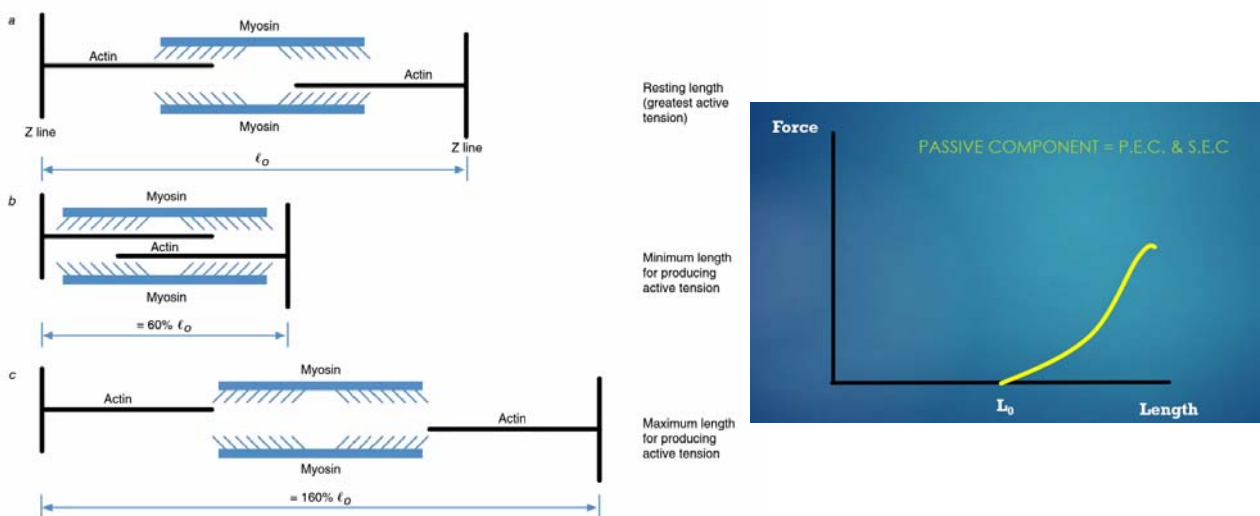
84

# Force-Length Principle



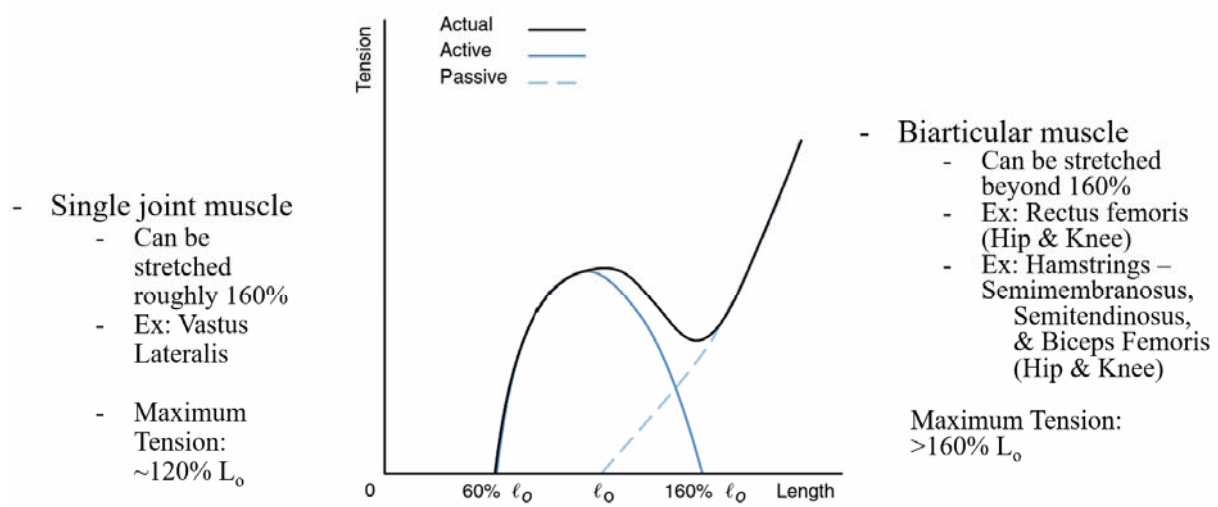
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# Force-Length Principle



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**Figure 11.14** The relationship between muscle length and tension.