Take Home Assignment 1 GE1358/MA1501

For each of the following questions, write down your solution with details of steps. Marks will not given if only final answers are provided.

1. Determine the products AB and BA for the following values of A and B (if possible):

   (a) $A = \begin{bmatrix} 1 & 3 & 6 \\ 2 & -1 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \\ -2 & 1 \end{bmatrix}$

   (b) $A = \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 3 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & -2 \end{bmatrix}$

2. Show that the inverse exists, of the following matrices (no marks will be awarded with a simple "yes" or "no"):

   (a) $\begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$

   (b) $\begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$

3. Perform Gaussian Elimination to solve for the following system of equations, (no marks will be awarded if other method(s) is/are used)

$$\begin{cases} 3x - 6y = & 11 \\ 2x - 4y = & 8 \end{cases}$$

4. Use Cramer's rule to solve the following system of equations:

   (a) $3x - 4y = -2$
   $x + y = 6$

   (b) $x - 2y - 2z = 3$
   $2x - 4y + 4z = 1$
   $3x - 3y - 3z = 4$

5. Show that

(a) $A = \begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \\ 3 & 5 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -a+2b+3c \\ 2a+b \\ 3a+5b-c \end{bmatrix}$

(b) $A = \begin{bmatrix} cos\,\theta & 0 & -sin\,\theta \\ 0 & 1 & 0 \\ sin\,\theta & 0 & cos\,\theta \end{bmatrix} \begin{bmatrix} cos\,\theta & 0 & +sin\,\theta \\ 0 & 1 & 0 \\ -sin\,\theta & 0 & cos\,\theta \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. Show that if $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ then

(a) $(A+B)(A+B) \neq \mathrm{A}^2 + 2AB + B^2$

(b) $(A+B)(A-B) \neq \mathrm{A}^2 - B^2$

7. Show that $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1x+d_1 & c_2x+d_2 & c_3x+d_3 \end{vmatrix} = x \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ d_1 & d_2 & d_3 \end{vmatrix}$

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