Tutorial 6 and 7 (Chapters 6 and 7)

1. If 50% of the population of a large community is in favour of a proposed rise in school taxes, approximate the probability that a random sample of 100 people will contain at least 60 who are in favour of the proposition.

2. The lifetimes of interactive computer chips produced by a certain semiconductor manufacturer are normally distributed with parameters $\mu = 1.4 \times 10^6$ hours and $\sigma = 4 \times 10^5$. What is the approximate probability that a batch of 100 chips will contain at least 90 whose lifetimes are less than 1.8×10^6?

3. (Optional) Suppose $X \sim Binomial(n, p)$. Use CLT to find the minimum value of n that satisfies

$$P(|\frac{X}{n} - p| < \frac{\sqrt{Var(X)}}{2}) \geq 0.99$$

4. Let $Y_1, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y|\theta) = \begin{cases} (\theta+1)y^\theta, & 0 < y < 1; \theta > -1 \\ 0 & \text{otherwise} \end{cases}$$

Find an estimator for θ by the method of moments and find the MLE.

5. If $Y_1, \ldots, Y_n$ denote a random sample from the normal distribution with known mean $\mu = 0$ and unknown variance σ^2, find the method-of-moments estimator of σ^2.

6. If $Y_1, \ldots, Y_n$ denote a random sample from the normal distribution with mean μ and variance σ^2, find the method-of-moments estimators of μ and σ^2.

7. Let $Y_1, \ldots, Y_n$ denote a random sample from the density function given by

$$f(y|\theta) = \begin{cases} \left(\frac{1}{\theta}\right) r y^{r-1} e^{-y^r/\theta}, & y > 0, \theta > 0 \\ 0 & \text{otherwise} \end{cases}$$

where r is a known positive constant. Find the MLE of θ.

8. Suppose that $Y_1, \ldots, Y_n$ constitute a random sample from a uniform distribution with probability density function

$$f(y|\theta) = \begin{cases} \frac{1}{2\theta+1} & 0 < y < 2\theta + 1 \\ 0 & \text{otherwise} \end{cases}$$

   (i) Obtain the MLE of θ.

   (ii) Obtain the MLE for the variance of the this distribution.

9. Suppose $Y_1, \ldots, Y_n$ is a random sample from the uniform distribution on $(\theta, \theta + 1)$.

   (i) Show that $\bar{Y}$ is a biased estimator and compute the bias (the bias is defined as $E(\hat{\theta}) - \theta$).

   (ii) Find a function of $\bar{Y}$ that is an unbiased estimator of θ.

   (iii) Find $MSE(\bar{Y})$ when $\bar{Y}$ is used as an estimator of θ.

10. Suppose $Y \sim Bin(n, p)$. Then Y/n is an unbiased estimator of p. To estimate the variance of Y, we can use $n(Y/n)(1 - Y/n)$.

   (i) Show that the suggested estimator is a biased estimator of $Var(Y)$.

   (ii) Modify $n(Y/n)(1 - Y/n)$ slightly to form an unbiased estimator of $Var(Y)$.

11. Let $Y_1, \ldots, Y_n$ be a random sample from a population with mean μ and variance σ^2. Consider the following estimators for μ:

$$\hat{\mu}_1 = \frac{1}{2}(Y_1 + Y_2),\ \hat{\mu}_2 = \frac{1}{4}Y_1 + \frac{Y_2 + \cdots + Y_{n-1}}{2(n-2)} + \frac{1}{4}Y_n,\ \hat{\mu}_3 = \bar{Y}.$$

(i) Show that all estimators defined above are unbiased.

(ii) Find the variances of the estimators.