Tutorial 1

1. How many different letter arrangements can be made from the letters
   (a) **M A N G O**
   (b) **N A N Y A N G**?

2. For years, telephone area codes in a certain country consisted of a sequence of three digits. The first digit was an integer between 1 and 9; the second digit was either 0 or 1; the third digit was any integer between 2 and 9.
   (a) How many area codes were possible?
   (b) How many area codes starting with 4 were possible?

3. Jimmy has 8 friends, of whom 5 will be invited to a party. How many choices are there if 2 of the friends are feuding and will not attend together?

4. From a group of n people, suppose that we want to choose a committee of k, $k \leq n$, one of whom is to be designated as chairperson.
   (a) By focusing first on the choice of the committee and then on the choice of the chair, argue that there are $\binom{n}{k} k$ possible choices.
   (b) By focusing first on the choice of the nonchair committee members and then on the choice of the chair, argue that there are $\binom{n}{k-1}(n-k+1)$ possible choices.
   (c) By focusing first on the choice of the chair and then on the choice of the other committee members, argue that there are $n\binom{n-1}{k-1}$ possible choices.
   (d) Conclude from parts (a), (b) and (c) that
   $$\binom{n}{k} k = \binom{n}{k-1}(n-k+1) = n\binom{n-1}{k-1}$$
   (e) Use the factorial definition of $\binom{m}{r}$ to verify the identity in part (d).

5. (a) What is the number of positive integer-valued solutions of
   $$x_1 + x_2 + \cdots + x_r = n?$$
   (b) What is the number of nonnegative integer-valued solutions of
   $$x_1 + x_2 + \cdots + x_r = n$$
   for which exactly k of the x_i are equal to 0?

6. A football team produced a record of 8 wins and 4 losses over its season.
   (a) How many different arrangements W (win) and L (loss) are possible?
   (b) How many different arrangements are possible if there are exactly 3 runs. (A run is a continuous stretch of W's or L's. For example, WWLLLWWLLW has 5 runs.)
   (c) What about 4 runs?

7. How many different ways to assign 9 students to 3 tutorial sections, such that
   (a) section one has 3 students (some other section may have no students)
   (b) each section has 3 students