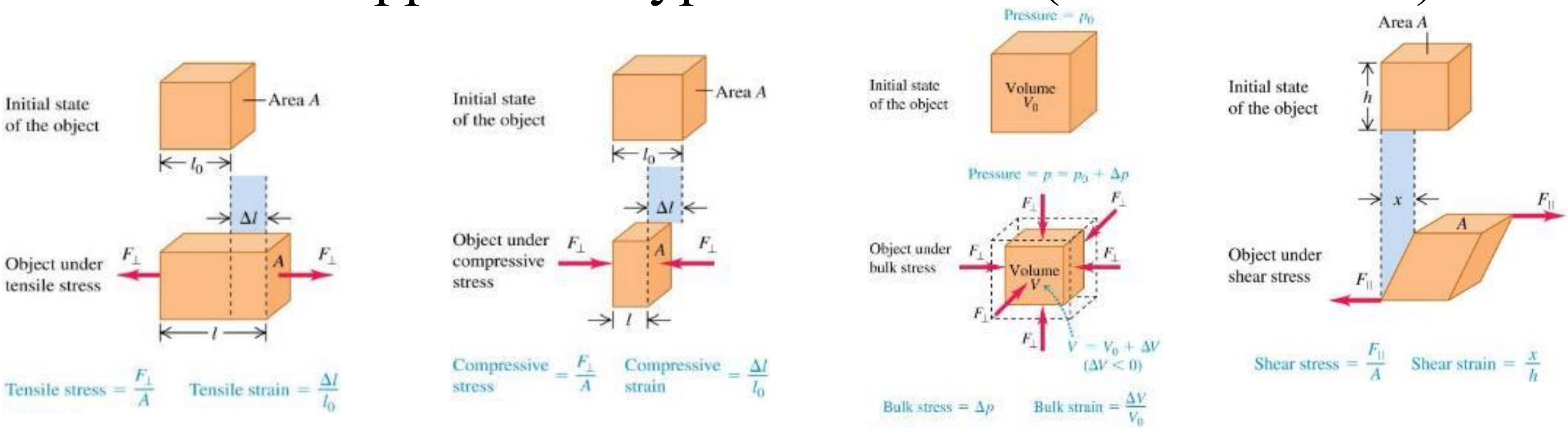
Chapter 12

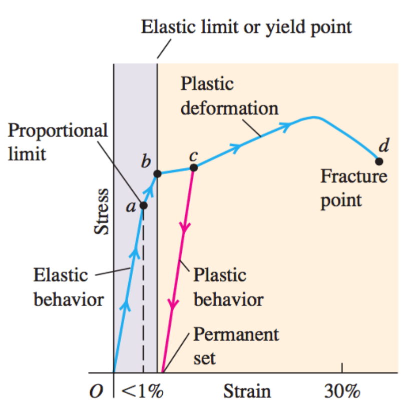
Fluid Mechanics

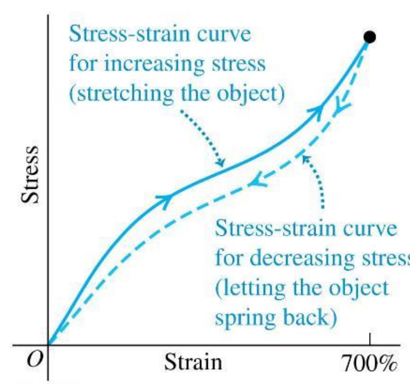
Review Chapter 11

- Solids in equilibrium:
  - $\sum \vec{F} = \vec{0}, \ \sum \vec{\tau} = \vec{0}$
- Solids can support four types of stresses (before fracture)



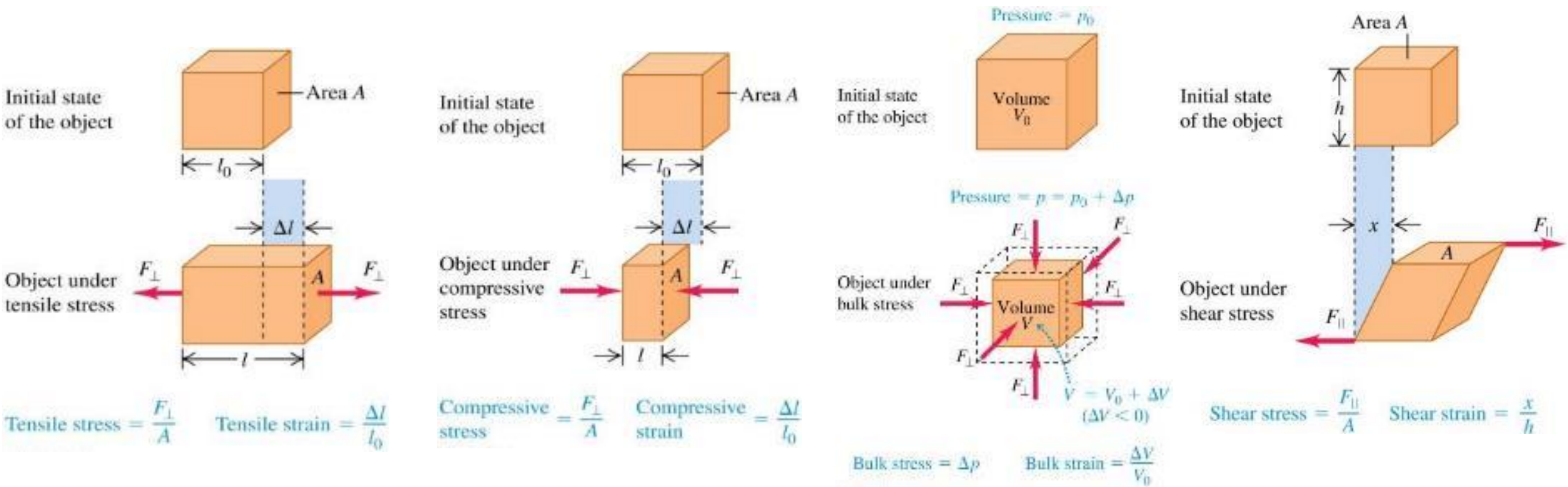
- Stress-strain curves for metals and rubber





From solids to liquids

- Solids can support all types of stresses (before breaking)



- Liquids can support only one type: bulk stress.
  - This is much simpler than solids.
  - However, we also need to study a more complicated phenomenon: flow.

Learning Goals for Chapter 12

- Stationary fluid:
  - Density 密度.
  - Pressure
  - Buoyancy, Archimedes' principle
  - Surface tension
- Flowing fluid: 流体.
  - Ideal fluid (not viscous, slow flowing, not compressible)
    - Continuity Equation
    - Bernoulli’s Equation
  - Viscous fluid
  - Turbulence

Red sections involve calculations.

Density

- An important property of any material, fluid or solid, is its **density**, defined as its mass per unit volume.
- A homogeneous material such as ice or iron has the same density throughout.
- For a homogeneous material,

Density of a homogeneous material

$$\rho = \frac{m}{V}$$

m: Mass of material

V: Volume occupied by material

- The SI unit of density is the kilogram per cubic meter (1 kg/m^3).

Example 12.1 The weight of a roomful of air

Find the mass and weight of the air at 20°C in a living room with a 4.0 m × 5.0 m floor and a ceiling 3.0 m high, and the mass and weight of an equal volume of water.

EXECUTE: We have $V = (4.0\ \text{m})(5.0\ \text{m})(3.0\ \text{m}) = 60\ \text{m}^3$, so from Eq. (12.1),

$$m_{\text{air}} = \rho_{\text{air}} V = (1.20\ \text{kg/m}^3)(60\ \text{m}^3) = 72\ \text{kg}$$

$$w_{\text{air}} = m_{\text{air}} g = (72\ \text{kg})(9.8\ \text{m/s}^2) = 700\ \text{N} = 160\ \text{lb}$$

The mass and weight of an equal volume of water are

$$m_{\text{water}} = \rho_{\text{water}} V = (1000\ \text{kg/m}^3)(60\ \text{m}^3) = 6.0 \times 10^4\ \text{kg}$$

$$\begin{aligned} w_{\text{water}} &= m_{\text{water}} g = (6.0 \times 10^4\ \text{kg})(9.8\ \text{m/s}^2) \\ &= 5.9 \times 10^5\ \text{N} = 1.3 \times 10^5\ \text{lb} = 66\ \text{tons} \end{aligned}$$

EVALUATE: A roomful of air weighs about the same as an average adult. Water is nearly a thousand times denser than air, so its mass and weight are larger by the same factor. The weight of a roomful of water would collapse the floor of an ordinary house.

Pressure

- Here are the forces acting on a small surface within a fluid at rest.

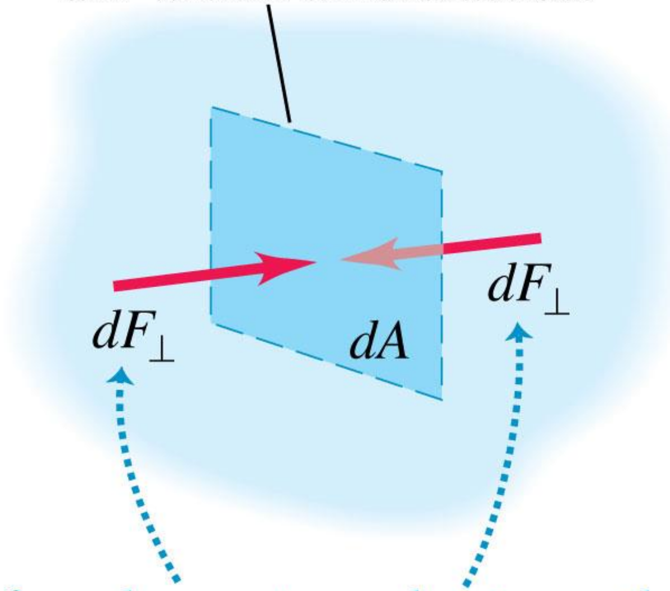
- We define the pressure p at that point as the normal force per unit area.

$$p = \frac{F_{\perp}}{A}$$

- The SI unit of pressure is the **pascal**

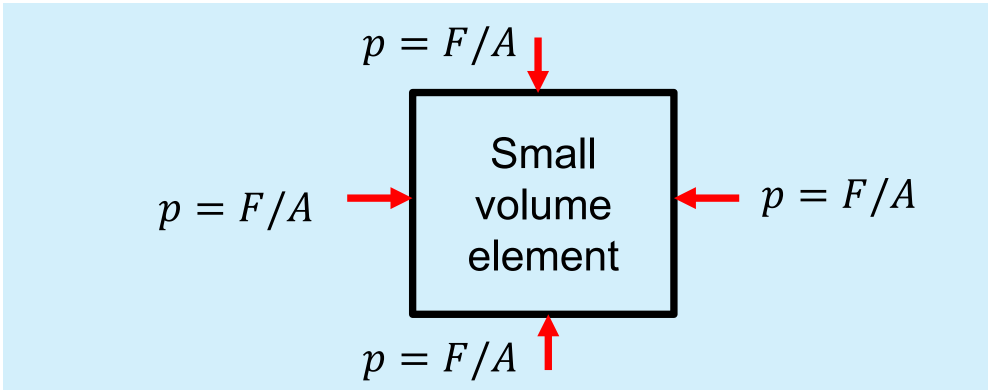
$$1 \text{ pascal} = 1 \text{ Pa} = 1 \text{ N/m}^2$$

A small surface of area dA within a fluid at rest

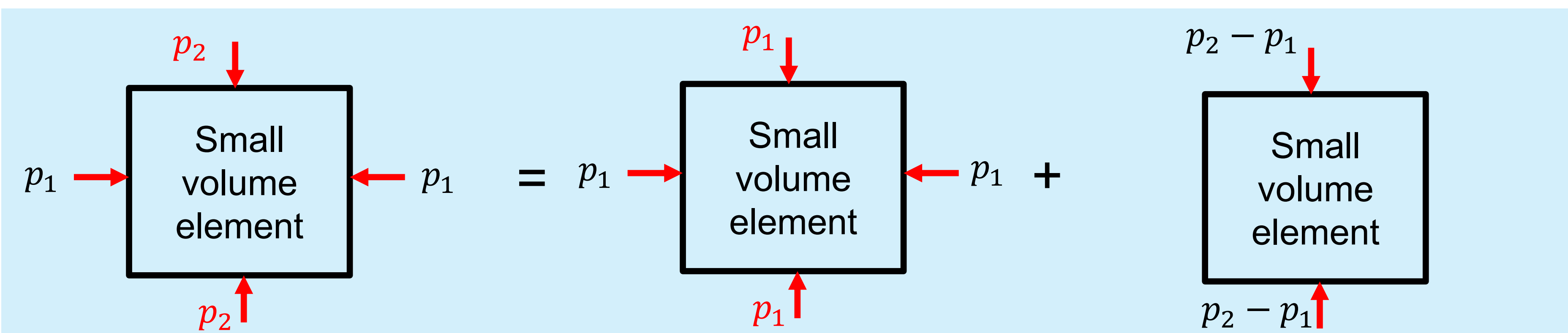


The surface does not accelerate, so the surrounding fluid exerts equal normal forces on both sides of it. (The fluid cannot exert any force parallel to the surface, since that would cause the surface to accelerate.)

Pressure is a scalar quantity in a liquid



- In a liquid, pressure for a small volume element are equal in all directions.
- Otherwise, the stress is decomposed into bulk and tensile/compressive components. Liquids cannot support the latter.



Direction-dependent pressure cannot exist because it can be decomposed into: = bulk stress and + tensile/compressive stress (depending on the sign of $p_2 - p_1$), which the liquid cannot support.

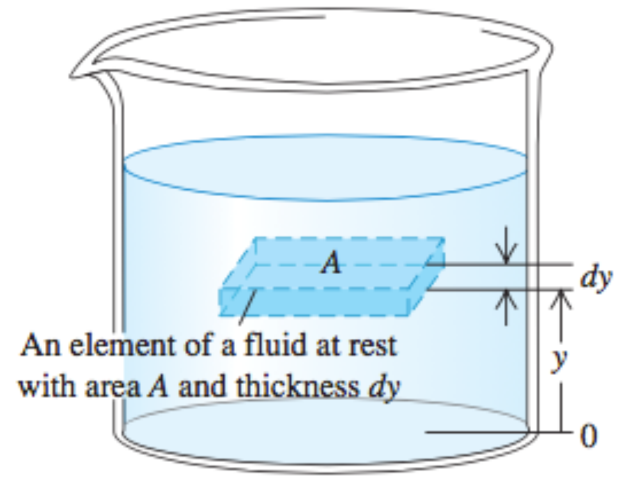
Pressure at depth in a fluid

- Consider a thin element of fluid with thickness dy. The bottom and top surfaces each have area A, and they are at elevations y and $y + dy$ above some reference level. The volume of the fluid element is $dV = A\,dy$, its mass is $dm = \rho dV = \rho A dy$, and its weight is $dw = dmg = \rho g A dy$.

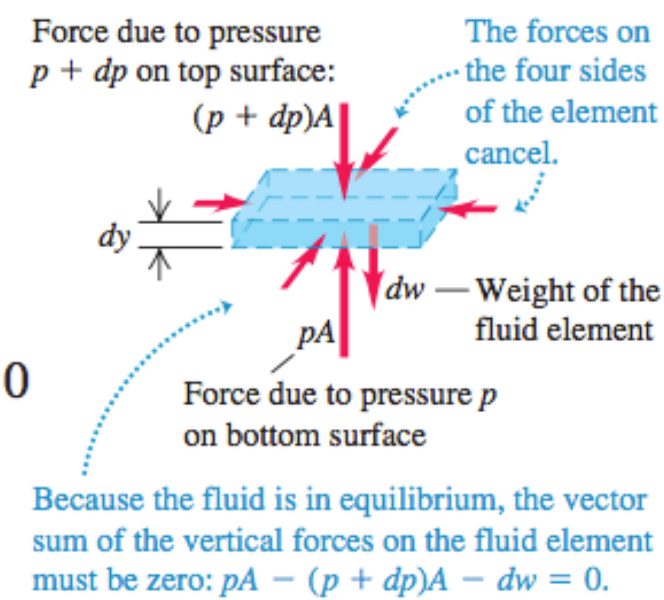
- Total y-component of upward force on the bottom surface is pA. Total y-component of (downward) force on the top surface is $-(p + dp)A$. The fluid element is in equilibrium, so the total y-component of force, including the weight and the forces at the bottom and top surfaces, must be zero:

$$\sum F_y = 0 \qquad \text{so} \qquad pA - (p + dp)A - \rho g A\,dy = 0$$

(a)



(b)



Pressure at depth in a fluid

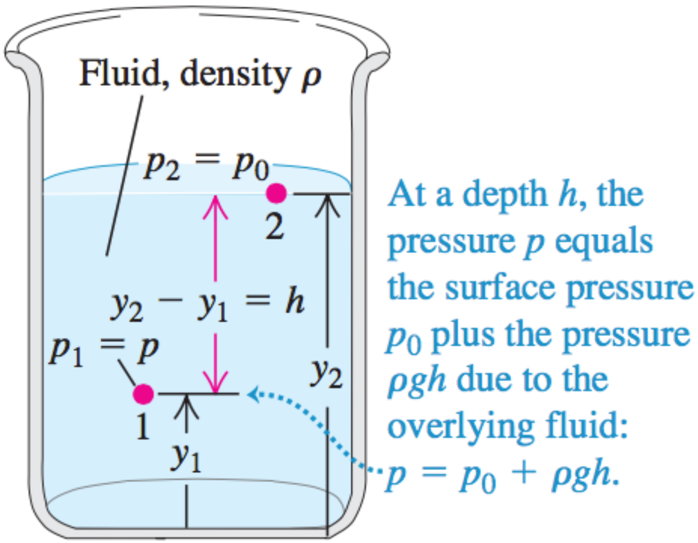
- Divide out the area A and rearrange, we get

$$\frac{dp}{dy} = -\rho g$$

- If ρ and g are constant, then

$$p_2 - p_1 = -\rho g(y_2 - y_1)$$

- Expressing the above equation in terms of the depth h in a fluid, we have



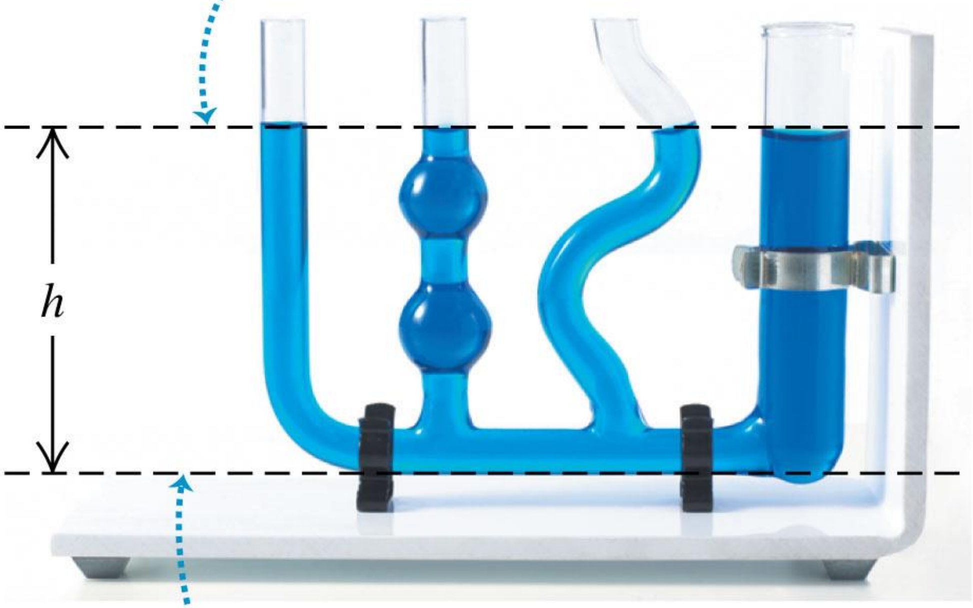
Pressure difference between levels 1 and 2:
$p_2 - p_1 = -\rho g(y_2 - y_1)$
The pressure is greater at the lower level.

Pressure at depth h in a **fluid of uniform density**

$$p = p_0 + \rho g h$$

- Uniform density of fluid (ρ)
- Depth below surface (h)
- Pressure at surface of fluid (p_0)
- Acceleration due to gravity ($g > 0$)

Pressure at depth in a fluid



- Each fluid column has the same height, no matter what its shape.

Q12.3

A cylinder is completely filled with water. The top of the cylinder is sealed with a tight-fitting lid. If you push down on the lid with a pressure of 1000 Pa, the water pressure at the bottom of the cylinder

A. increases by more than 1000 Pa.

B. increases by 1000 Pa.

C. increases by less than 1000 Pa.

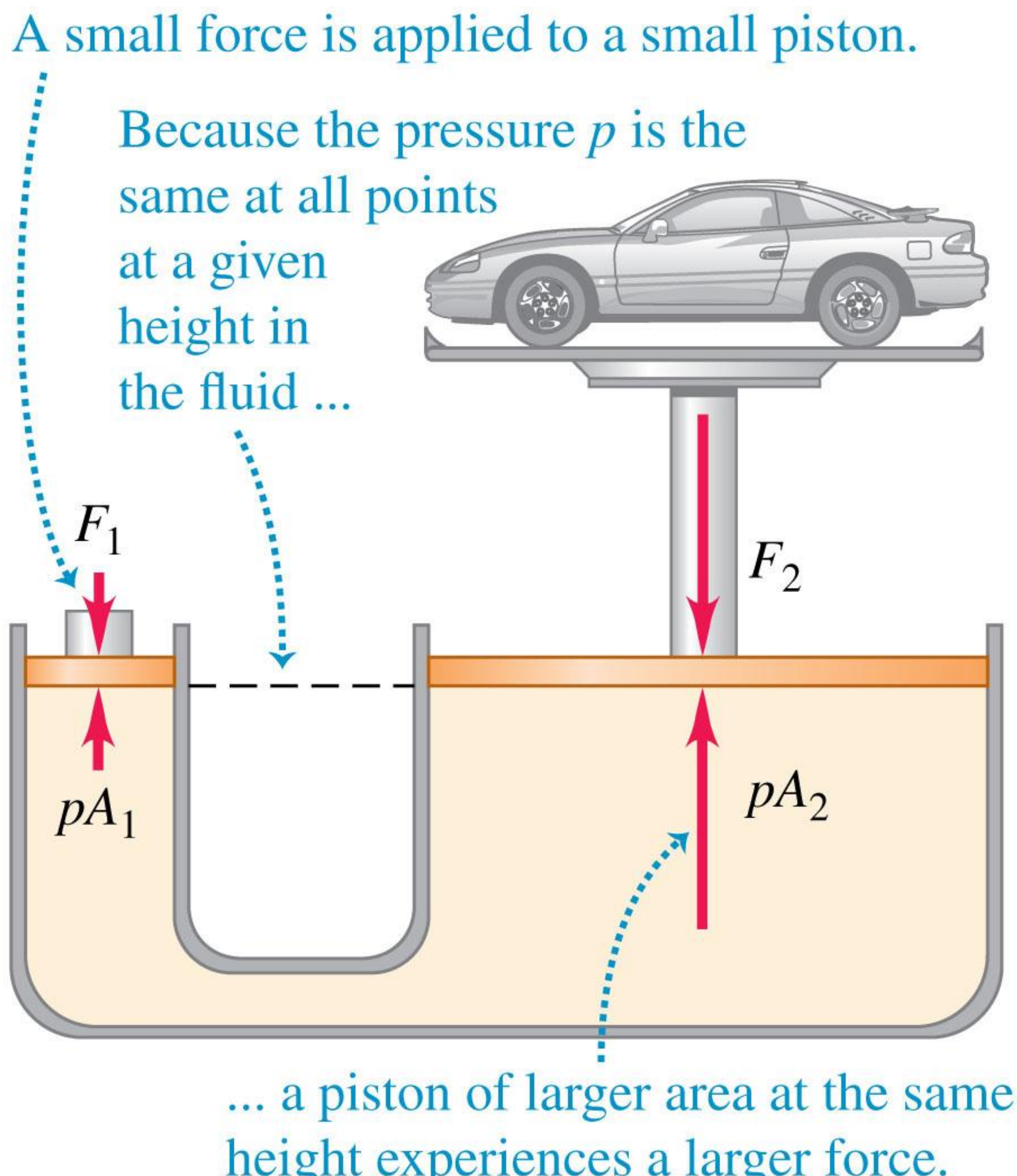
D. is unchanged.

E. increases by an amount determined by the height of the cylinder.

Pascal's law

- Pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel:

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$



Alternative units of pressure

- Two related units, used principally in meteorology, are the *bar,* equal to 10^5 Pa, and the *millibar,* equal to 100 Pa.

- **Atmospheric pressure** p_a is the pressure of the earth's atmosphere, the pressure at the bottom of this sea of air in which we live. This pressure varies with weather changes and with elevation. Normal atmospheric pressure at sea level (an average value) is 1 *atmosphere* (atm), defined to be exactly 101,325 Pa. To four significant figures,

$$\begin{aligned}(p_a)_{av} &= 1\ \text{atm} = 1.013 \times 10^5\ \text{Pa} \\ &= 1.013\ \text{bar} = 1013\ \text{millibar} = 14.70\ \text{lb/in.}^2\end{aligned}$$

Example 12.2 The force of air

In the room described in Example 12.1, what is the total downward force on the floor due to an air pressure of 1.00 atm?

SOLUTION

IDENTIFY and SET UP: This example uses the relationship among the pressure p of a fluid (air), the area A subjected to that pressure, and the resulting normal force $F_\perp$ the fluid exerts. The pressure is uniform, so we use Eq. (12.3), $F_\perp = pA$, to determine $F_\perp$. The floor is horizontal, so $F_\perp$ is vertical (downward).

EXECUTE: We have $A = (4.0 \text{ m})(5.0 \text{ m}) = 20 \text{ m}^2$, so from Eq. (12.3),

$$\begin{aligned} F_\perp = pA &= (1.013 \times 10^5 \text{ N/m}^2)(20 \text{ m}^2) \\ &= 2.0 \times 10^6 \text{ N} = 4.6 \times 10^5 \text{ lb} = 230 \text{ tons} \end{aligned}$$

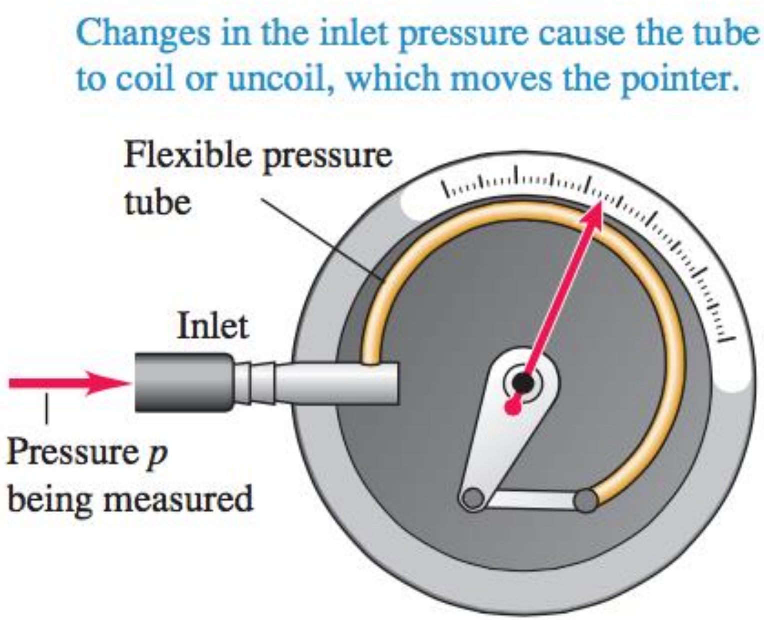
EVALUATE: Unlike the water in Example 12.1, $F_\perp$ will not collapse the floor here, because there is an *upward* force of equal magnitude on the floor's underside. If the house has a basement, this upward force is exerted by the air underneath the floor. In this case, if we neglect the thickness of the floor, the *net* force due to air pressure is zero.

Absolute pressure and gauge pressure

- When we say that the pressure in a car tire is “32 pounds per square inch (psi)”, (2.2×10^5 Pa), we mean that it is *greater* than atmospheric pressure by this amount.
- The *total* pressure in the tire is greater by p_{atm}.
- The excess pressure above atmospheric pressure is usually called **gauge pressure**.
- The total pressure is called **absolute pressure**.
- If the pressure is less than atmospheric, as in a partial vacuum, the gauge pressure is negative.

Pressure gauges

- This Bourdon-type pressure gauge is connected to a high-pressure gas line.
- The gauge pressure shown is just over 5 bars (1 bar = 10^5 Pa).



Example 12.3 Finding absolute and gauge Pressures

Water stands 12.0 m deep in a storage tank whose top is open to the atmosphere. What are the absolute and gauge pressures at the bottom of the tank?

EXECUTE: From Eq. (12.6), the pressures are

absolute:

$$\begin{aligned} p &= p_0 + \rho g h \\ &= (1.01 \times 10^5 \text{ Pa}) + (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(12.0 \text{ m}) \\ &= 2.19 \times 10^5 \text{ Pa} = 2.16 \text{ atm} = 31.8 \text{ lb/in.}^2 \end{aligned}$$

gauge:
$$\begin{aligned} p - p_0 &= (2.19 - 1.01) \times 10^5 \text{ Pa} \\ &= 1.18 \times 10^5 \text{ Pa} = 1.16 \text{ atm} = 17.1 \text{ lb/in.}^2 \end{aligned}$$

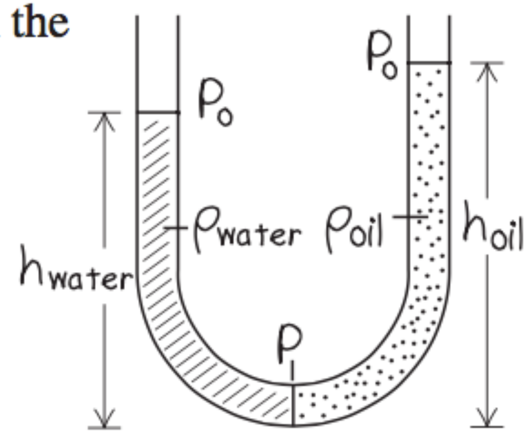
EVALUATE: A pressure gauge at the bottom of such a tank would probably be calibrated to read gauge pressure rather than absolute pressure.

Blood pressure

- Blood-pressure readings, such as 130/80, give the maximum and minimum gauge pressures in the arteries, measured in “mm Hg” or “torr.”
- Blood pressure varies with vertical position within the body.
- The standard reference point is the upper arm, level with the heart.

Example 12.4 A tale of two fluids

A manometer tube is partially filled with water. Oil (which does not mix with water) is poured into the left arm of the tube until the oil–water interface is at the midpoint of the tube as shown. Both arms of the tube are open to the air. Find a relationship between the heights h_{oil} and h_{water}.



SOLUTION

IDENTIFY and SET UP: Figure 12.10 shows our sketch. The relationship between pressure and depth given by Eq. (12.6) applies only to fluids of uniform density; we have two fluids of different densities, so we must write a separate pressure–depth relationship for each. Both fluid columns have pressure p at the bottom (where they are in contact and in equilibrium) and are both at atmospheric pressure p_0 at the top (where both are in contact with and in equilibrium with the air).

EXECUTE: Writing Eq. (12.6) for each fluid gives

$$p = p_0 + \rho_{water} g h_{water}$$
$$p = p_0 + \rho_{oil} g h_{oil}$$

Since the pressure p at the bottom of the tube is the same for both fluids, we set these two expressions equal to each other and solve for h_{oil} in terms of h_{water}. You can show that the result is

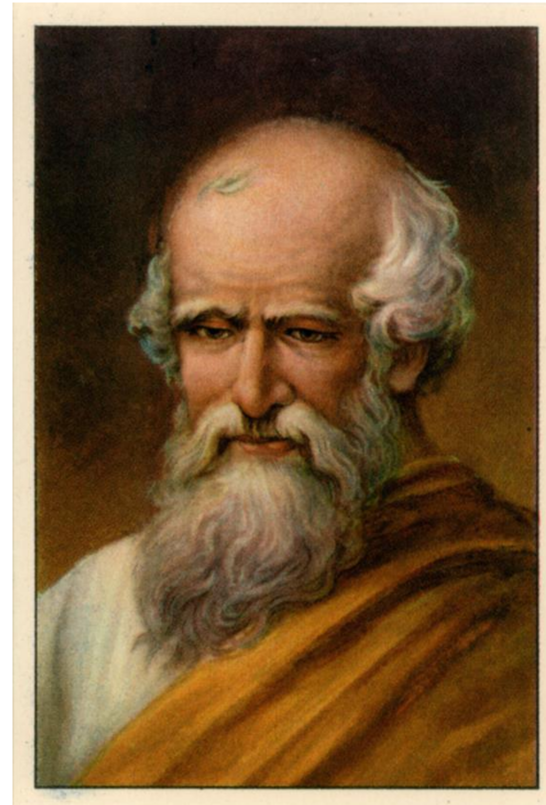
$$h_{oil} = \frac{\rho_{water}}{\rho_{oil}} h_{water}$$

EVALUATE: Water $(\rho_{water} = 1000\ \mathrm{kg/m^3})$ is denser than oil $(\rho_{oil} \approx 850\ \mathrm{kg/m^3})$, so h_{oil} is greater than h_{water} as Fig. 12.10 shows. It takes a greater height of low-density oil to produce the same pressure p at the bottom of the tube.

Buoyancy

- **Buoyancy** is a familiar phenomenon: A body immersed in water seems to weigh less than when it is in air. When the body is less dense than the fluid, it floats. The human body usually floats in water, and a helium-filled balloon floats in air.

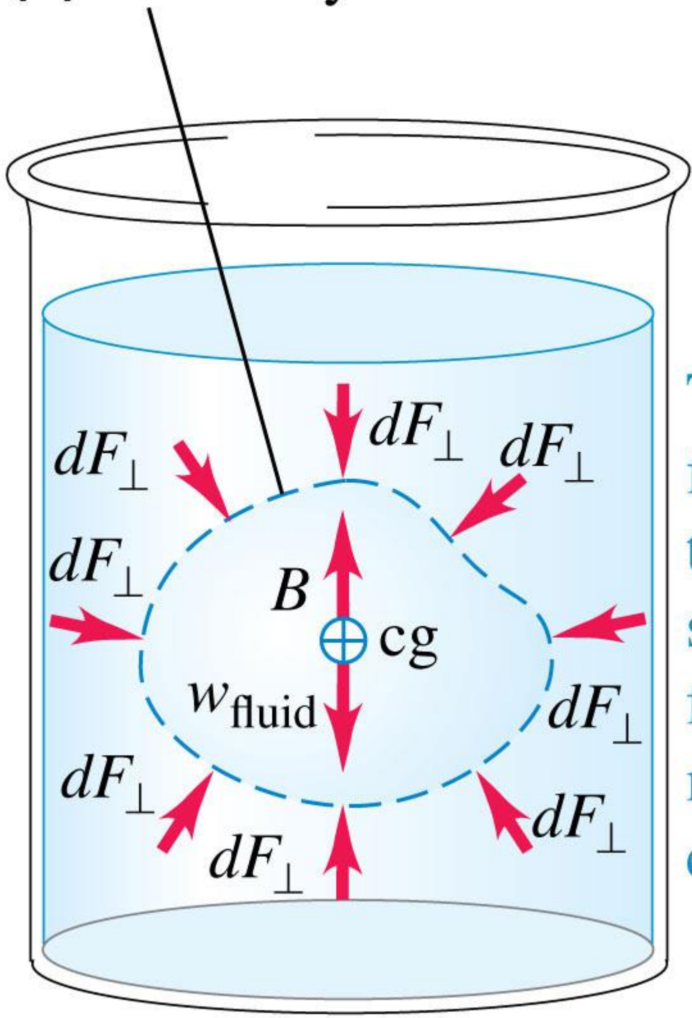
- **Archimedes' principle**: When a body is completely or partially immersed in a fluid, the fluid exerts an upward force on the body equal to the weight of the fluid displaced by the body.



Archimedes of Syracuse
(c. 287 BC – c. 212 BC)

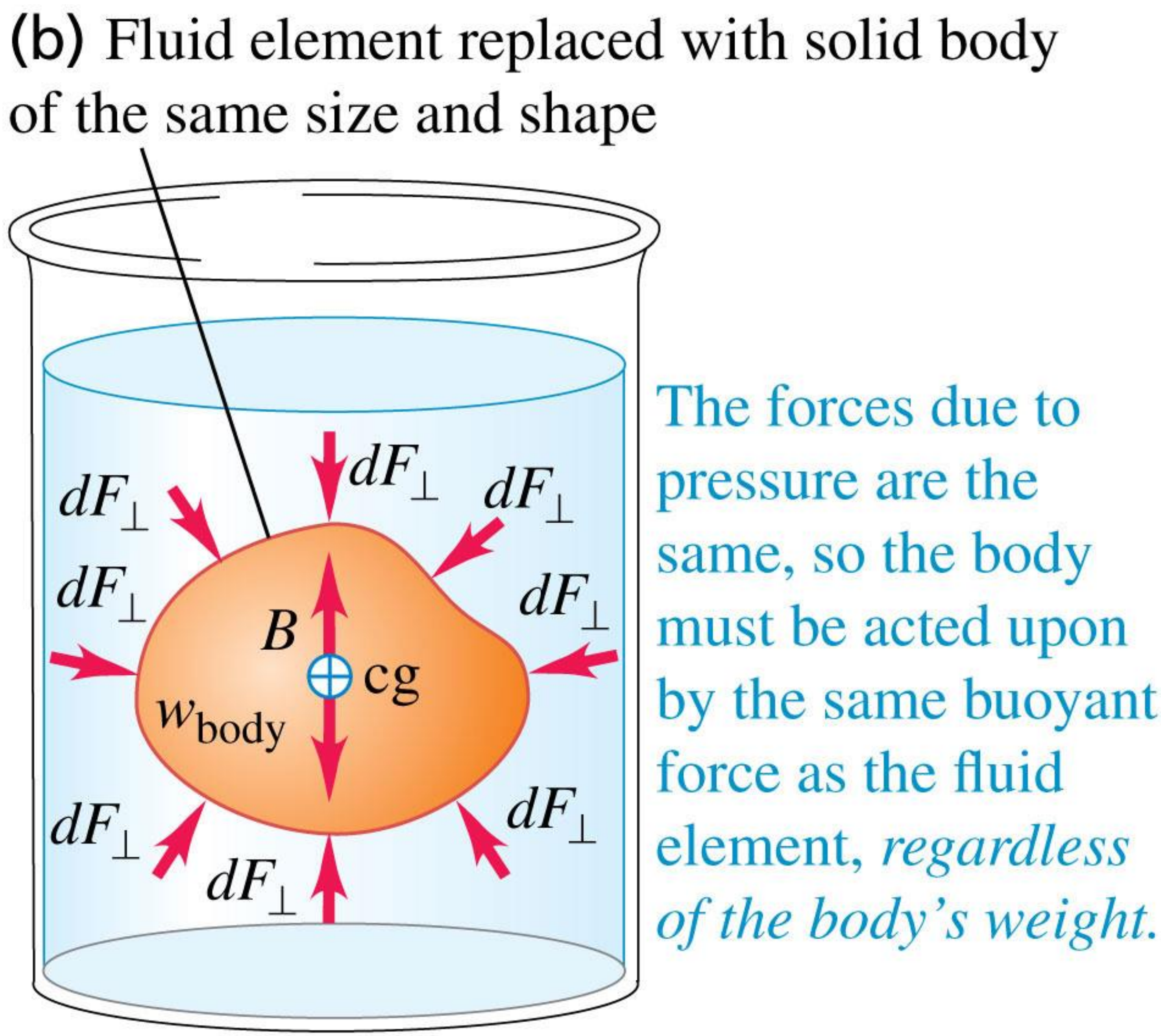
Archimedes's principle: Proof step 1

(a) Arbitrary element of fluid in equilibrium



The forces on the fluid element due to pressure must sum to a buoyant force equal in magnitude to the element's weight.

Archimedes's principle: Proof step 2



Q12.2

A block of ice (density 920 kg/m^3), a block of concrete (density 2000 kg/m^3), and a block of iron (density 7800 kg/m^3) are all submerged in the same fluid. All three blocks have the same volume. Which block experiences the greatest buoyant force?

A. The block of ice experiences the greatest buoyant force.

B. The block of concrete experiences the greatest buoyant force.

C. The block of iron experiences the greatest buoyant force.

D. All three experience the same buoyant force.

E. The answer depends on the density of the fluid.

Example 12.5 Buoyancy

A 15.0-kg solid gold statue is raised from the sea bottom (Fig. 12.13a). What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?

EXECUTE: (a) To find B_{sw}, we first find the statue's volume V using the density of gold from Table 12.1:

$$V = \frac{m_{\text{statue}}}{\rho_{\text{gold}}} = \frac{15.0\ \text{kg}}{19.3 \times 10^3\ \text{kg/m}^3} = 7.77 \times 10^{-4}\ \text{m}^3$$

The buoyant force B_{sw} equals the weight of this same volume of seawater. Using Table 12.1 again:

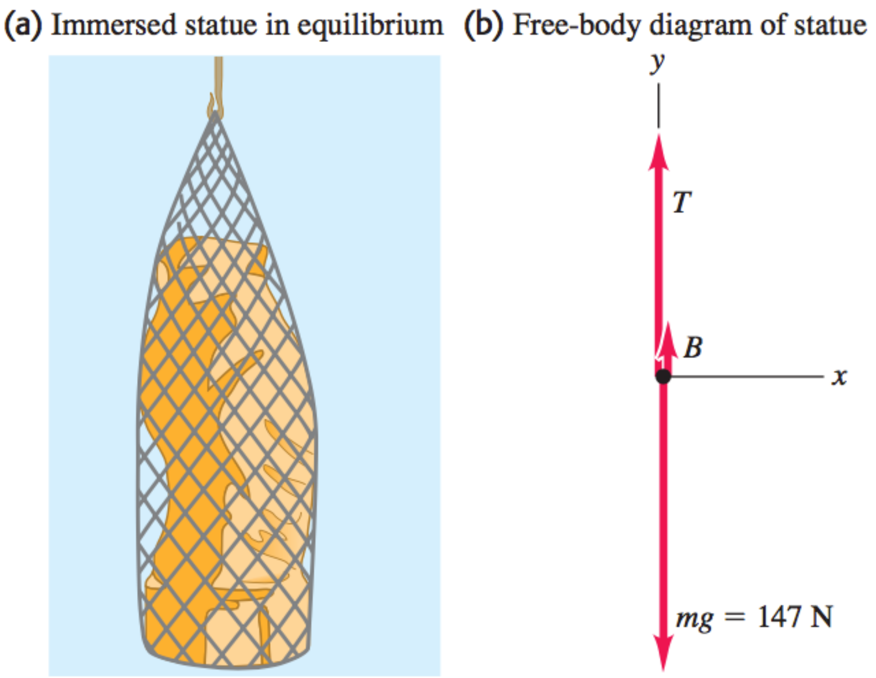
$$\begin{aligned} B_{sw} &= w_{sw} = m_{sw}g = \rho_{sw}Vg \\ &= (1.03 \times 10^3\ \text{kg/m}^3)(7.77 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) \\ &= 7.84\ \text{N} \end{aligned}$$

The statue is at rest, so the net external force acting on it is zero. From Fig. 12.13b,

$$\sum F_y = B_{sw} + T_{sw} + (-m_{\text{statue}}g) = 0$$

$$\begin{aligned} T_{sw} &= m_{\text{statue}}g - B_{sw} = (15.0\ \text{kg})(9.80\ \text{m/s}^2) - 7.84\ \text{N} \\ &= 147\ \text{N} - 7.84\ \text{N} = 139\ \text{N} \end{aligned}$$

A spring scale attached to the upper end of the cable will indicate a tension 7.84 N less than the statue's actual weight $m_{\text{statue}}g =$ 147 N.



Example 12.5 Buoyancy

A 15.0-kg solid gold statue is raised from the sea bottom (Fig. 12.13a). What is the tension in the hoisting cable (assumed massless) when the statue is (a) at rest and completely underwater and (b) at rest and completely out of the water?

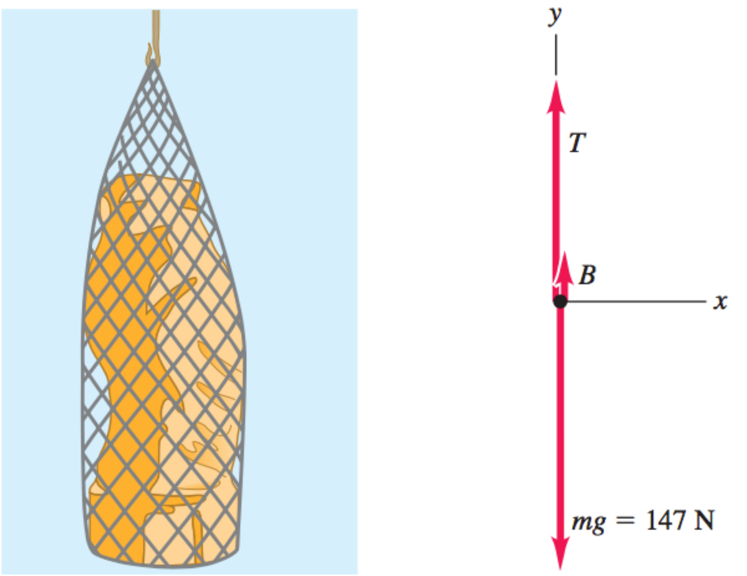
(b) The density of air is about 1.2 kg/m^3, so the buoyant force of air on the statue is

$$B_{\text{air}} = \rho_{\text{air}} V g = (1.2\ \text{kg/m}^3)(7.77 \times 10^{-4}\ \text{m}^3)(9.80\ \text{m/s}^2) = 9.1 \times 10^{-3}\ \text{N}$$

This is negligible compared to the statue's actual weight $m_{\text{statue}} g =$ 147 N. So within the precision of our data, the tension in the cable with the statue in air is $T_{\text{air}} = m_{\text{statue}} g = 147$ N.

EVALUATE: Note that the buoyant force is proportional to the density of the *fluid* in which the statue is immersed, *not* the density of the statue. The denser the fluid, the greater the buoyant force and the smaller the cable tension. If the fluid had the same density as the statue, the buoyant force would be equal to the statue's weight and the tension would be zero (the cable would go slack). If the fluid were denser than the statue, the tension would be *negative:* The buoyant force would be greater than the statue's weight, and a downward force would be required to keep the statue from rising upward.

(a) Immersed statue in equilibrium **(b)** Free-body diagram of statue

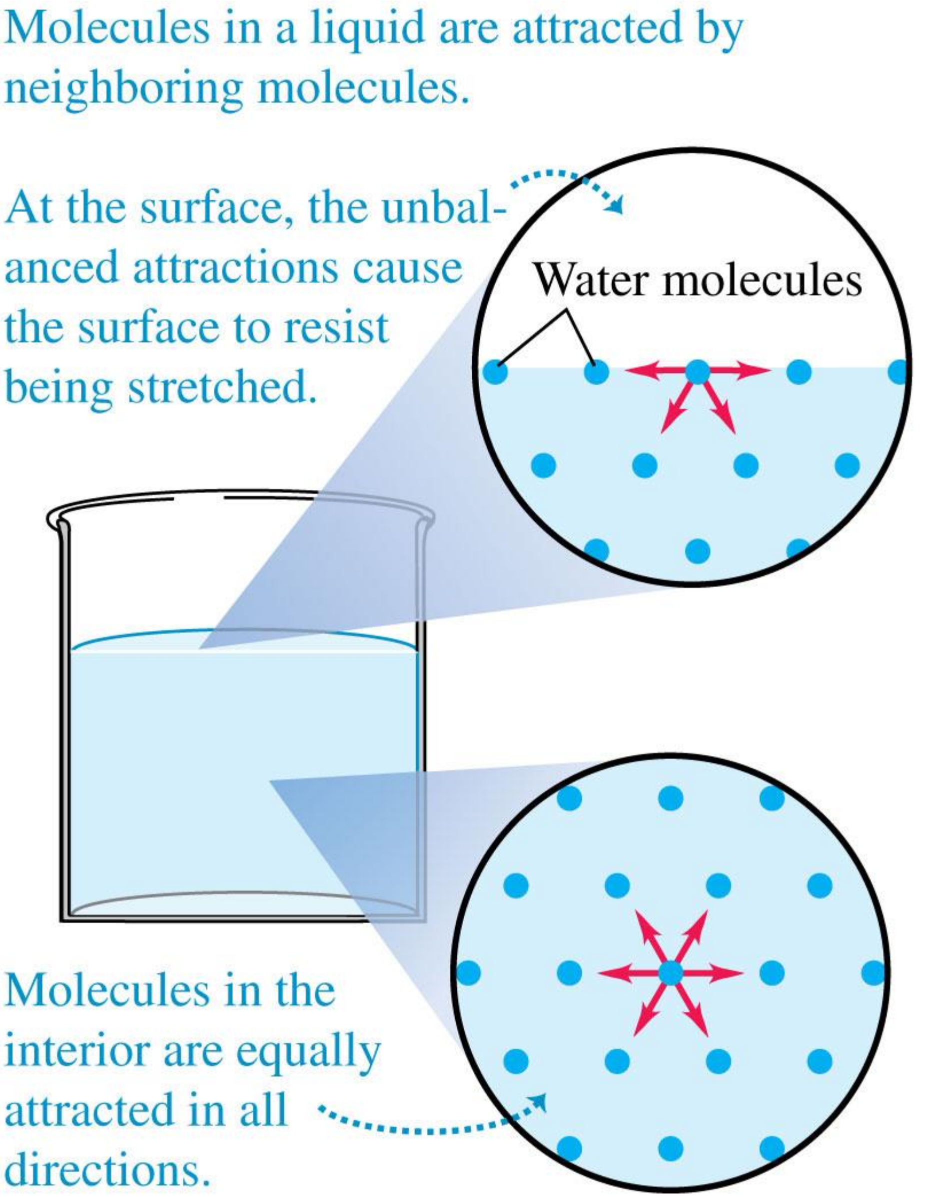


Surface tension

- The surface of the water acts like a membrane under tension, allowing this water strider to “walk on water.”

Surface tension

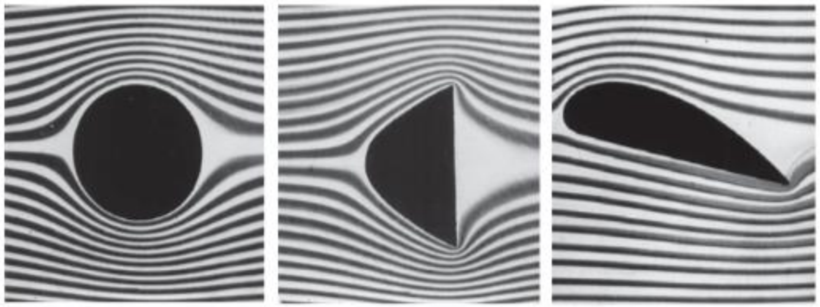
- A molecule at the surface of a liquid is attracted into the bulk liquid, which tends to *reduce* the liquid's surface area.

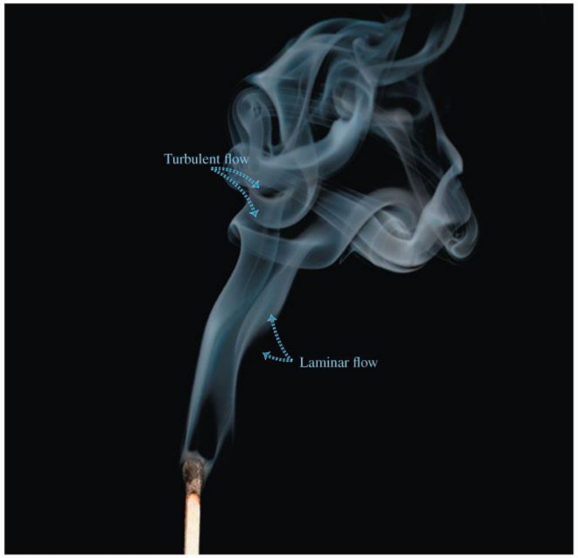


Fluid flow: laminar and turbulent types

- Slow flow is laminar, fast flow is turbulent.
- In laminar flow, adjacent layers of fluid slide smoothly past each other and the flow is steady.
- In turbulent flow there is no steady-state pattern; the flow pattern changes continuously.

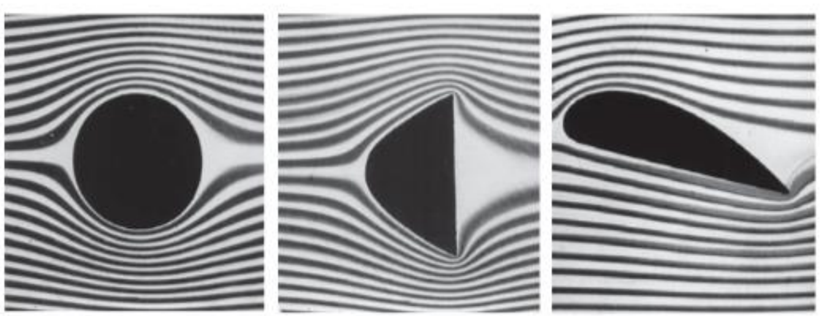
12.19 Laminar flow around obstacles of different shapes.

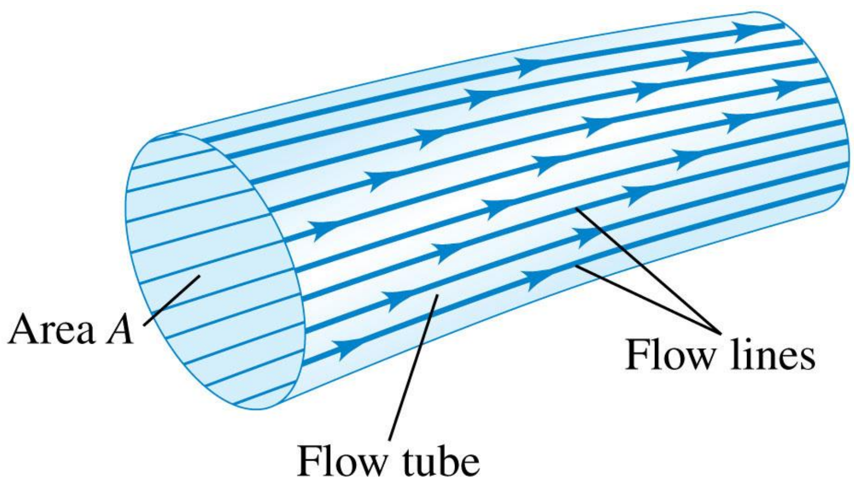




Laminar fluid flow

- The path of an individual particle in a moving fluid is called a **flow line**.
- In **steady flow**, the overall flow pattern does not change with time, so every element passing through a given point follows the same flow line.
- **Flow tubes** are imaginary areas encompassed by flow lines.
- In steady flow no fluid can cross the side walls of a given flow tube.





The continuity equation (matter conservation)

- The figure at the right shows a flow tube with changing cross-sectional area.

- The **continuity equation** for an incompressible fluid is.

Continuity equation for an **incompressible fluid**

$$A_1 v_1 = A_2 v_2$$

Cross-sectional area of flow tube at two points

Speed of flow at the two points

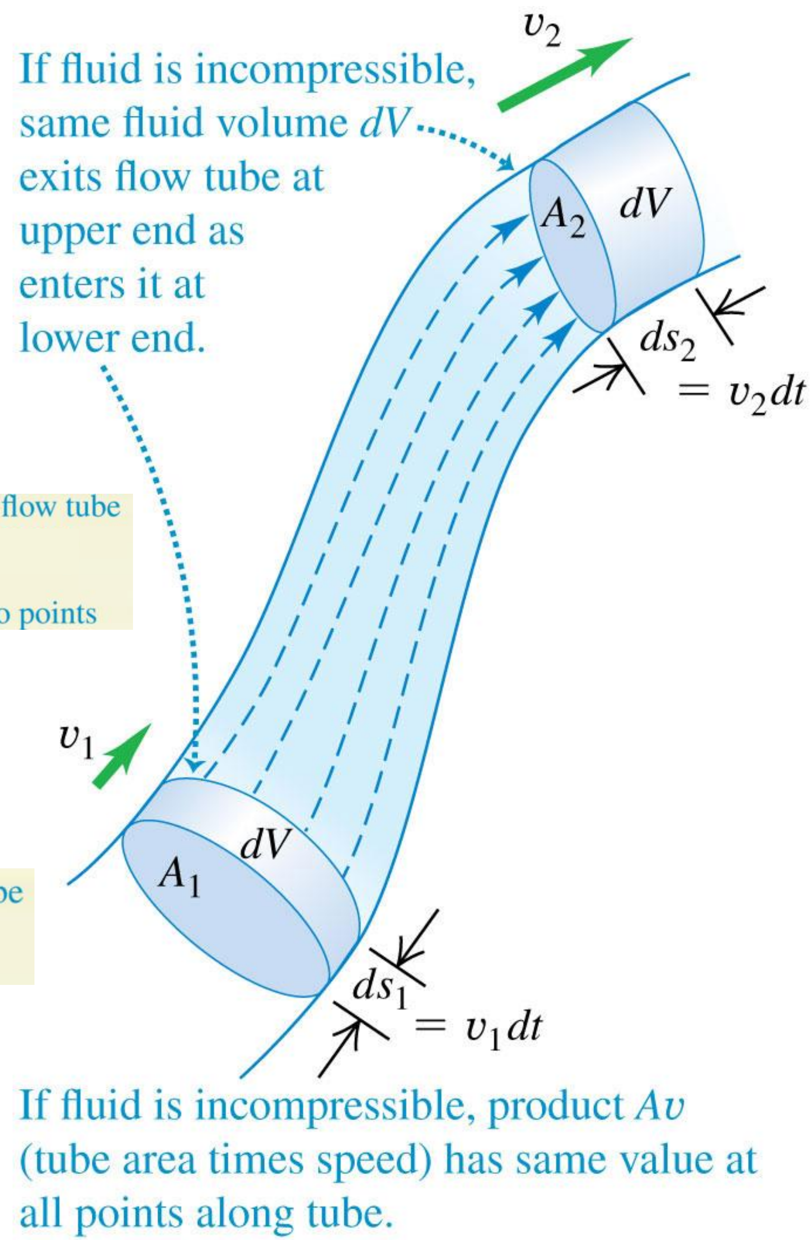
- The **volume flow rate** is.

Volume flow rate of a fluid

$$\frac{dV}{dt} = Av$$

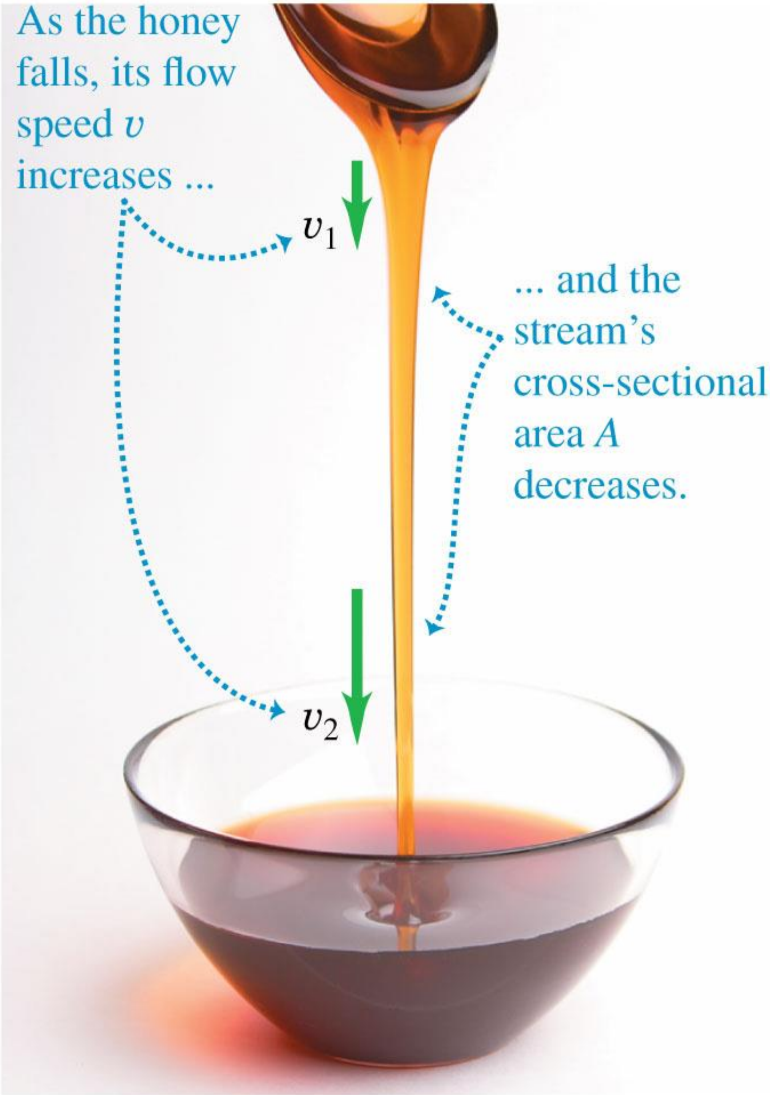
Cross-sectional area of flow tube

Speed of flow



The continuity equation

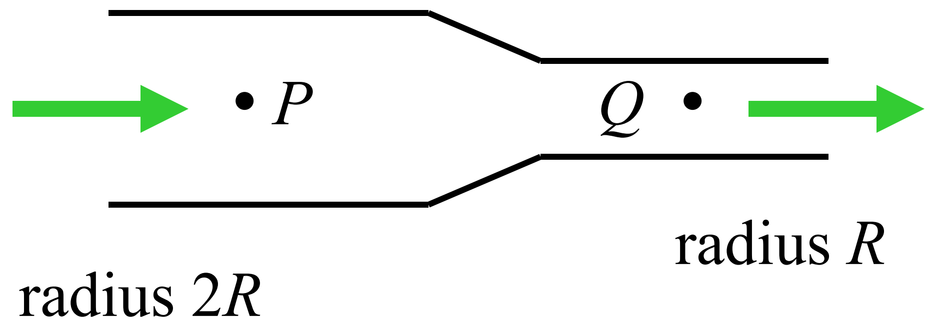
- The continuity equation helps explain the shape of a stream of honey poured from a spoon.



The volume flow rate $dV/dt = Av$ remains constant.

Q12.5

An incompressible fluid with zero viscosity flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point P, the fluid at point Q has



A. four times the fluid speed.

B. twice the fluid speed.

C. the same fluid speed.

D. half the fluid speed.

E. one-quarter the fluid speed.

Bernoulli's equation (energy conservation)

- Bernoulli's equation is:

$$p_1 + \rho g y_1 + 1/2\ \rho v_1^2 = p_2 + \rho g y_2 + 1/2\ \rho v_2^2$$

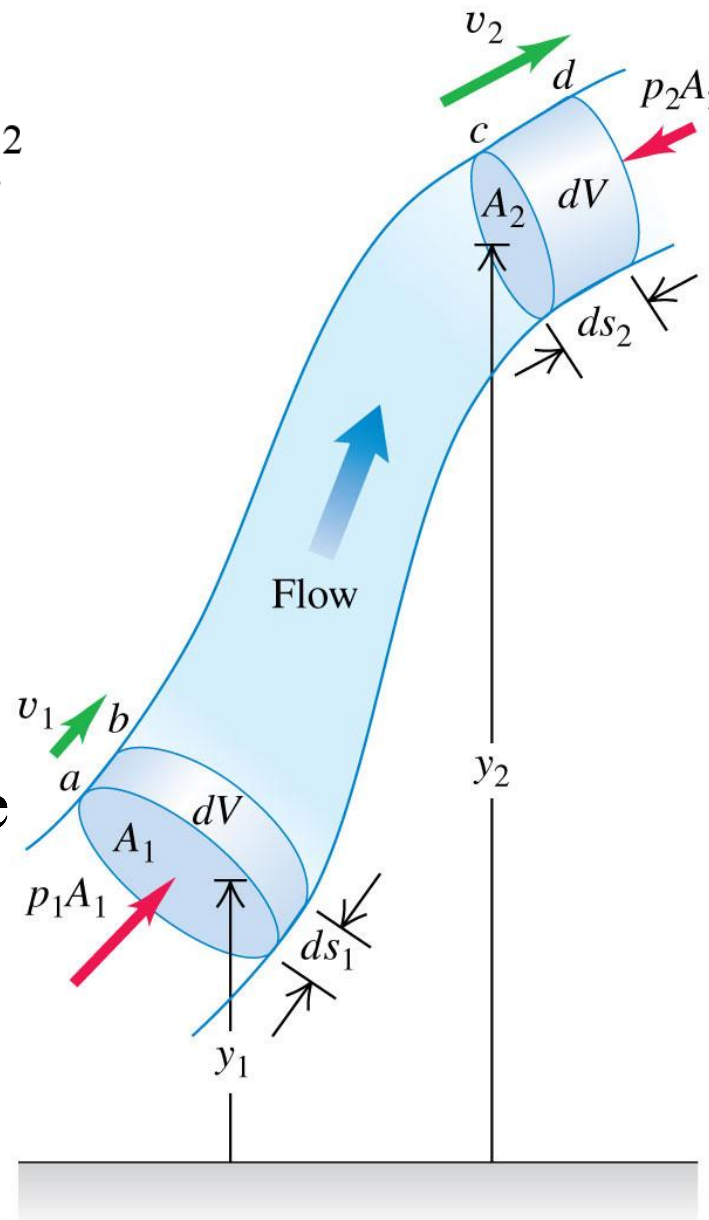
- or

Pressure Fluid density Value is **same** at all points in flow tube.

$$p + \rho g y + \tfrac{1}{2}\rho v^2 = \text{constant}$$

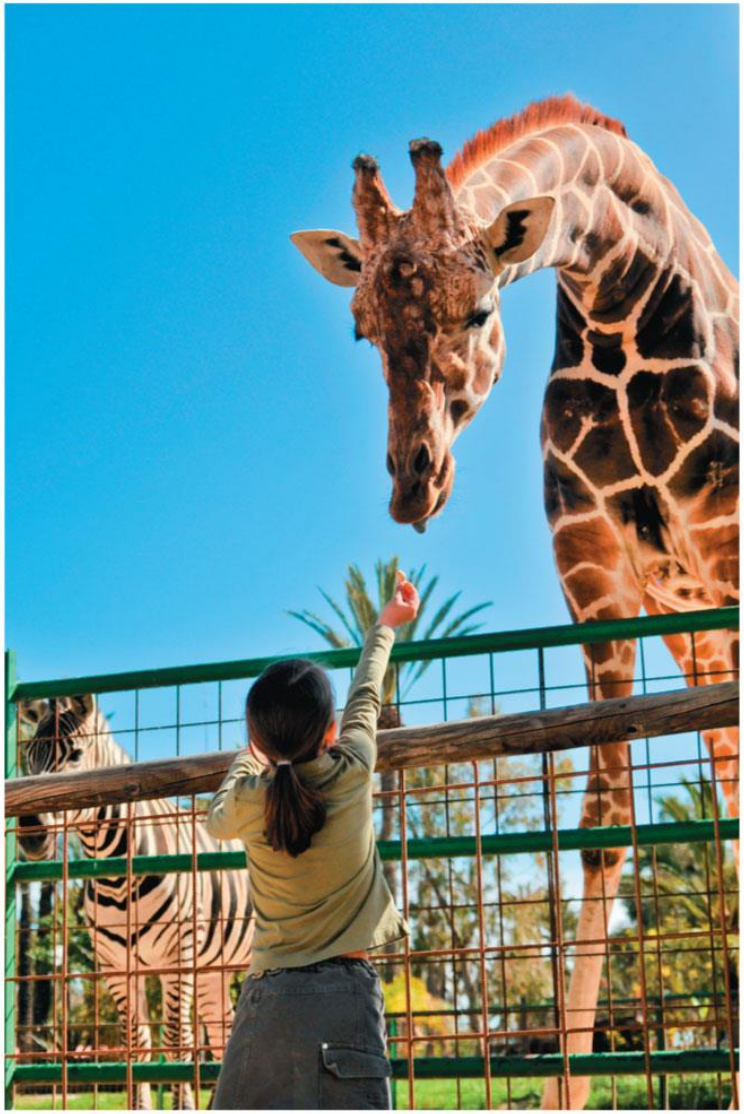
Acceleration due to gravity Elevation Flow speed

- It is due to the fact that the work done on a unit volume of fluid by the surrounding fluid is equal to the sum of the changes in kinetic and potential energies per unit volume that occur during the flow.



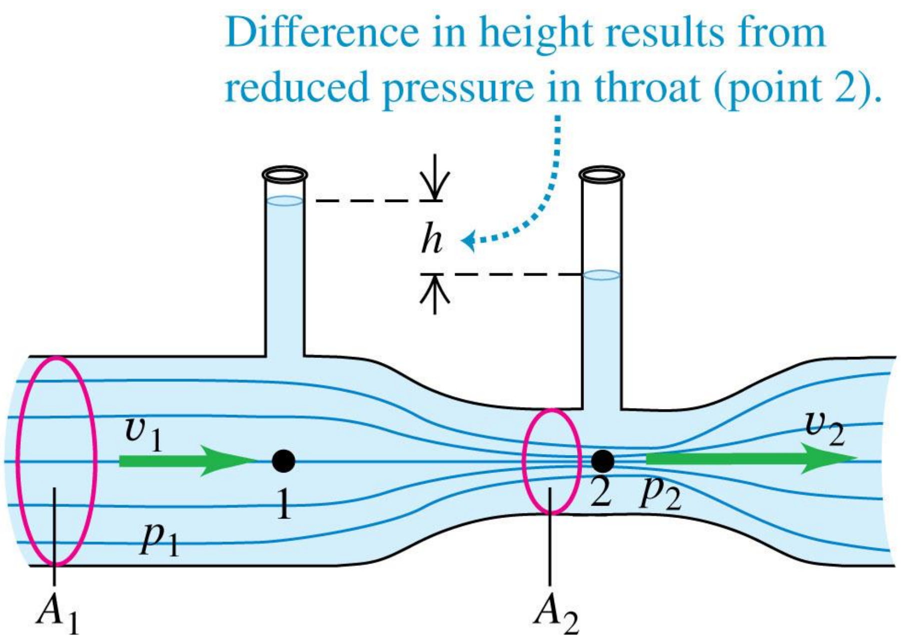
Why healthy giraffes have high blood pressure

- Bernoulli's equation suggests that as blood flows upward at roughly constant speed v from the heart to the brain, the pressure p will drop as the blood's height y increases.
- For blood to reach the brain with the required minimal pressure, the giraffe's maximum (systolic) blood pressure must be 280 mm Hg!



The Venturi meter

The Venturi meter used to measure flow speed in a pipe. Derive an expression for the flow speed v_1 in terms of the cross-sectional areas A_1 and A_2 and the difference in height h of the liquid levels in the two vertical tubes.



Points 1 and 2 have the same vertical coordinate, so

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

From the continuity equation, $v_2 = \left(\frac{A_1}{A_2}\right) v_1$.

Substituting this and rearranging, we get

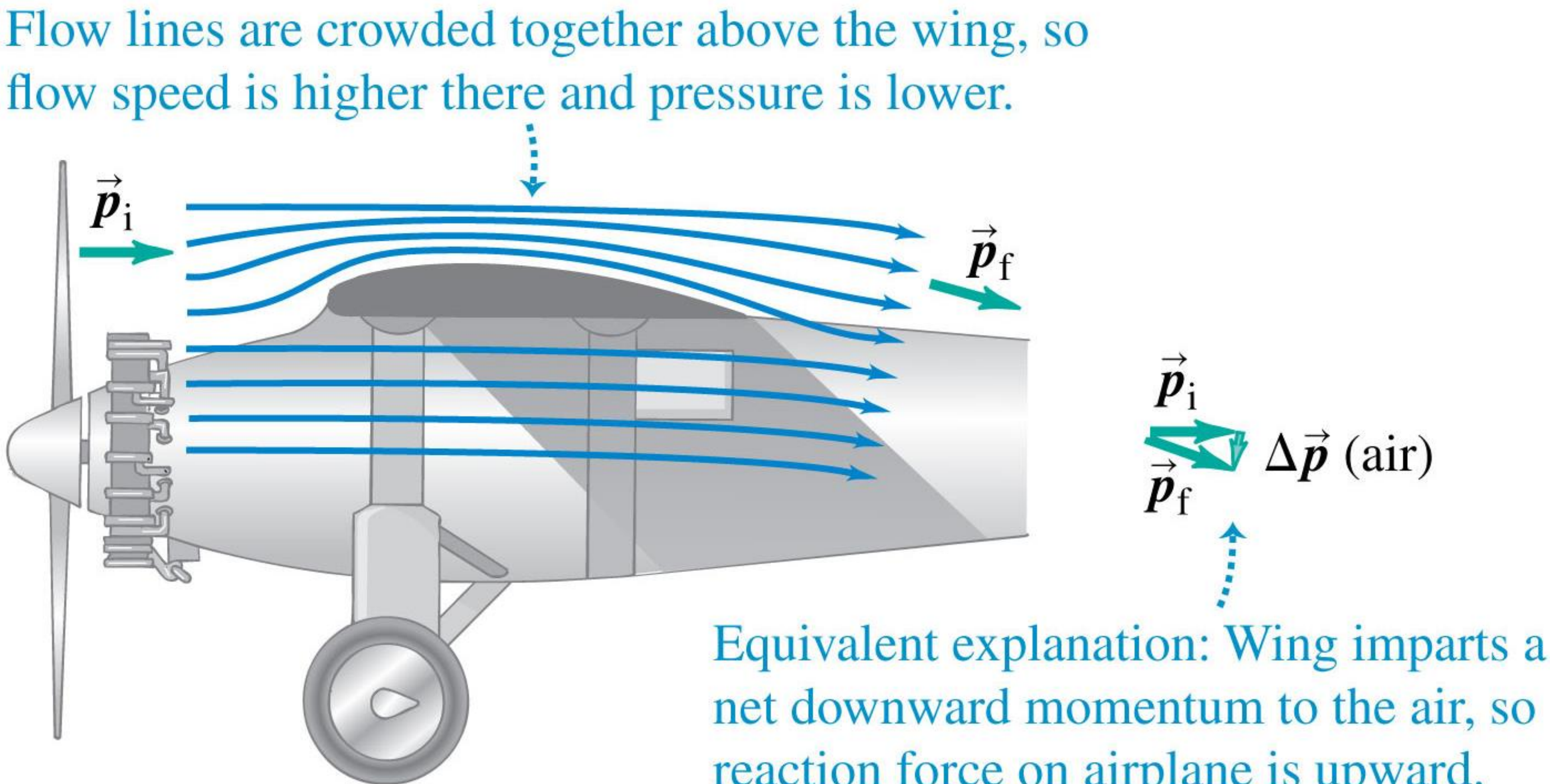
$$p_1 - p_2 = \frac{1}{2}\rho v_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right]$$

Since $p_1 - p_2 = \rho g h$, we get

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

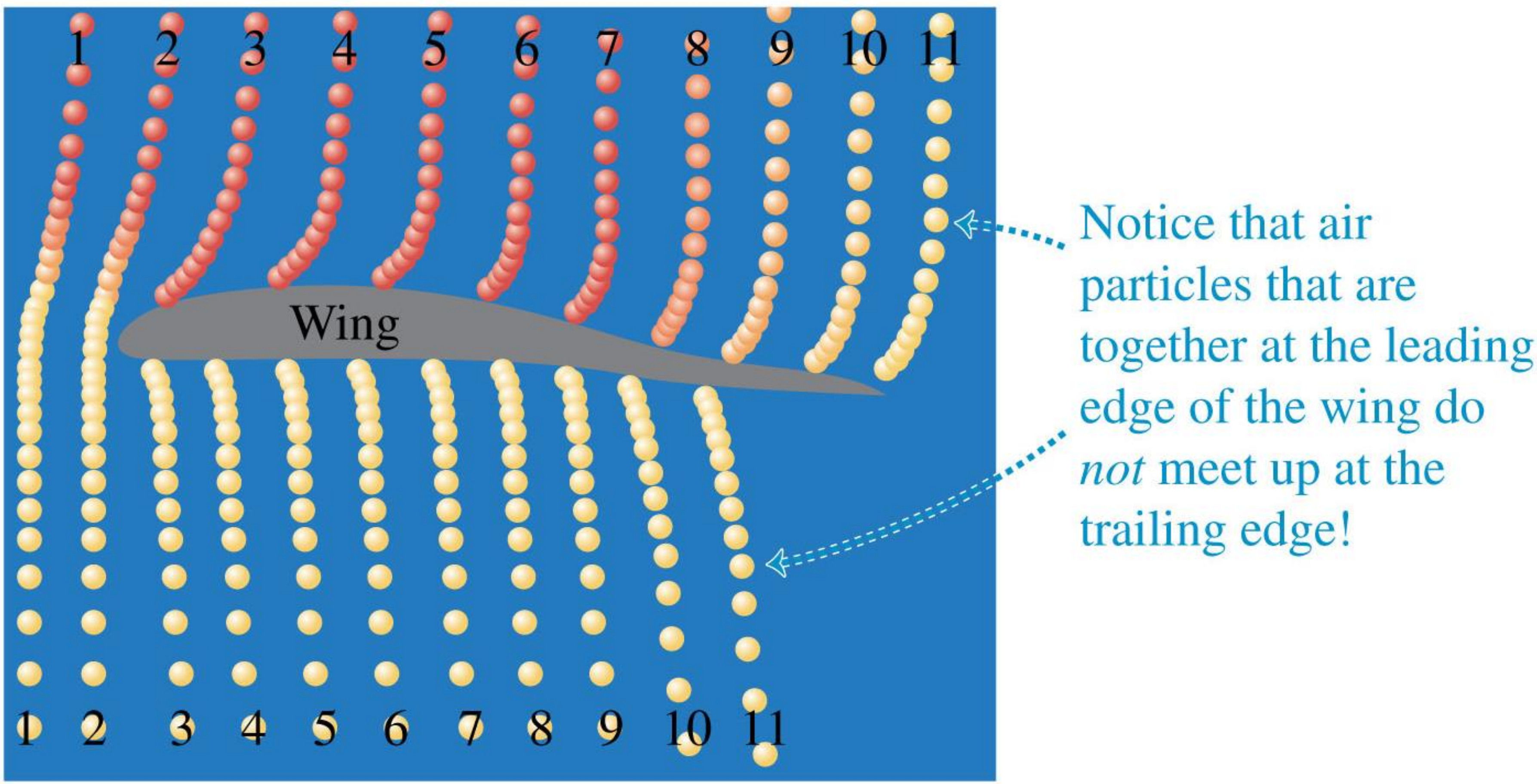
Lift on an airplane wing

- Bernoulli's principle helps to explain how airplanes fly.

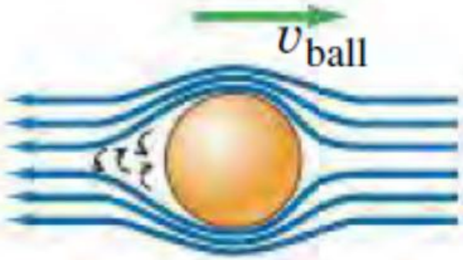


Lift on an airplane wing

- Computer simulation of air parcels flowing around a wing, showing that air moves much faster over the top than over the bottom.



Motion of a spinning ball



This side of the ball moves opposite to the airflow.

This side moves in the direction of the airflow.

A moving ball drags the adjacent air with it. So, when air moves past a spinning ball:

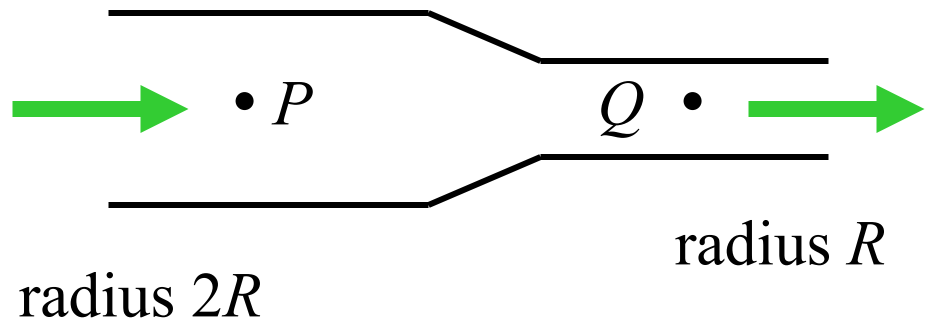
On one side, the ball **slows the air**, creating a region of **high pressure**.

On the other side, the ball **speeds the air**, creating a region of **low pressure**.

The resultant force points in the direction of the low-pressure side.

Q12.4

An incompressible fluid with zero viscosity flows through a pipe of varying radius (shown in cross-section). Compared to the fluid at point *P*, the fluid at point *Q* has



A. a greater pressure and a greater volume flow rate.

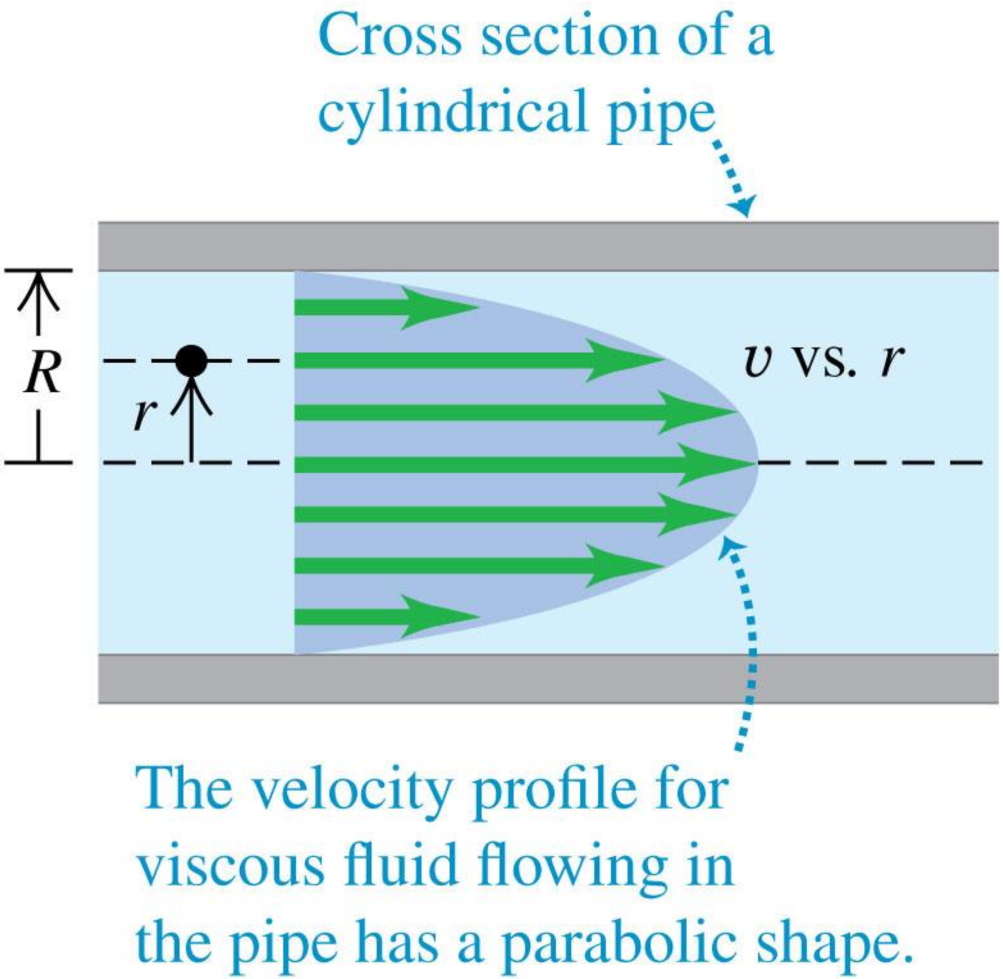
B. a greater pressure and the same volume flow rate.

C. the same pressure and a greater volume flow rate.

D. a lower pressure and the same volume flow rate.

E. none of the above.

Viscosity



- **Viscosity** is internal friction in a fluid. This makes Bernoulli's equation invalid.
- Due to viscosity, the speed is zero at the pipe walls (to which the fluid clings) and is greatest at the center of the pipe.

Viscosity

- Lava is an example of a viscous fluid.
- The viscosity decreases with increasing temperature: The hotter the lava, the more easily it can flow.

Turbulence

- At low flow speeds, the flow of a fluid is **laminar**.
- When a critical speed is exceeded, however, the flow pattern becomes **turbulent**. The threshold depends on viscosity and geometry.

Listening for turbulent flow

- Normal blood flow in the human aorta is laminar, but a small disturbance such as a heart pathology can cause the flow to become turbulent.
- Turbulence makes noise, which is why listening to blood flow with a stethoscope is a useful diagnostic technique.

Summary

- Stationary fluid:
  - Density, $\rho \equiv m/V$
  - Pressure, $p \equiv \frac{F}{A} = p_0 + \rho g h$
  - Archimedes' principle: Buoyancy force=weight of displaced water.
  - Surface tension
- Flowing fluid:
  - Ideal fluid (not viscous, slow flowing, not compressible)
    - Continuity Equation (matter conservation), $Av = \text{constant}$
    - Bernoulli's Equation (energy conservation), $p + \rho g y + \frac{1}{2}\rho v^2 = \text{constant}$
  - Viscous fluid
  - Turbulence