CITY UNIVERSITY OF HONG KONG

Course code & title	:	MA1501/GE1358 Coordinate Geometry
Session	:	Semester B 2020/21
Time allowed	:	1.5 hours

This paper has **ELEVEN** pages (including this cover page).

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1. Attempt all **NINE** questions in this paper.
2. Start each question on a new page.
3. **Show all steps clearly** in order to get full credits.

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*This is a **closed-book** examination.*

Students are allowed to use the following materials/aids:

Non-programmable calculators

Materials/aids other than those stated above are not permitted. Students will be subject to disciplinary action if any unauthorized materials or aids are found on them.

NOT TO BE TAKEN AWAY

1. Given the matrices

$$A = \begin{pmatrix} 2 & 3 \\ 0 & 4 \end{pmatrix}, \text{B=}\begin{pmatrix} \frac{1}{2} & x \\ 0 & \frac{1}{4} \end{pmatrix}, \text{C=}\begin{pmatrix} 12 & -4 \\ -8 & y \end{pmatrix}$$

(a) Evaluate $\mathbf{A}^2$ and $\mathbf{A}^{-1}$.
(b) Find the value of x such that **AB** equals the identity matrix. State the relationship between the matrices **A**, **B**, and $\mathbf{A}^{-1}$?
(c) Find the value of y such that det(**A**)=det(**C**).
(d) Calculate the determinant of the matrix **AC**, for the value of y found above.
(e) Find the transpose of **A.**

(a) $A^2 = \begin{pmatrix} 4 & 18 \\ 0 & 16 \end{pmatrix}$ $A^{-1} = \begin{pmatrix} \frac{1}{2} & -\frac{3}{8} \\ 0 & \frac{1}{4} \end{pmatrix}$

(b) $x = -\frac{3}{8}$ $AB = I_2$ where $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A \cdot B = AA^{-1}$

(c) $y = 10/3$

(d) $\det AC = 64$

(e) $\begin{pmatrix} 2 & 0 \\ 3 & 4 \end{pmatrix}$

2. Given the matrices $A=\begin{pmatrix}1 & 3\\0 & 1\end{pmatrix}$ $I=\begin{pmatrix}1 & 0\\0 & 1\end{pmatrix}$, find constants p and q such that $A^2 = pA + qI$. Hence find A^{-1}.

$p = 2 \quad q = -1$

$A^{-1} = \begin{pmatrix}1 & -3\\0 & 1\end{pmatrix}$

3. Let $\vec{A}=\begin{bmatrix}1\\0\\0\end{bmatrix}$ $\vec{B}=\begin{bmatrix}0\\1\\0\end{bmatrix}$. Show that $(\vec{A} \bullet \vec{B})\ x\ \vec{B} = 0$ **(YOU MUST SHOW ALL RELEVANT FORMULA AND WORK TO RECEIVE FULL MARKS.)**

$$\begin{bmatrix}1\\0\\0\end{bmatrix} \cdot \begin{bmatrix}0\\1\\0\end{bmatrix} = (1)(0) + (0)(1) + (0)(0)$$
$$= 0$$

$(\vec{A} \cdot \vec{B} \times \vec{B})$ is a scalar cross product
a vector
∴ void.

4. Prove that $\det\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix} = (a-b)(b-c)(c-a)$.

$$\det\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ a^2 & b^2 & c^2\end{vmatrix} \overset{R_3 \to R_3 + (-a)R_2}{=} \det\begin{vmatrix}1 & 1 & 1\\ a & b & c\\ 0 & b^2-ab & c^2-ac\end{vmatrix}$$

$$\Big\downarrow \; R_2 \to R_2 + (-a)R_1$$

$$\det\begin{bmatrix}1 & 1 & 1\\ 0 & b-a & c-a\\ 0 & 0 & (c-a)(c-b)\end{bmatrix} \xleftarrow{R_3 + (-b)R_2} \det\begin{vmatrix}1 & 1 & 1\\ 0 & b-a & c-a\\ 0 & b^2-ab & c^2-ac\end{vmatrix}$$

$$= 0\begin{vmatrix}1 & 1\\ b-a & c-a\end{vmatrix} + 0\begin{vmatrix}1 & 1\\ 0 & (c-a)(c-b)\end{vmatrix} - 1\begin{vmatrix}b-a & c-a\\ 0 & (c-a)(c-b)\end{vmatrix}$$

$$= (b-a)(c-a)(c-b)$$

$$= (a-b)(c-a)(b-c)$$

5. Find c so that the points (c, -2), (3,1), and (-2,4) shall be collinear, that is, lie on the line.

$c = 8$

6. Name the type of triangle (i.e. equilateral, isosceles, or scalene) with the vertices (6,-5), (2,-4) and (5,-1). Provide an explanation using relevant formula. **SKETCHING THE GRAPH ALONE IS INSUFFICENT AND RECEIVES A GRADE OF ZERO.**

$\sqrt{17}$, $\sqrt{17}$

59.0362° , 59.0362°

$\therefore$

7. (a) Find an equation of all lines parallel to the line $8x - 2y + 1 = -2$ and in particular of the one passing through the point $M(3,-1)$.

(b) Find an equation of all lines perpendicular to the line $8x - 2y + 1 = -2$, and in particular of the one passing through the point $N(-8, 10)$.

(a) $8x - 2y - 26 = 0$

(b) $x + 4y - 32 = 0$

8. Find an equation of the first degree in *x*, *y*, and *z* which has (1, -2, 3) for a solution but which has no solution in common with the equation $5x + 2y - 3z + 1 = 0$.

$5x + 2y - 3z + 8 = 0$

9. A plane is through the point (1, -1, 1) and is perpendicular to the line intersection of the two planes, namely, $2x - 3y + z + 2 = 0$ and $3x + 2y - z + 2 = 0$. Find the equation of such plane.

$x + 5y + 13z = 9$

Useful Trigonometric Identities

Pythagorean identities

1. $\sin^2\theta + \cos^2\theta = 1.$

2. $1 + \tan^2\theta = \sec^2\theta.$

3. $1 + \cot^2\theta = \csc^2\theta.$

Double-angle formulas

4. $\sin 2\theta = 2\sin\theta\cos\theta.$

5. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta.$

Half-angle formulas

6. $\sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta).$

7. $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta).$

Compound-angle formulas

8. $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B.$

9. $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B.$

10. $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}.$

Sum-to-product formulas

11. $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}.$

12. $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}.$

13. $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}.$

14. $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}.$

Product-to-sum formulas

15. $\sin A\cos B = \dfrac{1}{2}\big[\sin(A+B) + \sin(A-B)\big].$

16. $\cos A\sin B = \dfrac{1}{2}\big[\sin(A+B) - \sin(A-B)\big].$

17. $\cos A\cos B = \dfrac{1}{2}\big[\cos(A+B) + \cos(A-B)\big].$

18. $\sin A\sin B = -\dfrac{1}{2}\big[\cos(A+B) - \cos(A-B)\big].$

Euler's formulas

19. $\mathrm{e}^{\pm \mathrm{i}\theta} = \cos\theta \pm \mathrm{i}\sin\theta.$

20. $\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta} = 2\cos\theta,\ \cos\theta = \dfrac{1}{2}(\mathrm{e}^{\mathrm{i}\theta} + \mathrm{e}^{-\mathrm{i}\theta}).$

21. $\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta} = 2\mathrm{i}\sin\theta,\ \sin\theta = \dfrac{1}{2\mathrm{i}}(\mathrm{e}^{\mathrm{i}\theta} - \mathrm{e}^{-\mathrm{i}\theta}).$

Remark. Formulas of the form $A \pm B = C \pm D$ contain two separate formulas

$$A + B = C + D, \qquad \text{and} \qquad A - B = C - D.$$

Likewise, formulas of the form $A \pm B = C \mp D$ contain two separate formulas

$$A + B = C - D, \qquad \text{and} \qquad A - B = C + D.$$

Academic Honesty Pledge

I pledge that the answers in this exam/quiz are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

- I will not plagiarize (copy without citation) from any source;
- I will not communicate or attempt to communicate with any other person during the exam/quiz; neither will I give or attempt to give assistance to another student taking the exam/quiz; and
- I will use only approved devices (e.g., calculators) and/or approved device models.

I understand that any act of academic dishonesty can lead to disciplinary action.

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