Chapter 5

Applying Newton's Laws

Introduction

- Newton's three laws of motion can be stated very simply, but applying these laws to real-life situations requires analytical skills and problem-solving techniques.
- In this chapter we'll begin with equilibrium problems, in which we analyze the forces that act on a body that is at rest or moving with constant velocity.
- We'll then consider bodies that are not in equilibrium, for which we'll have to deal with the relationship between forces and motion.

Using Newton's first law when forces are in equilibrium

- A body is in *equilibrium* when it is at rest or moving with constant velocity in an inertial frame of reference.

- The essential physical principle is Newton's first law:

Newton's first law: *Net* force on a body ... $\sum \vec{F} = \mathbf{0}$ ... must be *zero* for a **body in equilibrium.**

Sum of x-components of force on body must be zero.

Sum of y-components of force on body must be zero.

$$\sum F_x = 0 \qquad \sum F_y = 0$$

Problem-solving strategy for equilibrium situations

- **Identify** the relevant concept: You must use Newton’s first law.
- **Set up** the problem by using the following steps:
  1. Draw a sketch of the physical situation.
  2. Draw a free-body diagram for each body that is in equilibrium.
  3. Ask yourself what is interacting with the body by contact or in any other way. If the mass is given, use $w = mg$ to find the weight.
  4. Check that you have only included forces that act *on* the body.
  5. Choose a set of coordinate axes and include them in your free-body diagram.

Problem-solving strategy for equilibrium situations

- **Execute** the solution as follows:
  1. Find the components of each force along each of the body's coordinate axes.
  2. Set the sum of all x-components of force equal to zero. In a separate equation, set the sum of all y-components equal to zero.
  3. If there are two or more bodies, repeat all of the above steps for each body. If the bodies interact with each other, use Newton's third law to relate the forces they exert on each other.
  4. Make sure that you have as many independent equations as the number of unknown quantities. Then solve these equations to obtain the target variables.
- **Evaluate** your answer.

Using Newton's second law: dynamics of particles

- In *dynamics* problems, we apply Newton's second law to bodies on which the net force is *not* zero.
- These bodies are *not* in equilibrium and hence are accelerating:

Newton's second law: If *net* force on a body is not zero ...

$$\sum \vec{F} = m\vec{a}$$

... body has *acceleration* in same direction as net force.

Mass of body

Each component of net force on body ...

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

... equals body's mass times corresponding acceleration component.

Problem-solving strategy for dynamics situations

- **Identify** the relevant concept: You must use Newton's second law.
- **Set up** the problem by using the following steps:
  1. Draw a simple sketch of the situation that shows each moving body. For each body, draw a free-body diagram that shows all the forces acting *on* the body.
  2. Label each force. Usually, one of the forces will be the body's weight $w = mg$.
  3. Choose your x- and y-coordinate axes for each body, and show them in your free-body diagram.
  4. Identify any other equations you might need. If more than one body is involved, there may be relationships among their motions; for example, they may be connected by a rope.

Problem-solving strategy for dynamics situations

- **Execute** the solution as follows:
  1. For each body, determine the components of the forces along each of the body’s coordinate axes.
  2. List all of the known and unknown quantities. In your list, identify the target variable or variables.
  3. For each body, write a separate equation for each component of Newton’s second law. Write any additional equations that you identified in step 4 of “Set Up.” (You need as many equations as there are target variables.)
  4. Do the easy part—the math! Solve the equations to find the target variable(s).
- **Evaluate** your answer.

Apparent weight and apparent weightlessness

- When a passenger with mass m rides in an elevator with y-acceleration a_y, a scale shows the passenger's apparent weight to be:

$$n = m(g + a_y)$$

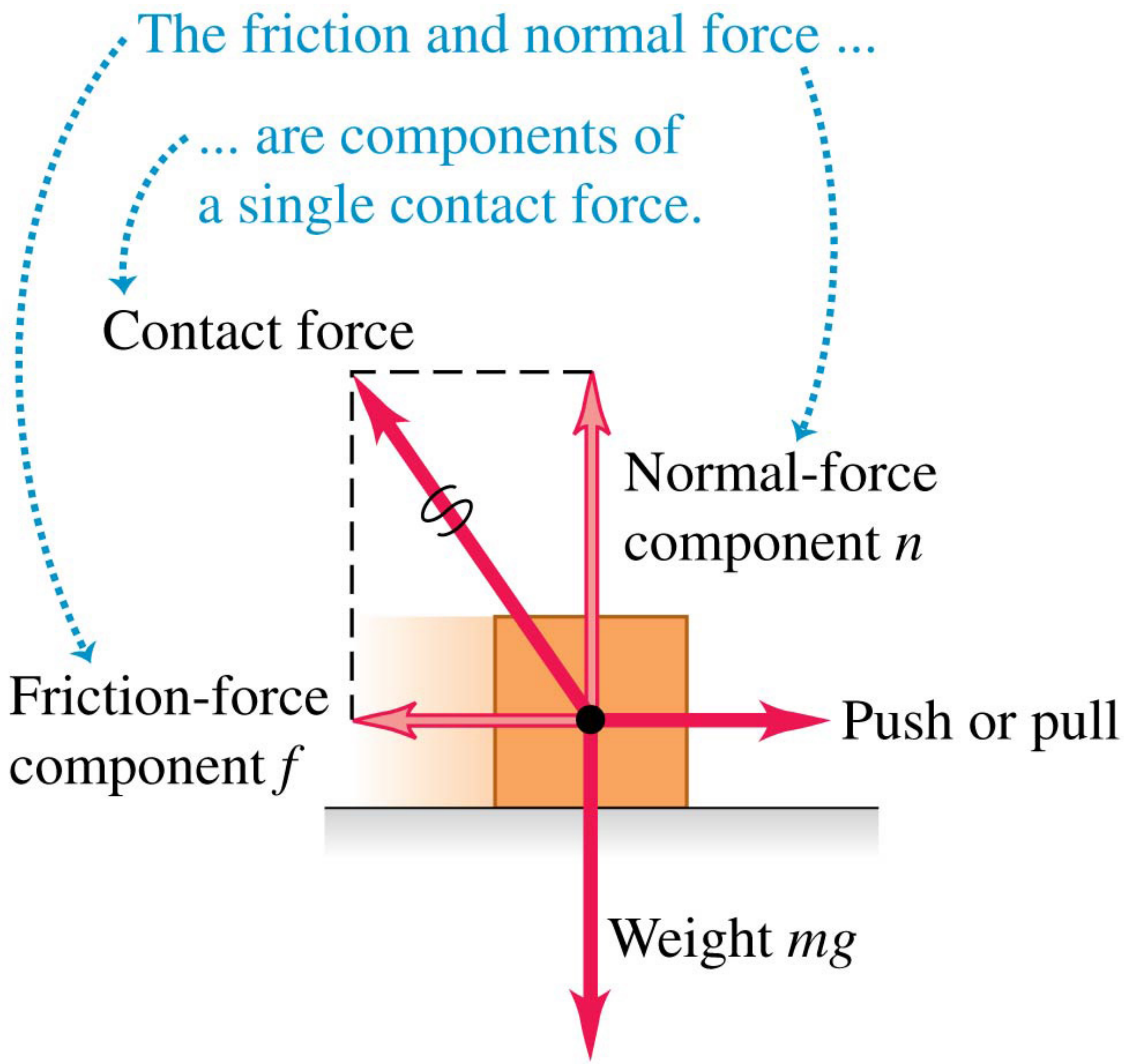
- The extreme case occurs when the elevator has a downward acceleration $a_y = -g$ — that is, when it is in free fall.
- In that case $n = 0$ and the passenger *seems* to be weightless.
- Similarly, an astronaut orbiting the earth with a spacecraft experiences *apparent weightlessness*.

Frictional forces

- There is friction between the feet of this caterpillar (the larval stage of a butterfly of the family Papilionidae) and the surfaces over which it walks.
- Without friction, the caterpillar could not move forward or climb over obstacles.

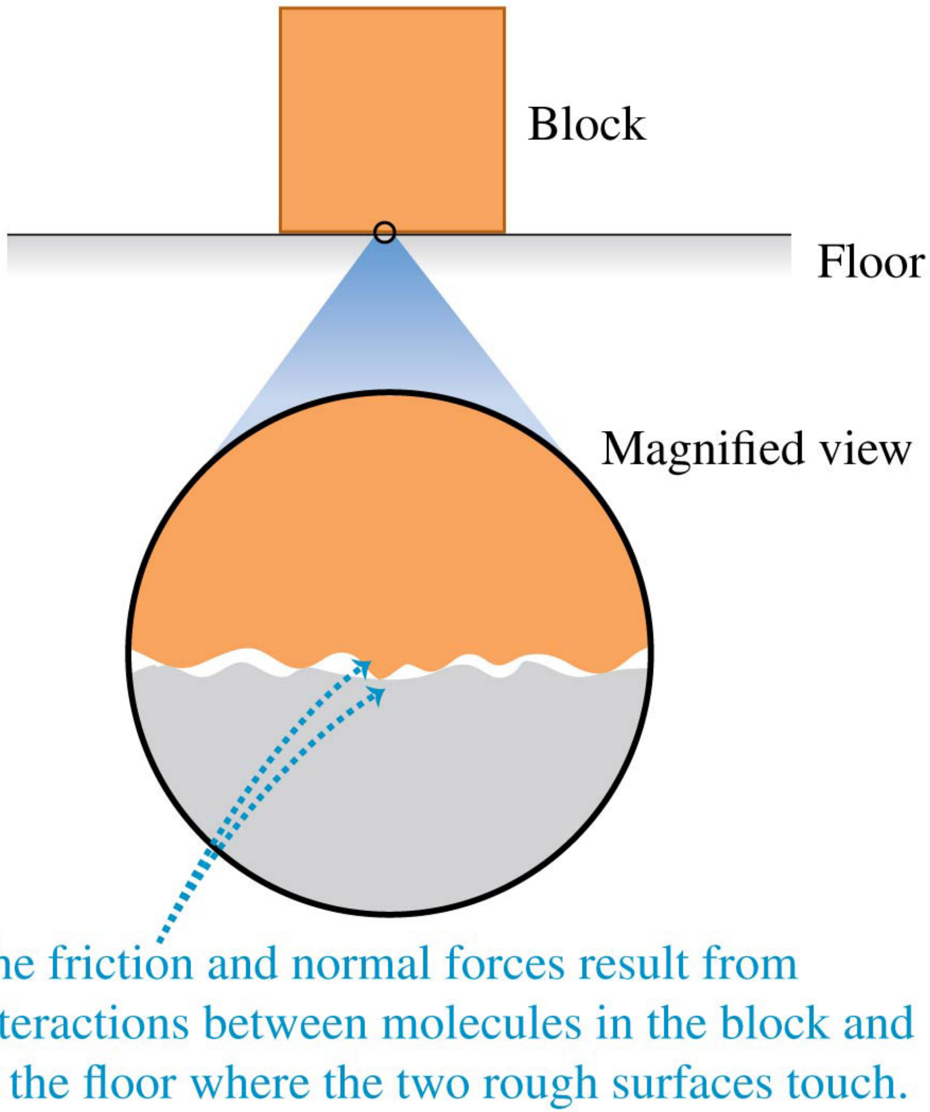
Frictional forces

- When a body rests or slides on a surface, the *friction force* is parallel to the surface.



Frictional forces

- Friction between two surfaces arises from interactions between molecules on the surfaces.

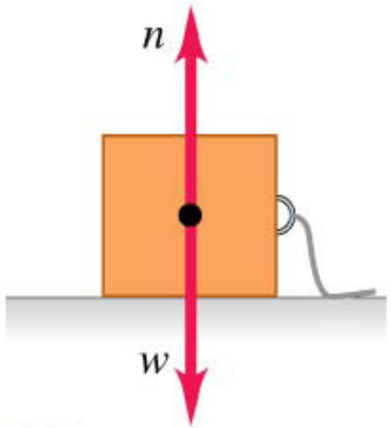


Kinetic and static friction

- *Kinetic friction* acts when a body slides over a surface.
- The *kinetic friction force* is $f_{\mathrm{k}} = \mu_{\mathrm{k}} n$.
- *Static friction* acts when there is no relative motion between bodies.
- The *static friction force* can vary between zero and its maximum value: $f_{\mathrm{s}} \leq \mu_{\mathrm{s}} n$.

Static friction followed by kinetic friction: Slide 1

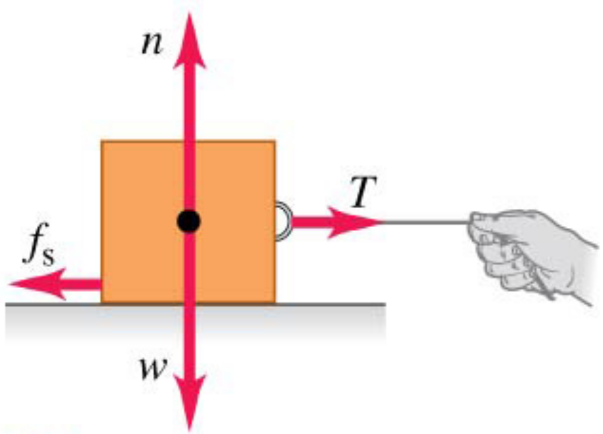
- Before the box slides, static friction acts. But once it starts to slide, kinetic friction acts.



(1) No applied force, box at rest.
No friction:
$f_s = 0$

Static friction followed by kinetic friction: Slide 2

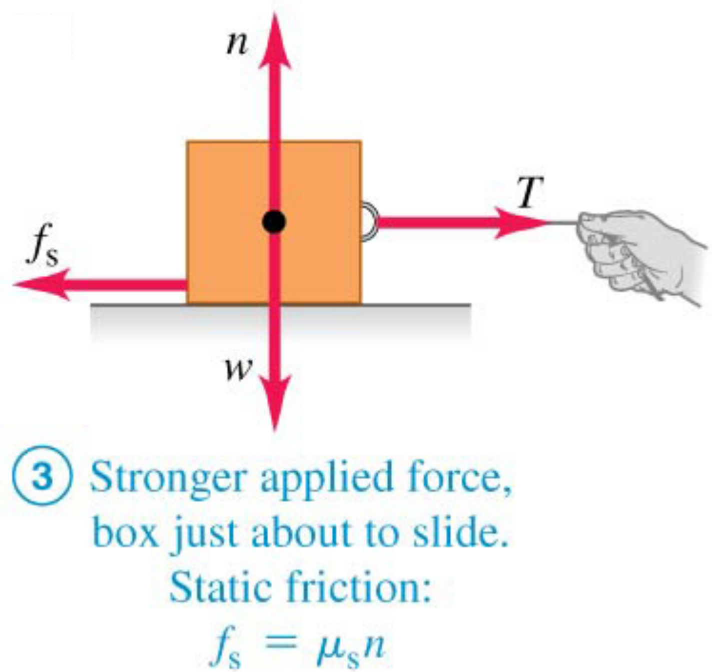
- Before the box slides, static friction acts. But once it starts to slide, kinetic friction acts.



2 Weak applied force, box remains at rest. Static friction: $f_s < \mu_s n$

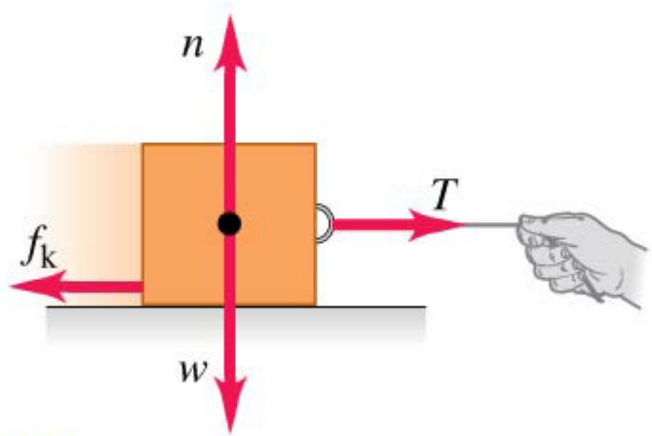
Static friction followed by kinetic friction: Slide 3

- Before the box slides, static friction acts. But once it starts to slide, kinetic friction acts.



Static friction followed by kinetic friction: Slide 4

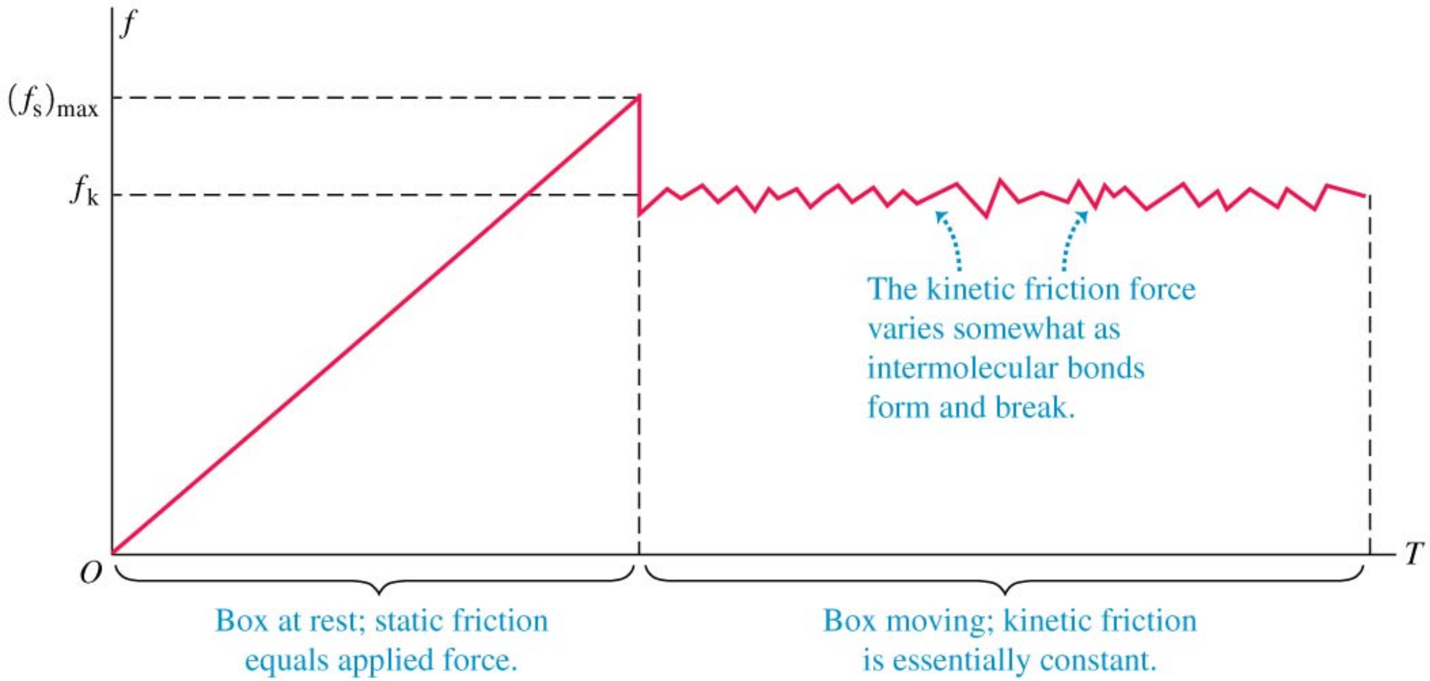
- Before the box slides, static friction acts. But once it starts to slide, kinetic friction acts.



4 Box sliding at constant speed. Kinetic friction: $f_k = \mu_k n$

Static friction followed by kinetic friction: Slide 5

- Before the box slides, static friction acts. But once it starts to slide, kinetic friction acts.



Some approximate coefficients of friction

Materials	Coefficient of Static Friction, μ_s	Coefficient of Kinetic Friction, μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Copper on steel	0.53	0.36
Brass on steel	0.51	0.44
Zinc on cast iron	0.85	0.21
Copper on cast iron	1.05	0.29
Glass on glass	0.94	0.40
Copper on glass	0.68	0.53
Teflon on Teflon	0.04	0.04
Teflon on steel	0.04	0.04
Rubber on concrete (dry)	1.0	0.8
Rubber on concrete (wet)	0.30	0.25

Static friction and windshield wipers

- The squeak of windshield wipers on dry glass is a stick-slip phenomenon.
- The moving wiper blade sticks to the glass momentarily, then slides when the force applied to the blade by the wiper motor overcomes the maximum force of static friction.
- When the glass is wet from rain or windshield cleaning solution, friction is reduced and the wiper blade doesn't stick.

Rectangular coordinates

The equation of motion, $\vec{F} = m\,\vec{a}$, is best used when the problem requires finding forces (especially forces perpendicular to the path), accelerations, velocities, or mass. Remember, unbalanced forces cause acceleration!

Three scalar equations can be written from this vector equation. The equation of motion, being a vector equation, may be expressed in terms of its three components in the Cartesian (rectangular) coordinate system as

$$\sum \vec{F} = m\,\vec{a} \quad \text{or} \quad \sum F_x\,\hat{\imath} + \sum F_y\,\hat{\jmath} + \sum F_z\,\hat{k} = m(a_x\,\hat{\imath} + a_y\,\hat{\jmath} + a_z\,\hat{k})$$

or, as scalar equations, $\sum F_x = ma_x$, $\sum F_y = ma_y$, and $\sum F_z = ma_z$.

Procedure for analysis

- **Free-Body Diagram (always critical!!)**

Establish your coordinate system and draw the particle's free-body diagram showing only external forces. These external forces usually include the weight, normal forces, friction forces, and applied forces. Show the 'm $\vec{a}$ ' vector (sometimes called the inertial force) on a separate diagram.

Make sure any friction forces act opposite to the direction of motion! If the particle is connected to an elastic linear spring, a spring force equal to '$k\ s$' should be included on the FBD.

Procedure for analysis

- **Equations of Motion**

If the forces can be resolved directly from the free-body diagram (often the case in 2-D problems), use the scalar form of the equation of motion. In more complex cases (usually 3-D), a Cartesian vector is written for every force and a vector analysis is often best.

A Cartesian vector formulation of the second law is

$$\sum \vec{\boldsymbol{F}} = m\,\vec{\boldsymbol{a}} \quad \text{or}$$

$$\sum F_x\,\hat{\boldsymbol{i}} + \sum F_y\,\hat{\boldsymbol{j}} + \sum F_z\,\hat{\boldsymbol{k}} = m(a_x\,\hat{\boldsymbol{i}} + a_y\,\hat{\boldsymbol{j}} + a_z\,\hat{\boldsymbol{k}})$$

Three scalar equations can be written from this vector equation. You may only need two equations if the motion is in 2-D.

Procedure for analysis

- **Kinematics**

The second law only provides solutions for forces and accelerations. If velocity or position have to be found, kinematics equations are used once the acceleration is found from the equation of motion.

Any of the kinematics tools learned before may be needed to solve a problem.

Make sure you use consistent positive coordinate directions as used in the equation of motion part of the problem!

Quiz

1. In dynamics, the friction force acting on a moving object is always ________

   A) in the direction of its motion. ✔B) a kinetic friction.

   C) a static friction. D) zero.

2. If a particle is connected to a spring, the elastic spring force is expressed by $F = ks$. The "s" in this equation is the

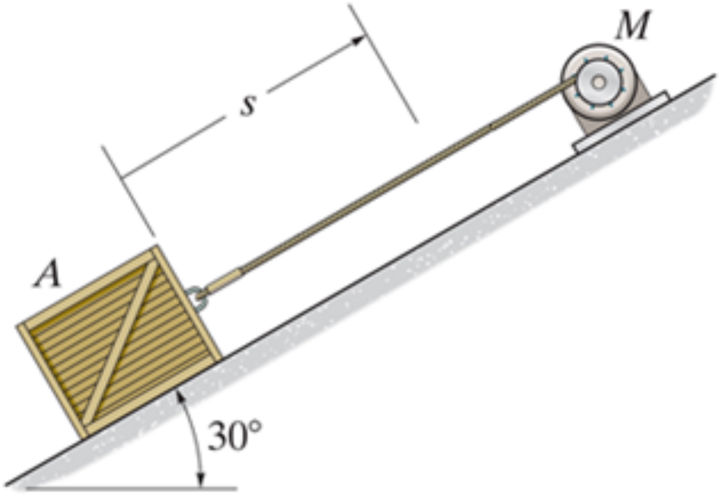
   A) spring constant.

   B) un-deformed length of the spring.

   ✔C) difference between deformed length and un-deformed length.

   D) deformed length of the spring.

Example 1



Given: The motor winds in the cable with a constant acceleration such that the 20-kg crate moves a distance s = 6 m in 3 s, starting from rest. $\mu_k = 0.3$.

Find: The tension developed in the cable.

Plan:

1) Draw the free-body and kinetic diagrams of the crate.

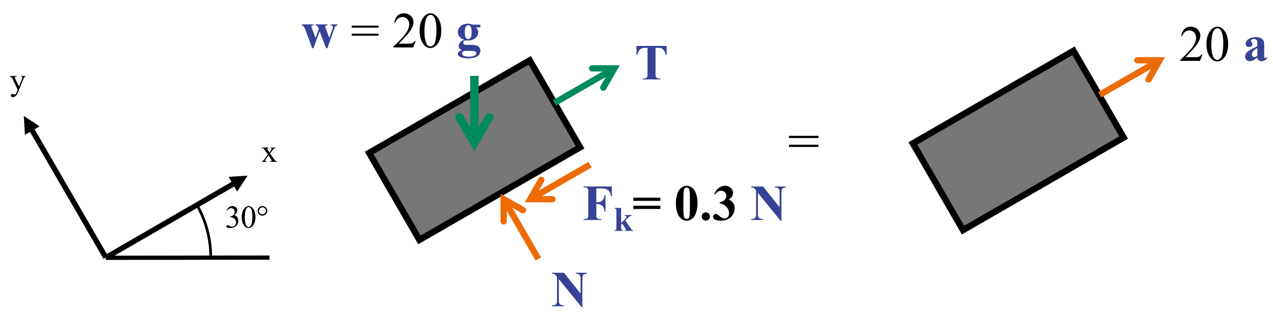
2) Using a kinematic equation, determine the acceleration of the crate.

3) Apply the equation of motion to determine the cable tension.

Example 1

Solution:

1) Draw the free-body and kinetic diagrams of the crate:



Since the motion is up the incline, rotate the x-y axes so the x-axis aligns with the incline. Then, motion occurs only in the x-direction.

There is a friction force acting between the surface and the crate. Why is it in the direction shown on the FBD?

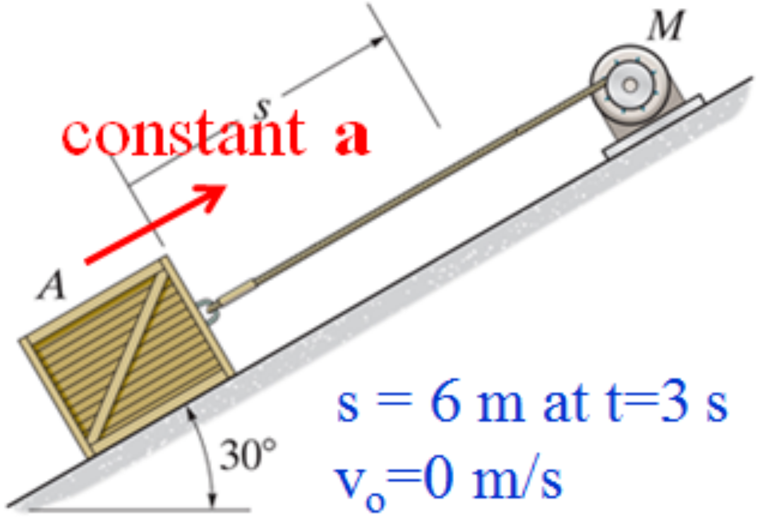
Example 1

2) Using kinematic equation

$$s = v_0\, t + \tfrac{1}{2}\, a\, t^2$$

$$\Rightarrow 6 = (0)\,3 + \tfrac{1}{2}\, a\, (3^2)$$

$$\Rightarrow a = 1.333 \text{ m/s}^2$$



3) Apply the equations of motion

$$+\nwarrow \sum F_y = 0 \Rightarrow -20\, g\, (\cos 30°) + N = 0$$

$$\Rightarrow N = 169.9 \text{ N}$$

$$+\nearrow \sum F_x = m\, a \Rightarrow T - 20g(\sin 30°) - 0.3\, N = 20\, a$$

$$\Rightarrow T = 20\,(9.81)\,(\sin 30°) + 0.3(169.9) + 20\,(1.333)$$

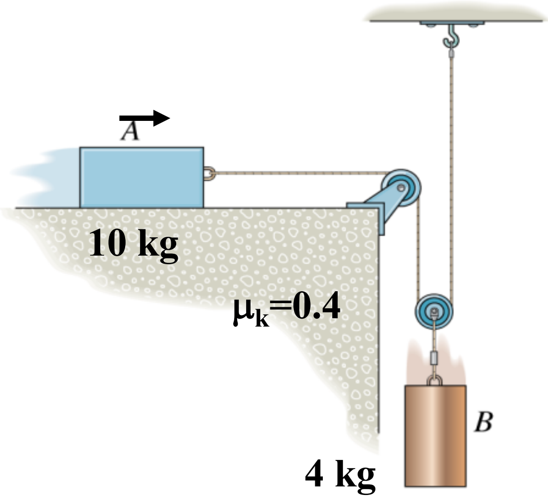
$$\Rightarrow T = 176 \text{ N}$$

Quiz

1. If the cable has a tension of 3 N, determine the acceleration of block B.

   A) 4.26 m/s² ↑ B) 4.26 m/s² ↓

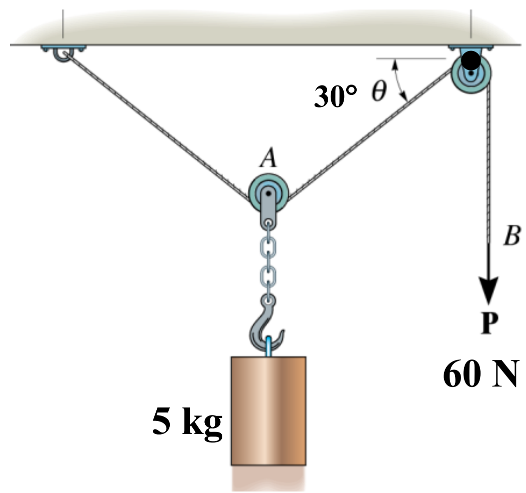
   C) 8.31 m/s² ↑ ✔D) 8.31 m/s² ↓



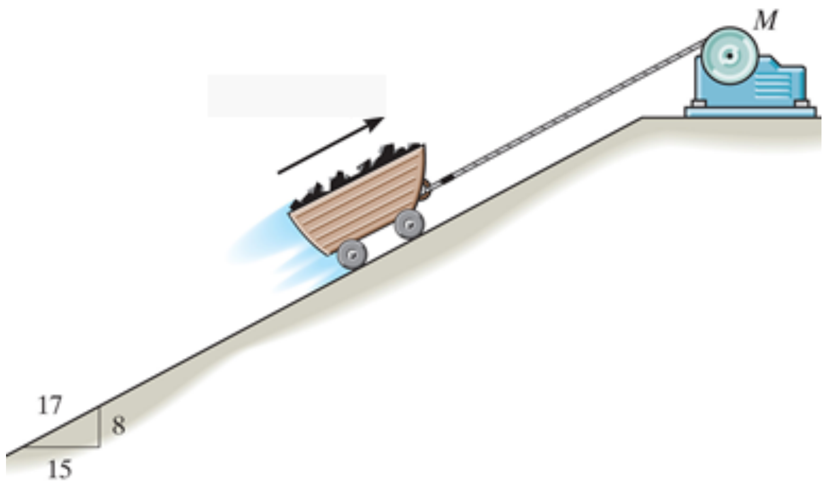
2. Determine the acceleration of the block.

   ✔ A) 2.20 m/s² ↑ B) 3.17 m/s² ↑

   C) 11.0 m/s² ↑ D) 4.26 m/s² ↑



Example 2



Given: Ore car's mass is 400 kg,
$v_o = 2$ m/s.
The force in the cable is
$F = (3200\ t^2)$ N,
where t is in seconds.

Find: v and s when t = 2 s.

Plan:

Since both forces and velocity are involved, this problem requires both kinematics and the equation of motion.

1) Draw the free-body and kinetic diagrams of the car.

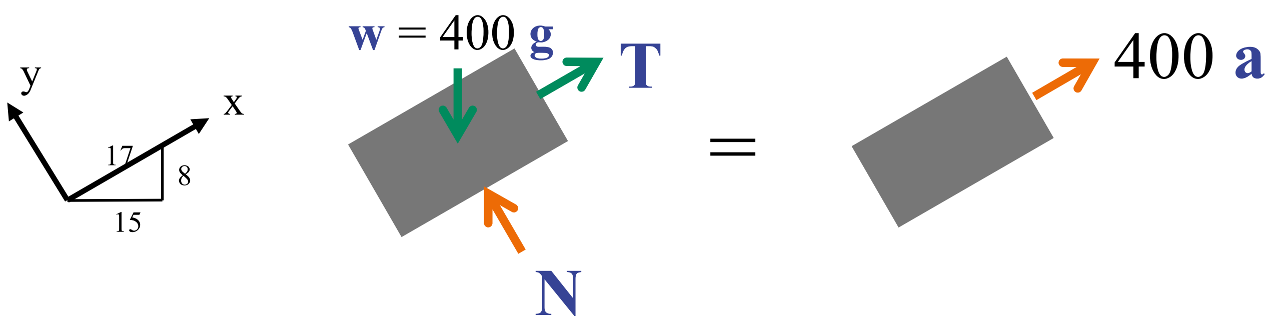
2) Apply the equation of motion to determine the acceleration.

3) Using a kinematic equation, determine velocity & distance.

Example 2

Solution:

1) Free-body and kinetic diagrams of the mine car:



Note that the mine car is moving along the x-axis.

2) Apply the equation of motion

$$+\nearrow \sum F_x = m\,a \Rightarrow T - 400g\,(8/17) = 400\,a$$

$$\Rightarrow 3200\,t^2 - 400\,(9.81)\,(8/17) = 400\,a$$

$$a = (8\,t^2 - 4.616)\ \text{m/s}^2$$

Example 2

3) Using kinematic equation to determine velocity and distance;

Since $a = (8\,t^2 - 4.616)\ \mathrm{m/s^2}$

$$v = v_0 + \int a\,dt = 2 + \int_0^t (8\,t^2 - 4.616)\,dt$$

$$\Rightarrow v = 2 + \frac{8}{3}t^3 - 4.616\,t$$

At $t = 2$ s,

$$v = 2 + \frac{8}{3}2^3 - 4.616\,(2) = 14.1\ \mathrm{m/s}$$

$$s = s_0 + \int v\,dt = 0 + \int_0^t \left(2 + \frac{8}{3}t^3 - 4.616\,t\right)dt$$

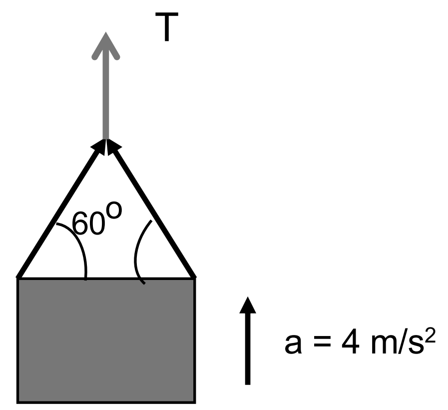
$$\Rightarrow s = 2\,t + \frac{8}{12}t^4 - \frac{4.616}{2}t^2$$

At $t = 2$ s,

$$s = 2\,(2) + \frac{8}{12}(2)^4 - \frac{4.616}{2}(2)^2 = 5.43\ \mathrm{m}$$

Quiz

1. Determine the tension in the cable when the 400-kg box is moving upward with a 4 m/s^2 acceleration.



A) 2265 N   B) 3365 N

✔ C) 5524 N   D) 6543 N

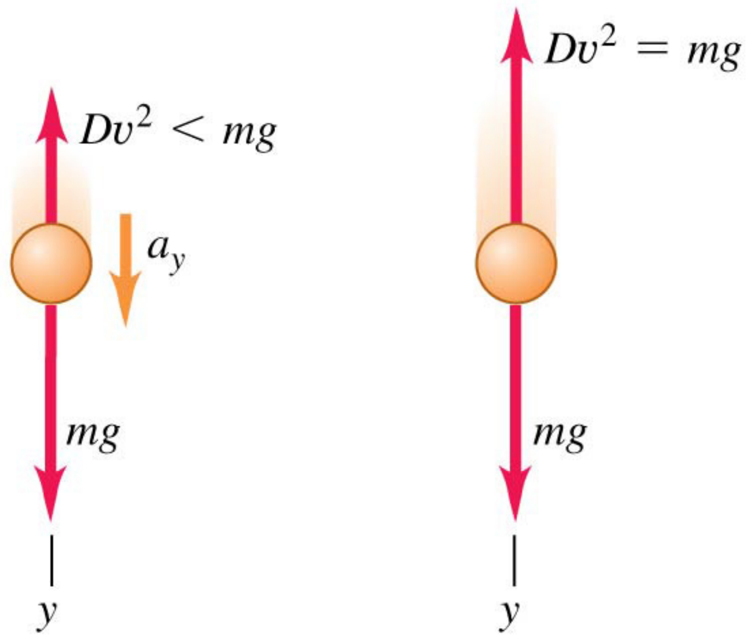
2. A 10-kg particle has forces of $\vec{F}_1 = (3\,\hat{\imath} + 5\,\hat{\jmath})$ N and $\vec{F}_2 = (-7\,\hat{\imath} + 9\,\hat{\jmath})$ N acting on it. Determine the acceleration of the particle.

✔ A) $(-0.4\,\hat{\imath} + 1.4\,\hat{\jmath})$ m/s^2 B) $(-4\,\hat{\imath} + 14\,\hat{\jmath})$ m/s^2

C) $(-12.9\,\hat{\imath} + 45\,\hat{\jmath})$ m/s^2 D) $(13\,\hat{\imath} + 4\,\hat{\jmath})$ m/s^2

Fluid resistance and terminal speed

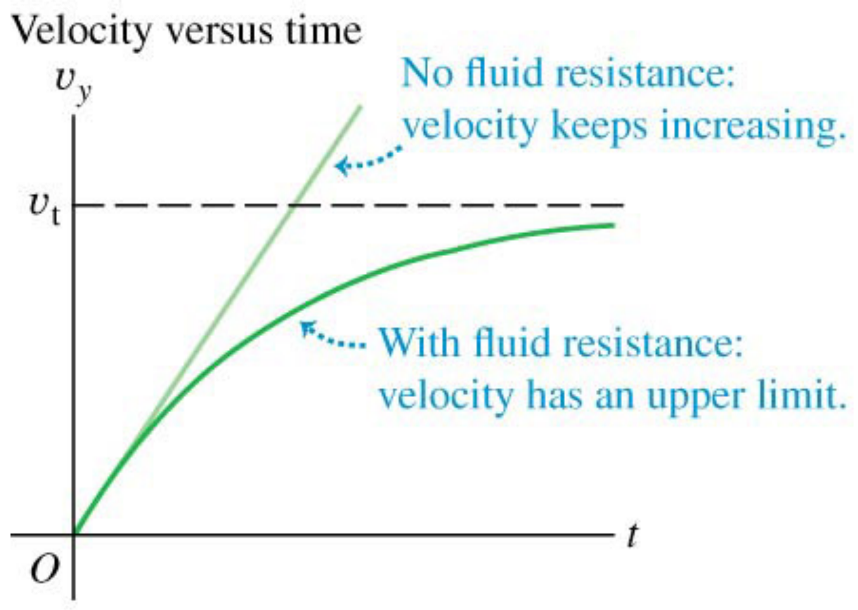
- The *fluid resistance* acting on a body depends on the speed of the body.
- A falling body reaches its *terminal speed* when the resisting force equals the weight of the body.
- The figures at the right illustrate the effects of air drag.



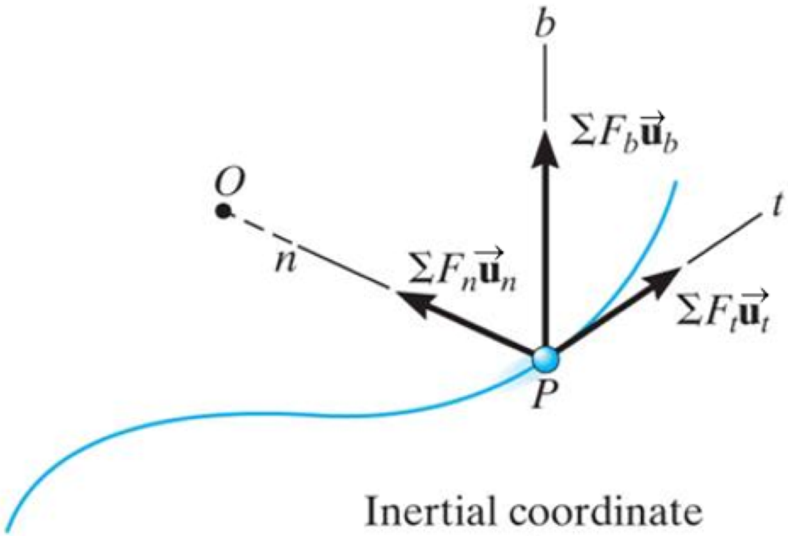
Before terminal speed: Object accelerating, drag force less than weight.

At terminal speed v_t: Object in equilibrium, drag force equals weight.

Fluid resistance and terminal speed



n-t coordinates



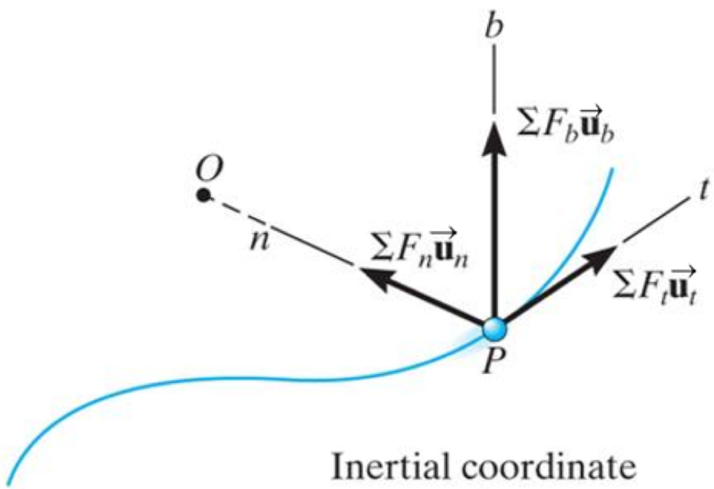
Inertial coordinate system

When a particle moves along a curved path, it may be more convenient to write the equation of motion in terms of normal and tangential coordinates.

The normal direction (n) always points toward the path's center of curvature. In a circle, the center of curvature is the center of the circle.

The tangential direction (t) is tangent to the path, usually set as positive in the direction of motion of the particle.

Equations of motion



Inertial coordinate system

Since the equation of motion is a vector equation, $\sum \mathbf{F} = m\mathbf{a}$, it may be written in terms of the n & t coordinates as

$$\sum F_t \vec{u}_t + \sum F_n \vec{u}_n + \sum F_b \vec{u}_b = ma_t + ma_n$$

Here $\sum F_t$ & $\sum F_n$ are the sums of the force components acting in the t & n directions, respectively.

This vector equation will be satisfied provided the individual components on each side of the equation are equal, resulting in the two scalar equations: $\sum F_t = ma_t$ and $\sum F_n = ma_n$.

Since there is no motion in the binormal (b) direction, we can also write $\sum F_b = 0$.

Normal and tangential accelerations

The tangential acceleration, $a_t = dv/dt$, represents the time rate of change in the magnitude of the velocity. Depending on the direction of $\sum F_t$, the particle's speed will either be increasing or decreasing.

The normal acceleration, $a_n = v^2/r$, represents the time rate of change in the direction of the velocity vector. Remember, a_n always acts toward the path's center of curvature. Thus, $\sum F_n$ will always be directed toward the center of the path.

Recall, if the path of motion is defined as $y = f(x)$, the radius of curvature at any point can be obtained from

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Solving problems with n-t coordinates

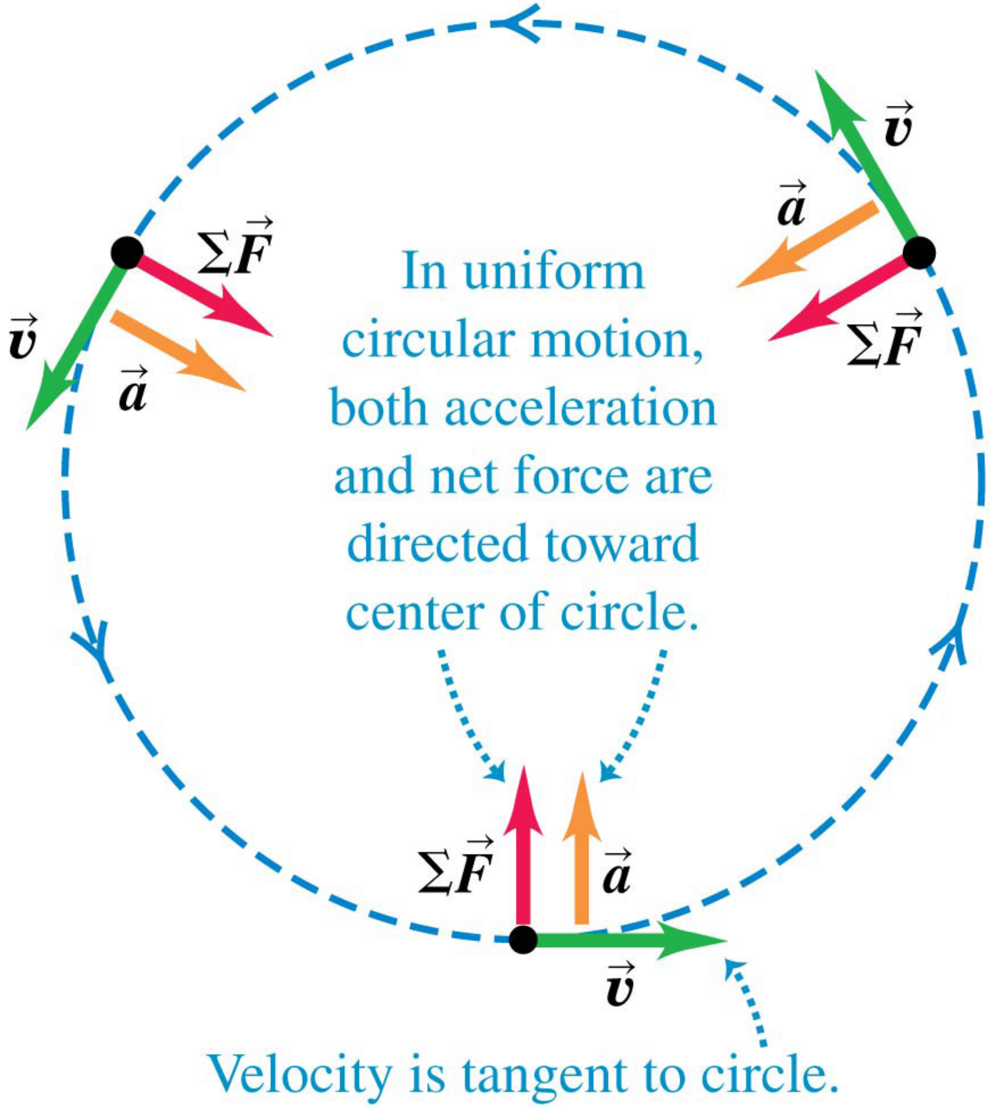
- Use n-t coordinates when a particle is moving along a known, curved path.
- Establish the n-t coordinate system on the particle.
- Draw free-body and kinetic diagrams of the particle. The normal acceleration (a_n) always acts "inward" (the positive n-direction). The tangential acceleration (a_t) may act in either the positive or negative t direction.
- Apply the equations of motion in scalar form and solve.
- It may be necessary to employ the kinematic relations:

$$a_t = dv/dt = v\, dv/ds \qquad a_n = v^2/\rho$$

Dynamics of circular motion

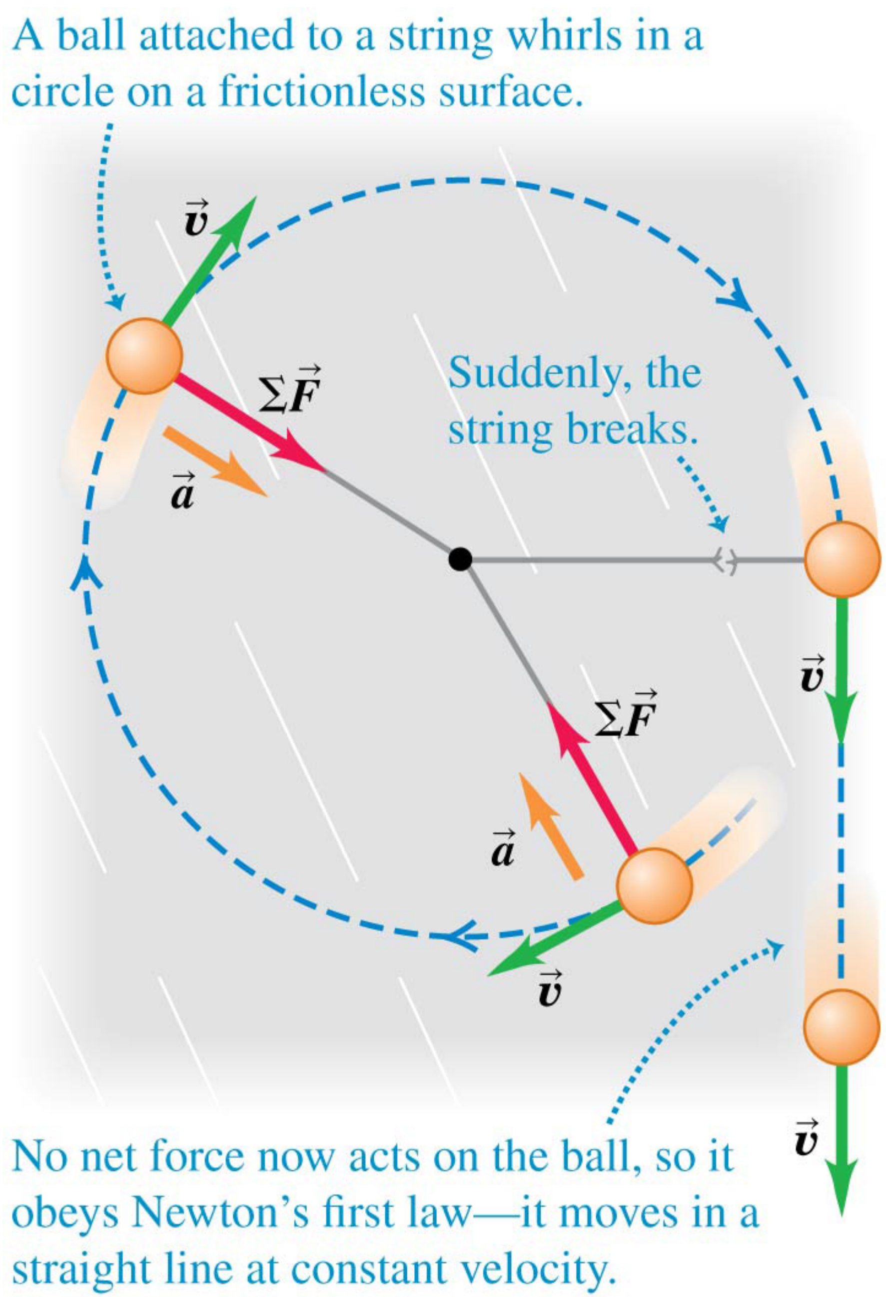
- If a particle is in uniform circular motion, both its acceleration and the net force on it are directed toward the center of the circle (centripetal acceleration/ centripetal force).

- The net force on the particle is $F_{\text{net}} = mv^2/R$.



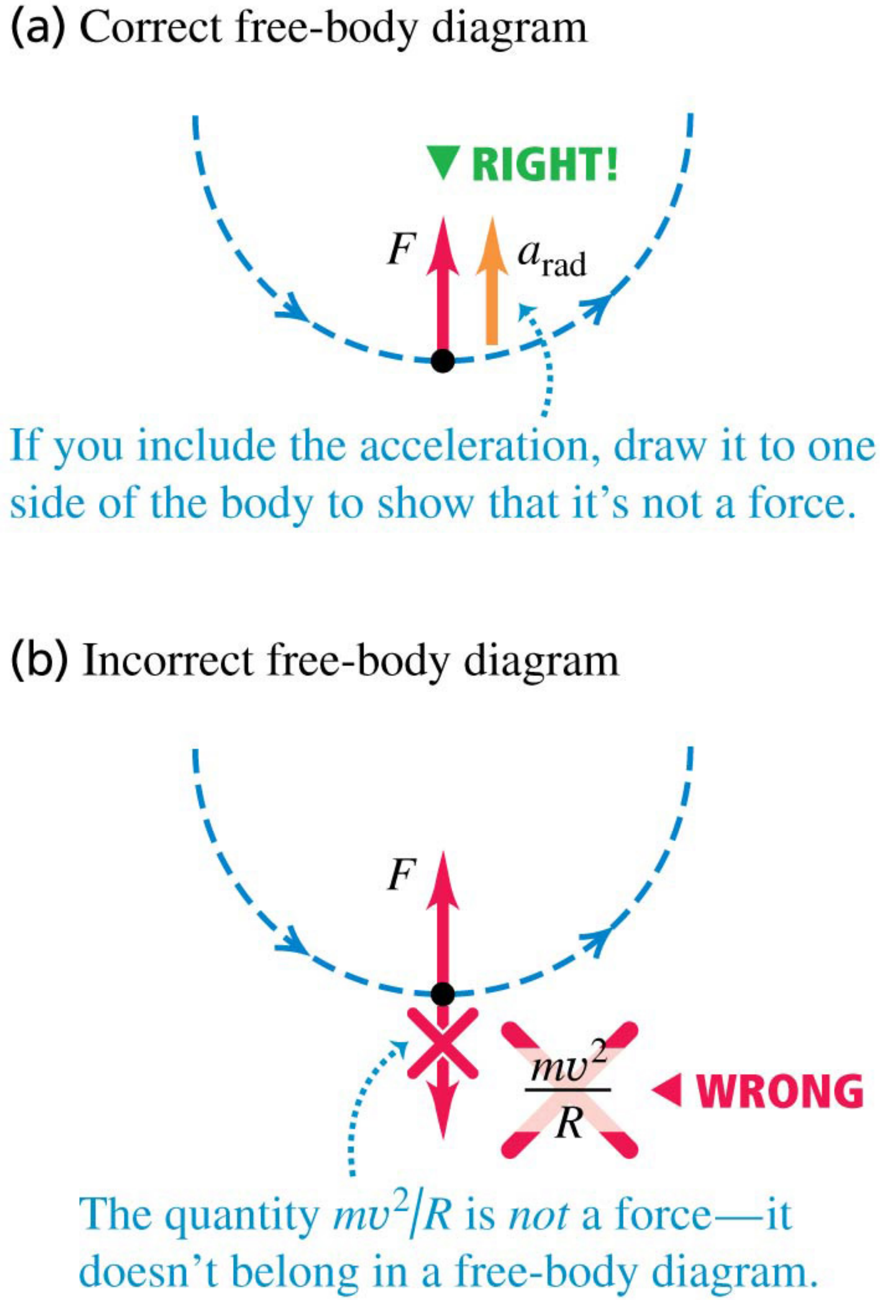
What if the string breaks?

- If the string breaks, no net force acts on the ball, so it obeys Newton's first law and moves in a straight line.



Avoid using "centrifugal force"

- Figure (a) shows the correct free-body diagram for a body in uniform circular motion.
- Figure (b) shows a common error.
- In an inertial frame of reference, there is no such thing as "centrifugal force."



Quiz

1. The "normal" component of the equation of motion is written as $\Sigma F_n = ma_n$, where ΣF_n is referred to as the ________.

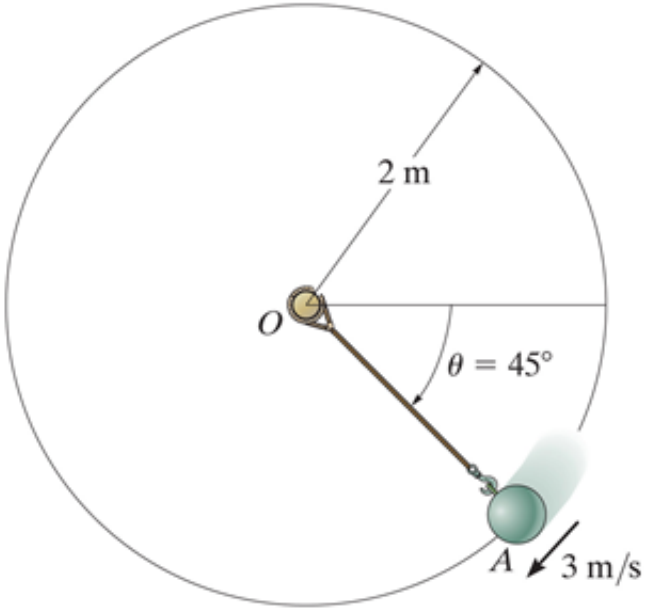
   A) impulse ✔B) centripetal force

   C) tangential force D) inertia force

2. The positive n direction of the normal and tangential coordinates is ____________.

   A) normal to the tangential component
   B) always directed toward the center of curvature
   C) normal to the binomial component
   ✔ D) All of the above.

Example 1



Given:

The 10-kg ball has a velocity of 3 m/s when it is at A, along the vertical path.

Find:

The tension in the cord and the increase in the speed of the ball.

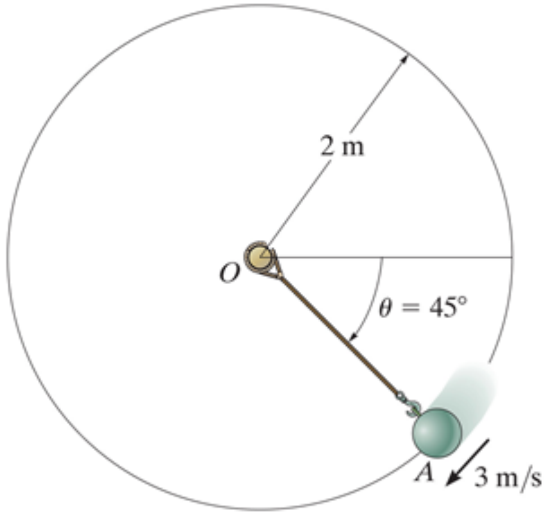
Plan:

1) Since the problem involves a curved path and requires finding the force perpendicular to the path, use n-t coordinates. Draw the ball's free-body and kinetic diagrams.
2) Apply the equation of motion in the n-t directions.

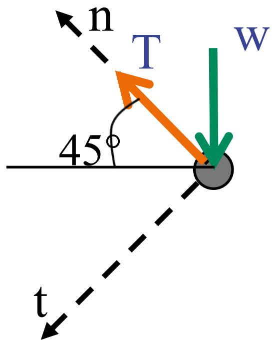
Example 1

Solution:

1) The n-t coordinate system can be established on the ball at Point A, thus at an angle of 45°.
   Draw the free-body and kinetic diagrams of the ball.



Free-body diagram



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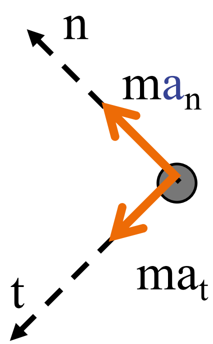
Kinetic diagram



Example 1

2)Apply the equations of motion in the n-t directions.

(a) $\sum F_n = ma_n \Rightarrow T - w \sin 45° = m\, a_n$

Using $a_n = v^2/\rho = 3^2/2$, $w = 10(9.81)$ N, and $m = 10$ kg

$\Rightarrow T - 98.1 \sin 45° = (10)(3^2/2)$

$\Rightarrow T = 114$ N

(b) $\sum F_t = ma_t \Rightarrow w \cos 45° = ma_t$

$\Rightarrow 98.1 \cos 45° = 10\, a_t$

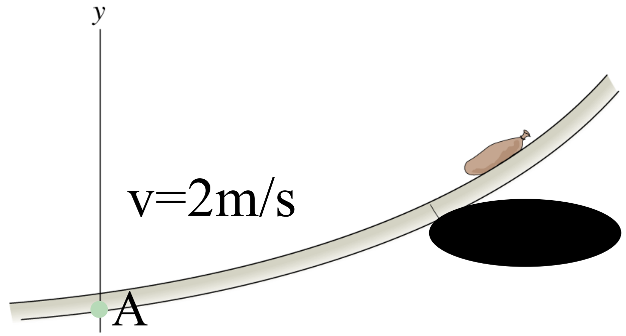
$\Rightarrow a_t = (dv/dt) = 6.94 \text{ m/s}^2$

Quiz

1. A 10-kg sack slides down a smooth surface. If the normal force on the surface at the flat spot, A, is 98.1 N (↑) , the radius of curvature is ____.

A) 0.2 m B) 0.4 m

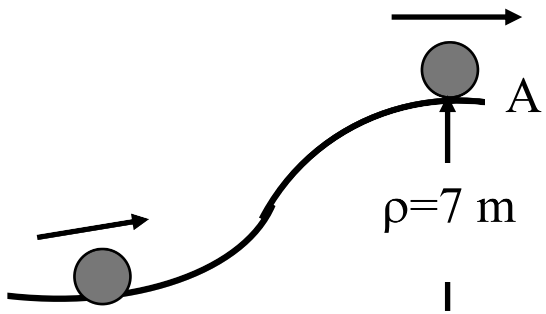
C) 1.0 m ✔ D) None of the above.



2. A 2-kg block is moving along a smooth surface. If the normal force on the surface at A is 10 N, the velocity is ________.

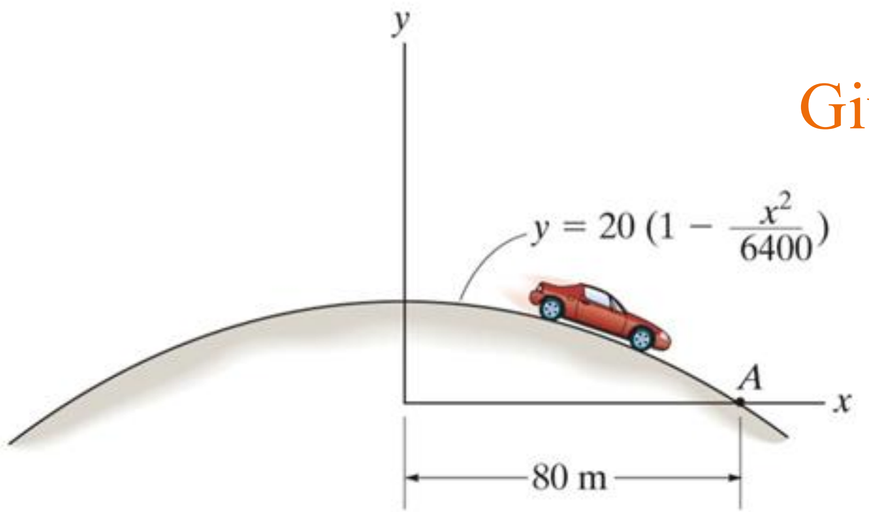
A) 7.6 m/s ✔B) 5.8 m/s

C) 10.6 m/s D) 12.6 m/s



(9.81-10/2)=v^2/7 v^2=7*(9.81-10/2)=33.67 v=sqrt(33.67)=5.8

Example 2



Given: A 800-kg car is traveling over a hill with the shape of a parabola. When the car is at point A, its v = 9 m/s and $a_t = 3\ m/s^2$. (Neglect the size of the car.)

Find: The resultant normal force and resultant frictional force exerted on the road at point A by the car.

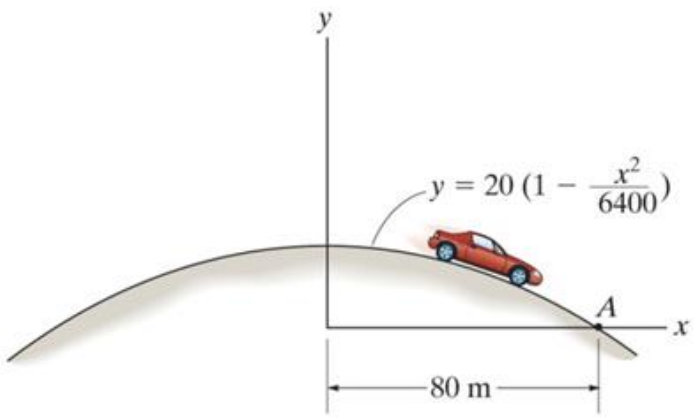
Plan:

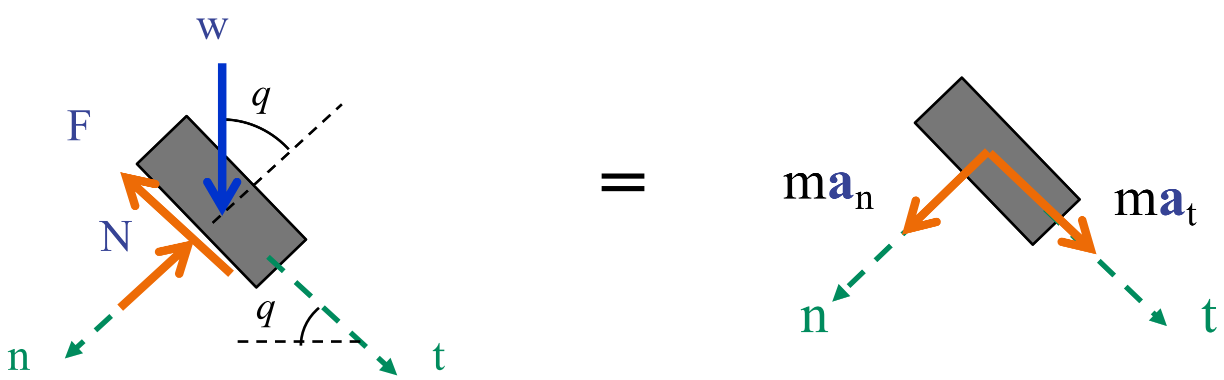
1) Treat the car as a particle. Draw its free-body and kinetic diagrams.
2) Apply the equations of motion in the n-t directions.
3) Use calculus to determine the slope and radius of curvature of the path at point A.

Example 2

Solution:

1) The n-t coordinate system can be established on the car at point A. Treat the car as a particle and draw the free-body and kinetic diagrams:





w = mg = weight of car

N = resultant normal force on road

F = resultant friction force on road

Example 2

2) Apply the equations of motion in the n-t directions:

$\Sigma F_n = ma_n \Rightarrow w \cos\theta - N = ma_n$

Using $w = mg$ and $a_n = v^2/\rho = (9)^2/\rho$

$\Rightarrow (800)(9.81) \cos q - N = (800)(81/\rho)$

$\Rightarrow N = 7848 \cos\theta - 64800/\rho$ (1)

$\Sigma F_t = ma_t \Rightarrow w \sin\theta - F = ma_t$

Using $w = mg$ and $a_t = 3\ m/s^2$ (given)

$\Rightarrow (800)(9.81) \sin\theta - F = (800)(3)$

$\Rightarrow F = 7848 \sin\theta - 2400$ (2)

Example 2

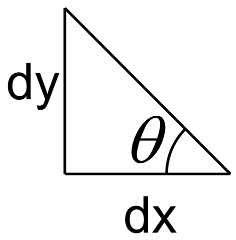
3) Determine ρ by differentiating y = f(x) at x = 80 m:

$$y = 20(1 - x^2/6400) \Rightarrow dy/dx = (-40)\,x / 6400$$

$$\Rightarrow d^2y/dx^2 = (-40) / 6400$$

$$\rho\Big|_{x = 80\text{ m}} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{[1 + (-0.5)^2]^{3/2}}{|\,0.00625\,|} = 223.6\text{ m}$$

Determine θ from the slope of the curve at A:



$$\tan\theta = dy/dx\Big|_{x = 80\text{ m}}$$

$$\theta = \left|\tan^{-1}(dy/dx)\right| = \left|\tan^{-1}(-0.5)\right| = 26.6^\circ$$

Example 2

From Eq.(1): $N = 7848 \cos\theta - 64800 / \rho$

$$= 7848 \cos(26.6^\circ) - 64800 / 223.6 = 6728 \text{ N}$$

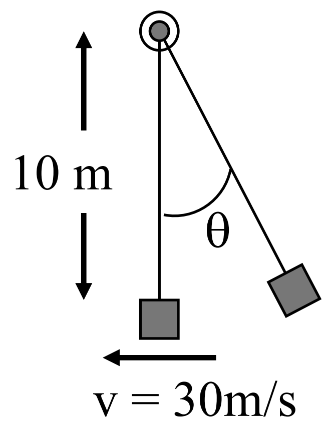
From Eq.(2): $F = 7848 \sin\theta - 2400$

$$= 7848 \sin(26.6^\circ) - 2400 = 1114 \text{ N}$$

Quiz

1. The tangential acceleration of an object
   A) represents the rate of change of the velocity vector's direction.
   ✔ B) represents the rate of change in the magnitude of the velocity.
   C) is a function of the radius of curvature.
   D) Both B and C.

2. The block has a mass of 20 kg and a speed of $v = 30$ m/s at the instant it is at its lowest point. Determine the tension in the cord at this instant.



A) 1596 N   B) 1796 N

✔ C) 1996 N   D) 2196 N

(30^2/10+9.81)*20=1996.2N

The fundamental forces of nature

- According to current understanding, all forces are expressions of four distinct *fundamental* forces:
  - *gravitational interactions*
  - *electromagnetic interactions*
  - the *strong interaction*
  - the *weak interaction*
- Physicists have taken steps to unify all interactions into a *theory of everything*.