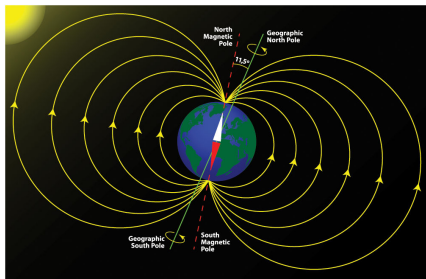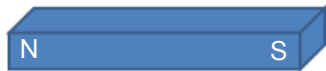# Lecture 9

Magnetic Field

Lecture Outline

- **Chapter 28**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Magnetic Field
  - Definition of **B**
  - Magnetic field lines
  - The Hall effect
  - Magnetic force on a current carrying wire
  - Torque on a current loop

Magnetism

- Properties of magnets:
  - All magnets have two poles: North and South.
  - Like poles repel each other.
  - Unlike poles attract each other.
  - Magnetic poles always found in pairs.
  - Isolated magnetic poles never been found.

N   S

Magnetic field of Earth

https://www.sott.net/article/248315-Scientists-Offer-Theory-For-Why-Earths-Magnetic-Field-Is-Wonky

Department of Physics
香港城市大學
City University of Hong Kong

Magnetism

- The (non)existence of magnetic monopoles:
  - In the study of theoretical physics, such as in grand unified theories and quantum gravity, one predicts that there exist magnetic charges, just like electric charges.
  - Such an entity would be called a magnetic monopole, having +ve or –ve magnetic charge.
  - However, if we break a piece of magnet into many small pieces, each individual piece still contains both N & S poles.

| N   S | ➡ | N   S   N   S | ➡ | N S   N S   N S |

  - As a matter of fact, even a single individual electron has a magnetic 'dipole'.
  - Although some experimental results said to have demonstrated the existence of magnetic monopoles, all were dismissed as inconclusive.
  - So far no monopoles have ever been found and their existence remains to be discovered.

Source of the Magnetic Field

- If there are no magnetic charges, then what is the source of the magnetic field?

  – The answer is **electric charges in motion**.

  – One way that magnetic fields are produced is to use moving electrically charged particles, such as a current in a wire, to make an **electromagnet.** The current produces a magnetic field that is utilizable.

  – Another way to produce a magnetic field is by means of elementary particles such as electrons, because these particles have an *intrinsic magnetic field* around them.

Permanent Magnet

- The magnetic fields of the electrons/molecules in certain materials add together to give a net magnetic field around the material. Such addition is the reason why a **permanent magnet**, has a permanent magnetic field.

- In other materials, the magnetic fields of the electrons/molecules cancel out, giving no net magnetic field surrounding the material.

Department of Physics
香港城市大學
City University of Hong Kong
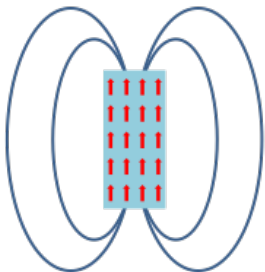
The definition of Magnetic Field **B**

● Recall that if we want to define the magnitude and direction of an unknown electric field distribution, we can place a stationary test charge q in the field and measure the force FE encountered by the charge due to the field and extract E from

$$\mathbf{F_E} = q\mathbf{E} \quad \Longrightarrow \quad \mathbf{E} = \mathbf{F_E}/q$$

● Unfortunately we cannot define the magnitude and direction of the magnetic field **B** using the same stationary test charge because there is no equivalent magnetic charge qm.

● In order to define the magnetic field **B**, we will need a moving test charge. Because the force from the **B** field **F**B is proportional to both the velocity and charge of the test charge.

The definition of Magnetic Field **B**

- Experimentally we find that when a charged particle moving with velocity **v** in a magnetic field, it encounters a force **FB** due to the field that acts on the particle.

$$\mathbf{F_B} = q\mathbf{v} \times \mathbf{B}$$ Experimental result

- Since **FB**, **v** can be measured, this proves to be a convenient way to define and measure **B**.

- Here q is the charge of the particle, v is its velocity, and **B** the magnetic field in the region. The magnitude of this force is then:

$$F_B = |q|vB\sin\phi$$

- Here f is the angle between vectors **v** and **B**.

PHY 1202

Magnetic Force: Observations

- The magnitude of the magnetic force is proportional to the charge q and the speed v of the particle.
- The direction of the force depend on the velocity of the particle and on the direction of the magnetic field.
- When the charged particle moves *parallel* to the magnetic field the magnetic force on the charge is *zero*.
- When the charged particle moves in a direction not parallel to the magnetic field the magnetic force is *perpendicular to both the velocity of the charge and the magnetic field*.
- Positive and negative charges moving in the same direction experience magnetic forces in opposite directions.
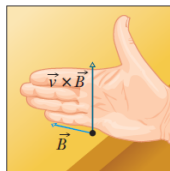- If the velocity vector makes an angle of *f* with the magnetic field vector, the magnetic force is proportional to sin*f*.

Cross $\vec{v}$ into $\vec{B}$ to get the new vector $\vec{v} \times \vec{B}$.
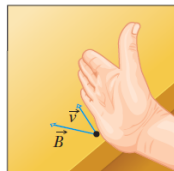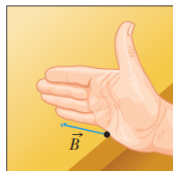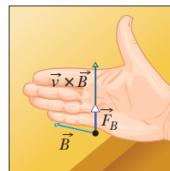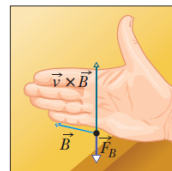
Force on positive particle

Force on negative particle

(a) (b) (c) (d) (e)

Fig. 28-2 (a) – (c) The right-hand rule (in which $\vec{v}$ is swept into $\vec{B}$ through the smaller angle ϕ between them) gives the direction of $\vec{v} \times \vec{B}$ as the direction of the thumb. (d) If q is positive, then the direction of $\vec{F}_B = q\vec{v} \times \vec{B}$ is in the direction of $\vec{v} \times \vec{B}$. (e) If q is negative, then the direction of $\vec{F}_B$ is opposite that of $\vec{v} \times \vec{B}$.

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

PHY 1202

CityU
Department of Physics
香港城市大學
City University of Hong Kong

Table 28-1
Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

The SI unit for **B** that follows is newton per coulomb-meter per second. For convenience, this is called the **tesla (T):**

$$1 \text{ tesla} = 1 \text{ T} = 1 \frac{\text{newton}}{(\text{coulomb})(\text{meter/second})}$$

$$= 1 \frac{\text{newton}}{(\text{coulomb/second})(\text{meter})} = 1 \frac{\text{N}}{\text{A} \cdot \text{m}}$$

An earlier (non-SI) unit for **B** is the *gauss (G),* and

$$1 \text{ tesla} = 10^4 \text{ gauss.}$$

CityU
Department of Physics
香港城市大學
City University of Hong Kong

We can represent magnetic fields with field lines, as we did for electric fields. Similar rules apply; that is,

1.   the direction of the tangent to a magnetic field line at any point gives the direction of **B** at that point, and

2.   the spacing of the lines represents the magnitude of **B** — the magnetic field is stronger where the lines are closer together, and conversely.
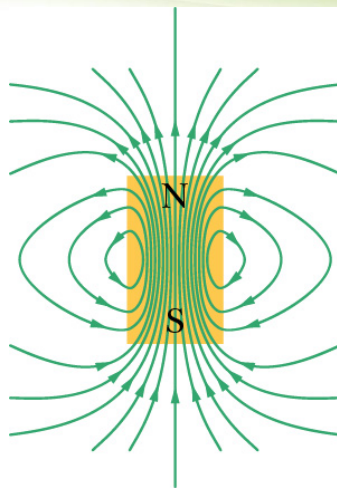

↪ Opposite magnetic poles attract each other, and like magnetic poles repel each other.

CityU
Department of Physics
香港城市大學
City University of Hong Kong

- The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where field lines enter the magnet, is called the *south pole.*

- Opposite magnetic poles attract each other, and like magnetic poles repel each other.

Department of Physics
香港城市大學
City University of Hong Kong
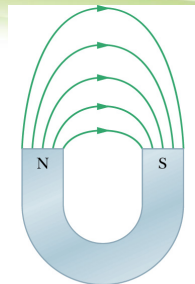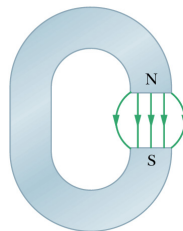
A uniform magnetic field $\vec{B}$, with magnitude 1.2 mT, is directed vertically upward throughout the volume of a laboratory chamber. A proton with kinetic energy 5.3 MeV enters the chamber, moving horizontally from south to north. What magnetic deflecting force acts on the proton as it enters the chamber? The proton mass is 1.67×10^{-27} kg. (Neglect Earth's magnetic field.)

KEY IDEAS

Because the proton is charged and moving through a magnetic field, a magnetic force $\vec{F}_B$ can act on it. Because the initial direction of the proton's velocity is not along a magnetic field line, $\vec{F}_B$ is not simply zero.
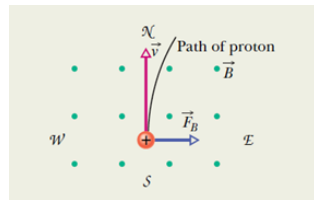
Magnitude: To find the magnitude of $\vec{F}_B$, we can use Eq. 28-3 ($F_B = |q|vB \sin \phi$) provided we first find the proton's speed v. We can find v from the given kinetic energy because $K = \frac{1}{2}mv^2$. Solving for v, we obtain
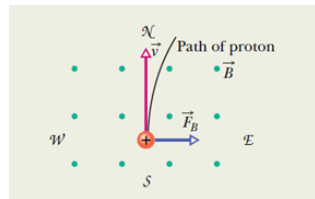
$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{(2)(5.3 \text{ MeV})(1.60 \times 10^{-13} \text{ J/MeV})}{1.67 \times 10^{-27} \text{ kg}}}$$

$$= 3.2 \times 10^7 \text{ m/s}.$$
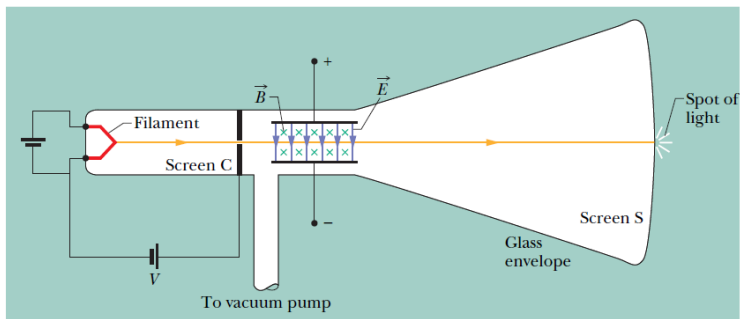
Equation 28-3 then yields

$$F_B = |q|vB \sin \phi$$
$$= (1.60 \times 10^{-19} \text{ C})(3.2 \times 10^7 \text{ m/s})$$
$$\times (1.2 \times 10^{-3} \text{ T})(\sin 90°)$$
$$= 6.1 \times 10^{-15} \text{ N}. \qquad \text{(Answer)}$$

This may seem like a small force, but it acts on a particle of small mass, producing a large acceleration; namely,

$$a = \frac{F_B}{m} = \frac{6.1 \times 10^{-15} \text{ N}}{1.67 \times 10^{-27} \text{ kg}} = 3.7 \times 10^{12} \text{ m/s}^2.$$

PHY 1202

Department of Physics
香港城市大學
City University of Hong Kong

Fig. 28-7 A modern version of J.J. Thomson's apparatus for measuring the ratio of mass to charge for the electron. An electric field **E** is established by connecting a battery across the deflecting-plate terminals. The magnetic field **B** is set up by mean of a current in a system of coils. The procedure first determine the velocity of the particle by adjusting the **E** and **B** field ratio, then the B field is turned off and the e/m ratio can then be determined from the amount of deflection measured on the screen.

When the two fields in Fig. 28-7 are adjusted so that the two deflecting forces acting on the charged particle cancel, we have

$$|q|E = |q|vB \sin(90°) = |q|vB \quad \Longrightarrow \quad v = \frac{E}{B}.$$

Thus, the crossed fields allow us to measure the speed of the charged particles passing through them.

PHY 1202

Determine the e/m ratio

- When the magnetic field is turned off, the electron experience a transverse force from the electric field=eE
- This gives a transverse acceleration=eE/m, which deflected the electron path and the spot the electron hits the screen.
- Deflection is related to the E and from the deflection, E filed, and the magnetic field the e/m ratio is found.

**Fig. 28-8** A strip of copper carrying a current *i is immersed in a magnetic field* .

(*a*)*The situation immediately after the* magnetic field is turned on. The curved path that will then be taken by *an electron* is shown.

(*b*) *The situation at equilibrium,* which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Thus, the left side is at a higher potential than the right side.

(*c*) *For the same* current direction, if the charge carriers were positively charged, *they would pile up* on the right side, and the right side would be at the higher potential.

PHY 1202

City University of Hong Kong
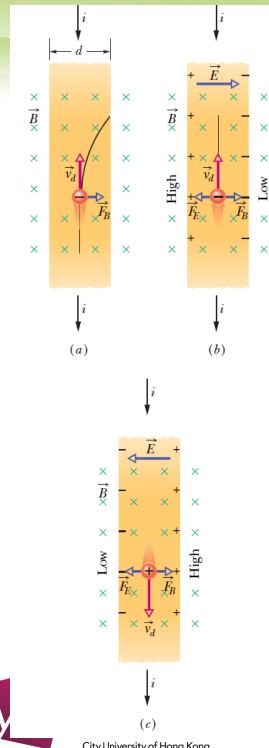
A Hall potential difference V is associated with the electric field across strip width d, and the magnitude of that potential difference is $V = Ed$. When the electric and magnetic forces are in balance (Fig. 28-8b),

$$F_E = F_B \Rightarrow eE = ev_d B$$

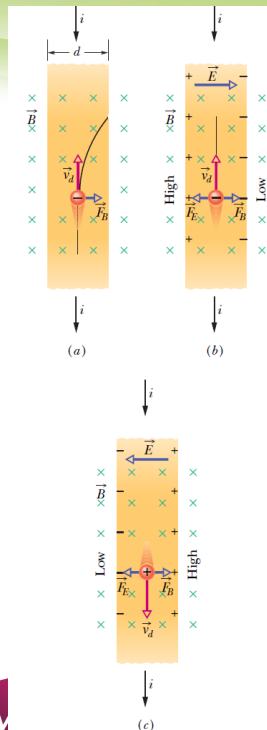where vd is the drift speed. But,

$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

Where J is the current density, A the cross-sectional area, e the electronic charge, and n the number of charges per unit volume, or commonly known as the carrier density.

Therefore, $n = \dfrac{Bi}{Vle}$,

Here, $l = (A/d)$, the thickness of the strip.

(a)   (b)

(c)

City University of Hong Kong

CS

Electrons circulating in a chamber containing gas at low pressure (their path is the glowing circle). A uniform magnetic field **B**, pointing directly out of the plane of the page, fills the chamber. Note the radially directed magnetic force FB; for circular motion to occur, FB *must* point toward the centre of the circle. Use the right-hand rule for cross products to confirm that FB= q**v x B** gives FB the proper direction. (Don't forget the sign of q.)

The magnetic force continuously deflects the particle, and since **B** and **v** are always perpendicular to each other, this deflection causes the particle to follow **a circular path.**

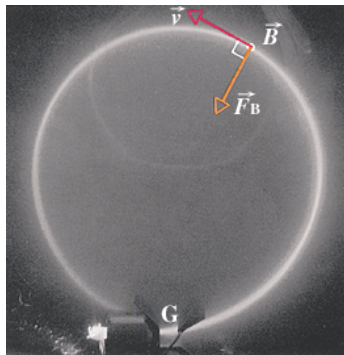The magnetic force acting on the particle has a magnitude of $|q|vB$.

For uniform circular motion $\quad F = m\dfrac{v^2}{r}, \quad \Longrightarrow \quad |q|vB = \dfrac{mv^2}{r}.$

$$r = \frac{mv}{|q|B} \quad \text{(radius).}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad \text{(period).} \quad \text{Eq. 28-17}$$

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \quad \text{(frequency).}$$

$$\omega = 2\pi f = \frac{|q|B}{m} \quad \text{(angular frequency).}$$

PHY 1202

- In the previous example, we see that a moving charged particle moves in a circulating path if it is in a uniform **B** field.

- If the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical (corkscrew) path about the direction of the field vector.

- Note that the **B** field only acts on the component of the velocity that is perpendicular to the **B** field to generate the force to move the particle in the circular motion.

The parallel component determines the *pitch p* of the helix—that is, the distance between adjacent turns

$$v_{II} = v\cos\phi \qquad v_{\perp} = v\sin\phi$$
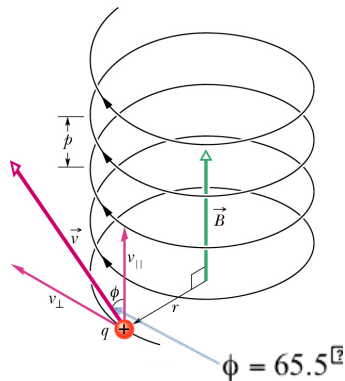
PHY 1202

Department of Physics

CityU
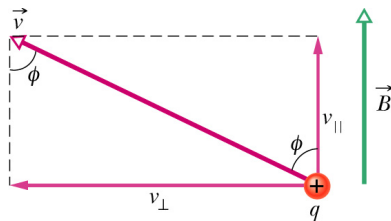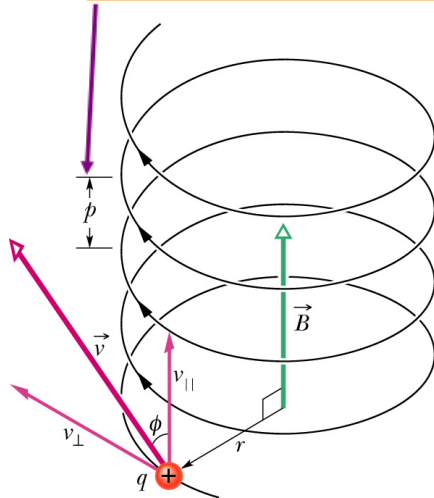
Example, Helical Motion of a Charged Particle in a Magnetic Field:

An electron with a kinetic energy of 22.5 eV moves into a region of uniform magnetic field $\vec{B}$ of magnitude 4.55×10^{-4} T. The angle between the directions of $\vec{B}$ and the electron's velocity $\vec{v}$ is 65.5°. What is the pitch of the helical path taken by the electron?

KEY IDEAS

(1) The pitch p is the distance the electron travels parallel to the magnetic field $\vec{B}$ during one period T of circulation. (2) The period T is given by Eq. 28-17 regardless of the angle between the directions of $\vec{v}$ and $\vec{B}$ (provided the angle is not zero, for which there is no circulation of the electron).

$$\phi = 65.5°$$

**Calculations:** Using Eqs. 28-20 and 28-17, we find

$$p = v_{\parallel} T = (v \cos \phi) \frac{2\pi m}{|q|B}. \qquad (28\text{-}21)$$

Calculating the electron's speed v from its kinetic energy, find that $v = 2.81 \times 10^6$ m/s. Substituting this and known data in Eq. 28-21 gives us

$$p = (2.81 \times 10^6 \text{ m/s})(\cos 65.5°)$$

$$\times \frac{2\pi(9.11 \times 10^{-31} \text{ kg})}{(1.60 \times 10^{-19} \text{ C})(4.55 \times 10^{-4} \text{ T})}$$

$$= 9.16 \text{ cm}. \qquad \text{(Answer)}$$

A force acts on a current through a B field.

Fig. 28-14 A flexible wire passes between the pole faces of a magnet (only the farther pole face is shown). (*a*) Without current in the wire, the wire is straight. (*b*) With upward current, the wire is deflected rightward. (*c*) With downward current, the deflection is leftward. The connections for getting the current into the wire at one end and out of it at the other end are not shown.
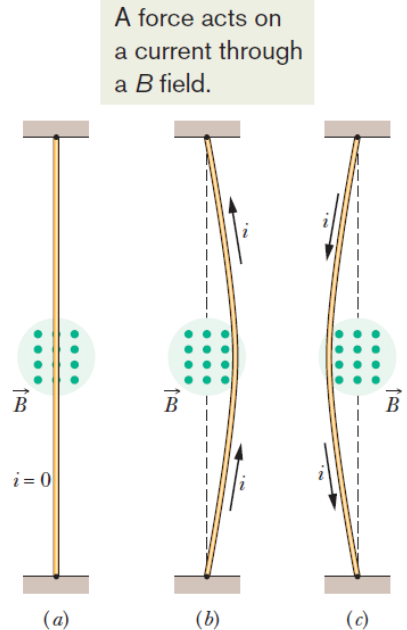
28.8: Magnetic Force on a Current-Carrying Wire:
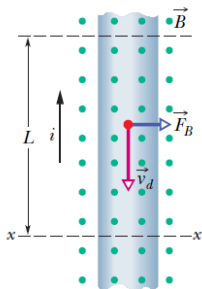
Fig. 28-15 A close-up view of a section of the wire of Fig. 28-14*b*. The current direction is upward, which means that electrons drift downward. A magnetic field that emerges from the plane of the page causes the electrons and the wire to be deflected to the right.

Consider a length L of the wire in the figure. All the conduction electrons in this section of wire will drift past plane xx in a time $t = L/vd$.

Thus, in that time a charge will pass through that plane that is given by

$$q = it = i\frac{L}{v_d}$$

$$F_B = qv_dB \sin\phi = \frac{iL}{v_d}v_dB \sin 90°$$

$$F_B = iLB.$$

$$\vec{F}_B = i\vec{L} \times \vec{B} \qquad \text{(force on a current).}$$

Here L is a length vector that has magnitude L and is directed along the wire segment in the direction of the (conventional) current.

If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments . The force on the wire as a whole is then the vector sum of all the forces on the segments that make it up. In the differential limit, we can write $d\vec{F}_B = i\,d\vec{L} \times \vec{B}$, ind the resultant force on any given arrangement of currents by integrating Eq. 28-28 over that arrangement.

PHY 1202

Example, Magnetic Force on a Wire Carrying Current:

A straight, horizontal length of copper wire has a current $i = 28$ A through it. What are the magnitude and direction of the minimum magnetic field $\vec{B}$ needed to suspend the wire — that is, to balance the gravitational force on it? The linear density (mass per unit length) of the wire is 46.6 g/m.

KEY IDEAS

(1) Because the wire carries a current, a magnetic force $\vec{F}_B$ can act on the wire if we place it in a magnetic field $\vec{B}$. To balance the downward gravitational force $\vec{F}_g$ on the wire, we want $\vec{F}_B$ to be directed upward (Fig. 28-17). (2) The direction of $\vec{F}_B$ is related to the directions of $\vec{B}$ and the wire's length vector $\vec{L}$ by Eq. 28-26 ($\vec{F}_B = i\vec{L} \times \vec{B}$).

Calculations: Because $\vec{L}$ is directed horizontally (and the current is taken to be positive), Eq. 28-26 and the right-hand rule for cross products tell us that $\vec{B}$ must be horizontal and rightward (in Fig. 28-17) to give the required upward $\vec{F}_B$.

The magnitude of $\vec{F}_B$ is $F_B = iLB \sin \phi$ (Eq. 28-27). Because we want $\vec{F}_B$ to balance $\vec{F}_g$, we want

$$iLB \sin \phi = mg, \qquad (28\text{-}29)$$

where mg is the magnitude of $\vec{F}_g$ and m is the mass of the wire.
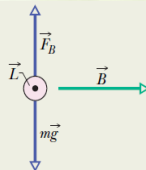
Fig. 28-17 A wire (shown in cross section) carrying current out of the page.

We also want the minimal field magnitude B for $\vec{F}_B$ to balance $\vec{F}_g$. Thus, we need to maximize $\sin \phi$ in Eq. 28-29. To do so, we set $\phi = 90°$, thereby arranging for $\vec{B}$ to be perpendicular to the wire. We then have $\sin \phi = 1$, so Eq. 28-29 yields

$$B = \frac{mg}{iL \sin \phi} = \frac{(m/L)g}{i}. \qquad (28\text{-}30)$$

We write the result this way because we know m/L, the linear density of the wire. Substituting known data then gives us

$$B = \frac{(46.6 \times 10^{-3} \text{ kg/m})(9.8 \text{ m/s}^2)}{28 \text{ A}}$$

$$= 1.6 \times 10^{-2} \text{ T.} \qquad (\text{Answer})$$

This is about 160 times the strength of Earth's magnetic field.

The elements of an electric motor. A rectangular loop of wire, carrying a current and free to rotate about a fixed axis, is placed in a magnetic field. Magnetic forces on the wire produce a torque that rotates it. A commutator (not shown) reverses the direction of the current every half-revolution so that the torque always acts in the same direction.
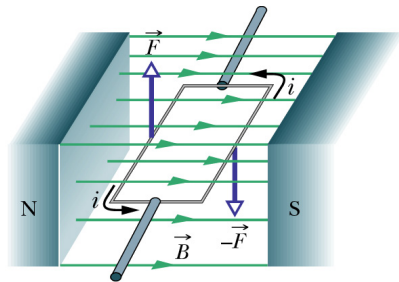
The two magnetic forces *F* and –*F* produce a torque on the loop, tending to rotate it about its central axis.
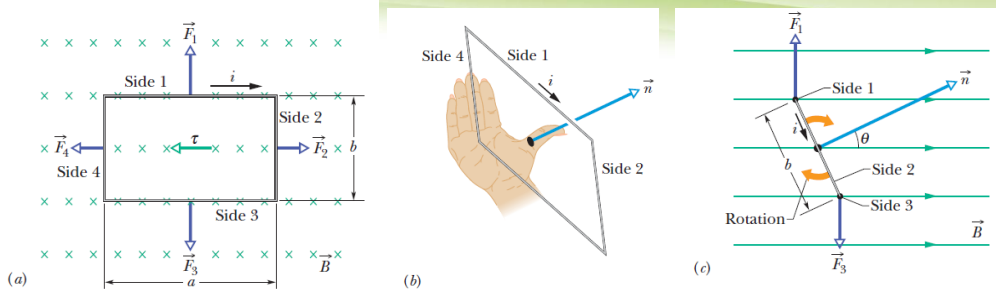
28.9: Torque on a Current Loop:

Figure 28-19

To define the orientation of the loop in the magnetic field, we use a normal vector n that is perpendicular to the plane of the loop. Along with the current i and the area A plane we can define a magnetic dipole moment for the single wire loop, similar to the electric dipole moment,

$$\mu = iA$$

Figure 28-19b shows a right-hand rule for finding the direction of n. In Fig. 28-19c, the normal vector of the loop is shown at an arbitrary angle q to the direction of the magnetic field.

PHY 1202

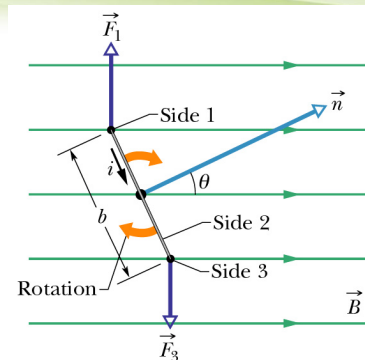For side 2 the magnitude of the force acting on this side is

F2=ibB sin(90°-q)=ibB cosq =F4.
F2 and *F4* cancel out exactly.

Forces **F1** and **F3** have the common magnitude *iaB.* As Fig. 28-19c shows, these two forces do not share the same line of action; so they produce a net torque.

a = length of sides 1 and 3

b = length of sides 2 and 4

A = a x b = area enclosed by the coil

$$\tau' = \left(iaB \frac{b}{2} \sin \theta \right) + \left(iaB \frac{b}{2} \sin \theta \right) = iabB \sin \theta.$$

PHY 1202

28.9: Torque on a Current Loop:

For *N* loops, when *A=ab*, the area of the loop, the total torque is:

$$\tau = N\tau' = NiabB \sin \theta = (NiA)B \sin \theta,$$

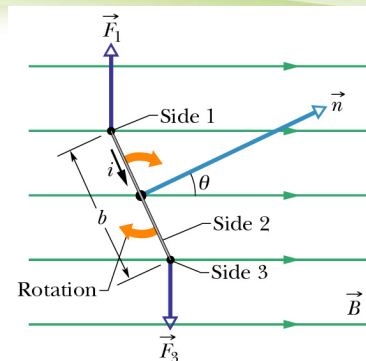Magnetic dipole moment of a coil with N loop:

$$\mu = NiA$$

The torque on a magnetic dipole moment due to a magnetic field:

$$\vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \tau = \mu B \sin \theta$$

Similar to $\vec{\tau} = \vec{p} \times \vec{E}$ for an electric dipole

Potential Energy of the Magnetic Dipole in a Magnetic Field $\quad U = -\vec{\mu} \cdot \vec{B}$

a = length of sides 1 and 3

b = length of sides 2 and 4

A = *a* x *b* = area enclosed by the coil

PHY 1202

Potential energy of a magnetic dipole in a magnetic field

-
  - The torque on the magneti
    is $-\mu B sin\theta$ (negative as it
  - Work done by force/torque
    potential energy, same as i
    potential. Change of poten
    $W = U_i - U_f$ or $-W = U_f$