Chapter 11

Equilibrium and Elasticity

Introduction

- This Roman aqueduct uses the principle of the arch to sustain the weight of the structure and the water it carries.

- When designing bridges, aqueducts, ladders etc., we're interested in making sure that they *don't* accelerate.

- Real materials are not truly rigid. They are *elastic* and do deform to some extent.

- We shall introduce concepts such as stress and strain to understand the deformation of real bodies.

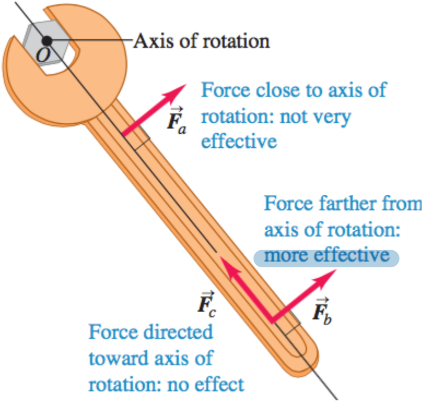
变形

Torque 扭转力.

Forces can affect a body's **translational** 平移 **motion** 运动 (moving as a whole).

Forces can also cause **rotations** (twist or turn) – **torque**

10.1 Which of these three equal-magnitude forces is most likely to loosen the tight bolt?



Torque

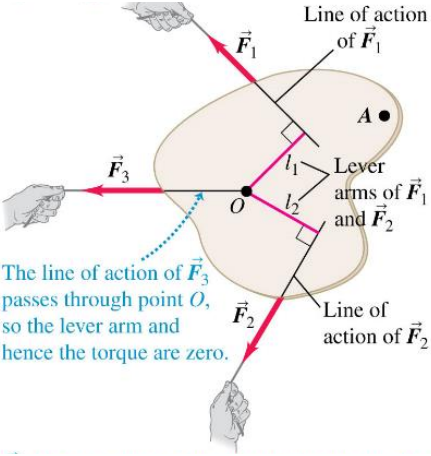
Consider rotation around *O:*

The **line of action** of a force is the line along which the force vector lies.

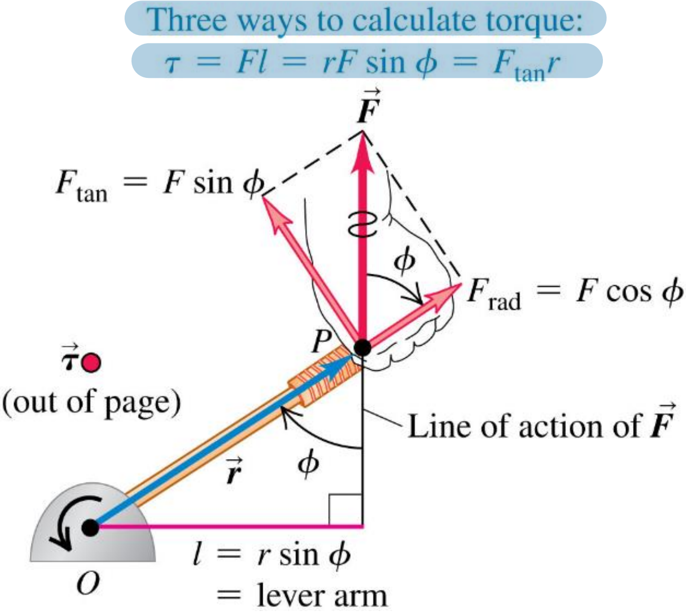
The **lever arm** for a force is the perpendicular distance from *O* to the line of action of the force.

The torque of a force is the product of the force and its lever arm.

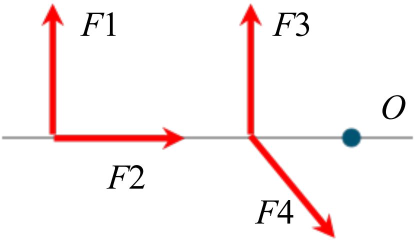
For rotation around A, torques are different.



Three ways to calculate torque



The four forces shown all have the same magnitude: $F1 = F2 = F3 = F4$. Which force produces the *greatest torque* about the point O (marked by the blue dot)?



A. $F1$

B. $F2$

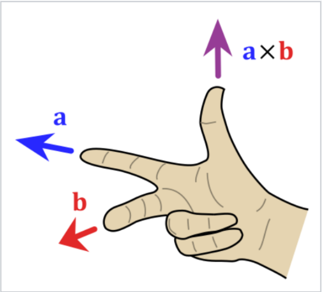
C. $F3$

D. $F4$

E. Not enough information is given to decide.

Torque as a vector

- Torque is $\tau =$
- Vector cross
- $\vec{a} \times \vec{b} = absi$



Finding the direction of the cross product by the right-hand rule.

Torque vector due to force $\vec{F}$ relative to point O

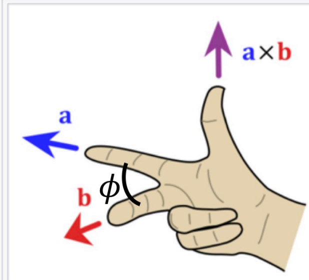
Vector from O to where $\vec{F}$ acts

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Force $\vec{F}$

Torque as a vector

- Torque is $\tau = rF\sin\phi$
- Vector cross product definition:
- $\vec{a}\times\vec{b} = ab\sin\phi\ \vec{n}$ with the direction of $\vec{n}$ determined by the right-hand rule
- Torque can be expressed as a vector using the vector product.

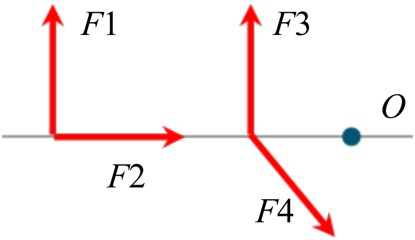


Finding the direction of the cross product by the right-hand rule.

Torque vector due to force $\vec{F}$ relative to point O → $\vec{\tau} = \vec{r}\times\vec{F}$ ← Vector from O to where $\vec{F}$ acts; Force $\vec{F}$

$$\vec{\tau} = \vec{r}\times\vec{F}$$

Which of the four forces shown here produces a torque about O that is directed *out of* the plane of the drawing?



A. $F1$

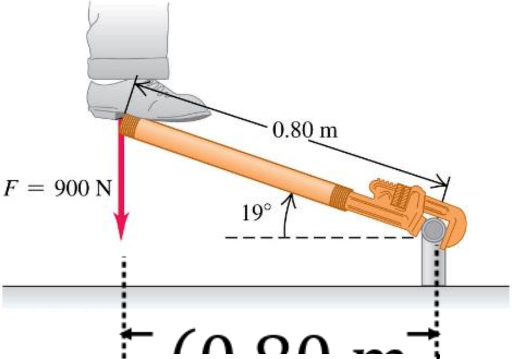
B. $F2$

C. $F3$

D. $F4$

E. more than one of these

A plumber pushes straight down on the end of a long wrench as shown. What is the magnitude of the torque he applies about the pipe at lower right?



(0.80 m) cos 19°

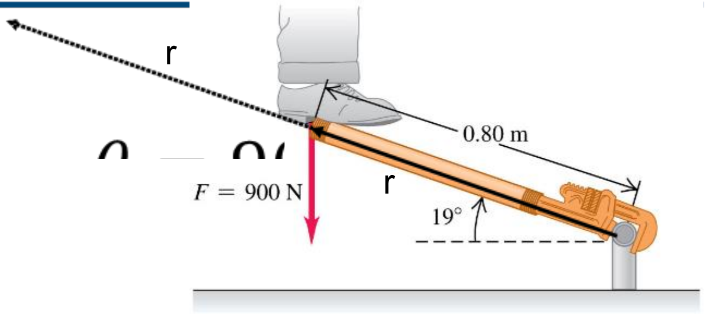
A. (0.80 m)(900 N)sin 19°

B. (0.80 m)(900 N)cos 19°

C. (0.80 m)(900 N)tan 19°

D. (0.80 m)(900 N)/(sin 19°)

E. none of the above



A. (0.80 m)(900 N)sin 19°

B. (0.80 m)(900 N)cos 19°

C. (0.80 m)(900 N)tan 19°

D. (0.80 m)(900 N)/(sin 19°)

E. none of the above

$\sin\theta = \sin(90^\circ -$

Conditions for equilibrium

- For an extended body to be in static equilibrium, two conditions must be satisfied.
- The first condition is that the vector sum of all external forces acting on the body must be zero:

First condition for equilibrium: For the center of mass of a body at rest to remain at rest ...

$$\sum \vec{F} = \mathbf{0}$$

... net external force on the body must be *zero*.

- The second condition is that the sum of external torques must be zero about any point:

Second condition for equilibrium: For a nonrotating body to remain nonrotating ...

$$\sum \vec{\tau} = \mathbf{0}$$

... net external torque around any point on the body must be *zero*.

Conditions for equilibrium: reference doesn't matter

First condition for equilibrium: For the center of mass of a body at rest to remain at rest ... $\sum \vec{F} = 0$ ... *net external force* on the body must be *zero*.

Second condition for equilibrium: For a nonrotating body to remain nonrotating ... $\sum \vec{\tau} = 0$ ... *net external torque around any point* on the body must be *zero*.

- Consider two reference point A and B.
- $\sum \overrightarrow{\tau_B} = \sum[(\vec{r} - \vec{r}_B) \times \vec{F}] =$

Conditions for equilibrium: reference doesn't matter

First condition for equilibrium: For the center of mass of a body at rest to remain at rest ... $\sum \vec{F} = 0$ ... *net external force* on the body must be *zero.*

Second condition for equilibrium: For a nonrotating body to remain nonrotating ... $\sum \vec{\tau} = 0$ ... *net external torque around any point* on the body must be *zero.*

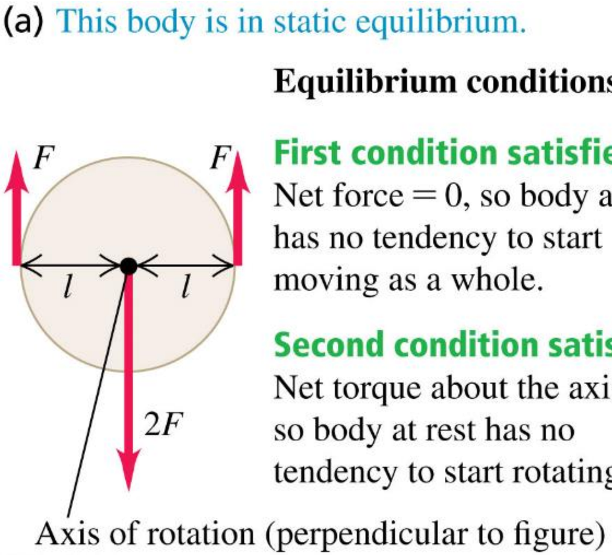
- Consider two reference points, A and B
- $\sum \overrightarrow{\tau_B} = \sum[(\vec{r} - \overrightarrow{r_B})\times\vec{F}] = \sum[(\vec{r} - \overrightarrow{r_A} + \overrightarrow{r_A} - \overrightarrow{r_B})\times\vec{F}]$

$$= \sum[(\vec{r} - \overrightarrow{r_A})\times\vec{F}] + \sum[(\overrightarrow{r_A} - \overrightarrow{r_B})\times\vec{F}]$$

$$= \sum\overrightarrow{\tau_A} + (\overrightarrow{r_A} - \overrightarrow{r_B})\times\sum\vec{F}$$

- If $\sum\vec{F} = 0$, then $\sum\overrightarrow{\tau_B} = \sum\overrightarrow{\tau_A}$

Conditions for equilibrium: Example 1



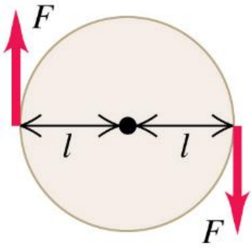
Equilibrium conditions:

First condition satisfied:
Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition satisfied:
Net torque about the axis = 0, so body at rest has no tendency to start rotating.

Conditions for equilibrium: Example 2

(b) This body has no tendency to accelerate as a whole, but it has a tendency to start rotating.

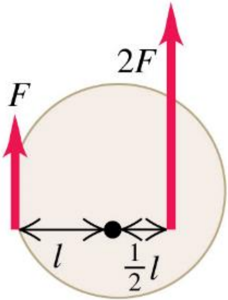


First condition satisfied: Net force = 0, so body at rest has no tendency to start moving as a whole.

Second condition NOT satisfied: There is a net clockwise torque about the axis, so body at rest will start rotating clockwise.

Conditions for equilibrium: Example 3

(c) This body has a tendency to accelerate as a whole but no tendency to start rotating.



First condition NOT satisfied: There is a net upward force, so body at rest will start moving upward.

Second condition satisfied: Net torque about the axis $= 0$, so body at rest has no tendency to start rotating.

Which of the following situations satisfies *both* the first condition for equilibrium (net force = 0) and the second condition for equilibrium (net torque = 0)?

A. an automobile crankshaft turning at an increasing angular speed in the engine of a parked car

B. a seagull gliding at a constant angle below the horizontal and at a constant speed

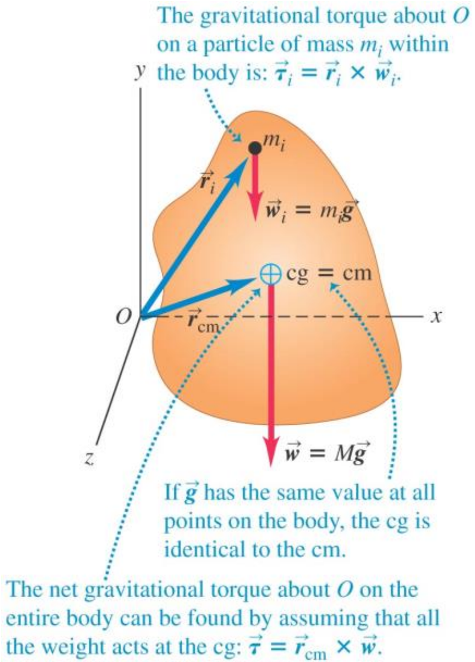
C. a thrown baseball that does not rotate as it sails through the air

D. more than one of the above

E. none of the above

Center of gravity

- We can treat a body's weight as though it all acts at a single point — the *center of gravity*.
- If we can ignore the variation of gravity with altitude, the center of gravity is the same as the center of mass.



Centre of Gravity vs. Centre of Mass

Centre of Mass is given by

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} = \frac{\sum_i m_i\vec{r}_i}{\sum_i m_i}$$

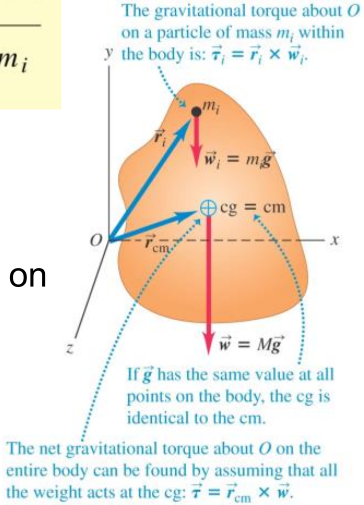
Torque of the weight of a particle

$$\vec{\tau}_i = \vec{r}_i \times m_i\vec{g}$$

Total torque due to gravitational force on all the particles (if *g* is constant) is

$$\vec{\tau} = \sum \vec{\tau}_i = \sum\left(\vec{r}_i \times m_i\vec{g}\right) = \left(\sum m_i\vec{r}_i\right)\times\vec{g}$$

$$= \frac{\left(\sum m_i\vec{r}_i\right)}{\sum m_i}\times\left(\sum m_i\right)\vec{g} = \vec{r}_{cm}\times M\vec{g}$$



Center of gravity

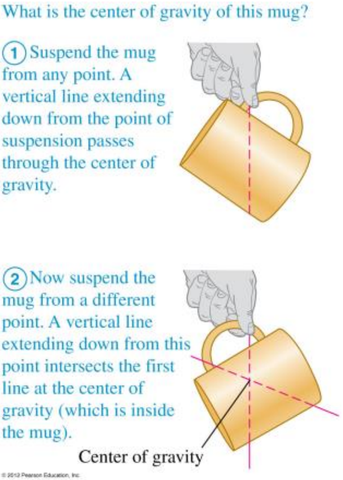
The acceleration due to gravity at the bottom of the 452-m-tall Petronas Towers in Malaysia is only 0.014% greater than at the top.

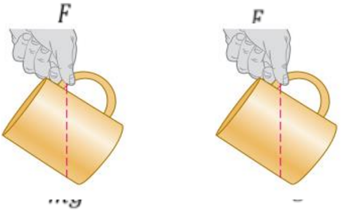
The center of gravity of the towers is only about 2 cm below the center of mass.

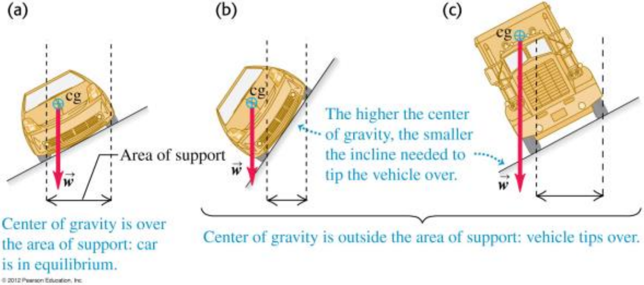
Finding Centre of Gravity

The center of gravity is always at or directly above or below the point of suspension.

Otherwise, net torque is nonzero.

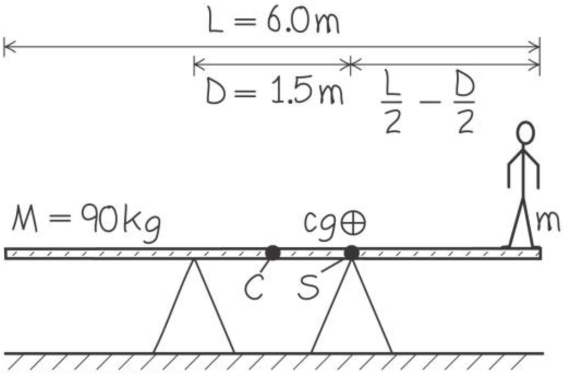




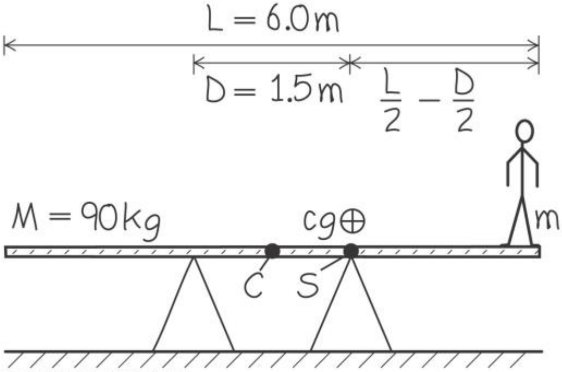


Walking the plank

A uniform plank of length $L = 6.0$ m and mass $M = 90$ kg rests on sawhorses separated by $D = 1.5$ m and equidistant from the center of the plank. Cousin Throckmorton wants to stand on the right-hand end of the plank. If the plank is to remain at rest, how massive can Throckmorton be?



Walking the plank



By taking moment about S

$$m\left(\frac{L}{2}-\frac{D}{2}\right)\leq M\left(\frac{D}{2}\right)$$

$$m\leq M\left(\frac{D}{L-D}\right)=30\,kg$$

Let point C be the origin of coordinate

$$x_{cg}=\frac{M(0)+m(L/2)}{M+m}=\frac{mL}{2(M+m)}$$

$$if\ \ x_{cg}=x_S$$

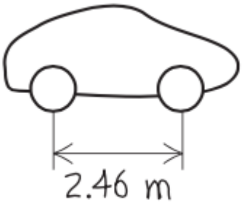
$$\frac{mL}{2(M+m)}=\frac{D}{2}$$

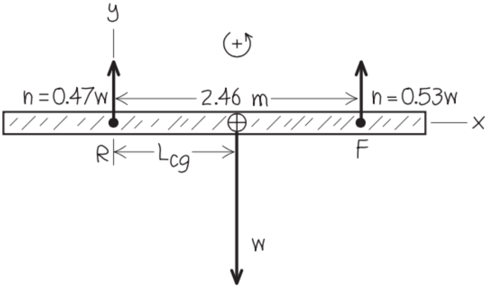
$$m=30\,kg$$

Solving rigid-body equilibrium problems

Example 11.2 Weight distribution for a car

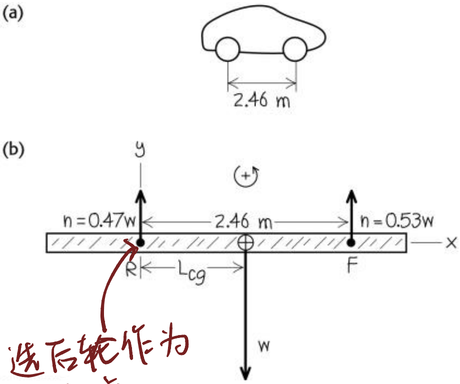
An auto magazine reports that a certain sports car has 53% of its weight on the front wheels and 47% on its rear wheels. (That is, the total normal forces on the front and rear wheels are $0.53w$ and $0.47w$, respectively, where w is the car's weight.) The distance between the axles is 2.46 m. How far in front of the rear axle is the car's center of gravity?





Solving rigid-body equilibrium problems

- How far in front of the rear axle is the car's centre of gravity?
- For static equilibrium, $\Sigma F = 0$, $\Sigma\tau = 0$



选后轮作为参考点

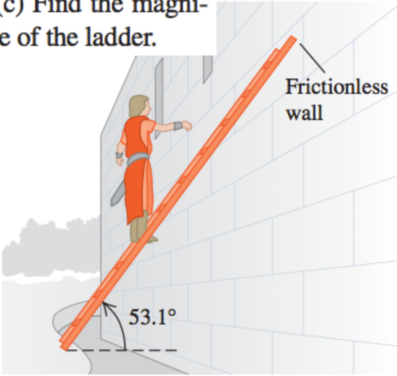
$$\Sigma F = 0.47w + 0.53w - w = 0$$

$$\Sigma\tau_{rear} = wL_{cg} - 2.46(0.53w) = 0$$

$$L_{cg} = 1.3\text{m}$$

Will the ladder slip?

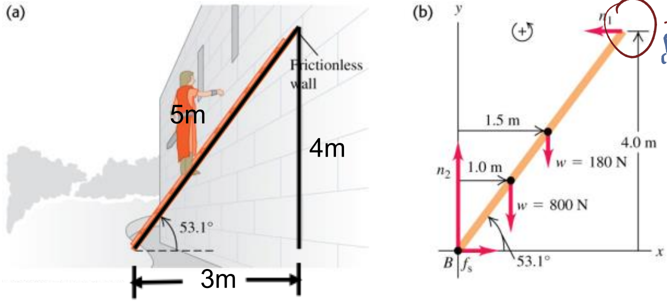
Sir Lancelot, who weighs 800 N, is assaulting a castle by climbing a uniform ladder that is 5.0 m long and weighs 180 N (Fig. 11.9a). The bottom of the ladder rests on a ledge and leans across the moat in equilibrium against a frictionless, vertical castle wall. The ladder makes an angle of 53.1° with the horizontal. Lancelot pauses one-third of the way up the ladder. (a) Find the normal and friction forces on the base of the ladder. (b) Find the minimum coefficient of static friction needed to prevent slipping at the base. (c) Find the magnitude and direction of the contact force on the base of the ladder.



Will the ladder slip?

- (a) Find the normal and friction forces on the base of the ladder. (b) Find the minimum coefficient of static friction needed to prevent slipping at the base. (c) Find the magnitude and direction of the contact force on the base of the ladder.

墙壁光滑无f.



思路：

静止 → $\Sigma F_y = 0.$

$\Sigma F_x = 0.$

找点 → $\Sigma \tau = 0.$

$\Sigma F_y = n2 + (-800) + (-180) = 0$

<u>n2 = 980 N</u>

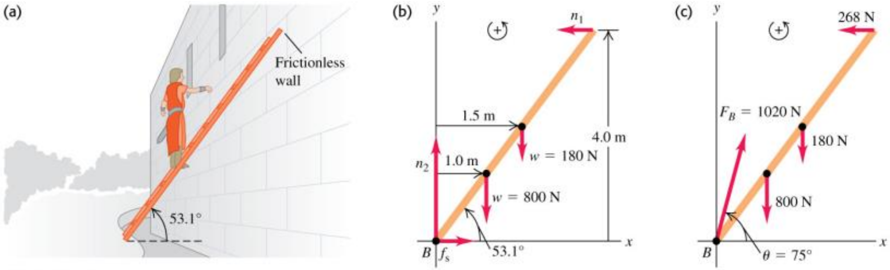
$\Sigma F_x = fs - n1 = 0$

But both fs and n1 are unknown

About point B, $\Sigma\tau = -800(1.0) - 180(1.5) + n1(4.0) = 0$

<u>n1 = 268 N = fs</u>

Will the ladder slip?



$$f_s \le \mu n_2$$

$$\mu \ge f_s / n_2 = 0.273$$

contact force on the base of the ladder, F_B,

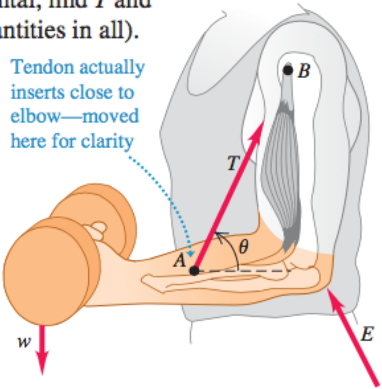
$$\vec{F}_B = \vec{n}_2 + \vec{f}_s$$

$$\left|\vec{F}_B\right| = \sqrt{268^2 + 980^2} = 1020\,N$$

$$\theta = \tan^{-1}\frac{980}{268} = 74.7°$$

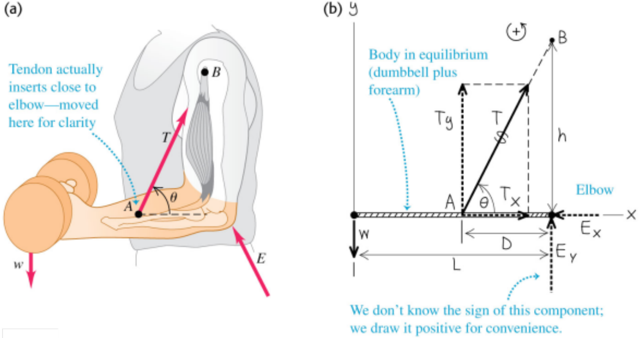
Equilibrium and pumping iron

Figure 11.10a shows a horizontal human arm lifting a dumbbell. The forearm is in equilibrium under the action of the weight $\vec{w}$ of the dumbbell, the tension $\vec{T}$ in the tendon connected to the biceps muscle, and the force $\vec{E}$ exerted on the forearm by the upper arm at the elbow joint. We neglect the weight of the forearm itself. (For clarity, the point A where the tendon is attached is drawn farther from the elbow than its actual position.) Given the weight w and the angle θ between the tension force and the horizontal, find T and the two components of $\vec{E}$ (three unknown scalar quantities in all).



Equilibrium and pumping iron

- Follow Example 11.4. Find T and E in terms of other variables.



$$\sum \tau_{elbow} = 0 = wL - T_y D$$

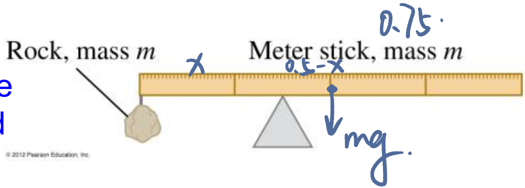
$$T_y = \frac{wL}{D}$$

$$T = \frac{wL}{D\sin\theta}$$

ΣFx = 0 = Tx – Ex
Ex = Tcosθ
= wL/Dtanθ
=wL/h

ΣFy = 0 = Ty + Ey – w
Ey = w – (wL/D)
= w(D-L)/D
[it is negative!]

A rock is attached to the left end of a uniform meter stick that has the same mass as the rock. How far from the left end of the stick should the triangular object be placed so that the combination of meter stick and rock is in balance?



$$mgx = mg(0.5-x)$$

$$2x = 0.5$$

$$x = 0.25$$

A. less than 0.25 m

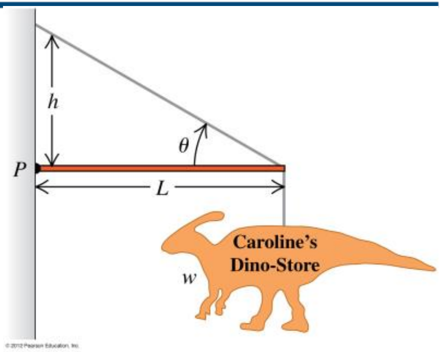
B. 0.25 m

C. between 0.25 m and 0.50 m

D. 0.50 m

E. more than 0.50 m

A metal advertising sign (weight w) is suspended from the end of a massless rod of length L. The rod is supported at one end by a hinge at point P and at the other end by a cable at an angle q from the horizontal.



What is the tension in the cable?

A. $T = w \sin q$

B. $T = w \cos q$

C. $T = w/(\sin q)$

D. $T = w/(\cos q)$

E. none of the above