MA1300 Self Practice # 2

1. (P70, #15) Evaluate the limit $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$

2. (P70, #19) Evaluate the limit $\lim_{x\to -2} \dfrac{x+2}{x^3+8}$

3. (P70, #26) Evaluate the limit $\lim_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+t}\right)$

4. (P70, #36) Use the Squeeze Theorem to show that

$$\lim_{x\to 0} \sqrt{x^3+x^2}\sin\frac{\pi}{x} = 0.$$

5. (P70, #38) If $2x \le g(x) \le x^4 - x^2 + 2$ for all x, evaluate $\lim_{x\to 1} g(x)$.

6. (P70, #40) Prove that $\lim_{x\to 0^+} \sqrt{x}[1+\sin^2(2\pi/x)] = 0$.

For Questions $7 \sim 9$*, find the limit, if it exists. If the limit does not exist, explain why.*

7. (P70, #41) $\lim_{x\to 3}(2x + |x-3|)$

8. (P70, #42) $\lim_{x\to -6} \dfrac{2x+12}{|x+6|}$

9. (P70, #46) $\lim_{x\to 0^-} \left(\dfrac{1}{x} - \dfrac{1}{|x|}\right)$

10. (P70, #47) The *signum* (or sign) *function*, denoted by sgn, is defined by

$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

(a) Sketch the graph of this function.

(b) Find each of the following limits or explain why it does not exist.

(i) $\lim_{x\to 0^+} \operatorname{sgn} x$ (ii) $\lim_{x\to 0^-} \operatorname{sgn} x$ (iii) $\lim_{x\to 0} \operatorname{sgn} x$ (iv) $\lim_{x\to 0} |\operatorname{sgn} x|$

11. (P71, #54) In the theory of relativity, the Lorentz contraction formula

$$L = L_0\sqrt{1 - v^2/c^2}$$

expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest and c is the speed of light. Find $\lim_{v\to c^-} L$ and interpret the result. Why is a left-hand limit necessary?

12. (P71, #59) If

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

prove that $\lim_{x \to 0} f(x) = 0$.

13. (P71, #62) Evaluate $\lim_{x \to 2} \dfrac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1}$.

14. (P81, #13)

(a) Find a number δ such that if $|x - 2| \leq \delta$, then $|4x - 8| < \varepsilon$, where $\varepsilon = 0.1$.

(b) Repeat part (a) with $\varepsilon = 0.01$.

15. (P81, #39) If the function f is defined by

$$f(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ 1 & \text{if } x \text{ is irrational} \end{cases}$$

prove that $\lim_{x \to 0} f(x)$ does not exist.

16. (P81, #41) How close to -3 do we have to take x so that

$$\frac{1}{(x+3)^4} > 10,000$$

17. (P81, #42, Optional) Prove that $\lim_{x \to -3} \dfrac{1}{(x+3)^4} = \infty$.