Chapter 5 Part B

Dynamics of Rotational Motion

Outline for Part B

- What causes rotation?
- Definition of torque
- Work done and power in rotation motion
- Angular momentum, relation to torque, conservation of angular momentum

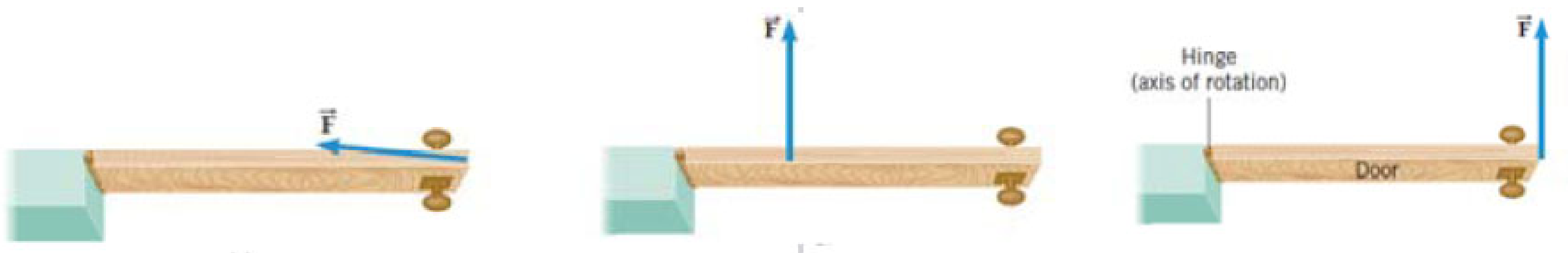
Introduction

- What causes the merry-go-round or the ice-skater to start or stop spinning?
- Introduce torque and angular momentum, to explain the cause of rotational motion and some rotation phenomena.

Torque

- Force cause change of motion along a straight line
- What causes change of rotation motion of a rigid body?

- Just force is not enough!



- Need a bit more!

- Needs forces applied at a distance from the axis of rotation

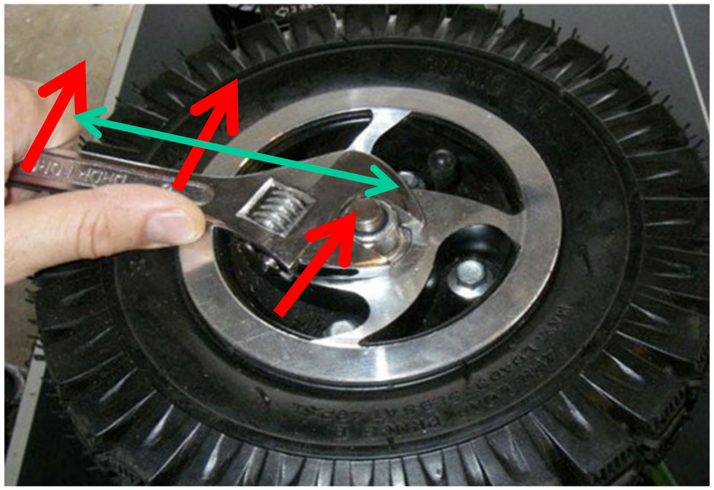
→ Answer: torque = Force × Arm (N·m)

Rotation effect of a force

- Which of the three equal-magnitude forces in the figure is most likely to rotate the bolt?
- The effectiveness of the three forces depends on the distance of the force from the rotation axis

The larger the distance, the stronger effect on rotation

Rotation effect is proportional to force and proportional to distance from axis of rotation

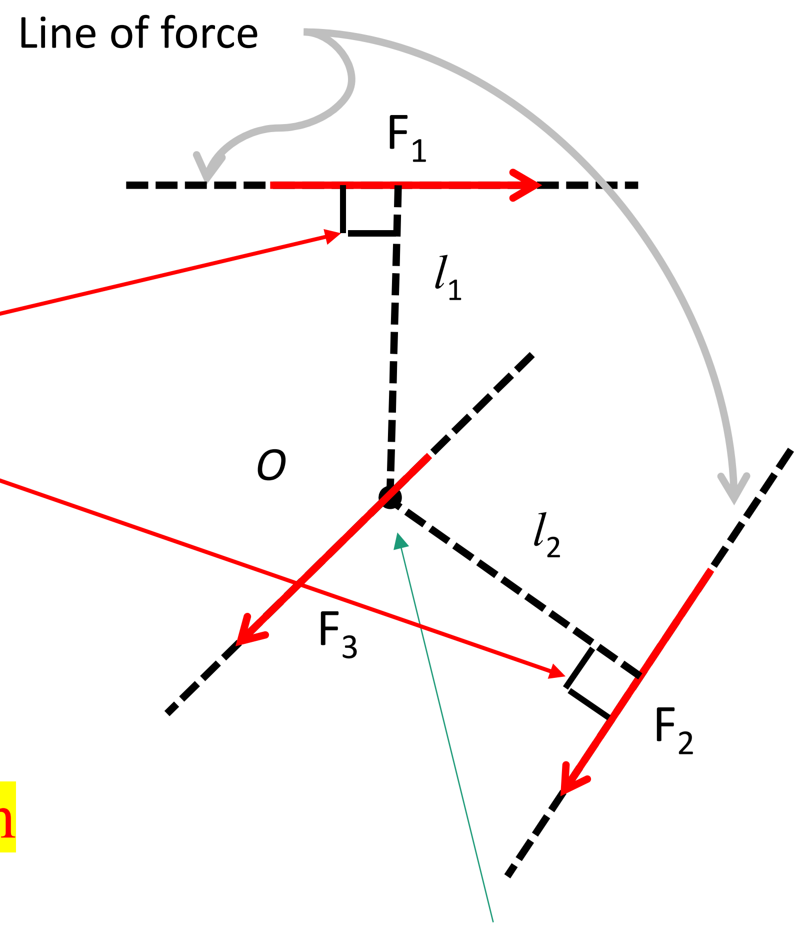


Definition of Torque

- The *line of action* of a force is the line along which the force vector lies.
- The *lever arm* (or *moment arm*) for a force is the perpendicular distance from *O (the axis of rotation)* to the line of action of the force (see l_1, l_2 in figure).
- The torque of a force with respect to *O* is the product of the force and its lever arm: torque = Force × lever arm

The three torques are:

$$\tau_1 = F_1 l_1 \,,\ \tau_2 = F_2 l_2 \qquad \tau_3 = F_3 \times 0 = 0$$



O is the axis of rotation

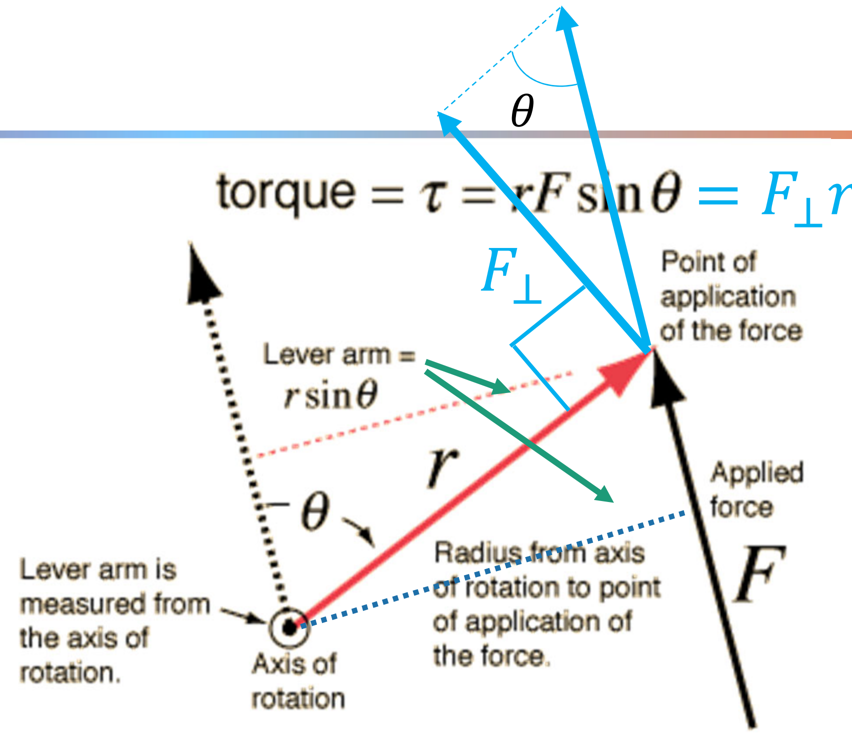
Torque is a vector

- Using axis of rotation as origin, we draw the position vector r for the point of action.

$\rightarrow \ \tau = Fl = Fr\sin\theta = F_{\perp}r$

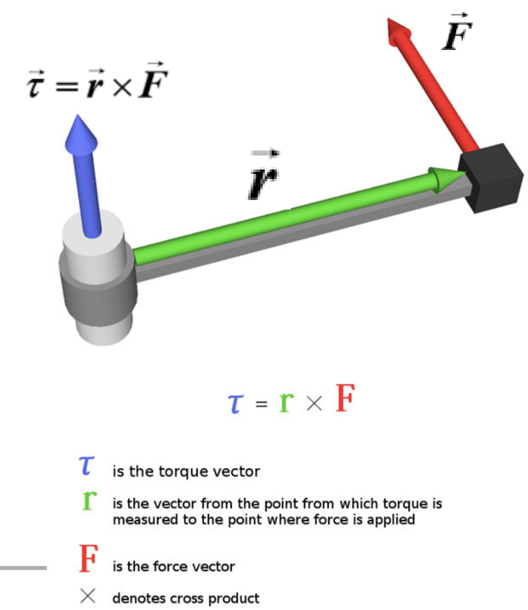
$= |\vec{r} \times \vec{F}|$

= magnitude of torque



$\rightarrow$ Torque can be expressed as a vector using the vector product:

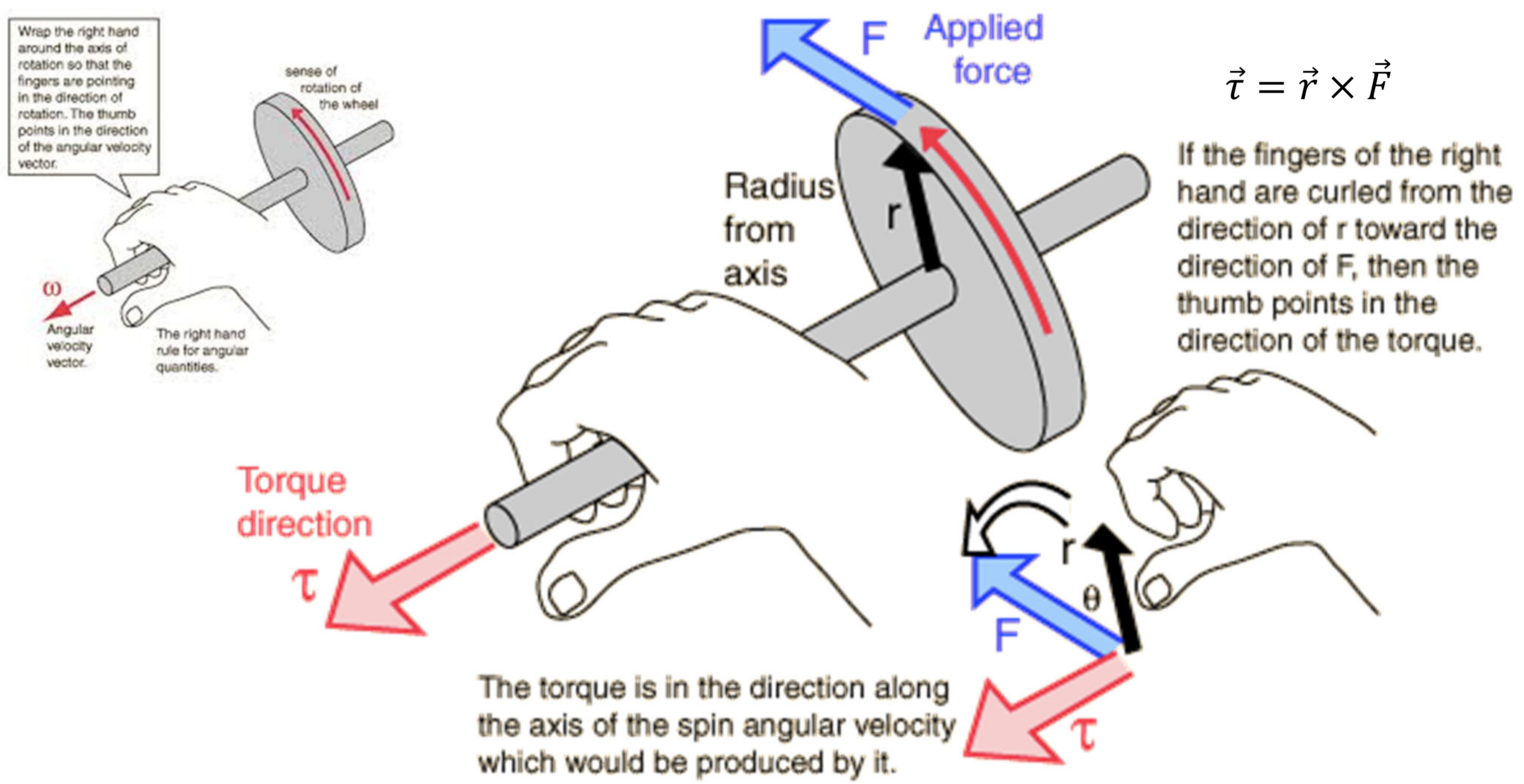
$\vec{\tau} = \vec{r} \times \vec{F}$ (definition of torque vector)



Torque is a vector along the axis of rotation and is perpendicular to $\vec{r}$ and $\vec{F}$,

Find the direction of torque

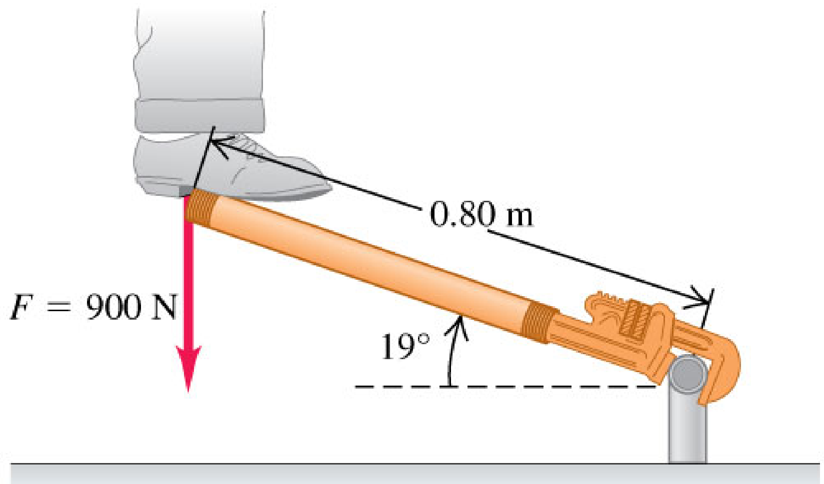
- The direction of the torque can be found using the right hand rule
- It is similar to the angular velocity $\vec{\omega}$



Example 10.1 Applying a torque

To loosen a pipe fitting, a weekend plumber slips a piece of scrap pipe (a "cheater") over his wrench handle. He stands on the end of the cheater, applying his full 900-N weight at a point 0.80 m from the center of the fitting (Fig. 10.5a). The wrench handle and cheater make an angle of 19° with the horizontal. Find the magnitude and direction of the torque he applies about the center of the fitting.

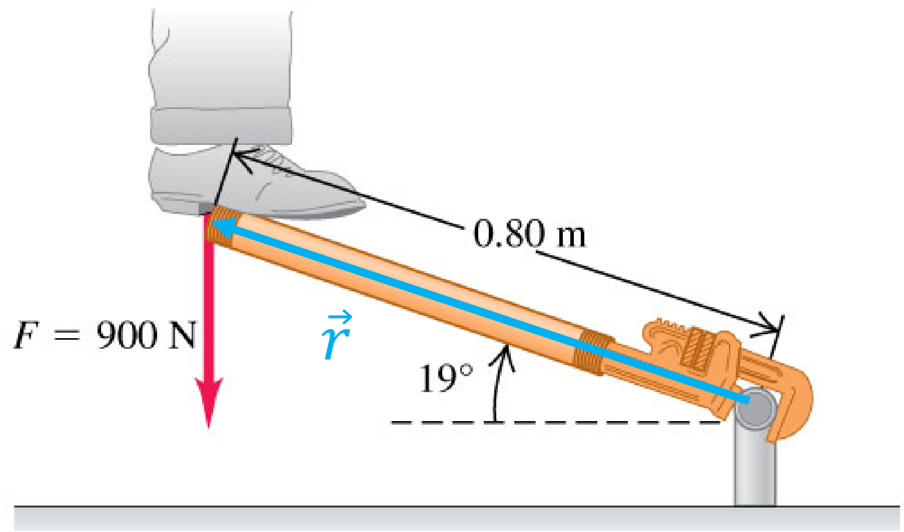
(a) Diagram of situation



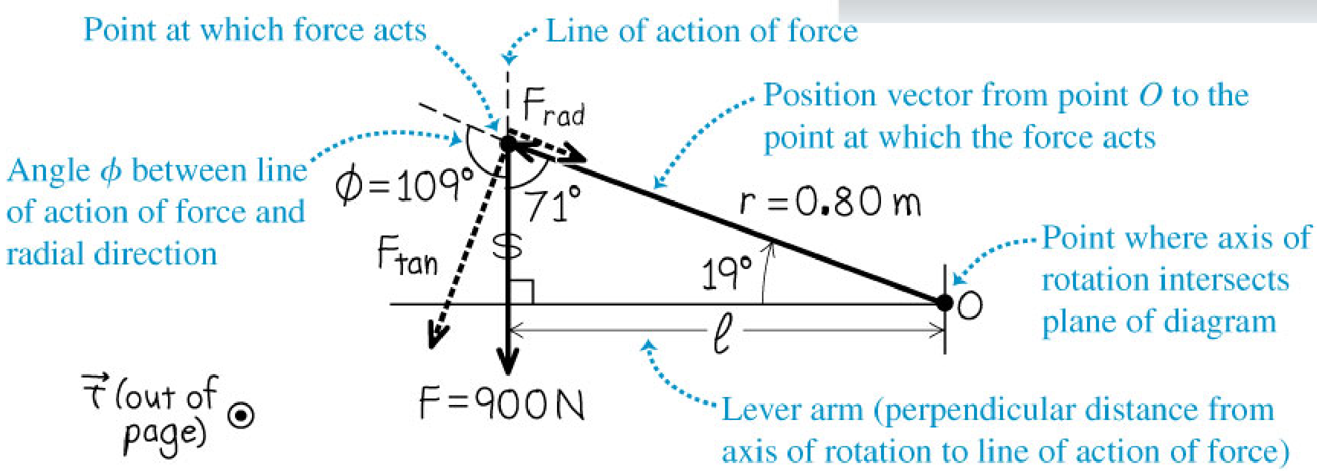
Applying a torque

- Follow Example 10.1 using Figure 10.5.

(a) Diagram of situation



(b) Free-body diagram



SOLUTION

IDENTIFY and SET UP: Figure 10.5b shows the vectors $\vec{r}$ and $\vec{F}$ and the angle between them ($\phi = 109°$). Equation (10.1) or (10.2) will tell us the magnitude of the torque. The right-hand rule with Eq. (10.3), $\vec{\tau} = \vec{r} \times \vec{F}$, will tell us the direction of the torque.

EXECUTE: To use Eq. (10.1), we first calculate the lever arm l. As Fig. 10.5b shows,

$$\boxed{l = r\sin\phi = (0.80\ \text{m})\sin 109° = (0.80\ \text{m})\sin 71° = 0.76\ \text{m}}$$

Then Eq. (10.1) tells us that the magnitude of the torque is

$$\boxed{\tau = Fl = (900\ \text{N})(0.76\ \text{m}) = 680\ \text{N}\cdot\text{m}}$$

We get the same result from Eq. (10.2):

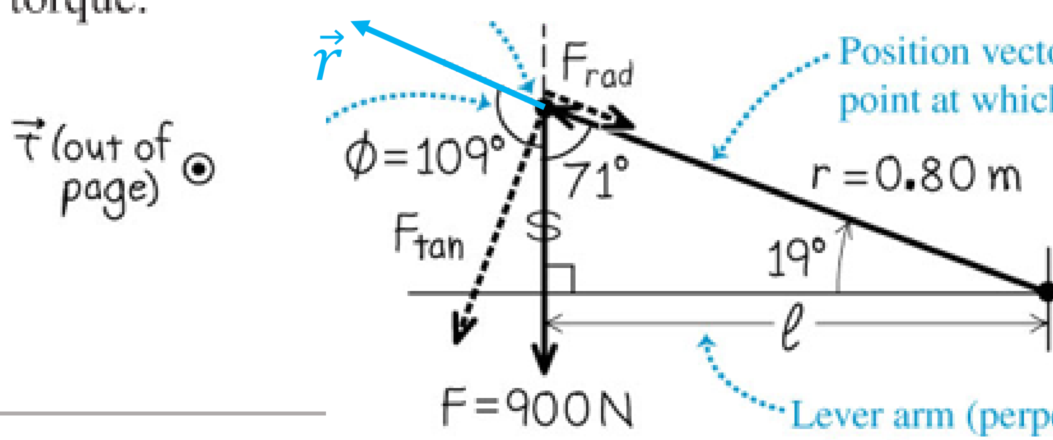
$$\boxed{\tau = rF\sin\phi = (0.80\ \text{m})(900\ \text{N})(\sin 109°) = 680\ \text{N}\cdot\text{m}}$$

Alternatively, we can find $F_{\tan}$, the tangential component of $\vec{F}$ that acts perpendicular to $\vec{r}$. Figure 10.5b shows that this component is at an angle of $109° - 90° = 19°$ from $\vec{F}$, so $F_{\tan} = F \sin\phi = F(\cos 19°) = (900\text{ N})(\cos 19°) = 851\text{ N}$. Then, from Eq. 10.2,

$$\tau = F_{\tan}r = (851\text{ N})(0.80\text{ m}) = 680\text{ N}\cdot\text{m}$$

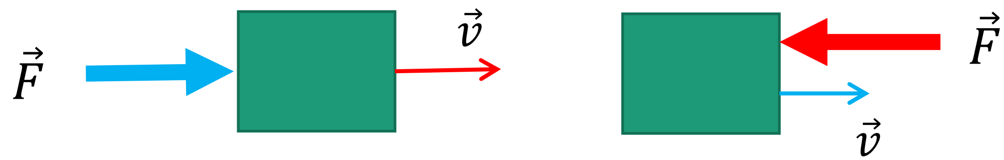
Curl the fingers of your right hand from the direction of $\vec{r}$ (in the plane of Fig. 10.5b, to the left and up) into the direction of $\vec{F}$ (straight down). Then your right thumb points out of the plane of the figure: This is the direction of $\vec{\tau}$.

EVALUATE: To check the direction of $\vec{\tau}$, note that the force in Fig. 10.5 tends to produce a counterclockwise rotation about O. If you curl the fingers of your right hand in a counterclockwise direction, the thumb points out of the plane of Fig. 10.5, which is indeed the direction of the torque.

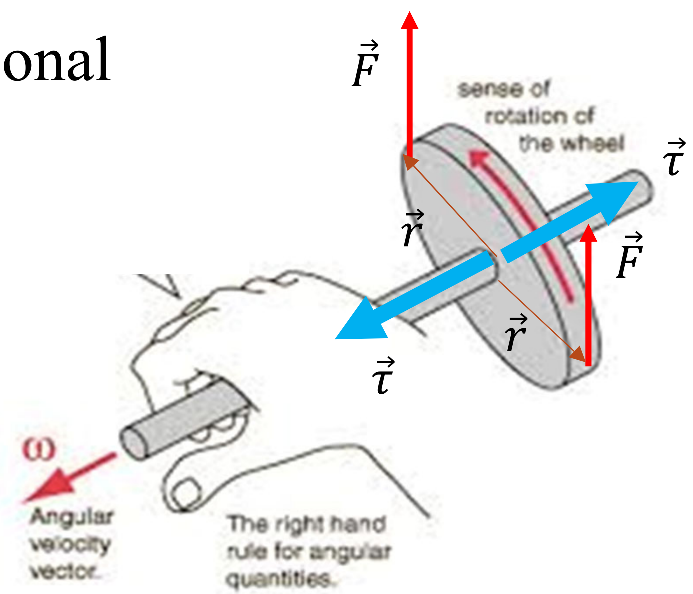


Effect of Torque

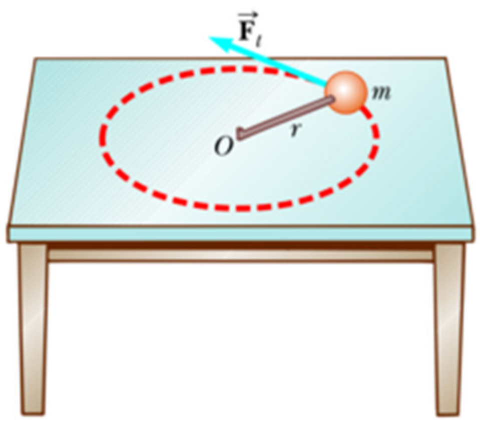
- When $\vec{F}$ is in the same direction as $\vec{v}$, v increases, but it decreases if they are opposite.



- Same thing happens to torque.
- Torque produces acceleration in rotational motion



- The ball will rotate faster under a torque



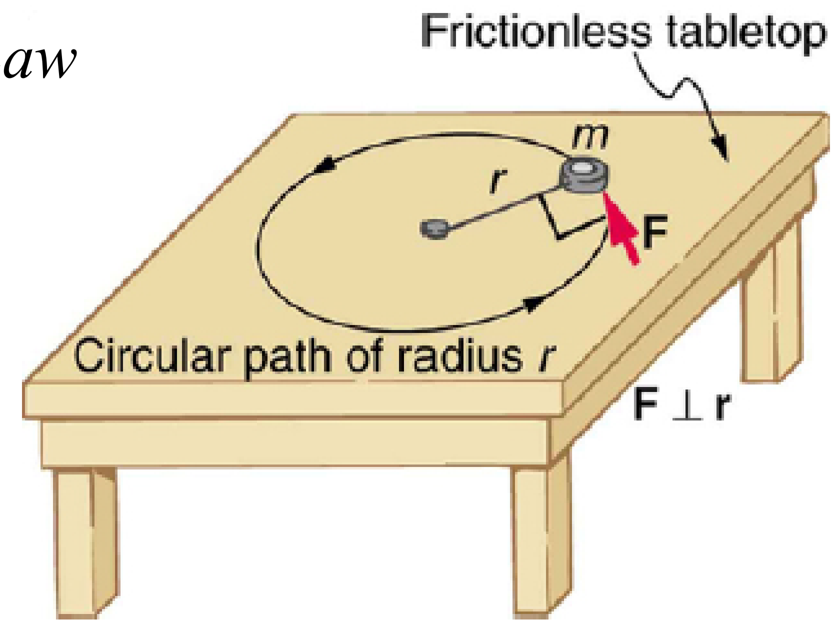
Newton's law for rotation motion

Consider a mass in circular motion with a radius r and it is accelerated by a tangential force F

$\boxed{F = ma_t,}$ *according to Newton's Law*

(a_t is tangential to the circle)

$$a_t = r\alpha$$
$$F = mr\alpha$$
$$rF = mr^2\alpha$$
$$\boxed{\tau = I\,\alpha}$$



Torque equals moment of inertia I times angular acceleration α

This is Newton's law for rotation motion

Many forces (torques)

Torques are like forces, we can add them together. We sum all the torques on all the particles in a rigid body as follows.

$$\tau_{1z} + \tau_{2z} + \cdots = I_1\alpha_z + I_2\alpha_z + \cdots = m_1 r_1^2 \alpha_z + m_2 r_2^2 \alpha_z + \cdots$$

$$\sum \tau_{iz} = \left(\sum m_i r_i^2 \right) \alpha_z$$

$$I = \sum m_i r_i^2,$$

$$\sum \tau_z = I\alpha_z$$

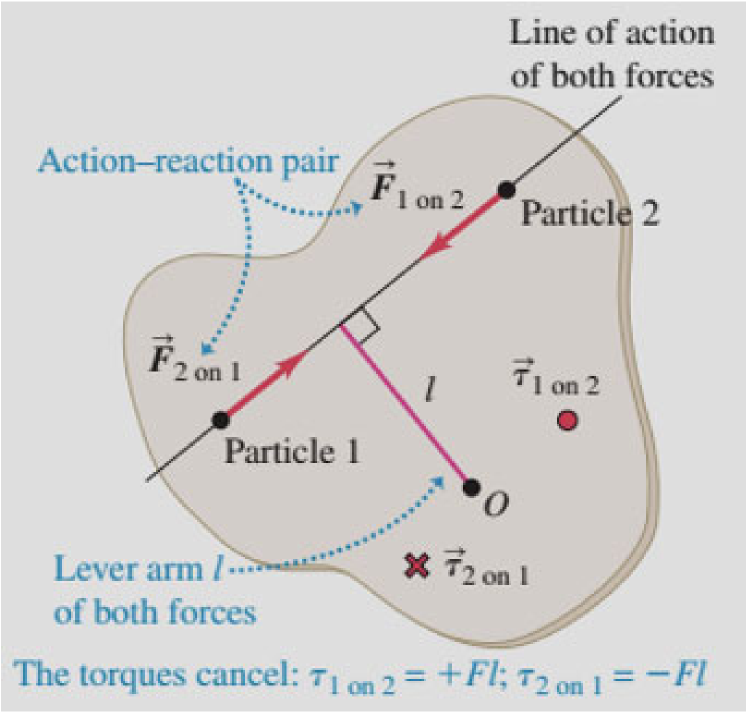
The torque and the angular acceleration is along the z direction so we add a subscript z

Sum of all the torque applied to an rigid object equals moment of inertia of the body times angular acceleration

Cancelation of internal forces (torques)

Internal forces between any two particles are a pair of action-reaction forces:

$$\vec{F}_{2\ on\ 1} = -\vec{F}_{1\ on\ 2}$$



They produce internal torques of equal magnitude but opposite directions:

$$\vec{\tau}_{2\ on\ 1} = -\vec{\tau}_{1\ on\ 2}$$

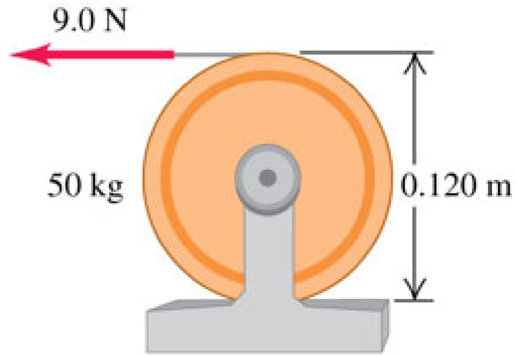
They will cancel each other in the total sum of torques: $\sum \tau_z = I\alpha_z$

$$\boxed{\sum \tau_{z(external)} = I\alpha_z}$$

Sum of all external torques applied to an rigid object equals its moment of inertia times angular acceleration

Torque and angular acceleration for a rigid body

- The rotational analog of Newton's second law for a rigid body is $\Sigma\tau_{z(external)} = I\alpha_z$.



Example 10.2 An unwinding cable I

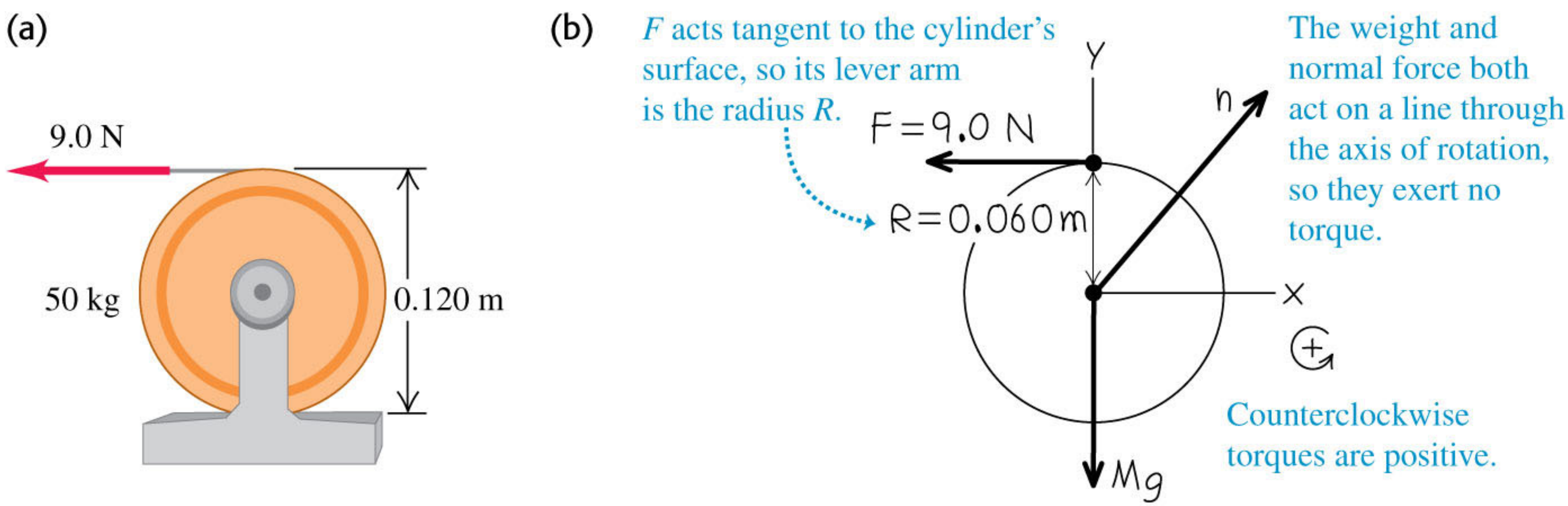
Figure 10.9a shows the situation analyzed in Example 9.7 using energy methods. What is the cable's acceleration?

Example 9.7 An unwinding cable I

We wrap a light, nonstretching cable around a solid cylinder of mass 50 kg and diameter 0.120 m, which rotates in frictionless bearings about a stationary horizontal axis (Fig. 9.16). We pull the free end of the cable with a constant 9.0-N force for a distance of 2.0 m; it turns the cylinder as it unwinds without slipping. The cylinder is initially at rest. Find its final angular speed and the final speed of the cable.

Example 10.2 An unwinding cable I

Figure 10.9a shows the situation analyzed in Example 9.7 using energy methods. What is the cable's acceleration?



SOLUTION

IDENTIFY and SET UP: We can't use the energy method of Section 9.4, which doesn't involve acceleration. Instead we'll apply rotational dynamics to find the angular acceleration of the cylinder (Fig. 10.9b). We'll then find a relationship between the motion of the cable and the motion of the cylinder rim, and use this to find the acceleration of the cable. The cylinder rotates counterclockwise when the cable is pulled, so we take counterclockwise rotation to be positive. The net force on the cylinder must be zero because its center of mass remains at rest. The force F exerted by the cable produces a torque about the rotation axis. The weight (magnitude Mg) and the normal force (magnitude n) exerted by the cylinder's bearings produce *no* torque about the rotation axis because they both act along lines through that axis.

EXECUTE: The lever arm of F is equal to the radius $R = 0.060$ m of the cylinder, so the torque is $\boxed{\tau_z = FR.}$ (This torque is positive, as it tends to cause a counterclockwise rotation.) From Table 9.2, case (f), the moment of inertia of the cylinder about the rotation axis is $\boxed{I = \frac{1}{2}MR^2.}$ Then Eq. (10.7) tells us that

$$\alpha_z = \frac{\tau_z}{I} = \frac{FR}{MR^2/2} = \frac{2F}{MR} = \frac{2(9.0\ \text{N})}{(50\ \text{kg})(0.060\ \text{m})} = 6.0\ \text{rad/s}^2$$

(We can add "rad" to our result because radians are dimensionless.)

To get the linear acceleration of the cable, recall from Section 9.3 that the acceleration of a cable unwinding from a cylinder is the same as the tangential acceleration of a point on the surface of the cylinder where the cable is tangent to it. This tangential acceleration is given by Eq. (9.14):

$$a_{\tan} = R\alpha_z = (0.060\ \text{m})(6.0\ \text{rad/s}^2) = 0.36\ \text{m/s}^2$$

EVALUATE: Can you use this result, together with an equation from Chapter 2, to determine the speed of the cable after it has been pulled 2.0 m? Does your result agree with that of Example 9.7?

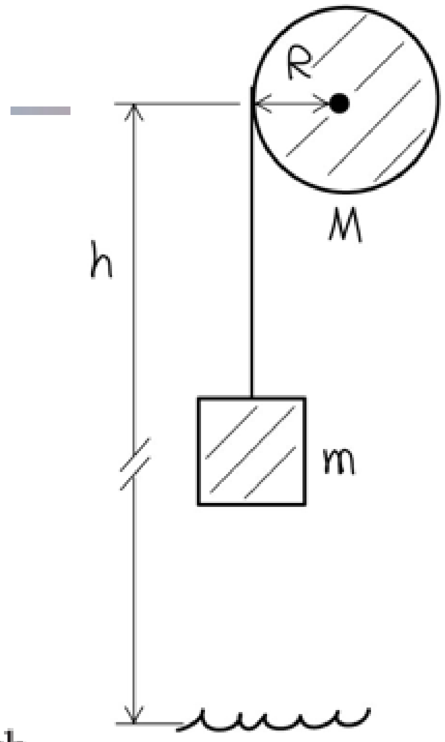
$$2a_x \Delta x = v_x^2 - v_{x0}^2$$

$$v_x = \sqrt{2a_t \Delta x} = \sqrt{2 \times 0.36 \times 2.0} = 1.2\mathrm{m/s}$$

$\omega = v_x/\mathrm{R} = 1.2/0.060$ = 20 rad/s

Example 10.3 An unwinding cable II

In Example 9.8 (Section 9.4), what are the acceleration of the falling block and the tension in the cable?

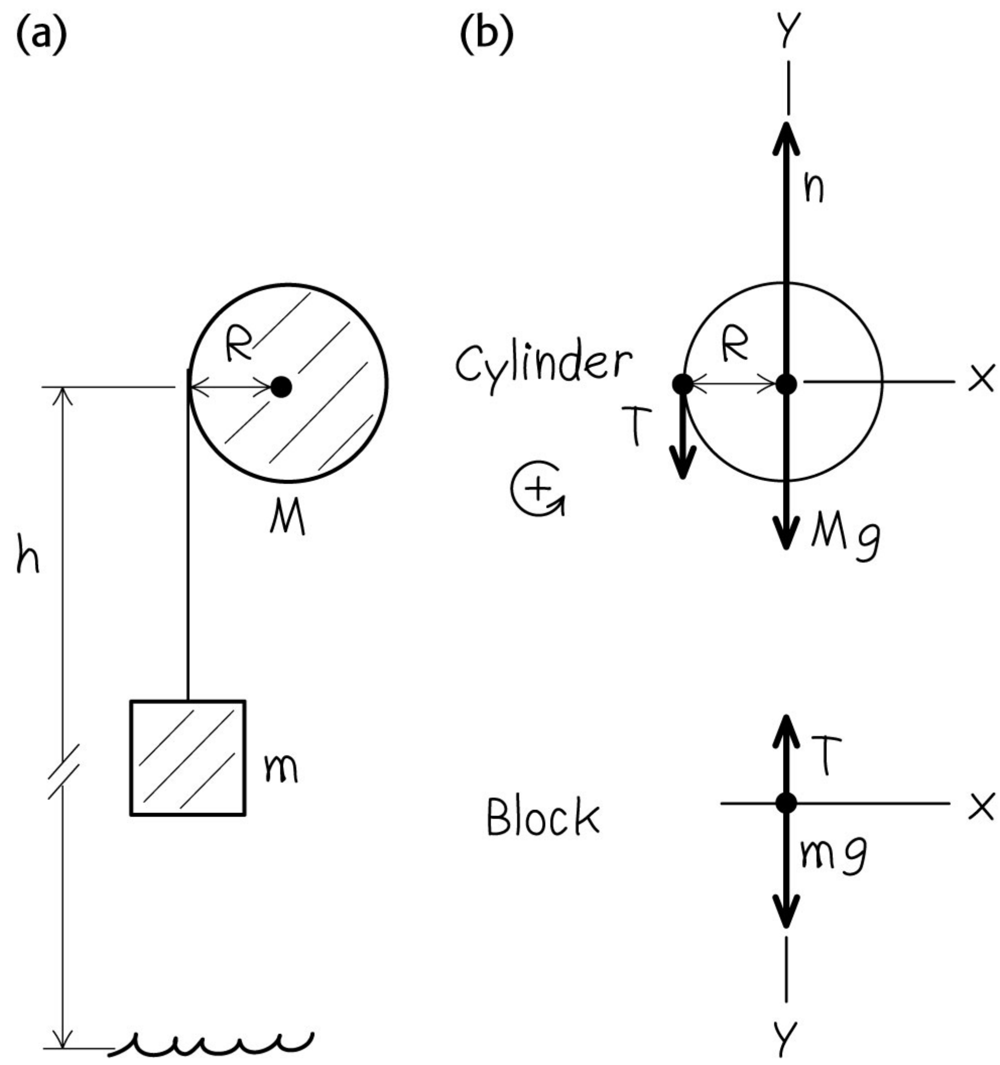


Example 9.8 An unwinding cable II

We wrap a light, nonstretching cable around a solid cylinder with mass M and radius R. The cylinder rotates with negligible friction about a stationary horizontal axis. We tie the free end of the cable to a block of mass m and release the block from rest at a distance h above the floor. As the block falls, the cable unwinds without stretching or slipping. Find expressions for the speed of the falling block and the angular speed of the cylinder as the block strikes the floor.

Another unwinding cable example

- We analyze the block and cylinder from Example 9.8 using torque.
- Follow Example 10.3 using Figure 10.10.



SOLUTION

IDENTIFY and SET UP: We'll apply translational dynamics to the block and rotational dynamics to the cylinder. As in Example 10.2, we'll relate the linear acceleration of the block (our target variable) to the angular acceleration of the cylinder. Figure 10.10 shows our sketch of the situation and a free-body diagram for each body. We take the positive sense of rotation for the cylinder to be counterclockwise and the positive direction of the y-coordinate for the block to be downward.

EXECUTE: For the block, Newton's second law gives

$$\sum F_y = mg + (-T) = ma_y$$

For the cylinder, the only torque about its axis is that due to the cable tension T. Hence Eq. (10.7) gives

$$\sum \tau_z = RT = I\alpha_z = \tfrac{1}{2}MR^2\alpha_z$$

As in Example 10.2, the acceleration of the cable is the same as the tangential acceleration of a point on the cylinder rim. From Eq. (9.14), this acceleration is $\boxed{a_y = a_{\tan} = R\alpha_z.}$ We use this to replace $R\alpha_z$ with a_y in the cylinder equation above, and then divide by R. The result is $T = \frac{1}{2}Ma_y$. Now we substitute this expression for T into Newton's second law for the block and solve for the acceleration a_y:

$$mg - \tfrac{1}{2}Ma_y = ma_y$$

$$a_y = \frac{g}{1 + M/2m}$$

To find the cable tension T, we substitute our expression for a_y into the block equation:

$$T = mg - ma_y = mg - m\left(\frac{g}{1 + M/2m}\right) = \frac{mg}{1 + 2m/M}$$

EVALUATE: The acceleration is positive (in the downward direction) and less than g, as it should be, since the cable is holding back the block. The cable tension is *not* equal to the block's weight mg; if it were, the block could not accelerate.

Let's check some particular cases. When M is much larger than m, the tension is nearly equal to mg and the acceleration is correspondingly much less than g. When M is zero, $T = 0$ and $a_y = g$; the object falls freely. If the object starts from rest ($v_{0y} = 0$) a height h above the floor, its y-velocity when it strikes the ground is given by $v_y^2 = v_{0y}^2 + 2a_yh = 2a_yh$, so

$$v_y = \sqrt{2a_yh} = \sqrt{\frac{2gh}{1 + M/2m}}$$

We found this same result from energy considerations in Example 9.8.

EVALUATE: No mechanical energy was lost or gained, so from the energy standpoint the string is merely a way to convert some of the gravitational potential energy (which is released as the cylinder falls) into rotational kinetic energy rather than translational kinetic energy. Because not all of the released energy goes into translation, v_{cm} is less than the speed $\sqrt{2gh}$ of an object dropped from height h with no strings attached.

Angular momentum and Linear Momentum

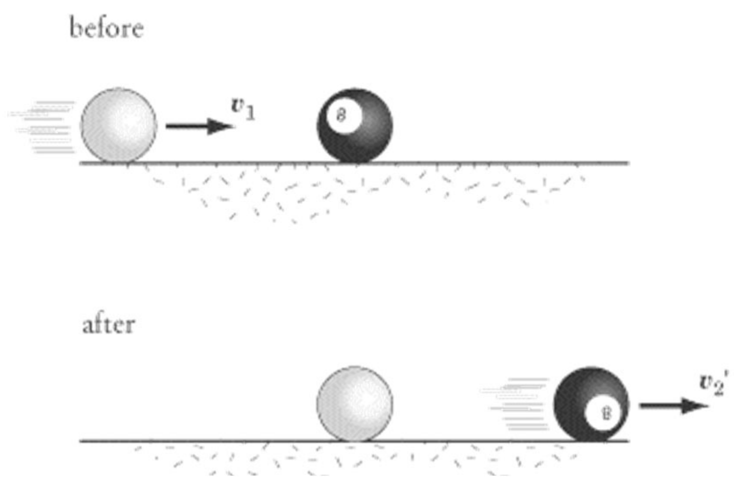
- Linear momentum is defined as the mass time velocity

$$\vec{p} = m\vec{v}$$

- Newton's 2nd Law can be rewritten as force equals rate of change of momentum

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d\vec{p}}{dt}$$

- Conservation of momentum: when no external force acts on the objects, total momentum is conserved.



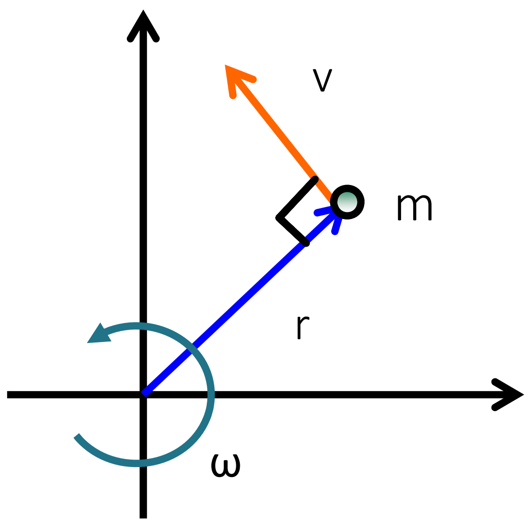
$$p_A + p_B = constant$$

What is the momentum of rotation? Single particle

Similar to linear momentum, we look at Newton's law for rotation:

$$\tau = I\alpha = I\frac{d\omega}{dt} = \frac{dI\omega}{dt} = \frac{dL}{dt}$$

$$L = I\omega = mr^2\omega$$



$L = I\omega$ is the angular momentum for a single particle

$p = mv$ → momentum for linear motion

Angular momentum can also be written as

$$L = mr\, r\omega = r\, mv = r\, p$$

This is the case when the particle is performing a circular motion so v and r are perpendicular to each other

When $\vec{v}$ and $\vec{r}$ are not perpendicular

If $\vec{v}$ and $\vec{r}$ are not perpendicular

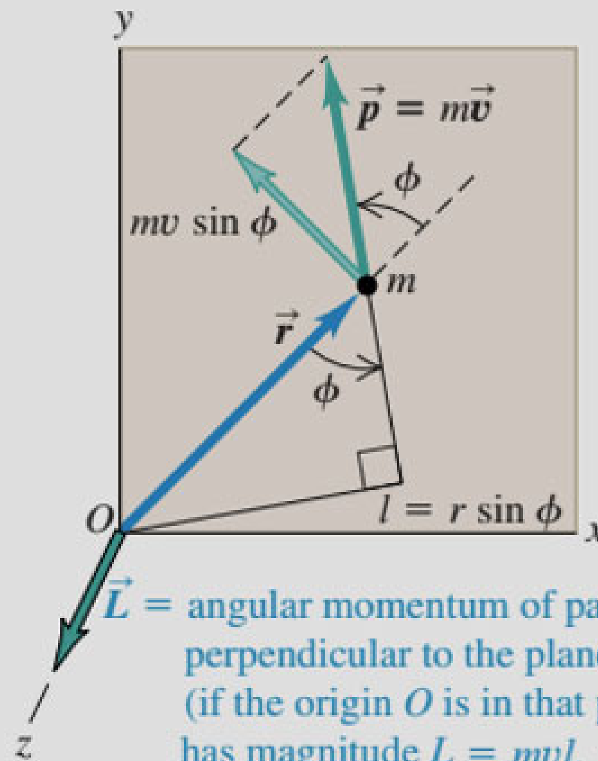
$$L = mrv\sin\phi$$

$$\rightarrow \vec{L} = \vec{r} \times m\vec{v}$$

Similar to torque:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

10.23 Calculating the angular momentum $\vec{L} = \vec{r} \times m\vec{v} = \vec{r} \times \vec{p}$ of a particle with mass m moving in the xy-plane.



$\vec{L}$ = angular momentum of particle. $\vec{L}$ is perpendicular to the plane of motion (if the origin O is in that plane) and has magnitude $L = mvl$.

Newton's law for angular momentum

Rate of change of angular momentum equals torque

a rigid body (an object) rotating about a fixed axis

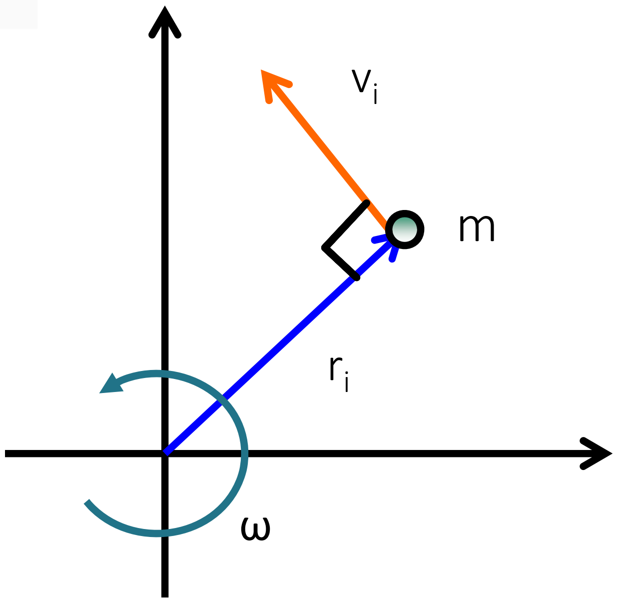
$$L = \sum_{i=1}^{n} r_i p_i = \sum_{i=1}^{n} r_i m v_i = \sum_{i=1}^{n} m r_i^2 \omega = I\omega$$

$$\frac{dL}{dt} = \frac{d(\sum_{i=1}^{n} m r_i^2 \omega)}{dt} = \sum_{i=1}^{n} m r_i^2 \frac{d\omega}{dt}$$

$$= \sum_{i=1}^{n} I_i \alpha = \sum_{i=1}^{n} \tau_i = I\alpha$$

$$\frac{dL}{dt} = \tau = I\alpha$$

v and r are perpendicular



Many particles mean adding all the particles together

Conservation of angular momentum

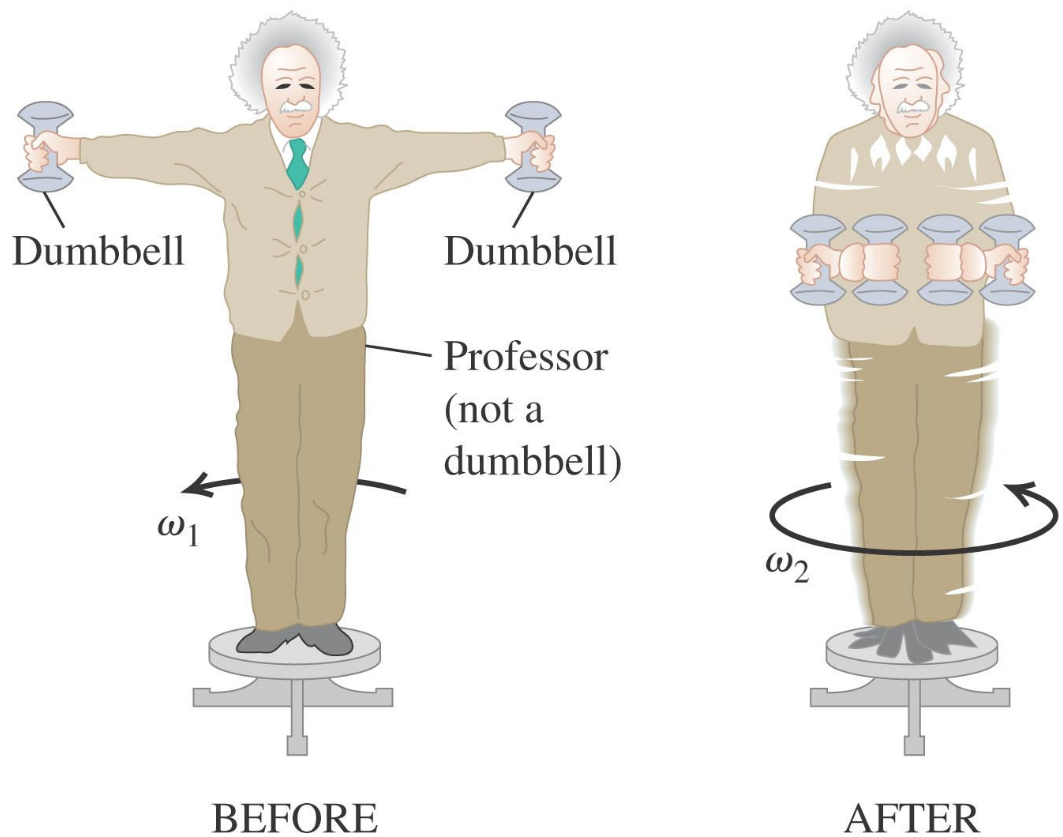
- If there is no torque, the rate of change of angular momentum is zero
- The angular momentum is a constant.
- Angular momentum is conserved.

Conservation of angular momentum

- When the net external torque acting on a system is zero, the total angular momentum of the system is constant (conserved).
- Follow Example 10.10 using Figure 10.29 below.

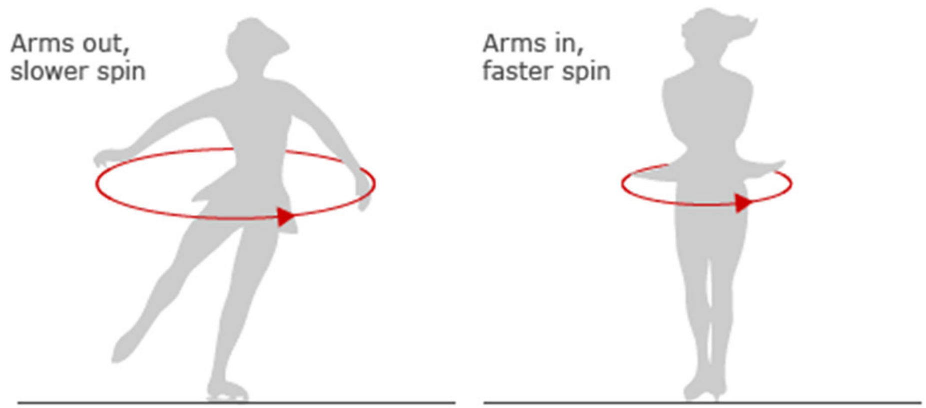
- When the professor pulls in his arms, his moment of inertia decreases.

- Since angular momentum is constant, $I_i \omega_i = I_f \omega_f$. If $I_f < I_i$ then $\omega_f > \omega_i$. The professor spins faster after pulling in his arms.



Conservation of angular momentum

- The same thing occurs for a skater:

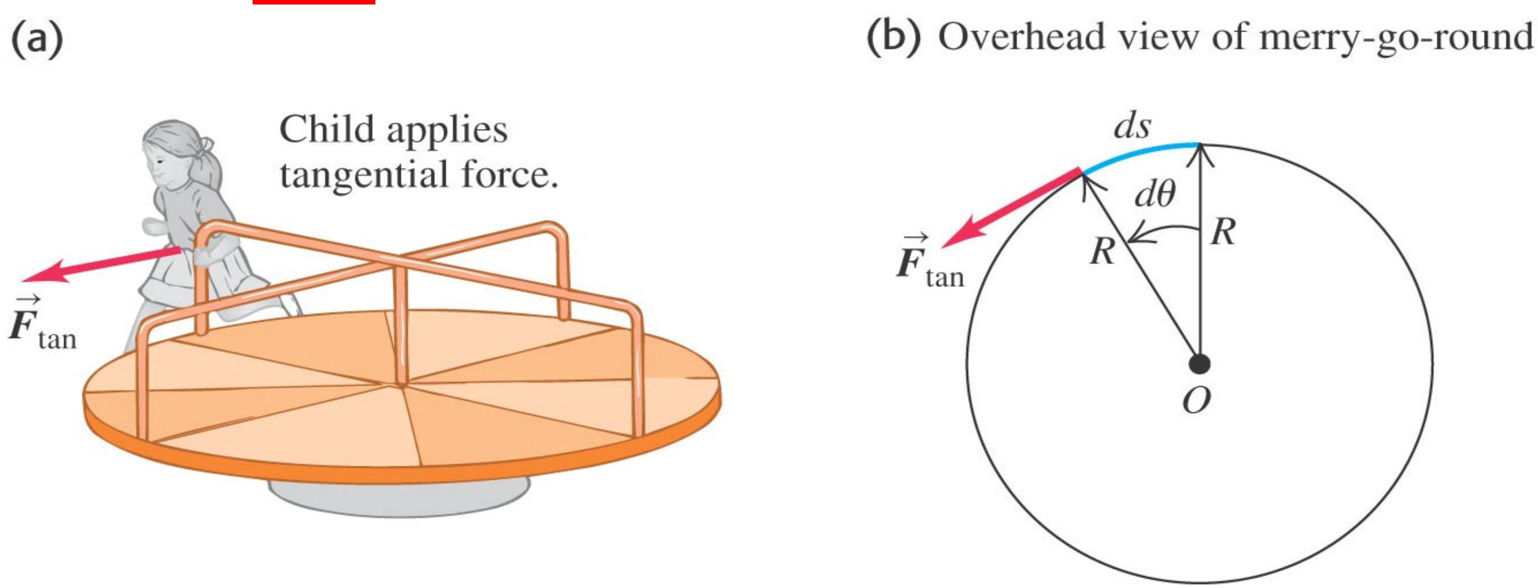


- Quad jump:

Work and power in rotational motion

- Figure 10.21 below shows that a tangential force applied to a rotating body does work on it.
- The total work done on the body $= \boxed{F\Delta s} = Fr\Delta\theta$

$$= \boxed{\tau\Delta\theta} = torque\ \times angular\ displacement$$



Power

$$\text{Power} = \text{Work output rate} = \text{Work/Time}$$

- In dt, the distance travelled is ds, the work done is

$$\mathrm{W} = \mathrm{Fd}s \qquad \rightarrow \text{Power} = \mathrm{Fd}s/\mathrm{d}t = \boxed{\mathrm{F}v}$$

- For rotation, if the angle is dθ in dt time, then

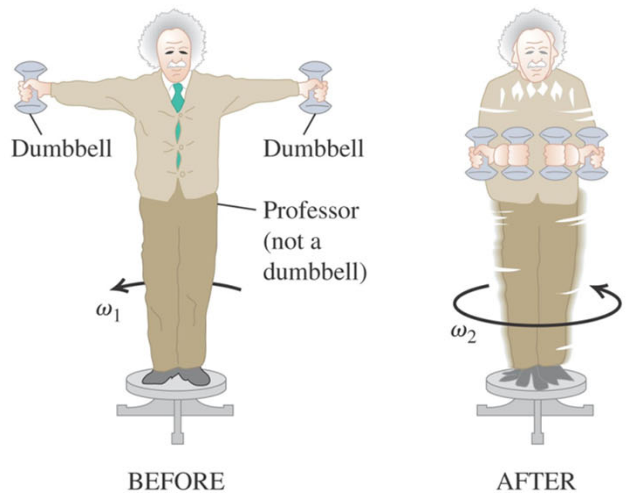
$$\text{work done} = Fds = FRd\theta$$

$$\text{power} = F\frac{ds}{dt} = FR\frac{d\theta}{dt} = FR\omega_z = \tau_z\omega_z$$

$$P = \boxed{\tau\omega}$$

Example 10.10 Anyone can be a ballerina

A physics professor stands at the center of a frictionless turntable with arms outstretched and a 5.0-kg dumbbell in each hand (Fig. 10.29). He is set rotating about the vertical axis, making one revolution in 2.0 s. Find his final angular velocity if he pulls the dumbbells in to his stomach. His moment of inertia (without the dumbbells) is $3.0\ \mathrm{kg \cdot m^2}$ with arms outstretched and $2.2\ \mathrm{kg \cdot m^2}$ with his hands at his stomach. The dumbbells are 1.0 m from the axis initially and 0.20 m at the end.

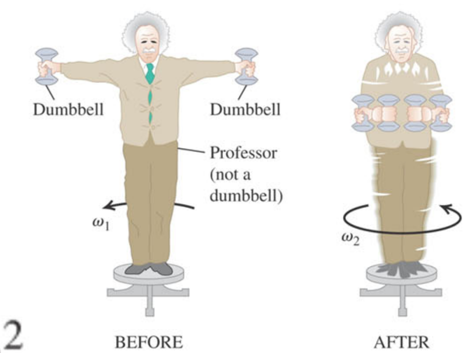


SOLUTION

IDENTIFY, SET UP, and EXECUTE: No external torques act about the z-axis, so L_z is constant. We'll use Eq. (10.30) to find the final angular velocity ω_{2z}. The moment of inertia of the system is $I = I_{prof} + I_{dumbbells}$. We treat each dumbbell as a particle of mass m that contributes mr^2 to $I_{dumbbells}$, where r is the perpendicular distance from the axis to the dumbbell. Initially we have

$$I_1 = 3.0\ \text{kg}\cdot\text{m}^2 + 2(5.0\ \text{kg})(1.0\ \text{m})^2 = 13\ \text{kg}\cdot\text{m}^2$$

$$\omega_{1z} = \frac{1\ \text{rev}}{2.0\ \text{s}} = 0.50\ \text{rev/s}$$



The final moment of inertia is

$$I_2 = 2.2\ \text{kg}\cdot\text{m}^2 + 2(5.0\ \text{kg})(0.20\ \text{m})^2 = 2.6\ \text{kg}\cdot\text{m}^2$$

From Eq. (10.30), the final angular velocity is

$$\omega_{2z} = \frac{I_1}{I_2}\omega_{1z} = \frac{13\ \text{kg}\cdot\text{m}^2}{2.6\ \text{kg}\cdot\text{m}^2}(0.50\ \text{rev/s}) = 2.5\ \text{rev/s} = 5\omega_{1z}$$

Can you see why we didn't have to change "revolutions" to "radians" in this calculation?

EVALUATE: The angular momentum remained constant, but the angular velocity increased by a factor of 5, from $\omega_{1z} = (0.50\ \text{rev/s})(2\pi\ \text{rad/rev}) = 3.14\ \text{rad/s}$ to $\omega_{2z} = (2.5\ \text{rev/s})(2\pi\ \text{rad/rev}) = 15.7\ \text{rad/s}$. The initial and final kinetic energies are then

$$K_1 = \tfrac{1}{2}I_1\omega_{1z}{}^2 = \tfrac{1}{2}(13\ \text{kg}\cdot\text{m}^2)(3.14\ \text{rad/s})^2 = 64\ \text{J}$$

$$K_2 = \tfrac{1}{2}I_2\omega_{2z}{}^2 = \tfrac{1}{2}(2.6\ \text{kg}\cdot\text{m}^2)(15.7\ \text{rad/s})^2 = 320\ \text{J}$$

The fivefold increase in kinetic energy came from the work that the professor did in pulling his arms and the dumbbells inward.

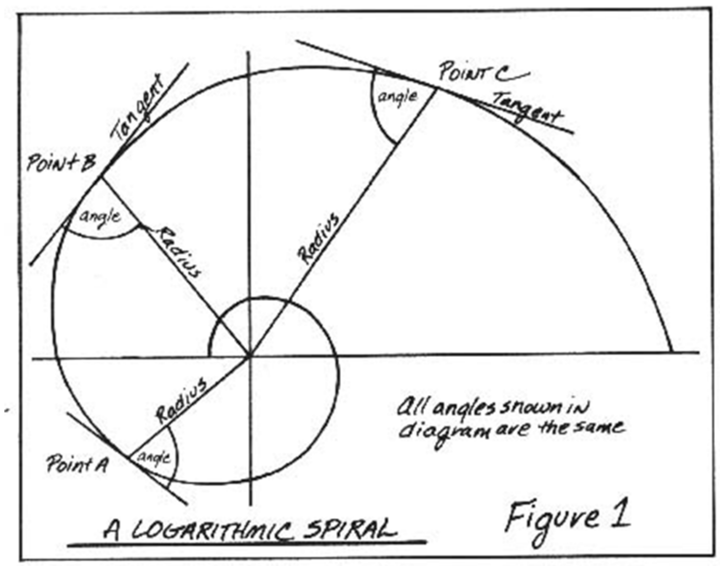
Why KE is increased

- When the professor pulls in his arms, the dumbells do not follow a circular path. The path is like a spiral.

- The velocity is along the tangent
- The force is along the radius

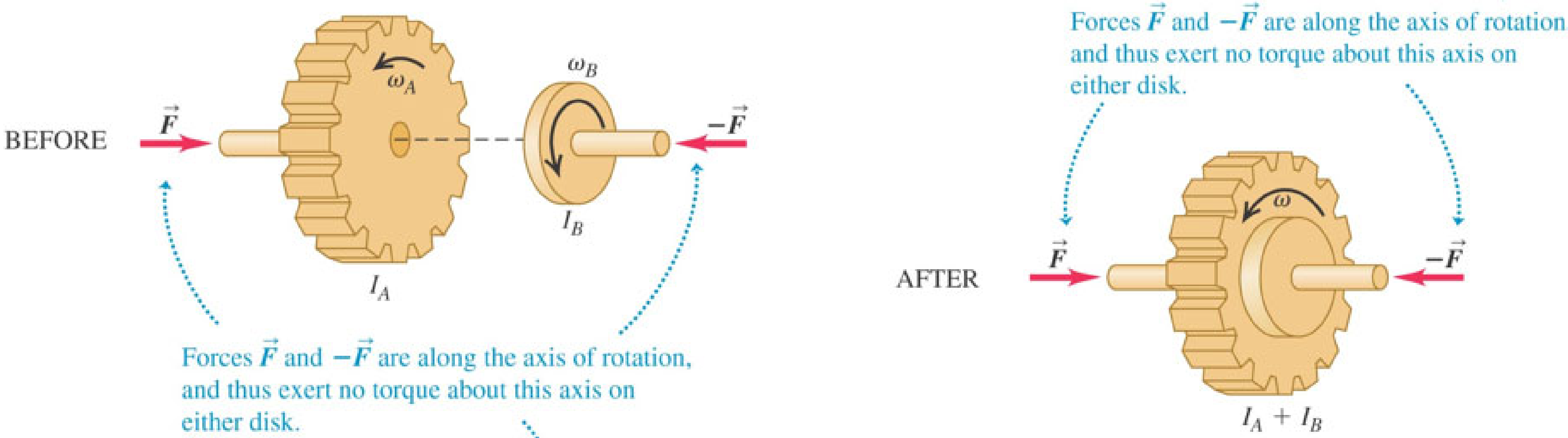
- The angle between motion and force is less than 90 degree

→ Force from professor does positive work. → KE increased!



Example 10.11 A rotational "collision"

Figure 10.30 shows two disks: an engine flywheel (A) and a clutch plate (B) attached to a transmission shaft. Their moments of inertia are I_A and I_B; initially, they are rotating with constant angular speeds ω_A and ω_B, respectively. We push the disks together with forces acting along the axis, so as not to apply any torque on either disk. The disks rub against each other and eventually reach a common angular speed ω. Derive an expression for ω.



SOLUTION

IDENTIFY, SET UP, and EXECUTE: There are no external torques, so the only torque acting on either disk is the torque applied by the other disk. Hence the total angular momentum of the system of two disks is conserved. At the end they rotate together as one body with total moment of inertia $I = I_A + I_B$ and angular speed ω.

When the net external torque is zero, angular momentum is conserved.

Figure 10.30 shows that all angular velocities are in the same direction, so we can regard ω_A, ω_B, and ω as components of angular velocity along the rotation axis. Conservation of angular momentum gives

$$I_A\omega_A + I_B\omega_B = (I_A + I_B)\omega$$

$$\omega = \frac{I_A\omega_A + I_B\omega_B}{I_A + I_B}$$

Energy change:

$$KE_i = \frac{1}{2}I_A\omega_A^2 + \frac{1}{2}I_B\omega_B^2$$

$$KE_f = \frac{1}{2}(I_A+I_B)\omega^2 = \frac{(I_A\omega_A + I_B\omega_B)^2}{2(I_A + I_B)}$$

$$\Delta KE = KE_f - KE_i = -\frac{I_A I_B(\omega_A - \omega_B)^2}{2(I_A + I_B)} < 0 \;\rightarrow \text{energy loss}$$

EVALUATE: This "collision" is analogous to a completely inelastic collision (see Section 8.3). When two objects in translational motion along the same axis collide and stick, the linear momentum of the system is conserved. Here two objects in *rotational* motion around the same axis "collide" and stick, and the *angular* momentum of the system is conserved.

The kinetic energy of a system decreases in a completely inelastic collision. Here kinetic energy is lost because nonconservative (frictional) internal forces act while the two disks rub together. Suppose flywheel A has a mass of 2.0 kg, a radius of 0.20 m, and an initial angular speed of 50 rad/s (about 500 rpm), and clutch plate B has a mass of 4.0 kg, a radius of 0.10 m, and an initial angular speed of 200 rad/s. Can you show that the final kinetic energy is only two-thirds of the initial kinetic energy?

Example 10.8 Calculating power from torque

An electric motor exerts a constant 10-$N \cdot m$ torque on a grindstone, which has a moment of inertia of 2.0 $kg \cdot m^2$ about its shaft. The system starts from rest. Find the work W done by the motor in 8.0 s and the grindstone kinetic energy K at this time. What average power P_{av} is delivered by the motor?

SOLUTION

IDENTIFY and SET UP: The only torque acting is that due to the motor. Since this torque is constant, the grindstone's angular acceleration α_z is constant. We'll use Eq. (10.7) to find α_z, and then use this in the kinematics equations from Section 9.2 to calculate the angle $\Delta\theta$ through which the grindstone rotates in 8.0 s and its final angular velocity ω_z. From these we'll calculate W, K, and P_{av}.

EXECUTE: We have $\sum \tau_z = 10\ \text{N} \cdot \text{m}$ and $I = 2.0\ \text{kg} \cdot \text{m}^2$, so $\sum \tau_z = I\alpha_z$ yields $\alpha_z = 5.0\ \text{rad/s}^2$. From Eq. (9.11),

$$\Delta\theta = \tfrac{1}{2}\alpha_z t^2 = \tfrac{1}{2}(5.0\ \text{rad/s}^2)(8.0\ \text{s})^2 = 160\ \text{rad}$$

$$W = \tau_z \Delta\theta = (10\ \text{N} \cdot \text{m})(160\ \text{rad}) = 1600\ \text{J}$$

From Eqs. (9.7) and (9.17),

$$\omega_z = \alpha_z t = (5.0\ \text{rad/s}^2)(8.0\ \text{s}) = 40\ \text{rad/s}$$

$$K = \tfrac{1}{2} I \omega_z^2 = \tfrac{1}{2}(2.0\ \text{kg} \cdot \text{m}^2)(40\ \text{rad/s})^2 = 1600\ \text{J}$$

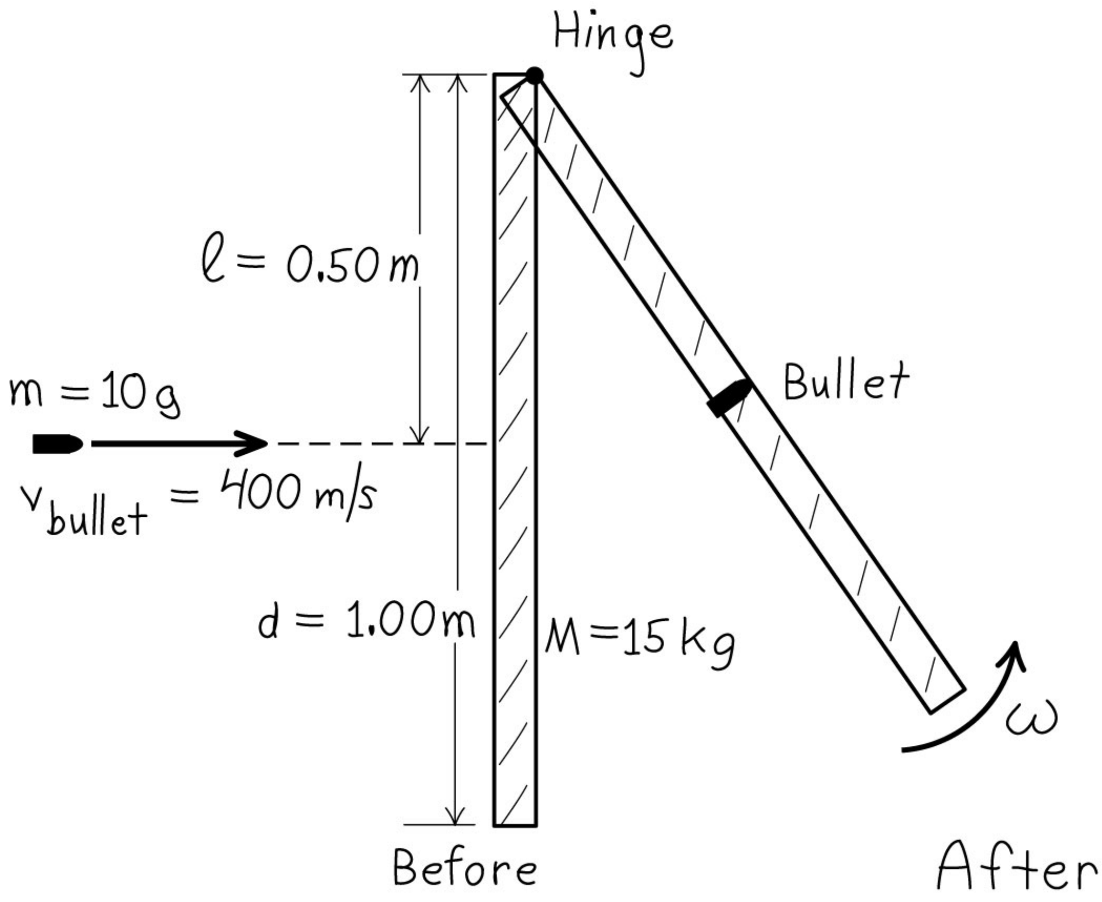
The average power is the work done divided by the time interval:

$$P_{\text{av}} = \frac{1600\ \text{J}}{8.0\ \text{s}} = 200\ \text{J/s} = 200\ \text{W}$$

EVALUATE: The initial kinetic energy was zero, so the work done W must equal the final kinetic energy K [Eq. (10.22)]. This is just as we calculated. We can check our result $P_{av} = 200\ \text{W}$ by considering the *instantaneous* power $P = \tau_z \omega_z$. Because ω_z increases continuously, P increases continuously as well; its value increases from zero at $t = 0$ to $(10\ \text{N} \cdot \text{m})(40\ \text{rad/s}) = 400\ \text{W}$ at $t = 8.0\ \text{s}$. Both ω_z and P increase *uniformly* with time, so the *average* power is just half this maximum value, or 200 W.

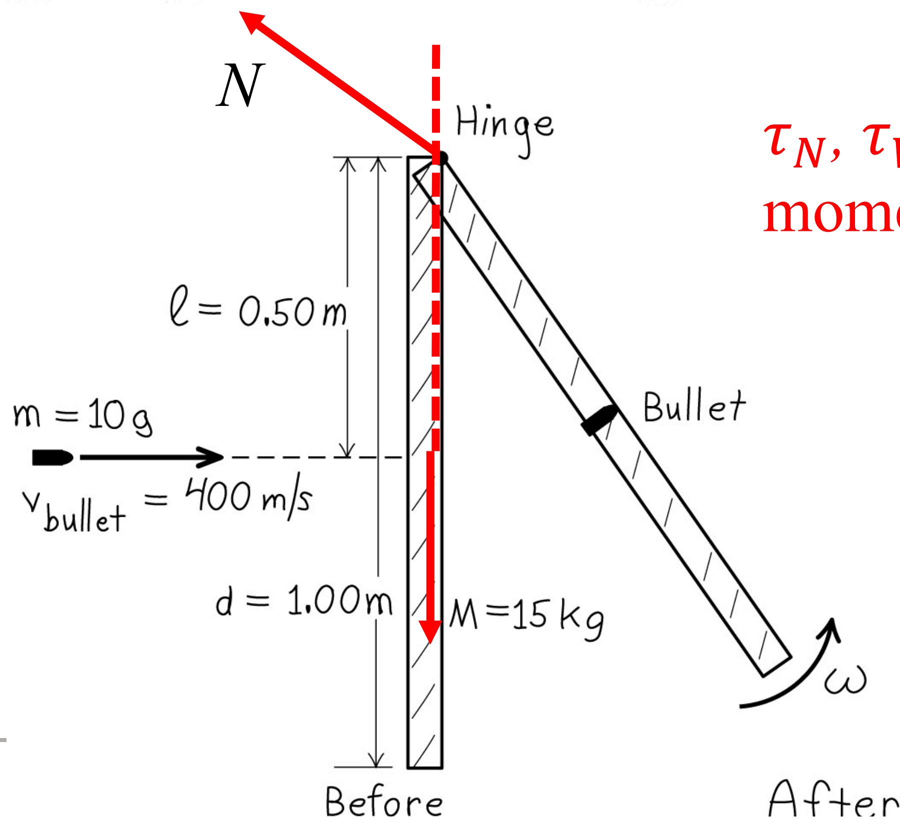
Angular momentum in a crime bust

- A bullet hits a door causing it to swing.
- Follow Example 10.12 using Figure 10.31 below.



Example 10.12 Angular momentum in a crime bust

A door 1.00 m wide, of mass 15 kg, can rotate freely about a vertical axis through its hinges. A bullet with a mass of 10 g and a speed of 400 m/s strikes the center of the door, in a direction perpendicular to the plane of the door, and embeds itself there. Find the door's angular speed. Is kinetic energy conserved?



τ_N, $\tau_W = 0$ at the moment of collision

SOLUTION

IDENTIFY and SET UP: We consider the door and bullet as a system. There is no external torque about the hinge axis, so angular momentum about this axis is conserved. Figure 10.31 shows our sketch. The initial angular momentum is that of the bullet, as given by Eq. (10.25). The final angular momentum is that of a rigid body composed of the door and the embedded bullet. We'll equate these quantities and solve for the resulting angular speed ω of the door and bullet.

$$L = mvr\sin\phi = mvl \tag{10.25}$$

EXECUTE: From Eq. (10.25), the initial angular momentum of the bullet is

$$L = mvl = (0.010\ \text{kg})(400\ \text{m/s})(0.50\ \text{m}) = 2.0\ \text{kg} \cdot \text{m}^2/\text{s}$$

The final angular momentum is $I\omega$, where $I = I_{\text{door}} + I_{\text{bullet}}$. From Table 9.2, case (d), for a door of width $d = 1.00$ m,

$$I_{\text{door}} = \frac{Md^2}{3} = \frac{(15\ \text{kg})(1.00\ \text{m})^2}{3} = 5.0\ \text{kg} \cdot \text{m}^2$$

The moment of inertia of the bullet (with respect to the axis along the hinges) is

$$I_{\text{bullet}} = ml^2 = (0.010\ \text{kg})(0.50\ \text{m})^2 = 0.0025\ \text{kg} \cdot \text{m}^2$$

Conservation of angular momentum requires that $mvl = I\omega$, or

$$\omega = \frac{mvl}{I} = \frac{2.0\ \text{kg}\cdot\text{m}^2/\text{s}}{5.0\ \text{kg}\cdot\text{m}^2 + 0.0025\ \text{kg}\cdot\text{m}^2} = 0.40\ \text{rad/s}$$

The initial and final kinetic energies are

$$K_1 = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.010\ \text{kg})(400\ \text{m/s})^2 = 800\ \text{J}$$

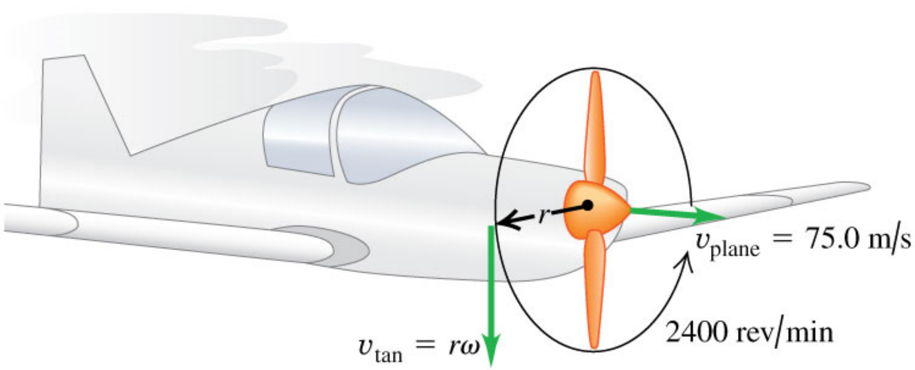
$$K_2 = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(5.0025\ \text{kg}\cdot\text{m}^2)(0.40\ \text{rad/s})^2 = 0.40\ \text{J}$$

EVALUATE: The final kinetic energy is only $\frac{1}{2000}$ of the initial value! We did not expect kinetic energy to be conserved: The collision is inelastic because nonconservative friction forces act during the impact. The door's final angular speed is quite slow: At 0.40 rad/s, it takes 3.9 s to swing through 90° ($\pi/2$ radians).

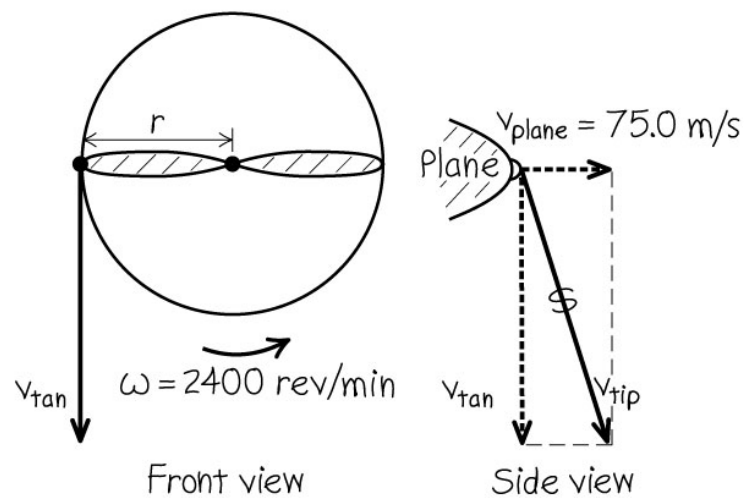
Example 9.5 Designing a propeller

You are designing an airplane propeller that is to turn at 2400 rpm (Fig. 9.13a). The forward airspeed of the plane is to be 75.0 m/s, and the speed of the tips of the propeller blades through the air must not exceed 270 m/s. (This is about 80% of the speed of sound in air. If the speed of the propeller tips were greater than this, they would produce a lot of noise.) (a) What is the maximum possible propeller radius? (b) With this radius, what is the acceleration of the propeller tip?

(a)



(b)



SOLUTION

IDENTIFY and SET UP: We consider a particle at the tip of the propeller; our target variables are the particle's distance from the axis and its acceleration. The speed of this particle through the air, which cannot exceed 270 m/s, is due to both the propeller's rotation and the forward motion of the airplane. Figure 9.13b shows that the particle's velocity $\vec{v}_{tip}$ is the vector sum of its tangential velocity due to the propeller's rotation of magnitude $v_{tan} = \omega r$, given by Eq. (9.13), and the forward velocity of the airplane of magnitude $v_{plane} = 75.0$ m/s. The propeller rotates in a plane perpendicular to the direction of flight, so $\vec{v}_{tan}$ and $\vec{v}_{plane}$ are perpendicular to each other, and we can use the Pythagorean theorem to obtain an expression for v_{tip} from v_{tan} and v_{plane}. We will then set $v_{tip} = 270$ m/s and solve for the radius r. The angular speed of the propeller is constant, so the acceleration of the propeller tip has only a radial component; we'll find it using Eq. (9.15).

EXECUTE: We first convert ω to rad/s (see Fig. 9.11):

$$\omega = 2400 \text{ rpm} = \left(2400\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)$$
$$= 251 \text{ rad/s}$$

(a) From Fig. 9.13b and Eq. (9.13),

$$v_{\text{tip}}{}^2 = v_{\text{plane}}{}^2 + v_{\tan}{}^2 = v_{\text{plane}}{}^2 + r^2\omega^2 \qquad \text{so}$$

$$r^2 = \frac{v_{\text{tip}}{}^2 - v_{\text{plane}}{}^2}{\omega^2} \qquad \text{and} \qquad r = \frac{\sqrt{v_{\text{tip}}{}^2 - v_{\text{plane}}{}^2}}{\omega}$$

If $v_{\text{tip}} = 270$ m/s, the maximum propeller radius is

$$r = \frac{\sqrt{(270 \text{ m/s})^2 - (75.0 \text{ m/s})^2}}{251 \text{ rad/s}} = 1.03 \text{ m}$$

(b) The centripetal acceleration of the particle is, from Eq. (9.15),

$$a_{\text{rad}} = \omega^2 r = (251\ \text{rad/s})^2(1.03\ \text{m})$$
$$= 6.5 \times 10^4\ \text{m/s}^2 = 6600g$$

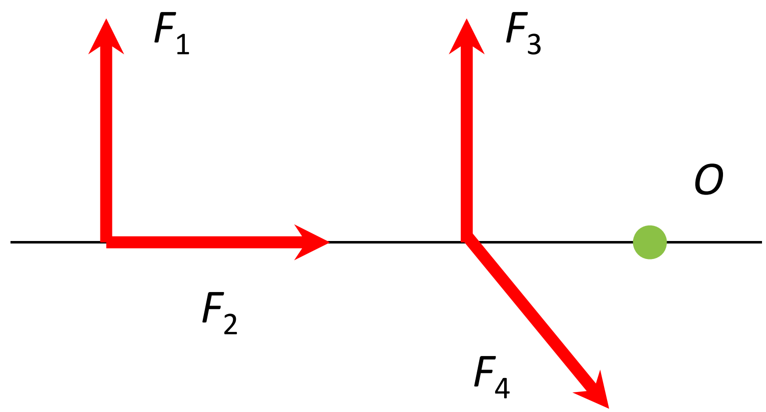
The tangential acceleration $a_{\tan}$ is zero because ω is constant.

EVALUATE: From $\sum \vec{F} = m\vec{a}$, the propeller must exert a force of 6.5×10^4 N on each kilogram of material at its tip! This is why propellers are made out of tough material, usually aluminum alloy.

Think Questions

TQ5B.1

Which of the four forces shown here produces a torque about *O* that is directed *out of* the plane of the drawing?



A. F_1

B. F_2

C. F_3

D. F_4

E. more than one of these

TQ5B.2

A force $\vec{F} = 5\hat{\imath}$ acts on an object at a point

located at the position $\vec{r} = 4\hat{\imath} + 8\hat{\jmath}$

What is the magnitude of the torque that this force applies about the origin?

A. zero

B. 40

C. 20

D. 25

E. 30

TQ5B.3

A glider of mass m_1 on a frictionless horizontal track is connected to an object of mass m_2 by a massless string. The glider accelerates to the right, the object accelerates downward, and the string rotates the pulley. What is the relationship among T_1 (the tension in the horizontal part of the string), T_2 (the tension in the vertical part of the string), and the weight m_2g of the object? (the pulley has non-zero mass at a distance from the center)

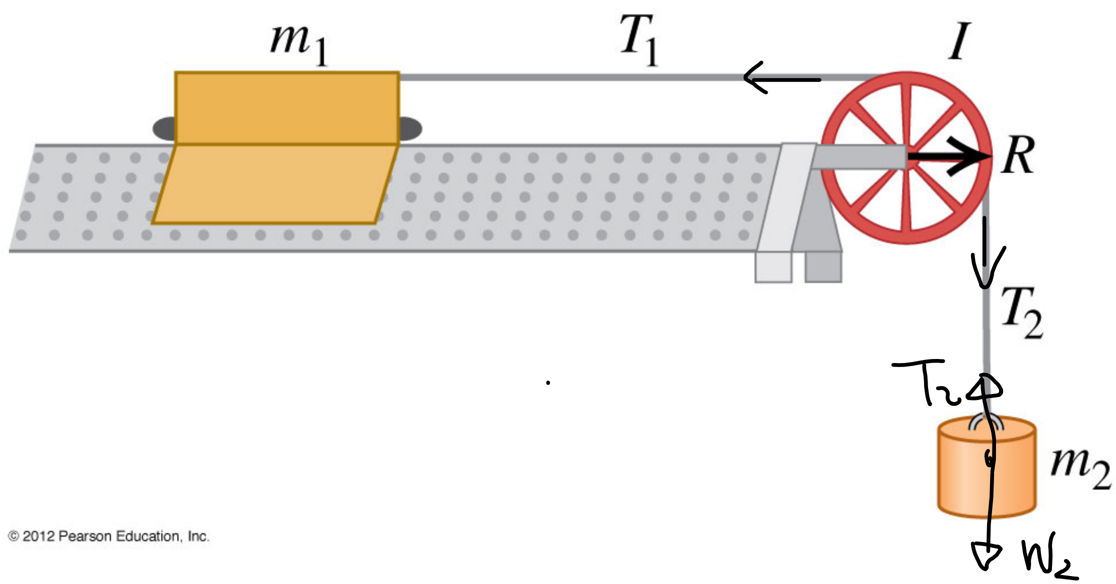
A. $m_2g = T_2 = T_1$

B. $m_2g > T_2 = T_1$

C. $m_2g > T_2 > T_1$

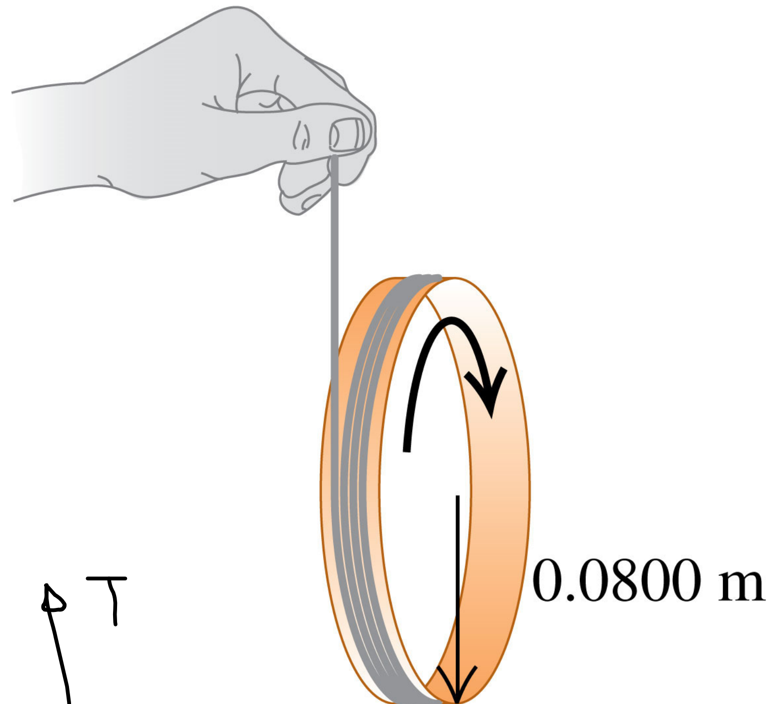
D. $m_2g = T_2 > T_1$

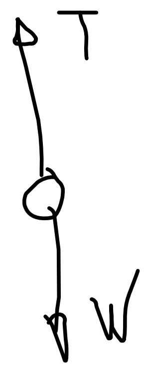
E. none of the above

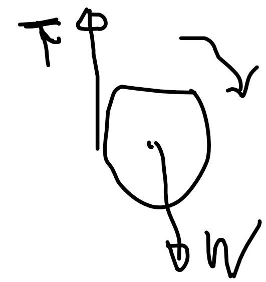


TQ5B.4

A lightweight string is wrapped several times around the rim of a small hoop. If the free end of the string is held in place and the hoop is released from rest, the string unwinds and the hoop descends. How does the tension in the string (T) compare to the weight of the hoop (W)?







A. $T = w$

B. $T > w$

C. $T < w$

D. not enough information given to decide