Chapter 4

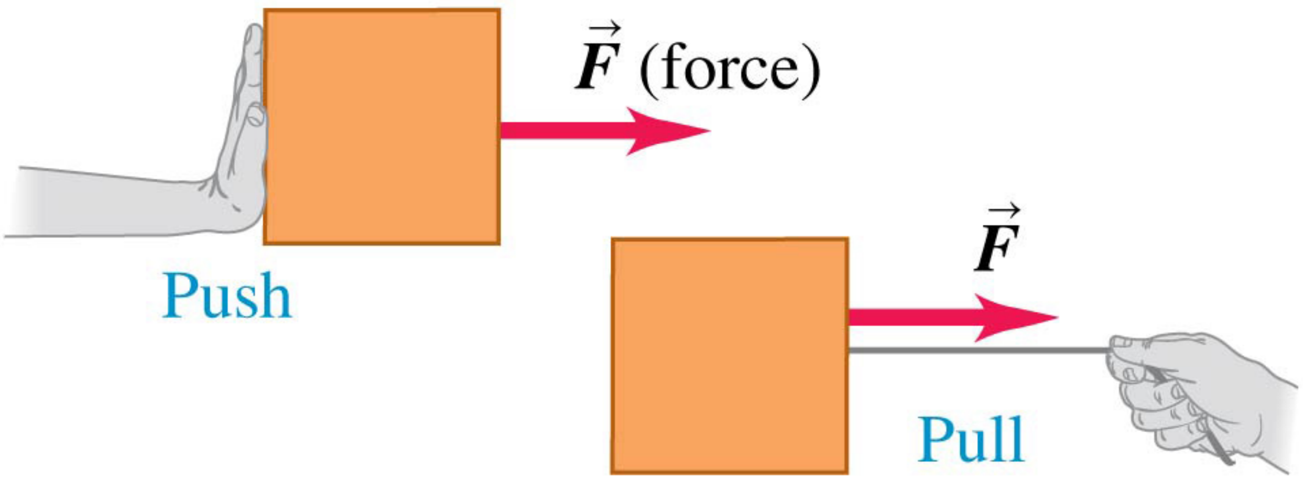
Newton’s Laws of Motion

Introduction

- We've seen how to use *kinematics* to describe motion in one, two, or three dimensions.
- But what *causes* bodies to move the way that they do?
- The answer takes us into the subject of **dynamics**, the relationship of motion to the forces that cause it.
- The principles of dynamics were clearly stated for the first time by Sir Isaac Newton; today we call them Newton's laws of motion.
- Newton did not *derive* these laws, but rather deduced them from a multitude of experiments performed by other scientists.

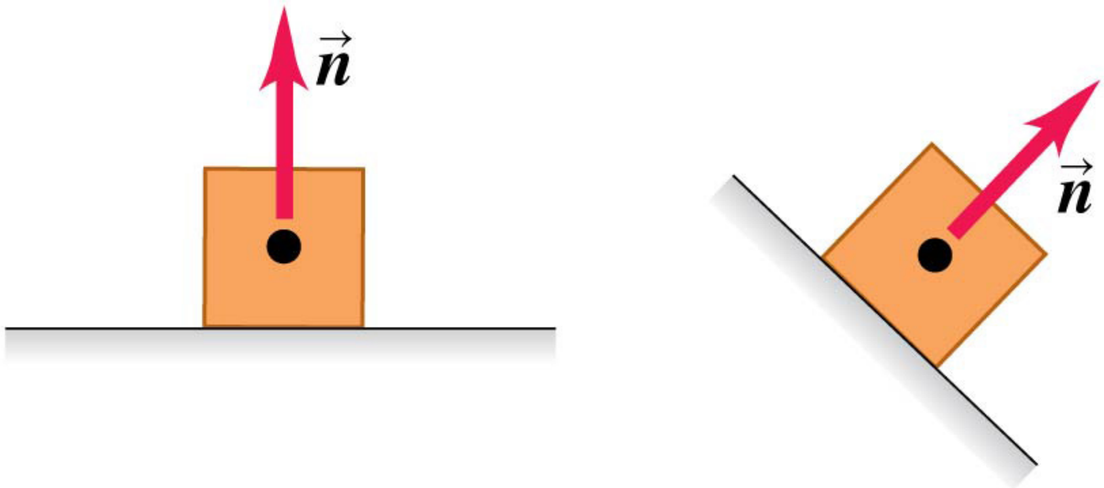
What are some properties of a force?

- A force is a push or a pull.
- A force is an interaction between two objects or between an object and its environment.
- A force is a vector quantity, with magnitude and direction.



There are four common types of forces: Normal

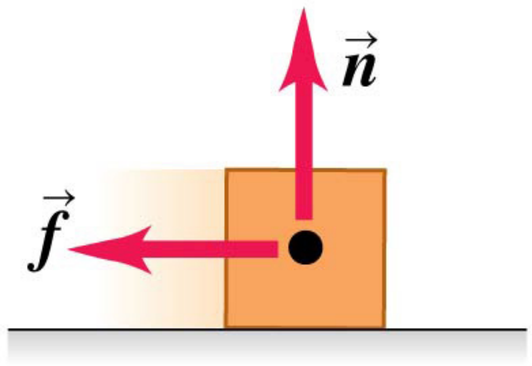
Normal force $\vec{n}$: When an object rests or pushes on a surface, the surface exerts a push on it that is directed perpendicular to the surface.



- The normal force is a contact force.

There are four common types of forces: Friction

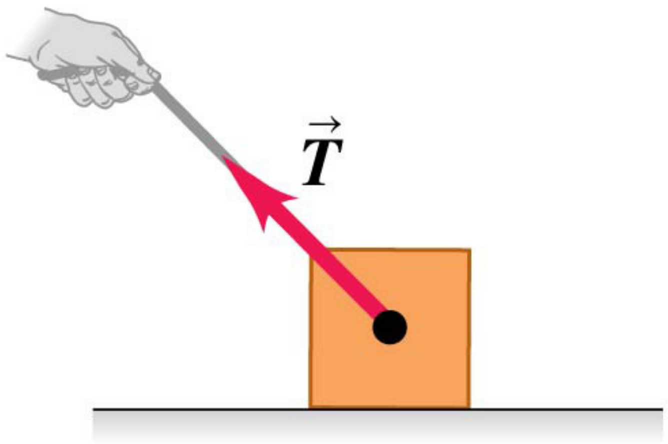
Friction force $\vec{f}$: In addition to the normal force, a surface may exert a friction force on an object, directed parallel to the surface.



- Friction is a contact force.

There are four common types of forces: Tension

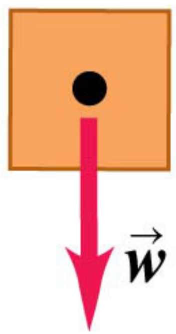
Tension force $\vec{T}$: A pulling force exerted on an object by a rope, cord, etc.



- Tension is a contact force.

There are four common types of forces: Weight

Weight $\vec{w}$: The pull of gravity on an object is a long-range force (a force that acts over a distance).



- Weight is a long-range force.

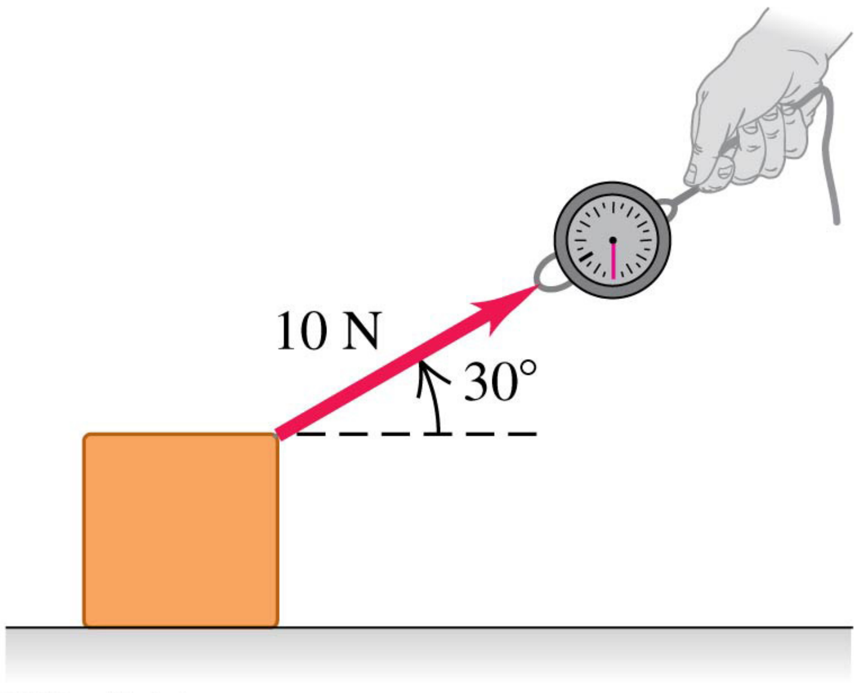
What are the magnitudes of common forces?

- The SI unit of the magnitude of force is the newton, abbreviated N. Some typical force magnitudes are:

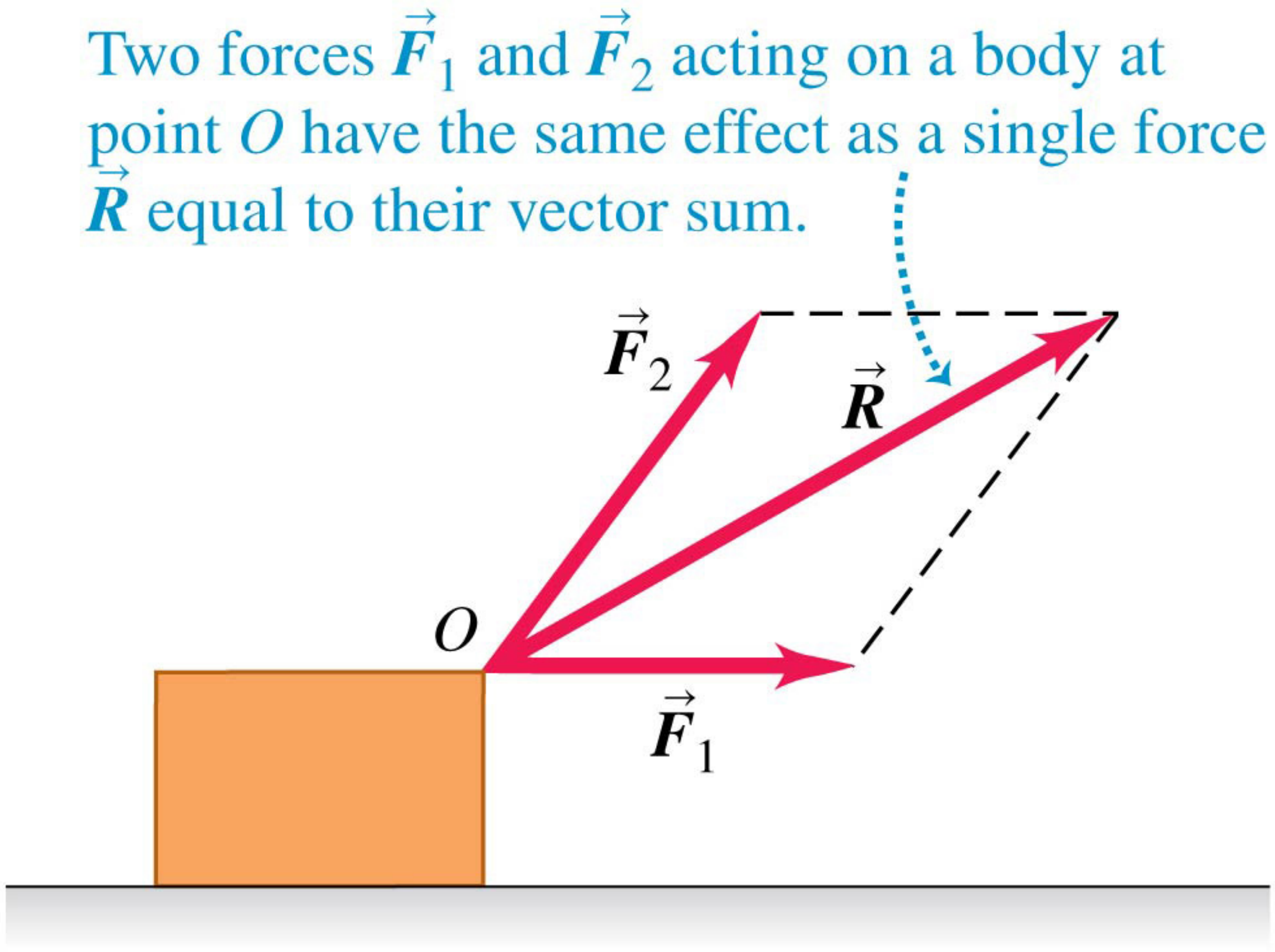
Weight of a large blue whale	1.9×10^{6} N
Maximum pulling force of a locomotive	8.9×10^{5} N
Weight of a 250-lb linebacker	1.1×10^{3} N
Weight of a medium apple	1 N
Electric attraction between the proton and the electron in a hydrogen atom	8.2×10^{-8} N
Gravitational attraction between the proton and the electron in a hydrogen atom	3.6×10^{-47} N

Drawing force vectors

- The figure shows a spring balance being used to measure a pull that we apply to a box.
- We draw a vector to represent the applied force.
- The length of the vector shows the magnitude; the longer the vector, the greater the force magnitude.

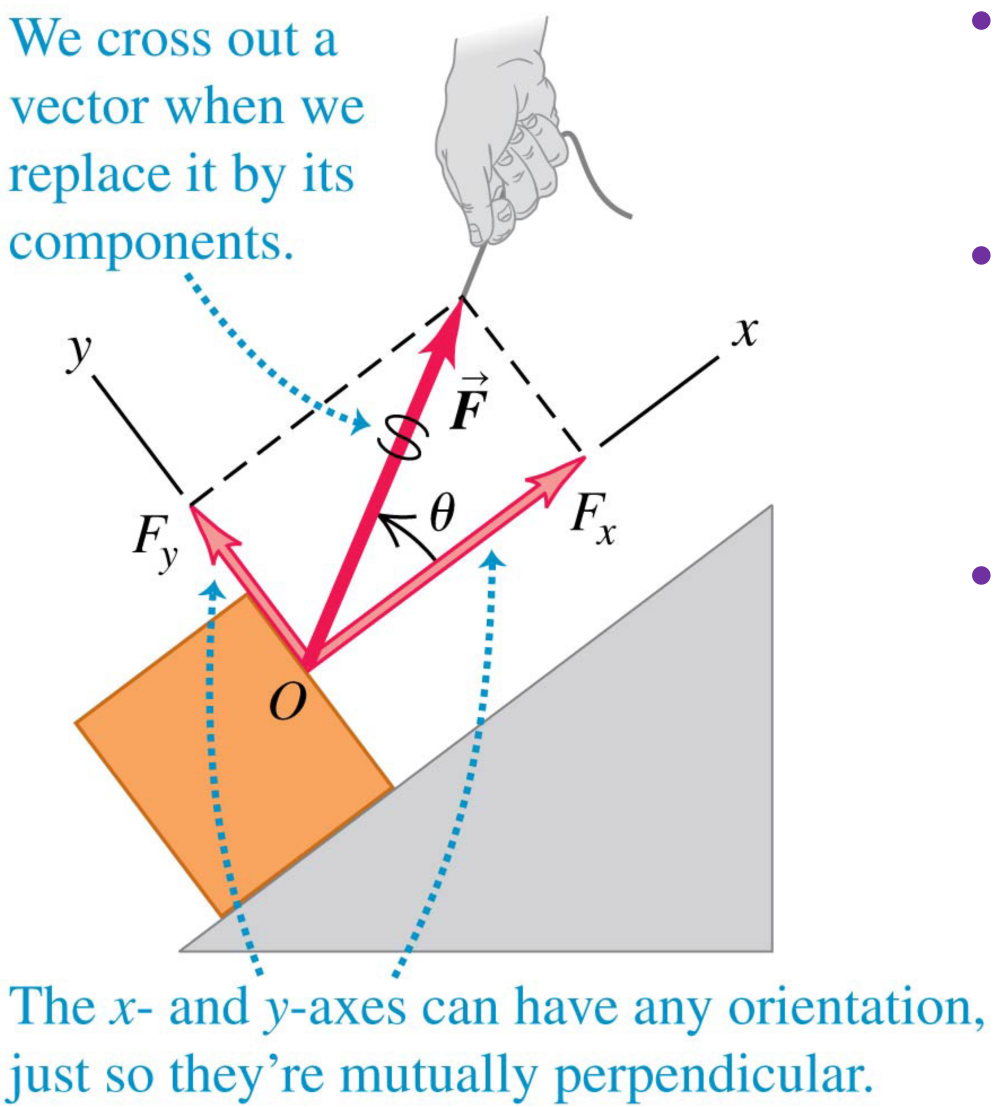


Superposition of forces



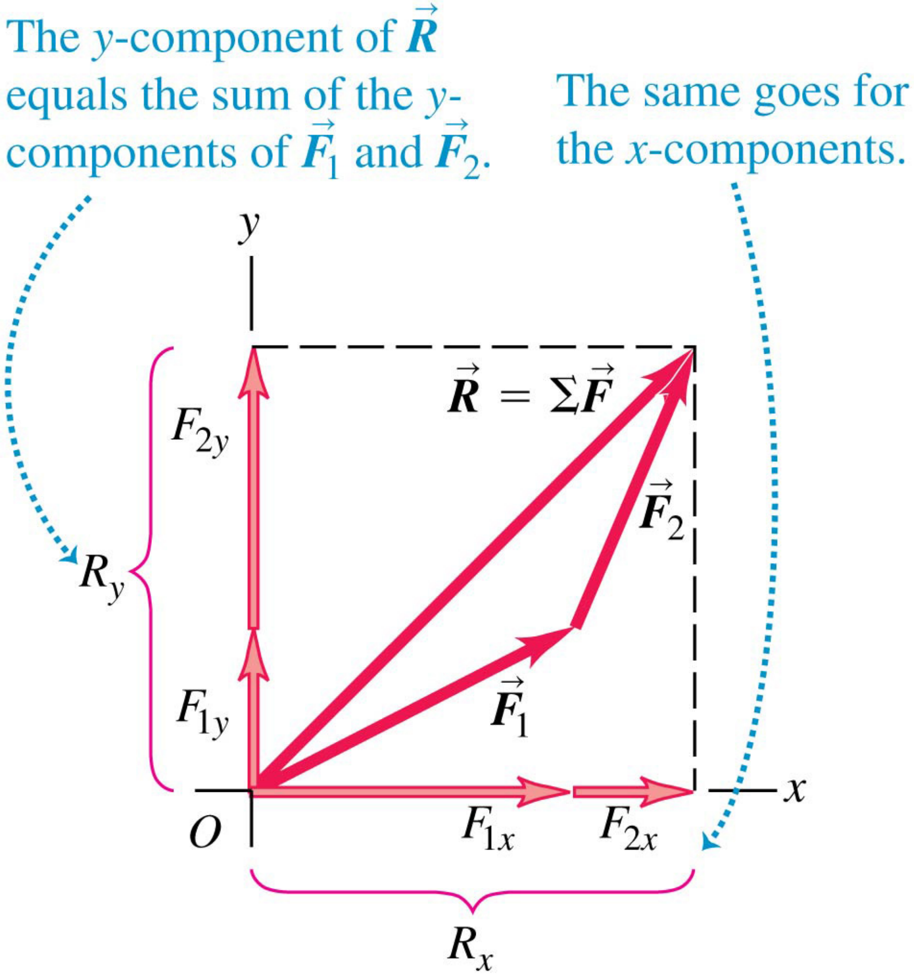
- Several forces acting at a point on an object have the same effect as their vector sum acting at the same point.

Decomposing a force into its component vectors



- Choose perpendicular x- and y-axes.
- F_x and F_y are the components of a force along these axes.
- Use trigonometry to find these force components.

Notation for the vector sum



- The vector sum of all the forces on an object is called the **resultant** of the forces or the **net force**:

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \Sigma\vec{F}$$

Newton’s first law

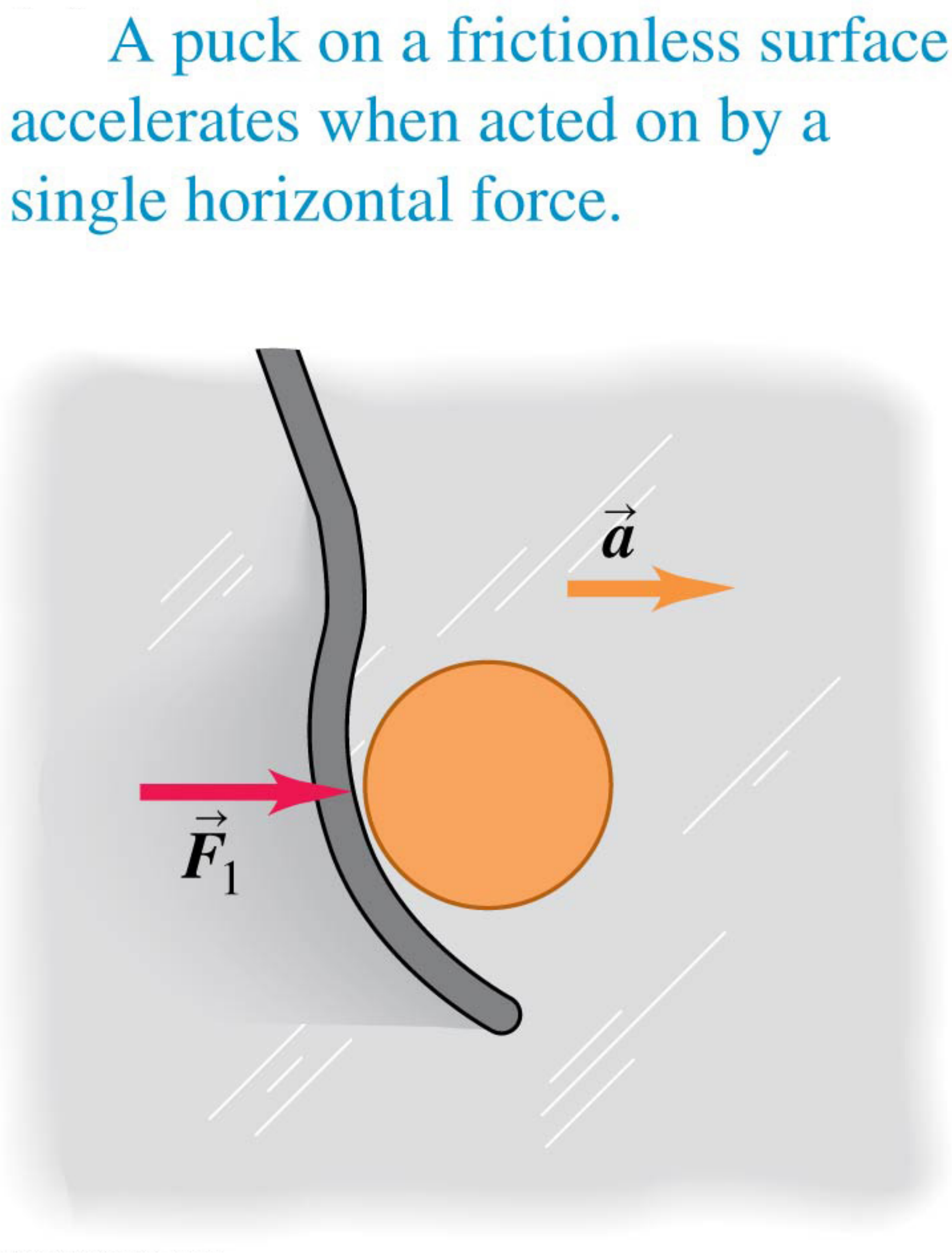
- When a body is either at rest or moving with constant velocity (in a straight line with constant speed), we say that the body is in **equilibrium**.

- For a body to be in equilibrium, it must be acted on by no forces, or by several forces such that their vector sum—that is, the net force—is zero:

Newton’s first law: Net force on a body ... $\sum \vec{F} = \mathbf{0}$ ... must be zero if body is in **equilibrium**.

Net force causes acceleration

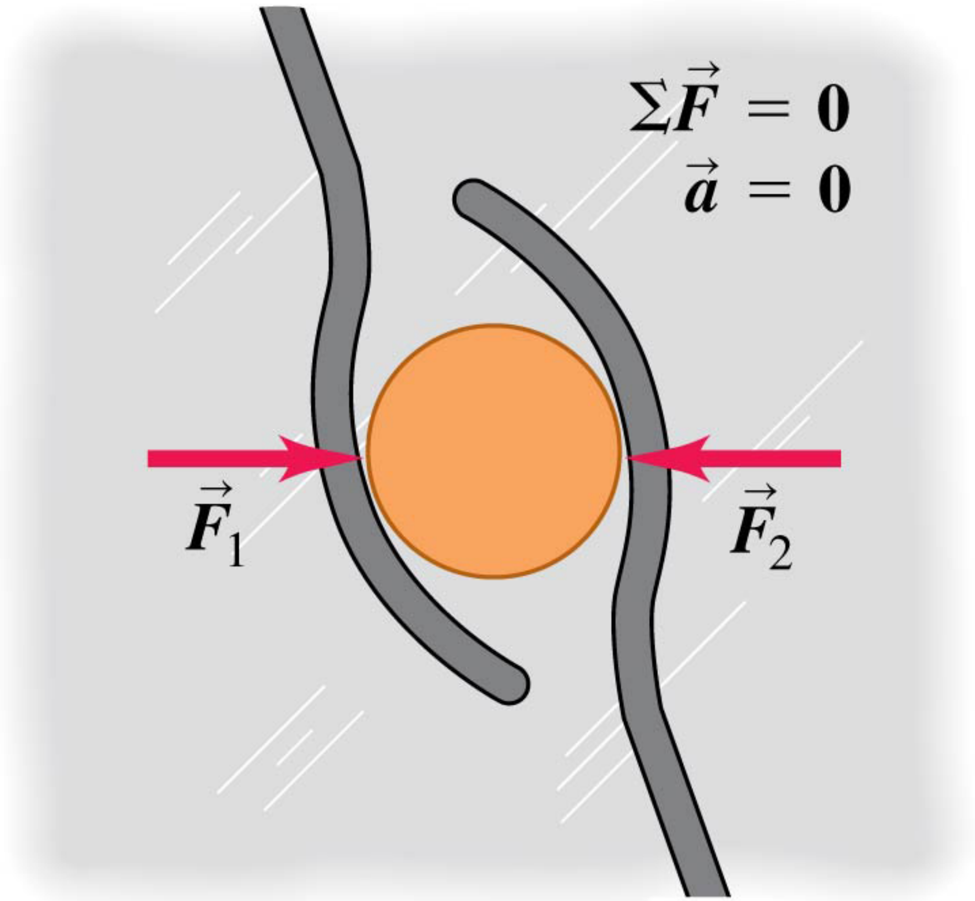
- A hockey puck accelerates in the direction of a net applied force.



Newton's first law

- When the net force is zero, the acceleration is zero, and the puck is in equilibrium.

This puck is acted on by two horizontal forces whose vector sum is zero. The puck behaves as though no forces act on it.



Sledding with Newton's first law

- The downward force of gravity acting on the child and sled is balanced by an upward normal force exerted by the ground.
- The adult's foot exerts a forward force that balances the backward force of friction on the sled.

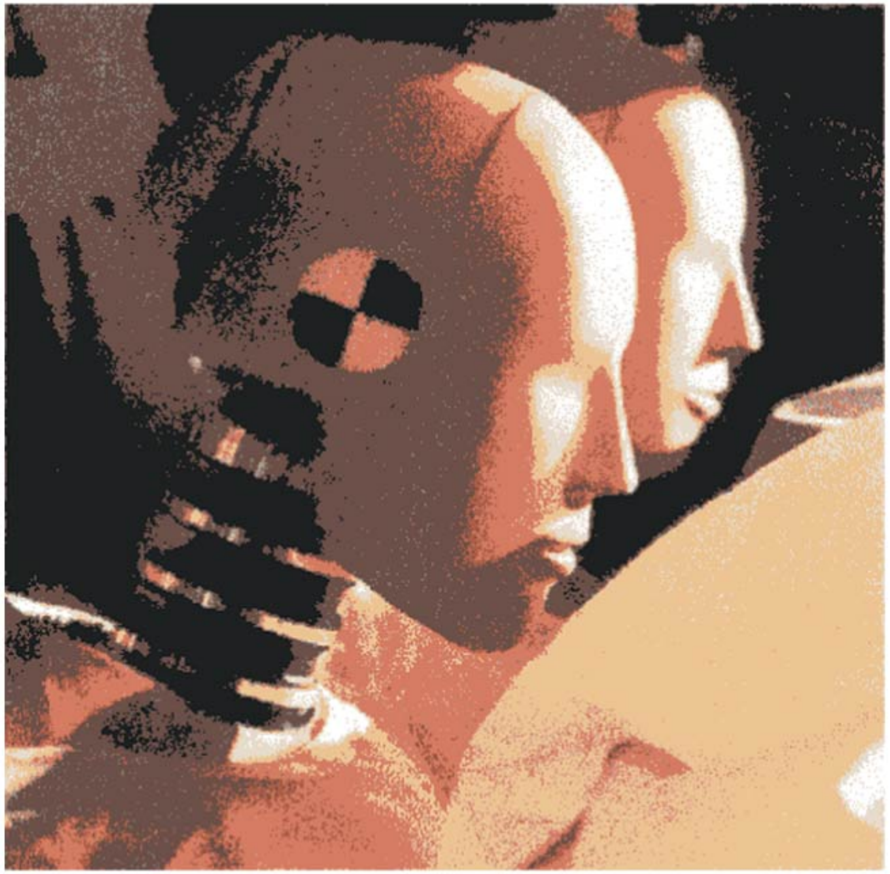
- Hence there is no net force on the child and sled, and they slide with a constant velocity.

When is Newton's first law valid?

- Suppose you are in a bus that is traveling on a straight road and speeding up.
- If you could stand in the aisle on roller skates, you would start moving *backward* relative to the bus as the bus gains speed.
- It looks as though Newton's first law is not obeyed; there is no net force acting on you, yet your velocity changes.
- The bus is accelerating with respect to the earth and is not a suitable frame of reference for Newton's first law.
- A frame of reference in which Newton's first law is valid is called an **inertial frame of reference**.

Crash test dummies

- From the frame of reference of the car, it seems as though a force is pushing the crash test dummies forward as the car comes to a sudden stop.

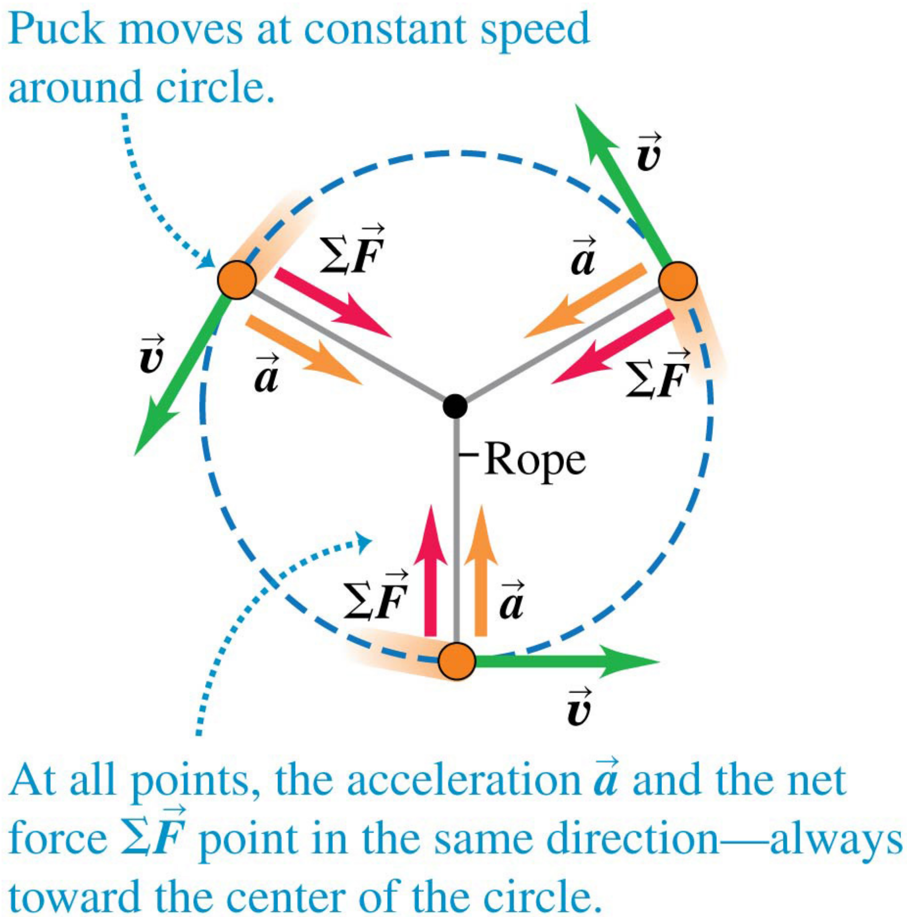


- But there is really no such force: As the car stops, the dummies keep moving forward as a consequence of Newton's first law.
- the frame of reference of the car: **non-inertial** frame of reference

An object undergoing uniform circular motion

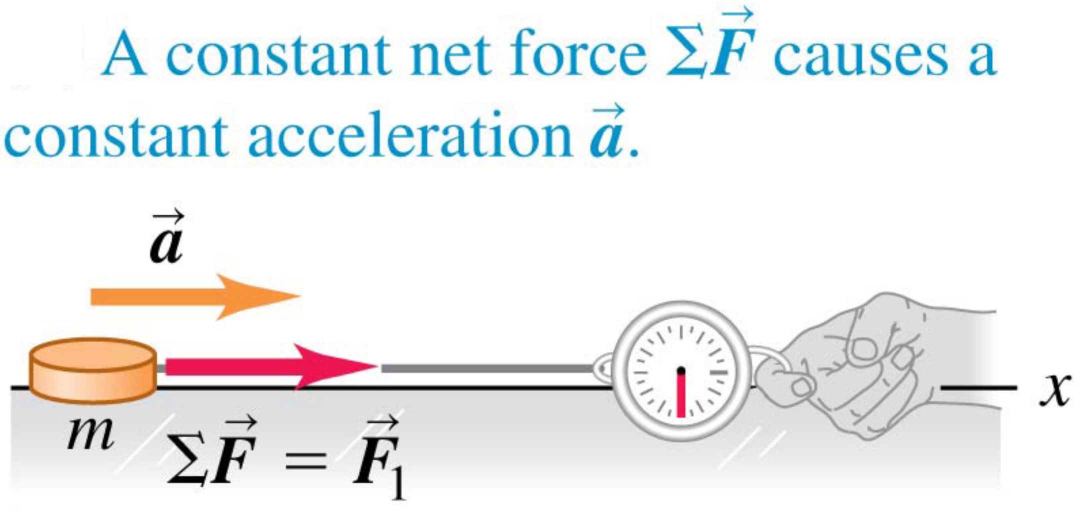
- As we have already seen, an object in uniform circular motion is accelerated toward the center of the circle. So the net force on the object must point toward the center of the circle.

- The frame following the object is also a non-inertial frame.



Force and acceleration

- The acceleration $\vec{a}$ of an object is directly proportional to the net force $\Sigma\vec{F}$ on the object.



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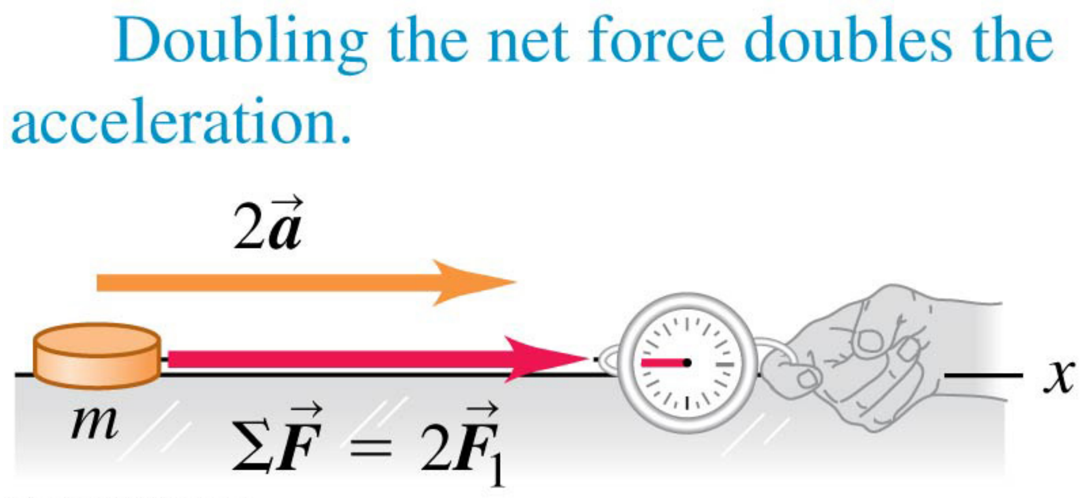

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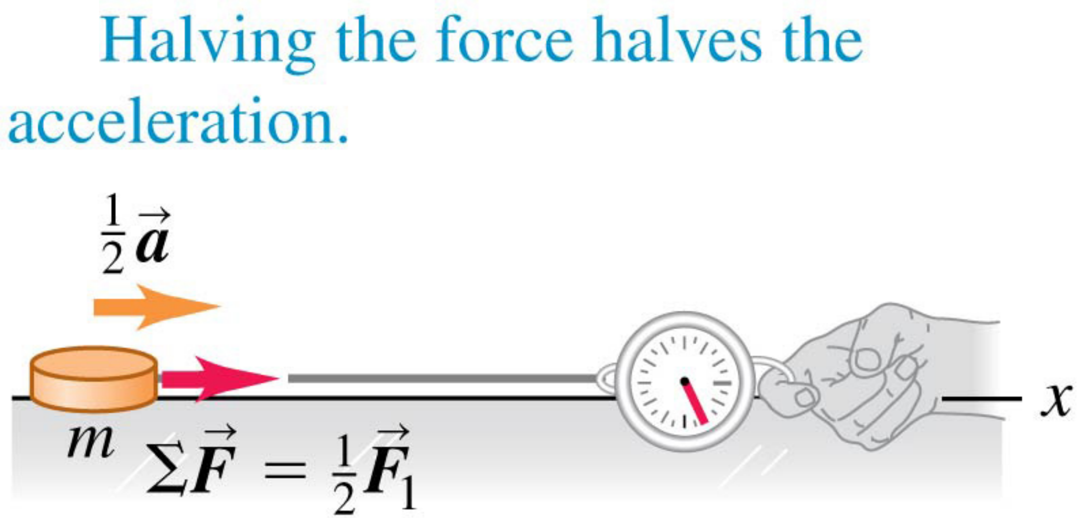
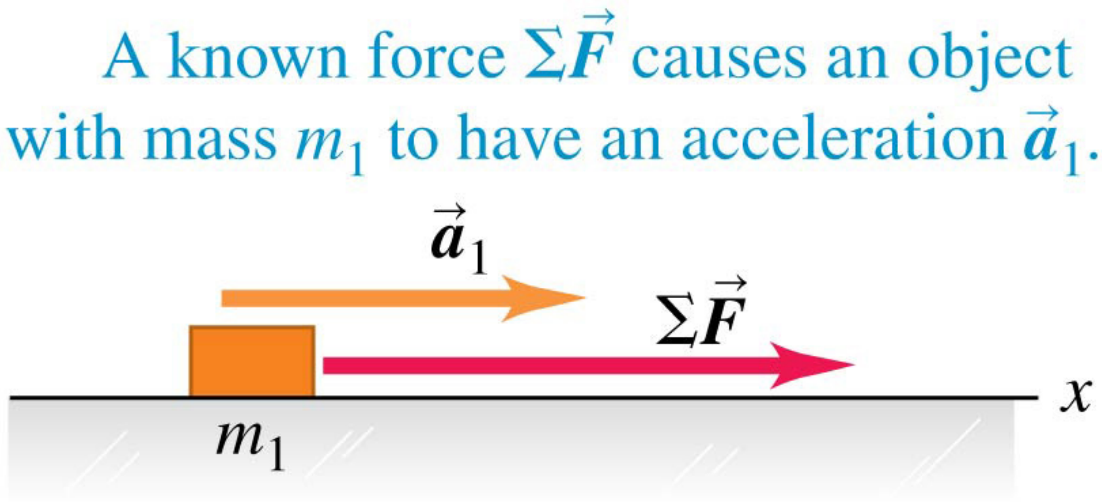


Mass and acceleration

- The acceleration of an object is inversely proportional to the object's mass if the net force remains fixed.



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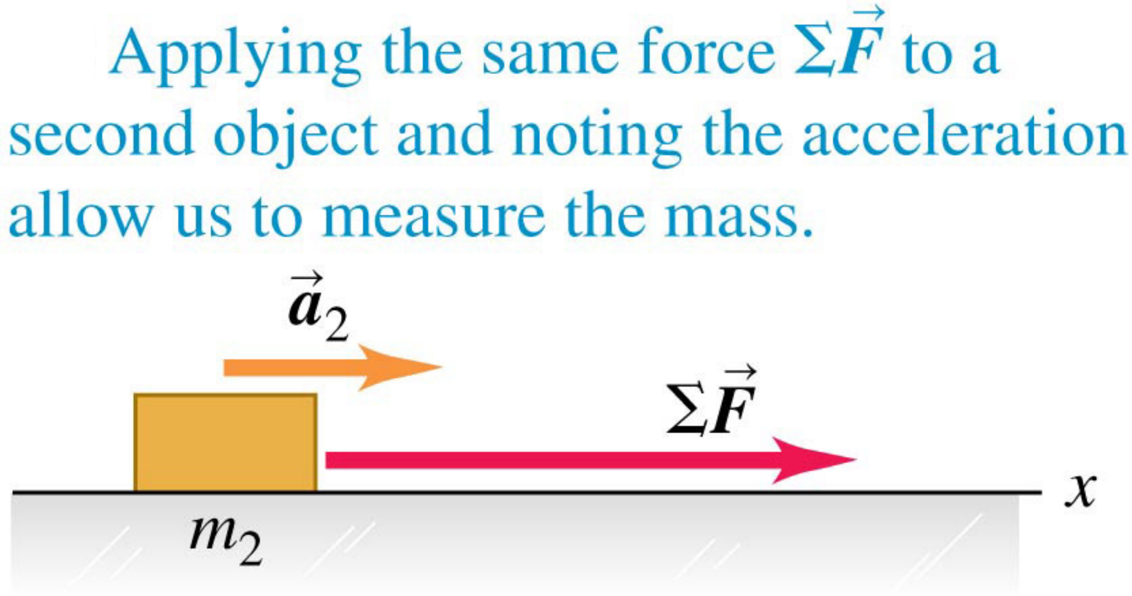

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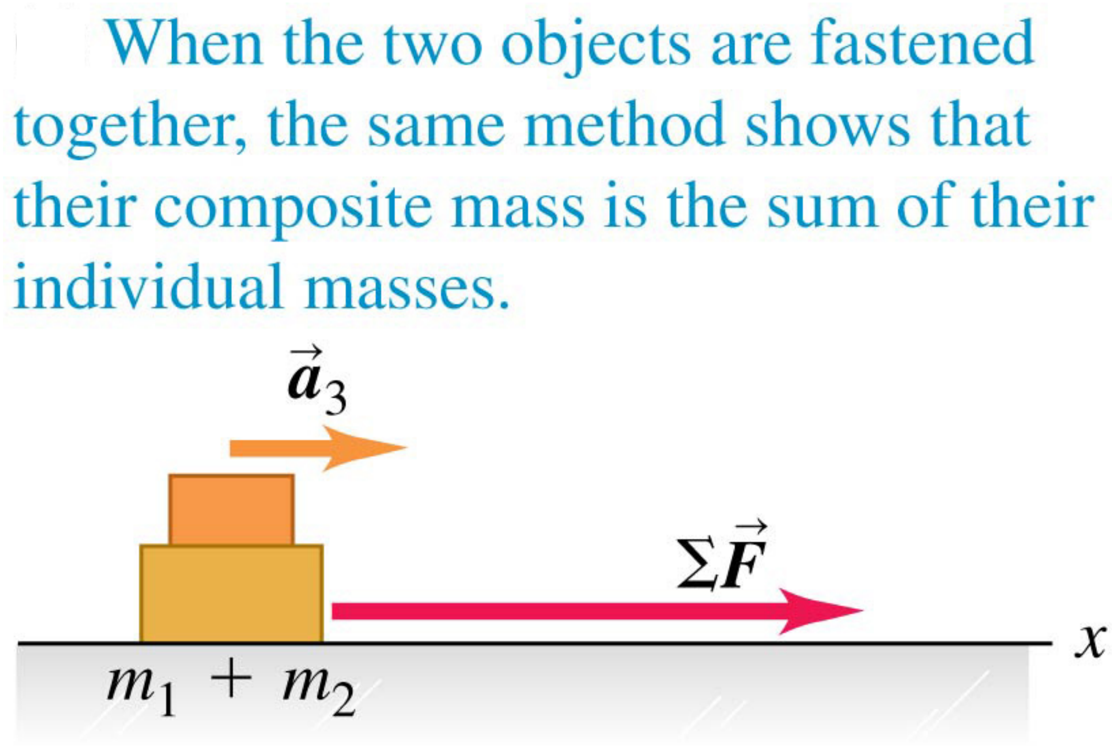


Newton's second law of motion

- The acceleration of an object is directly proportional to the net force acting on it, and inversely proportional to the mass of the object.

Newton's second law:
If there is a net force on a body ...

$$\sum \vec{F} = m\vec{a}$$

... the body accelerates in same direction as the net force.

Mass of body

- The SI unit for force is the newton (N).

$$1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$$

Mass and weight

- It is important to understand the difference between the mass and weight of a body!

- Mass is an absolute property of a body. It is independent of the gravitational field in which it is measured. The mass provides a measure of the resistance of a body to a change in velocity, as defined by Newton's second law of motion (m = $\vec{F}/\vec{a}$).

$$\underbrace{w}_{\text{Magnitude of weight of a body}} = \underbrace{m}_{\text{Mass of body}}\,\underbrace{g}_{\text{Magnitude of acceleration due to gravity}}$$

Mass and weight

- The *weight* of an object (on the earth) is the gravitational force that the earth exerts on it.
- The weight of a body is not absolute, since it depends on the gravitational field in which it is measured.
- The weight w of an object of mass m is:

Magnitude of weight of a body

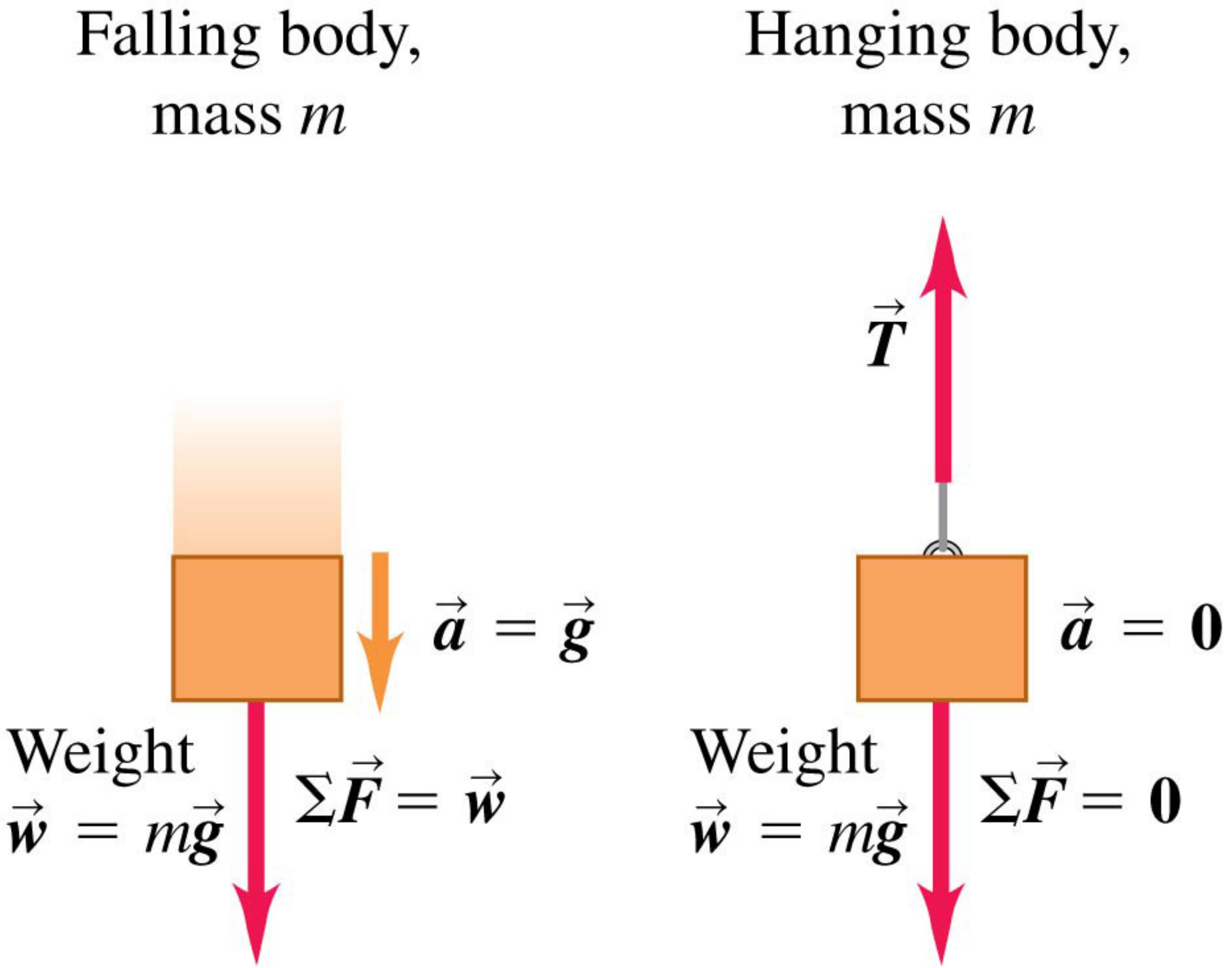
$$w = mg$$

Mass of body

Magnitude of acceleration due to gravity

- The value of g depends on altitude.
- On other planets, g will have an entirely different value than on the earth.

Relating the mass and weight of a body



- The relationship of mass to weight: $\vec{w} = m\vec{g}$.
- This relationship is the same whether a body is falling or stationary.

Systems of units

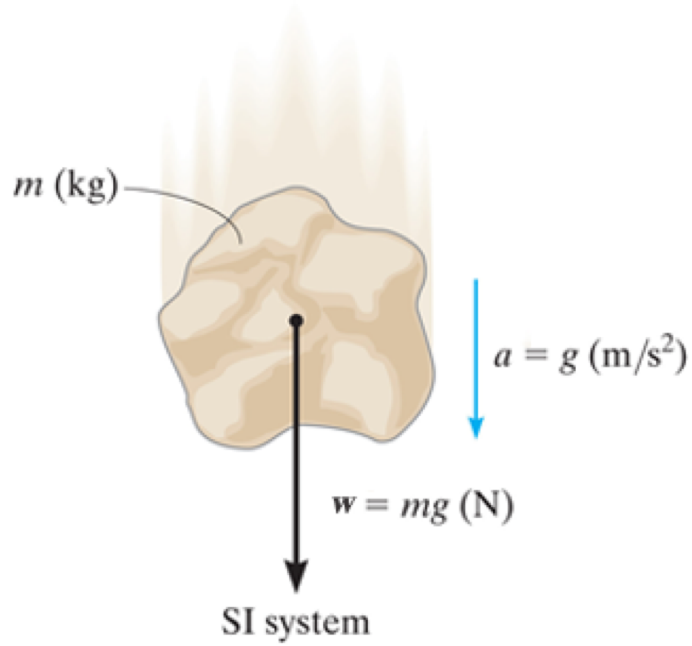
- We will use the SI system.
- In the British system, force is measured in *pounds*, distance in feet, and mass in *slugs*.
- In the cgs system, mass is in grams, distance in centimeters, and force in *dynes*.

System of Units	Force	Mass	Acceleration
SI	newton (N)	kilogram (kg)	m/s^2
cgs	dyne (dyn)	gram (g)	cm/s^2
British	pound (lb)	slug	ft/s^2

Systems of units: SI system

SI system: In the SI system of units, mass is a base unit and weight is a derived unit.

Typically, mass is specified in kilograms (kg), and weight is calculated from w = mg.



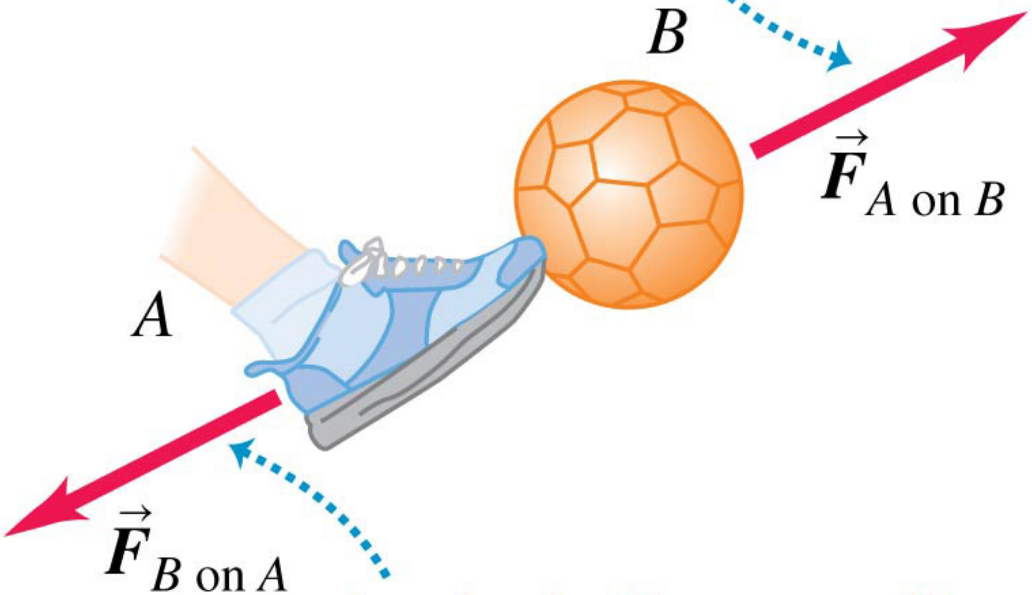
If the gravitational acceleration (g) is specified in units of m/s^2, then the weight is expressed in newtons (N).

On the earth's surface, g can be taken as g = 9.81 m/s^2.

w (N) = m (kg) g (m/s^2) => N = kg·m/s^2

Newton's third law

If body A exerts force $\vec{F}_{A \text{ on } B}$ on body B (for example, a foot kicks a ball) ...

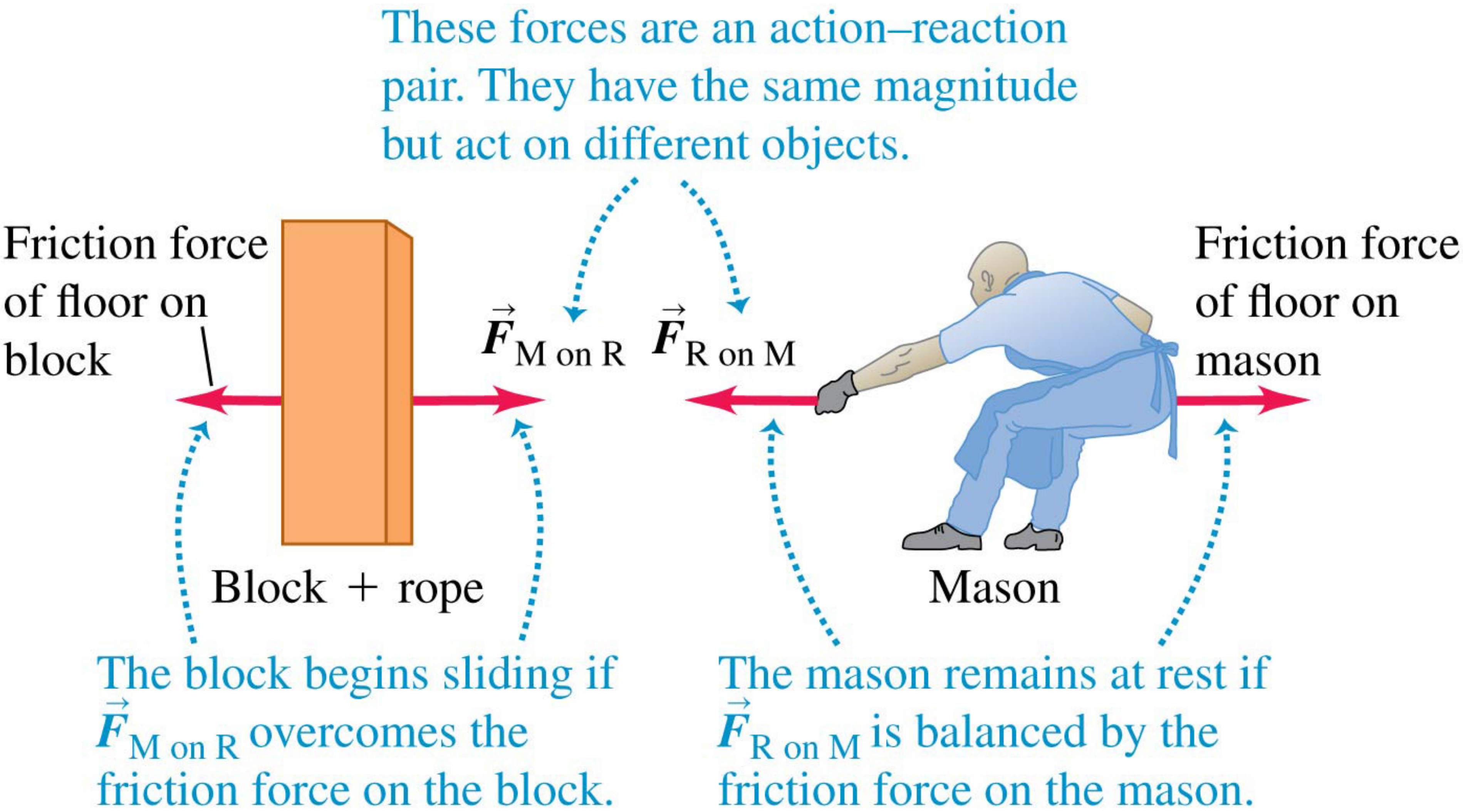


... then body B necessarily exerts force $\vec{F}_{B \text{ on } A}$ on body A (ball kicks back on foot).

The two forces have same magnitude but opposite directions: $\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$.

A paradox?

- If an object pulls back on you just as hard as you pull on it, how can it ever accelerate?



Newton’s third law

- The simple act of walking depends crucially on Newton’s third law.

- To start moving forward, you push backward on the ground with your foot.
- As a reaction, the ground pushes forward on your foot (and hence on your body as a whole) with a force of the same magnitude.
- This external force provided by the ground is what accelerates your body forward.

Newton's law of motion: summary

The motion of a particle is governed by Newton's three laws of motion.

First Law: A particle originally at rest, or moving in a straight line at constant velocity, will remain in this state if the resultant force acting on the particle is zero.

Second Law: If the resultant force on the particle is not zero, the particle experiences an acceleration in the same direction as the resultant force. This acceleration has a magnitude proportional to the resultant force.

Third Law: Mutual forces of action and reaction between two particles are equal, opposite, and collinear.

Newton's law of motion: summary

The first and third laws were used in developing the concepts of statics. Newton's second law forms the basis of the study of dynamics.

Mathematically, Newton's second law of motion can be written as

$$\vec{\boldsymbol{F}} = m\vec{\mathbf{a}}$$

where $\vec{\boldsymbol{F}}$ is the resultant unbalanced force acting on the particle, and $\vec{\mathbf{a}}$ is the acceleration of the particle. The positive scalar m is called the mass of the particle.

Newton's second law cannot be used when the particle's speed approaches the speed of light, or if the size of the particle is extremely small (~ size of an atom).

Newton's law of gravitational attraction (more in Chap 11)

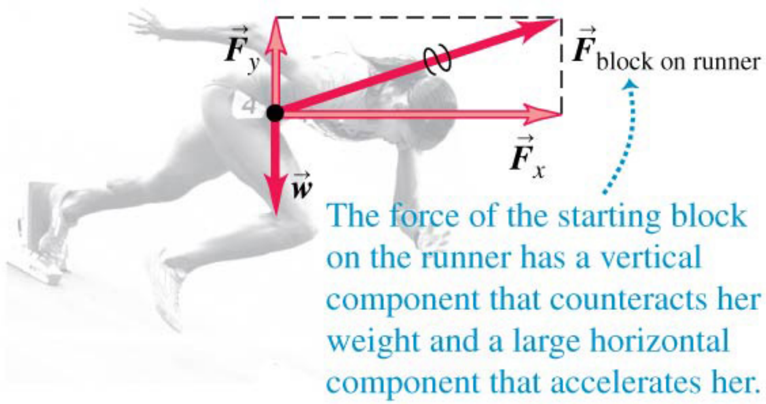
Any two particles or bodies have a mutually attractive gravitational force acting between them. Newton postulated the law governing this gravitational force as

$$F = G(m_1 m_2 / r^2)$$

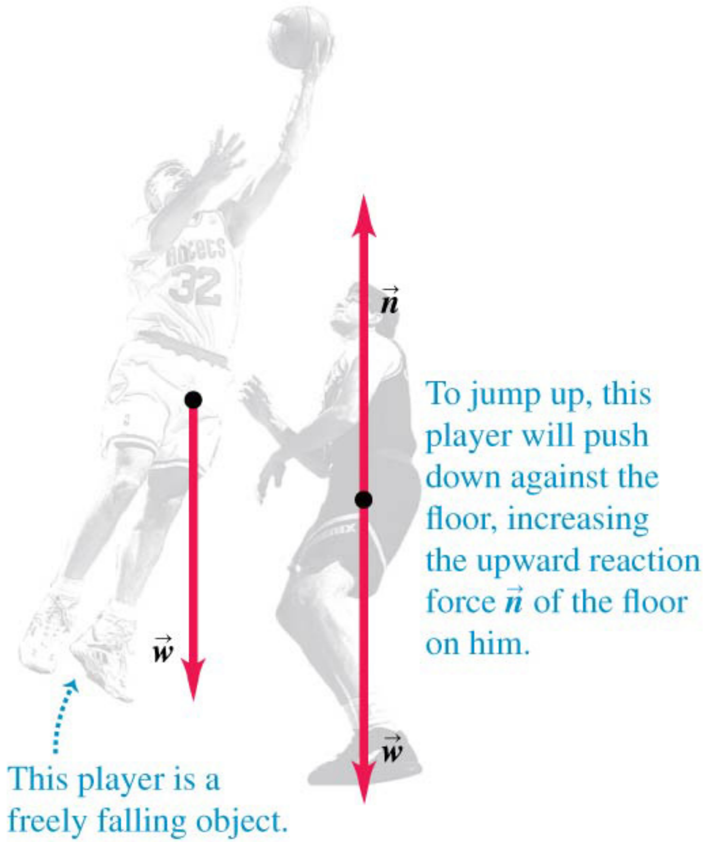
where F = force of attraction between the two bodies,
G = universal constant of gravitation ,
m_1, m_2 = mass of each body, and
r = distance between centers of the two bodies.

When near the surface of the earth, the only gravitational force having any sizable magnitude is that between the earth and the body. This force is called the weight of the body.

Free-body diagrams

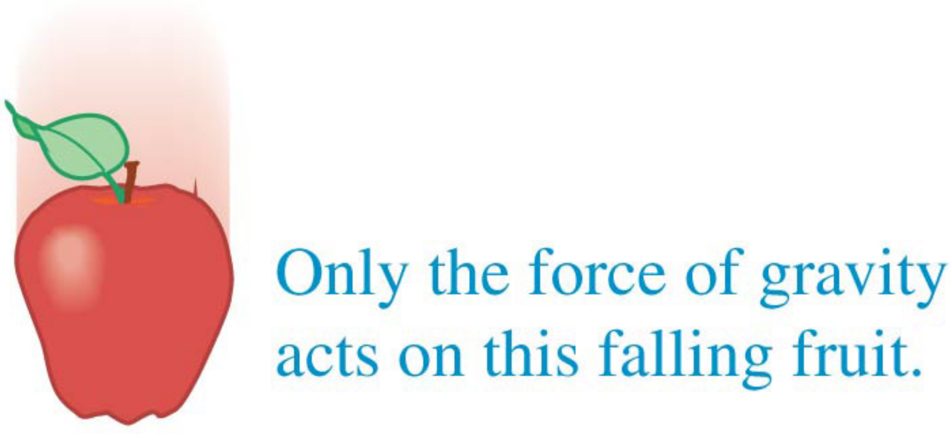


Free-body diagrams



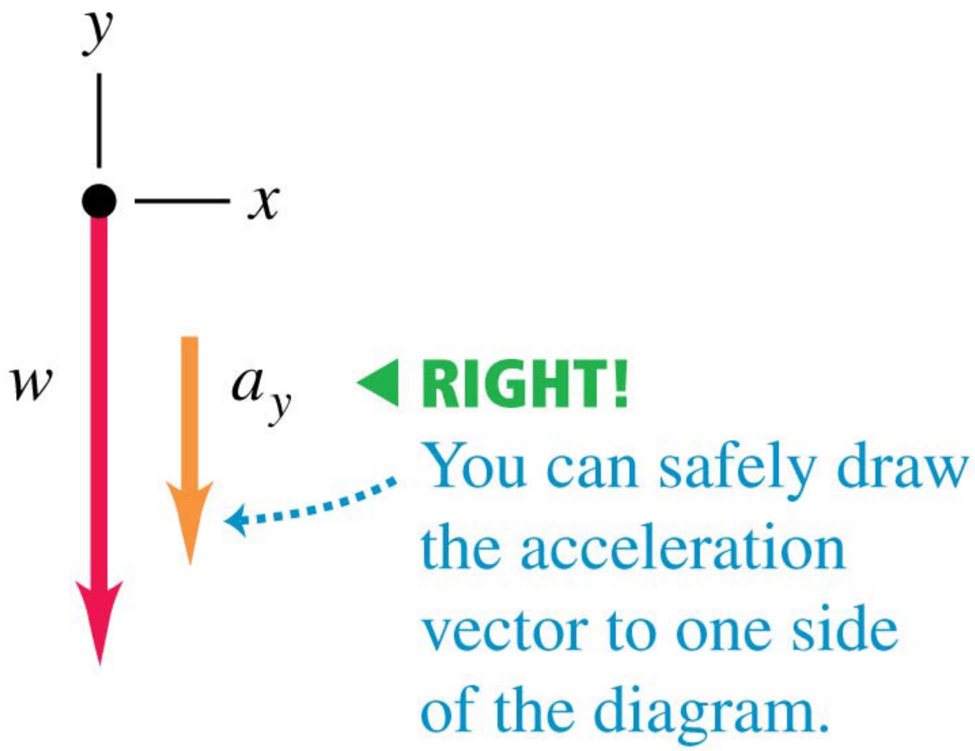
A note on free-body diagrams

- $m\vec{a}$ does *not* belong in a free-body diagram.



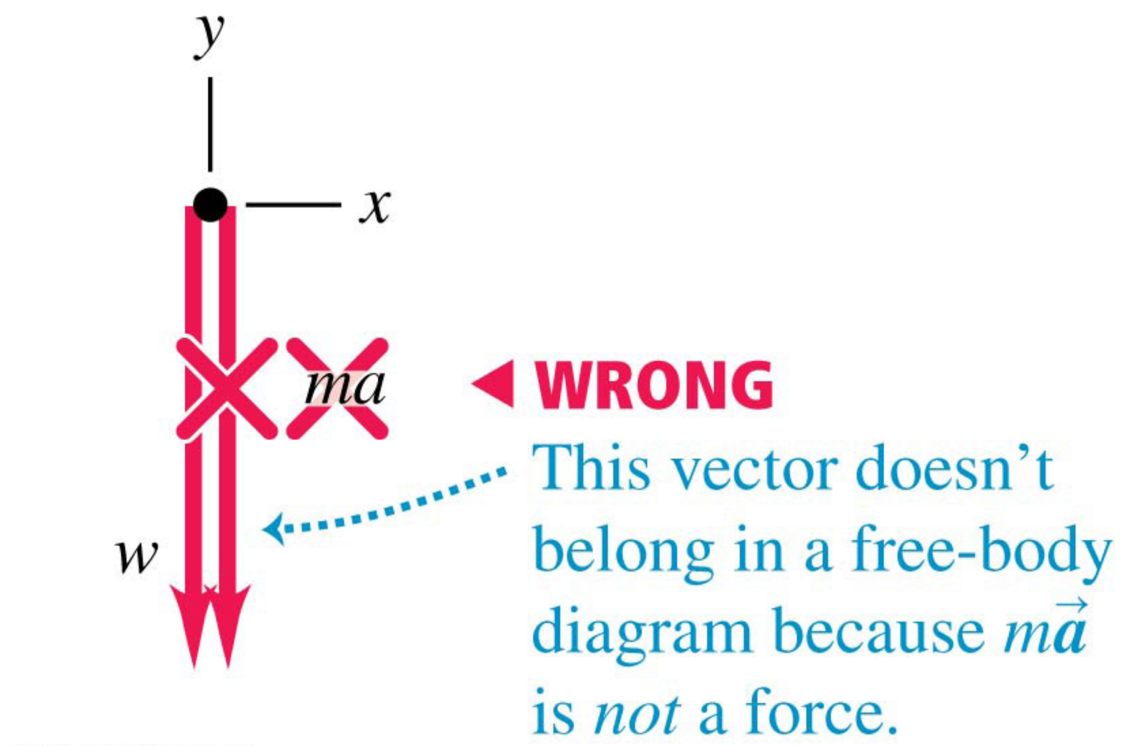
A note on free-body diagrams

- Correct free-body diagram



A note on free-body diagrams

- Incorrect free-body diagram

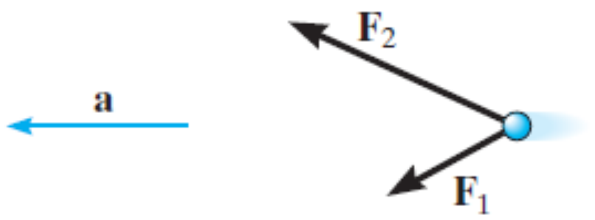


Equation of motion

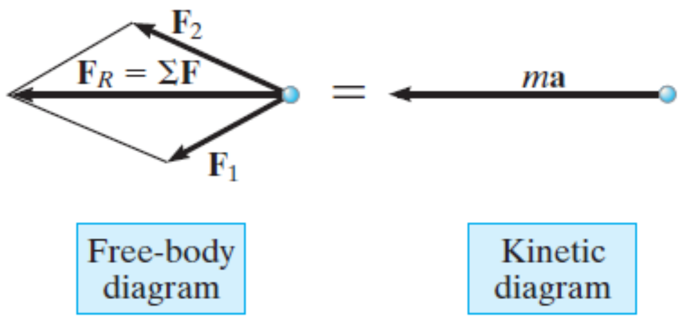
The motion of a particle is governed by Newton's second law, relating the unbalanced forces on a particle to its acceleration. If more than one force acts on the particle, the equation of motion can be written

$$\sum \vec{\boldsymbol{F}} = \vec{\boldsymbol{F}}_{\mathrm{R}} = \mathrm{m}\, \vec{\boldsymbol{a}}$$

where $\vec{\boldsymbol{F}}_{\mathrm{R}}$ is the resultant force, which is a vector summation of all the forces.



To illustrate the equation, consider a particle acted on by two forces.



First, draw the particle's free-body diagram, showing all forces acting on the particle. Next, draw the kinetic diagram, showing the inertial force m**a** acting in the same direction as the resultant force $\boldsymbol{F}_{\mathrm{R}}$.

Inertial frame of reference

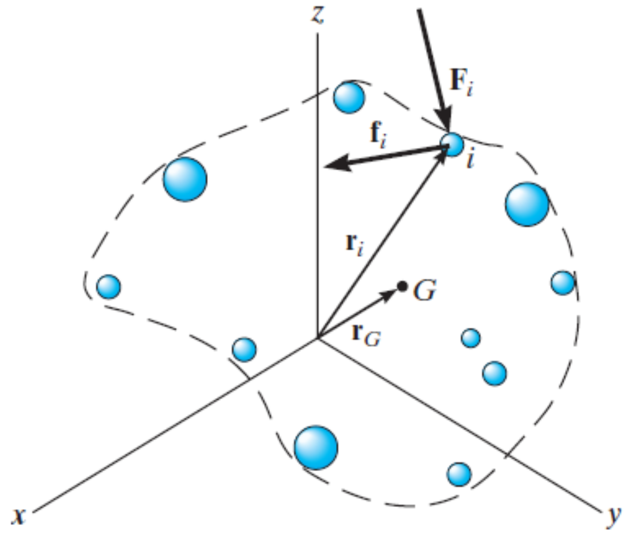
This equation of motion is only valid if the acceleration is measured in a Newtonian or inertial frame of reference. What does this mean?

For problems concerned with motions at or near the earth's surface, we typically assume our "inertial frame" to be fixed to the earth. We neglect any acceleration effects from the earth's rotation.

For problems involving satellites or rockets, the inertial frame of reference is often fixed to the stars.

Equation of motion for a system of particles

The equation of motion can be extended to include systems of particles. This includes the motion of solids, liquids, or gas



Inertial coordinate system

As in statics, there are internal forces and external forces acting on the system. What is the difference between them?

Using the definitions of $m = \sum m_i$ as the total mass of all particles and $\vec{\mathbf{a}}_G$ as the acceleration of the center of mass G of the particles, then $m\,\vec{\mathbf{a}}_G = \sum m_i\,\vec{\mathbf{a}}_i$.

For a system of particles: $\sum \vec{\mathbf{F}} = m\,\vec{\mathbf{a}}_G$ where $\sum \vec{\mathbf{F}}$ is the sum of the external forces acting on the entire system.

Key points

1) Newton's second law is a "law of nature"-- experimentally proven, not the result of an analytical proof.

2) Mass (property of an object) is a measure of the resistance to a change in velocity of the object.

3) Weight (a force) depends on the local gravitational field. Calculating the weight of an object is an application of $F = m\ a$, i.e., $w = m\ g$.

4) Unbalanced forces cause the acceleration of objects. This condition is fundamental to all dynamics problems!

Procedure for the application of the equation of motion

1) Select a convenient coordinate system. Rectangular, normal/tangential, or cylindrical coordinates may be used.

2) Draw a free-body diagram showing all external forces applied to the particle. Resolve forces into their appropriate components.

3) Draw the kinetic diagram, showing the particle's inertial force, m $\vec{\mathbf{a}}$. Resolve this vector into its appropriate components.

4) Apply the equations of motion in their scalar component form and solve these equations for the unknowns.

5) It may be necessary to apply the proper kinematic relations to generate additional equations.

Quiz

1. Newton's second law can be written in mathematical form as $\Sigma\,\vec{\mathbf{F}} = m\,\vec{\mathbf{a}}$. Within the summation of forces, $\Sigma\vec{\mathbf{F}}$, ________ are(is) not included.

   A) external forces   B) weight

   ✔ C) internal forces   D) All of the above.

2. The equation of motion for a system of n-particles can be written as $\Sigma\,\vec{\mathbf{F}}_i = \Sigma\, m_i\,\vec{\mathbf{a}}_i = m\,\vec{\mathbf{a}}_G$, where $\vec{\mathbf{a}}_G$ indicates ________.

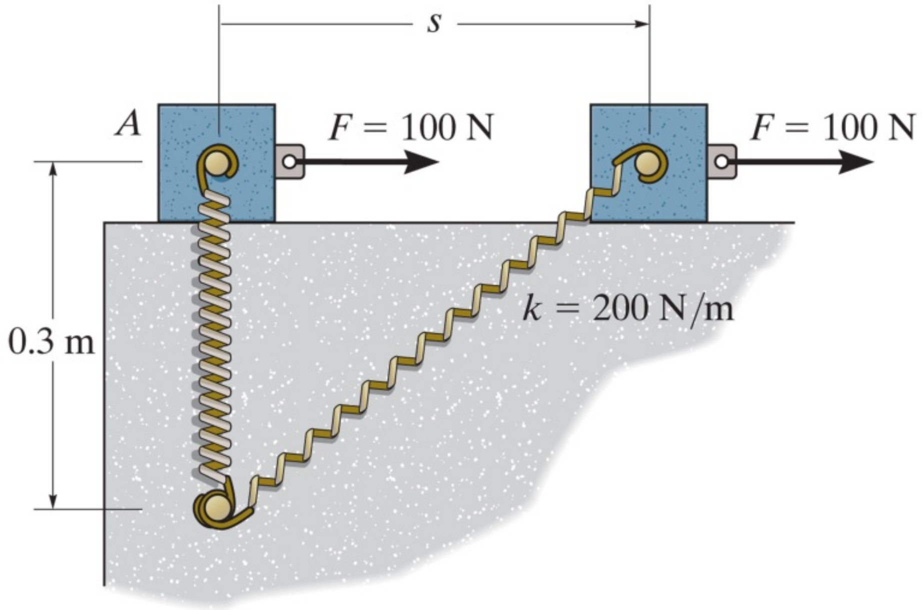
   A) summation of each particle's acceleration

   ✔ B) acceleration of the center of mass of the system

   C) acceleration of the largest particle

   D) None of the above.

Example 1



Given: A 25-kg block is subjected to the force F=100 N. The spring has a stiffness of k = 200 N/m and is un-stretched when the block is at A. The contact surface is smooth.

Find: Draw the free-body and kinetic diagrams of the block when s=0.4 m.

Plan:

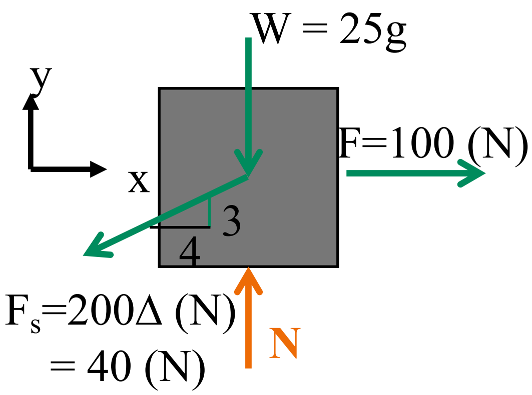
1) Define an inertial coordinate system.
2) Draw the block's free-body diagram, showing all external forces.
3) Draw the block's kinetic diagram, showing the inertial force vector in the proper direction.

Example 1

Solution:

1) An inertial x-y frame can be defined as fixed to the ground.

2) Draw the free-body diagram of the block:



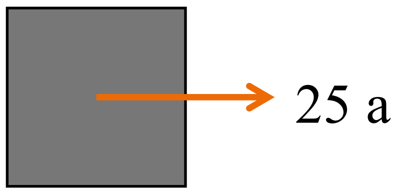
The weight force (w) acts through the block's center of mass. F is the applied load and F_s = 200Δ (N) is the spring force, where Δ is the spring deformation. When s = 0.4,

$$\Delta = 0.5 – 0.3 = 0.2 \text{ m.}$$

The normal force (N) is perpendicular to the surface. There is no friction force since the contact surface is smooth.

Example 1

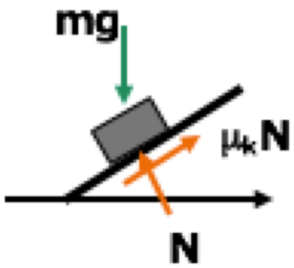
3) Draw the kinetic diagram of the block.

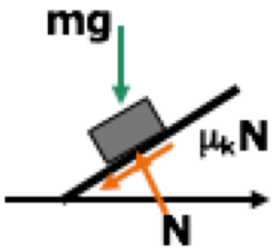


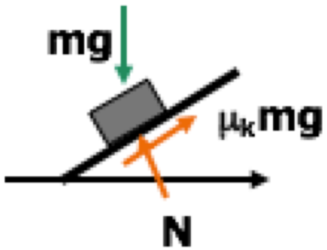
The block will be moved to the right. The acceleration can be directed to the right if the block is speeding up or to the left if it is slowing down.

Quiz

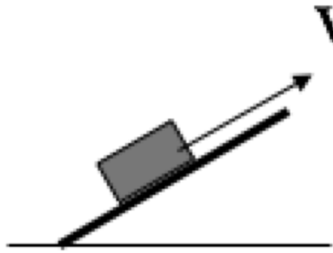
1. The block (mass = m) is moving upward with a speed v. Draw the FBD if the kinetic friction coefficient is μ_k.

A) 

B) 

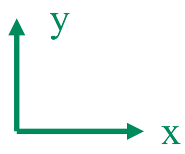
C) 

D) None of the above.

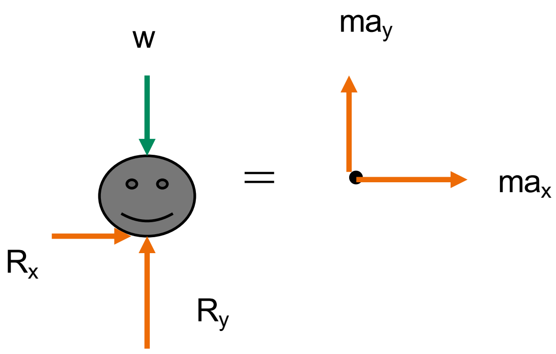


Quiz

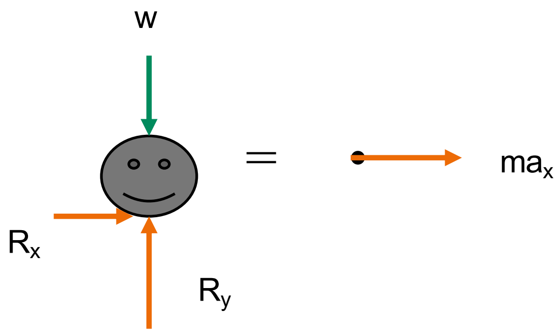
2. Packaging for oranges is tested using a machine that exerts $a_y = 20$ m/s^2 and $a_x = 3$ m/s^2, simultaneously. Select the correct FBD and kinetic diagram for this condition.



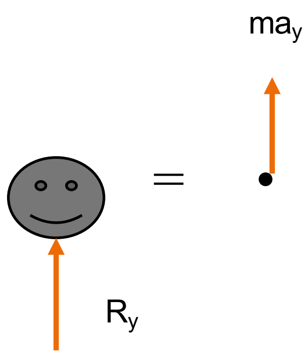
✔ A)



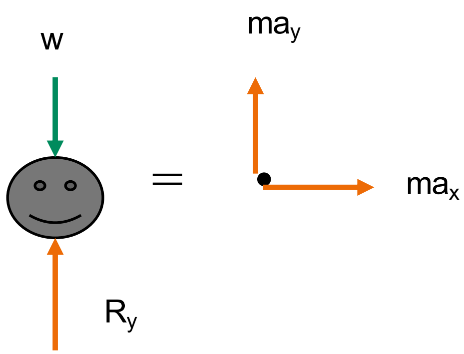
B)



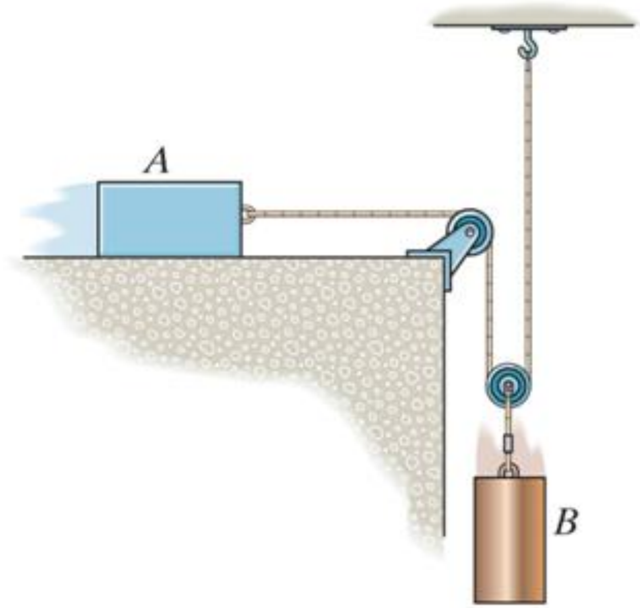
C)



D)



Example 2



Given: The block and cylinder have a mass of m. The coefficient of kinetic friction at all surfaces of contact is μ. Block A is moving to the right.

Find: Draw the free-body and kinetic diagrams of each block.

Plan:

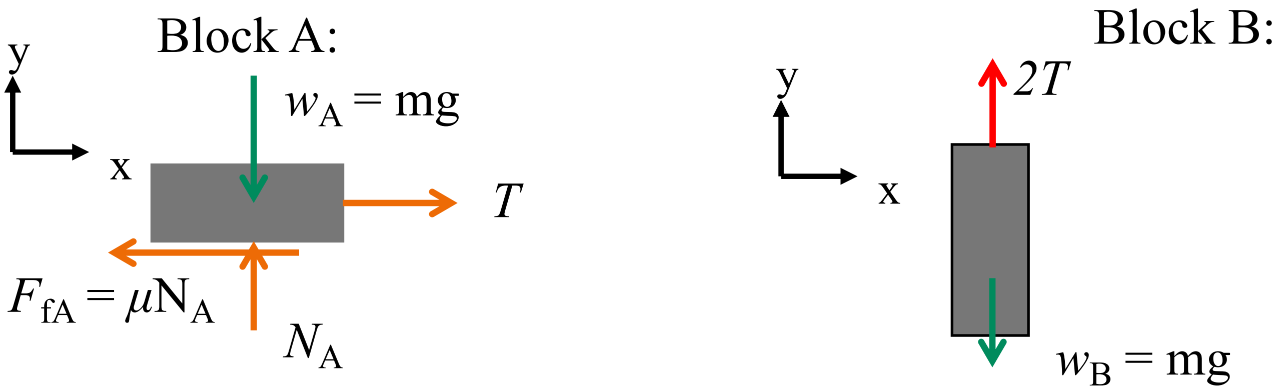
1) Define an inertial coordinate system.
2) Draw the free-body diagrams for each block, showing all external forces.
3) Draw the kinetic diagrams for each block, showing the inertial forces.

Example 2

Solution:

1) An inertial x-y frame can be defined as fixed to the ground.

2) Draw the free-body diagram of each block:



The friction force opposes the motion of block A relative to the surfaces on which it slides.

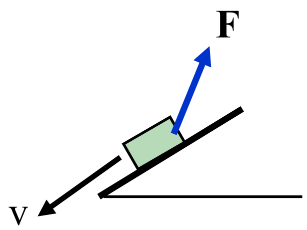
3) Draw the kinetic diagram of each block:

Quiz

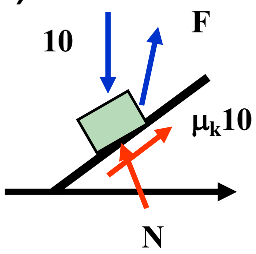
1. Internal forces are not included in an equation of motion analysis because the internal forces are_____

   A) equal to zero.
   B) equal and opposite and do not affect the calculations.
   C) negligibly small.
   D) not important.

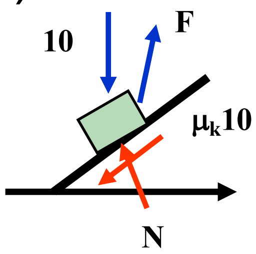
2. A 10-kg block is initially moving down a ramp with a velocity of **v**. The force **F** is applied to bring the block to rest. Select the correct FBD.



A)



B)



C)