| Entered | Answer Preview | Result |
|---|---|---|
| 0.656986598718789 | $\sin(7)$ | correct |
| 0 | $0$ | correct |
| 7744.75603317678 | $\frac{3\sqrt{3}e^{8}}{2}$ | correct |
| 4471.43698056259 | $\frac{3e^{8}}{2}$ | correct |
| -3.83585729543983 | $e^{\frac{3}{2}}\cos\left(\frac{3\sqrt{3}}{2}\right)$ | correct |
| 2.31770052686035 | $e^{\frac{3}{2}}\sin\left(\frac{3\sqrt{3}}{2}\right)$ | correct |

All of the answers above are correct.

(1 point) Write each of the given numbers in the form $a + bi$ :

(a) $\frac{e^{7i} - e^{-7i}}{2i}$

sin7 + 0 $i$,

(b) $3e^{\left(8+\frac{i\pi}{6}\right)}$

3*sqrt(3)*e^8/2 + 3*e^8/2 $i$,

(c) $e^{\left(3e^{\left(\frac{i\pi}{3}\right)}\right)}$

e^(3/2)*cos(3*sqrt(3) + e^(3/2)*sin(3*sqrt(3)/ $i$.

$\theta =$ tan^-1(1/5)

B. Find $r$ and $\theta$ (use an angle between $-\pi$ and $\pi$) such that the blue point $z_2 = re^{i\theta}$:

$r =$ sqrt(2)

$\theta =$ pi+tan^-1(-1)

Using the law of exponents it is really easy to multiply complex numbers represented in polar coordinates -- the angles just add!

$$(r_1 e^{i\theta_1})(r_2 e^{i\theta_2}) = r_1 r_2 e^{i(\theta_1+\theta_2)}.$$

C. Find $z_1 \cdot z_2 = re^{i\theta}$ using polar coordinates and your answer above:

$r =$ 2*sqrt(13)

$\theta =$ tan^-1(1/5)+pi+

Check your answer by doing the standard multiplication and then converting to polar coordinates. Can you plot this number on the graph?

(1 point) For some practice working with complex numbers:

Calculate

$(4 + 6i) + (4 + 5i) =$ 8+11i ,

$(4 + 6i) - (4 + 5i) =$ i ,

$(4 + 6i)(4 + 5i) =$ -14+44i .

The complex conjugate of $(1 + i)$ is $(1 - i)$. In general to obtain the complex conjugate reverse the sign of the imaginary part. (Geometrically this corresponds to finding the "mirror image" point in the complex plane by reflecting through the $x$-axis. The complex conjugate of a complex number $z$ is written with a bar over it: $\overline{z}$ and read as "z bar".

Notice that if $z = a + ib$, then

$(z)\left(\overline{z}\right) = |z|^2 = a^2 + b^2$

which is also the square of the distance of the point $z$ from the origin. (Plot $z$ as a point in the "complex" plane in order to see this.)

If $z = 4 + 6i$ then $\overline{z} =$ 4-6i and $|z| =$ sqrt(52) .

You can use this to simplify complex fractions. Multiply the numerator and denominator by the complex conjugate of the denominator to make the denominator real.

$\frac{4 + 6i}{4 + 5i} =$ 46/41 $+i$ 4/41 .

Two convenient functions to know about pick out the real and imaginary parts of a complex number.

$Re(a + ib) = a$ (the real part (coordinate) of the complex number), and
$Im(a + ib) = b$ (the imaginary part (coordinate) of the complex number. $Re$ and $Im$ are linear functions -- now that you know about linear behavior you may start noticing it often.

(1 point) Evaluate the following expressions and write them in the form $a + bi$.

$(-2 - 9i)(-3 + 3i) =$ 33+21i .

$(-2 - 9i)\overline{(-3 + 3i)} =$ -21+33i .

$\overline{(-2 - 9i)(-3 + 3i)} =$ 33-21i .

$(-2 - 9i)\overline{(-2 - 9i)} =$ 85 .

$|-2 - 9i| =$ sqrt(85) .

(1 point) Place the following in order:

(a) $|z_1| + |z_2|$

(b) $|z_2| - |z_1|$

(c) $|z_1 + z_2|$

(d) $\left||z_2| - |z_1|\right|$

**Answer:** b $\leq$ d $\leq$ c $\leq$ a

(1 point) Calculate:

(a) $\left|\frac{4 + 3i}{-3 - 4i}\right| =$ 1 ,

(b) $\left|\overline{(1 + i)(4 - i)(3 - i)}\right| =$ 2sqrt(85) ,

(c) $\left|\frac{i(2 + i)^3}{(1 - 2i)^2}\right| =$ sqrt(5) ,

(d) $\left|\frac{(\pi + i)^{100}}{(\pi - i)^{100}}\right| =$ 1 .

(1 point)

Write the complex number $z = (1 + \sqrt{3}\,i)^{14}$ in polar form: $z = r(\cos\theta + i\sin\theta)$ where

$r =$ 2^14 and $\theta =$ (2pi)/3

The angle should satisfy $0 \leq \theta < 2\pi$.

(1 point) Write each of the given numbers in the form $a + bi$ :

(a) $e^{-\frac{i\pi}{4}}$

sqrt(2)/2 + -sqrt(2)/2 $i$,

(b) $\frac{e^{(1+i2\pi)}}{e^{\left(-1+\frac{i\pi}{2}\right)}}$

0 + -e^2 $i$,

(c) $e^{e^{i}}$

e^(cos1)cos(sin + e^(cos1)sin(sin $i$.