PHY1203: General Physics III

Chapter 39
Atomic structure; matter wave; blackbody radiation; uncertainty principle – *Part 2*

Recap: The Bohr model of hydrogen

- Bohr found that the magnitude of the electron's angular momentum is quantized; that is, this magnitude must be an integral multiple of $h/2\pi$.
- Let's number the orbits by the **principal quantum number** n, where $n = 1, 2, 3, \ldots$, and call the radius of orbit n r_n and the speed of the electron in that orbit v_n.
- The magnitude of the angular momentum of an electron of mass m in such an orbit is:

Quantization of angular momentum:

Orbital angular momentum

Principal quantum number ($n = 1, 2, 3, \ldots$)

$$L_n = mv_n r_n = n\frac{h}{2\pi}$$

Planck's constant

Electron mass

Electron speed

Electron orbital radius

Recap: The Bohr model of hydrogen

- The orbital speed of the electron in Bohr's model of a hydrogen atom is:

Orbital speed in nth orbit in the Bohr model

$$v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh}$$

Magnitude of electron charge

Planck's constant

Electric constant

Principal quantum number ($n = 1, 2, 3, \ldots$)

- The radius of this orbit is:

Radius of nth orbit in the Bohr model

$$r_n = n^2 a_0$$

Bohr radius

Principal quantum number ($n = 1, 2, 3, \ldots$)

where the Bohr radius is $a_0 = 5.29 \times 10^{-11}$ m.

- Total energies in the Bohr model:

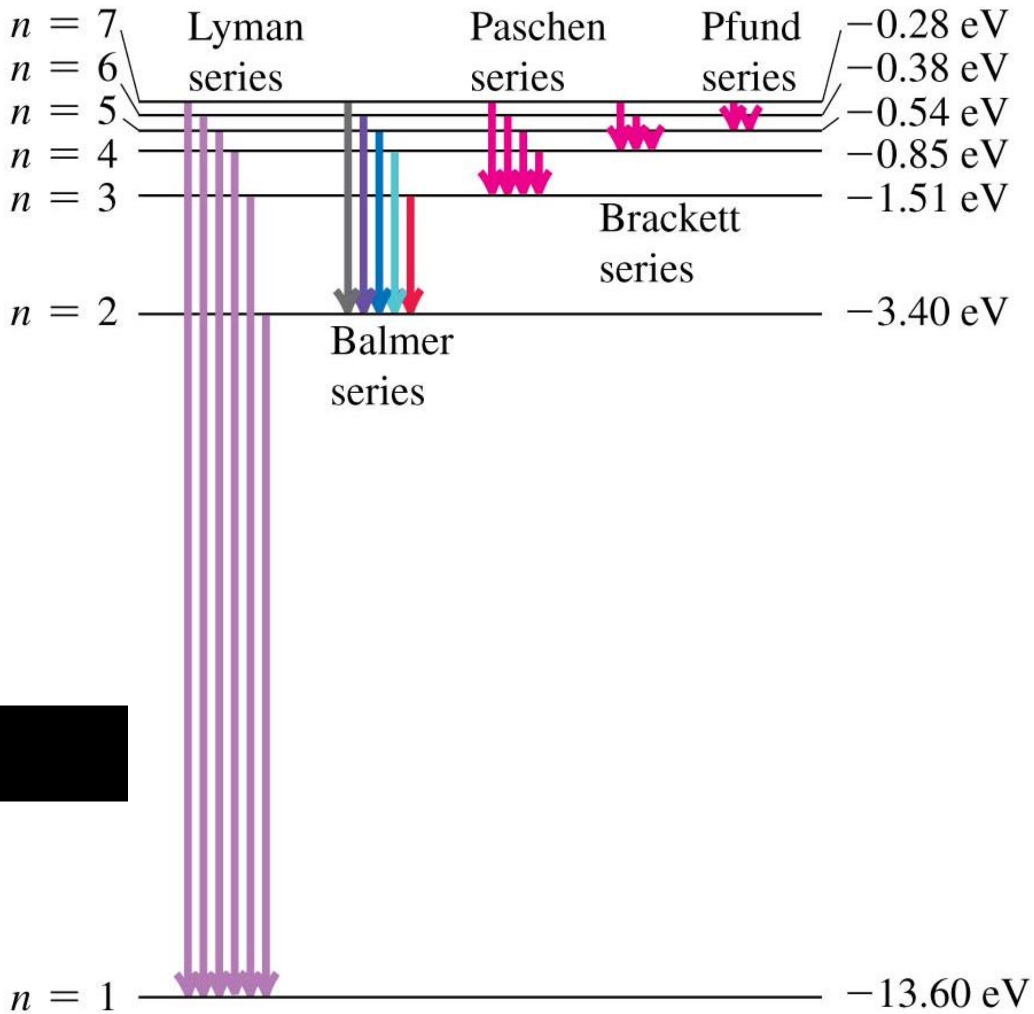
$$K_n = \tfrac{1}{2}mv_n^2 = \frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}$$ (kinetic energies in the Bohr model)

$$U_n = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_n} = -\frac{1}{\epsilon_0^2}\frac{me^4}{4n^2h^2}$$ (potential energies in the Bohr model)

$$E_n = K_n + U_n = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2}$$ (total energies in the Bohr model)

Hydrogen spectrum in more detail

- The Balmer series is not the entire spectrum of hydrogen; it's just the visible-light portion.
- Hydrogen also has a series of spectral lines in the ultraviolet (Lymann), and several series of spectral lines in the infrared.



Rydberg formula

- The wavelength of the photon emitted in a transition from level n_U to level n_L:

$$\frac{hc}{\lambda} = E_{n_U} - E_{n_L} = \left(-\frac{hcR}{n_U^{\,2}}\right) - \left(-\frac{hcR}{n_L^{\,2}}\right) = hcR\left(\frac{1}{n_L^{\,2}} - \frac{1}{n_U^{\,2}}\right)$$

$$\frac{1}{\lambda} = R\left(\frac{1}{n_L^{\,2}} - \frac{1}{n_U^{\,2}}\right) \qquad \text{(hydrogen wavelengths in the Bohr model, } n_L < n_U\text{)}$$

- **Rydberg constant** $R = 1.097 * 10^{7}\ \mathrm{m}^{-1}$

Q-RT39.1

An atom has four energy levels, with energies as shown. The figure shows six possible transitions (A through F) between these energy levels. In each transition the atom emits a photon. ***Rank*** the transitions in order of the *wavelength* of the emitted photon, from longest to shortest.

Answer: AFCBED

A. $n = 4 \rightarrow n = 3$

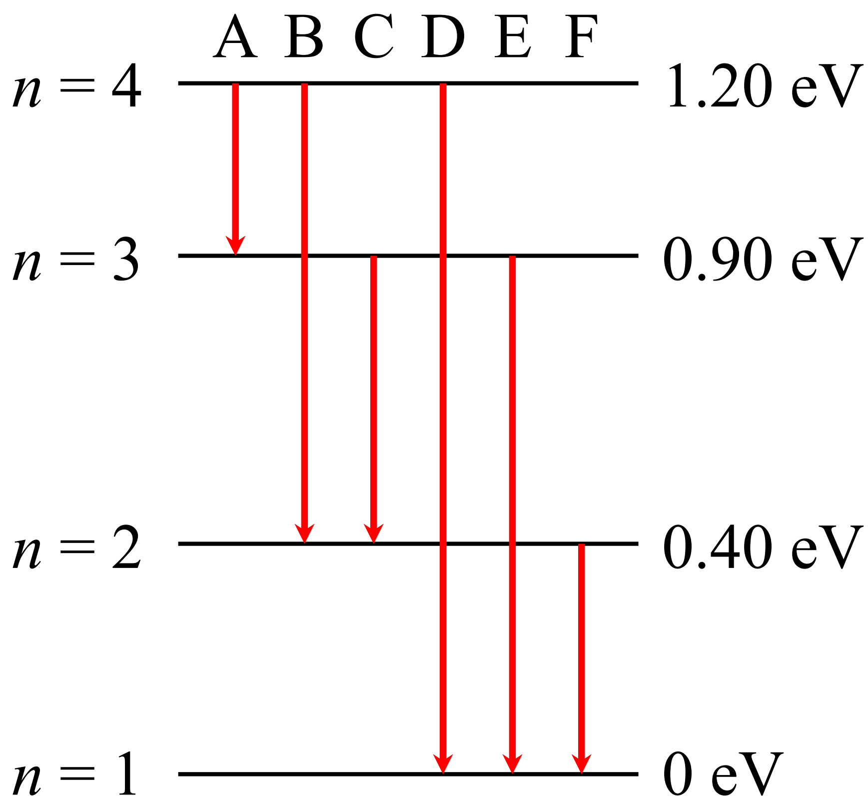
B. $n = 4 \rightarrow n = 2$

C. $n = 3 \rightarrow n = 2$

D. $n = 4 \rightarrow n = 1$

E. $n = 3 \rightarrow n = 1$

F. $n = 2 \rightarrow n = 1$



Example 39.6: Exploring the Bohr model

Find the kinetic, potential, and total energies of the hydrogen atom in the first excited level, and find the wavelength of the photon emitted in a transition from that level to the ground level.

EXECUTE: We could evaluate Eqs. (39.12), (39.13), and (39.14) for the nth level by substituting the values of m, e, ϵ_0, and h. But we can simplify the calculation by comparing with Eq. (39.15), which shows that the constant $me^4/8\epsilon_0^2h^2$ that appears in Eqs. (39.12), (39.13), and (39.14) is equal to hcR:

$$\begin{aligned}\frac{me^4}{8\epsilon_0^2h^2} &= hcR \\ &= (6.626 \times 10^{-34}\ \mathrm{J \cdot s})(2.998 \times 10^8\ \mathrm{m/s}) \\ &\quad \times (1.097 \times 10^7\ \mathrm{m^{-1}}) \\ &= 2.179 \times 10^{-18}\ \mathrm{J} = 13.60\ \mathrm{eV}\end{aligned}$$

This allows us to rewrite Eqs. (39.12), (39.13), and (39.14) as

$$K_n = \frac{13.60\ \mathrm{eV}}{n^2} \qquad U_n = \frac{-27.20\ \mathrm{eV}}{n^2} \qquad E_n = \frac{-13.60\ \mathrm{eV}}{n^2}$$

For the first excited level ($n = 2$), we have $K_2 = 3.40$ eV, $U_2 = -6.80$ eV, and $E_2 = -3.40$ eV. For the ground level ($n = 1$), $E_1 = -13.60$ eV. The energy of the emitted photon is then $E_2 - E_1 = -3.40\ \mathrm{eV} - (-13.60\ \mathrm{eV}) = 10.20$ eV, and

$$\begin{aligned}\lambda &= \frac{hc}{E_2 - E_1} = \frac{(4.136 \times 10^{-15}\ \mathrm{eV \cdot s})(3.00 \times 10^8\ \mathrm{m/s})}{10.20\ \mathrm{eV}} \\ &= 1.22 \times 10^{-7}\ \mathrm{m} = 122\ \mathrm{nm}\end{aligned}$$

This is the wavelength of the Lyman-alpha (L_α) line, the longest-wavelength line in the Lyman series of ultraviolet lines in the hydrogen spectrum (see Fig. 39.24).

Another test of Bohr's theory

- One additional test of the Bohr model is its predicted value of the *ionization energy* of the hydrogen atom. This is the energy required to remove the electron completely from the atom.

- Ionization corresponds to a transition from the ground level $n = 1$ to an infinitely large orbit radius $n = \infty$, so the energy that must be added to the atom is $E_\infty - E_1 = 0 - E_1 = 13.606$ eV.

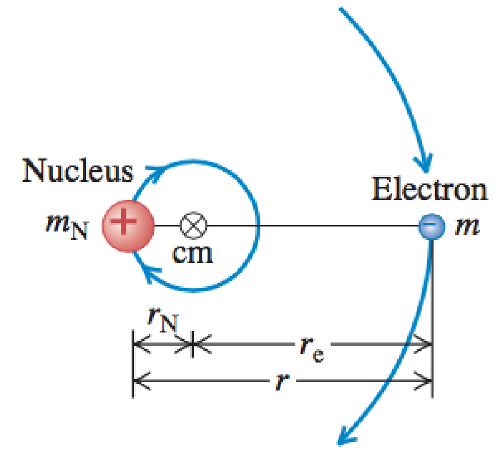
- The ionization energy can also be measured directly; the result is 13.60 eV. These two values agree within 0.1%.

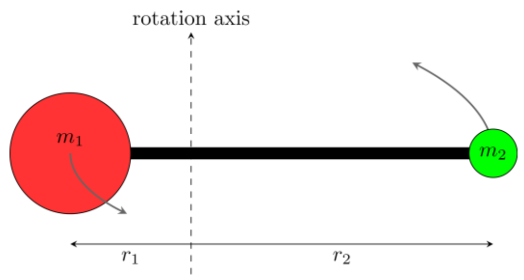
Nuclear motion and reduced mass of an atom

- Why the difference 0.1%?
- The explanation is that we assumed that the nucleus (a proton) remains at rest. However, the proton and electron *both* revolve in circular orbits about their common center of mass.
- It turns out that we can take this motion into account very simply by using in Bohr's equations not the electron rest mass m but a quantity called the **reduced mass** m_r of the system.
- For a system composed of two bodies of masses m_1 and m_2, the reduced mass is

$$m_r = \frac{1}{\frac{1}{m_1} + \frac{1}{m_2}} = \frac{m_1 m_2}{m_1 + m_2}.$$

39.26 The nucleus and the electron both orbit around their common center of mass. The distance r_N has been exaggerated for clarity; for ordinary hydrogen it actually equals $r_e/1836.2$.



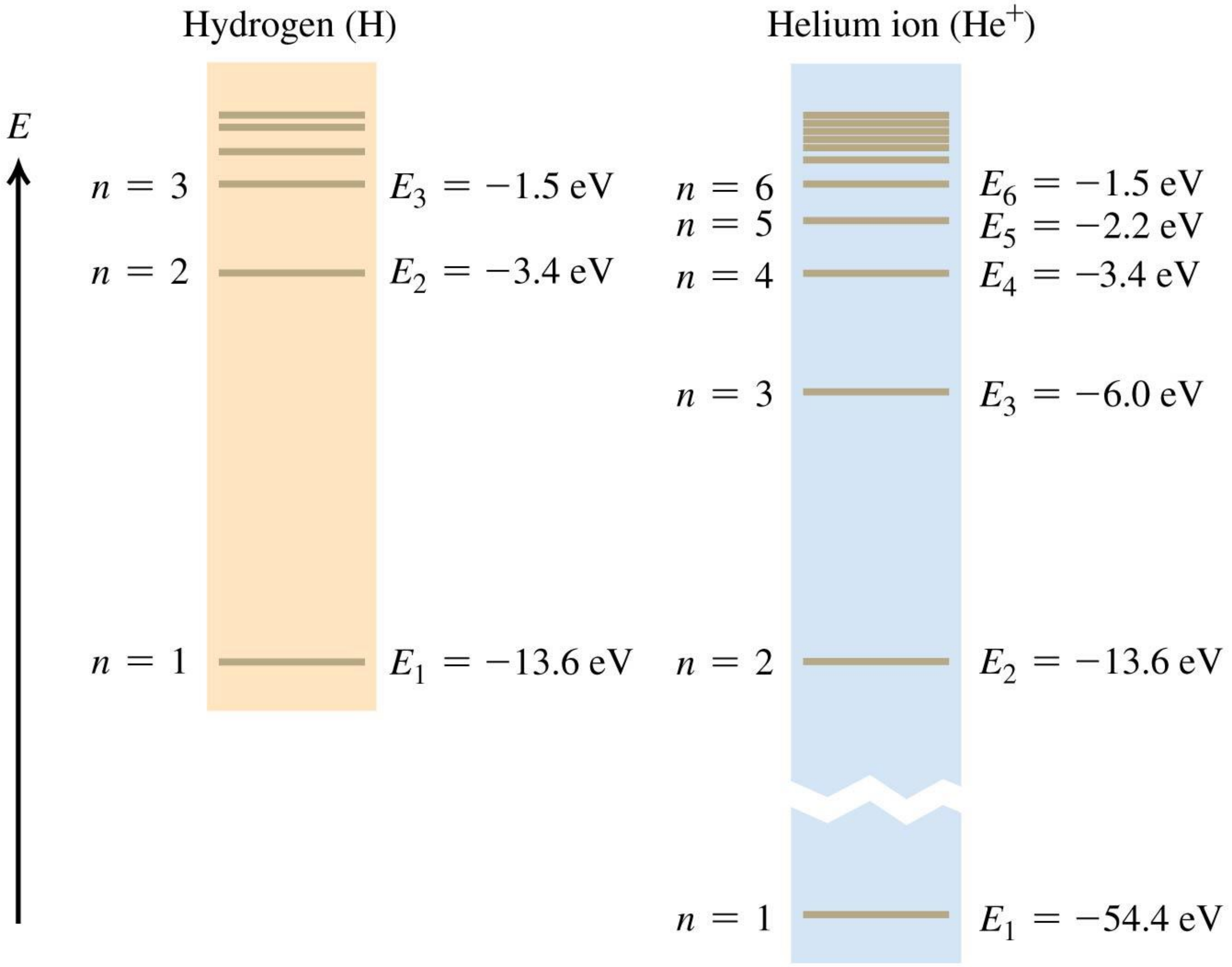


Hydrogen-like atoms

- We can extend the Bohr model to other one-electron atoms, such as singly ionized helium He^+, doubly ionized lithium Li^{2+}, and so on.
- Such atoms are called *hydrogenlike* atoms.
- In such atoms, the nuclear charge is not e but Ze, where Z is the *atomic number*, equal to the number of protons in the nucleus.
- The effect in the previous analysis is to replace e^2 everywhere by Ze^2.
- In particular, the orbital radii r_n given for H become smaller by a factor of Z, and the energy levels E_n are multiplied by Z^2.
- You can verify these statements by yourself.

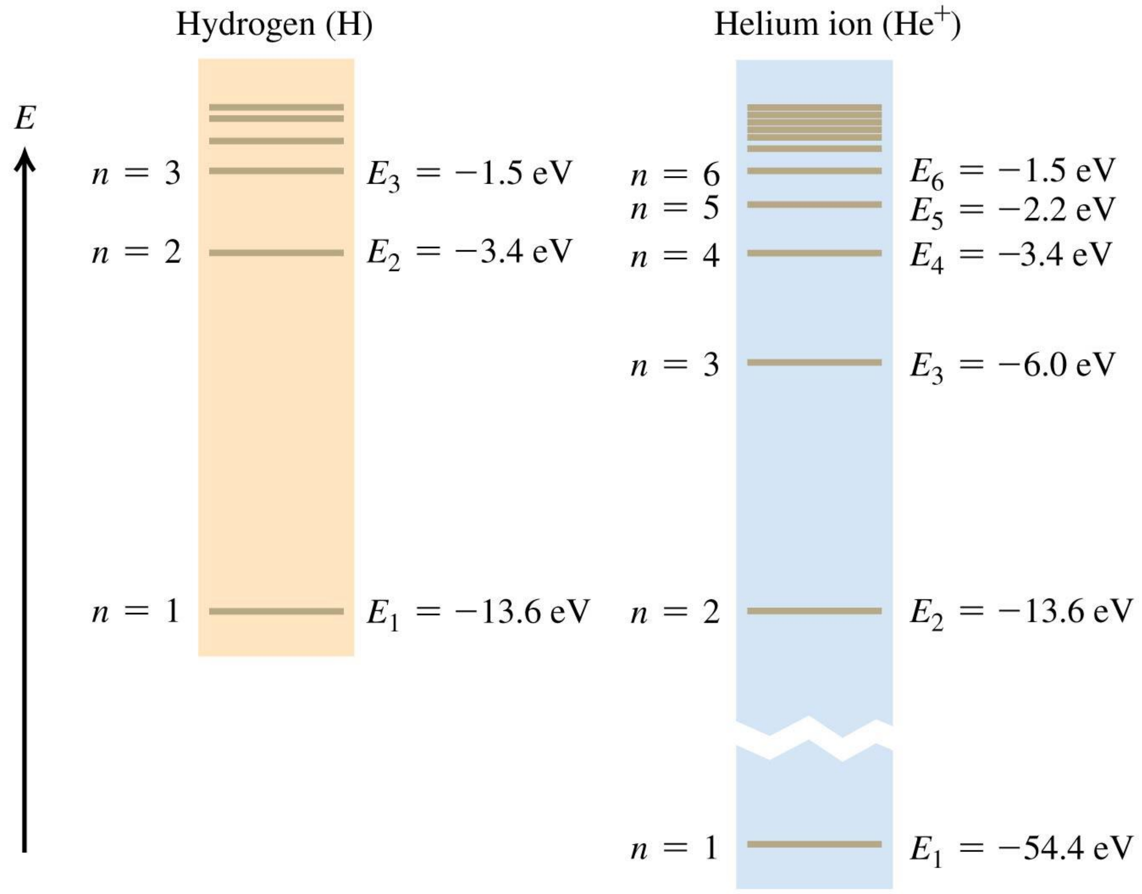
Hydrogen-like atoms

- He$^+$ has $Z = 2$.



Example

Test Your Understanding of Section 39.3 Consider the possible transitions between energy levels in a He^+ ion. For which of these transitions in He^+ will the wavelength of the emitted photon be nearly the same as one of the wavelengths emitted by excited H atoms? (i) $n = 2$ to $n = 1$; (ii) $n = 3$ to $n = 2$; (iii) $n = 4$ to $n = 3$; (iv) $n = 4$ to $n = 2$; (v) more than one of these; (vi) none of these.

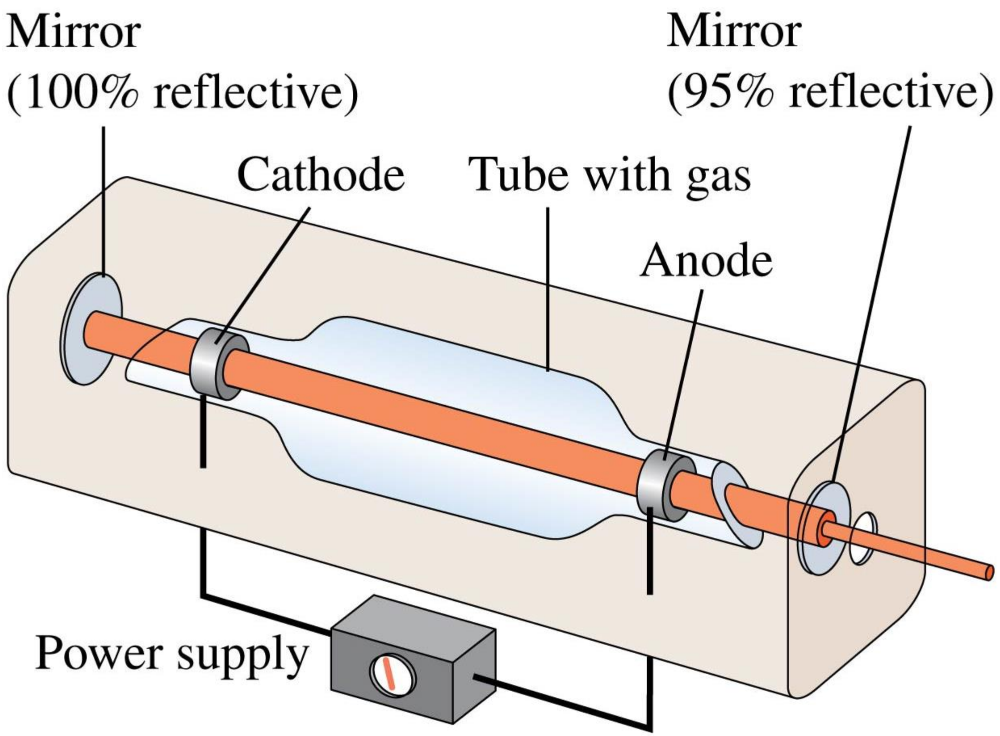


Outlook from the Bohr model

- Although the Bohr model predicted the energy levels of the hydrogen atom correctly, it raised as many questions as it answered.
- It combined elements of classical physics with new postulates that were inconsistent with classical ideas.
  - The model provided no insight into what happens during a transition from one orbit to another;
  - The angular speeds of the electron motion were not in general the angular frequencies of the emitted radiation, a result that is contrary to classical electrodynamics.
  - Attempts to extend the model to atoms with two or more electrons were not successful.
  - An electron moving in one of Bohr's circular orbits forms a current loop and should produce a magnetic dipole moment. However, a hydrogen atom in its ground level has *no* magnetic moment due to orbital motion.
- In Chapters 40 we will find that an even more radical departure from classical concepts was needed before the understanding of atomic structure could progress further.

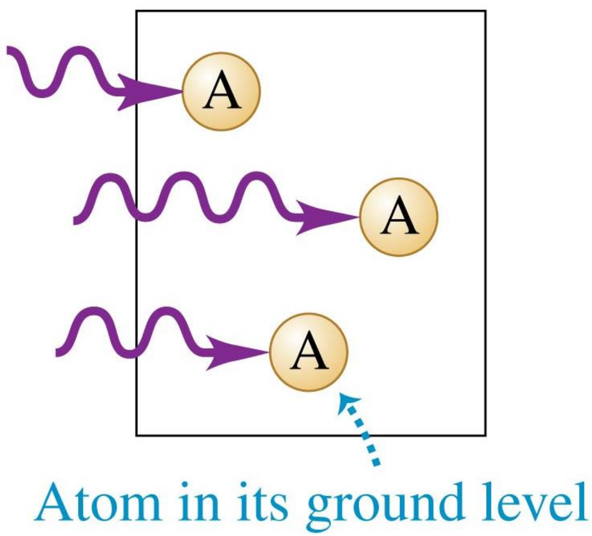
The laser

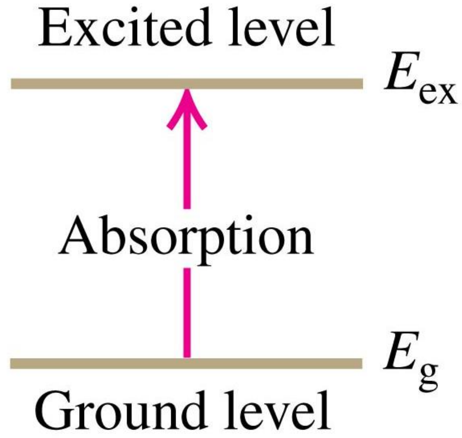
- The laser is a light source that produces a beam of highly coherent and very nearly monochromatic light as a result of cooperative emission from many atoms.
- The name “laser” is an acronym for “light amplification by stimulated emission of radiation.”



Absorption

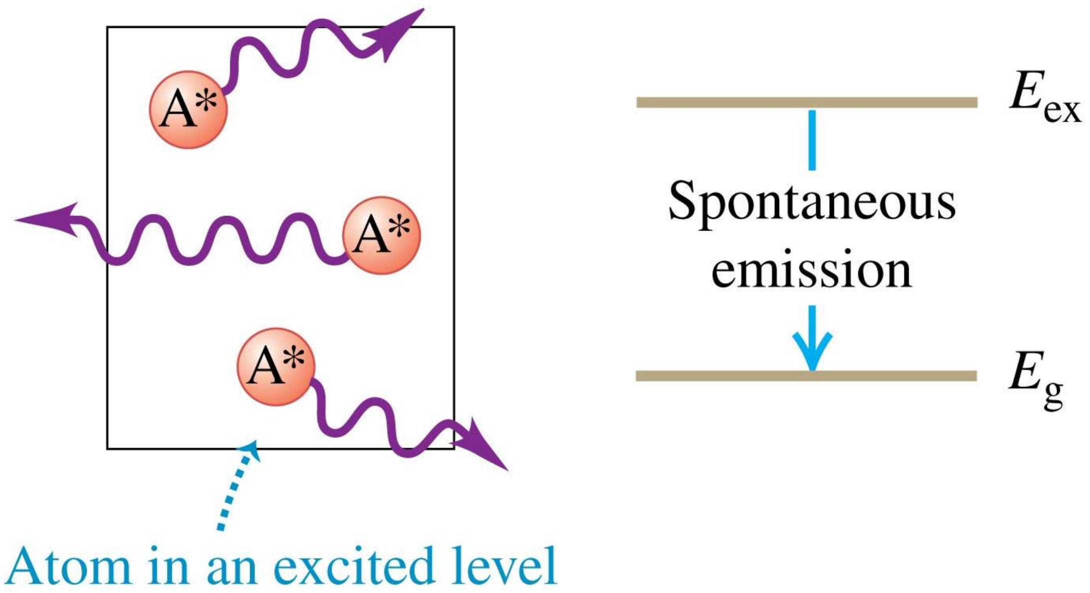
- Consider a gas of atoms in a transparent container.
- Each atom is initially in its ground level of energy E_g and also has an excited level of energy E_{ex}.
- If we shine light of frequency f on the container, an atom can **absorb** one of the photons provided the photon energy $E = hf$ equals the energy difference $E_{ex} - E_g$ between the levels.
- The figure shows this process, in which three atoms A each absorb a photon and go into the excited level.





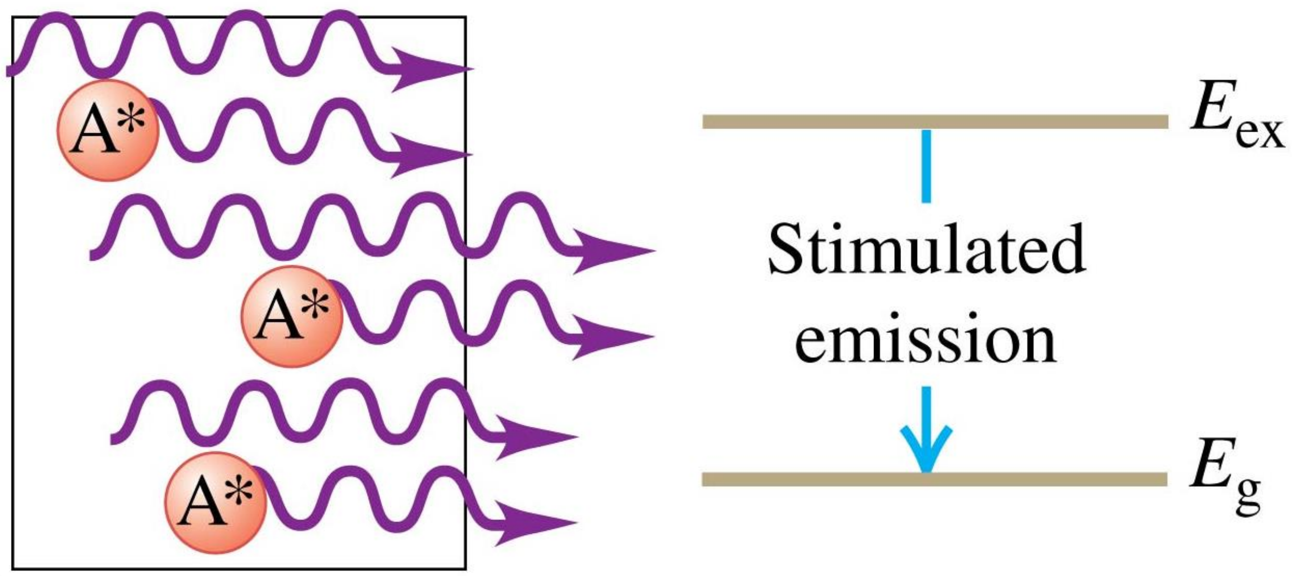
Spontaneous emission

- Excited atoms (which we denote as A*) can return to the ground level by each emitting a photon with the same frequency as the one originally absorbed.
- This process is called **spontaneous emission**.
- The direction and phase of each spontaneously emitted photon are random.



Stimulated emission

- In **stimulated emission**, each incident photon encounters a previously excited atom.
- A kind of resonance effect induces each atom to emit a second photon with the same frequency, direction, phase, and polarization as the incident photon, which is not changed by the process.



Populations of atomic states

- Typically the number of atoms in excited states is much smaller than that in the ground state.

- According to the Maxwell–Boltzmann distribution, when the gas is in thermal equilibrium at absolute temperature T, the number n of atoms in a state with energy E is proportional to $e^{-E/kT}$, where k is the Boltzmann constant.

- If E_g is a ground-state energy and E_{ex} is the energy of an excited state, then the ratio of numbers of atoms in the two states is

$$\frac{n_{\mathrm{ex}}}{n_{\mathrm{g}}} = \frac{Ae^{-E_{\mathrm{ex}}/kT}}{Ae^{-E_{\mathrm{g}}/kT}} = e^{-(E_{\mathrm{ex}}-E_{\mathrm{g}})/kT}$$

Populations of atomic states

- Suppose E_{ex} - E_g = 2.0 eV = 3.2 * 10^{-19} J, the energy of a 620-nm visible-light photon. At T = 3000 K (the temperature of the filament in an incandescent light bulb)

$$\frac{E_{ex} - E_g}{kT} = \frac{3.2 \times 10^{-19}\ \mathrm{J}}{(1.38 \times 10^{-23}\ \mathrm{J/K})(3000\ \mathrm{K})} = 7.73$$

$$e^{-(E_{ex}-E_g)/kT} = e^{-7.73} = 0.00044$$

- That is, the fraction of atoms in a state 2.0 eV above a ground state is extremely small, even at this high temperature.

- The point is that at any reasonable temperature there aren't enough atoms in excited states for any appreciable amount of stimulated emission from these states to occur. Rather, a photon emitted by one of the rare excited atoms will almost certainly be absorbed by an atom in the ground state rather than encountering another excited atom.

How can we make laser happen?

- We need to promote stimulated emission by increasing the number of atoms in excited states.
- Can we do that simply by illuminating the container with radiation of frequency $f = E/h$ corresponding to the energy difference $E = E_{ex} - E_g$?
- Some of the atoms absorb photons of energy E and are raised to the excited state, and the population ratio $n_{ex}>n_g$ momentarily increases.
- But because n_g is originally so much larger than n_{ex}, an enormously intense beam of light would be required to momentarily increase n_{ex} to a value comparable to n_g.
- The rate at which energy is *absorbed* from the beam by the n_g ground-state atoms far exceeds the rate at which energy is added to the beam by stimulated emission from the relatively rare (n_{ex}) excited atoms.

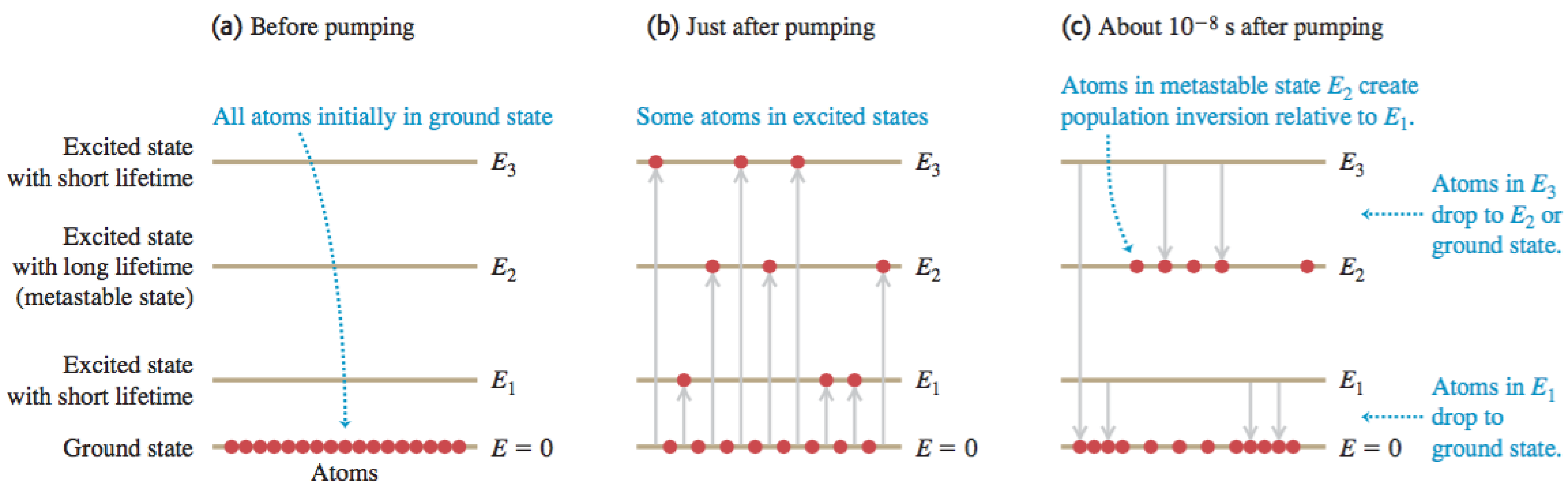
Population inversion

- We need to create a *non-equilibrium* situation in which the number of atoms in a higher-energy state is greater than the number in a lower-energy state.
- Such a situation is called a **population inversion.**
- Then the rate of energy radiation by stimulated emission can *exceed* the rate of absorption, and the system will act as a net *source* of radiation with photon energy E.
- It turns out that we can achieve a population inversion by starting with atoms that have the right kinds of excited states.

Population inversion

- We need an atom with a ground state and *three* excited states of energies E_1, E_2, and E_3. States of energies E_1 and E_3 must have ordinary short lifetimes of about 10^{-8} s, while the state of energy E_2 must have an unusually long lifetime of 10^{-3} s or so.
- Such a long-lived **metastable state** can occur if, for instance, there are restrictions imposed by conservation of angular momentum that hinder photon emission from this state.

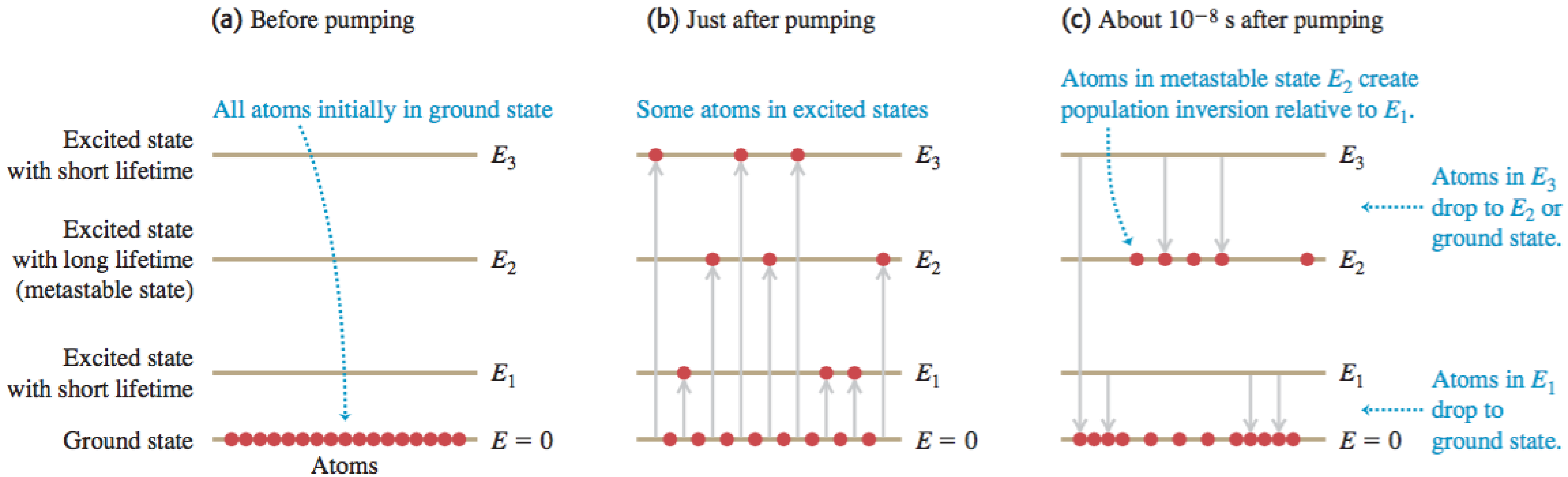
39.29 (a), (b), (c) Stages in the operation of a four-level laser. (d) The light emitted by atoms making spontaneous transitions from state E_2 to state E_1 is reflected between mirrors, so it continues to stimulate emission and gives rise to coherent light. One mirror is partially transmitting and allows the high-intensity light beam to escape.



Population inversion

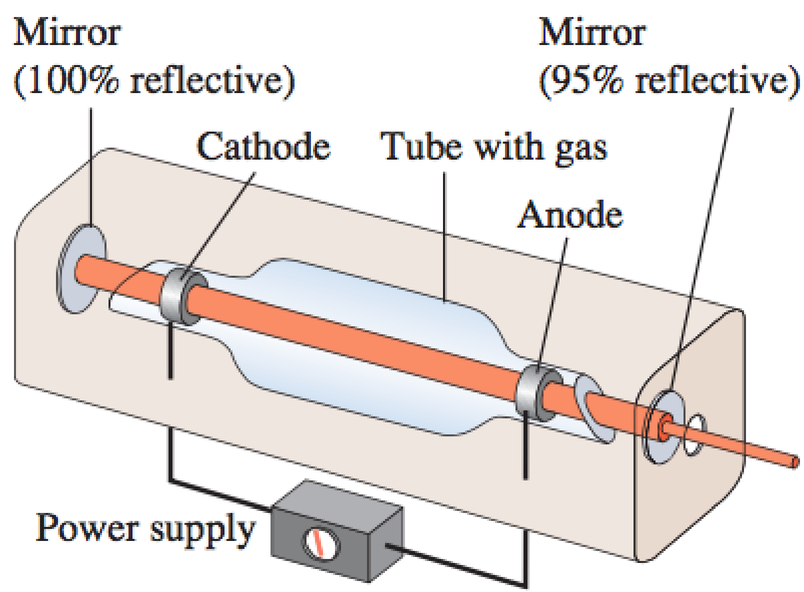
- After pumping, the number of atoms in the metastable state is *less* than the number in the ground state, but is *much greater* than in the nearly unoccupied state of energy E_1.
- Hence there is a population inversion of state E_2 relative to state E_1.
- Atoms that undergo spontaneous emission from the E_3 level help to populate the E_2 level, and the presence of the E_1 level makes a population inversion possible.

39.29 (a), (b), (c) Stages in the operation of a four-level laser. (d) The light emitted by atoms making spontaneous transitions from state E_2 to state E_1 is reflected between mirrors, so it continues to stimulate emission and gives rise to coherent light. One mirror is partially transmitting and allows the high-intensity light beam to escape.



Laser production

- 10^{-3} s after the population has been inverted, some of the atoms in the long-lived metastable state E_2 transition to state E_1 by spontaneous emission.
- The emitted photons of energy $hf = E_2 - E_1$ are sent back and forth through the gas many times by a pair of parallel mirrors, so that they can *stimulate* emission from as many of the atoms in state E_2 as possible.
- The net result of all these processes is a beam of light of frequency f that can be quite intense, has parallel rays, is highly monochromatic, and is spatially *coherent* at all points within a given cross section — that is, a laser beam.
- One of the mirrors is partially transparent, so a portion of the beam emerges.



Inventors of laser

Charles H. Townes
(1915-2015)

Nobel Prize in
Physics 1964
Prize share ½

Nicolay G. Basov
(1922-2001)

Nobel Prize in
Physics 1964
Prize share ¼

Aleksandr M. Prokhorov
(1916-2002)

Nobel Prize in
Physics 1964
Prize share ¼

“for fundamental work in the field of quantum electronics, which has led to the construction of oscillators and amplifiers based on the maser-laser principle "

Curiosity: Basov

Dimitri N. Basov

Welcome to the Basov group @ Columbia

Dimitri Basov is a Professor of Physics in the Department of Physics at Columbia University. His research focuses on electronic phenomena in quantum materials that he investigates using a variety of nano-optical techniques developed in his laboratory.

PUBLICATIONS

HONORS

National Academy of Sciences 2020

Gordon and Betty Moore Investigator in Quantum Materials, 2014-2019 and 2020-2025

Vannevar Bush Faculty Fellowship, 2019

Lebedev Physical Institute

Nicolay G. Basov
(1922-2001)

Nobel Prize in
Physics 1964
Prize share ¼

Applications of laser

- The *pulsed* laser we just studied are used in, for example, the LASIK eye surgery (*laser-assisted in situ keratomileusis*) to reshape the cornea and correct for nearsightedness, farsightedness, or astigmatism.

- In a *continuous* laser, such as those found in the barcode scanners used at retail checkout counters, energy is supplied to the atoms continuously.

- Some types of laser use materials other than a gas.

- The most common kind of laser—used in laser printers, laser pointers, and to read the data on the disc in a DVD player or Blu-ray player—is a *semiconductor laser*, which uses the energy levels of electrons that are free to roam throughout the volume of the semiconductors.

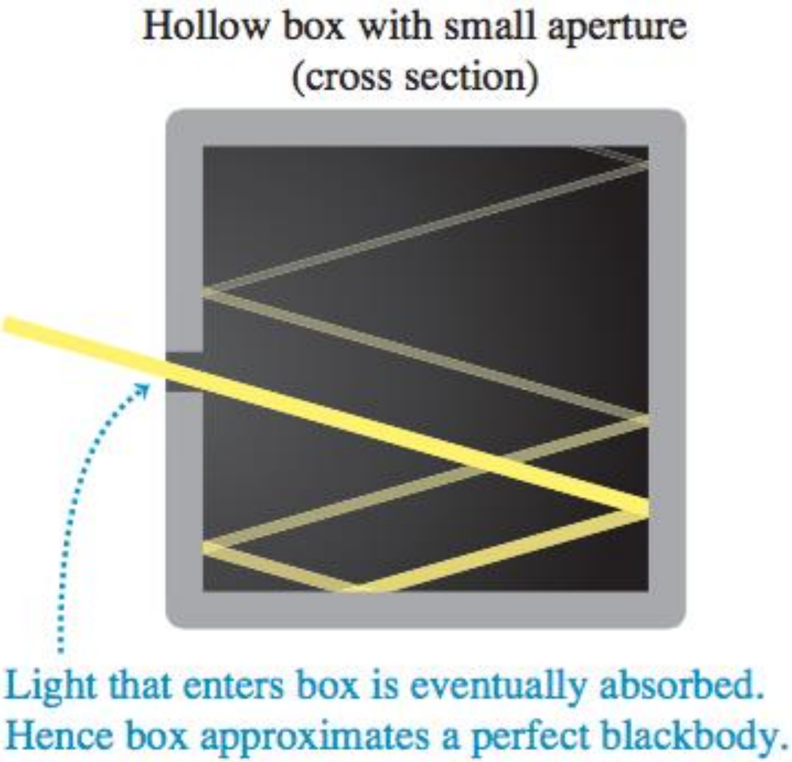
Continuous spectra

- Emission line spectra come from matter in the gaseous state, in which the atoms are so far apart that interactions between them are negligible and each atom behaves as an isolated system.
- By contrast, a heated solid or liquid (in which atoms are close to each other) nearly always emits radiation with a *continuous* distribution of wavelengths rather than a line spectrum.
- Think about many tuning forks in a box: they emit sounds with a single frequency when individually struck, but if you shake the box they collide with each other and what you hear is going to be noise.
- In this part we’ll study an idealized case of continuous-spectrum radiation from a hot, dense object. We’ll find that we can understand the continuous spectrum only if we use the ideas of energy levels and photons.

The Blackbody

- In the same way that an atom's emission spectrum has the same lines as its absorption spectrum, the ideal surface for *emitting* light with a continuous spectrum is one that also *absorbs* all wavelengths of electromagnetic radiation.
- Such an ideal surface is called a *blackbody* because it would appear perfectly black when illuminated; it would reflect no light at all.

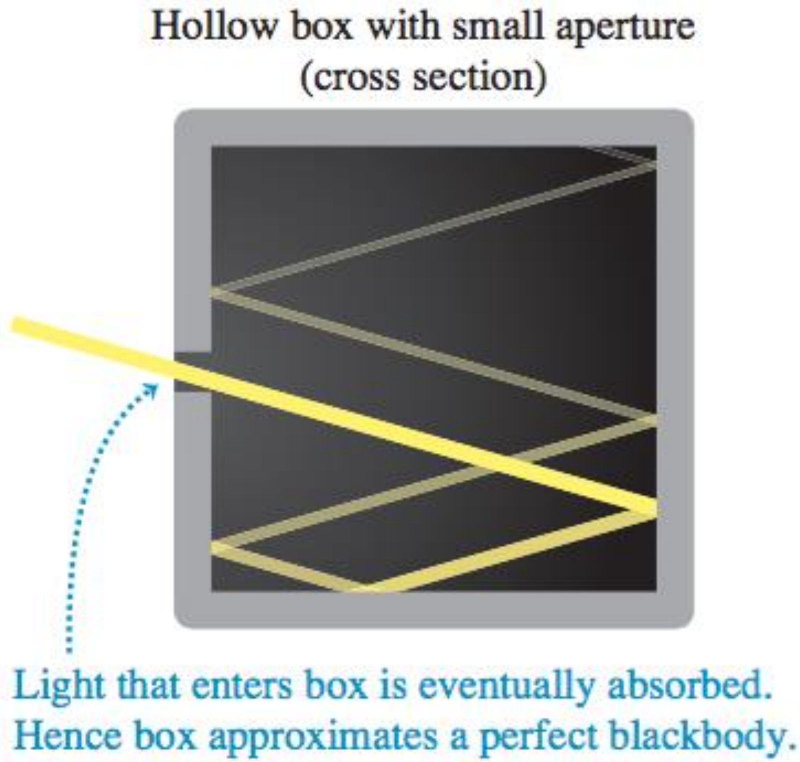
39.30 A hollow box with a small aperture behaves like a blackbody. When the box is heated, the electromagnetic radiation that emerges from the aperture has a blackbody spectrum.



The Blackbody

- The continuous-spectrum radiation that a blackbody emits is called **blackbody radiation.**
- Like a perfectly frictionless incline or a massless rope, a perfect blackbody does not exist but is nonetheless a useful idealization.
- By 1900 blackbody radiation had been studied extensively, and three characteristics had been established (the next slide) .

39.30 A hollow box with a small aperture behaves like a blackbody. When the box is heated, the electromagnetic radiation that emerges from the aperture has a blackbody spectrum.



Blackbody radiation I: Stefan–Boltzmann law

- The total intensity *I* (the average rate of radiation of energy per unit surface area or average power per area) emitted from the surface of an ideal radiator is proportional to the fourth power of the absolute temperature (**Stefan–Boltzmann law)**:

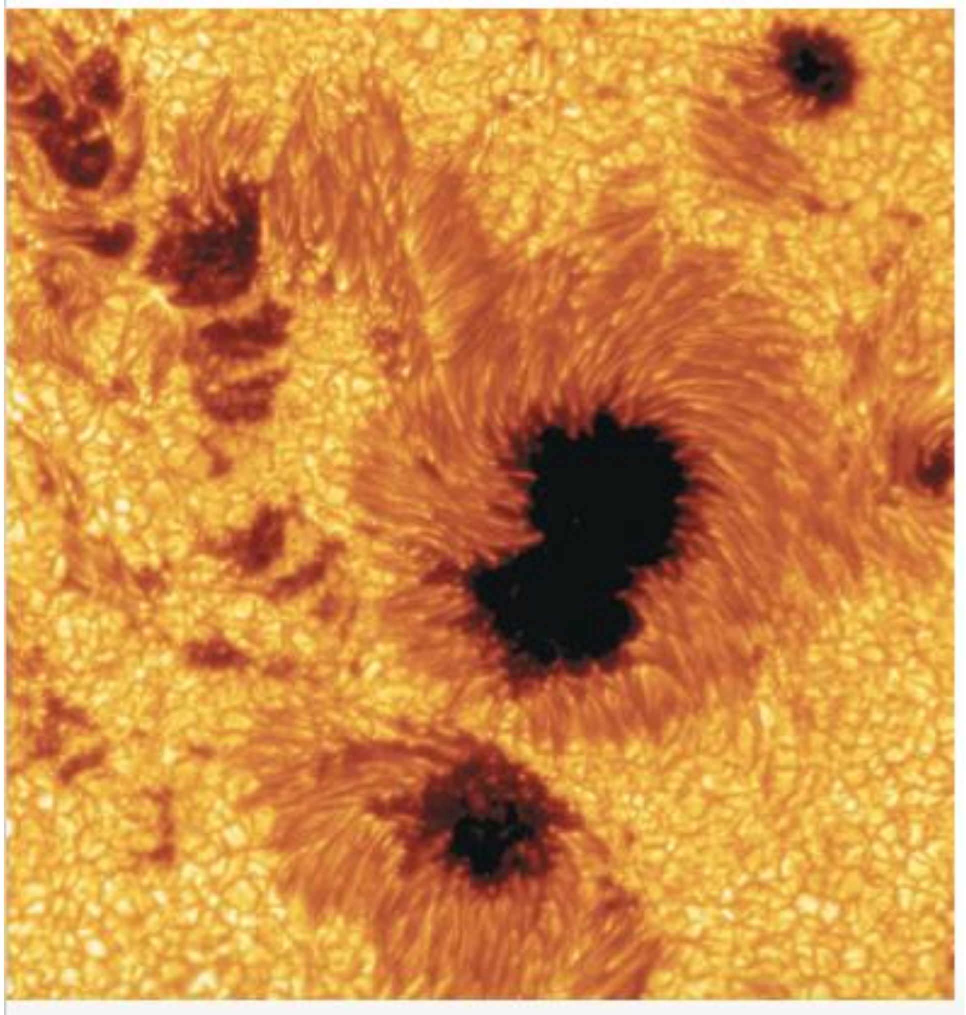
$$I = \sigma T^4 \qquad \text{(Stefan–Boltzmann law for a blackbody)}$$

where σ is a fundamental physical constant called the *Stefan–Boltzmann constant.*

$$\sigma = 5.670400(40) \times 10^{-8} \frac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{K}^4}$$

Example of the Stefan–Boltzmann law

39.31 This close-up view of the sun's surface shows two dark sunspots. Their temperature is about 4000 K, while the surrounding solar material is at T = 5800 K. From the Stefan–Boltzmann law, the intensity from a given area of sunspot is only $(4000\ \mathrm{K}/5800\ \mathrm{K})^4 = 0.23$ as great as the intensity from the same area of the surrounding material—which is why sunspots appear dark.



Blackbody radiation II

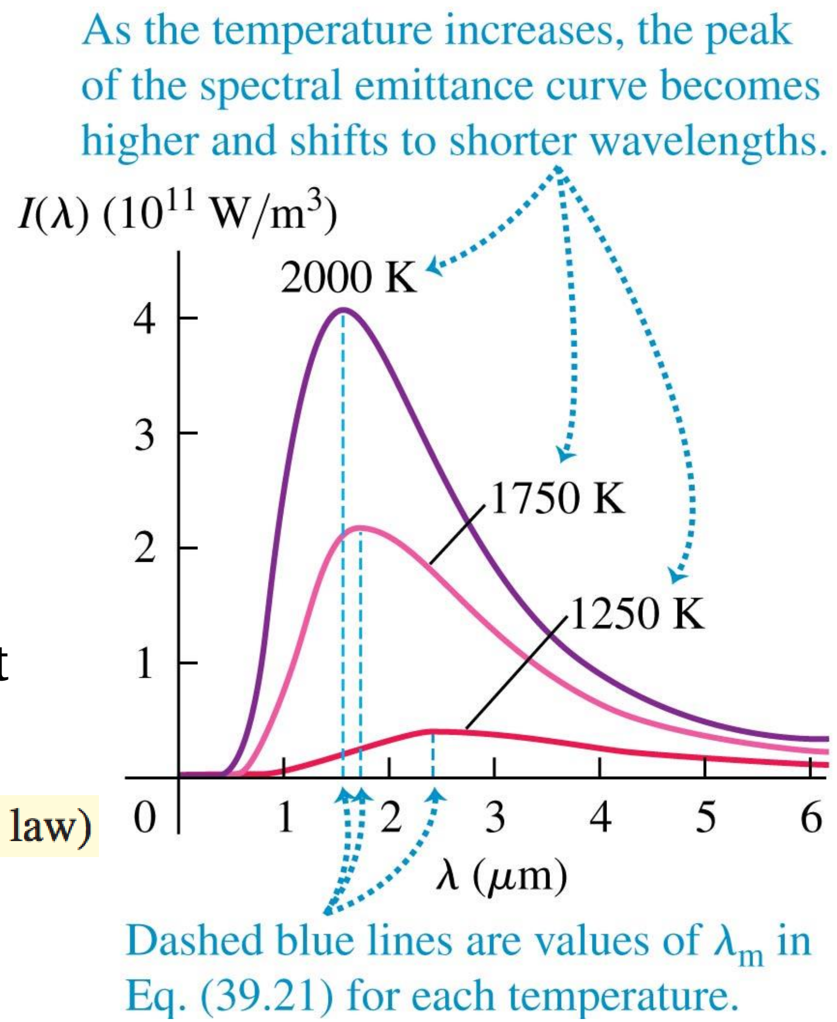
- The intensity is not uniformly distributed over all wavelengths. Its distribution can be measured and described by the intensity per wavelength interval *I(λ)*, called the *spectral emittance.*

$$I = \int_0^\infty I(\lambda)\, d\lambda$$

- Experiment shows that λ_m is inversely proportional to T (**Wien displacement law**):

$$\lambda_m T = 2.90 \times 10^{-3}\ \mathrm{m \cdot K} \qquad \text{(Wien displacement law)}$$

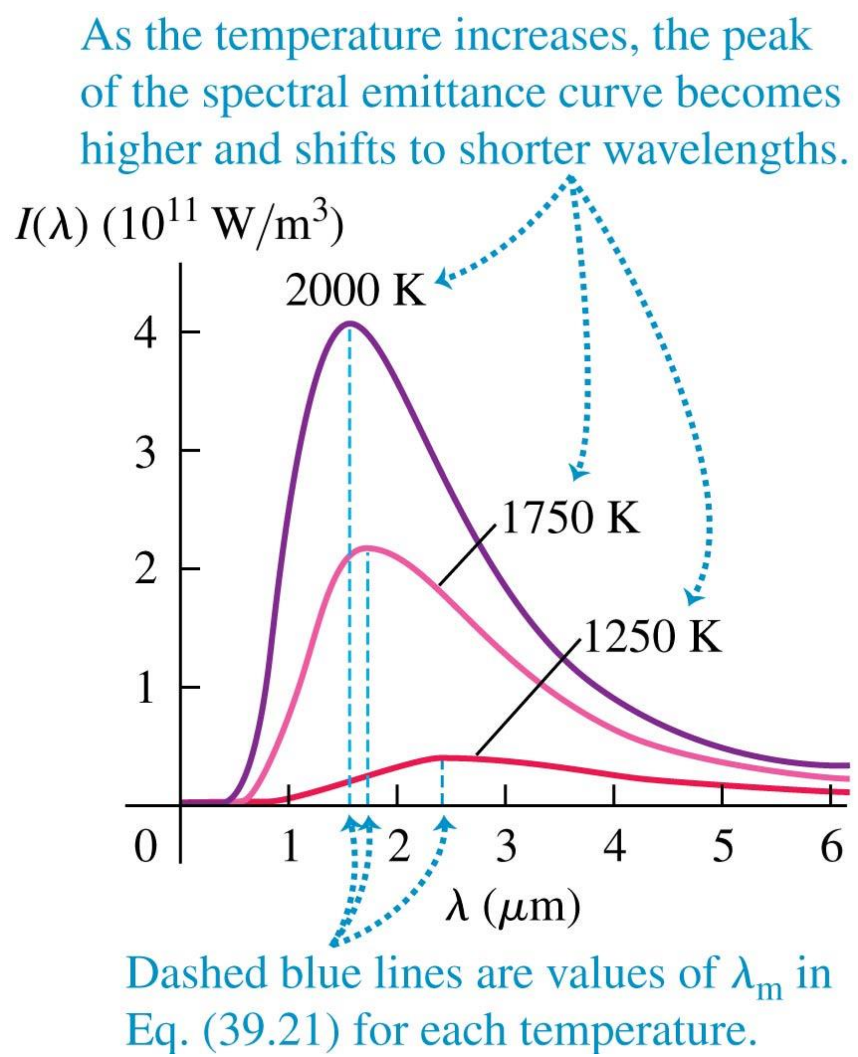
- Yellow light has shorter wavelengths than red light, so a body that glows yellow is hotter and brighter than one of the same size that glows red.



Blackbody radiation III

- Experiments also show that the *shape* of the distribution function is the same for all temperatures.

- We can make a curve for one temperature fit any other temperature by simply changing the scales on the graph.



Q39.8

Complete the sentence: "If you increase the temperature of a blackbody, it will emit __________ radiation at very short wavelengths and __________ radiation at very long wavelengths."

A. more, more

B. more, less

C. less, more

D. less, less

E. none of the above

The Rayleigh–Jeans law

- In 1900 and 1905, the British physicists Lord Rayleigh and Sir James Jeans made an explanation of the blackbody radiation based on classical physics, which is called the Rayleigh–Jeans law.

John William Strutt, 3rd Baron Rayleigh
(1842-1919)

Nobel Prize in Physics 1904

“for his investigations of the densities of the most important gases and for his discovery of argon in connection with these studies"

Sir James H. Jeans, *OM FRS*
(1877-1946)

Adam Prize (1917)
Royal Prize (1919)

The Rayleigh–Jeans law

- We will not go into details of how the law was derived. We just note that it used the classical physics, and in particular the Equipartition Theorem, namely the total energy of each “normal mode” should be equal to kT

- The Rayleigh-Jeans law is

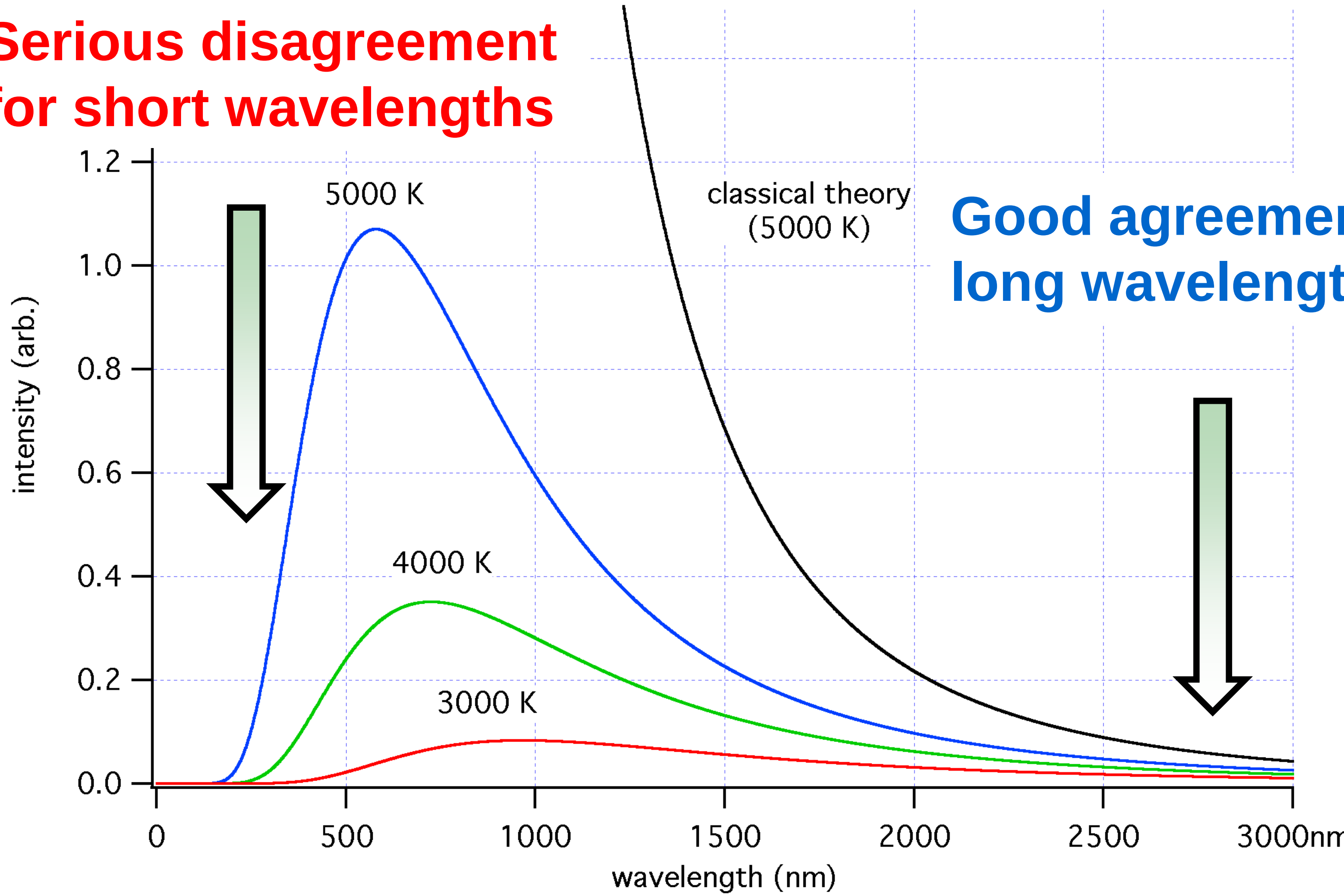
$$I(\lambda) = \frac{2\pi ckT}{\lambda^4}$$

- So that the spectral emittance is directly proportional to T and inversely proportional to λ^4

The ultraviolet catastrophe

Serious disagreement for short wavelengths

Good agreement for long wavelengths



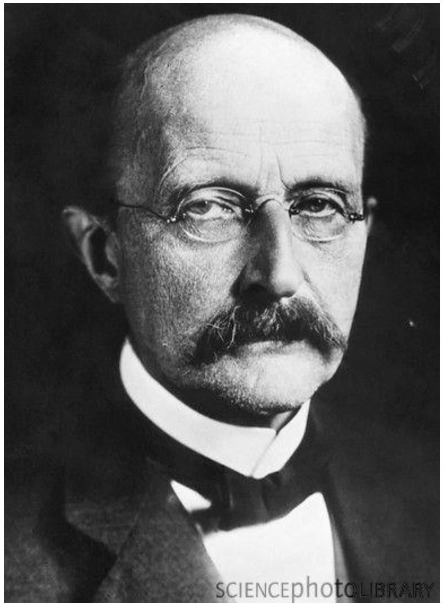
https://upload.wikimedia.org/wikipedia/commons/a/a1/Blackbody-lg.png

The ultraviolet catastrophe

- At large wavelengths, the Rayleigh-Jeans law agrees quite well with the experiments.
- However, there is serious disagreement at small wave-lengths: The experimental curves fall toward zero at small λ. By contrast, Rayleigh's goes in the opposite direction, approaching infinity.
- This result was called the "ultraviolet catastrophe."
- Even worse, the integral of Rayleigh's spectral emittance over all λ is infinite, indicating an infinitely large *total* radiated intensity.
- **Clearly, something is wrong.**

Planck and the Quantum Hypothesis

- In 1900, the German physicist Max Planck succeeded in deriving a function, now called the **Planck radiation law,** that agreed very well with experimental intensity distribution curves.
- In his derivation he made what seemed at the time to be a crazy assumption—the energy quanta—which revolutionized the physics .



Max Planck (1858-1947)

Nobel Prize in Physics 1918

"in recognition of the services he rendered to the advancement of Physics by his discovery of energy quanta".

Planck and the Quantum Hypothesis

- Planck assumed that electromagnetic oscillators (electrons) in the walls of the blackbody (the "box") vibrating at a frequency f could have only certain values of energy equal to nhf, where $n = 0, 1, 2, 3, \ldots$ and h turned out to be the constant that now bears Planck's name.

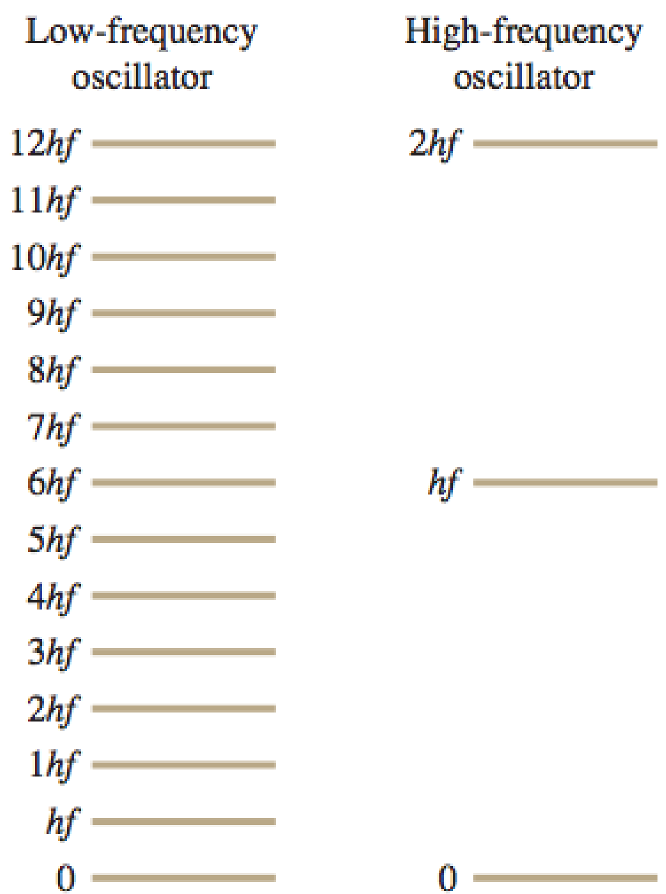
- These oscillators were in equilibrium with the electromagnetic waves in the box, so they both emitted and absorbed light.

- His assumption gave quantized energy levels and said that the energy in each "normal mode" was also a multiple of hf. This was in sharp contrast to Rayleigh's point of view that each normal mode could have any amount of energy (kT).

Planck and the Quantum Hypothesis

- We look at two oscillators with different frequency in the box.
- According to Rayleigh's picture, both of these oscillators have the same amount of energy *kT* and are equally effective at emitting radiation.
- In Planck's model, however, the high-frequency oscillator is very ineffective as a source of light.

39.33 Energy levels for two of the oscillators that Planck envisioned in the walls of a blackbody like that shown in Fig. 39.30. The spacing between adjacent energy levels for each oscillator is hf, which is smaller for the low-frequency oscillator.



Planck and the Quantum Hypothesis

- The ratio of the number of oscillators in the first excited state (n = 1, energy hf) to the number of oscillators in the ground state (n = 0, energy zero) is

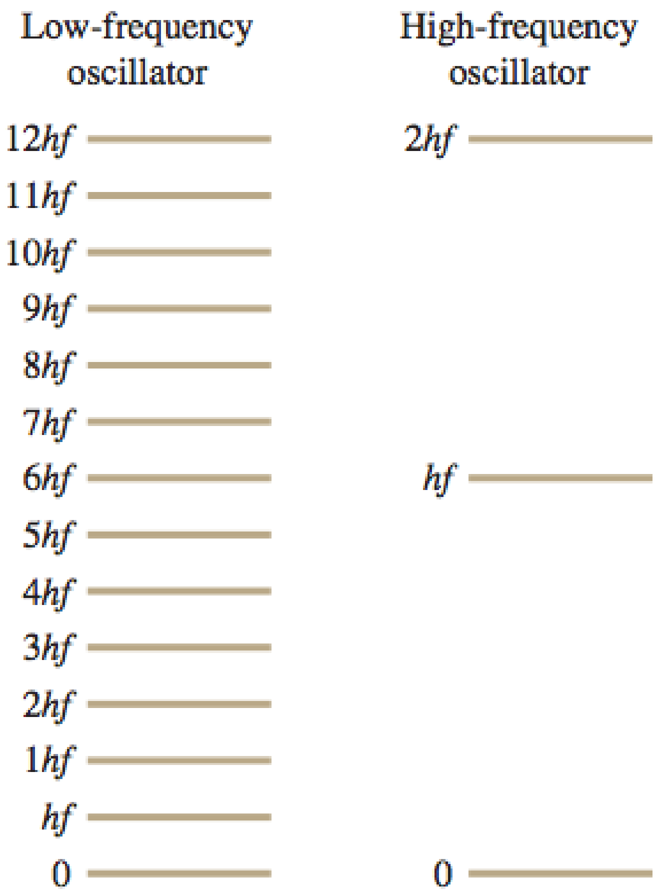
$$\frac{n_1}{n_0} = \frac{Ae^{-hf/kT}}{Ae^{-(0)/kT}} = e^{-hf/kT}$$

- Let us do a calculation with T=2000 K, λ_1=3.00μm, λ_2=0.50μm

$$\frac{n_1}{n_0} = e^{-hf/kT} = 0.0909 \text{ for } \lambda = 3.00\ \mu\text{m}$$

$$\frac{n_1}{n_0} = e^{-hf/kT} = 5.64 \times 10^{-7} \text{ for } \lambda = 0.500\ \mu\text{m}$$

39.33 Energy levels for two of the oscillators that Planck envisioned in the walls of a blackbody like that shown in Fig. 39.30. The spacing between adjacent energy levels for each oscillator is hf, which is smaller for the low-frequency oscillator.



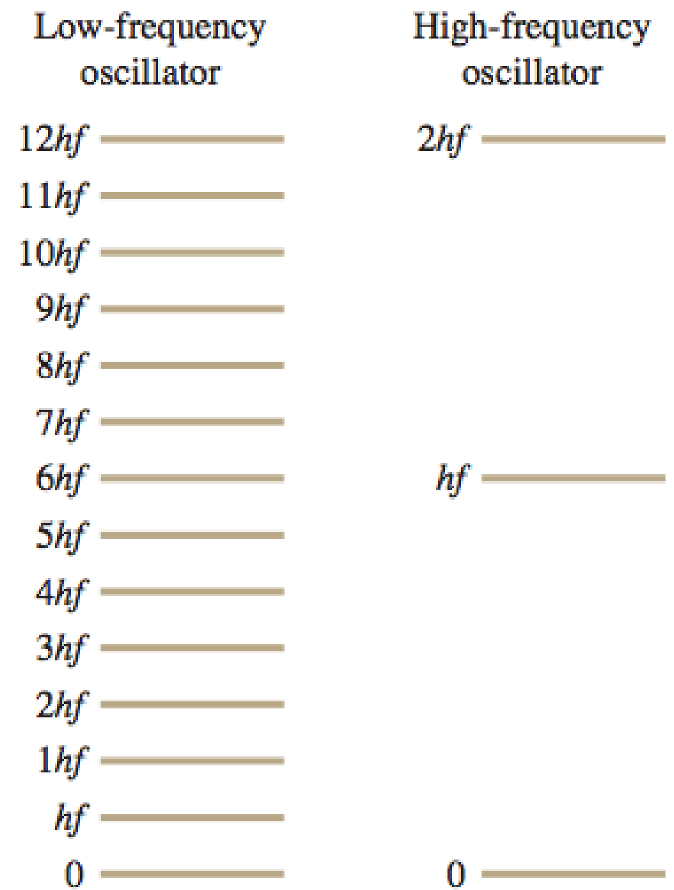
Planck and the Quantum Hypothesis

$$\frac{n_1}{n_0} = e^{-hf/kT} = 0.0909 \text{ for } \lambda = 3.00\ \mu\text{m}$$

$$\frac{n_1}{n_0} = e^{-hf/kT} = 5.64 \times 10^{-7} \text{ for } \lambda = 0.500\ \mu\text{m}$$

- The results mean that radiations with λ_1=3.00μm are rather plentiful, but those with λ_2=0.50μm are *tremendously* suppressed compared to Rayleigh's prediction.
- So Planck's quantum hypothesis provided a natural way to suppress the spectral emittance of a blackbody at short wavelengths, and hence averted the ultraviolet catastrophe.

39.33 Energy levels for two of the oscillators that Planck envisioned in the walls of a blackbody like that shown in Fig. 39.30. The spacing between adjacent energy levels for each oscillator is hf, which is smaller for the low-frequency oscillator.



Planck radiation law

- We will again not going into details of the derivation, but will give the result:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)} \qquad \text{(Planck radiation law)}$$

- We can obtain the Stefan–Boltzmann law for a blackbody by integrating $I(\lambda)$ over all wavelengths to find the total radiated intensity:

$$I = \int_0^\infty I(\lambda)\, d\lambda = \frac{2\pi^5 k^4}{15c^2h^3}T^4 = \sigma T^4$$

- You can also derive the Wien displacement law by finding the maximum of $I(\lambda)$

Example 39.7: Light from the sun

To a good approximation, the sun's surface is a blackbody with a surface temperature of 5800 K. (We are ignoring the absorption produced by the sun's atmosphere, shown in Fig. 39.9.) (a) At what wavelength does the sun emit most strongly? (b) What is the total radiated power per unit surface area?

EXECUTE: (a) From Eq. (39.21),

$$\lambda_m = \frac{2.90 \times 10^{-3}\ \mathrm{m \cdot K}}{T} = \frac{2.90 \times 10^{-3}\ \mathrm{m \cdot K}}{5800\ \mathrm{K}}$$

$$= 0.500 \times 10^{-6}\ \mathrm{m} = 500\ \mathrm{nm}$$

(b) From Eq. (39.19),

$$I = \sigma T^4 = (5.67 \times 10^{-8}\ \mathrm{W/m^2 \cdot K^4})(5800\ \mathrm{K})^4$$

$$= 6.42 \times 10^7\ \mathrm{W/m^2} = 64.2\ \mathrm{MW/m^2}$$

EVALUATE: The 500-nm wavelength found in part (a) is near the middle of the visible spectrum. This should not be a surprise: The human eye evolved to take maximum advantage of natural light.

The enormous value $I = 64.2\ \mathrm{MW/m^2}$ found in part (b) is the intensity at the *surface* of the sun, a sphere of radius 6.96×10^8 m. When this radiated energy reaches the earth, 1.50×10^{11} m away, the intensity has decreased by the factor $[(6.96 \times 10^8\ \mathrm{m})/(1.50 \times 10^{11}\ \mathrm{m})]^2 = 2.15 \times 10^{-5}$ to the still-impressive $1.4\ \mathrm{kW/m^2}$.

Example 39.8: A slice of sunlight

Find the power per unit area radiated from the sun's surface in the wavelength range 600.0 to 605.0 nm.

EXECUTE: To obtain the height of the $I(\lambda)$ curve at $\lambda = 602.5\ \text{nm} = 6.025 \times 10^{-7}$ m, we first evaluate the quantity $hc/\lambda kT$ in Eq. (39.24) and then substitute the result into Eq. (39.24):

$$\frac{hc}{\lambda kT} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})(2.998 \times 10^{8}\ \text{m/s})}{(6.025 \times 10^{-7}\ \text{m})(1.381 \times 10^{-23}\ \text{J/K})(5800\ \text{K})} = 4.116$$

$$I(\lambda) = \frac{2\pi(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})(2.998 \times 10^{8}\ \text{m/s})^2}{(6.025 \times 10^{-7}\ \text{m})^5(e^{4.116} - 1)}$$
$$= 7.81 \times 10^{13}\ \text{W/m}^3$$

The intensity in the 5.0-nm range from 600.0 to 605.0 nm is then approximately

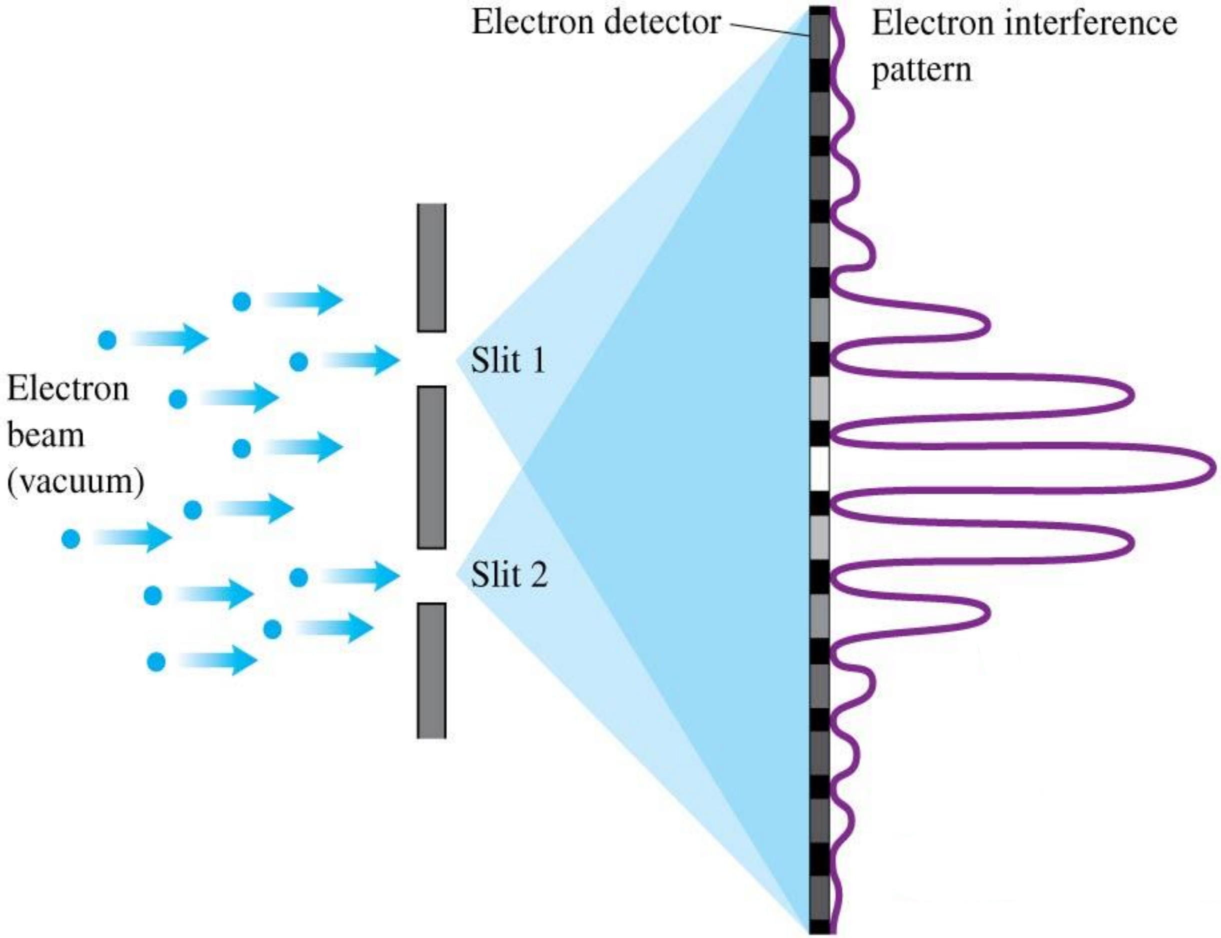
$$I(\lambda)\Delta\lambda = (7.81 \times 10^{13}\ \text{W/m}^3)(5.0 \times 10^{-9}\ \text{m})$$
$$= 3.9 \times 10^{5}\ \text{W/m}^2 = 0.39\ \text{MW/m}^2$$

The uncertainty principle revisited

- The discovery of the dual wave–particle nature of matter forces us to reevaluate the kinematic language we use to describe the position and motion of a particle.
- In classical Newtonian mechanics we think of a particle as a point. We can describe its location and state of motion at any instant with three spatial coordinates and three components of velocity.
- But because matter also has a wave aspect, when we look at the behavior on a small enough scale—comparable to the de Broglie wavelength of the particle—we can no longer use the Newtonian description. Certainly no Newtonian particle would undergo diffraction like electrons do.

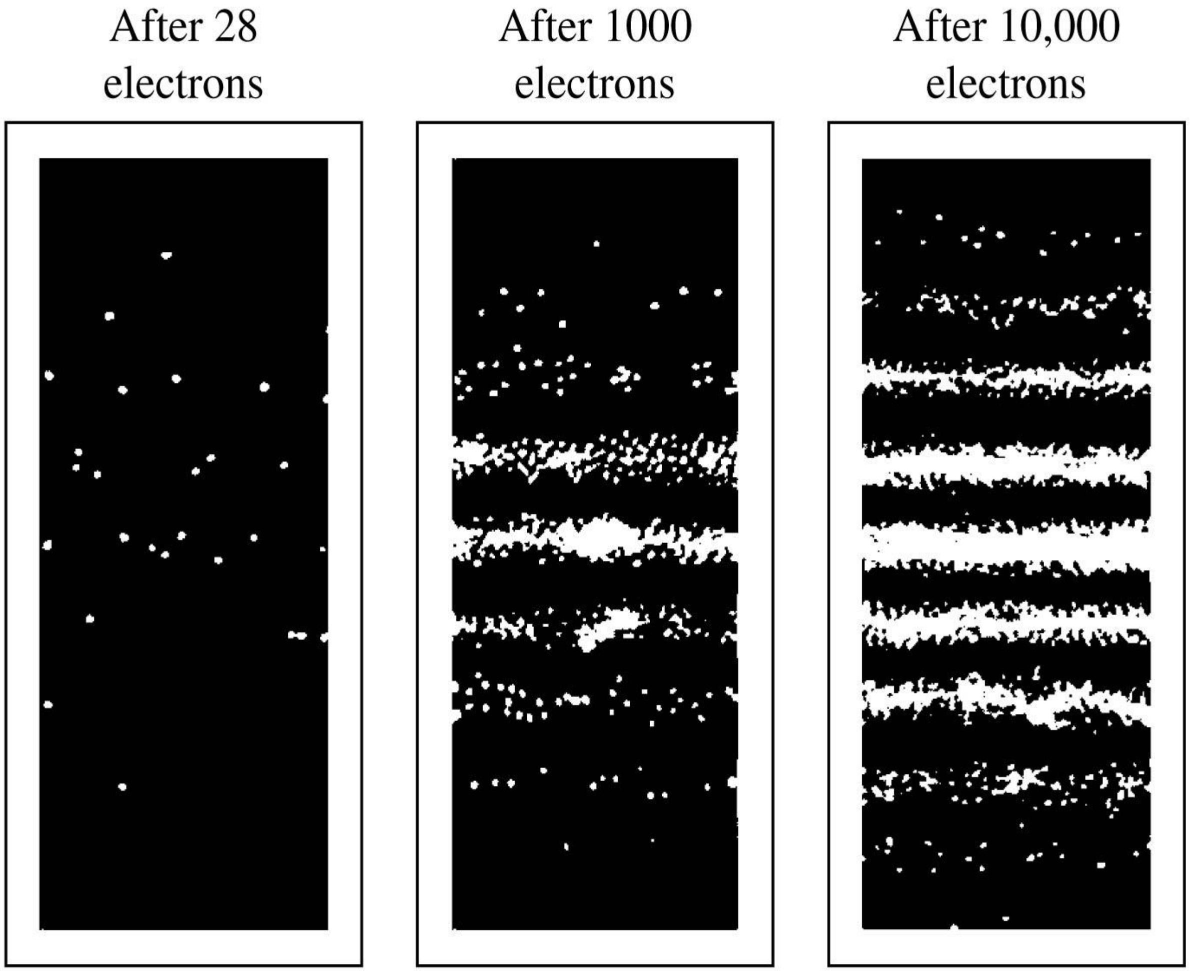
A two-slit interference experiment for electrons

What kind of pattern appears on the detector on the other side of the slits?



A two-slit interference experiment for electrons

Answer: *exactly the same* kind of interference pattern we saw for photons!



The uncertainty principle revisited

- The principle of complementarity tells us that we cannot apply the wave and particle models simultaneously to describe any single element of this experiment.
- Thus we cannot predict exactly where in the pattern (a wave phenomenon) any individual electron (a particle) will land.
- We can’t even ask which slit an individual electron passes through.
- If we tried to look at where the electrons were going by shining a light on them—that is, by scattering photons off them—the electrons would recoil, which would modify their motions so that the two-slit interference pattern would not appear.

Caution

CAUTION **Electron two-slit interference is not interference between two electrons** It's a common misconception that the pattern in Fig. 39.34b is due to the interference between *two* electron waves, each representing an electron passing through one slit. To show that this cannot be the case, we can send just one electron at a time through the apparatus. It makes no difference; we end up with the same interference pattern. In a sense, each electron wave interferes with itself.

The Heisenberg Uncertainty Principles for Matter

- Just as electrons and photons show the same behavior in a two-slit interference experiment, electrons and other forms of matter obey the same Heisenberg uncertainty principles as photons do:

$$\Delta x \Delta p_x \geq \hbar/2$$
$$\Delta y \Delta p_y \geq \hbar/2$$
$$\Delta z \Delta p_z \geq \hbar/2$$

(Heisenberg uncertainty principle for position and momentum)

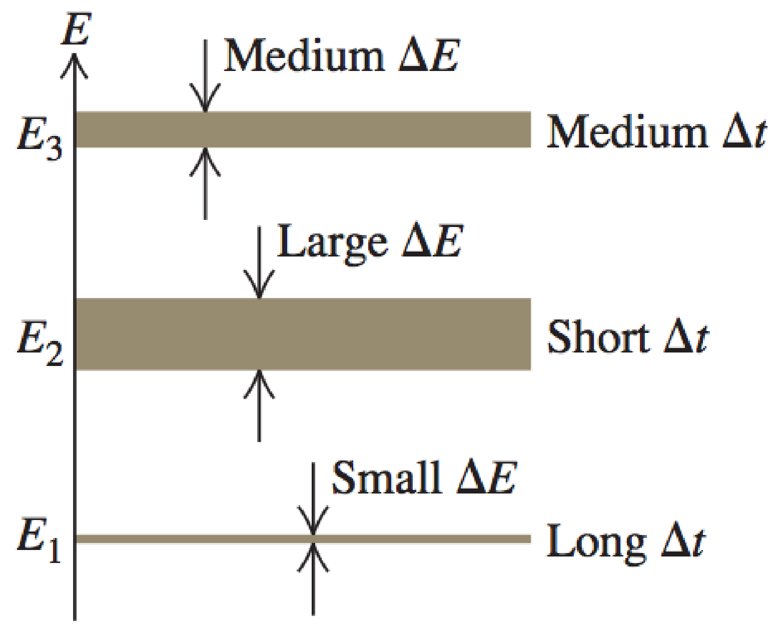
$$\Delta t \, \Delta E \geq \hbar/2$$

(Heisenberg uncertainty principle for energy and time interval)

The Uncertainty Principle and energy levels

- The uncertainty principle for energy and time interval has a direct application to energy levels.
- We have assumed that each energy level in an atom has a very definite energy.
- However, the uncertainty principle says this is not true for all energy levels. A system that remains in a metastable state for a very long time (large Δt) can have a very well-defined energy (small ΔE), but if it remains in a state for only a short time (small Δt) the uncertainty in energy must be correspondingly greater (large ΔE).

39.35 The longer the lifetime Δt of a state, the smaller is its spread in energy (shown by the width of the energy levels).



Example 39.9: The uncertainty principle: position and momentum

An electron is confined within a region of width 5.000×10^{-11} m (roughly the Bohr radius). (a) Estimate the minimum uncertainty in the x-component of the electron's momentum. (b) What is the kinetic energy of an electron with this magnitude of momentum? Express your answer in both joules and electron volts.

EXECUTE: (a) From Eqs. (39.29), for a given value of Δx, the uncertainty in momentum is minimum when the product $\Delta x \, \Delta p_x$ equals $\hbar$. Hence

$$\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \mathrm{J \cdot s}}{2(5.000 \times 10^{-11}\ \mathrm{m})} = 1.055 \times 10^{-24}\ \mathrm{J \cdot s/m}$$

$$= 1.055 \times 10^{-24}\ \mathrm{kg \cdot m/s}$$

(b) We can rewrite the nonrelativistic expression for kinetic energy as

$$K = \tfrac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

Hence an electron with a magnitude of momentum equal to Δp_x from part (a) has kinetic energy

$$K = \frac{p^2}{2m} = \frac{(1.055 \times 10^{-24}\ \mathrm{kg \cdot m/s})^2}{2(9.11 \times 10^{-31}\ \mathrm{kg})}$$

$$= 6.11 \times 10^{-19}\ \mathrm{J} = 3.81\ \mathrm{eV}$$

Example 39.9: The uncertainty principle: position and momentum

EVALUATE: This energy is typical of electron energies in atoms. This agreement suggests that the uncertainty principle is deeply involved in atomic structure.

A similar calculation explains why electrons in atoms do not fall into the nucleus. If an electron were confined to the interior of a nucleus, its position uncertainty would be $\Delta x \approx 10^{-14}$ m. This would give the electron a momentum uncertainty about 5000 times greater than that of the electron in this example, and a kinetic energy so great that the electron would immediately be ejected from the nucleus.

Example 39.10: The uncertainty principle: energy and time

A sodium atom in one of the states labeled "Lowest excited levels" in Fig. 39.19a remains in that state, on average, for 1.6×10^{-8} s before it makes a transition to the ground level, emitting a photon with wavelength 589.0 nm and energy 2.105 eV. What is the uncertainty in energy of that excited state? What is the wavelength spread of the corresponding spectral line?

EXECUTE: From Eq. (39.30),

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{1.055 \times 10^{-34}\ \mathrm{J \cdot s}}{2(1.6 \times 10^{-8}\ \mathrm{s})}$$
$$= 3.3 \times 10^{-27}\ \mathrm{J} = 2.1 \times 10^{-8}\ \mathrm{eV}$$

The atom remains in the ground level indefinitely, so that level has *no* associated energy uncertainty. The fractional uncertainty of the *photon* energy is therefore

$$\frac{\Delta E}{E} = \frac{2.1 \times 10^{-8}\ \mathrm{eV}}{2.105\ \mathrm{eV}} = 1.0 \times 10^{-8}$$

You can use some simple calculus and the relation $E = hc/\lambda$ to show that $\Delta\lambda/\lambda \approx \Delta E/E$, so that the corresponding spread in wavelength, or "width," of the spectral line is approximately

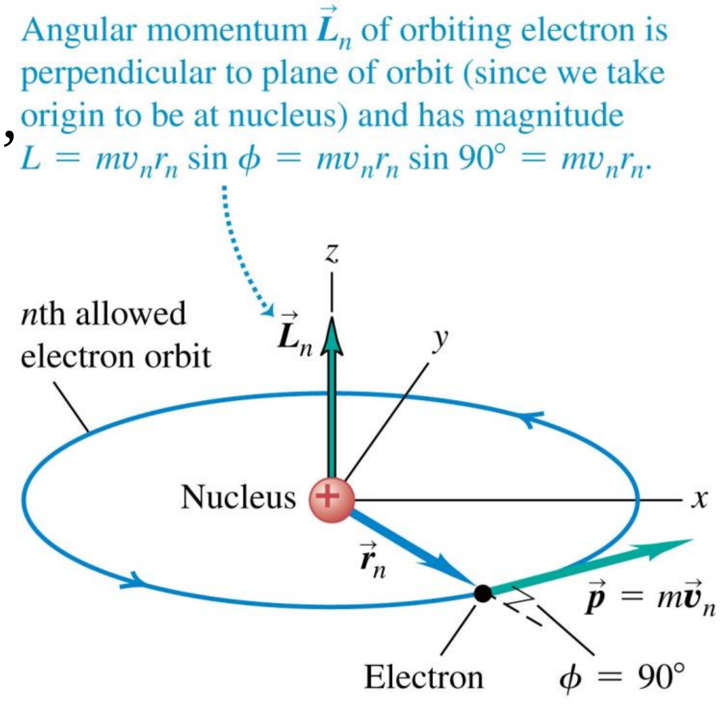
$$\Delta\lambda = \lambda\frac{\Delta E}{E} = (589.0\ \mathrm{nm})(1.0 \times 10^{-8}) = 0.0000059\ \mathrm{nm}$$

Example 39.10: The uncertainty principle: energy and time

EVALUATE: This irreducible uncertainty $\Delta\lambda$ is called the *natural line width* of this particular spectral line. Though very small, it is within the limits of resolution of present-day spectrometers. Ordinarily, the natural line width is much smaller than the line width arising from other causes such as the Doppler effect and collisions among the rapidly moving atoms.

The Uncertainty Principle and the Limits of the Bohr Model

- We saw that the Bohr model of the hydrogen atom was tremendously successful.
- However, the Heisenberg uncertainty principle for position and momentum shows that this model *cannot* be a correct description of how an electron in an atom behaves.
- Take the electron orbit to be the *xy*-plane, Bohr's theory says you can always find an electron at z=0, and its z-momentum is always p_z=0
- That means there is no uncertainties in both z and p_z, in contradiction to the uncertainty principle!



The Uncertainty Principle and the Limits of the Bohr Model

- This conclusion isn’t too surprising, since the electron in the Bohr model is a mix of particle and wave ideas (the electron moves in an orbit like a miniature planet, but has a wavelength).
- To get an accurate picture of how electrons behave inside an atom and elsewhere, we need a description that is based *entirely* on the electron’s wave properties.
- Our goal in the next chapter will be to develop this description, which we call ***quantum mechanics***.
- To do this we’ll introduce the *Schrödinger equation*, the fundamental equation that describes the dynamics of matter waves.
- The Schrödinger equation is as fundamental to quantum mechanics as Newton’s laws are to classical mechanics or as Maxwell’s equations are to electromagnetism.

Bridging problem: Hot stars and Hydrogen Clouds

Figure 39.36 shows a cloud, or *nebula*, of glowing hydrogen in interstellar space. The atoms in this cloud are excited by short-wavelength radiation emitted by the bright blue stars at the center of the nebula. (a) The blue stars act as blackbodies and emit light with a continuous spectrum. What is the wavelength at which a star with a surface temperature of 15,100 K (about $2\frac{1}{2}$ times the surface temperature of the sun) has the maximum spectral emittance? In what region of the electromagnetic spectrum is this? (b) Figure 39.32 shows that most of the energy radiated by a blackbody is at wavelengths between about one half and three times the wavelength of maximum emittance. If a hydrogen atom near the star in part (a) is initially in the ground level, what is the principal quantum number of the highest energy level to which it could be excited by a photon in this wavelength range? (c) The red color of the nebula is primarily due to hydrogen atoms making a transition from $n = 3$ to $n = 2$ and emitting photons of wavelength 656.3 nm. In the Bohr model as interpreted by de Broglie, what are the *electron* wavelengths in the $n = 2$ and $n = 3$ levels?

39.36 The Rosette Nebula.

The 5th Solvay Conference (1927)