Problem Set Solutions Week 10 and 11

1. Find the equation of the circle with given centre and radius:
   a. $x^2 + y^2 - 6x - 10y + 25 = 0$
   b. $x^2 + y^2 + 4x - 6y + 12 = 0$
   c. $x^2 + y^2 + 2x + 6y + 6 = 0$
   d. $x^2 + y^2 - 4x + 4y - 17 = 0$
   e. $x^2 + y^2 - 10y + 9 = 0$

2. Identify the centre and radius oof the following circles:
   f. center (1,2), radius = 5
   g. center (2,-3), radius = 3
   h. center (-1,0), radius = 2
   i. center (-3,-3.5), radius = 6
   j. center (1,-1.5), radius = $\sqrt{\dfrac{19}{12}}$

3. Find the equation of the tangent to each circle at the point specified.
   k. $4y = 3x - 20$
   l. $4y + 3x = 15$
   m. $4y + 3x + 49 = 0$
   n. $5y + 12x = 61$
   o. $5y + 12x = 184$

4. Intersection (1,3) and (-1,-1). The mid-point (0,1), $y = 2x + 1$ is a diameter. The tangents are $2y + x = 7$ and $2y + x = 3$ respectively.

5. Intersection (1,-2) and (2,-1). The tangents are $y = -2$ and $x = 2$ respectively. They intersect at the point (2,-2)

6. Intersection (4,2) and (2,6). The tangents are $3y + x = 10$ and $y = 3x$ respectively. They intersect at the point (1,3).

Ellipse

1. Vertices $(\sqrt{7}, 0)$ and $-\sqrt{7}, 0)$
   Foci $(\sqrt{2}, 0)$ and $-\sqrt{2}, 0)$

2. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

3. Center (3,5)
   Foci $(3+\sqrt{7}, 5)$ and $(3-\sqrt{7}, 5)$
   Vertices (7,5) and $(-1,5)$

4. $\dfrac{(y-3)^2}{16} + \dfrac{(x+2)^2}{12} = 1$

5. $\dfrac{(x-2)^2}{9} + \dfrac{(y-1)^2}{5} = 1$

6. Center (1,-2)

   Foci $(1,-2-2\sqrt{3})$ and $(1,-2+2\sqrt{3})$

   Vertices $(1,-6)$ and $(1,2)$

7. $\dfrac{x^2}{9} + \dfrac{y^2}{5} = 1$, for graph, you may refer to lecture notes

Parabola

1. $x^2 = -20y$

   Focus (0,-5)

   Directrix y=5

Hyperbola

1. $\dfrac{y^2}{64} - \dfrac{x^2}{16} = 1$

2. Center (-5,2)

   Horizontal hyperbola a=3, b=6

   Transverse axis length 6

   Vertices will be at (-2,2) and (-8,2)

   Asymptote slope will be 2

   Asymptote $y - 2 = \pm 2(x + 5)$

3. $\dfrac{(y-4)^2}{16} - \dfrac{(x-1)^2}{9} = 1$

4. $\dfrac{(x-2)^2}{4} - \dfrac{(y-2)^2}{5} = 1$

5. Center (-1,2)

   Foci (-1,12) and (-1,-8)

   Vertices (-1,10) and (-1,-6)

   Equation of asymptotes $y - 2 = \pm\dfrac{4}{3}(x + 1)$

6. $\dfrac{(x+5)^2}{20} - \dfrac{(y-1)^2}{\frac{125}{4}} = 1$

   Center (-5,1)

   Foci $\left(1 - 5 \pm \dfrac{\sqrt{205}}{2}, 12\right)$

   Vertices $(-5 \pm 2\sqrt{5}, 1)$

   Equation of asymptotes $y - 1 = \pm\dfrac{5}{4}(x + 5)$

7. $\dfrac{(y-1)^2}{9} - \dfrac{(x-2)^2}{7} = 1$