CITY UNIVERSITY OF HONG KONG

Course code & title	:	MA1501/GE1358 Coordinate Geometry
Session	:	Semester B 2020/21
Time allowed	:	Two hours

This paper has **FOUR** pages (including this cover page).

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1. Attempt all **NINE** questions in this paper.
2. Start each question on a new page.
3. Show all steps clearly in order to get full credits.

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*This is a **closed-book** examination.*

Students are allowed to use the following materials/aids:

Non-programmable calculators

Materials/aids other than those stated above are not permitted. Students will be subject to disciplinary action if any unauthorized materials or aids are found on them.

NOT TO BE TAKEN AWAY

Question 1 [10]

(a) Given the points A(-2,4,3), B(1,-2,5), C(0,4,-1) and knowing that the quadrilateral *ABCD* is a parallelogram, **use vectors** to find the coordinates of *D*.

(b) **Use midpoints** of the diagonals to check your answer in part (a).

Question 2 [5]

Show that an equation of a plane containing the line $\frac{x-x_1}{u_1} = \frac{y-y_1}{v_1} = \frac{z-z_1}{w_1}$ and perpendicular to the plane $ax + by + cz + d = 0$ is $\det\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ u_1 & v_1 & w_1 \\ a & b & c \end{vmatrix} = 0.$

Question 3 [5]

Show that the plane $x + 2z = 0$ is parallel to a plane containing the line determined by two equations:

$$x - 2y + 4z + 4 = 0$$

$$x + y + z - 8 = 0$$

And consequently, is parallel to the line.

Question 4 [10]

(a) Find the vertices and foci of the ellipse:

$$6x^2 - 100y + 4 = -25y^2 - 36x$$

(b) Determine the type of curve represented by the equation:

$$\frac{x^2}{n} + \frac{y^2}{n - 5} = 1$$

In each of the following cases:

$$n > 5 \,,\ 0 < n < 5 \,,\ n < 0$$

Show that all the curves with:

$$n > 5 \text{ or } 0 < n < 5$$

have the same foci, no matter what the value of *n* is.

<u>Question 5 [20]</u>

(a) The parabola $z = 16y^2$ is rotated about the z- axis. Write the equation of the resulting surface in rectangular and in cylindrical coordinates.

(b) Identify the surface whose equation in spherical coordinates is given by:

$$\rho^2(\sin^2\varphi\cos^2\theta + \cos^2\varphi) = 16$$

And determine its equation in rectangular coordinates.

<u>Question 6 [20]</u>

For (a) and (b) separately, reduce the equation to one of the standard forms, identify the quadric surface and then sketch it (label key points):

(a) $16x^2 + 16y^2 - 32y + z^2 = 0$

(b) $-16x^2 + y^2 - 16z^2 = 16$

<u>Question 7 [5]</u>

Find an equation for the surface consisting of all points P for which the distance from P to the *x*-axis is twice the distance from P to the *yz*-plane. Identify the surface.

<u>Question 8 [15]</u>

Find a vector function that represents the curve of intersection of the two surfaces:

(a) The cylinder $x^2 + y^2 = 16$ and the surface $z=xy$

(b) The paraboloid $z = 16x^2 + y^2$ and the parabolic cylinder $y = x^2$

(c) The cone $z = \sqrt{x^2 + y^2}$ and the plane $z = 1 + y$

<u>Question 9 [10]</u>

One of the world's largest solar parabolic dishes is located at Coordinate Geometry Village. As its name suggested, the reflective surface of the parabolic dish is utilized to collect or project energy such as light, sounds or radio waves for the Coordinate Geometrians residing in Coordinate Geometry Village. One day, the Head Chief described the surface of a parabolic reflector as

$$\frac{x^2}{100} + \frac{y^2}{100} = \frac{z}{4}$$

Please identify the focal point of the given parabolic reflector in the above.

- END -

Useful Trigonometric Identities

Pythagorean identities

1. $\sin^2\theta + \cos^2\theta = 1.$

2. $1 + \tan^2\theta = \sec^2\theta.$

3. $1 + \cot^2\theta = \csc^2\theta.$

Double-angle formulas

4. $\sin 2\theta = 2\sin\theta\cos\theta.$

5. $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta.$

Half-angle formulas

6. $\sin^2\theta = \dfrac{1}{2}(1 - \cos 2\theta).$

7. $\cos^2\theta = \dfrac{1}{2}(1 + \cos 2\theta).$

Compound-angle formulas

8. $\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B.$

9. $\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B.$

10. $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}.$

Sum-to-product formulas

11. $\sin A + \sin B = 2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}.$

12. $\sin A - \sin B = 2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}.$

13. $\cos A + \cos B = 2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2}.$

14. $\cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2}.$

Product-to-sum formulas

15. $\sin A\cos B = \dfrac{1}{2}\big[\sin(A+B) + \sin(A-B)\big].$

16. $\cos A\sin B = \dfrac{1}{2}\big[\sin(A+B) - \sin(A-B)\big].$

17. $\cos A\cos B = \dfrac{1}{2}\big[\cos(A+B) + \cos(A-B)\big].$

18. $\sin A\sin B = -\dfrac{1}{2}\big[\cos(A+B) - \cos(A-B)\big].$

Euler's formulas

19. $e^{\pm i\theta} = \cos\theta \pm i\sin\theta.$

20. $e^{i\theta} + e^{-i\theta} = 2\cos\theta,\ \cos\theta = \dfrac{1}{2}(e^{i\theta} + e^{-i\theta}).$

21. $e^{i\theta} - e^{-i\theta} = 2i\sin\theta,\ \sin\theta = \dfrac{1}{2i}(e^{i\theta} - e^{-i\theta}).$

Remark. Formulas of the form $A \pm B = C \pm D$ contain two separate formulas

$$A + B = C + D, \qquad \text{and} \qquad A - B = C - D.$$

Likewise, formulas of the form $A \pm B = C \mp D$ contain two separate formulas

$$A + B = C - D, \qquad \text{and} \qquad A - B = C + D.$$