MA1300 Self Practice # 8

1. (P174, #18) If a tank holds 5000 liters of water, which drains from the bottom of the tank in 40 minutes, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as

$$V = 5000\left(1 - \frac{t}{40}\right)^2 \qquad 0 \le t \le 40.$$

Find the rate at which water is draining from the tank after (a) 5 min, (b) 10 min, (c) 20 min, and (d) 40 min. At what time is the water flowing out the fastest? The slowest? Summarize your findings.

2. (P174, #19) The quantity of charge Q in coulombs (C) that has passed through a point in a wire up to time t (measured in seconds) is given by $Q(t) = t^3 - 2t^2 + 6t + 2$. Find the current when **(a)** $t = 0.5$ s, and **(b)** $t = 1$ s. (The unit of current is an ampere (1 A = 1 C/s).) At what time is the current lowest?

3. (P174, #20) Newton's Law of Gravitation says that the magnitude F of the force exerted by a body of mass m on a body of mass M is

$$F = \frac{GmM}{r^2},$$

where G is the gravitational constant and r is the distance between the bodies.

a Find dF/dr and explain its meaning. What does the minus sign indicate?

b Suppose it is known that the earth attracts an object with a force that decreases at the rate of 2 N/km when $r = 20,000$ km. How fast does this force change when $r = 10,000$ km?

4. (P180, #2)

a If A is the area of a circle with radius r and the circle expands as time passes, find dA/dt in terms of dr/dt.

b Suppose oil spills from a ruptured tanker and spreads in a circular pattern. If the radius of the oil spill increases at a constant rate of 1 m/s, how fast is the area of the spill increasing when the radius is 30 m?

5. (P182, #28) A kite 50 m above the ground moves horizontally at a speed of 2 m/s. At what rate is the angle between the string and the horizontal decreasing when 100 m of string has been let out?

6. (P182, #33) Boyle's Law states that when a sample of gas is compressed at a constant temperature, the pressure P and volume V satisfy the equation $PV = C$, where C is a constant. Suppose that at a certain instant the volume is 600 cm^3, the pressure is 150 kPa, and the pressure is increasing at a rate of 20 kPa/min. At what rate is the volume decreasing at this instant?

7. (P204, #1) Explain the difference between an absolute minimum and a local minimum.

8. (P204, #2) Suppose f is a continuous function defined on a closed interval $[a, b]$.

a What theorem guarantees the existence of an absolute maximum value and an absolute minimum value for f?

b What steps would you take to find those maximum and minimum values?

9. (P204, #9, 10) Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties respectively.

a Absolute maximum at 5, absolute minimum at 2, local maximum at 3, local minima at 2 and 4.

b f has no local maximum or minimum in $(1, 5)$, but 2 and 4 are critical numbers.

10. (P204, #11 (a)(b))

a Sketch the graph of a function that has a local maximum at 2 and is differentiable at 2.

b Sketch the graph of a function that has a local maximum at 2 and is continuous but not differentiable at 2.