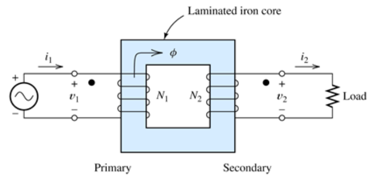
Chapter 14 Transformers

1. Ideal transformers and their functions.

2. Real transformers: equivalent circuits, regulations and power efficiencies.

Ideal Transformers



$$v_1 = N_1 \frac{d\phi}{dt}$$

$$v_2 = N_2 \frac{d\phi}{dt}$$

$$v_2(t) = \frac{N_2}{N_1} v_1(t) \qquad i_2(t) = \frac{N_1}{N_2} i_1(t)$$

$$p_2(t) = p_1(t)$$

Symbol and analog

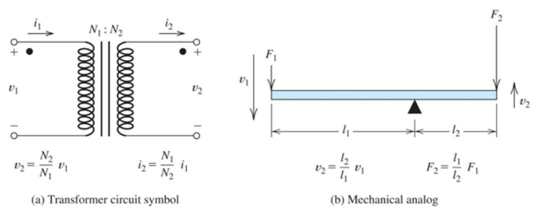


Figure 14.23 The circuit symbol for a transformer and its mechanical analog.

Example 14.10

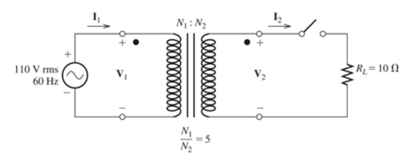


Figure 14.24 Circuit of Example 14.10.

$$V_{2rms} = \frac{N_2}{N_1} V_{1rms} = \frac{1}{5} \times 110 = 22V$$

Impedance Transformations

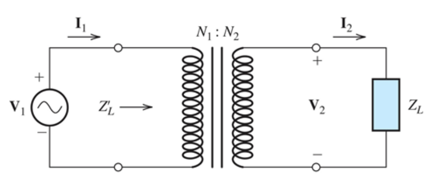


Figure 14.25 The impedance seen looking into the primary is $Z'_L = (N_1 / N_2)^2 \times Z_L$.

$$Z'_L = \frac{\mathbf{V}_1}{\mathbf{I}_1} = \left(\frac{N_1}{N_2}\right)^2 Z_L$$

Example 14.11

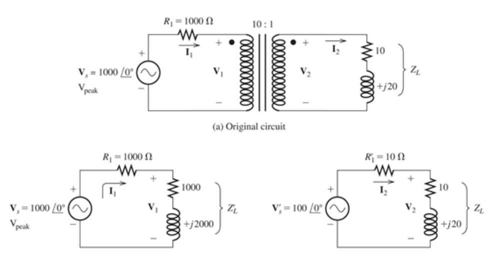


Figure 14.26 The circuit of Examples 15.11 and 15.12.

$$Z_L' = \left(\frac{N_1}{N_2}\right)^2 Z_L = 10^2(10 + j20) = 1000 + j2000$$

$$Z_s = R_1 + Z_L' = 2000 + j2000 = 2828\angle 45°$$

$$I_1 = \frac{V_s}{Z_s} = 0.3536\angle -45°$$

$$V_1 = I_1 Z_L' = 790.6\angle 18.43°$$

$$I_2 = \frac{N_1}{N_2} I_1 = 3.536\angle -45°$$

Exercise 14.18: find the turn ratio for maximum power delivery

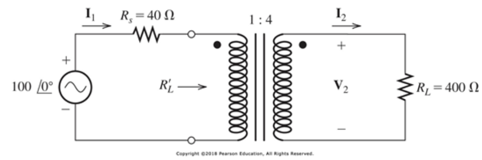
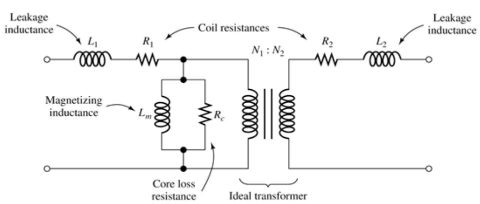


Figure 14.27 Circuit of Exercises 15.16 and 15.17.

Real Transformers



Variations of the Transformer Model

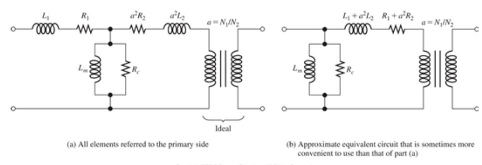


Figure 14.29 Variations of the transformer equivalent circuit. The circuit of (b) is not exactly equivalent to that of (a), but is sufficiently accurate for practical applications.

Table 14.1 Circuit Values of a 60-Hz 20-kVA 2400/240-V Transformer Compared with Those of an Ideal Transformer

Element Name	Symbol	Ideal	Real
Primary resistance	R_1	0	3.0 Ω
Secondary resistance	R_2	0	0.03 Ω
Primary leakage reactance	$X_1 = \omega L_1$	0	6.5 Ω
Secondary leakage reactance	$X_2 = \omega L_2$	0	0.07 Ω
Magnetizing reactance	$X_m = \omega L_m$	∞	15 kΩ
Core-loss resistance	R_c	∞	100 kΩ

Regulation and Efficiency

$$\text{percent regulation} = \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} \times 100\%$$

$$\text{power efficiency} = \frac{P_{\text{load}}}{P_{\text{in}}} \times 100\% = \left(1 - \frac{P_{\text{loss}}}{P_{\text{in}}}\right) \times 100\%$$

Example 14.13

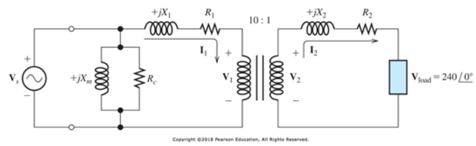


Figure 14.30 Circuit of Example 14.13.

$$V_{load} = 240\angle 0°V, \quad P_{app} = 20kVA$$

$$I_2 = 83.33\angle -36.87°A$$

$$I_1 = \frac{N_2}{N_1} I_2 = 8.333\angle -36.87°A$$

$$V_2 = V_{load} + I_2(R_2 + jX_2) = 245.50 + j3.166A$$

$$V_1 = \frac{N_1}{N_2} V_2 = 2455 + j31.66V$$

$$P_{in} = P_{load} + P_{loss} = 16479.5W$$

$$efficiency = (1 - \frac{P_{loss}}{P_{in}}) \times 100\% = 97.09\%$$

No-load case:

$$I_1 = I_2 = 0$$

$$V_1 = V_s = 2508.2V$$