

City University of Hong Kong
Department of Mathematics 3442 8646

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Course code and title: MA1501 Coordinate Geometry

Session: Semester B, 2019-2020

Time allowed: Two hours

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This paper has 3 pages (including this cover page)

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Instructions to students:

1. This exam paper consists of four independent questions.
2. Answer all of the following four questions and show all your steps and reasoning in order to justify your answers.
3. Present your solutions neatly and logically.
4. The full mark for this paper is 100.
5. This is an open-book examination. Calculators and lecture notes are allowed.

Question 1 (25 marks).

(a) Prove that for any vectors $\mathbf{a}, \mathbf{b}, \mathbf{c} \in \mathbf{R}^3$ we have the following property

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$$

(b) Find two unit vectors that are orthogonal to both vectors

$$\mathbf{a} = \mathbf{j} + 2\mathbf{k} \text{ and } \mathbf{b} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}.$$

Question 2 (25 marks).

(a) Determine the parametric and symmetric equations for the line through the point $P(0,1,2)$ that is perpendicular to the line

$$x = 1 + t, y = 1 - t, z = 2t, \quad t \in \mathbf{R}$$

and is parallel to the plane

$$x + y + z = 2.$$

(b) Find the foci and vertices of the conic section whose equation is given by:

$$y^2 + 2y = 4x^2 + 3$$

Then, sketch the graph.

Question 3 (25 marks).

(a) Write the equation of the surface consisting in all points that are equidistant from the point $M(-1,0,0)$ and the plane $x = 1$. Identify the surface.

(b) Identify the surface whose equation in spherical coordinates is given by:

$$\rho \sin \varphi = 2$$

and determine its equation in rectangular and in cylindrical coordinates.

Question 4 (25 marks).

For (a) and (b) separately, reduce the equation to one of the standard forms, identify the quadric surface, and then sketch it:

$$(a) \quad 4x^2 + 4y^2 - 8y + z^2 = 0$$

$$(b) \quad -4x^2 + y^2 - 4z^2 = 4$$