BME2102：

Introduction to Biomechanics

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Scaling Laws

Physical quantity	Scaling exponent	Typical magnitude	Scaling accuracy	Comments
Area	2	10^{-18} m^2	Definitional	Surface forces domain
Volume	3	10^{-27} m^3	Definitional	Volume forces ignorable
Mass	3	ag-zg	Good	
vdW Forces	2 (Area)	pN-nN	Good	
vdW Forces	-7(Dis., Attractive) -13 (Dis., Repulsive)	pN-nN	Good	AFM, manipulation, Supermolecules
Electrostatic force	2	pN-nN	Good at small scale	Actuation
Magnetic force	2(Dis.) 6(Sizes)	10^{-23} N	Good	
Friction	2		Moderate to inapplicable	Motion
Oscillating frequency	-3/2	GHz-THz	Good	Motion, sensing

Summary

- Scaling to micro- and nano-scale is typically not intuitive. Use scaling laws to estimate the effects.
- Depending on the size-scale different physical effects become more or less important
  - Going down from macro to micro inertia forces become less important while surface forces such as adhesive forces become more important
- Moreover, when the size of a structure is comparable to the size of its atoms there are quantum effects

Further reading

- K. E. Drexler "Classical Magnitudes and Scaling Laws" from "Nanosystems: Molecular Machinery, Manufacturing and Computation"
  http://www.e-drexler.com/d/06/00/Nanosystems/toc.html
- M. Wautelet "Scaling laws in the macro-, micro- and nanoworlds" Eur. J. Phys. 22, 601-611, 2001

VII. Viscoelasticity

Time-dependent behaviour

Review: A simple one-dimensional model of a skeletal muscle

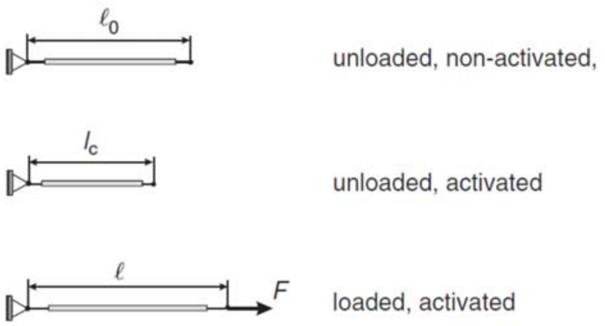


Figure 4.6

Different reference and current lengths of a muscle.

- l_0: the length of the muscle in the non-activated state
- l_c: the length of the muscle in the activated or contracted but unloaded state.
- l: the length of the muscle in the activated and loaded state.

- Now, in contrast with a simple elastic spring, the contracted length l_c serves as the reference length, such that the force in the muscle may be expressed as:

$$F = c\left(\frac{\ell}{\ell_c} - 1\right) = c\,\varepsilon \qquad \lambda_c = \frac{\ell_c}{\ell_0} \qquad \lambda = \frac{\ell}{\ell_0}$$

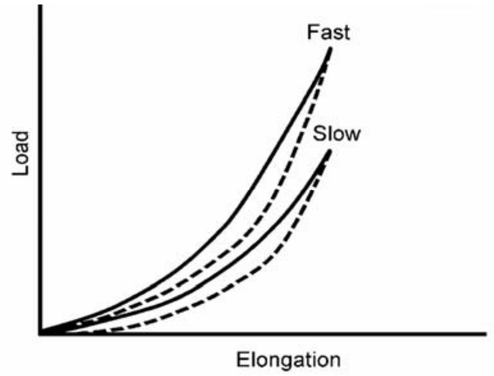
activation or contraction stretch

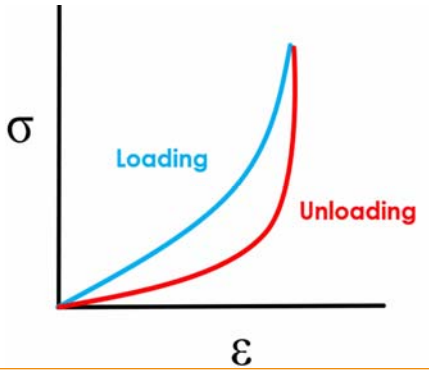
Time-dependent elasticity

- In Part IV, muscle fibers were considered to be elastic, meaning that a unique relation exists between the extensional force and the deformation of the fiber.
- This implies that the force versus stretch curves for the loading and unloading path are identical. There is no history dependency and all energy that is stored into the fiber during deformation is regained during the unloading phase.
- This also implies that the rate of loading or unloading does not affect the force versus stretch curves. However, most biological materials do not behave elastically!

Review

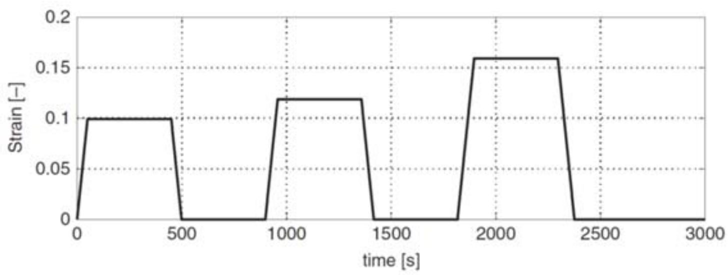
- Structure-Property Correlation
  - Viscoelastic properties occur when the stress and strain on a materials are dependent on how quickly or slowly the load applied
  - Hysteresis is an elastic property describing a material that has a different stress-strain curve when being unloaded compared to loaded





Relaxation and Creep

- An example of a loading history and a typical response of a biological material.



(a)

- In Fig. 5.1(a) a deformation history is given that might be used in an experiment to mechanically characterize some material specimen. The specimen is stretched fast to a certain value, then the deformation is fixed and after a certain time restored to zero. After a short resting period, the stretch is applied again but to a higher value of the stretch. This deformation cycle is repeated several times. In this case the length change is prescribed and the associated force is measured.

Relaxation and Creep

- Fig. 5.1(b) shows the result of such a measurement. When the length of the fiber is kept constant, the force decreases in time. This phenomenon is called **relaxation**.
- Reversely, if a constant load is applied, the length of the fiber will increase. This is called **creep**.

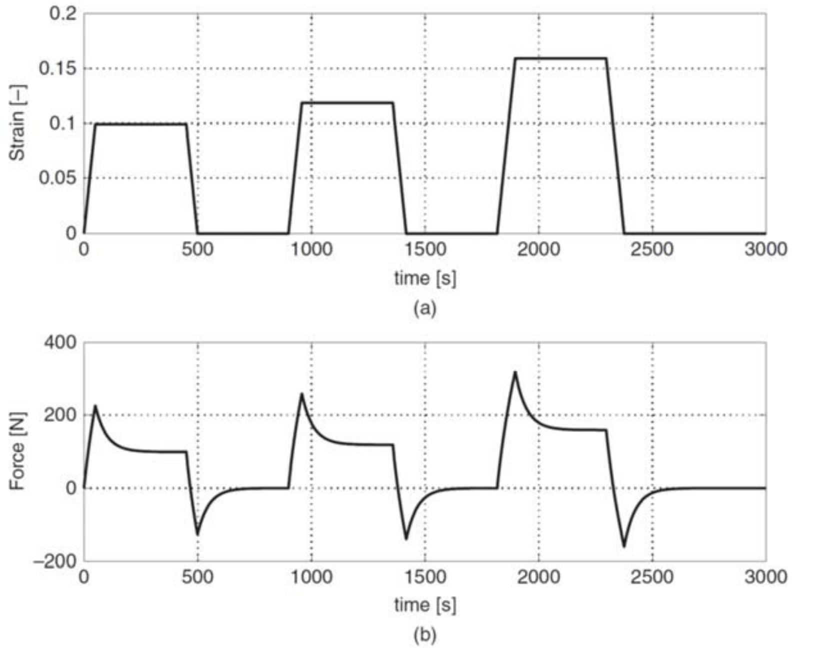
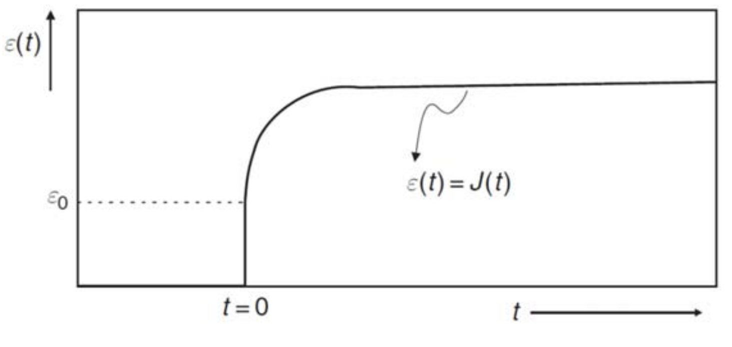


Figure 5.1

Loading history in a relaxation experiment. The deformation of the tissue specimen is prescribed.

Creep compliance

- Reversely, if a **constant load** is applied, the **length** of the fiber will **increase**. This is called **creep**.
- The response ε(t) might have an evolution as given in Fig. 5.7. This response denoted by $J(t)$ is called the **creep compliance** or **creep function**.



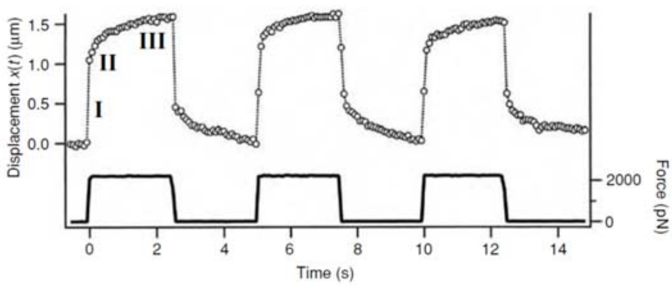


Figure 5.7

Typical example of the strain response after a unit-step in the force

Hysteresis and Viscoelasticity

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- When the material is subjected to cyclic loading, the force versus stretch relation in the loading process is usually somewhat different from that in the unloading process. This is called hysteresis and is demonstrated in Fig. 5.2.
- The difference in the response paths during loading and unloading implies that energy is dissipated, usually in the form of heat, during the process.
- Most biological materials show more or less the above given behavior, which is called viscoelasticity.

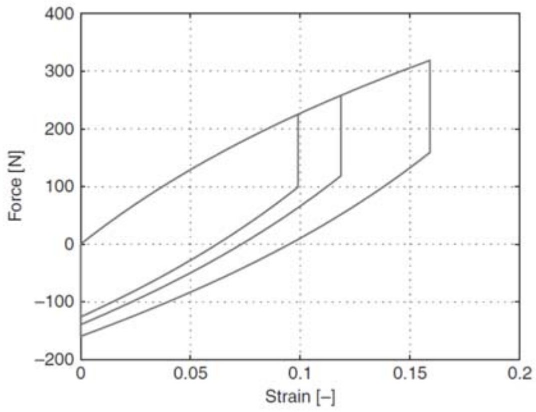

Figure 5.2
Force/strain curve for cyclic loading of a biological material.

Hysteresis and Viscoelasticity

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- The present part discusses how to describe this behavior mathematically. Pure viscous behavior, as can be attributed to an ideal fluid is considered first.
- The description will then be extended to linear viscoelastic behavior, followed by a discussion on harmonic excitation, a technique that is often used to determine material properties of viscoelastic materials.

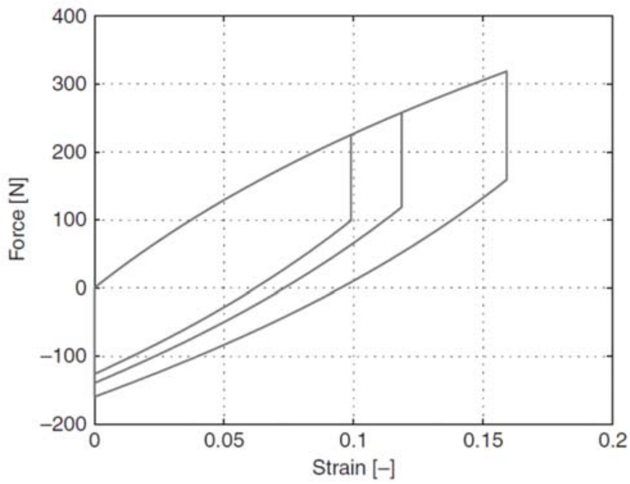


Figure 5.2
Force/strain curve for cyclic loading of a biological material.

Viscous behavior

- It is not surprising that biological tissues do not behave purely elastically, since a large percentage of most tissues is water. The behavior of water can be characterized as ‘viscous’.
- Cast in a one-dimensional format, viscous behavior during elongation (as in a fiber) may be represented by

$$F = c_\eta \boxed{\frac{1}{\ell}\frac{d\ell}{dt}},$$

- where c_η is the damping coefficient in [Ns] and *d/dt* measures the rate of change of the length of the fiber.

Dashpot

- Mechanically this force-elongational rate relation may be represented by a **dashpot** (see Fig. 5.3).
- Generally:

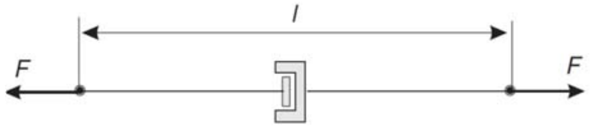


Figure 5.3

Mechanical representation of a viscous fibre by means of a dashpot.

$$D = \frac{1}{\ell}\frac{d\ell}{dt}$$

- is called the rate of deformation, which is related to the stretch parameter *λ*. Recall that

$$\lambda = \frac{\ell}{\ell_0}, \qquad \frac{dl}{dt} = l_0\frac{d\lambda}{dt}$$

- Such that

$$D = \frac{1}{\ell}\frac{d\ell}{dt} = \frac{1}{\lambda}\frac{d\lambda}{dt}.$$

Dashpot

- Ideally, a fluid stretching experiment should create a deformation pattern as visualized in Fig. 5.4(a).
- In practice this is impossible, because the fluid has to be spatially fixed and loaded, for instance via end plates, as depicted in Fig. 5.4(b). In this experiment a fluid is placed between two parallel plates at an initial distance l_0. Next, the end plates are displaced and the force on the end plates is measured.

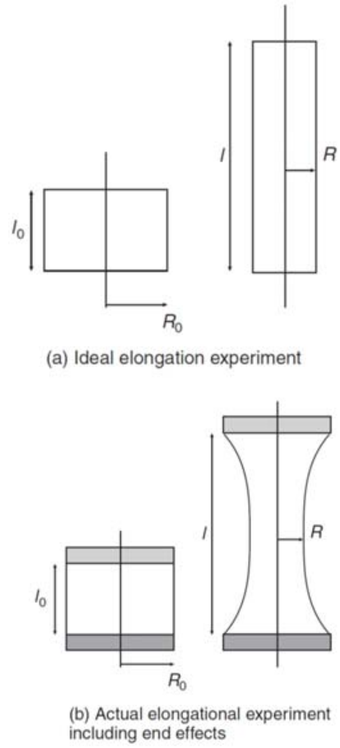


Figure 5.4

Schematic representation of an elongation experiment for fluids.

Dashpot

- A typical example of a stretched filament is shown in Fig. 5.5. Although this seems to be a simple experiment, it is rather difficult to perform in practice.
- Fig. 5.5 shows an experiment, where the fluid is a little extended initially, after which gravitational sag continues the filament stretching process. Ideally, a filament stretching experiment should be performed at a constant elongational rate. This is not trivial to achieve.

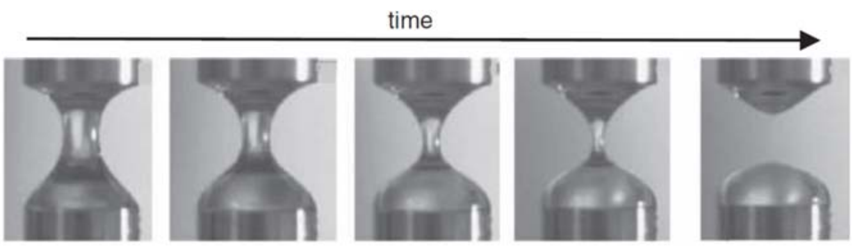


Figure 5.5

Example of uniaxial testing experiment with a fluid.

Dashpot

- For instance, let one end of the dashpot, say point A positioned at the origin (i.e. $x_A = 0$), be fixed in space, while the other end, point B is displaced with a constant velocity v, as depicted in Fig. 5.6. In that case the position of point B is given by

$$x_\mathrm{B} = \ell_0 + vt,$$

- with l_0 the initial length of the dashpot.

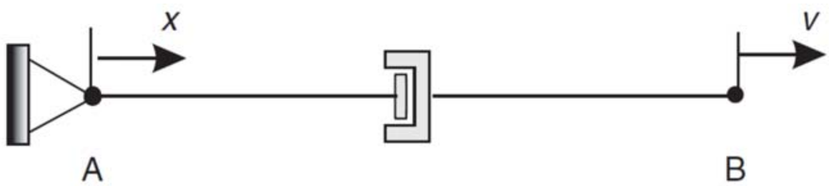


Figure 5.6

Point B is moved with a constant velocity *v*.

Dashpot

- The actual length *l* at time *t* is given by:

$$\ell = x_\mathrm{B} - x_\mathrm{A} = \ell_0 + vt.$$

- Hence, the elongational rate is given by

$$D = \frac{1}{\ell}\frac{d\ell}{dt} = \frac{v}{\ell_0 + vt}.$$

- This shows, that if one end is moved with a constant velocity, the elongational rate decays with increasing time *t*. Maintaining a constant elongational rate is possible if the velocity of point B is adjusted as a function of time.

Dashpot

- Indeed, a constant elongational rate D implies that the length must satisfy

$$\frac{1}{\ell}\frac{d\ell}{dt} = D,$$

- subject to the initial condition $l = l_0$ at $t = 0$, while D is constant. Since

$$\frac{d\ln(\ell)}{dt} = \frac{1}{\ell}\frac{d\ell}{dt} = D, \qquad \ln\left(\frac{l}{l_0}\right) = Dt$$

- the solution is given by

$$\ell = \ell_0\, \mathrm{e}^{Dt}.$$

Small stretches: linearization

- If u_B and u_A denote the end point displacements, introduce:

$$\Delta\ell = u_B - u_A.$$

- The stretch λ may be expressed as

$$\lambda = \frac{\ell_0 + \Delta\ell}{\ell_0}.$$

- Introducing the strain ε as

$$\varepsilon = \frac{\Delta\ell}{\ell_0},$$

- the stretch is written as

$$\lambda = 1 + \varepsilon.$$

Small stretches: linearization

- For sufficiently small strain levels, i.e. |ε| << 1, and using the notation $\dot{\varepsilon} = d\varepsilon/dt$ it can be written:

$$D = \frac{1}{\lambda}\frac{d\lambda}{dt} = \frac{1}{1+\varepsilon}\dot{\varepsilon} \approx (1-\varepsilon)\dot{\varepsilon} \approx \dot{\varepsilon}.$$

- Consequently, if |ε| << 1 then

$$D = \frac{1}{\ell}\frac{d\ell}{dt} \approx \dot{\varepsilon} = \frac{1}{\ell_0}\frac{d\ell}{dt},$$

- Recall that

$$F = c_\eta \frac{1}{\ell}\frac{d\ell}{dt},$$

so we have

$$\boxed{F = c_\eta \dot{\varepsilon}}.$$

Linear viscoelastic behavior

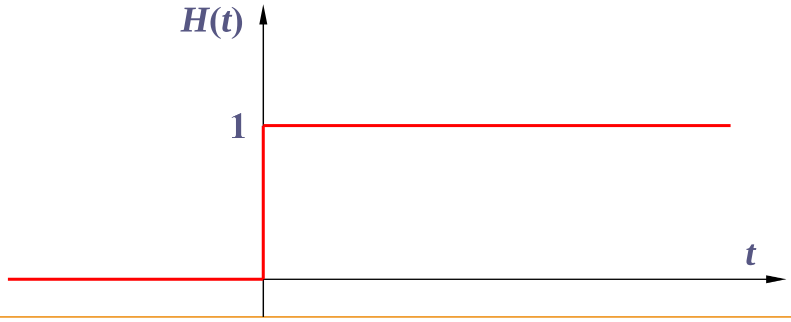
- Biological tissues usually demonstrate a combined viscous-elastic behavior as described in Part IV. In the present section we assume geometrically and physically linear behavior of the material. This means that the theory leads to linear relations, expressing the force in the deformation(-rate), and that the constitutive description satisfies two conditions:
  - **superposition** The response on combined loading histories can be described as the summation of the responses on the individual loading histories.
  - **proportionality** When the strain is multiplied by some factor the force is multiplied by the same factor (in fact proportionality is a consequence of superposition).

Unit-step function (Heaviside function)

- To study the effect of these conditions a unit-step function for the force is introduced, defined as $H(t)$ (Heaviside function):

$$\mathrm{H}(t) = \begin{cases} 0 & \text{if} \quad t < 0 \\ 1 & \text{if} \quad t \geq 0 \end{cases}.$$

- Assume that a unit-step in the force $F(\mathrm{t}) = H(\mathrm{t})$ is applied to a linear viscoelastic material.



Creep compliance

- The response $\varepsilon(\mathrm{t})$ might have an evolution as given in Fig. 5.7. This response denoted by $J(t)$ is called the **creep compliance** or **creep function**.

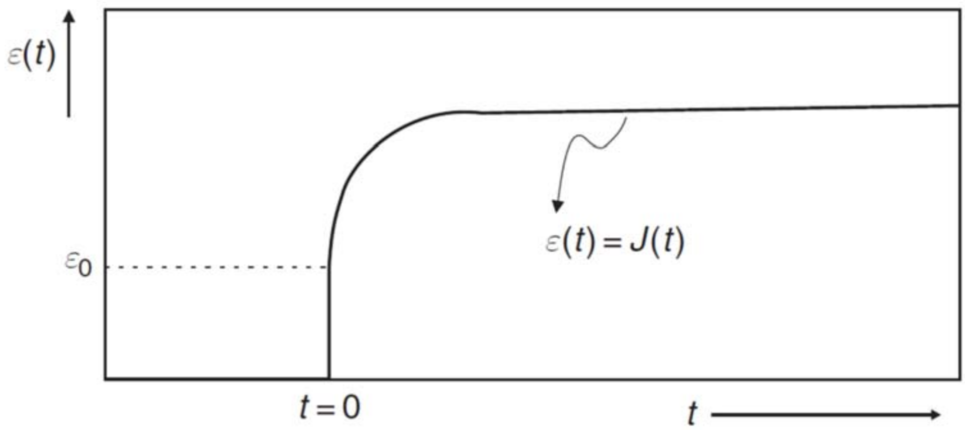


Figure 5.7

Typical example of the strain response after a unit-step in the force

Proportionality and Superposition

- Proportionality means, that increasing the force with some factor F_0 leads to a proportional increase in the strain:

$$F(t) = \mathrm{H}(t)\ F_0 \quad \rightarrow \quad \varepsilon(t) = J(t)\ F_0.$$

- Superposition implies, that applying a load step F_0 at t = 0, with response:

$$\varepsilon_0(t) = J(t)\,F_0,$$

- followed by a load step F_1 at $t = t_1$ with individual response:

$$\varepsilon_1(t) = J(t - t_1)\,F_1,$$

Superposition

- This is graphically shown in Fig. 5.8.

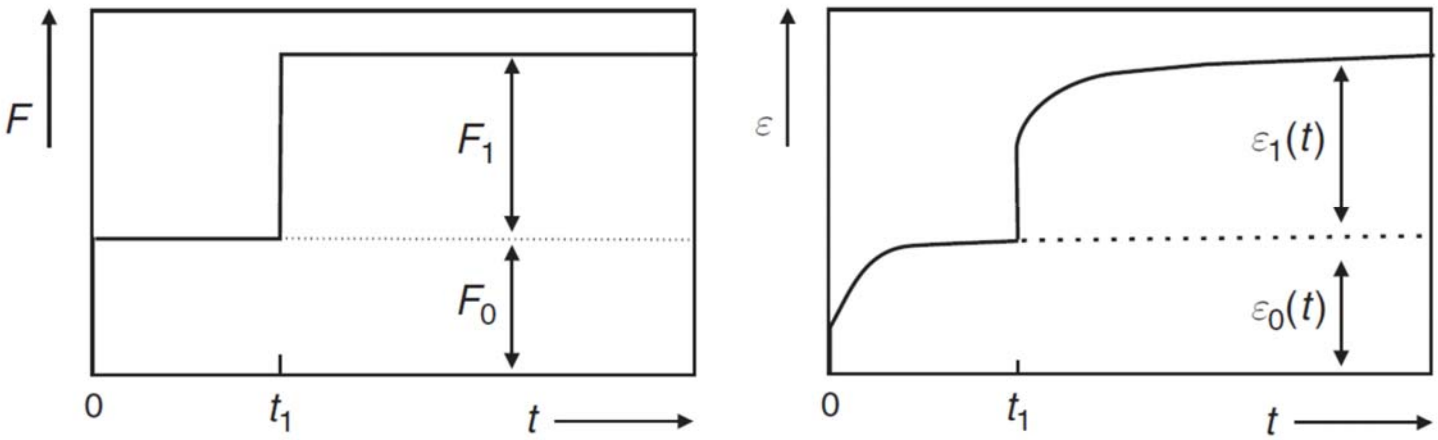


Figure 5.8

Superposition of responses for a linear visco-elastic material.

Creep

- Both principles can be used to derive a more general constitutive equation for linear viscoelastic materials. Assume, we have an arbitrary excitation as sketched in Fig. 5.9. This excitation can be considered to be built up by an infinite number of small steps in the force.

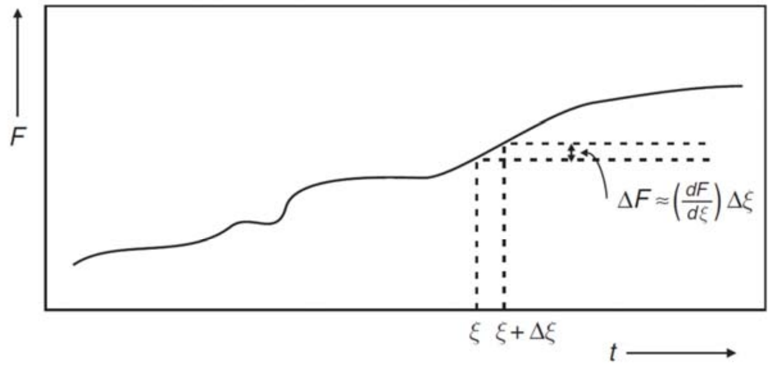


Figure 5.9

An arbitrary force history in a creep test.

Creep

- The increase ΔF of the force F between time steps $t = \xi$ and $t = \xi + \Delta\xi$ is equal to

$$\Delta F \approx \left(\frac{dF}{d\xi}\right)\Delta\xi = \dot{F}(\xi)\,\Delta\xi\,.$$

- The response at time t as a result of this step at time ξ is given by

$$\Delta\varepsilon(t) = \dot{F}(\xi)\,\Delta\xi J(t-\xi)\,.$$

- The time-dependent force $F(t)$ as visualized in Fig. 5.9 can be considered as a composition of sequential small steps.

Creep function

- By using the superposition principle we are allowed to add the responses on all these steps in the force (for each ξ). This will lead to the following integral expression, with all intervals $\Delta\xi$ taken as infinitesimally small:

$$\varepsilon(t) = \int_{\xi=-\infty}^{t} J(t-\xi)\,\dot{F}(\xi)\,d\xi .$$

- This integral was derived first by Boltzmann in 1876.
- Assuming, that the creep function is zero for $t < 0$, the Laplace transform is

$$\hat{\varepsilon}(s) = \hat{J}(s)\ s\,\hat{F}(s)$$

Relaxation

- In the creep experiment the load is prescribed and the resulting strain is measured. Often, the experimental set-up is designed to prescribe the strain and to measure the associated, required force. If the strain is applied as a step, this is called a **relaxation experiment**, because after a certain initial increase the force will gradually decrease in time. The same strategy as used to derive the previous equation can be pursued for an imposed strain history, leading to

$$F(t) = \int_{\xi=-\infty}^{t} G(t-\xi)\,\dot{\varepsilon}(\xi)\,d\xi ,$$

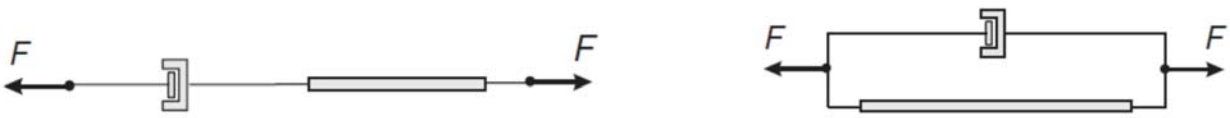
- with $G(t)$ the relaxation function. Assuming, that the relaxation function is zero for $t < 0$, the Laplace transform is

$$\hat{F}(s) = \hat{G}(s)\ s\,\hat{\varepsilon}(s) .$$

Viscoelastic models based on springs and dashpots: Maxwell model

- An alternative way of describing linear viscoelastic materials is by assembling a model using the elastic and viscous components as discussed before. Two examples are given, while only small stretches are considered. In that case the constitutive models for the elastic spring and viscous dashpot are given by

$$F = c\,\varepsilon, \qquad F = c_\eta \dot{\varepsilon}.$$



(a) Spring-dashpot in series (b) Spring-dashpot in parallel

Figure 5.10

A Maxwell (a) and Kelvin-Voigt (b) arrangement of the spring and dashpot.

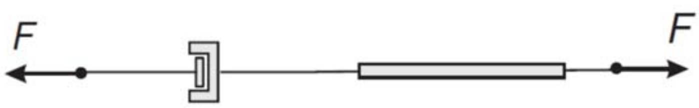
Maxwell model

- In the Maxwell model according to the set-up of Fig. 5.10(a) the strain ε is additionally composed of the strain in the spring (ε_s) and the strain in the damper (ε_d):

$$\varepsilon = \varepsilon_{\mathrm{s}} + \varepsilon_{\mathrm{d}},$$

- implying that

$$\dot{\varepsilon} = \dot{\varepsilon}_{\mathrm{s}} + \dot{\varepsilon}_{\mathrm{d}}.$$



$$\boxed{\dot{F} + \frac{1}{\tau}F = c\dot{\varepsilon}.}$$

Maxwell model

Apparently, the integral equation as introduced in the previous section,

$$F(t) = \int_{\xi=-\infty}^{t} G(t-\xi)\,\dot{\varepsilon}(\xi)\,d\xi$$

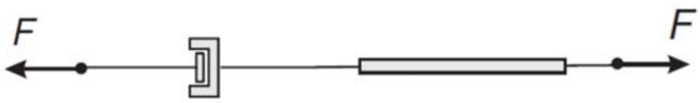
can be considered as a general solution of a differential equation. In the present case the relaxation spectrum, as defined in

$$G(t) = G_\infty + \sum_{j=1}^{M} g_j\, \mathrm{e}^{-t/\tau_j}$$

is built up by just one single Maxwell element and in this case: $G(t) = \mathrm{e}^{-t/\tau}$.

$$\boxed{F(t) = c\int_0^t \mathrm{e}^{-(t-\xi)/\tau}\,\dot{\varepsilon}(\xi)\ d\xi}$$

Maxwell model: Summary



$$F = c_\eta \dot{\varepsilon} \qquad F = c\,\varepsilon$$

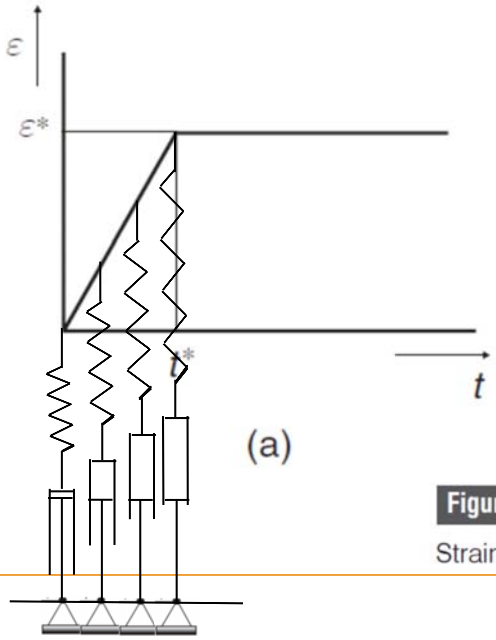
$$\boxed{\dot{F} + \frac{1}{\tau}F = c\dot{\varepsilon}} \qquad \boxed{\tau = \frac{c_\eta}{c}}$$

- The solution *F* is given by

$$\boxed{F(t) = c\int_0^t \mathrm{e}^{-(t-\xi)/\tau}\,\dot{\varepsilon}(\xi)\ d\xi}$$

Maxwell model

- To understand the implications of this model, consider a strain history as specified in Fig. 5.11, addressing a spring-dashpot system in which one end point is fixed while the other end point has a prescribed displacement history.



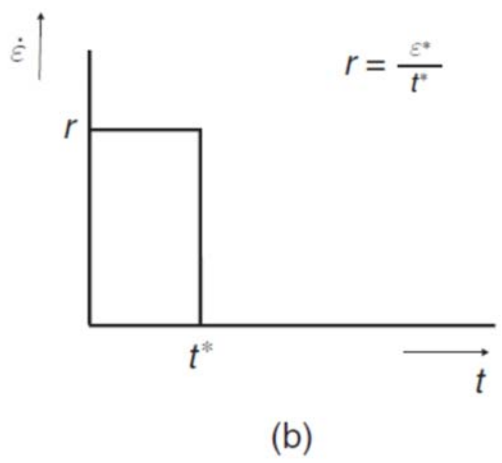
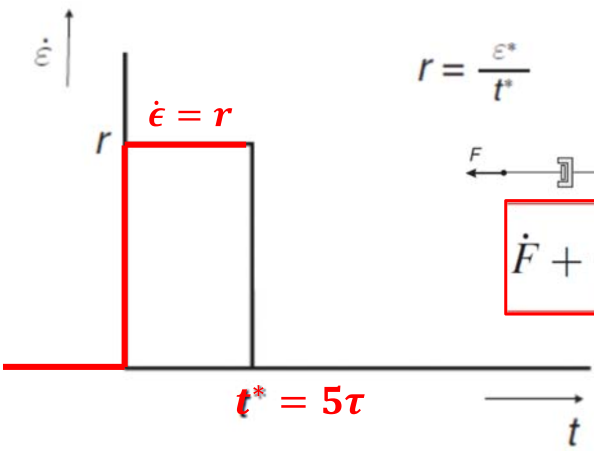


Figure 5.11

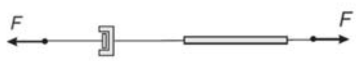
Strain(a) and strain rate (b) as a function of time.

Maxwell model

- The force response is given in Fig. 5.12 in case $t^* = 5\tau$. Notice that in this figure the time has been scaled with the relaxation time τ, while the force has been scaled with $c_\eta r$, with r the strain rate, see Fig. 5.11. Two regimes may be distinguished.



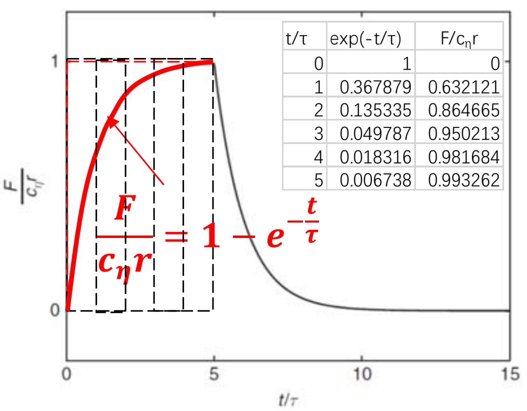
$r = \frac{\varepsilon^*}{t^*}$



$$\boxed{\dot{F} + \frac{1}{\tau}F = c\dot{\varepsilon}}$$

Figure 5.11

Strain(a) and strain rate (b) as a function of time.



t/τ	exp(-t/τ)	F/c$_\eta$r
0	1	0
1	0.367879	0.632121
2	0.135335	0.864665
3	0.049787	0.950213
4	0.018316	0.981684
5	0.006738	0.993262

Figure 5.12

Force response of the Maxwell model.

Maxwell model

- (i) For $t < t^*$ the strain proceeds linearly in time leading to a constant strain rate r. In this case the force response is given by (recall that $c_\eta = \tau c$)

$$F = c_\eta r\left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right).$$

- For $t << \tau$ it holds that

$$\mathrm{e}^{-\frac{t}{\tau}} \approx 1 - \frac{t}{\tau},$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = \frac{x^0}{0!} + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \cdots$$
$$\boxed{= 1 + x} + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots.$$

- such that the force in that case is given by

$$F = c_\eta \frac{t}{\tau} r = c\,t\,r. \qquad \boxed{\tau = \frac{c_\eta}{c}}$$

Maxwell model

- So, for relatively small times t, and constant strain rate, the response is dominantly elastic. This is consistent with the spring-dashpot configuration. For small t only the spring is extended while the dashpot is hardly active. Furthermore, at t = 0:

$$\dot{F} = cr,$$

- which implies that the line tangent to the force versus time curve should have a slope of cr.

Maxwell model

- For larger values of t, but still smaller than t^* we have

$$e^{-\frac{t}{\tau}} \to 0,$$

- such that the force is given by

$$F = c_\eta r,$$

- which is a purely viscous response. In this case the spring has a constant extension, and the force response is dominated by the dashpot. This explains why, at a constant strain rate, the force curve tends towards an asymptote in Fig. 5.12 for sufficiently large t.

Maxwell model

- (ii) For times $t > t^*$ the strain rate is zero. In that case, the force decreases exponentially in time. This is called **relaxation**. If F^* denotes the force $t = t^*$, the force for $t > t^*$ is given by

$$F = F^* e^{-(t-t^*)/\tau}.$$

- The rate of force **relaxation** is determined by τ, which explains why τ is called a relaxation time. At $t = t^*$, the slope of the tangent to the force curve equals

$$\dot{F}\big|_{t=t^*} = -\frac{F^*}{\tau}.$$

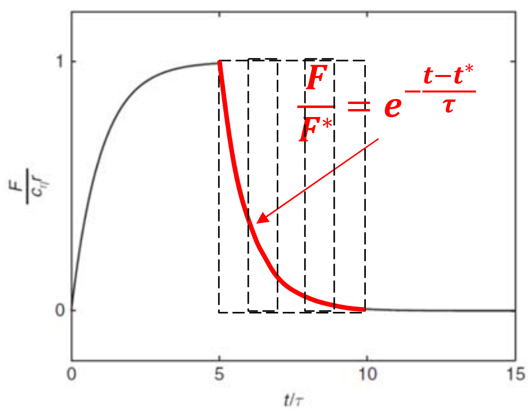
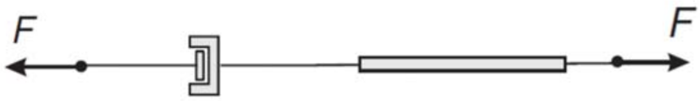


Figure 5.12

Force response of the Maxwell model.

Maxwell model: Summary



$$F = c_\eta \dot{\varepsilon} \qquad F = c\,\varepsilon$$

$$\boxed{\dot{F} + \frac{1}{\tau}F = c\dot{\varepsilon}} \qquad \boxed{\tau = \frac{c_\eta}{c}}$$

- The solution F is given by

$$\boxed{F(t) = c\int_0^t e^{-(t-\xi)/\tau}\,\dot{\varepsilon}(\xi)\;d\xi}$$

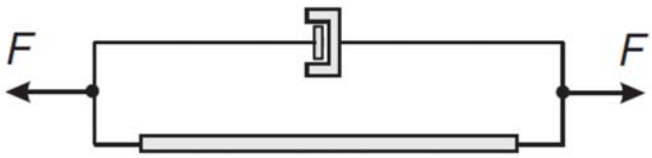
- Special cases

$$\dot{F} = cr \qquad F = c_\eta r\left(1 - e^{-\frac{t}{\tau}}\right) \qquad F = c_\eta r$$

t* = 0** | ***t* < *t* | **For larger values of *t*, but *t* < *t****

Viscoelastic models based on springs and dashpots: Kelvin–Voigt model

- A second example of combined viscous and elastic behaviour is obtained for the set-up of Fig. 5.10(b). In this case the total force F equals the sum of the forces due to the elastic spring and the viscous damper:



(b) Spring-dashpot in parallel

$$F = c\,\varepsilon + c_\eta \dot{\varepsilon},$$

- or, alternatively after dividing by c_η:

$$\frac{F}{c_\eta} = \frac{c}{c_\eta}\varepsilon + \dot{\varepsilon}.$$

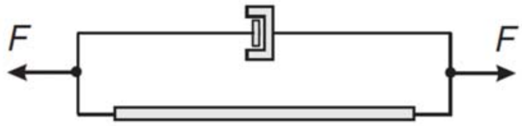
- Introducing the retardation time:

$$\tau = \frac{c_\eta}{c},$$

Kelvin–Voigt model

- $\dfrac{F}{c_\eta} = \dfrac{c}{c_\eta}\varepsilon + \dot{\varepsilon}$ may also be written as

$$\frac{F}{c_\eta} = \frac{1}{\tau}\varepsilon + \dot{\varepsilon}, \qquad \dot{F} + \frac{1}{\tau}F = c\dot{\varepsilon}$$



(b) Spring-dashpot in parallel

- The set-up according to Fig. 5.10(b) is known as the Kelvin–Voigt model. In analogy with the Maxwell model, the solution of this differential equation is given by

$$\varepsilon(t) = \frac{1}{c_\eta}\int_0^t \mathrm{e}^{-(t-\xi)/\tau} F(\xi)\, d\xi . \qquad F(t) = c\int_0^t \mathrm{e}^{-(t-\xi)/\tau} \dot{\varepsilon}(\xi)\, d\xi$$

- In the case of a constant force *F*, the strain response is given by

$$\varepsilon(t) = \frac{F}{c}\left(1 - \mathrm{e}^{-t/\tau}\right). \qquad F = c_\eta r\left(1 - \mathrm{e}^{-\frac{t}{\tau}}\right)$$

Kelvin–Voigt model

- In the case of a constant force *F*, the strain response is given by

$$\varepsilon(t) = \frac{F}{c}\left(1 - \mathrm{e}^{-t/\tau}\right).$$

- This phenomenon of an increasing strain with a constant force (up to a maximum of *F*/*c*) is called **creep**. For *t* <<*τ*:

$$\varepsilon \approx \frac{Ft}{c_\eta},$$

- corresponding to a viscous response, while for *t* >> *τ* :

$$\varepsilon \approx \frac{F}{c},$$

- reflecting a purely elastic response.

- The standard linear model can be represented with one dashpot and two springs, as shown in Fig. 5.14. The upper part is composed of a linear spring, the lower part shows a Maxwell element.

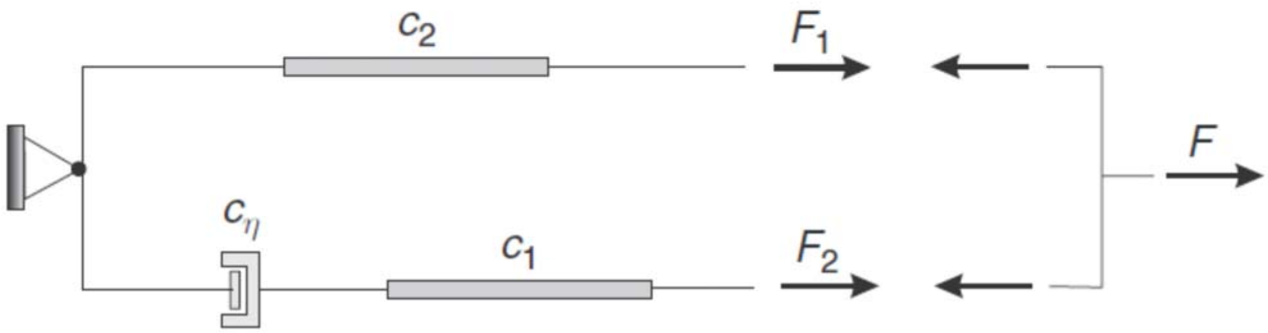


Figure 5.14

The 3-parameter standard linear visco-elastic model.

The standard linear model

- The total strain of the Maxwell element is considered as an addition of the strain in the dashpot (ε_d) and the strain in the spring ($\varepsilon_s = \varepsilon - \varepsilon_d$). The following relations can be proposed for variables that determine the standard linear model:

$$F = F_1 + F_2$$
$$F_1 = c_2 \varepsilon$$
$$F_2 = c_\eta \dot{\varepsilon}_d$$
$$F_2 = c_1(\varepsilon - \varepsilon_d)\,.$$

- Elimination of $\dot{\varepsilon}_d$, F_1 and F_2 from this set of equations leads to

$$F + \tau_R \dot{F} = c_2 \varepsilon + (c_1 + c_2)\,\tau_R \dot{\varepsilon},$$

- with $\tau_R = c_\eta / c_1$ the characteristic relaxation time.

The standard linear model

- The force response to a step $\varepsilon(t) = \varepsilon_0 H(t)$ in the strain yields

$$F(t) = F(t)_{\text{hom}} + F(t)_{\text{part}} = \alpha e^{-t/\tau_R} + c_2 \varepsilon_0,$$

- with α an integration constant to be determined from the initial conditions. Determining the initial condition at $t = 0$ for this problem is not trivial.

$$F(t) = \int_{-\infty}^{t} \left(c_2 + c_1 e^{-(t-\xi)/\tau_R} \right) \dot{\varepsilon}(\xi)\, d\xi .$$

$$\varepsilon(t) = \int_{-\infty}^{t} \frac{1}{c_2} \left(1 - \frac{c_1}{c_1 + c_2} e^{-(t-\xi)/\tau_K} \right) \dot{F}(\xi)\, d\xi .$$

Lect 1: Assessment Tasks/Activities (ATs)

- Continuous Assessment: 40%
  - In-class Test: 20%
  - Laboratory Reports : 20%--3 reports to be submitted
- Examination: 60 %
  - Duration: 2 hours
- **For a student to pass the course, at least 30% of the maximum mark for both coursework and examination should be obtained.**

Final Exam

- Comprehensive but focus more on the 2nd part of the course (75%, everything learned after the In-Class Test)
  - ~25%
    - I. Introduction
    - II. Rigid-body Mechanics: Linear Motion and Newton's Laws
    - III. Angular Motion and Euler's Laws
    - IV. Mechanics of Biomaterials part 1
    - HW1-3
  - **~75%**
    - **IV. Mechanics of Biomaterials part 2**
    - **V. Fluid Mechanics**
    - **VI. Cellular Biomechanics**
    - **VII. Scaling Laws**
    - **VIII. Viscoelasticity**
    - **Labs 1-3 (Theoretic parts on 3-point bending and electrophoresis)**
    - **HW4-7**
  - 4 questions (25% each)

Schedule (Final Exam)

- Final Exam: Dec. 16, 9:30-11:30 am
  - YEUNG: LT-12 Mr & Mrs Ho Chun Hung Lecture Theatre (LT12)
  - YEUNG: B5-210 Classroom

  - 60% of your final grade
  - Comprehensive but focus more on the 2nd part of the course

  - This is a closed-book and closed-notes examination. Formula sheet provided.
  - Justify your answers and show the details of your calculations.