Saturday, March 5, 2022 12:21 AM

Ex Find intersection of $x+y+z=1$ ① and $x+2y+2z=1$ ②

Step 1 let $z=0$ Find x and y and z $(1,0,0)$

$$(-)\begin{cases} x+y+0=1 & \cdots ① \\ x+2y+0=1 & \cdots ② \end{cases}$$

$-y=0$

$y=0\cdots$ plug back into $\cdots ①$

$x+0+0=1$

$x=1$

Step 2 $\cdots ①\ \vec{a}=\vec{i}+\vec{j}+\vec{k}$ $\cdots ②\ \vec{b}=\vec{i}+2\vec{j}+2\vec{k}$

$\vec{n}=\vec{a}\times\vec{b}=0\vec{i}-\vec{j}+\vec{k}$

Step 3

$x=x_0+at \rightarrow x=1$ $P(1,0,0)$

$y=y_0+bt \rightarrow y=-t$

$z=z_0+ct \rightarrow z=t$

Ex. point of intersection of plane $3x-y+2z-3=0$ and line $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z-1}{-2}$

Step. 1 $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z-1}{-2}=t$

$\frac{x+1}{3}=t$ $\frac{y+1}{2}=t$ $\frac{z-1}{-2}=t$

$x+1=3t$ $y+1=2t$ $z-1=-2t$

$x=3t-1$ $y=2t-1$ $z=-2t+1$

Step 2. System of equations (substitution)

$3(3t-1)-(2t-1)+2(-2t+1)-3=0$

$9t-3-2t+1-4t+2-3=0$

$3t-3=0$

$t=1$

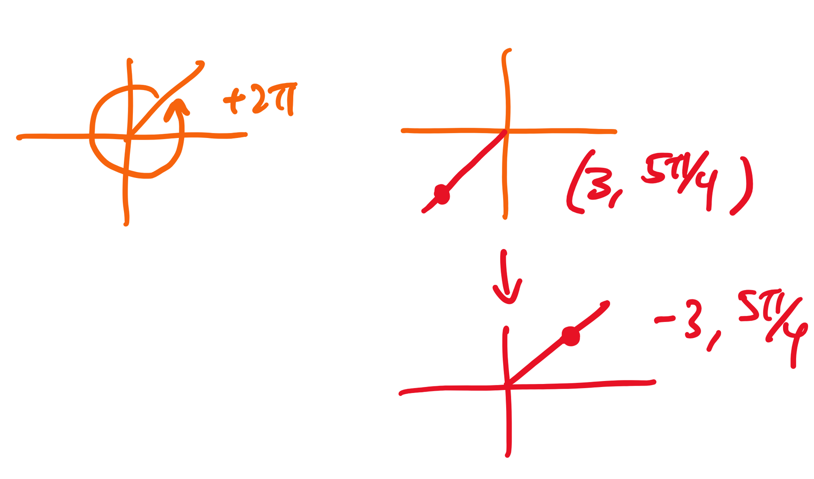
Step 3. $x=3(1)-1$ $y=2(1)-1$ $z=-2(1)+1$

$x=2$ $y=1$ $z=-1$ or $(2,1,-1)$

Ex Another way to write this $(3, \pi/4)$

① $(3, 9\pi/4)$

② $(-3, 5\pi/4)$



Ex $(-8, 2\pi/3)$ polar $\rightarrow$ cartesian

r θ (step 3) x, y (step 1) $\cdots$ $x=r\cos\theta$, $y=r\sin\theta$ (step 2)

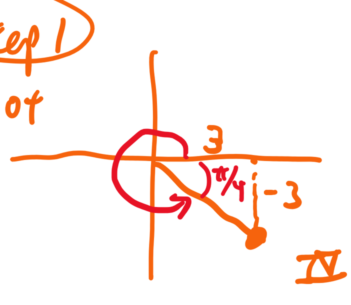
$x=-8\cos 2\pi/3=(-8)(-\frac{1}{2})=4$

$y=-8\sin 2\pi/3=(-8)(\sqrt{3}/2)=-4\sqrt{3}$

$(4, -4\sqrt{3})$

Ex. $(3,-3)$ with $r>0$ $0\le\theta\le 2\pi$

step 1 plot



step 2 find r

$r=\sqrt{x^2+y^2}$

$r=\sqrt{3^2+(-3)^2}$

$r=\sqrt{9+9}$

$r=\sqrt{18}$

$r=3\sqrt{2}$

step 3 Find θ

$\tan\theta=\frac{y}{x}$

$\tan\theta=-3/3$

$\tan\theta=-1$

$\theta=\tan^{-1}(-1)$

$\theta=\pi/4$

$\theta=2\pi-\pi/4$

$\theta=7\pi/4$

$\therefore (r,\theta)=(3\sqrt{2}, 7\pi/4)$

If restriction is missing then $(3\sqrt{2}, 7\pi/4)$ is one of the answer.

Ex. A pt that divides internally the line segment joining the pt $(8,9)$ and $(-7,4)$ in the ratio $2:3$. Find the coordinates of the point.

step 1 recall: $x=\frac{mx_2+nx_1}{m+n}$ $y=\frac{my_2+ny_1}{m+n}$

step 2

$x_1=8$ $y_1=9$

$x_2=-7$ $y_2=4$

$m=2$ $n=3$

$\left(\frac{(2)(-7)+3(8)}{2+3}, \frac{2(4)+3(9)}{2+3}\right)$

$=(2,5)$

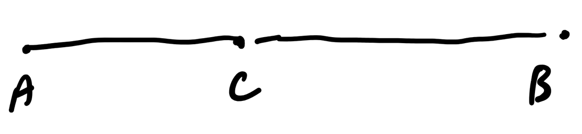
Ex. $A(4,5)$ and $B(7,-1)$ 2 given pts and point C divides the AB externally in ratio of $4:3$. Find the coordinates of C.

step 1 recall $x=\frac{mx_2-nx_1}{m-n}$ $y=\frac{my_2-ny_1}{m-n}$

$x=\frac{4(7)-3(4)}{4-3}$ $y=\frac{4(-1)-3(5)}{4-3}$

$x=16$ $y=-19$ $(16,-19)$

Ex.



Find ratio = ?

Ex Distance between parallel lines $3x+4y+7=0$ and $3x+4y-5=0$

$d=\frac{|c_1-c_2|}{\sqrt{a^2+b^2}}$ $c_1=7$ $c_2=-5$ $a=3$ $b=4$

$d=\frac{7-(-5)}{\sqrt{3^2+4^2}}=\frac{12}{5}$

Ex. Find the distance from the line $6x-4y+36=0$ to point $(0,0)$

$d=\frac{|a_1x_1+b_1y_1+c_1|}{\sqrt{a_1^2+b_1^2}}$ $a_1=6$ $b_1=-4$ $c_1=36$ $x_1=0$ $y_1=0$

$d=\frac{|6(0)+(-4)(0)+36|}{\sqrt{6^2+(-4)^2}}=\frac{36}{\sqrt{52}}$

Ex $\vec{a}=(3,-3,1)$ $\vec{b}=(4,9,2)$

Ans:

$$\vec{a}\times\vec{b}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -3 & 1 \\ 4 & 9 & 2 \end{vmatrix}=\vec{i}\begin{vmatrix} -3 & 1 \\ 9 & 2 \end{vmatrix}-\vec{j}\begin{vmatrix} 3 & 1 \\ 4 & 2 \end{vmatrix}+\vec{k}\begin{vmatrix} 3 & -3 \\ 4 & 9 \end{vmatrix}$$

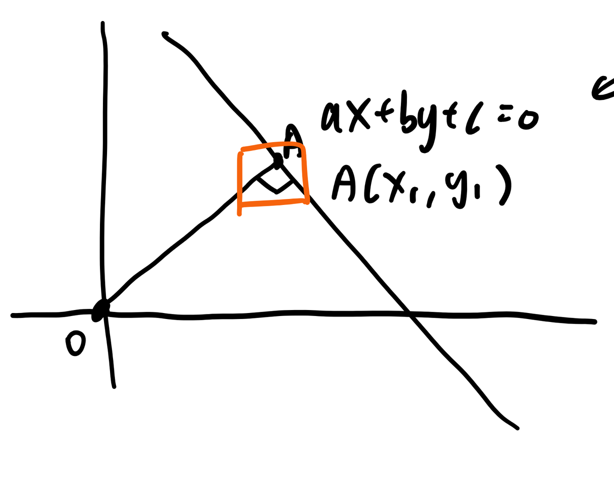
$=((-3\cdot 2)-(9)(1))\vec{i}-((3)(2)-(1)(4))\vec{j}+((3)(9)-(4)(-1))\vec{k}$

$\vec{a}\times\vec{b}=-15\vec{i}-2\vec{j}+39\vec{k}$

$\cdots$ Apply to different shapes :)

Assignment 2.

Ex. 1



Find slope

$by=-ax-c$

$y=-\frac{a}{b}x-\frac{c}{b}$

$m=-a/b\cdot\frac{y_1}{x_1}=-1$

$x_1=\frac{-ac}{a^2+b^2}$ $y=\frac{-bc}{a^2+b^2}$

$d=\sqrt{x_1^2+y_1^2}=\frac{|c|}{\sqrt{a^2+b^2}}$

Ex 2. $P(2,-1,3)$ ⊥ $\frac{(x+1)}{2}=\frac{(y-1)}{3}=\frac{(z-0)}{-4}=t$

$x=2t-1$

$y=2t+1$

$z=-4t$

$2x+3y-4z+D=0$

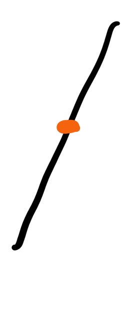
$2(2)+3(-1)-4(3)+D=0$

$D=11$

$\therefore 2x+3y-4z+11=0$

Ex 4. $A(3,-1,4)$ $\boxed{x=-2+3t \quad y=-2t \quad z=1+4t}$

$\vec{u}=\langle 3,-2,4\rangle$ $B(-2,0,1)$ $t=0$



$\vec{AB}=(5,-1,3)$

$$\vec{n}=\vec{u}\times\vec{AB}=\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -2 & 4 \\ 5 & -1 & 3 \end{vmatrix}=\langle -2,11,7\rangle$$

$d=\frac{|\vec{u}\times\vec{AB}|}{|\vec{u}|}=\frac{\sqrt{174}}{\sqrt{29}}=\sqrt{6}$

Ex 5. $t(1,1,-1)$ pt $(1,-2,4)$

$x=1+t$

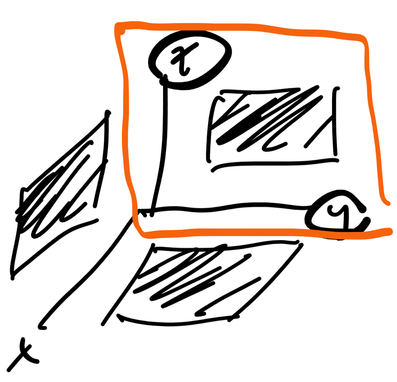
$y=-2+t$

$z=4-t$

Ex. 6. (a) $\vec{u}=\vec{P_0P_1}=\langle 1,-5,4\rangle$ $P_0(2,4,-3)$ $P_1(3,-1,1)$

$x=2+t$ $y=4-5t$ $z=-3+4t$

(b)



$x=0$

$0=2+t$

$-2=t$

$x=0$ $y=4-5(-2)$ $z=-3+4(-2)$

$y=14$ $z=-11$