MA1300 Self Practice # 14

1. (P750, #5, 12, 15) Determine whether the series converges or diverges.

$$\text{(a).}\ \sum_{n=1}^{\infty} \frac{n+1}{n\sqrt{n}}, \quad \text{(b).}\ \sum_{k=1}^{\infty} \frac{(2k-1)(k^2-1)}{(k+1)(k^2+4)^2}, \quad \text{(c).}\ \sum_{n=1}^{\infty} \frac{4^{n+1}}{3^n-2}.$$

2. (P755, #7, 11, 19, 20) Test the series for convergence or divergence.

$$\text{(a).}\ \sum_{n=1}^{\infty}(-1)^n\frac{3n-1}{2n+1}, \quad \text{(b).}\ \sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{n^3+4}, \quad \text{(c).}\ \sum_{n=1}^{\infty}(-1)^n\frac{n^n}{n!}, \quad \text{(d).}\ \sum_{n=1}^{\infty}(-1)^n\left(\sqrt{n+1}-\sqrt{n}\right).$$

3. (P755, #32) For what values of p is the series convergent?

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p}.$$

4. (P761-762, #5, 9, 29) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\text{(a).}\ \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[4]{n}}, \quad \text{(b).}\ \sum_{n=1}^{\infty}(-1)^n\frac{(1.1)^n}{n^4}, \quad \text{(c).}\ \sum_{n=1}^{\infty} \frac{2\cdot 4\cdot 6\cdots(2n)}{n!}.$$

5. (P765, #21, 30) Test the series for convergence or divergence.

$$(a)\ \sum_{n=1}^{\infty}(-1)^n\cos(1/n^2), \qquad (b)\ \sum_{j=1}^{\infty}(-1)^j\frac{\sqrt{j}}{j+5}.$$

6. (P769-770, #5, 20, 23) Find the radius of convergence and interval of convergence of the series.

$$(a)\ \sum_{n=1}^{\infty} \frac{x^n}{2n-1}, \quad (b)\ \sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}}, \quad (c)\ \sum_{n=1}^{\infty} n!(2x-1)^n.$$

7. (P789, #3) If $f^{(n)}(0) = (n+1)!$ for $n = 0, 1, 2, \ldots$, find the Maclaurin series for f and its radius of convergence.

8. (P789, #6) Find the Maclaurin series for f using the definition of a Maclaurin series. Also, find the associated radius of convergence.

$$f(x) = \ln(1+x).$$

9. (P789, #38) Use a Maclaurin series in Table 1 on page 786 of the textbook to obtain the Maclaurin series for the given function

$$f(x) = \begin{cases} \frac{x-\sin x}{x^3} & \text{if } x \neq 0, \\ \frac{1}{6} & \text{if } x = 0. \end{cases}$$

10. (P790, #72) If $f(x) = (1+x^3)^{30}$, what is $f^{(58)}(0)$?

11. (P790, #74 (a)) Show that the function defined by

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

is not equal to its Maclaurin series.

12. (Extra question) Let f be a continuous function on $[a, \infty)$. If f is differentiable on (a, ∞) and $f(a) = 0$, $\lim_{x\to\infty} f(x) = 0$, prove that there exists some $\xi \in (a, \infty)$ such that $f'(\xi) = 0$.

13. (Extra question) Let f be a continuous function on $[0, \infty)$. If f is differentiable on $(0, \infty)$, $f(0) = 0$, and f' is increasing on $(0, \infty)$, prove that the function g defined by $g(x) = \frac{f(x)}{x}$ is increasing on $(0, \infty)$.

14. (Extra question) Prove that the function f defined by $f(x) = \begin{cases} \frac{3-x^2}{2} & \text{if } x \in [0,1] \\ \frac{1}{x} & \text{if } x \in (1,2] \end{cases}$ on the interval $[0, 2]$ satisfies the condition of the mean value theorem. Then find a number $\xi \in (0, 2)$ such that $f'(\xi) = \frac{f(2)-f(0)}{2-0}$.