BME2102：

Introduction to Biomechanics

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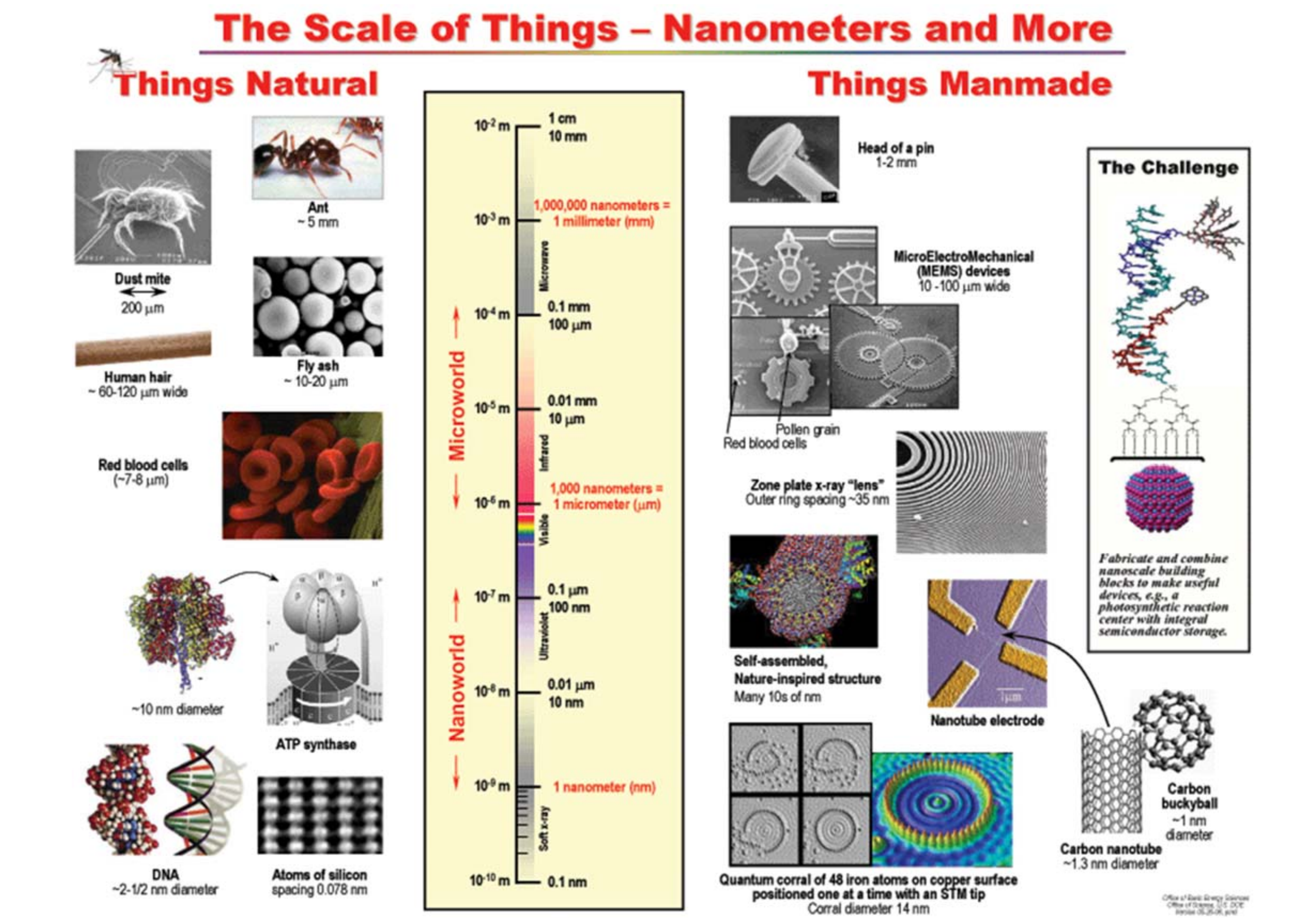
VII. Scaling Laws



Can I build a magnetic levitated micro-/nanotrain?

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- Magnetic levitation, or magnetic suspension, is a method by which an object is suspended with no support other than magnetic fields. If a magnetic levitation system composing with two cubic permanent magnets (side length a = 1 mm) achieves balance as shown in Fig. 1, and the separation d = 100 μm. Can the upper magnet suspend if the side length scales down to 100 nm supposing the separation scales along with it? Why? Analyze the problem with scaling laws.

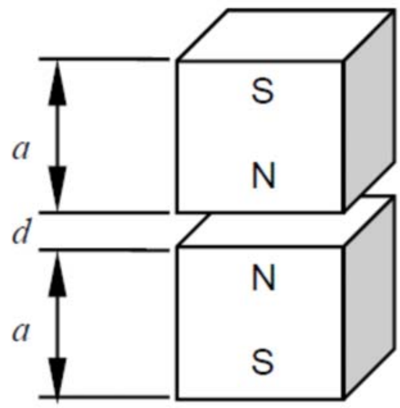


Fig. 1 Magnetic levitation

Can I build a magnetic levitated micro-/nanotrain?

- Magnetic levitation, or magnetic suspension, is a method by which an object is suspended with no support other than magnetic fields. Assuming the magnetic field generated by a large base permanent magnet is uniform in the central area, where a cubic permanent magnet A (side length a = 100 mm) achieves balance as shown in Fig. 1, and the separation d = 10 mm.

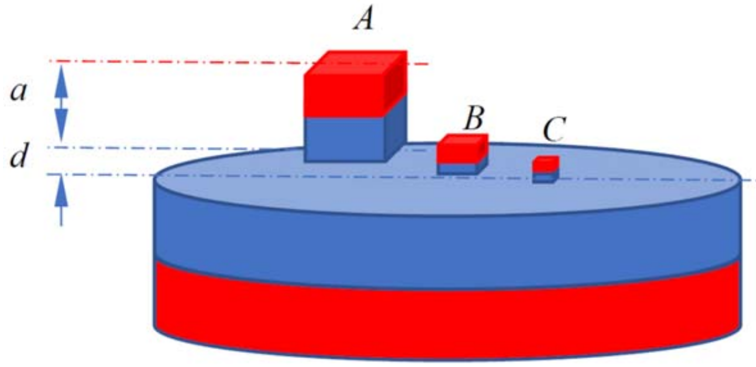
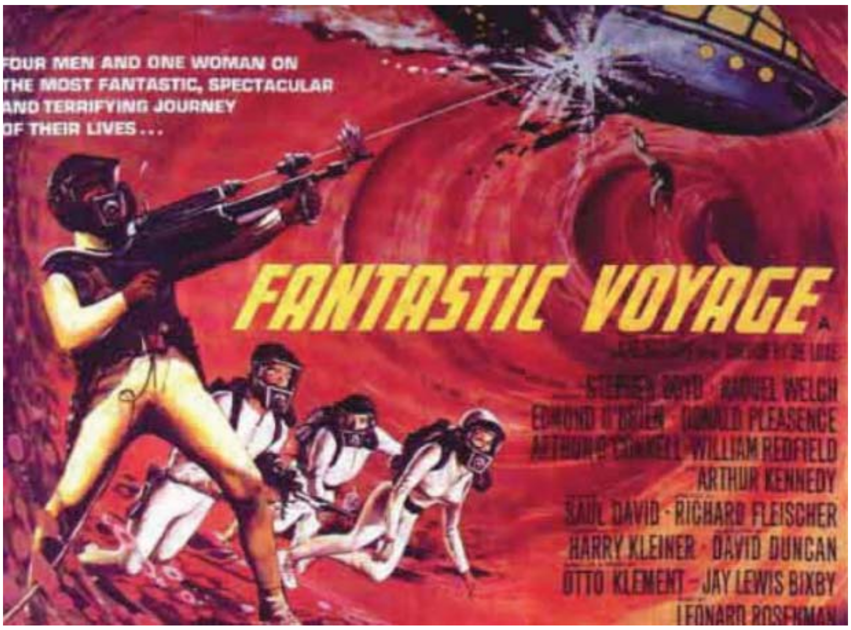


Fig. 1 Magnetic levitation

Will the miniaturized submarine (or micro-/nanorobots) or "swallowed doctors" work?



SCUBA, Film, Harry Kleiner

Book, I. Asimov, 1966

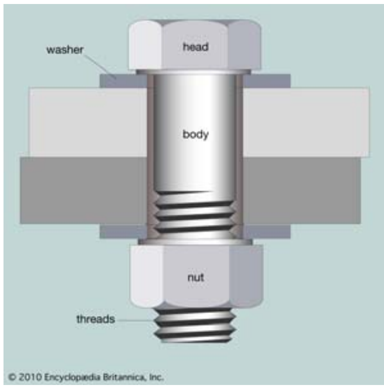
In Fantastic Voyage a team of scientists and their submarine are miniaturized using a top-secret technology, enabling them to navigate through the human blood stream to save the life of a dying man.

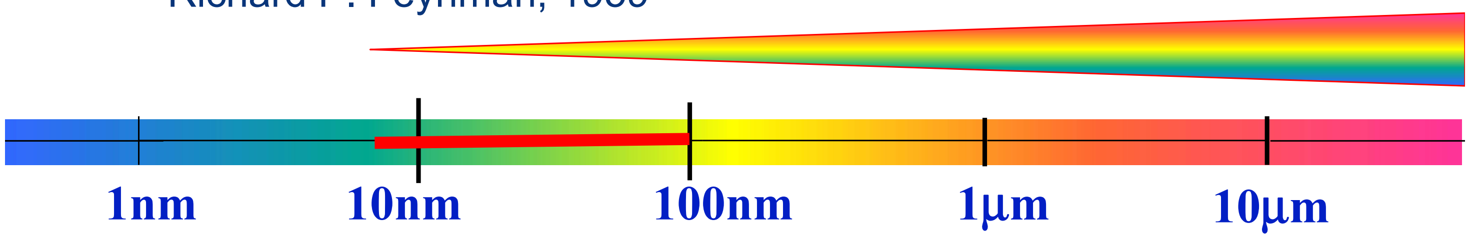
Why can a gecko climb glass?

Gecko climbing glass (Wiki)

Scaling down

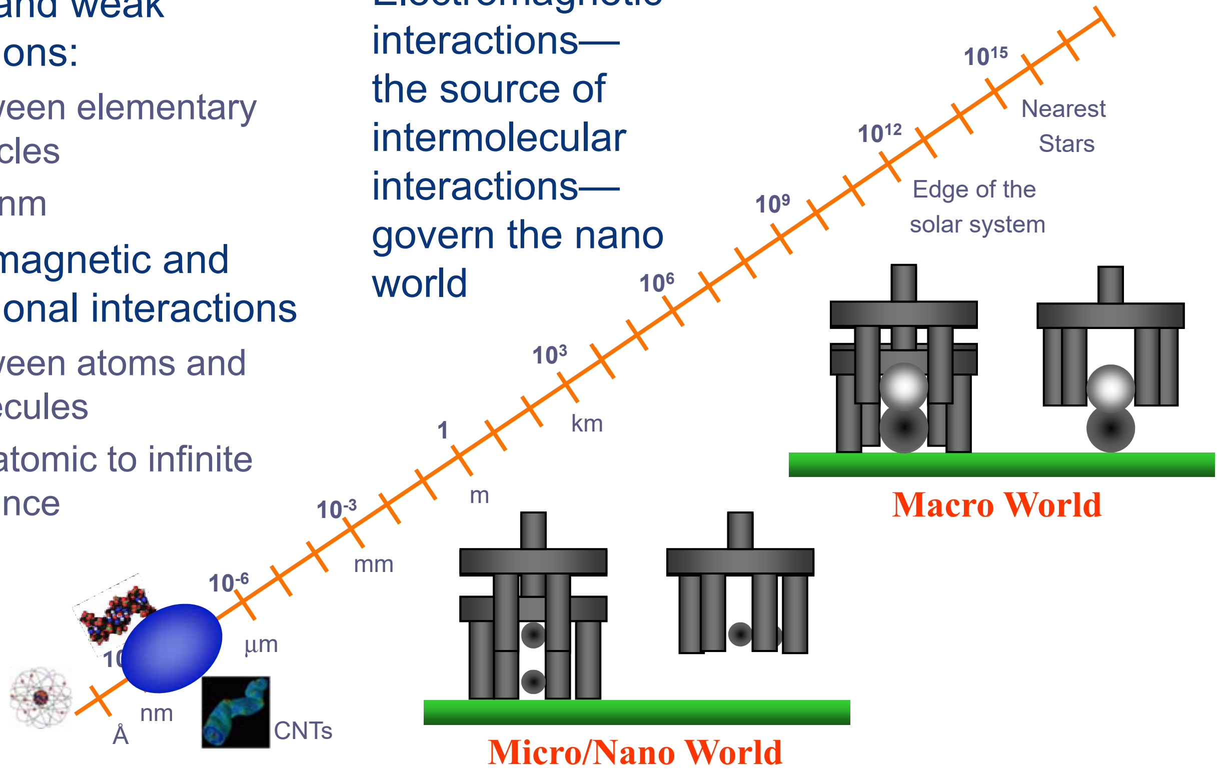
- As we go down in size, there are a number of interesting problems that arise. **All things do not simply scale down in proportion.** There is the problem that materials stick together by the molecular (Van der Waals) attractions. It would be like this: After you have made a part and you unscrew the nut from a bolt, it isn't going to fall down because the gravity isn't appreciable; it would even be hard to get it off the bolt. …There will be several problems of this nature that we will have to be ready to design for.
  --Richard P. Feynman, 1959





Small is different

- Strong and weak interactions:
  - Between elementary particles
  - 10^{-5} nm
- Electromagnetic and gravitational interactions
  - Between atoms and molecules
  - Subatomic to infinite distance
- Electromagnetic interactions—the source of intermolecular interactions—govern the nano world



Van der Waals forces become important at small scale

- In physical chemistry, the van der Waals force (or van der Waals interaction), named after Dutch scientist Johannes Diderik van der Waals, is the sum of the attractive or repulsive forces between molecules (or between parts of the same molecule) other than those due to covalent bonds, the hydrogen bonds, or the electrostatic interaction of ions with one another or with neutral molecules or charged molecules.

Gecko climbing glass (Wiki)

Scaling Effects

- The problem is that even if this amazing shrinking technology existed, the miniature submarine and its crew would be totally ineffective.
  - **The submarine's propellers would not function at that scale.**
  - The crew would not be able to walk around inside the submarine because of the **adhesive forces** between their feet and the floor.
  - Even if the crew could make it out of the submarine, the tiny SCUBA divers could kick their legs all day and go nowhere.

Scaling Laws

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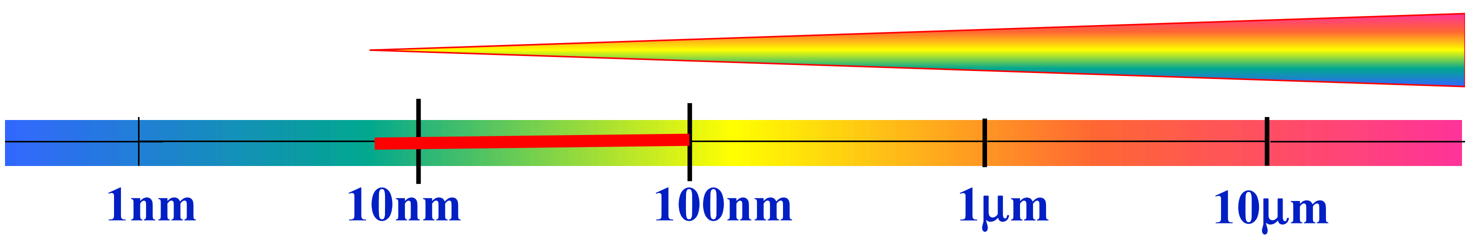
- Most physical magnitudes characterizing nanoscale systems differ enormously from those familiar in macroscale systems. **Some of these magnitudes can, however, be estimated by applying scaling laws to the values for macroscale systems.** …it can provide orientation, preliminary estimates, and a means for testing whether answers derived by more sophisticated methods are in fact reasonable.

--K. Eric Drexler,
Nanosystems, 1992

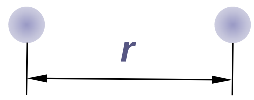


Van der Waals Forces

- The generalized interaction between molecules is given by the Mie pair potential

$$E(r) = -\frac{A}{r^n} + \frac{B}{r^m}$$

attractive repulsive



- A specific case of the Mie potential is the Lennard-Jones potential

$$E(r) = -\frac{A}{r^6} + \frac{B}{r^{12}}$$

where A and B are constants, e.g., for solid argon, $A = 8.0 \times 10^{-77}$ Jm^6 and $B = 1.12 \times 10^{-133}$ Jm^{12}.

Van der Waals Forces

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- The net van der Waals force is given by

$$F_{\mathrm{vdW}} = -\frac{dE}{dr} = -\frac{6A}{r^7} + \frac{12B}{r^{13}}$$

- If r is scaled as $\sim L$, the attractive force scales as $\sim L^{-7}$, and thus its importance dramatically increases at the nanoscale. The repulsive force scales as $\sim L^{-13}$, which is important only at subnanometer scales.

Electrostatic Forces

- Let us consider the electrostatic force between two parallel plates and examine how that force is affected by scaling. Let *A* denote the surface area of the plates, and let *x* be the separation distance. Assume *x* is small relative to the dimensions of the plates. The capacitance is given by

$$C = \varepsilon \frac{A}{x}$$

where ε is the permittivity of the dielectric material separating the plates. The capacitance relates a voltage *U* that is applied to the plates to the charge Q that is accumulated on each plate: $Q = CU$. The electrostatic co-energy stored in the capacitor can be expressed by

$$W = \frac{1}{2} C U^2$$

and the attractive force between the plates is computed as $F = -dW/dx$.

Electrostatic Forces

- In the case of constant voltage

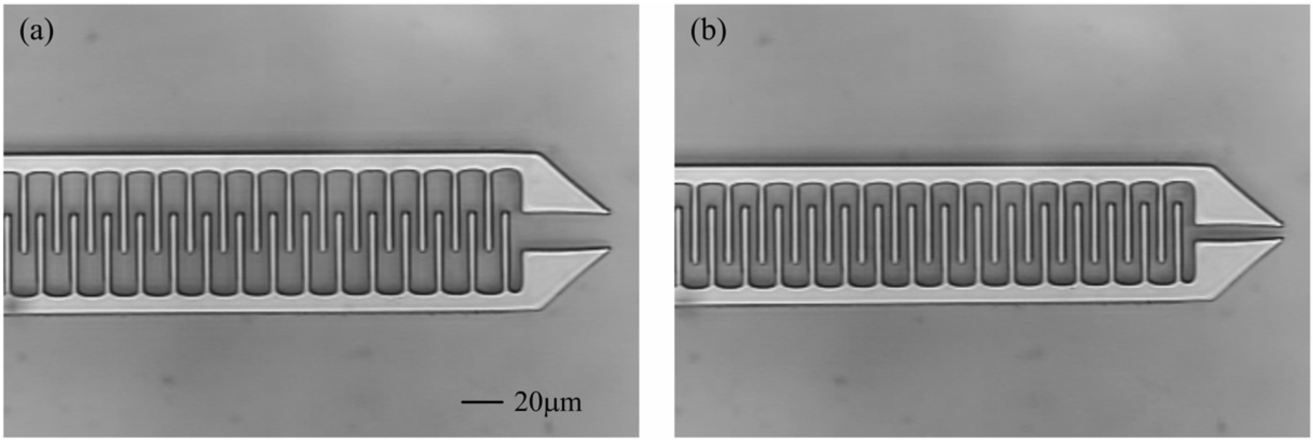
$$F_U = \frac{\varepsilon A U^2}{2x^2}$$

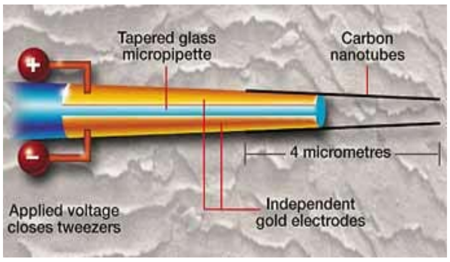
- In the case of constant charge:

$$F_Q = \frac{Q^2}{2\varepsilon A}$$

- For example, consider scaling only the dimensions of the plate as $\sim L$. The area *A* is scaled as $\sim L^2$, thus, F_U scales as $\sim L^2$ and F_Q scales as $\sim L^{-2}$. If we scale the gap distance *x* as well, F_U remains constant whereas F_Q still scales as $\sim L^{-2}$.

Electrostatic Gripper





P. Kim, C. M. Lieber, Nanotube nanotweezers. Science 286, 2148-2150 (1999).

Friction

- When two surfaces are sliding about each other, at the macroscopic level, the friction force is given by

$$F_{fr} = \mu F_{gr} = \mu mg$$

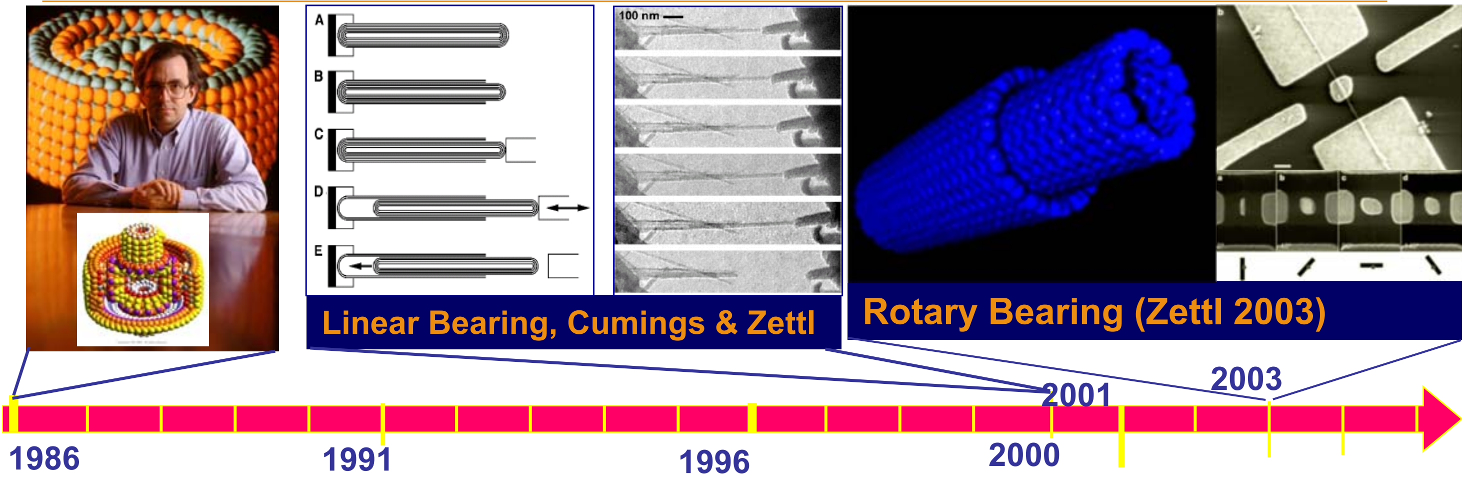
where μ is the friction coefficient. Provided μ is constant,

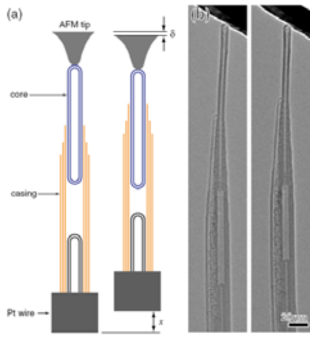
$$F_{fr} \sim L^3$$

- At the microscopic level, things are different due to surface roughness. As seen earlier, the adhesive forces are very large. The striction (i.e. the combination of adhesion and friction) forces F_{str}, have to be taken into account. These forces scale like the contact area

$$F_{str} \sim L^2$$

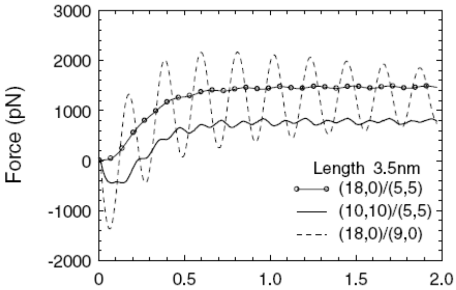
Telescoping CNT-based NEMS





$$
\begin{aligned}
F &= F_{\text{friction}} + F_{\text{cohesion}} \\
&= \underbrace{-\operatorname{sgn}(\dot{x})\pi D\tau L(x)}_{\text{dissipative}} \underbrace{-\gamma\pi D + f_{\text{corr}}(x)}_{\text{conservative}},
\end{aligned}
$$

Zettl, PRL, 97, 025501



Guo, PRL, 91, 125501

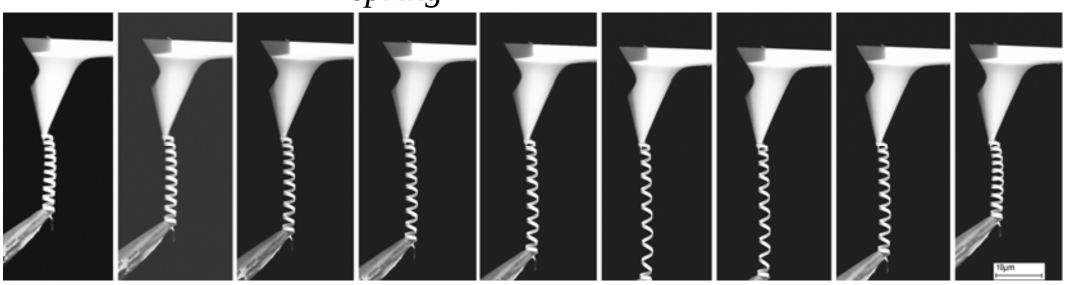
Spring Forces

- Springs exist in all dimensions. The spring force F_{spring} = $-k\delta L$, where k is the stiffness or spring constant and δL is the elongation from the equilibrium position. Providing k remains constant, one obtains

$$F_{spring} \sim L$$

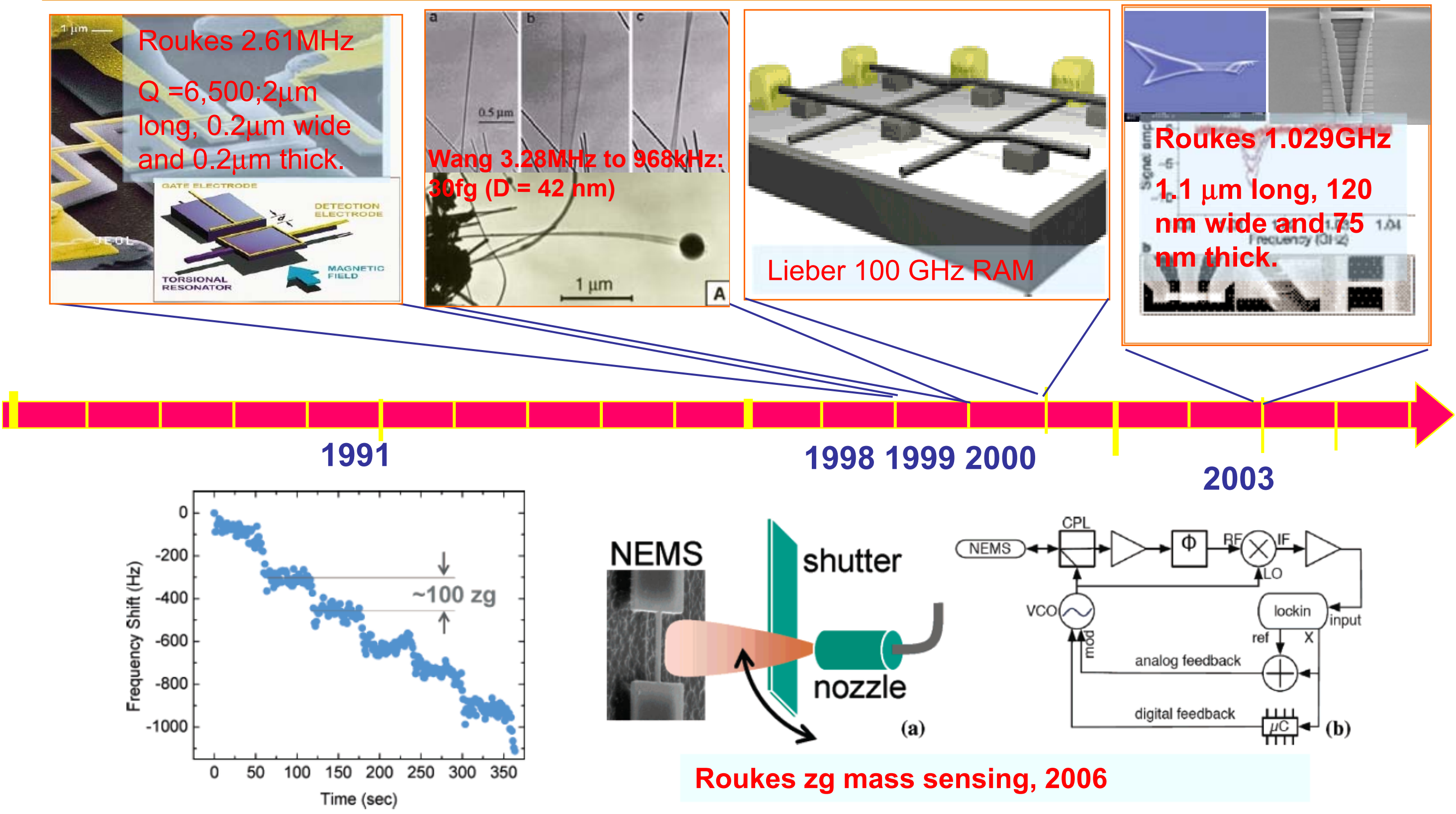
- The corresponding oscillating frequency $f_{spring} = (1/2\pi)(k/m)^{1/2}$ *and*

$$f_{spring} \sim L^{-3/2}$$



L. X. Dong, L. Zhang, D. J. Bell, D. Grützmacher, and B. J. Nelson, ***Journal of Physics: Conference Series*** 61, 257, 2007.

GHz Resonators based NEMS



Magnetic Forces

- Consider the force between two identical magnets with magnetization *M and volume v,* aligned along their dipole axes, and separated by a distance *x. The field created by one magnet* along its axis is expressed using the point dipole model as

$$H(x) = \frac{Mv}{2\pi x^3}$$

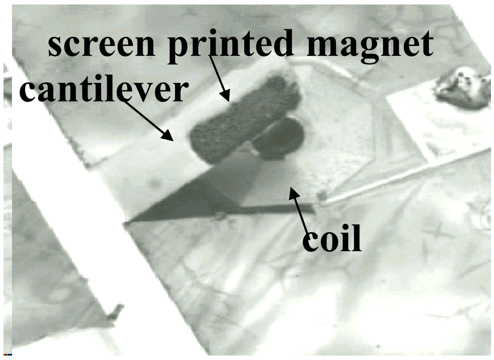
- The magnitude of the attractive/repulsive force on the other magnet is then given by

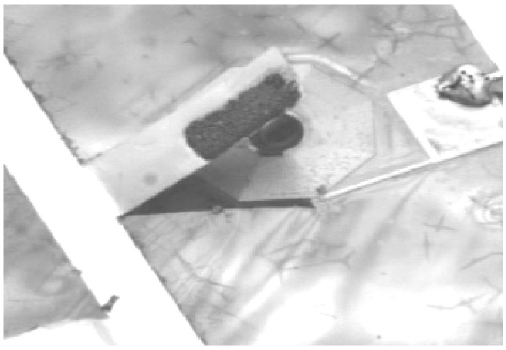
$$F_m = \mu_0 M v \left| \frac{\partial H}{\partial x} \right| = \frac{3\mu_0 M^2 v^2}{2\pi x^4}$$

where μ_0 is the permeability of free space. Note that the magnetization *M* remains constant for scaling as it is an intrinsic physical property of the magnets.

Magnetic Forces

$$F_m = \mu_0 M v \left| \frac{\partial H}{\partial x} \right| = \frac{3 \mu_0 M^2 v^2}{2 \pi x^4}$$





- If we scale the magnets down as ~L but hold the distance between the magnets constant, the force will be scaled as ~L^6, resulting in a very poor scaling.
- However, if we also scale the distance between magnets as ~L , the force is scaled as ~L^2, which is significantly better.

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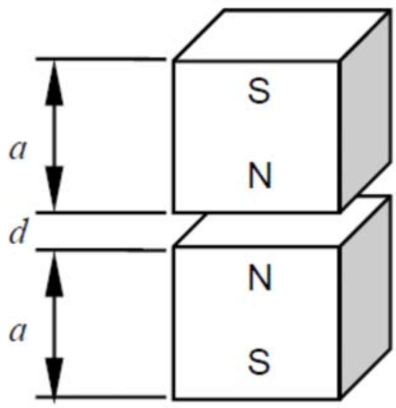
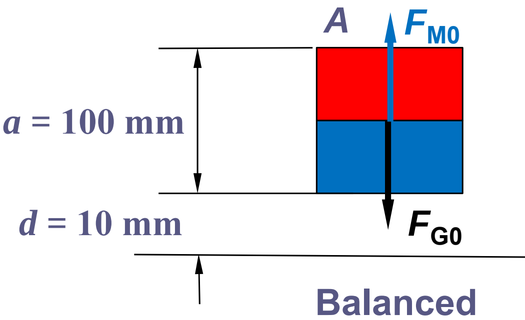


Fig. 1 Magnetic levitation

Magnetic levitation

- If the density of the materials of the magnet A is 7.8 g/cm^3, how large is the magnetic force?

$F_{M0} - F_{G0} = 0$



Balanced

$$F_{M0} = \frac{3\mu_0 M^2 v^2}{2\pi x^4} = \frac{3\mu_0 M^2 a^6}{2\pi d^4}$$

$$\parallel$$

$$F_{G0} = \rho v g = \rho g a^3$$

Magnetic levitation

- If the density of the materials of the magnet A is 7.8 g/cm^3, how large is the magnetic force?

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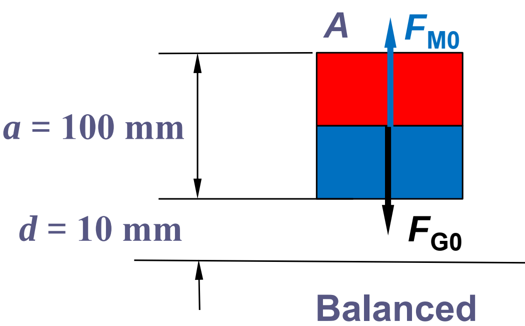


Balanced

$$F_{M0} = \frac{3\mu_0 M^2 v^2}{2\pi x^4} = \frac{3\mu_0 M^2 a^6}{2\pi d^4}$$

$$\parallel$$

$$F_{G0} = \rho v g = \rho g a^3$$

$$= 7.8 \times \frac{10^{-3}}{10^{-6}} \times 9.81 \times 0.1^3$$

$$= \boxed{76.518[N]}$$

Magnetic levitation

- Scaling of magnetic forces and gravitation forces

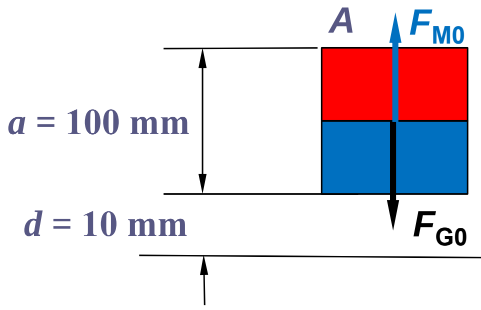
If $k_a = k_d = k_{ad}$

If both a and d changes $F_{Md} = \dfrac{3\mu_0 M^2 (k_a a)^6}{2\pi (k_d d)^4} = (k_a)^6 (k_d)^{-4} F_{Mo} = (k_{ad})^2 F_{Mo}$

If only a changes $F_{Ma} = \dfrac{3\mu_0 M^2 (k_a a)^6}{2\pi d^4} = (k_a)^6 F_{Mo}$

If only d changes $F_{Md} = \dfrac{3\mu_0 M^2 a^6}{2\pi (k_d d)^4} = (k_d)^{-4} F_{Mo}$

$F_{M0} - F_{G0} = 0$



$$F_{M0} = \frac{3\mu_0 M^2 v^2}{2\pi x^4} = \frac{3\mu_0 M^2 a^6}{2\pi d^4}$$

$$\parallel$$

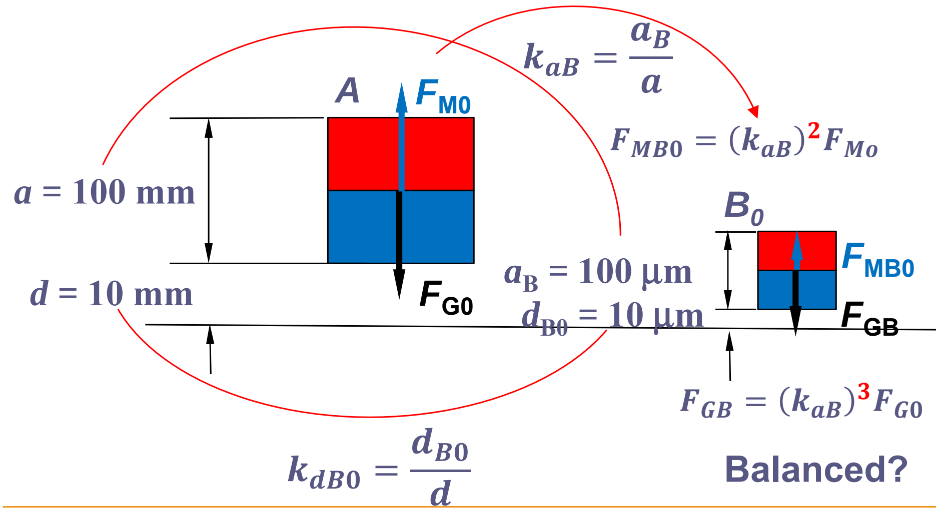
$$F_{G0} = \rho v g = \rho g a^3$$

$$F_{Ga} = \rho g (k_a a)^3 = (k_a)^3 F_{G0}$$

Magnetic levitation

- Can a smaller cubic permanent magnet B (side length a = 100 mm) made from the same materials suspend? If yes, what will the separation be?
- Consider both the side length and the separation decreases equally

If balanced, the separation is d_{B0} = 10 µm.
If unbalanced, the separation will increase (net force is upward) or decrease (net force is upward) . If the separation increases, is there an upper limit (if not, the magnet will become a satellite or escape from the earth gravitation)?
If the separation decreases is there a lower limit (if not, the magnet will make a contact to the base, and suspension fails)?



Balanced?

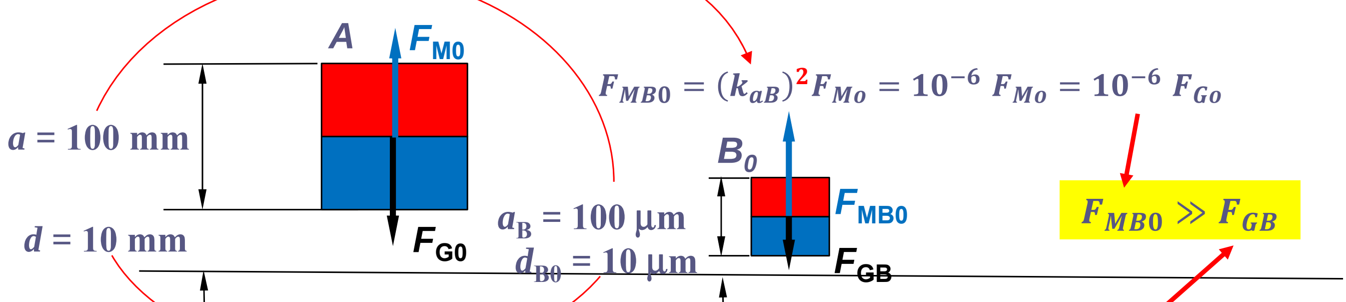
Magnetic levitation

- Can a smaller cubic permanent magnet B (side length *a* = 100 mm) made from the same materials suspend? If yes, what will the separation be?
- Consider both the side length and

  the separation decreases equally

$$k_{aB} = \frac{a_B}{a} = \frac{100\times10^{-6}}{100\times10^{-3}} = 10^{-3}$$

$$F_{MB0} = (k_{aB})^2 F_{Mo} = 10^{-6}\ F_{Mo} = 10^{-6}\ F_{Go}$$



$$F_{GB} = (k_{aB})^3 F_{G0} = 10^{-9}\ F_{Mo} = 10^{-9}\ F_{Go}$$

$$k_{dB0} = \frac{d_{B0}}{d}$$

Balanced? **Not balanced**

Life at Low Reynolds Number

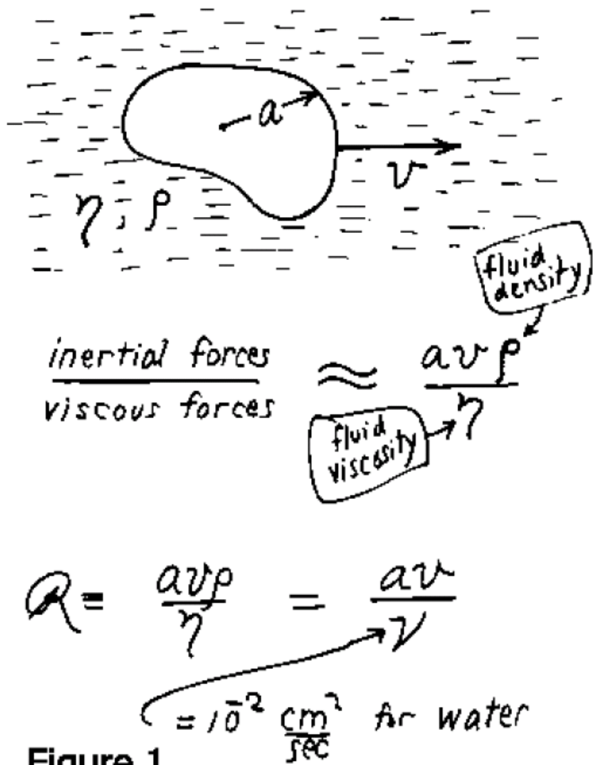


Figure 1

- The ratio of the inertial forces to the viscous forces is called the Reynolds number
- When that number is small the viscous forces dominate.

E. M. Purcell, Am. J. Phys. 45, 3 (1977).

Life at Low Reynolds Number

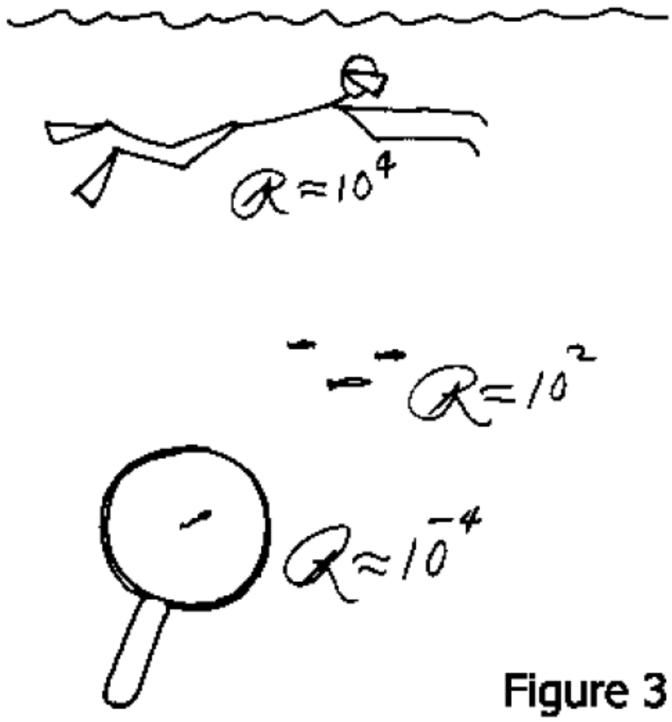


Figure 3

- For animals which are the order of a micron in size, inertia is totally irrelevant.
- If I have to push that animal to move it, and suddenly I stop pushing, how far will it coast before it slows down? The answer is, about 0.1 angstrom. And it takes it about 0.6 μs to slow down.

E. M. Purcell, Am. J. Phys. 45, 3 (1977).

Life at Low Reynolds Numbers

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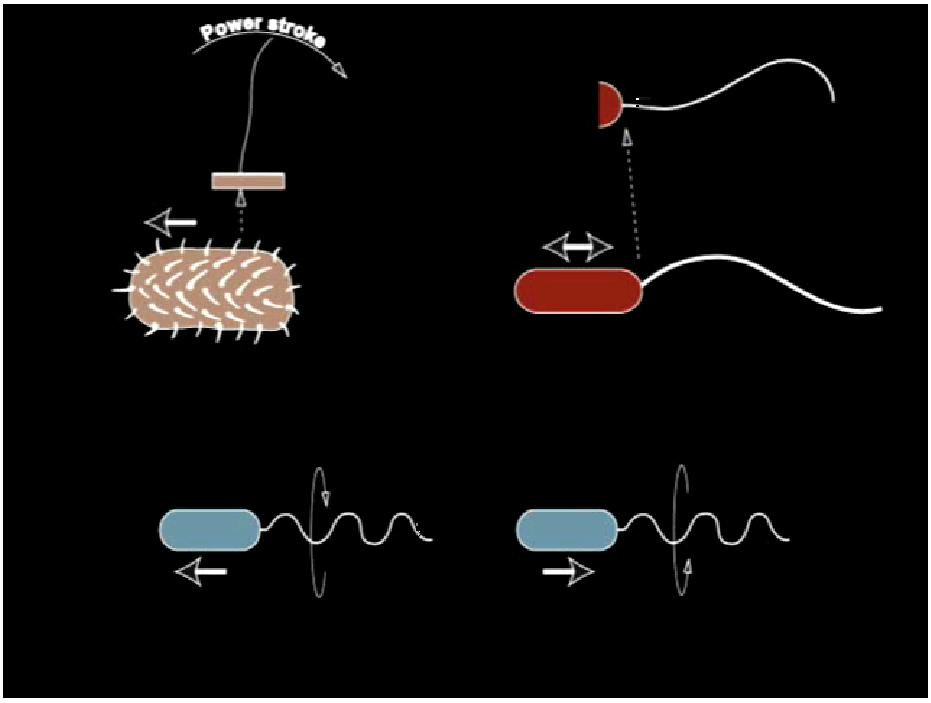
Cilia **Eukariotic flagella**

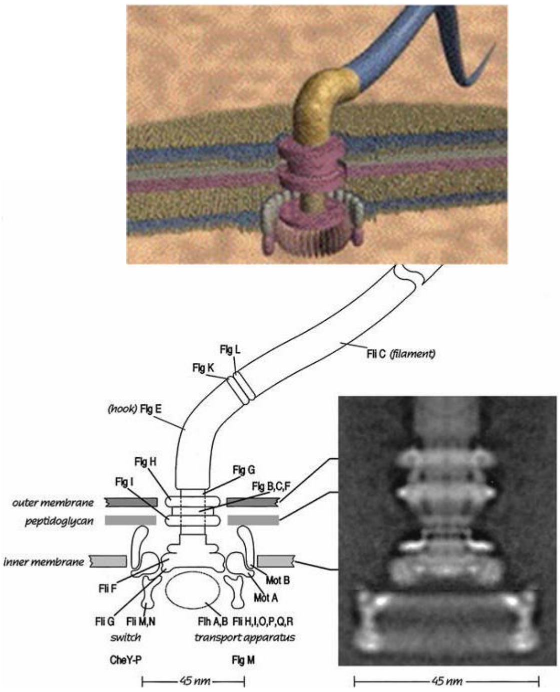


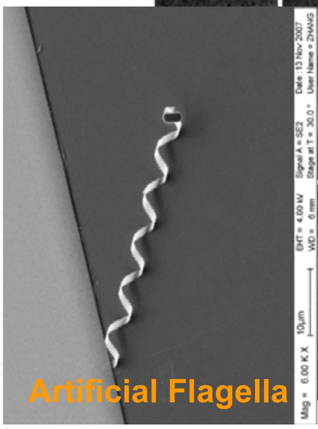
Bacterial flagella spun by molecular motors

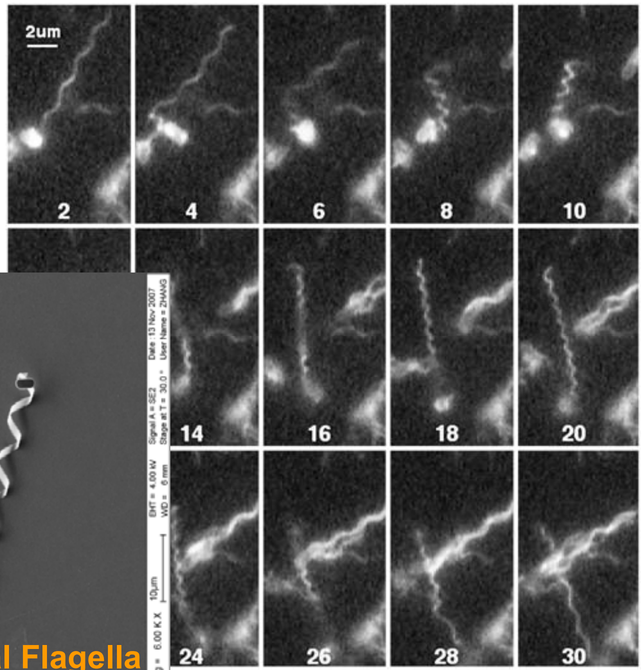
Bacteria Flagella
J.P. Armitage

J. J. Abbott, K. E. Peyer, L. X. Dong, and B. J. Nelson, "How Should Microrobots Swim?" in *Springer Tracts in Advanced Robotics: Robotics Research*, 2008.

Bacterial Flagella







H.C. Berg, R.A. Anderson, Bacteria swim by rotating their flagellar filaments**,** Nature 245, 380–382, 1973.

L. Zhang, J. J. Abbott, L. X. Dong, B. E. Kratochvil, D. J. Bell, and B. J. Nelson, Artificial Bacterial Flagella: Fabrication and Magnetic Control, Applied Physics Letters 94, 064107, 2009.

Swimming Micro-/NanoRobots

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- The problem is that even if this amazing shrinking technology existed, the miniature submarine and its crew would be totally ineffective.
  - **The submarine's propellers would not function at that scale.**
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Scaling Laws

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Physical quantity	Scaling exponent	Typical magnitude	Scaling accuracy	Comments
Area	2	10^{-18} m^2	Definitional	Surface forces domain
Volume	3	10^{-27} m^3	Definitional	Volume forces ignorable
Mass	3	ag-zg	Good	
vdW Forces	2 (Area)	pN-nN	Good	
vdW Forces	-7(Dis., Attractive) -13 (Dis., Repulsive)	pN-nN	Good	AFM, manipulation, Supermolecules
Electrostatic force	2	pN-nN	Good at small scale	Actuation
Magnetic force	2(Dis.) 6(Sizes)	10^{-23} N	Good	
Friction	2		Moderate to inapplicable	Motion
Oscillating frequency	-3/2	GHz-THz	Good	Motion, sensing

Summary

- Scaling to micro- and nano-scale is typically not intuitive. Use scaling laws to estimate the effects.

- Depending on the size-scale different physical effects become more or less important

  – Going down from macro to micro inertia forces become less important while surface forces such as adhesive forces become more important

- Moreover, when the size of a structure is comparable to the size of its atoms there are quantum effects

Further reading

- K. E. Drexler “Classical Magnitudes and Scaling Laws” from “Nanosystems: Molecular Machinery, Manufacturing and Computation”
  http://www.e-drexler.com/d/06/00/Nanosystems/toc.html
- M. Wautelet “Scaling laws in the macro-, micro- and nanoworlds” Eur. J. Phys. 22, 601-611, 2001