CITY UNIVERSITY OF HONG KONG

Department of Mathematics

Course Code & Title	:	MA1301 Enhanced Calculus and Linear Algebra II
Session	:	Semester B, 2016-2017
Time Allowed	:	Three Hours

This paper has **Three** pages. (including this cover page)

Instructions to candidates:

1. This paper has **eight** questions.
2. Answer **ALL** questions.
3. Start each main question on a new page.
4. Show all steps.

---

*This is a **closed-book** examination.*

Candidates are allowed to use the following materials/aids:

Non-programmable portable battery operated calculator.

Materials/aids other than those stated above are not permitted. Candidates will be subject to disciplinary action if any unauthorized materials or aids are found on them.

1. (a) [6 marks] Show that $\int_0^{\infty} e^{x^2} dx$ is divergent. Then evaluate

$$\lim_{x \to +\infty} \frac{\int_0^x e^{t^2} dt}{e^{x^2}}.$$

(b) [5 marks] $\int x \cos^2 x dx$

(c) [6 marks] $\int \frac{e^x - 1}{e^x + 1} dx.$

(d) [7 marks] $\int \sqrt{x^2 - 2x - 1}\, dx.$

2. A wire has the form of a curve $y = f(x)$ for $a \leq x \leq b$, where $f'(x)$ is continuous on $[a, b]$. Suppose that the density (mass per unit length) of the wire is $\rho = \rho(x)$, derive the formula for (show all steps)

(a) [6 marks] the mass of the wire;

(b) [5 marks] the center of mass of the wire.

3. [7 marks] Show that

$$|\vec{a} \cdot \vec{b}|^2 + |\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2.$$

4. (a) [7 marks] Let $A = (1, -2, 1)$, $B = (1, 3, 1)$ and $C = (2, 1, 1)$ three points on a plane π. Find the shortest distance from $D = (-4, -1, 2)$ to the plane π.

(b) [6 marks] Use the concept of vectors to determine all possible values of k such that the points $E(1, 1, 1)$, $F(1, 1, 4)$, $G(k, 2, 7)$, $H(3, k, 3)$ are located in the same plane.

5. (a) [5 marks] Solve the equation $z^4 + 8\sqrt{2}\, i = -8\sqrt{2}$ in the set of all complex numbers and express your answer in **Euler's form with principal value of argument**.

(b) [5 marks] Use de Moivre's theorem to show that

$$\cos(3x) = 4\cos^3 x - 3\cos x, \quad \sin(3x) = 3\sin x - 4\sin^3 x.$$

6. Consider the linear system

$$\begin{cases} \lambda x_1 + x_2 + x_3 = 0, \\ x_1 + \mu x_2 + x_3 = 0, \\ x_1 + 2\mu x_2 + x_3 = 0. \end{cases}$$

(a) [6 marks] Find all possible values of λ and μ such that the system have non-zero solutions.

(b) [10 marks] Find all the non-zero solutions.

7. Let

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ k & 1 & 1 & 1 \\ 1 & 2 & 1 & 2k \end{pmatrix}.$$

Using elementary row operations,

(a) [5 marks] When $k = 1$, find the rank of A;

(b) [5 marks] When $k = 2$, find the inverse of A.

8. (a) [5 marks] Let A and B be 2×2 matrices and there exists positive integers k and l such that $A^k = 0$ and $B^l = 0$. Prove or disprove (provide a counterexample) that you can always find a positive integer m such that $(AB)^m = 0$.

(b) [4 marks] Let M and N be 3×3 matrices and the determinants $\det(N) = 5$ and $\det(MN) = 10$. Compute the determinants: $\det(N^T M^2)$ and $\det(2M^{-1})$.

- End -