1. A muscle-tendon complex is marked on two sides with small dots. The initial position of these dots is measured in an unloaded reference configuration (Fig. 1). From these measurements it appears that the positions are given as

$$\vec{x}_{0,A} = -\vec{e}_x + 3\vec{e}_y$$
$$\vec{x}_{0,B} = 2\vec{e}_x + 3\vec{e}_y$$

The muscle-tendon complex moves and in the current (deformed) configuration the positions of points A and B are measured again:

$$\vec{x}_A = 4\vec{e}_x + 3\vec{e}_y$$
$$\vec{x}_B = 8\vec{e}_x - 1.i\vec{e}_y$$

where, i = $|i_4 - i_5|$, **i.e. the difference between the 4th and 5th digits of your Student ID counted from the left.**

a. The constant in the force versus extension relation is c = 300 [N]. Determine the force vectors $\vec{F}_A$ and $\vec{F}_B$ in the deformed configuration.

b. At the initial position, the muscle and the tendon have the same length. If the constants in the force versus extension relation for the tendon c_t is known as twice that of the muscle c_m, i.e. $c_t = 2\,c_m$. What are the current length of the muscle and the tendon?

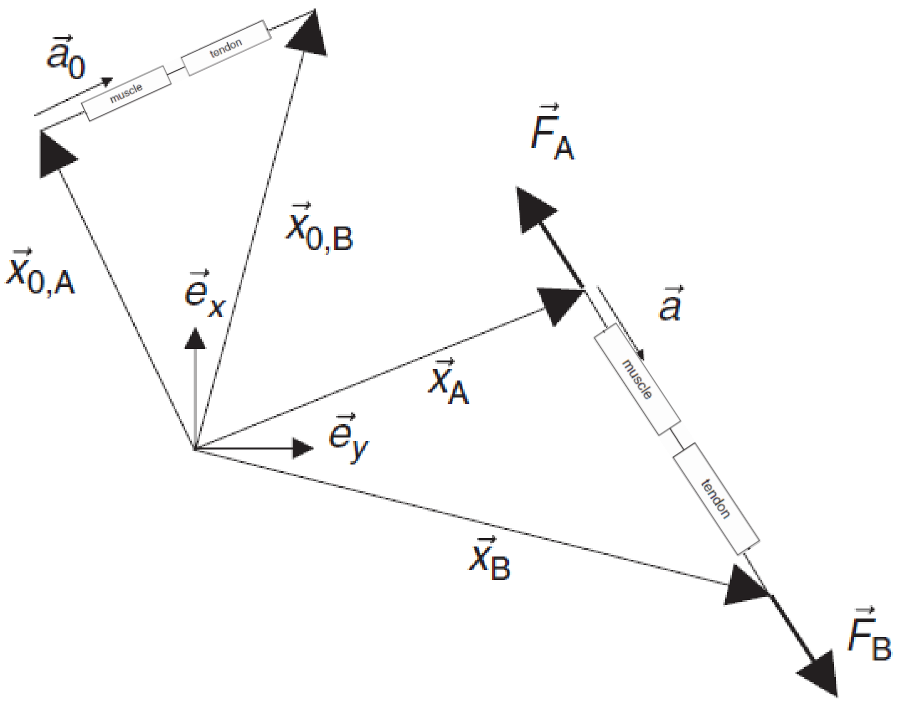

Fig. 1 Question 1

2. As shown in Fig. 2, a board with length 5.ia (where, i = $|i_5 - i_6|$, **i.e. the difference between the 5th and 6th digits of your Student ID counted from the left.)** is fixed to the wall in point A with a hinge. The board is able to rotate freely around the joint. In a point B at a distance 3a of A, the board is kept in horizontal position by means of a cable, tied to the board in B and fixed to the rigid wall in point C. The mass of the board is M_P. Cable and board can be assumed to be rigid (undeformable) structures. A box, with length 2a and mass M_K is placed on the board. The front of the box is exactly placed at the front edge of the board. The gravitation acceleration is g.

a. Draw a free body diagram of the construction to enable the calculation of the reaction forces in points A and C.
b. Calculate the reaction forces in point A and C.

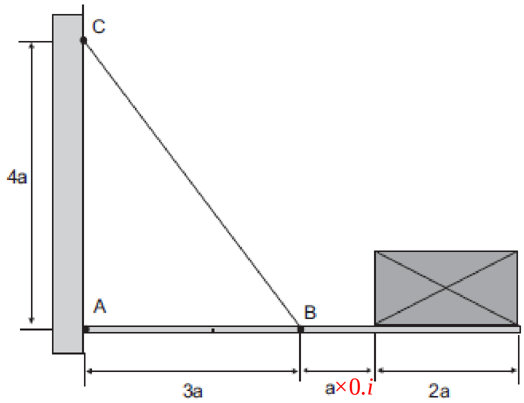

Fig. 2 Question 2

3. Lateral force microscopy (LFM) is derived from the contact mode atomic force microscopy. Whereas in contact mode AFM we measure the deflection of the cantilever in the vertical direction to gather sample surface information, we measure the deflection of the cantilever in the horizontal direction in LFM. The lateral deflection of the cantilever is a result of the force applied to the cantilever when it scans horizontally across the sample surface. The horizontal component of van der Waals attractive force between the shaded tip atoms 1-2 on the cantilever and the surface atoms A-C as shown in Fig. 3 contributes to the friction even if the surface is atomically flat. Supposing $h = 2a = b = 0.4\boldsymbol{i}$ nm (where, $\boldsymbol{i} = |\boldsymbol{i_6} - \boldsymbol{i_7}|$, **i.e. the difference between the 6th and 7th digits of your Student ID**), for the interaction between two atoms the value of A is known to be $A = 10^{-77}$ J m^6.
   a. Calculate the van der Waals attractive forces $F(r) = -dw(r)/dr$ using the Lennard-Jones potential (repulsive term ignored)
   $$w(r) = -A/r^6$$
   between tip atoms 1-2 and surface atoms A-C as the tip scans from position P_0 to $P_1, P_2, \ldots, P_{12}$ as shown in Fig. 3. Use the symmetricity and scaling laws wherever possible.
   b. Find the horizontal components of van der Waals forces between the tip atoms 1-2 and the surface atoms A-C as the tip scans from position P_0 to $P_1, P_2, \ldots, P_{12}$ as shown in Fig. 4, then sum them up and sketch them in Fig. 3.

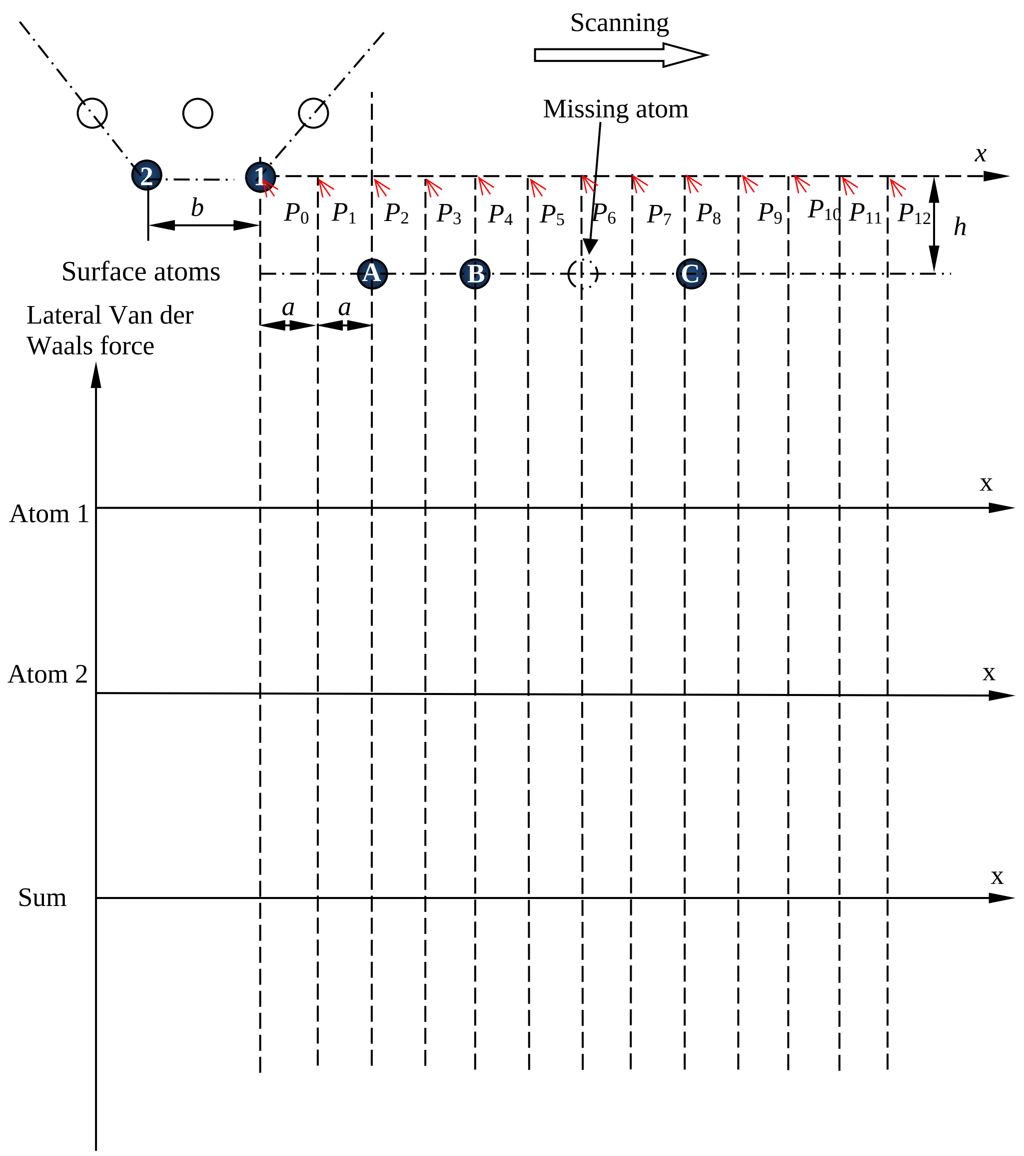

Fig. 3 Question 3

4. Electrostatic forces are widely used in biomedical devices. Figure 4 shows the model of an electrostatic gripper for grasping single cells, where the side length of the cube (mass is *M*) and the plate are a = 100 μm and the spring constant is *K*. The system achieves a balance as the separation d = 10 μm. Suppose we are going to design a gripper for grasping viruses, which is 1000 times smaller than a cell. (**25 points**)
   a. Find the conditions that make the cube in a static equilibrium.
   b. How do the electrostatic forces, gravitational forces, spring forces, and van der Waals surface forces scale with the side length and the separation?
   c. Will the gripper still work if the side length scales down to a' = 0.***i*** μm (where, ***i*** **=** $|i_7 - i_8|$**, i.e. the difference between the 7th and 8th digits of your Student ID**. If ***i*** = 0, take ***a'*** = **1** μm) supposing the separation scales along with it? If the gripper does not work, how to make it work? Analyse the problems with scaling laws.

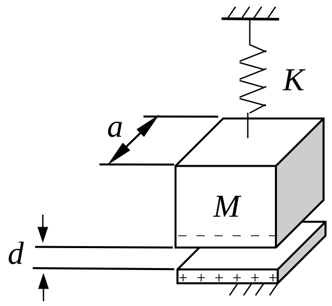

Fig. 4 Question 4

- END -

Formula Sheet

MATHEMATICAL FORMULAS

Pythagorean theorem

$A^2 + B^2 = C^2$ **(1.5)**

Trigonometric functions

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$ **(1.6)**

$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$ **(1.7)**

$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$ **(1.8)**

$\theta = \arcsin\left(\dfrac{\text{opposite side}}{\text{hypotenuse}}\right)$ **(1.9)**

$\theta = \arccos\left(\dfrac{\text{adjacent side}}{\text{hypotenuse}}\right)$ **(1.10)**

$\theta = \arctan\left(\dfrac{\text{opposite side}}{\text{adjacent side}}\right)$ **(1.11)**

LINEAR KINEMATICS

Average speed

$\bar{s} = \dfrac{\ell}{\Delta t}$ **(2.5)**

Average velocity

$\bar{v} = \dfrac{d}{\Delta t}$ **(2.6)**

Average acceleration

$\bar{a} = \dfrac{v_f - v_i}{\Delta t}$ **(2.9)**

PROJECTILE EQUATIONS

Vertical motion (y)

Vertical position:

$y_f = y_i + v_i\Delta t + \frac{1}{2}g(\Delta t)^2$ **(2.14)**

$y_f = \frac{1}{2}g(\Delta t)^2$ if $y_i = 0$ and $v_i = 0$ **(2.16)**

Vertical velocity:

$v_f = v_i + g\Delta t$ **(2.11)**

$v^2 = v^2 + 2g\Delta y$ **(2.15)**

$v_{peak} = 0$ **(2.19)**

$v_f = g\Delta t$ if yi = 0 and vi = 0 **(2.17)**

$v^2 = 2g\Delta y$ if $v_i = 0$ **(2.18)**

Vertical acceleration:

$a = g = -9.81 \text{ m/s}^2$ **(2.10)**

Horizontal motion (x)

Horizontal position:

$x = v\Delta t$ **(2.26)**

Horizontal velocity:

$v = v_f = v_i =$ constant **(2.22)**

Horizontal acceleration:

$a = 0$ **(2.23)**

Other equations governing projectile motion

Time of flight:

$\Delta t_{up} = \Delta t_{down}$ if $y_f = y_i$ **(2.20)**

$\Delta t_{flight} = 2\Delta t_{up}$ if $y_f = y_i$ **(2.21)**

Parabolic equation:

$y_f = y_i + v_{y_i}\left(\dfrac{x}{v_x}\right) + \dfrac{1}{2}g\left(\dfrac{x}{v_x}\right)^2$ **(2.27)**

LINEAR KINETICS

Weight

$W = mg$ **(1.2)**

Static and dynamic friction

$F_s = \mu_s R$ **(1.3)**

$F_d = \mu_d R$ **(1.4)**

Static equilibrium

$\Sigma F = 0$ **(1.12)**

$\Sigma F_x = 0$ **(1.13)**

$\Sigma F_y = 0$ **(1.14)**

Newton's 1st law: law of inertia

$v =$ constant if $\Sigma F = 0$ **(3.1a)**

or

$\Sigma F = 0$ if $v =$ constant **(3.1b)**

Linear momentum

$L = mv$ **(3.6)**

Conservation of momentum

$L =$ constant if $\Sigma F = 0$ **(3.7)**

$L_x =$ constant if $\Sigma F_x = 0$ **(3.8)**

$L_y =$ constant if $\Sigma F_y = 0$ **(3.9)**

$L_i = \Sigma(mu) = m_1u_1 + m_2u_2 + m_3u_3 + \ldots = m_1v_1 + m_2v_2 + m_3v_3 + \ldots = \Sigma(mv) = L_f =$ constant if $\Sigma F = 0$ **(3.11)**

Perfectly elastic collision of two objects

$v_1 = \dfrac{2m_2u_2 + (m_1 - m_2)u_1}{m_1 + m_2}$ **(3.17)**

Perfectly inelastic collision of two objects

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$ **(3.19)**

Coefficient of restitution

$e = \left|\dfrac{v_1 - v_2}{u_1 - u_2}\right| = \left|\dfrac{v_2 - v_1}{u_1 - u_2}\right|$ **(3.20)**

Newton's 2nd law: law of acceleration

$\Sigma F = ma$ **(3.22)**

$\Sigma F_x = ma_x$ **(3.23)**

$\Sigma F_y = ma_y$ **(3.24)**

Impulse–momentum equation

$\Sigma\bar{F}\,\Delta t = m(v_f - v_i)$ **(3.29)**

Universal law of gravitation: gravitational force

$F = G\left(\dfrac{m_1m_2}{r^2}\right)$ **(3.30)**

WORK, POWER, AND ENERGY

Work

$U = \bar{F}(d)$ **(4.2)**

Kinetic energy

$KE = \frac{1}{2}mv^2$ **(4.4)**

Gravitational potential energy

$PE = Wh$ **(4.5)**

Strain energy

$SE = \frac{1}{2}k\,\Delta x^2$ **(4.7)**

Work–energy principle

$U = \Delta E$ **(4.8)**

Power

$P = \dfrac{U}{\Delta t}$ **(4.12)**

$P = \bar{F}\bar{v}$ **(4.13)**

ANGULAR KINEMATICS

Angular position measured in radians

$\theta = \dfrac{arc\ length}{r} = \dfrac{\ell}{r}$ **(6.1)**

Angular displacement and arc length

$\ell = \Delta\theta r$ **(6.4)**

Average angular velocity

$\bar{\omega} = \dfrac{\Delta\theta}{\Delta t} = \dfrac{\theta_f - \theta_i}{\Delta t}$ **(6.6)**

Angular velocity and linear velocity

$v_T = \omega r$ **(6.8)**

Average angular acceleration

$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t}$ **(6.9)**

Tangential acceleration

$a_T = \alpha r$ **(6.10)**

Centripetal acceleration

$a_r = \frac{v_T^2}{r}$ **(6.11)**

$a_r = \omega^2 r$ **(6.12)**

ANGULAR KINETICS

Torque

$T = F \times r$ **(5.1)**

Static equilibrium

$\Sigma T = 0$ **(5.2)**

Center of gravity

$\Sigma(W \times r) = (\Sigma W) \times r_{cg}$ **(5.3)**

Moment of inertia

$I_a = \Sigma m_i r_i^2$ **(7.1)**

$I_a = mk_a^2$ **(7.2)**

Moment of inertia: parallel axis theorem

$I_b = I_{cg} + mr^2$ **(7.3)**

Angular momentum

$H_a = I_a \omega_a$ **(7.4)**

Angular momentum of the human body

$H_a = \Sigma(I_i \omega_i + m_i r_{i/cg}^2 \omega_{i/cg})$ **(7.5)**

Conservation of angular momentum

$H_i = I_i\omega_i = I_f\omega_f = H_f =$ constant if $\Sigma T = 0$ **(7.7)**

Angular version of Newton's 2nd law

$\Sigma T_a = I_a \alpha_a$ **(7.9)**

$\Sigma \overline{T}_a = \frac{\Delta H_a}{\Delta t} = \frac{(H_f - H_i)}{\Delta t}$ **(7.10)**

Angular impulse-momentum

$\Sigma \overline{T}_a \Delta t = (H_f - H_i)_a$ **(7.11)**

FLUID MECHANICS

Pressure

$P = \frac{F}{A}$

Density

$\rho = \frac{m}{V}$ **(8.3)**

Drag force

$F_D = \frac{1}{2} C_D \rho A v^2$ **(8.5)**

Lift force

$F_L = \frac{1}{2} C_L \rho A v^2$ **(8.6)**

MECHANICS OF MATERIALS

Stress

$\sigma = \frac{F}{A}$ **(9.1)**

Shear stress

$\tau = \frac{F}{A}$ **(9.2)**

Strain

$\varepsilon = \frac{\ell - \ell_o}{\ell_o}$ **(9.4)**

Elastic modulus

$E = \frac{\Delta\sigma}{\Delta\varepsilon}$ **(9.5)**

Abbreviations for Variables and Subscripts Used in Equations

Variables

a = instantaneous linear acceleration
$\bar{a}$ = average linear acceleration
A = area
C_D = coefficient of drag
C_L = coefficient of lift
d = displacement
e = coefficient of restitution
E = energy
E = elastic modulus or Young's modulus
F = force
$\bar{F}$ = average force
F_d = dynamic friction force
F_s = static friction force
ΣF = net force = sum of forces
g = acceleration due to gravity
G = gravitational constant
h = height
H = angular momentum
I = moment of inertia
k = radius of gyration
k = stiffness or spring constant
KE = kinetic energy
ℓ = distance traveled or length
L = linear momentum
m = mass
P = power
P = pressure
P = force
PE = gravitational potential energy
r = radius
r = moment arm
R = normal contact force
s = instantaneous linear speed
$\bar{s}$ = average linear speed
t = time
T = torque
u = pre-impact velocity
U = work done
v = instantaneous linear velocity
v = post-impact velocity
$\bar{v}$ = average linear velocity
V = volume
W = weight
x = horizontal position
y = vertical position
α = instantaneous angular acceleration
$\bar{\alpha}$ = average angular acceleration
Δ = change in … = final – initial
ε = strain
μ = coefficient of friction
ρ = density
σ = stress
Σ = sum of …
τ = shear stress
θ = angular position
ω = instantaneous angular velocity
$\bar{\omega}$ = average angular velocity

Subscripts

a = axis
b = axis
d = dynamic
cg = center of gravity
D = drag
f = final or ending
i = initial or starting
i = one of a number of parts
L = lift
o = original or undeformed
r = radial
s = static
T = tangential
x = horizontal
y = vertical