I. Current and Kirchhoff's Current Law

- Charge is a fundamental electrical quantity
- Typically denoted by the symbol Q, and its SI unit is the Coulomb (C)
- The smallest amount of charge that exists is the charge that is carried by an electron:

$$Q_e = 1.602 \times 10^{-19}\ \text{C}$$

Always include units, otherwise there is no meaning

Current: Free electrons on the move

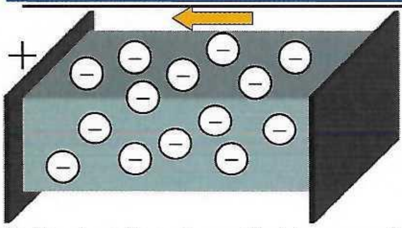


Fig 2a. A voltage is applied to move the charges

Direction of current

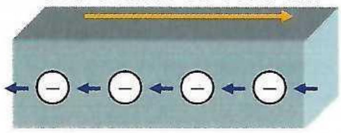


Fig 2b. Movement of electrons gives rise to current

Key Concept 1
For current to flow, there must be mobile charges

- Direction of electron flows is from negative to positive
- Direction of current flows (positive charges) is from positive to negative
- Typically denoted by the symbol, I
- SI unit is the Ampere (A)

'0

Current must run in loops

- If a current flows out of a given point, then it must return to that point with the same amount
- Current must flow in loops
- What goes around comes around

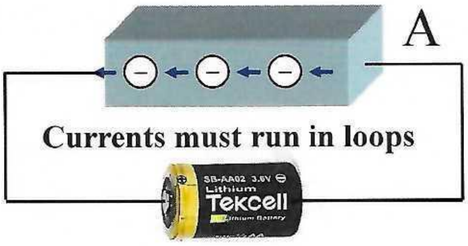


Fig 3. Current must run in loops

Key Concept 2
Currents must run in loops (otherwise charge is either destroyed or created which cannot happen)

Fundamental law for charge

- Current has to flow in closed loop
- No current flows if there is a break in the path
- Underlying physical law: Charge cannot be created or destroyed ("what goes in must also come out")
- This is the basis of Kirchhoff's Current Law

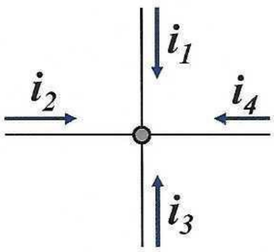


Fig 4.

Kirchhoff's current law

Sum of <u>currents</u> at a **<u>node</u>** must equal to zero:

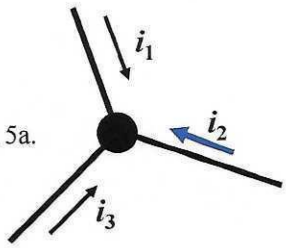
$i_1 + i_2 + i_3 + i_4 = 0$

Kirchhoff's current law

Does the direction of current matter? **YES!!**

I_2 running **into** node — I_2 running **out of** node

Fig 5a.



Different

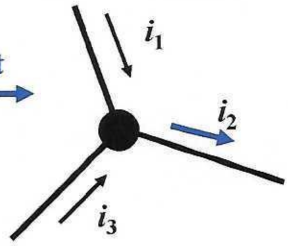


Fig 5b.

$$i_1 + i_2 + i_3 = 0$$

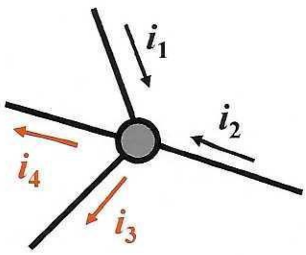
$$i_1 + i_2 + i_3 = 0 \textbf{ (WRONG)}$$
$$i_1 + i_3 = i_2 \textbf{ (CORRECT)}$$

Kirchhoff's current law

Sign convention when applying KCL

Currents flowing IN - Positive

Currents flowing OUT - Negative



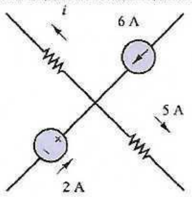
Entering: I_1 & I_2 (+ve)

Exiting: I_3 & I_4 (-ve)

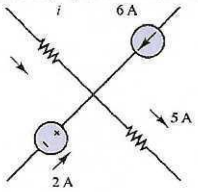
$$\boxed{i_1 + i_2 - i_3 - i_4 = 0}$$

Worked Example on KCL

Find the unknown current within each of the following circuit networks



i=3A



i=-3A

$-i + 6 - 5 + 2 = 0$

$i = 3\ A$ #

$i + 6 - 5 + 2 = 0$

$i = -3\ A$ #

Voltage and Kirchhoff's Voltage Law

- Energy is required to move electrons between 2 points
- This energy could come from the battery cell connected to the circuit
- As the current goes across the cell, energy is pumped in
- As the current goes through a light bulb, energy is consumed
- The amount of energy required to move a unit charge is given by the voltage
- SI unit is Voltage (V)

Potential Difference

$v_x = v_A - v_B$

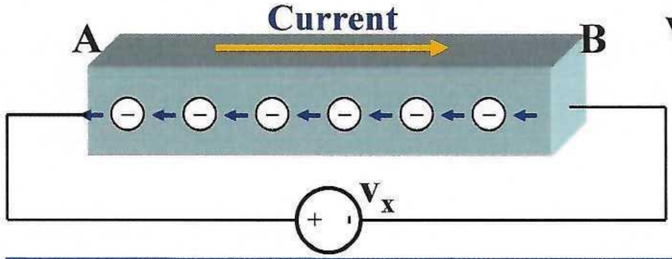


Fig 6

Sign Convention

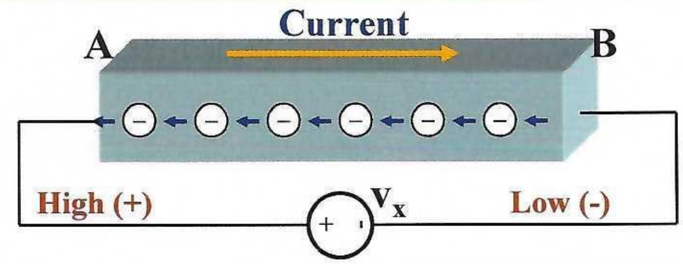


Fig 6

Sign Convention for Voltage

Voltage **rises** (negative to positive) along the direction of the current for an element that is **generating** power

Voltage **drops** (positive to negative) along the direction of the current for an element that is **consuming** power

Kirchhoff's Voltage Law (KVL)

- KVL states that the sum of voltages around a loop must equal to zero
- If a voltage starts at any given point, no gain or loss of any voltage when it returns to the same point
- Whatever energy is pumped into the loop must also be consumed

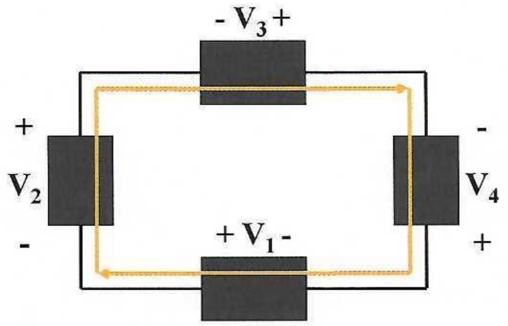


Fig 7: Voltages around a closed loop

Kirchhoff's voltage law

Net voltage around a closed circuit is zero:

$$v_1 + v_2 + v_3 + v_4 = 0$$

Kirchhoff's voltage law

- First, you need to define the direction
- V_1 and V_3 are voltage rises (positive) while V_2 is a voltage drop (negative)
- In the other direction, V_2 is voltage rise (positive), while V_1 and V_3 are voltage drops (negative)

  $V_2 - V_1 - V_3 = 0$

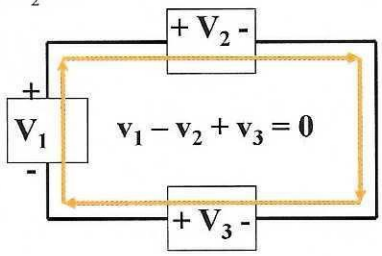
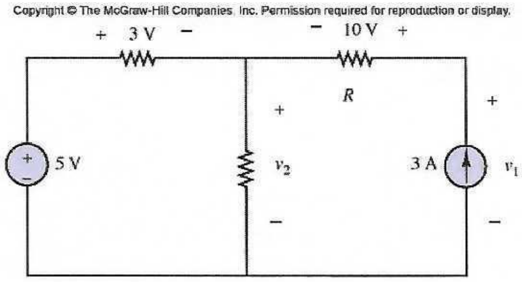


Fig 8: Defined voltage drops and rises

Sign Convention for KVL
Voltage RISE – Positive
Voltage DROP – Negative

Worked Example on KVL

Apply KVL to find voltage V_1 and V_2 $V_1 = 12V, V_2 = 2V$



V_1 :

$5 - 3 + 10 - V_1 = 0$

$V_1 = 12\text{ V}$ #

V_2 :

$5 - 3 - V_2 = 0$

$V_2 = 2V$ #

II. Resistance and Ohm's Law

- When current flows through a conductor, it will always experience some resistance
- Resistance is given by the change in voltage over change in current
- SI unit is Ohm (Ω)
- If the voltage is linear with current, then resistance is said to be linear and obeys Ohm's law

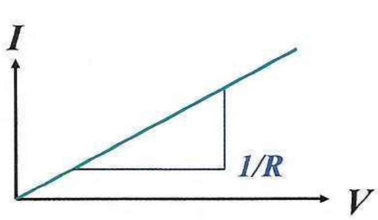


Fig 9a: Linear resistance that obeys Ohm's law

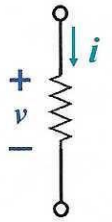


Fig 9b

Ohm's law is given as:

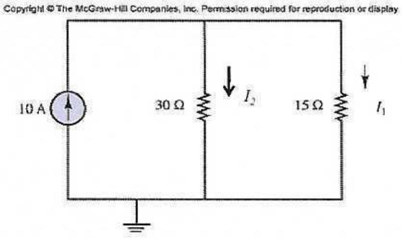
V = IR

Important point to note:

If an unknown current is defined to flow from A to B, the voltage at A is assumed to be higher than B.

Worked Example Applying 3 Laws

Find the current through the 15Ω resistor $I_1 = 6.67A$, $I_2 = 3.33A$



KCL :

$10 - I_2 - I_1 = 0$ —(1)

30-Ω & 15-Ω resistors have same voltage

$I_2 \cdot 30 = I_1 \cdot 15$

$I_1 = 2 I_2$ —(2)

From (1) & (2) $I_1 = 6.67\ A$

$I_2 = 3.33\ A$

III. Electrical Power

- A voltage drop in the direction of the current indicates the power is consumed
- Conversely, a voltage rise in the direction of the current indicates that power is generated
- If the voltage across a resistor is V, and current through it is I, then the power consumed is given by:

  $P = VI$

  Ohm's law → $P = I^2R$

  Ohm's law → $P = V^2/R$

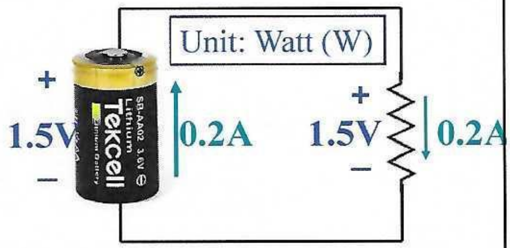


Fig 10: Power generated and consumed in a circuit

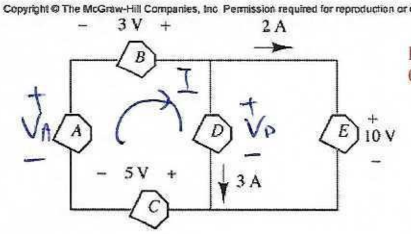
Power generated by source

MUST EQUAL

Power dissipated in the load

Worked Example on Power

Determine which components are absorbing power and which are delivering power



Delivering: A (60W), B (15W)
Consuming: C (25W), D (30W), E (20W)

To find V_A (consider outer loop)

$V_A + 3 - 10 - 5 = 0$

$V_A = +12\ V$, $P_A = V_A \cdot I_A = 12 \times (2A + 3A)$

$\Rightarrow P_A = 60\ W$ (delivering power)

delivering power because the current is leaving +Ve voltage.

To find V_D

Consider the right loop: $10 - V_D = 0$

$V_D = 10\ V$

$P_D = V_D I_D = 10 \times 3$

$\Rightarrow P_D = 30\ W$ (consuming power)

Consuming power because the current flows into +Ve voltage

Terminology: Branch and Node

BRANCH: Any **path** of a circuit with 2 TERMINALS connected to it

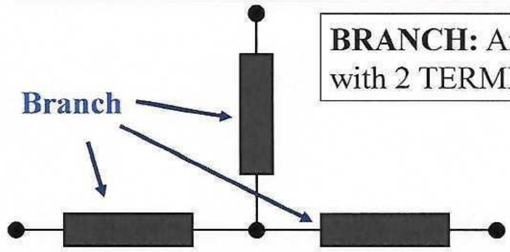
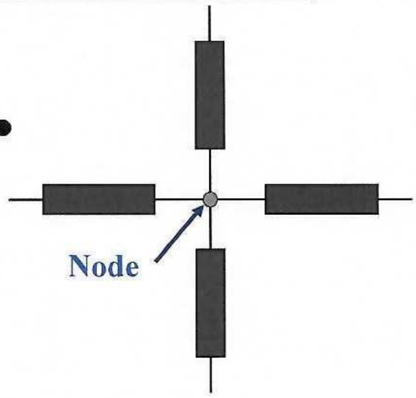


Fig 11a: 3 branches



NODE: **Junction** of 2 or more branches

Fig 11b: 1 node

Terminology: Loop and Mesh

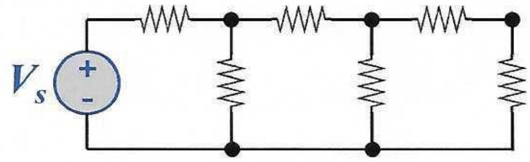


Fig 12: Loops (6) and meshes (3)

- **Loop**: Any closed connection of branches
- **Mesh**: Loop that does not contain any loops
- In Fig 12, there are 6 loops and 3 meshes.
- We will use the term mesh more than loop in this course

IV. Sources, Short Circuit and Open Circuit

- Loads (e.g. resistors) consume power
- Sources on the other hand deliver power

Ideal Voltage Source

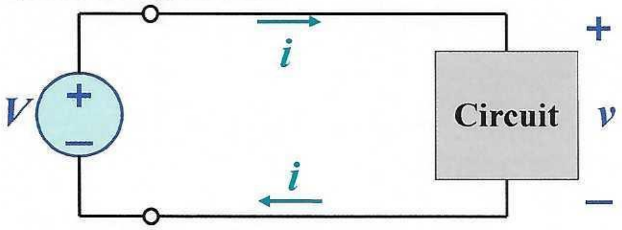


Fig 13a: Ideal voltage source in a circuit

- The purpose of a voltage source is to keep the voltage across its terminals unchanged
- Current through a voltage source is allowed to change in order to maintain the voltage at V with reference to Fig 13a (i.e., a different i for a different circuit loading)

Ideal Current Source

Ideal Current Source

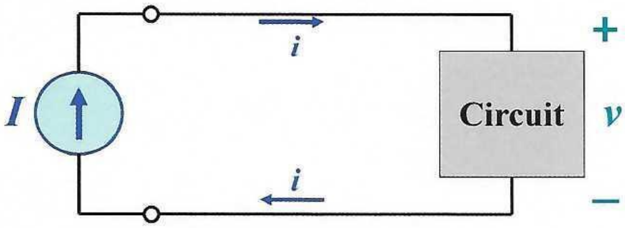


Fig 13b: Ideal current source in a circuit

- The purpose of a current source is to keep the current flowing through it unchanged
- Voltage across a current source is allowed to change in order to maintain the current at I with reference to Fig 13b

Recognize the differences between an ideal voltage and current source in their functions and symbols.

Short Circuit

- A short circuit means connecting 2 or more terminals together so that the voltage between them is zero
- It is typically associated with currents rather than voltage, e.g. short circuit current

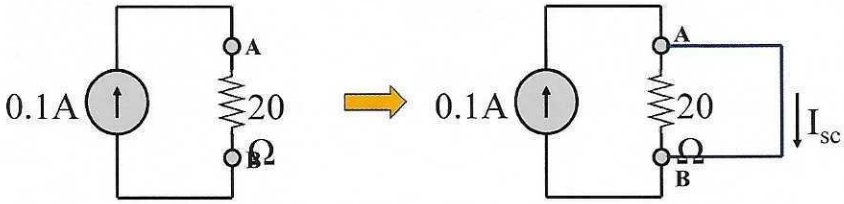


Fig 14a: Short circuiting the resistor

- All the current from the source flows through the short circuit, bypassing the resistor
- Short circuit current I_{SC} = 0.1 A

Open Circuit

- A open circuit means no extra connections are imposed across two terminals
- It is typically associated with voltage rather than current, e.g. open circuit voltage

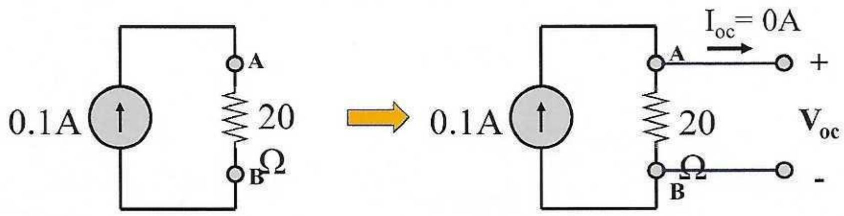
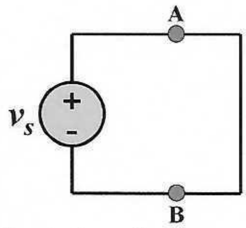


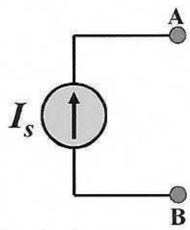
Fig 14b: Open circuiting the resistor

- In Fig 14b, the open circuit voltage of the resistor is simply the voltage across the resistor
- Open circuit Voltage V_{OC} = 2 V

Self-contradictory circuits



What is the voltage across A and B?



What is the current through the source?

Prefixes

- As engineers, it is important that we replace exponents with prefixes
- For example:
  0.00215 A → 2.15 mA
  never $2.15\text{x}10^{-3}$A
- Make sure you memorize the prefixes from nano (n) to mega (M)

Prefixes: Memorize and **apply** them!

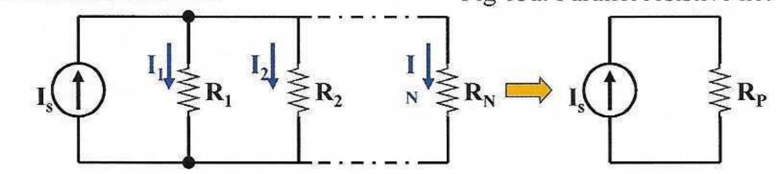
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}

V. Resistive Networks

- Resistors are either arranged in parallel or in series or as combination of both

Parallel Network

Fig 15a: Parallel resistive network



Equivalent Resistance *Rp*: $$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N}$$

Current Divider Rule Proof: $I_s = I_1 + I_2 + I_3 + \dots + I_N \Rightarrow \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \dots + \frac{V}{R_N}$

$$I_k = \frac{R_P}{R_k} I_S$$

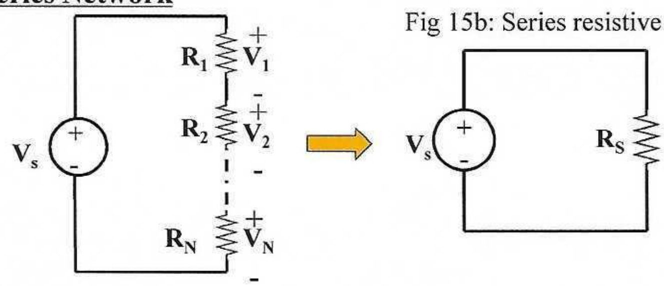
- Current (I_s) from the source will have to be shared between the resistors (R_k) in each branch

$\Rightarrow \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$

Series network (Highlights)

Series Network

Fig 15b: Series resistive network



Equivalent Resistance *Rp*: $$R_S = R_1 + R_2 + \dots + R_N$$

Voltage Divider Rule

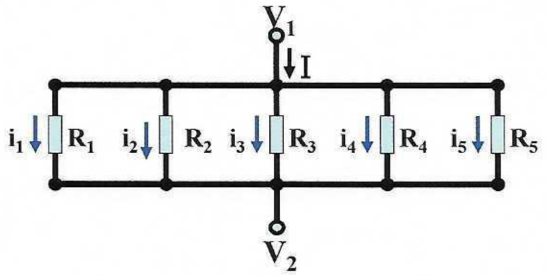
$$V_k = \frac{R_k}{R_S} V_S$$

- Total voltage drop across all the resistors (V_s) in series is the sum total of voltage (V_k) drops across each resistor

Worked Example on Current Divider Rule

Rank the currents from largest to smallest if $\mathbf{R_2 > R_4 > R_1 > R_5 > R_3}$

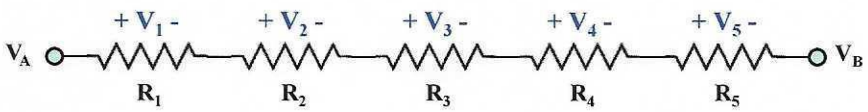
$I_3 > I_5 > I_1 > I_4 > I_2$



Worked Example on Voltage Divider Rule

Rank the potential difference from largest to smallest if $\mathbf{R_2 > R_4 > R_1 > R_5 > R_3}$

$V_2 > V_4 > V_1 > V_5 > V_3$

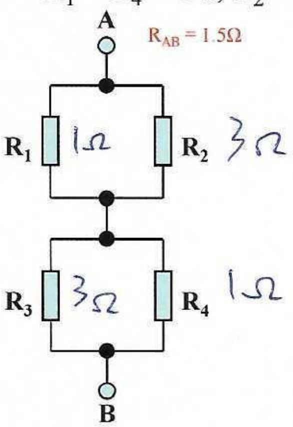


Worked Example 1 on Resistive Networks

Find the effective resistance across A and B

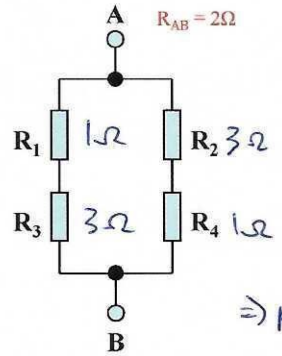
$R_1 = R_4 = 1\Omega$, $R_2 = R_3 = 3\Omega$

$R_{AB} = 1.5\Omega$



$$R_{AB} = R_1 // R_2 + R_3 // R_4$$
$$= \frac{R_1 R_2}{R_1 + R_2} \times 2$$
$$= \frac{1 \times 3}{1+3} \times 2$$
$$R_{AB} = 1.5\Omega$$

$R_{AB} = 2\Omega$



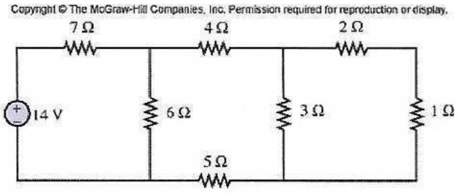
$$R_{AB} = (R_1 + R_3) // (R_2 + R_4)$$
$$= 4 // 4$$
$$= \frac{4 \times 4}{4+4}$$
$$\Rightarrow R_{AB} = 2\Omega$$

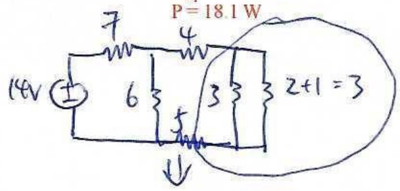
Worked Example 2 on Resistive Networks

What is the equivalent resistance seen by the source?

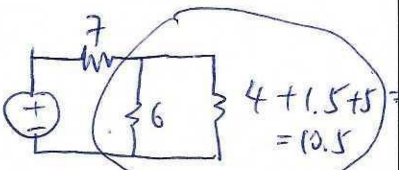
What is power supplied by the source?

$R_{eq} = 10.818\,\Omega$
$P = 18.1$ W





$$\Rightarrow 3 // 3 = \frac{3 \times 3}{3+3} = 1.5\Omega$$



$$\Rightarrow \frac{6 \times 10.5}{6 + 10.5} = \frac{6 \times 10.5}{16.5} = 3.818\,\Omega$$

$$R_{eq} = 7 + 3.818 = 10.818\,\Omega \ \#$$

$$P = \frac{V^2}{R_{eq}} = \frac{14^2}{10.818} = 18.1\,W \ \#$$

VI. Measuring Instruments - Voltmeter

- Voltmeter measures voltage across a circuit element

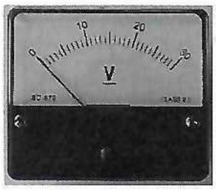


Fig 16: Image of typical voltage with a measuring range up to 30V

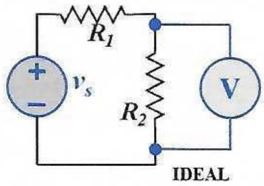


Fig 17a: Placement of an ideal voltmeter to measure the PD across R_2

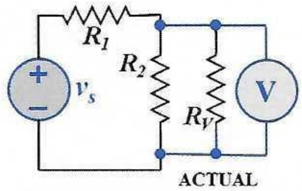


Fig 17b: Circuit model of how a real voltmeter behaves, which looks like a large parallel resistor (R_v)c

How would you place the voltmeter in the circuits?

- Connected in **parallel** with the element being measured

In Fig 17a, voltmeter has no effect on original circuit.
In Fig 17b, voltmeter will steal current from R_2. This will change the voltage measured across R_2. **Thus, the PD measured with a voltmeter is lower than the true value.**

Worked Example on Voltmeters

Referring to Fig 17b, if $V_s = 5V$, and $R_V = 1M\Omega$, find the voltage across R_2 when
(i)$R_1 = R_2 = 1k\Omega$ V = 2.5V
(ii)$R_1 = R_2 = 1M\Omega$ V = 1.67V

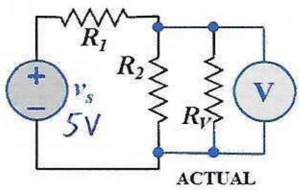


Fig 17b

(i) $R_1 = R_2 = 1k$, $R_V = 1M = 1000k$

$R_p = R_2 // R_V = \frac{1 \times 1000}{1+1000} \simeq 1k$

$V_{R_2} = \frac{R_p}{R_1 + R_p} \times V_s = \frac{1}{1+1} \times 5 = 2.5V$ #

(ii) $R_1 = R_2 = 1M$, $R_V = 1M$

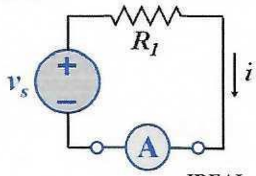
$R_p = R_2 // R_V = \frac{1 \times 1}{1+1} = 0.5M$

$V_{R_2} = \frac{R_p}{R_1 + R_p} V_s = \frac{0.5}{1+0.5} \times 5 = 1.67V$ #

Ammeter

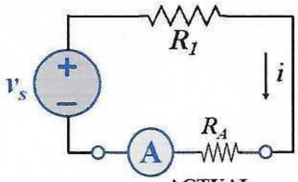
- Ammeter measures current

Fig 18: Image of typical current with a measuring range up to 20A



IDEAL

Fig 19a: Placement of an ideal ammeter to measure current through R_1



ACTUAL

Fig 19b: Circuit model of how a real ammeter behaves, which looks like a small series resistor (R_A)

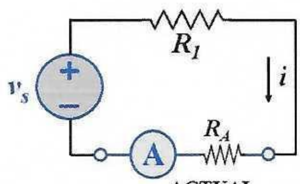
How would you place the ammeter in the circuits?

- Connected in **series** with the element being measured

In Fig 19a, ammeter has no effect on original circuit.
In Fig 19b, ammeter contains some resistance (R_A), adding to the total resistance, lowering the measured current. **Thus, the current measured with an ammeter is lower than the true value.**

Ammeter Example

Referring to Fig 19b, if $V_s = 5V$, and $R_A = 1\Omega$, find the loop current when
(i)$R_1 = 1k\Omega$ I = 4.995 mA
(ii)$R_1 = 2\Omega$ I = 1.67A



ACTUAL

Fig 19b

(i) $R_1 = 1k\Omega$

$$i = \frac{5}{1+10^{-3}} = \frac{5}{1.001} = 4.995A \#$$

(ii) $R_1 = 2\Omega$

$$i = \frac{5}{1+2} = \frac{5}{3} = 1.67 A \#$$