Chapter 2 Resistive Circuits

1. Simple circuits analysis.
2. Voltage-division and current-division principles.
3. Node-voltage analysis.
4. Mesh-current analysis.
5. Thévenin and Norton equivalents.
6. Superposition principle.
7. Wheatstone bridge.

Simple circuits analysis

Resistors in series

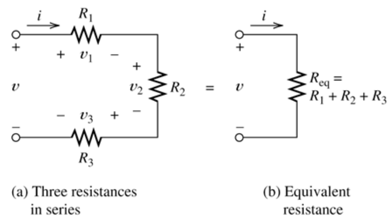


Figure 2.1 Series resistances can be combined into an equivalent resistance.

Resistors in parallel

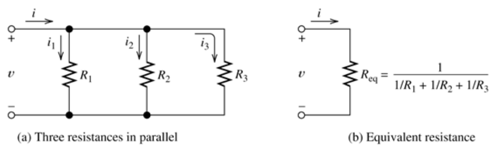


Figure 2.2 Parallel resistances can be combined into an equivalent resistance.

Example 2.1

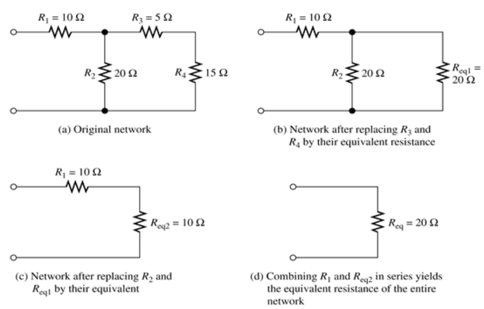


Figure 2.3 Resistive network for Example 2.1.

Circuit Analysis using Series/Parallel Equivalents

1. Begin by locating a combination of resistances that are in series or parallel. Often the place to start is farthest from the source.
2. Redraw the circuit with the equivalent resistance for the combination found in step 1.
3. Repeat steps 1 and 2 until the circuit is reduced as far as possible. Often (but not always) we end up with a single source and a single resistance.
4. Solve for the currents and voltages in the final equivalent circuit.

Example 2.2

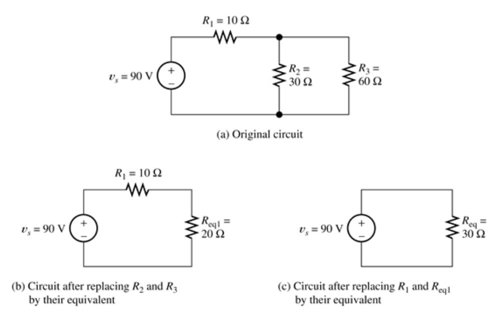
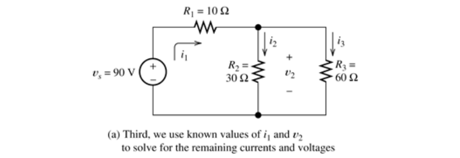


Figure 2.5 A circuit and its simplified versions. See Example 2.2.



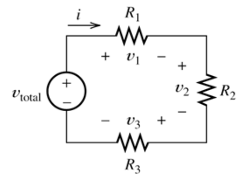
(a) Third, we use known values of i_1 and v_2 to solve for the remaining currents and voltages

(b) Second, we find $v_2 = R_{eq1}\, i_1 = 60$ V

(c) First, we solve for $i_1 = \frac{v_s}{R_{eq}} = 3$ A

Figure 2.6 After reducing the circuit to a source and an equivalent resistance, we solve the simplified circuit. Then, we transfer results back to the original circuit. Notice that the logical flow in solving for currents and voltages starts from the simplified circuit in (c).

Voltage Division



$$v_{total} = v_1 + v_2 + v_3$$

Example 2.3: Application of the Voltage-Division Principle

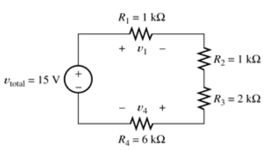
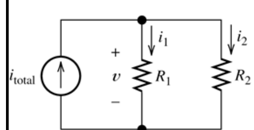


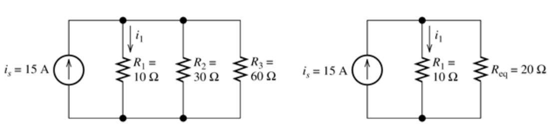
Figure 2.9 Circuit for Example 2.3.

Current Division



$$i_{total} = i_1 + i_2$$

Example 2.5



(a) Original circuit

(b) Circuit after combining R_2 and R_3

Figure 2.12 Circuit for Example 2.5.

Application of the Current-Division Principle

Exercise 2.3

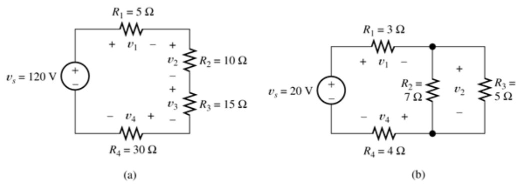


Figure 2.14 Circuits for Exercise 2.3.

Exercise 2.4

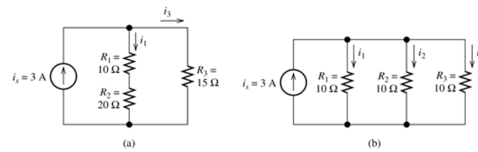


Figure 2.15 Circuits for Exercise 2.4.

Although they are very important concepts, series/parallel equivalents and the current/voltage division principles are NOT sufficient to solve all circuit problems.

Example

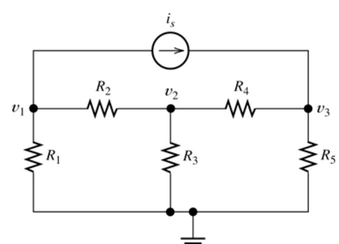


Figure 2.18 Circuit for Example 2.6.

Node Voltage Analysis

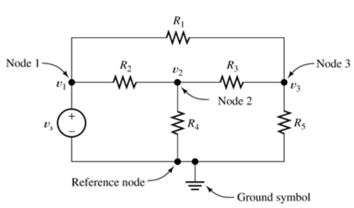


Figure 2.16 The first step in node analysis is to select a reference node and label the voltages at each of the other nodes.

- One of two key methods, the other is mesh analysis
- Based on KCL, more systematic
- All voltages have a common reference point

Writing KCL Equations in Terms of the Node Voltages

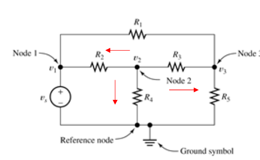


Figure 2.16 The first step in node analysis is to select a reference node and label the voltages at each of the other nodes.

Example 2.6 Write equations

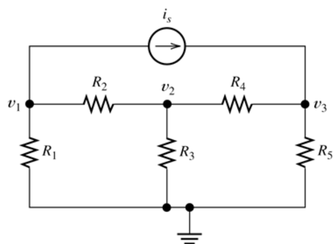
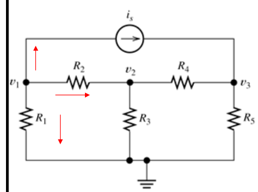


Figure 2.18 Circuit for Example 2.6.



Example 2.8

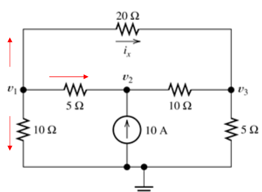


Figure 2.21 Circuit for Example 2.8.

Different reference node

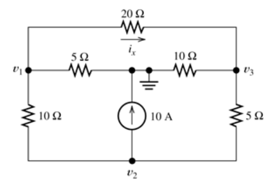


Figure 2.23 Circuit of Example 2.8 with a different choice for the reference node. See Exercise 2.9.

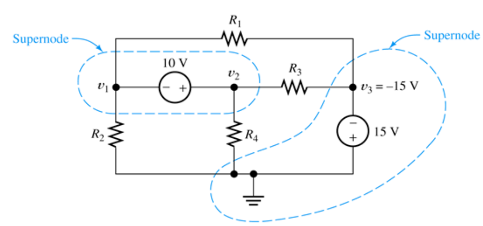
$$i_x = ?$$

$$How\ about\ v_1, v_2, v_3\ ?$$

Circuits with Voltage Sources

- We obtain dependent equations if we use all of the nodes in a network to write KCL equations

Introducing ***super-nodes***



Exercise 2.13

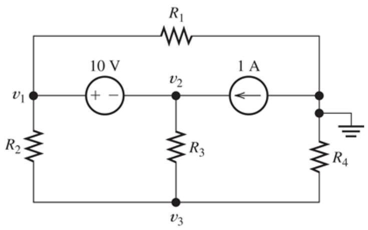
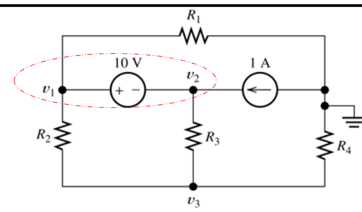


Figure 2.27 Circuit for Exercise 2.13.



Summary: Node-Voltage Analysis

1. Select a reference node and assign variables for the unknown node voltages.
2. Write current equations for nodes and supernodes using KCL.
3. Put the equations into standard form and solve for the node voltages.
4. Use the values found for the node voltages to calculate any other currents or voltages of interest.

Example 2.10

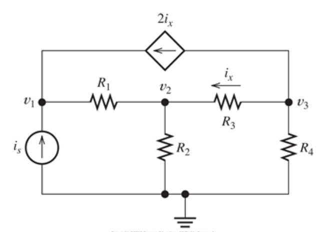
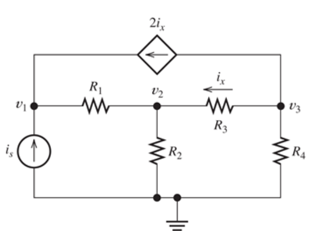


Figure 2.28 Circuit containing a current-controlled current source. See Example 2.10.



Exercise 2.14

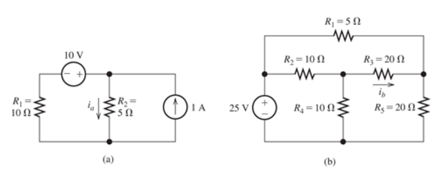


Figure 2.31 Circuits for Exercise 2.14.

Review – Chapter 2

1. Series and parallel Resistors
2. Voltage and current division
3. Node Voltage Approach
   KCL, Node voltages, voltage sources (super case)

Mesh Current Analysis

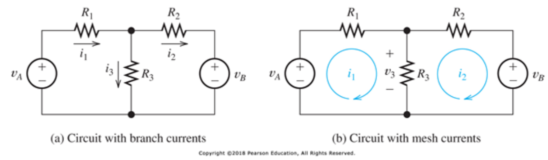


Figure 2.33 Circuit for illustrating the mesh-current method of circuit analysis.

- One of two key methods, the other is Nodal analysis
- Based on KVL, more systematic
- Work with imaginary currents

Step 1: Choosing the Mesh Currents

When several mesh currents flow through one element, we consider the current in that element to be the algebraic sum of the mesh currents.

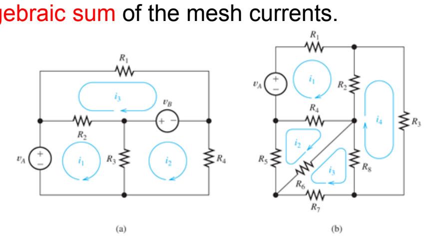
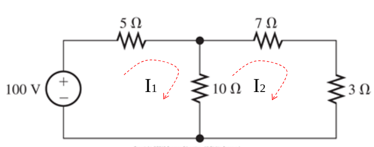


Figure 2.34 Two circuits and their mesh-current variables.

Step 2: Writing Equations to Solve for Mesh Currents via KVL

Figure 2.34a

Exercise 2.19



$i_{10\Omega} = ?$

Figure 2.36 Circuit of Exercise 2.19.

Mesh Currents in Circuits Containing Current Sources

A common mistake made by beginning students is to assume that the voltages across current sources are zero. In Figure 2.38, we have:

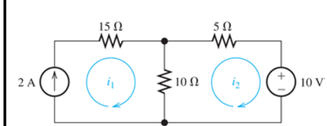


Figure 2.38 In this circuit, we have $i_1 = 2$ A.

More complex example

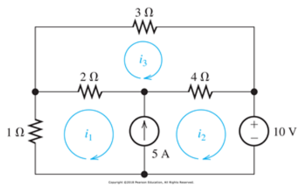


Figure 2.39 A circuit with a current source common to two meshes.

Combine meshes 1 and 2 into a **supermesh**. In other words, we write a KVL equation around the periphery of meshes 1 and 2 combined.

Summary: Mesh-Current Analysis

1. Define the mesh currents flowing around each of the open areas defined by the network. For consistency, we usually select a clockwise direction for each of the mesh currents, but this is not a requirement.
2. Write voltage equations for the mesh loop by using the KVL.
3. Put the equations into standard form. Solve for the mesh currents.
4. Use the values found for the mesh currents to calculate any other currents or voltages of interest.

Example 2.16

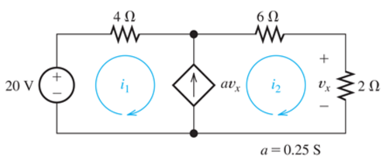


Figure 2.42 A circuit with a voltage-controlled current source. See Example 2.16.

Nodal versus Mesh analysis

- Both are efficient
- Which is more efficient depends on the problem
- Nodal analysis used more often
- Nodal: KCL, Node voltages, voltage sources (super case)
- Mesh: KVL, Mesh currents, current sources (super case)

Thévenin Equivalent Circuits

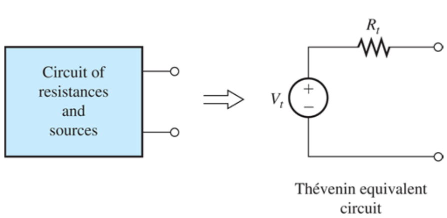


Figure 2.44 A two-terminal circuit consisting of resistances and sources can be replaced by a Thévenin equivalent circuit.

How to find equivalent circuit?

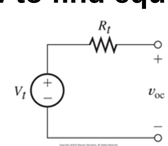


Figure 2.45 Thévenin equivalent circuit with open-circuited terminals. The open-circuit voltabe v_{OC} is equal to the Thévenin voltage v_t.

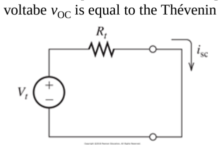


Figure 2.46 Thévenin equivalent circuit with short-circuited terminals. The short-circuit current is $i_{sc} = V_t/R_t$.

Example 2.18

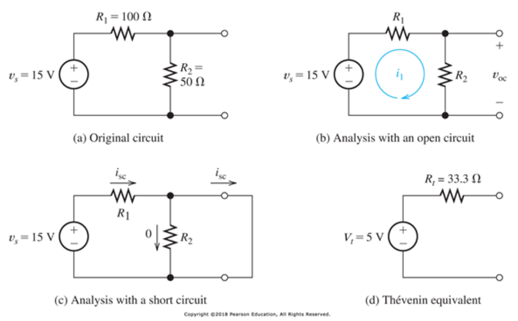


Figure 2.47 Circuit for Example 2.18.

Finding the Thévenin Resistance Directly

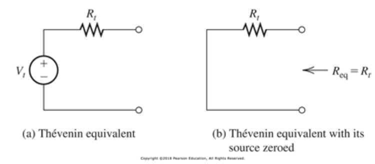


Figure 2.49 When the source is zeroed, the resistance seen from the circuit terminals is equal to the Thévenin resistance.

We can find the Thévenin resistance by zeroing the sources in the original network and then computing the resistance between the terminals.

When zeroing a voltage source, it becomes a short circuit.

When zeroing a current source, it becomes an open circuit.

Example 2.19

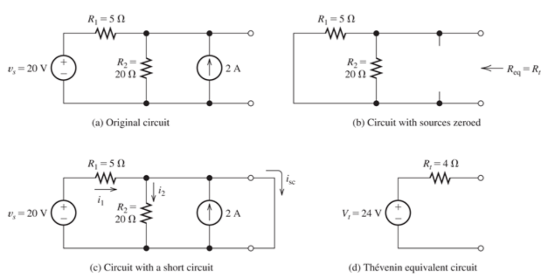


Figure 2.50 Circuit for Example 2.19.

Norton Equivalent Circuit

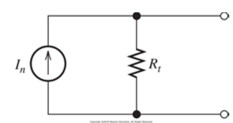


Figure 2.53 The Norton equivalent circuit consists of an independent current source I_n in parallel with the Thévenin resistance R_t.

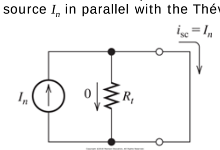


Figure 2.54 The Norton equivalent circuit with a short circuit across its terminals.

Step-by-step Thévenin/Norton-Equivalent-Circuit Analysis

1. Perform two of these:

a. Determine the open-circuit voltage $V_t = v_{oc}$.

b. Determine the short-circuit current $I_n = i_{sc}$.

c. Zero the sources and find the Thévenin resistance R_t looking back into the terminals.

2. Use the equation $V_t = R_t I_n$ to compute the remaining value.

3. The Thévenin equivalent consists of a voltage source V_t in series with R_t .

4. The Norton equivalent consists of a current source I_n in parallel with R_t .

Source Transformations

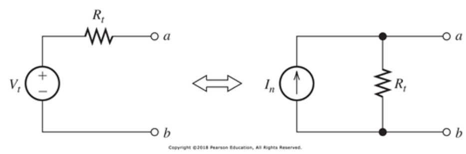


Figure 2.57 A voltage source in series with a resistance is externally equivalent to a current source in parallel with the resistance, provided that $I_n = V_t / R_t$.

The two circuits are equivalent if they have the same current-voltage relationship at their terminals

Example 2.22

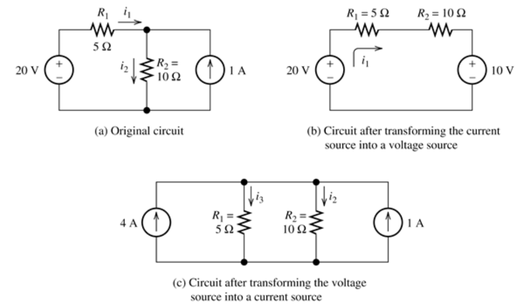


Figure 2.54 Circuit for Example 2.18.

Maximum Power Transfer

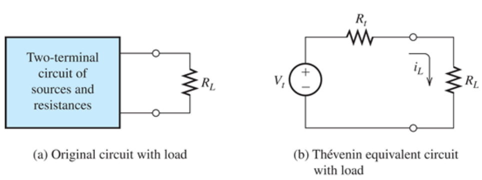


Figure 2.60 Circuits for analysis of maximum power transfer.

The load resistance that absorbs the maximum power from a two-terminal circuit is equal to the Thévenin resistance.

Example 2.23

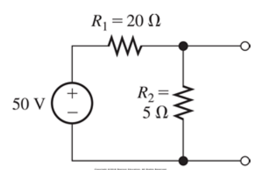


Figure 2.61 Circuit for Example 2.23.

Superposition Principle

The **superposition principle** states that the total response is the algebric sum of the responses to each of the independent sources acting individually. In equation form, this is

$$r_T = r_1 + r_2 + \cdots + r_n$$

Foundation of the principle: linearity

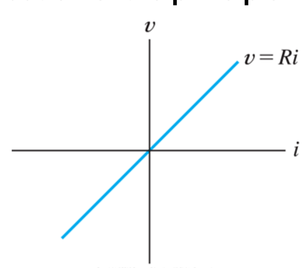


Figure 2.63 A resistance that obeys Ohm's law is linear.

Example 2.24

$$V_T = V_1 + V_2 = 5 + 6.66 = 11.66V$$

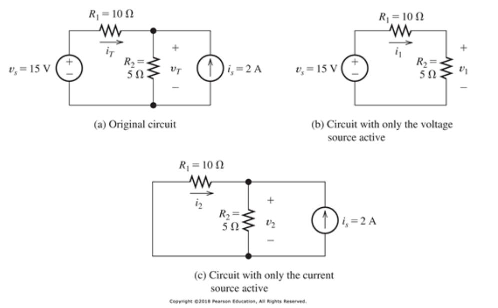
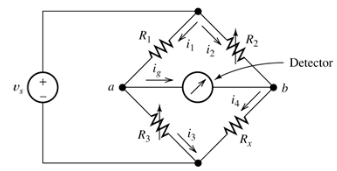


Figure 2.64 Circuit for Example 2.24 and Exercise 2.31.

Wheatstone Bridge

The Wheatstone bridge is used by mechanical and civil engineers to measure the resistances of strain gauges in experimental stress studies of machines and buildings.



$$i_g = 0$$

$$R_x = \frac{R_2}{R_1} R_3$$