BME2102：Introduction to Biomechanics

DONG Lixin (董立新)
Professor
Department of Biomedical Engineering
City University of Hong Kong
香港城市大學 生物醫學工程學系
http://www.cityu.edu.hk/bme/lixidong/

Instructors

- Lixin Dong, Ph.D., Professor
  - Department of Biomedical Engineering (BME)
  - **Room B7456, 7/F, Yellow Zone**
  - **Yeung Kin Man Academic Building**
  - City University of Hong Kong
  - 83 Tat Chee Avenue
  - Kowloon Tong, Hong Kong
  - Phone: +852-3442-9545
  - **Email: L.X.Dong@cityu.edu.hk**
- Office hours
  - **14:30-16:30 F**
  - **By appointment**

BME2102 -- Introduction to Biomechanics

- Credit Units: 3 credits
- Level: B2
- Course duration: 1 semester (Semester A 2021/2022)
- Prerequisites: BCH1200 Discovery in Biology or AP1201/PHY1201 General Physics I
- C01
  - Date: F 30/08/2021 - 27/11/2021
  - 17:00-17:50 Lecture
  - 18:00-18:50 Lecture
  - Classroom: YEUNG LT-18
- T01
  - Date: F 08/10/2021, 19/11/2021
  - 10:00-11:50 Tutorial
  - Classroom: YEUNG LT-18

Lab Instructors

- Updates: AIMS and/or Canvas
- Labs (Students in HK must attend on campus):
  - Schedule:
    - L01 : wk 5, 8, 11 T (Y1501, B1667)
    - L02 : wk 6, 9, 12 T (Y1501, B1667)
    - L03 : wk 5, 8, 11 W (Y1501, B1667)
    - L04 : wk 5, 8, 11 R (Y1501, B1667)
    - L05 : wk 6, 9, 12 R (Y1501, B1667)
  - Y1501, Yeung Kin Man Academic Building
- TBD:
  - L01 & L02
  - Email: ???@my.cityu.edu.hk
- TBD:
  - L03 & L04
  - E-mail: ???@my.cityu.edu.hk
- TBD:
  - L05
  - Email: ???@my.cityu.edu.hk

Schedule

			15331	15332	15333	15334	15335	15336	15337
			C01	**T01**	**L01**	**L02**	**L03**	**L04**	**L05**
			17:00 - 18:50	10:00 - 11:50	09:00 - 11:50	09:00 - 11:50	14:00-16:50	09:00 - 11:50	09:00 - 11:50
			YEUNG	YEUNG	YEUNG	YEUNG	YEUNG	YEUNG	YEUNG
			LT-18	LT-18	Y1501	Y1501	Y1501	Y1501	Y1501
Week	Date	Instructor	DONG Lixin	DONG Lixin					
1	3/9	F	Lect 1						
2	10/9	F	Lect 2						
3	16/9	R							
	17/9	F	Lect 3						
4	23/9	R							
	24/9	F	Lect 4						
5	28/9	T			Lab				
	29/9	W					Lab		
	30/9	R						Lab	
	1/10	F	National Day						
6	5/10	T				Lab			
	7/10	R							Lab
	8/10	F	Lect 5	Tutorial 1					
7	15/10	F	In-class Test						
8	19/10	T			Lab				
	20/10	W					Lab		
	21/10	R						Lab	
	22/10	F	Lect 6						
9	26/10	T				Lab			
	28/10	R							Lab
	29/10	F	Lect 7						
10	4/11	R							
	5/11	F	Lect 8						
11	9/11	T			Lab				
	10/11	W					Lab		
	11/11	R						Lab	
	12/11	F	Lect 9						
12	16/11	T				Lab			
	18/11	R							Lab
	19/11	T	Lect 10	Tutorial 2					
13	26/11	F	Lect 11						

Abstract

- This course aims to introduce students to the **fundamental concepts** that are required for the development of biomedical prosthetic devices in the human body; to provide a supportive, directed experiential and cooperative learning environment for students to acquire and develop **technique skills** to solve diverse engineering problems in various biomedical products.

Course Intended Learning Outcomes (CILOs)

- **Describe** the fundamental concepts of biomechanics and their impacts on the behavior of physical bodies subject to forces or displacements.
- **Identify** the mechanical engineering problems in biomaterials and biomedical devices, **explain** the problems with critical thinking generated from mechanics concepts, and **calculate** the problems with mechanics theory.
- **Apply** the biomechanics knowledge to explain structural and functional behavior of biological systems such as humans, animals, plants, organs, cells.
- **Present** the procedure, results and analysis of the lab experiments in scientific written reports.

Teaching and Learning Activities (TLAs)

- Lecture
  - Take place in classroom setting which consist of lectures and student activities in between.
  - 2 hrs/week
- Tutorial/Laboratory Sessions
  - Take place in classroom and laboratory, with assignments towards developing laboratory reports.
  - Tutorials: 2 hrs/week for 2 weeks
  - Labs: 3 hrs/week for 3 weeks

Assessment Tasks/Activities (ATs)

- Continuous Assessment: 40%
  - In-class Test: 20%
  - Laboratory Reports : 20%--3 reports to be submitted
- Examination: 60 %
  - Duration: 2 hours
- **For a student to pass the course, at least 30% of the maximum mark for both coursework and examination should be obtained.**

Assessment Rubrics

Assessment Task	Criterion	Excellent (A+, A, A-)	Good (B+, B, B-)	Fair (C+, C, C-)	Marginal (D)	Failure (F)
1. In-class test	Describe the mechanical design concepts and principles and provide solution to related design problems.	High	Significant	Moderate	Basic	Not even reaching marginal levels
2. Laboratory Reports	Attendance of the lab/demo session; ABILITY to EXPLAIN the methodology and procedure and ANALYSE the lab data.	High	Significant	Moderate	Basic	Not even reaching marginal levels
3. Examination	Explain the fundamental concepts and working principles, select proper machine elements and solve problems in the design process.	High	Significant	Moderate	Basic	Not even reaching marginal levels

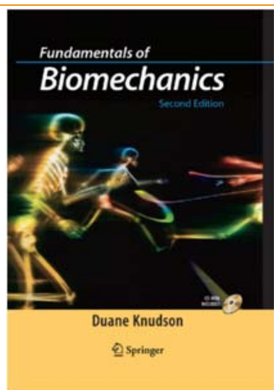
Keyword Syllabus

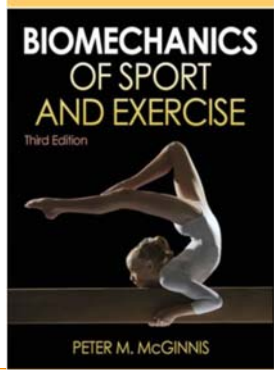
- Biomechanics, biomaterials, cells, tissues, organs, implants, human musculoskeletal system, biomedical devices, cell/surface interactions, endovascular system, drug delivery, dental implants, hip/knee implants, doctor and patients, ethical issues
- Solid mechanics, fluid mechanics, physical bodies, vector, force, displacement, moment, mechanical properties, Hooke's law, stress, strain, elasticity, plasticity, viscoelasticity, fracture, fatigue, wear, corrosion, toughening of materials, composites
- Problem identification and solving techniques, lab planning and control, reporting and presentation

In addition to the examination and in-class test, students are required to learn through collaborative lab sessions in order to improve their understanding on strategic thinking, problem solving, team working processes, the relationships and interactions between the fields of knowledge that they have learnt in this and other courses.

Reading List

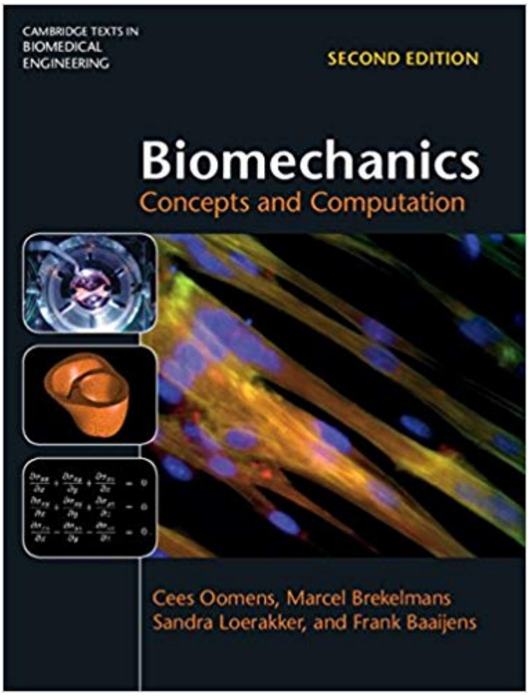
- Lecture notes and other teaching materials posted in on-line learning system.
- Fundamentals of Biomechanics, Duane Knudson, Springer, 2007 (2nd Edition)
- Biomechanics of Sport and Exercise, Peter M. McGinnis, Human Kinetics, 2013 (3rd Edition)





Reading List—Additional

- Biomechanics: Concepts and Computation (Cambridge Texts in Biomedical Engineering), Cees Oomens, Marcel Brekelmans and Frank Baaijens, Cambridge University Press, 2009

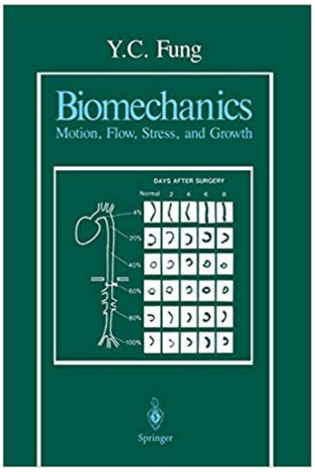


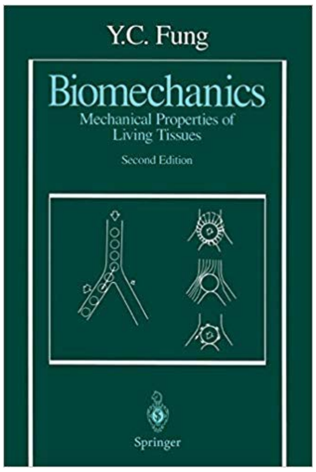
Reading List—Additional

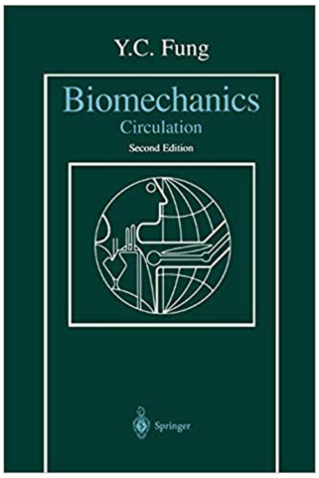
- Additional Readings
  - Introductory Biomechanics: from Cells to Organisms, C. Ross Ethier and Craig A. Simmons, Cambridge University Press, 2007
  - Biomechanics: Principles and Applications, D.R. Peterson and J.D. Bronzino, Editors, CRC Press, 2008
  - Biomaterials Science: An Introduction to Materials in Medicine, B.D. Ratner, A.S. Hoffman, F.J. Schoen and J.E. Lemons, Editors, Academic Press, 2004 (Second Edition)
  - Biomechanics in the Musculoskeletal System, M. Panjabi & A.A. White II, Philadelphia, PA, 2001
  - Basic Orthopedic Biomechanics, V.C. Mow and W.C. Hayes, Lippincott-Williams & Wilkins Press, 1997
  - An Introduction to Tissue-Biomaterials Interactions, K.C. Dee, D.A. Puleo and R. Bizios, Wiley-Liss, John Wiley & Sons, 2002

Reading List—Classical

- Biomechanics: Motion, Flow, Stress, and Growth, **Y.C. Fung**, Springer, 1990
- Biomechanics: Mechanical Properties of Living Tissues, **Y.C. Fung**, Springer, 1993 (2nd Edition)
- Biomechanics: Circulation, **Y.C. Fung**, Springer, 2010 (2nd Edition)







Yuan-Cheng “Bert” Fung (馮元楨)

- Sept. 15, 1919 (Changzhou, Jiangsu) – Dec. 15, 2019 (San Diego)
- an American bioengineer
- He is regarded as a founding figure of bioengineering, tissue engineering, and the “Founder of Modern Biomechanics”.

Contents

I. Introduction

What is biomechanics?

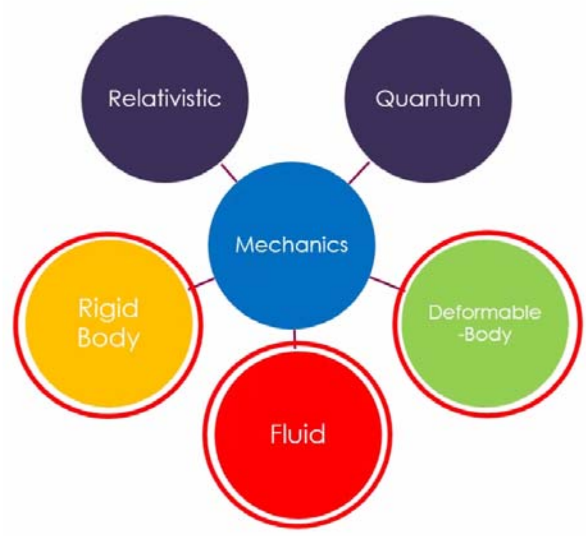
香港城市大學
City University of Hong Kong

- The study of the movement of living things using the science of mechanics.

What is biomechanics?

香港城市大學
City University of Hong Kong

- The study of the movement of living things using the science of mechanics.
- Mechanics?
  - Physics – Motion & Cause of Motion



What is biomechanics?

- The study of the movement of living things using the science of mechanics.
- Mechanics?
  - Physics – Motion & Cause of Motion
- Cause of Motion: Force
  - Forces act on body and body segments
  - Internal force & external force
- Consequences of Force
  - Movement/Deformation
  - Growth
  - Injury

What is biomechanics?

- Biomechanics:
  - Is the study of forces acting on the body & body segments; and the consequences of those forces.
- Two main areas of biomechanics
  - Performance improvement
    - ➢ Techniques
    - ➢ Training
    - ➢ Equipment
  - Injury prevention
- Goal in sports and exercise biomechanics

Analyzing Movement

- Fundamental measurement
  - Time (Duration)
  - Length
  - Mass
  - Weight
- International Systems of Units (SI)
  - Time: Seconds (s)
  - Length: Meters (m)
  - Mass: Kilograms (kg)
  - Weight: Newtons (N)

Time

- Duration $\Delta t = t_{\text{Final}} - t_{\text{Initial}}$
- Example
  - Initial (Starting) Time = 0.0 s
  - Final Time = 9.2 s
  - Duration $\Delta t = t_{\text{Final}} - t_{\text{Initial}}$ = 9.2 s - 0.0 s = 9.2 s

Length

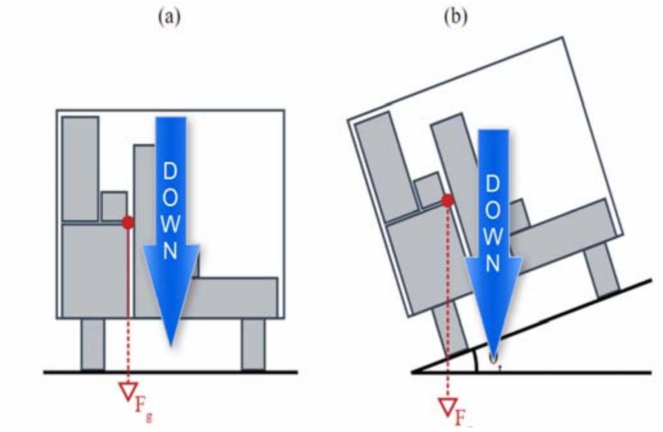
- Length of an object/body or change in length
- Change in position: Displacement
- Units
  - Meters (m)
  - Centimeters (cm) – 100 cm in 1 m
  - Kilometers (km) – 1000 m in 1 km
- Length = $L_{Final} - L_{Initial}$
- Displacement = $P_{Final} - P_{Initial}$
- Example
  - Initial (Starting) Position = 10.0 m
  - Final Position = 50.0 m
  - Displacement = $P_F - P_I$ = 50.0 m - 10.0 m = 40.0 m

Mass & Weight

- Mass: Quantity of matter of an object
- Mass would be the same on earth vs space
  - Unit: Kilograms (kg)
- Weight is the combination of mass and the pull of gravity
  - Weight is a force
  - Unit: Newtons (N)
- Conversions
  - 1 kg = 9.81 N

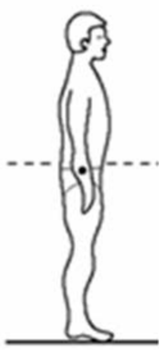
Center of Gravity (CoG)

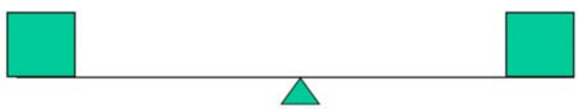
- Imaginary point where the mass / weight of an object / body is evenly distributed around
- Center of Mass (CoM)
  - CoM / CoG interchangeable on Earth
- Weight (Gravity) vector acts through the CoG



Center of Gravity (CoG)

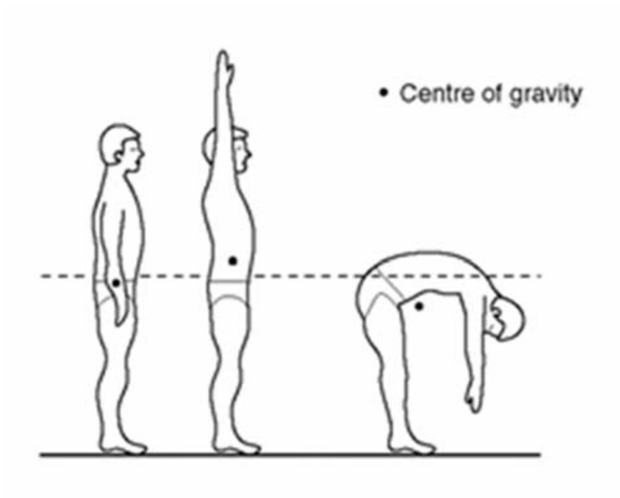
- Theoretical point at which all of the body's weight is considered to be concentrated
- Point about which a body will balance
- It is not necessarily the point about which there are equal amounts of weight. Rather, it is a "point" about which these weights are "balanced".
- CoG location is dependent on the weight and the distribution of this weight within the body.



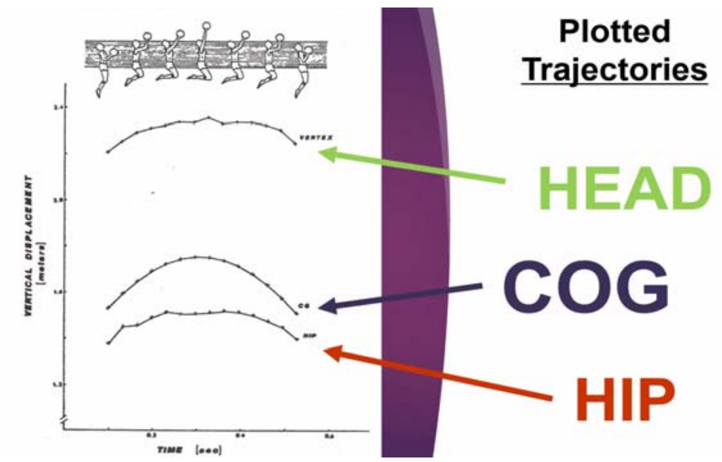


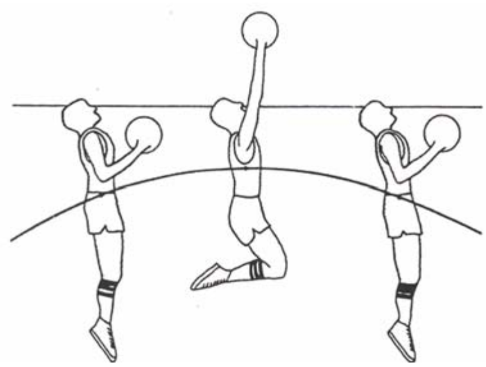
Center of Gravity (CoG)

- In the anatomical position, the CoG is near the waist.
  - Females: **53-56%** of standing height
  - Males: **54-57%** of standing height
- Is the CoG of the human body always in the same place?
- The CoG does NOT have to lie within the physical matter of the body.



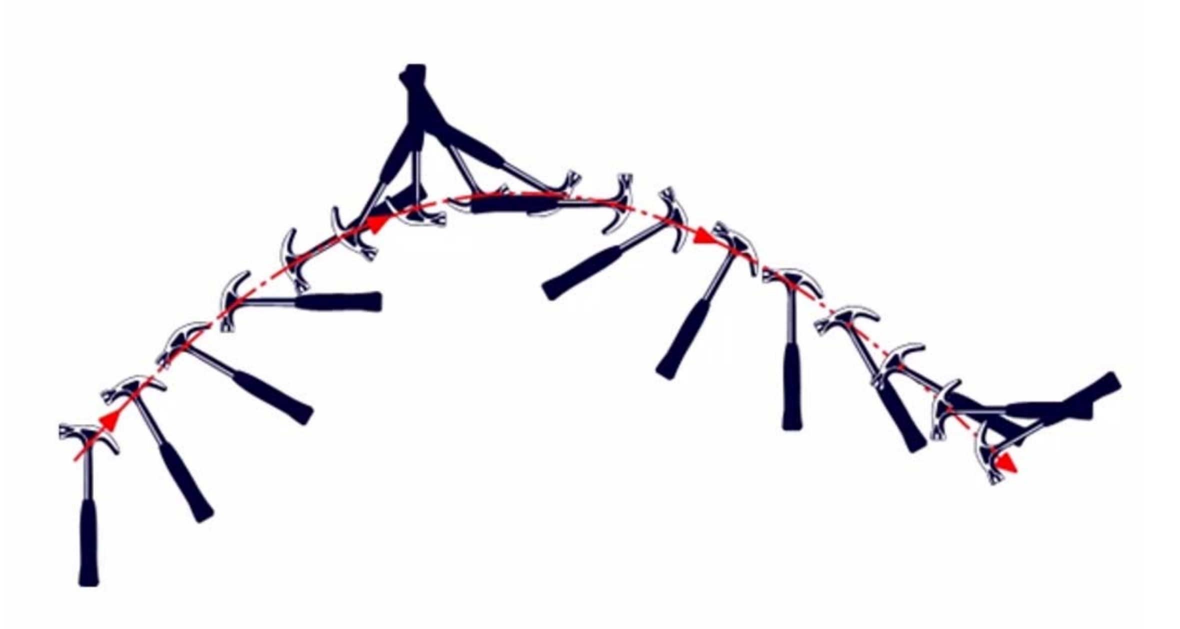
Floating or flying illusion





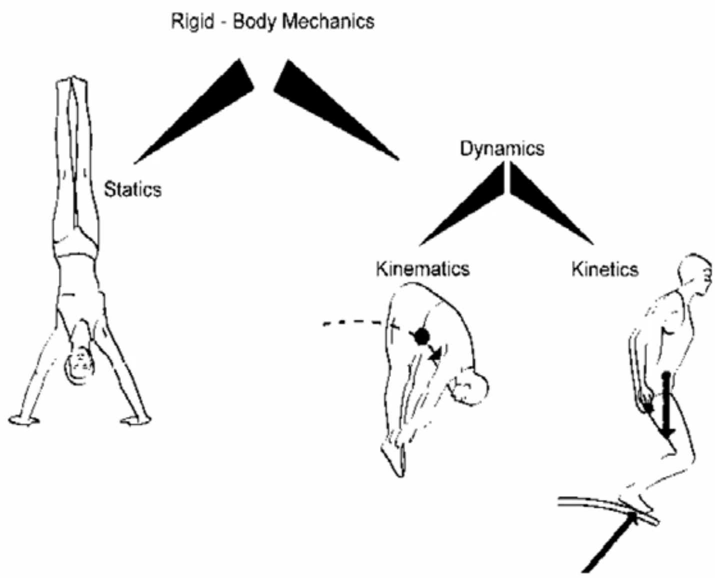
CoG is Axis of Rotation in Air

- A projectile's axis



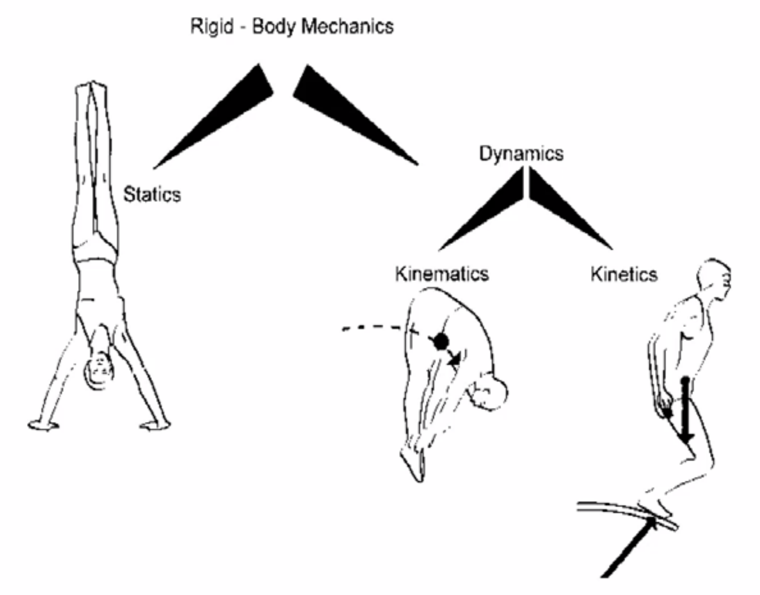
Rigid Body Mechanics

- Divided into Statics & Dynamics
- Statics
  - Study of objects at rest or in CONSTANT motion
- Dynamics
  - Study of objects being accelerated by forces



Two Branches of Dynamics

- Kinematics
  - Is the description of motion
  - Usually in linear or angular terms
  - Without Forces
- Kinetics
  - Determining/Examining the cause of motion
  - Cause of motion = Forces
  - With Forces



Movement Systems

- Fundamental ways we examine and study motion and the causes of motion
- Scalar Measures
  - Measurements that have magnitude but NOT direction
- Vector Measures
  - Measurements that have magnitude AND direction
- Key Terms
  - “Magnitude”—The SIZE of a value. Larger values have a greater Magnitude.
  - Direction: Positive & negative

Vector vs Scalar Measures

- Scalar Examples
  - Time
  - Temperature
  - Volume
  - Mass
  - Speed
- Vector Examples
  - Force
  - Velocity
  - Torque
  - Acceleration

Definition of a Vector

- A **vector** is a physical entity having both a magnitude (length or size) and a direction. For a vector $\vec{a}$ it holds, see Fig. 1.1:

$$\vec{a} = a\vec{e}.$$

- The **length** of the vector $\vec{a}$ is denoted by $|\vec{a}|$ and is equal to the length of the arrow. The length is equal to a, when a is positive, and equal to $-a$ when a is negative.
- The **direction** of $\vec{a}$ is given by the unit vector $\vec{e}$ combined with the sign of a. The unit vector $\vec{e}$ has length 1. The vector $\vec{0}$ has length zero.

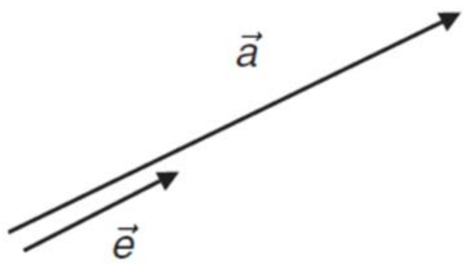


Figure 1.1

The vector $\vec{a} = a\vec{e}$ with $a > 0$.

Vector Operations: Multiplication

- **Multiplication** of a vector $\vec{a} = a\vec{e}$ by a positive scalar α yields a vector $\vec{b}$ having the same direction as $\vec{a}$ but a different magnitude $\alpha|\vec{a}|$:

$$\vec{b} = \alpha\vec{a} = \alpha a\vec{e}.$$

- This makes sense: pulling twice as hard on a wire creates a force in the wire having the same orientation (the direction of the wire does not change), but with a magnitude that is twice as large.

Vector Operations: Sum

- The **sum** of two vectors $\vec{a}$ and $\vec{b}$ is a new vector $\vec{c}$, equal to the diagonal of the parallelogram spanned by $\vec{a}$ and $\vec{b}$:

$$\vec{c} = \vec{a} + \vec{b}.$$

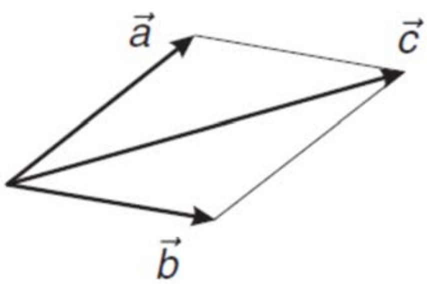


Figure 1.2

Graphical representation of the sum of two vectors: $\vec{c} = \vec{a} + \vec{b}$.

Vector Operations: Inner Product or Dot Product

- The **inner product** or **dot product** of two vectors is a scalar quantity, defined as

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos(\phi),$$

where ϕ is the smallest angle between $\vec{a}$ and $\vec{b}$.

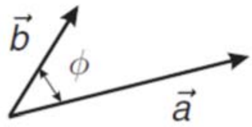


Figure 1.3

Definition of the angle ϕ.

- The inner product is **commutative**, i.e.

$$\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}.$$

Vector Operations: Inner Product or Dot Product

- The inner product can be used to define the length of a vector, since the inner product of a vector with itself yields (ϕ = 0):

$$\vec{a} \cdot \vec{a} = |\vec{a}||\vec{a}| \cos(0) = |\vec{a}|^2.$$

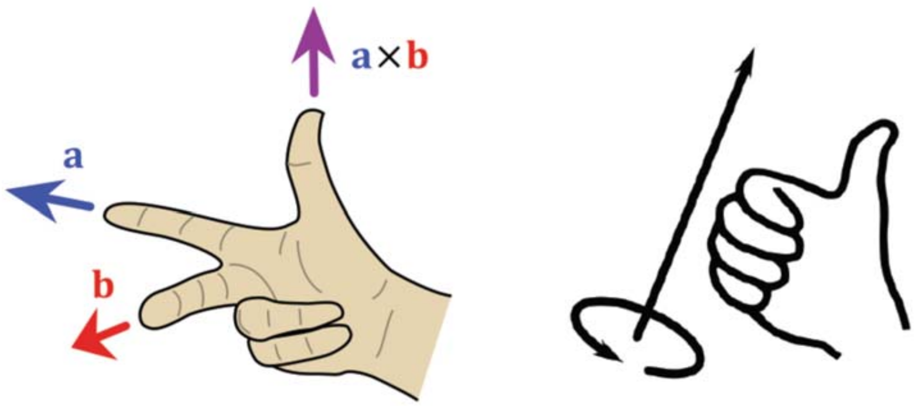
- If two vectors are perpendicular to each other the inner product of these two vectors is equal to zero, since in that case $\phi = \frac{\pi}{2}$:

$$\vec{a} \cdot \vec{b} = 0, \text{ if } \phi = \frac{\pi}{2}.$$

Vector Operations: Cross Product or Vector Product

- The **cross product** or **vector product** of two vectors $\vec{a}$ and $\vec{b}$ yields a new vector $\vec{c}$ that is perpendicular to both $\vec{a}$ and $\vec{b}$ such that $\vec{a}$, $\vec{b}$ and $\vec{c}$ form a right-handed system. The vector $\vec{c}$ is denoted as

$$\vec{c} = \vec{a} \times \vec{b} \,.$$



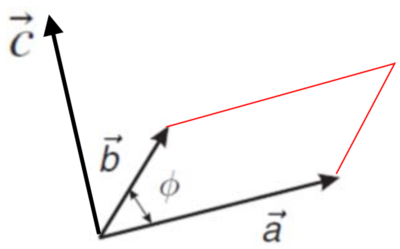
Vector Operations: Cross Product or Vector Product

$$\vec{c} = \vec{a} \times \vec{b} \,.$$

- The length of the vector $\vec{c}$ is given by

$$|\vec{c}| = |\vec{a}||\vec{b}| \sin(\phi) \,,$$

- where ϕ is the smallest angle between $\vec{a}$ and $\vec{b}$.



Area of the parallelogram

Vector Operations: Cross Product or Vector Product

- The vector product of a vector $\vec{a}$ with itself yields the zero vector since in that case ϕ = 0:

$$\vec{a} \times \vec{a} = \vec{0}.$$

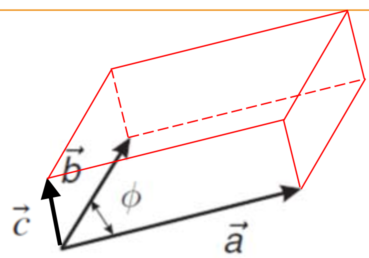
- The vector product is **not** commutative, since the vector product of $\vec{b}$ and $\vec{a}$ yields a vector that has the opposite direction of the vector product of $\vec{a}$ and $\vec{b}$:

$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}.$$

Triple Product of Three Vectors

- The triple product of $\vec{a}$, $\vec{b}$ and $\vec{c}$ is a scalar defined by

$$\vec{a} \times \vec{b} \cdot \vec{c} = (\vec{a} \times \vec{b}) \cdot \vec{c}$$



- If all three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are non-zero vectors, while the triple product is equal to zero then the vector $\vec{c}$ lies in the plane spanned by the vectors $\vec{a}$ and $\vec{b}$. This can be explained by the fact that the vector product of $\vec{a}$ and $\vec{b}$ yields a vector perpendicular to the plane spanned by $\vec{a}$ and $\vec{b}$. Reversely, this implies that if the triple product is nonzero then the three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are not in the same plane. In that case the absolute value of the triple product of the vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ equals the volume of the parallelepiped spanned by $\vec{a}$, $\vec{b}$ and $\vec{c}$.

Basis, Orthogonal Basis, and Cartesian basis

- In the three-dimensional space a set of three vectors $\vec{c}_1$, $\vec{c}_2$ and $\vec{c}_3$ is called a **basis** if the triple product of the three vectors is non-zero, hence if all three vectors are non-zero vectors and if they do not lie in the same plane:

$$\vec{c}_1 \times \vec{c}_2 \cdot \vec{c}_3 = 0.$$

- The three vectors $\vec{c}_1$, $\vec{c}_2$ and $\vec{c}_3$, composing the basis, are called basis vectors.
- If the basis vectors are mutually perpendicular vectors the basis is called an **orthogonal basis**. If such basis vectors have unit length, then the basis is called **orthonormal**.
- A **Cartesian basis** is an orthonormal, right-handed basis with basis vectors independent of the location in the three-dimensional space. In the following we will indicate the Cartesian basis vectors with $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$.

Decomposition of a Vector with respect to a Basis

- As stated above, a Cartesian vector basis is an orthonormal basis. Any vector can be decomposed into the sum of, at most, three vectors parallel to the three basis vectors $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$:

$$\vec{c} = a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z.$$

- The components a_x, a_y and a_z can be found by taking the inner product of the vector $\vec{a}$ with respect to each of the basis vectors:

$$a_x = \vec{a} \cdot \vec{e}_x$$
$$a_y = \vec{a} \cdot \vec{e}_y$$
$$a_z = \vec{a} \cdot \vec{e}_z$$

- where the basis vectors $\vec{e}_x$, $\vec{e}_y$ and $\vec{e}_z$ have unit length and are mutually orthogonal.

Decomposition of a Vector with respect to a Basis

- The components, say a_x, a_y and a_z, of a vector $\vec{a}$ with respect to the Cartesian vector basis, may be collected in a column, denoted by $\tilde{a}$:

$$\tilde{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

or

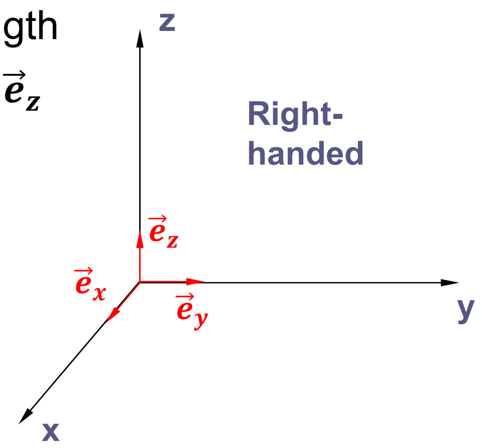
$$\underset{\sim}{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

So, with respect to a Cartesian vector basis any vector $\vec{a}$ may be decomposed in components that can be collected in a column:

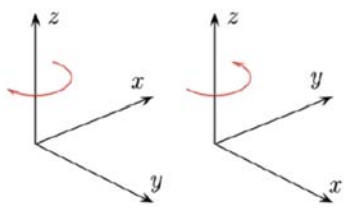
$$\vec{a} \longleftrightarrow \underset{\sim}{a} \,.$$

Decomposition of a Vector with respect to a Basis

- $|\vec{e}_x| = |\vec{e}_y| = |\vec{e}_z| = 1$ or $|\vec{e}_i| = 1$ for $i = x, y, z$
  - $\vec{e}_x \cdot \vec{e}_y = |\vec{e}_x||\vec{e}_y| \cos(90^\circ) = 0$ or $\vec{e}_i \cdot \vec{e}_j = 0$ for $i, j = x, y, z$ but $i \neq j$
  - $\vec{e}_x \cdot \vec{e}_x = |\vec{e}_x||\vec{e}_x| \cos(0^\circ) = 1$ or $\vec{e}_i \cdot \vec{e}_j = 1$ for $i, j = x, y, z$ but $i = j$
- $\vec{e}_x \cdot \vec{e}_y \times \vec{e}_z = \vec{e}_x \cdot (\vec{e}_y \times \vec{e}_z) = 1$
- $\vec{e}_x \times \vec{e}_y$ forms a vector with a unit length and pointing to the same direction of $\vec{e}_z$



Left-
handed



Right-
handed

Decomposition of a Vector with respect to a Basis

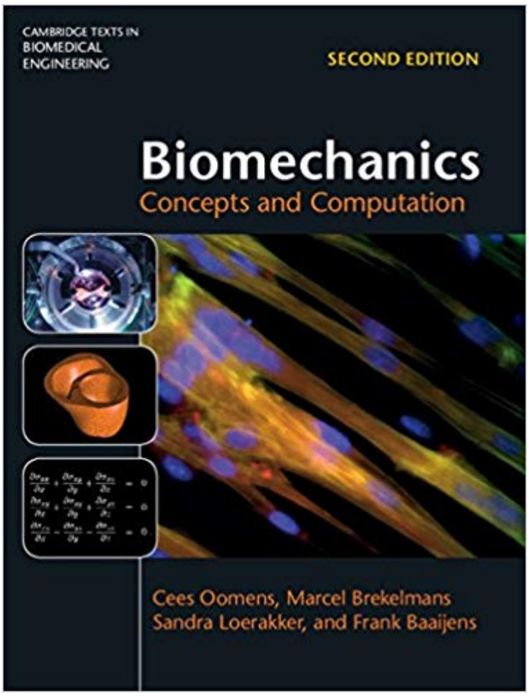
$$\vec{a}\cdot\vec{b} = (a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z)\cdot(b_x\vec{e}_x + b_y\vec{e}_y + b_z\vec{e}_z)$$
$$= a_x b_x + a_y b_y + a_z b_z.$$

$$\underset{\sim}{a}^{\mathrm{T}}\underset{\sim}{b} = [a_x\ a_y\ a_z]\begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = a_x b_x + a_y b_y + a_z b_z.$$

$$\vec{a}\times\vec{b} = (a_x\vec{e}_x + a_y\vec{e}_y + a_z\vec{e}_z)\times(b_x\vec{e}_x + b_y\vec{e}_y + b_z\vec{e}_z)$$
$$= (a_y b_z - a_z b_y)\vec{e}_x + (a_z b_x - a_x b_z)\vec{e}_y + (a_x b_y - a_y b_x)\vec{e}_z.$$
$$= \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

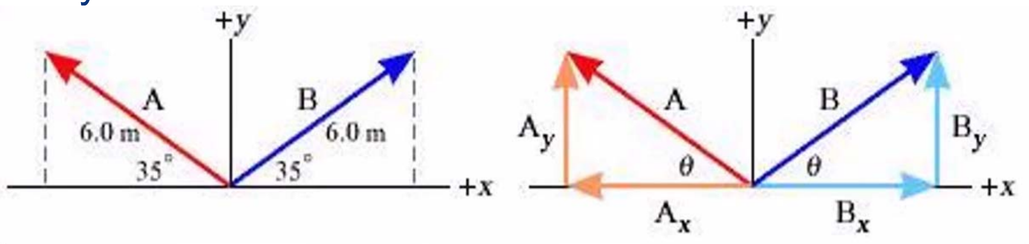
$$\begin{array}{lll} \vec{e}_x\times\vec{e}_x = \vec{0} & \vec{e}_y\times\vec{e}_x = -\vec{e}_z & \vec{e}_z\times\vec{e}_x = \vec{e}_y \\ \vec{e}_x\times\vec{e}_y = \vec{e}_z & \vec{e}_y\times\vec{e}_y = \vec{0} & \vec{e}_z\times\vec{e}_y = -\vec{e}_x \\ \vec{e}_x\times\vec{e}_z = -\vec{e}_y & \vec{e}_y\times\vec{e}_z = \vec{e}_x & \vec{e}_z\times\vec{e}_z = \vec{0}, \end{array}$$

Read more about vectors



Vector Resolution & Addition

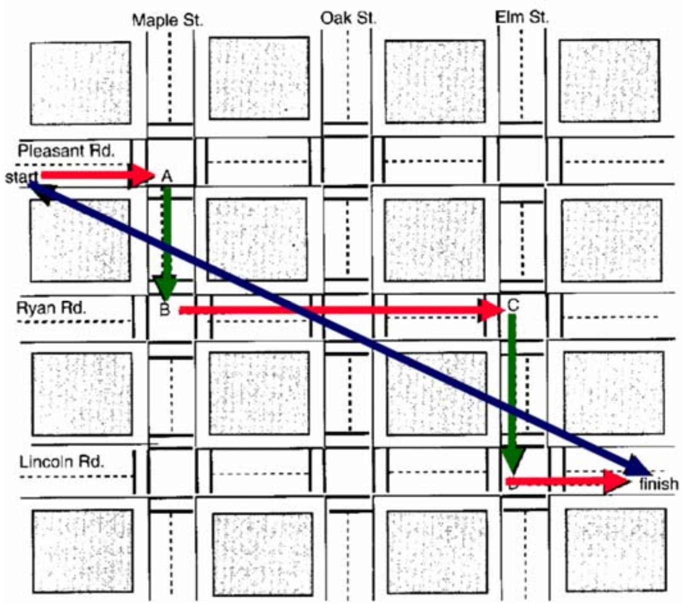
- We break vectors down to horizontal and vertical components to analyze



- Resultant—The resultant Vector is the combination of the horizontal vector and the vertical vector
  - A is the Resultant of A_x and A_y

Distance & Displacement

- Describing change in linear or angular position
- Distance (scalar): length of path
- Displacement (vector): difference between starting and finishing positions; independent of path
- Symbols
  - linear - *d* (m, cm, km)
  - angular – *θ* (degrees, radians, revolutions)



1 rev = 2π rad (6.28 rad) = 360°

1 rad = 57.3°

Speed & Velocity

- Describing the rate of change of linear or angular position with respect to time
- Speed or velocity: Rate at which a body moves from one position to another
  - Speed (scalar)
  - Velocity (vector)
- Linear (m/s, km/hr, ft/s, mph)

$$\overline{v} = \frac{\Delta d}{\Delta t}$$

- Angular (deg/s, rad/s, rpm)

$$\overline{\omega} = \frac{\Delta \theta}{\Delta t}$$