(1 point)

Which of the following correctly expresses the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^4}{n^5}$, as a definite integral?

- [ ] A. $\int_1^2 x^2\,dx$
- [ ] B. $\int_0^1 x^3\,dx$
- [ ] C. $\int_1^2 x^3\,dx$
- [x] D. $\int_0^1 x^4\,dx$
- [ ] E. $\int_0^1 x^2\,dx$
- [ ] F. $\int_1^2 x^4\,dx$

(1 point) For this problem, you will need to use the Desmos Riemann Sum Calculator. (This link opens a new tab/window.)

Initially, the calculator shows a left Riemann sum with $n = 5$ subintervals for the function $f(x) = 2x + 1$ on the interval $[1, 4]$. Use the applet to compute the following sums for this function on this interval.

$L_5 =$ 81/5 , $M_5 =$ 18 , $R_5 =$ 99/5

$L_{25} =$ 441/25 , $M_{25} =$ 18 , $R_{25} =$ 459/25

$L_{100} =$ 1791/100 , $M_{100} =$ 18 , $R_{100} =$ 1809/100

Now use basic geometry to determine the exact area bounded by $f(x) = 2x + 1$ and the x-axis on the interval $[1, 4]$.

Exact Area = 18

Make a note of any patterns you observe.

(1 point)

Consider the integral $\int_2^6 \frac{x}{1+x^5}\,dx$. Which of the following expressions represents the integral as a limit of Riemann sums?

- [ ] A. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \frac{2+\frac{4i}{n}}{1+\left(2+\frac{4i}{n}\right)}$
- [x] B. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{4}{n} \frac{2+\frac{4i}{n}}{1+\left(2+\frac{4i}{n}\right)^5}$
- [ ] C. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2+\frac{4i}{n}}{1+\left(2+\frac{4i}{n}\right)^5}$
- [ ] D. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2+\frac{6i}{n}}{1+\left(2+\frac{6i}{n}\right)^5}$
- [ ] E. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n} \frac{2+\frac{6i}{n}}{1+\left(2+\frac{6i}{n}\right)^5}$
- [ ] F. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{6}{n} \frac{2+\frac{6i}{n}}{1+\left(2+\frac{6i}{n}\right)}$

(1 point)

The limit

$$\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{2x_i^* + (x_i^*)^2}\,\Delta x$$

can be expressed as a definite integral on the interval $[1, 8]$ of the form

$$\int_a^b f(x)\,dx$$

Determine a, b, and $f(x)$.

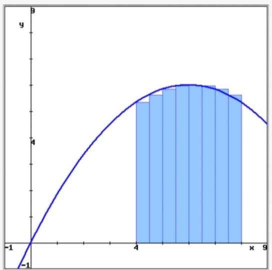
$a =$ 1

$b =$ 8

$f(x) =$ sqrt(2x+x^2)

(1 point) The rectangles in the graph below illustrate a left endpoint Riemann sum for $f(x) = \frac{-x^2}{6} + 2x$ on the interval $[4, 8]$.

The value of this left endpoint Riemann sum is 277/12 , and it is there is ambiguity the area of the region enclosed by $y = f(x)$, the x-axis, and the vertical lines x = 4 and x = 8.



Left endpoint Riemann sum for $y = \frac{-x^2}{6} + 2x$ on $[4, 8]$

Using left and right Riemann sums based on the diagrams above, we definitively conclude that

273/24 $\leq \int_4^6 \frac{-x^2}{6} + 2x\,dx \leq$ 281/24

273/24 $\leq \int_6^8 \frac{-x^2}{6} + 2x\,dx \leq$ 281/24

Using left and right Riemann sums based on the diagrams above, we definitively conclude that

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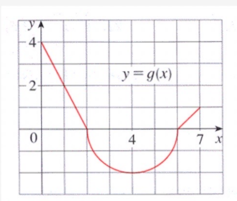
273/24 $\leq \int_6^8 \frac{-x^2}{6} + 2x\,dx \leq$ 281/24

273/12 $\leq \int_4^8 \frac{-x^2}{6} + 2x\,dx \leq$ 281/12

Hint: For the last integral, you should consistently choose either to underestimate or overestimate the area. This may require that you use the left Riemann sum for some x-intervals and the right Riemann sum for other x-intervals.

(1 point)

Consider the graph of the function $g(x)$:



The graph from $x = 2$ to $x = 6$ is a semicircle. Evaluate the following integrals by interpreting them in terms of areas:

(a) $\int_0^2 g(x)\,dx =$ 4

(b) $\int_2^6 g(x)\,dx =$ -2pi

(c) $\int_0^7 g(x)\,dx =$ 4.5-2pi