Copyright

I. Nodal Voltage Analysis (NVA)

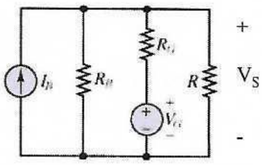
KCL & Ohm's law in action

- The method of nodal voltage analysis is an application of KCL and Ohm's law together
- The unknown variables that you will solve for are **node voltages**
- We will apply KCL at a mode and express the unknown currents as unknown node voltages

Example on how to do NVA

Find the voltage across the current source, V_S:

Given: $I_B = 12A$, $V_G = 12V$, $R_G = 0.3\Omega$, $R_B = 1\Omega$, $R = 0.23\Omega$



Apply KCL at V_S:

$$I_B = \frac{V_S}{R_B} + \frac{V_S - V_G}{R_G} + \frac{V_S}{R}$$

$$12 = \frac{V_S}{1} + \frac{V_S - 12}{0.3} + \frac{V_S}{0.23}$$

$$V_S = 5.99V$$

- Note that the negative terminal of V_G is used as a reference (0V) in the example
- The value of V_S is also referenced to this node
- The value of the reference node is not important since we are only interested in the voltage difference

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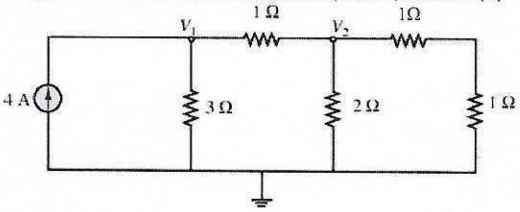
Worked Example on NVA 1

Use nodal voltage analysis to find V_1 and V_2. (Answer: $V_1 = 4.8V$, $V_2 = 2.4V$)



First method:

-Apply KCL at V_1 (1)

-Apply KCL at V_2 (2)

-Solve equations (1) and (2)

Alternate method:

-Combine all the resistors to find V_1 first

-Then combine the 3 resistors on the right followed by voltage divider rule to find V_2

At node V_1: $4 = \frac{V_1}{3} + \frac{V_1 - V_2}{1}$ —(1)

At node V_2: $\frac{V_1 - V_2}{1} = \frac{V_2}{2} + \frac{V_2}{1+1}$ (2)

Solve (1) & (2): $V_1 = 4.8V$; $V_2 = 2.4V$ #

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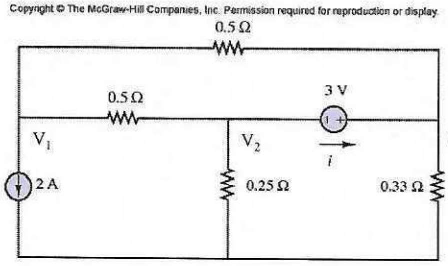
Worked Example on NVA 2

Use nodal voltage analysis to find the current through the 3V source.

(Answer: i = 8.31 A)



Apply KCL at V_1:

$$2 = \frac{V_2 - V_1}{0.5} + \frac{V_3 - V_1}{0.5}$$

$$V_2 + V_3 - 2V_1 = 1 \quad (1)$$

Apply KCL at V_2:

$$\frac{V_1 - V_2}{0.5} = \frac{V_2}{0.25} + i$$

$$2V_1 - 6V_2 = i \quad (2)$$

Apply KCL at V_3:

$$\frac{V_3}{0.33} + \frac{V_3 - V_1}{0.5} = i$$

$$\frac{166}{33}V_3 - 2V_1 = i \quad (3)$$

Since a new variable is introduced in (2), we need one more equation.

$$V_3 - V_2 = 3 \quad (4)$$

II. Mesh Current Analysis (MCA)

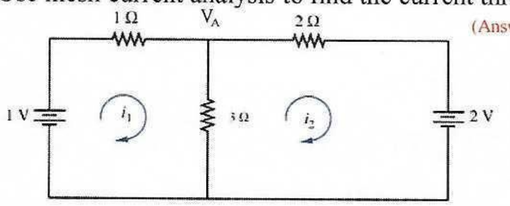
KVL & Ohm's law in action

- The method of mesh current analysis is an application of KVL and Ohm's law together
- The unknown variables that you will solve for are **mesh currents**
- From KVL, the sum of voltage drops and rise must equal zero
- The voltage differences are expressed as the current going through each branch in the mesh

Example on how to do MCA

Use mesh current analysis to find the current through the 3Ω resistor.

(Answer: $i_{3\Omega} = 4/11$A)



-There are 2 meshes and hence 2 mesh currents

-Apply KVL to each of these 2 meshes

Apply KVL to mesh 1:

$$1 = i_1(1+3) - 3i_2$$

$$4i_1 - 3i_2 = 1 \qquad (1)$$

Useful tip:

- keep voltages of sources on one side of the equation, and keep voltages of resistors on the other side

Apply KVL to mesh 2:

Follow the defined direction of the mesh current

$$-2 = i_2(2+3) - 3i_1$$

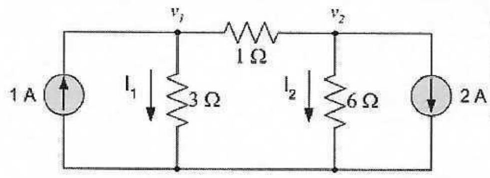
$$5i_2 - 3i_1 = -2 \qquad (2)$$

Now, solve equation (1) and (2):

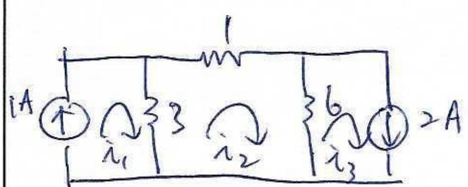
$i_{3\Omega} = i_1 - i_2$

Worked Example on MCA 1

Use mesh current analysis to find currents I_1 and I_2. (Answer: $I_1 = I_2 = -0.5$A)



-You see 3 meshes, but only the middle mesh is unknown, the other 2 are defined by the current sources.



$i_1 = 1A$; $i_3 = 2A$

For the middle mesh

$-(i_2 - i_1) \times 3 - i_2 - 6(i_2 - i_3) = 0$

$\because i_1 = 1,\ i_3 = 2$

$\Rightarrow -10 i_2 + 15 = 0$

$\Rightarrow \quad i_2 = 1.5$

$I_1 = i_1 - i_2 = 1 - 1.5 = -0.5\ A$

$I_2 = i_2 - i_3 = 1.5 - 2 = -0.5\ A$

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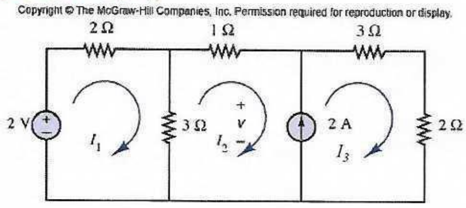
Worked Example on MCA 2

Use mesh current analysis to find the voltage across the current source.

(Answer: V = 3.89 V)



-In the process of deriving equations (2) and (3), a new variable V is introduced.

-Hence, we need one more equation, (4)

Apply KVL at mesh 1:
$2 = I_1(2+3) - I_2(3)$
⇨ $5I_1 - 3I_2 = 2$ (1)

Apply KVL at mesh 2:
$-V = I_2(3+1) - I_1(3)$
⇨ $4I_2 - 3I_1 = -V$ (2)

Apply KVL at mesh 3:
$V = I_3(3+2)$
⇨ $5I_3 = V$ (3)

One more equation:

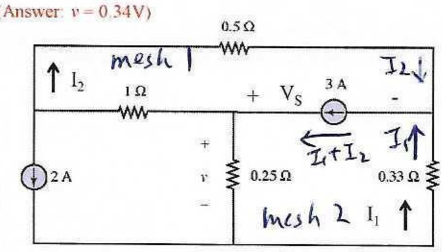
$I_3 - I_2 = 2A$ (4)

Solve for V, verify by V = (I_3)(3+2)

Worked Example on MCA 3

Use mesh current analysis, find the voltage across the 0.25 Ω resistor.

(Answer: v = 0.34V)



Method:

-Find the current through 0.25Ω resistor first, then use this to find the voltage.

-Although there are 3 meshes, the bottom left mesh is already known (2A going anti-clockwise).

Apply KVL at mesh 1:
$Vs = I_2(0.5 + 1) + (2)(1)$
⇨ $Vs = 1.5I_2 + 2$ (1)

Apply KVL at mesh 2:
$Vs = I_1(0.25 + 0.33) - (2)(0.25)$
⇨ $Vs = 0.58I_1 - 0.5$ (2)

Consider the current source:
$I_1 + I_2 = 3$ (3)

Solve for I_1 and I_2

Current through 0.25 Ω resistor: $I_1 - 2$

$V = (I_1 - 2)(0.25)$

Quick note on choosing NVA and MCA

How do you choose between NVA and MCA?

-Your choice should not be due to level of familiarity between the methods.

- One consideration is whichever is simpler to use for a given circuit.

- How then do you decide what is simpler?

- For a start, having fewer equations certainly makes solving easier.

III. Superposition

The method

- This method applies only to circuits that have **multiple sources**
- In such case, it can come in handy (or not)
- In a circuit with multiple sources, superposition considers the current or voltage associated with a given branch for one of the sources, while turning the rest off

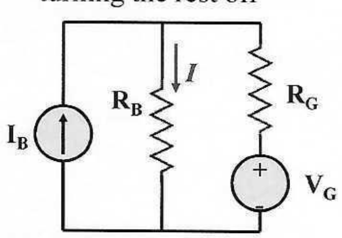


Fig 1: Basic circuit with 2 sources

Aim: Find the current through R_B.

1. **Find the current (I_1) through R_B when only I_B is present (V_G is removed)**
2. **Find the current (I_2) through R_B when only V_G is present (I_B is removed)**
3. **The net current $I = I_1 + I_2$**

How do we "remove" a current source or voltage source?

Killing sources

Voltage source: If the voltage source does not exist, the voltage across the terminals would be zero. It looks like a short circuit. (see Fig 2a)

Current source: If the current source does not exist, the current through it would be zero. It looks like an open circuit. (see Fig 2b)

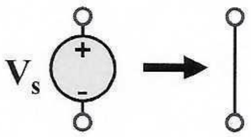


Fig 2a: Disable a voltage source by Replacing with a short circuit.

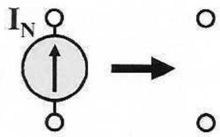


Fig 2b: Disable a current source by Replacing with an open circuit.

Apply the tips to Fig 1

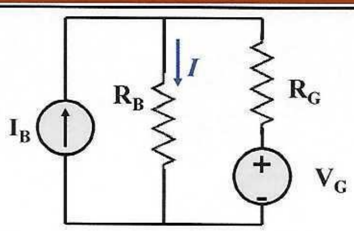
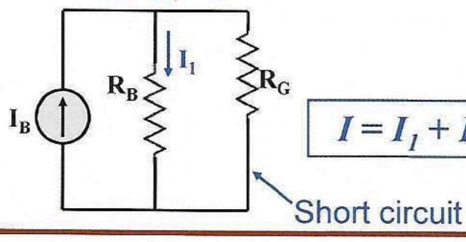


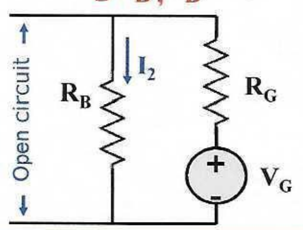
Fig 1: Basic circuit with 2 sources

Killing V_{G}, $V_G = 0$



$$\boxed{I = I_1 + I_2}$$

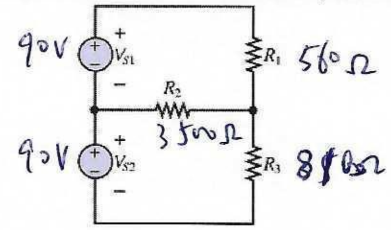
Killing I_B, $I_B = 0$



Worked Example on Superposition

Determine the current through R_1 using superposition.

Given $R_1 = 560\Omega$, $R_2 = 3.5k\Omega$, $R_3 = 810\Omega$, $V_{S2} = V_{S1} = 90V$



a. First, short V_{S1}
Voltage across R_1:

$$V_{R1a} = \frac{R_1 \parallel R_2}{(R_1 \parallel R_2) + R_3} V_{S2}$$

$$= \left(\frac{560 \parallel 3500}{(560 \parallel 3500) + 810} \right) 90$$

$$= 33.61V$$

Current through R_1:

$$I_{R1a} = V_{R1a} / R_{1a}$$
$$= 33.61 / 560$$
$$= 0.06A$$

Worked Example on Superposition

b. Next, short V_{S2}

Current through R_1:

$$I_{R1b} = \frac{V_{S1}}{R_1 + (R_2 \parallel R_3)} = \frac{90}{560 + (3500 \parallel 810)}$$
$$= 0.074A$$

Take careful note of the directions defined for each of these currents.

Finally,

$I_{R1} = I_{R1a} + I_{R1b} = 0.134A$

One-Port Network

Introduction

- A **one port network** is simply a **two terminal device** (See Fig below)

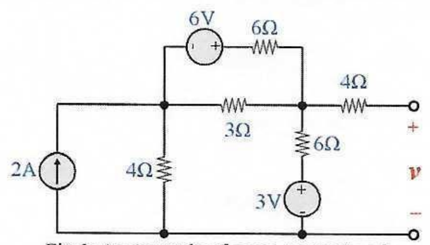


Fig 1: An example of a one port network

- Given a number of different resistor loads, if you were asked to find the current and voltage at the terminals for each resistor, you would have to recalculate the whole circuit
- A different load will give you different output current & voltage

One-Port Network

Simpler analysis

- Transform any 2 terminal circuit into a circuit that is as simple as having 1 resistor and 1 source (see Fig 2)

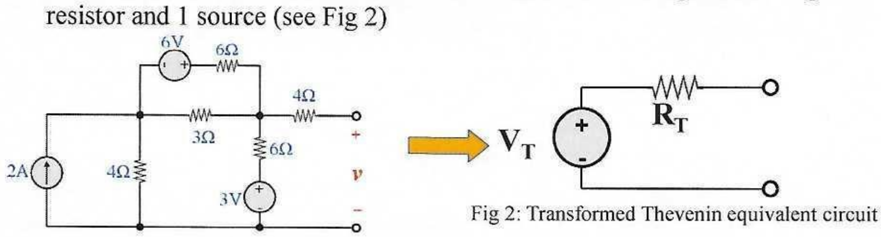


Fig 2: Transformed Thevenin equivalent circuit

The first part of this part is organized into the following 3 sections:

- Thevenin equivalent
- Norton equivalent
- Source transformation

I. Thevenin

Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **voltage source** V_T in **series** with an equivalent **resistance** R_T

Deriving the Thevenin equivalent circuit

Step 1: Remove the load from the rest of the one port network. This is the first most fundamental step.

Step 2: Find the equivalent resistance R_T. Kill all ideal sources then find the resistance across the terminals.

For voltage source – replace with short circuit

For current source – replace with open circuit

Step 3: Find the Thevenin voltage source. The Thevenin voltage source is equal to the open circuit voltage seen across the terminals (with no load). Solve for this voltage using any preferred method (NVA, MCA, superposition).

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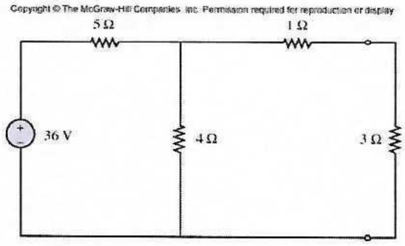
Worked Example on Thevenin 1

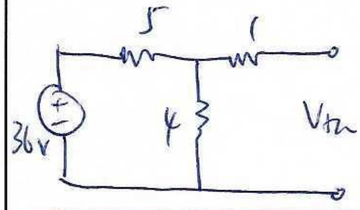
Derive the Thevenin equivalent circuit seen by the 3Ω load.

Answer: $R_{th} = 3.22\ \Omega$, $V_{th} = 16$ V



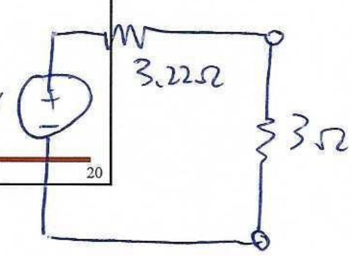
Steps:

1. Remove the 3Ω load
2. Find R_T (disable the voltage source)
3. Find V_T (with no load)
4. Draw the Thevenin circuit



$$V_{th} = \frac{4}{5+4} \times 36 = 16\ V$$

Thevenin Circuit



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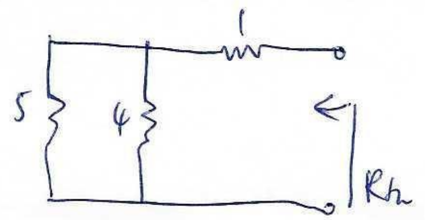


$$R_{th} = 1 + (5 \parallel 4)$$

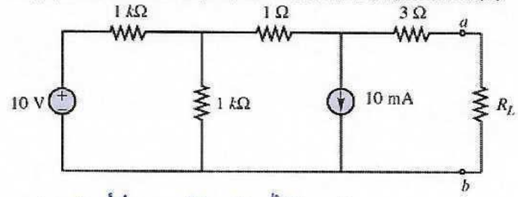
$$= 1 + \frac{5 \times 4}{5+4}$$

$$\Rightarrow R_{th} = 3.22\ \Omega$$

Worked Example on Thevenin 2

Derive the Thevenin equivalent circuit as seen by the resistor R_L.

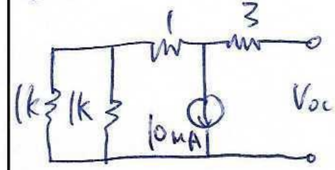
Answer: $R_{th} = 504\ \Omega$, $V_{th} = -10$ mV



Steps:

1. Remove the load
2. Find R_T (kill the voltage and current sources)
3. Find V_T (with no load)
4. Draw the Thevenin circuit

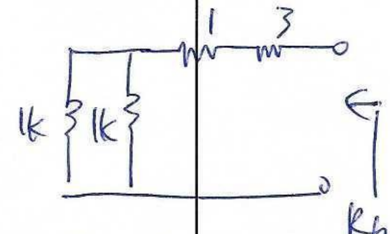
① Kill the voltage source



$V_{oc1} = -[(1k \parallel 1k) + 1] \times 10mA$

$= -501 \times 10 \times 10^{-3}$

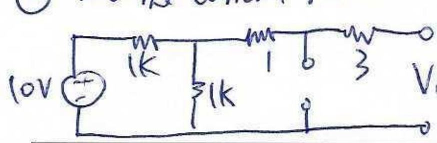
$= -5.01\ V$



$R_{th} = 3 + 1 + (1k \parallel 1k) = 504\ \Omega$ #

② kill the current source

No current in 1Ω & 3Ω



$V_{oc2} = \frac{1}{1+1} \times 10 = 5V$

$V_{th} = V_{oc1} + V_{oc2}$

$= -5.01 + 5$

$= -0.01\ V$

$V_{th} = -10\ mV$ #

II. Norton

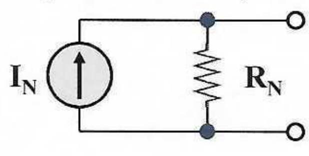
Definition

- Any one-port network composed of **ideal** voltage and current sources, and **linear** resistors, can be represented by an equivalent circuit consisting of an ideal **current source** I_N in **parallel** with an equivalent **resistance** R_N

Deriving the Norton equivalent circuit

Steps 1 and 2 for deriving the Norton equivalent circuit are exactly the same as that for Thevenin.

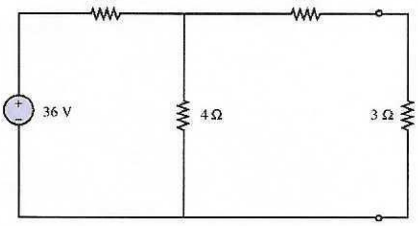
Step 3: Find the Norton current source. The Norton current source is equal to the short circuit current seen across the terminals (with no load). Solve for this ~~voltage~~ current using any preferred method (NVA, MCA, superposition).



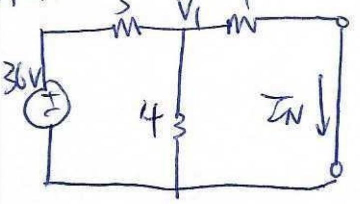
Worked Example on Norton 1

Derive the Norton equivalent circuit seen by the 3Ω load.

Answer: $R_N = 3.22\ \Omega$, $I_N = 4.96$ A



Norton's source :



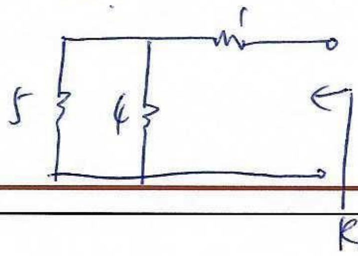
$$V_1 = \frac{4/\!/1}{(4/\!/1)+5} \times 36$$

$$= \frac{0.8}{5.8} \times 36$$

$\Rightarrow V_1 = 4.96$V

$$I_N = \frac{V_1}{1} = 4.96A$$

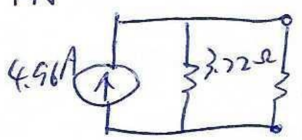
Norton resistance :



$R_N = 1 + (5/\!/4)$

$= 1 + 2.22$

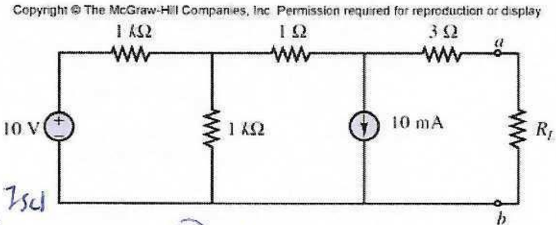
$\Rightarrow R_N = 3.22\ \Omega$ #

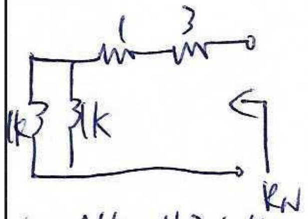


Worked Example on Norton 2

Derive the Norton equivalent circuit seen by the load R_L.

Answer: $R_N = 504\ \Omega$, $I_N = -19.8\ \mu$A



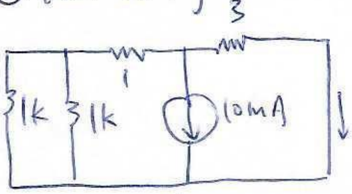


$R_N = (1k /\!/ 1k) + 1 + 3$

$= 500 + 4$

$= 504\ \Omega$ #

① kill voltage source



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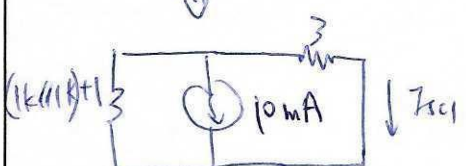
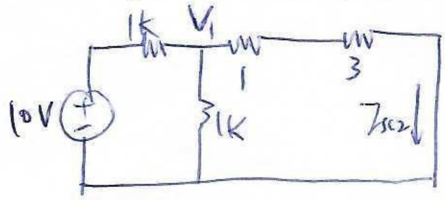


② kill current source



$$I_{SC2} = \frac{V_1}{4}$$

$$= \frac{0.0396825}{4} (A)$$

$= 9.92063$ mA

$I_{SC} = I_{SC1} + I_{SC2}$

$= (-9.94048 + 9.92063)$ mA

$= -19.85\ \mu$A

$$I_{SC1} = \frac{(1k /\!/ 1k)+1}{(1k /\!/ (1k+1))+3} \times (-10)$$

$$= \frac{-501}{504} \times 10$$

$= -9.94048$ mA

$$V_1 = \frac{1k /\!/ 4}{(1k /\!/ 4)+1k} \times 10$$

$$1k /\!/ 4 = \frac{1000 \times 4}{1000+4} = 3.984064$$

$$\therefore V_1 = \frac{3.984064}{3.984064+1000} \times 10 = 0.0396825V$$

III. Source Transformation

Definition

- This trick rests on the fact that the Thevenin and Norton forms are equivalent to each other and therefore are also interchangeable

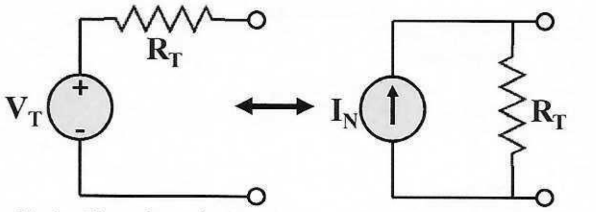


Fig 4a: Thevenin equivalent circuit   Fig 4b: Norton equivalent circuit

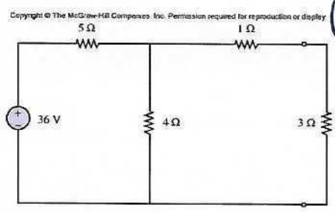
We can transform between the two equivalent circuits, observing each time that:

$V_T = I_N R_T$

- Rather than transform the whole circuit in one go to Thevenin or Norton equivalent forms, we can instead transform part of the circuit
- The process of merge transform and merge again can be repeated

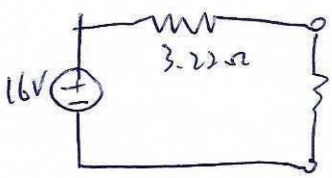
Worked Example on Source Transformation 1

Derive either of the equivalent circuit forms seen by the 3Ω load.



(1) From Worked Example on Thevenin 1 (p. 20)

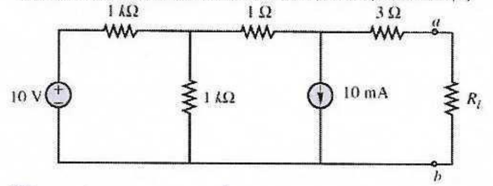
The Thevenin Circuit is



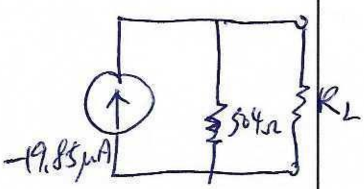
(2) From Worked Example on Norton 1, the Norton Circuit is

Worked Example on Source Transformation 2

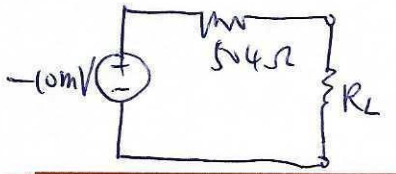
Derive either of the equivalent circuit forms seen by the load R_L.



② From Worked Example on Norton 2



① From Worked Example on Thevenin 2 (P. 21)



III. Dependent Sources

- All the sources we have come across are independent source
  - > For a voltage source, the voltage maintained across the source is fixed
  - > For a current source, the current through the source is fixed
- There also exist dependent sources in circuit theory
- Unlike independent sources, the value of a dependent source is not predetermined, but is set by the current or voltage through a specific branch

Symbol

- The symbol for an independent source is a circle
- For a dependent source, the symbol is a diamond

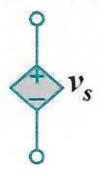
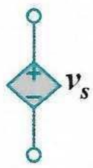


Fig 5a: Dependent voltage source

Fig 5b: Dependent current source

Dependent Sources

Dependent Voltage Source



v_S is the voltage source across the terminals.

Current Controlled Voltage Source

For example, $v_S = 5i_X$. The current i_X is not the current through the source but belongs to another branch.

Voltage Controlled Voltage Source

For example, $v_S = 7v_X$. The voltage v_X is not the source voltage. As v_X changes, so also v_S according to the above relations.

Dependent Current Source

i_S is the current source through the terminals.

Current Controlled Current Source

For example, $i_S = 5i_X$. The current i_X is not the current through the source but belongs to another branch.

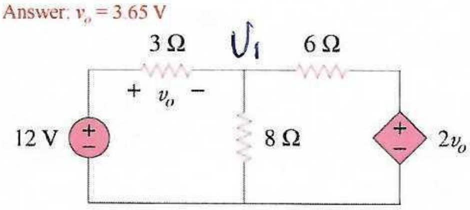
Voltage Controlled Current Source

For example, $i_S = 7v_X$. The voltage v_X is not the source voltage. As v_X changes, so also i_S according to the above relations.

Worked Example on Dependent Source 1

Calculate v_o in the circuit.

Answer: $v_o = 3.65$ V



Hints:

1. The dependent source here is a voltage controlled voltage source
2. Apply KCL

Apply KCL :

$$\frac{V_1 - 12}{3} + \frac{V_1}{8} + \frac{V_1 - 2V_0}{6} = 0 \quad -①$$

But at 3-Ω resistor $V_o = 12 - V_1$ — ②

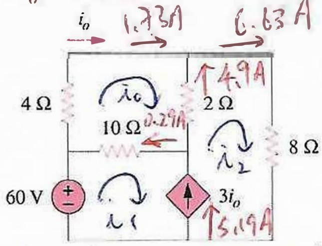
Two equations for two unknowns (V_1, V_o)

$\Rightarrow$ $V_1 = 8.3478$ V & $V_o = 12 - V_1 = 3.65$ V #

Worked Example on Dependent Source 2

Calculate i_o in the circuit.

Answer: i_o = 1.73 A



Hints:

1. The dependent source here is a current controlled current source
2. Apply KVL

① Consider loop 0 : $-4i_0 - 2(i_0 - i_2) - 10(i_0 - i_1) = 0$

$\Rightarrow -8i_0 + 5i_1 + i_2 = 0$ —①

② Consider (loop 1 + loop 2) and use KVL :

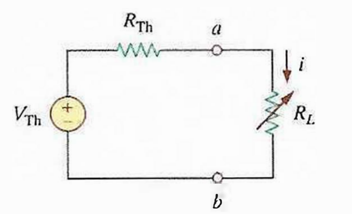
$60 - 10(i_1 - i_0) - 2(i_2 - i_0) - 8i_2 = 0$

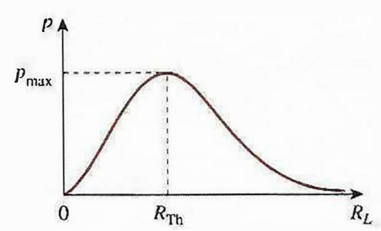
$\Rightarrow 6i_0 - 5i_1 - 5i_2 = -30$ —②

③ At the current source : $i_2 - i_1 = 3i_0$ —③

solving ①, ②, ③ : $i_0 = 1.73A$; $i_1 = 1.44A$; $i_2 = 6.63A$

Maximum Power Transfer





$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

$$\frac{dp}{dR_L} = 0 \rightarrow R_L = R_{Th}$$

$$p_{max} = \frac{V_{Th}^2}{4R_{Th}}$$