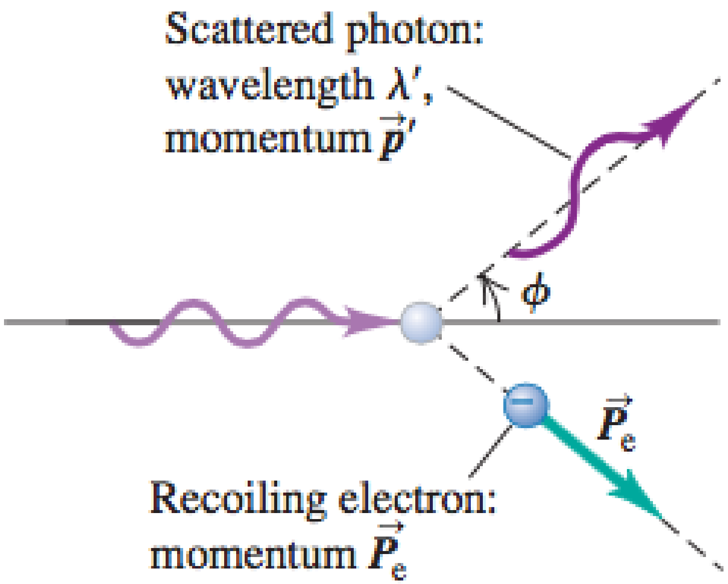
PHY1203: General Physics III

Chapter 38
Photons: Light Waves Behaving as Particles – *Part 2*

Light scattered as photons: Compton scattering and pair production

- The final aspect of light that we must test against the photon model is its behavior after the light is produced and before it is eventually absorbed.
- We can do this by considering the scattering of light.
- Scattering is what happens when light bounces off particles such as molecules in the air or electrons.



Predictions

Classical E&M:

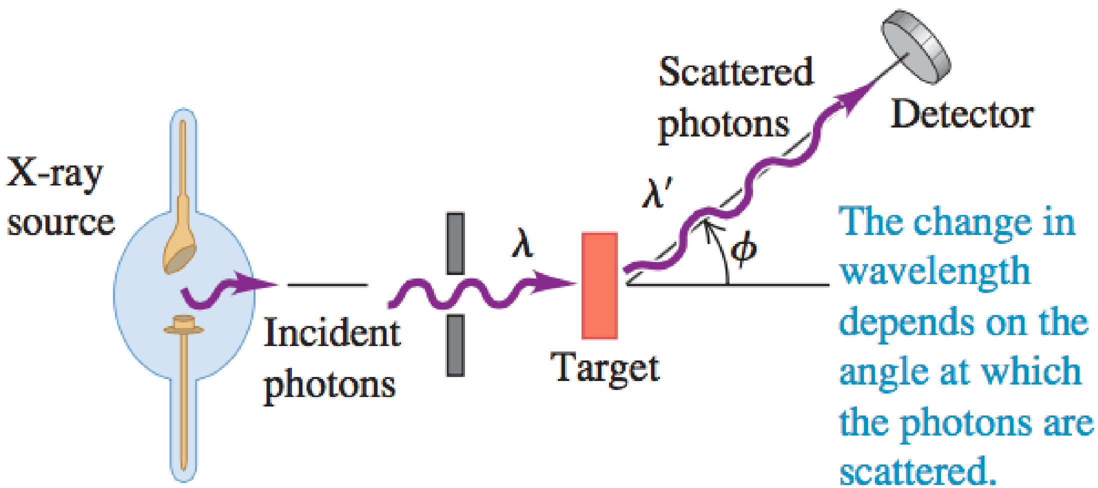
- Scattering is a process of absorption and re-radiation.
- Part of the energy of the light wave would be absorbed by the electrons, which oscillate in response to the oscillating electric field of the wave. Then they re-radiate the energy as scattered waves in a variety of directions.
- The scattered light and incident light have the **same frequency** and **same wavelength**.

Photon model:

- Scattering process is a collision of two particles.
- The incident photon would give up part of its energy and momentum to the electron, which recoils as a result. The scattered photon that remains can fly off at a variety of angles, but it has less energy and less momentum than the incident photon
- The scattered light has a **lower frequency** and **longer wavelength** than the incident light.

Compton's experiment

- In 1922, American physicist Compton carried out the scattering experiment which tests these predictions.
- In the Compton experiment, x rays are scattered from electrons. The wavelength of the scattered x rays are measured.



Arthur H. Compton
(1892-1962)

Nobel Prize in Physics 1927
(Prize share: 1/2)

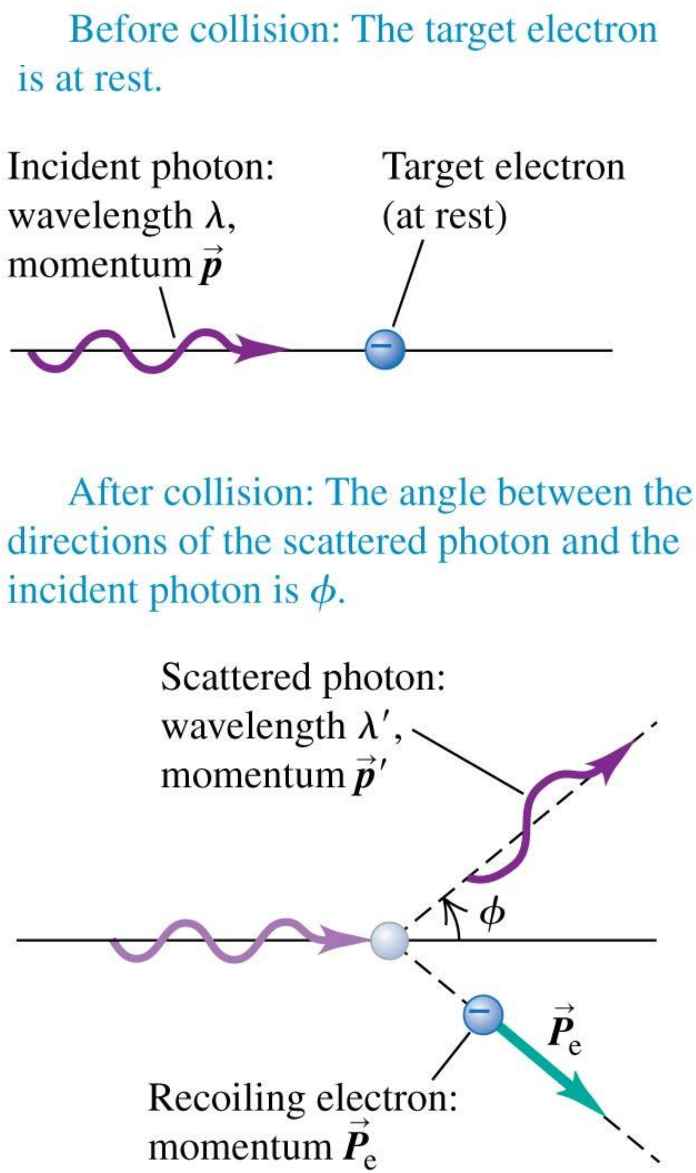
"for his discovery of **the effect named after him**".

Compton’s experiment

AIP Emilio Segrè Visual Archives

X-ray scattering: The Compton experiment

- Compton has found that the scattered x rays have a longer wavelength than the incident x rays, and the scattered wavelength depends on the scattering angle ϕ.
- This is consistent with the photon model.
- This process is now called Compton effect or Compton scattering.



Physical Review 21, 483 (1923)

PHYSICAL REVIEW

A QUANTUM THEORY OF THE SCATTERING OF X-RAYS BY LIGHT ELEMENTS

By Arthur H. Compton

Abstract

A quantum theory of the scattering of X-rays and γ-rays by light elements.—The hypothesis is suggested that when an X-ray quantum is scattered it spends all of its energy and momentum upon some particular electron. This electron in turn scatters the ray in some definite direction. The change in momentum of the X-ray quantum due to the change in its direction of propagation results in a recoil of the scattering electron. The energy in the scattered quantum is thus less than the energy in the primary quantum by the kinetic energy of recoil of the scattering electron. The corresponding *increase in the wave-length of the scattered beam* is $\lambda_\theta - \lambda_0 = (2h/mc)\sin^2\frac{1}{2}\theta = 0.0484\sin^2\frac{1}{2}\theta$, where h is the Planck constant, m is the mass of the scattering electron, c is the velocity of light, and θ is the angle between the incident and the scattered ray. Hence the increase is independent of the wave-length. *The distribution of the scattered radiation* is found, by an indirect and not quite rigid method, to be concentrated in the forward direction according to a definite law (Eq. 27).

- You can access the paper from here.
- See also a Focus story: Landmarks: Photons are Real

Compton scattering

- In **Compton scattering**, an incident photon collides with an electron that is initially at rest.
- The photon gives up part of its energy and momentum to the electron, which recoils as a result of this impact.
- The scattered photon flies off at an angle ϕ with respect to the incident direction, but it has less energy and less momentum than the incident photon.
- Therefore, the wavelength of the scattered photon λ' is longer than the wavelength λ of the incident photon.

Wavelength of scattered radiation

Wavelength of incident radiation

Planck's constant

Compton scattering:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$$

Scattering angle

Electron rest mass

Speed of light in vacuum

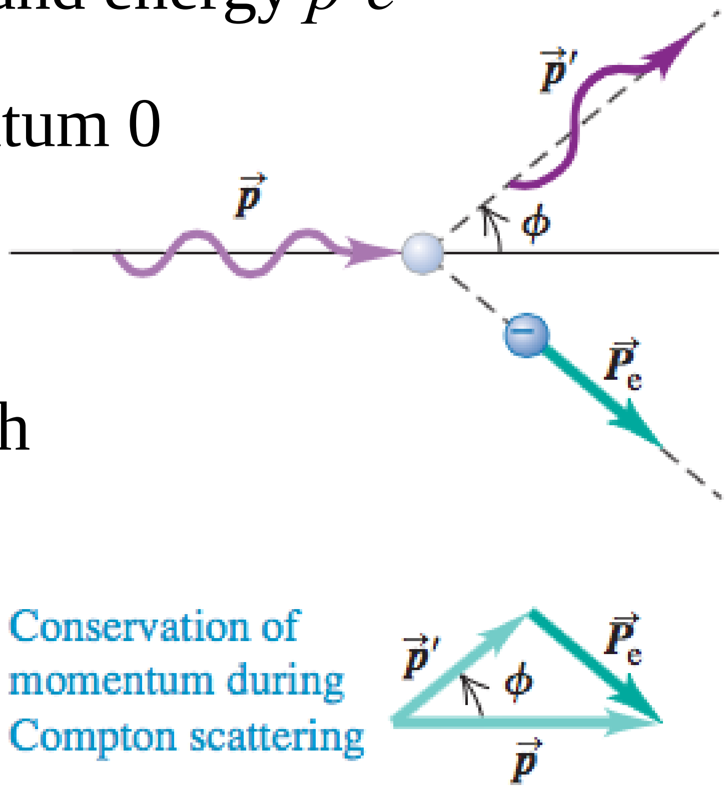
Derivation from conservation laws

- Incident photon: momentum ***p*** and energy *pc*
- Scattered photon: momentum ***p'*** and energy *p'c*
- Electron before collision: momentum 0 and energy mc^2
- Electron after collision: momentum $\boldsymbol{P_e}$ and energy E_e, with

$$E_e^2 = (mc^2)^2 + (P_e c)^2$$

- Energy conservation:

$$pc + mc^2 = p'c + E_e$$

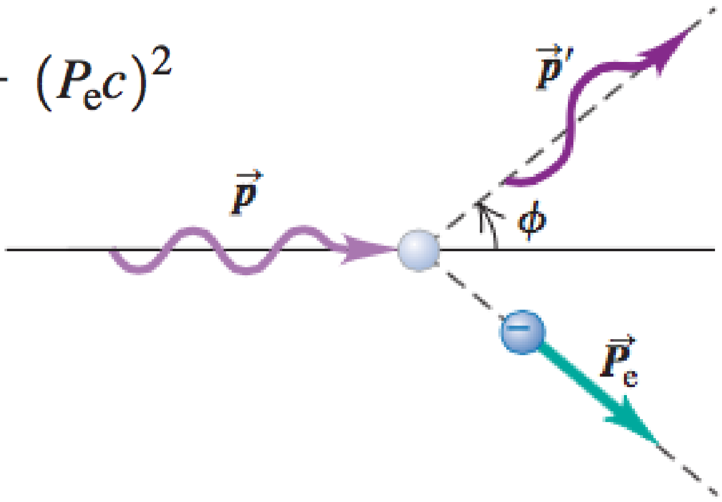


Derivation from conservation laws

- Energy conservation:

$$pc + mc^2 = p'c + E_e$$

$$(pc - p'c + mc^2)^2 = E_e{}^2 = (mc^2)^2 + (P_e c)^2$$



- Momentum conservation:

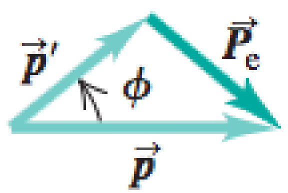
$$\vec{P}_e = \vec{p} - \vec{p}'$$

$$P_e{}^2 = p^2 + p'^2 - 2pp'\cos\phi$$

- Combining the two gives

$$\frac{mc}{p'} - \frac{mc}{p} = 1 - \cos\phi$$

Conservation of momentum during Compton scattering



- Using $p' = h/\lambda'$ and $p = h/\lambda$,
  We arrive at the results shown before.

Experimental Results

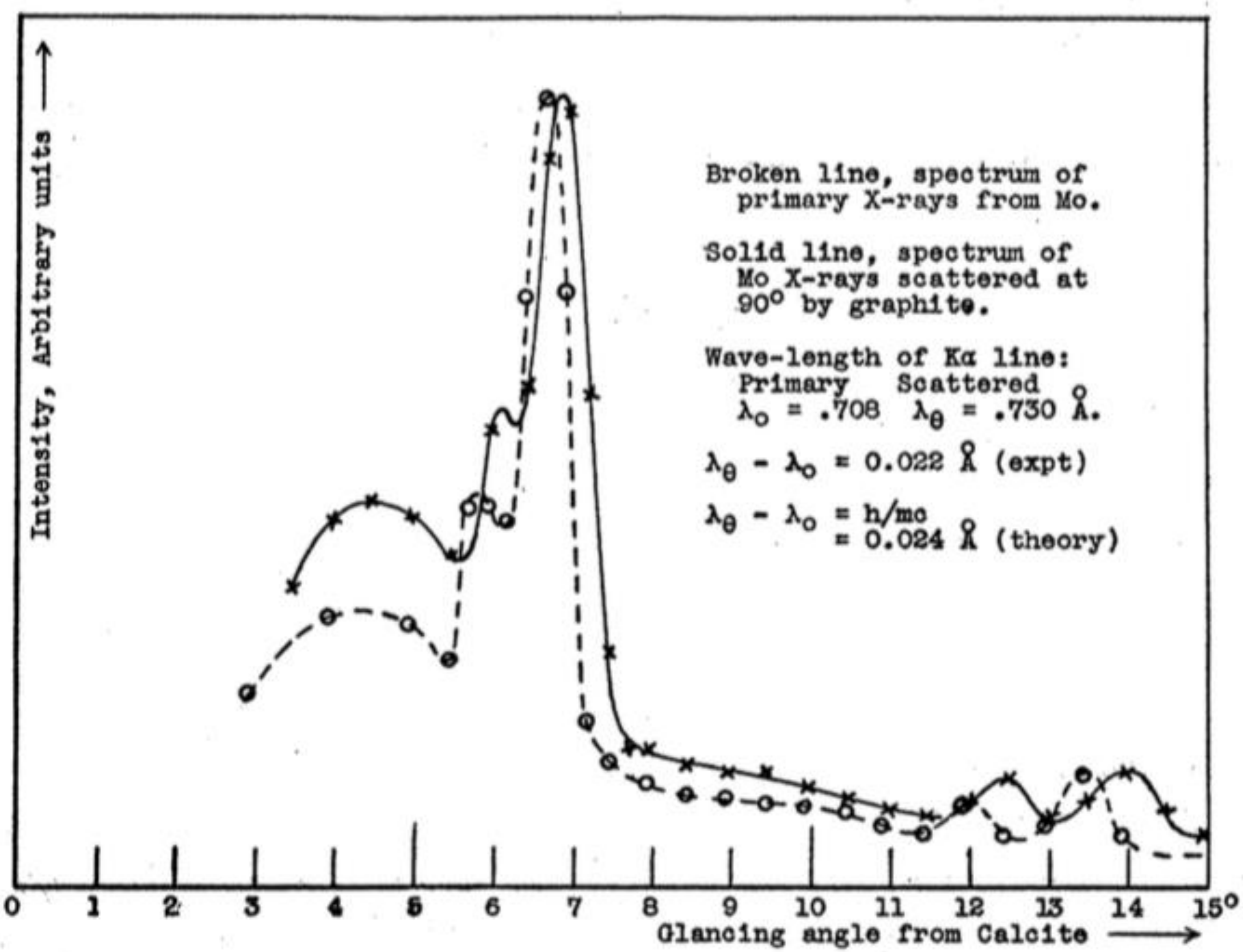


Fig. 4. Spectrum of molybdenum X-rays scattered by graphite, compared with the spectrum of the primary X-rays, showing an increase in wave-length on scattering.

Q38.4

When an x-ray photon bounces off an electron,

A. the photon wavelength decreases and the photon frequency decreases.

B. the photon wavelength decreases and the photon frequency increases.

C. the photon wavelength increases and the photon frequency decreases.

D. the photon wavelength increases and the photon frequency increases.

E. none of the above

Example 38.5: Compton scattering

You use 0.124-nm x-ray photons in a Compton-scattering experiment. (a) At what angle is the wavelength of the scattered x rays 1.0% longer than that of the incident x rays? (b) At what angle is it 0.050% longer?

EXECUTE: (a) In Eq. (38.7) we want $\Delta\lambda = \lambda' - \lambda$ to be 1.0% of 0.124 nm, so $\Delta\lambda = 0.00124 \text{ nm} = 1.24 \times 10^{-12}$ m. Using the value $h/mc = 2.426 \times 10^{-12}$ m, we find

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\phi)$$

$$\cos\phi = 1 - \frac{\Delta\lambda}{h/mc} = 1 - \frac{1.24 \times 10^{-12} \text{ m}}{2.426 \times 10^{-12} \text{ m}} = 0.4889$$

$$\phi = 60.7°$$

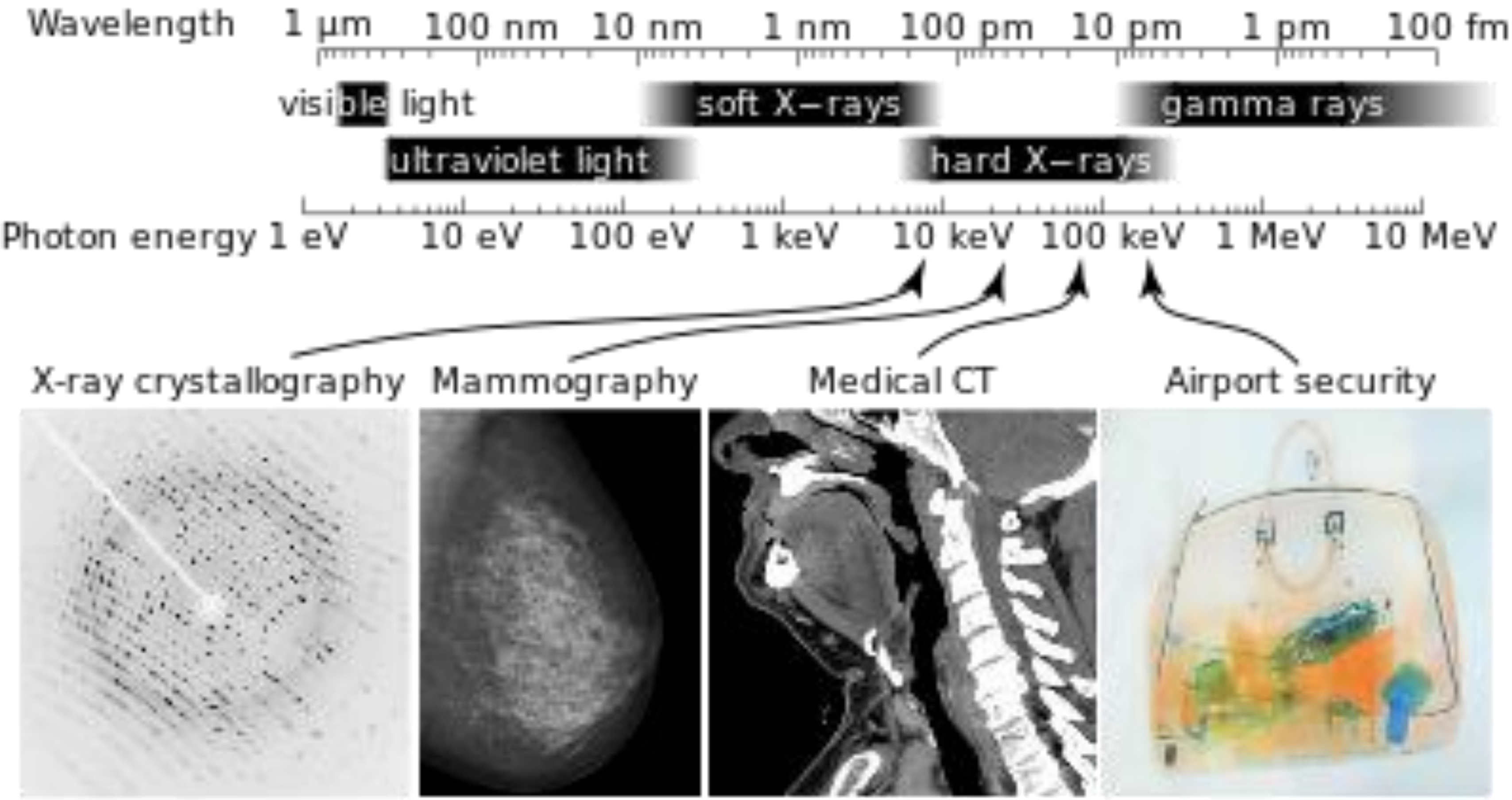
(b) For $\Delta\lambda$ to be 0.050% of 0.124 nm, or 6.2×10^{-14} m,

$$\cos\phi = 1 - \frac{6.2 \times 10^{-14} \text{ m}}{2.426 \times 10^{-12} \text{ m}} = 0.9744$$

$$\phi = 13.0°$$

EVALUATE: Our results show that smaller scattering angles give smaller wavelength shifts. Thus in a grazing collision the photon energy loss and the electron recoil energy are smaller than when the scattering angle is larger. This is just what we would expect for an elastic collision, whether between a photon and an electron or between two billiard balls.

X-rays as part of the electromagnetic spectrum



A-RT38.1

Rank the following photons in order of their energy, from highest to lowest. (Recall that $c = 3.00 \times 10^8$ m/s.)

A. a visible-light photon of wavelength 550 nm

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{550 * 10^{-9} m} = 5.45 \times 10^{14} Hz$$

B. an infrared photon of wavelength 10.6 µm

$$f = \frac{c}{\lambda} = \frac{3 \times 10^8 m/s}{10.6 * 10^{-6} m} = 2.83 \times 10^{13} Hz$$

C. an ultraviolet photon of frequency $4*10^{15}$ Hz

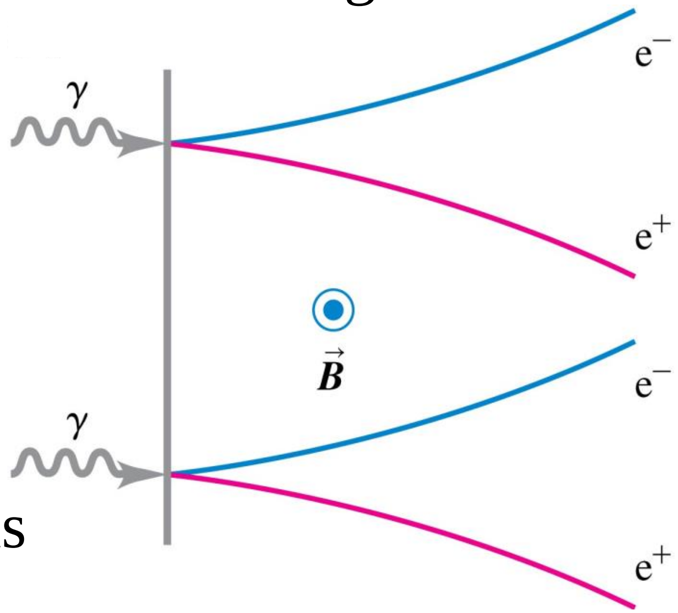
D. a radio photon of frequency 900 kHz

✔ Answer: CABD

Pair production

- When gamma rays of sufficiently short wavelength are fired into a metal plate, they can convert into an electron and a **positron**, each of mass m and rest energy mc^2.

- The photon model explains this: The photon wavelength must be so short that the photon energy is at least $2mc^2$.



- The minimum energy is

$$E_{\text{min}} = 2mc^2 = 2(9.109 \times 10^{-31}\ \text{kg})(2.998 \times 10^8\ \text{m/s})^2$$
$$= 1.637 \times 10^{-13}\ \text{J} = 1.022\ \text{MeV}$$

- The corresponding photon wavelength is

$$\lambda_{\text{max}} = \frac{hc}{E_{\text{min}}} = \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(2.998 \times 10^8\ \text{m/s})}{1.637 \times 10^{-13}\ \text{J}}$$
$$= 1.213 \times 10^{-12}\ \text{m} = 1.213 \times 10^{-3}\ \text{nm} = 1.213\ \text{pm}$$

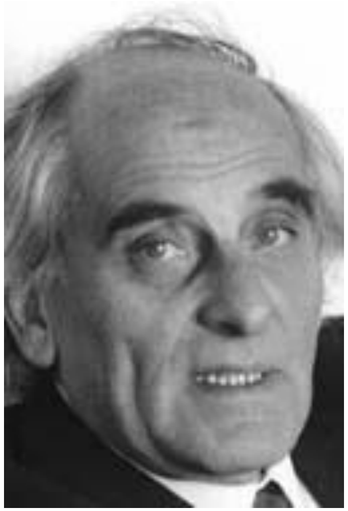
Observation of Pair Production

- The pair production was first observed in 1933 by the English physicist Patrick Blackett and the Italian physicist Giuseppe Occhialini

Patrick M. S. Blackett
(1897-1974)

Nobel Prize in Physics 1948

"for his development of the Wilson cloud chamber method, and his discoveries therewith in the fields of nuclear physics and cosmic radiation".



Giuseppe P. S. Occhialini
(1907-1993)

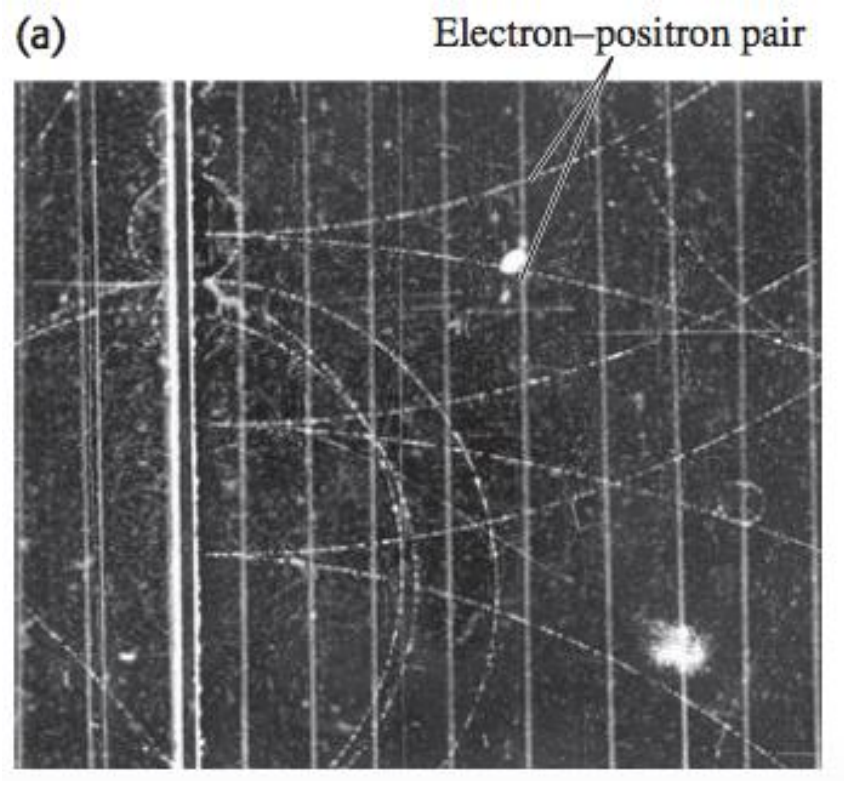
Wolf Prize in Physics 1979

“for his contributions to the discovery of **electron pair production** and of the charged pion“。

Observation of Pair Production

- Blackett and Occhialini's original work can be found at "Some Photographs of the Tracks of Penetrating Radiation" [PMS Blackett and GPS Occhialini, Proc. Roy. Soc. Lond. Ser. A **139**, 699 (1933).]
- Here we show an observation in a bubble chamber

Photograph of bubble-chamber tracks of electron–positron pairs that are produced when 300-MeV photons strike a lead sheet. A magnetic field directed out of the photograph made the electrons (e-) and positrons (e+) curve in opposite directions.



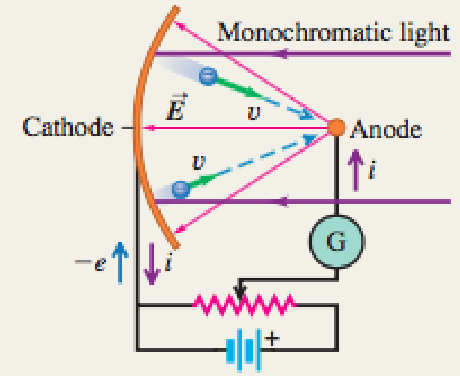
Photons: Electromagnetic radiation behaves as both waves and particles. The energy in an electromagnetic wave is carried in units called photons. The energy E of one photon is proportional to the wave frequency f and inversely proportional to the wavelength λ, and is proportional to a universal quantity h called Planck's constant. The momentum of a photon has magnitude E/c.

$$E = hf = \frac{hc}{\lambda} \tag{38.2}$$

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda} \tag{38.5}$$

The photoelectric effect: In the photoelectric effect, a surface can eject an electron by absorbing a photon whose energy hf is greater than or equal to the work function ϕ of the material. The stopping potential V_0 is the voltage required to stop a current of ejected electrons from reaching an anode. (See Examples 38.2 and 38.3.)

$$eV_0 = hf - \phi \tag{38.4}$$



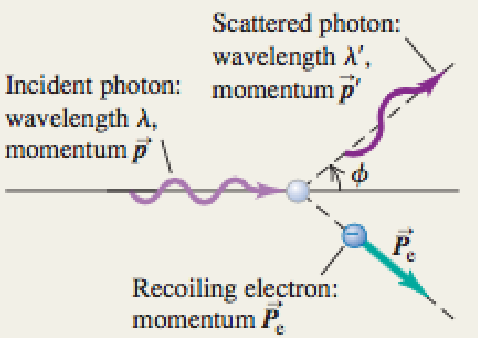
Photon production, photon scattering, and pair production: X rays can be produced when electrons accelerated to high kinetic energy across a potential increase V_{AC} strike a target. The photon model explains why the maximum frequency and minimum wavelength produced are given by Eq. (38.6). (See Example 38.4.) In Compton scattering a photon transfers some of its energy and momentum to an electron with which it collides. For free electrons (mass m), the wavelengths of incident and scattered photons are related to the photon scattering angle ϕ by Eq. (38.7). (See Example 38.5.) In pair production a photon of sufficient energy can disappear and be replaced by an electron–positron pair. In the inverse process, an electron and a positron can annihilate and be replaced by a pair of photons. (See Example 38.6.)

$$eV_{AC} = hf_{max} = \frac{hc}{\lambda_{min}} \tag{38.6}$$

(bremsstrahlung)

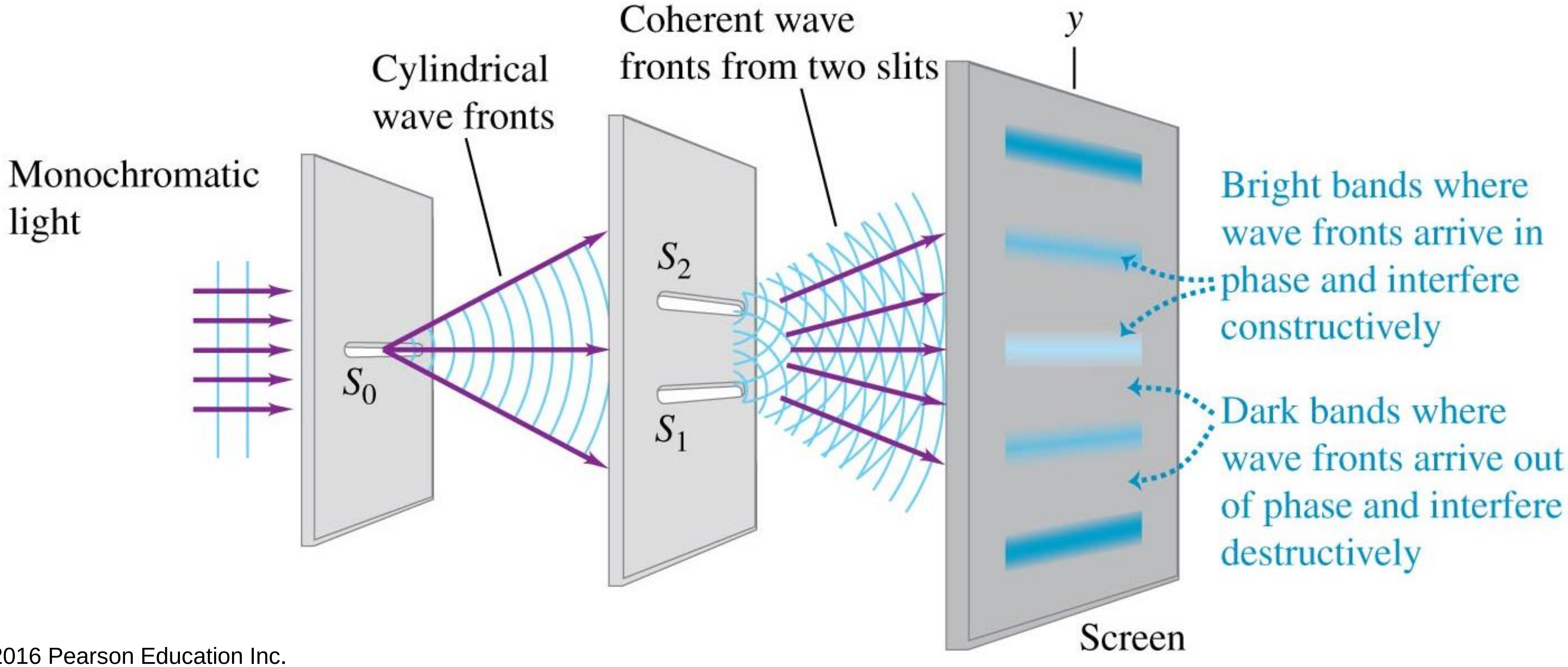
$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi) \tag{38.7}$$

(Compton scattering)

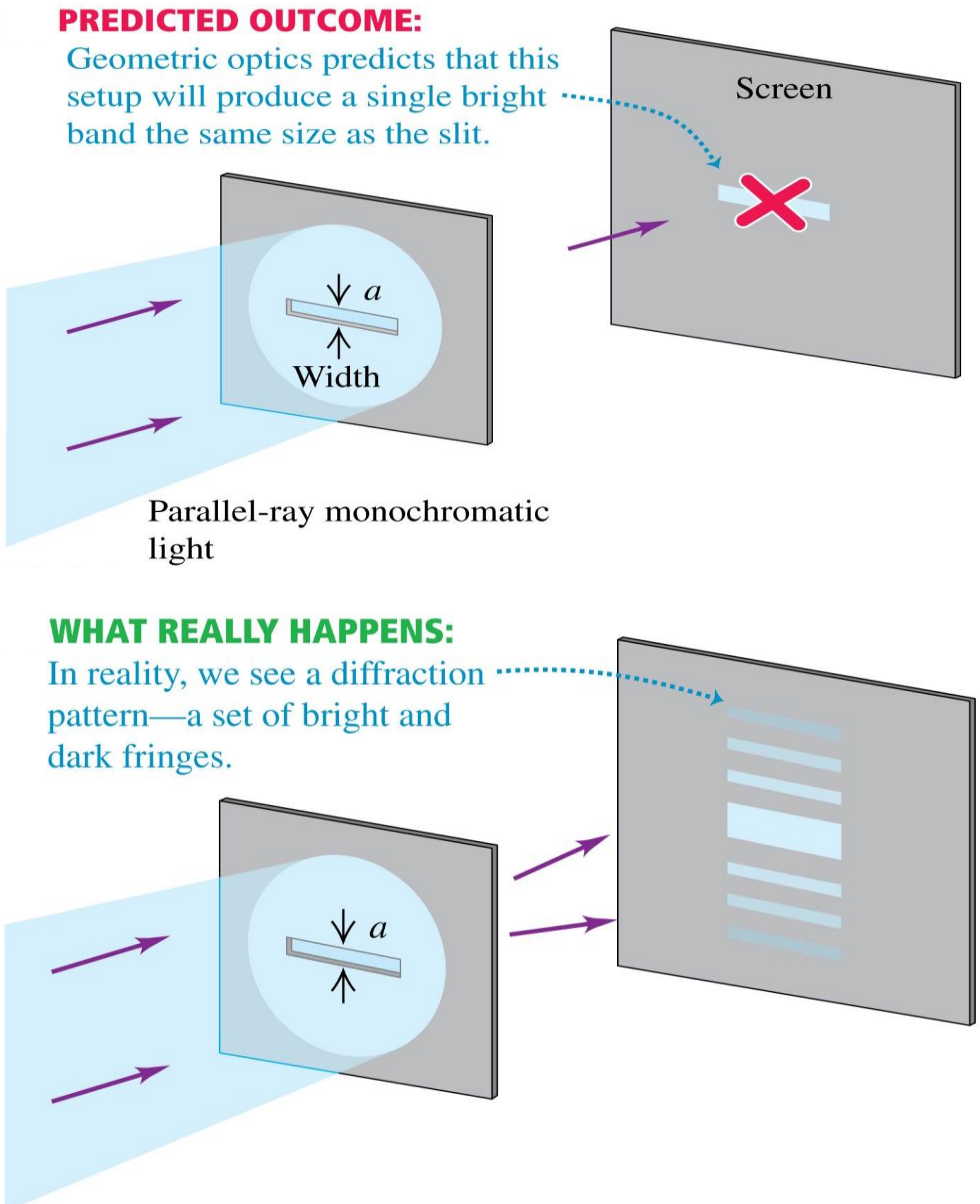


Review: Two-source interference of light

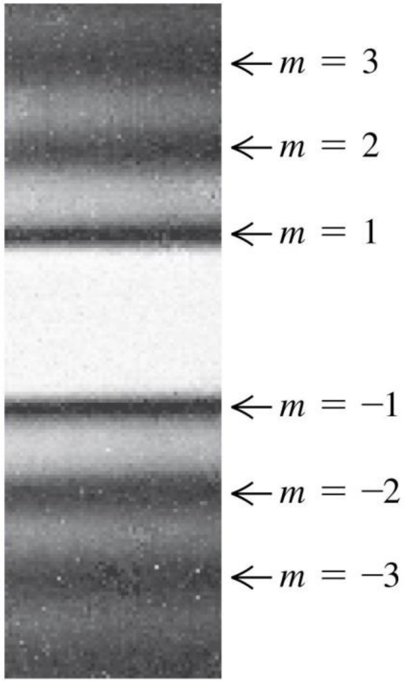
- Shown below is one of the earliest quantitative experiments to reveal the interference of light from two sources, first performed by Thomas Young.
- The interference of waves from slits S_1 and S_2 produces a pattern on the screen.

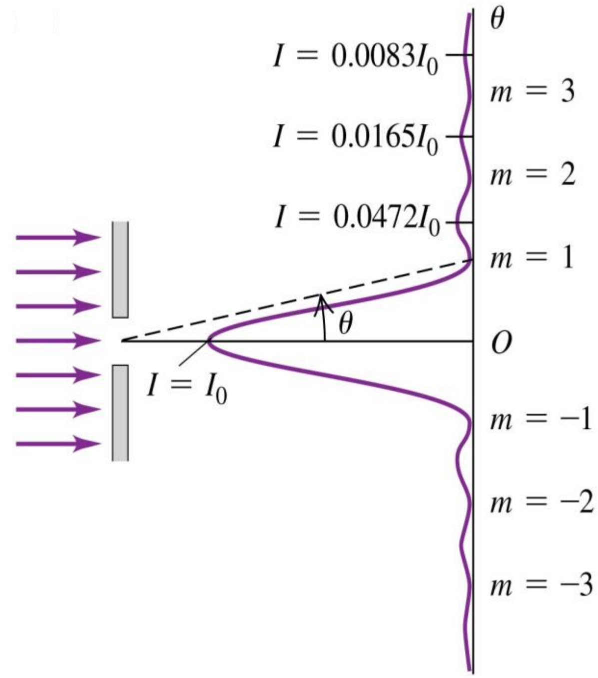


Review: Diffraction from a single slit



Review: Diffraction from a single slit





Dark fringes, single-slit diffraction:

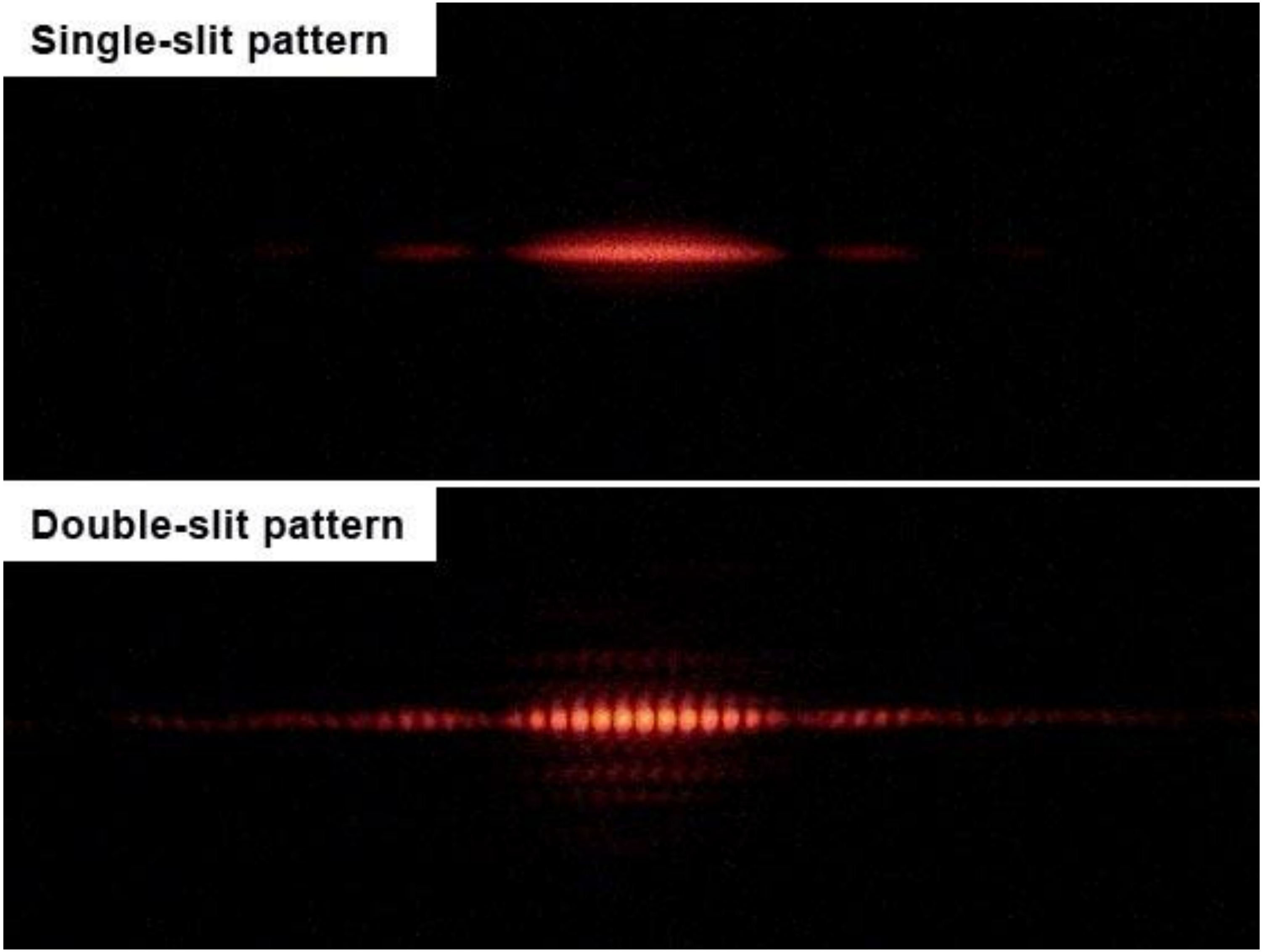
Angle of line from center of slit to *m*th dark fringe on screen

$$\sin\theta = \frac{m\lambda}{a} \quad (m = \pm1, \pm2, \pm3, \ldots)$$

Slit width

Wavelength

Review: single-slit and double slit patterns



The nature of light

- We have by far studied many examples of the behavior of light and other electromagnetic radiation.
- The wave nature of light explain very well the interference and diffraction.
- The particle nature of light explains the photoelectric effect and the Compton scattering.
- But.. How can light be both wave and particle at the same time? It just seems counterintuitive.
- **What is the nature of light?**

Principle of complementarity

- We can find the answer to the wave-particle conflict in the **principle of complementarity**.
- It is first stated by the Danish physicist Niels Bohr in 1928.
- Original literature can be found at Nature **121**, 580 (1928) or "Discussions with Einstein on Epistemological Problems in Atomic Physics"
- Idea: The wave descriptions and the particle descriptions are complementary.
- We need both to complete our model of nature, but we will never need to use both at the same time to describe a single part of an occurrence.

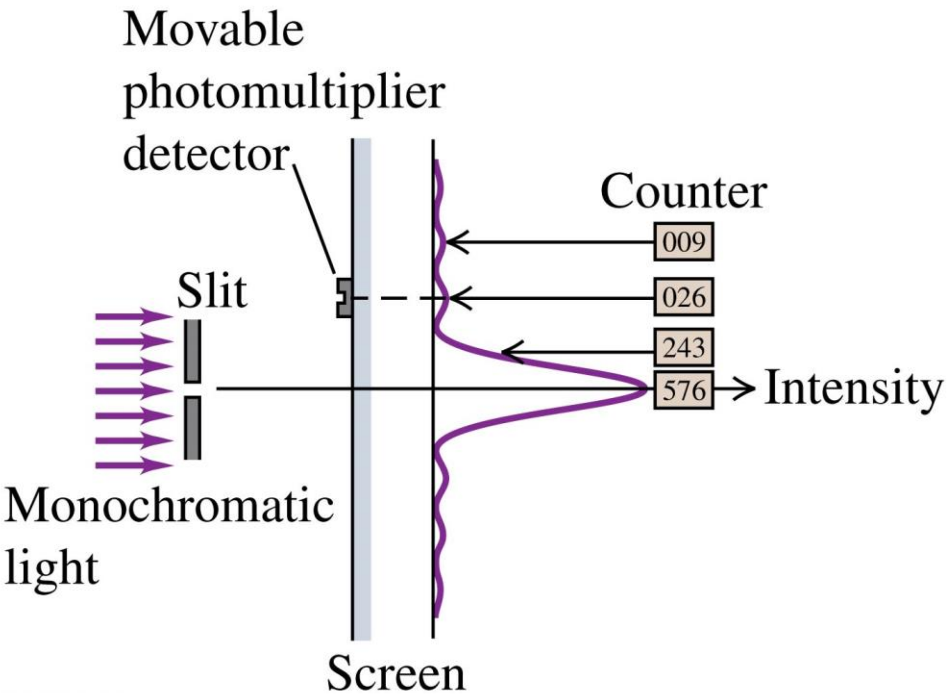
Niels H. D. Bohr (1885-1962)

Nobel Prize in Physics 1922

"for his services in the investigation of the structure of atoms and of the radiation emanating from them".

Diffraction from a single slit

- Instead of using a film, we use a photomultiplier to detect individual photons
- Suppose we reduce the intensity to allow only a few photons per second pass through the slit. We now record a series of discrete strikes, each representing a single photon.



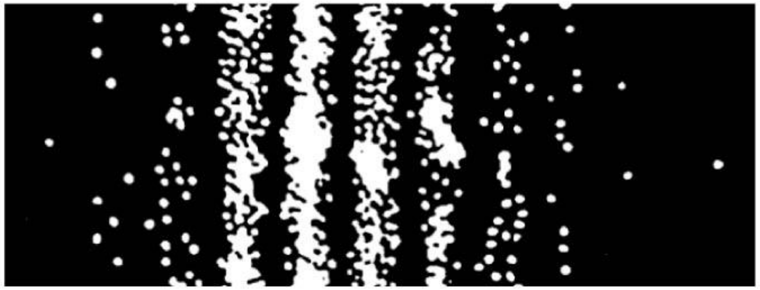
- Over time the accumulating strikes build up the familiar diffraction pattern we expect for a wave.
- To reconcile the wave and particle aspects, we have to regard the pattern as a **statistical distribution** that tells us how many photons, on average, go to each spot. Equivalently, the pattern tells us the **probability** that any individual photon will land at a given spot.

Interference from a double-slit experiment

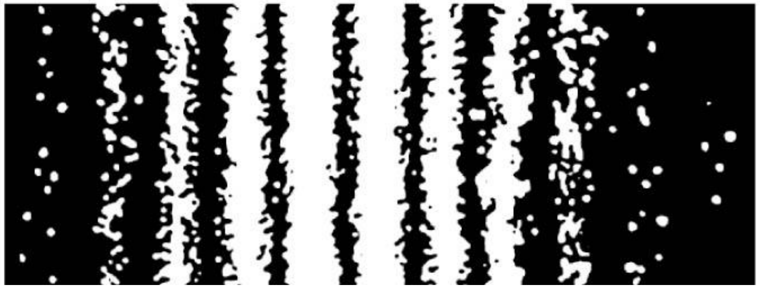
- We get an analogous result for a double-slit.
- Again we can’t predict exactly where an individual photon will go; the interference pattern is a **statistical distribution**.

After 21 photons reach the screen

After 1000 photons reach the screen



After 10,000 photons reach the screen



Wave-particle duality

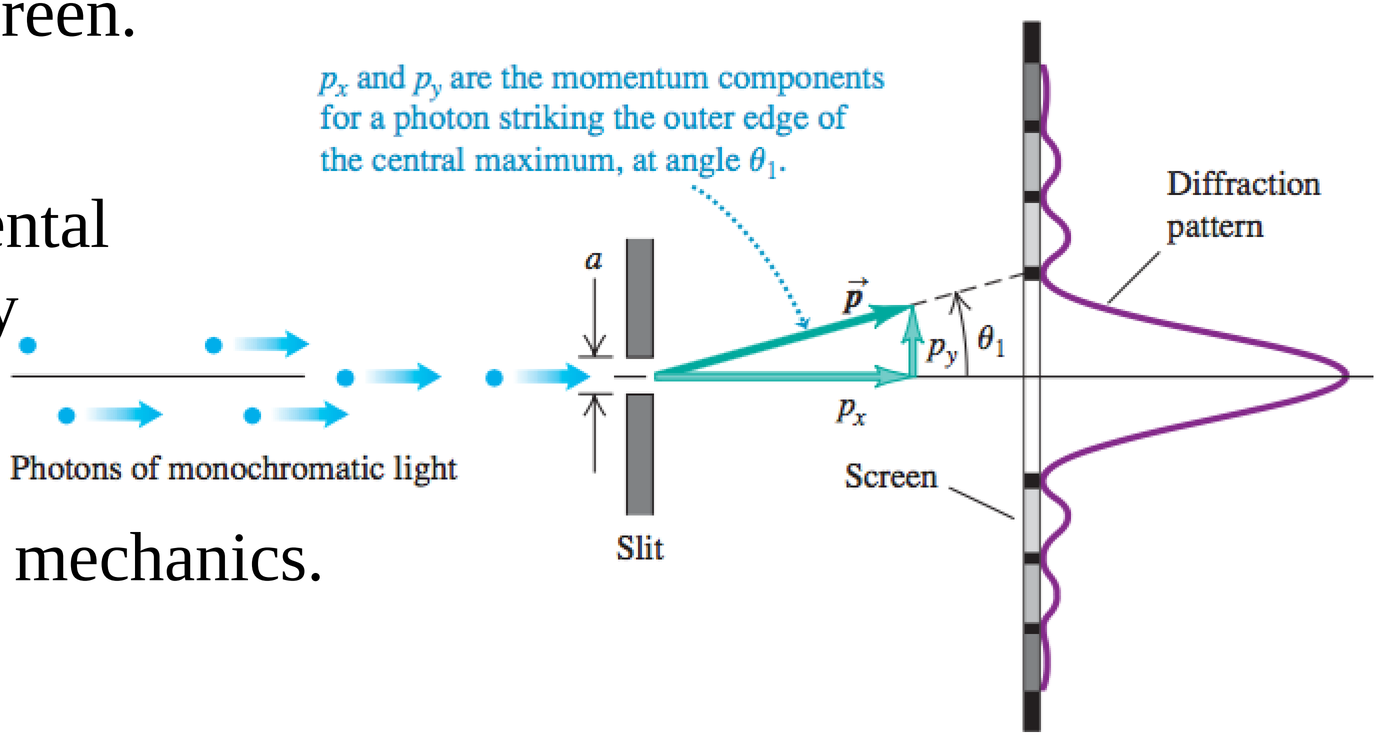
- The wave description, not the particle description, explains the single- and double-slit patterns.
- The particle description, not the wave description, explains why the photomultiplier records discrete packages of energy.
- The two descriptions complete our understanding of the results.
- Suppose we consider an individual photon and ask how it knows “which way to go” when passing through the slit. This question seems like a conundrum, but that is because it is framed in terms of a particle description—whereas it is the wave nature of light that determines the distribution of photons.
- Conversely, the fact that the photomultiplier detects faint light as a sequence of individual “spots” can’t be explained in wave terms.
- The nature of the light is therefore the **wave-particle duality.**

Photons are not particles in "common sense"

- Although photons have energy and momentum, they are very different from the particle model we used for Newtonian mechanics.
- The Newtonian particle model treats an object as a point mass. We can describe the location and state of motion of such a particle at any instant with three spatial coordinates (x, y, z) and three components of momentum ($\boldsymbol{p}_x$, $\boldsymbol{p}_y$, $\boldsymbol{p}_z$), and we can then predict the particle's future motion.
- This model doesn't work for photons. This is because there are **fundamental limitations on the precision** with which we can **simultaneously determine the position and momentum of a photon**. Many aspects of a photon's behavior can be stated only in terms of **probabilities**.

Single-slit diffraction revisited

- Even though the photons all have the same initial state of motion, they don't all follow the same path.
- We can't predict the exact trajectory of any individual photon from knowledge of its initial state; we can only describe the probability that an individual photon will strike a given spot on the screen.
- This fundamental indeterminacy has no counterpart in Newtonian mechanics.



Q38.6

A beam of photons passes through a narrow slit. The photons land on a distant screen, forming a diffraction pattern. In order for a particular photon to land at the center of the diffraction pattern, it must pass

A. through the center of the slit.

B. through the upper half of the slit.

C. through the lower half of the slit.

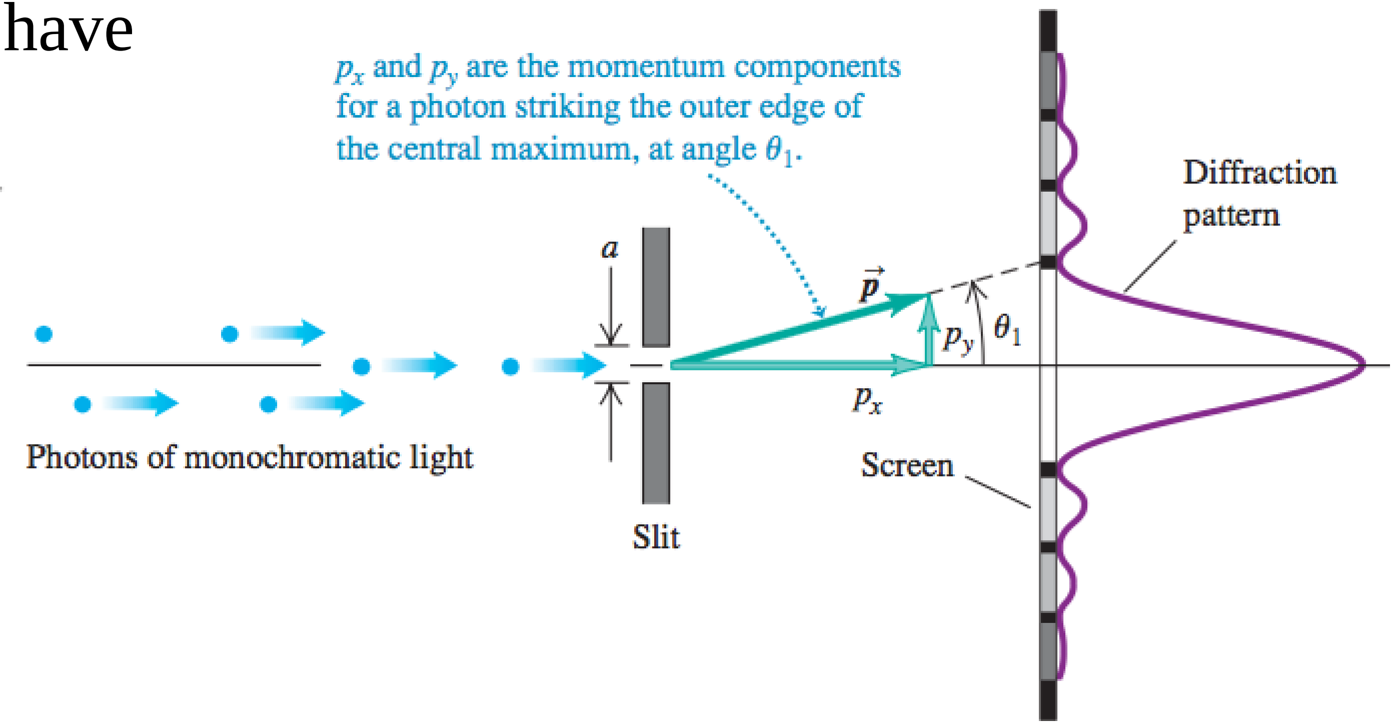
D. either B or C.

E. through the slit, but it is impossible to tell exactly where.

Quantifying the uncertainties

- There are fundamental uncertainties in both the position and the momentum of an individual particle, and these uncertainties are related inseparably.
- Suppose $\lambda << a$, we have $\sin\theta = \theta = \lambda/a$
- We also have $\tan\theta = \theta = p_y/p_x$
- Together we have

$$p_y = p_x \frac{\lambda}{a}$$

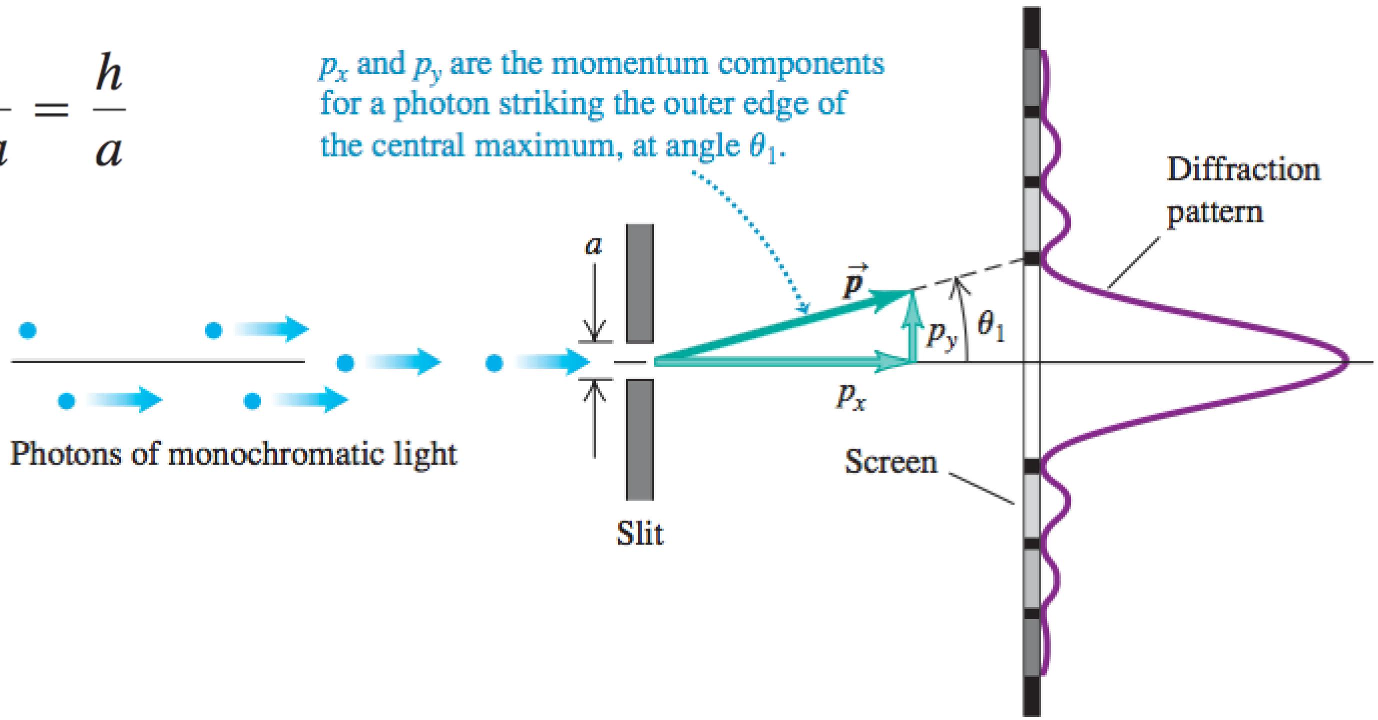


Quantifying the uncertainties

- We have $$p_y = p_x \frac{\lambda}{a}$$
- However, photons spread out over the entire screen, so there is an uncertainty in p_y: $$\Delta p_y \geq p_x \frac{\lambda}{a}$$
- Using $\lambda = h/p_x$ we have

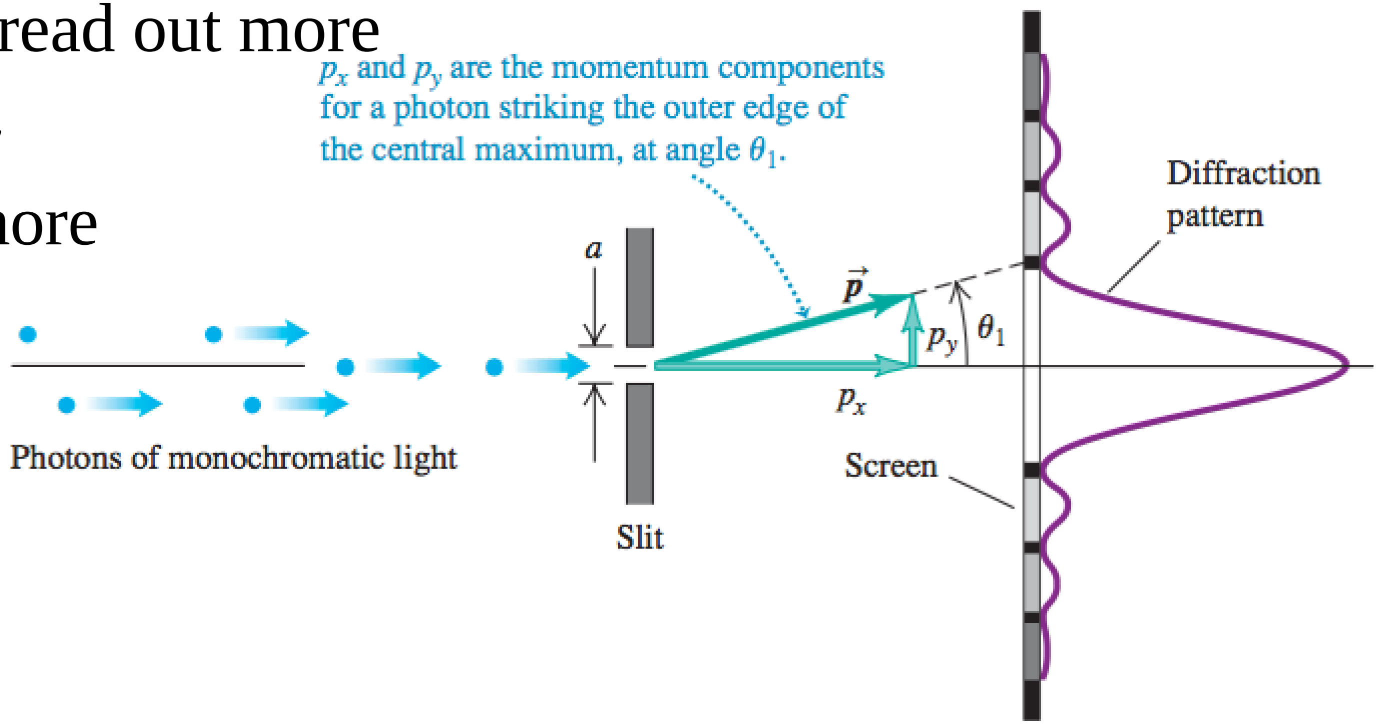
$$\Delta p_y \geq p_x \frac{h}{p_x a} = \frac{h}{a}$$

$$\Delta p_y a \geq h$$



Quantifying the uncertainties

- What does $\Delta p_y a \geq h$ mean?
- a represents the uncertainty in the y-component of the *position* as a photon passes through the slit.
- The uncertainties of the *position* and *momentum* are related!
- To make our position more accurate, a must be smaller
  → photon spread out more
  → larger Δp_y
- To make p_y more accurate
  → a must be larger



Intuition v.s. reality

- You may protest that it doesn't seem to be consistent with common sense for a photon not to have a definite position and momentum.
- What we call *common sense* is based on familiarity gained through experience.
- Our usual experience includes very little contact with the microscopic behavior of particles like photons.
- **Sometimes we have to accept conclusions that violate our intuition** when we are dealing with areas that are far removed from everyday experience.

The Heisenberg uncertainty principle

- The uncertainty principle was first discovered by the German physicist Werner Heisenberg.

Werner K. Heisenberg
(1901-1976)

Nobel Prize in Physics 1932

" for the creation of quantum mechanics, the application of which has, inter alia, led to the discovery of the allotropic forms of hydrogen ".

The Heisenberg uncertainty principle

- You cannot simultaneously know the position and momentum of a photon, or any other particle, with arbitrarily great precision.
- The better you know the value of one quantity, the less well you know the value of the other.

Heisenberg uncertainty principle for position and momentum:

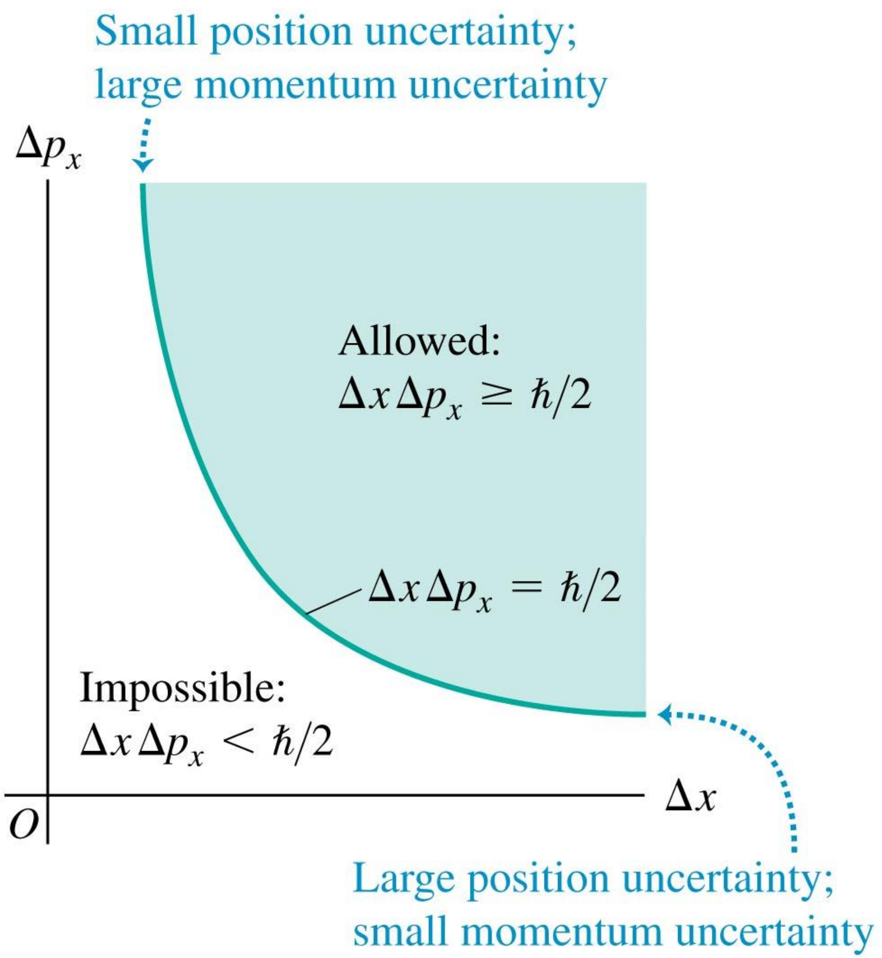
$$\overbrace{\Delta x}^{\text{Uncertainty in coordinate } x}\ \underbrace{\Delta p_x}_{\text{Uncertainty in corresponding momentum component } p_x} \geq \overset{\text{Planck's constant divided by } 2\pi}{\hbar}/2$$

$$\hbar = \frac{h}{2\pi} = 1.054571628(53) \times 10^{-34}\ \mathrm{J \cdot s}$$

- There is a similar uncertainty relationship for the *y*- and *z*-coordinate axes and their corresponding momentum components.

The Heisenberg uncertainty principle

- Shown is a graphical representation of the Heisenberg uncertainty principle.
- A measurement with uncertainties whose product puts them to the left of or below the blue line is not possible to make.



Q38.7

A photon has a position uncertainty of 2.00 mm. If you decrease the position uncertainty to 1.00 mm, how does this change the momentum uncertainty of the photon?

A. The momentum uncertainty becomes one-quarter as large.

B. The momentum uncertainty becomes one-half as large.

C. The momentum uncertainty is unchanged.

D. The momentum uncertainty becomes twice as large.

E. The momentum uncertainty becomes four times larger.

Q38.8

A photon has a momentum uncertainty of 2.00×10^{-28} kg • m/s. If you increase the momentum uncertainty to 4.00×10^{-28} kg • m/s, how does this change the position uncertainty of the photon?

A. The position uncertainty becomes one-quarter as large.

B. The position uncertainty becomes one-half as large.

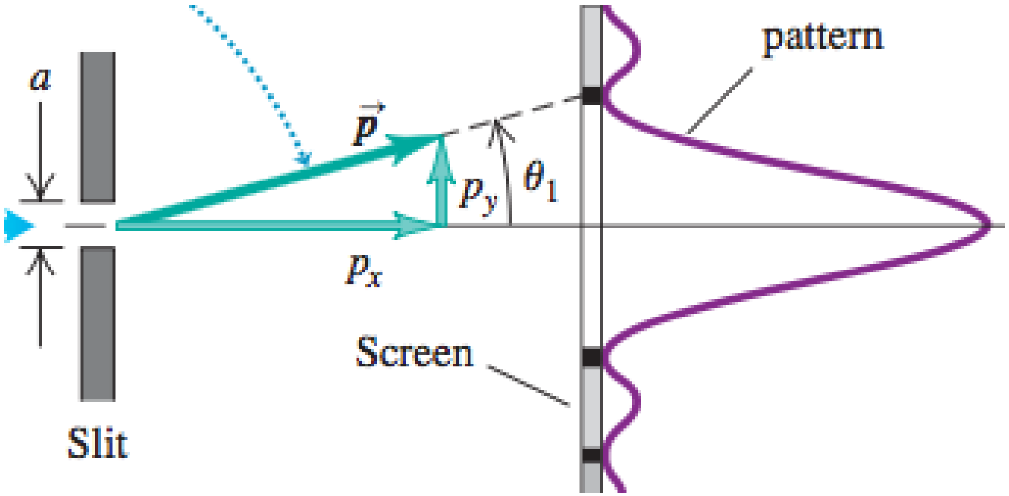
C. The position uncertainty is unchanged.

D. The position uncertainty becomes twice as large.

E. The position uncertainty becomes four times larger.

What if we have a better detector?

- We cannot circumvent the uncertainty principle by using a more sophisticated detector.
- To detect a particle, the detector must interact with it, and this interaction unavoidably changes the state of motion of the particle, introducing uncertainty about its original state.
- The uncertainties we have described are fundamental and intrinsic.
- They cannot be circumvented even in principle by any experimental technique, no matter how sophisticated.



Uncertainty in energy

- There is also an uncertainty principle that involves energy and time.

- The better we know a photon’s energy, the less certain we are of when we will observe the photon:

Heisenberg uncertainty principle for energy and time:

$$\overbrace{\Delta t}^{\text{Time uncertainty of a phenomenon}}\ \underbrace{\Delta E}_{\text{Energy uncertainty of same phenomenon}} \geq \overbrace{\hbar}^{\text{Planck's constant divided by }2\pi}/2$$

- This relation holds true for other kinds of particles as well.

Example 38.7: Ultrashort laser pulses and the uncertainty principle

Many varieties of lasers emit light in the form of pulses rather than a steady beam. A tellurium–sapphire laser can produce light at a wavelength of 800 nm in ultrashort pulses that last only 4.00×10^{-15} s (4.00 femtoseconds, or 4.00 fs). The energy in a single pulse produced by one such laser is $2.00\ \mu\text{J} = 2.00 \times 10^{-6}$ J, and the pulses propagate in the positive x-direction. Find (a) the frequency of the light; (b) the energy and minimum energy uncertainty of a single photon in the pulse; (c) the minimum frequency uncertainty of the light in the pulse; (d) the spatial length of the pulse, in meters and as a multiple of the wavelength; (e) the momentum and minimum momentum uncertainty of a single photon in the pulse; and (f) the approximate number of photons in the pulse.

Example 38.7: Ultrashort laser pulses and the uncertainty principle

EXECUTE: (a) From the relationship $c = \lambda f$, the frequency of 800-nm light is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \mathrm{m/s}}{8.00 \times 10^{-7}\ \mathrm{m}} = 3.75 \times 10^{14}\ \mathrm{Hz}$$

(b) From Eq. (38.2) the energy of a single 800-nm photon is

$$\begin{aligned} E = hf &= (6.626 \times 10^{-34}\ \mathrm{J \cdot s})(3.75 \times 10^{14}\ \mathrm{Hz}) \\ &= 2.48 \times 10^{-19}\ \mathrm{J} \end{aligned}$$

The time uncertainty equals the pulse duration, $\Delta t = 4.00 \times 10^{-15}\ \mathrm{s}$. From Eq. (38.24) the minimum uncertainty in energy corresponds to the case $\Delta t\,\Delta E = \hbar/2$, so

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{1.055 \times 10^{-34}\ \mathrm{J \cdot s}}{2(4.00 \times 10^{-15}\ \mathrm{s})} = 1.32 \times 10^{-20}\ \mathrm{J}$$

This is 5.3% of the photon energy $E = 2.48 \times 10^{-19}\ \mathrm{J}$, so the energy of a given photon is uncertain by at least 5.3%. The uncertainty could be greater, depending on the shape of the pulse.

Example 38.7: Ultrashort laser pulses and the uncertainty principle

(c) From the relationship $f = E/h$, the minimum frequency uncertainty is

$$\Delta f = \frac{\Delta E}{h} = \frac{1.32 \times 10^{-20}\ \text{J}}{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}} = 1.99 \times 10^{13}\ \text{Hz}$$

This is 5.3% of the frequency $f = 3.75 \times 10^{14}$ Hz we found in part (a). Hence these ultrashort pulses do not have a definite frequency; the average frequency of many such pulses will be 3.75×10^{14} Hz, but the frequency of any individual pulse can be anywhere from 5.3% higher to 5.3% lower.

(d) The spatial length Δx of the pulse is the distance that the front of the pulse travels during the time $\Delta t = 4.00 \times 10^{-15}$ s it takes the pulse to emerge from the laser:

$$\begin{aligned} \Delta x = c\Delta t &= (3.00 \times 10^{8}\ \text{m/s})(4.00 \times 10^{-15}\ \text{s}) \\ &= 1.20 \times 10^{-6}\ \text{m} \end{aligned}$$

$$\Delta x = \frac{1.20 \times 10^{-6}\ \text{m}}{8.00 \times 10^{-7}\ \text{m/wavelength}} = 1.50\ \text{wavelengths}$$

This justifies the term *ultrashort.* The pulse is less than two wavelengths long!

Example 38.7: Ultrashort laser pulses and the uncertainty principle

(e) From Eq. (38.5), the momentum of an average photon in the pulse is

$$p_x = \frac{E}{c} = \frac{2.48 \times 10^{-19}\ \text{J}}{3.00 \times 10^{8}\ \text{m/s}} = 8.28 \times 10^{-28}\ \text{kg} \cdot \text{m/s}$$

The spatial uncertainty is $\Delta x = 1.20 \times 10^{-6}$ m. From Eq. (38.17) minimum momentum uncertainty corresponds to $\Delta x\, \Delta p_x = \hbar/2$, so

$$\Delta p_x = \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{2(1.20 \times 10^{-6}\ \text{m})} = 4.40 \times 10^{-29}\ \text{kg} \cdot \text{m/s}$$

This is 5.3% of the average photon momentum p_x. An individual photon within the pulse can have a momentum that is 5.3% greater or less than the average.

(f) To estimate the number of photons in the pulse, we divide the total pulse energy by the average photon energy:

$$\frac{2.00 \times 10^{-6}\ \text{J/pulse}}{2.48 \times 10^{-19}\ \text{J/photon}} = 8.06 \times 10^{12}\ \text{photons/pulse}$$

The energy of an individual photon is uncertain, so this is the *average* number of photons per pulse.

Example 38.7: Ultrashort laser pulses and the uncertainty principle

EVALUATE: The percentage uncertainties in energy and momentum are large because this laser pulse is so short. If the pulse were longer, both Δt and Δx would be greater and the corresponding uncertainties in photon energy and photon momentum would be smaller.

Our calculation in part (f) shows an important distinction between photons and other kinds of particles. In principle it is possible to make an exact count of the number of electrons, protons, and neutrons in an object such as this book. If you repeated the count, you would get the same answer as the first time. By contrast, if you counted the number of photons in a laser pulse you would *not* necessarily get the same answer every time! The uncertainty in photon energy means that on each count there could be a different number of photons whose individual energies sum to 2.00×10^{-6} J. That's yet another of the many strange properties of photons.

Bridging Problem: Compton Scattering and Electron Recoil

An incident x-ray photon is scattered from a free electron that is initially at rest. The photon is scattered straight back at an angle of 180° from its initial direction. The wavelength of the scattered photon is 0.0830 nm. (a) What is the wavelength of the incident photon? (b) What are the magnitude of the momentum and the speed of the electron after the collision? (c) What is the kinetic energy of the electron after the collision?