Lecture 10

Magnetic Field due to Current and Inductance and Inductors

PHY 1202

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Lecture 09 Review

- All magnets have two poles: North and South with like poles repel each other and unlike poles attract each other.
- Magnetic poles are always found in pairs and isolated magnetic poles, or magnetic monopoles, have never been found. Although they are predicted to exist in theory.
- The source of the magnetic field is from moving electric charges. In fact, even the electron has a magnetic dipole that can generate tiny magnetic field due to its spin.
- Permanent magnets have their electrons/molecules add together to give a net magnetic field.
- To define the magnitude and direction of an unknown magnetic field, we will need a moving test charge.

Lecture 09 Review

- Magnetic field **B** is defined in terms of the force **FB** acting on a test particle with charge *q* moving through the field with velocity **v**.

$$\vec{F}_B = q\vec{v} \times \vec{B}$$

- The SI unit of magnetic field is tesla, where 1 T= 1 N/(A-m).
- The magnetic field of a magnet starts from the North pole and ends in the South pole.
- When a conducting strip carrying a current i is placed in a uniform magnetic field B, some charge carries build up on one side of the conductor, creating a potential difference V, the Hall potential difference, across the strip. The polarities of the sides indicate the sign of the charge carriers. This is refers to as the Hall Effect.
- We can measure the drift velocity and the carrier density with the Hall effect.

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Lecture 09 Review

- When a charged particle moving with a constant velocity perpendicular to a uniform magnetic field will circulate in the field with a angular frequency

$$\omega = 2\pi f = \frac{|q|B}{m}$$

- A straight wire carrying a current i in a uniform magnetic field experiences a sideway force $\vec{F}_B = i\vec{L} \times \vec{B}$
- A coil carrying a current in a uniform magnetic field will experience a torque given by

$$\vec{\tau} = \vec{\mu} \times \vec{B} \rightarrow \tau = \mu B \sin\theta$$

- Here m is the magnetic dipole moment of the current loop, that is a function of the current i, area A and the number of loops N.

$$\mu = NiA$$

- This is similar to the electric dipole moment in an electric field.

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Lecture Outline

- **Chapter 29 & 30**, D Halliday, R Resnick, and J Walker, "Fundamentals of Physics" 9th Edition, Wiley (2005).

- Magnetic Field due to Current
  - Biot-Savart Law
  - Magnetic Field due to a long straight wire
  - Magnetic field due to a current in a circular arc
  - force between two parallel wires
  - Ampere's Law
  - Solenoids & Toriods
  - Induction and Inductors

Source of magnetic field

- A moving charge produces a magnetic field

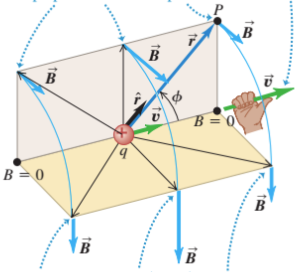
$$B = \frac{\mu_0}{4\pi}\frac{|q|v\sin\phi}{r^2}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}$$ (magnetic field of a point charge with constant velocity)

$$\mu_0 = 4\pi\times10^{-7}\,\mathrm{T\cdot m/A} \approx 1.26\times10^{-6}\,\mathrm{T\cdot m/A}.$$

Right-hand rule for the magnetic field due to a positive charge moving at constant velocity: Point the thumb of your right hand in the direction of the velocity. Your fingers now curl around the charge in the direction of the magnetic field lines. (If the charge is negative, the field lines are in the opposite direction.)

For these field points, $\vec{r}$ and $\vec{v}$ both lie in the beige plane, and $\vec{B}$ is perpendicular to this plane.



For these field points, $\vec{r}$ and $\vec{v}$ both lie in the gold plane, and $\vec{B}$ is perpendicular to this plane.

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29.2: Calculating the Magnetic Field due to a Current
Biot-Savart Law

The Biot-Savart Law in electromagnetism describes the magnetic field generated by an electric current.

A small current element as shown will generate a circulating magnetic field around the element. The magnitude of the field is proportional to the current and the length of the element. Similar to the electric field it is also inversely proportional to the square of the distance.

This element of current creates a magnetic field at *P*, into the page.

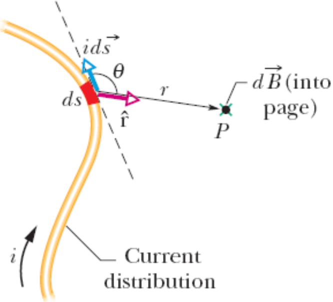

Fig. 29-1 A current-length element $i\,d\vec{s}$ produces a differential magnetic field $d\vec{B}$ at point P. The green $\times$ (the tail of an arrow) at the dot for point P indicates that $d\vec{B}$ is directed *into* the page there.

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$$B = \frac{\mu_0}{4\pi}$$

$$dQ =$$

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29.2: Calculating the Magnetic Field due to a Current
Biot-Savart Law

The magnitude of the field ***dB*** produc length element ***i ds*** turns out to be

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

This law is experimentally deduced

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T\cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T\cdot m/A}.$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2}$$ *(Biot-Savart Law)*

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29.2: Magnetic Field due to a Long Straight Wire:

Fig. 29-3 Iron filings that have been sprinkled onto cardboard collect in concentric circles when current is sent through the central wire. The alignment, which is along magnetic field lines, is caused by the magnetic field produced by the current. *(Courtesy Education Development Center)*

The magnetic field vector at any point is tangent to a circle.

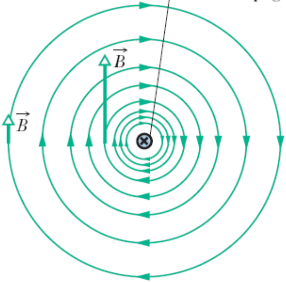

Fig. 29-2 The magnetic field lines produced by a current in a long straight wire form concentric circles around the wire. Here the current is into the page, as indicated by the ×.

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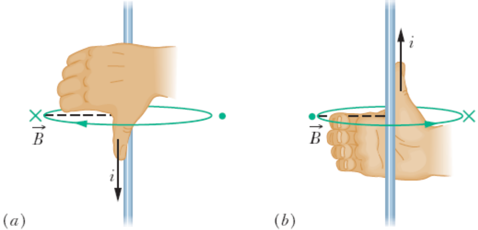
29.2: Magnetic Field due to a Long Straight Wire:

Right-hand rule: Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.



The thumb is in the current's direction. The fingers reveal the field vector's direction, which is tangent to a circle.

Fig. 29-4 A right-hand rule gives the direction of the magnetic field due to a current in a wire. *(a)* The magnetic field ***B*** at any point to the left of the wire is perpendicular to the dashed radial line and directed into the page, in the direction of the fingertips, as indicated by the x. *(b) If the* current is reversed, at any point to the left is still perpendicular to the dashed radial line but now is directed out of the page, as indicated by the dot.

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29.2: Magnetic Field due to a Long Straight Wire:

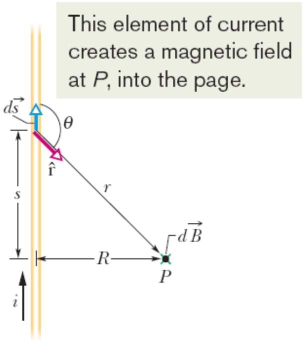

Fig. 29-5 Calculating the magnetic field produced by a current i in a long straight wire. The field $d\vec{B}$ at P associated with the current-length element $i\,d\vec{s}$ is directed into the page, as shown.

B due to the current element *ids Biot-Savart Law.*

$$dB = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

B due to the long straight wire by summing up (integrate) all the current elements from –inf to inf.

$$B = \int_{-\infty}^{\infty} dB = 2\int_0^{\infty} dB$$

$$= \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{\sin\theta\,ds}{r^2}$$

$$\sin\theta = \sin(\pi - \theta) = \frac{R}{\sqrt{s^2 + R^2}}.$$

$$r = \sqrt{s^2 + R^2}$$

$$B = \frac{\mu_0 i}{2\pi}\int_0^{\infty}\frac{R\,ds}{\left(s^2 + R^2\right)^{3/2}} = \frac{\mu_0 i}{2\pi R}\left[\frac{s}{\left(s^2 + R^2\right)^{1/2}}\right]_0^{\infty}$$

$$= \frac{\mu_0 i}{2\pi R}\left[\frac{\lim\limits_{s\to\infty} s}{\left(\lim\limits_{s\to\infty} s^2 + R^2\right)^{1/2}} - 0\right] = \frac{\mu_0 i}{2\pi R}$$

B due to a long straight wire	$B = \dfrac{\mu_0 i}{2\pi R}$

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Integral for ~~Ste~~ 29.2 Magnetic field due to a long straight wire

$$\int \frac{ds}{(s^2+R^2)^{3/2}} = \int \frac{ds}{R^3\left(1+\left(\frac{s}{R}\right)^2\right)^{3/2}}$$

$$= \frac{1}{R^2}\int \frac{du}{(1+u^2)^{3/2}} \qquad u = \frac{s}{R}$$

$$u = \tan\theta$$

$$= \frac{1}{R^2}\int \frac{\sec^2\theta \, d\theta}{(\sec^2\theta)^{3/2}}$$

$$= \frac{1}{R^2}\int \frac{1}{\sec\theta}\, d\theta$$

$$= \frac{1}{R^2}\int \cos\theta\, d\theta = \frac{1}{R^2}\sin\theta$$

$$\tan\theta = \frac{s}{R} \quad \cancel{\cot\theta =} \quad \cot\theta = \frac{R}{s}$$

$$1+\cot^2\theta = \frac{1}{\sin^2\theta}$$

$$\sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+R^2/s^2}}$$

$$= \frac{s}{\sqrt{s^2+R^2}}$$

$$\int \frac{ds}{(s^2+R^2)^{3/2}} = \frac{1}{R^2}\sin\theta$$

$$= \frac{1}{R^2}\frac{s}{\sqrt{s^2+R^2}}$$

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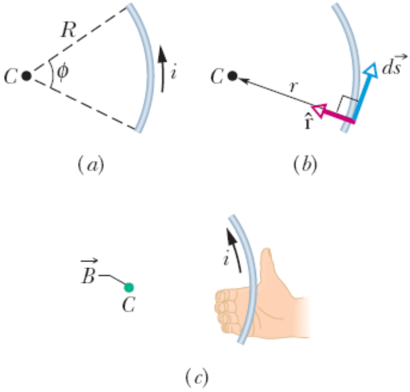
29.2: Magnetic Field due to a Current in a Circular Arc of Wire:



The right-hand rule reveals the field's direction at the center.

B field at the center of the circle due to the current element *ids* of the circular arc of current:

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2} \qquad d\vec{s} \perp \hat{r}$$

$$B = \int dB = \int_0^\phi \frac{\mu_0}{4\pi}\frac{i\,R\,d\phi}{R^2} = \frac{\mu_0 i}{4\pi R}\int_0^\phi d\phi \qquad ds = R\,d\phi$$

$$B = \frac{\mu_0 i\phi}{4\pi R}$$

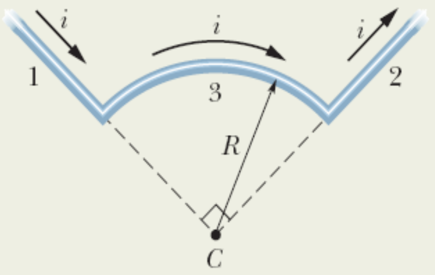
B due to the circular current arc of angle *f*

$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

B at center of a full circle of current.

Example, Magnetic field at the center of a circular arc of a circle.:

The wire in Fig. 29-7a carries a current i and consists of a circular arc of radius R and central angle $\pi/2$ rad, and two straight sections whose extensions intersect the center C of the arc. What magnetic field $\vec{B}$ (magnitude and direction) does the current produce at C?



(a)

Straight sections: For any current-length element in section 1, the angle θ between $d\vec{s}$ and $\hat{r}$ is zero (Fig. 29-7b); so Eq. 29-1 gives us

$$dB_1 = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2} = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 0}{r^2} = 0.$$

Thus, the current along the entire length of straight section 1 contributes no magnetic field at C:

$$B_1 = 0.$$

The same situation prevails in straight section 2, where the angle θ between $d\vec{s}$ and $\hat{r}$ for any current-length element is 180°. Thus,

$$B_2 = 0.$$

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Example, Magnetic field at the center of a circular arc of a circle.:

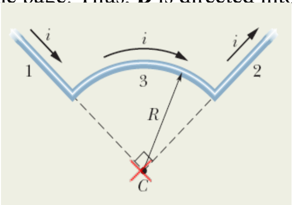
Circular arc: Application of the Biot–Savart law to evaluate the magnetic field at the center of a circular arc leads to Eq. 29-9 ($B = \mu_0 i\phi/4\pi R$). Here the central angle ϕ of the arc is $\pi/2$ rad. Thus from Eq. 29-9, the magnitude of the magnetic field $\vec{B}_3$ at the arc's center C is

$$B_3 = \frac{\mu_0 i(\pi/2)}{4\pi R} = \frac{\mu_0 i}{8R}.$$

Net field: Generally, when we must combine two or more magnetic fields to find the net magnetic field, we must combine the fields as vectors and not simply add their magnitudes. Here, however, only the circular arc produces a magnetic field at point C. Thus, we can write the magnitude of the net field $\vec{B}$ as

$$B = B_1 + B_2 + B_3 = 0 + 0 + \frac{\mu_0 i}{8R} = \frac{\mu_0 i}{8R}. \qquad \text{(Answer)}$$

To find the direction of **B**, we apply the right-hand rule displayed in the Fig. 29-4. Mentally grasp the circular arc with your right hand as in Fig. 29-7c, with your thumb in the direction of the current. The direction in which your fingers curl around the wire indicates the direction of the magnetic field lines around the wire. They form circles around the wire, coming our of the page above the arc and going into the page inside the arc. In the region of point C (inside the arc), your fingertips point into the plane of the page. Thus, **B** is directed into that plane.



(a)

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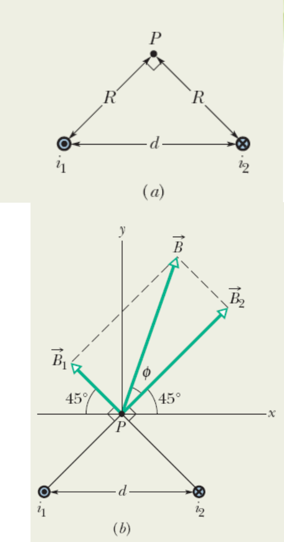
Example, Magnetic field off to the side of two long straight currents:

Figure 29-8*a* shows two long parallel wires carrying currents i_1 and i_2 in opposite directions. What are the magnitude and direction of the net magnetic field at point P? Assume the following values: $i_1 = 15$ A, $i_2 = 32$ A, and $d = 5.3$ cm.



Finding the vectors: In Fig. 29-8*a*, point P is distance R from both currents i_1 and i_2. Thus, Eq. 29-4 tells us that at point P those currents produce magnetic fields $\vec{B}_1$ and $\vec{B}_2$ with magnitudes

$$B_1 = \frac{\mu_0 i_1}{2\pi R} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi R}.$$

In the right triangle of Fig. 29-8*a*, note that the base angles (between sides R and d) are both 45°. This allows us to write $\cos 45° = R/d$ and replace R with $d \cos 45°$. Then the field magnitudes B_1 and B_2 become

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$

The two currents create magnetic fields that must be added as vectors to get the net field.

Example, Magnetic field off to the side of two long straight currents:

Adding the vectors: We can now vectorially add $\vec{B}_1$ and $\vec{B}_2$ to find the net magnetic field $\vec{B}$ at point P, either by using a vector-capable calculator or by resolving the vectors into components and then combining the components of $\vec{B}$. However, in Fig. 29-8*b*, there is a third method: Because $\vec{B}_1$ and $\vec{B}_2$ are perpendicular to each other, they form the legs of a right triangle, with $\vec{B}$ as the hypotenuse. The Pythagorean theorem then gives us

$$B_1 = \frac{\mu_0 i_1}{2\pi d \cos 45°} \quad \text{and} \quad B_2 = \frac{\mu_0 i_2}{2\pi d \cos 45°}.$$

$$B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0}{2\pi d(\cos 45°)}\sqrt{i_1^2 + i_2^2}$$

$$= \frac{(4\pi \times 10^{-7}\,\mathrm{T \cdot m/A})\sqrt{(15\,\mathrm{A})^2 + (32\,\mathrm{A})^2}}{(2\pi)(5.3 \times 10^{-2}\,\mathrm{m})(\cos 45°)}$$

$$= 1.89 \times 10^{-4}\,\mathrm{T} \approx 190\,\mu\mathrm{T}. \qquad \text{(Answer)}$$

The angle ϕ between the directions of $\vec{B}$ and $\vec{B}_2$ in Fig. 29-8*b* follows from

$$\phi = \tan^{-1}\frac{B_1}{B_2},$$

which, with B_1 and B_2 as given above, yields

$$\phi = \tan^{-1}\frac{i_1}{i_2} = \tan^{-1}\frac{15\,\mathrm{A}}{32\,\mathrm{A}} = 25°.$$

The angle between the direction of $\vec{B}$ and the x axis shown in Fig. 29-8*b* is then

$$\phi + 45° = 25° + 45° = 70°. \qquad \text{(Answer)}$$

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29.3: Force Between Two Parallel Wires:

B field due to wire a at a distance d.

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

Force on wire b due to the **B** field of a.

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a,$$

$$F_{ba} = i_b L B_a \sin 90° = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

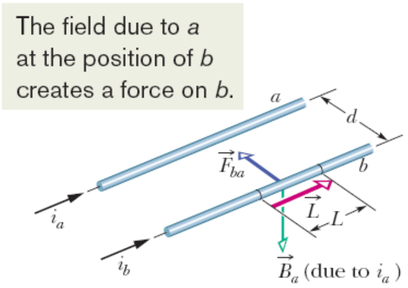

Fig. 29-9 Two parallel wires carrying currents in the same direction attract each other. $\vec{B}_a$ is the magnetic field at wire b produced by the current in wire a. $\vec{F}_{ba}$ is the resulting force acting on wire b because it carries current in $\vec{B}_a$.

To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

Parallel currents attract each other, and antiparallel currents repel each other.

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29.3: Force Between Two Parallel Wires, Rail Gun:

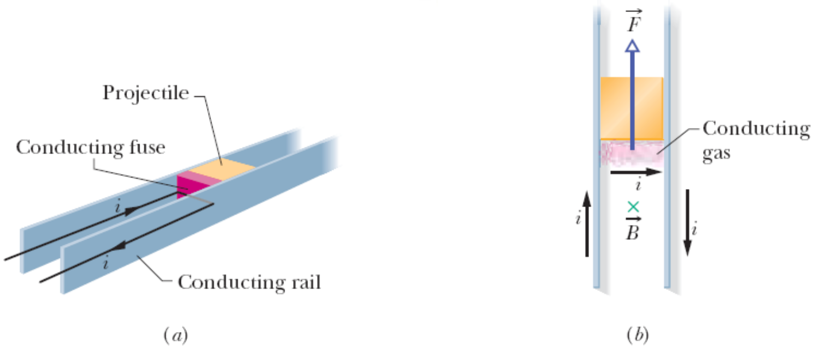

Fig. 29-10 (*a*) A rail gun, as a current i is set up in it. The current rapidly causes the conducting fuse to vaporize. (*b*) The current produces a magnetic field $\vec{B}$ between the rails, and the field causes a force $\vec{F}$ to act on the conducting gas, which is part of the current path. The gas propels the projectile along the rails, launching it.

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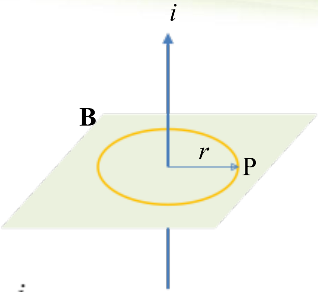
29.4: Ampere's Law:

Consider the magnetic field at point P from the long straight wire carrying current i.

$$B = \frac{\mu_0 i}{2\pi r}$$



Since **B** is tangent to the circle the dot product between **B** and d**s** is simply Bds.

$$\vec{B} \cdot d\vec{s} = Bds$$

If we integrate the dot product over the complete circle, then

$$\oint \vec{B} \cdot d\vec{s} = \frac{\mu_0 i}{2\pi r} \times 2\pi r = \mu_0 i$$

*The line integral of the **B** field around any closed loop is proportional to the electric current passing through the area enclosed by the loop.*

Ampere's Law: $$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

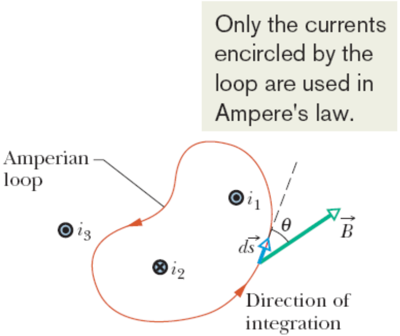


Fig. 29-11 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

29.4: Ampere's Law, Magnetic Field Outside a Long Straight Wire Carrying Current:

All of the current is encircled and thus all is used in Ampere's law.

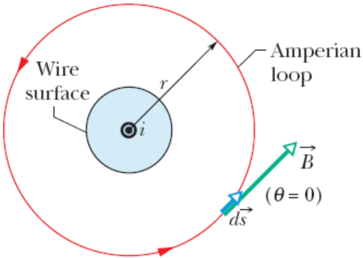


Fig. 29-13 Using Ampere's law to find the magnetic field that a current i produces outside a long straight wire of circular cross section. The Amperian loop is a concentric circle that lies outside the wire.

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = B \oint ds = B(2\pi r).$$

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r} \quad \text{(outside straight wire).}$$

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29.4: Ampere's Law, Magnetic Field Inside a Long Straight Wire Carrying Current:

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r).$$

$$i_{\text{enc}} = i \frac{\pi r^2}{\pi R^2}.$$

$$B(2\pi r) = \mu_0 i \frac{\pi r^2}{\pi R^2}$$

$$B = \left(\frac{\mu_0 i}{2\pi R^2} \right) r \qquad \text{(inside straight wire).}$$

Only the current encircled by the loop is used in Ampere's law.

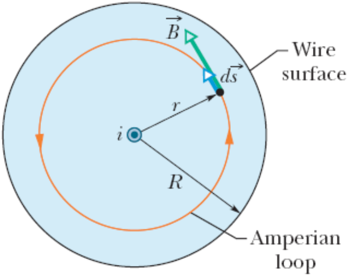
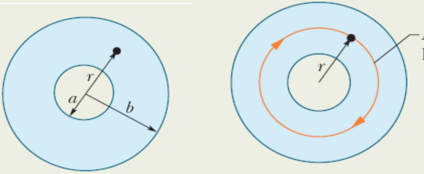


Fig. 29-14 Using Ampere's law to find the magnetic field that a current i produces inside a long straight wire of circular cross section. The current is uniformly distributed over the cross section of the wire and emerges from the page. An Amperian loop is drawn inside the wire.

Example, Ampere's Law to find the magnetic field inside a long cylinder of current.

Figure 29-15*a* shows the cross section of a long conducting cylinder with inner radius $a = 2.0$ cm and outer radius $b = 4.0$ cm. The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by $J = cr^2$, with $c = 3.0 \times 10^6$ A/m^4 and r in meters. What is the magnetic field $\vec{B}$ at the dot in Fig. 29-15*a*, which is at radius $r = 3.0$ cm from the central axis of the cylinder?



(*a*) (*b*)

Calculations: We write the integral as

$$i_{enc} = \int J\,dA = \int_a^r cr^2(2\pi r\,dr)$$

$$= 2\pi c \int_a^r r^3\,dr = 2\pi c\left[\frac{r^4}{4}\right]_a^r$$

$$= \frac{\pi c(r^4 - a^4)}{2}.$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc},$$

gives us

$$B(2\pi r) = -\frac{\mu_0 \pi c}{2}(r^4 - a^4).$$

Solving for B and substituting known data yield

$$B = -\frac{\mu_0 c}{4r}(r^4 - a^4)$$

$$= -\frac{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(3.0 \times 10^6\ \mathrm{A/m^4})}{4(0.030\ \mathrm{m})}$$

$$\times [(0.030\ \mathrm{m})^4 - (0.020\ \mathrm{m})^4]$$

$$= -2.0 \times 10^{-5}\ \mathrm{T}.$$

Thus, the magnetic field $\vec{B}$ at a point 3.0 cm from the central axis has magnitude

$$B = 2.0 \times 10^{-5}\ \mathrm{T} \qquad \text{(Answer)}$$

29.5: Solenoids and Toroids:

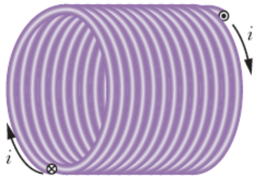


Fig. 29-16 A solenoid carrying current i.

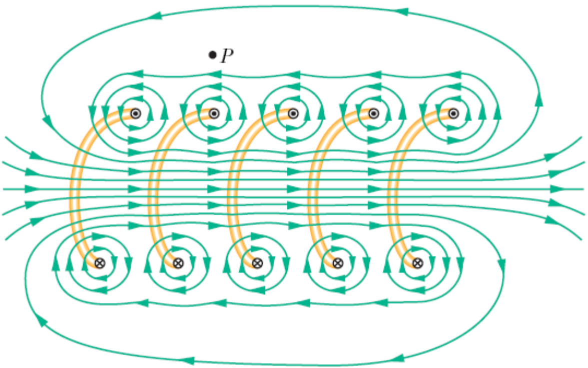
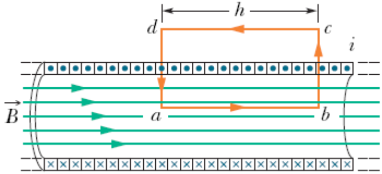


Fig. 29-17 A vertical cross section through the central axis of a "stretched-out" solenoid. The back portions of five turns are shown, as are the magnetic field lines due to a current through the solenoid. Each turn produces circular magnetic field lines near itself. Near the solenoid's axis, the field lines combine into a net magnetic field that is directed along the axis. The closely spaced field lines there indicate a strong magnetic field. Outside the solenoid the field lines are widely spaced; the field there is very weak.

29.5: Solenoids:

Fig. 29-19 Application of Ampere's law to a section of a long ideal solenoid carrying a current *i*. The Amperian loop is the rectangle *abcda*.



$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\mathrm{enc}},$$

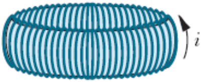
$$\oint \vec{B} \cdot d\vec{s} = \int_a^b \vec{B} \cdot d\vec{s} + \int_b^c \vec{B} \cdot d\vec{s} + \int_c^d \vec{B} \cdot d\vec{s} + \int_d^a \vec{B} \cdot d\vec{s}.$$

$i_{\mathrm{enc}} = i(nh).$ Here n be the number of turns per unit length of the solenoid

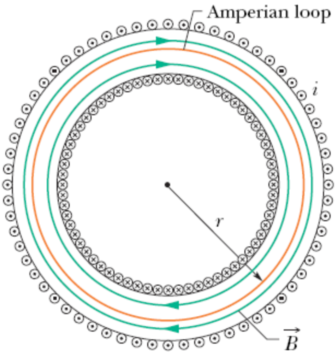
$$Bh = \mu_0 inh$$

$$B = \mu_0 in \qquad \text{(ideal solenoid).}$$

29.5: Magnetic Field of a Toroid:



(a)



(b)

Fig. 29-20 (*a*) A toroid carrying a current *i*. (*b*) A horizontal cross section of the toroid. The interior magnetic field (inside the bracelet-shaped tube) can be found by applying Ampere's law with the Amperian loop shown.

$$(B)(2\pi r) = \mu_0 i N,$$

where *i* is the current in the toroid windings (and is positive for those windings enclosed by the Amperian loop) and *N* is the total number of turns. This gives

$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r} \qquad \text{(toroid).}$$

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Example, The field inside a solenoid:

A solenoid has length $L = 1.23$ m and inner diameter $d = 3.55$ cm, and it carries a current $i = 5.57$ A. It consists of five close-packed layers, each with 850 turns along length L. What is B at its center?

KEY IDEA

The magnitude B of the magnetic field along the solenoid's central axis is related to the solenoid's current i and number of turns per unit length n by Eq. 29-23 ($B = \mu_0 in$).

Calculation: Because B does not depend on the diameter of the windings, the value of n for five identical layers is simply five times the value for each layer. Equation 29-23 then tells us

$$B = \mu_0 in = (4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(5.57\ \mathrm{A})\frac{5 \times 850\ \text{turns}}{1.23\ \mathrm{m}}$$

$$= 2.42 \times 10^{-2}\ \mathrm{T} = 24.2\ \mathrm{mT}. \qquad \text{(Answer)}$$

To a good approximation, this is the field magnitude throughout most of the solenoid.

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29.6: A Current Carrying Coil as a Magnetic Dipole:

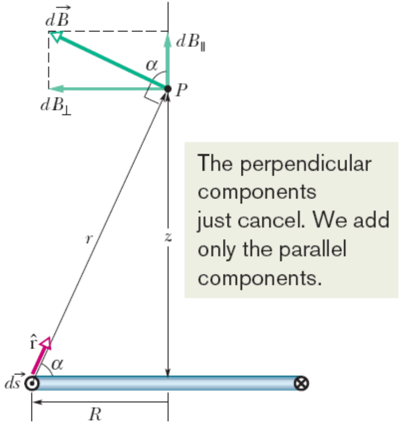

Fig. 29-22 Cross section through a current loop of radius R. The plane of the loop is perpendicular to the page, and only the back half of the loop is shown. We use the law of Biot and Savart to find the magnetic field at point P on the central perpendicular axis of the loop.

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{r^2}.$$

$$dB_{\parallel} = dB\cos\alpha. \quad = \frac{\mu_0 i\cos\alpha\,ds}{4\pi r^2}.$$

$$r = \sqrt{R^2 + z^2}$$

$$\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}.$$

$$dB_{\parallel} = \frac{\mu_0 iR}{4\pi(R^2 + z^2)^{3/2}}\,ds.$$

$$B = \int dB_{\parallel}$$

$$= \frac{\mu_0 iR}{4\pi(R^2 + z^2)^{3/2}}\int ds$$

$$B(z) = \frac{\mu_0 iR^2}{2(R^2 + z^2)^{3/2}}$$

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29.6: A Current Carrying Coil as a Magnetic Dipole:

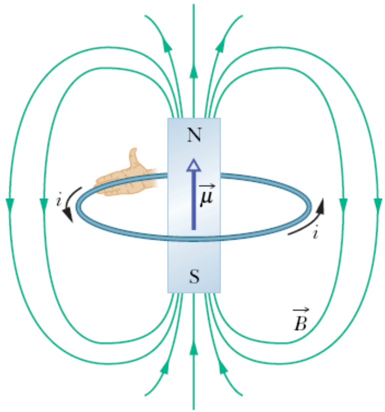

Fig. 29-21 A current loop produces a magnetic field like that of a bar magnet and thus has associated north and south poles. The magnetic dipole moment $\vec{\mu}$ of the loop, its direction given by a curled–straight right-hand rule, points from the south pole to the north pole, in the direction of the field $\vec{B}$ within the loop.

$$B(z) = \frac{\mu_0 i R^2}{2(R^2 + z^2)^{3/2}},$$

$$B(z) \approx \frac{\mu_0 i R^2}{2z^3}. \qquad z \gg R$$

$$B(z) = \frac{\mu_0}{2\pi}\frac{NiA}{z^3}.$$

Magnetic Dipole Moment of N current loops: $\mu = NiA = Ni\pi R^2$

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$

$$\vec{\mu} = Ni\vec{A}$$

B **due to the magnetic dipole**

29.7: Summary:

- Biot-Savart Law:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\, d\vec{s} \times \hat{r}}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

- **B** field of a long straight wire

$$B = \frac{\mu_0 i}{2\pi R}$$

- **B** field of a circular arc

$$B = \frac{\mu_0 i \phi}{4\pi R}$$

B due to the circular current arc of angle ***f***

$$B = \frac{\mu_0 i(2\pi)}{4\pi R} = \frac{\mu_0 i}{2R}$$

B at center of a full circle of current.

- Force between parallel currents

$$F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}$$

- Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$$

- **B** field of solenoid & Toroid

$$B = n\mu_0 i$$

solenoid

$$B = \frac{N\mu_0 i}{2\pi}\frac{1}{r}$$

toroid

- Magnetic Dipole

$$\vec{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$

Inductance: The First Experiment

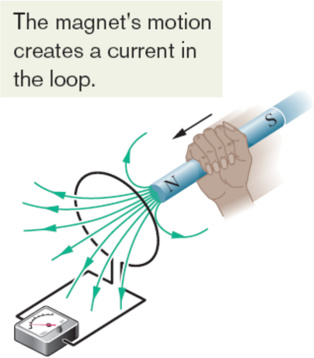

Fig. 30-1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

1. A current appears only if there is relative motion between the loop and the magnet (one must move relative to the other); the current disappears when
the relative motion between them ceases.

2. Faster motion produces a greater current.

3. If moving the magnet's north pole toward the loop causes, say, clockwise current, then moving the north pole away causes counterclockwise current.
Moving the south pole toward or away from the loop also causes currents, but in the reversed directions.

Inductance: The First Experiment

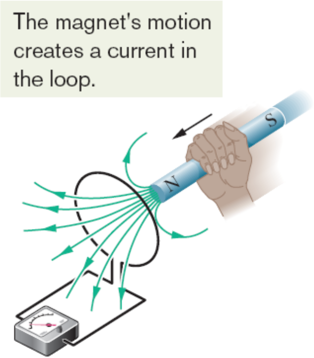

Fig. 30-1 An ammeter registers a current in the wire loop when the magnet is moving with respect to the loop.

1. The current thus produced in the loop is called ***induced current***.

2. The work done per unit charge to produce that current (to move the conduction electron that constitute the current) is called an ***induced emf***. *The same emf as in Lecture 8.*

3. ***emf*** – electromotive force and have the unit of volt (V)

4. The process of producing the current and emf is called ***induction***.

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For this experiment we use the apparatus of Fig. 30-2, with the two conducting loops close to each other but not touching. If we close switch S, to turn on a current in the right-hand loop, the meter suddenly and briefly registers a current—an induced current—in the left-hand loop. If we then open the switch, another sudden and brief induced current appears in the left hand loop, but in the opposite direction.

We get an induced current (and thus an induced emf) only when the current in the right-hand loop is changing (either turning on or turning off) and not when it is constant (even if it is large).

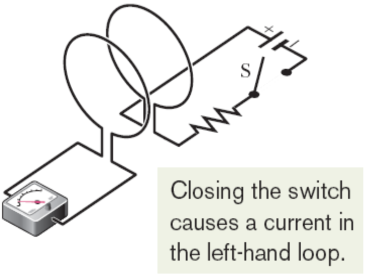

Fig. 30-2 An ammeter registers a current in the left-hand wire loop just as switch S is closed (to turn on the current in the right-hand wire loop) or opened (to turn off the current in the right-hand loop). No motion of the coils is involved.

Faraday's Law of Induction:

The magnitude of the emf $\mathscr{E}$ induced in a conducting loop is equal to the rate at which the magnetic flux Φ_B through that loop changes with time.

The ***magnetic flux*** is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} \qquad \text{(magnetic flux through area } A\text{)}.$$

If the loop lies in a plane and the magnetic field is perpendicular to the plane of the loop, and if the magnetic field is constant, then

$$\Phi_B = BA \qquad (\vec{B} \perp \text{area } A, \vec{B} \text{ uniform}).$$

The SI unit for magnetic flux is the tesla–square meter, which is called the *weber* (abbreviated *Wb*):

$$1 \text{ weber} = 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2.$$

$$\mathscr{E} = -\frac{d\Phi_B}{dt} \qquad \text{(Faraday's law)},$$

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Faraday's Law of Induction:

If we change the magnetic flux through a coil of N turns, an induced emf appears in every turn and the total emf induced in the coil is the sum of these individual induced emfs. If the coil is tightly wound (*closely packed*), so that the same magnetic flux Φ_B passes through all the turns, the total emf induced in the coil is

$$\mathscr{E} = -N\frac{d\Phi_B}{dt} \quad \text{(coil of } N \text{ turns).} \tag{30-5}$$

Here are the general means by which we can change the magnetic flux through a coil:

1. Change the magnitude B of the magnetic field within the coil.
2. Change either the total area of the coil or the portion of that area that lies within the magnetic field (for example, by expanding the coil or sliding it into or out of the field).
3. Change the angle between the direction of the magnetic field $\vec{B}$ and the plane of the coil (for example, by rotating the coil so that field $\vec{B}$ is first perpendicular to the plane of the coil and then is along that plane).

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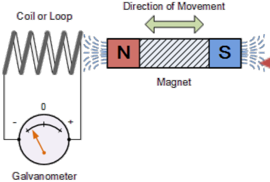
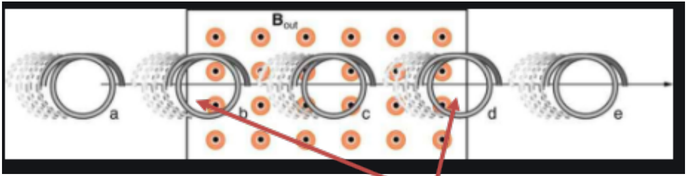
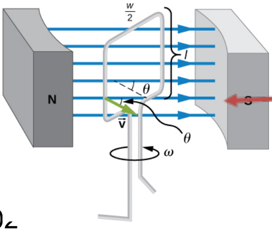


Change magnetic field



Move a coil into/out of a field

Only b and d has an induced current



Rotate a coil in a field

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30.4: Lenz's Law:

An induced current has a direction such that the magnetic field due to *the current* opposes the change in the magnetic flux that induces the current.

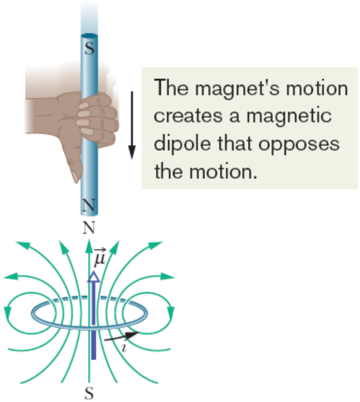
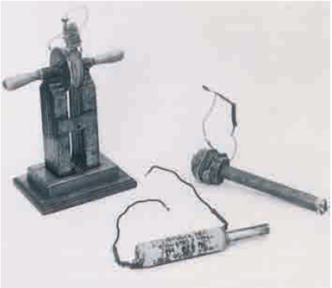


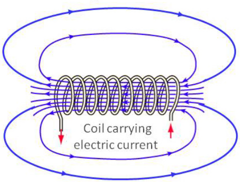
Fig. 30-4 Lenz's law at work. As the magnet is moved toward the loop, a current is induced in the loop. The current produces its own magnetic field, with magnetic dipole moment $\vec{\mu}$ oriented so as to oppose the motion of the magnet. Thus, the induced current must be counterclockwise as shown.

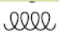
Opposition to Pole Movement. The approach of the magnet's north pole in Fig. 30-4 increases the magnetic flux through the loop, inducing a current in the loop. To oppose the magnetic flux increase being caused by the approaching magnet, the loop's north pole (and the magnetic moment ***m***) must face toward the approaching north pole so as to repel it. The current induced in the loop must be counterclockwise in Fig. 30-4. If we next pull the magnet away from the loop, a current will again be induced in the loop. Now, the loop will have a south pole facing the retreating north pole of the magnet, so as to oppose the retreat. Thus, the induced current will be clockwise.

Inductors and Inductance:



The crude inductors with which Michael Faraday discovered the law of induction. In those days amenities such as insulated wire were not commercially available. It is said that Faraday insulated his wires by wrapping them with strips cut from one of his wife's petticoats. *(The Royal Institution/Bridgeman Art Library/NY)*



An **inductor** (symbol ) can be used to *produce a desired magnetic field.*

If we establish a current *i* in the windings (turns) of the solenoid which can be treated as our inductor, the current produces a magnetic flux *FB* through the central region of the inductor.

The inductance of the inductor is then

$$L = \frac{N\Phi_B}{i} \qquad \text{(inductance defined)}$$

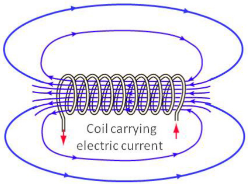
The SI unit of inductance is the tesla–square meter per ampere (T m2/A). We call this the henry (H), after American physicist Joseph Henry.

Inductance of a Solenoid:

Consider a long solenoid of cross-sectional area *A*, with number of turns *N*, and of length *l*. The flux is $N\Phi_B = (nl)(BA),$

Here *n* is the number of turns per unit length.

The magnitude of *B* is given by: $B = \mu_0 in,$



Therefore,

$$L = \frac{N\Phi_B}{i} = \frac{(nl)(BA)}{i} = \frac{(nl)(\mu_0 in)(A)}{i}$$
$$= \mu_0 n^2 lA.$$

The *inductance per unit length near the center* is therefore:

$$\frac{L}{l} = \mu_0 n^2 A \qquad \text{(solenoid).}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$
$$= 4\pi \times 10^{-7}\ \mathrm{H/m}.$$

Self-Induction:

An induced emf $\mathscr{E}_L$ appears in any coil in which the current is changing.

This process (see Fig. 30-13) is called **self-induction,** and the emf that appears is called a **self-induced emf.** It obeys Faraday's law of induction just as other induced emfs do.

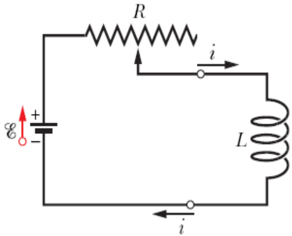


Fig. 30-13 If the current in a coil is changed by varying the contact position on a variable resistor, a self-induced emf $\mathscr{E}_L$ will appear in the coil *while the current is changing.*

$$N\Phi_B = Li.$$

$$\mathscr{E}_L = -\frac{d(N\Phi_B)}{dt}.$$

$$\mathscr{E}_L = -L\frac{di}{dt} \qquad \text{(self-induced emf)}.$$

This is defined in the direction of the current.

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