WEEK 3/4

CHAPTER ONE

- Points and Lines in the Plane

OUTLINE

- Foundations of Geometry
- Points on Cartesian Plane
- The gradient/slope of a line
- Distance between two points
- Internal and external division of a line segment
- An equation of a line
- Distance from a line to a point
- Parametric equation of a line
- Distance between two points and angle between direct line segments

FOUNDATIONS OF GEOMETRY

- Introduction to Proof
- Axioms, Axiomatic Systems
- Incidence Axioms for Geometry
- Axioms for points, lines and planes

INTRODUCTION TO PROOF

EXAMPLE

IF-THEN STATEMENTS, CONDITIONALS

EXAMPLE

EXAMPLE

LOGICALLY EQUIVALENT

DIRECT PROOFS

INDIRECT PROOFS

AXIOMS, AXIOMATIC SYSTEMS

- An axiomatic system always contains statements which are assumed without proof-the axioms. These axioms are chosen
  (a) for their convenience and efficiency
  (b) fir their consistency and, in some cases, (but not always)
  (c) for their plausibility
- Undefined terms
  Every axiom must, of necessity, contain some terms that have been purposely left without definitions- the undefined terms.

For example, in geometry, the most common undefined terms are “point” and “line.”
In reality, a point is a dot with physical dimension, but ideally in geometry, it has no dimension.
A line is that has length without width.

MODELS FOR AXIOMATIC SYSTEMS

- A Model for an axiomatic system is a realization of the axioms in some mathematical setting. All undefined terms are interpreted, and all the axioms are true.
- Independence and consistency in axiomatic systems
  An axiomatic system must be independent (every axiom is essential, none is a logical consequence of the others). and consistent (freedom from contradictions).

EXAMPLE

INCIDENCE AXIOMS FOR GEOMETRY

AXIOMS FOR POINTS, LINES AND PLANES

- Axiom I: Each two distinct points determine a line.

EXAMPLE TO PROOF A THEOREM BY AXIOM

FAMOUS AXIOMS

- Axiom II: Three noncollinear points determine a (unique) plane.
- Axiom III: If two points lie in a plane, then any line containing those two points lies in that plane.
- Axiom IV: If two distinct planes meet, their intersection is a line.

THE CARTESIAN PLANE

LENGTH OF A STRAIGHT LINE

EXAMPLE

MIDPOINT OF A STRAIGHT LINE

EXAMPLE

GRADIENT OF A STRAIGHT LINE

CALCULATING GRADIENT USING MEASUREMENT

CALCULATING GRADIENT FROM COORDINATES

EXAMPLE

POSITIVE AND NEGATIVE GRADIENT

GRADIENT OF HORIZONTAL AND VERTICAL LINES

GRADIENT OF PARALLEL LINES AND PERPENDICULAR LINES

EXAMPLE

DISTANCE BETWEEN TWO POINTS

DISTANCE FORMULA

DIVISION OF A LINE SEGMENT

FIRST CASE

INTERNAL DIVISION POINT

SECOND CASE

EXTERNAL DIVISION POINT

EXAMPLE

RECAP

LINES ON CARTESIAN PLANE

VERTICAL AND HORIZONTAL LINES

GRADIENT OF A STRAIGHT LINE

POSITIVE AND NEGATIVE GRADIENT

CALCULATING GRADIENT

GRADIENT OF HORIZONTAL AND VERTICAL LINES

GRADIENT OF PARALLEL AND PERPENDICULAR LINES

THE EQUATION OF A STRAIGHT LINE

EXAMPLE

HORIZONTAL AND VERTICAL LINES

LINEAR AND NON-LINEAR EQUATION

GRAPHING LINEAR EQUATION

TO DETERMINE THE EQUATION OF A STRAIGHT LINE

- Given the values of m and c
- Given the gradient and a point on the line
- Given two points on the line

GIVEN THE VALUES OF M AND C

GIVEN THE GRADIENT AND A POINT ON THE LINE

GIVEN TWO POINTS ON THE LINE

STRAIGHT LINE IN VARIOUS FORMS

- Point-Slope
- Slope-Intercept
- Line through two points (intercept-intercept form)

POINT-SLOPE

SLOPE-INTERCEPT

LINE THROUGH TWO POINTS (INTERCEPT-INTERCEPT)

EXAMPLE

PARALLEL STRAIGHT LINES

PERPENDICULAR STRAIGHT LINES (PROOF)

EXAMPLE

INTERSECTION OF TWO LINES

- 3 cases...

CASE 1

CASE 2

CASE 3

EXAMPLE

PUTTING IT ALL TOGETHER QUESTION

THE DISTANCE FROM A POINT TO A LINE

- Class activity…

EXAMPLE (PART A)

EXAMPLE (PART B)

THE CONSTRUCTION OF A FORMULA FOR THE DISTANCE TO A POINT FROM THE LINE

EXAMPLE (PART A)

EXAMPLE (PART B)

DEFINITION AND PROPERTIES OF BETWEENNESS

- Definition 3.15 (Betweenness) For any three points A,B and C, we say that B is between A and C, and we write A − B − C, iff A, B, and C are distinct, collinear points, and AB + BC = AC.
- Theorem 3.16 If A − B − C then C − B − A, and neither A − C − B nor B − A − C.

PROOF

PARAMETRIC EQUATION OF A LINE

EXAMPLE

VECTOR EQUATION OF A LINE AND DIRECTION OF A VECTOR

EXAMPLE

THE SCALAR PRODUCT OF TWO VECTORS

EXAMPLE

EXAMPLE

EXAMPLE