MA1301 **Name:** ____________________

Semester B 2021-22

HW 3

1. Let $A = (0, 1, -1)$ and $B = (1, 2, 0)$ be two points in a plane. Let X be a point between A and B such that $AX : XB = 2 : 1$.
   (a) Find $\vec{AB}$ and $\vec{AX}$.
   (b) Hence, find the coordinate of X by finding its position vector $\vec{OX}$.

2. Let A, B and C be three points in a plane such that $|\vec{AB}| = |\vec{AC}| = 4$ and $\vec{AB} \cdot \vec{AC} = 2$. Find the length of BC.

3. Find the value of $\vec{a} \times \vec{b}$ for each of following set of the vectors $\vec{a}$ and $\vec{b}$.
   (a) $\vec{a} = \vec{i} + 3\vec{j}$ and $\vec{b} = -2\vec{j} + 5\vec{k}$;
   (b) $\vec{a} = \vec{i} + \vec{j} - 2\vec{k}$ and $\vec{b} = -3\vec{i} + 2\vec{j} + 5\vec{k}$;
   (c) $\vec{a} = -3\vec{i} + \vec{j} + 3\vec{k}$ and $\vec{b} = 6\vec{j} + \vec{k}$;
   (d) $\vec{a} = \vec{j} + \vec{k}$ and $\vec{b} = 3\vec{i} - \vec{j} + 2\vec{k}$.

4. Let $A = (1, 2, 0)$, $B = (3, -1, -2)$ and $C = (-2, 0, 1)$ be three points in the plane.
   (a) Find a vector which is perpendicular to both $\vec{AB}$ and $\vec{AC}$;
   (b) Let $\vec{a}$ be a vector with same magnitude as that of $\vec{BC}$ and it is perpendicular to both vectors $\vec{AB}$ and $\vec{AC}$. Find the vector $\vec{a}$;
   (c) Find the equation of the plane containing the points A, B and C.

5. Determine if each of the following set of vectors are linearly independent.
   (a) $\vec{a} = \vec{i} - 2\vec{j}$ and $\vec{b} = 2\vec{i} + \vec{j}$;
   (b) $\vec{a} = \vec{i} - 2\vec{j} + 3\vec{k}$, $\vec{b} = 2\vec{i} + 5\vec{j} + \vec{k}$ and $\vec{c} = 3\vec{i} + 2\vec{j} - 3\vec{k}$;
   (c) $\vec{a} = \vec{i} + 2\vec{j} - 5\vec{k}$, $\vec{b} = -\vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{c} = 3\vec{i} - 2\vec{j} + \vec{k}$.

6. Let z_1 and z_2 be two complex numbers, show that
$$|z_1 + z_2|^2 - |z_1 - z_2|^2 = 4\mathrm{Re}(\bar{z}_1 z_2).$$

7. Solve the following equations:
   (a) $z^6 = -3 + \sqrt{3}i$;
   (b) $(1 - z)^7 + (1 + z)^7 = 0$;
   (c) $z^{10} - 5z^5 - 6 = 0$.

8. (a) By considering the expression $(\cos\theta + i\sin\theta)^5$, show that
$$\cos 5\theta = 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta;$$
   (b) Using similar techniques as in (a), show that
$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

9. Using Gaussian Elimination, solve the following system:

(a)

$$\begin{cases} x - y + 3z = 15 \\ -3x + 2y + z = 4 \\ 2x - 3y + 2z = 9 \end{cases}$$

(b)

$$\begin{cases} 2x + y - 3z = 12 \\ 4x + z = 5 \\ 3x - y + 2z = 1 \end{cases}$$

10. Consider the system

$$\begin{cases} 2x + y - bz = 3 \\ ay - z = 2 \\ -2x + 5y = 1 \end{cases} \tag{0.1}$$

Find all possible values of a and b such that the system

(a) has unique solution;

(b) has infinitely many solutions;

(c) has no solution.

11. Find the inverse (if exists) of each of the following matrix using Gauss-Jordan method:

(a)

$$A = \begin{bmatrix} 1 & 2 \\ -3 & 4 \end{bmatrix}$$

(b)

$$A = \begin{bmatrix} 3 & 1 \\ -6 & 4 \end{bmatrix}$$

(c)

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 3 \\ 4 & 2 & 1 \end{bmatrix}$$

(d)

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 5 & 5 & 1 \end{bmatrix}$$