Significant digits and Angles in Physics

Significant digits

- All measurement have accuracy which depends on the instruments you use.
- For example, this ruler gives $\Delta x = 2.89 - 0.20 = 2.69$

The last digit is about accuracy

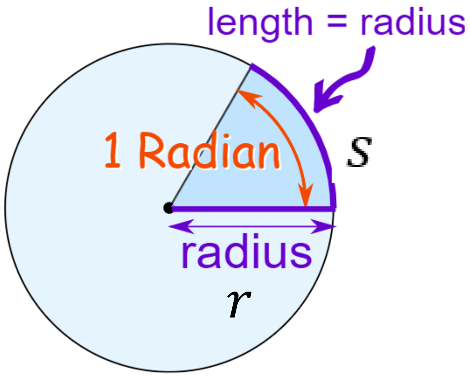
- In Physics, we only write down digits that have significance.
- We keep the same number of significant digits after calculation by carrying over the extra digits: $(\Delta x)^2 = (2.69)^2 = 7.2361 = 7.24$

Angles in Physics

- In physics, the angle is measured in radian not in degree

$$\theta \equiv \frac{Arc\ length}{radius} = \frac{s}{r}$$

$$s = r\theta$$



Degrees	Radians (exact)	Radians (approx)
30°	$\pi/6$	0.524
45°	$\pi/4$	0.785
60°	$\pi/3$	1.047
90°	$\pi/2$	1.571
180°	π	3.142
270°	$3\pi/2$	4.712
360°	2π	6.283

- One round turn = 360 degree
- One round turn $= \frac{2\pi r}{r} = 2\pi = 6.283$ rad

$$1 \text{ rad} = \frac{360}{6.283} = 57.3 \text{ degree}$$

Chapter 2 Motion

Part1: Motion Along a Straight Line

Topics for Chapter 2 part 1

- What is motion? Why do we need to understand motion?
- How do we describe motion? Position, velocity, acceleration
- One dimensional motion in terms of velocity and acceleration
- Instantaneous and average velocity (acceleration). Differences.

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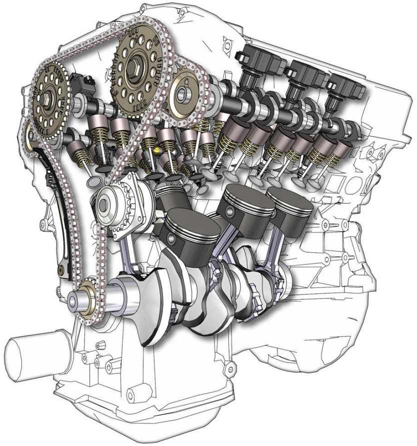
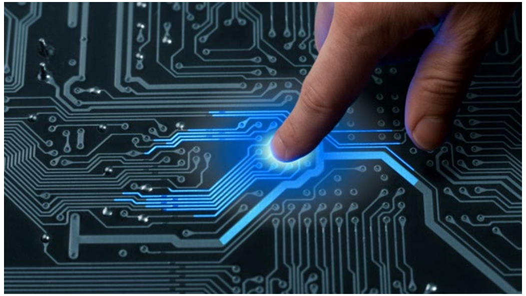
What is motion?

- Motion is: change of position with time (position as function of time)
- Motion of basket ball, missile & train : change of the basket ball position as a function of time
- Change of missile and train position with time

Why do we need to study motion/mechanics?

- Design of engine, moving components, car, trains,
- Understand human motion in sport
- Design robots, artificial limbs
- Describe electron motion in electronic devices



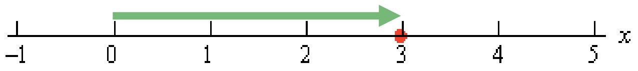


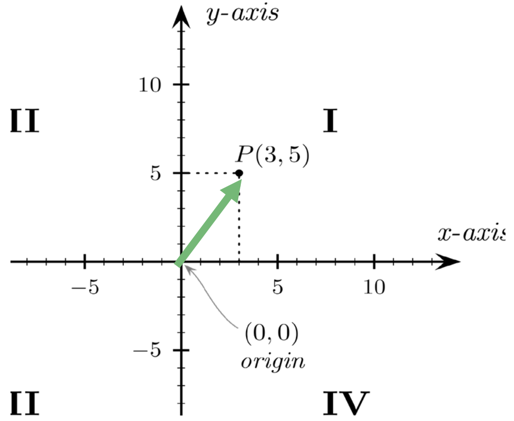
Description of Motion by physical quantities

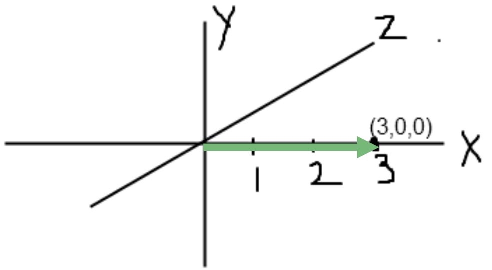
- Mechanics is the study of motion and the cause of motion (change of position with time).
- The objective is to find the position as a function of time
- *Velocity* and *acceleration* are other physical quantities used to describe motion apart from position and time
- The reason we study *velocity* and *acceleration* is because they are directly related to the cause of motion – forces/energy
- Then, from velocity and acceleration, we can deduce the position (coordinates) as a function of time.

How is position described?

- First thing to do is to set up a coordinate system (origin and rulers).
- Position is described by coordinates or position vector (a vector that takes you from the origin to the position)
- One dimension: x coordinate on the x-axis
- Two dimension: two coordinates: x, y and position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath}$
- Three dimension: three coordinates: x, y, z, position vector $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$







How do we describe motion?

- Motion is change of position with time,

- Write coordinates as a function of time describes the motion

  1Dimension: x as a function of t, $x = x(t)$

  2Dimensin: x(t) and y(t); $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath}$

  3Dimension: x(t), y(t) and z(t); $\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$

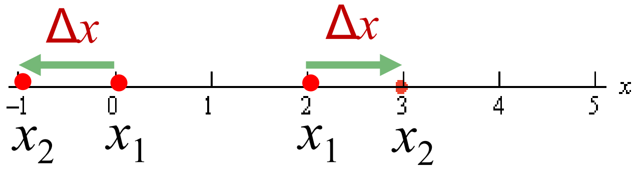
- Apart from position, we also want to know: how fast is the motion, how fast motion is changed, so we need…

  Velocity (how fast the motion is),

  Acceleration (how fast the motion is changed)

Displacement (vector)

- A particle moving along the x-axis, its coordinate $x(t)$ *changes with time* t.



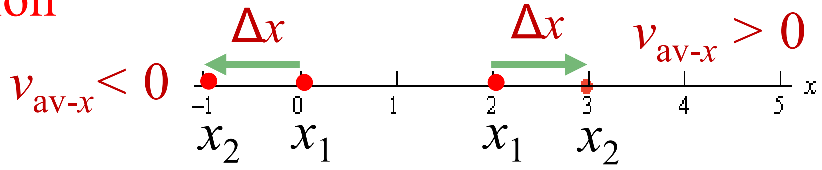
- The change in the particle's position (coordinate) is $\Delta x = x_2 - x_1$ in time Δt.

- Δx is called the displacement. Δx can be positive, moving to the right or negative, moving to the left. Thus displacement is a vector

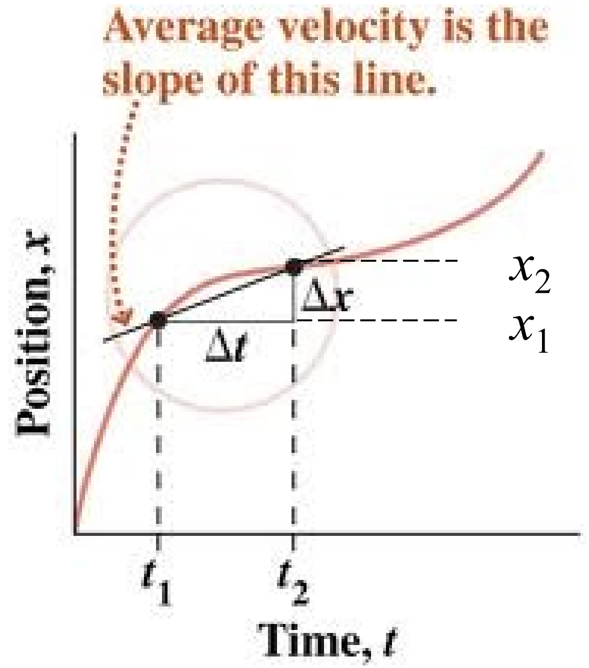
Displacement and Average Velocity (vector)

- *x(t) is a function of time*, as shown in the *x-t* graph.
- The average *x*-velocity of the particle in time Δt is

  $v_{av\text{-}x} = \Delta x/\Delta t$ = displacement divided by time.
- In the *x-t* graph, the average *x*-velocity $v_{av\text{-}x}$ is the slope of the line connecting two positions
- It describes how fast (rate) the particle moves in time Δt and direction



- You can find displacement/position from average velocity using $\Delta x = v_{av\text{-}x}\Delta t$ or $x_2 = x_1 - v_{av\text{-}x}\Delta t$



A position-time graph (an *x-t* graph) shows the particle's position *x* as a function of time *t*.

Instantaneous velocity

- Average velocity is for a time interval Δt, so it is not accurate enough, we want velocity at an instance (at a particular time t)

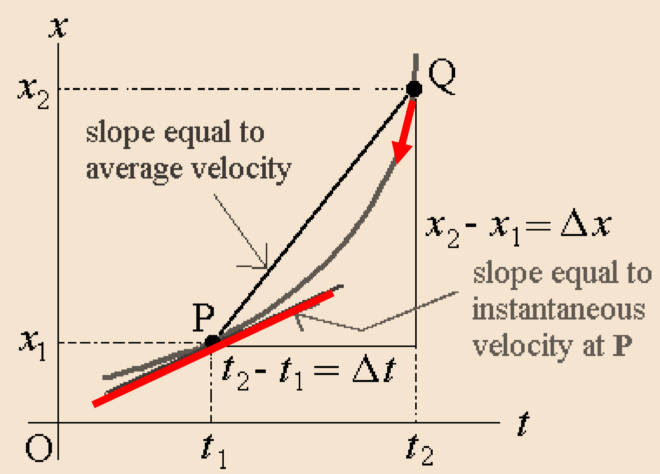
- The *instantaneous velocity* is the velocity at a specific instant of time or specific point along the path and is given by $v_x = dx/dt$. *(differentiation of x with respect to t)*

Uniform velocity, Constant velocity, Uniform motion means the instantaneous velocity is a constant. So, the average velocity equals the instantaneous velocity and is also a constant.

Finding instantaneous velocity on an *x-t* graph

- Let us look at the average velocity in a small time interval Δt:
  Slope of line PQ is the average velocity $v_x = \Delta x / \Delta t$
- Now let $\Delta t \to 0$, or Q approach P. PQ becomes the tangent at P, and

$$v_{\mathrm{av}-x} = \frac{\Delta x}{\Delta t} \Rightarrow \frac{dx}{dt} \equiv v_x \text{ when } \Delta t \to 0$$



$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The average velocity becomes the instantaneous velocity, which is the differentiation of $x(t)$ with respect to t:
- Instantaneous velocity is the slope of the tangent at P.

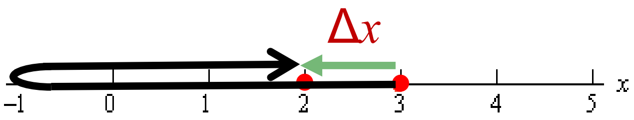
Distance and Average Speed (scalar)

- Displacement Δx can be positive or negative, depending on the direction of travel, but distance D is the total length of travel and is the sum of the lengths of each segment.

- Distance D is always positive and is a scalar.

$$\Delta x = 2 - 3 = -1$$

$$D = 4 + 3 = 7$$



- Average Speed v_{av} (scalar) = $D/\Delta t$.

- Instantaneous Speed v (scalar) = lim $|\Delta x|/\Delta t = |v_x|$ = magnitude of velocity.

Average acceleration and Instantaneous acceleration

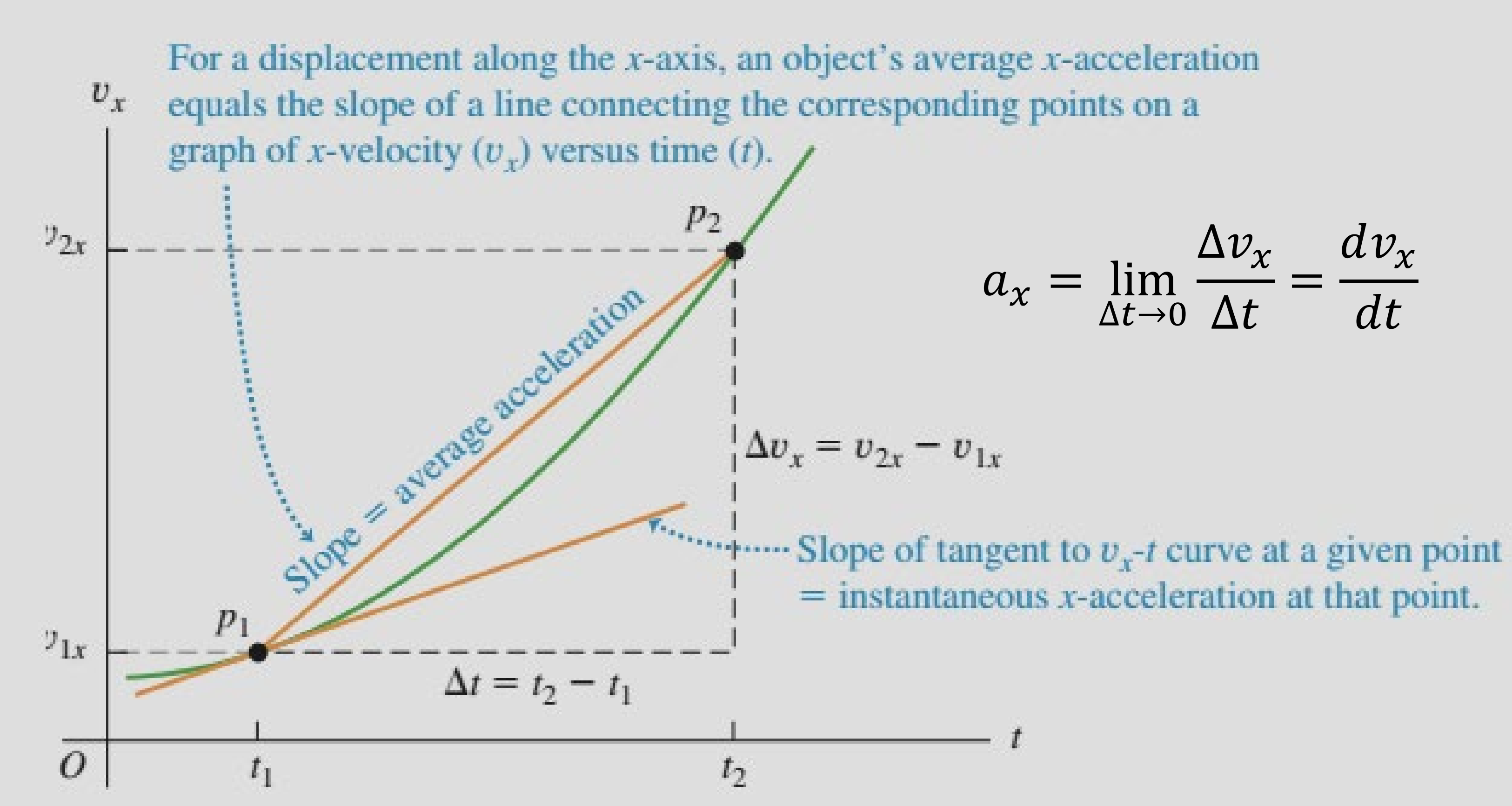
- Average acceleration describes the average rate of change of velocity with time in a time interval Δt, *describes how fast the motion is changed*
- The *average x*-acceleration is $a_{av\text{-}x} = \Delta v_x/\Delta t$.
- The *instantaneous* acceleration is $a_x = dv_x/dt$ or *average acceleration when* Δt *go to zero:*

$$a_x = \frac{dv_x}{dt} = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t}$$

You can find the change in velocity from the average acceleration: $\Delta v_x = a_{av\text{-}x}\Delta t$

Uniform acceleration, constant acceleration means the instantaneous and average acceleration are constants.

Find acceleration from velocity-time graph



$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

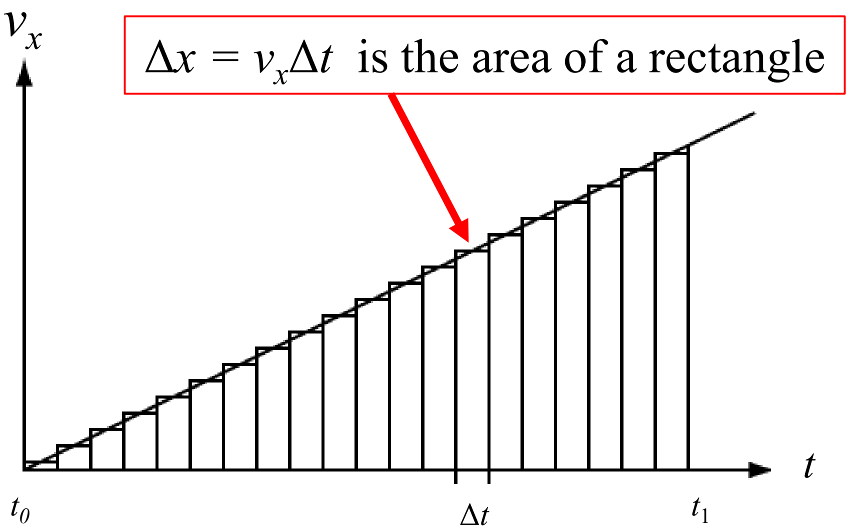
Find x from v_x: Area under the v_x- t curve gives the displacement x

- Divide the area under the curve into small slices, the area of a small slice is approximately by the area of a rectangle

- $\Delta x = v_x \Delta t$, is the area of a rectangle $=$ the displacement in time Δt

- Sum of the area of all the rectangles $=$ approximately the area under the curve, when slices are thin.

- $\sum \Delta x = \sum v_x \Delta t$ = total displacement = area under the curve:

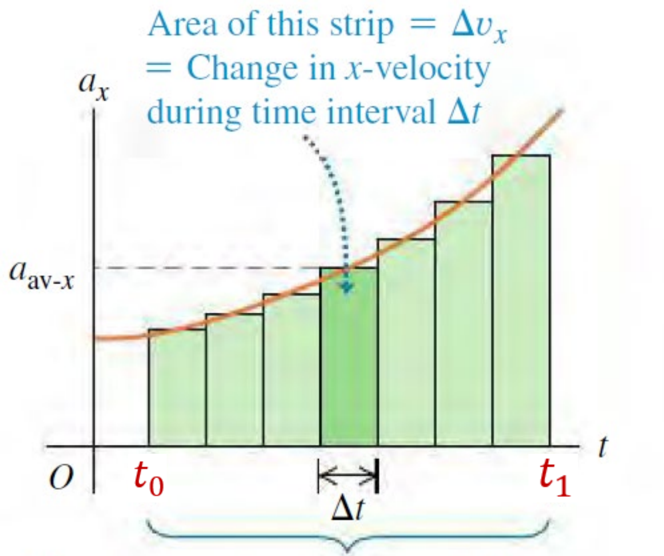


$$x_1 - x_0 = \int_{t_0}^{t_1} v_x dt$$

Find v_x from a_x: Area under the a_x- t curve gives the velocity v_x

- Likewise, area under a_x - t curve is the change in velocity

$$v_{x1} - v_{x0} = \int_{t_0}^{t_1} a_x dt$$



Total area under the x-t graph from t_0 to t_1
= Net change in x-velocity from t_0 to t_1

Special Case: constant acceleration motion

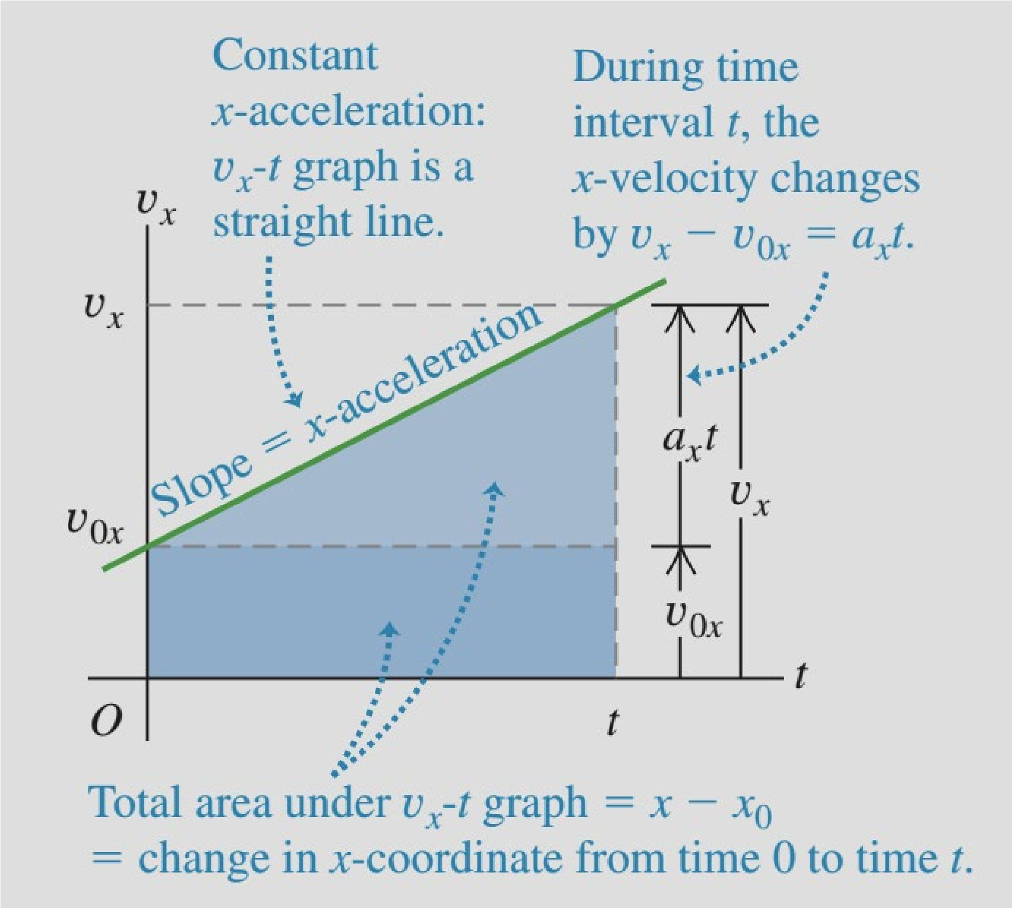
Motion with constant acceleration

- For a particle with constant acceleration, the velocity changes at the same rate throughout the motion:

$$\frac{\Delta v_x}{\Delta t} = a_x = \text{const.}$$

- Velocity-time graph is a straight line

$$v_x - v_{0x} = a_x t$$



The equations of motion with constant acceleration

- There are four equations shown to the right for any straight-line motion with constant acceleration a_x.
- First equation gives the velocity at time t
- Second equation gives the position of the particle at time t

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x \left(x - x_0 \right)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t$$

- Third equation gives you the velocity at t in terms of the initial velocity, acceleration and the distance travelled (displacement between t=0 and t).

- Fourth equation gives the distance travelled in terms of initial velocity, final velocity and time spent.

$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t$$

Derivation of the four equations

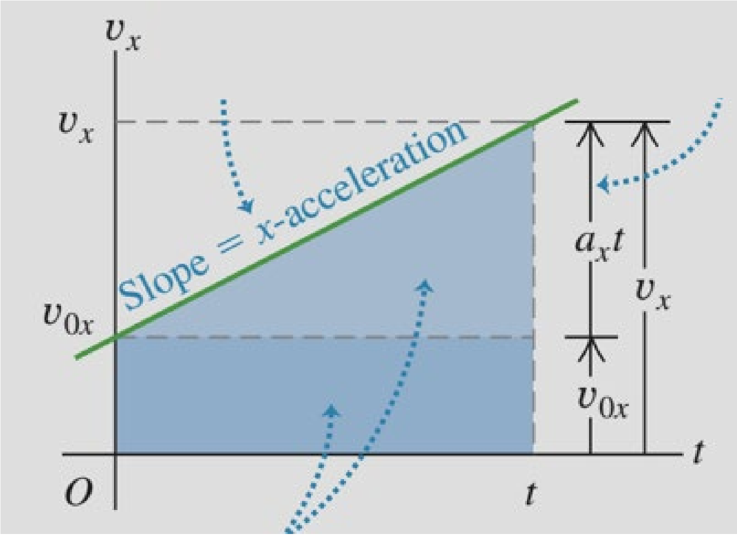
- The 1st , 2nd and 4th equations can be derived using the velocity-time graph.
- 3rd can be derived from 1st and 2nd or 4th.

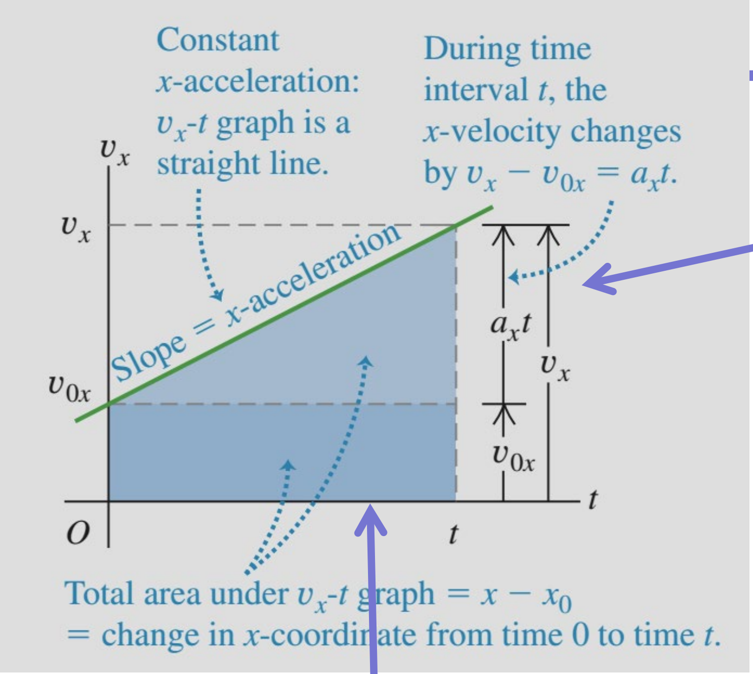
$$v_x = v_{0x} + a_x t$$

$$x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$$

$$v_x^2 = v_{0x}^2 + 2a_x (x - x_0)$$

$$x - x_0 = \left(\frac{v_{0x} + v_x}{2} \right) t$$





4th & 2nd eqt.

displacement = $x - x_0$ = Area = $t\,(v_x + v_{0x})/2$

(use expression for area of a trapezoid)

$x - x_0 = (v_{0x} + \boxed{v_{0x} + a_x t})\,t/2 = v_{0x} t + a_x t^2/2$

1st eqt.

$a_x t = v_x - v_{0x}$

$v_x = v_{0x} + a_x t$

3rd eqt.: $v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

1st: $v_x = v_{0x} + a_x t$

$\Rightarrow t = (v_x - v_{0x})/a_x$

Substitute it into

2nd : $x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2$

$$\Rightarrow x - x_0 = \frac{v_{0x}(v_x - v_{0x})}{a_x} + \frac{1}{2}(v_x - v_{0x})^2 / a_x$$

Multiply both sides by $2a_x$

$\Rightarrow 2a_x(x - x_0) = v_x^2 - v_{0x}^2$

$\Rightarrow v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

Another way

1st: $v_x = v_{0x} + a_x t$

$\Rightarrow a_x t = v_x - v_{0x}$

4th : $2(x - x_0) = (v_x + v_{0x})t$

Multiply both sides together

$2a_x(x - x_0)t = (v_x^2 - v_{0x}^2)t$

Cancel t:

$\Rightarrow v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$

Example of using calculus in constant acceleration

The example below show how calculus is used to find velocity of constant acceleration motion

Differentiation is used to find velocity from position, integration is used to find position vs time from acceleration

Differentiation is needed in exam, integration not needed

Use differentiation

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$$

$$v_x = \frac{dx}{dt};\ \frac{dx_0}{dt} = 0,\ \frac{d(v_{0x}t)}{dt} = v_{0x},\ \frac{d(a_x t^2/2)}{dt} = a_x t$$

$$v_x = \frac{dx}{dt} = v_{0x} + a_x t$$

needed in exam,

Use integration

$$v_x = \int a_x\, dt = v_{0x} + a_x t \quad \text{because } a_x = \frac{dv_x}{dt}$$

$$x = \int v_x\, dt = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 \quad \text{because } v_x = \frac{dx}{dt}$$

no need for examination and exercise, just understand it

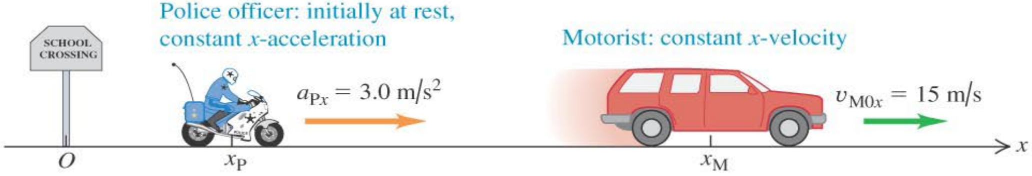
Two bodies with different accelerations

- Follow Example 2.5 in which the police officer and motorist have different accelerations.

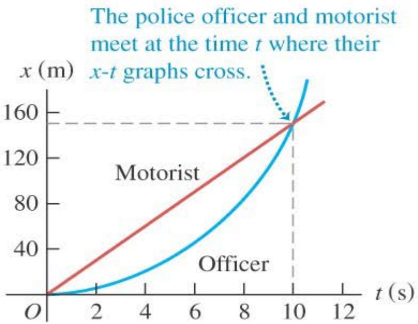
Example 2.5 **Two bodies with different accelerations**

A motorist traveling with a constant speed of 15 m/s (about 34 mi/h) passes a school-crossing corner, where the speed limit is 10 m/s (about 22 mi/h). Just as the motorist passes the school-crossing sign, a police officer on a motorcycle stopped there starts in pursuit with a constant acceleration of 3.0 m/s^2 (Fig. 2.21a). (a) How much time elapses before the officer passes the motorist? (b) What is the officer's speed at that time? (c) At that time, what distance has each vehicle traveled?

(a)



(b)



SOLUTION

IDENTIFY and SET UP: The officer and the motorist both move with constant acceleration (equal to zero for the motorist), so we can use the constant-acceleration formulas. We take the origin at the sign, so $x_0 = 0$ for both, and we take the positive direction to the right. Let x_P and x_M represent the positions of the officer and the motorist at any time; their initial velocities are $v_{P0x} = 0$ and $v_{M0x} = 15\ \text{m/s}$, and their accelerations are $a_{Px} = 3.0\ \text{m/s}^2$ and $a_{Mx} = 0$. Our target variable in part (a) is the time when the officer passes the motorist—that is, when the two vehicles are at the same position x; Table 2.4 tells us that Eq. (2.12) is useful for this part. In part (b) we're looking for the officer's speed v (the magnitude of his velocity) at the time found in part (a). We'll use Eq. (2.8) for this part. In part (c) we'll use Eq. (2.12) again to find the position of either vehicle at this same time.

Figure 2.21b shows an x-t graph for both vehicles. The straight line represents the motorist's motion, $x_M = x_{M0} + v_{M0x}t = v_{M0x}t$. The graph for the officer's motion is the right half of a concave–up parabola:

$$x_P = x_{P0} + v_{P0x}t + \frac{1}{2}a_{Px}t^2 = \frac{1}{2}a_{Px}t^2$$

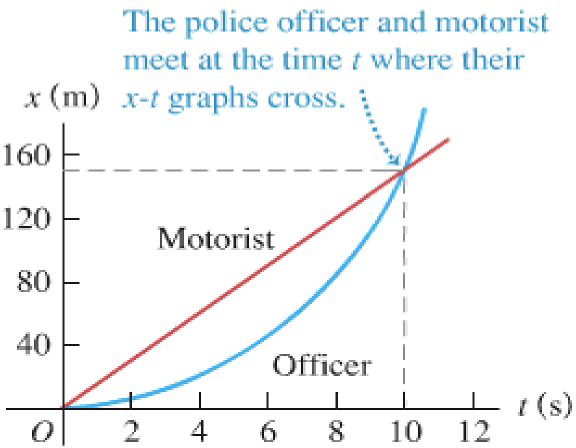
A good sketch will show that the officer and motorist are at the same position ($x_P = x_M$) at about $t = 10$ s, at which time both have traveled about 150 m from the sign.

$$x_{M0} = 0$$

$$x_{P0} = 0$$

$$v_{P0} = 0$$

(b)



EXECUTE: (a) To find the value of the time t at which the motorist and police officer are at the same position, we set $x_P = x_M$ by equating the expressions above and solving that equation for t:

$$v_{M0x}t = \frac{1}{2}a_{Px}t^2$$

$$t = 0 \qquad \text{or} \qquad t = \frac{2v_{M0x}}{a_{Px}} = \frac{2(15\ \text{m/s})}{3.0\ \text{m/s}^2} = 10\ \text{s}$$

Both vehicles have the same x-coordinate at *two* times, as Fig. 2.21b indicates. At $t = 0$ the motorist passes the officer; at $t = 10$ s the officer passes the motorist.

(b) We want the magnitude of the officer's x-velocity v_{Px} at the time t found in part (a). Substituting the values of v_{P0x} and a_{Px} into Eq. (2.8) along with $t = 10$ s from part (a), we find

$$v_{Px} = v_{P0x} + a_{Px}t = 0 + (3.0 \text{ m/s}^2)(10 \text{ s}) = 30 \text{ m/s}$$

The officer's speed is the absolute value of this, which is also 30 m/s.

(c) In 10 s the motorist travels a distance

$$x_M = v_{M0x}t = (15 \text{ m/s})(10 \text{ s}) = 150 \text{ m}$$

and the officer travels

$$x_P = \tfrac{1}{2}a_{Px}t^2 = \tfrac{1}{2}(3.0 \text{ m/s}^2)(10 \text{ s})^2 = 150 \text{ m}$$

This verifies that they have gone equal distances when the officer passes the motorist.

EVALUATE: Our results in parts (a) and (c) agree with our estimates from our sketch. Note that at the time when the officer passes the motorist, they do *not* have the same velocity. At this time the motorist is moving at 15 m/s and the officer is moving at 30 m/s. You can also see this from Fig. 2.21b. Where the two x-t curves cross, their slopes (equal to the values of v_x for the two vehicles) are different.

Is it just coincidence that when the two vehicles are at the same position, the officer is going twice the speed of the motorist? Equation (2.14), $x - x_0 = [(v_{0x} + v_x)/2]t$, gives the answer. The motorist has constant velocity, so $v_{M0x} = v_{Mx}$, and the distance $x - x_0$ that the motorist travels in time t is $v_{M0x}t$. The officer has zero initial velocity, so in the same time t the officer travels a distance $\frac{1}{2}v_{Px}t$. If the two vehicles cover the same distance in the same amount of time, the two values of $x - x_0$ must be the same. Hence when the officer passes the motorist $v_{M0x}t = \frac{1}{2}v_{Px}t$ and $v_{Px} = 2v_{M0x}$—that is, the officer has exactly twice the motorist's velocity. Note that this is true no matter what the value of the officer's acceleration.

Chapter 2 part 2

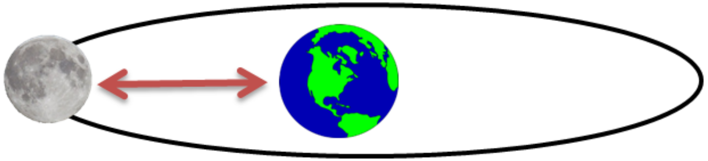
Motion in Two or Three Dimensions

Topics for chapter 2 part 2

- How do we describe motion in three or two dimension space? Use of vectors.
- The velocity vector of an object
- The acceleration vector of an object
- An example of two dimensional motion: the curved path of projectile
- Circular motion

Introduction

- In reality objects move in three dimension
- We need to extend our description of motion to two and three dimensions.



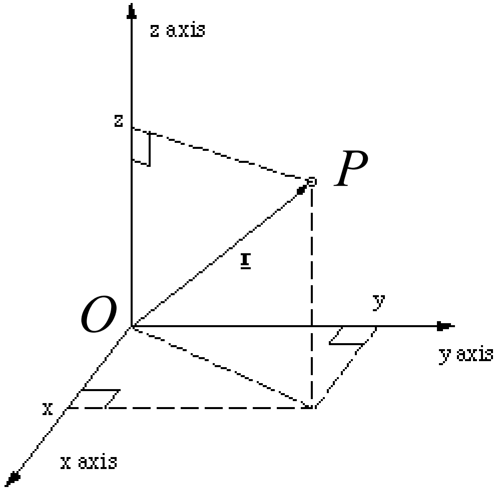
Position vector and coordinates

- Position of a point P (an object) is given by the coordinates or the position vector. Position vector tells you the position of the particle relative to the origin O.

- Position vector from the origin to point P has components x, y, and z. These components x, y, z are the coordinates of P

- Motion of P is just the position vector $\overrightarrow{OP} = \vec{r}$ or coordinates as a function of time:

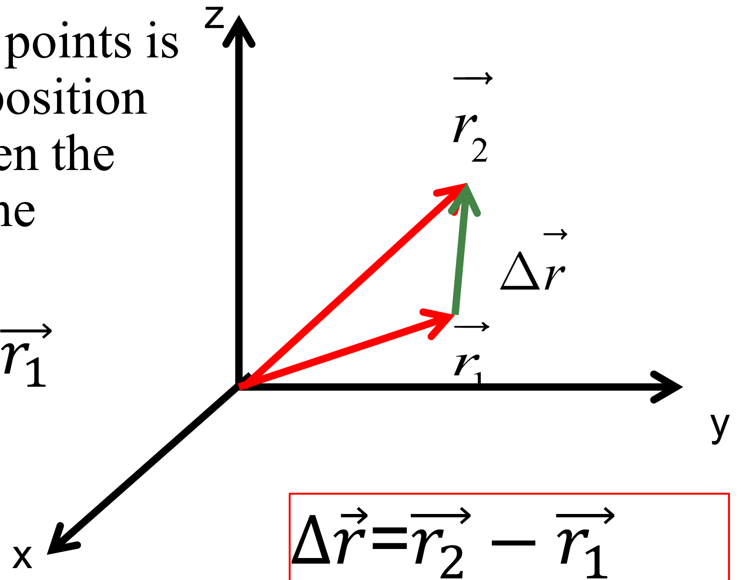
$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$



Average velocity—Figure 3.2

- The average velocity (a vector) between two points is the displacement (a vector) or difference of position vector $\Delta\vec{r}$ divided by the time interval between the two points, and it has the same direction as the displacement:

$$\vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} \parallel \Delta\vec{r} = \overrightarrow{r_2} - \overrightarrow{r_1}$$



- Write $\Delta\vec{r}$ in its components:

$$\Delta\vec{r} = \Delta x\hat{\imath} + \Delta y\hat{\jmath} + \Delta z\hat{k}$$

$$\rightarrow \quad \vec{v}_{av} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j} + \frac{\Delta z}{\Delta t}\hat{k}$$

$$\boxed{\begin{aligned} \Delta\vec{r} &= \overrightarrow{r_2} - \overrightarrow{r_1} \\ \Delta x &= x_2 - x_1 \\ \Delta y &= y_2 - y_1 \\ \Delta z &= z_2 - z_1 \end{aligned}}$$

Instantaneous velocity

- The *instantaneous velocity* is the instantaneous rate of change of position vector with respect to time. It is a vector.

The instantaneous velocity vector of a particle ...

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

... equals the limit of its average velocity vector as the time interval approaches zero ...

... and equals the instantaneous rate of change of its position vector.

Each component of a particle's instantaneous velocity vector ...

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

... equals the instantaneous rate of change of its corresponding coordinate.

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} + \frac{dz}{dt}\hat{k}$$

$$|\vec{v}| = v = \sqrt{{v_x}^2 + {v_y}^2 + {v_z}^2}$$

Instantaneous velocity (Graph)

Instantaneous velocity is always tangent to the path

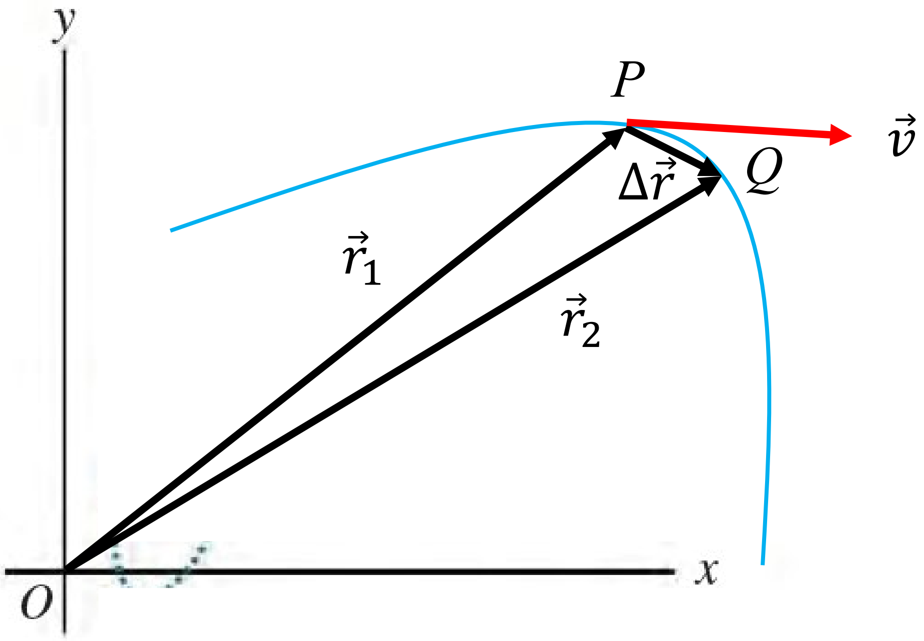
Consider a 2-D example of the blue path on the right:

Displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \overrightarrow{PQ}$

$\vec{v}_{av} = \Delta\vec{r}\ /\Delta t \parallel \Delta\vec{r}$

As $\Delta t \rightarrow 0$, Q → P and $\vec{v}_{av} \rightarrow \vec{v}$

But as $\Delta t \rightarrow 0$, $\overrightarrow{PQ}$ becomes tangent to the path



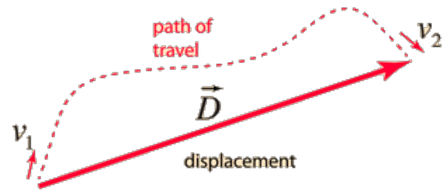
→Instantaneous velocity is always tangent to the path

Difference between speed and velocity

- Speed is a scalar, which is distance travelled divided by time
- Velocity is a vector = displacement/time

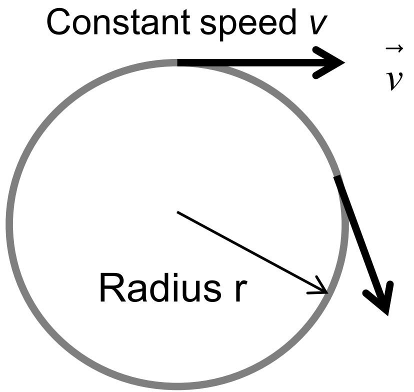
- Distance travelled can be larger than the displacement length
- Average speed may not equal average velocity
- Instantaneous speed is the magnitude of the velocity,

  i.e. the length of the velocity vector.



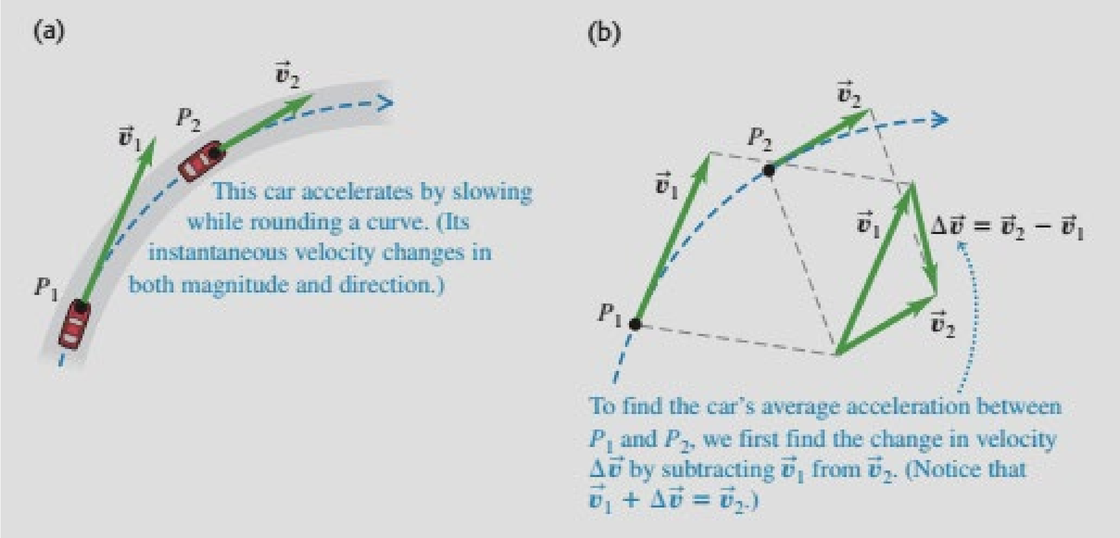
- Constant speed does not mean constant velocity

because velocity direction can change, while speed is constant (circular motion, come back to this later)



Average acceleration

- The *average acceleration* during a time interval Δt is defined as the velocity change during Δt divided by Δt.



$$\Delta\vec{v}=\overrightarrow{v_2}-\overrightarrow{v_1}$$
$$\Delta v_x = v_{x2} - v_{x1}$$
$$\Delta v_y = v_{y2} - v_{y1}$$
$$\Delta v_z = v_{z2} - v_{z1}$$

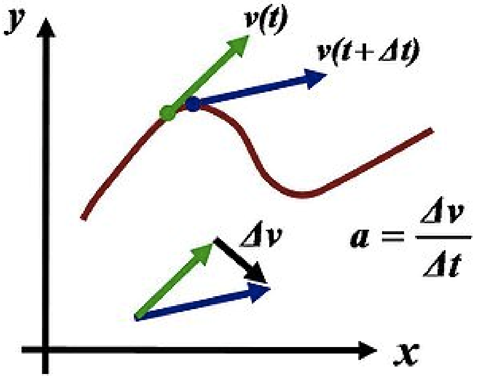
It is a vector with three components:

$$\overrightarrow{a_{av}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j} + \frac{\Delta v_z}{\Delta t}\hat{k}$$

Instantaneous acceleration

- The *instantaneous acceleration* is the instantaneous rate of change of the velocity with respect to time.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$



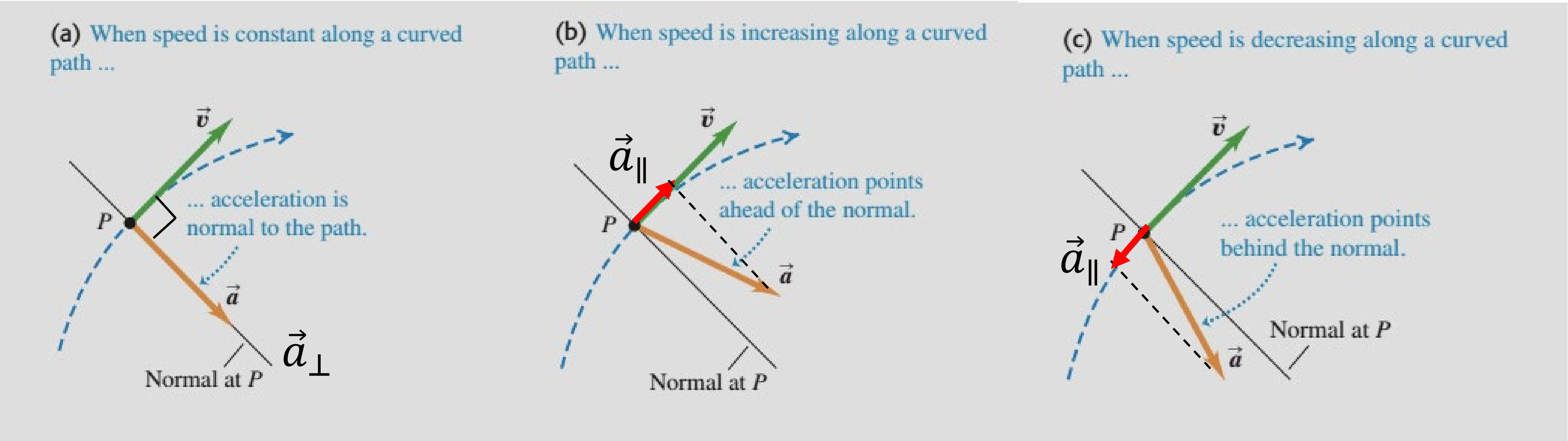
- The components of the instantaneous acceleration are
- $a_x = dv_x/dt$,
- $a_y = dv_y/dt$,
- $a_z = dv_z/dt$.

$$\vec{a} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} + \frac{dv_z}{dt}\hat{k}$$

Any particle following a curved path is accelerating, even if it has constant speed, because the **direction** is changing

Direction of the acceleration vector

- Velocity is always tangential to the path.
- But the direction of the acceleration vector is more or less arbitrary, depending on whether the speed is constant, increasing, or decreasing, as shown:



$\vec{a}_\perp$ → Velocity direction change. $\vec{a}_\parallel$ → Velocity magnitude change.

Working on motion in 2 and 3 dimensions.

- For 2-D and 3-D cases, we need to use vectors to describe motion
- It is convenient to express the vectors in terms of the unit vectors, i.e. their components.
- Each component can be treated as one-dimension motion.
- When you calculate the velocity and acceleration using the components, you are actually doing one dimensional problems for each components independent of others.

$$v_x = \frac{dx}{dt}, v_y = \frac{dy}{dt}, v_z = \frac{dz}{dt} \qquad a_x = \frac{dv_x}{dt}, a_y = \frac{dv_y}{dt}, a_z = \frac{dv_z}{dt}$$

Total: $|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2} \qquad a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

Calculating average and instantaneous velocity

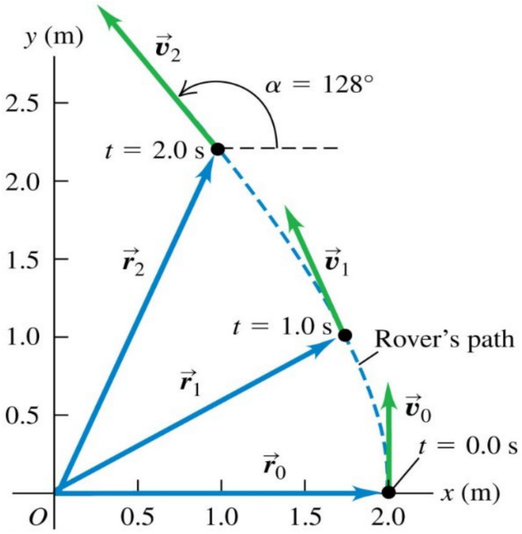
- A rover vehicle moves on the surface of Mars.
- Follow Example 3.1.

Example 3.1 **Calculating average and instantaneous velocity**

A robotic vehicle, or rover, is exploring the surface of Mars. The stationary Mars lander is the origin of coordinates, and the surrounding Martian surface lies in the *xy*-plane. The rover, which we represent as a point, has *x*- and *y*-coordinates that vary with time:

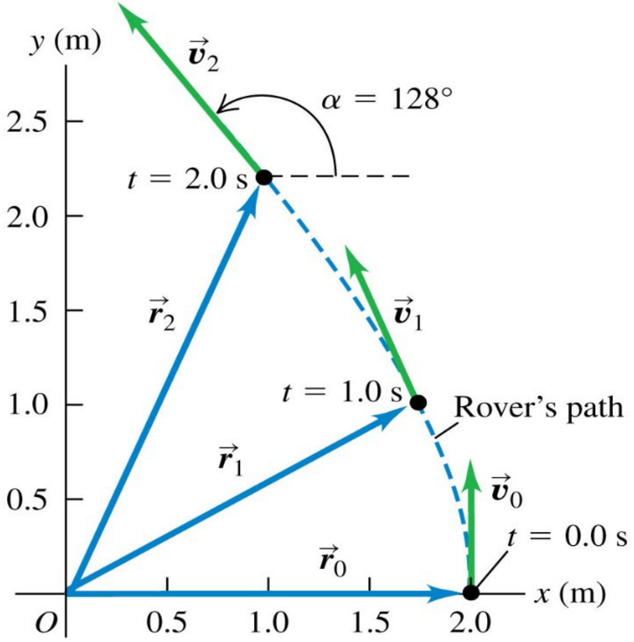
$$x = 2.0\ \text{m} - (0.25\ \text{m/s}^2)t^2$$
$$y = (1.0\ \text{m/s})t + (0.025\ \text{m/s}^3)t^3$$

(a) Find the rover's coordinates and distance from the lander at $t = 2.0$ s. (b) Find the rover's displacement and average velocity vectors for the interval $t = 0.0$ s to $t = 2.0$ s. (c) Find a general expression for the rover's instantaneous velocity vector $\vec{v}$. Express $\vec{v}$ at $t = 2.0$ s in component form and in terms of magnitude and direction.



Enlarge the figure for clarity

At $t = 0.0$ s the rover has position vector $\vec{r}_0$ and instantaneous velocity vector $\vec{v}_0$. Likewise, $\vec{r}_1$ and $\vec{v}_1$ are the vectors at $t = 1.0$ s; $\vec{r}_2$ and $\vec{v}_2$ are the vectors at $t = 2.0$ s.



SOLUTION

IDENTIFY and SET UP: This problem involves motion in two dimensions, so we must use the vector equations obtained in this section. Figure 3.5 shows the rover's path (dashed line). We'll use Eq. (3.1) for position $\vec{r}$, the expression $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ for displacement, Eq. (3.2) for average velocity, and Eqs. (3.5), (3.6), and (3.7) for instantaneous velocity and its magnitude and direction. The target variables are stated in the problem.

EXECUTE: (a) At $t = 2.0$ s the rover's coordinates are

$$x = 2.0\ \text{m} - (0.25\ \text{m/s}^2)(2.0\ \text{s})^2 = 1.0\ \text{m}$$
$$y = (1.0\ \text{m/s})(2.0\ \text{s}) + (0.025\ \text{m/s}^3)(2.0\ \text{s})^3 = 2.2\ \text{m}$$

The rover's distance from the origin at this time is

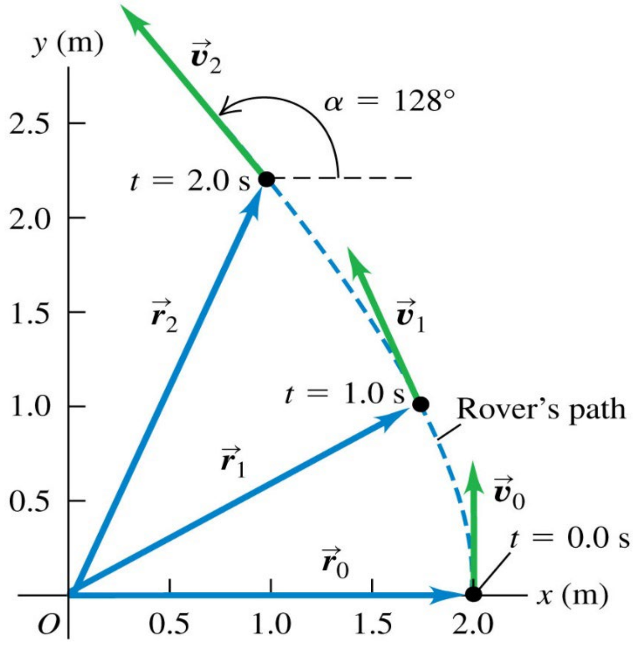
$$r = \sqrt{x^2 + y^2} = \sqrt{(1.0\ \text{m})^2 + (2.2\ \text{m})^2} = 2.4\ \text{m}$$

(b) To find the displacement and average velocity over the given time interval, we first express the position vector $\vec{r}$ as a function of time t. From Eq. (3.1) this is

$$\begin{aligned}\vec{r} &= x\hat{\imath} + y\hat{\jmath} \\ &= [2.0\ \text{m} - (0.25\ \text{m/s}^2)t^2]\hat{\imath} \\ &\quad + [(1.0\ \text{m/s})t + (0.025\ \text{m/s}^3)t^3]\hat{\jmath}\end{aligned}$$

At $t = 0.0$ s the position vector $\vec{r}_0$ is

$$\vec{r}_0 = (2.0\ \text{m})\hat{\imath} + (0.0\ \text{m})\hat{\jmath}$$



From part (a), the position vector $\vec{r}_2$ at $t = 2.0$ s is

$$\vec{r}_2 = (1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}$$

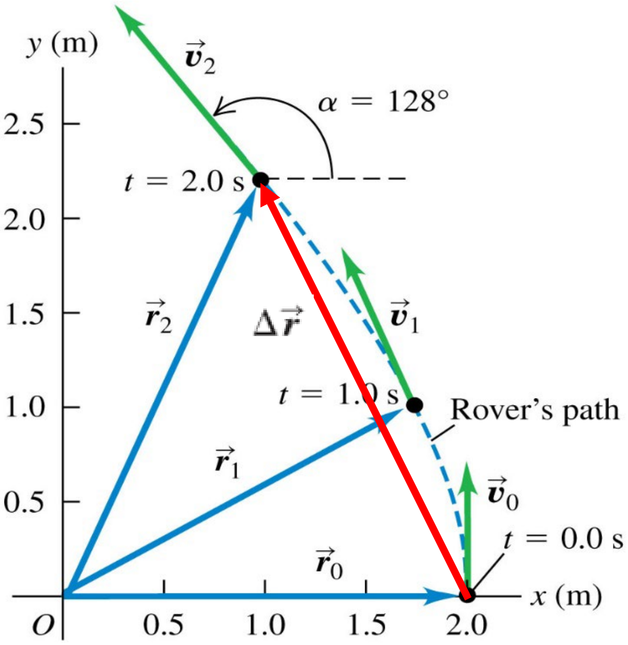
The displacement from $t = 0.0$ s to $t = 2.0$ s is therefore

$$\begin{aligned}\Delta\vec{r} = \vec{r}_2 - \vec{r}_0 &= (1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath} - (2.0\ \text{m})\hat{\imath}\\ &= (-1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}\end{aligned}$$

During this interval the rover moves 1.0 m in the negative x-direction and 2.2 m in the positive y-direction. From Eq. (3.2), the average velocity over this interval is the displacement divided by the elapsed time:

$$\begin{aligned}\vec{v}_{\text{av}} = \frac{\Delta\vec{r}}{\Delta t} &= \frac{(-1.0\ \text{m})\hat{\imath} + (2.2\ \text{m})\hat{\jmath}}{2.0\ \text{s} - 0.0\ \text{s}}\\ &= (-0.50\ \text{m/s})\hat{\imath} + (1.1\ \text{m/s})\hat{\jmath}\end{aligned}$$

The components of this average velocity are $v_{\text{av-}x} = -0.50$ m/s and $v_{\text{av-}y} = 1.1$ m/s.



(c) From Eq. (3.4) the components of *instantaneous* velocity are the time derivatives of the coordinates:

$$v_x = \frac{dx}{dt} = (-0.25\ \text{m/s}^2)(2t)$$

$$v_y = \frac{dy}{dt} = 1.0\ \text{m/s} + (0.025\ \text{m/s}^3)(3t^2)$$

Hence the instantaneous velocity vector is

$$\vec{\boldsymbol{v}} = v_x\hat{\boldsymbol{\imath}} + v_y\hat{\boldsymbol{\jmath}} = (-0.50\ \text{m/s}^2)t\hat{\boldsymbol{\imath}} + [1.0\ \text{m/s} + (0.075\ \text{m/s}^3)t^2]\hat{\boldsymbol{\jmath}}$$

At $t = 2.0$ s the velocity vector $\vec{\boldsymbol{v}}_2$ has components

$$v_{2x} = (-0.50\ \text{m/s}^2)(2.0\ \text{s}) = -1.0\ \text{m/s}$$

$$v_{2y} = 1.0\ \text{m/s} + (0.075\ \text{m/s}^3)(2.0\ \text{s})^2 = 1.3\ \text{m/s}$$

The magnitude of the instantaneous velocity (that is, the speed) at $t = 2.0$ s is

$$v_2 = \sqrt{v_{2x}^{\ 2} + v_{2y}^{\ 2}} = \sqrt{(-1.0\ \text{m/s})^2 + (1.3\ \text{m/s})^2} = 1.6\ \text{m/s}$$

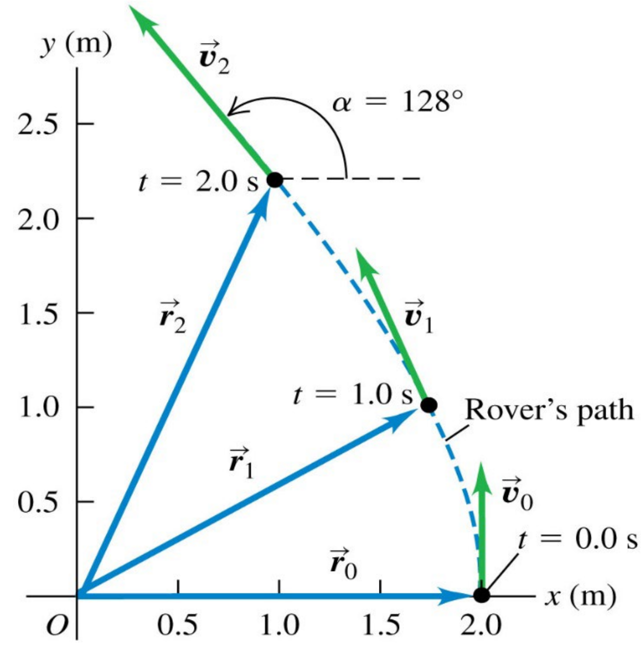


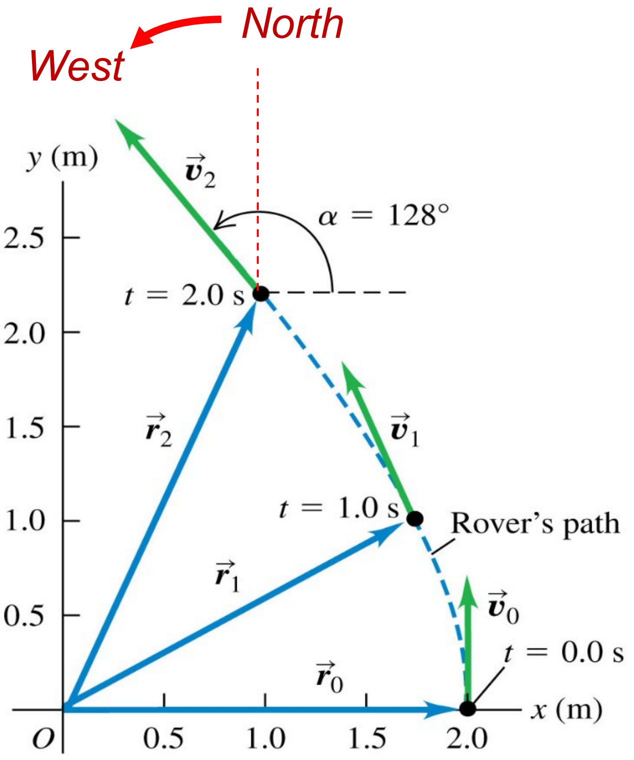
Figure 3.5 shows the direction of the velocity vector $\vec{v}_2$, which is at an angle α between 90° and 180° with respect to the positive x-axis. From Eq. (3.7) we have

$$\arctan\frac{v_y}{v_x} = \arctan\frac{1.3\ \text{m/s}}{-1.0\ \text{m/s}} = -52^\circ$$

This is off by 180°; the correct value of the angle is $\alpha = 180^\circ - 52^\circ = 128^\circ$, or 38° west of north.

EVALUATE: Compare the components of *average* velocity that we found in part (b) for the interval from $t = 0.0$ s to $t = 2.0$ s ($v_{\text{av-}x} = -0.50$ m/s, $v_{\text{av-}y} = 1.1$ m/s) with the components of *instantaneous* velocity at $t = 2.0$ s that we found in part (c) ($v_{2x} = -1.0$ m/s, $v_{2y} = 1.3$ m/s). The comparison shows that, just as in one dimension, the average velocity vector $\vec{v}_{\text{av}}$ over an interval is in general *not* equal to the instantaneous velocity $\vec{v}$ at the end of the interval (see Example 2.1).

Figure 3.5 shows the position vectors $\vec{r}$ and instantaneous velocity vectors $\vec{v}$ at $t = 0.0$ s, 1.0 s, and 2.0 s. (You should calculate these quantities for $t = 0.0$ s and $t = 1.0$ s.) Notice that $\vec{v}$ is tangent to the path at every point. The magnitude of $\vec{v}$ increases as the rover moves, which means that its speed is increasing.

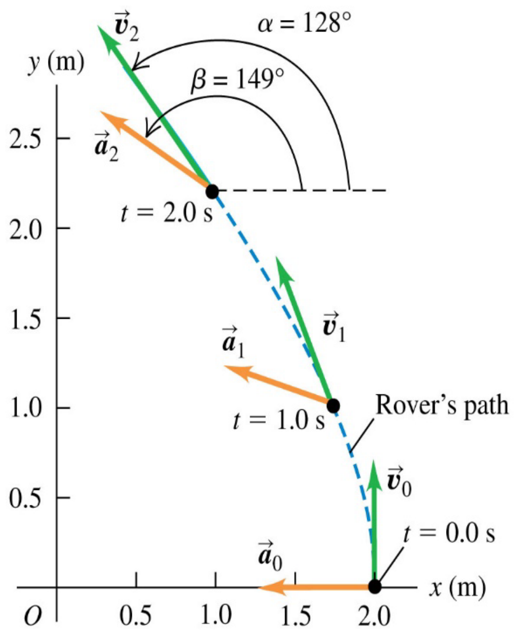


Calculating average and instantaneous acceleration

Example 3.2 **Calculating average and instantaneous acceleration**

Let's return to the motions of the Mars rover in Example 3.1. (a) Find the components of the average acceleration for the interval $t = 0.0$ s to $t = 2.0$ s. (b) Find the instantaneous acceleration at $t = 2.0$ s.

3.9 The path of the robotic rover, showing the velocity and acceleration at $t = 0.0$ s ($\vec{v}_0$ and $\vec{a}_0$), $t = 1.0$ s ($\vec{v}_1$ and $\vec{a}_1$), and $t = 2.0$ s ($\vec{v}_2$ and $\vec{a}_2$).



SOLUTION

IDENTIFY and SET UP: In Example 3.1 we found the components of the rover's instantaneous velocity at any time t:

$$v_x = \frac{dx}{dt} = (-0.25\ \text{m/s}^2)(2t) = (-0.50\ \text{m/s}^2)t$$

$$v_y = \frac{dy}{dt} = 1.0\ \text{m/s} + (0.025\ \text{m/s}^3)(3t^2)$$

$$= 1.0\ \text{m/s} + (0.075\ \text{m/s}^3)t^2$$

We'll use the vector relationships among velocity, average acceleration, and instantaneous acceleration. In part (a) we determine the values of v_x and v_y at the beginning and end of the interval and then use Eq. (3.8) to calculate the components of the average acceleration. In part (b) we obtain expressions for the instantaneous acceleration components at any time t by taking the time derivatives of the velocity components as in Eqs. (3.10).

EXECUTE: (a) In Example 3.1 we found that at $t = 0.0$ s the velocity components are

$$v_x = 0.0 \text{ m/s} \qquad v_y = 1.0 \text{ m/s}$$

and that at $t = 2.00$ s the components are

$$v_x = -1.0 \text{ m/s} \qquad v_y = 1.3 \text{ m/s}$$

Thus the components of average acceleration in the interval $t = 0.0$ s to $t = 2.0$ s are

$$a_{\text{av-}x} = \frac{\Delta v_x}{\Delta t} = \frac{-1.0 \text{ m/s} - 0.0 \text{ m/s}}{2.0 \text{ s} - 0.0 \text{ s}} = -0.50 \text{ m/s}^2$$

$$a_{\text{av-}y} = \frac{\Delta v_y}{\Delta t} = \frac{1.3 \text{ m/s} - 1.0 \text{ m/s}}{2.0 \text{ s} - 0.0 \text{ s}} = 0.15 \text{ m/s}^2$$

(b) Using Eqs. (3.10), we find

$$a_x = \frac{dv_x}{dt} = -0.50 \text{ m/s}^2 \qquad a_y = \frac{dv_y}{dt} = (0.075 \text{ m/s}^3)(2t)$$

Hence the instantaneous acceleration vector $\vec{a}$ at time t is

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} = (-0.50 \text{ m/s}^2)\hat{\imath} + (0.15 \text{ m/s}^3)t\hat{\jmath}$$

At $t = 2.0$ s the components of acceleration and the acceleration vector are

$$a_x = -0.50 \text{ m/s}^2 \qquad a_y = (0.15 \text{ m/s}^3)(2.0 \text{ s}) = 0.30 \text{ m/s}^2$$

$$\vec{a} = (-0.50 \text{ m/s}^2)\hat{\imath} + (0.30 \text{ m/s}^2)\hat{\jmath}$$

The magnitude of acceleration at this time is

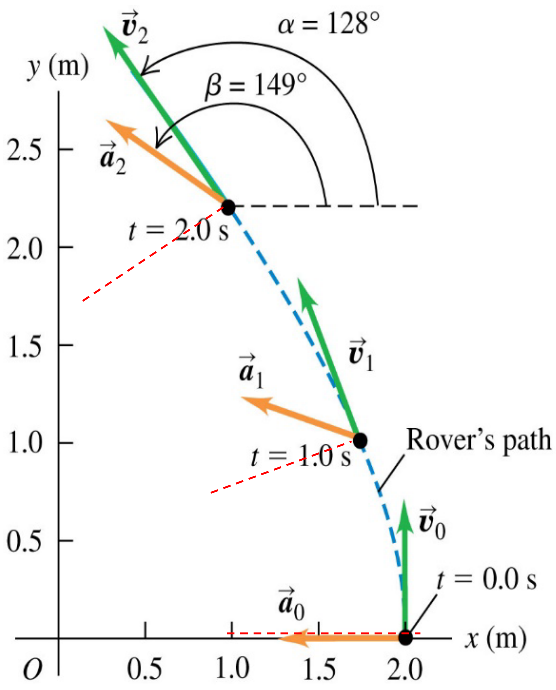
$$\begin{aligned} a &= \sqrt{a_x{}^2 + a_y{}^2} \\ &= \sqrt{(-0.50 \text{ m/s}^2)^2 + (0.30 \text{ m/s}^2)^2} = 0.58 \text{ m/s}^2 \end{aligned}$$

A sketch of this vector (Fig. 3.9) shows that the direction angle β of $\vec{a}$ with respect to the positive x-axis is between 90° and 180°. From Eq. (3.7) we have

$$\arctan\frac{a_y}{a_x} = \arctan\frac{0.30\ \text{m/s}^2}{-0.50\ \text{m/s}^2} = -31^\circ$$

Hence $\beta = 180^\circ + (-31^\circ) = 149^\circ$.

EVALUATE: Figure 3.9 shows the rover's path and the velocity and acceleration vectors at $t = 0.0$ s, 1.0 s, and 2.0 s. (You should use the results of part (b) to calculate the instantaneous acceleration at $t = 0.0$ s and $t = 1.0$ s for yourself.) Note that $\vec{v}$ and $\vec{a}$ are *not* in the same direction at any of these times. The velocity vector $\vec{v}$ is tangent to the path at each point (as is always the case), and the acceleration vector $\vec{a}$ points toward the concave side of the path.



Typical 2-dimension motion

-- projectile motion

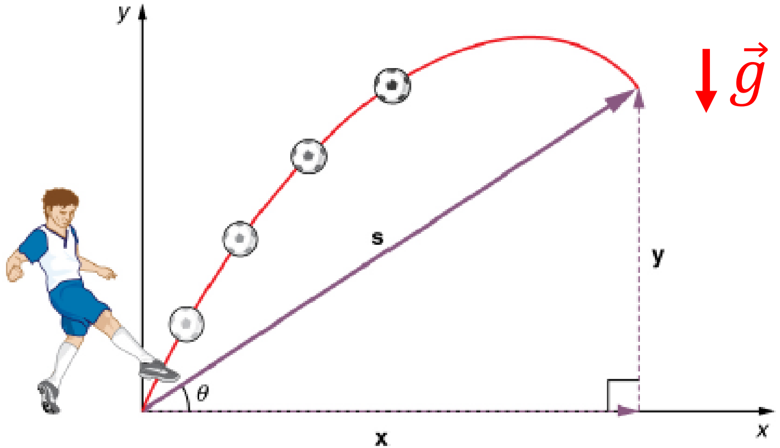
Projectile motion

- The projectile motion is a typical 2 dimension motion that happens on the surface of earth
- We learn more how to handle 2 dimension motion using projectile motion
- The trick: separate the motion into the motion along x and y directions (the x and y components of the position vector) and treat them individually as 1-D

What is a projectile motion?

- A projectile is any body given an initial velocity that then follows a path determined by the effects of gravity and air resistance.

- We usually neglect air resistance and the curvature and rotation of the earth. We consider constant gravity effect only. Gravity gives an acceleration $-g$ perpendicular to the surface and downward.



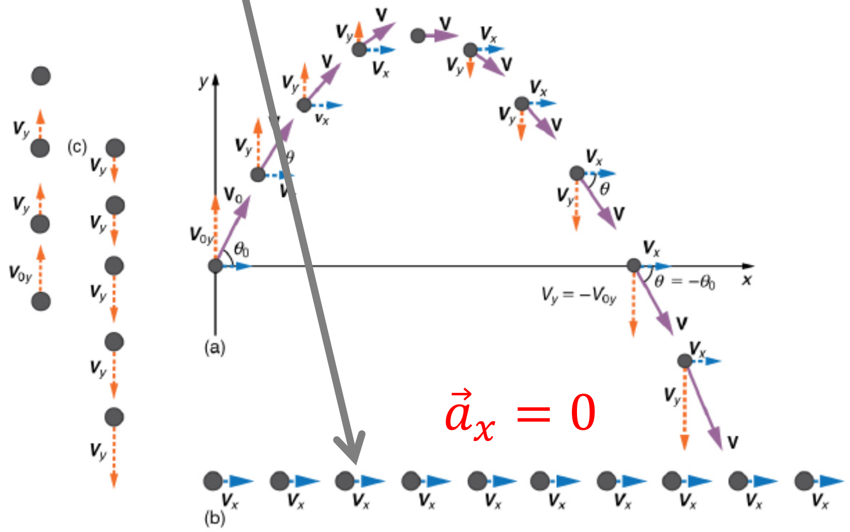
$g = 9.8 m/s^2$

The *x* and *y* motion are separable

- In projectile motion, the horizontal motion is not affected by vertical motion. The two motions can be separated and independent
- The horizontal position of the ball is the same as a ball moving with a constant velocity with zero acceleration

The vertical position is the same as that of one thrown up with velocity v_{0y} and influenced by the gravitational acceleration $-g = -9.8 m/s^2$

$\vec{a}_y = \vec{g}$



$\vec{a}_x = 0$

Interesting Video quiz

If they are not shown in the lecture, you can watch the videos on the web.

http://media.pearsoncmg.com/aw/aw_0media_physics/vtd/video2.html

http://media.pearsoncmg.com/aw/aw_0media_physics/vtd/video5.html

http://media.pearsoncmg.com/aw/aw_0media_physics/vtd/video7.html#

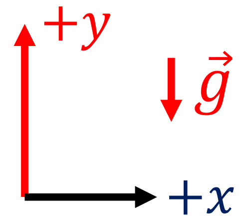
http://media.pearsoncmg.com/aw/aw_0media_physics/vtd/video9.html

Equations for projectile motion

We apply the four equations to the x direction motion (constant velocity) and the y direction motion (constant acceleration) independently

$v_x = v_{0x}$	$v_y = v_{0y} + a_y t$
$x = x_0 + v_{0x} t$	$y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2$
$v_x^2 = v_{0x}^2$	$v_y^2 = v_{0y}^2 + 2a_y (y - y_0)$
$x - x_0 = \left(\frac{v_{0x} + v_{0x}}{2} \right) t = v_{0x} t$	$y - y_0 = \left(\frac{v_{0y} + v_y}{2} \right) t$

$a_y = -g = -9.8 m/s^2$



but with a common thread: time t (both motions occur at the same time)

The equations for projectile motion

- If we set $x_0 = y_0 = 0$, the equations describing projectile motion are shown at the right.

$$v_{0x} = v_0 \cos\theta_0$$
$$v_{0y} = v_0 \sin\theta_0$$

$$x = v_0\cos\theta_0\, t$$
$$y = v_0\sin\theta_0\, t - \frac{1}{2}gt^2$$

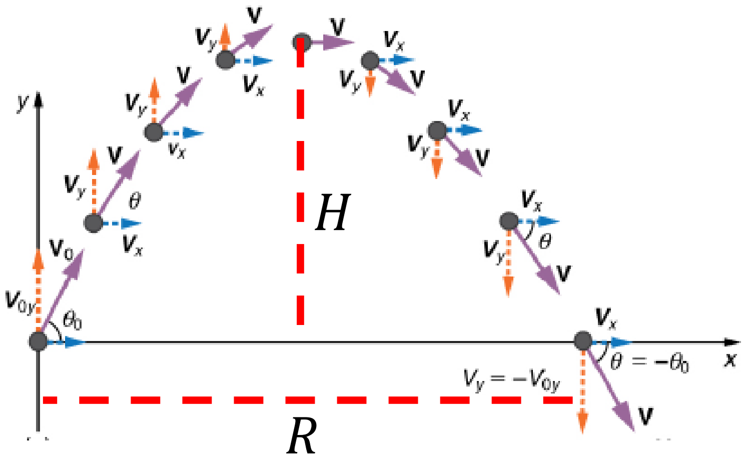
$$v_x = v_0\cos\theta_0$$
$$v_y = v_0\sin\theta_0 - gt$$

Eliminate parameter $t = x/v_0\cos\theta_0$:

$$y = x\tan\theta_0 - \frac{gx^2}{2v_0^2\cos^2\theta_0}$$

- The trajectory is a parabola.

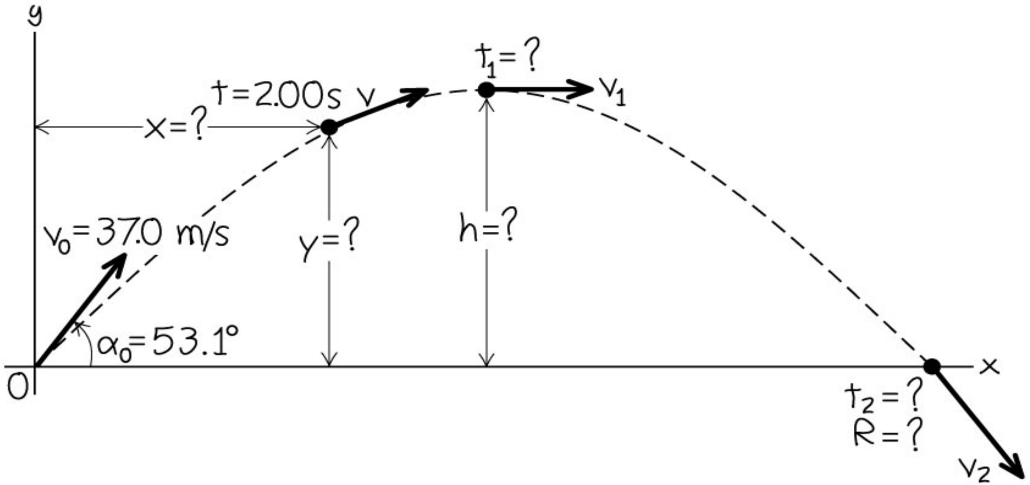
$$H = y_M = \frac{v_0^2\sin^2\theta_0}{2g}, \qquad R = \frac{v_0^2\sin 2\theta_0}{g}$$



Height and range of a projectile

Example 3.7 Height and range of a projectile I: A batted baseball

A batter hits a baseball so that it leaves the bat at speed $v_0 = 37.0\ \text{m/s}$ at an angle $\alpha_0 = 53.1°$. (a) Find the position of the ball and its velocity (magnitude and direction) at $t = 2.00\ \text{s}$. (b) Find the time when the ball reaches the highest point of its flight, and its height h at this time. (c) Find the *horizontal range* R—that is, the horizontal distance from the starting point to where the ball hits the ground.



SOLUTION

IDENTIFY and SET UP: As Fig. 3.20 shows, air resistance strongly affects the motion of a baseball. For simplicity, however, we'll ignore air resistance here and use the projectile-motion equations to describe the motion. The ball leaves the bat at $t = 0$ a meter or so above ground level, but we'll neglect this distance and assume that it starts at ground level ($y_0 = 0$). Figure 3.23 shows our sketch of the ball's trajectory. We'll use the same coordinate system as in Figs. 3.17 and 3.18, so we can use Eqs. (3.20) through (3.23). Our target variables are (a) the position and velocity of the ball 2.00 s after it leaves the bat, (b) the time t when the ball is at its maximum height (that is, when $v_y = 0$) and the y-coordinate at this time, and (c) the x-coordinate when the ball returns to ground level ($y = 0$).

EXECUTE: (a) We want to find x, y, v_x, and v_y at $t = 2.00$ s. The initial velocity of the ball has components

$$v_{0x} = v_0 \cos \alpha_0 = (37.0 \text{ m/s}) \cos 53.1° = 22.2 \text{ m/s}$$
$$v_{0y} = v_0 \sin \alpha_0 = (37.0 \text{ m/s}) \sin 53.1° = 29.6 \text{ m/s}$$

From Eqs. (3.20) through (3.23),

$$x = v_{0x}t = (22.2 \text{ m/s})(2.00 \text{ s}) = 44.4 \text{ m}$$

$$\begin{aligned} y &= v_{0y}t - \tfrac{1}{2}gt^2 \\ &= (29.6 \text{ m/s})(2.00 \text{ s}) - \tfrac{1}{2}(9.80 \text{ m/s}^2)(2.00 \text{ s})^2 \\ &= 39.6 \text{ m} \end{aligned}$$

$$v_x = v_{0x} = 22.2 \text{ m/s}$$

$$\begin{aligned} v_y &= v_{0y} - gt = 29.6 \text{ m/s} - (9.80 \text{ m/s}^2)(2.00 \text{ s}) \\ &= 10.0 \text{ m/s} \end{aligned}$$

The y-component of velocity is positive at $t = 2.00$ s, so the ball is still moving upward (Fig. 3.23). From Eqs. (3.25) and (3.26), the magnitude and direction of the velocity are

$$\begin{aligned} v &= \sqrt{v_x^2 + v_y^2} = \sqrt{(22.2\ \text{m/s})^2 + (10.0\ \text{m/s})^2} \\ &= 24.4\ \text{m/s} \end{aligned}$$

$$\alpha = \arctan\left(\frac{10.0\ \text{m/s}}{22.2\ \text{m/s}}\right) = \arctan 0.450 = 24.2^\circ$$

The direction of the velocity (the direction of the ball's motion) is 24.2° above the horizontal.

(b) At the highest point, the vertical velocity v_y is zero. Call the time when this happens t_1; then

$$v_y = v_{0y} - gt_1 = 0$$

$$t_1 = \frac{v_{0y}}{g} = \frac{29.6\ \text{m/s}}{9.80\ \text{m/s}^2} = 3.02\ \text{s}$$

The height h at the highest point is the value of y at time t_1:

$$\begin{aligned} h &= v_{0y}t_1 - \tfrac{1}{2}gt_1^2 \\ &= (29.6\ \text{m/s})(3.02\ \text{s}) - \tfrac{1}{2}(9.80\ \text{m/s}^2)(3.02\ \text{s})^2 \\ &= 44.7\ \text{m} \end{aligned}$$

(c) We'll find the horizontal range in two steps. First, *when* does the ball hit the ground? This occurs when $y = 0$. Call this time t_2; then

$$y = 0 = v_{0y}t_2 - \frac{1}{2}gt_2^2 = t_2\left(v_{0y} - \frac{1}{2}gt_2\right)$$

This is a quadratic equation for t_2. It has two roots:

$$t_2 = 0 \qquad \text{and} \qquad t_2 = \frac{2v_{0y}}{g} = \frac{2(29.6\ \text{m/s})}{9.80\ \text{m/s}^2} = 6.04\ \text{s}$$

There are two times at which $y = 0$; $t_2 = 0$ is the time the ball *leaves* the ground, and $t_2 = 2v_{0y}/g = 6.04$ s is the time of its return. This is exactly twice the time to reach the highest point that we found in part (b), $t_1 = v_{0y}/g = 3.02$ s, so the time of descent equals the time of ascent. This is *always* true if the starting and end points are at the same elevation and air resistance can be neglected.

The horizontal range R is the value of x when the ball returns to the ground—that is, at $t = 6.04$ s:

$$R = v_{0x}t_2 = (22.2\ \text{m/s})(6.04\ \text{s}) = 134\ \text{m}$$

The vertical component of velocity when the ball hits the ground is

$$\begin{aligned} v_y &= v_{0y} - gt_2 = 29.6\ \text{m/s} - (9.80\ \text{m/s}^2)(6.04\ \text{s}) \\ &= -29.6\ \text{m/s} \end{aligned}$$

That is, v_y has the same magnitude as the initial vertical velocity v_{0y} but the opposite direction (down). Since v_x is constant, the

angle $\alpha = -53.1°$ (below the horizontal) at this point is the negative of the initial angle $\alpha_0 = 53.1°$.

EVALUATE: It's often useful to check results by getting them in a different way. For example, we can also find the maximum height in part (b) by applying the constant-acceleration formula Eq. (2.13) to the y-motion:

$$v_y{}^2 = v_{0y}{}^2 + 2a_y(y - y_0) = v_{0y}{}^2 - 2g(y - y_0)$$

At the highest point, $v_y = 0$ and $y = h$. You should solve this equation for h; you should get the same answer that we obtained in part (b). (Do you?)

Note that the time to hit the ground, $t_2 = 6.04\ \text{s}$, is exactly twice the time to reach the highest point, $t_1 = 3.02\ \text{s}$. Hence the time of descent equals the time of ascent. This is *always* true if the starting and end points are at the same elevation and if air resistance can be neglected.

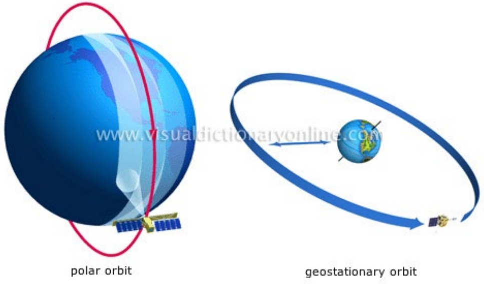
Note also that $h = 44.7\ \text{m}$ in part (b) is comparable to the 52.4-m height above the playing field of the roof of the Hubert H. Humphrey Metrodome in Minneapolis, and the horizontal range $R = 134\ \text{m}$ in part (c) is greater than the 99.7-m distance from home plate to the right-field fence at Safeco Field in Seattle. In reality, due to air resistance (which we have neglected) a batted ball with the initial speed and angle we've used here won't go as high or as far as we've calculated (see Fig. 3.20).

Another example of two dimension motion
– Uniform Circular Motion

Uniform circular motion

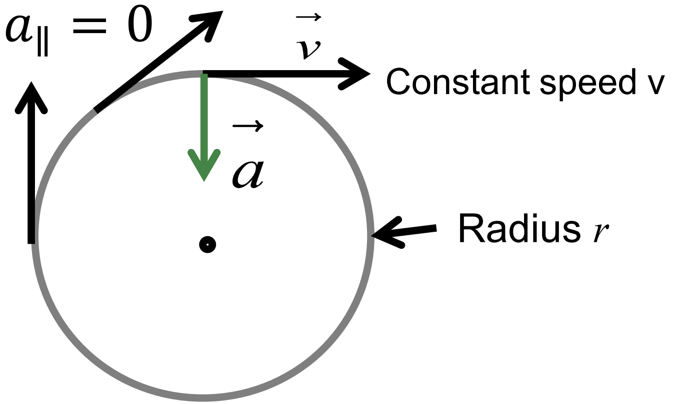
Another important example of two dimension motion is the uniform circular motion

Examples: the orbit of satellite around the earth, a car goes around a circular roundabout or corner, a merry-go-around



Uniform circular motion—Figure 3.27

- For *uniform circular motion*, the speed is constant but the direction of velocity vector is constantly changing
  → changing velocity → acceleration
- No change in speed → $a_{\parallel} = 0$
- The acceleration is perpendicular to the velocity and pointing to the center: It is called centripetal acceleration, a_c



$$a_c = \frac{v^2}{r}$$

v is the constant speed
r is the radius of circle

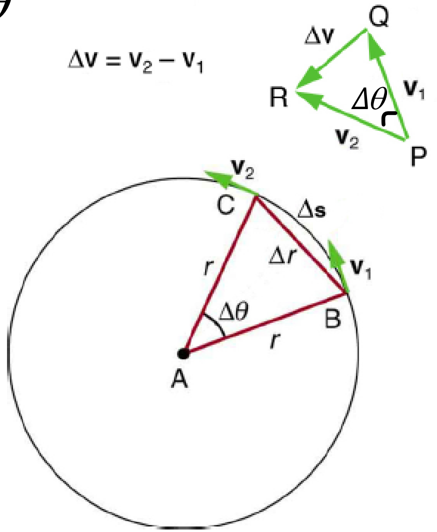
The *period* T is the time for one revolution, so

$v = 2\pi r/T$, and $a_c = 4\pi^2 r/T^2$.

Derivation of centripetal acceleration

Assume the object rotates an angle $\Delta\theta$ in Δt: $\Delta\theta = \Delta s / r = v\Delta t / r$

By geometry, the v-vector (tangent) also rotates an angle of $\Delta\theta$



The velocity changes from $\vec{v}_1$ to $\vec{v}_2$ is $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$

As $\Delta t \to 0, \Delta\theta \to 0, \Delta\vec{v} \perp \vec{v} \Rightarrow \vec{a} \equiv \dfrac{\Delta\vec{v}}{\Delta t} \perp \vec{v}$,

or $\vec{a}_c$ is perpendicular to $\vec{v}$

Magnitude of $|\vec{a}_c| \equiv \dfrac{|\Delta\vec{v}|}{\Delta t}$, but $\left|\Delta\vec{v}\right| = \Delta v = v\Delta\theta = v(v\Delta t) / r$

$$when \;\; \Delta t \to 0, \;\; \frac{\Delta v}{\Delta t} = \frac{v^2}{r} = average\ acceleration$$

instaneous acceleration $\boxed{a = v^2 / r}$

Magnitude of $\vec{a}_c$ in uniform circular motion is a constant $= a_c = v^2/r$

Centripetal acceleration on a curved road

Example 3.11 Centripetal acceleration on a curved road

An Aston Martin V8 Vantage sports car has a "lateral acceleration" of $0.96g = (0.96)(9.8\ \text{m/s}^2) = 9.4\ \text{m/s}^2$. This is the maximum centripetal acceleration the car can sustain without skidding out of a curved path. If the car is traveling at a constant 40 m/s (about 89 mi/h, or 144 km/h) on level ground, what is the radius R of the tightest unbanked curve it can negotiate?

SOLUTION

IDENTIFY, SET UP, and EXECUTE: The car is in uniform circular motion because it's moving at a constant speed along a curve that is a segment of a circle. Hence we can use Eq. (3.28) to solve for the target variable R in terms of the given centripetal acceleration

a_{rad} and speed v:

$$R = \frac{v^2}{a_{rad}} = \frac{(40 \text{ m/s})^2}{9.4 \text{ m/s}^2} = 170 \text{ m (about 560 ft)}$$

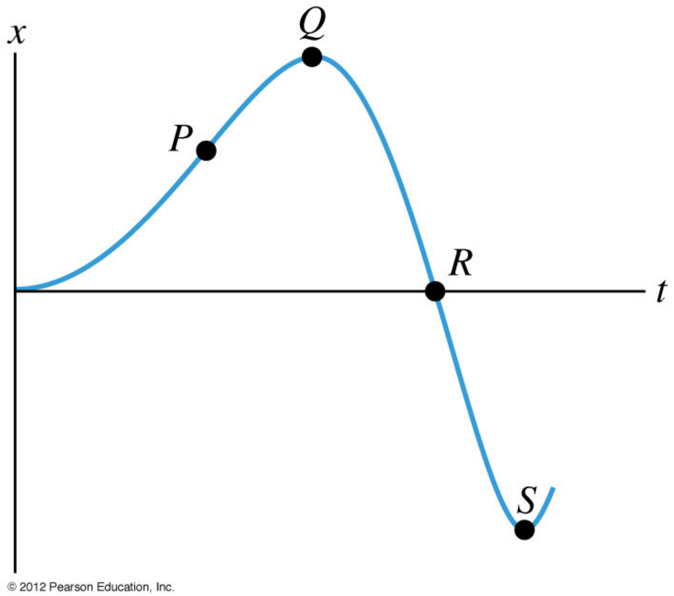
This is the *minimum* radius because a_{rad} is the *maximum* centripetal acceleration.

EVALUATE: The minimum turning radius R is proportional to the *square* of the speed, so even a small reduction in speed can make R substantially smaller. For example, reducing v by 20% (from 40 m/s to 32 m/s) would decrease R by 36% (from 170 m to 109 m).

Another way to make the minimum turning radius smaller is to *bank* the curve. We'll investigate this option in Chapter 5.

Thinking questions

Thinking question TQ2.1



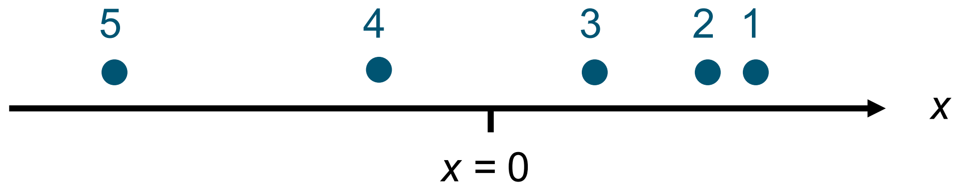
This is the *x*-*t* graph of the motion of a particle. Of the four points *P, Q, R,* and *S*, the acceleration a_x is greatest (most positive) at

A. point *P.* B. point *Q.* C. point *R.* D. point *S.*

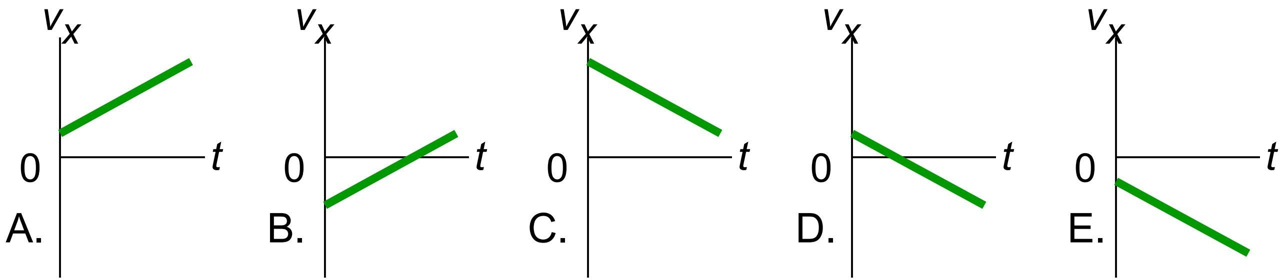
E. not enough information in the graph to decide

TQ2.2

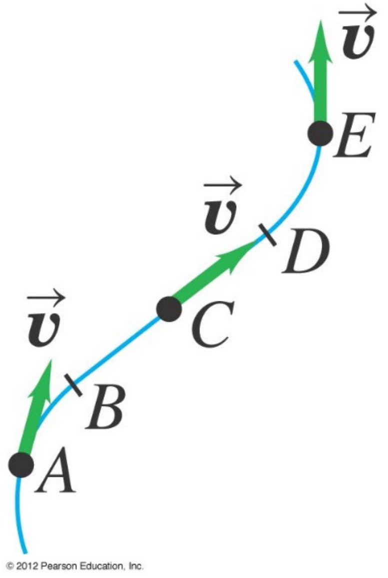
This is a motion diagram of an object moving along the *x*-direction with constant acceleration. The dots 1, 2, 3, … show the position of the object at equal time intervals Δt.



Which of the following v_x-t graphs best matches the motion shown in the motion diagram?



The motion diagram shows an object moving along a curved path at constant speed. At which of the points *A, C,* and *E* does the object have *zero* acceleration?



A. point *A* only

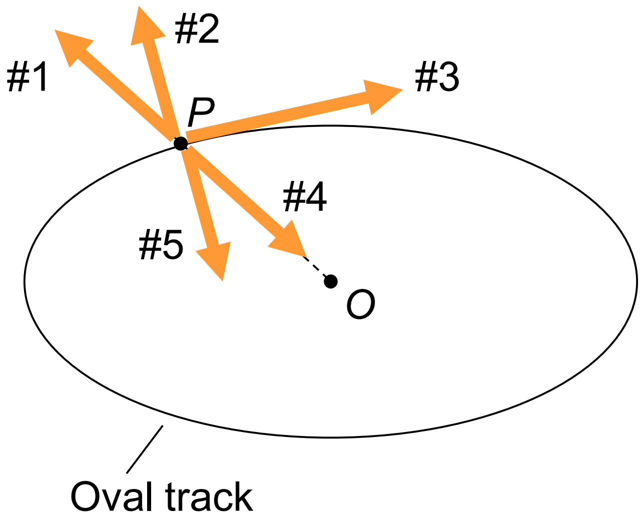
B. point *C* only

C. point *E* only

D. points *A* and *C* only

E. points *A, C,* and *E*

An object moves at a constant speed in a clockwise direction around an oval track. The geometrical center of the track is at point *O*. When the object is at point *P*, which arrow shows the direction of the object's acceleration vector?



A. #1 (directly away from *O*)

B. #2 (perpendicular to the track)

C. #3 (in the direction of motion)

D. #4 (directly toward *O*)

E. #5 (perpendicular to the track)

A zookeeper fires a tranquilizer dart directly at a monkey. The monkey lets go at the same instant that the dart leaves the gun barrel. The dart reaches a maximum height *P* before striking the monkey. Ignore air resistance.

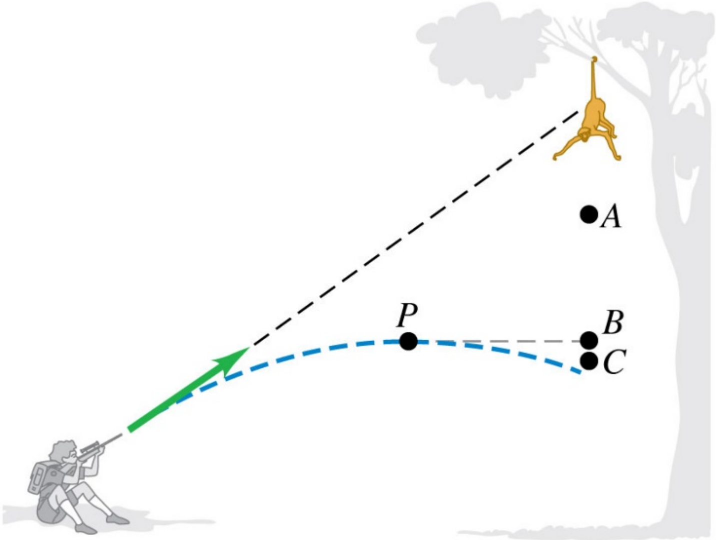
When the dart is at *P*, the monkey

A. is at *A* (higher than *P*).

B. is at *B* (at the same height as *P*).

C. is at *C* (lower than *P*).

D. not enough information given to decide



Hint: what would happen if there were no gravity? What does gravity do?

You drive a race car around a circular track of radius 100 m at a constant speed of 100 km/h. If you then drive the same car around a different circular track of radius 200 m at a constant speed of 200 km/h, your acceleration will be

A. 8 times greater.

B. 4 times greater.

C. twice as great.

D. the same.

E. half as great.

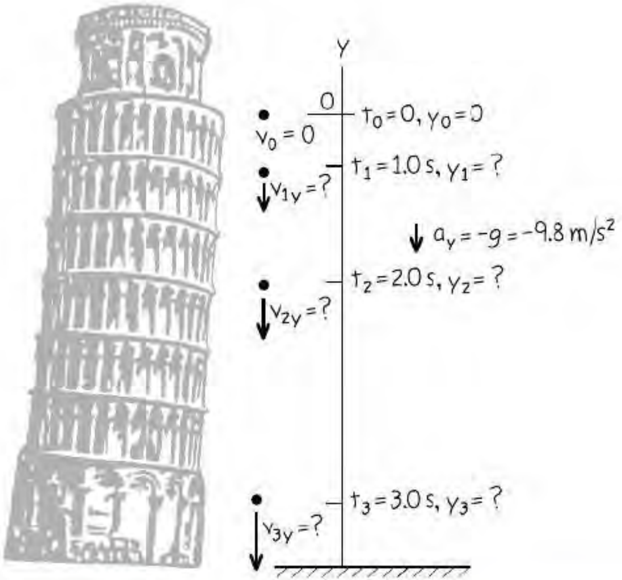
EXAMPLE 2.6 A FREELY FALLING COIN

A one-euro coin is dropped from the Leaning Tower of Pisa and falls freely from rest. What are its position and velocity after 1.0 s, 2.0 s, and 3.0 s?

SOLUTION

IDENTIFY and SET UP: "Falls freely" means "falls with constant acceleration due to gravity," so we can use the constant-acceleration equations. The right side of **Fig. 2.23** shows our motion diagram for the coin. The motion is vertical, so we use a vertical coordinate axis and call the coordinate y instead of x. We take the origin O at the starting point and the *upward* direction as positive. Both the initial coordinate y_0 and initial y-velocity v_{0y} are zero. The y-acceleration is downward (in the negative y-direction), so $a_y = -g = -9.8\ \text{m/s}^2$. (Remember that, by definition, g is a positive quantity.) Our target variables are the values of y and v_y at the three given times. To find these, we use Eqs. (2.12) and (2.8) with x replaced by y. Our choice of the upward direction as positive means that all positions and velocities we calculate will be negative.

2.23 A coin freely falling from rest.



EXECUTE: At a time t after the coin is dropped, its position and y-velocity are

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = 0 + 0 + \tfrac{1}{2}(-g)t^2 = (-4.9\ \text{m/s}^2)t^2$$

$$v_y = v_{0y} + a_y t = 0 + (-g)t = (-9.8\ \text{m/s}^2)t$$

When $t = 1.0\ \text{s}$, $y = (-4.9\ \text{m/s}^2)(1.0\ \text{s})^2 = -4.9\ \text{m}$ and $v_y = (-9.8\ \text{m/s}^2)(1.0\ \text{s}) = -9.8\ \text{m/s}$; after 1.0 s, the coin is 4.9 m below the origin (y is negative) and has a downward velocity (v_y is negative) with magnitude 9.8 m/s.

We can find the positions and y-velocities at 2.0 s and 3.0 s in the same way. The results are $y = -20\ \text{m}$ and $v_y = -20\ \text{m/s}$ at $t = 2.0\ \text{s}$, and $y = -44\ \text{m}$ and $v_y = -29\ \text{m/s}$ at $t = 3.0\ \text{s}$.

EVALUATE: All our answers are negative, as we expected. If we had chosen the positive y-axis to point downward, the acceleration would have been $a_y = +g$ and all our answers would have been positive.

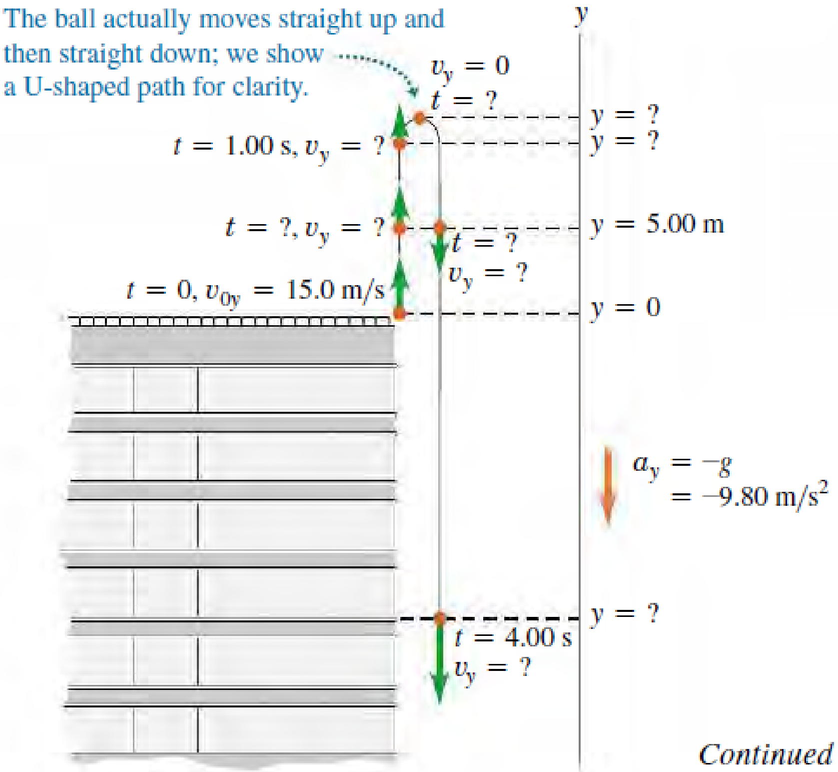
EXAMPLE 2.7 UP-AND-DOWN MOTION IN FREE FALL

You throw a ball vertically upward from the roof of a tall building. The ball leaves your hand at a point even with the roof railing with an upward speed of 15.0 m/s; the ball is then in free fall. On its way back down, it just misses the railing. Find (a) the ball's position and velocity 1.00 s and 4.00 s after leaving your hand; (b) the ball's velocity when it is 5.00 m above the railing; (c) the maximum height reached; (d) the ball's acceleration when it is at its maximum height.

SOLUTION

IDENTIFY and SET UP: The words "in free fall" mean that the acceleration is due to gravity, which is constant. Our target variables are position [in parts (a) and (c)], velocity [in parts (a) and (b)], and acceleration [in part (d)]. We take the origin at the point where the ball leaves your hand, and take the positive direction to be upward (**Fig. 2.24**). The initial position y_0 is zero, the initial y-velocity v_{0y} is $+15.0$ m/s, and the y-acceleration is $a_y = -g = -9.80\ \text{m/s}^2$. In part (a), as in Example 2.6, we'll use Eqs. (2.12) and (2.8) to find the position and velocity as functions of time. In part (b) we must find the velocity at a given *position* (no time is given), so we'll use Eq. (2.13).

2.24 Position and velocity of a ball thrown vertically upward.



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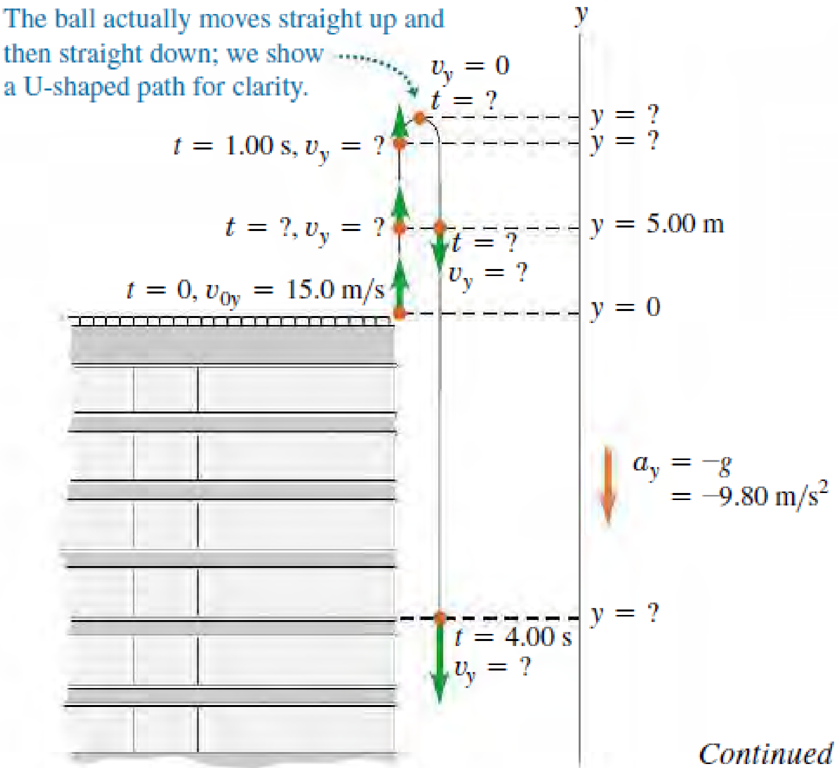
SOLUTION

IDENTIFY and SET UP: The words "in free fall" mean that the acceleration is due to gravity, which is constant. Our target variables are position [in parts (a) and (c)], velocity [in parts (a) and (b)], and acceleration [in part (d)]. We take the origin at the point where the ball leaves your hand, and take the positive direction to be upward (**Fig. 2.24**). The initial position y_0 is zero, the initial y-velocity v_{0y} is $+15.0$ m/s, and the y-acceleration is $a_y = -g = -9.80\ \mathrm{m/s^2}$. In part (a), as in Example 2.6, we'll use Eqs. (2.12) and (2.8) to find the position and velocity as functions of time. In part (b) we must find the velocity at a given *position* (no time is given), so we'll use Eq. (2.13).

2.24 Position and velocity of a ball thrown vertically upward.



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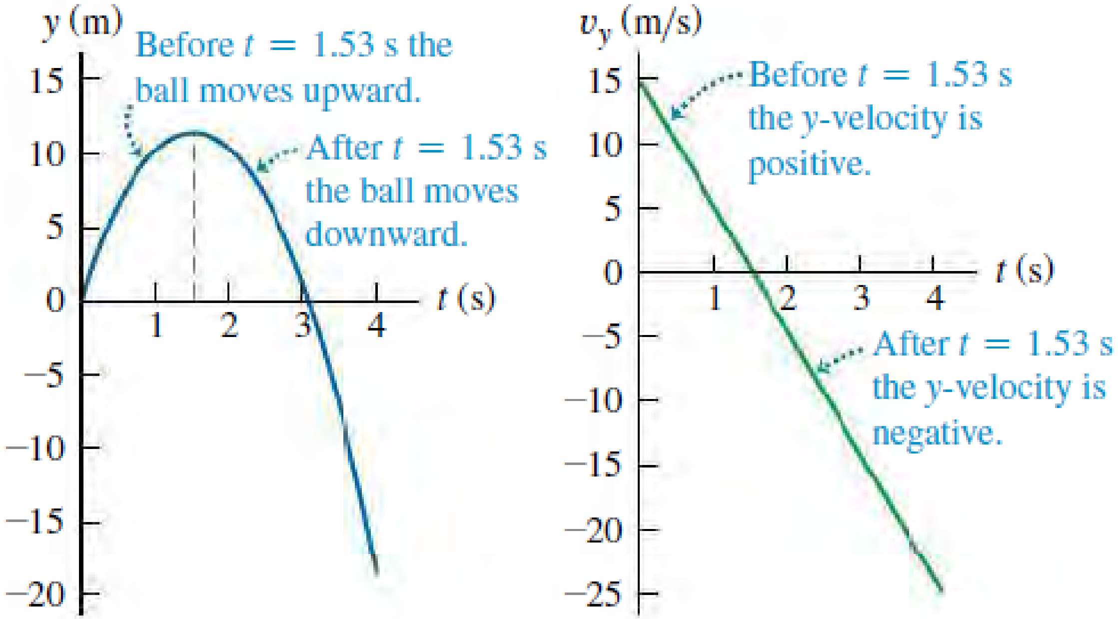
2.25 (a) Position and (b) velocity as functions of time for a ball thrown upward with an initial speed of 15.0 m/s.

(a) y-t graph (curvature is downward because $a_y = -g$ is negative)

(b) v_y-t graph (straight line with negative slope because $a_y = -g$ is constant and negative)



EXECUTE: (a) The position and y-velocity at time t are given by Eqs. (2.12) and (2.8) with x's replaced by y's:

$$y = y_0 + v_{0y}t + \tfrac{1}{2}a_y t^2 = y_0 + v_{0y}t + \tfrac{1}{2}(-g)t^2$$
$$= (0) + (15.0\ \mathrm{m/s})t + \tfrac{1}{2}(-9.80\ \mathrm{m/s^2})t^2$$
$$v_y = v_{0y} + a_y t = v_{0y} + (-g)t$$
$$= 15.0\ \mathrm{m/s} + (-9.80\ \mathrm{m/s^2})t$$

When $t = 1.00$ s, these equations give $y = +10.1$ m and $v_y = +5.2$ m/s. That is, the ball is 10.1 m above the origin (y is positive) and moving upward (v_y is positive) with a speed of 5.2 m/s. This is less than the initial speed because the ball slows as it ascends. When $t = 4.00$ s, those equations give $y = -18.4$ m and $v_y = -24.2$ m/s. The ball has passed its highest point and is 18.4 m *below* the origin (y is negative). It is moving *downward* (v_y is negative) with a speed of 24.2 m/s. Equation (2.13) tells us that the ball is moving at the initial 15.0-m/s speed as it moves downward past the launching point and continues to gain speed as it descends further.

(b) The y-velocity at any position y is given by Eq. (2.13) with x's replaced by y's:

$$v_y^2 = v_{0y}^2 + 2a_y(y - y_0) = v_{0y}^2 + 2(-g)(y - 0)$$
$$= (15.0\ \mathrm{m/s})^2 + 2(-9.80\ \mathrm{m/s^2})y$$

When the ball is 5.00 m above the origin we have $y = +5.00$ m, so

$$v_y^2 = (15.0\ \mathrm{m/s})^2 + 2(-9.80\ \mathrm{m/s^2})(5.00\ \mathrm{m}) = 127\ \mathrm{m^2/s^2}$$
$$v_y = \pm 11.3\ \mathrm{m/s}$$

We get *two* values of v_y because the ball passes through the point $y = +5.00$ m twice, once on the way up (so v_y is positive) and once on the way down (so v_y is negative) (see Figs. 2.24 and 2.25a).

(c) At the instant at which the ball reaches its maximum height y_1, its y-velocity is momentarily zero: $v_y = 0$. We use Eq. (2.13) to find y_1. With $v_y = 0$, $y_0 = 0$, and $a_y = -g$, we get

$$0 = v_{0y}^2 + 2(-g)(y_1 - 0)$$
$$y_1 = \frac{v_{0y}^2}{2g} = \frac{(15.0\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s^2})} = +11.5\ \mathrm{m}$$

(d) Caution It's a common misconception that at the highest point of free-fall motion, where the velocity is zero, the acceleration is also zero. Wrong! The acceleration in free fall is always $a_y = -g = -9.80\ \mathrm{m/s^2}$.

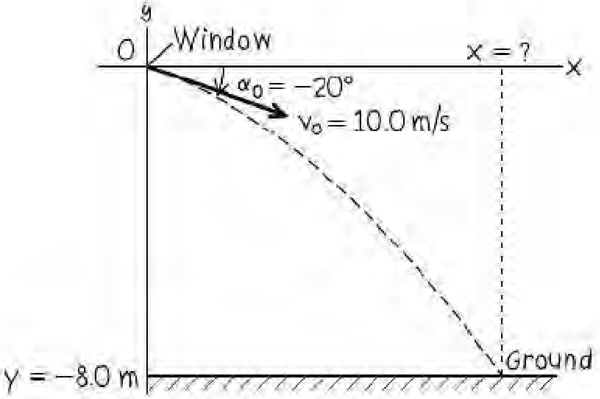
EXAMPLE 3.9 DIFFERENT INITIAL AND FINAL HEIGHTS

You throw a ball from your window 8.0 m above the ground. When the ball leaves your hand, it is moving at 10.0 m/s at an angle of 20° below the horizontal. How far horizontally from your window will the ball hit the ground? Ignore air resistance.

SOLUTION

IDENTIFY and SET UP: As in Examples 3.7 and 3.8, we want to find the horizontal coordinate of a projectile when it is at a given y-value. The difference here is that this value of y is *not* the same as the initial value. We again choose the x-axis to be horizontal and the y-axis to be upward, and place the origin of coordinates at the point where the ball leaves your hand (**Fig. 3.25**). We have $v_0 = 10.0\ \text{m/s}$ and $\alpha_0 = -20°$ (the angle is negative because the initial velocity is below the horizontal). Our target variable is the value of x when the ball reaches the ground at $y = -8.0\ \text{m}$. We'll use Eq. (3.20) to find the time t when this happens and then use Eq. (3.19) to find the value of x at this time.

3.25 Our sketch for this problem.



EXECUTE: To determine t, we rewrite Eq. (3.20) in the standard form for a quadratic equation for t:

$$\tfrac{1}{2}gt^2 - (v_0 \sin\alpha_0)t + y = 0$$

The roots of this equation are

$$t = \frac{v_0 \sin\alpha_0 \pm \sqrt{(-v_0 \sin\alpha_0)^2 - 4\left(\tfrac{1}{2}g\right)y}}{2\left(\tfrac{1}{2}g\right)}$$

$$= \frac{v_0 \sin\alpha_0 \pm \sqrt{v_0^2 \sin^2\alpha_0 - 2gy}}{g}$$

$$= \frac{\left[\begin{array}{l}(10.0\ \text{m/s})\sin(-20^\circ) \\ \pm \sqrt{(10.0\ \text{m/s})^2 \sin^2(-20^\circ) - 2(9.80\ \text{m/s}^2)(-8.0\ \text{m})}\end{array}\right]}{9.80\ \text{m/s}^2}$$

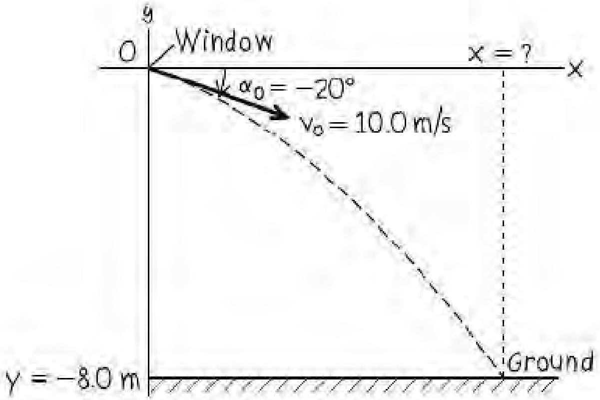
$$= -1.7\ \text{s} \qquad \text{or} \qquad 0.98\ \text{s}$$

We discard the negative root, since it refers to a time before the ball left your hand. The positive root tells us that the ball reaches the ground at $t = 0.98$ s. From Eq. (3.19), the ball's x-coordinate at that time is

$$x = (v_0 \cos\alpha_0)t = (10.0\ \text{m/s})[\cos(-20^\circ)](0.98\ \text{s}) = 9.2\ \text{m}$$

The ball hits the ground a horizontal distance of 9.2 m from your window.

3.25 Our sketch for this problem.



EXECUTE: To determine t, we rewrite Eq. (3.20) in the standard form for a quadratic equation for t:

$$\tfrac{1}{2}gt^2 - (v_0 \sin\alpha_0)t + y = 0$$

The roots of this equation are

$$t = \frac{v_0 \sin\alpha_0 \pm \sqrt{(-v_0 \sin\alpha_0)^2 - 4\left(\frac{1}{2}g\right)y}}{2\left(\frac{1}{2}g\right)}$$

$$= \frac{v_0 \sin\alpha_0 \pm \sqrt{v_0^2 \sin^2\alpha_0 - 2gy}}{g}$$

$$= \frac{\left[\begin{array}{l}(10.0\text{ m/s})\sin(-20°) \\ \pm \sqrt{(10.0\text{ m/s})^2 \sin^2(-20°) - 2(9.80\text{ m/s}^2)(-8.0\text{ m})}\end{array}\right]}{9.80\text{ m/s}^2}$$

$$= -1.7\text{ s} \qquad \text{or} \qquad 0.98\text{ s}$$

We discard the negative root, since it refers to a time before the ball left your hand. The positive root tells us that the ball reaches the ground at $t = 0.98$ s. From Eq. (3.19), the ball's x-coordinate at that time is

$$x = (v_0 \cos\alpha_0)t = (10.0\text{ m/s})[\cos(-20°)](0.98\text{ s}) = 9.2\text{ m}$$

The ball hits the ground a horizontal distance of 9.2 m from your window.

3.25 Our sketch for this problem.