MA1301 Semester B 2021-22
Midterm Exam (CB1)

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14/03/2022

Name: _____
Student ID: _____

This exam contains 3 pages (including this page) and 4 questions. Total of points is 100.

Grade Table (for instructor use only)

Question	Points	Score
1	15	
2	50	
3	15	
4	20	
Total:	100	

1. (15 points) (i) [5pts] State the definition of $\int_a^b f(x)dx$ (taking equal sub-intervals and the sample point to be the right endpoint).
(ii) [5pts] Show that

$$\int_0^1 \frac{x}{\sqrt{x^2+4}}dx = \lim_{n\to+\infty}\Big[\frac{1}{\sqrt{1^2\times n^2+4n^4}}+\frac{2}{\sqrt{2^2\times n^2+4n^4}}+\frac{3}{\sqrt{3^2\times n^2+4n^4}}+\cdots+\frac{n}{\sqrt{n^2\times n^2+4n^4}}\Big].$$

(iii)[5pts] Evaluate the limit in (ii).

2. (50 points) Evaluate the following integrals: (i)[15pts]

$$\int e^{-x}\sin(2x)dx$$

(ii)[10pts]

$$\int \frac{1}{\sqrt{4x^2+1}}dx.$$

(iii)[15pts]

$$\int \frac{x^2-4x-2}{(x-2)(x^2+2x+2)}dx$$

(iv)[10pts] Determine whether the following improper integral is convergent or divergent. If it converges, please find the value.

$$\int_0^{+\infty} \frac{x+2}{(4+x^2)^{\frac{3}{2}}}dx.$$

3. (15 points) Find the surface area of the surface generated by rotating the region in the first quadrant bounded by the curve $y^2 = 9 - x$ and x-axis about x-axis for one complete revolution.

4. (20 points) (i)[10pts] Let $f(x)$ be a continuously differentiable function on $[a, b]$ such that $\int_a^b f(x)dx$=0 and $f(a) = f(b) = 0$. Find the value of

$$\int_a^b xf'(x)dx$$

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(ii)[10pts] Let $f(x)$ be twice continuously differentiable function on $[0, 2]$ such that $f(0) = f(2) = 1$ and $\int_0^2 f(x)dx = 1$. Using integration by parts, find the value of

$$\int_0^2 x(2-x)f''(x)dx.$$