Chapter 7

Potential Energy and Energy Conservation

Introduction

- How do energy concepts apply to the descending sandhill crane?
- We will see that we can think of energy as being stored and *transformed* from one form to another.

Gravitational potential energy

- When a particle is in the gravitational field of the earth, there is a gravitational potential energy associated with the particle:

Gravitational potential energy associated with a particle

$$U_{grav} = mgy$$

Vertical coordinate of particle (y increases if particle moves upward)

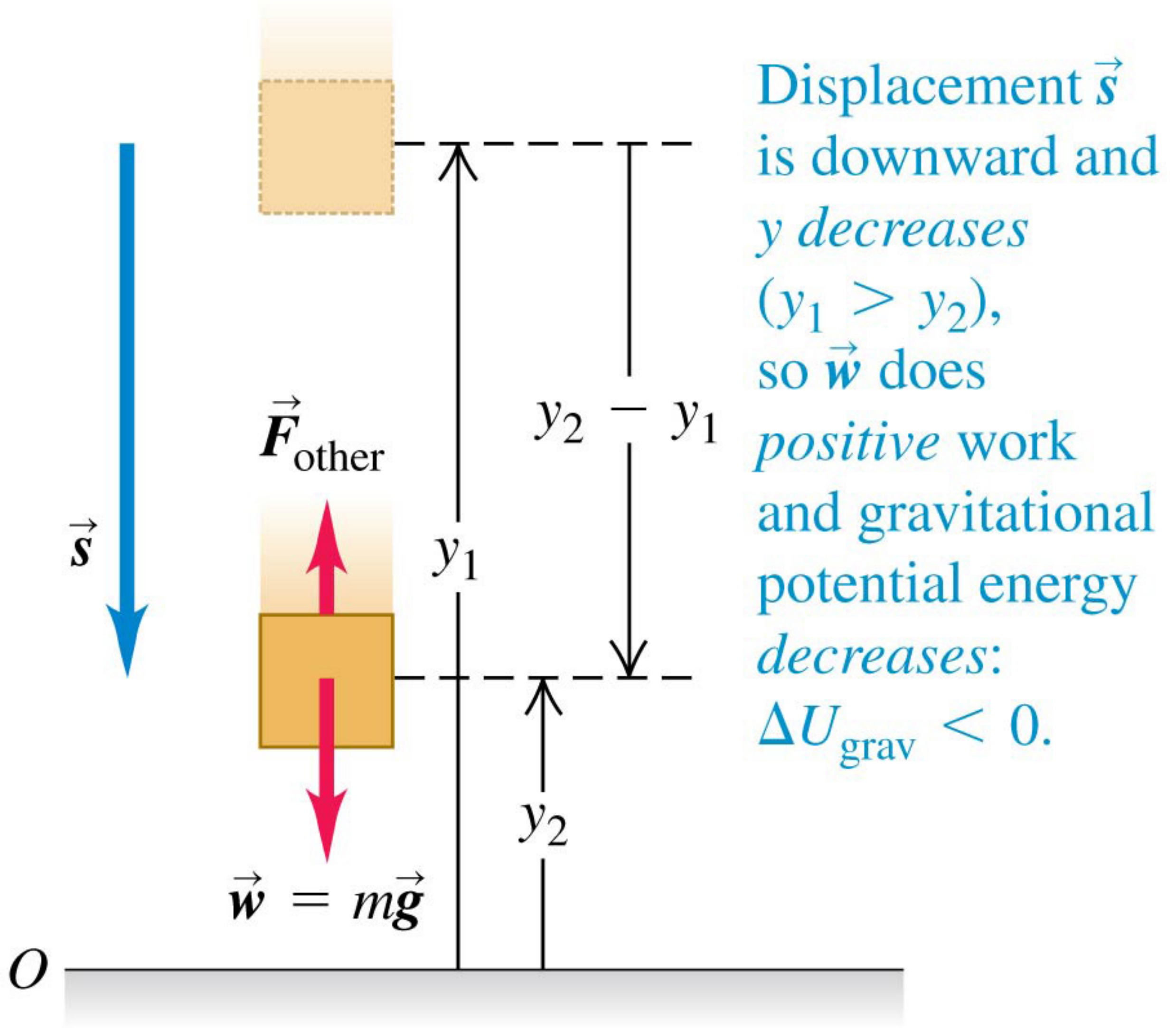
Mass of particle

Acceleration due to gravity

- As the basketball descends, gravitational potential energy is converted to kinetic energy and the basketball's speed increases.

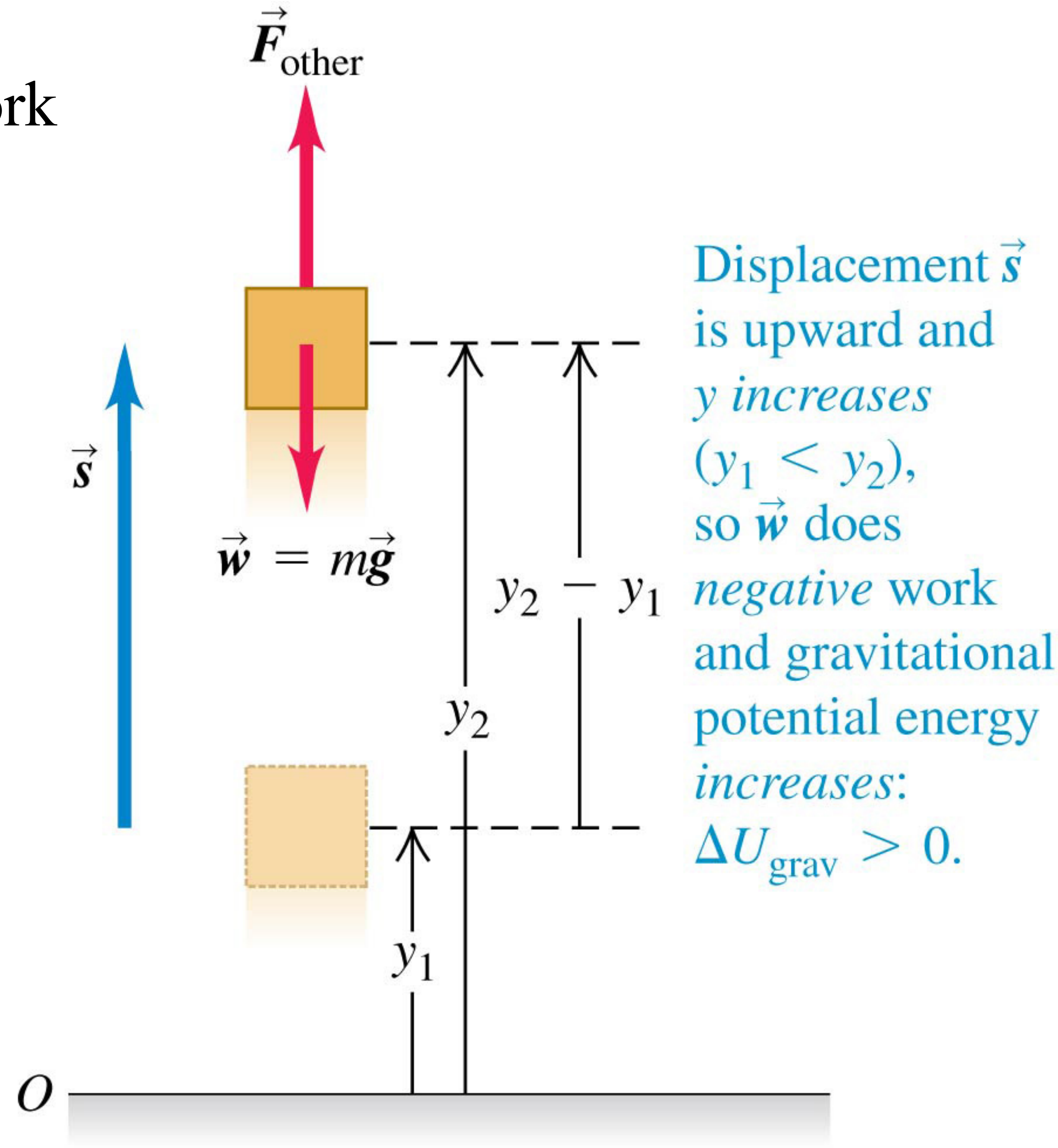
Gravitational potential energy

- The change in gravitational potential energy is related to the work done by gravity.



Gravitational potential energy

- When the body moves up, y increases, the work done by the gravitational force is negative, and the gravitational potential energy increases.



The conservation of mechanical energy

- The total **mechanical energy** of a system is the sum of its kinetic energy and potential energy.
- A quantity that always has the same value is called a **conserved** quantity.
- When only the force of gravity does work on a system, the total mechanical energy of that system is conserved.
- This is an example of the **conservation of mechanical energy**.

If only the gravitational force does work, total mechanical energy is conserved:

Initial kinetic energy $K_1 = \frac{1}{2}mv_1^2$

Initial gravitational potential energy $U_{grav,1} = mgy_1$

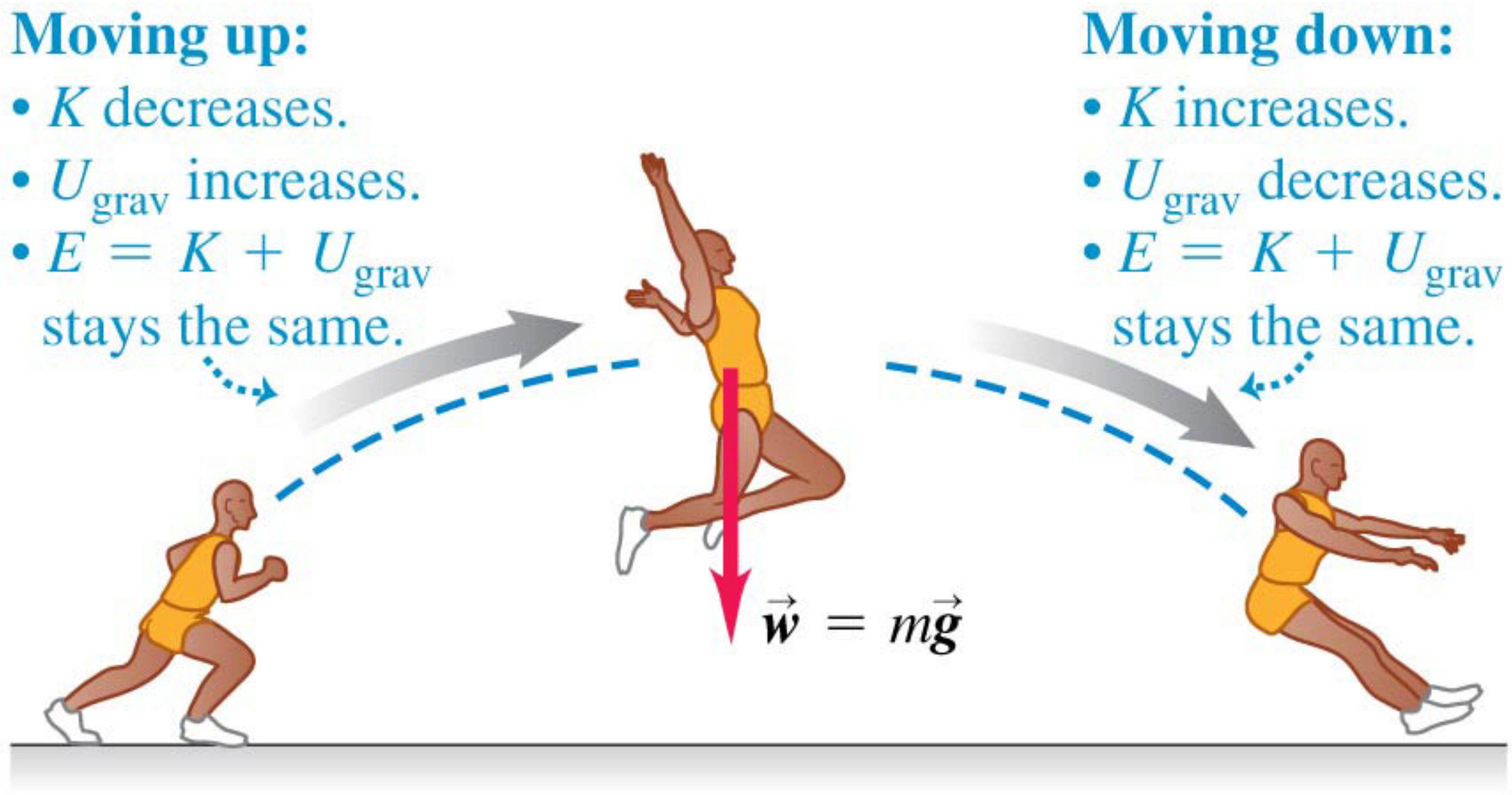
$$K_1 + U_{grav,1} = K_2 + U_{grav,2}$$

Final kinetic energy $K_2 = \frac{1}{2}mv_2^2$

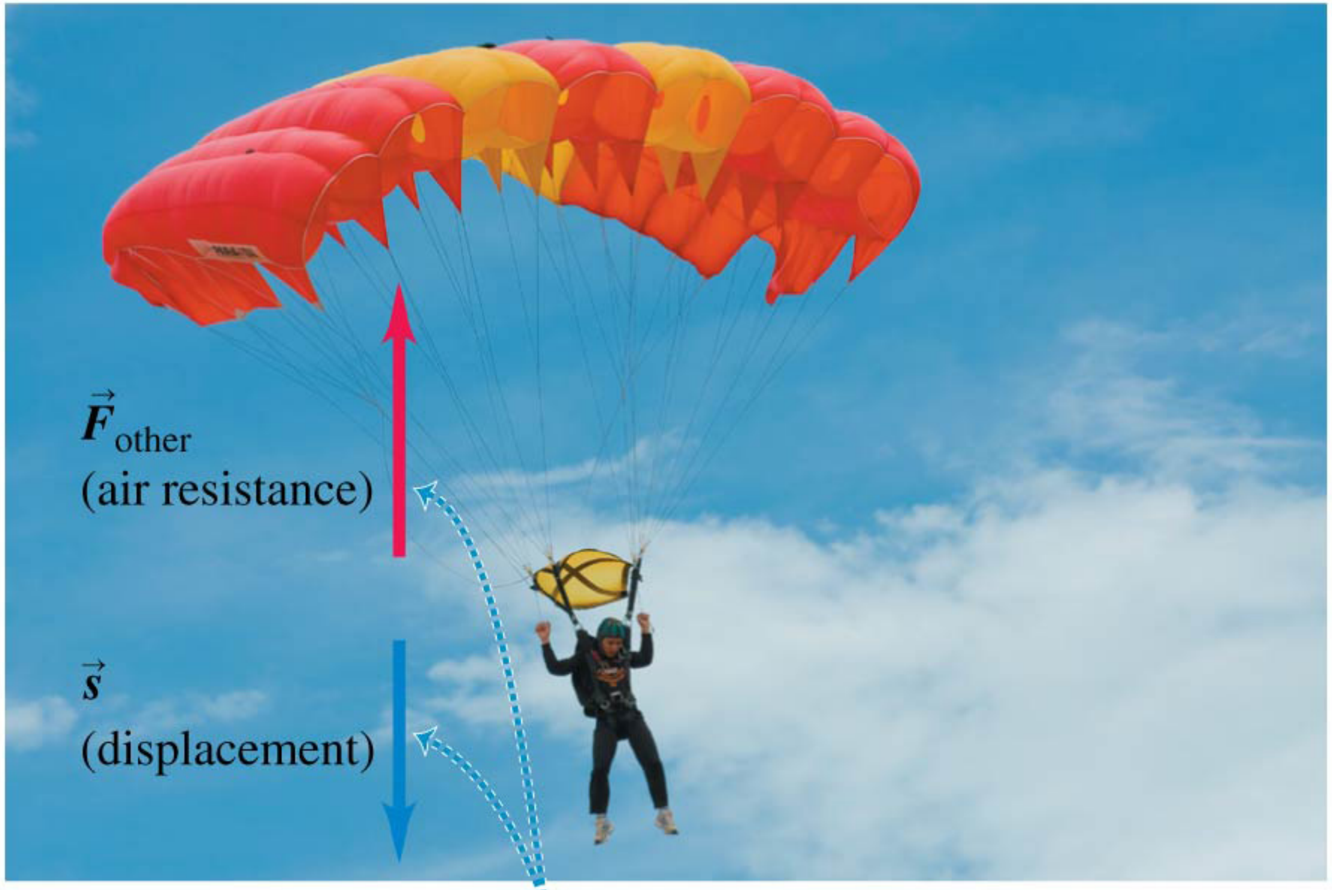
Final gravitational potential energy $U_{grav,2} = mgy_2$

The conservation of mechanical energy

- When only the force of gravity does work on a system, the total mechanical energy of that system is conserved.



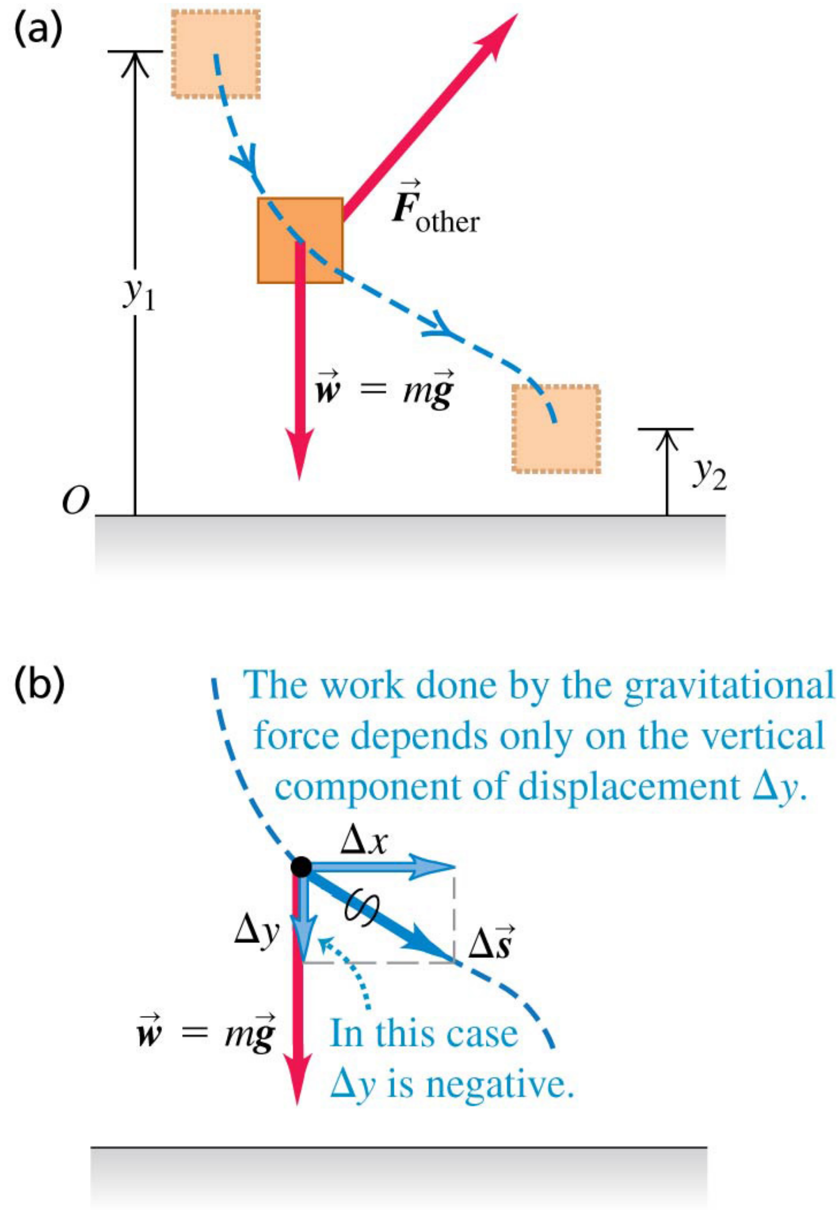
When forces other than gravity do work



- $\vec{F}_{\text{other}}$ and $\vec{s}$ are opposite, so $W_{\text{other}} < 0$.
- Hence $E = K + U_{\text{grav}}$ must decrease.
- The parachutist's speed remains constant, so K is constant.
- The parachutist descends, so U_{grav} decreases.

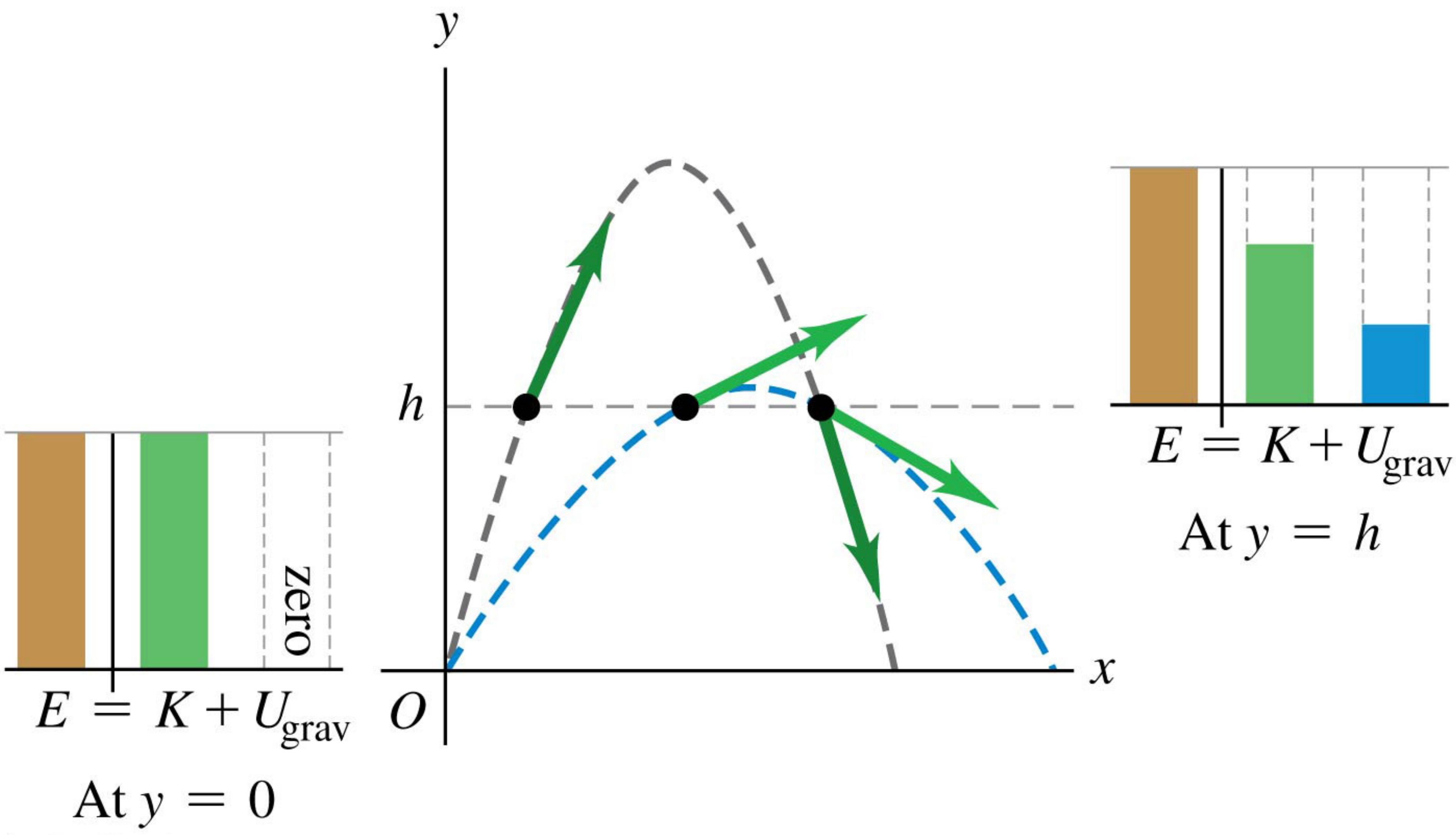
Work and energy along a curved path

- We can use the same expression for gravitational potential energy whether the body's path is curved or straight.
- $W_{grav} = mgy_1 - mgy_2$



Conceptual Example

- Two identical balls leave from the same height with the same speed but at different angles.



Elastic potential energy

- A body is **elastic** if it returns to its original shape after being deformed.

- **Elastic potential energy** is the energy stored in an elastic body, such as a spring:

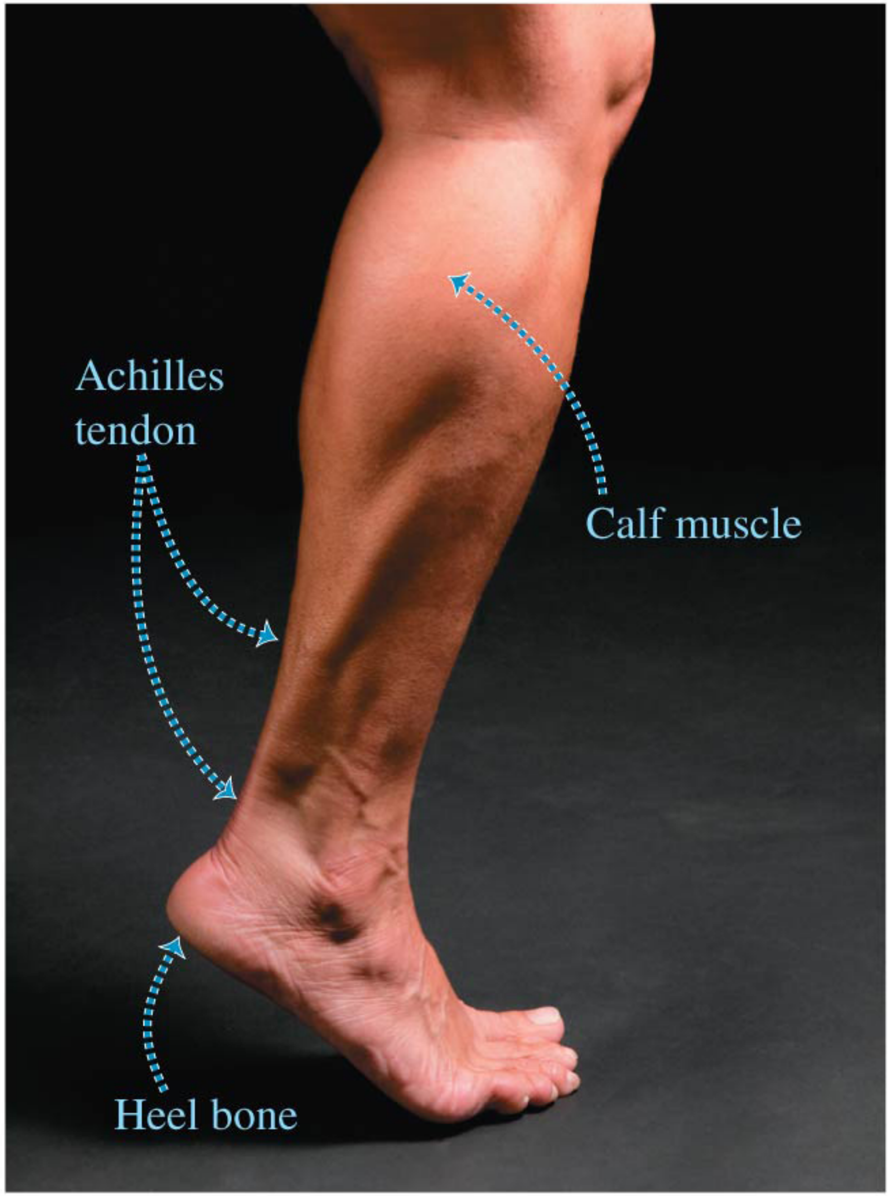
Elastic potential energy stored in a spring

$$U_{\mathrm{el}} = \frac{1}{2}kx^2$$

Force constant of spring

Elongation of spring ($x > 0$ if stretched, $x < 0$ if compressed)

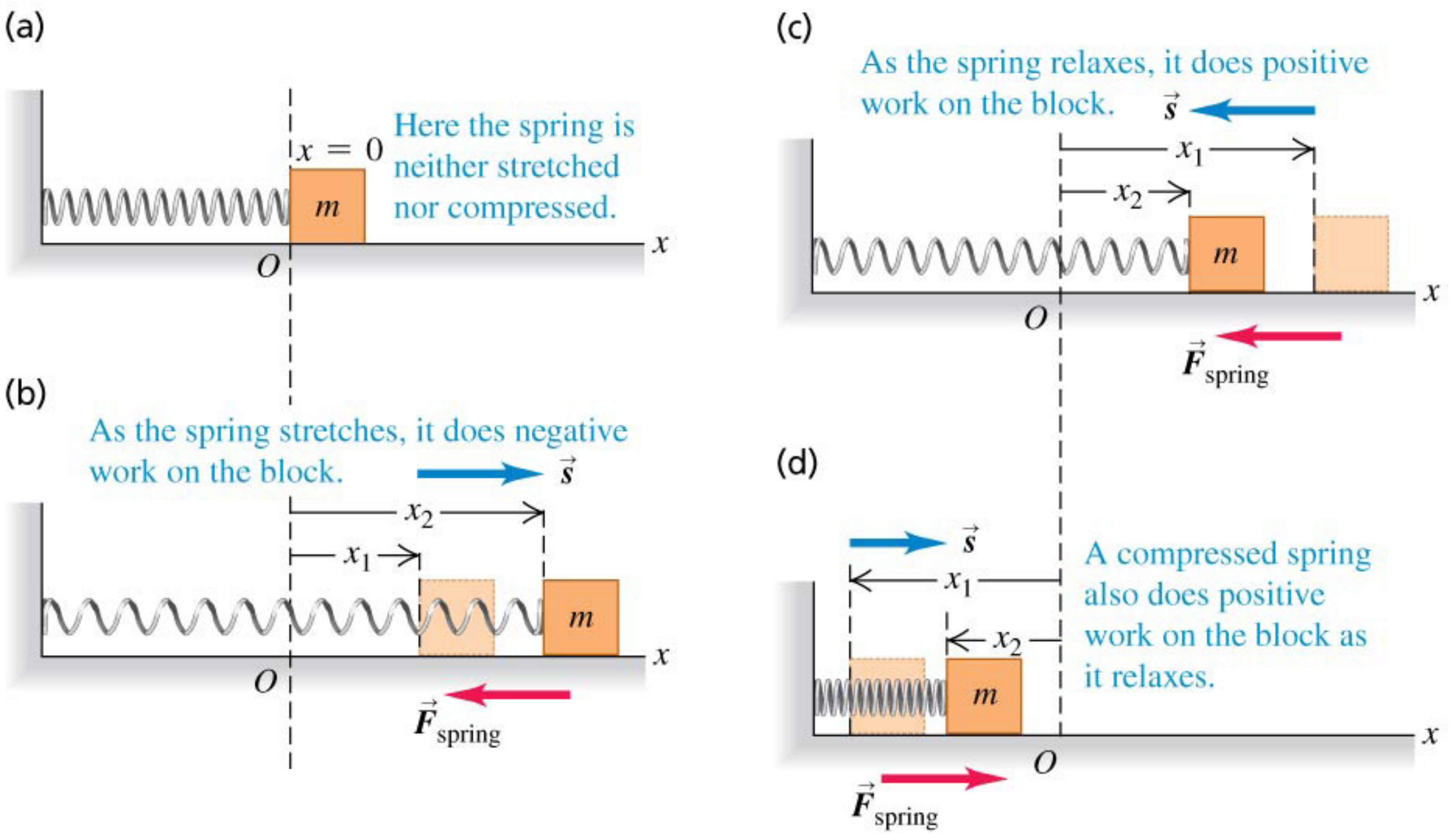
Elastic potential energy



- The Achilles tendon acts like a natural spring.
- When it stretches and then relaxes, this tendon stores and then releases elastic potential energy.
- This spring action reduces the amount of work your leg muscles must do as you run.

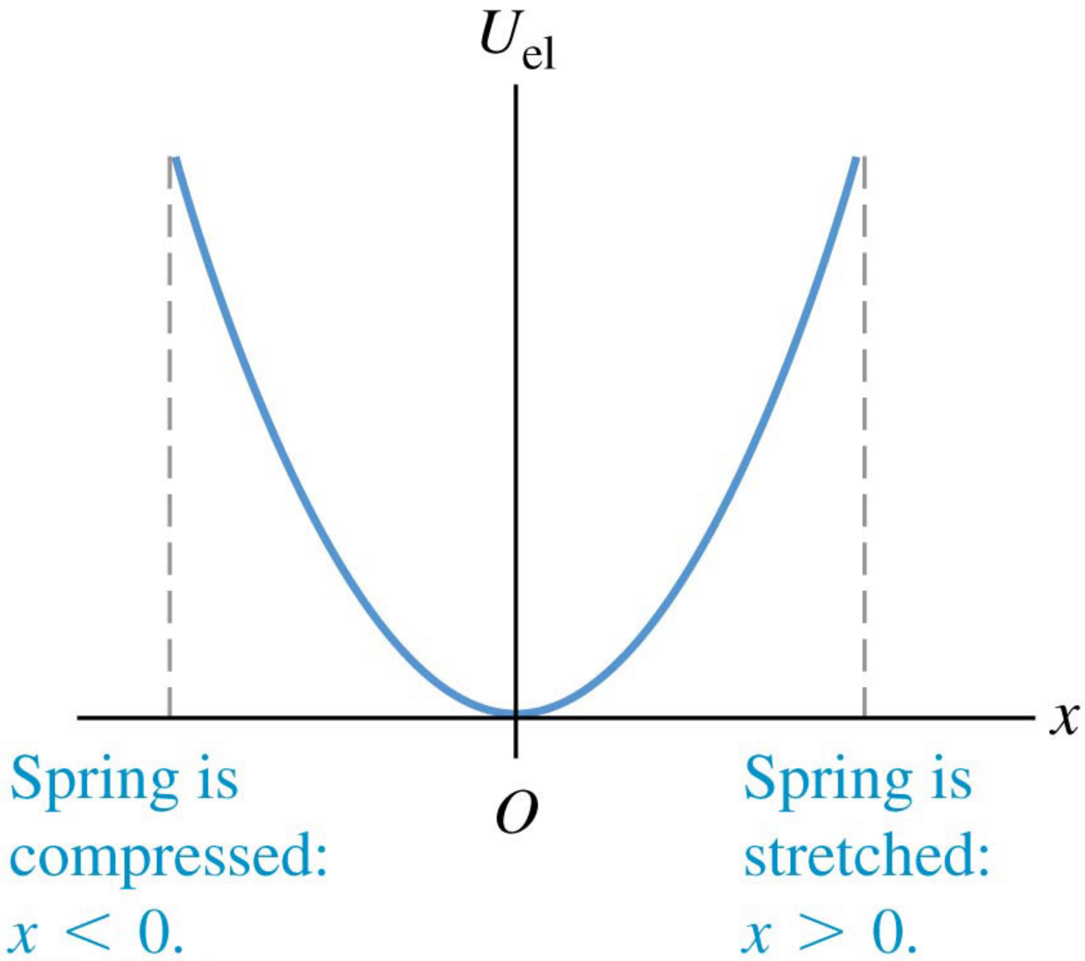
Work done by a spring

- The figure below shows how a spring does work on a block as it is stretched and compressed.



Elastic potential energy

- The graph of elastic potential energy for an ideal spring is a parabola.
- x is the extension or compression of the spring.
- Elastic potential energy is never negative.



Situations with both gravitational and elastic forces

- When a situation involves both gravitational and elastic forces, the total potential energy is the *sum* of the gravitational potential energy and the elastic potential energy:

$$U = U_{\text{grav}} + U_{\text{el}}$$

Conservative and nonconservative forces

- A **conservative force** allows conversion between kinetic and potential energy. Gravity and the spring force are conservative.

- The work done between two points by any conservative force

  a) can be expressed in terms of a *potential energy function*.

  b) is reversible.

  c) is independent of the path between the two points.

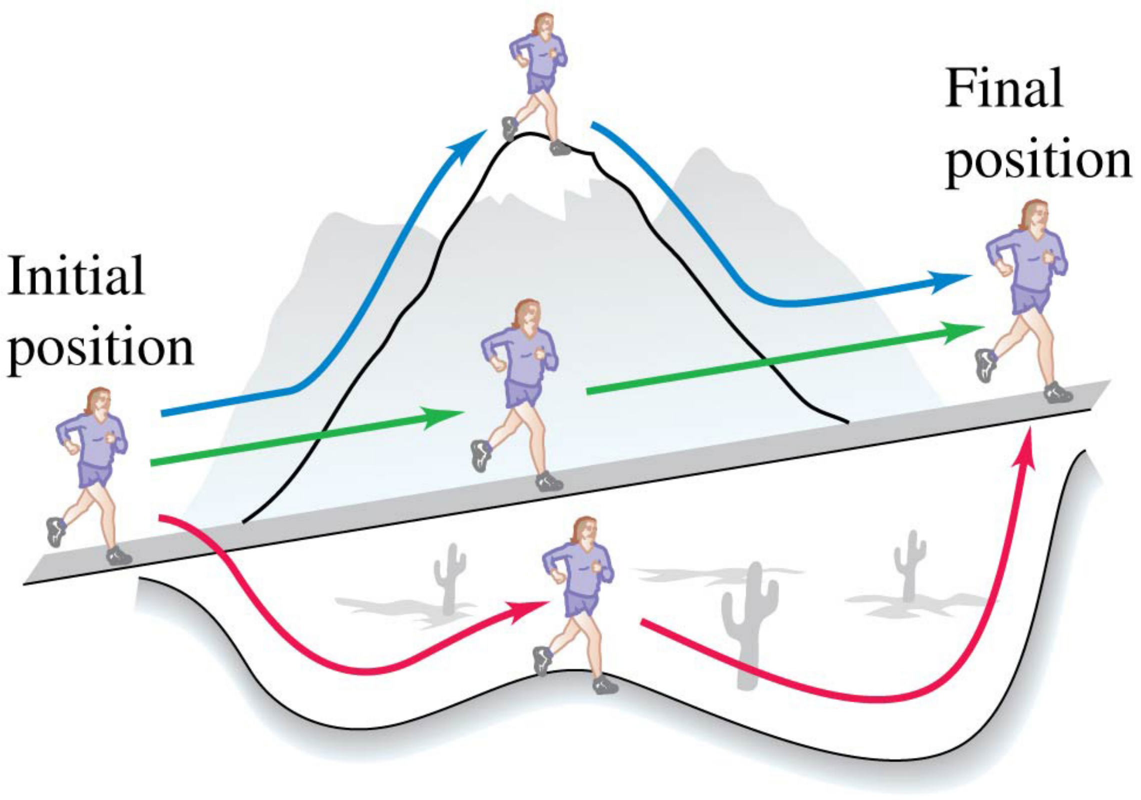
  d) is zero if the starting and ending points are the same.

- A force (such as friction) that is not conservative is called a **nonconservative force**, or a **dissipative force**.

Conservative forces

- The work done by a conservative force such as gravity depends on only the endpoints of a path, not the specific path taken between those points.

Because the gravitational force is conservative, the work it does is the same for all three paths.



Nonconservative forces

- As an automobile tire flexes as it rolls, nonconservative internal friction forces act within the rubber.
- Mechanical energy is lost and converted to internal energy of the tire.

- This causes the temperature and pressure of a tire to increase as it rolls.
- That's why tire pressure is best checked before the car is driven, when the tire is cold.

Conservation of energy

- Nonconservative forces do not store potential energy, but they do change the *internal energy* of a system.
- **The law of conservation of energy** means that energy is never created or destroyed; it only changes form.
- This law can be expressed as $\Delta K + \Delta U + \Delta U_{\text{int}} = 0$.

Force and potential energy in one dimension

- In one dimension, a conservative force can be obtained from its potential energy function using:

Force from potential energy: In **one-dimensional motion**, the value of a conservative force at point x ... is the negative of the derivative at x of the associated potential-energy function.

$$F_x(x) = -\frac{dU(x)}{dx}$$

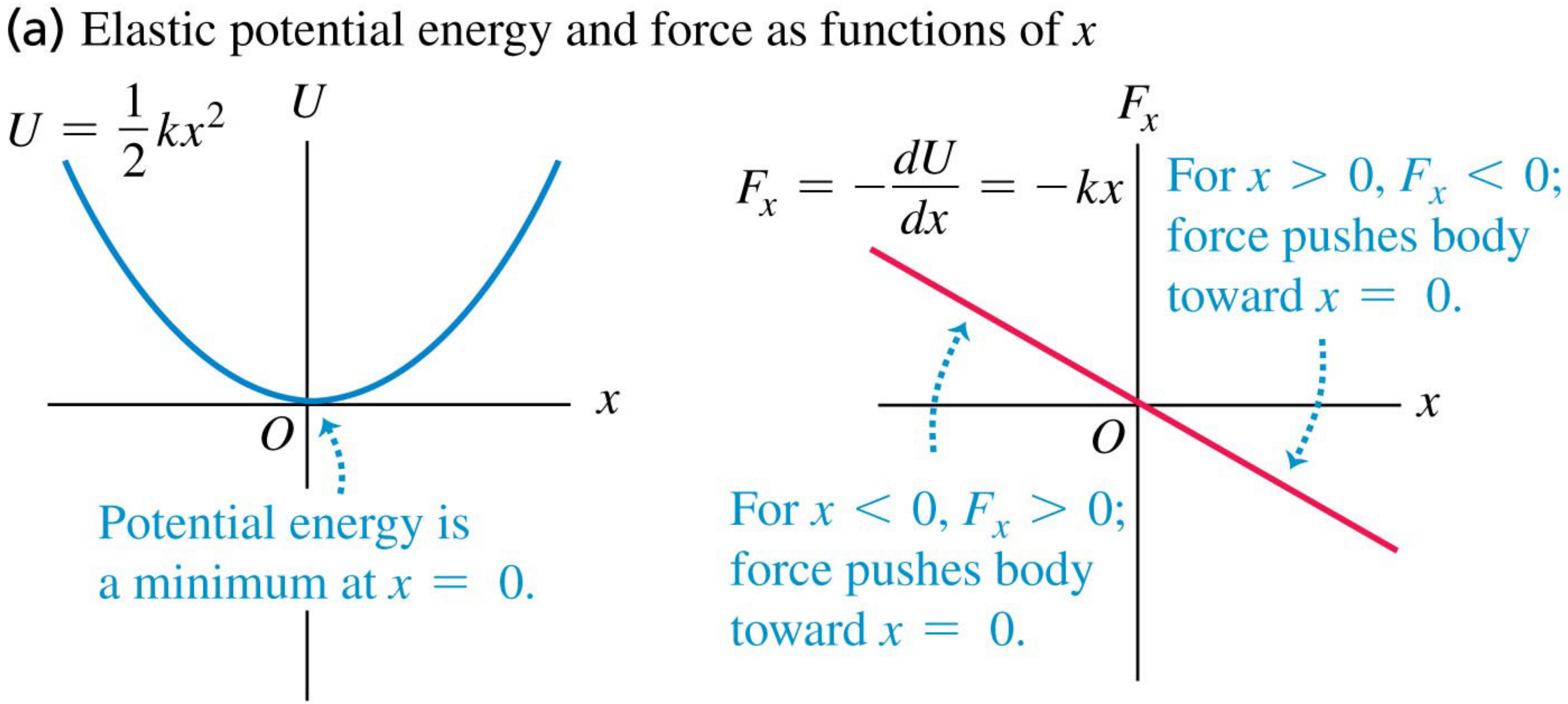
- In regions where $U(x)$ changes most rapidly with x, this corresponds to a large force magnitude.

- Also, when $F_x(x)$ is in the positive x-direction, $U(x)$ decreases with increasing x.

- A conservative force always acts to push the system toward *lower* potential energy.

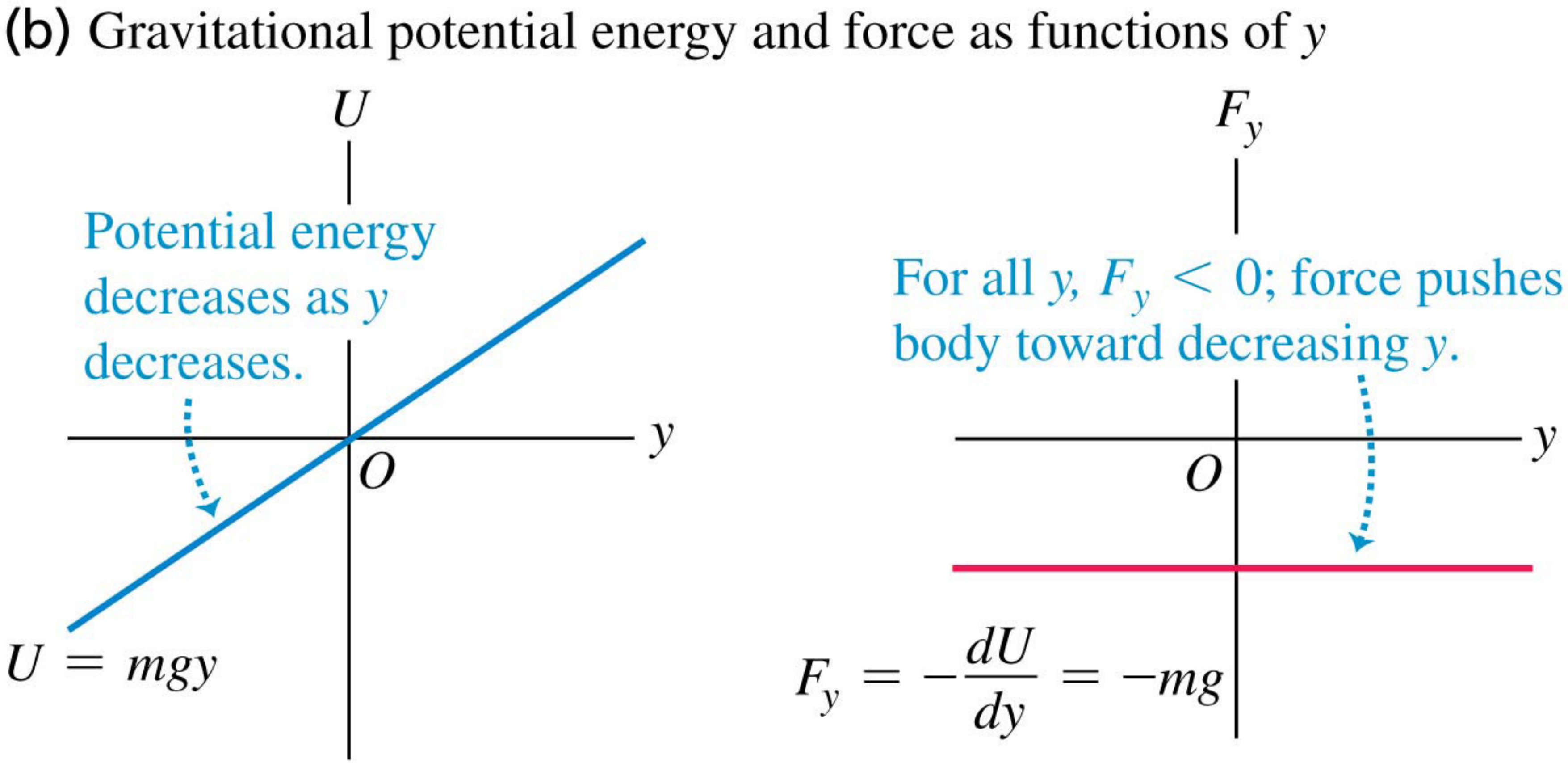
Force and potential energy in one dimension

- Elastic potential energy and force as functions of x for an ideal spring.



Force and potential energy in one dimension

- Gravitational potential energy and the gravitational force as functions of y.



Force and potential energy in three dimensions

- In three dimensions, the components of a conservative force can be obtained from its potential energy function using partial derivatives:

Force from potential energy: In **three-dimensional motion,** the value at a given point of each component of a conservative force ...

$$F_x = -\frac{\partial U}{\partial x} \qquad F_y = -\frac{\partial U}{\partial y} \qquad F_z = -\frac{\partial U}{\partial z}$$

... is the negative of the partial derivative at that point of the associated potential-energy function.

Force and potential energy in three dimensions

- When we take the partial derivative of U with respect to each coordinate, multiply by the corresponding unit vector, and then take the vector sum, this is called the **gradient** of U:

Force from potential energy: The vector value of a conservative force at a given point ...

$$\vec{F} = -\left(\frac{\partial U}{\partial x}\hat{\imath} + \frac{\partial U}{\partial y}\hat{\jmath} + \frac{\partial U}{\partial z}\hat{k}\right) = -\vec{\nabla} U$$

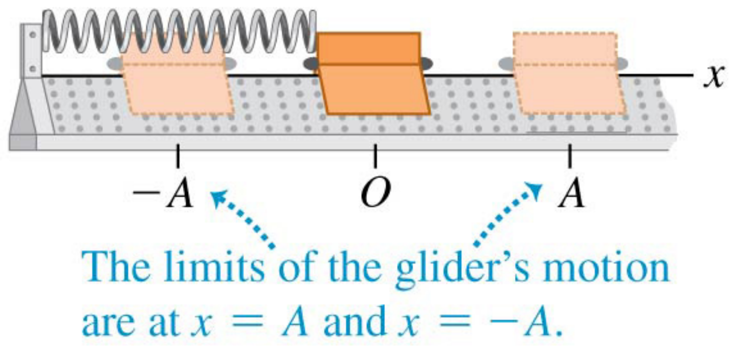
... is the negative of the gradient at that point of the associated potential-energy function.

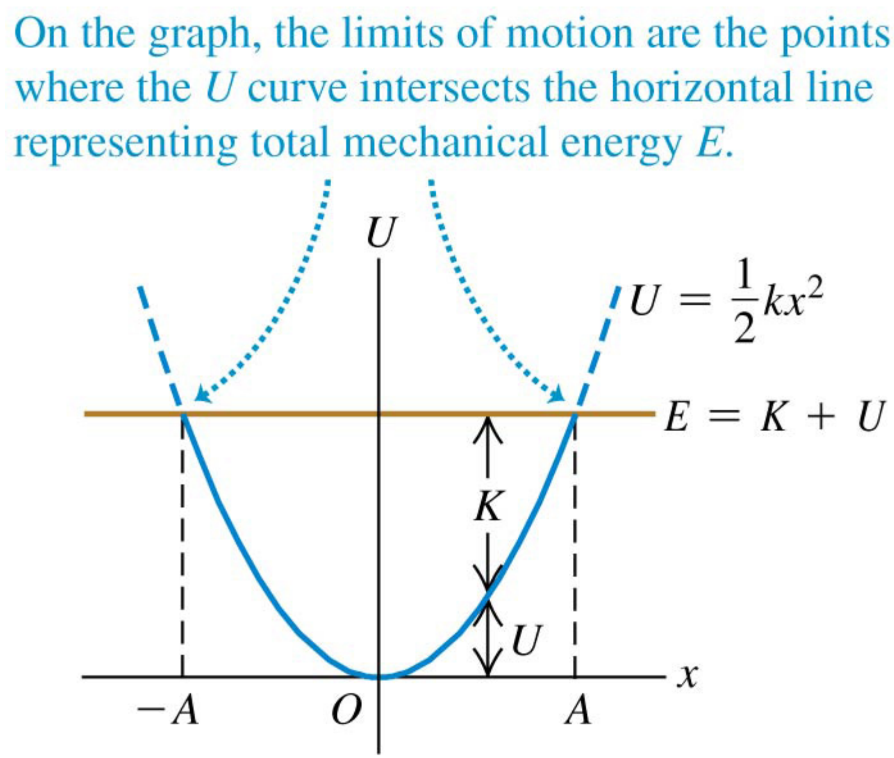
Force and potential energy

- The greater the elevation of a hiker in Canada's Banff National Park, the greater the gravitational potential energy U_{grav}.
- Where the mountains have steep slopes, U_{grav} has a large gradient and there's a strong force pushing you along the mountain's surface toward a region of lower elevation (and hence lower U_{grav}).
- There's zero force along the surface of the lake, which is all at the same elevation.

Energy diagrams

- An **energy diagram** is a graph that shows both the potential-energy function $U(x)$ and the total mechanical energy E.
- The figure illustrates the energy diagram for a glider attached to a spring on an air track.





Force and a graph of its potential-energy function

- For any graph of potential energy versus x, the corresponding force is $F_x = -dU/dx$.
- Whenever the slope of U is zero, the force there is zero, and this is a point of equilibrium.
- When U is at a minimum, the force near the minimum draws the object *closer* to the minimum, so it is a *restoring force*. This is called **stable equilibrium**.
- When U is at a maximum, the force near the maximum draws the object *away* from the maximum. This is called **unstable equilibrium**.

Unstable equilibrium

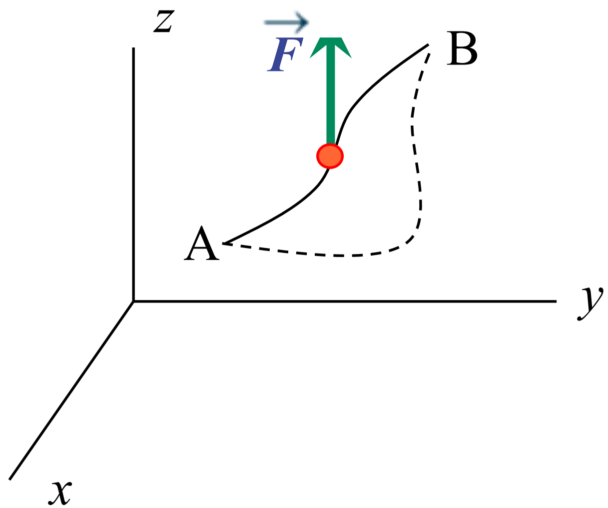
- Each of these acrobats is in unstable equilibrium.
- The gravitational potential energy is lower no matter which way an acrobat tips, so if she begins to fall she will keep on falling.
- Staying balanced requires the acrobats' constant attention.

Conservative force (general formalism)

A force F is said to be conservative if the work done is independent of the path followed by the force acting on a particle as it moves from A to B. This also means that the work done by the force F in a closed path (*i.e.*, from A to B and then back to A) is zero.

$$\oint \vec{F} \cdot d\vec{r} = 0$$

Thus, we say the work is conserved.



The work done by a conservative force depends only on the positions of the particle, and is independent of its velocity or acceleration.

Conservative force (general formalism)

The "conservative" potential energy of a particle/system is typically written using the potential function U. There are two major components to U commonly encountered in mechanical systems, the potential energy from gravity and the potential energy from springs or other elastic elements.

$$U_{total} = U_{gravity} + U_{springs}$$

Potential energy

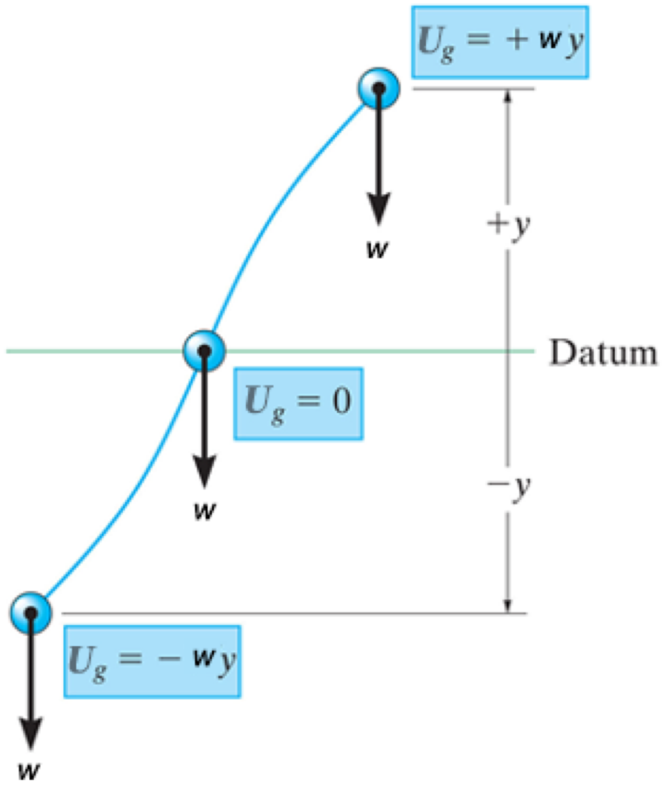
Potential energy is a measure of the amount of work a conservative force will do when a body changes position.

In general, for any conservative force system, we can define the potential function (V) as a function of position. The work done by conservative forces as the particle moves equals the change in the value of the potential function (e.g., the sum of $U_{gravity}$ and $U_{springs}$).

It is important to become familiar with the two types of potential energy and how to calculate their magnitudes.

Potential energy due to gravity

The potential function (formula) for a gravitational force, e.g., weight ($W = mg$), is the force multiplied by its elevation from a datum. The datum can be defined at any convenient location.

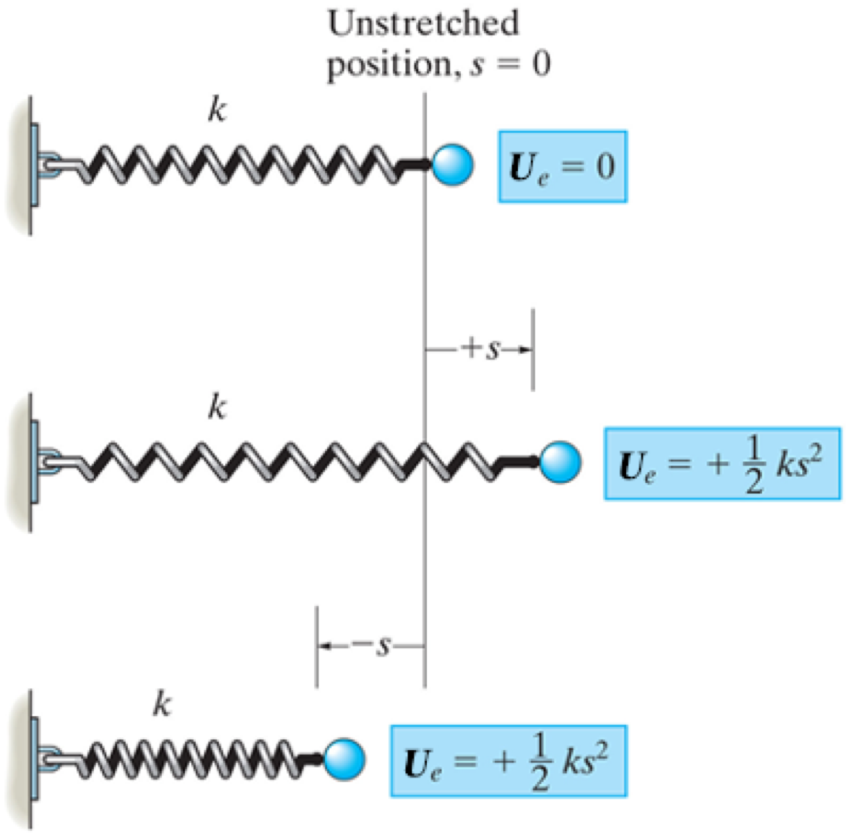


$$U_g = \pm w\, y$$

U_g is positive if y is above the datum and negative if y is below the datum. Remember, YOU get to set the datum.

Elastic potential energy

Recall that the force of an elastic spring is $F = ks$. It is important to realize that the potential energy of a spring, while it looks similar, is a different formula.



U_e (where 'e' denotes an elastic spring) has the distance "s" raised to a power (the result of an integration) or

$$U_e = \frac{1}{2} k s^2$$

Notice that the potential function U_e always yields positive energy.

Conservation of energy

When a particle is acted upon by a system of conservative forces, the work done by these forces is conserved and the sum of kinetic energy and potential energy remains constant. In other words, as the particle moves, kinetic energy is converted to potential energy and vice versa. This principle is called the principle of conservation of energy and is expressed as

$$K_1 + U_1 = K_2 + U_2 = \text{Constant}$$

K_1 stands for the kinetic energy at state 1 and U_1 is the potential energy function for state 1. K_2 and U_2 represent these energy states at state 2. Recall, the kinetic energy is defined as $K = \frac{1}{2} mv^2$.

Quiz

1. The potential energy of a spring is _________

   A) always negative. ✔B) always positive.

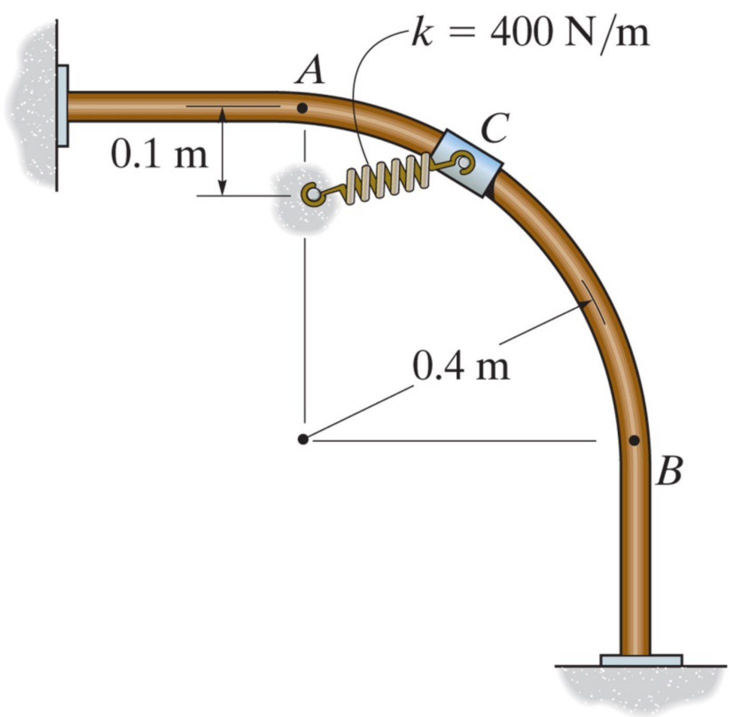
   C) positive or negative. D) equal to ks.

2. When the potential energy of a conservative system increases, the kinetic energy __________

   ✔A) always decreases. B) always increases.

   C) could decrease or increase. D) does not change.

Example



Given: The 4-kg collar, C, has a velocity of 2 m/s at A. The spring constant is 400 N/m. The unstretched length of the spring is 0.2 m.

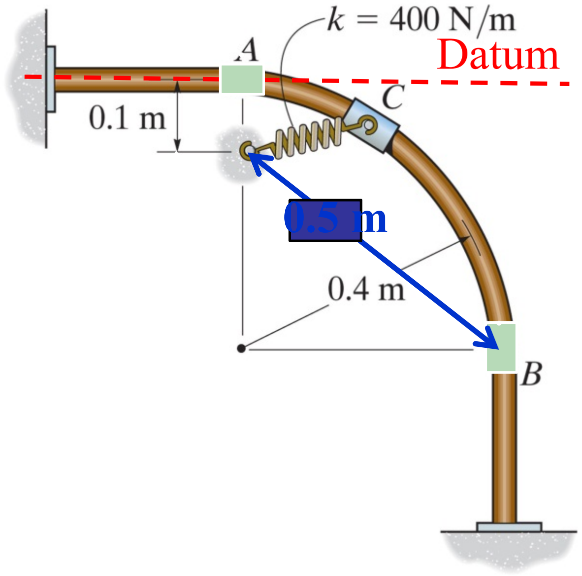
Find: The velocity of the collar at B.

Plan:

Apply the conservation of energy equation between A and B. Set the gravitational potential energy datum at point A or point B (in this example, choose point A—why?).

Example

Solution:



Note that the potential energy at *B* has two parts.

$U_B = (U_B)_e + (U_B)_g$

$U_B = \ 0.5\ (400)\ (0.5 - 0.2)^2 - 4\ (9.81)\ 0.4$

The kinetic energy at B is

$K_B = \ 0.5\ (4)\ v_B^{\,2}$

Similarly, the potential and kinetic energies at A will be

$U_A = \ 0.5\ (400)\ (0.1 - 0.2)^2, \quad K_A = \ 0.5\ (4)\ 2^2$

The energy conservation equation becomes $K_A + U_A = K_B + U_B$.

$$[\ 0.5(400)\ (0.5 - 0.2)^2 - 4(9.81)0.4\ ] + 0.5\ (4)\ v_B^{\,2}$$

$$= [0.5\ (400)\ (0.1 - 0.2)^2\ ] + 0.5\ (4)\ 2^2$$

$\Rightarrow \ v_B = 1.96$ m/s

Quiz

1. If the work done by a conservative force on a particle as it moves between two positions is –10 N·m, the change in its potential energy is ________.

   A) 0 N·m B) -10 N·m

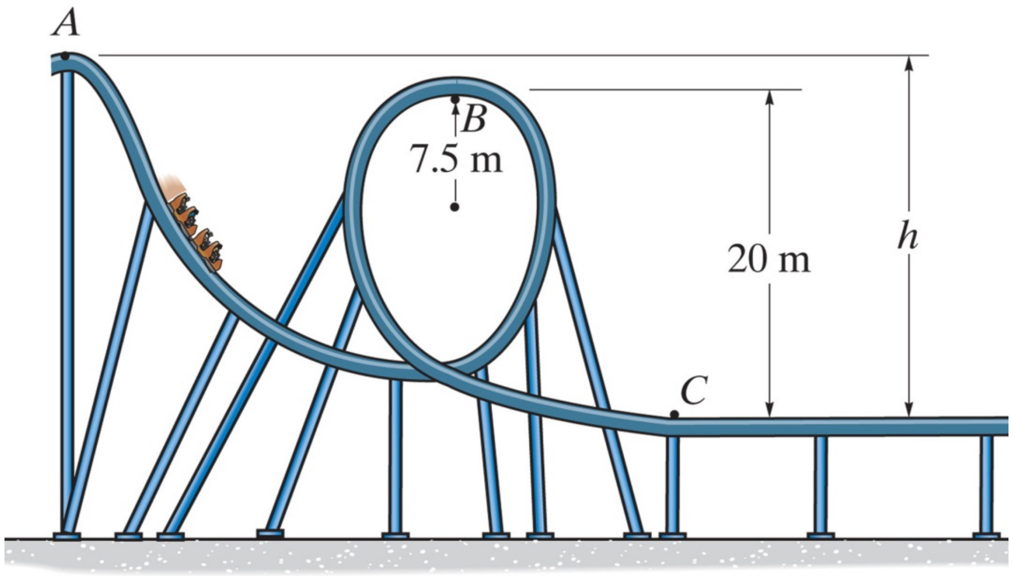
   ✔ C) +10 N·m D) None of the above.

2. Recall that the work of a spring is $W_{1\text{-}2} = -\frac{1}{2}k(s_2^2 - s_1^2)$ and can be either positive or negative. The potential energy of a spring is $U = \frac{1}{2}ks^2$. Its value is ____________.

   A) always negative B) either positive or negative

   ✔ C) always positive D) an imaginary number!

Example



Given: The 800-kg roller coaster car is released from rest at A.

Find: The minimum height, h, of Point A so that the car travels around inside loop at B without leaving the track. Also find the velocity of the car at C for this height, h, of A.

Plan:

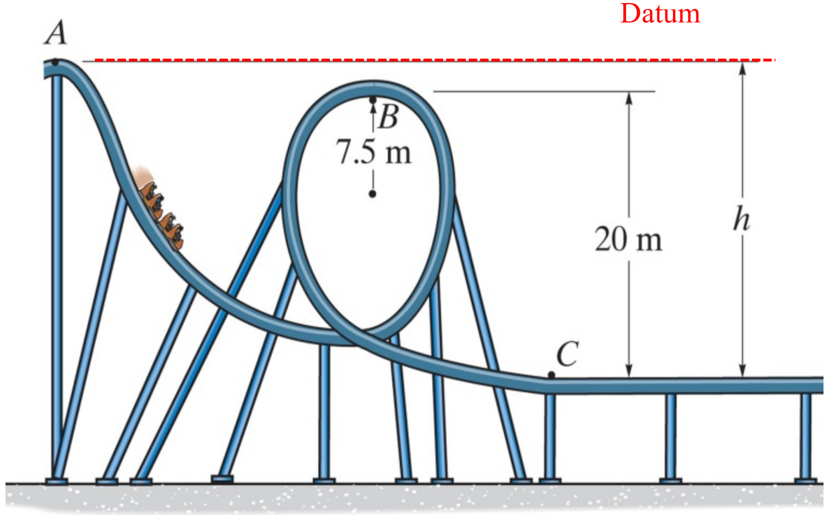
Note that only kinetic energy and potential energy due to gravity are involved. Determine the velocity at B using the equation of motion and then apply the conservation of energy equation to find minimum height h.

Example

Solution:

1) Placing the datum at A:

$$K_A + U_A = K_B + U_B$$



$$\Rightarrow 0.5\,(800)\,0^2 + 0$$
$$= 0.5\,(800)\,(v_B)^2 - 800(9.81)\,(h - 20)$$

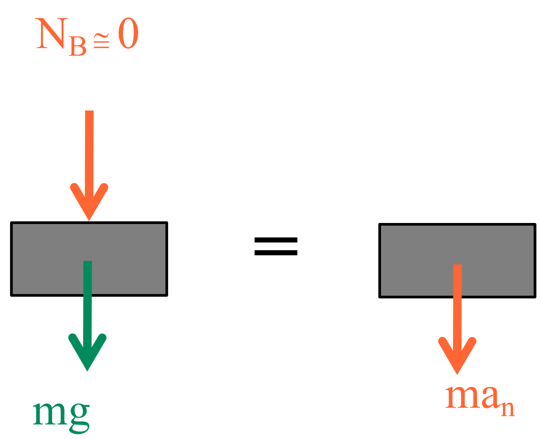
2) Find the required velocity of the coaster at B so it doesn't leave the track.

Equation of motion applied at B:

$$\Sigma F_n = ma_n = m\frac{v^2}{\rho}$$

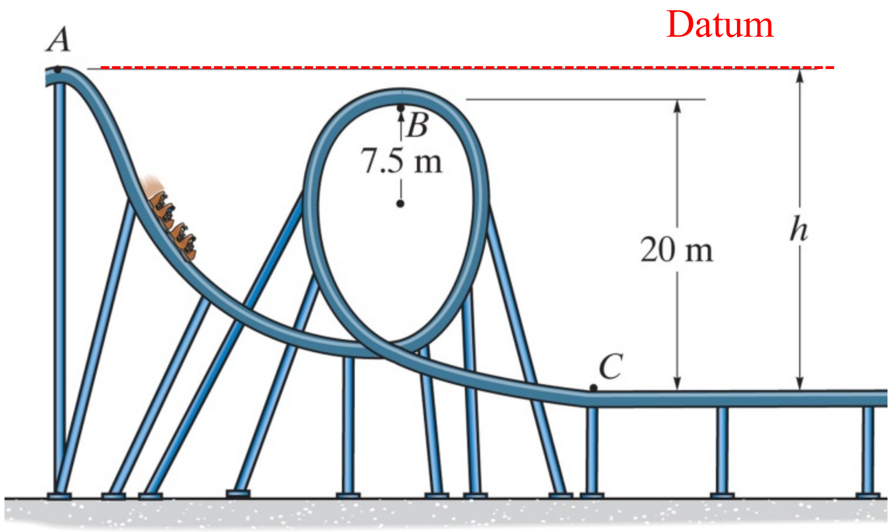
$$800\,(9.81) = 800\,\frac{(v_B)^2}{7.5}$$

$$\Rightarrow v_B = 8.578 \text{ m/s}$$



Example

Now using the energy conservation, eq.(1), the minimum h can be determined.



$$0.5\,(800)\,0^2 + 0 = 0.5\,(800)\,(8.578)^2 - 800(9.81)\,(h - 20)$$

$\Rightarrow$ $h = 23.75$ m

3) Find the velocity at C applying the energy conservation.

$$K_A + U_A = K_C + U_C$$

$$\Rightarrow 0.5\,(800)\,0^2 + 0 = 0.5\,(800)\,(v_C)^2 - 800(9.81)\,(23.75)$$

$$\Rightarrow U_C = 21.6 \text{ m/s}$$

Quiz

1. The principle of conservation of energy is usually ______ to apply than the principle of work & energy.

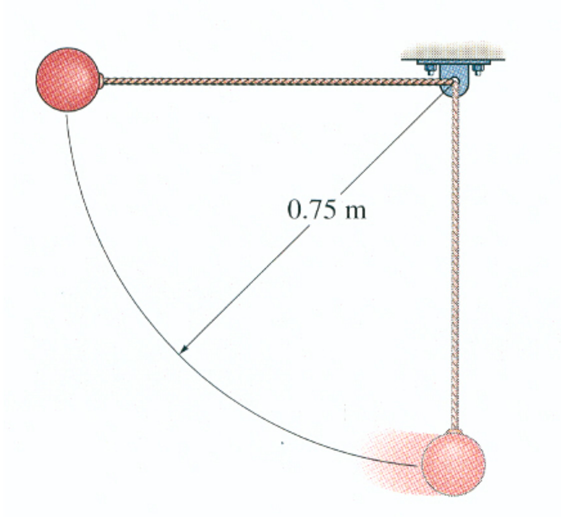
   A) harder ✔B) easier

   C) the same amount of work D) It is a mystery!

2. If the pendulum is released from the horizontal position, the velocity of its bob in the vertical position is _____.

   ✔A) 3.8 m/s B) 6.9 m/s

   C) 14.7 m/s D) 21 m/s



$$mgh=1/2\ mv^2 \quad v=sqrt(2gh)$$