MA1300 Self Practice # 3

1. (P91, #12, 14) Use the definition of continuity and the properties of limits to show that the function is continuous at the given number a.

$$(1) \qquad f(x) = x^2 + \sqrt{7-x}, \qquad a = 4.$$
$$(2) \qquad f(t) = \frac{2t - 3t^2}{1 + t^3}, \qquad a = 1.$$

2. (P91, #15) Use the definition of continuity and the properties of limits to show that the function is continuous on the given interval:

$$f(x) = \frac{2x+3}{x-2}, \qquad (2, \infty).$$

For Questions $3 \sim 4$*, explain why the function is discontinuous at the given number* 1*. Sketch the graph of the function.*

3. (P91, #19) $\quad f(x) = \begin{cases} 1 - x^2, & \text{if } x < 1 \\ \frac{1}{x}, & \text{if } x \geq 1 \end{cases} \qquad a = 1.$

4. (P91, #20) $\quad f(x) = \begin{cases} \frac{x^2 - x}{x^2 - 1}, & \text{if } x \neq 1 \\ 1, & \text{if } x = 1 \end{cases} \qquad a = 1.$

5. (P91, #36, 38) Use continuity to evaluate the limit.

$$(1) \quad \lim_{x \to \pi} \sin(x + \sin x)$$
$$(2) \quad \lim_{x \to 2} (x^3 - 3x + 1)^{-3}$$

6. (P92, #39) Show that f is continuous on $(-\infty, \infty)$:

$$f(x) = \begin{cases} x^2, & \text{if } x < 1, \\ \sqrt{x}, & \text{if } x \geq 1. \end{cases}$$

7. (P92, #44) The gravitational force exerted by the earth on a unit mass at a distance r from the center of the planet is

$$F(r) = \begin{cases} \frac{GMr}{R^3}, & \text{if } r < R \\ \frac{GM}{r^2}, & \text{if } r \geq R \end{cases}$$

where M is the mass of the earth, R is its radius, and G is the gravitational constant. Is F a continuous function of r? Explain why.

8. (P92, #45) For what values of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x, & \text{if } x < 2, \\ x^3 - cx, & \text{if } x \geq 2. \end{cases}$$

9. (P92, #49) If $f(x) = x^2 + 10 \sin x$, show that there is a number c such that $f(c) = 1000$.

10. (P92, #50) Suppose f is continuous on $[1, 5]$ and the only solutions to the equation $f(x) = 6$ are $x = 1$ and $x = 4$. If $f(2) = 8$, explain why $f(3) > 6$.