MA2507 Computing Mathematics Laboratory: Final test

This is a closed-book test. Duration of the test: 12:00 - 1:45pm. Submission period: 1:45pm-1:50pm. Late submission will Not be marked. Submit PDF files produced by `publish` as attachments to Canvas.

MATLAB script files should start with a line like `%% Your name, student ID, and question number`. Figures should have a title generated by `title('your name, student ID, and question number')`.

============= Honesty Pledge============

I pledge that the answers in this examination are my own and that I will not seek or obtain an unfair advantage in producing these answers. Specifically,

(i). I will not plagiarize (copy without citation) from any source;

(ii). I will not communicate or attempt to communicate with any other person during the examination; neither will I give or attempt to give assistance to another student taking the examination;

(iii). I understand that any act of academic dishonesty can lead to disciplinary action.

Please write "I pledge to follow the Rules on Academic Honesty and understand that violations may lead to severe penalties" onto the first page.

=================================

1. (15 points). Write a code to find the solution of the following equation

$$g(x) = x^5 + x - 10000$$

based on the algorithm described as follows:

$$\begin{aligned} y_n &= x_n - \frac{g(x_n)}{g'(x_n)}, \\ z_n &= y_n - \frac{g(y_n)}{g'(y_n)} - \frac{g(y_n)^2 P(y_n)}{2g'(y_n)^3}, \\ x_{n+1} &= z_n - \frac{g(z_n)}{g[z_n, y_n] + (z_n - y_n) g[z_n, y_n, y_n]}, \end{aligned}$$

where

$$\begin{aligned} P(y_n) &= \frac{2}{x_n - y_n}\Big(3\frac{g(x_n) - g(y_n)}{x_n - y_n} - 2g'(y_n) - g'(x_n)\Big), \\ g[z_n, y_n] &= \frac{g(z_n) - g(y_n)}{z_n - y_n}, \\ g[z_n, y_n, y_n] &= \frac{g[z_n, y_n] - g'(y_n)}{z_n - y_n}. \end{aligned}$$

The iteration will terminate when the tolerance is less than 10^{-8}, where the tolerance is defined as the error between x_{n+1} and x_n, i.e., tolerance $= |x_{n+1} - x_n|$.

Please output your solution.

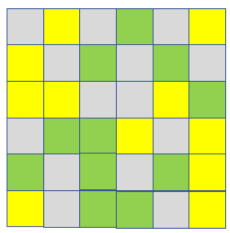
2. (20 points). The probability is defined as follows

$$p_i = P(X = x_i) = e^{-\lambda}\frac{\lambda^i}{i!}, \quad i = 0, 1, 2, \cdots, 11, \tag{1}$$

where $\lambda = 2$ and $x_i = i$.

Generate 1000 random numbers belonging to $[0, 1]$ and compute the probability based on (1). Write your own source codes to describe the above probability. In the codes, you should define p_i and x_i. No need to output x_i and p_i.

3. (15 points).



(a) Enter the above image into a 6×6 matrix A. Assume the shades you see are only 0, 0.5 and 1.

(b) Find the singular value decomposition (SVD) for A.

(c) Simplify the matrix preserving 2,3 and 4 singular values and plot the corresponding images using subplot.

4. (25 points). Given a data set as follows:

x	2	3	4	5
y	4	5	7	10

We will use a quadratic spline, called F, defined on $2 \le x \le 5$ to fit the given data, satisfying the conditions below:

- It is a quadratic polynomial on each subinterval ($[2, 3]$, $[3, 4]$, $[4, 5]$);
- It passes through all the data points;
- The first derivative is continuous at all interior points;

(a) Determine the quadratic spline, so that the following quantity

$$F'(2) = F'(5)$$

is satisfied. You should write your own code to find F, and the main steps for the codes should be clearly written. Please print the 9 coefficents for the quadratic polynomial.

(b) Plot the curve F and the given data points in one graph with x-label, y-label and title.

(c) Compare your result with **spline** and **polyfit**. What is your conclusion?

5. (25 points). Solve the differential equations

$$\begin{aligned} \sin(u_1)u_1'' + \cos(u_2)u_2'' + u_1 + u_3 &= 1, \\ -\cos(u_2)u_1'' + \sin(u_1)u_2'' + u_2 &= 0, \\ u_1 + u_2 + u_3 &= 2 \end{aligned}$$

over the interval $[-1, 1]$, i.e., $x \in [-1, 1]$. The boundary conditions are given as $u_1(-1) = 2, u_2(-1) = 0, u_1(1) = 0, u_2(1) = 0$.

Plot u_1, u_2 and u_3 with respect to x using subplot, where you should specify x-label, y-label and title.

====end of test