Tutorial 5 (Chapter 5)

1. Choose a number X at random from the set of numbers $\{1, 2, 3, 4, 5\}$. Now choose a second number Y at random from the subset no larger than X. Find the joint mass function of X and Y.

2. A television store owner figures that 45 percent of the customers entering this store will purchase an ordinary television set, 15 percent will purchase a plasma television set, and 40 percent will just be browsing. If 5 customers enter his store on a given day, what is the probability that he will sell 2 ordinary sets and 1 plasma set on the day?

3. A segment AC has length $2l$. B is the midpoint of AC. Pick a point D randomly on the segment AB. Pick a point E randomly on the segment BC. What is the probability that AD,DE and EC can form a triangle?

4. Two random variables X and Y are identically distributed, with density

$$f(x) = \begin{cases} \frac{3}{8}x^2 & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Suppose the events $P(X \le C)$ and $P(Y \le C)$ are independent, and $P(X \le C, Y \le C) = 1/4$. Find C.

5. Suppose two continuous random variables X and Y have joint CDF F, and marginal CDF F_X and F_Y respectively, Find the following probabilities in terms of the CDF's.

   (a) $P(X > a, Y < b)$

   (b) $P(X > a, Y > b)$

   (c) $P(X < a \text{ or } Y < b)$

   (d) $P(X < a \text{ or } Y > b)$

6. Suppose the joint PMF of (X, Y) is $p(0, 0) = p_1, p(0, 1) = p_2, p(1, 0) = p_3, p(1, 1) = p_4$ (thus $\sum p_i = 1$). Find the joint CDF.

7. Suppose the joint probability mass function for (X,Y) is

	-1	1	2
-1	$1/4$	$1/10$	$3/10$
2	$3/20$	$3/20$	$1/20$

Find the pmf of the following r.v: (a)$X + Y$ (b)$\max\{X, Y\}$ (c)$\sin \frac{\pi(XY)}{2}$

8. The joint probability mass function of X and Y is given by $p(1, 1) = 1/8, p(1, 2) = 1/4, p(2, 1) = 1/8, p(2, 2) = 1/2$.

   (a) Compute the conditional mass function of X given $Y = i, i = 1, 2$.

   (b) Are X and Y independent?

   (c) Compute $P\{XY \le 3\}$ and $P\{X + Y > 2\}$.

9. Joint density for (X, Y) is given by

$$f(x, y) = \begin{cases} 1 & |y| < x, 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional densities $f_{X|Y}$ and $f_{Y|X}$.

10. Suppose the variance of X exists and is nonzero. Let $Y = kX + a, k > 0$, find $\rho(X, Y)$.

11. For n random variables $X_1, \ldots, X_n$, the covariance matrix is defined as the $n \times n$ symmetric matrix

$$\begin{bmatrix} Var(X_1) & Cov(X_1, X_2) & Cov(X_1, X_3) & \cdots & Cov(X_1, X_n) \\ Cov(X_2, X_1) & Var(X_2) & Cov(X_2, X_3) & \cdots & Cov(X_2, X_n) \\ \vdots & & \vdots & & \vdots \\ Cov(X_n, X_1) & Cov(X_n, X_2) & \cdots & & Var(X_n) \end{bmatrix}$$

Suppose X and Y are i.i.d. $Pois(\lambda)$. Find the covariance matrix of $(2X + Y, 2X - Y)$.

12. Suppose X and Y are identically distributed, but not necessarily independent, show that $X + Y$ and $X - Y$ are uncorrelated.