Chapter 3
Inductance and Capacitance

1. Current –voltage relationship for a capacitance or inductance.
2. Stored energy in a capacitance or inductance.

Capacitors: Concept

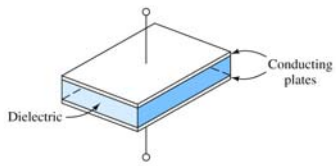
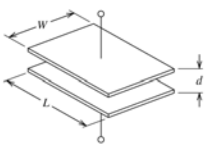


Figure 3.1 A parallel-plate capacitor consists of two conductive plates separated by a dielectric layer.

- A capacitor consists of two conducting plates
- Charge builds up on each of the plates
- Energy storage element via electrical field

Capacitance of the Parallel-Plate Capacitor



$$C = \frac{\varepsilon A}{d} \qquad A = WL$$

$$\varepsilon_0 \cong 8.85\times10^{-12}\ \text{F/m}$$

$$\varepsilon = \varepsilon_r \varepsilon_0$$

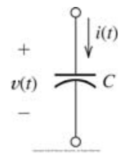
Table 3.1: Dielectric constants
Example: Air – 1.0

Relationship: Voltage-current

$q = Cv$ C: Capacitance in Farads

$$i(t) = \frac{dq}{dt}$$

$$i = C\frac{dv}{dt}$$



Passive Configuration

$$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)dt + v(t_0)$$

Example 3.1

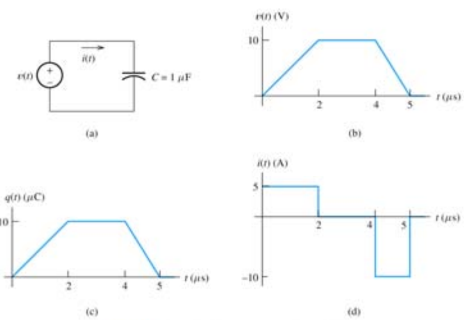


Figure 3.4 Circuit and waveforms for Example 3.1.

Example 3.2

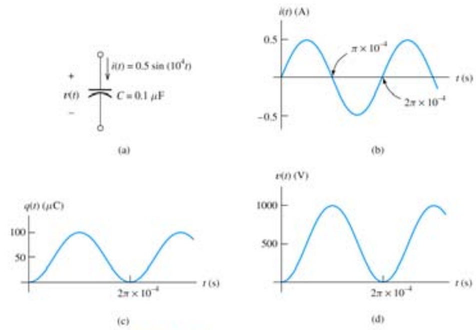


Figure 3.5 Waveforms for Example 3.2.

Power $p(t) = v(t)i(t) \quad p(t) = Cv(t)\dfrac{dv}{dt}$

Stored Energy

$$w(t) = \int_{t_0}^{t} p(t)dt$$

$$w(t) = \int_{t_0}^{t} Cv(t)\frac{dv}{dt}dt$$

$$= \int_{0}^{v(t)} Cv(t)dv \qquad v(0) = 0$$

$$= \frac{1}{2}Cv(t)^2$$

Example 3.3

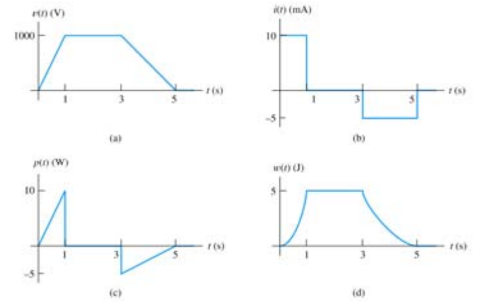


Figure 3.6 Waveforms for Example 3.3.

Exercise 3.2
Find out charge, voltage, power and energy

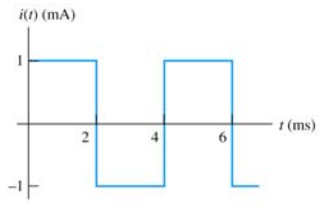


Figure 3.7 Square-wave current for Exercise 3.2.

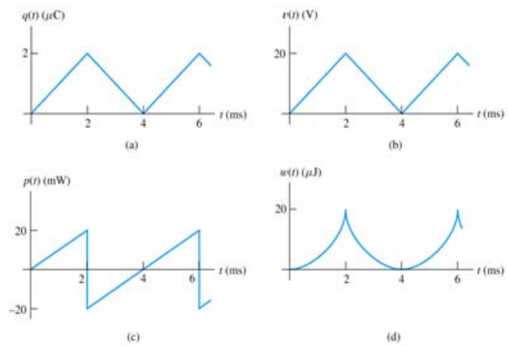


Figure 3.8 Answers for Exercise 3.2.

Questions

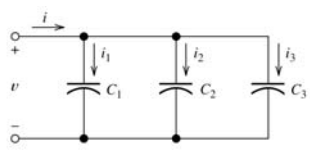
If a DC voltage is applied, what is the current?

Can the current change *instantly*?

Can the voltage change *instantly*?

How much energy does a capacitor dissipate?

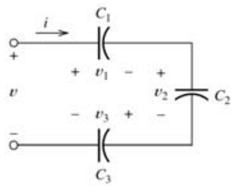
Capacitance in parallel



$C_{eq} = C_1 + C_2 + C_3$

Figure 3.9 Three capacitances in parallel.

Capacitance in series



$$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3}$$

Figure 3.11 Three capacitances in series.

Real Model of practical capacitors

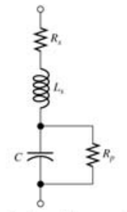


Figure 3.14 The circuit model for a capacitor including the parasitic elements R_s, L_s, and R_p.

Inductors

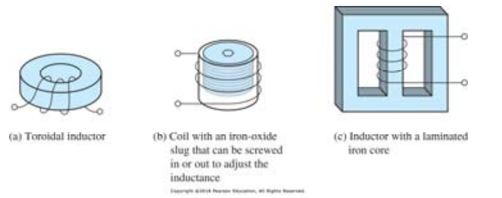
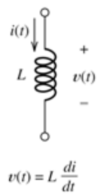


Figure 3.16 An inductor is constructed by coiling a wire around some type of form.

- An inductor is a coil of conducting wire wrapped around some magnetic material
- Energy storage element via magnetic field

Relationship: Voltage-current



$$v(t) = L\frac{di}{dt}$$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v(t)dt + i(t_0)$$

$$w(t) = \frac{1}{2}Li^2(t)$$

Example 3.7

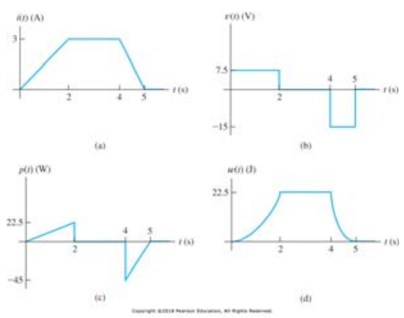


Figure 3.18 Waveforms for Example 3.7.

Example 3.8

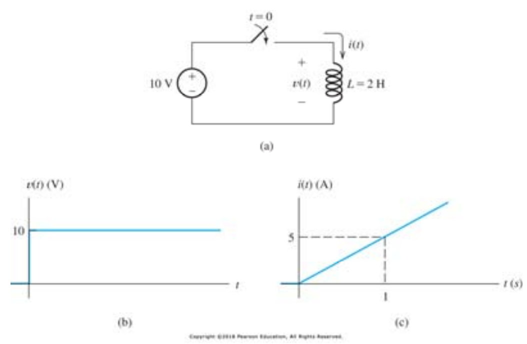


Figure 3.19 Circuit and waveforms for Example 3.8.

Exercise 3.7: Find current

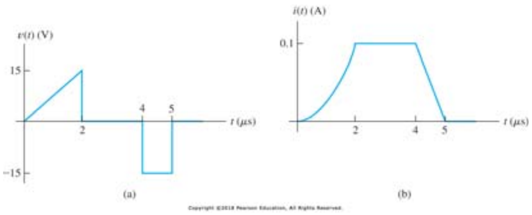


Figure 3.20 See Exercise 3.7.

Questions

If a DC current is applied, what is the voltage?

Can the current change *instantly*?

Can the voltage change *instantly*?

How much energy does an inductor dissipate?

Inductance in series and parallel

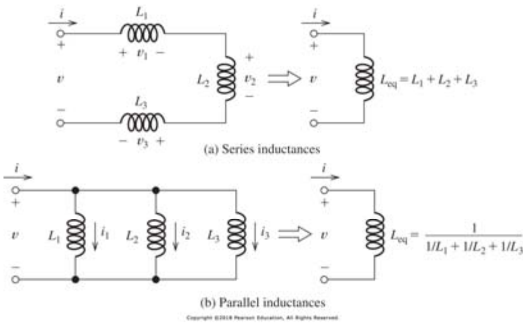


Figure 3.21 Inductances in series and parallel are combined in the same manner as resistances.

Proof for parallel inductance

$$v = L\frac{di}{dt} = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt} = L_3\frac{di_3}{dt}$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} = \frac{v}{L_1} + \frac{v}{L_2} + \frac{v}{L_3} = \frac{v}{L}$$

$$\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{L}$$

Exercise 3.10: Find equivalent inductance

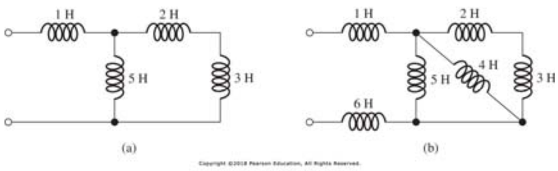


Figure 3.23 See Exercise 3.10.

Real model of practical inductors

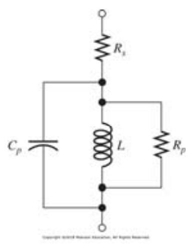


Figure 3.24 Circuit model for real inductors including several parasitic elements.