

## 时间复杂度

### Time complexity of Insertion Sort

Insertion-Sort (A)	cost	# of times
for j = 2 to A.length	$c_1$	$n$
key = A[j]	$c_2$	$n - 1$
i = j - 1	$c_3$	$n - 1$
while (i > 0 and A[i] > key)	$c_4$	$\sum_{j=2}^n t_j$
A[i+1] = A[i]	$c_5$	$\sum_{j=2}^n (t_j - 1)$
i = i - 1	$c_6$	$\sum_{j=2}^n (t_j - 1)$
A[i+1] = key	$c_7$	$n - 1$

Assume  $A.length = n$ , then total running time  $T(n)$  is:

$$c \cdot n + c' \cdot \left( \sum_{j=2}^n t_j \right) - c''$$

If  $t_j = 1$ , then  $T(n) \approx cn + c'n - c''$

**Best case:** A is sorted

If  $t_j = j$ , then  $T(n) \approx cn + (c'/2)n^2 - c''$

**Worst case:** A is reversely sorted

**Average case???**

$$T(n) = O(n^2)$$

$$\text{Runtime } T(n) = c \cdot n + c' \cdot \left( \sum_{j=2}^n t_j \right) - c''$$

**Best case:**  $T(n) = \Theta(n)$

$$t_j = 1 \text{ and } T(n) \approx cn + c'n - c''$$

**Worst case:**  $T(n) = \Theta(n^2)$

$$t_j = j \text{ and } T(n) \approx cn + (c'/2)n^2 - c''$$

## 5记号

$O$ : 与 $\omega$ 逻辑反, 渐进意义上的线性上界, 算法很好

$\Omega$ : 与 $O$ 意义相反, 与 $o$ 逻辑反, 代表算法时间下界, 算法不太好

$\Theta$ : 区间

$o$ : 任意系数 $c$

$\omega$ : 任意系数 $c$

- 有自反性, 对称性

## 渐进记号比较

- 洛必达
- 斯特勒公式: 估计阶乘

$$\sqrt{cn} \left(\frac{n}{e}\right)^n \leq n! \leq \sqrt{c_1 n} \left(\frac{n}{e}\right)^n$$