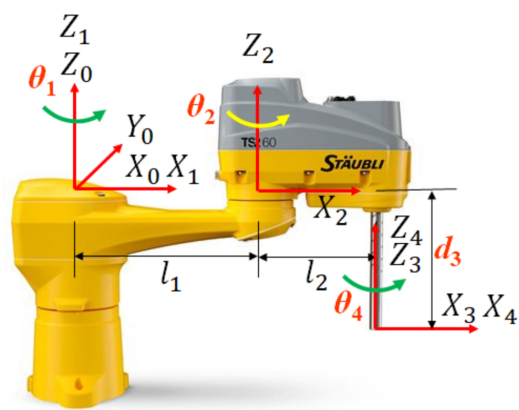


SCARA教学机器人(四个自由度)，机器人的末端装置即为连杆4的坐标系，根据给出的坐标系关系，建立个连杆坐标的D-H参数表，求解运动学正逆解方程。



本题解中为方便书写，用c1代表 θ_1

正解：

- 首先列出D-H表
- 注意d3的位移方向与z3方向相反，所以取负

	θ	α	l	d
01	c_1	0	0	0
12	c_2	0	l_1	0
23	0	0	l_2	0
34	c_4	0	0	$-d_3$

- 由matlab计算出正解
- A04如图

```
>> simplify(A01*A12*A23*A34)

ans =

[cos(c1 + c2 + c4), -sin(c1 + c2 + c4), 0, l2*cos(c1 + c2) + l1*cos(c1)]
[sin(c1 + c2 + c4),  cos(c1 + c2 + c4), 0, l2*sin(c1 + c2) + l1*sin(c1)]
[          0,          0, 1,          -d3]
[          0,          0, 0,          1]
```

因此正解为：

-

$$\begin{aligned} P_x &= l_2 * \cos(c_1 + c_2) + l_1 * \cos(c_1) \\ P_y &= l_2 * \sin(c_1 + c_2) + l_1 * \sin(c_1) \\ P_z &= -d_3 \end{aligned}$$

逆解

- 首先设出P:

```
>> P=simplify(Px^2+Py^2+Pz^2)
```

- P =

$$d3^2 + l1^2 + 2*\cos(c2)*l1*l2 + l2^2$$

直接从P解出c2:

$$c_2 = \arctg\left(\frac{\sqrt{1-M^2}}{M}\right)$$

$$M = \frac{P_x^2 + P_y^2 - l_1^2 - l_2^2}{2l_1l_2}$$

- 为了解出c1和c4, 我们从inv(A01)*A04=A14中发现了两个等式关系:
- inv (A01) 和A14如图

```
>> simplify(inv(A01))
```

```
ans =
```

- $$\begin{bmatrix} \cos(c1), & \sin(c1), & 0, & 0 \\ -\sin(c1), & \cos(c1), & 0, & 0 \\ 0, & 0, & 1, & 0 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

```
>> simplify(A12*A23*A34)
```

```
ans =
```

$$\begin{bmatrix} \cos(c2 + c4), & -\sin(c2 + c4), & 0, & l1 + l2*\cos(c2) \\ \sin(c2 + c4), & \cos(c2 + c4), & 0, & l2*\sin(c2) \\ 0, & 0, & 1, & -d3 \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

$$1^{st}: -\sin(c_1) * P_x + \cos(c_1) * P_y = l_2 * \sin(c_2)$$

$$2^{nd}: -\sin(c_1) * N_x + \cos(c_1) * N_y = l_2 * \sin(c_2 + c_4)$$

解出c1和c4:

$$c_1 = \arctg\left(\frac{\sqrt{1-N^2}}{N}\right) - \varphi$$

$$N = \frac{l_2 * \sin(c_2)}{\sqrt{P_x^2 + P_y^2}} \varphi = \arctg\left(\frac{P_x}{P_y}\right)$$

$$c_4 = \arctg\left(\frac{-\sin(c_1) * N_x + \cos(c_1) * N_y}{\cos(c_1) * N_x + \sin(c_1) * N_y}\right) - c_2$$

d3直接表示:

$$d_3 = -P_z$$