MOEA：演化算法解决多目标优化问题

一、pareto前沿

> 目标之间存在冲突
> MOEAs多目标演化算法

- 目的为了找到Pareto front
- 冲突举例：XM-1

$$\text{Minimize } f_1(\mathbf{x}) = \frac{1}{2}x_1x_2\cdots x_{M-1}(1+g(\mathbf{x}_M)),$$
$$\text{Minimize } f_2(\mathbf{x}) = \frac{1}{2}x_1x_2\cdots(1-x_{M-1})(1+g(\mathbf{x}_M)),$$

x比y好：那就是在每个目标上都更好（或至少一样weakly dominates）

- x weakly dominates y, denoted as $x \succcurlyeq y$, if
  $$\forall i \in \{1,2,\ldots,m\}\colon\ f_i(x) \geq f_i(y)$$
- x dominates y, denoted as $x \succ y$, if
  $$\forall i \in \{1,2,\ldots,m\}\colon\ f_i(x) \geq f_i(y) \text{ and } \exists i \in \{1,2,\ldots,m\}\colon\ f_i(x) > f_i(y)$$
- x is incomparable with y, if neither $x \succcurlyeq y$ nor $y \succcurlyeq x$

Pareto optima：最优解

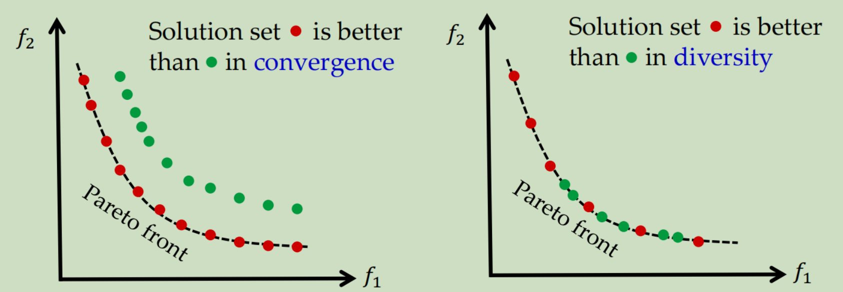
Pareto front：The collection of objective vectors of all Pareto optimal solutions 最优曲线

Pareto front两种评价标准

- Convergence (to the Pareto front)
- Diversity (along the Pareto front)

- rank+distance

> 先比较rank，后比较distance



Bi-objective minimization

解的比较方法

Crowded comparison employed by NSGA-II

Given a set P of solutions, for any two solutions x, y in P,
x is better than y, if

- rank(x) < rank(y)
- or rank(x) = rank(y) but distance(x) > distance(y)

Convergence

Diversity

二、NSGA-II

> 效果最差

1、父辈选择

> Binary tournament selection

Fast non-dominated sorting

> 针对rank

nx：比当前好的解个数

Sx：比当前差的解的集合

对每个解，遍历一遍，计算上述两个参数，再根据rank，存储进Fi（nx=0的所有解是rank=1）

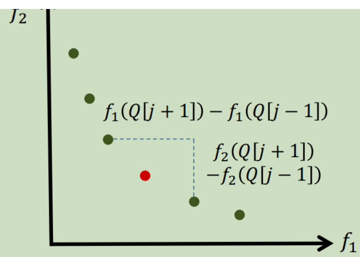
复杂度O（nn）

Crowding distance assignment

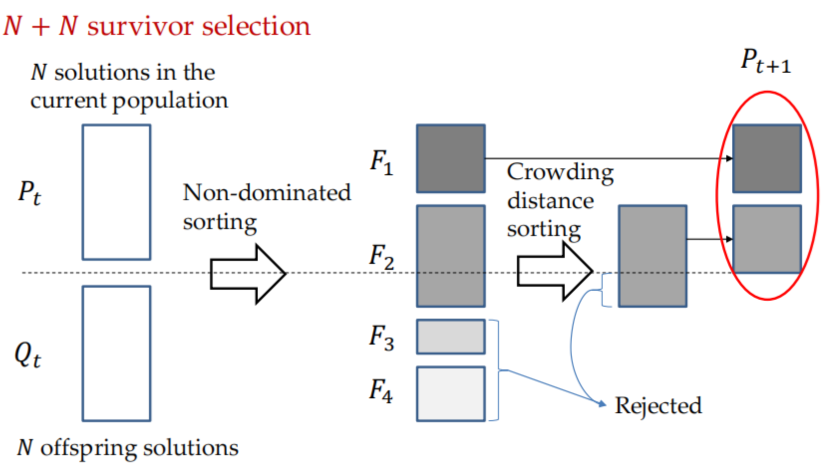
> 拥挤度，针对diversity

- 首尾设为无穷，因为肯定满足diversity
- Q越大越好
- 需要归一化作为评价尺度

Input: $Q = \{x_1, x_2, \ldots, x_l\}$ with the same rank;
for each j, set $Q[j]_{distance} = 0$ the j-th solution in Q
for each objective f_i
    $Q = sort(Q, f_i)$; in ascending order
    $Q[1]_{distance} = \infty$;
    $Q[l]_{distance} = \infty$; boundary solutions
    for $j = 2$ to $l - 1$
        $Q[j]_{distance} = Q[j]_{distance} + \frac{f_i(Q[j+1]) - f_i(Q[j-1])}{f_i.max - f_i.min}$ normalization
    end for



2、幸存者选择

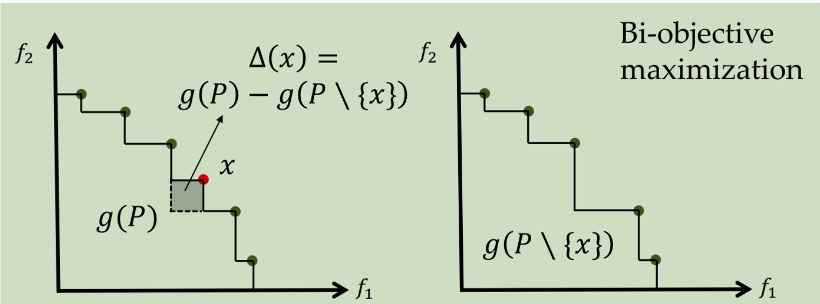


三、SMS-EMOA

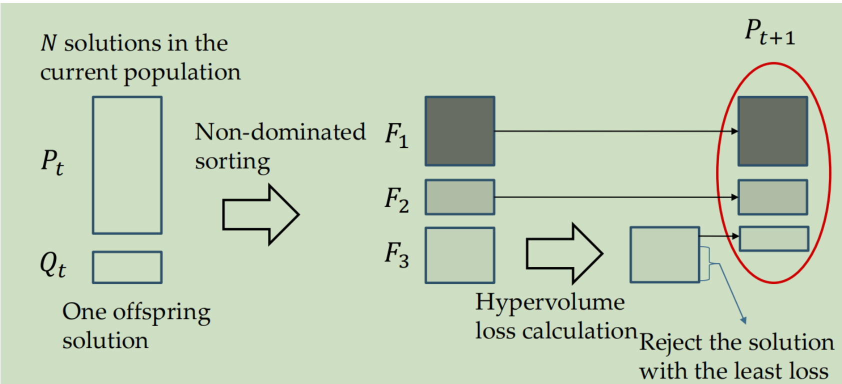
> 计算开销大，但收敛性好

- 增加对相同rank的解评估
- 利用quality indicators，去掉一个解，看损失多大
- 指标g(P)

比如 the hypervolume indicator：支配区域面积的大小（二维好理解）



只产生一个子代offspring



四、MOEA/D

> 基于分解
> 分解+解
> diversity好

分解方法

- 加权法（1范数）

> 算出每个单目标最优解，然后拟合成曲

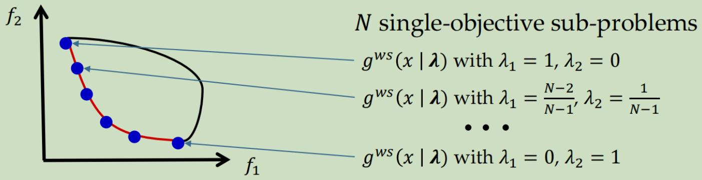
- Weighted sum approach

$$min_{x\in\mathcal{X}} \ \left(f_1(x), f_2(x)\right)$$

$$\Downarrow$$

$$min_{x\in\mathcal{X}} \ g^{ws}(x \mid \boldsymbol{\lambda}) = \lambda_1 f_1(x) + \lambda_2 f_2(x)$$

where $\lambda_1 + \lambda_2 = 1, \lambda_1, \lambda_2 \geq 0$



- 切比雪夫方法（无穷范数）

- Tchbycheff approach

$$min_{x\in\mathcal{X}} \ \left(f_1(x), f_2(x)\right)$$

$$\Downarrow$$

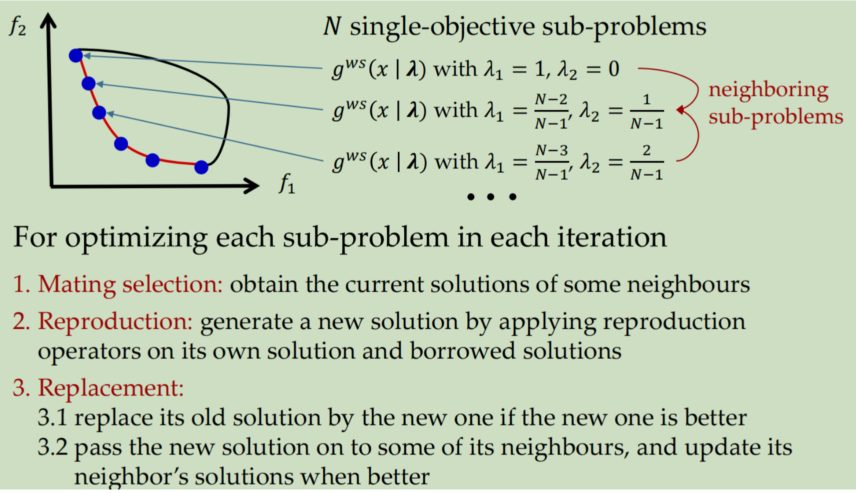
$$min_{x\in\mathcal{X}} \ g^{t}(x \mid \boldsymbol{\lambda}, \boldsymbol{z}^*) = \max\{\lambda_1 |f_1(x) - z_1^*|, \lambda_2 |f_2(x) - z_2^*|\}$$

where $\lambda_1 + \lambda_2 = 1, \lambda_1, \lambda_2 \geq 0$

$\boldsymbol{z}^*$ is an Utopian point,
where $z_1^* < \min\{f_1(x)\}$ and $z_2^* < \min\{f_2(x)\}$

For any Pareto optimal solution x^*, there is a $\boldsymbol{\lambda}$ such that x^* is optimal to $g^t(x \mid \boldsymbol{\lambda}, \boldsymbol{z}^*)$

求解



串并行有区别，串行在遍历时，可能之前已经算过了