2.13 Determination of the coefficient of viscosity of a liquid by its flow through a capillary tube :

• **Apparatus : First arrangement :** The apparatus employed to determine the coefficient of viscosity of a liquid is shown in Fig. 2.13-1. It consists of a constant-level tank N in which there is the inflow pipe I and outflow pipe O. A capillary tube C of about 40 to 50 cm long is attached to a glass tube G by a rubber tube K. The capillary tube C is almost of uniform bore and is kept *slightly inclined* to the horizontal. The tube G ends above the rubber cork R which closes the bottom of the tank N. The supply of liquid through I is slightly in excess of that flowing out through the capillary tube C and this excess liquid flows out through the tube O. The height of the free surface of the liquid in the tank N above the outflow end E of the tube C can be measured by a cathetometer.

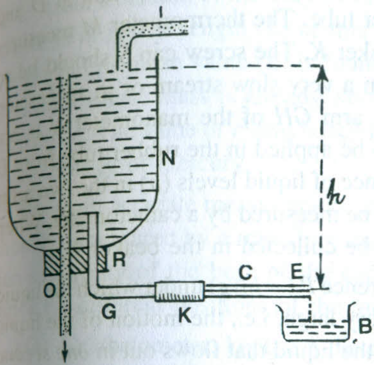

Fig. 2.13-1

Second arrangement : Another arrangement shown in Fig. 2.13-2 may be conveniently employed in the laboratory to find the coefficient of viscosity (η)

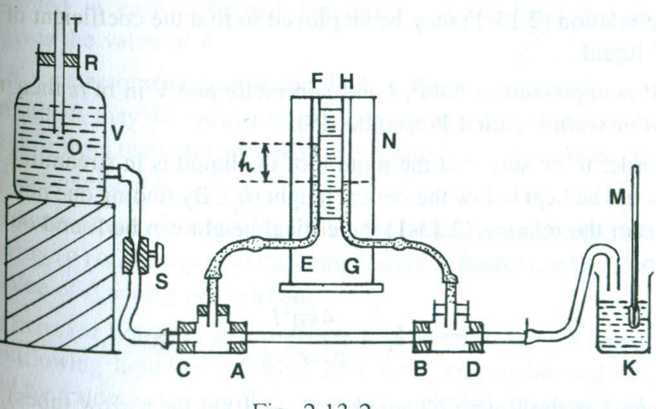

Fig. 2.13-2

of a liquid. V is a glass vessel which contains the experimental liquid. The mouth of the vessel is closed by a rubber cork R through which passes a glass tube TO, open at both ends. The lower end O remains well below the surface of the liquid in V. As the liquid flows out of the vessel V, air bubbles from the atmosphere enter the vessel through the tube T, by which the end O of it always remains at atmospheric pressure, so long as the liquid level in V remains above O.

The liquid from V flows to the junction at C through a screw cap S thence through the capillary tube AB of known length (l) to the junction at D and thence to the beaker K through a rubber tube. The thermometer M measures the temperature of the liquid in the beaker K. The screw cap S should be so adjusted that the liquid may issue out in a very slow stream or in succession of drops. If the height of liquid in the arm GH of the manometer N is not appreciable then second screw cap is to be applied in the rubber tube from D which goes to the beaker K. The difference of liquid levels (h) in the two arms (EF and GH) of the manometer (N) can be measured by a cathetometer. When everything is steady, the liquid should be collected in the beaker K.

• **Theory :** When the pressure difference $P\ (= h\rho g)$ under which the liquid flows through a capillary tube, is not very large, i.e., the motion of the liquid in it is in steamlines, the volume (V) of the liquid that flows out in *one second* through a capillary tube of radius r (radius of the bore of the tube) and length l is given by,

$$V = \frac{\pi P r^4}{8\eta l}\ ; \quad \text{or,}\ \eta = \frac{\pi P r^4}{8Vl} \qquad \text{... (2.13-1)}$$

Here η is the coefficient of viscosity of the liquid.

The relation (2.13-1) may be employed to find the coefficient of viscosity (η) of liquid.

If P is expressed in N/m^2, l and r in metre and V in m^3/s then η will be in newton-sec/m^2, called Poiseuille (Pl).

In order to be sure that the motion of the liquid is in streamline the value of h should be kept below the critical height (h_c). By finding one nominal value of η from the relation (2.13-1) the critical height can be found out from the relation.

$$h_c = \frac{4k\eta^2 l}{\rho^2 . r^3 . g} \qquad \text{... (2.13-2)}$$

where k is the Reynold's number ($k \approx 1000$ for narrow tubes) and ρ is the density of the liquid.

• **Procedure :** (i) The length l of the capillary tube (which is about 40 cm long and 1 mm in internal diameter) is measured thrice by a metre scale and its mean value (l) is found out.

(ii) To find the radius of the bore of the capillary, the bore of the capillary tube is first washed with dilute caustic soda, then with dilute nitric acid and finally in a stream of cold water. The tube is then made dry by passing hot air in it and a column of mercury covering almost the entire length of the tube is sucked in it. The length (L) of this mercury column is measured thrice by a scale and its mean value (L) is found out. This mercury is then weighed in a crucible whose mass is already known. From this, the mass m of mercury is obtained. The value of r^2 and hence r^4 is determined from the relation $m = \pi r^2 L \times$ density of mercury ... (2.13-3)

For more accurate measurement of r, the length L of the mercury column should be measured by a travelling microscope. To minimise the error due to non-uniformity of the bore of the capillary 'L' should be measured at least thrice for different positions of the mercury column along the capillary.

(iii) A cathetometer, kept at a distance, is adjusted in such a way that its pillar is vertical and the axis of the telescope is horizontal at its every position. The cross-wire of the telescope is then focussed by avoiding parallax. The telescope is then focussed successively on the surface of the liquid in the tank N and the axis of the outflow end E of the capillary tube C [or the telescope is successively focussed on the surfaces of liquid in the two arms EF and GH of the manometer N, Fig. 2.13-2]. The readings R_1 and R_2 of the vernier and scale are noted in each case and the difference of these two readings ($R_1 \sim R_2$) gives the value of h.

[**N.B.** If a cathetometer is not supplied in the laboratory then for less accurate result, h may be measured directly by a metre scale in the first arrangement or from the scale attached by the side of the manometer tubes in the second arrangement.]

(iv) The temperature of the liquid (T°C) in the tank is noted by a thermometer and the density (ρ) of the liquid at this temperature is **found out from a table.** Then the value of $P = h\rho g$ is found out.

(v) The mass (M_1) of an empty beaker is first determined by a balance and then the outflowing liquid is collected in it for a known interval of time t (determined by a stop-watch) until the beaker is half full. The mass ($M_1 + M$) of the beaker with the liquid in it is found out and the mass of the liquid collected in t second is thus given by M. Hence the volume (V) of the liquid collected in one sec. is $V = M/\rho t$ ...(2.13-4)

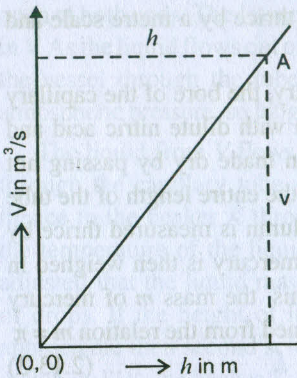

Fig. 2.13-3

[**N.B.** If a **measuring cylinder** with fine graduation is supplied then V can also be determined by collecting sufficient volume (V') of liquid for an appreciable length of time (t). Then V will be given by $V = V'/t$ ...(2.13-4)]

(*vi*) The experiment may be at first performed for one nominal value of h (say 2 or 3 cm) to find one nominal value of η. Using the relation (2.13-2) the value of the critical height h_c can be calculated. Now the experiment is performed for at least five different values of h, taking care that h remains well below h_c (usually below $\frac{h_c}{2}$).

(*vii*) A graph is then to be drawn with h along the x-axis and the corresponding value of V (volume of liquid flowing out per second) along the y-axis. The graph will be a straight line passing through origin so long as the motion of the liquid in the tube is in streamlines (Fig. 2.13-3).

By taking the coordinates (h, V) of a suitable point A on the graph η can be calculated.

• Experimental data :

(A) *Measurement of length (l) of capillary tube :*

TABLE I

No. of obs.	Measured length in cm	Mean length (l) in cm	l in metre
1.	...		
2.	...		
3.	...		

(B) *Measurement of radius (r) of the capillary tube :*

[**N.B.** If the length (L) of mercury column in the capillary is measured by *a travelling microscope* only then make the following TABLE]

TABLE II A

(Calculate the vernier constant as in Art. 2.1)

Position of Hg column	Reading for left end. Scale (S) in cm	Vernier ($v.r.$)	Total in cm $R_1 = S + v.r. \times v.c.$	Reading for right end Scale (S) in cm	Vernier ($v.r.$)	Total in cm $R_2 = S + v.r. \times v.c.$	Length L in cm $= R_1 \sim R_2$
1.							
2.							
3.							

TABLE II B

Length of Hg column in cm (L)	Mean L	Mass of empty crucible (m_1) in g	Mass of crucible + Hg ($m_1 + m$) in g	Mass of Hg (m)	$r^2 = m/(\pi L \times 13{\cdot}6 \times 10^3)$ in m²	r^4 in m⁴
(i)...	... cm	... + ...	... + ...	... g		
(ii)...	= ...	+ ... + ...	+ ... + ...	=	...	...
(iii)...	metre	= ... g	= ... g	... kg		

(C) *Data for nominal value of* η *and estimation of critical height* h_c *:*

TABLE III A

Diff. of liquid level by a metre scale h in cm	Vol. of liquid collected by a measuring cylinder V' in c.c.	Time of collection t in sec.	Vol. of liquid collected in m³/s $\frac{V'}{t}\times10^{-6}$
...	...	...	...

TABLE III B

h in m	V in m³/s	r in m	l in m	density ρ in kg/m³	k	$\eta = \frac{\pi h\rho g r^4}{8Vl}$ in N-s / m² or Pl	h_c in m $= \frac{4k\eta^2 l}{\rho^2 r^3 g}$	h_c in cm
...	...	...	...	...	1000	...	...	...

(D) *Cathetometer reading to find P in terms of 'h' :*

Value of 1 small division of main scale = s = ... cm.

n number of vernier division (v.d.) = m divisions of main scale (s.d.)

$\therefore$ 1.v.d. = m/n s.d.

Vernier constant (v.c.) = 1 s.d. – 1 v.d. = $\left(1-\frac{m}{n}\right)$ s.d. = $\left(1-\frac{m}{n}\right)\times$ s cm

TABLE IV A

No. of obs.	Reading in liq. level in the tank or in the arm EF of manometer (R_1): Scale (S) in cm	Vernier (v.r.)	Total in cm $R_1 = S + v.r. \times v.c.$	Reading of the axis of tube C at E or of liq. level in the arm GH of manometer (R_2): Scale (S) in cm	Vernier (v.r.)	Total in cm $R_2 = S + v.r. \times v.c.$	Difference in cm $h = (R_1 \sim R_2)$	h in metre
1.	...		...	...		...	...	...
2.	...		...	...		...	...	....
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.
5.	...		...	...		...	...	....

Or, make the following Table when h is determined by metre scale.

(D) *Data for pressure difference in terms of 'h' :*

TABLE IV B

No. of obs.	Reading of liquid level in the tank or reading of liquid level in the left arm of manometer (h_1) in cm	Reading of the axis of the tube near the beaker or reading of liquid level in the right arm of manometer (h_2) in cm	$h = h_1 \sim h_2$ in cm	h in metre
1.	...	...	...	...
2.	...	...	...	...
3.	...	...	...	...
4.	...	...	...	...
5.	...	...	...	...

Temperature of the liquid in the tank T°C = ...°C

Density (ρ) of the liquid at T°C = ... kg/m³.

(E) *To find volume (V) of liquid collected in one second :*

TABLE V A

No. of obs.	Mass of empty beaker (M_1)	Mass of beaker + liquid collected in t sec. ($M_1 + M$)	Mass of liquid in gms collected in t secs. (M)	Time of collection t sec.	Volume of liquid collected $V = \frac{M\times10^{-3}}{\rho t}$ in m³/s
1.	...g + ...g +...mg+...mg +... = ...g	...g + ...g +...mg + ...mg +... = ...g		... min ...sec. = ...sec.	
2.		...	...	...	...
etc.	etc.	etc.	etc.	etc.	etc.
5	etc.	...	...		...

Or, make the following Table if measuring cylinder is used to find 'V'

(E) *To find volume (V) of liquid collected in one second :*

TABLE V B

No. of obs.	h in metre from TABLE IV	Time of collection of liquid = t sec.	Volume of liquid collected in c.c. (V')	Mean V' in c.c.	Vol. (V) of liquid collected in m^3 per sec. $(V'/t)\times10^{-6}$
1.	...	...	...	...	...
2.	...	...	...	...	...
etc.	etc.	etc.	etc.	etc.	etc.

(F) *Calculation of* η *at ..... °C :*

h in m from graph	Corresponding V in m^3/s from graph	l in m from TABLE I	r^4 in m^4 from TABLE II B	g in m/s²	ρ in kg/m³	$\eta = \dfrac{\pi pr^4}{8\eta l}$ in N-s/m² or Poiseuille (Pl)

• **Precautions and Discussions :** (*i*) Since radius r occurs in the fourth power, the bore of the capillary tube should be as uniform as possible and this can be previously tested by introducing a short column of mercury in the tube and measuring its length at various parts.

(*ii*) Owing to capillarity the outflowing liquid from the end E of the tube C may run back along the outside of the tube. To prevent this, either the tube should be kept slightly inclined to the horizontal or a little vaseline should be smeared on the outside of the tube near its free end, when the capillary tube is kept horizontal [for first arrangement only].

(*iii*) The pressure difference, under which the liquid flows must not be *high* to make the flow turbulent which will be evident from the bending of the straight line h vs. V curve at higher values of h. The critical height ($h = h_c$) beyond which the motion of the liquid becomes turbulent should be first estimated (or known otherwise). During experiment h should be kept less than $h_c/2$.

(*iv*) If the error due to kinetic energy of the liquid and its acceleration near the entrance end of the capillary tube is to be eliminated, the following relation is to be employed to find η,

$$\eta = \frac{\pi r^4}{8V(l+1.64r)}\rho g\left(h - \frac{k'V^2}{g\pi^2 r^4}\right)$$

The value of k' is approximately equal to unity but for accurate work its value should be determined by calibration.

(*v*) For greater accuracy, the quantity of liquid collected should be appreciable.

(*vi*) When the volume of liquid collected and the time of such collection are both small, the volume should be determined by weighing otherwise large proportional error in η will occur.

(*vii*) The temperature of the liquid should be noted carefully, for the value of the coefficient of viscosity changes rapidly with temperature.

• **Maximum percentage error :**

$$\eta = \frac{\pi h \rho g r^4}{8Vl};\ V = \frac{V'}{t},\ \text{where } V' \text{ volume of water collected in } t \text{ sec.}$$

$$\therefore \left(\frac{\delta\eta}{\eta}\right)_{max} = \frac{\delta h}{h} + 4\frac{\delta r}{r} + \frac{\delta V'}{V'} + \frac{\delta t}{t} + \frac{\delta l}{l} \quad ...(2.13\text{-}4)$$

where $\delta h = 0\cdot2$ cm [two divs. of a metre scale, when such a scale is used to measure h ; if cathetometer is used $\delta h = 2\times v.c.$]

$\delta l = 0\cdot2$ cm (2 div. of metre scale)

$\delta t \approx 0\cdot5$ s (error in a stop-watch)

δr and $\delta V'$ are to be determined as follows :

From the relation (2.13-3) $m = \pi r^2 L \times \rho$

$$\therefore \left.\frac{2\delta r}{r}\right|_{max} = \frac{\delta m}{m} + \frac{\delta L}{L} \quad ... (2.13\text{-}5)$$

where $\delta m = 2$ mg $= 0{\cdot}002$ g (for a balance weighing down to 2 mg)

$\delta L = 0{\cdot}2$ cm (if measured by metre scale)

$= 2 \times v.c.$ (if measured by travelling microscope)

If V' is measured by a measuring cylinder then

$\delta V'$ = minimum graduation of the cylinder

If V' is measured by weighing then

$$V' = \frac{M}{\rho}$$

$$\frac{\delta V'}{V'} = \frac{\delta M}{M} \qquad \text{... (2.13-6)}$$

where $\delta M = 2$ mg $= 0.002$ g

Now, substituting a set of measured values of h, l, t, m, L and M we can calculate $\left.\frac{\delta\eta}{\eta}\right|_{max}$ and then multiplying it by 100 we can get % error in η.

□ Oral Questions and Answers □

1. What do you mean by the term 'viscosity' and 'coefficient of viscosity' of liquid ?

Ans. Whenever there is a relative motion between two layers of a liquid a tangential opposition force is set up between the layers to destroy this relative motion. This property of the liquid is called viscosity and it is analogous to friction. This tangential force per unit area on either of the two liquid surfaces, when there is unit velocity gradient between them, is known as the coefficient of viscosity.

2. What do you mean by streamline motion and turbulent motion ?

Ans. When the pressure difference, under which the liquid flows in a capillary tube is small, the particles of the liquid move in ordered continuous paths and this kind of motion of liquid is known as streamline motion. When the pressure difference is made large, the particles flow in zigzag paths and this motion is called turbulent motion.

3. How does the coefficient of viscosity change with temperature?

Ans. In the case of liquids viscosity diminishes with temperature, while in the case of gases it increases with temperature.

4. Discuss the nature of flow of liquid in capillary tube.

Ans. When streamline motion occurs, the layer of liquid in contact with capillary tube is at rest while the velocity of the other layers increases as we go towards the axis of tube at which the velocity is maximum.

5. What is the harm if, instead of capillary tube, a tube of wide bore is employed?

Ans. When the bore of the tube is large, the increase of velocity towards the axis of the tube will be so great that the motion of liquid near the axis of the tube becomes turbulent and hence the formula employed to find η will fail.

6. Which quantity would you measure with greater care?

Ans. As the radius of capillary tube occurs in fourth power, it should be measured with great accuracy and proper care should be taken to select a tube of uniform bore (see precautions).

7. What is the harm if the capillary tube is not horizontal?

Ans. If the tube is horizontal then the height h of liquid level is to be measured from liquid in the tank to the axis of the capillary tube. When the capillary tube is inclined, the height of h liquid is to be measured from the liquid level in the tank to the free end E of the capillary tube (for first arrangement).

8. What is the unit of the coefficient of viscosity?

Ans. N-s/m^2 or Poiseuille (Pl) in SI, poise in C.G.S., 1 Pl = 10 poise.

9. What do you mean by 'critical height' and 'critical velocity'?

Ans. When the value of h (and hence pressure difference) exceeds certain value h_c, the motion of the liquid becomes turbulent. This 'h' is called critical height.

There is a particular velocity of flow below which the motion is streamline and beyond which the motion is turbulent. This particular velocity is called critical velocity.

10. What type of measuring cylinder is suited in measuring 'V'?

Ans. Measuring cylinder having finer graduations.

11. Can you perform the experiment with a tube of wider bore?

Ans. No. In that case a small pressure difference will cause the motion to be turbulent.

12. Can you measure 'r' by focussing one end of the capillay tube and using a travelling microscope?

Ans. In this case we get the value of r at a point. But the capillary bore may not be perfectly uniform. Measuring 'r' with Hg column we get an average value of 'r'.

13. Do you know any other method of finding η ?

Ans. This method is suitable only for liquids of low viscosity. Rotating viscometer or Stoke's law can be used to find η of highly viscous liquids.

14. Do you think that the streamline motion is always in straight line?

Ans. No, it can also be curved.

15. What's Reynold's number?

Ans. According to Reynold the critical velocity $v_c = \frac{k\eta}{\rho r}$ where k is a number, called Reynold's number.

16. What is a streamline?

Ans. A streamline is a line in a fluid such that the tangent to it at any point is in the direction of the velocity of the fluid particle at that point.