Stability Analysis of Device-to-Device Relay-Assisted Cellular Networks: Proofs

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APPENDIX A
FLUID LIMITS

In this section, we present the basic concepts on fluid limits for the queues in our system and its associated lemmas with respect to our system model. For further study, kindly refer to [2]. These lemmas are crucial in proving [1, Theorem 1].

Before, we proceed further, we introduce some notations. [1, Equation 2], [1, Equation 3] and [1, Equation 4] can be respectively rephrased as follows:

$$X_i(n) = X_i(0) + \mathscr{A}_i(n) - \sum_{j\in[N],j\neq i} \mathscr{D}_{ij}(n) - \mathscr{D}_{i0}(n), \quad (1)$$

$$Y_i(n) = Y_i(0) + \sum_{j\in[N],j\neq i} \mathscr{D}_{ji}(n) - \mathscr{D}_{iR}(n), \quad (2)$$

$$U_i(n) = U_i(0) + \mathscr{A}_{Ui}(n) - \mathscr{D}_{Ui}(n), \quad (3)$$

where $\mathscr{A}_i(n)$ is the total exogenous arrivals in MS i till slot n, $\mathscr{D}_{ij}(n)$ is the total number of packets transmitted by MS i to MS j until slot n, $\mathscr{D}_{i0}(n)$ ($\mathscr{D}_{iR}(n)$) denotes the number of departures to BS from the *own* (*relay*) *queue* until slot n. $\mathscr{A}_{Ui}(n)$ and $\mathscr{D}_{Ui}(n)$ be the total arrivals and departures, respectively, that takes place in the *virtual queue* of MS i till slot n.

Suppose the set of non-negative integers and reals are denoted by $\mathbb{N}$ and $\mathbb{R}$, respectively. The domain of the functions $X_i(n), Y_i(n), \mathscr{A}_i(n), \mathscr{D}_{iO}(n), \mathscr{D}_{iR}(n), \mathscr{D}_{ij}(n), U_i(n), \mathscr{A}_{Ui}(n)$, $\mathscr{D}_{Ui}(n)$ is $\mathbb{N}$. We define these functions for arbitrary $t \in \mathbb{R}$ by using a piecewise linear interpolation. The piecewise linear interpolation of a function is defined as: for any function f with $\mathbb{N}$ as its domain, $t \in (n, n+1]$,

$$f(t) = f(n) + (t-n)(f(n+1) - f(n)).$$

Suppose that for any scheduling policy, the number of packets that can be transmitted across any link in a slot is bounded by μ_{max}. Also, as mentioned before, the maximum number of exogenous arrivals at each MS i in a slot is bounded by A_{max}. Further, each MS i can utilize at most P_{max} power in each slot. For a random process $f(t)_{t\geq 0}$, we show the randomness of the sampling path ω by denoting its value at time t along the sample path ω by $f(t,\omega)$. For every ω, $\forall i \in [N]$ and $t \geq 0$, we can easily show that $X_i(t,\omega), Y_i(t,\omega), \mathscr{A}_i(t,\omega), \mathscr{D}_{iO}(t,\omega), \mathscr{D}_{iR}(t,\omega), \mathscr{D}_{ij}(t,\omega)$, $U_i(t,\omega), \mathscr{A}_{Ui}(t,\omega), \mathscr{D}_{Ui}(t,\omega)$ are all Lipschitz functions. Now, let us define fluid scaling of any given functions $f(.)$ as follows:

$$f^r(t,\omega) \overset{\text{def}}{=} \frac{f(rt,\omega)}{r}, \qquad \text{for every } r > 0.$$

It follows that for every $r > 0$

$$X_i^r(t+\delta,\omega) - X_i^r(t,\omega) \leq A_{max}\delta.$$

We have similar bounds on $Y_i^r(t,\omega), \mathscr{A}_i^r(t,\omega), \mathscr{D}_{iO}^r(t,\omega), \mathscr{D}_{iR}^r(t,\omega), \mathscr{D}_{ij}^r(t,\omega), U_i^r(t,\omega)$, $\mathscr{A}_{Ui}^r(t,\omega), \mathscr{D}_{Ui}^r(t,\omega)$.

Thus, all the above functions are Lipschitz continuous, and hence uniformly continuous on any compact interval. Clearly, the above functions are also bounded on any compact interval. Fix a compact interval $[0,t]$. Now, consider any sequence r_n such that $r_n \to \infty$ as $n \to \infty$. Then, by Arzela-Ascoli theorem [4], there exists a subsequence r_{n_k} and continuous functions $\widetilde{X}_i(.)$ such that for every i, ω

$$\lim_{k\to\infty} \sup_{\hat{t}\in[0,t]} |X_i^{r_{n_k}}(\hat{t},\omega) - \widetilde{X}_i(\hat{t})| = 0.$$

Similar convergence results for other system processes exist. Let the functions to which $Y_i^r(t,\omega), \mathscr{A}_i^r(t,\omega), \mathscr{D}_{i0}^r(t,\omega), \mathscr{D}_{iR}^r(t,\omega), \mathscr{D}_{ij}^r(t,\omega), U_i^r(t,\omega)$, $\mathscr{A}_{Ui}^r(t,\omega), \mathscr{D}_{Ui}^r(t,\omega)$ converge, for some subsequence, be denoted by $\widetilde{Y}_i(t), \widetilde{\mathscr{A}_i}(t), \widetilde{\mathscr{D}}_{i0}(t), \widetilde{\mathscr{D}}_{iR}(t), \widetilde{\mathscr{D}}_{ij}(t), \widetilde{U}_i(t), \widetilde{\mathscr{A}}_{Ui}(t)$, $\widetilde{\mathscr{D}}_{Ui}(t)$, respectively. We will now define fluid limits.

Definition 1. $\widetilde{X}_i(.), \widetilde{Y}_i(.), \widetilde{\mathscr{A}_i}(.), \widetilde{\mathscr{D}}_{i0}(.), \widetilde{\mathscr{D}}_{iR}(.), \widetilde{\mathscr{D}}_{ij}(.), \widetilde{U}_i(.)$, $\widetilde{\mathscr{A}}_{Ui}(.), \widetilde{\mathscr{D}}_{Ui}(.)$ *are called fluid limits if there exists r_{n_k} such that all the aforementioned convergence relations are satisfied.*

Now, we state some important properties of the fluid limits.

Lemma 1. *Every fluid limit satisfies, $\widetilde{\mathscr{A}_i}(t) = \lambda_i t$ w.p. 1 for every MS i and $t \geq 0$.*

Proof: Since $\widetilde{\mathscr{A}_i}(t)$ is a fluid limit, thus, by Definition 1, there exists a sequence r_{n_k} such that $\lim_{k\to\infty} r_{n_k} = \infty$ and

$$\begin{aligned}\widetilde{\mathscr{A}_i}(t) &= \lim_{k\to\infty} \mathscr{A}_i^{r_{n_k}}(t) \\ &= \lim_{k\to\infty} \frac{\mathscr{A}_i(r_{n_k}t)}{r_{n_k}} \\ &= \lim_{k\to\infty} \frac{\mathscr{A}_i(r_{n_k}t)}{r_{n_k}t}t \\ &= \lambda_i t \text{ w.p. 1 (by strong law of large numbers (SLLN)).}\end{aligned}$$

■

Lemma 2. *Any fluid limit $\widetilde{X}_i(.), \widetilde{\mathscr{A}_i}(.), \widetilde{\mathscr{D}}_{i0}(.), \widetilde{\mathscr{D}}_{ij}(.)$ satisfies the following equality for every MS i and $t \geq 0$ w.p. 1: $\widetilde{X}_i(t) = \lambda_i t - \sum_{j\in[N],j\neq i} \widetilde{\mathscr{D}}_{ij}(t) - \widetilde{\mathscr{D}}_{iD}(t)$.*

Proof: Since $\widetilde{X}_i(.), \widetilde{\mathscr{A}_i}(.), \widetilde{\mathscr{D}}_{i0}(.)$ and $\widetilde{\mathscr{D}}_{ij}(.)$ are fluid limits, there exists a sequence r_{n_k} such that $\lim_{k\to\infty} r_{n_k} =$

∞ and they are obtained as a uniform limits of functions $X_i^{r_{n_k}}(.), \mathscr{A}_i^{r_{n_k}}(.), \mathscr{D}_{i0}^{r_{n_k}}(.), \mathscr{D}_{ij}^{r_{n_k}}(.)$, respectively. Now, from (1) it follows that for every r_{n_k} and $t \geq 0$

$$X_i^{r_{n_k}}(t) = X_i^{r_{n_k}}(0) + \mathscr{A}_i^{r_{n_k}}(t) - \sum_{j\in[N]} \mathscr{D}_{ij}^{r_{n_k}}(t) - \mathscr{D}_{i0}^{r_{n_k}}(t).$$

The results follow from Lemma 1 after taking the limit $k \to \infty$ on both sides of the above equality. ■

Lemma 3. *Any fluid limit $\widetilde{Y}_i(.), \widetilde{\mathscr{A}_i}(.), \widetilde{\mathscr{D}}_{ji}(.), \widetilde{\mathscr{D}}_{iR}(.)$ satisfies the following equality for every MS i and $t \geq 0$ w.p. 1:* $\widetilde{Y}_i(t) = \sum_{j\in[N], j\neq i} \widetilde{\mathscr{D}}_{ji}(t) - \widetilde{\mathscr{D}}_{iR}(t)$.

Proof: The lemma can be proved in a similar way as Lemma 2. ■

Lemma 4. *Any fluid limit $\widetilde{U}_i(.), \widetilde{\mathscr{A}_{Ui}}(.), \widetilde{\mathscr{D}}_{Ui}(.)$ satisfies the following equality for every MS i and $t \geq 0$ w.p. 1:* $\widetilde{U}_i(t) = \widetilde{\mathscr{A}_{Ui}}(t) - \widetilde{\mathscr{D}}_{Ui}(t)$.

Proof: The lemma can be proved in a similar way as Lemma 2. ■

Note that $\widetilde{\mathscr{D}}_{Ui}(t) \leq \overline{P}_i t$, as shown below:

$$\begin{aligned}
\widetilde{\mathscr{D}}_{Ui}(t) &= \lim_{k\to\infty} \frac{\mathscr{D}_{Ui}(r_{n_k} t)}{r_{n_k}} \\
&= \lim_{k\to\infty} \frac{\mathscr{D}_{Ui}(r_{n_k} t)t}{r_{n_k} t} \\
&\leq \overline{P}_i t \quad \text{(By SLLN and } \mathscr{D}_{Ui}(n) \leq n\overline{P}_i). \qquad (4)
\end{aligned}$$

Appendix B
Proof of Theorem 1

The proof is done in two stages: (a) $\overline{\mathcal{C}}_{IID} \subseteq \Lambda_{IID}$ and (b) $\Lambda_{IID} \subseteq \overline{\mathcal{C}}_{IID}$ using the supplementary results from Appendix A.

First we prove stage (a). Suppose, there exists $\boldsymbol{\lambda} \in \overline{\mathcal{C}}_{IID}$. By [1, Definition 3], there exists a randomized policy Δ such that $\boldsymbol{\lambda}$ is rate stable under Δ. Fix any such Δ and let us define the following quantities:

- Define indicator $\mathbb{I}_s(n) = 1$ if $s_{IID}(n) = s$, and 0 otherwise.
- Define indicator $\mathbb{I}_k^{\Delta}(n) = 1$ when $\boldsymbol{\ell}^{\Delta}(n) = \boldsymbol{\ell}_k$, and 0 otherwise.
- $\mathbb{I}_{i_X}^{\Delta}(n) = 1$, when the packets from the own queue of MS i is served on link $(i, 0)$ in slot n, and 0 otherwise.

Additionally,

$$\lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} \mathbb{I}_s(n)\mathbb{I}_k^{\Delta}(n) = x_{sk}^{\Delta} \text{ w.p. } 1, \qquad (5)$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} \mathbb{I}_s(n)\mathbb{I}_k^{\Delta}(n)\mathbb{I}_{i_X}^{\Delta}(n) = y_{i_X sk}^{\Delta} \text{ w.p. } 1, \qquad (6)$$

$$\lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} \mathbb{I}_s(n)\mathbb{I}_k^{\Delta}(n)(1 - \mathbb{I}_{i_X}^{\Delta}(n)) = y_{i_Y sk}^{\Delta} \text{ w.p. } 1, \qquad (7)$$

$$\overline{P}_i^{\Delta} = \lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} \sum_{j\in[N], j\neq i} \ell_{ij}^{\Delta}(n). \qquad (8)$$

Assume that the system is ergodic such that the above limits exist. Equation (5) refers to fraction of time network topology of system is s and power vector $\boldsymbol{\ell}_k$ has been selected by policy Δ. Equation (6) refers to fraction of time own queue of MS i has been selected when topology is s and power vector $\boldsymbol{\ell}_k$ is selected by policy Δ. Equation (7) refers to similar fraction for relay queue of MS i. Equation (8) refers to power consumed at MS i under policy Δ.

Let w'^{Δ}_{sk} be the fraction of time power vector $\boldsymbol{\ell}_k$ has been selected under policy Δ given that network topology is s. Thus,

$$w'^{\Delta}_{sk} = \frac{x_{sk}^{\Delta}}{\pi_s} = \lim_{T\to\infty} \frac{\frac{1}{T}\sum_{n=1}^{T} \mathbb{I}_s(n)\mathbb{I}_k^{\Delta}(n)}{\frac{1}{T}\sum_{n=1}^{T} \mathbb{I}_s(n)}. \qquad (9)$$

Similarly, let q'_{isk} be the fraction of time own queue has been selected for transmission given that power vector is $\boldsymbol{\ell}_k$ and topology is at s.

$$q'_{isk} = \frac{y_{i_X sk}^{\Delta}}{x_{sk}^{\Delta}} = \frac{\lim_{T\to\infty} \frac{1}{T}\sum_{n=1}^{T} \mathbb{I}_s(n)\mathbb{I}_k^{\Delta}(n)\mathbb{I}_{i_X}^{\Delta}(n)}{\lim_{T\to\infty} \frac{1}{T}\sum_{n=1}^{T} \mathbb{I}_s(n)\mathbb{I}_k^{\Delta}(n)}. \qquad (10)$$

Then, $1 - q'_{isk}$ the fraction of time packets from relay queue has been selected for transmission under same condition.

Recall that we assumed that $\boldsymbol{\lambda} \in \overline{\mathcal{C}}_{IID}$. So, from [1, Definition 2] and [1, Definition 3], the policy Δ is feasible, i.e., $\overline{P}_i^{\Delta} \leq \overline{P}_i$. Now,

$$\begin{aligned}
\overline{P}_i^{\Delta} &= \lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} \sum_{j\in[N], j\neq i} \ell_{ij}^{\Delta}(n) \\
&= \lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} \sum_{s\in\mathcal{S}} \sum_{k=1}^{M} \sum_{j\in[N], j\neq i} \ell_{ijk}\mathbb{I}_s(n)\mathbb{I}_k^{\Delta}(n) \\
&= \sum_{s\in\mathcal{S}} \sum_{k=1}^{M} \sum_{j\in[N], j\neq i} \ell_{ijk} \Big(\lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} \mathbb{I}_s(n)\mathbb{I}_k^{\Delta}(n) \Big) \\
&= \sum_{s\in\mathcal{S}} \sum_{k=1}^{M} \sum_{j\in[N], j\neq i} \ell_{ijk} x_{sk}^{\Delta} \\
&= \sum_{s\in\mathcal{S}} \sum_{k=1}^{M} \pi_s w'^{\Delta}_{sk} \sum_{j\in[N], j\neq i} \ell_{ijk} \text{ w.p. } 1.
\end{aligned}$$

Then, $\sum_{s\in\mathcal{S}} \sum_{k=1}^{M} \pi_s w'^{\Delta}_{sk} \sum_{j\in[N], j\neq i} \ell_{ijk} \leq \overline{P}_i$. Similarly, we have

$$\begin{aligned}
\overline{\mu}_{ij}^{\Delta} &= \lim_{T\to\infty} \frac{1}{T} \sum_{n=1}^{T} \mu_{ij}^{\Delta}(n) \\
&= \sum_{s\in\mathcal{S}} \sum_{k=1}^{M} \mu_{ij}(s, \ell_k)\pi_s w'^{\Delta}_{sk} w.p.1,
\end{aligned}$$

$$\begin{aligned}
\overline{\mu}_{iR}^{\Delta} &= \liminf_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} \mu_{i0}^{\Delta}(n)[1 - \mathbb{I}_i^{\Delta}(n)] \\
&= \sum_{s\in\mathcal{S}} \sum_{k=1}^{M} \mu_{i0}(s, \ell_k)(1 - q'_{isk})\pi_s w'^{\Delta}_{sk} w.p.1.
\end{aligned}$$

Recall that from [1, Definition 2], $\sum_{j\in[N],j\neq i}\overline{\mu}_{ji}^{\Delta} \leq \overline{\mu}_{iR}^{\Delta}$. Then,

$$\sum_{j\in[N],j\neq i}\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\mu_{ji}(s,\ell_k)\pi_s w_{sk}'^{\Delta} \leq \sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\mu_{i0}(s,\ell_k)(1-q_{isk}')\pi_s w_{sk}'^{\Delta}.$$

Also, from [1, Definition 2], $\lambda_i \leq \overline{\mu}_{i0}^{\Delta} + \sum_{j\in[N],j\neq i}\overline{\mu}_{ij}^{\Delta}$. Doing the analysis in a similar way, we get

$$\sum_s\sum_k \pi_s w_{sk}'^{\Delta}[\mu_{i0}(s,\ell_k)q_{isk}' + \sum_{j\in[N],j\neq i}\mu_{ij}(s,\ell_k)] \geq \lambda_i.$$

Hence, we have shown that there exists $w_{sk}'^{\Delta}$ and q_{isk}' which satisfy the constraints for Λ_{IID} and thus $\boldsymbol{\lambda} \in \Lambda_{IID}$. So, $\overline{\mathcal{C}}_{IID} \subseteq \Lambda_{IID}$.

Now, we have to prove $\Lambda_{IID} \subseteq \overline{\mathcal{C}}_{IID}$. For any $\boldsymbol{\lambda} \in \Lambda_{IID}$, we have a randomized resource allocation policy with appropriate w_{sk} and $q_{isk}, \forall s \in \mathcal{S}, \boldsymbol{\ell}_k \in \mathcal{L}$ and $i \in [N]$ satisfying [1, Equation 9, Equation 10, Equation 11, Equation 12, Equation 13]. We will try to show that this randomized policy is rate stable. Assume that the fluid limits $\widetilde{X}_i(0) = \widetilde{Y}_i(0) = \widetilde{U}_i(0) = 0$. Then, we will try to show that $\widetilde{X}_i(t) = \widetilde{Y}_i(t) = \widetilde{U}_i(t) = 0$ for every $t > 0$. Towards this end, in order to measure the aggregate network congestion, we define a Lyapunov function $V(t)$ as a sum of the squares of the fluid limits of the individual queue lengths:

$$V(t) = \sum_i \widetilde{X}_i^2(t) + \sum_i \widetilde{Y}_i^2(t) + \sum_i \widetilde{U}_i^2(t). \tag{11}$$

From our assumption, $V(0) = 0$. Our task will be to show $V(t) = 0, \forall t > 0$. To achieve that, we just need to show that $V'(t) \leq 0, \forall t > 0$ after which we appeal to the [3, Lemma 1]. This lemma is provided here for reference.

Lemma 5. *Let $f : [0,\infty) \to [0,\infty)$ be an absolutely continuous function with $f(0) = 0$. Assume that whenever $f(t) > 0$ and f is differentiable at t, $f'(t) \leq 0$ for almost every t (wrt Lebesgue measure). Then $f(t) = 0$ for almost every $t \geq 0$.*

The fact that fluid limits are Lipschitz continuous guarantees $V(t)$ to be differentiable almost everywhere. Applying Leibniz's rule for differentiation to (11) and taking only right-hand limit, we have

$$\begin{aligned} V'(t) = &\sum_i \widetilde{X}_i(t)\lim_{\epsilon\to 0^+}\frac{\widetilde{X}_i(t+\epsilon)-\widetilde{X}_i(t)}{\epsilon} \\ &+ \sum_i \widetilde{Y}_i(t)\lim_{\epsilon\to 0^+}\frac{\widetilde{Y}_i(t+\epsilon)-\widetilde{Y}_i(t)}{\epsilon} \\ &+ \sum_i \widetilde{U}_i(t)\lim_{\epsilon\to 0^+}\frac{\widetilde{U}_i(t+\epsilon)-\widetilde{U}_i(t)}{\epsilon}. \end{aligned} \tag{12}$$

Suppose $V(0) = 0$. We will try to prove that $V(t) = 0$ for some $t > 0$. Assume that $V(t) > 0$ for some $t > 0$. Now, we have the following cases:

1) Only one of the three queues is non-zero.
2) Any two of the three queues is non-zero.
3) All of the three queues are non-zero.

1) Case 1: Assume that $\widetilde{X}_i(t) > 0$ for some $t > 0$. Without loss of generality, we take

$$\lim_{\epsilon\to 0^+}\frac{\widetilde{X}_i(t+\epsilon)-\widetilde{X}_i(t)}{\epsilon} > 0, \tag{13}$$

for some $\epsilon > 0$. Let $a = \min_{t'\in[t,t+\epsilon]}\widetilde{X}_i(t') > 0$. Thus, for large enough k, we have $\frac{X_i(r_{n_k}t')}{r_{n_k}} \geq a \;\; \forall t' \in [t, t+\epsilon]$ and $ar_{n_k} > \mu_{max}$. Then, $X_i(r_{n_k}t') \geq \mu_{max} \;\; \forall t' \in [t, t+\epsilon]$. Since we are guaranteed that minimum queue length in this interval is μ_{max}, so we can write

$$\begin{aligned} &\lim_{\epsilon\to 0^+}\frac{\widetilde{X}_i(t+\epsilon)-\widetilde{X}_i(t)}{\epsilon} \\ &= \lim_{\epsilon\to 0^+}\frac{\widetilde{\mathscr{A}_i}(\epsilon)-\sum_{j\in[N]}\widetilde{\mathscr{D}}_{ij}(\epsilon)-\widetilde{\mathscr{D}}_{i0}(\epsilon)}{\epsilon} \\ &= \lambda_i - \\ &\left[\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s w_{sk}\left[\mu_{i0}(s,\ell_k)q_{isk} + \sum_{j\in[N],j\neq i}\mu_{ij}(s,\ell_k)\right]\right]. \end{aligned}$$

Then, by (13)

$$\begin{aligned} &\lambda_i \geq \\ &\left[\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s w_{sk}\left[\mu_{i0}(s,\ell_k)q_{isk} + \sum_{j\in[N],j\neq i}\mu_{ij}(s,\ell_k)\right]\right], \end{aligned}$$

which is a contradiction as we have assumed that $\boldsymbol{\lambda} \in \Lambda_{IID}$. Similarly, we can prove that $\widetilde{Y}_i(t')$ is decreasing for every t'.

Suppose

$$\lim_{\epsilon\to 0^+}\frac{\widetilde{U}_i(t+\epsilon)-\widetilde{U}_i(t)}{\epsilon} > 0. \tag{14}$$

Same as before, for large enough k, we can have $U_i(r^{n_k}t') \geq \overline{P}_i \;\; \forall t' \in [t, t+\epsilon]$. So, we can always expect departure from token queue to take place at maximum capacity every instant in this interval. Then,

$$\begin{aligned} &\lim_{\epsilon\to 0^+}\frac{\widetilde{U}_i(t+\epsilon)-\widetilde{U}_i(t)}{\epsilon} \\ &= -\overline{P}_i + \sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s w_{sk}\sum_{j\in[N],j\neq i}\ell_{ijk}. \end{aligned}$$

Then, by (14), we have

$$\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s w_{sk}\sum_{j\in[N],j\neq i}\ell_{ijk} \geq \overline{P}_i,$$

which is a contradiction as we assumed $\boldsymbol{\lambda} \in \Lambda_{IID}$. Thus, $V'(t) \leq 0 \;\; \forall t > 0$.

2) Case 2: Again, w.l.o.g, assume that queues $\widetilde{X}_i(t)$ and $\widetilde{Y}_i(t)$ are non-zero for some $t > 0$. From case 1, we have

$$\lim_{\epsilon\to 0^+} \frac{\widetilde{X}_i(t+\epsilon) - \widetilde{X}_i(t)}{\epsilon} \le 0,$$
$$\lim_{\epsilon\to 0^+} \frac{\widetilde{Y}_i(t+\epsilon) - \widetilde{Y}_i(t)}{\epsilon} \le 0.$$

So, $V'(t) \le 0 \;\; \forall t > 0$. Similarly, we can take other possible combinations of queues, two at a time, and we can show the same result.

3) Case 3: Again, without loss of generality, assume that queues $\widetilde{X}_i(t)$, $\widetilde{Y}_i(t)$ and $\widetilde{U}_i(t)$ are non-zero for some $t > 0$. From case 1, we have

$$\lim_{\epsilon\to 0^+} \frac{\widetilde{X}_i(t+\epsilon) - \widetilde{X}_i(t)}{\epsilon} \le 0,$$
$$\lim_{\epsilon\to 0^+} \frac{\widetilde{Y}_i(t+\epsilon) - \widetilde{Y}_i(t)}{\epsilon} \le 0,$$
$$\lim_{\epsilon\to 0^+} \frac{\widetilde{U}_i(t+\epsilon) - \widetilde{U}_i(t)}{\epsilon} \le 0.$$

Thus, $V'(t) \le 0 \;\; \forall t > 0$ for every case.

Hence, by Lemma 5, $V(t) = 0$ for $t \ge 0$. However, $\widetilde{X}_i(t)$, $\widetilde{Y}_i(t)$ and $\widetilde{U}_i(t)$ are non-negative reals. Therefore, $\widetilde{X}_i(t) = 0$, $\widetilde{Y}_i(t) = 0$ and $\widetilde{U}_i(t) = 0 \;\; \forall i \in [1, .., N]$. Then using Lemma 1, 2, 3 and 4, we get

$$\begin{aligned}
\widetilde{\mathscr{A}_i}(t) &= \sum_{j\in[N], j\ne i} \widetilde{\mathscr{D}}_{ij}(t) + \widetilde{\mathscr{D}}_{i0}(t),\\
\sum_{j\in[N], j\ne i} \widetilde{\mathscr{D}}_{ji}(t) &= \widetilde{\mathscr{D}}_{iR}(t),\\
\widetilde{\mathscr{A}_{U}}_i(t) &= \widetilde{\mathscr{D}}_{Ui}(t) \le \overline{P}_i t.
\end{aligned} \tag{15}$$

Writing the expression for the fluid limits and making appropriate manipulations, we get

$$\lim_{k\to\infty} \frac{\sum_{j\in[N], j\ne i} \mathscr{D}_{ij}(r_{n_k}t) + \mathscr{D}_{i0}(r_{n_k}t)}{r_{n_k}t} = \lambda_i,$$
$$\lim_{k\to\infty} \frac{\sum_{j\in[N], j\ne i} \mathscr{D}_{ji}(r_{n_k}t)}{r_{n_k}t} = \lim_{k\to\infty} \frac{\mathscr{D}_{iR}(r_{n_k}t)}{r_{n_k}t}.$$

We have *Average Departure rate* = *Average Arrival rate* for every queue. But, its a fact that *Average Departure rate* $\le$ *Average Service rate* for any queue. Let $\overline{\mu}_{ij}^{\Delta}, \overline{\mu}_{i0}^{\Delta}$ and $\overline{\mu}_{iR}^{\Delta}$ be the average service rate of link (i, j), and of link $(i, 0)$ for *own queue* and *relay queue* of MS i, respectively as defined in [1, Equation 6, Equation 7, Equation 8] under the randomised resource allocation policy Δ. Then, by strong law of large numbers and assuming large queue backlog, we have

$$\sum_{j\in[N], j\ne i} \overline{\mu}_{ij}^{\Delta} + \overline{\mu}_{i0}^{\Delta} \ge \lambda_i,$$
$$\sum_{j\in[N], j\ne i} \overline{\mu}_{ji}^{\Delta} \le \overline{\mu}_{iR}^{\Delta}.$$

Thus, conditions (a) and (b) of [1, Definition 2] are met. Furthermore, (15) can be written as:

$$\frac{\widetilde{A}_{Ui}(t)}{t} = \overline{P}_i^{\Delta} \le \overline{P}_i \;\; \text{(by SLLN)}.$$

This implies the randomized policy is feasible. Hence, for any $\boldsymbol{\lambda} \in \Lambda_{IID}$, we have shown the existence of a randomized policy which is rate stable for that $\boldsymbol{\lambda}$. Then using [1, Definition 3], $\boldsymbol{\lambda} \in \overline{\mathcal{C}}_{IID}$ and hence, $\Lambda_{IID} \subseteq \overline{\mathcal{C}}_{IID}$. Thus, $\Lambda_{IID} = \overline{\mathcal{C}}_{IID}$.

Appendix C
Proof of Lemma 1

We assume that $\boldsymbol{\lambda}$ is stabilizable. Note that for policy Δ_{IID}^{o}, $\{\mathbf{X}(n), \mathbf{Y}(n), \mathbf{U}(n)\}_{n\ge 1}$ is an irreducible, aperiodic and countable Markov chain. Now, define the following Lyapunov function:

$$f(n) = \sum_{i=1\in[N]} \Big[(X_i(n))^2 + (Y_i(n))^2 + (U_i(n))^2\Big]. \tag{16}$$

Recall that μ_{max} be the maximum possible transmission rate across any link in the network at any topology and P_{max} be the maximum transmit power for any MS in the network. Let $M \overset{\text{def}}{=} NA_{max}^2 + N(N\mu_{max})^2 + \sum_{i\in[N]} \overline{P}_i + NP_{max}^2$. It follows that

$$\begin{aligned}
&f(n+1) - f(n)\\
&\le M - 2\sum_{i\in[N]} U_i(n)\overline{P}_i + 2\sum_{i\in[N]} X_i(n)A_i(n)\\
&- 2\sum_{i\in[N]} \Bigg[\sum_{\substack{j\in[N]\\ j\ne i}} \Big(X_i(n) - Y_i(n)\Big)\mu_{ij}(s_{IID}(n), \boldsymbol{\ell}^{\Delta_{IID}^{o}}(n))\\
&\qquad + \max(X_i(n), Y_i(n))\mu_{i0}(s_{IID}(n), \boldsymbol{\ell}^{\Delta_{IID}^{o}}(n))\\
&\qquad - U_i(n)\sum_{\substack{j=0\\ j\ne i}}^{N} \ell_{ij}^{\Delta_{IID}^{o}}(n)\Bigg].
\end{aligned}$$

Thus,

$$\begin{aligned}
&\mathbb{E}[f(n+1) - f(n) | \mathbf{X}(n), \mathbf{Y}(n), \mathbf{U}(n)]\\
&\le M - 2\sum_{i\in[N]} U_i(n)\overline{P}_i + 2\sum_{i\in[N]} X_i(n)\lambda_i\\
&- 2\sum_{i\in[N]} \mathbb{E}\Bigg[\sum_{\substack{j\in[N]\\ j\ne i}} \Big(X_i(n) - Y_i(n)\Big)\mu_{ij}(s_{IID}(n), \boldsymbol{\ell}^{\Delta_{IID}^{o}}(n))\\
&+ \max\{X_i(n), Y_i(n)\}\mu_{i0}(s_{IID}(n), \boldsymbol{\ell}^{\Delta_{IID}^{o}}(n))\\
&- U_i(n)\sum_{j=0, j\ne i}^{N} \ell_{ij}^{\Delta_{IID}^{o}}(n) \mid \mathbf{X}(n), \mathbf{Y}(n), \mathbf{U}(n)\Bigg]\\
&= M - 2\sum_{i\in[N]} U_i(n)\overline{P}_i + 2\sum_{i\in[N]} X_i(n)\lambda_i\\
&\quad - 2\,\mathbb{E}[W(\mathbf{X}(n), \mathbf{Y}(n), \mathbf{U}(n), s_{IID}(n), \boldsymbol{\ell}^{\Delta_{IID}^{o}}(n)) \mid\\
&\qquad \mathbf{X}(n), \mathbf{Y}(n), \mathbf{U}(n)],
\end{aligned} \tag{17}$$

where,

$$
\begin{aligned}
&W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta^o_{IID}}(n))\\
&=\sum_{i\in[N]}\Bigg[\sum_{\substack{j\in[N]\\ j\neq i}}\Big(X_i(n)-Y_i(n)\Big)\mu_{ij}(s_{IID}(n),\boldsymbol{\ell}^{\Delta^o_{IID}}(n))\\
&+\max\{X_i(n),Y_i(n)\}\mu_{i0}(s_{IID}(n),\boldsymbol{\ell}^{\Delta^o_{IID}})\\
&-U_i(n)\sum_{\substack{j=0\\ j\neq i}}^{N}\ell_{ij}^{\Delta^o_{IID}}(n)\Bigg].
\end{aligned}
$$

Now, since $\boldsymbol{\lambda}$ is stabilizable, we can obtain a randomized policy Δ_{IID} satisfying [1, Equation (9)-(13)]. From definition of Δ^o_{IID} ([1, Equation 14]), for every Δ_{IID} and $n\geq 0$

$$
\begin{aligned}
&\mathbb{E}[W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta^o_{IID}}(n))\\
&\qquad\qquad\mid \mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)]\\
&\geq\mathbb{E}[W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta_{IID}}(n))\\
&\qquad\qquad\mid \mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)].
\end{aligned}\tag{18}
$$

Thus, from (17),

$$
\begin{aligned}
&\mathbb{E}[f(n+1)-f(n)|\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)]\\
&\leq M-2\sum_{i\in[N]}U_i(n)\overline{P}_i+2\sum_{i\in[N]}X_i(n)\lambda_i\\
&\quad-2\,\mathbb{E}[W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta_{IID}}(n))\mid\\
&\qquad\qquad\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)].
\end{aligned}\tag{19}
$$

there exists some $\delta>0$ such that in [1, Equation (11)-(13)], we can write

$$
\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s w_{sk}\Bigg[\mu_{i0}(s,\ell_k)q_{isk}+\sum_{j\in[N],j\neq i}\mu_{ij}(s,\ell_k)\Bigg]\geq\lambda_i+\delta,\tag{20}
$$

$$
\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s(1-q_{isk})w_{sk}\mu_{i0}(s,\ell_k)\geq\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\sum_{j\in[N],j\neq i}\pi_s w_{sk}\mu_{ji}(s,\ell_k)+\delta,\tag{21}
$$

$$
\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s w_{sk}\sum_{j\in[N],j\neq i}\ell_{ijk}+\delta\leq\overline{P}_i.\tag{22}
$$

Thus, we have

$$
\begin{aligned}
&\mathbb{E}[W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta_{IID}}(n))\mid\\
&\qquad\qquad\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)]\\
&\geq\sum_{i\in[N]}\Bigg[X_i(n)(\lambda_i+\delta)+Y_i(n)\delta-U_i(n)(\overline{P}_i-\delta)\Bigg].
\end{aligned}
$$

Hence, from (19)

$$
\begin{aligned}
&\mathbb{E}[f(n+1)-f(n)|\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)]\\
&\leq M-2\delta\sum_{i\in[N]}(X_i(n)+Y_i(n)+U_i(n)).
\end{aligned}\tag{23}
$$

Let $\mathcal{A}=\{\{\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)\}:\sum_{i\in[N]}(X_i(n)+Y_i(n)+U_i(n))\leq(M+1)/2\delta\}$. Then,

$$
\begin{aligned}
&\mathbb{E}[f(n+1)-f(n)|\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)]\\
&\quad<\begin{cases}\infty, & \forall\ \{\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)\}\\ -1, & \text{if } \{\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)\}\notin\mathcal{A}.\end{cases}
\end{aligned}\tag{24}
$$

Thus, since $|\mathcal{A}|$ is finite, by Foster's Theorem [5, Theorem 2.2.3], $\{\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)\}_{n\geq 0}$ is positive recurrent, and for each queue the expected queue length under its stationary distribution is finite. Thus, the system is stable under Δ^o_{IID}.

APPENDIX D
PROOF OF LEMMA 2

Let $\boldsymbol{\lambda}$ be stabilizable and

$$
f(n)=\sum_{i=1\in[N]}\Bigg[(X_i(n))^2+(Y_i(n))^2+(U_i(n))^2\Bigg].\tag{25}
$$

Let $M_1\overset{\text{def}}{=}N[(TA_{max})^2+(T\mu_{max})^2+(N^2+1)T^2\mu_{max}^2+2T^2\overline{P}_i^2]$. Using the analysis similar to the one used for obtaining (17)

$$
\begin{aligned}
&\mathbb{E}[f((K+1)T)-f(KT)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)]\\
&\leq M_1-\\
&2\sum_{n=KT}^{(K+1)T-1}\mathbb{E}\Bigg[\sum_{i,j}(X_i(KT)-Y_j(KT)\mu_{ij}(s_{IID}(n),\boldsymbol{\ell}^{\Delta^T_{IID}}(n))\\
&+\sum_{i\in[N]}\max\left(X_i(KT),Y_i(KT)\right)\mu_{i0}(s_{IID}(n),\boldsymbol{\ell}^{\Delta^T_{IID}}(n))-\\
&\sum_{i\in[N]}\sum_{\substack{j=0\\ j\neq i}}^{N}U_i(KT)\ell_{ij}^{\Delta^T_{IID}}(n)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)\Bigg]\\
&+2T\sum_{i\in[N]}\lambda_i X_i(KT)-2T\sum_{i\in[N]}\overline{P}_iU_i(KT)\\
&=M_1-2\times\\
&\sum_{n=KT}^{(K+1)T-1}\mathbb{E}\Bigg[W(\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT),s_{IID}(n),\boldsymbol{\ell}^{\Delta^T_{IID}}(n))\\
&\qquad\qquad\mid\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)\Bigg]\\
&+2T\sum_{i\in[N]}\lambda_i X_i(KT)-2T\sum_{i\in[N]}\overline{P}_iU_i(KT),
\end{aligned}\tag{26}
$$

where,

$$
\begin{aligned}
&W(\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT),s_{IID}(n),\boldsymbol{\ell}^{\Delta^T_{IID}}(n))=\\
&\sum_{i\in[N]}\Bigg[\sum_{\substack{j\in[N]\\ j\neq i}}\Big(X_i(KT)-Y_i(KT)\Big)\mu_{ij}(s_{IID}(n),\boldsymbol{\ell}^{\Delta^T_{IID}}(n))\\
&+\max(X_i(KT),Y_i(KT))\mu_{i0}(s_{IID}(n),\boldsymbol{\ell}^{\Delta^T_{IID}}(n))\\
&-U_i(KT)\sum_{j=0,j\neq i}^{N}\ell_{ij}^{\Delta^T_{IID}}(n)\Bigg].
\end{aligned}
$$

Our next step is to find a relationship between $W(\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT),s_{IID}(n),\boldsymbol{\ell}^{\Delta^T_{IID}}(n))$ and $W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta^o_{IID}}(n))$. Towards that end, we have the following inequalities

$$X_i(n) \leq X_i(KT) + \sum_{t=KT}^{n-1} A_i(t) \leq X_i(KT) + TA_{max}, \tag{27}$$

$$\begin{aligned} X_i(n) \geq X_i(KT) &- \sum_{t=KT}^{n-1}\sum_{j\in[N]} \mu_{ij}(s(t),\boldsymbol{\ell}(t) \\ &- \sum_{t=KT}^{n-1} \mu_{i0}(s(t),\boldsymbol{\ell}(t)) \\ &\geq X_i(KT) - T\mu_{max}, \end{aligned} \tag{28}$$

$$Y_i(n) \leq Y_i(KT) + T(N-1)\mu_{max}, \tag{29}$$

$$Y_i(n) \geq Y_i(KT) - T\mu_{max}, \tag{30}$$

$$\begin{aligned} &\max(X_i(n),Y_i(n))\mu_{i0}(s^{IID}(n),\boldsymbol{\ell}(n)) \\ &\quad\leq \max(X_i(KT),Y_i(KT))\mu_{i0}(s^{IID}(n),\boldsymbol{\ell}(n)) \\ &\qquad + T\mu_{max}[A_{max}+N\mu_{max}], \end{aligned} \tag{31}$$

$$\begin{aligned} &\max(X_i(n),Y_i(n))\mu_{i0}(s^{IID}(n),\boldsymbol{\ell}(n)) \\ &\geq \max(X_i(KT),Y_i(KT))\mu_{i0}(s^{IID}(n),\boldsymbol{\ell}(n)) - 2T\mu_{max}^2, \end{aligned} \tag{32}$$

$$U_i(n) \leq U_i(KT) + T\overline{P}_i, \tag{33}$$

$$U_i(n) \geq U_i(KT) - T\overline{P}_i, \tag{34}$$

where we have used the identities $\max(a+b,c) \leq \max(a,c)+b$ and $\max(a-b,c) \geq \max(a,c)-b$ for obtaining (31) and (32).

With $\boldsymbol{\lambda}$ being stabilizable, [1, Lemma 1] guarantees that policy $\boldsymbol{\lambda} \in \overline{\mathcal{C}}^{\Delta^o_{IID}}_{IID}$. Let $\frac{M_2}{2} \stackrel{\text{def}}{=} TN^2(A_{max}+\mu_{max})\mu_{max} + NT\mu_{max}[A_{max}+N\mu_{max}] + T\sum_{i\in[N]}\overline{P}_i^2$ From definition of Δ^T_{IID} ([1, Equation 16]) and from (27)-(34), for every $n \in [KT,(K+1)T-1]$

$$\begin{aligned} &W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta^o_{IID}}(n)) \\ &\leq W(\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT),s_{IID}(n),\boldsymbol{\ell}^{\Delta^T_{IID}(n)}) + \frac{M_2}{2}. \end{aligned} \tag{35}$$

From (26) and (35), and using law of iterated expectations, we have

$$\begin{aligned} &\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)] \\ &\leq M_1 + M_2 - \\ &2\sum_{n=KT}^{(K+1)T-1} \mathbb{E}[\mathbb{E}[W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta^o_{IID}}(n)) \\ &\mid \mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)]] \\ &+ 2T\sum_{i\in[N]}\lambda_i X_i(KT) - 2T\sum_{i\in[N]}\overline{P}_i U_i(KT) \\ &\leq M_1 + M_2 - \\ &2\sum_{n=KT}^{(K+1)T-1} \mathbb{E}[\mathbb{E}[W(\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),s_{IID}(n),\boldsymbol{\ell}^{\Delta_{IID}}(n)) \\ &\mid \mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)]] \\ &+ 2T\sum_{i\in[N]}\lambda_i X_i(KT) - 2T\sum_{i\in[N]}\overline{P}_i U_i(KT). \end{aligned} \tag{36}$$

Note that (36) was obtained using (18). For every $\delta > 0$ in (20)-(22), we have

$$\begin{aligned} &\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)] \\ &\leq M_1 + M_2 - \\ &2\sum_{n=KT}^{(K+1)T-1} \mathbb{E}\Bigg[\sum_{i\in[N]}[X_i(n)(\lambda_i+\delta) + Y_i(n)\delta - U_i(n)(\overline{P}_i-\delta)] \\ &\mid \mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n),\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)]\Bigg] \\ &+ 2T\sum_{i\in[N]}\lambda_i X_i(KT) - 2T\sum_{i\in[N]}\overline{P}_i U_i(KT). \end{aligned}$$

Let $\frac{M_3}{2} \stackrel{\text{def}}{=} T^2\mu_{max}\sum i\in[N](\lambda_i+\delta) + T^2\sum_{i\in[N]}\overline{P}_i(\overline{P}_i-\delta)$. Using (28) and (33), we get

$$\begin{aligned} &\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)] \\ &\leq M_1 + M_2 + M_3 - 2\delta\sum_{i\in[N]}(X_i(KT)+Y_i(KT)+U_i(KT)). \end{aligned} \tag{37}$$

Let $\mathcal{B} = \{\{\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)\} : \sum_{i\in[N]}(X_i(KT) + Y_i(KT) + U_i(KT)) \leq (M_1+M_2+M_3+1)/2\delta\}$. Then,

$$\begin{aligned} &\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)] \\ &\quad < \begin{cases} \infty, & \forall\ \{\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)\} \\ -1, & \text{if } \{\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)\} \notin \mathcal{B}. \end{cases} \end{aligned} \tag{38}$$

Thus, since $|\mathcal{B}|$ is finite, by Foster's Theorem [5, Theorem 2.2.3], $\{\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)\}_{KT\geq 1}$ is positive recurrent, and for each queue the expected queue length under its stationary distribution is finite. Thus, the system is stable under Δ^T_{IID}.

APPENDIX E
PROOF OF THEOREM 2

Let $\boldsymbol{\lambda}$ be stabilizable and

$$f(n) = \sum_{i=1\in[N]} \Big[(X_i(n))^2 + (Y_i(n))^2 + (U_i(n))^2\Big]. \quad (39)$$

Let $M_4 \overset{\text{def}}{=} N[(TA_{max})^2 + (T\mu_{max})^2 + (N^2+1)T^2\mu_{max}^2 + 2T^2\overline{P}_i^2]$. Using an analysis similar to that used for obtaining (26)

$$\begin{aligned}
&\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT)] \\
&\leq M_4 - 2\times \\
&\sum_{n=KT}^{(K+1)T-1} \mathbb{E}\Big[W(\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT), s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)) \\
&\qquad\qquad | \ \mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT)\Big] \\
&+ 2T\sum_{i\in[N]} \lambda_i X_i(KT) - 2T\sum_{i\in[N]} \overline{P}_i U_i(KT), \quad (40)
\end{aligned}$$

where,

$$\begin{aligned}
&W(\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT), s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)) = \\
&\sum_{i\in[N]} \Bigg[\sum_{\substack{j\in[N]\\ j\neq i}} \Big(X_i(KT) - Y_i(KT)\Big)\mu_{ij}(s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)) \\
&+ \max(X_i(KT), Y_i(KT))\mu_{i0}(s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)) \\
&- U_i(KT)\sum_{j=0, j\neq i}^{N} \ell_{ij}^{\Delta^T_{RW}}(n)\Bigg].
\end{aligned}$$

Consider two parallel networks with one network having topology evolving in IID fashion and the other network having topology evolving as generalized random walk. Assume that the stationary distribution of the network with topology evolving as generalized random walk is same as IID distribution of the first network. Let $s_{IID}(n)$ and $s_{RW}(n)$ be the state of the grid under IID and Markov mobility process in slot n, respectively. If in a slot n, $s_{IID}(n) = s_{RW}(n)$, then

$$\begin{aligned}
&W(\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT), s_{IID}(n), \boldsymbol{\ell}^{\Delta^T_{IID}}(n)) \\
&= W(\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT), s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)). \quad (41)
\end{aligned}$$

Suppose in slot n, $s_{IID}(n) \neq s_{RW}(n)$. Towards that end, consider μ_{max} be the maximum transmission rate achievable across any pair of transmitter and receiver under any network topology. Also, let P_{max} be the maximum power allowed at any MS in the network. Then, considering the assumption that each MS can transmit to only one receiver in maximum, we have the following relations:

$$\mu_{ij}(s_{IID}(n), \boldsymbol{\ell}^{\Delta^T_{IID}}(n)) \leq \mu_{ij}(s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)) + \mu_{max}, \quad (42)$$

$$\mu_{ij}(s_{IID}(n), \boldsymbol{\ell}^{\Delta^T_{IID}}(n)) + \mu_{max} \geq \mu_{ij}(s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)), \quad (43)$$

$$\mu_{i0}(s_{IID}(n), \boldsymbol{\ell}^{\Delta^T_{IID}}(n)) \leq \mu_{i0}(s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)) + \mu_{max}, \quad (44)$$

$$\max(X_i(KT), Y_i(KT)) \leq X_i(KT) + Y_i(KT), \quad (45)$$

$$\sum_{j=0, j\neq i}^{N} \ell_{ij}^{\Delta^T_{RW}}(n) + P_{max} \geq \sum_{j=0, j\neq i}^{N} \ell_{ij}^{\Delta^T_{IID}}(n), \quad (46)$$

$$\sum_{j=0, j\neq i}^{N} \ell_{ij}^{\Delta^T_{RW}}(n) \leq \sum_{j=0, j\neq i}^{N} \ell_{ij}^{\Delta^T_{IID}}(n) + P_{max}. \quad (47)$$

Thus, for slots n with $s_{IID}(n) \neq s_{RW}(n)$, using (42)-(47), we have

$$\begin{aligned}
&W(\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT), s_{IID}(n), \boldsymbol{\ell}^{\Delta^T_{IID}}(n)) \\
&\leq W(\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT), s_{RW}(n), \boldsymbol{\ell}^{\Delta^T_{RW}}(n)) \\
&+ \mu_{max}(N+1)\Bigg(\sum_{i\in[N]} X_i(KT) + \sum_{i\in[N]} Y_i(KT)\Bigg) \\
&+ P_{max}\sum_{i\in N} U_i(KT). \quad (48)
\end{aligned}$$

For tractability, suppose there are two possible states, s_1 and s_2, that network topology can assume. Let number of mismatches in number of states when network evolves in IID fashion and when network topology evolves as a generalized random walk in a single time interval of T slots be τ. That is,

$$\begin{aligned}
\tau = |\ &\text{number of states } s_1 \text{ in IID process} \\
&- \text{number of states } s_1 \text{ in Markov process}\ |.
\end{aligned}$$

Let $\gamma = (N+1)\mu_{max}\sum_{i\in[N]} X_i(KT) + (N+1)\mu_{max}\sum_{i\in[N]} Y_i(KT) + P_{max}\sum_{i\in N} U_i(KT)$. Thus, in (40), we have

$$\begin{aligned}
&\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT)] \\
&\leq M_4 - 2\times \\
&\sum_{n=KT}^{(K+1)T-1} \mathbb{E}[W(\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT), s_{IID}(n), \boldsymbol{\ell}^{\Delta^T_{IID}}(n)) \\
&\qquad\qquad |\mathbf{X}(KT), \mathbf{Y}(KT), \mathbf{U}(KT)] \\
&+ \gamma\,\mathbb{E}[\tau] + 2T\sum_{i\in[N]} \lambda_i X_i(KT) - 2T\sum_{i\in[N]} \overline{P}_i U_i(KT). \quad (49)
\end{aligned}$$

Now for some $\epsilon > 0$ and $\delta > 0$, we can write

$$\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s w_{sk}\left[\mu_{i0}(s,\ell_k)q_{isk} + \sum_{j\in[N],j\neq i}\mu_{ij}(s,\ell_k)\right] \geq \lambda_i + \delta + \epsilon, \tag{50}$$

$$\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s(1-q_{isk})w_{sk}\mu_{i0}(s,\ell_k) \geq \sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\sum_{j\in[N],j\neq i}\pi_s w_{sk}\mu_{ji}(s,\ell_k) + \delta + \epsilon, \tag{51}$$

$$\sum_{s\in\mathcal{S}}\sum_{k=1}^{M}\pi_s w_{sk}\sum_{j\in[N],j\neq i}\ell_{ijk} + \delta + \epsilon \leq \overline{P}_i. \tag{52}$$

Using (50)-(52) and with similar steps as taken in obtaining (37) from (26), we can show that

$$\begin{aligned}
&\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)]\\
&\leq M_4 + M_2 + M_3\\
&+ \big((N+1)\mu_{max}\,\mathbb{E}[\tau] - 2\epsilon\big)\sum_{i\in[N]}X_i(KT)\\
&+ \big((N+1)\mu_{max}\,\mathbb{E}[\tau] - 2\epsilon\big)\sum_{i\in[N]}Y_i(KT)\\
&+ \big(P_{max}\,\mathbb{E}[\tau] - 2\epsilon\big)\sum_{i\in[N]}U_i(KT)\\
&- 2\delta\sum_{i\in[N]}(X_i(KT) + Y_i(KT) + U_i(KT)).
\end{aligned} \tag{53}$$

Note that if $\epsilon > \frac{1}{2}\max\{(N+1)\mu_{max}\,\mathbb{E}[\tau], P_{max}\,\mathbb{E}[\tau]\}$, then, we have

$$\begin{aligned}
&\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)]\\
&\leq M_4 + M_2 + M_3\\
&- 2\delta\sum_{i\in[N]}(X_i(KT) + Y_i(KT) + U_i(KT)).
\end{aligned} \tag{54}$$

Using exponentially fast convergence of empirical distribution to the unique stationary distribution for ergodic Markov chains [6], we have $\mathbb{E}[\tau] \rightarrow 0$ as $T \rightarrow \infty$. Let $\mathcal{C}_\epsilon = \{\{\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)\} : \sum_{i\in[N]}(X_i(KT)+Y_i(KT)+U_i(KT)) \leq (M_1+M_2+M_3+1)/2\delta\}$. Then,

$$\begin{aligned}
&\mathbb{E}[f((K+1)T) - f(KT)|\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)]\\
&\quad< \begin{cases}\infty, & \forall\ \{\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)\}\\ -1, & \text{if } \{\mathbf{X}(n),\mathbf{Y}(n),\mathbf{U}(n)\} \notin \mathcal{C}_\epsilon.\end{cases}
\end{aligned} \tag{55}$$

For any $\epsilon > \frac{1}{2}\max\{(N+1)\mu_{max}\,\mathbb{E}[\tau], P_{max}\,\mathbb{E}[\tau]\}$, we have finite $|\mathcal{C}_\epsilon|$. By Foster's Theorem [5, Theorem 2.2.3], $\{\mathbf{X}(KT),\mathbf{Y}(KT),\mathbf{U}(KT)\}_{KT\geq 1}$ is positive recurrent, and for each queue the expected queue length under its stationary distribution is finite. Summarizing, depending on size of the interval T, we decide on some $\epsilon > \frac{1}{2}\max\{(N+1)\mu_{max}\,\mathbb{E}[\tau], P_{max}\,\mathbb{E}[\tau]\}$ which guarantees that the system is stable under policy Δ_{RW}^{T} $\forall\boldsymbol{\lambda}\in\Lambda_\epsilon$.

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