1.

Krihk 9

a) $f(x,y) = 3x^2y + y^3 - 2x$

$$\frac{\partial f}{\partial x} = 3x^2y + y^3 - 2x$$
$$= 3y(2x) + 0 - 2$$
$$= 6xy - 2$$

$$\frac{\partial f}{\partial y} = 3x^2y + y^3 - 2x$$
$$= 3x^2 + 3y^2 - 0$$
$$= 3(x^2 + y^2)$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}\left(\frac{\partial}{\partial x}\right)$$
$$= 6xy - 2\left(\frac{\partial}{\partial x}\right)$$
$$= 6y$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial f}{\partial y}\left(\frac{\partial}{\partial y}\right)$$
$$= 3x^2 + 3y^2\left(\frac{\partial}{\partial y}\right)$$
$$= 6y$$

$$\frac{\partial^2 f}{\partial xy} = \frac{\partial f}{\partial x}\left(\frac{\partial}{\partial y}\right)$$
$$= (6xy - 2)\left(\frac{\partial}{\partial y}\right)$$
$$= 6x$$

$$\frac{\partial^2 f}{\partial yx} = \frac{\partial f}{\partial y}\left(\frac{\partial}{\partial x}\right)$$
$$= 3x^2 + 3y^2\left(\frac{\partial}{\partial x}\right)$$
$$= 6x$$

b) $g(x,y) = e^{xy}\sin(x) + \ln(y+1)$

$$\frac{\partial g}{\partial x} = e^{xy}\sin(x) + \ln(y+1)$$
$$= \frac{\partial g}{\partial x}(e^{xy}\cdot\sin(x)) + \frac{\partial g}{\partial x}(\ln(y+1))$$
$$= e^{xy}\cos(x) + ye^{xy}\sin(x)$$
$$= e^{xy}(y\sin(x) + \cos(x))$$

$$\frac{\partial g}{\partial y} = e^{xy}\sin(x) + \ln(y+1)$$
$$= \frac{\partial g}{\partial y}(e^{xy}\sin(x)) + \frac{\partial g}{\partial y}(\ln(y+1))$$
$$= \sin(x)\,e^{xy}\cdot x + \frac{1}{(y+1)}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}\left(\frac{\partial}{\partial x}\right)$$
$$= e^{xy}(y\sin(x) + \cos(x))\left(\frac{\partial}{\partial x}\right)$$
$$= e^{xy}\cdot y(\cos(x) - \sin(x)) + e^{xy}\cdot y(y\sin(x) + \cos(x))$$
$$= e^{xy}\left[y(-\sin(x) + \cos(x)) + y(y\sin(x) + \cos(x))\right]$$
$$= e^{xy}\left[-y\sin(x) + y\cos(x) + y^2\sin(x) + y\cos(x)\right]$$
$$= e^{xy}\left[(y^2 - 1)\sin(x) + 2y\cos(x)\right]$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial f}{\partial y}\left(\frac{\partial}{\partial y}\right)$$
$$= x\cdot\sin(x)\,e^{xy} + \frac{1}{y+1}\left(\frac{\partial}{\partial y}\right)$$
$$= \sin(x)\cdot x(e^{xy})\cdot x - \frac{1}{(y+1)^2}$$
$$= x^2\sin(x)\,e^{xy} - \frac{1}{(y+1)^2}$$

$$\frac{\partial^2 f}{\partial yx} = \frac{\partial f}{\partial x}\left(\frac{\partial}{\partial y}\right)$$
$$= e^{xy}(y\sin(x) + \cos(x))\left(\frac{\partial}{\partial y}\right)$$
$$= x(y\sin(x) + \cos(x))\,e^{xy} + e^{xy}\sin(x)$$
$$= e^{xy}\left[x(y\sin(x) + \cos(x)) + \sin(x)\right]$$

$$\frac{\partial^2 f}{\partial yx} = \frac{\partial f}{\partial y}\left(\frac{\partial}{\partial x}\right)$$
$$= xe^{xy}\sin(x) + \frac{1}{(y+1)}\left(\frac{\partial}{\partial x}\right)$$

goes to zero

$$= 1e^{xy}\sin x + xe^{xy}\cdot y\sin x + xe^{xy}\cos x$$
$$= e^{xy}\left[\sin(x) + xy\sin(x) + x\cos(x)\right]$$

c) $f(x,y) = \dfrac{x^3 - y^2}{xy+1}$

$$\frac{\partial f}{\partial x} = \frac{-(x^3 - y^2)(y) + (xy+1)(3x^2)}{(xy+1)^2}$$
$$= \frac{(-x^3 + y^2)(y) + (xy+1)(3x^2)}{(xy+1)^2}$$
$$= \frac{-x^3y + y^3 + 3x^3y + 3x^2}{(xy+1)^2}$$
$$= \frac{2x^3y + y^3 + 3x^2}{(xy+1)^2}$$

$$\frac{\partial f}{\partial y} = \frac{(-2y)(xy+1) - (x^3 - y^2)(x)}{(xy+1)^2}$$
$$= \frac{-2xy^2 - 2y(-x^3 + y^2)(x)}{(xy+1)^2}$$
$$= \frac{-2xy^2 - 2y - x^4 + xy^2}{(xy+1)^2}$$
$$= \frac{x^4 - xy^2 - 2y}{(xy+1)^2}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial f}{\partial x}\left(\frac{\partial}{\partial x}\right)$$
$$= \frac{2x^3y + y^3 + 3x^2}{(xy+1)^2}\left(\frac{\partial}{\partial x}\right)$$
$$= -2y(2x^3y + 3x^2 + y^3)/(xy+1)^3 + \left(\frac{6x^2y + 6x}{(xy+1)^2}\right)$$

$$\frac{\partial^2 f}{\partial xy} = \frac{\partial f}{\partial x}\left(\frac{\partial f}{\partial y}\right)$$
$$= \frac{2x^3y + y^3 + 3x^2}{(xy+1)^2}\left(\frac{\partial f}{\partial y}\right)$$
$$= -2x(2x^3y + y^3 + 3x^2)/(xy+1)^3 + \left(\frac{2x^3 + 3y^2}{(xy+1)^2}\right)$$

$$\frac{\partial^2 f}{\partial yx} = \frac{\partial f}{\partial y}\left(\frac{\partial}{\partial x}\right)$$
$$= \frac{x^4 - xy^2 - 2y}{(xy+1)^2}\left(\frac{\partial}{\partial x}\right)$$
$$= -2y(x^4 - xy^2 - 2y)/(xy+1)^3 + \frac{2x^3 + 3y^2}{(xy+1)^2}$$

$$\frac{\partial^2 f}{\partial y^2} = \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial y}\right)$$
$$= \frac{x^4 - xy^2 - 2y}{(xy+1)^2}\left(\frac{\partial f}{\partial y}\right)$$
$$= -2x(x^4 - xy^2 - 2y)/(xy+1)^3 + \left(\frac{-2xy - 2}{(xy+1)^2}\right)$$

2. $L = f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) + f_z(a,b,c)(z-c) + f(a,b,c)$

$f_x = xyz + e^{x+y}$
$= yz + e^{x+y}$
$f_x(1,0,-1) = e$

$f_y = xyz + e^{x+y}$
$= xz + e^{x+y}$
$f_y(1,0,-1) = -1 + e$

$f_z = xyz + e^{x+y}$
$= xy$
$f_z(1,0,-1) = 0$

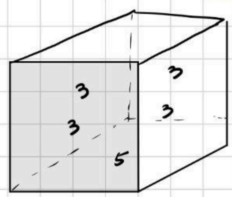
$f(a,b,c) = xyz + e^{xy}$
$= 0 \quad e^{(1+0)}$
$= e$

a) $L = yz + e^{x+y}(x-a) + xz + e^{x+y}(y-b) + xy(z-c) + f(a,b,c)$

b) $L = e(x-1) + (-1+e)(y) + e$
$= ex \cancel{-e} - y + ey \cancel{+e}$
$= ex + ey - y$

c) $L(0.9, 0.1, -1.1) = e(0.9) + e(0.1) - (0.1)$
$= 2.62$

3.



$V = 32\ m^3$

$V = xyz$

$32 = xyz$

$z = \frac{32}{xy}$

SA = base + 4x Walls

$= xy + 2zy + 2zx$
$= 5xy + 6zy + 6zx$
$= 5xy + 6\left(\frac{32}{xy}\right)y + 6\left(\frac{32}{xy}\right)x$
$= 5xy + \frac{192y}{xy} + \frac{192x}{xy}$
$= 5xy + \frac{192}{x} + \frac{192}{y}$

$\frac{\partial s}{\partial x} = 5y - \frac{192}{x^2}$

$0 = 5y - \frac{192}{x^2}$

$\frac{192}{x^2} = 5y$

$y = \frac{192}{5x^2}$

$\frac{\partial s}{\partial y} = 5x - \frac{192}{y^2}$
$= 5x - \frac{192}{\left(\frac{192}{5x^2}\right)^2}$
$= 5x - \frac{25x^4}{192}$
$0 = 5x - \frac{25x^4}{192}$

$\boxed{x = 3.374}$

$0 = 5y - \frac{192}{(3.374)^2}$
$0 = 5y - 16.87$
$\boxed{y = 3.373\ m}$

$z = \frac{32}{(3.37)^2}$
$\boxed{= 2.81}$

In [4]:
```python
import sympy as sp
import matplotlib as plt
```

In [5]:
```python
x, y = sp.symbols('x y')

# Define a function
f = x**2 + sp.sin(y)
# Differentiate f with respect to x
df_dx = sp.diff(f, x)
# Differentiate f with respect to y
df_dy = sp.diff(f, y)
```

In [6]:
```python
h = sp.exp(x)*sp.sin(y)+y**3

dh_dx = sp.diff(h, x)

dh_dy = sp.diff(h, y)

print (dh_dx)

print (dh_dy)
```

```
exp(x)*sin(y)
3*y**2 + exp(x)*cos(y)
```

In [17]:
```python
import sympy as sp

x, y = sp.symbols('x y')
g = x**2 * y + x * y**2

partial_x = sp.diff(g, x)
partial_y = sp.diff(g, y)

point = {x: 1, y: -1}
gradient_vector = [partial_x.subs(point), partial_y.subs(point)]

# Compute the magnitude of the gradient vector
gradient_magnitude = sp.sqrt(sum(component**2 for component in gradient_vector))

print("Gradient vector at the point (1, -1):", gradient_vector)
print("Magnitude of the gradient vector:", gradient_magnitude.evalf())
```

```
Gradient vector at the point (1, -1): [-1, -1]
Magnitude of the gradient vector: 1.41421356237310
```

In [8]:
```python
f = sp.log(x**2 + y**2)

d2f_dx2 = sp.diff (f, x, x)

d2f_dy2 = sp.diff (f, y, y)

d2f_dxdy = sp.diff (f, x, y)

print (d2f_dx2)

print (d2f_dy2)

print (d2f_dxdy)
```

```
2*(-2*x**2/(x**2 + y**2) + 1)/(x**2 + y**2)
2*(-2*y**2/(x**2 + y**2) + 1)/(x**2 + y**2)
-4*x*y/(x**2 + y**2)**2
```

In [16]:

```
import sympy as sp
import numpy as np
import matplotlib.pyplot as plt

# Define the symbolic variables and function
x, y = sp.symbols('x y')
w = 2*x**2 + x*y**3 - y**3*x

# Convert the SymPy expression to a numerical function
w_func = sp.lambdify((x, y), w, 'numpy')

# Create a grid of x and y values and evaluate w on this grid
x_vals = np.linspace(-2, 3, 400)
y_vals = np.linspace(-2, 3, 400)
X, Y = np.meshgrid(x_vals, y_vals)
Z = w_func(X, Y)

# Plot the contour
plt.contourf(X, Y, Z, levels=50, cmap='viridis')
plt.colorbar()
plt.title('Contour plot of $w(x, y) = 2x^2 + xy^3 - y^3x$')
plt.xlabel('$x$')
plt.ylabel('$y$')
plt.show()
```