cyclic derivatives: Symmetry of Fractional and complex

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1 Introduction

2 The symmetry of fractional cyclics and the complex plane

2.1 Overlapping circle in the complex

3 Mittag-Leffler and cyclic derivatives

3.1 Mittage-Leffler is somewhat a cyclic derivative

In the previous cyclic derivatives research, I pointed out that Cyclic derivatives and Mittag-Leffler are connected, and after some research, I can say that we can express cyclic functions in terms of Mittag-Leffler functions like this $\cosh_{\mathrm{n}}(x) = E_n(x^n)$
which can be seen in the result of $E_2(x)$ being $\cosh(\sqrt{x})$, when squared we get $\cosh(x) = E_2(x^2)$
this also allows us to get the integerral defnition of $\cosh_{\mathrm{n}}(x)$ using the $E_n(x^n)$

$$E_{\alpha,\beta}(x) = \frac{1}{2i\pi} \oint_C \frac{t^{\alpha-\beta} e^t}{t^\alpha - x} dt \quad \mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0$$

$$\cosh_{\mathrm{n}}(\,x) = E_{n,1}(x^n) = \frac{1}{2i\pi} \oint_C \frac{t^{n-1} e^t}{t^n - x^n} dt \quad \mathrm{Re}(n) > 0$$

The same way we can get $\sinh_{\mathrm{n}} \mathrm{K}(x)$ as the K-th deriavtive of $E_n(x^n)$, and if we assume that this function converge and the integral is fixed, we can get the derivative inside of the integral too

$$\sinh_{\mathrm{n}} \mathrm{K}(x) = D^K E_n(x^n) = \frac{1}{2i\pi} \oint_C D^K \left(\frac{t^{n-1} e^t}{t^n - x^n} \right) dt \quad \mathrm{Re}(n) > 0, K \in \mathbb{N}$$

using this we can get te first and the second $\sinh_{\mathrm{n}} \mathrm{K}(x)$ functions

$$\sinh_{\mathrm{n}}(x) = \frac{1}{2i\pi} \oint_C t^{n-1} e^t \frac{n x^{n-1}}{(t^n - x^n)^2} dt \quad \mathrm{Re}(n) > 0$$

$$\sinh_{\mathrm{n}}\mathrm{II}(x)=\frac{1}{2i\pi}\oint_C t^{n-1}e^t\frac{n(n-1)x^{n-2}(t^n-x^n)+2n^nx^{2n-2}}{(t^n-x^n)^2}dt \quad \mathrm{Re}(n)>0$$

we can also find $\cos_{\mathrm{2q|q}}(x)$ where $\cos_{\mathrm{2q|q}}(x)=\sum_{n=0}^{\infty}\frac{(-1)^n x^{2q}}{(2q)!}$