Appendix I

Fluid-Particle Momentum Transfer

The trajectory of a Lagrangian particle is governed by the momentum conservation equation:

$$m_{\rm p}\frac{{\rm d}\mathbf{u}_{\rm p}}{{\rm d}t} = -\frac{1}{2}\rho C_{\rm d} A_{\rm p,c} (\mathbf{u}_{\rm p}-\mathbf{u})\|\mathbf{u}_{\rm p}-\mathbf{u}\| + m_{\rm p}\mathbf{g} \tag{I.1}$$

where $m_{\rm p}$ is the particle mass, $\mathbf{u}_{\rm p}(t)$ the particle velocity, $A_{\rm p,c}$ the particle cross-sectional area, $C_{\rm d}$ the drag coefficient, ρ the gas density, $\mathbf{u}$ the gas velocity in the vicinity of the particle, and $\mathbf{g}$ the gravity vector. There is no analytical solution to this equation, but its linearized form:

$$\frac{{\rm d}\mathbf{u}_{\rm p}}{{\rm d}t} = -\beta(\mathbf{u}_{\rm p}-\mathbf{u}) + \mathbf{g} \qquad \beta = \frac{1}{2m_{\rm p}}\rho C_{\rm d} A_{\rm p,c}\|\mathbf{u}_{\rm p}(0)-\mathbf{u}\| \tag{I.2}$$

has the solution:

$$\mathbf{u}_{\rm p}(t) = \mathbf{u} + \left(\mathbf{u}_{\rm p}(0) - \mathbf{u} - \frac{\mathbf{g}}{\beta}\right){\rm e}^{-\beta t} + \frac{\mathbf{g}}{\beta} \tag{I.3}$$

assuming that the gas velocity $\mathbf{u}$ is unchanging over the short duration for which this solution is valid.

In FDS, the particle position is advanced over the course of a gas-phase time step, δt, by a series of sub-time steps, $\delta t_{\rm p}$, that are determined so as to ensure that the particle does not traverse the width of a grid cell in one sub-time step:

$$\delta t_{\rm p} = \frac{\delta t}{\lceil 0.9\,\mathrm{CFL}\rceil} \quad ; \quad \mathrm{CFL} = \delta t \max\left(\frac{|u_{\rm p}^n|}{\delta x}, \frac{|v_{\rm p}^n|}{\delta y}, \frac{|w_{\rm p}^n|}{\delta z}\right) \tag{I.4}$$

Note that the *ceiling* function, $\lceil \mathrm{CFL} \rceil$, denotes the least integer greater than the CFL. For a given time step, denoted by n, the particle position is advanced according to:

$$\mathbf{x}_{\rm p}^{n+1} = \mathbf{x}_{\rm p}^n + \frac{\delta t_{\rm p}}{2}\left(\mathbf{u}_{\rm p}^{n+1} + \mathbf{u}_{\rm p}^n\right) \tag{I.5}$$

where $\mathbf{u}_{\rm p}^{n+1}$ is given by Eq. (I.3).