MAT240 LEC0101 Notes

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1 Syllabus

The course syllabus can be found at this link: `http://www.math.toronto.edu/mgualt/courses/20-240/`

2 Sets

A **set** is a collection of objects, and viewed as an object in its own right. If some object x is contained in the set S, we say that x is an **element** of S, using this notation:

$$x \in S.$$

The arrangement (order) and repetition of elements in a set are ignored.

Example 0

$\emptyset$ is a set that has no elements.

Example 1

Suppose $P = \{R, G, B\}$. P is a set of three letters. Since the order and repetition of the elements in P are not important, we can say that

$$\begin{aligned} P &= \{R, G, B\} \\ &= \{G, B, R\} \\ &= \{R, R, B, G, B\}. \end{aligned}$$

More examples

$\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$ is the set of integers.

$\mathbb{Q} = \{p/q : p \in \mathbb{Z}, q \in \mathbb{Z}\backslash\{0\}\}$ is the set of rational numbers.

$\mathbb{N} = \{0, 1, 2, 3, ...\}$ is the set of natural numbers.

$\mathbb{R}$ and $\mathbb{C}$ are the sets of rational numbers and complex numbers, respectively.

3 Finite sets

A set is **finite** when it has a **finite** number of elements. The number $|S| \in \mathbb{N}$ describes the **cardinality**, or **size** of S.

4 Maps between sets

Consider a set A consisting of 5 elements, and another set B consisting of 3 elements. A mapping is a way of "sending" elements between sets. The set "sending" the elements is known as the **domain**, while the set "receiving" the elements is known as the **codomain**. Every element in the **domain** must be mapped to an element in the **codomain**. The domain and codomain do not have to be symmetrical, meaning that elements in the codomain do not have to be attached to an element in the domain.

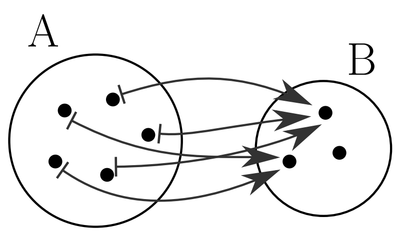

Figure 1: An example mapping of elements from set A to set B.

A **map** from set A to set B assigns to each $x \in A$ a unique element $f(x) \in B$.

This can be notated as:

$$A \xrightarrow{f} B$$
$$x \mapsto f(x)$$

In this case, we say that x **maps to** $f(x)$.

RGB Example

A colour in RGB system is a map:

$$P = \{R, G, B\} \to \{0, 1, ..., 255\}$$

The color white would be represented as:

$$
\begin{aligned}
R &\mapsto 255 \\
G &\mapsto 255 \\
B &\mapsto 255
\end{aligned}
$$

While red would be represented as:

$$
\begin{aligned}
R &\mapsto 255 \\
G &\mapsto 0 \\
B &\mapsto 0
\end{aligned}
$$

The total number of these maps is the total number of colours in the RGB system.

Functions

Maps such as $\mathbb{R} \xrightarrow{f} \mathbb{R}$ were taught in high school, such as $f(x) = x^2 + x$. Such maps were typically graphed by putting the domain on one axis, and the codomain on another. For every domain element, a point based on the codomain would be graphed. These maps are typically called **functions**, where the codomain is typically equal to some number system, such as $\mathbb{R}$.

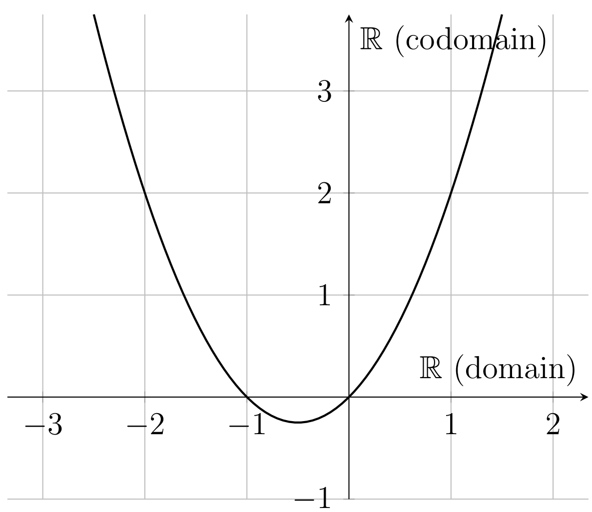

Figure 2: A graph of $f(x) = x^2 + x$.