

## Time Complexity & Big O notation

This morning I wanted to eat some pizzas; so I asked my brother to get me some from Dominos (3 km far)

He got me the pizza and I was happy only to realize it was too less for 29 friends who came to my house for a surprise visit!

My brother can get 2 pizzas for me on his bike but pizza for 29 friends is too huge of an input for him which he cannot handle.

2 pizzas → okay! not a big deal!

68 pizzas → Not possible!  
in short time

What is Time Complexity?

Time Complexity is the study of efficiency of algorithms.

① Time Complexity = How time taken to execute an algorithm grows with the size of the input!

Consider two developers who created an algorithm to sort  $n$  numbers. Shubham and Rohan did this independently.

When ran for input size  $n$ , following results were recorded.

no of elements ( $n$ )	Shubham's Algo	Rohan's Algo
10 elements	90 ms	122 ms
70 elements	110 ms	124 ms
110 elements	180 ms	131 ms
1000 elements	250 ms	800 ms

We can see that initially Shubham's algorithm was shining for smaller input but as the number of elements increases Rohan's algorithm looks good!

Quick Quiz : Who's Algorithm is better ?

Time Complexity : Sending GTA V to a friend

Let us say you have a friend living 5 kms away from your place. You want to send him a game.

Final exams are over and you want him to get this 60 GB file from you. How will you send it to him?

Note that both of you are using JIO 4G with 1 Gb/day data limit.

The best way to send him the game is by delivering it to his house.

Copy the game to a Hard disk and send it!

Will you do the same thing for sending a game like minesweeper which is in KB's of size?

No because you can send it via internet.

As the file size grows, time taken by online sending increases linearly  $\rightarrow O(n^1)$

As the file size grows, time taken by physical sending remains constant.  $O(n^0)$  or  $O(1)$

Calculating Order in terms of Input size

In order to calculate the order, most impactful term containing  $n$  is taken into account.

$\hookrightarrow$  Size of input

Let us assume that formula of an algorithm in terms of input size  $n$  looks like this:

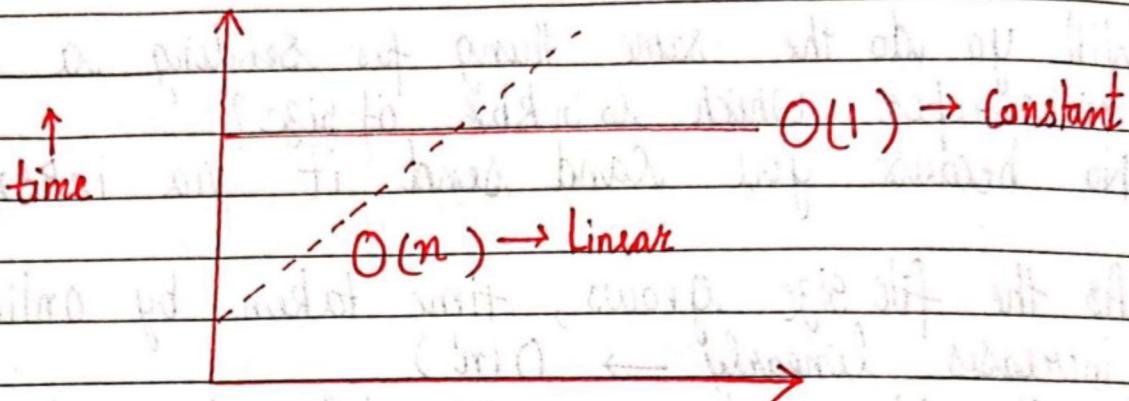
$$\text{Algo 1} \rightarrow \underbrace{k_1 n^2}_{\substack{\text{Highest} \\ \text{order term}}} + \underbrace{k_2 n + 36}_{\substack{\text{can ignore} \\ \text{lower} \\ \text{order terms}}} \Rightarrow O(n^2)$$

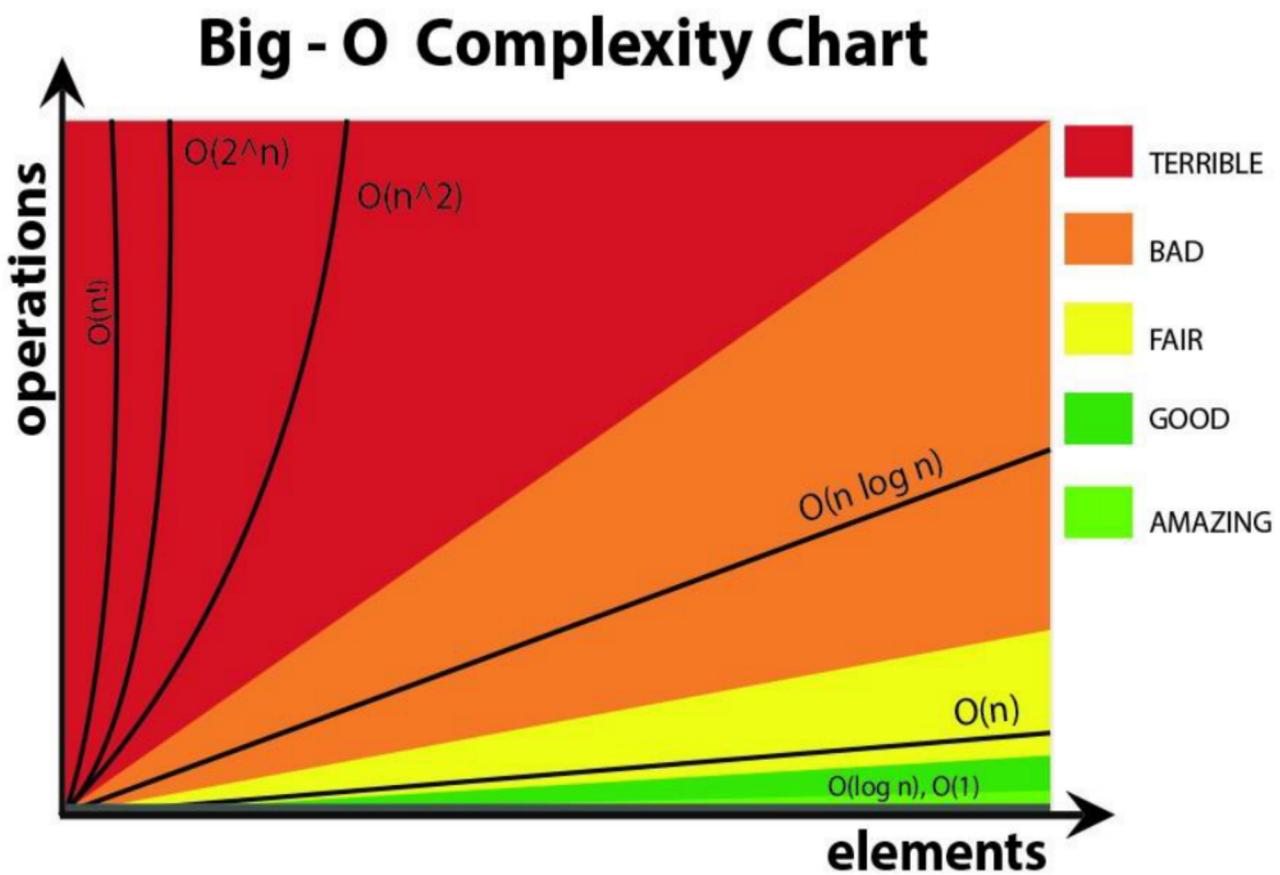
$$\begin{aligned} \text{Algo 2} \rightarrow & k_1 k_2^2 + k_3 k_2 + 8 \\ & \Downarrow \\ & k_1 k_2^2 n^0 + k_3 k_2 + 8 \Rightarrow O(n^0) \text{ or } O(1) \end{aligned}$$

Note that these are the formulas for time taken by them.

## Visualising Big O

If we were to plot  $O(1)$  and  $O(n)$  on a graph, they will look something like this:





Source: <https://stackoverflow.com/questions/3255/big-o-how-do-you-calculate-approximate-it>