MA323 Lab 3

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Question 1

Since it is a uniform distribution, we have all p_i equal. Thus

$$q_0 = 0,\ q_1 = \frac{1}{N},\ q_2 = \frac{2}{N} \ \dots\ q_N = 1$$

where N is the size of the sequence $S = 1, 3, 5,\ \dots\ , 9999$ \ So for every $u \in \mathcal{U}[0,1]$ if the resulting number is S_k then $q_{k-1} \leq u < q_k$ or $k - 1 \leq uN < k$. Therefore,

$$\lfloor uN \rfloor = k - 1$$

Experimentally we get the following distribution (see Figure 1) for 100000 samples

Question 2

(a)

$f(x) = 20x(1-x)^3$ attains its maximum value of $\frac{135}{64}$ at $x = 1/4$. As we're using $\mathcal{U}[0,1]$ so $g(x) = 1$ and thus $c = \frac{135}{64}$ is the minimum value of c satisfying $f(x) \leq cg(x)$.

(b)

upon generating the values, we get the following histogram (density).

On average we need 2.108 iterations for $c = \frac{135}{64}$

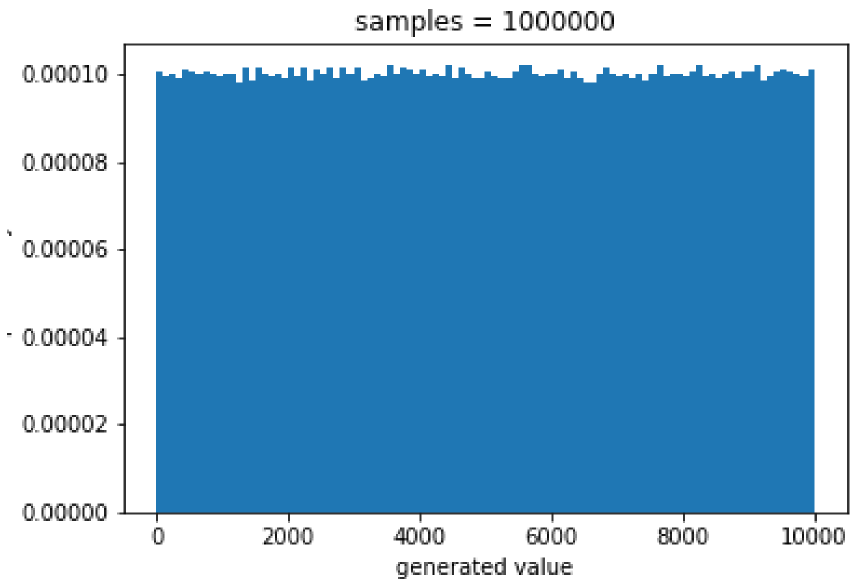


Figure 1: number of 10^6

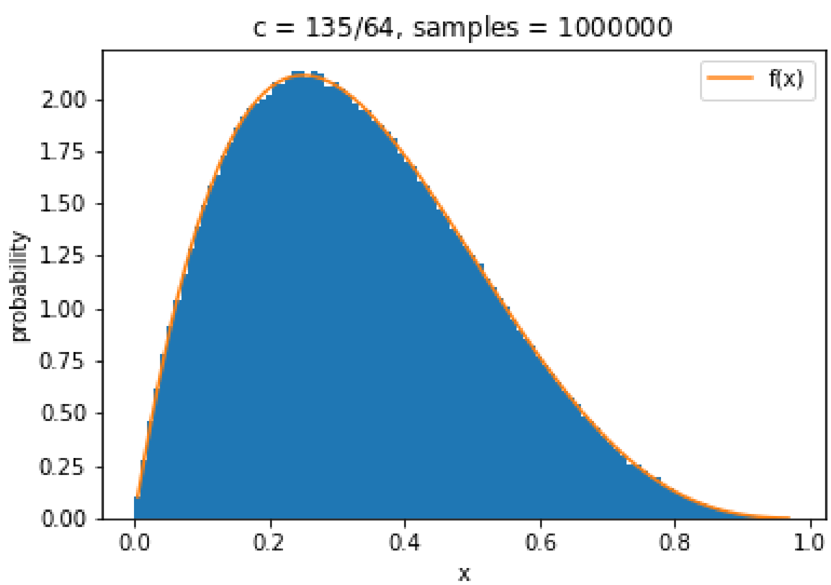


Figure 2: number of samples $= 10^6$

sample size	1000	10000	100000	500000	1000000
max $\mid p(x) - f(x) \mid$	0.997	0.322	0.149	0.058	0.028

Seeing the results above, we can see that $p(x)$ converges to $f(x)$. Here $p(x)$ is the PDF of our experimentally generated random variables

(c)

The following is the table for the number of iterations required on average to generate one random variable.

sample size	1000	10000	100000	500000	1000000
avg iterations	2.175	2.118	2.099	2.112	2.108

We expect average number of iterations to converge to

$$\frac{1}{P(U \leq f(X)/cg(X))} = c = \frac{135}{64} = 2.109$$

which is indeed the case.

(d)

Table 1: c=4

sample size	1000	10000	100000	500000	1000000
avg. iterations	4.115	3.995	4.021	4.006	4.001

Table 2: c=10

sample size	1000	10000	100000	500000	1000000
avg. iterations	9.737	9.985	10.011	9.997	10.004

We can see that in both cases average number of iterations converges to the value of c.

Question 3

The minimum values of c will be 1.2 as $g(x) = 0.1$ for all $x \in \{1, 2, ..., 10\}$. We can see that the distribution of values generated converge to required distribution.

Also, the average number of iterations converges to the values of c. In this case, c is taken to be 1.5 and 10

Table 3: $c = 1.5$

samples	1000	10000	100000	500000	1000000
avg iterations	1.4640	1.4984	1.4979	1.5001	1.4999

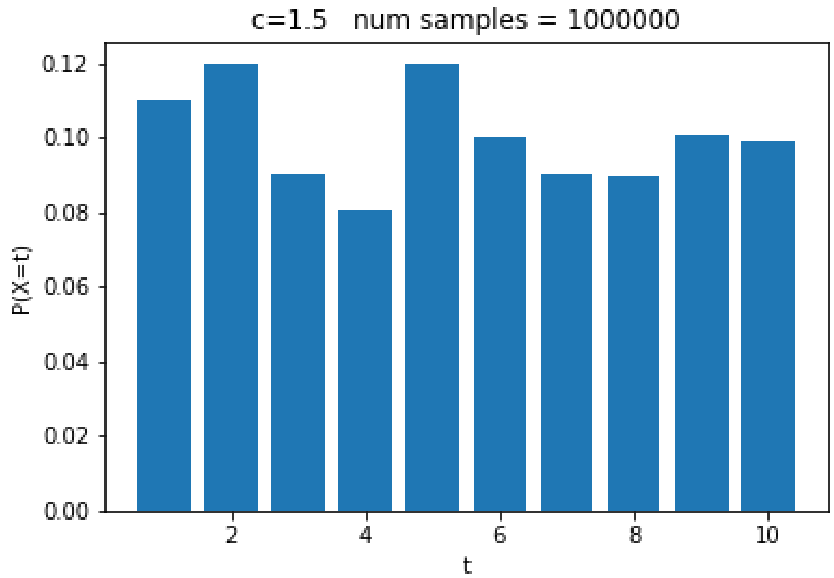
Table 4: $c = 10$

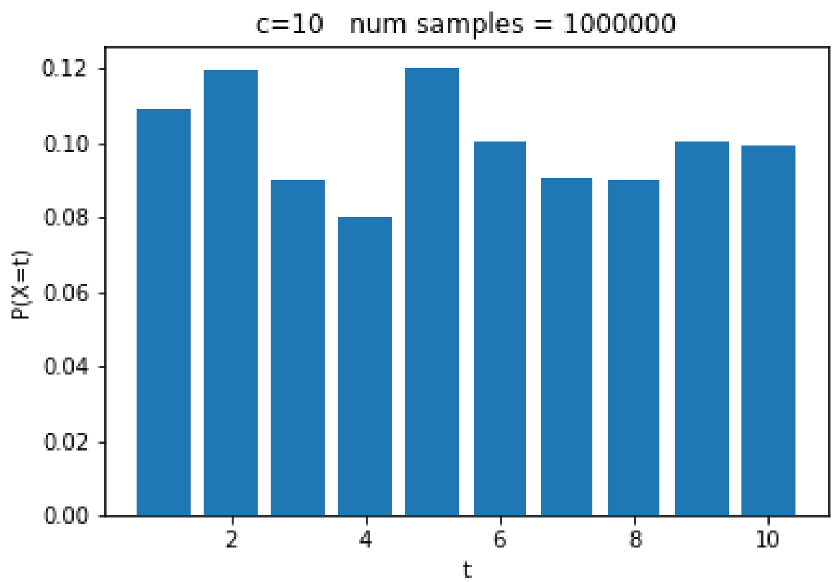
samples	1000	10000	100000	500000	1000000
avg iterations	10.184	10.058	9.996	10.009	9.9998

Table 5: $c = 1.5$

samples	1000	10000	100000	500000	1000000
max $\mid P(x = t) - f(t) \mid$	0.0171	0.0070	0.0008	0.0006	0.0005

Here are the graphs showing the probabilities of our randomly generated variable.


c=1.5   num samples = 1000000
0.12
0.10
0.08
0.06
0.04
0.02
0.00
P(X=t)
2
4
6
8
10
t


c=10   num samples = 1000000
0.12
0.10
0.08
0.06
0.04
0.02
0.00
P(X=t)
2
4
6
8
10
t