MA323 Lab 2

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Question 1

The recursion formula

$$U_{i+1} = (U_{i-17} - U_{i-5}), \qquad U_i := U_{i+1} \quad \text{if} \quad U_i < 0$$

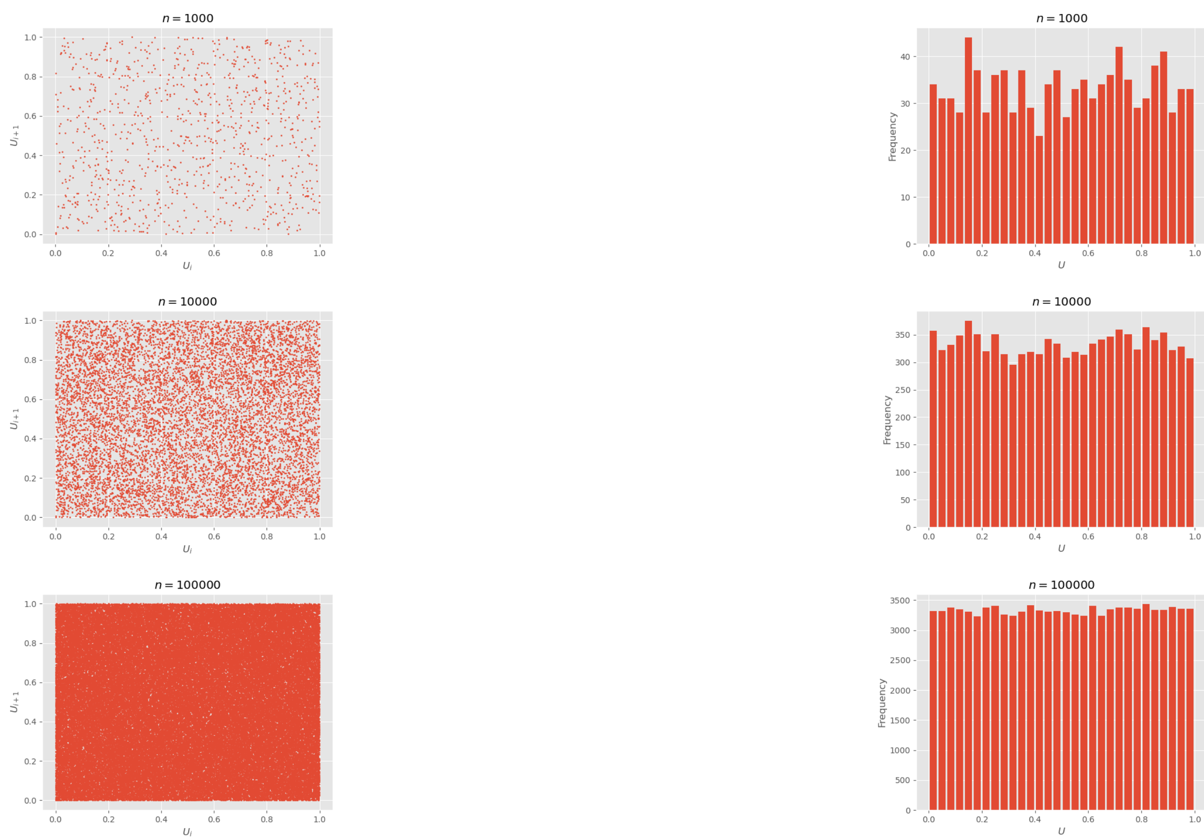
is a similar to the lagged Fibonacci generator given by

$$N_{i+1} = (N_{i-\mu} - N_{i-\eta}) \bmod M$$

Here the assignment ($U_i := U_{i+1} \quad \text{if} \quad U_i < 0$) is equivalent to the mod operation in the lagged Fibonacci generator.

From the theory course, lagged Fibonacci generators are used for generating uniformly distributed psuedo-random numbers.
So we should expect a uniform distribution, and a random looking plot of (U_i, U_{i+1}), which is what we are getting in our result.

Figure 1: N indicates number of terms taken



Question 2

Since using the inverse transform for the function

$$F(x) = 1 - e^{-x/\theta}. \quad x \geq 0 \tag{1}$$

gives $X = -\theta \log(1 - U)$ where U is from a uniform distribution. We should expect that the distribution of X converges to F.

From the results, we can see that the CDF, Mean and Variance converge to that distribution (1)

Table 1: When $\theta = 5.0$

		number of samples			
	Expected	100	1000	10000	100000
mean	5.0000	5.5723	4.9230	4.9457	4.9850
variance	25.0000	34.3488	24.3864	24.3278	24.7453

Figure 2: Distribution when $\theta = 5.0$

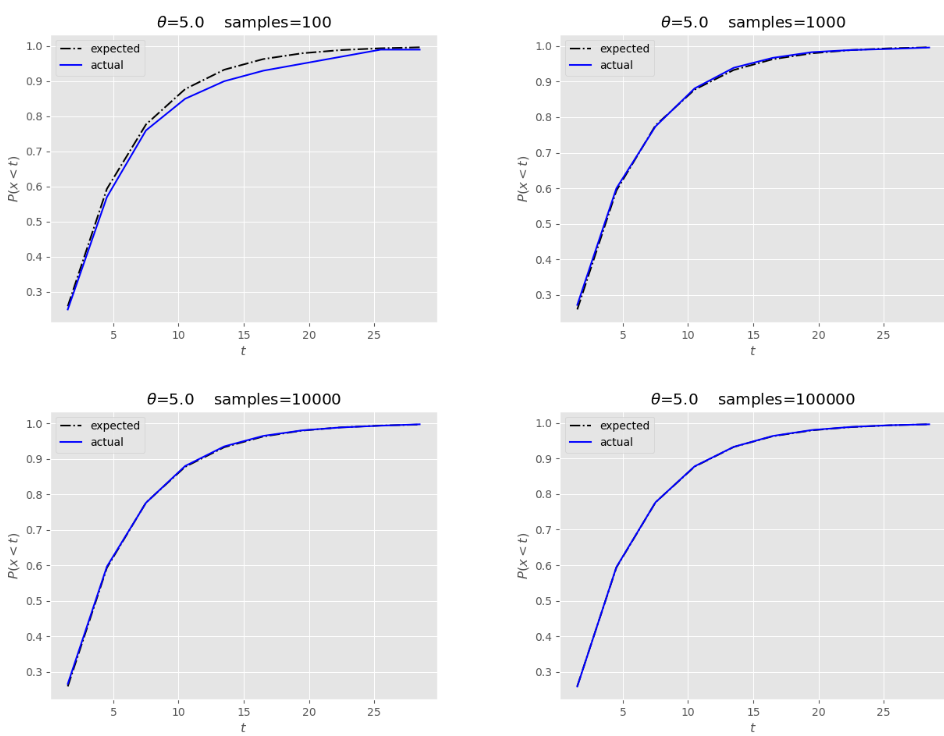
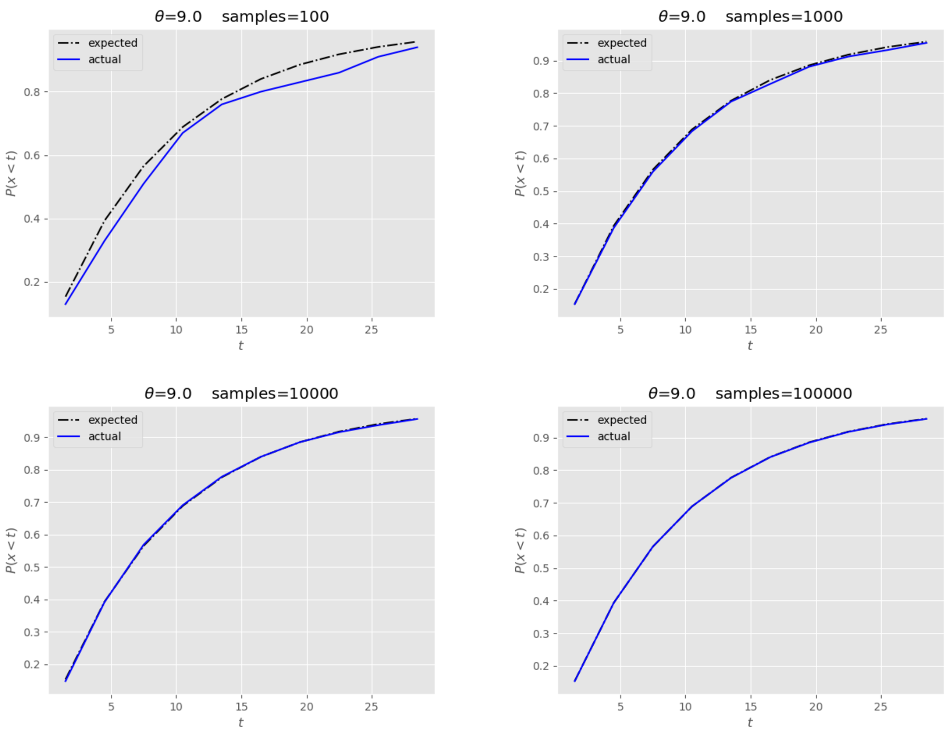


Table 2: When $\theta = 9.0$

		number of samples			
	Expected	100	1000	10000	100000
mean	9.0000	10.2750	9.1454	9.0215	9.0270
variance	81.0000	102.1820	78.8661	82.4015	82.0918

Figure 3: Distribution when $\theta = 9.0$



Question 3

Again, using the inverse transform method for function

$$F(x) = \frac{2}{\pi} \arcsin \sqrt{x}, \quad 0 \leq x \leq 1 \tag{2}$$

we get $X = \frac{1}{2} - \frac{1}{2}\cos(U\pi)$ where $U \sim \mathcal{U}[0, 1]$. So we should expect that the distribution, mean and variance of our randomly generated sequence to converge to that from the distribution (2).

We can see that as number of samples increases, the CDF, mean and variance get closer and closer to that of distribution (2)

Table 3: Mean and variance for various sample sizes

	number of samples					
	100	500	1000	5000	10000	100000
mean	0.46483	0.49270	0.49352	0.50414	0.49993	0.49914
variance	0.12327	0.12587	0.12115	0.12532	0.12545	0.12515