MA323 Lab 6

Ashish Kumar Barnawal - 180123006

October 16, 2020

Question 1

The values were generated using the following formula:

$$X = \mu + AZ$$

where $Z \sim \mathcal{N}(0, I_d)$ Here, A is calculated using the Cholesky factorization:

$$A = \begin{pmatrix} \sigma_1 & 0 \\ \sigma_2\rho & \sigma_2\sqrt{1-\rho^2} \end{pmatrix}$$

here $\sigma_1 = \sqrt{\Sigma_{11}}$ and $\sigma_2 = \sqrt{\Sigma_{22}}$ and $\rho = \frac{\Sigma_{12}}{\sigma_1\sigma_2}$

Question 2

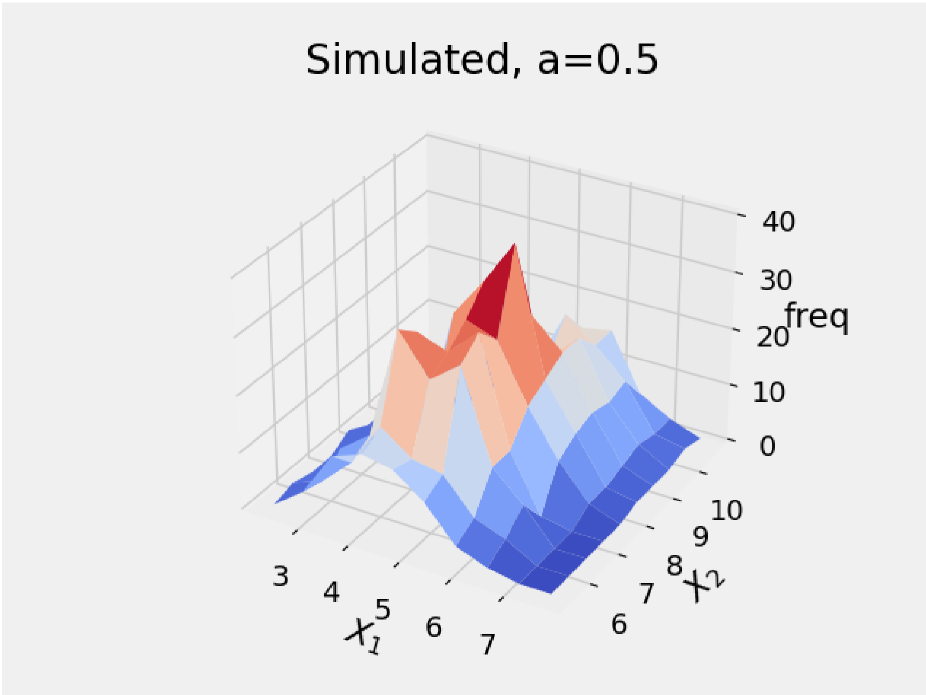
The following plots were generated in the simulation for different values of a From the plots we can see that actual marginal densities don't change with a as a only affects the covariance.

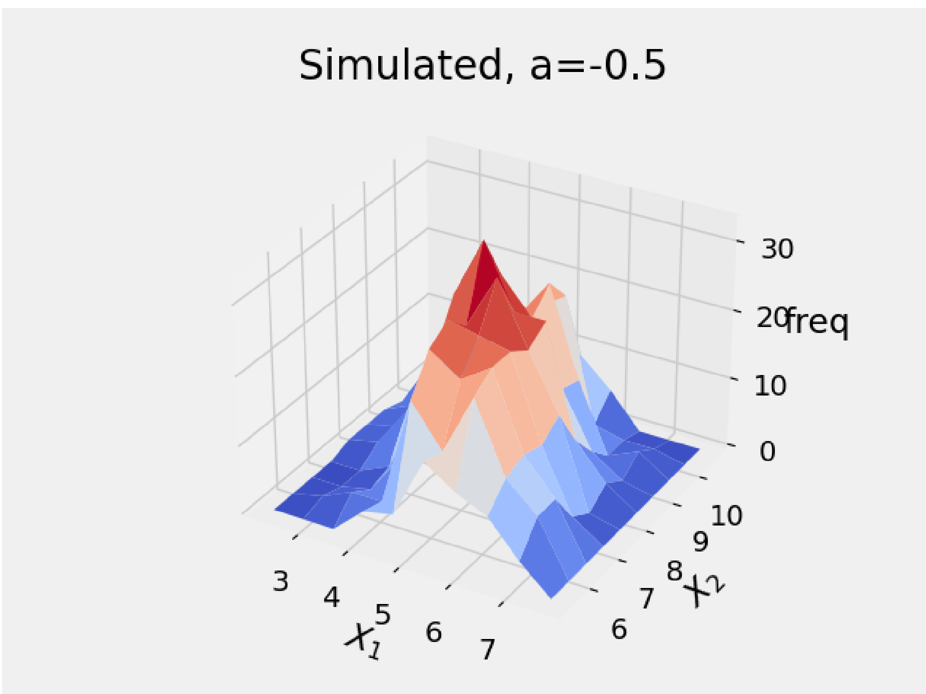
Question 3

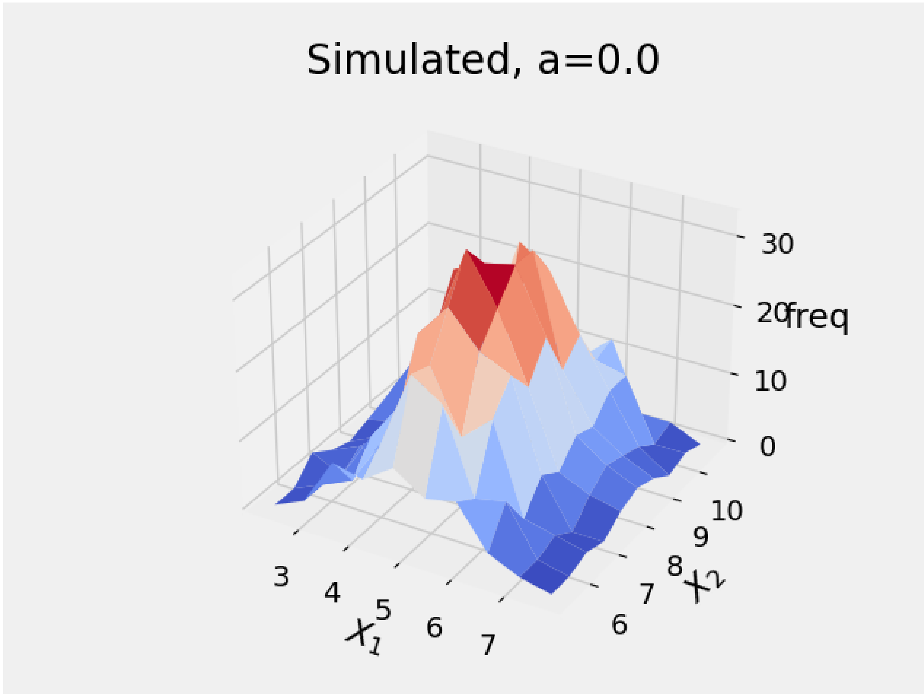
The joint density for the case $a = 1$ doesn't exist. In that case

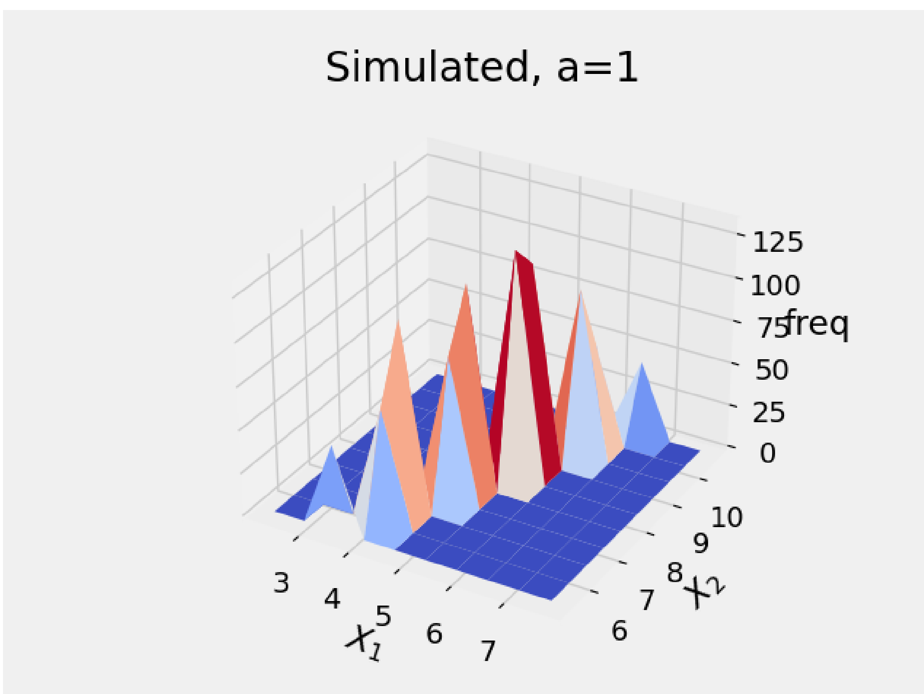
$$A = \begin{pmatrix} 1 & 0 \\ 2 & 0 \end{pmatrix}$$

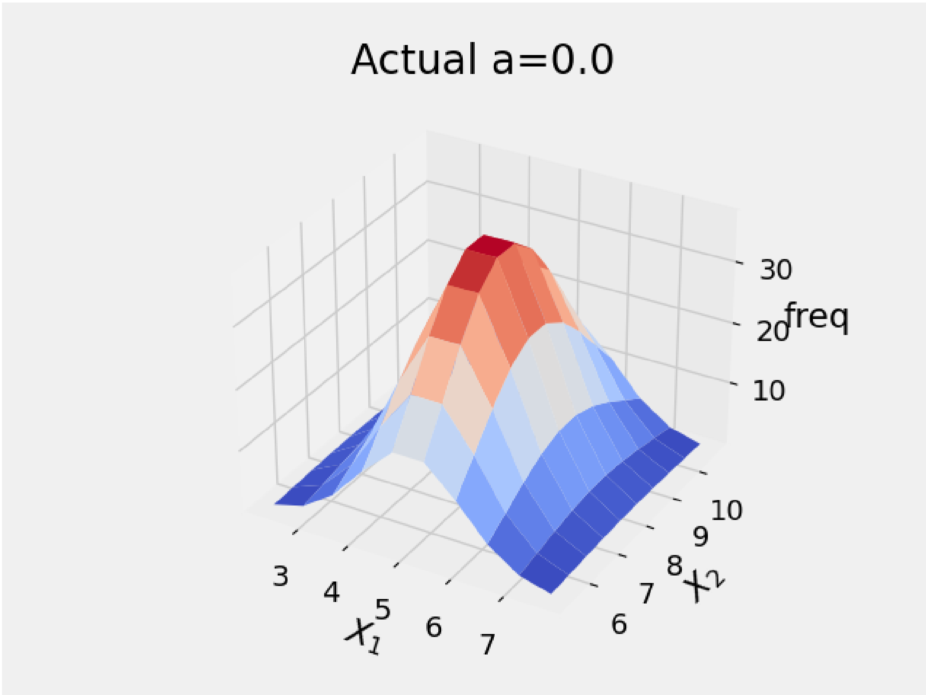
Thus. we'll have the relation $X_2 = 2X_1 + \mu_2 - \mu_1$. Thus joint density won't exist. We can also see from the simulated joint density that there are sharp peaks at this straight line, while everywhere else the frequency is zero.

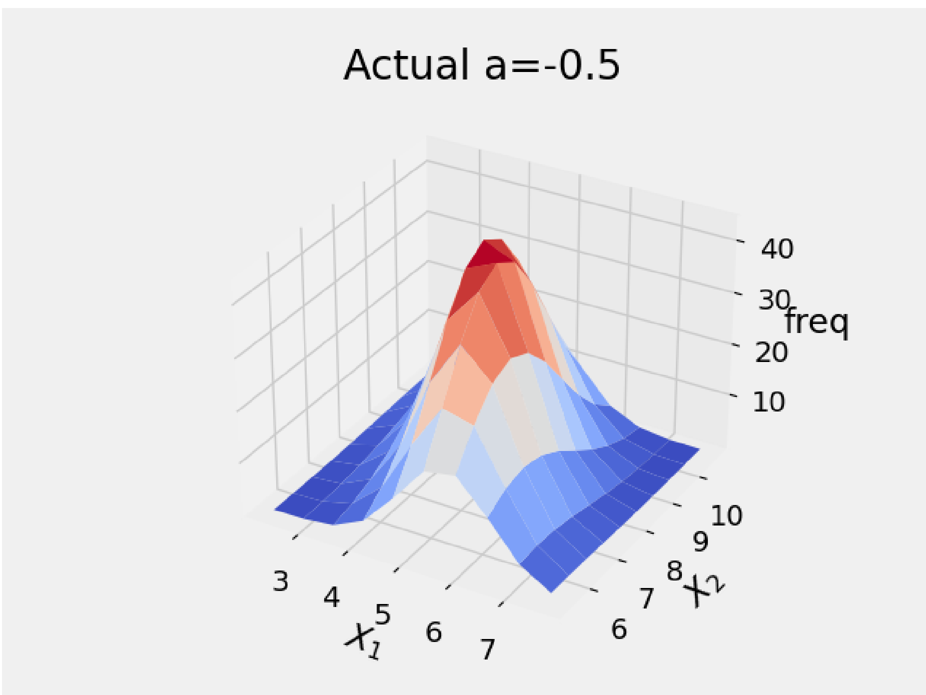

Simulated, a=0.5
40
30
20
10
0
freq
3
4
5
6
7
x_1
6
7
8
9
10
x_2

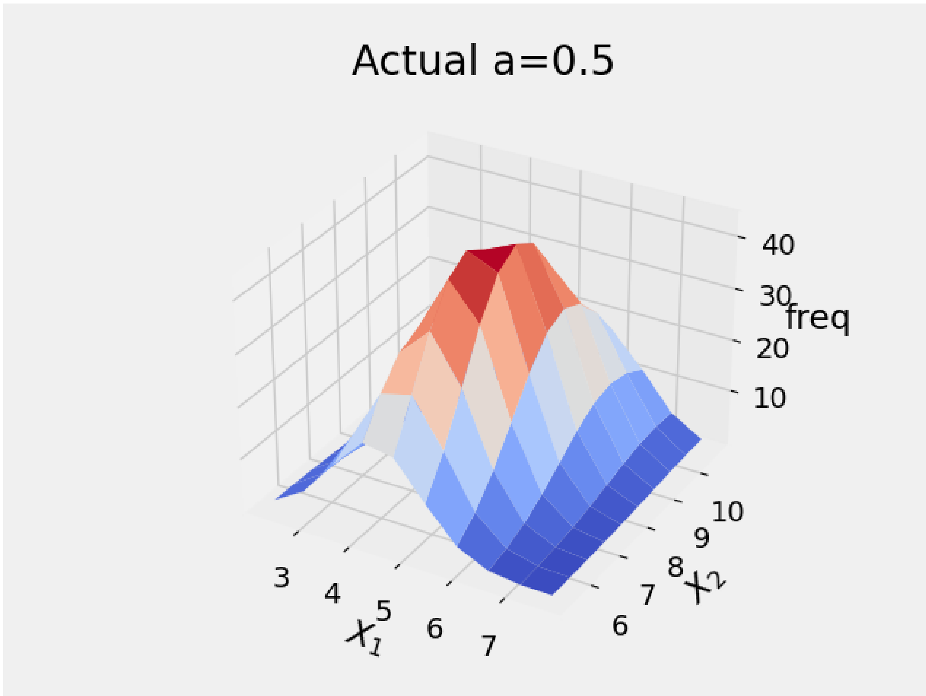

Simulated, a=-0.5
30
20
10
0
freq
3
4
5
6
7
x_1
6
7
8
9
10
x_2


Simulated, a=0.0
30
20
10
0
freq
3
4
5
6
7
x_1
6
7
8
9
10
x_2

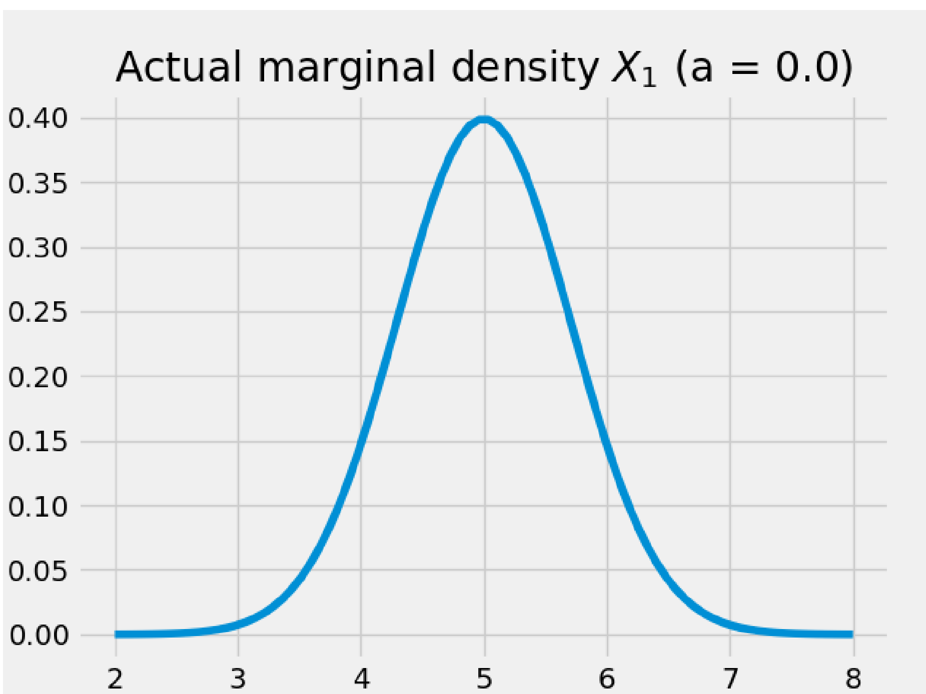

Simulated, a=1
125
100
75
50
25
0
freq
3
4
5
6
7
x_1
6
7
8
9
10
x_2


Actual a=0.0
freq
30
20
10
3
4
5
6
7
X_1
6
7
8
9
10
X_2

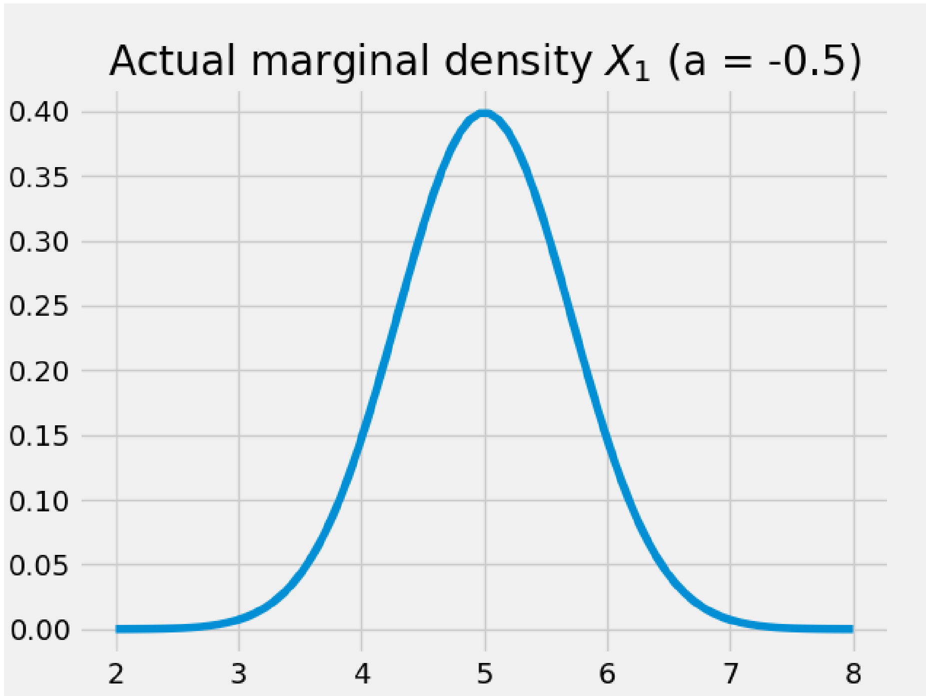

Actual a=-0.5
freq
40
30
20
10
3
4
5
6
7
X_1
6
7
8
9
10
X_2

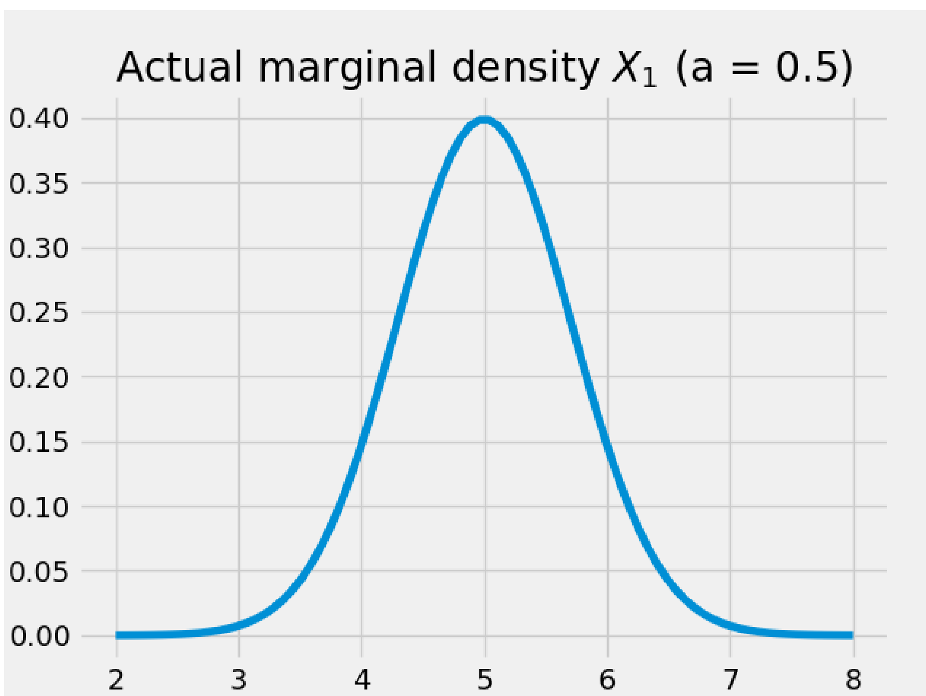


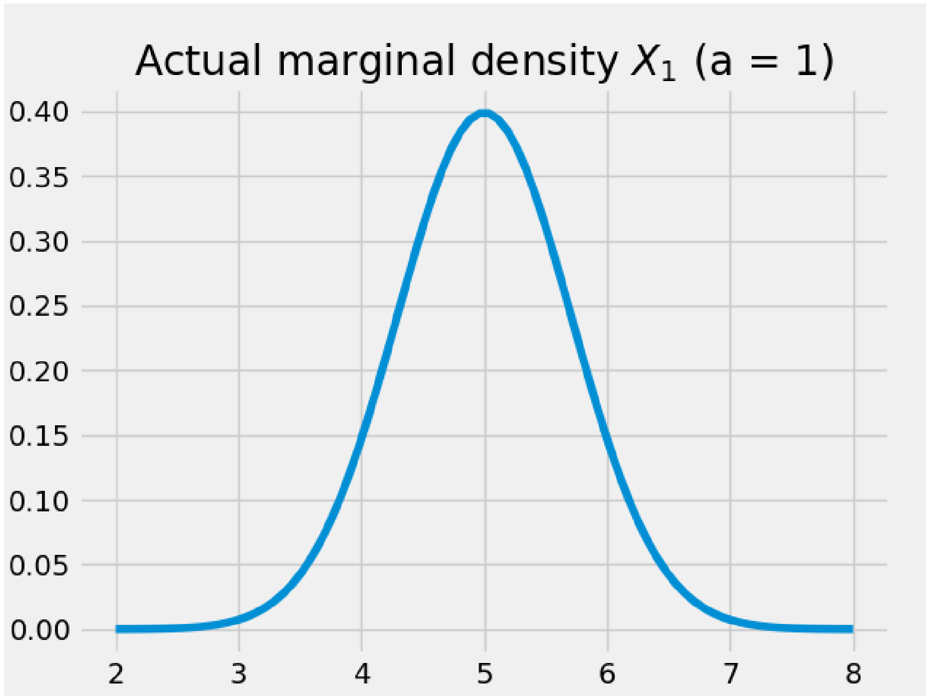
Actual a=0.5
40
30
freq
20
10
10
9
8
7
6
X_2
3
4
5
6
7
X_1

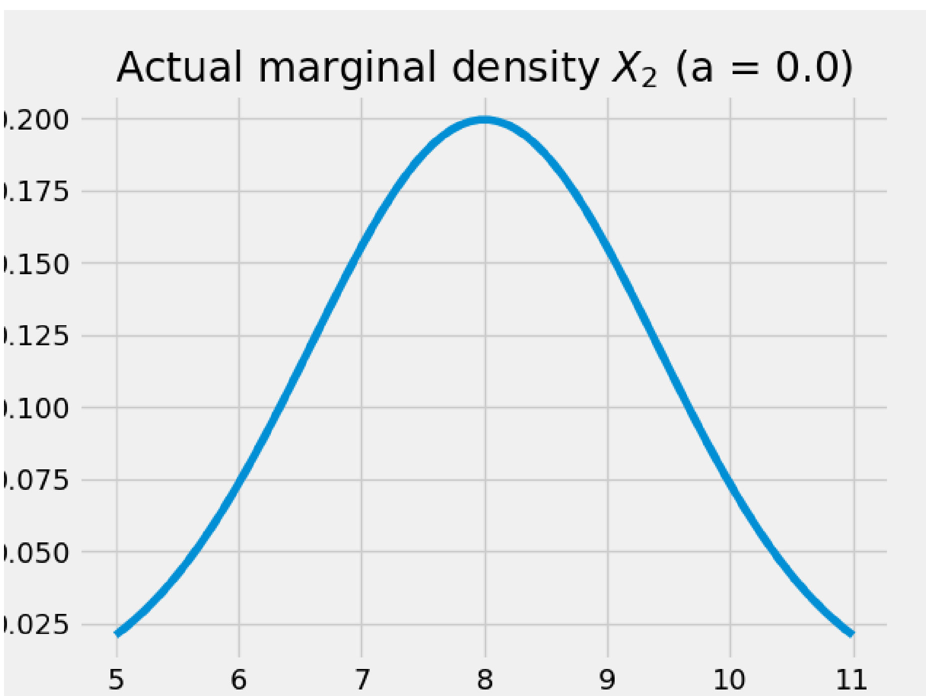


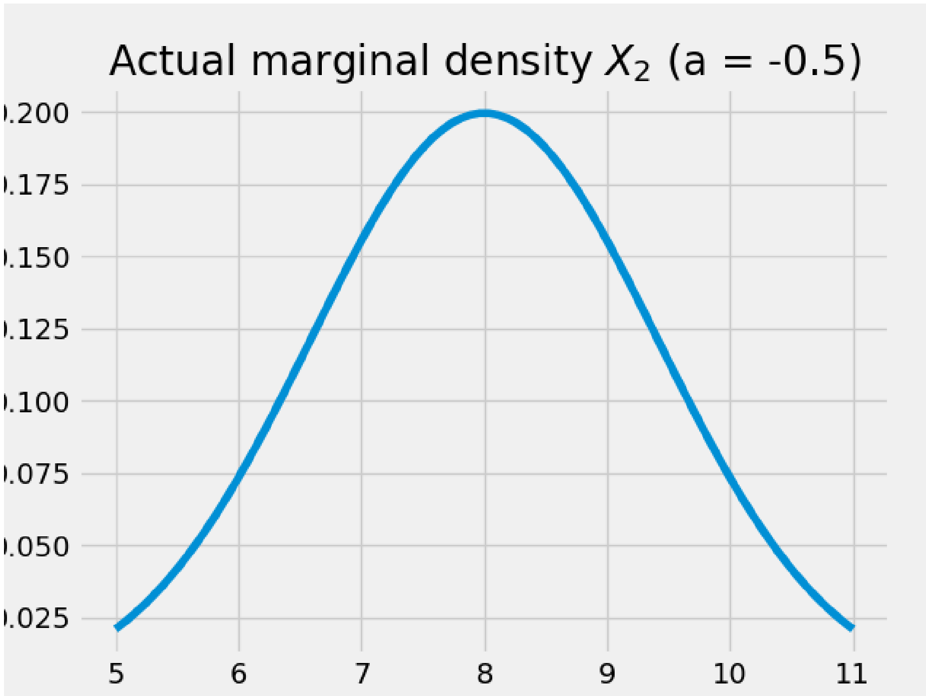
Actual marginal density X_1 (a = 0.0)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2
3
4
5
6
7
8


Actual marginal density X_1 (a = -0.5)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2
3
4
5
6
7
8

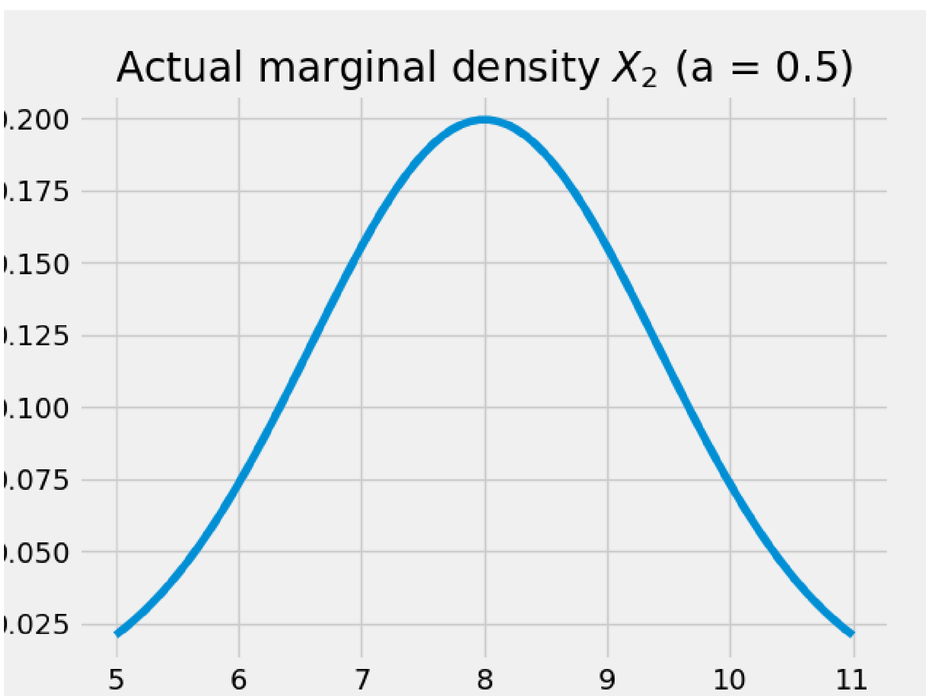

Actual marginal density X_1 (a = 0.5)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2
3
4
5
6
7
8


Actual marginal density X_1 (a = 1)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2
3
4
5
6
7
8

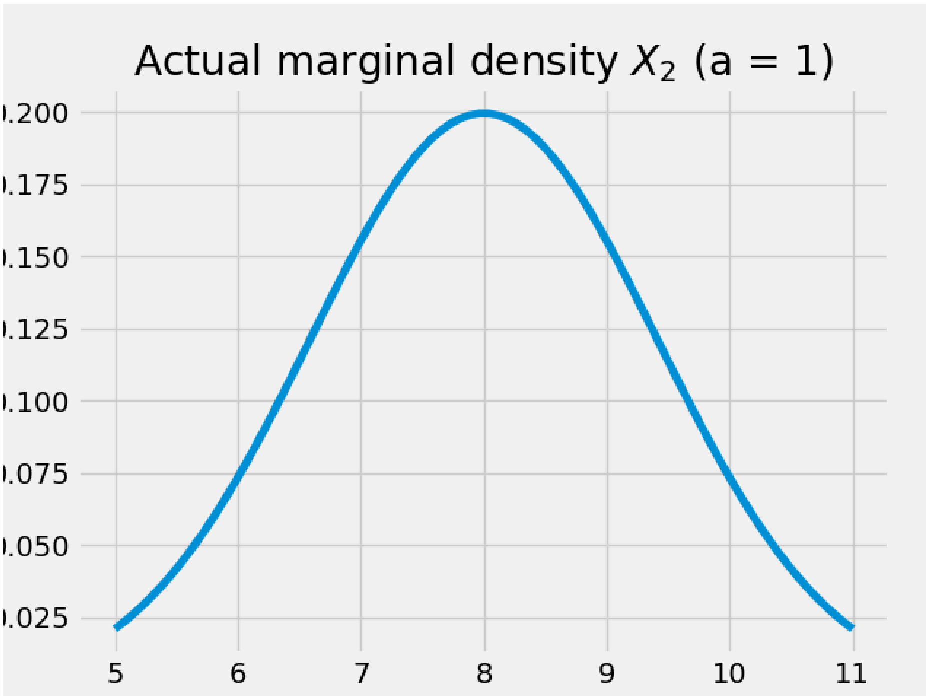

Actual marginal density X_2 (a = 0.0)
.200
.175
.150
.125
.100
.075
.050
.025
5
6
7
8
9
10
11

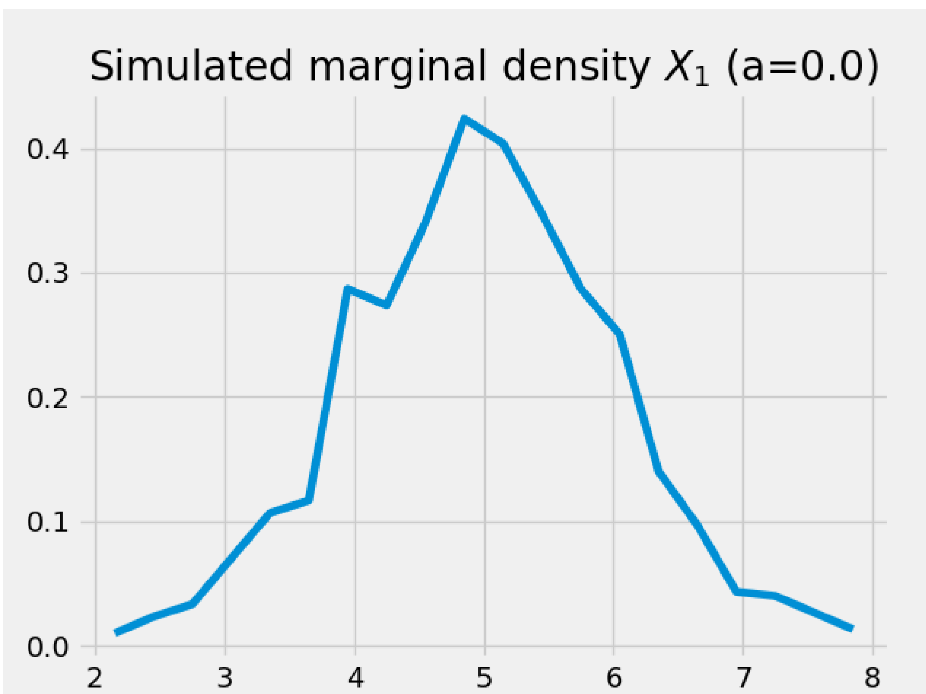


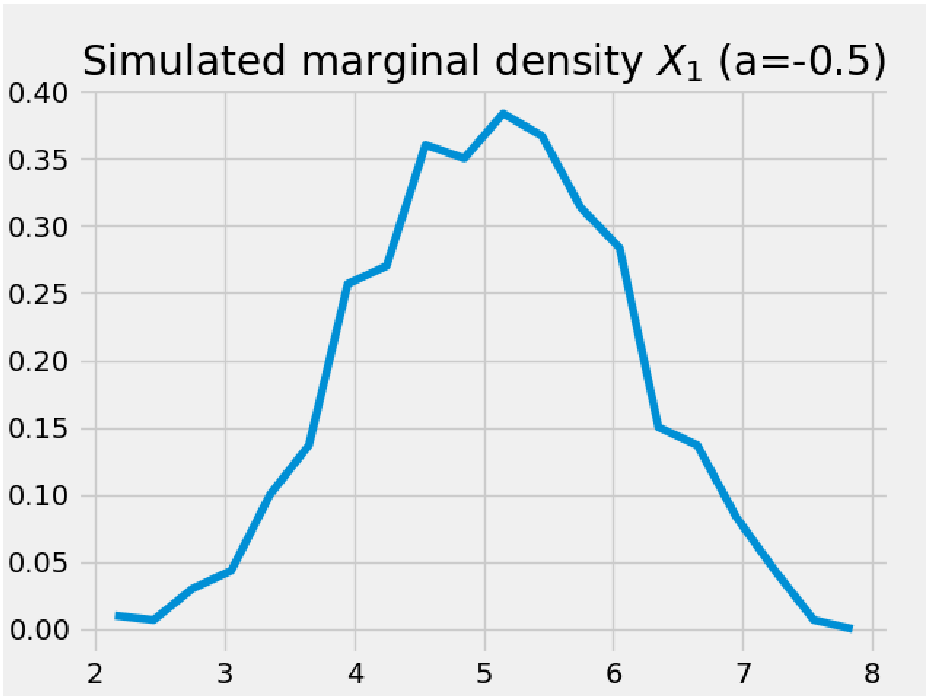
Actual marginal density X_2 (a = -0.5)
.200
.175
.150
.125
.100
.075
.050
.025
5
6
7
8
9
10
11

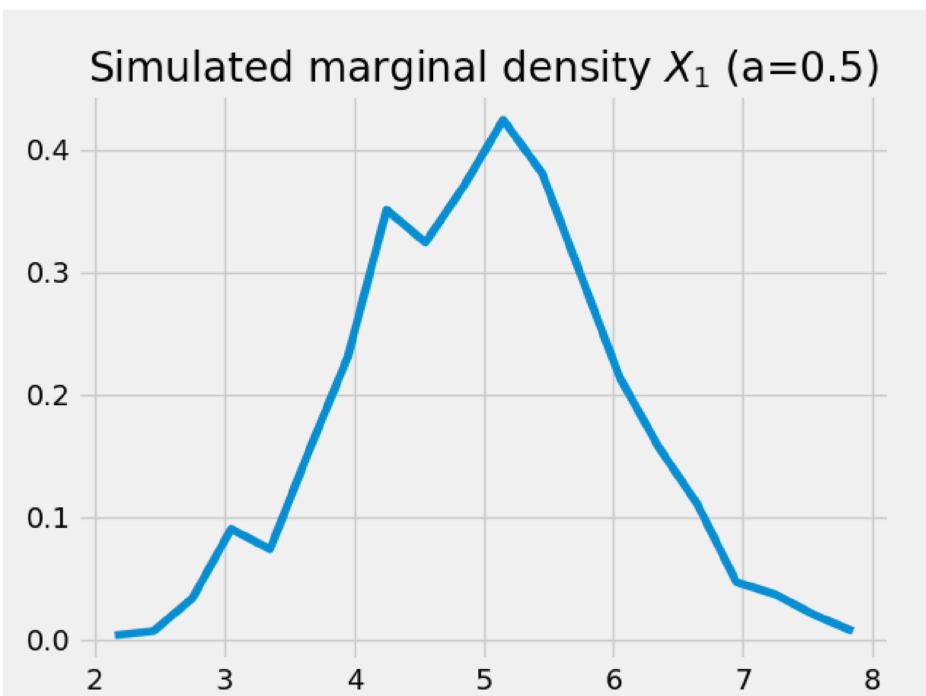


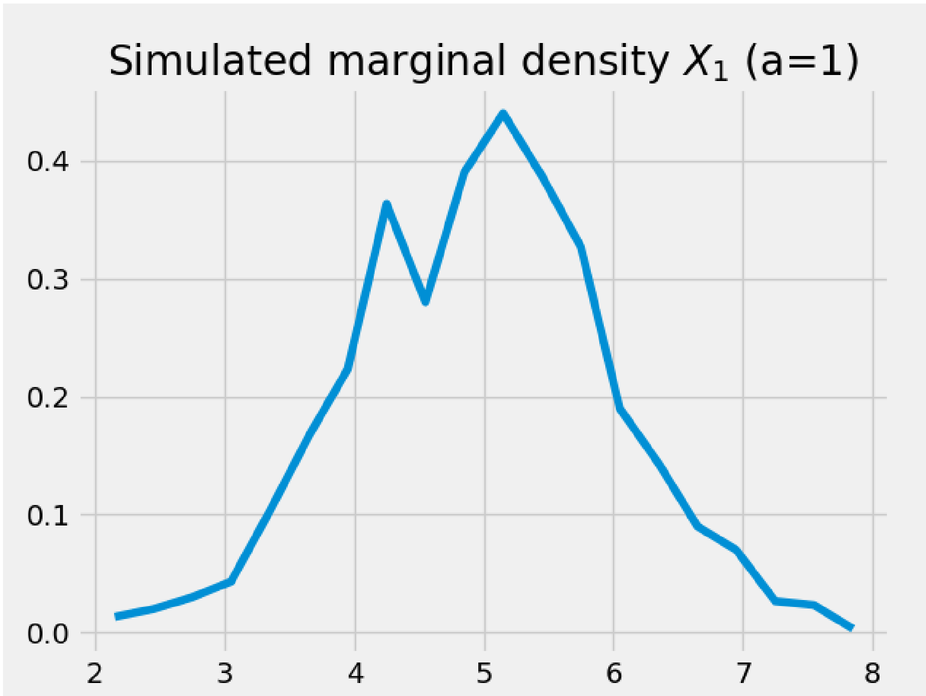
Actual marginal density X_2 (a = 0.5)
.200
.175
.150
.125
.100
.075
.050
.025
5
6
7
8
9
10
11


Actual marginal density X_2 (a = 1)
.200
.175
.150
.125
.100
.075
.050
.025
5
6
7
8
9
10
11

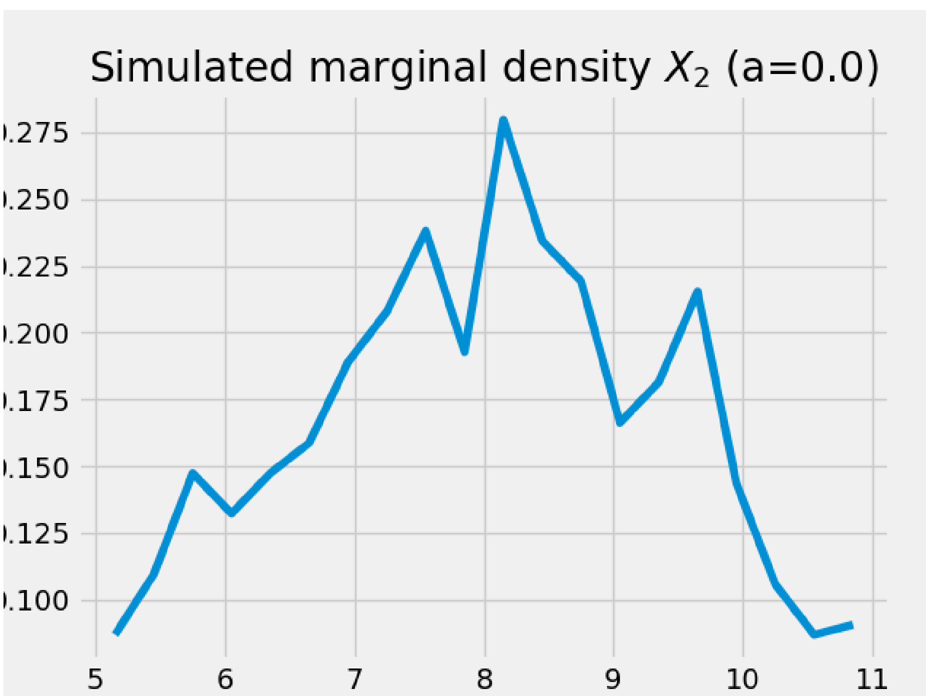

Simulated marginal density X_1 (a=0.0)
0.4
0.3
0.2
0.1
0.0
2
3
4
5
6
7
8


Simulated marginal density X_1 (a=-0.5)
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
2
3
4
5
6
7
8

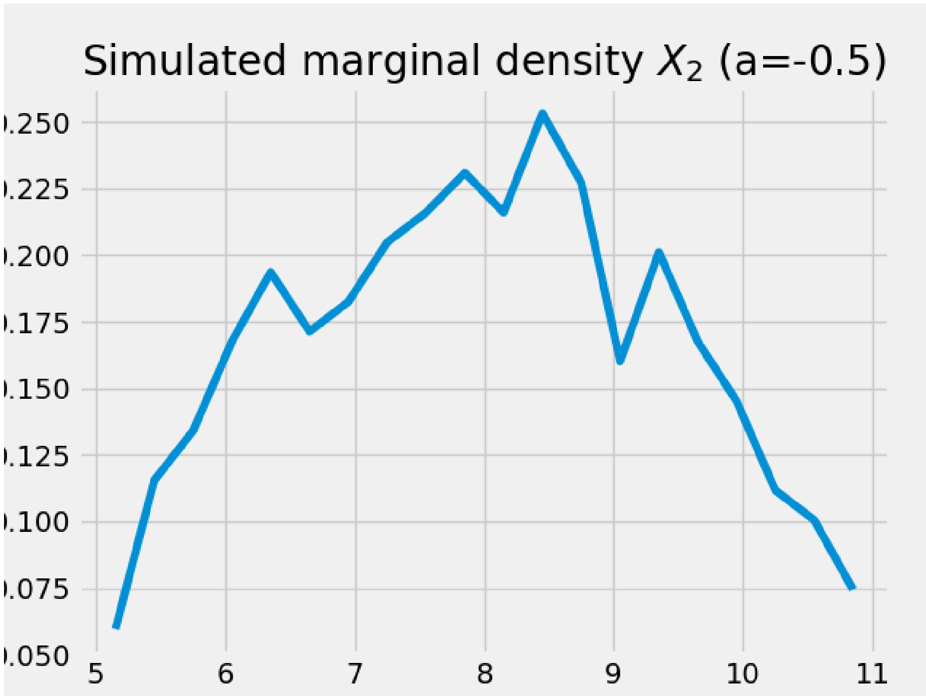

Simulated marginal density X_1 (a=0.5)
0.4
0.3
0.2
0.1
0.0
2
3
4
5
6
7
8

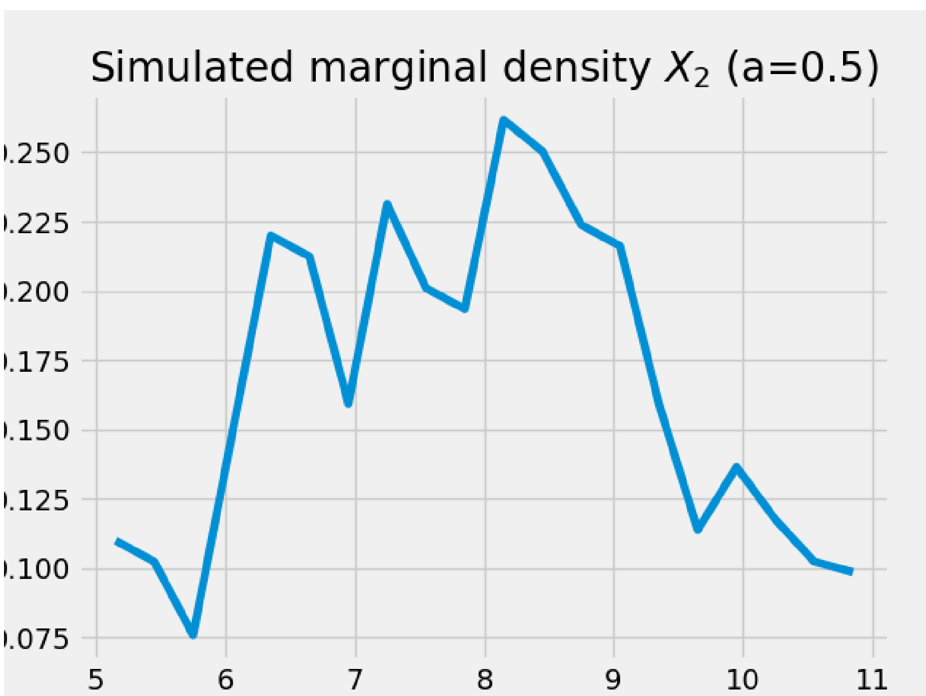


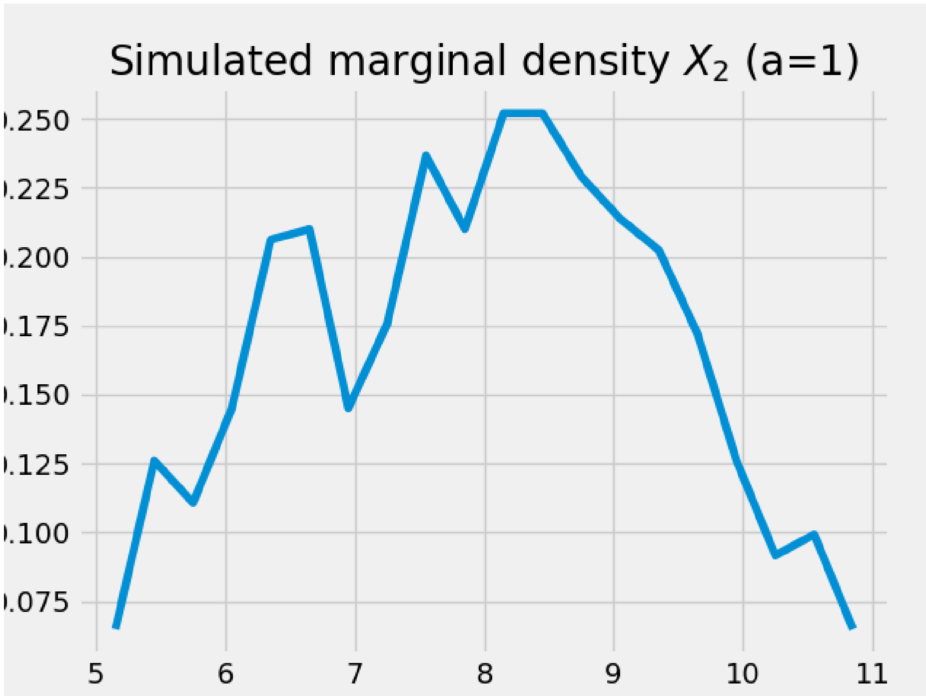
Simulated marginal density X_1 (a=1)
0.0
0.1
0.2
0.3
0.4
2
3
4
5
6
7
8



Simulated marginal density X_2 (a=0.0)
.100
.125
.150
.175
.200
.225
.250
.275
5
6
7
8
9
10
11


Simulated marginal density X_2 (a=-0.5)
.250
.225
.200
.175
.150
.125
.100
.075
.050
5
6
7
8
9
10
11


Simulated marginal density X_2 (a=0.5)
.250
.225
.200
.175
.150
.125
.100
.075
5
6
7
8
9
10
11


Simulated marginal density X_2 (a=1)
.250
.225
.200
.175
.150
.125
.100
.075
5
6
7
8
9
10
11