High order method for Black-Scholes PDE

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Overview

Central Finite Difference

$$f(x+\Delta x) = f(x) + \Delta x f'(x) + \Delta x^2 \frac{f''(x)}{2!} + \Delta x^3 \frac{f'''(x)}{3!} + \Delta x^4 \frac{f^{(4)}(x)}{4!} + \Delta x^5 \frac{f^{(5)}(\xi_1)}{5!}$$

$$f(x-\Delta x) = f(x) - \Delta x f'(x) + \Delta x^2 \frac{f''(x)}{2!} - \Delta x^3 \frac{f'''(x)}{3!} + \Delta x^4 \frac{f^{(4)}(x)}{4!} - \Delta x^5 \frac{f^{(5)}(\xi_2)}{5!}$$

$$f(x+2\Delta x) = f(x) + 2\Delta x f'(x) + 4\Delta x^2 \frac{f''(x)}{2!} + 8\Delta x^3 \frac{f'''(x)}{3!} + 16\Delta x^4 \frac{f^{(4)}(x)}{4!} + 32\Delta x^5 \frac{f^{(5)}(\xi_3)}{5!}$$

$$f(x-2\Delta x) = f(x) - 2\Delta x f'(x) + 4\Delta x^2 \frac{f''(x)}{2!} - 8\Delta x^3 \frac{f'''(x)}{3!} + 16\Delta x^4 \frac{f^{(4)}(x)}{4!} - 32\Delta x^5 \frac{f^{(5)}(\xi_4)}{5!}$$

- Eliminate $f''(x)$, $f'''(x)$ and $f^{(4)}(x)$ terms to get the $O(\Delta x^4)$ approximation to $f'(x)$

$$f'(x) = \frac{-f(x+2\Delta x) + 8f(x+\Delta x) - 8f(x-\Delta x) + f(x-2\Delta x)}{12\Delta x} + O(\Delta x^4)$$

- Similar for $f''(x)$

$$f''(x) = \frac{-f(x+2\Delta x) + 16f(x+\Delta x) - 30f(x) + 16f(x-\Delta x) - f(x-2\Delta x)}{12\Delta x^2} + O(\Delta x^4)$$

Central Finite Difference

Considering set of $N-1$ points

For $2 \leq i \leq N-2$

$$\frac{\partial V_i}{\partial S} = \frac{-V_{i+2} + 8V_{i+1} - 8V_{i-1} + V_{i-2}}{12h}$$

$$\frac{\partial^2 V_i}{\partial S^2} = \frac{-V_{i+2} + 16V_{i+1} - 30V_i + 15V_{i-1} - V_{i-2}}{12h^2}$$

For $i = 1, N-1$,

$$\frac{\partial V_1}{\partial S} = \frac{-3V_0 - 10V_1 + 18V_2 - 6V_3 + V_4}{12h}$$

$$\frac{\partial^2 V_1}{\partial S^2} = \frac{10V_0 - 15V_1 - 4V_2 + 14V_3 - 6V_4 + V_5}{12h^2}$$

$$\frac{\partial V_{N-1}}{\partial S} = \frac{-3V_N - 10V_{N-1} + 18V_{N-2} - 6V_{N-3} + V_{N-4}}{12h}$$

$$\frac{\partial^2 V_{N-1}}{\partial S^2} = \frac{10V_N - 15V_{N-1} - 4V_{N-2} + 14V_{N-3} - 6V_{N-4} + V_{N-5}}{12h^2}$$

The End