MA691 : COBRA-18 Project Report

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Abstract

In this project we tried to predict the survival function of patient diagnosed with Primary biliarycholangitis (PBC) using COBRA with Survival Trees as weak learners.

Theory

We denote T to be the time at which an event occurs (in our case a patient dies). T is also called the response variable.
The survival function at time t is defined to be the probability of survival at that time. In other words,

$$S(t) = Pr\{t > T\} = 1–F(t)$$

where $F(t)$ is the Cumulative Distribution Function of variable T.

From lifetime data, we can estimate the survival function using the Kaplan-Meier estimator:

$$\widehat{S}(t) = \prod_{i:\ t_i \leq t} \left(1 - \frac{d_i}{n_i}\right)$$

with t_i a time when at least one event happened, d_i the number of events that happened at time t_i, and n_i the individuals known to have survived (have not yet had an event or been censored) up to time t_i

In this project we use Survival Trees [4] which use the Kaplan Meier estimator at its leaf node for estimating the survival function. Then we use COBRA [1] as Ensembling algorithm over the Survival Trees to get the survival time and the survival function.

Implementation

For the implementation we made use of scikit-survival [5] library. The library contains the class `SurvivalTree` which uses Survival Tree [4] for predicting the survival function after training it with training data.

We create a class `CobraSurvivalTree` which inherits from `SurvivalTree` , the difference is that the predict function returns a single value: the expected survival time rather than returning the survival function. This is useful for the COBRA implementation.

The expected survival time is calculated as follows:

$$
\begin{aligned}
E[T] &= \int_0^\infty t\, dF(t) = -\int_0^\infty t\, dS(t) \\
&\approx -\int_0^{T_{\max}} t\, dS(t) + T_{\max} S(T_{\max}) \\
&\approx \sum_{i=1}^{n-1} t_i\, \Delta S_i \; + \; t_n S_n
\end{aligned}
$$

Here n is the number of elements in `sksurv.SurvivalTree.event_times_` array.

Now, we apply COBRA on the survival time of the dataset with $\epsilon = 3$ and using `CobraSurvivalTree` the weak learner. This model will give predictions for the survival time.

To predict survival function, we will take the average of survival function over the set of selected observations $\{(x_i, y_i)\}_{i \in \mathcal{D}}$ and over the set of selected machines α_m (A datapoint (x_i, y_i) is selected if it in within ϵ range of all selected machines i.e. $i \in \mathcal{D}$ if $|r_j(\mathbf{x}_i) - r_j(\mathbf{x})| < \epsilon \; \forall \; r_j \in \alpha_m$). Here $\mathcal{D}$ denotes the set of indices of selected datapoints and α_m is the set of selected machines in the COBRA algorithm. The survival function is then calculated as follows

$$S(t) = \frac{1}{|\alpha_m| N} \sum_{r_j \in \alpha_m} \sum_{i \in \mathcal{D}} S_{r_j}(t \,|\, \mathbf{x}_i)$$

Here $S_{r_j}(t \,|\, \mathbf{x}_i)$ is the predicted survival probability of $\mathbf{x}_i$ at time t by machine r_j

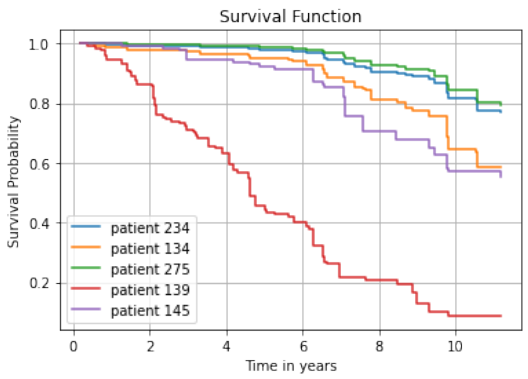
Results

After running our implementation of COBRA with survival tree we get the following result

```
Mean Absolute Error : 2.31 years

Concordance index: 0.68
```

And the following survival function prediction:



References

[1] G´erard Biau, Aur´elie Fischer, Benjamin Guedj, and James Malley. COBRA: A Combined Regression Strategy.Journal of Multivariate Analysis, 2016.

[2] Fleming and Harrington. Primary Biliary Cirrhosis (PBC) Data, Appendix D, 1991.

[3] Benjamin Guedj and Bhargav Srinivasa Desikan. Pycobra: A python toolbox for ensemblelearning and visualisation, 2019.

[4] Hemant Ishwaran, Udaya B. Kogalur, Eugene H. Blackstone, and Michael S. Lauer. Randomsurvival forests.The Annals of Applied Statistics, 2(3):841 – 860, 2008

[5] Sebastian P¨olsterl. scikit-survival: A library for time-to-event analysis built on top of scikit-learn.Journal of Machine Learning Research, 21(212):1–6, 2020