Elektromagnetika — PR V

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1. Menurunkan persamaan gelombang EM dengan kehadiran sumber, untuk medan $\vec{E}$.

Dari persamaan maxwell ke-3

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Curl-kan kedua sis persamaan maxwell diatas,

$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t} \right)$$
$$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{B} \right)$$

Dengan identitas vektor, bahwa $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$ dan karena $\vec{B} = \mu_0 \vec{H}$ maka

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{H} \right)$$

karena $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$ dengan $\vec{D} = \varepsilon_0 \vec{E}$, maka

$$\begin{aligned}
\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} &= -\mu_0 \frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{H} \right) \\
&= -\mu_0 \frac{\partial}{\partial t} \left(\frac{\partial \vec{D}}{\partial t} + \vec{J} \right) \\
&= -\mu_0 \frac{\partial}{\partial t} \left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J} \right) \\
\vec{\nabla}(\vec{\nabla} \cdot \underbrace{\vec{E}}_{\frac{\vec{D}}{\varepsilon_0}}) - \nabla^2 \vec{E} &= -\mu_0 \varepsilon \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \frac{\partial}{\partial t} \vec{J}
\end{aligned}$$

Dalam kasus ini $\vec{\nabla} \cdot \vec{D} = \rho;$ dimana $\rho \neq 0$, sehingga

$$\vec{\nabla}(\vec{\nabla} \cdot \frac{1}{\varepsilon_0}\vec{D}) - \nabla^2\vec{E} = -\mu_0\varepsilon\frac{\partial^2}{\partial t^2}\vec{E} - \mu_0\frac{\partial}{\partial t}\vec{J}$$

$$\frac{1}{\varepsilon_0}\vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{D}}_{\rho}) - \nabla^2\vec{E} = -\mu_0\varepsilon\frac{\partial^2}{\partial t^2}\vec{E} - \mu_0\frac{\partial}{\partial t}\vec{J}$$

karena $c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$, maka

$$\frac{1}{\varepsilon_0}\vec{\nabla}\rho - \nabla^2\vec{E} = -\underbrace{\mu_0\varepsilon}_{\frac{1}{c^2}}\frac{\partial^2}{\partial t^2}\vec{E} - \mu_0\frac{\partial}{\partial t}\vec{J}$$

rearrange persamaan diatas sehingga menjadi

$$\nabla^2\vec{E} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = \mu_0\frac{\partial}{\partial}\vec{J} + \frac{1}{\varepsilon_0}\nabla\rho$$

atau

$$\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\vec{E} = \mu_0\frac{\partial}{\partial}\vec{J} + \frac{1}{\varepsilon_0}\nabla\rho$$

2. Menurunkan persamaan gelombang EM dalam medium pengahantra, untuk medan $\vec{E}$ dan $\vec{H}$.

- Untuk medan $\vec{E}$
  Dari persamaan maxwell ke-3

  $$\vec{\nabla} \times \vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

  Curl-kan kedua sisi persamaan maxwell diatas,

  $$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = \vec{\nabla} \times \left(-\frac{\partial\vec{B}}{\partial t}\right)$$

  $$\vec{\nabla} \times \vec{\nabla} \times \vec{E} = -\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{B}\right)$$

  Dengan identitas vektor, bahwa $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2\vec{A}$ dan karena $\vec{B} = \mu_0\vec{H}$ maka

  $$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2\vec{E} = -\mu_0\frac{\partial}{\partial t}\left(\vec{\nabla} \times \vec{H}\right)$$

karena $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$ dengan $\vec{D} = \varepsilon_0 \vec{E}$, maka

$$\begin{aligned}
\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} &= -\mu_0 \frac{\partial}{\partial t} \left(\vec{\nabla} \times \vec{H}\right) \\
&= -\mu_0 \frac{\partial}{\partial t} \left(\frac{\partial \vec{D}}{\partial t} + \vec{J}\right) \\
&= -\mu_0 \frac{\partial}{\partial t} \left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J}\right) \\
\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} &= -\mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} - \mu \frac{\partial}{\partial t} \vec{J}
\end{aligned}$$

Dari hukum Ohm $\vec{J} = \sigma \vec{E} \neq 0$, untuk $\vec{E} \neq 0$, sehingga persamaan diatas menjadi

$$\begin{aligned}
\vec{\nabla}(\vec{\nabla} \cdot \underbrace{\vec{E}}_{\frac{\vec{D}}{\varepsilon_0}}) - \nabla^2 \vec{E} &= -\mu_0 \varepsilon \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E} \\
\frac{1}{\varepsilon_0} \vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{D}}_{\rho}) - \nabla^2 \vec{E} &= -\mu_0 \varepsilon \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E}
\end{aligned}$$

Dalam medium konduktor, resistivitas $\rho = 0$ dan konduktivitas $\sigma \neq 0$, maka tersisa

$$-\nabla^2 \vec{E} = -\mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E}$$

rearrange persamaan diatas menjadi,

$$\begin{aligned}
\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E} &= 0 \\
\left(\nabla^2 - \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} - \mu_0 \sigma \frac{\partial}{\partial t}\right) \vec{E} &= 0
\end{aligned}$$

- Untuk medan $\vec{H}$
  Dari persamaan Maxwell ke-4

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Curl-kan kedua ruas persamaan

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \vec{\nabla} \times \left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

Dengan identitas vektor, bahwa $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$ dan karena $\vec{B} = \mu_0 \vec{H}$ maka

$$\vec{\nabla}(\vec{\nabla} \cdot \underbrace{\vec{H}}_{\frac{1}{\mu_0}\vec{B}}) - \nabla^2 \vec{H} = \vec{\nabla} \times \vec{J} + \vec{\nabla} \times \frac{\partial \vec{D}}{\partial t}$$

$$\frac{1}{\mu_0}\vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{H} = \vec{\nabla} \times \vec{J} + \vec{\nabla} \times \frac{\partial \vec{D}}{\partial t}$$

Berdasarkan persamaan Maxwell ke-2 $\vec{\nabla} \cdot \vec{B} = 0$, maka tersisa

$$-\nabla^2 \vec{H} = \vec{\nabla} \times \vec{J} + \vec{\nabla} \times \frac{\partial \vec{D}}{\partial t}$$

Karena $\vec{D} = \varepsilon_0 \vec{E}$, maka

$$\begin{aligned}
-\nabla^2 \vec{H} &= \vec{\nabla} \times \vec{J} + \frac{\partial}{\partial t}\vec{\nabla} \times (\varepsilon_0 \vec{E}) \\
&= \vec{\nabla} \times \vec{J} + \varepsilon_0 \frac{\partial}{\partial t} \underbrace{\vec{\nabla} \times \vec{E}}_{\substack{-\frac{\partial \vec{B}}{\partial t} \\ \text{Pers. Ke-3} \\ \text{Maxwell}}} \\
&= \vec{\nabla} \times \vec{J} + \varepsilon_0 \frac{\partial}{\partial t}\left(-\frac{\partial \vec{B}}{\partial t}\right) \\
&= \vec{\nabla} \times \vec{J} - \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t}
\end{aligned}$$

karena $\vec{B} = \mu_0 \vec{H}$, maka

$$-\nabla^2 \vec{H} = \vec{\nabla} \times \vec{J} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t}$$

Dari hukum Ohm $\vec{J} = \sigma \vec{E} \neq 0$, untuk $\vec{E} \neq 0$, sehingga persamaan diatas menjadi

$$\begin{aligned}
-\nabla^2 \vec{H} &= \vec{\nabla} \times (\sigma \vec{E}) - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t} \\
&= \sigma \underbrace{\vec{\nabla} \times \vec{E}}_{\substack{-\frac{\partial \vec{B}}{\partial t} \\ \text{Pers. Ke-3} \\ \text{Maxwell}}} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t} \\
-\nabla^2 \vec{H} &= -\sigma \frac{\partial \vec{B}}{\partial t} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t}
\end{aligned}$$

karena $\vec{B} = \mu_0 \vec{H}$, maka

$$-\nabla^2 \vec{H} = -\sigma\mu_0 \frac{\partial \vec{H}}{\partial t} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{H}}{\partial t}$$

rearrange persamaan diatas menjadi

$$\begin{aligned}\nabla^2 \vec{H} - \sigma\mu_0 \frac{\partial \vec{H}}{\partial t} - \varepsilon_0\mu_0 \frac{\partial^2 \vec{H}}{\partial t} &= 0 \\ \left(\nabla^2 - \sigma\mu_0 \frac{\partial}{\partial t} - \varepsilon_0\mu_0 \frac{\partial^2}{\partial t}\right) \vec{H} &= 0\end{aligned}$$

3. Diketahui konduktivitas perak $\sigma = 3 \times 10^7$ S/m pada frekuensi gelombang mikro. Tentukan *skin depth* pada frekuensi 10^{10}Hz.

skin depth didefinisikan sebagai jarak untuk mengurangi amplitudo Gelombang EM dengan faktor $1/e$, yakni:

$$\delta \equiv \frac{1}{\kappa}; \text{ dimana } \kappa \equiv \omega\sqrt{\frac{\varepsilon\mu}{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\varepsilon\omega}\right)^2} - 1\right]^{1/2}$$

Untuk konduktivitas tinggi $(\sigma \gg \omega\varepsilon) \Rightarrow \sigma/\varepsilon \gg 1$, sehingga

$$\begin{aligned}\delta &= \frac{1}{\omega\sqrt{\frac{\mu\varepsilon}{2}}\left[\frac{\sigma}{\varepsilon\omega}\right]^{1/2}} \\ &= \frac{1}{\sqrt{\frac{\omega^{\cancel{2}}\mu\cancel{\varepsilon}\sigma}{2\cancel{\varepsilon}\cancel{\omega}}}} \\ &= \frac{1}{\sqrt{\frac{\omega\mu\sigma}{2}}} \\ &= \sqrt{\frac{2}{\omega\mu\sigma}} \\ &= \sqrt{\frac{2}{2\pi f\mu\sigma}}\end{aligned}$$

diketahui nilai $f = 10^{10}$Hz, $\sigma = 3 \times 10^7$S/m dan permeabilitas material $\mu = \mu_0(1 + \chi_m)$, dimana suseptabilitas material perak $\chi_m = -2,4 \times 10^{-5}$, atau

$$\begin{aligned}\mu &= \mu_0(1 + \chi_m) \\ &= 4\pi \times 10^{-7}(1 + -2.4 \times 10^{-5}) \\ &= 1,26 \times 10^{-6}\text{N/A}^2\end{aligned}$$

subtitusi pada persamaan *skin depth* sebelumnya, didapat

$$\begin{aligned}
\delta &= \sqrt{\frac{2}{2\pi(10^{10})(1,26\times10^{-6})(3\times10^{7})}} \\
&= \sqrt{\frac{2}{2.37\times10^{12}}} \\
&= 9,18\times10^{-7}\text{m} \\
&= 0,918\ \mu\text{m}
\end{aligned}$$

4. Air laut memiliki konduktivitas $\sigma = 3\times10^7$ S/m dan $\mu = \mu_0$. Tentukan nilai frekuensi ketika *skin depth*-nya bernilai satu meter.

5. Intensitas medan listrik yang berbentuk gelombang bidang dalam vakum dinyatakan dengan persamaan sebagai berikut:

$$\vec{E} = 100\cos(\omega t + 8z)\hat{i}\ \text{V/m}$$

maka tentukan:

a) Kecepatan jalar gelombang
b) Frekuensi gelombang EM
c) Panjang gelombang
d) Intensitas medan magnet

$$\iiint_{-\infty}^{\infty} \nabla\times E = \frac{1}{4\pi\varepsilon_0}$$