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## Elektromagnetika — PR V

Nama : Firman Qashdus Sabil  
NIM : 210321606892

Offering: AC

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1. Menurunkan persamaan gelombang EM dengan kehadiran sumber, untuk medan  $\vec{E}$ .

Dari persamaan maxwell ke-3

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Curl-kan kedua sis persamaan maxwell diatas,

$$\begin{aligned}\vec{\nabla} \times \vec{\nabla} \times \vec{E} &= \vec{\nabla} \times \left( -\frac{\partial \vec{B}}{\partial t} \right) \\ \vec{\nabla} \times \vec{\nabla} \times \vec{E} &= -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})\end{aligned}$$

Dengan identitas vektor, bahwa  $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$  dan karena  $\vec{B} = \mu_0 \vec{H}$  maka

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

karena  $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$  dengan  $\vec{D} = \epsilon_0 \vec{E}$ , maka

$$\begin{aligned}\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} &= -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H}) \\ &= -\mu_0 \frac{\partial}{\partial t} \left( \frac{\partial \vec{D}}{\partial t} + \vec{J} \right) \\ &= -\mu_0 \frac{\partial}{\partial t} \left( \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J} \right) \\ \vec{\nabla}(\vec{\nabla} \cdot \underbrace{\vec{E}}_{\frac{\vec{D}}{\epsilon_0}}) - \nabla^2 \vec{E} &= -\mu_0 \epsilon \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \frac{\partial}{\partial t} \vec{J}\end{aligned}$$

Dalam kasus ini  $\vec{\nabla} \cdot \vec{D} = \rho$ ; dimana  $\rho \neq 0$ , sehingga

$$\begin{aligned}\vec{\nabla}(\vec{\nabla} \cdot \frac{1}{\epsilon_0} \vec{D}) - \nabla^2 \vec{E} &= -\mu_0 \epsilon \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \frac{\partial}{\partial t} \vec{J} \\ \frac{1}{\epsilon_0} \vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{D}}_{\rho}) - \nabla^2 \vec{E} &= -\mu_0 \epsilon \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \frac{\partial}{\partial t} \vec{J}\end{aligned}$$

karena  $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ , maka

$$\frac{1}{\epsilon_0} \vec{\nabla} \rho - \nabla^2 \vec{E} = - \underbrace{\mu_0 \epsilon}_{\frac{1}{c^2}} \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \frac{\partial}{\partial t} \vec{J}$$

rearrange persamaan diatas sehingga menjadi

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = \mu_0 \frac{\partial}{\partial t} \vec{J} + \frac{1}{\epsilon_0} \nabla \rho$$

atau

$$\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) \vec{E} = \mu_0 \frac{\partial}{\partial t} \vec{J} + \frac{1}{\epsilon_0} \nabla \rho$$

2. Menurunkan persamaan gelombang EM dalam medium pengahantra, untuk medan  $\vec{E}$  dan  $\vec{H}$ .

- Untuk medan  $\vec{E}$

Dari persamaan maxwell ke-3

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Curl-kan kedua sisi persamaan maxwell diatas,

$$\begin{aligned}\vec{\nabla} \times \vec{\nabla} \times \vec{E} &= \vec{\nabla} \times \left( -\frac{\partial \vec{B}}{\partial t} \right) \\ \vec{\nabla} \times \vec{\nabla} \times \vec{E} &= -\frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})\end{aligned}$$

Dengan identitas vektor, bahwa  $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$  dan karena  $\vec{B} = \mu_0 \vec{H}$  maka

$$\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H})$$

karena  $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$  dengan  $\vec{D} = \epsilon_0 \vec{E}$ , maka

$$\begin{aligned}\vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} &= -\mu_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{H}) \\ &= -\mu_0 \frac{\partial}{\partial t} \left( \frac{\partial \vec{D}}{\partial t} + \vec{J} \right) \\ &= -\mu_0 \frac{\partial}{\partial t} \left( \epsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J} \right) \\ \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} &= -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \frac{\partial}{\partial t} \vec{J}\end{aligned}$$

Dari hukum Ohm  $\vec{J} = \sigma \vec{E} \neq 0$ , untuk  $\vec{E} \neq 0$ , sehingga persamaan diatas menjadi

$$\begin{aligned}\vec{\nabla}(\vec{\nabla} \cdot \underbrace{\vec{E}}_{\frac{\vec{D}}{\epsilon_0}}) - \nabla^2 \vec{E} &= -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E} \\ \frac{1}{\epsilon_0} \vec{\nabla}(\underbrace{\vec{\nabla} \cdot \vec{D}}_{\rho}) - \nabla^2 \vec{E} &= -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E}\end{aligned}$$

Dalam medium konduktor, resistivitas  $\rho = 0$  dan konduktivitas  $\sigma \neq 0$ , maka tersisa

$$-\nabla^2 \vec{E} = -\mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E}$$

*rearrange* persamaan diatas menjadi,

$$\begin{aligned}\nabla^2 \vec{E} - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} \vec{E} - \mu_0 \sigma \frac{\partial}{\partial t} \vec{E} &= 0 \\ \left( \nabla^2 - \mu_0 \epsilon_0 \frac{\partial^2}{\partial t^2} - \mu_0 \sigma \frac{\partial}{\partial t} \right) \vec{E} &= 0\end{aligned}$$

- Untuk medan  $\vec{H}$   
Dari persamaan Maxwell ke-4

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Curl-kan kedua ruas persamaan

$$\vec{\nabla} \times \vec{\nabla} \times \vec{H} = \vec{\nabla} \times \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right)$$

Dengan identitas vektor, bahwa  $\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) - \nabla^2 \vec{A}$  dan karena  $\vec{B} = \mu_0 \vec{H}$  maka

$$\begin{aligned}\vec{\nabla}(\vec{\nabla} \cdot \underbrace{\vec{H}}_{\frac{1}{\mu_0} \vec{B}}) - \nabla^2 \vec{H} &= \vec{\nabla} \times \vec{J} + \vec{\nabla} \times \frac{\partial \vec{D}}{\partial t} \\ \frac{1}{\mu_0} \vec{\nabla}(\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{H} &= \vec{\nabla} \times \vec{J} + \vec{\nabla} \times \frac{\partial \vec{D}}{\partial t}\end{aligned}$$

Berdasarkan persamaan Maxwell ke-2  $\vec{\nabla} \cdot \vec{B} = 0$ , maka tersisa

$$-\nabla^2 \vec{H} = \vec{\nabla} \times \vec{J} + \vec{\nabla} \times \frac{\partial \vec{D}}{\partial t}$$

Karena  $\vec{D} = \varepsilon_0 \vec{E}$ , maka

$$\begin{aligned}-\nabla^2 \vec{H} &= \vec{\nabla} \times \vec{J} + \frac{\partial}{\partial t} \vec{\nabla} \times (\varepsilon_0 \vec{E}) \\ &= \vec{\nabla} \times \vec{J} + \varepsilon_0 \frac{\partial}{\partial t} \underbrace{\vec{\nabla} \times \vec{E}}_{\substack{-\frac{\partial \vec{B}}{\partial t} \\ \text{Pers. Ke-3} \\ \text{Maxwell}}} \\ &= \vec{\nabla} \times \vec{J} + \varepsilon_0 \frac{\partial}{\partial t} \left( -\frac{\partial \vec{B}}{\partial t} \right) \\ &= \vec{\nabla} \times \vec{J} - \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}\end{aligned}$$

karena  $\vec{B} = \mu_0 \vec{H}$ , maka

$$-\nabla^2 \vec{H} = \vec{\nabla} \times \vec{J} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

Dari hukum Ohm  $\vec{J} = \sigma \vec{E} \neq 0$ , untuk  $\vec{E} \neq 0$ , sehingga persamaan diatas menjadi

$$\begin{aligned}-\nabla^2 \vec{H} &= \vec{\nabla} \times (\sigma \vec{E}) - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} \\ &= \sigma \underbrace{\vec{\nabla} \times \vec{E}}_{\substack{-\frac{\partial \vec{B}}{\partial t} \\ \text{Pers. Ke-3} \\ \text{Maxwell}}} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} \\ -\nabla^2 \vec{H} &= -\sigma \frac{\partial \vec{B}}{\partial t} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2}\end{aligned}$$

karena  $\vec{B} = \mu_0 \vec{H}$ , maka

$$-\nabla^2 \vec{H} = -\sigma \mu_0 \frac{\partial \vec{H}}{\partial t} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2}$$

rearrange persamaan diatas menjadi

$$\begin{aligned} \nabla^2 \vec{H} - \sigma \mu_0 \frac{\partial \vec{H}}{\partial t} - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{H}}{\partial t^2} &= 0 \\ \left( \nabla^2 - \sigma \mu_0 \frac{\partial}{\partial t} - \varepsilon_0 \mu_0 \frac{\partial^2}{\partial t^2} \right) \vec{H} &= 0 \end{aligned}$$

3. Diketahui konduktivitas perak  $\sigma = 3 \times 10^7$  S/m pada frekuensi gelombang mikro. Tentukan *skin depth* pada frekuensi  $10^{10}$ Hz.

*skin depth* didefinisikan sebagai jarak untuk mengurangi amplitudo Gelombang EM dengan faktor  $1/e$ , yakni:

$$\delta \equiv \frac{1}{\kappa}; \text{ dimana } \kappa \equiv \omega \sqrt{\frac{\varepsilon \mu}{2} \left[ \sqrt{1 + \left( \frac{\sigma}{\varepsilon \omega} \right)^2} - 1 \right]^{1/2}}$$

Untuk konduktivitas tinggi ( $\sigma \gg \omega \varepsilon$ )  $\Rightarrow \sigma/\varepsilon \gg 1$ , sehingga

$$\begin{aligned} \delta &= \frac{1}{\omega \sqrt{\frac{\mu \varepsilon}{2} \left[ \frac{\sigma}{\varepsilon \omega} \right]^{1/2}}} \\ &= \frac{1}{\sqrt{\frac{\omega^2 \mu \varepsilon \sigma}{2 \varepsilon \omega}}} \\ &= \frac{1}{\sqrt{\frac{\omega \mu \sigma}{2}}} \\ &= \sqrt{\frac{2}{\omega \mu \sigma}} \\ &= \sqrt{\frac{2}{2\pi f \mu \sigma}} \end{aligned}$$

diketahui nilai  $f = 10^{10}$ Hz,  $\sigma = 3 \times 10^7$ S/m dan permeabilitas material  $\mu = \mu_0(1 + \chi_m)$ , dimana suseptabilitas material perak  $\chi_m = -2,4 \times 10^{-5}$ , atau

$$\begin{aligned} \mu &= \mu_0(1 + \chi_m) \\ &= 4\pi \times 10^{-7}(1 + -2.4 \times 10^{-5}) \\ &= 1,26 \times 10^{-6} \text{N/A}^2 \end{aligned}$$

substitusi pada persamaan *skin depth* sebelumnya, didapat

$$\begin{aligned}\delta &= \sqrt{\frac{2}{2\pi(10^{10})(1,26 \times 10^{-6})(3 \times 10^7)}} \\ &= \sqrt{\frac{2}{2,37 \times 10^{12}}} \\ &= 9,18 \times 10^{-7} \text{ m} \\ &= 0,918 \mu\text{m}\end{aligned}$$

4. Air laut memiliki konduktivitas  $\sigma = 3 \times 10^7 \text{ S/m}$  dan  $\mu = \mu_0$ . Tentukan nilai frekuensi ketika *skin depth*-nya bernilai satu meter.

Untuk bahan dengan konduktifitas tinggi maka *skin depth* nya adalah

$$\begin{aligned}\delta &= \sqrt{\frac{2}{\omega\mu\sigma}} \\ &= \sqrt{\frac{2}{2\pi f\mu\sigma}}\end{aligned}$$

untuk mencari nilai frekuensi, maka berdasarkan persamaan diatas

$$\begin{aligned}\sqrt{2\pi f\mu\sigma} &= \frac{\sqrt{2}}{\delta} \\ \sqrt{f} &= \frac{\sqrt{2}}{\delta\sqrt{2\pi\mu\sigma}} \\ f &= \frac{2}{\delta^2 2\pi\mu\sigma}\end{aligned}$$

diketahui bahwa konduktifitas  $\sigma = 3 \times 10^7$ ,  $\mu = \mu_0$ , dan  $\delta = 1 \text{ m}$ , maka

$$\begin{aligned}f &= \frac{2}{1^2(2\pi)(4\pi \times 10^{-7})(3 \times 10^7)} \\ &= 0,00844 \text{ Hz} \\ &= 8,44 \times 10^{-3} \text{ Hz}\end{aligned}$$

5. Intensitas medan listrik yang berbentuk gelombang bidang dalam vakum dinyatakan dengan persamaan sebagai berikut:

$$\vec{E} = 100 \cos(\omega t + 8z) \hat{i} \text{ V/m}$$

maka tentukan:

- Kecepatan jalar gelombang
- Frekuensi gelombang EM

- c) Panjang gelombang
- d) Intensitas medan magnet

$$\iint_{-\infty}^{\infty} \nabla \times E = \frac{1}{4\pi\epsilon_0}$$