**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans : **B**

μ = 55 minutes (mean)

σ = 8 minutes (standard deviation)

z = (X - μ) / σ

z = (60 - 55) / 8

z = 5 / 8

z = 0.625

Now, using the z-table or statistical software, we find the cumulative probability corresponding to z = 0.625.

The cumulative probability for z = 0.625 is approximately 0.734, which represents the probability that the service time is less than or equal to 60 minutes.

Probability = 1 - 0.734

Probability = 0.266

Therefore, the probability that the service manager cannot meet his commitment is approximately 0.266, or 26.6%.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans :

A) **False**.

Explaination :

Since the age distribution is normal with a mean of 38 and a standard deviation of 6, we can use the properties of the normal distribution to analyze the statement.

To determine the proportion of employees older than 44, we need to calculate the area under the normal curve to the right of 44. Similarly, to determine the proportion of employees between 38 and 44, we need to calculate the area under the normal curve between those two values.

However, without knowing the exact values of the z-scores corresponding to 44, 38, and their respective probabilities, we cannot make a definitive comparison. We would need additional information to determine the proportions and make a comparison between the two statements.

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans : **B** True.

q2b = stats.norm.cdf(30, loc = mean, scale = std1)\*100

print('A training program for employees under the age of 30 at the center would be expected to attract about',np.round((q2b\*400)/100,0),'employees')

A training program for employees under the age of 30 at the center would be expected to attract about 36.0 employees

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans :

The Normal Distribution has its link with the Central Limit Theorem, which states that ‘Any large sum of independent identically distribution random variables are approximately Normal then

(X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The Difference between 2X1 and (X1 + X2) is the magnitude they hold of two different sample subsets (X1 and X2) from the same source(population).

X1 and X2 can be a different subset of a sample from a similar source (population) but

If X1 ~ N(μ, σ2) then, 2 X1 ~ N(2 μ, 4 σ2 )

If X1 ~ N(μ, σ2) and X2 ~ N(μ, σ2) are iid normal random variables then

(X1 + X2)~N(μ+ μ, σ2+ σ2)~(2 μ, 2 σ2)

Hence, 2X1 – (X1+X2) ~(2 μ – 2 μ, 4 σ2 + 2σ2 )

The distribution remains the same for every sample subset of similar source, it tends to fall under Normal distribution and slight deviations in parameters.

The Normal distribution has two parameters, the mean, µ, and the variance, σ2.

µ and σ2satisfy −∞ < µ < ∞, σ2> 0. We write X ∼ Normal (µ, σ2) or X ∼ N(µ, σ2 ).

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans : **D**

import scipy.stats as stats

mean = 100

std\_dev = 20

# Calculate the z-scores corresponding to the lower and upper probabilities

lower\_prob = (1 - 0.99) / 2

upper\_prob = 1 - lower\_prob

z\_lower = stats.norm.ppf(lower\_prob)

z\_upper = stats.norm.ppf(upper\_prob)

# Calculate the values of a and b

a = mean + (z\_lower \* std\_dev)

b = mean + (z\_upper \* std\_dev)

a, b =(48.5, 151.5)

The two values of a and b, symmetric about the mean, are such that the probability of the random variable taking a value between them is 0.99: [ 48.5 151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans :

Rupee ranges in between [9.9 to 98.1] Crore Rupees, 95% of the time for the Annual Profit of the Company.

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans :

The 5TH Percentile of profit for the company is 17 Crore Rupees

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans :

The Division #2 (Profit2 ~ N(7, 42) ) has a larger probability of making a loss in a given year