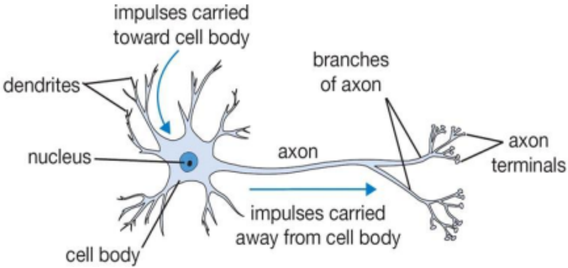
Motivation

Historic aim (McCulloch& Pitts, 1943):
Mimic the biological processes of real neurons for Machine Learning.



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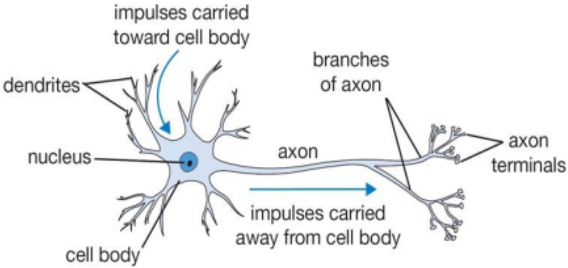
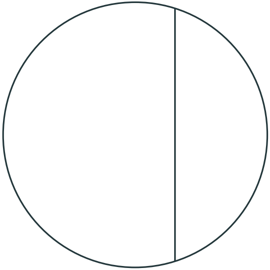


Key observation: Biological neurons transmit signals **only** if the required activation energy is reached by all incoming signals.

(Simple) Perceptron - an artificial neuron



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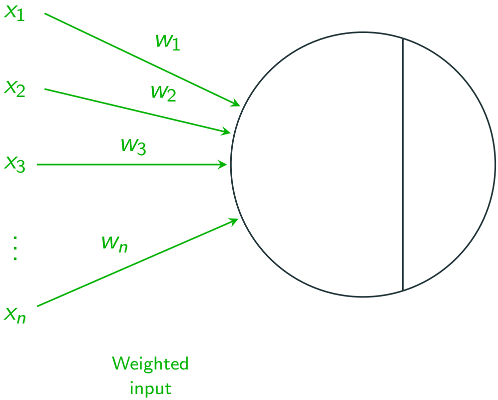

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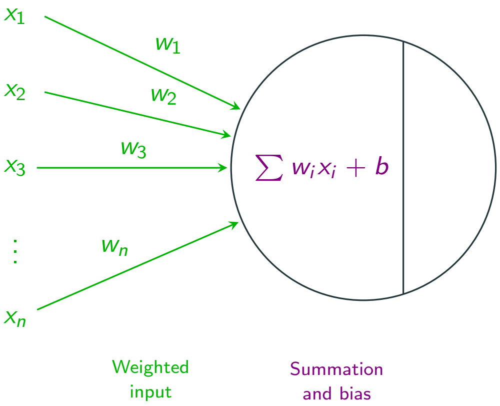

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$$\sum_{i=1}^{n} w_i x_i + b$$

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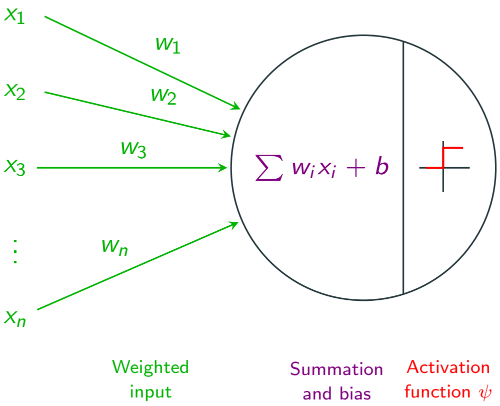

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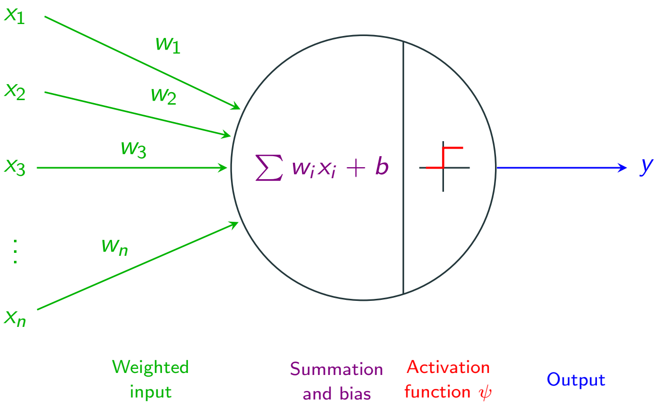

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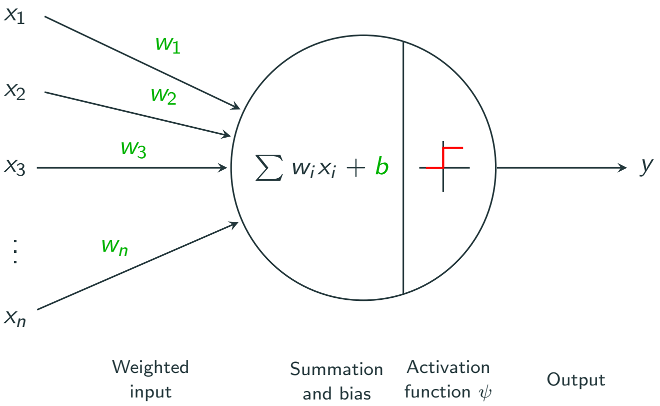

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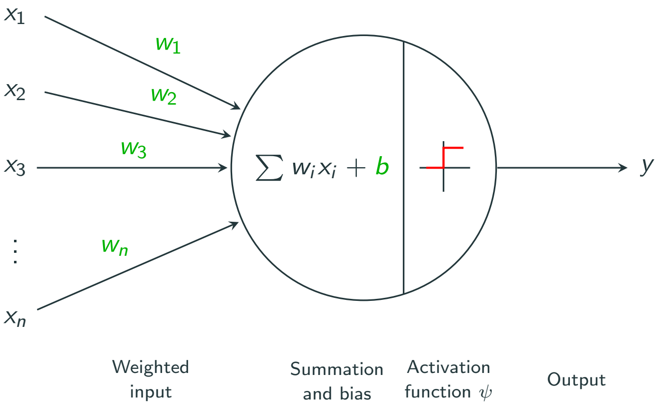
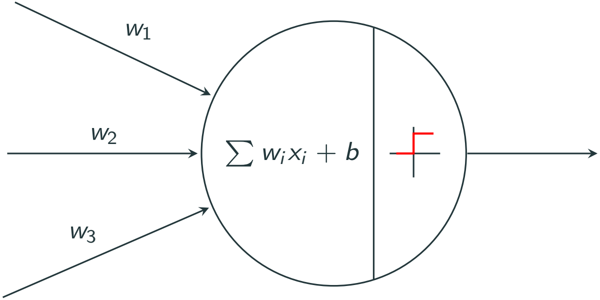


For a fixed activation function $\psi\colon \mathbb{R} \to \mathbb{R}$ the behaviour of the perceptron is defined by the free parameters $(w_1, \ldots, w_n, b) = (\vec{w}, b) =: \theta \in \mathbb{R}^{n+1}$. Thus, the perceptron realizes a parametrized map $f_\theta\colon \mathbb{R}^n \to \mathbb{R}$ with $f_\theta(\vec{x}) := f(\vec{x}; \theta) = f(x_1, \ldots, x_n; \theta)$.

An example perceptron

We analyze a perceptron with 3 fixed input signals $(x_1, x_2, x_3) = (0, 1, 1)$. We use the Heavyside function $H \colon \mathbb{R} \to \{0, 1\}$ as activation function and set the free parameters as $\theta = (w_1, w_2, w_3, b) = (1, 0, 1, -1)$.



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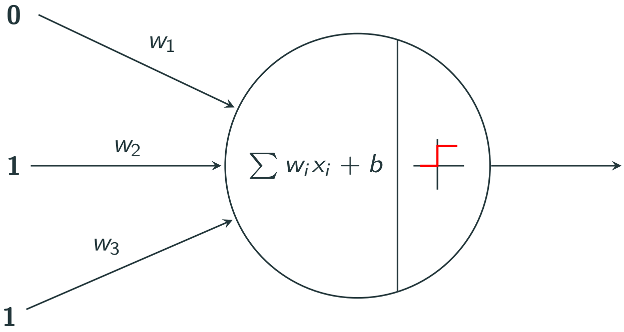

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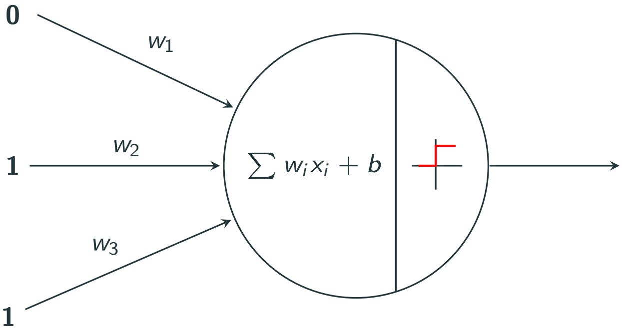

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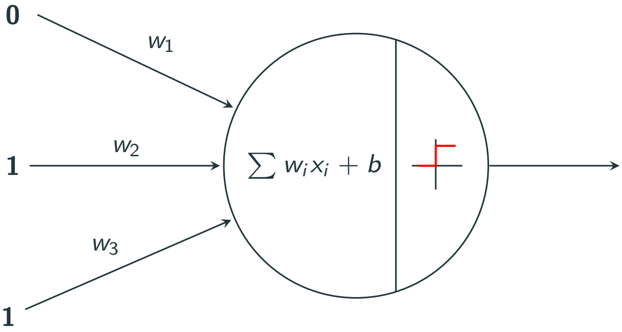

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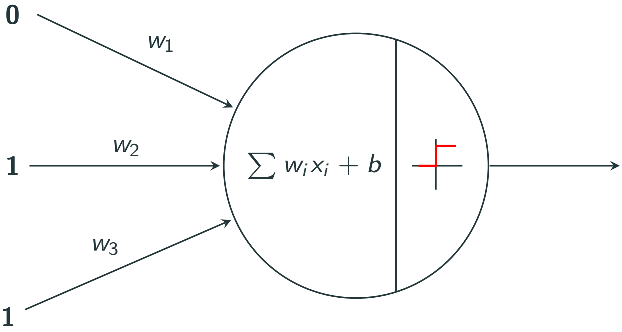

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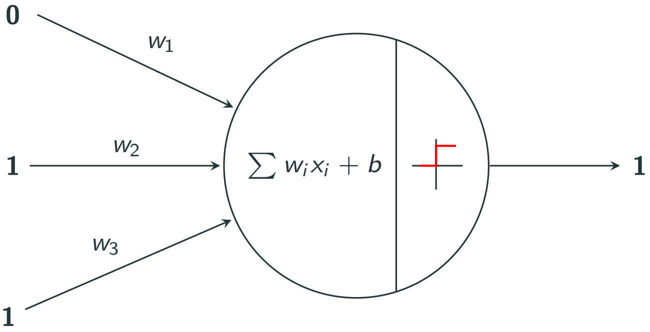

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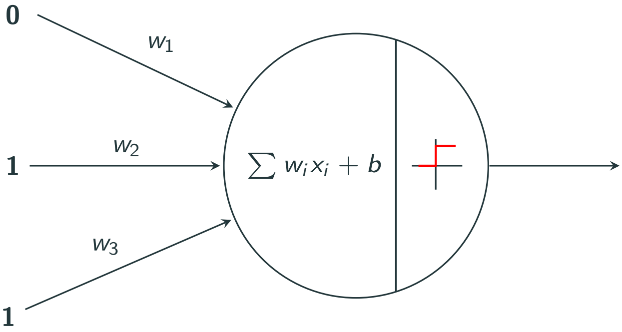

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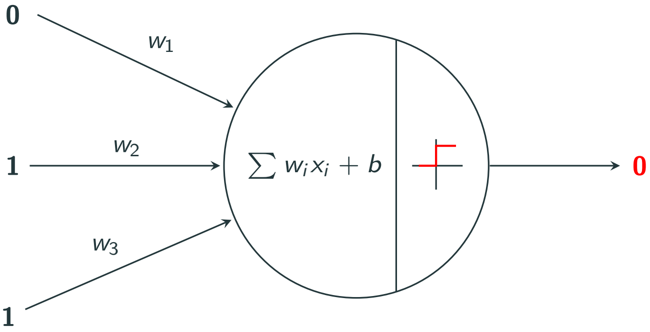

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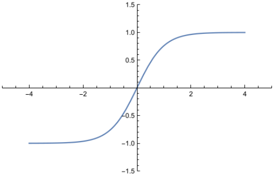
Continuous activation functions

The following **continuous activation functions** are commonly used in artificial neurons due to their nice analytic properties:

Tanh



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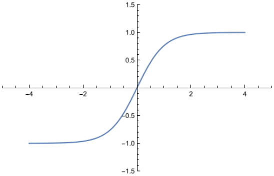

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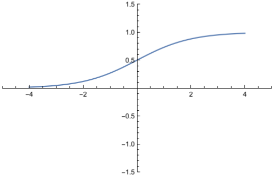

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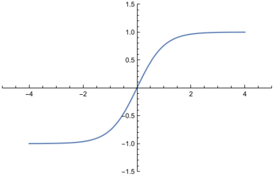

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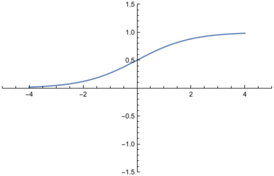

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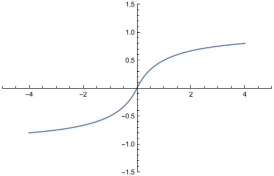

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Observations on the perceptron

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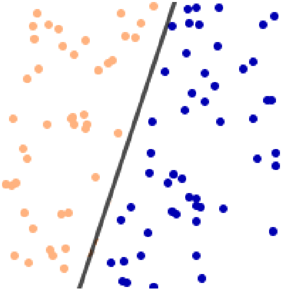

Given a set of input data $\{\vec{x}^{(1)}, \ldots, \vec{x}^{(N)}\}$ with $x^{(i)} \in \mathbb{R}^n$, the free parameters $\theta \in \mathbb{R}^{n+1}$ induce a hyperplane that **linearly** separates the data in two classes.

$$f_\theta(\vec{x}^{(i)}) := \begin{cases} 1, & \text{if } \langle \vec{w}, \vec{x}^{(i)} \rangle + b > 0, \\ 0, & \text{otherwise.} \end{cases}$$

Artificial neural networks

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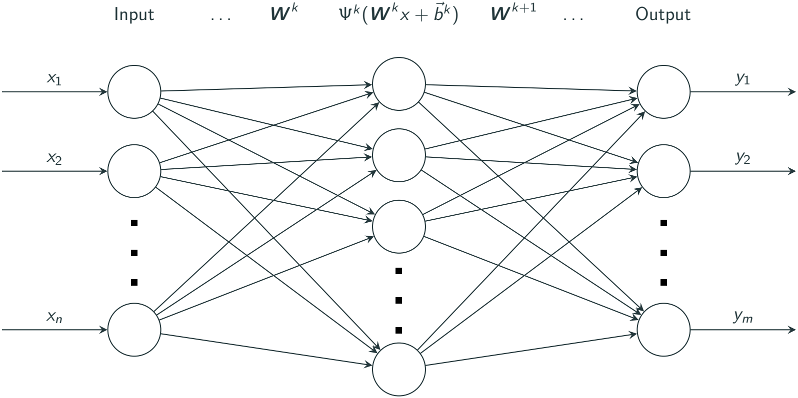
- connections between neurons can be (almost) **arbitrary**

  $\rightarrow$ often there are no connections within same layer (except in recurrent neural networks)

  $\rightarrow$ certain network structures have proved to be successful for different applications, e.g., convolutional neural networks

Fully-connected feedforward neural network

Classical representation: Mappings from kth to $(k+1)$st layer:



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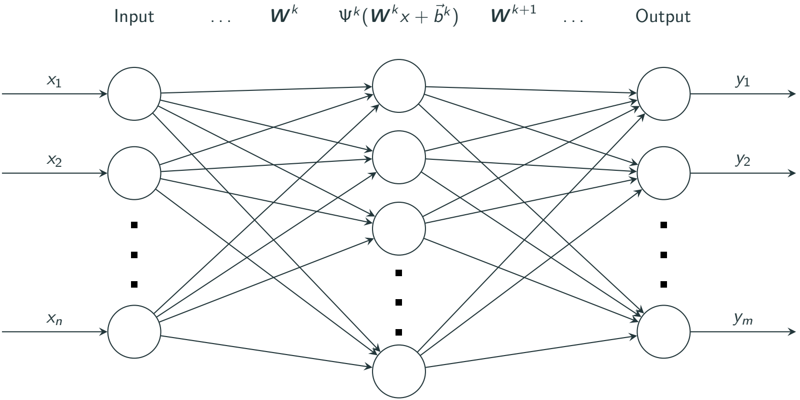
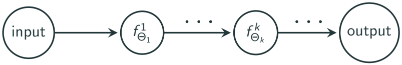


Compact representation: 

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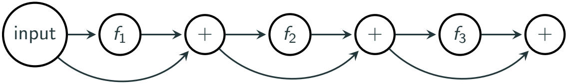
- the network is fully-connected if each weight matrix $\boldsymbol{W}^k$ is dense

Non-sequential artificial neural networks

- **Residual network:** Popular architecture involving *residual connections*. Can be interpreted as forward Euler method.



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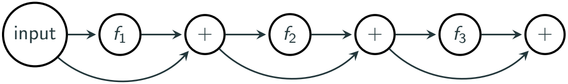

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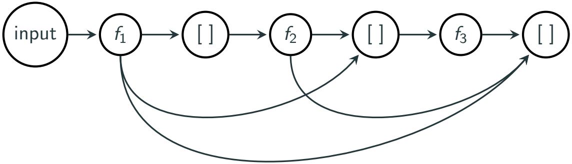

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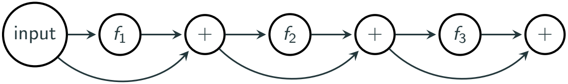

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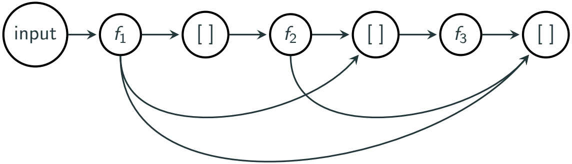

- **Sparse network:** Network architectures where most weights are zero, i.e., the connectivity is small relative to the number of connections possible.

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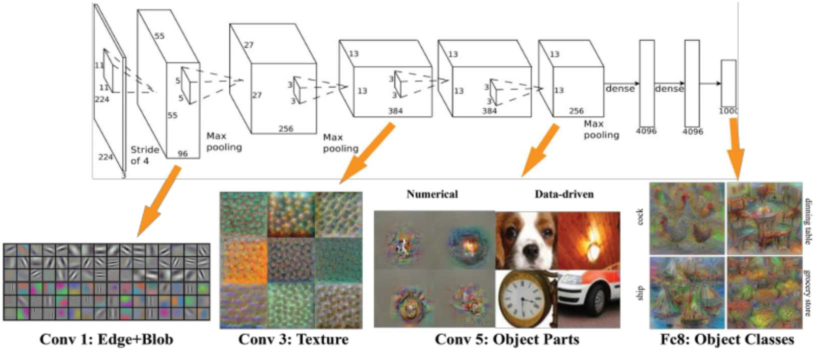
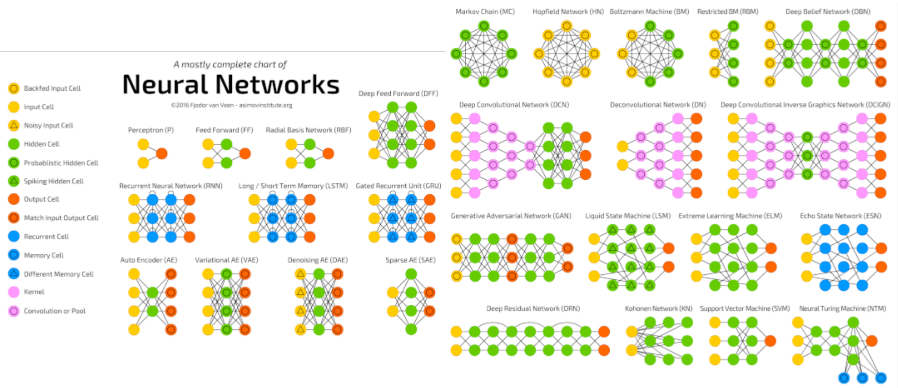


Zoo of architectures



Open question

Machine learning task:

Given pairs of input/output data $(x^{(1)}, y^{(1)}), \ldots, (x^{(N)}, y^{(N)})$. How can we build an artificial neural network f_Θ such that

$$f_\Theta(x^{(k)}) \approx y^{(k)}, \; k = 1, \ldots, N.$$

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Given pairs of input/output data $(x^{(1)}, y^{(1)}), \dots, (x^{(N)}, y^{(N)})$. How can we build an artificial neural network f_Θ such that

$$f_\Theta(x^{(k)}) \approx y^{(k)},\ \ k = 1, \dots, N.$$

Example:

Imagine an artificial neural network with an input layer (10 neurons), 5 hidden layers (10 neurons each), and a single output neuron. This leads to $10 * 10 + 4 * (10 * 10) + 10 =$ 510 free parameters for the *weights* and 51 free parameters for the *biases*.

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Solution: Obtain good parameters by **training** the neural network!

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Thank you for your attention!