Report on Project 4, Part 5

Parallelization of Matrix Multiplication

CS415

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**Overview:**

Computing matrix multiplication is known to have a long calculation time. Matrix multiplication itself is not necessarily difficult but as the size of the matrix grows, more calculations are needed. Computing matrix multiplication has a run time of n^3. The purpose of this experiment is to show the speedup achieved with parallelization.

**Test Methodology:**

Two different programs are written to test the matrix multiplication time, sequential and parallel. The sequential code will be used as a controlled variable. The code will be tested at various sizes starting at 360 by 360 sized matrix. For more detail of the sequential and parallel code, please see below.

Sequential

The sequential code first allocates spaces for three matrixes, 2 for multiplicand and 1 for product. The two multiplicand matrixes are filled. After being filled, the two matrixes are multiplied together and the results are stored in the product matrix. The time starts when the calculation starts and ends when the calculation ends.

Parallel

The parallel code first allocates spaces for three tiles in all the cores, 2 for multiplicand and 1 for product. The size of the tiles is determined by the size of the matrix and the number of cores. The two multiplicand matrixes are filled. After being filled, all the cores will follow Cannon’s algorithm to multiple the two matrixes (spread across all the tiles) together. The time starts when the calculation starts and ends when the calculation ends.

**Code Revisions:**

Addressing Comments:

I have received the following concerns over my initial parallel implementation and addressed them accordingly. I grouped similar concerns together so that I can address them all at once.

* Code is very dense. Spacing out code statements with new line would make is easier to read.
* Code is very dense and bit difficult to read. This is partly due to it being written in C but there is a lack of auxiliary functions to make the code much more readable. Also line spacing would be nice to see.  
    
  **Resulting Changes:**  
  I added more line space between different lines of code. I also added additional functions for complex row and column id calculations. I added section headers for long functions to add readability.
* The code would also benefit from some more functions to make the code easier to read. For instance, a allocateMatrix function would be nice to see. It would also be nice to see a functions for sending and receiving.
* Although the functionality could be split into subroutines a little more, the overall code structure is solid.
* Everything was nice and understandable. I might put some of the repeated things in a function(matrix memory allocation, for example).  
    
  **Resulting Changes:**  
  I added an allocate matrix function, free matrix function and calculate neighbor function to reduce the amount of repeating code. I did not added send and receive functions because the function would only be used once at the beginning with a very specific set of circumstances to work.
* Instead of the switch statement it would nice to see just the sqrt() function in math.h and then some checking. It is possible to run this code on the cluster with 49 cores.
* On a more style-related note, you could make your switch statement at the beginning check for a remainder when you divide the size by it's square root, which would make it a bit less verbose and help it handle the general case.  
    
  **Resulting Changes:**  
  I change the switch statement to a square root function
* I really like the print flag.

**Resulting Changes:**  
I added more flags for printing matrix and printing time

* I'm not sure what the output time means. I'm assuming its in micro seconds. That would be nice to see in the documentation.  
    
  **Resulting Changes:**  
  I added additional notes in the documentation explicitly stating that the time printout is in microseconds. I did not change the printout format because I wanted to keep the output in csv format
* The only markdown is mostly from not collecting the final result into one matrix.   
    
  **Resulting Changes:**  
  I added a function that collects all the data from the tiles and puts them into one matrix
* Not that it's super important, but you might clean this[makefile] up a bit(there's still stuff that looks like it's from PA1). It's really just for your own benefit, but it might help if you have to debug something in you makefile.  
    
  **Resulting Changes:**  
  I glad this person noticed the relics from previous projects in my makefile. I am not making any immediate changes to the makefile in case of the off chance of needing to use old code again.
* You might speed up your initial matrix communication time by transposing things to begin with to avoid the whole doubly-nested-for-loop inside a triply-nested-for-loop thing.  
    
  **Resulting Changes:**  
  I did not change my matrix communication method because I do not possess the knowledge to implement the feature of transposing matrices.

**Data Analysis:**

The raw data from the tests can be found in the file project4Analysis.

Sequential

The run time of the sequential code exhibits a runtime of O(n^3) as shown in Graph 1.1. The execution time grows exponentially as the length of the matrix grows linearly.

Parallel

The parallel implementation of matrix multiplication showed a decrease of overall runtime compared to the sequential implementation. The speedup calculation indicated that there was super linear speed up as the values were significantly greater than the cores used as shown in graph 2.1, graph 2.2 and graph 3.1. The super linear speedup did not show up in larger matrix sizes as shown in graph 5.1, and graph 5.2. The efficiency appears to decrease as the size of the matrix grows. The decrease is due to the increase communication time as the cores had to send larger matrices to each other. The decreases can be seen in graph 5.3. The various dips and peaks in these graphs can be attributed to the number of users on the cluster. The communication time increased as more users ran their application on the shared cluster.

Increasing the number of cores also increases the largest matrix that can be calculated within a given amount of time as shown on graph 4.1. The matrix size appears to grow logarithmically as the number of cores increase. These results indicate that increasing the number of cores would be give diminishing returns on very large matrices (beyond 10,000 matrix length). The sudden jump length jump from 4 to 9 cores is due to the introduction of another box. The 9th core utilize all the resources of an additional box.

Super Linear Speedup Explanation

The super linear speedup is likely due to the core’s cache. On larger matrixes, the sequential implementation had more cache misses which resulted in more memory fetch time. The parallel implementation used the cache resource of multiple cores instead of one.

**Conclusion:**

Matrix multiplication has a runtime of O(n^3). Adding more cores would decrease the overall runtime. For larger matrices, each addition core would give diminishing returns on both speedup and efficiency. Thus, a different parallel matrix multiplication algorithm should be developed so that communication time is less.

Parallelization of application appears to benefit more from higher memory than higher clock speeds. The cache of the CPU was major influence on the runtime of matrix multiplication. High performance computers should focus on improving the memory size and access time.

**Graphs and Tables**

Graph 1.1

The graph shows the runtime of matrix multiplication is O(n^3). The orange line is a prediction of the runtime or matrix multiplication. The predicted data was produced by Microsoft Excel’s Forecast function. The upper bound was used as a control for this experiment.

Table 1.1



The table is the average sequential runtime of matrix multiplication. The runtime is O(n^3) as the length increases, the runtime increases exponentially. The predicted data was produced by Microsoft Excel’s Forecast function. The table shown only goes to 4320 matrix length, the remaining table can be found in the file Project4Analysis.

Graph 1.2

The graph indicates that as the size of the matrix increases, the run time increases exponentially, O(n^3). As the number of cores increases, the run time decreases.

Graph1.3

The graph indicates that as the size of the matrix increases, the run time increases exponentially, O(n^3). As the number of cores increases, the run time decreases.

Table 1.2



The table shows that as the size of the matrix increases, the run time increases exponentially, O(n^3). As the number of cores increases, the run time decreases.

Graph 2.1

The speedup increased as the matrix length increases. The speedup of matrix multiplication had super linear speedup due to cache hits and misses. Before 720 matrix length, the speedup was caused by the cannons algorithm. The valleys and peeks are caused by variations in communication time.

Graph 2.2

The speedup increased as the matrix length increases. The speedup of matrix multiplication had super linear speedup due to cache hits and misses. Before 720 matrix length, the speedup was caused by the cannons algorithm. The valleys and peeks are caused by variations in communication time.

Table 2.1



The speedup increased as the matrix length increases. The speedup of matrix multiplication had super linear speedup due to cache hits and misses. Before 720 matrix length, the speedup was caused by the cannons algorithm. After 2520 matrix length, the speedup began to decrease for 4 cores, 9 cores, 16 cores.

Graph 3.1

The efficiency increased as the matrix length increases. The efficiency of matrix multiplication was beyond 100% at certain points due to cache hits and misses. The valleys and peeks are caused by variations in communication time.

Table 3.2



The efficiency increased as the matrix length increases. The efficiency of matrix multiplication was beyond 100% at certain points due to cache hits and misses.

Graph 4.1

Given a finite amount of time, the matrix sized increased logarithmically as the number of cores increased. The spike from 4 cores to 9 cores is caused by the additional resources of another box.

Graph 5.1

The speedup increased as the matrix length increases. The speedup of matrix multiplication had super linear speedup due to cache hits and misses. For very large matrixes, the speedup dropped significantly as all the cores were experiencing cache misses. The valleys and peeks are caused by variations in communication time.

Graph 5.2

The efficiency increased as the matrix length increases. The efficiency of matrix multiplication was beyond 100% at certain points due to cache hits and misses. For very large matrixes, the efficiency dropped significantly as all the cores were experiencing cache misses. The valleys and peeks are caused by variations in communication time.

Graph 5.3

The efficiency increased as the matrix length increases. The efficiency of matrix multiplication was beyond 100% at certain points due to cache hits and misses. For very large matrixes, the efficiency dropped significantly as all the cores were experiencing cache misses. The efficiency dropped below 100% for large matrices which implies that the parallel implementation was not able to utilize the cache similar to smaller matrices. The valleys and peeks are caused by variations in communication time.

Table 6.1



Raw statistical data for 4 core matrix multiplication

Table 6.2



Raw statistical data for 9 core matrix multiplication

Table 6.3



Raw statistical data for 16 core matrix multiplication

Table 6.4



Raw statistical data for 25 core matrix multiplication