DSO windowed optimization 公式

这里有一个细节，我想了很久才想明白，DSO 中的 residual 联系了两个关键帧之间的相对位姿，但是最终需要优化帧的绝对位姿，中间的导数怎么转换？这里使用的是李群、李代数中的Adjoint。
参考 http://ethaneade.com/lie.pdf 。

需要变通一下，字母太多，表达不方便。此处 ξ 表示 se(3) 和 affLight 参数。

Adjoint 在其中的使用如下（根据代码推断，具体数学推导看我的博客《Adjoint of SE(3)》）：

$$
\begin{aligned}
{\frac{\partial r_{th}^{(i)}}{\partial \xi_h}}^T \frac{\partial r_{th}^{(i)}}{\partial \xi_h} &= \left(\frac{\partial r_{th}^{(i)}}{\partial \xi_{th}} \frac{\partial \xi_{th}}{\partial \xi_h}\right)^T \frac{\partial r_{th}^{(i)}}{\partial \xi_{th}} \frac{\partial \xi_{th}}{\partial \xi_h} \\
&= {\frac{\partial \xi_{th}}{\partial \xi_h}}^T {\frac{\partial r_{th}^{(i)}}{\partial \xi_{th}}}^T \frac{\partial r_{th}^{(i)}}{\partial \xi_{th}} \frac{\partial \xi_{th}}{\partial \xi_h} \\
\frac{\partial \xi_{th}}{\partial \xi_h} &= -\mathrm{Ad}_{T_{th}} \\
\frac{\partial \xi_{th}}{\partial \xi_t} &= I
\end{aligned}
$$

复习一下 Schur Complement：

$$
\begin{bmatrix} H_{\rho\rho} & H_{\rho X} \\ H_{X\rho} & H_{XX} \end{bmatrix} \begin{bmatrix} \delta\rho \\ \delta X \end{bmatrix} = -\begin{bmatrix} J_\rho^T r \\ J_X^T r \end{bmatrix}
$$

$$
\begin{bmatrix} H_{\rho\rho} & H_{\rho X} \\ 0 & H_{XX} - H_{X\rho}H_{\rho\rho}^{-1}H_{\rho X} \end{bmatrix} \begin{bmatrix} \delta\rho \\ \delta X \end{bmatrix} = -\begin{bmatrix} J_\rho^T r \\ J_X^T r - H_{X\rho}H_{\rho\rho}^{-1}J_\rho^T r \end{bmatrix}
$$

EnergyFuntional::accumulateAF_MT 和 EnergyFunctional::accumulateLF_MT 的目标是计算$H_{XX}, J_X^T r$，EnergyFunctional::accumulateSCF_MT 的目标是计算$H_{X\rho}H_{\rho\rho}^{-1}H_{\rho X}, H_{X\rho}H_{\rho\rho}^{-1}J_\rho^T r$。

X是68维的，4个相机参数加上8*8个帧状态量。就不写出来了。

这里需要注意一下，r是Nx1，ρ是Mx1，$M \le N$，即 residual 的数目与需要优化的逆深度的数目不一定相等。J_ρ是NxM，J_X 是Nx68。

非 Schur Complement 部分

$$
H_{XX} = \begin{bmatrix} {\frac{\partial r}{\partial C}}^T \frac{\partial r}{\partial C} & {\frac{\partial r}{\partial C}}^T \frac{\partial r}{\partial \xi} \\ {\frac{\partial r}{\partial \xi}}^T \frac{\partial r}{\partial C} & {\frac{\partial r}{\partial \xi}}^T \frac{\partial r}{\partial \xi} \end{bmatrix}
$$

$$
\begin{aligned}
{\frac{\partial r}{\partial C}}^T \frac{\partial r}{\partial C} &= \begin{bmatrix} {\frac{\partial r^{(1)}}{\partial C}}^T & {\frac{\partial r^{(2)}}{\partial C}}^T & \dots & {\frac{\partial r^{(N)}}{\partial C}}^T \end{bmatrix} \begin{bmatrix} \frac{\partial r^{(1)}}{\partial C} \\ \frac{\partial r^{(2)}}{\partial C} \\ \dots \\ \frac{\partial r^{(N)}}{\partial C} \end{bmatrix} \\
&= \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial C}
\end{aligned}
$$

$$
\begin{aligned}
{\frac{\partial r}{\partial C}}^T \frac{\partial r}{\partial \xi} &= \begin{bmatrix} {\frac{\partial r^{(1)}}{\partial C}}^T & {\frac{\partial r^{(2)}}{\partial C}}^T & \dots & {\frac{\partial r^{(N)}}{\partial C}}^T \end{bmatrix} \begin{bmatrix} \frac{\partial r^{(1)}}{\partial \xi_1} & \frac{\partial r^{(1)}}{\partial \xi_2} & \dots & \frac{\partial r^{(1)}}{\partial \xi_8} \\ \frac{\partial r^{(2)}}{\partial \xi_1} & \frac{\partial r^{(2)}}{\partial \xi_2} & \dots & \frac{\partial r^{(2)}}{\partial \xi_8} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial r^{(N)}}{\partial \xi_1} & \frac{\partial r^{(N)}}{\partial \xi_2} & \dots & \frac{\partial r^{(N)}}{\partial \xi_8} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \xi_1} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \xi_2} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \xi_8} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
{\frac{\partial r}{\partial \xi}}^T \frac{\partial r}{\partial \xi} &= \begin{bmatrix} {\frac{\partial r^{(1)}}{\partial \xi_1}}^T & {\frac{\partial r^{(2)}}{\partial \xi_1}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_1}}^T \\ {\frac{\partial r^{(1)}}{\partial \xi_2}}^T & {\frac{\partial r^{(2)}}{\partial \xi_2}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_2}}^T \\ \vdots & \vdots & \ddots & \vdots \\ {\frac{\partial r^{(1)}}{\partial \xi_8}}^T & {\frac{\partial r^{(2)}}{\partial \xi_8}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_8}}^T \end{bmatrix} \begin{bmatrix} \frac{\partial r^{(1)}}{\partial \xi_1} & \frac{\partial r^{(1)}}{\partial \xi_2} & \dots & \frac{\partial r^{(1)}}{\partial \xi_8} \\ \frac{\partial r^{(2)}}{\partial \xi_1} & \frac{\partial r^{(2)}}{\partial \xi_2} & \dots & \frac{\partial r^{(2)}}{\partial \xi_8} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial r^{(N)}}{\partial \xi_1} & \frac{\partial r^{(N)}}{\partial \xi_2} & \dots & \frac{\partial r^{(N)}}{\partial \xi_8} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \xi_1} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \xi_2} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \xi_8} \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \xi_1} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \xi_2} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \xi_8} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \xi_1} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \xi_2} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \xi_8} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
J_X^T r &= \begin{bmatrix} {\frac{\partial r}{\partial C}}^T \\ {\frac{\partial r}{\partial \xi}}^T \end{bmatrix} r \\
&= \begin{bmatrix} {\frac{\partial r^{(1)}}{\partial C}}^T & {\frac{\partial r^{(2)}}{\partial C}}^T & \dots & {\frac{\partial r^{(N)}}{\partial C}}^T \\ {\frac{\partial r^{(1)}}{\partial \xi_1}}^T & {\frac{\partial r^{(2)}}{\partial \xi_1}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_1}}^T \\ {\frac{\partial r^{(1)}}{\partial \xi_2}}^T & {\frac{\partial r^{(2)}}{\partial \xi_2}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_2}}^T \\ \vdots & \vdots & \ddots & \vdots \\ {\frac{\partial r^{(1)}}{\partial \xi_8}}^T & {\frac{\partial r^{(2)}}{\partial \xi_8}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_8}}^T \end{bmatrix} \begin{bmatrix} r^{(1)} \\ r^{(2)} \\ \vdots \\ r^{(N)} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T r^{(i)} \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T r^{(i)} \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T r^{(i)} \\ \vdots \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T r^{(i)} \end{bmatrix}
\end{aligned}
$$

所以算这些矩阵就是遍历每一个 residual，累加求和。

Schur Complement 部分

$$
\begin{bmatrix} H_{\rho\rho} & H_{\rho X} \\ H_{X\rho} & H_{XX} \end{bmatrix} \begin{bmatrix} \delta\rho \\ \delta X \end{bmatrix} = -\begin{bmatrix} J_\rho^T r \\ J_X^T r \end{bmatrix}
$$

$$
\begin{bmatrix} H_{\rho\rho} & H_{\rho X} \\ 0 & H_{XX} - H_{X\rho}H_{\rho\rho}^{-1}H_{\rho X} \end{bmatrix} \begin{bmatrix} \delta\rho \\ \delta X \end{bmatrix} = -\begin{bmatrix} J_\rho^T r \\ J_X^T r - H_{X\rho}H_{\rho\rho}^{-1}J_\rho^T r \end{bmatrix}
$$

Hsc：

$$
H_{X\rho}H_{\rho\rho}^{-1}H_{\rho X}
$$

bsc:

$$
H_{X\rho}H_{\rho\rho}^{-1}J_\rho^T r
$$

$$
\begin{aligned}
J_\rho^T J_\rho &= {\frac{\partial r}{\partial \rho}}^T \frac{\partial r}{\partial \rho} \\
&= \begin{bmatrix} {\frac{\partial r^{(1)}}{\partial \rho^{(1)}}}^T & {\frac{\partial r^{(2)}}{\partial \rho^{(1)}}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \rho^{(1)}}}^T \\ {\frac{\partial r^{(1)}}{\partial \rho^{(2)}}}^T & {\frac{\partial r^{(2)}}{\partial \rho^{(2)}}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \rho^{(2)}}}^T \\ \vdots & \vdots & \ddots & \vdots \\ {\frac{\partial r^{(1)}}{\partial \rho^{(M)}}}^T & {\frac{\partial r^{(2)}}{\partial \rho^{(M)}}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \rho^{(M)}}}^T \end{bmatrix} \begin{bmatrix} \frac{\partial r^{(1)}}{\partial \rho^{(1)}} & \frac{\partial r^{(1)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(1)}}{\partial \rho^{(M)}} \\ \frac{\partial r^{(2)}}{\partial \rho^{(1)}} & \frac{\partial r^{(2)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(2)}}{\partial \rho^{(M)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial r^{(N)}}{\partial \rho^{(1)}} & \frac{\partial r^{(N)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(N)}}{\partial \rho^{(M)}} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(M)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(M)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(M)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & 0 & \dots & 0 \\ 0 & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(M)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
{\frac{\partial r}{\partial C}}^T \frac{\partial r}{\partial \rho} &= \begin{bmatrix} {\frac{\partial r^{(1)}}{\partial C}}^T & {\frac{\partial r^{(2)}}{\partial C}}^T & \dots & {\frac{\partial r^{(N)}}{\partial C}}^T \end{bmatrix} \begin{bmatrix} \frac{\partial r^{(1)}}{\partial \rho^{(1)}} & \frac{\partial r^{(1)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(1)}}{\partial \rho^{(M)}} \\ \frac{\partial r^{(2)}}{\partial \rho^{(1)}} & \frac{\partial r^{(2)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(2)}}{\partial \rho^{(M)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial r^{(N)}}{\partial \rho^{(1)}} & \frac{\partial r^{(N)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(N)}}{\partial \rho^{(M)}} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
{\frac{\partial r}{\partial \xi}}^T \frac{\partial r}{\partial \rho} &= \begin{bmatrix} {\frac{\partial r^{(1)}}{\partial \xi_1}}^T & {\frac{\partial r^{(2)}}{\partial \xi_1}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_1}}^T \\ {\frac{\partial r^{(1)}}{\partial \xi_2}}^T & {\frac{\partial r^{(2)}}{\partial \xi_2}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_2}}^T \\ \vdots & \vdots & \ddots & \vdots \\ {\frac{\partial r^{(1)}}{\partial \xi_8}}^T & {\frac{\partial r^{(2)}}{\partial \xi_8}}^T & \dots & {\frac{\partial r^{(N)}}{\partial \xi_8}}^T \end{bmatrix} \begin{bmatrix} \frac{\partial r^{(1)}}{\partial \rho^{(1)}} & \frac{\partial r^{(1)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(1)}}{\partial \rho^{(M)}} \\ \frac{\partial r^{(2)}}{\partial \rho^{(1)}} & \frac{\partial r^{(2)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(2)}}{\partial \rho^{(M)}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial r^{(N)}}{\partial \rho^{(1)}} & \frac{\partial r^{(N)}}{\partial \rho^{(2)}} & \dots & \frac{\partial r^{(N)}}{\partial \rho^{(M)}} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \end{bmatrix}
\end{aligned}
$$

$$
\begin{aligned}
H_{X\rho}H_{\rho\rho}^{-1}H_{\rho X} &= J_X^T J_\rho (J_\rho^T J_\rho)^{-1} J_\rho^T J_X \\
&= \begin{bmatrix} {\frac{\partial r}{\partial C}}^T \\ {\frac{\partial r}{\partial \xi}}^T \end{bmatrix} \frac{\partial r}{\partial \rho} \left({\frac{\partial r}{\partial \rho}}^T \frac{\partial r}{\partial \rho}\right)^{-1} {\frac{\partial r}{\partial \rho}}^T \begin{bmatrix} \frac{\partial r}{\partial C} & \frac{\partial r}{\partial \xi} \end{bmatrix} \\
&= \begin{bmatrix} {\frac{\partial r}{\partial C}}^T \frac{\partial r}{\partial \rho} \\ {\frac{\partial r}{\partial \xi}}^T \frac{\partial r}{\partial \rho} \end{bmatrix} \begin{bmatrix} \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & 0 & \dots & 0 \\ 0 & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(M)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right)^{-1} \end{bmatrix} \\
&\quad \begin{bmatrix} {\frac{\partial r}{\partial \rho}}^T \frac{\partial r}{\partial C} & {\frac{\partial r}{\partial \rho}}^T \frac{\partial r}{\partial \xi} \end{bmatrix} \\
&= \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \\ \vdots & \vdots & \ddots & \vdots \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}} & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}} & \dots & \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}} \end{bmatrix} \\
&\quad \begin{bmatrix} \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & 0 & \dots & 0 \\ 0 & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(M)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right)^{-1} \end{bmatrix} \\
&\quad \begin{bmatrix} \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^T & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^T & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^T \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^T & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^T & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^T \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(3)}}\right)^T & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(3)}}\right)^T & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(3)}}\right)^T \\ \vdots & \vdots & \ddots & \vdots \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right)^T & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right)^T & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right)^T \end{bmatrix} \\
&= \begin{bmatrix} \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right) \cdots \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right) \cdots \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right) \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right) \cdots \end{bmatrix} \\
&\quad \begin{bmatrix} \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^T & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^T & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^T \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^T & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^T & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^T \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(3)}}\right)^T & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(3)}}\right)^T & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(3)}}\right)^T \\ \vdots & \vdots & \ddots & \vdots \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right)^T & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right)^T & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right)^T \end{bmatrix}
\end{aligned}
$$

同理

$$
H_{X\rho}H_{\rho\rho}^{-1}J_\rho^T r = \begin{bmatrix} \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial C}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right) \cdots \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_1}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right) \cdots \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_2}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right) \cdots \\ \vdots & \vdots & \ddots & \vdots \\ \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(1)}}\right)^{-1} & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)\left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T \frac{\partial r^{(i)}}{\partial \rho^{(2)}}\right)^{-1} & \dots & \left(\sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \xi_8}}^T \frac{\partial r^{(i)}}{\partial \rho^{(M)}}\right) \cdots \end{bmatrix} \begin{bmatrix} \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(1)}}}^T r^{(i)} \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(2)}}}^T r^{(i)} \\ \vdots \\ \sum_{i=1}^N {\frac{\partial r^{(i)}}{\partial \rho^{(M)}}}^T r^{(i)} \end{bmatrix}
$$

分类: SLAM

« 上一篇： DSO 优化代码中的 Schur Complement
» 下一篇： DSO 代码框架

posted @ 2018-01-17 23:09 JingeTU 阅读(2477) 评论(9) 编辑 收藏

评论列表

#1楼 2018-05-11 02:18 zoloo

请问DSO中关于零空间计算那部分的内容，博主有什么相关理论材料可以参考学习吗？那个orthogonize函数的作用不是特别理解

支持(0) 反对(0)

#2楼 [楼主] 2018-05-11 16:04 JingeTU

@ zoloo
斯密特正交化，目的是使 Hx=b 方程有唯一解，也就是解空间为一维。这个代码我还没有仔细推导。推荐你看一看 YouTube 上 cvprtum 的 MVG 课程，这课程的主线就是解空间。

支持(1) 反对(0)

#3楼 2018-05-12 03:19 zoloo

@ JingeTU
谢谢推荐，另外非常感谢博主的DSO系列，帮我扫清了很多细节方面的疑惑，期待博主更多精彩博文！

支持(0) 反对(0)

#4楼 2018-05-21 10:52 接住这个柿子

博主您好，请问X为何是68维的，X的维度不应该是和关键帧的数量n成正比的吗？

支持(0) 反对(0)

#5楼 [楼主] 2018-05-21 11:41 JingeTU

@ 接住这个柿子
setting_maxFrames 设置的值是 7，也就是保持窗口中关键帧的数量不超过7。你应该理解系统稳定时，有 8 个关键帧参与窗口优化。每个关键帧有 8 个参数（se(3) 有 6 个，AffLight 有 2 个）。68 = 4 + 8*8。剩余的 4 个参数是相机内参数 fx, fy, cx, cy，也就是窗口优化也优化相机内参数。相机内参数的数量“4”在代码中用宏 CPARS 表示。

支持(1) 反对(0)

#6楼 2020-03-20 14:23 daichujiang

@JingeTU
楼主好，关于矩阵Hxx的维数应该是64*64维的吧，因为X是窗口内所有帧的位姿变量，是64维的

支持(0) 反对(0)

#7楼 [楼主] 2020-03-22 09:16 JingeTU

@daichujiang
还有 4 维度的相机参数，“X是68维的，4个相机参数加上8*8个帧状态量”。

支持(0) 反对(0)

#8楼 2020-03-24 17:05 daichujiang

@JingeTU
是我理解有问题，这里ksi包含8个参数（se(3)和两个光度参数）。这里还有个问题，相对光度参数delta(bij)对host帧光度参数的伴随，代码中只求取了对bi的导数，而没有求对ai的导数，请问楼主这是为什么呢？

支持(0) 反对(0)

#9楼 [楼主] 2020-04-07 22:43 JingeTU

@daichujiang
没有计算吗？你看这一行。具体的你去确认这附近的代码，我很久没看了。

支持(0) 反对(0)