Teaching Statement

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My Teaching Philosophy

It is certain that one of the most important factors which can either inspire or discourage a person from pursuing a career in a field is the teacher or lecturer. I've experienced the former, and wanted to become a mathematician thanks to mainly my math olympiad teacher that I met on the 8th grade; on the other hand I've also seen a lot of people discouraged from being a mathematician or a scientist because of the toxic environment in the classrooms. One of the reasons that I am pursuing an academic career is to be an inspiration to people from younger generations, and encourage more people to become mathematicians or at least have fun while learning mathematics and appreciate the beauty of it. As a Ph.D. student who has been holding tutorials and problem sessions for almost 6 years, this has been my primary goal in teaching.

Creating the environment which will allow students to not be afraid of asking questions, and not be ashamed of it even if they may seem like "wrong" questions or have "obvious" looking answer, is an essential ingredient to creating a non-toxic environment. Although asking the right questions is very important in mathematics, one should encourage their students to do this by positive reinforcement without judging the ones which do not fall in this category. In my tutorials and problem sessions, I regularly remind them to not hesitate asking questions, either at the moment or in private after the tutorials, and try to do my best with helping them to understand what is going on.

This does not only apply to questions but also to the answers to the questions asked by the lecturers. Even the experts in a field are making a lot of mistakes, so we should convince the students that it is ok to make mistakes, and it is an integral part of the learning process. It is also worthwhile to note the importance of encouraging students to notice the lecturer's mistakes and typos, and to give feedbacks about it. As a student, getting a positive response during such situations increased both my self esteem and my respect for the instructor. Now as a tutor and prospective lecturer, I thank my students for noticing my mistakes or providing an alternative better solution to the problem.

Providing a non-toxic environment to the students is one necessary part of the solution however it is not sufficient. One should also inspire the students for continuing their career in the field, or at least make them enjoy the lecture material and maintain their interests in the topic. Throughout my experience both as a student and a tutor, I have realized that there are several key steps for achieving that.

The reason I liked mathematics is that unlike how it has been perceived by most of the students, it is not just about applying formulas and calculations, but also about understanding the reasoning behind them and the relationships between different concepts. Mathematics is very undogmatic by

its nature; if something is a theorem, then it should follow by applying the rules of logic either from definitions and axioms, or other theorems. Therefore, I would be very happy if my prospective students could also appreciate this as I did. However, one should also realize that technical proofs can also sometimes be very scary especially for the students of mathematics modules from different majors. And in that case, their point of view can change from memorizing the formulas to memorizing proofs. I believe the secret to the solution of this problem lies in the balance between rigor and intuition.

In mathematical research, rigor and formality is extremely important. Indeed, an ambiguity in the argument of a proof or a tiny mistake in details could lead to wrong results. However, focusing on rigor too much and not giving enough priority to intuition would lead the students not appreciate the material, internalize the concepts, and understand why the arguments are the way they are. Therefore, the lecturer's priority should be giving the students a good intuition and a motivation. After the students really have a good "picture" on their mind of what is going on, the importance of the rigor and formal proof could be much easier for them to appreciate. As Ravi Vakil said on his online lectures on algebraic geometry - AGITTOC, the most important question is "why is a thing" instead of "what is a thing". Learning the proof should be a tool for answering this question, it should not be the end goal.

To give intuition to the students, choosing good examples and counter examples is essential. Indeed, applying the arguments in the proof of a statement to some non-trivial examples, the students might already have a guess about what the general argument should be. This does not only apply to the proof, but also to a general method for solution of a problem, and more abstract concepts. On the other hand, counter examples is helpful for students to understand the importance of details, or common mistakes. For instance, if I notice a common mistake done in the previous years, I warn them in my tutorials about it and show them why it leads to a wrong result by choosing a counter example.

In addition, starting to the semester with some motivation about the topic in the broader sense will also get the students' attention to the lecture material throughout the semester. This includes why the subject is invented, or why it is important in terms of its applications to other areas, or even examples from future content. This will also allow the student to realize that learning is not always linear. I believe that the syllabus of the lecture should be self contained as much as possible. However, although sometimes providing first examples or applications from the future material, then teaching the content might make the students get a bit out of their comfort zone, it will help them to internalize the material much better when the time comes. I also tend to do the opposite of this, by reminding them a bit of the previous content if I need to use it in my arguments. I also start my tutorials by telling them what we have covered the last time, and how we will continue this week, if the contents of the last two lectures are relevant.

I have also observed as a student that how a lecturer writes their lecture notes on the blackboard affect the student's learning. In my opinion, the notes should be as organized as possible; the closer it is to a good structured math book (or an online lecture note) the better. It is even better if it includes drawings and diagrams to improve the students' intuition. This method has also have some advantages for the people who could not attend the lectures. However this definitely should not mean that the lecture must consist purely of writing a book to thre blackboard, as this would make the students get lost, and lose interest to the module in general. The lecture should include in-class

discussions about the topic, interaction with students, etc. The notes should be written clearly, and be visible from the backseats in the classroom. Last but not least, using beamer slides, and using animations (like the ones in 3B1B youtube channel), programming simulations could boost the understanding of the student and make the lecture much more interesting.

Finally, I should note that as a tutor, knowing and learning more in abstract mathematics have affected my teaching simpler topics to the students in a much better way. I first realized it while I was a teaching assistant of Multivariable Calculus. Prior to that year, I had a chance to audit Differential Geometry module and learn about tangent spaces and what a differential is in a much more abstract way than we learned at Multivariable Calculus. I felt that I was much more confident with the material, and thanks to the knowledge I had, I could provide a much better intuition about these concepts, for example the multivariable chain rule, to the students. In the Mathematics: an Introduction module that I have been tutoring for the last 4 years, I integrate my intuition from abstract algebra to explain the students how to solve basic first order equations.

My Teaching Experience and Future Plans

I have been a teaching assistant and a tutor since 2017 - the last semester of my undergraduate at Koç University, and currently at UCD, I had a lot of experience with different groups of students. The problem sessions that I gave in Koç University were mainly aimed at mathematics students; which were Real Analysis I, Complex Analysis, Introduction to Abstract Mathematics, Discrete Mathematics; and also engineering and science students; such as Differential Equations, Multivariable Calculus.

In UCD, my tutorials were aimed at non-mathematicians, which were Mathematics for Agriculture, Mathematics: an Introduction, Intro to Analysis for Economics & Finance, and Foundations of Maths for Computer Science I.

Outside of my compulsory teaching duties, in 2017 I also tutored Abstract Algebra II (Galois Theory) in Koç Office of Learning and Teaching (KOLT), and I was a section leader at CS-Bridge that took place at Koç University. CS-Bridge was a programme which was co-organized by Stanford University and Koç University, aimed at high school students that were interested in but had no prior knowledge on programming. Lecturers from Stanford University taught the students the essentials of programming via using Stanford's ACM library in JAVA.

In the following years, if I have the chance, I would like to organize modules on various topics aimed both at undergraduate students and graduate students. Most of them would be closely related to Algebraic Number Theory and Arithmetic Geometry, but also include topics as Homological Algebra, Category Theory, etc. Personally, I believe that exams are not the best way to measure a student's performance and knowledge, hence in the modules I would organize, I would like to depend more on homeworks, group projects, in class presentations, and so on.

I am also planning to give lectures related to my research in Nesin Mathematics Village in Şirince, a village in İzmir, Turkey, after I complete my Ph.D. dissertation. It is a mathematics village, founded by Ali Nesin who is a mathematician (and the son of the Turkish author Aziz Nesin) who dedicated himself to make people ranging from elementary school students to graduate students love mathematics by showing them the real essence of mathematics, teaching it the way it is. Many of my mathematician friends realized they loved mathematics in that village. Therefore it

would be an incredible opportunity for me to get closer to my goal on inspiring and inviting people to see the beauty of mathematics, and the area that I am working on.