3.4

3. The characteristic equation is $4r^2 - 8r - 5 = 0$, with roots $r = -1/2\,, 5/2$. The general solution is $y(t) = c_1 e^{-t/2} + c_2 e^{5t/2}$.

6. The characteristic equation is $r^2 - 10r + 25 = 0$, with the double root $r = 5$. The general solution is $y(t) = c_1 e^{5t} + c_2 t\, e^{5t}$.

Problem 9 : The characteristic equation is

$25r^2 - 30r + 9 = 0$, with roots r_1 and r_2.

$\Delta = 30^2 - 4 \cdot 25 \cdot 9 = 0$

$r_1 = r_2 = \frac{30}{2 \cdot 25} = \frac{3}{5}$

The general solution is $y(t) = c_1 e^{+\frac{3}{5}t} + t \cdot c_2 \cdot e^{+3/5 t}$.

Problem 12: The characteristic equation is

$r^2 - 6r + 9 = 0$ with roots $r_1 = r_2 = 3$.

The general solution is,

$$\boxed{y(t) = c_1 e^{+3t} + t \cdot c_2 \cdot e^{+3t}}$$

From $y(0) = 0$ we get that,

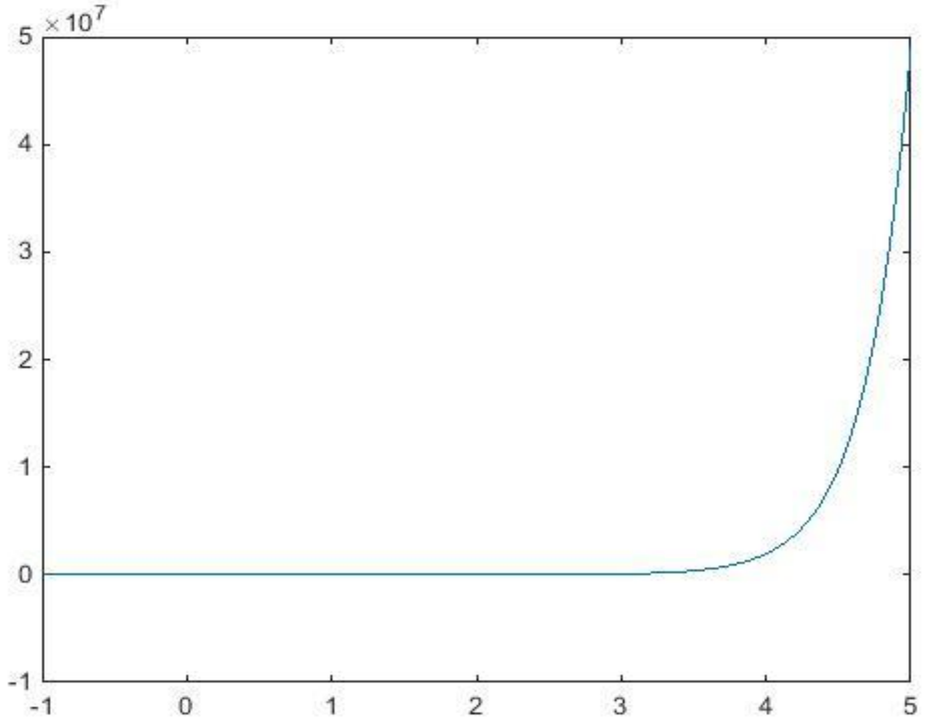
$\boxed{0 = c_1}$ So, $y(t) = c_2 t e^{+3t}$.

The derivative is $y'(t) = c_2 e^{+3t} + c_2 \cdot t \cdot 3 \cdot e^{+3t}$ and invoking the initial condition $y'(0) = 3$ we obtain,

$3 = c_2 e^0 + 0 \rightarrow \boxed{c_2 = 3}$

The solution is $\boxed{y(t) = 3te^{+3t}}$

As $t \to \infty$, we have $y(t) \to \infty$.



Problem 14 : the characteristic equation is

$r^2 + 4r + 4 = 0$ with roots $r_1 = r_2 = -2$

the general solution is

$$y(t) = c_1 e^{-2t} + c_2 t e^{-2t}$$

$$y'(t) = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$$

From, $y(-1) = 2$ and $y'(-1) = 3$ we've

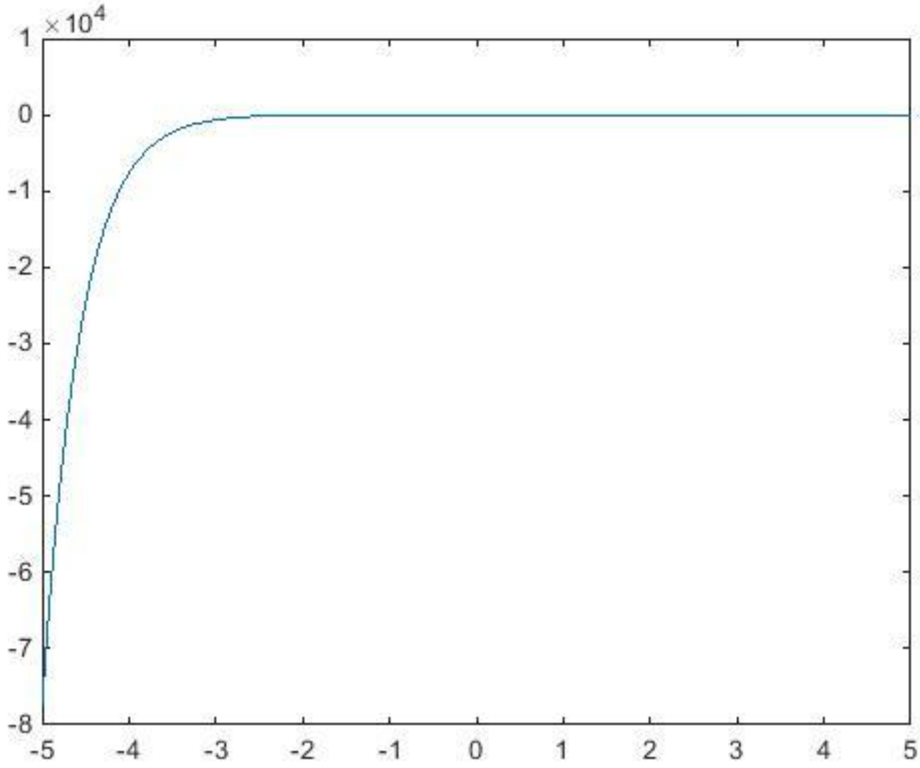
$$\begin{cases} 2 = c_1 e^2 - c_2 e^2 \\ 3 = -2c_1 e^2 + c_2 e^2 + 2c_2 e^2 \end{cases}$$

So that $c_2 = e^{-2} \cdot 7$ and $c_1 = 9e^{-2}$

Hence,

$$y(t) = 9e^{-2(t+1)} + 7t\, e^{-2(t+1)}$$

As $t \to \infty$ we have (clear) $y(t) \to 0$.



16. The characteristic roots are $r_1 = r_2 = 1/2$. Hence the general solution is given by $y(t) = c_1 e^{t/2} + c_2 t\, e^{t/2}$. Invoking the initial conditions, we require that $c_1 = 2$, and that $1 + c_2 = b$. The specific solution is $y(t) = 2e^{t/2} + (b-1)t\, e^{t/2}$. Since the second term dominates, the long-term solution depends on the sign of the coefficient $b - 1$. The critical value is $b = 1$.

23. Set $y_2(t) = t^3\, v(t)$. Substitution into the differential equation results in

$$t^2(t^3 v'' + 6t^2 v' + 6tv) - 4t(t^3 v' + 3t^2 v) + 6t^3 v = 0\,.$$

After collecting terms, we end up with $t^5 v'' + 2t^4 v' = 0$. Hence $v(t) = c_1 + c_2/t$, and thus $y_2(t) = c_1 t^3 + c_2 t^2$. Setting $c_1 = 0$ and $c_2 = 1$, we obtain $y_2(t) = t^2$.

25. Set $y_2(t) = t^{-1}v(t)$. Substitution into the differential equation into the differential equation results in

$$t^2(2t^{-3}v - t^{-2}v' + v''t^{-1} - t^{-2}v') + 3t(v't^{-1} - t^{-2}v) + t^{-1}v = 0.$$

After collecting terms, we end up with $tv'' + v' = 0$. This equation is linear in variable $w = v'$. It follows that $v'(t) = t^{-1} + c_1$, and $v(t) = \ln(t) + c_1 t + c_2$. Thus $y_2(t) = t^{-1}\ln(t) + c_1 t t^{-1} + c_2 t^{-1} = t^{-1}\ln(t) + c_1 + c_2 t^{-1}$. Setting $c_1 = 0$ and $c_2 = 0$, we obtain $y_2(t) = t^{-1}\ln(t)$.