6.6

By definition of Convolution,

$$
\begin{aligned}
((f * g) * h)\ (u) &= \int_{\mathbb{R}} (f * g)\ (x)\, h\,(u - x)\, dx \\
&= \int_{\mathbb{R}} \left[\int_{\mathbb{R}} f\,(y)\, g\,(x - y)\, dy \right] h\,(u - x)\, dx \\
&= \int_{\mathbb{R}} \int_{\mathbb{R}} f\,(y)\, g\,(x - y)\, h\,(u - x)\, dy\, dx.
\end{aligned}
$$

By Fubini's theorem we can switch the integration,

$$
\begin{aligned}
((f * g) * h)\ (u) &= \int_{\mathbb{R}} \int_{\mathbb{R}} f\,(y)\, g\,(x - y)\, h\,(u - x)\, dx\, dy \\
&= \int_{\mathbb{R}} f\,(y) \left[\int_{\mathbb{R}} g\,(x - y)\, h\,(u - x)\, dx \right] dy.
\end{aligned}
$$

Look at the inner integral, by translation invariant

$$\begin{aligned}\int_{\mathbb{R}} g(x-y)\,h(u-x)\,dx &= \int_{\mathbb{R}} g((x+y)-y)\,h(u-(x+y))\,dx \\ &= \int_{\mathbb{R}} g(x)\,h((u-y)-x)\,dx \\ &= (g*h)\ (u-y)\,.\end{aligned}$$

So we have shown that

$$((f*g)*h)\ (u) = \int_{\mathbb{R}} f(y)\ (g*h)\ (u-y)\,dy,$$

which by definition is $(f*(g*h))\ (u)$. Hence convolution is associative

3. It follows directly that

$$(f*f)(t) = \int_0^t \cos(t-\tau)\cos(\tau)\,d\tau = \frac{1}{2}\int_0^t [\cos(t-2\tau)+\cos(t)]d\tau = \frac{1}{2}(\sin t + t\cos t).$$

The range of the resulting function is $\mathbb{R}$.

5. We have $\mathcal{L}[e^{-t}] = 1/(s+1)$ and $\mathcal{L}[\sin 2t] = 2/(s^2+4)$. Based on Theorem 6.6.1,

$$\mathcal{L}\left[\int_0^t e^{-(t-\tau)}\sin(2\tau)\,d\tau\right] = \frac{1}{s+1}\cdot\frac{2}{s^2+4} = \frac{2}{(s+1)(s^2+4)}\,.$$

7. We have $f(t) = (g*h)(t)$, in which $g(t) = \sin t$ and $h(t) = \cos 2t$. The transform of the convolution integral is

$$\mathcal{L}\left[\int_0^t g(t-\tau)h(\tau)\,d\tau\right] = \frac{1}{s^2+1}\cdot\frac{s}{s^2+4} = \frac{s}{(s^2+1)(s^2+4)}\,.$$

Problem 8 : $F(s) = \dfrac{1}{s^4(s^2+4)}$

note that,

$\mathcal{L}^{-1}\left(\dfrac{1}{s^4}\right) = \dfrac{1}{6}\cdot t^3$ and

$\mathcal{L}^{-1}\left(\dfrac{1}{s^2+4}\right) = \dfrac{1}{2}\cdot \sin 2t$

Based on the convolution theorem,

$$\mathcal{L}^{-1}\left(\frac{1}{s^4}\cdot\frac{1}{s^2+4}\right) = \frac{1}{6}\cdot\frac{1}{2}\cdot\int_0^t (t-\tau)^3\cdot\sin 2\tau\, d\tau$$

10. We first note that

$$\mathcal{L}^{-1}\left[\frac{1}{(s+1)^3}\right] = \frac{1}{2}t^2\, e^{-t} \quad \text{and} \quad \mathcal{L}^{-1}\left[\frac{1}{s^2+4}\right] = \frac{1}{2}\sin 2t\,.$$

Based on the convolution theorem,

$$\begin{aligned}\mathcal{L}^{-1}\left[\frac{1}{(s+1)^3(s^2+4)}\right] &= \frac{1}{4}\int_0^t (t-\tau)^2 e^{-(t-\tau)} \sin 2\tau\, d\tau \\ &= \frac{1}{4}\int_0^t \tau^2\, e^{-\tau} \sin(2t-2\tau)\, d\tau\,.\end{aligned}$$

13. Taking the initial conditions into consideration, the transform of the ODE is

$$s^2\,Y(s) - s - 1 + \omega^2\,Y(s) = G(s).$$

Solving for the transform of the solution,

$$Y(s) = \frac{s+1}{s^2+\omega^2} + \frac{G(s)}{s^2+\omega^2}.$$

As shown in a related situation, Problem 11,

$$\mathcal{L}^{-1}\left[\frac{G(s)}{s^2+\omega^2}\right] = \frac{1}{\omega}\int_0^t \sin(\omega(t-\tau))\,g(\tau)\,d\tau\,.$$

Hence the solution of the IVP is

$$y(t) = \cos(\omega t) + \frac{1}{\omega}\,\sin(\omega t) + \frac{1}{\omega}\int_0^t \sin(\omega(t-\tau))\,g(\tau)\,d\tau\,.$$

17. Taking the initial conditions into consideration, the transform of the ODE is

$$s^2\,Y(s) - s + 2 + 4\,[s\,Y(s) - 1] + 4\,Y(s) = G(s).$$

Solving for the transform of the solution,

$$Y(s) = \frac{s+2}{(s+2)^2} + \frac{G(s)}{(s+2)^2}.$$

We can write

$$\frac{s+2}{(s+2)^2} = \frac{1}{s+2}.$$

It follows that

$$\mathcal{L}^{-1}\left[\frac{1}{s+2}\right] = e^{-2t}.$$

Based on the convolution theorem, the solution of the IVP is

$$y(t) = e^{-2t} + \int_0^t (t-\tau)e^{-2(t-\tau)}g(\tau)\,d\tau\,.$$

19. The transform of the ODE (given the specified initial conditions) is

$$s^4\,Y(s) - Y(s) = G(s).$$

Solving for the transform of the solution,

$$Y(s) = \frac{G(s)}{s^4-1}.$$

First write

$$\frac{1}{s^4-1} = \frac{1}{2}\left[\frac{1}{s^2-1} - \frac{1}{s^2+1}\right].$$

It follows that

$$\mathcal{L}^{-1}\left[\frac{1}{s^4-1}\right] = \frac{1}{2}\left[\sinh t - \sin t\right]\,.$$

Based on the convolution theorem, the solution of the IVP is

$$y(t) = \frac{1}{2}\int_0^t \left[\sinh(t-\tau) - \sin(t-\tau)\right]g(\tau)\,d\tau\,.$$

7.1

1. Introduce the variables $x_1 = u$ and $x_2 = u'$. It follows that $x_1' = x_2$ and

$$x_2' = u'' = -2u - 3\,u'.$$

In terms of the new variables, we obtain the system of two first order ODEs

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -2x_1 - 3\,x_2\,. \end{aligned}$$

3. First divide both sides of the equation by t^3, and write

$$u'' = -\frac{1}{t^2}\,u' - (\frac{1}{t} - \frac{1}{4t^3})u\,.$$

Set $x_1 = u$ and $x_2 = u'$. It follows that $x_1' = x_2$ and

$$x_2' = u'' = -\frac{1}{t^2}\,u' - (\frac{1}{t} - \frac{1}{4t^3})u\,.$$

We obtain the system of equations

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -(\frac{1}{t} - \frac{1}{4t^3})x_1 - \frac{1}{t^2}\,x_2\,. \end{aligned}$$

5. Let $x_1 = u$ and $x_2 = u'$; then $u'' = x_2'$. In terms of the new variables, we have

$$x_2' + 2x_2 + 4\,x_1 = 2\,\cos 3t$$

with the initial conditions $x_1(0) = 1$ and $x_2(0) = -2$. The equivalent first order system is

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -4\,x_1 - 2\,x_2 + 2\,\cos 3t \end{aligned}$$

with the above initial conditions.

7.(a) Solving the first equation for x_2, we have $x_2 = x_1' + 2x_1$. Substitution into the second equation results in $(x_1' + 2x_1)' = x_1 - 2(x_1' + 2x_1)$. That is, $x_1'' + 4\,x_1' + 3\,x_1 = 0$. The resulting equation is a second order differential equation with constant coefficients. The general solution is $x_1(t) = c_1 e^{-t} + c_2 e^{-3t}$. With x_2 given in terms of x_1, it follows that $x_2(t) = c_1 e^{-t} - c_2 e^{-3t}$.

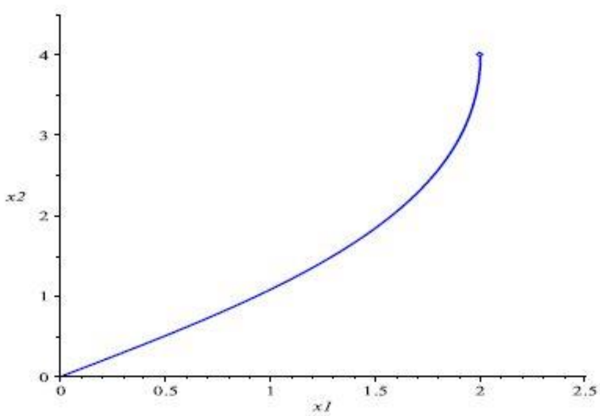
(b) Imposing the specified initial conditions, we obtain

$$c_1 + c_2 = 2, \qquad c_1 - c_2 = 4,$$

with solution $c_1 = 3$ and $c_2 = -1$. Hence

$$x_1(t) = 3e^{-t} - e^{-3t} \text{ and } x_2(t) = 3e^{-t} + e^{-3t}.$$

(c)



1. **(a)** $\begin{pmatrix} 6 & -6 & 3 \\ 5 & 9 & -2 \\ 2 & 3 & 8 \end{pmatrix}$ **(b)** $\begin{pmatrix} -15 & 6 & -12 \\ 7 & -18 & -1 \\ -26 & -3 & -5 \end{pmatrix}$

(c) $\begin{pmatrix} 6 & -12 & 3 \\ 4 & 3 & 7 \\ 9 & 12 & 0 \end{pmatrix}$ **(d)** $\begin{pmatrix} -8 & -9 & 11 \\ 14 & 12 & -5 \\ 5 & -8 & 5 \end{pmatrix}$

7.2

3.(c,d)

$$\begin{aligned} \mathbf{A}^T + \mathbf{B}^T &= \begin{pmatrix} -2 & 3 & 2 \\ 1 & 0 & -1 \\ 3 & -3 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 1 & -2 \\ 2 & -1 & 1 \\ 2 & -1 & 0 \end{pmatrix} \\ &= \begin{pmatrix} -1 & 4 & 0 \\ 3 & -1 & 0 \\ 5 & -4 & 1 \end{pmatrix} = (\mathbf{A}+\mathbf{B})^T. \end{aligned}$$

4.(b)

$$\overline{\mathbf{A}} = \begin{pmatrix} 3+2i & 1-2i \\ 2+i & -2-3i \end{pmatrix}.$$

(c) By definition,

$$\mathbf{A}^* = \overline{\mathbf{A}^T} = (\overline{\mathbf{A}})^T = \begin{pmatrix} 3+2i & 2+i \\ 1-2i & -2-3i \end{pmatrix}.$$

10. $\begin{pmatrix} \frac{3}{11} & -\frac{4}{11} \\ \frac{2}{11} & \frac{1}{11} \end{pmatrix}$

8.(a) $\mathbf{x}^T\mathbf{y}= 2(-1+i)+2(4i)+(1-i)(2+i) = 1+9i.$

(b) $\mathbf{y}^T\mathbf{y}= (-1+i)^2+2^2+(2+i)^2 = 7+2i.$

(c) $(\mathbf{x},\mathbf{y}) = 2(-1-i)+2(4i)+(1-i)(2-i) = -1+3i.$

(d) $(\mathbf{y},\mathbf{y}) = (-1+i)(-1-i)+2^2+(2+i)(2-i) = 11.$

12. $\begin{pmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{pmatrix}$

8.(a) $\mathbf{x}^T\mathbf{y}= 2(-1+i)+2(4i)+(1-i)(2+i)=1+9i.$

(b) $\mathbf{y}^T\mathbf{y}= (-1+i)^2+2^2+(2+i)^2=7+2i.$

(c) $(\mathbf{x},\mathbf{y})=2(-1-i)+2(4i)+(1-i)(2-i)=-1+3i.$

(d) $(\mathbf{y},\mathbf{y})=(-1+i)(-1-i)+2^2+(2+i)(2-i)=11.$

7.3

4. The augmented matrix is

$$\begin{pmatrix} 1 & 2 & -1 & | & 0 \\ 2 & 1 & 1 & | & 0 \\ 1 & -1 & 2 & | & 0 \end{pmatrix}.$$

Adding -2 times the first row to the second row and subtracting the first row from the third row results in

$$\begin{pmatrix} 1 & 2 & -1 & | & 0 \\ 0 & -3 & 3 & | & 0 \\ 0 & -3 & 3 & | & 0 \end{pmatrix}.$$

Adding the negative of the second row to the third row results in

$$\begin{pmatrix} 1 & 2 & -1 & | & 0 \\ 0 & -3 & 3 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{pmatrix}.$$

We evidently end up with an equivalent system of equations

$$\begin{aligned} x_1+2x_2-x_3&=0 \\ -x_2+x_3&=0\,. \end{aligned}$$

Since there is no unique solution, let $x_3=\alpha$, where α is arbitrary. It follows that $x_2=\alpha$, and $x_1=-\alpha$. Hence all solutions have the form

$$x=\alpha\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}.$$

13. By inspection, we find that

$$\mathbf{x}^{(1)}(t) - 2\mathbf{x}^{(2)}(t) = \begin{pmatrix} -e^{-t} \\ 0 \end{pmatrix}.$$

Hence $4\,\mathbf{x}^{(1)}(t) - 8\,\mathbf{x}^{(2)}(t) + \mathbf{x}^{(3)}(t) = \mathbf{0}$, and the vectors are linearly dependent.

18. The eigenvalues λ and eigenvectors $\mathbf{x}$ satisfy the equation

$$\begin{pmatrix} -3-\lambda & 1 \\ 1 & -3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

For a nonzero solution, we must have $(-3-\lambda)(-3-\lambda) - 1 = 0$, that is,

$$\lambda^2 + 6\lambda + 8 = 0\,.$$

The eigenvalues are $\lambda_1 = -4$ and $\lambda_2 = -2$. For $\lambda_1 = -4$, the system of equations becomes

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix},$$

which reduces to $x_1 + x_2 = 0$. A solution vector is given by $\mathbf{x}^{(1)} = (1\,,-1)^T$. Substituting $\lambda = \lambda_2 = -2$, we have

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

The equations reduce to $x_1 = x_2$. Hence a solution vector is given by $\mathbf{x}^{(2)} = (1\,,1)^T$.

7.1.) 9a. $\begin{cases} x_1' = 1.25 x_1 + 0.75 x_2 \\ x_2' = 0.75 x_1 + 1.25 x_2 \end{cases}$ $\quad x_1(0) = -2$, $\quad x_2(0) = 3$

Solve the first equation for x_2 first: $x_2 = \frac{x_1'}{0.75} - \frac{5}{3} x_1$

Substitute into the second eqn: $\quad 0.75 = \frac{3}{4}$, $1.25 = \frac{5}{4}$

$$\Rightarrow \frac{4}{3} x_1'' - \frac{5}{3} x_1' = \frac{3}{4} x_1 + \frac{5}{4}\left(\frac{4}{3} x_1' - \frac{5}{3} x_1\right)$$

$$\Rightarrow \frac{4}{3} x_1'' - \frac{10}{3} x_1' + \frac{4}{3} x_1 = 0 \quad // . \quad \equiv (2x_1'' - 5x_1' + 2x_1 = 0).$$

9.b. The general solution is $x_1(t) = c_1 e^{t/2} + c_2 e^{2t}$.

x_2 in terms of x_1:

$$x_2 = \frac{4}{3} \cdot \frac{c_1}{2} e^{t/2} + \frac{4}{3} \cdot 2 c_2 e^{2t} - \frac{5}{3} c_1 e^{t/2} - \frac{5}{3} c_2 e^{2t}.$$

$$x_2 = -c_1 e^{t/2} + c_2 e^{2t}.$$

$\begin{cases} x_1(0) = -2 \\ x_2(0) = 3 \end{cases} \Rightarrow \begin{cases} c_1 + c_2 = -2 \\ -c_1 + c_2 = 3 \end{cases} \Rightarrow c_2 = 1/2, \; c_1 = -\frac{5}{2}.$

Hence, $x_1 = -\frac{5}{2} e^{t/2} + \frac{1}{2} e^{2t}$, $\quad x_2 = \frac{5}{2} e^{t/2} + \frac{1}{2} e^{2t}$.

7.3) 1. $\begin{cases} x_1 - x_3 = 0 \\ 3x_1 + x_2 + x_3 = 1 \\ -x_1 + x_2 + 2x_3 = -1 \end{cases}$

The augmented matrix is $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 3 & 1 & 1 & 1 \\ -1 & 1 & 2 & -1 \end{array}\right)$

Adding -3 times the first row to the second row and adding the first row to the third row: $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 1 & 1 & -1 \end{array}\right)$

Substracting the second row from the third row: $\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & -3 & -2 \end{array}\right)$

We end up with an equivalent system of eqns:

$$\left.\begin{array}{r} x_1 - x_3 = 0 \\ x_2 + 4x_3 = 1 \\ -3x_3 = -2 \end{array}\right\} \Rightarrow \begin{array}{l} x_3 = 2/3 \\ x_2 = -5/3 \\ x_1 = 2/3 \end{array}$$

2. The augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 2 & 1 & 1 & 2 \\ 1 & -1 & 2 & 3 \end{array}\right) \xrightarrow[-1\times\text{ — } + 3^{rd}\text{ row}]{-2\times 1^{st}\text{ row} + 2^{nd}\text{ row}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & -3 & 3 & 2 \end{array}\right) \xrightarrow{2^{nd} + 3^{rd}} \left(\begin{array}{ccc|c} 1 & 2 & -1 & 1 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 2 \end{array}\right)$$

$\Rightarrow$ From the third row, observe $0.x_1 + 0.x_2 + 0.x_3 = 2$!

So no solution.

7.3) 7. $x^{(1)} = (1,1,1)$, $x^{(2)} = (0,1,1)$, $x^{(3)} = (1,0,1)$.

column vector

$$X = (x^{(1)} \;\; x^{(2)} \;\; x^{(3)}) = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

$\det X = 1(1-0) + 1(1-1) = 1 \neq 0$. So they are linearly independent.

9. $X = \begin{pmatrix} 1 & -1 & -2 & -3 \\ 2 & 0 & -1 & 0 \\ 2 & 3 & 1 & -1 \\ 3 & 1 & 0 & 3 \end{pmatrix} \Rightarrow \det X = 0.$

Hence the vectors are linearly dependent. In order to find a linear relationship between them, write $c_1 x^{(1)} + c_2 x^{(2)} + c_3 x^{(3)} + c_4 x^{(4)} = 0$. Let $c = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{pmatrix}$

$\Rightarrow X c = 0$, $0 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$.

Perform row operations on X

$\begin{cases} -2 \times 1^{st} \text{ row add to 2nd and } 3^{rd} \text{ row} \\ -3 \times \text{ ——— last row} \end{cases}$

$$\Rightarrow \left(\begin{array}{cccc|c} 1 & -1 & -2 & -3 & 0 \\ 0 & 2 & 3 & 6 & 0 \\ 0 & 5 & 5 & 5 & 0 \\ 0 & 4 & 6 & 12 & 0 \end{array}\right) \to \left(\begin{array}{cccc|c} 1 & -1 & -2 & -3 & 0 \\ 0 & 2 & 3 & 6 & 0 \\ 0 & 0 & -5 & -10 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

$-5/2 \times 2^{nd}$ row $+ 3^{rd}$ row

$-2 \times$ ——— $+$ last row

$$\Rightarrow \begin{aligned} c_1 - c_2 - 2c_3 - 3c_4 &= 0 \\ 2c_2 + 3c_3 + 6c_4 &= 0 \\ -5c_3 - 10c_4 &= 0 \end{aligned}$$

Set $c_4 = 1$, $\Rightarrow c_3 = 4$, $c_2 = -3$, $c_1 = 2$,

Hence $2x^{(1)} - 3x^{(2)} + 4x^{(3)} + x^{(4)} = 0$.

7.3.) 16. The eigenvalues λ and eigenvectors x satisfy the equation
$$\begin{pmatrix} 5-\lambda & 3 \\ -1 & 1-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

For a non-zero solution, we must have
$$(5-\lambda)(1-\lambda)+3=0$$
$$\lambda^2-6\lambda+8=0 \Rightarrow \lambda_{1,2}=2,4$$

For $\lambda_1=2$ we have:
$$\begin{pmatrix} 3 & 3 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0 \qquad 3x_1+3x_2=0 \Rightarrow x_1=-x_2$$

Hence $x^{(1)} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

For $\lambda=4$ we have:
$$\begin{pmatrix} 1 & 3 \\ -1 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow x_1=-3x_2$$

Hence $x^{(2)} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$

22. $$\begin{pmatrix} 1-\lambda & 0 & 0 \\ 2 & 1-\lambda & -2 \\ 3 & 2 & 1-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

For a non-zero sol. we must have:
$$\underbrace{(1-\lambda)}_{0}\ \text{or}\ \underbrace{((1-\lambda)^2+4)}_{0}=0$$
$$\Rightarrow \lambda_1=1, \quad \lambda^2-2\lambda+5=0$$
$$\lambda_{2,3}=\frac{+2\mp\sqrt{-16}}{2}=\frac{2\mp4i}{2}$$
$$\lambda_{2,3}=1\mp2i$$

22. cont. For $\lambda_1 = 1$ we have:

$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & 0 & -2 \\ 3 & 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$2x_1 - 2x_3 = 0 \Rightarrow \begin{cases} x_1 = x_3 \\ 2x_2 = -3x_1 \end{cases}$$
$$3x_1 + 2x_2 = 0 \Rightarrow$$

$$\underset{-3}{2 x_2} = \underset{2}{-3x_1}$$

Hence $x^{(1)} = \begin{pmatrix} 2 \\ -3 \\ 2 \end{pmatrix}$

$\lambda_2 = 1 - 2i$ we have:

$$\begin{pmatrix} 2i & 0 & 0 \\ 2 & 2i & -2 \\ 3 & 2 & 2i \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{cases} 2ix_1 = 0 \\ 2x_1 + 2i \cdot x_2 - 2x_3 = 0 \\ 3x_1 + 2x_2 + 2i x_3 = 0 \end{cases}$$

$\Rightarrow \quad x_1 = 0 \quad \Rightarrow \quad 2ix_2 - 2x_3 = 0.$

Hence $x^{(2)} = \begin{pmatrix} 0 \\ 2 \\ 2i \end{pmatrix}$. $\quad x^{(3)}$ is $\overline{x^{(2)}} = \begin{pmatrix} 0 \\ 2 \\ -2i \end{pmatrix}$.