1.3

3. The differential equation is fourth order, since the highest derivative of the function y is of order four. The equation is also linear, since the terms containing the dependent variable is linear in y and its derivatives.

5. The differential equation is second order. Furthermore, the equation is nonlinear, since the dependent variable y is an argument of the sine function, which is not a linear function.

7. $y_1(t) = e^t$, so $y_1'(t) = y_1''(t) = e^t$. Hence $y_1'' - y_1 = 0$. Also, $y_2(t) = \sinh t$, so $y_1'(t) = \cosh t$ and $y_2''(t) = \sinh t$. Thus $y_2'' - y_2 = 0$.

9. $y(t) = 4t + t^2$, so $y'(t) = 4 + 2t$. Substituting into the differential equation, we have $t(4 + 2t) - (4t + t^2) = 4t + 2t^2 - 4t - t^2 = t^2$. Hence the given function is a solution.

21. The order of the partial differential equation is two, since the highest derivative, in fact each one of the derivatives, is of second order. The equation is linear, since the left hand side is a linear function of the partial derivatives.

2.1

17. The integrating factor is $\mu(t) = e^{-4t}$, and the differential equation can be written as $(e^{-4t}\, y)' = 1$. Integrating, we obtain $e^{-4t}\, y(t) = t + c$. Invoking the specified initial condition results in the solution $y(t) = (t + 2)e^{4t}$.

19. After writing the equation in standard form, we find that the integrating factor is $\mu(t) = e^{\int (5/t)\, dt} = t^5$. Multiplying both sides by $\mu(t)$, the equation can be written as $(t^5\, y)' = t\, e^{-t}$. Integrating both sides results in $t^5 y(t) = -(t + 1)e^{-t} + c$. Letting $t = -1$ and setting the value equal to zero gives $c = 0$. Hence the specific solution of the initial value problem is $y(t) = -(t^{-4} + t^{-5})e^{-t}$.

29.(a) The integrating factor is $\mu(t) = e^{t/4}$, and the differential equation can be written as $(e^{t/4}\, y)' = 3\, e^{t/4} + 2\, e^{t/4} \cos 2t$. After integration, we get that the general solution is $y(t) = 12 + (8 \cos 2t + 64 \sin 2t)/65 + ce^{-t/4}$. Invoking the initial condition, $y(0) = 0$, the specific solution is $y(t) = 12 + (8 \cos 2t + 64 \sin 2t - 788\, e^{-t/4})/65$. As $t \to \infty$, the exponential term will decay, and the solution will oscillate about an average value of 12, with an amplitude of $8/\sqrt{65}$.

(b) Solving $y(t) = 12$, we obtain the desired value $t \approx 10.0658$.

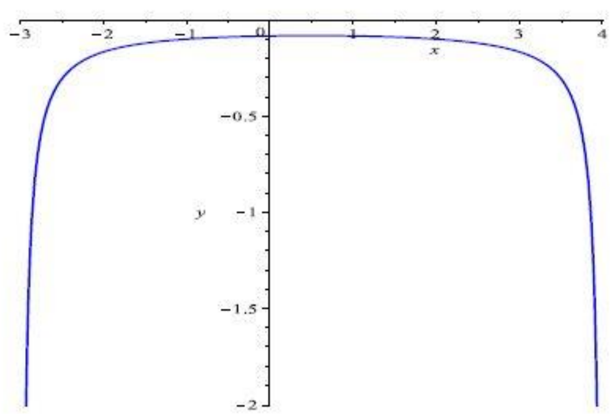
2.2

3. The differential equation may be written as $y^{-2}dy = -\cos x\,dx$. Integrating both sides of the equation, with respect to the appropriate variables, we obtain the relation $-y^{-1} = -\sin x + c$. That is, $(c + \sin x)y = 1$, in which c is an arbitrary constant. Solving for the dependent variable, explicitly, $y(x) = 1/(c + \sin x)$.

5. Write the differential equation as $\cos^{-2} 4y\,dy = \cos^2 x\,dx$, which also can be written as $\sec^2 4y\,dy = \cos^2 x\,dx$. Integrating both sides of the equation, with respect to the appropriate variables, we obtain the relation $\tan 4y = 2\sin x\,\cos x + 2x + c$.

9.(a) The differential equation is separable, with $y^{-2}dy = (1-2x)dx$. Integration yields $-y^{-1} = x - x^2 + c$. Substituting $x = 0$ and $y = -1/12$, we find that $c = 12$. Hence the specific solution is $y = 1/(x^2 - x - 12)$.

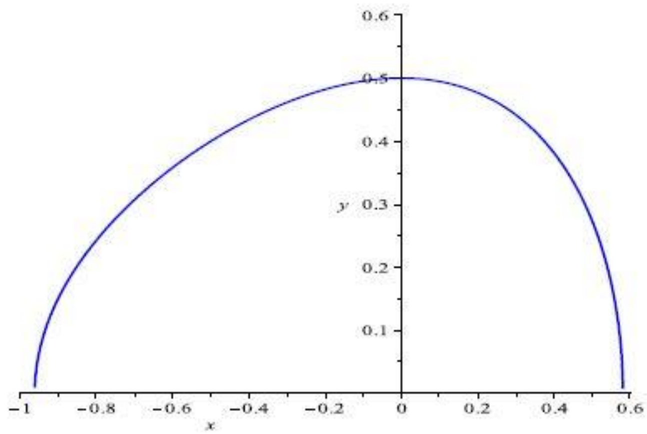
(b)



(c) Note that $x^2 - x - 12 = (x+3)(x-4)$. Hence the solution becomes singular at $x = -3$ and $x = 4$, so the interval of existence is $(-3, 4)$.

11.(a) Rewrite the differential equation as $x\,e^x dx = -2y\,dy$. Integrating both sides of the equation results in $x\,e^x - e^x = -y^2 + c$. Invoking the initial condition, we obtain $c = -3/4$. Hence $y^2 = e^x - x\,e^x - 3/4$. The explicit form of the solution is $y(x) = \sqrt{e^x - x\,e^x - 3/4}$. The positive sign is chosen, since $y(0) = 1/2$.

(b)



(c) The function under the radical becomes negative near $x \approx -0.96$ and $x \approx 0.58$.

MATH 204 - HW #1

2.1: 13) Find the solution of the given initial value problems:

$$y' - y = 2te^{2t}, \quad y(0) = 1$$

The integrating factor is $\mu(t) = e^{-t}$. Multiplying both sides by $\mu(t)$, the equation can be written as $(e^{-t}y)' = 2te^{t}$.

Integrating both sides of the equation results in the general solution $y(t) = 2te^{2t} - 2e^{2t} + ce^{t} = 2(t-1)e^{2t} + ce^{t}$.

Invoking the specified initial condition results in the equation $y(t) = 2(t-1)e^{2t} + 3e^{t}$.

2.1: 15) $ty' + 2y = t^2 - t + 1, \quad y(1) = \frac{1}{2}, \quad t > 0.$

After writing the equation in standard form,

$$y' + \frac{2}{t}y = t - 1 + \frac{1}{t}$$

we find that the integrating factor is $\mu(t) = e^{\int \frac{2}{t}dt} = t^2$.

Multiplying both sides results in $t^2y' + 2ty = t^3 - t^2 + t$, which can be written as $(t^2y)' = t^3 - t^2 + t$.

Integrating both sides results in $t^2y = \frac{t^4}{4} - \frac{t^3}{3} + \frac{t^2}{2} + c$.

letting $t = 1$ and setting the initial value equal to $\frac{1}{2}$, gives $c = \frac{1}{12}$. Hence the specific solution of the initial value problem is $y(t) = \frac{t^2}{4} - \frac{t}{3} + \frac{1}{2} + \frac{1}{12t^2}$

2.2 : 1) Solve the given differential equation: $y' = \frac{3x^2}{y}$

The differential equation may be written as $y\,dy = 3x^2\,dx$. Integrating both sides, with respect to appropriate variables, we obtain the relation $\frac{y^2}{2} = x^3 + c$. That is,

$$y^2(x) - 2x^3 = 2c \quad , \; y \neq 0 .$$

2.2) 15 Find the solution of the given initial value problem. Plot the graph of the solution. Determine (approximately) the interval in which solution is defined.

$$y' = 2x/(1+2y) \; , \; y(1) = 0 .$$

$\Rightarrow (1+2y)\,dy = 2x\,dx$ (integrate both sides)

$\Rightarrow y + y^2 = x^2 + C$,

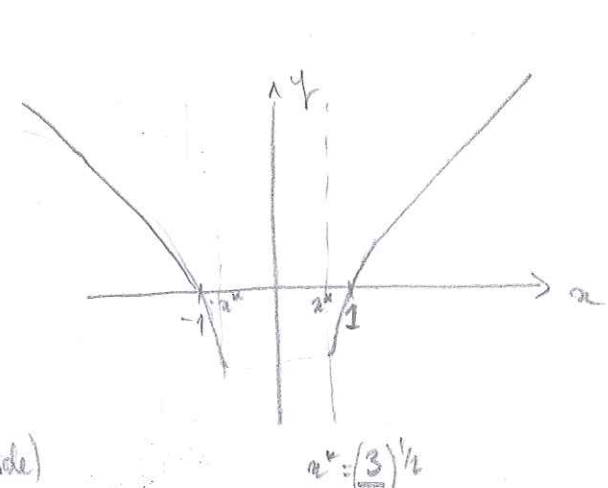
$\Rightarrow x = 1, y = 0 \Rightarrow C = -1$. (initial value)

$\Rightarrow y + y^2 = x^2 - 1$, (multiply by 4 both sides)

$\Rightarrow 4y^2 + 4y + 1 = 4x^2 - 3$ (-1 shifts to left side)

$\Rightarrow (2y+1)^2 = 4x^2 - 3$.

$\Rightarrow y(x) = \frac{1}{2}\left(\sqrt{4x^2-3} - 1\right)$, $|x| > \left(\frac{3}{4}\right)^{1/2}$



$x^* = \left(\frac{3}{4}\right)^{1/2}$

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HW 1

Section 1.2 - Problem 7 : $dp/dt = 0.5p - 450.$

(a) Solving the diff. eqn

$$\frac{dp}{dt} = \frac{p-900}{2}$$

$$\frac{dp}{p-900} = \frac{dt}{2}$$ integrate both sides,

$$\int \frac{dp}{p-900} = \int \frac{1}{2} dt$$ so that $\ln|p-900| = \frac{t}{2} + C$ where $C \in \mathbb{R}$.

$$\Rightarrow |p-900| = \exp\left(\frac{t}{2}\right) \cdot \exp(C)$$

$$\boxed{p = 900 \pm e^{\frac{t}{2}} \cdot e^{C}}$$ induce initial value $p(0) = 800$

to get $p(0) = 900 \pm e^{0} \cdot e^{C} = 800$ then $e^{C} = 100$ or $C = \ln(100)$.

So that $\boxed{p = 900 - e^{\frac{t}{2}} \cdot 100}$

Find t such that $p(t) = 0$ i.e

$$900 - 100 \cdot e^{t/2} = 0 \rightarrow e^{\frac{t}{2}} = +9 \quad \text{OR} \quad \frac{t}{2} = \ln(9)$$

OR $t \approx 2\ln(9) \approx 4.39$ months

(b) $p(0) = p_0$ where $0 < p_0 < 900$.

From (a) $p(t) = 900 - e^{t/2} \cdot e^{C}$ since $0 < p_0 < 900$

$$p_0 = p(0) = 900 - e^{C} \rightarrow e^{C} = (900 - p_0)$$

$$p(t) = 900 - (900 - p_0) e^{t/2}$$

if $p(t)=0$ we have extinction

$$0 = p(t) = 900 - (900-p_0)e^{t/2}$$

i.e $\quad e^{t/2} = \frac{900}{900-p_0}$

OR $\quad t = 2 \cdot \ln\left(\frac{900}{900-p_0}\right)$ months

(C) Find $p(0)$ if $p(t)$ becomes extinct in 1 year = 12 months.

From (b) $\quad$ 12 months $= 2\ln\left(\frac{900}{900-p_0}\right)$

$$\Rightarrow \quad 6 = \ln\left(\frac{900}{900-p_0}\right)$$

$\frac{900}{900-p_0} = e^6$ OR $\frac{900}{e^6} = 900 - p_0$

from which we obtain

$$p_0 = 900 - \frac{900}{e^6} = 900 \cdot (1 - e^{-6}) \approx 897.8$$

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Section 1.2 – Problem 9 :

$$\frac{dv}{dt} = 9.8 - \frac{v}{5} \quad \text{and} \quad v(0) = 0$$

(a) From example 2, page 13-14 we already have that,

$$v(t) = 49\left(1 - e^{-t/5}\right) \quad \text{eqn 26, page 14.}$$

~~We'll find max velocity,~~

$\cancel{\frac{dv}{dt} = 0 \Rightarrow (49 - 49 e^{-t/5})' = -49\cdot\left(-\frac{1}{5}\right) e^{-t/5} = 0.}$

It's limiting velocity is, $v_* = \lim_{t \to \infty} (49)\left(1 - e^{-t/5}\right) = 49$

and 95% of limiting velocity is, $v = (0.95) \cdot 49 = 46.55$

So, $46.55 = 49 \cdot \left(1 - e^{-t/5}\right)$ yields,

$$0.95 = 1 - e^{-t/5}$$

$$e^{-t/5} = 1 - 0.95 = 0.05 = \frac{1}{20}$$

$$-\frac{t}{5} = \ln(1/20) = -\ln(20)$$

OR $t = 5 \cdot \ln(20) \approx 14.98 \text{ sec}$

(b) From distance eqn 29, page 14 we already have

$x(t) = 49t + 245 e^{-t/5} - 245$ thus at $t = 14.98\,s$

we get $x = 49(5 \ln 20) + 245 \cdot e^{-\ln(20)} - 245 \cong 501.20\,m$

Section 1.3 – Problem 11 :

Verify that $y_1(t) = t^{1/2}$ and $y_2(t) = t^{-1}$ are solutions of $2t^2 y'' + 3ty' - y = 0 \quad t>0$

$\underline{y_1 = t^{1/2}}$; $\quad y_1' = +\frac{1}{2} \cdot t^{-\frac{1}{2}}$; $\quad y_1'' = -\frac{1}{4} \cdot t^{-3/2}$

so $\quad 2t^2 \cdot (y_1'') + 3ty_1' - y_1 = 2t^2 \cdot \left(-\frac{1}{4} \cdot t^{-\frac{3}{2}}\right) + 3t\left(\frac{1}{2} t^{-\frac{1}{2}}\right) - t^{1/2}$

$$= -\frac{1}{2} t^{1/2} + \frac{3}{2} \cdot t^{1/2} - t^{1/2} = 0$$

$\underline{y_2 = t^{-1}}$; $\quad y_2' = -1 \cdot t^{-2}$ and $y_2'' = 2 \cdot t^{-3}$

so $\quad 2t^2 (y_2'') + 3t(y_2') - y_2 = 2t^2 \cdot (2t^{-3}) + 3t(-t^{-2}) - t^{-1}$

$$= 4t^{-1} - 3t^{-1} - t^{-1} = 0$$

as desired

Section 2.1 – Problem 21

$y' - 2y = 2\cos t \; , \; y(0) = a.$

(b) I.F is $\mu(t) = \exp(\int -2\,dt) = \exp(-2t) = e^{-2t}$

then multiply by e^{-2t} both sides of eqn to get,

$$e^{-2t} \cdot y' - 2y e^{-2t} = 2\cos t \cdot e^{-2t}$$

$$(ye^{-2t})' = 2\cos t \, e^{-2t} \quad \text{integrate,}$$

$y \cdot e^{-2t} = \int 2\cos t \, e^{-2t} dt$, Now compute $2 \cdot \int \cos t \, e^{-2t} dt$

call $A = \int \underbrace{2\cos t}_{u} \underbrace{e^{-2t} dt}_{dv} = \underbrace{2\cos t}_{u} \cdot \underbrace{\frac{e^{-2t}}{-2}}_{v} - \int \underbrace{\frac{e^{-2t}}{-2}}_{v} \cdot \underbrace{(-2\sin t)\,dt}_{du}$

$= -\cos t \, e^{-2t} - \int \underbrace{\sin t}_{u} \underbrace{e^{-2t} dt}_{dv} = -\cos t \, e^{-2t} - \left(\sin t \cdot \frac{e^{-2t}}{-2} - \int \frac{e^{-2t}}{-2} \cos t \, dt\right)$

$= -\cos t \, e^{-2t} + \frac{1}{2} \sin t \, e^{-2t} - \frac{1}{2} \int \cos t \, e^{-2t} dt$

$A/2$

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So that,

$$A = -\cos t\, e^{-2t} + \frac{1}{2}\sin t e^{-2t} - \frac{A}{4}$$

$$\frac{5A}{4} = \frac{1}{2}\sin t e^{-2t} - \cos t \cdot e^{-2t}$$

$$\boxed{A = \frac{2}{5}\sin t e^{-2t} - \frac{4}{5}\cos t e^{-2t}}$$ thus,

$$y \cdot e^{-2t} = A = \frac{2}{5}\sin t e^{-2t} - \frac{4}{5}\cos t\, e^{-2t} + C$$

$$\boxed{y(t) = \frac{2}{5}\sin t - \frac{4}{5}\cos t + C \cdot e^{2t}}$$

$y(0) = a$ yields,

$$a = y(0) = \frac{2}{5} \cdot 0 - \frac{4}{5} + C \Rightarrow C = a + \frac{4}{5}$$

$$\boxed{y(t) = \frac{2}{5}\sin t - \frac{4}{5}\cos t + \left(a + \frac{4}{5}\right)e^{2t}}, \quad y(0) = a.$$

(c) if $y(0) = y_0 = a_0$ then

$$y(t) = \frac{2}{5}\sin t - \frac{4}{5}\cos t + \left(a_0 + \frac{4}{5}\right)e^{2t}$$ and

the solution oscillates for $a = a_0$.