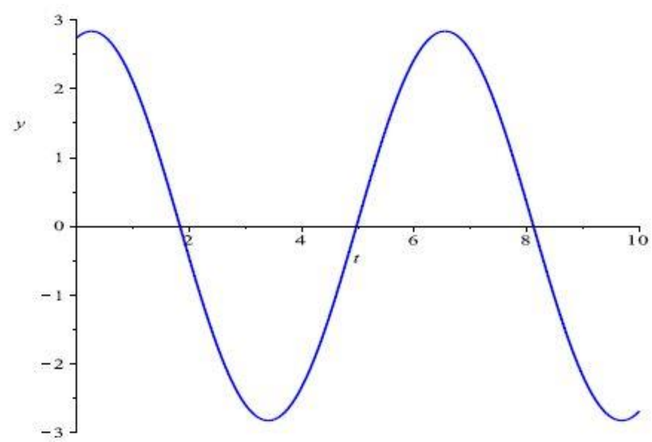
2. $e^{2-i} = e^2 e^{-i} = e^2(\cos 1 - i \sin 1)$.

3. $e^{3i\pi} = \cos 3\pi + i \sin 3\pi = -1$.

10. The characteristic equation is $r^2 + 4r + 5 = 0$, with roots $r = -2 \pm i$. Hence the general solution is $y = c_1 e^{-2t} \cos t + c_2 e^{-2t} \sin t$.

20. The characteristic equation is $r^2 + 1 = 0$, with roots $r = \pm i$. Hence the general solution is $y = c_1 \cos t + c_2 \sin t$. Its derivative is $y' = -c_1 \sin t + c_2 \cos t$. Based on the first condition, $y(\pi/3) = 2$, we require that $c_1 + \sqrt{3}\, c_2 = 4$. In order to satisfy the condition $y'(\pi/3) = -2$, we find that $-\sqrt{3}\, c_1 + c_2 = -4$. Solving these for the constants, $c_1 = 1 + \sqrt{3}$ and $c_2 = \sqrt{3} - 1$. Hence the specific solution is a steady oscillation, given by $y(t) = (1 + \sqrt{3}) \cos t + (\sqrt{3} - 1) \sin t$.



Math 204 HW#4

3.3.5. $2^{2-i} = e^{(2-i)\ln 2} = e^{2\ln 2} e^{-i\ln 2}$

$= e^{2\ln 2}(\cos(\ln 2) - i\sin(\ln 2))$.

$= 4\cos(\ln 2) - 4i\sin(\ln 2)$

$\cong 3.0770 - 2.558i$ //

3.3.7. $y'' - 4y' + 5y = 0$.

$r^2 - 4r + 5 = 0$, $\quad r_{1,2} = \dfrac{4 \mp \sqrt{-4}}{2} = 2 \mp i$.

$\Delta = 16 - 20 = -4$

Hence the general solution:

$y(t) = c_1 e^{2t}\cos t + c_2 e^{2t}\sin t$ //

3.3.11. $y'' + 6y' + 10y = 0$.

$r^2 + 6r + 10 = 0 \Rightarrow r_{1,2} = \dfrac{-6 \mp \sqrt{-4}}{2} = -3 \mp i$

$\Delta = 36 - 40 = -4$

$\Rightarrow y(t) = c_1 e^{-3t}\cos t + c_2 e^{-3t}\sin t$ //

3.3.14. $9y'' + 3y' - 2y = 0$.

$y'' + \frac{1}{3}y' - \frac{2}{9}y = 0$

$r^2 + \frac{1}{3}r - \frac{2}{9} = 0$

$(r + \frac{2}{3})(r - \frac{1}{3}) = 0 \Rightarrow r_1 = \frac{1}{3}, r_2 = -\frac{2}{3}$

$\Rightarrow y(t) = c_1 e^{t/3} + c_2 e^{-2/3 t}$ //

3.3.18. $y'' + 4y' + 5y = 0, \quad y(0) = 1, \quad y'(0) = 0$

$r^2 + 4r + 5 = 0 \qquad \Rightarrow r_{1,2} = \dfrac{-4 \mp \sqrt{-4}}{2} = -2 \mp i$

$\Delta = 16 - 20$
$= -4$

$\Rightarrow y(t) = c_1 e^{-2t} \cos t + c_2 e^{-2t} \sin t$

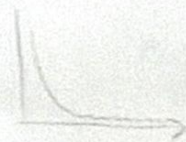
$y'(t) = -2c_1 e^{-2t} \cos t - c_1 e^{-2t} \sin t - 2c_2 e^{-2t} \sin t + \cos t \, c_2 e^{-2t}.$

$y(0) = 1 = c_1 \cdot 1 \cdot 1 + c_2 \cdot 1 \cdot 0 = c_1 \quad \Rightarrow c_1 = 1$

$y'(0) = 0 = -2c_1 + c_2 \qquad \Rightarrow c_2 = 2$

$\Rightarrow y(t) = e^{-2t} \cos t + 2e^{-2t} \sin t.$

y decays oscillating as t increases.



3.3.23. $3u'' - u' + 2u = 0, \quad u(0) = 2, \quad u'(0) = 0.$

a) $3r^2 - r + 2 = 0 \qquad r_{1,2} = \dfrac{1 \mp \sqrt{23}}{2.3} = \dfrac{1}{6} \mp i \dfrac{\sqrt{23}}{6}$

$\Delta = 1 - 4.3.2$
$= -23$

$u(t) = c_1 e^{t/6} \cos(\sqrt{23}/6 \, t) + c_2 e^{t/6} \sin\left(\frac{\sqrt{23}}{6} t\right).$

$u(0) = 2 = c_1 + 0 \quad \Rightarrow c_1 = 2$

$u'(t) = \dfrac{c_1}{6} e^{t/6} \cos\left(\frac{\sqrt{23}}{6} t\right) - c_1 e^{t/6} \frac{\sqrt{23}}{6} \sin\left(\frac{\sqrt{23}}{6} t\right) + \frac{c_2}{6} e^{t/6} \sin\left(\frac{\sqrt{23}}{6} t\right)$
$+ \cos\left(\frac{\sqrt{23}}{6} t\right) \frac{\sqrt{23}}{6} c_2 e^{t/6}.$

$u'(0) = 0 = \dfrac{c_1}{6} + \dfrac{\sqrt{23}}{6} c_2 \quad \Rightarrow c_2 = -\dfrac{2}{6} \cdot \dfrac{6}{\sqrt{23}} = \dfrac{-2}{\sqrt{23}}$

$\Rightarrow u(t) = 2e^{t/6} \cos\left(\frac{\sqrt{23}}{6} t\right) - \frac{2}{\sqrt{23}} e^{t/6} \sin\left(\frac{\sqrt{23}}{6} t\right).$

3.3.23.

b) $|u(t)| = 10 = 2e^{t/6}\cos\left(\frac{\sqrt{23}}{6}t\right) - \frac{2}{2\sqrt{3}}e^{t/6}\sin\left(\frac{\sqrt{23}}{6}t\right)$

$\rightarrow t = 10.7598$

3.3.25. $y'' + 2y' + 6y = 0$, $y(0) = 2$, $y'(0) = \alpha \geq 0$.

a) $r^2 + 2r + 6 = 0$ $\quad r_{1,2} = \frac{-2 \mp \sqrt{-20}}{2} = -1 \mp \sqrt{5}$

$\Delta = 4 - 24 = -20$

$\Rightarrow y(t) = c_1 e^{-t}\cos(\sqrt{5}t) + c_2 e^{-t}\sin\sqrt{5}t)$

$y(0) = 2 = c_1$

$y'(t) = -c_1 e^{-t}\cos(\sqrt{5}t) - c_1\sqrt{5}e^{-t}\sin(\sqrt{5}t) - c_2 e^{-t}\sin(\sqrt{5}t) + c_2\sqrt{5}e^{-t}\cos(\sqrt{5}t)$

$y'(0) = \alpha = -c_1 + c_2 \cdot \sqrt{5} \Rightarrow c_2 = \frac{\alpha + 2}{\sqrt{5}}$

$\Rightarrow y(t) = 2e^{-t}\cos\sqrt{5}t + \left(\frac{\alpha+2}{\sqrt{5}}\right)e^{-t}\sin\sqrt{5}t.$

b) $y(1) = \boxed{0 = 2e^{-1}\cos\sqrt{5} + \left(\frac{\alpha+2}{\sqrt{5}}\right)e^{-1}\sin\sqrt{5}}$

$\Rightarrow \frac{\alpha+2}{\sqrt{5}} = -\frac{2}{e}\cos\sqrt{5} / \frac{\sin}{e}\sqrt{5} = -2\cot\sqrt{5} = +1.5692$

$\Rightarrow \alpha = 1.5088$

c) $y = 0$, $t = ?$

$0 = 2e^{-t}\cos\sqrt{5}t + \left(\frac{\alpha+2}{\sqrt{5}}\right)e^{-t}\sin\sqrt{5}t. \Rightarrow 2 + ((\alpha+2)/\sqrt{5})\tan\sqrt{5}t = 0$

$\tan(\sqrt{5}t) = \frac{-2\sqrt{5}}{(\alpha+2)}$

$t = \{\pi - \arctan(2\sqrt{5}/\alpha+2)\}/\sqrt{5}$

d) $\lim_{\alpha\to\infty}\{\pi - \arctan[\underbrace{+2\sqrt{5}/(2+\alpha)}_{\to 0}]\}/\sqrt{5} = \pi/\sqrt{5} - 0 = \frac{\pi}{\sqrt{5}}$