
Definition of state and cost matrices (A,B,C,D,Q,R) and solving DARE

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In[1]:= ClearAll["Global`*"];
(*Define the matrices A,B,Q,R,and P*)
A = {{A11, A12, A13}, {A21, A22, A23}, {0, 0, 0}};
B = {{0}, {0}, {1}};
Q = {{q2 * f^4 + q1, 0, -q2 * f^4}, {0, 0, 0}, {-q2 * f^4, 0, q2 * f^4}};
R = {{q3}};
P = {{p11, p12, p13}, {p12, p22, p23}, {p13, p23, p33}};

ATPA = Transpose[A].P.A;
ATPB = Transpose[A].P.B;
BTPB = Transpose[B].P.B;
invTerm = Inverse[R + BTPB];
BTPA = Transpose[B].P.A;
dare = ATPA - ATPB.invTerm.BTPA + Q - P;
solution = Solve[dare == 0, {p11, p12, p13, p22, p23, p33}];
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In[14]:= substitutionRules = {A11 → Cos[f], A12 → Sin[f] / f,
    A13 → 1 - Cos[f], A21 → -f * Sin[f], A22 → Cos[f], A23 → f * Sin[f]};
solutionWithSubstitution = solution[[4]] /. substitutionRules;
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In[16]:= (*Extract p13,p23,and p33 from the solution*)
p13Value = p13 /. solutionWithSubstitution;
p23Value = p23 /. solutionWithSubstitution;
p33Value = p33 /. solutionWithSubstitution;
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Analytical p, d, c gains (from DARE)

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In[22]:= p = Simplify[-(f p23Value Sin[f]) / (p33Value + q3) + (p13Value Cos[f]) / (p33Value + q3)]
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Out[22]=

$$\begin{aligned} & - \left(\left(\csc\left[\frac{f}{2}\right]^2 \left(-28 q_1^2 - 50 f^4 q_1 q_2 - 2 f^8 q_2^2 - 22 q_1 q_3 - 42 f^4 q_2 q_3 + 16 q_3^2 + 64 q_1^2 \cos[f] + \right. \right. \right. \\ & 84 f^4 q_1 q_2 \cos[f] - 4 f^8 q_2^2 \cos[f] + 44 q_1 q_3 \cos[f] + 84 f^4 q_2 q_3 \cos[f] - 16 q_3^2 \cos[f] - \\ & 71 q_1^2 \cos[2f] - 56 f^4 q_1 q_2 \cos[2f] - f^8 q_2^2 \cos[2f] - 54 q_1 q_3 \cos[2f] - \\ & 74 f^4 q_2 q_3 \cos[2f] + 56 q_1^2 \cos[3f] + 38 f^4 q_1 q_2 \cos[3f] + 2 f^8 q_2^2 \cos[3f] + \\ & 66 q_1 q_3 \cos[3f] + 30 f^4 q_2 q_3 \cos[3f] - 28 q_1^2 \cos[4f] - 22 f^4 q_1 q_2 \cos[4f] + \\ & 2 f^8 q_2^2 \cos[4f] - 50 q_1 q_3 \cos[4f] - 14 f^4 q_2 q_3 \cos[4f] - 16 q_3^2 \cos[4f] + \\ & 8 q_1^2 \cos[5f] + 6 f^4 q_1 q_2 \cos[5f] + 2 f^8 q_2^2 \cos[5f] + 18 q_1 q_3 \cos[5f] + \\ & 14 f^4 q_2 q_3 \cos[5f] + 16 q_3^2 \cos[5f] - q_1^2 \cos[6f] + f^8 q_2^2 \cos[6f] - \\ & 2 q_1 q_3 \cos[6f] + 2 f^4 q_2 q_3 \cos[6f] - 64 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3)} \sin[f]^4 - \\ & 64 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3)} \sin[f]^4 + 120 q_1 \cos[f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3)} \sin[f]^4 + \\ & 8 f^4 q_2 \cos[f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3)} \sin[f]^4 + \\ & \left. \left. 128 q_3 \cos[f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3)} \sin[f]^4 - 88 q_1 \cos[2f] \right) \right) \end{aligned}$$

$$\begin{aligned}
& \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 24 f^4 q2 \cos[2 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} - \\
& 64 q3 \cos[2 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 40 q1 \cos[3 f] \\
& \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 24 f^4 q2 \cos[3 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\
& 64 q3 \cos[3 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} - 8 q1 \cos[4 f] \\
& \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 8 f^4 q2 \cos[4 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} - \\
& \sqrt{2} q1 \sqrt{(35 q1^2 + 50 f^4 q1 q2 + 3 f^8 q2^2 + 40 q1 q3 + 40 f^4 q2 q3 - 8 q3^2 - 8 q1^2 \cos[3 f] - \\
& 8 f^4 q1 q2 \cos[3 f] - 16 q1 q3 \cos[3 f] + q1^2 \cos[4 f] + 6 f^4 q1 q2 \cos[4 f] + \\
& f^8 q2^2 \cos[4 f] + 8 q1 q3 \cos[4 f] + 8 f^4 q2 q3 \cos[4 f] + 8 q3^2 \cos[4 f] + \\
& 48 q1 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 48 f^4 q2 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\
& 32 q3 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 \cos[2 f] (7 q1^2 + 2 f^4 q1 q2 - f^8 q2^2 + 8 q3 \\
& \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 q1 (q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4}) + \\
& 4 f^4 q2 (q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4})) - 8 \cos[f] \\
& (7 q1^2 + 8 f^4 q2 q3 + q1 (7 f^4 q2 + 6 q3 + 8 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4})) - \\
& \sqrt{2} f^4 q2 \sqrt{(35 q1^2 + 50 f^4 q1 q2 + 3 f^8 q2^2 + 40 q1 q3 + 40 f^4 q2 q3 - 8 q3^2 - 8 q1^2 \cos[3 f] - \\
& 8 f^4 q1 q2 \cos[3 f] - 16 q1 q3 \cos[3 f] + q1^2 \cos[4 f] + 6 f^4 q1 q2 \cos[4 f] + \\
& f^8 q2^2 \cos[4 f] + 8 q1 q3 \cos[4 f] + 8 f^4 q2 q3 \cos[4 f] + 8 q3^2 \cos[4 f] + \\
& 48 q1 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 48 f^4 q2 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\
& 32 q3 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 \cos[2 f] (7 q1^2 + 2 f^4 q1 q2 - f^8 q2^2 + 8 q3 \\
& \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 q1 (q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4}) + \\
& 4 f^4 q2 (q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4})) - 8 \cos[f] \\
& (7 q1^2 + 8 f^4 q2 q3 + q1 (7 f^4 q2 + 6 q3 + 8 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4})) - \\
& 2 \sqrt{2} q3 \sqrt{(35 q1^2 + 50 f^4 q1 q2 + 3 f^8 q2^2 + 40 q1 q3 + 40 f^4 q2 q3 - 8 q3^2 - 8 q1^2 \cos[3 f] - \\
& 8 f^4 q1 q2 \cos[3 f] - 16 q1 q3 \cos[3 f] + q1^2 \cos[4 f] + 6 f^4 q1 q2 \cos[4 f] + \\
& f^8 q2^2 \cos[4 f] + 8 q1 q3 \cos[4 f] + 8 f^4 q2 q3 \cos[4 f] + 8 q3^2 \cos[4 f] + \\
& 48 q1 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 48 f^4 q2 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\
& 32 q3 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 \cos[2 f] (7 q1^2 + 2 f^4 q1 q2 - f^8 q2^2 + 8 q3 \\
& \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 q1 (q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4}) + \\
& 4 f^4 q2 (q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4})) - 8 \cos[f] \\
& (7 q1^2 + 8 f^4 q2 q3 + q1 (7 f^4 q2 + 6 q3 + 8 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4})) + \\
& 4 \sqrt{2} q1 \cos[f] \sqrt{(35 q1^2 + 50 f^4 q1 q2 + 3 f^8 q2^2 + 40 q1 q3 + 40 f^4 q2 q3 - 8 q3^2 - \\
& 8 q1^2 \cos[3 f] - 8 f^4 q1 q2 \cos[3 f] - 16 q1 q3 \cos[3 f] + q1^2 \cos[4 f] + 6 f^4 q1 q2 \\
& \cos[4 f] + f^8 q2^2 \cos[4 f] + 8 q1 q3 \cos[4 f] + 8 f^4 q2 q3 \cos[4 f] + 8 q3^2 \cos[4 f] + \\
& 48 q1 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 48 f^4 q2 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\
& 32 q3 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 \cos[2 f] (7 q1^2 + 2 f^4 q1 q2 - f^8 q2^2 + 8 q3 \\
& \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 q1 (q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4}) + \\
& 4 f^4 q2 (q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4})) - 8 \cos[f] \\
& (7 q1^2 + 8 f^4 q2 q3 + q1 (7 f^4 q2 + 6 q3 + 8 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4})) -
\end{aligned}$$

$$\begin{aligned}
& 2 \sqrt{2} f^4 q_2 \cos[f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right.} \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \quad \left. 4 q_1 (q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4}) + \right. \\
& \quad \quad \left. 4 f^4 q_2 (q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4})\right) - 8 \cos[f] \\
& \quad \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 (7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4})\right) + \\
& 2 \sqrt{2} q_3 \cos[f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right.} \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \quad \left. 4 q_1 (q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4}) + \right. \\
& \quad \quad \left. 4 f^4 q_2 (q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4})\right) - 8 \cos[f] \\
& \quad \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 (7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4})\right) - \\
& 6 \sqrt{2} q_1 \cos[2 f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right.} \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \quad \left. 4 q_1 (q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4}) + \right. \\
& \quad \quad \left. 4 f^4 q_2 (q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4})\right) - 8 \cos[f] \\
& \quad \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 (7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4})\right) - \\
& 6 \sqrt{2} q_3 \cos[2 f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right.} \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \quad \left. 4 q_1 (q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4}) + \right. \\
& \quad \quad \left. 4 f^4 q_2 (q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4})\right) - 8 \cos[f] \\
& \quad \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 (7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4})\right) + \\
& 4 \sqrt{2} q_1 \cos[3 f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right.}
\end{aligned}$$

[illegible]

$$\begin{aligned}
& 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - 8 \cos[f] \\
& \quad \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 4 \sqrt{2} \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + \right. \\
& \quad 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - 8 \cos[f] \\
& \quad \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 8 \sqrt{2} \cos[f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + \right. \\
& \quad 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - 8 \cos[f] \\
& \quad \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) \Big) \Big) / \\
& \left(4 \left(-4 (q_1 + f^4 q_2 + 2 q_3) \cos[f] + (q_1 - f^4 q_2) (3 + \cos[2 f]) \right) \right) \\
& \left(6 q_1 + \right. \\
& \quad 6 f^4 q_2 + \\
& \quad 4 q_3 - \\
& \quad 8 q_1 \cos[f] + \\
& \quad 2 q_1 \cos[2 f] + \\
& \quad 2 f^4 q_2 \cos[2 f] + \\
& \quad 4 q_3 \cos[2 f] + \\
& \quad 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad \sqrt{2} \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3 f] - \right. \\
& \quad \left. 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + \right. \\
& \quad \left. f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + \right. \\
& \quad \left. 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \right. \right.
\end{aligned}$$

$$\begin{aligned} & \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 q1 \left(q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} \right) + \\ & 4 f^4 q2 \left(q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} \right) - 8 \cos[f] \\ & \left(7 q1^2 + 8 f^4 q2 q3 + q1 \left(7 f^4 q2 + 6 q3 + 8 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} \right) \right) \end{aligned}$$

In[23]:= **d = Simplify[(p23Value Cos[f]) / (p33Value + q3) + (p13Value Sin[f]) / (f (p33Value + q3))]**

Out[23]=

$$\begin{aligned} & \left(\csc\left[\frac{f}{2}\right] \sec\left[\frac{f}{2}\right] \left(14 q1^2 + 18 f^4 q1 q2 - 16 f^8 q2^2 + 10 q1 q3 - 10 f^4 q2 q3 - 16 q3^2 - 20 q1^2 \cos[f] - \right. \right. \\ & 12 f^4 q1 q2 \cos[f] - 16 f^8 q2^2 \cos[f] - 12 q1 q3 \cos[f] - 52 f^4 q2 q3 \cos[f] - \\ & 16 q3^2 \cos[f] + q1^2 \cos[2 f] - 8 f^4 q1 q2 \cos[2 f] + 7 f^8 q2^2 \cos[2 f] + 10 q1 q3 \cos[2 f] - \\ & 10 f^4 q2 q3 \cos[2 f] + 14 q1^2 \cos[3 f] + 6 f^4 q1 q2 \cos[3 f] + 12 f^8 q2^2 \cos[3 f] - \\ & 2 q1 q3 \cos[3 f] + 34 f^4 q2 q3 \cos[3 f] - 14 q1^2 \cos[4 f] - 10 f^4 q1 q2 \cos[4 f] + \\ & 8 f^8 q2^2 \cos[4 f] - 18 q1 q3 \cos[4 f] + 18 f^4 q2 q3 \cos[4 f] + 16 q3^2 \cos[4 f] + \\ & 6 q1^2 \cos[5 f] + 6 f^4 q1 q2 \cos[5 f] + 4 f^8 q2^2 \cos[5 f] + 14 q1 q3 \cos[5 f] + \\ & 18 f^4 q2 q3 \cos[5 f] + 16 q3^2 \cos[5 f] - q1^2 \cos[6 f] + f^8 q2^2 \cos[6 f] - \\ & 2 q1 q3 \cos[6 f] + 2 f^4 q2 q3 \cos[6 f] + 64 f^4 q2 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\ & 64 q3 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 8 q1 \cos[f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\ & 120 f^4 q2 \cos[f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\ & 128 q3 \cos[f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} - 24 q1 \cos[2 f] \\ & \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 88 f^4 q2 \cos[2 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\ & 64 q3 \cos[2 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 24 q1 \cos[3 f] \\ & \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 40 f^4 q2 \cos[3 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\ & 64 q3 \cos[3 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} - 8 q1 \cos[4 f] \\ & \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 8 f^4 q2 \cos[4 f] \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\ & \sqrt{2} q1 \sqrt{\left(35 q1^2 + 50 f^4 q1 q2 + 3 f^8 q2^2 + 40 q1 q3 + 40 f^4 q2 q3 - 8 q3^2 - 8 q1^2 \cos[3 f] - \right.} \\ & 8 f^4 q1 q2 \cos[3 f] - 16 q1 q3 \cos[3 f] + q1^2 \cos[4 f] + 6 f^4 q1 q2 \cos[4 f] + \\ & f^8 q2^2 \cos[4 f] + 8 q1 q3 \cos[4 f] + 8 f^4 q2 q3 \cos[4 f] + 8 q3^2 \cos[4 f] + \\ & 48 q1 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 48 f^4 q2 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\ & 32 q3 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 \cos[2 f] \left(7 q1^2 + 2 f^4 q1 q2 - f^8 q2^2 + 8 q3 \right. \\ & \left. \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 q1 \left(q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} \right) + \right. \\ & \left. 4 f^4 q2 \left(q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} \right) \right) - \\ & 8 \cos[f] \left(7 q1^2 + 8 f^4 q2 q3 + q1 \left(7 f^4 q2 + 6 q3 + 8 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} \right) \right) \left. \right) + \\ & \sqrt{2} f^4 q2 \sqrt{\left(35 q1^2 + 50 f^4 q1 q2 + 3 f^8 q2^2 + 40 q1 q3 + 40 f^4 q2 q3 - 8 q3^2 - 8 q1^2 \cos[3 f] - \right.} \\ & 8 f^4 q1 q2 \cos[3 f] - 16 q1 q3 \cos[3 f] + q1^2 \cos[4 f] + 6 f^4 q1 q2 \cos[4 f] + \\ & f^8 q2^2 \cos[4 f] + 8 q1 q3 \cos[4 f] + 8 f^4 q2 q3 \cos[4 f] + 8 q3^2 \cos[4 f] + \\ & 48 q1 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 48 f^4 q2 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + \\ & 32 q3 \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 \cos[2 f] \left(7 q1^2 + 2 f^4 q1 q2 - f^8 q2^2 + 8 q3 \right. \\ & \left. \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} + 4 q1 \left(q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} \right) + \right. \\ & \left. 4 f^4 q2 \left(q3 + \sqrt{(q1 + q3) (f^4 q2 + q3) \sin[f]^4} \right) \right) - \end{aligned}$$

$$\begin{aligned}
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 2 \sqrt{2} q_3 \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3 f] - \right.} \\
& \quad 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + \\
& \quad f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + \\
& \quad 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \right. \\
& \quad \left. \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 2 \sqrt{2} q_1 \cos[f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - \right.} \\
& \quad 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \\
& \quad \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + \\
& \quad 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \right. \\
& \quad \left. \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 4 \sqrt{2} f^4 q_2 \cos[f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right.} \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 2 \sqrt{2} q_3 \cos[f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right.} \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 6 \sqrt{2} f^4 q_2 \cos[2 f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right.} \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] +
\end{aligned}$$

$$\begin{aligned}
& 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& \sqrt{2} q_1 \cos[4 f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right. \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& \sqrt{2} f^4 q_2 \cos[4 f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right. \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 4 \sqrt{2} \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + \right. \\
& \quad 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& \quad q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 8 \sqrt{2} \cos[f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \\
& \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3 f] - \right. \\
& \quad 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + \\
& \quad f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + \\
& \quad \left. 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right.
\end{aligned}$$

$$\begin{aligned}
& 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \right. \\
& \quad \left. \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - 8 \cos[f] \\
& \quad \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) \Bigg) \Bigg/ \\
& \left(4 f \left(4 (q_1 + f^4 q_2 + 2 q_3) \cos[f] - (q_1 - f^4 q_2) (3 + \cos[2 f]) \right) \right. \\
& \quad \left(6 q_1 + \right. \\
& \quad 6 f^4 q_2 + \\
& \quad 4 q_3 - \\
& \quad 8 q_1 \cos[f] + \\
& \quad 2 q_1 \cos[2 f] + \\
& \quad 2 f^4 q_2 \cos[2 f] + \\
& \quad 4 q_3 \cos[2 f] + \\
& \quad 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad \sqrt{2} \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3 f] - \right.} \\
& \quad 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + \\
& \quad f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + \\
& \quad 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \right. \\
& \quad \left. \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& \quad \left. \left. 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) \right) \right) \Bigg)
\end{aligned}$$

In[24]:= **c = Simplify[(f p23Value Sin[f]) / (p33Value + q3) + (p13Value (1 - Cos[f])) / (p33Value + q3)]**

Out[24]=

$$\begin{aligned}
& - \left(\left(21 q_1^2 + 34 f^4 q_1 q_2 - 7 f^8 q_2^2 + 16 q_1 q_3 + 16 f^4 q_2 q_3 - 16 q_3^2 - 42 q_1^2 \cos[f] - \right. \right. \\
& \quad 48 f^4 q_1 q_2 \cos[f] - 6 f^8 q_2^2 \cos[f] - 28 q_1 q_3 \cos[f] - 68 f^4 q_2 q_3 \cos[f] + \\
& \quad 36 q_1^2 \cos[2 f] + 24 f^4 q_1 q_2 \cos[2 f] + 4 f^8 q_2^2 \cos[2 f] + 32 q_1 q_3 \cos[2 f] + \\
& \quad 32 f^4 q_2 q_3 \cos[2 f] - 21 q_1^2 \cos[3 f] - 16 f^4 q_1 q_2 \cos[3 f] + 5 f^8 q_2^2 \cos[3 f] - \\
& \quad 34 q_1 q_3 \cos[3 f] + 2 f^4 q_2 q_3 \cos[3 f] + 7 q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + \\
& \quad 3 f^8 q_2^2 \cos[4 f] + 16 q_1 q_3 \cos[4 f] + 16 f^4 q_2 q_3 \cos[4 f] + 16 q_3^2 \cos[4 f] - \\
& \quad q_1^2 \cos[5 f] + f^8 q_2^2 \cos[5 f] - 2 q_1 q_3 \cos[5 f] + 2 f^4 q_2 q_3 \cos[5 f] + \\
& \quad 32 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 64 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} - 56 q_1 \cos[f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 56 f^4 q_2 \cos[f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_1 \cos[2 f] \\
& \quad \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 f^4 q_2 \cos[2 f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 64 q_3 \cos[2 f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} - 8 q_1 \cos[3 f] \\
& \quad \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 8 f^4 q_2 \cos[3 f] \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad \left. \left. \sqrt{2} q_1 \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3 f] - \right. \right. \right. \\
& \quad 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + \\
& \quad \left. \left. \left. f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + \right. \right. \right.
\end{aligned}$$

$$\begin{aligned}
& 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 \cos[2f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \right. \\
& \quad \left. \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& \sqrt{2} f^4 q_2 \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3f] - \right. \\
& \quad 8 f^4 q_1 q_2 \cos[3f] - 16 q_1 q_3 \cos[3f] + q_1^2 \cos[4f] + 6 f^4 q_1 q_2 \cos[4f] + \\
& \quad f^8 q_2^2 \cos[4f] + 8 q_1 q_3 \cos[4f] + 8 f^4 q_2 q_3 \cos[4f] + 8 q_3^2 \cos[4f] + \\
& 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 \cos[2f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \right. \\
& \quad \left. \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 2 \sqrt{2} q_3 \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - 8 q_3^2 - 8 q_1^2 \cos[3f] - \right. \\
& \quad 8 f^4 q_1 q_2 \cos[3f] - 16 q_1 q_3 \cos[3f] + q_1^2 \cos[4f] + 6 f^4 q_1 q_2 \cos[4f] + \\
& \quad f^8 q_2^2 \cos[4f] + 8 q_1 q_3 \cos[4f] + 8 f^4 q_2 q_3 \cos[4f] + 8 q_3^2 \cos[4f] + \\
& 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 \cos[2f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \right. \\
& \quad \left. \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 3 \sqrt{2} q_1 \cos[f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right. \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3f] - 8 f^4 q_1 q_2 \cos[3f] - 16 q_1 q_3 \cos[3f] + \\
& \quad q_1^2 \cos[4f] + 6 f^4 q_1 q_2 \cos[4f] + f^8 q_2^2 \cos[4f] + 8 q_1 q_3 \cos[4f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4f] + 8 q_3^2 \cos[4f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 3 \sqrt{2} f^4 q_2 \cos[f] \sqrt{\left(35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 - \right. \\
& \quad 8 q_3^2 - 8 q_1^2 \cos[3f] - 8 f^4 q_1 q_2 \cos[3f] - 16 q_1 q_3 \cos[3f] + \\
& \quad q_1^2 \cos[4f] + 6 f^4 q_1 q_2 \cos[4f] + f^8 q_2^2 \cos[4f] + 8 q_1 q_3 \cos[4f] + \\
& \quad 8 f^4 q_2 q_3 \cos[4f] + 8 q_3^2 \cos[4f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& \quad 4 \cos[2f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) +
\end{aligned}$$

$$\begin{aligned}
& 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 3 \sqrt{2} q_1 \cos[2 f] \sqrt{35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 -} \\
& 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 3 \sqrt{2} f^4 q_2 \cos[2 f] \sqrt{35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 -} \\
& 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) + \\
& 6 \sqrt{2} q_3 \cos[2 f] \sqrt{35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 -} \\
& 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& 8 \cos[f] \left(7 q_1^2 + 8 f^4 q_2 q_3 + q_1 \left(7 f^4 q_2 + 6 q_3 + 8 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) - \\
& \sqrt{2} q_1 \cos[3 f] \sqrt{35 q_1^2 + 50 f^4 q_1 q_2 + 3 f^8 q_2^2 + 40 q_1 q_3 + 40 f^4 q_2 q_3 -} \\
& 8 q_3^2 - 8 q_1^2 \cos[3 f] - 8 f^4 q_1 q_2 \cos[3 f] - 16 q_1 q_3 \cos[3 f] + \\
& q_1^2 \cos[4 f] + 6 f^4 q_1 q_2 \cos[4 f] + f^8 q_2^2 \cos[4 f] + 8 q_1 q_3 \cos[4 f] + \\
& 8 f^4 q_2 q_3 \cos[4 f] + 8 q_3^2 \cos[4 f] + 48 q_1 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 48 f^4 q_2 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + 32 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \\
& 4 \cos[2 f] \left(7 q_1^2 + 2 f^4 q_1 q_2 - f^8 q_2^2 + 8 q_3 \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} + \right. \\
& \quad \left. 4 q_1 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) + \right. \\
& \quad \left. 4 f^4 q_2 \left(q_3 + \sqrt{(q_1 + q_3) (f^4 q_2 + q_3) \sin[f]^4} \right) \right) -
\end{aligned}$$


```
In[48]:= limitD = FullSimplify[Limit[d /. {q3 → 1, q2 → 1}, q1 → Infinity]]
```

```
Out[48]=
```

$$\frac{2 \cos[f] \cot\left[\frac{f}{2}\right]}{f}$$

```
In[49]:= limitC = Simplify[Limit[c /. {q3 → 1, q2 → 1}, q1 → Infinity]]
```

```
Out[49]=
```

$$1 + 2 \cos[f]$$

Dimensionless p, d, c gains in case of $q_2 \rightarrow \infty$

```
In[50]:= limitP = Simplify[Limit[p /. {q3 → 1, q1 → 1}, q2 → Infinity]];
Assuming[f > 0, FullSimplify[PowerExpand[limitP]]]
```

```
Out[51]=
```

$$-\cos[f]$$

```
In[52]:= limitD = Simplify[Limit[d /. {q3 → 1, q1 → 1}, q2 → Infinity]];
Assuming[f > 0, FullSimplify[PowerExpand[limitD]]]
```

```
Out[53]=
```

$$-\frac{\sin[f]}{f}$$

```
In[54]:= limitC = Simplify[Limit[c /. {q3 → 1, q1 → 1}, q2 → Infinity]];
Assuming[f > 0, FullSimplify[PowerExpand[limitC]]]
```

```
Out[55]=
```

$$-1 + \cos[f]$$
