FACULTY OF COMPUTING

SEMESTER 1

2023/2024

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SECI1013 – DISCRETE STRUCTURE

SECTION 02

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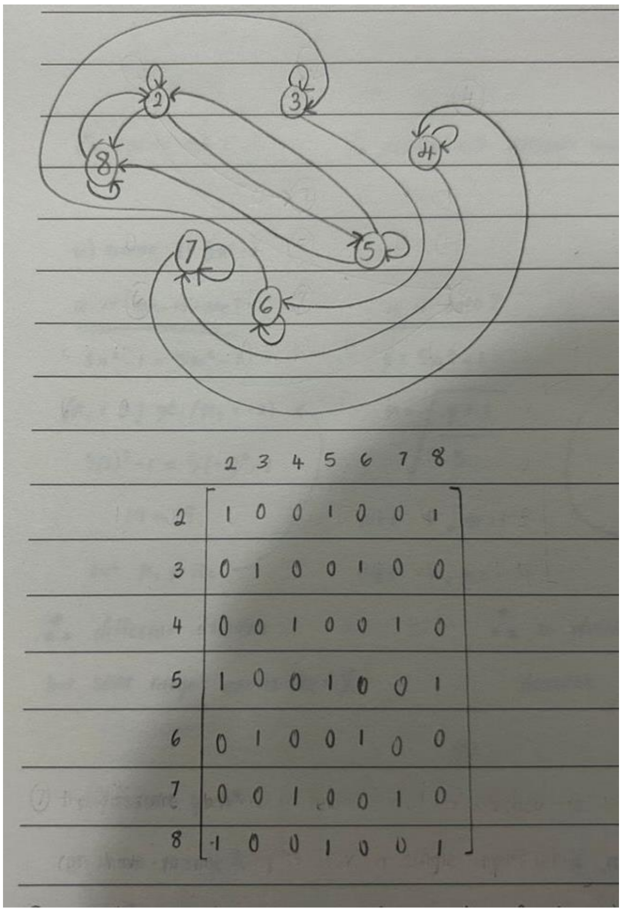
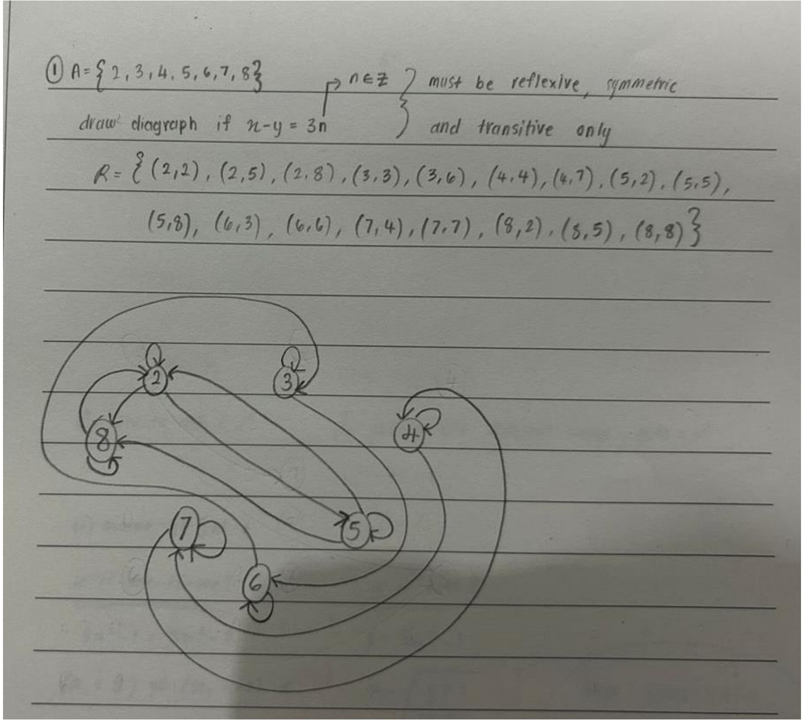
## Q1. Relation

1. Given A = {2, 3, 4, 5, 6, 7, 8} and R a relation over A. Draw the directed graph of R after realising that xRy iff x−y = 3n for some n ∈ Z. Find all possible equivalence relations for R.

(5 marks)



$$\begin{array}{c|ccccccc} & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 2 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 3 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 6 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 8 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \end{array}$$



① $A = \{2,3,4,5,6,7,8\}$

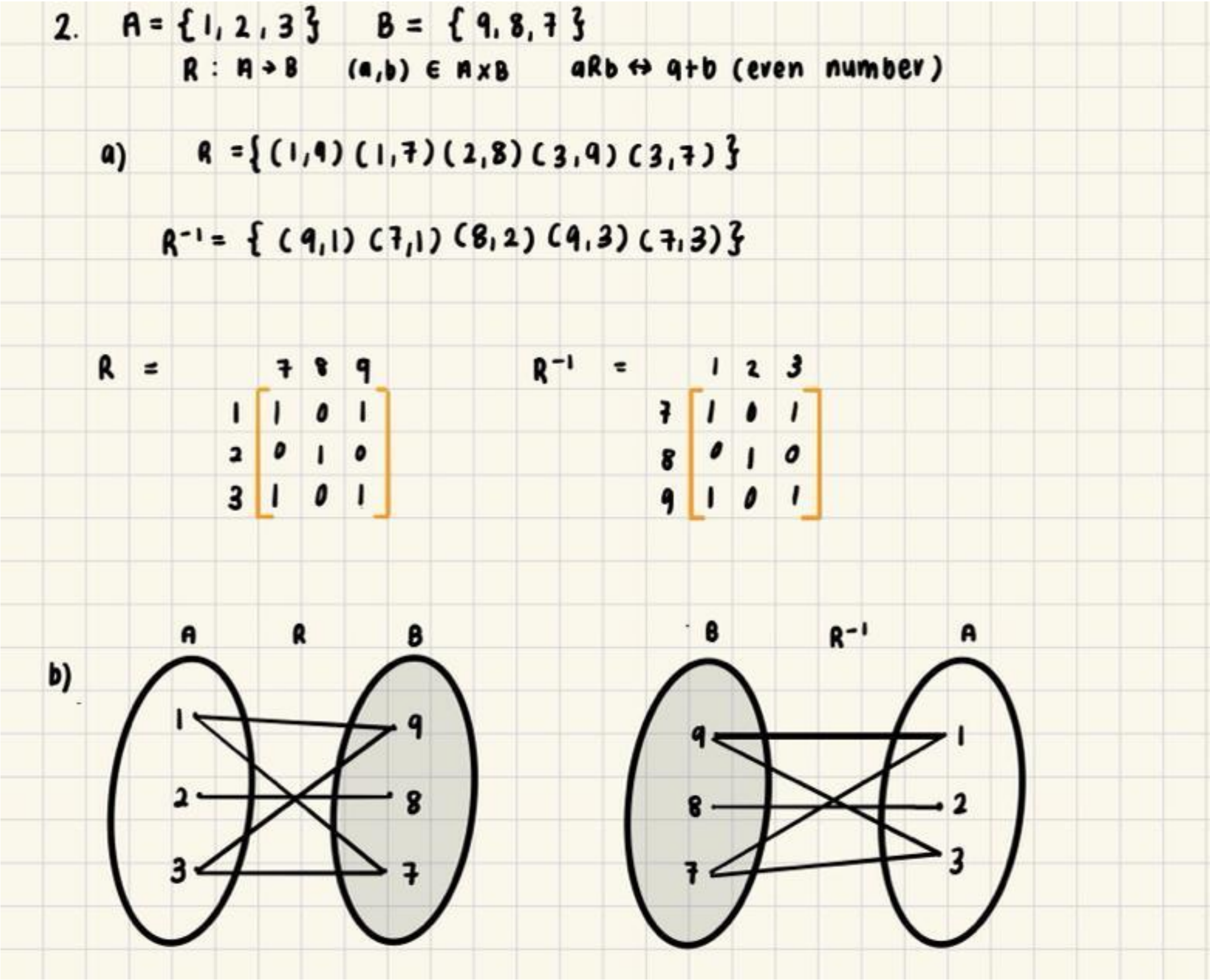
draw diagraph if $x-y = 3n$ → $n \in \mathbb{Z}$ } must be reflexive, symmetric and transitive only

$R = \{(2,2),(2,5),(2,8),(3,3),(3,6),(4,4),(4,7),(5,2),(5,5),$
$(5,8),(6,3),(6,6),(7,4),(7,7),(8,2),(8,5),(8,8)\}$

2. Let A = {1, 2, 3} and B = {9, 8, 7}.
Let R: A to B. For all (a, b) ∈ A × B, and given a R b ⇔ a+b is an even number,

   a. Determine R and $R^{-1}$.
   b. Draw arrow diagrams for both.
   c. Describe R−1 in words.

(10 marks)



2. A = {1, 2, 3}  B = {9, 8, 7}

R : A → B  (a,b) ∈ A×B  aRb ⇔ a+b (even number)

a) R = {(1,9) (1,7) (2,8) (3,9) (3,7)}

$R^{-1}$ = {(9,1) (7,1) (8,2) (9,3) (7,3)}

$$R = \begin{array}{c|ccc} & 7 & 8 & 9 \\ \hline 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array} \qquad R^{-1} = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 7 & 1 & 0 & 1 \\ 8 & 0 & 1 & 0 \\ 9 & 1 & 0 & 1 \end{array}$$

b)

3. Let $A$ = {1, 2, 3, 4, 5}, and let $R$ be the relation on $A$ that has the matrix (given below)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

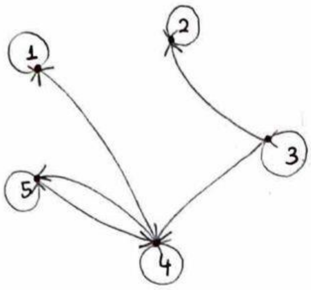
Construct the digraph of $R$, and list in-degrees and out-degrees of all vertices.

(6 marks)

3. Let A = {1,2,3,4,5} , and let R be the relation on A that has the matrix

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

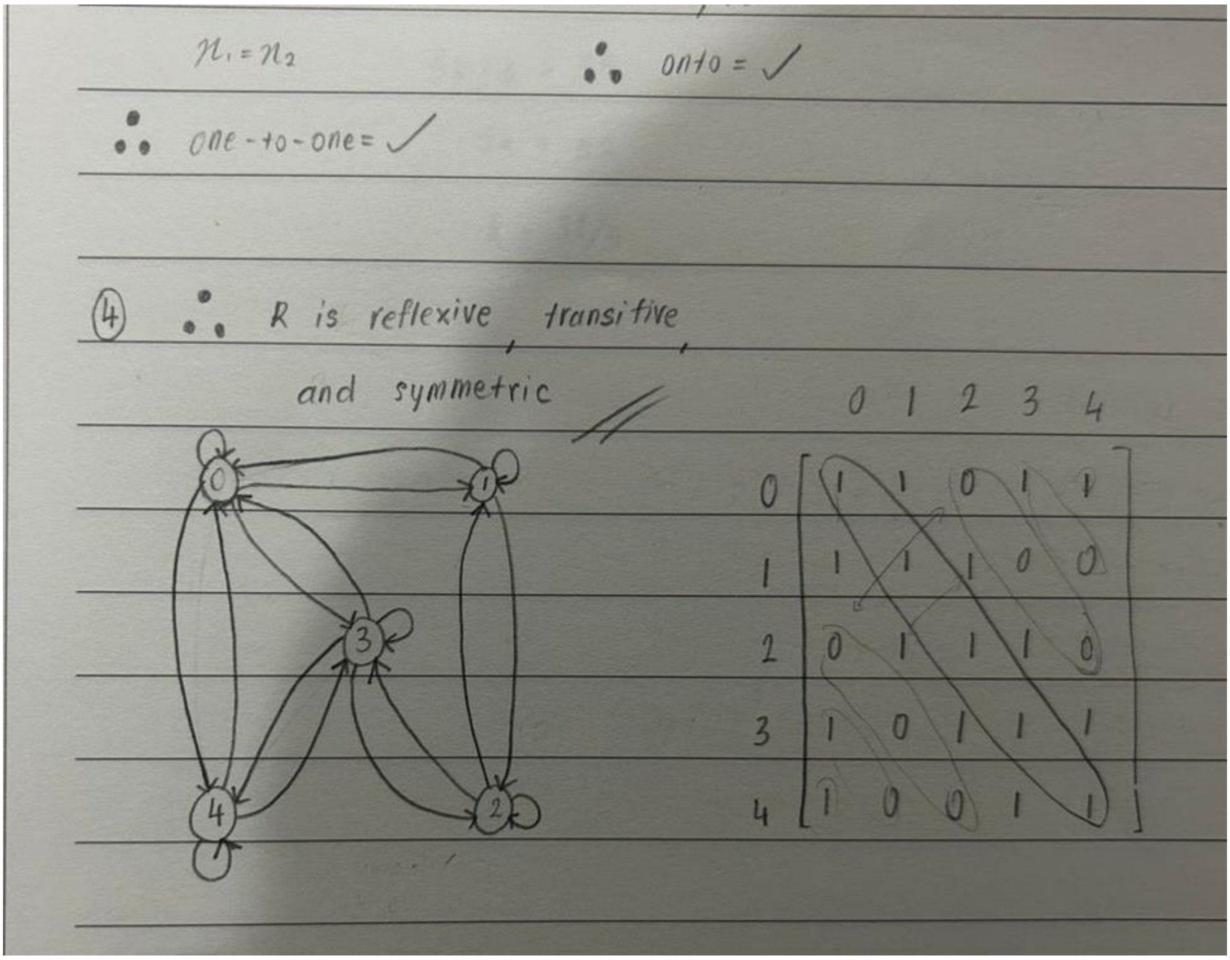
Construct the diagraph of R, and list in-degrees and out-degrees of all verticles.



| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| In-degrees | 2 | 2 | 1 | 3 | 2 |
| Out-degrees | 1 | 1 | 3 | 3 | 2 |

4. Given A = {0, 1, 2, 3, 4}, and R = {(0, 0),(0, 1),(0, 3),(0, 4),(1, 0),(1,1), (1,2), (2,1), (2, 2),(2,3), (3, 0),(3, 2),(3,3),(3,4),(4,0),(4,3),(4,4)}. Draw the digraph and find is R reflexive, symmetric, or transitive?

(12 marks)



$n_1 = n_2$ ∴ onto = ✓

∴ one-to-one = ✓

④ ∴ R is reflexive, transitive, and symmetric //

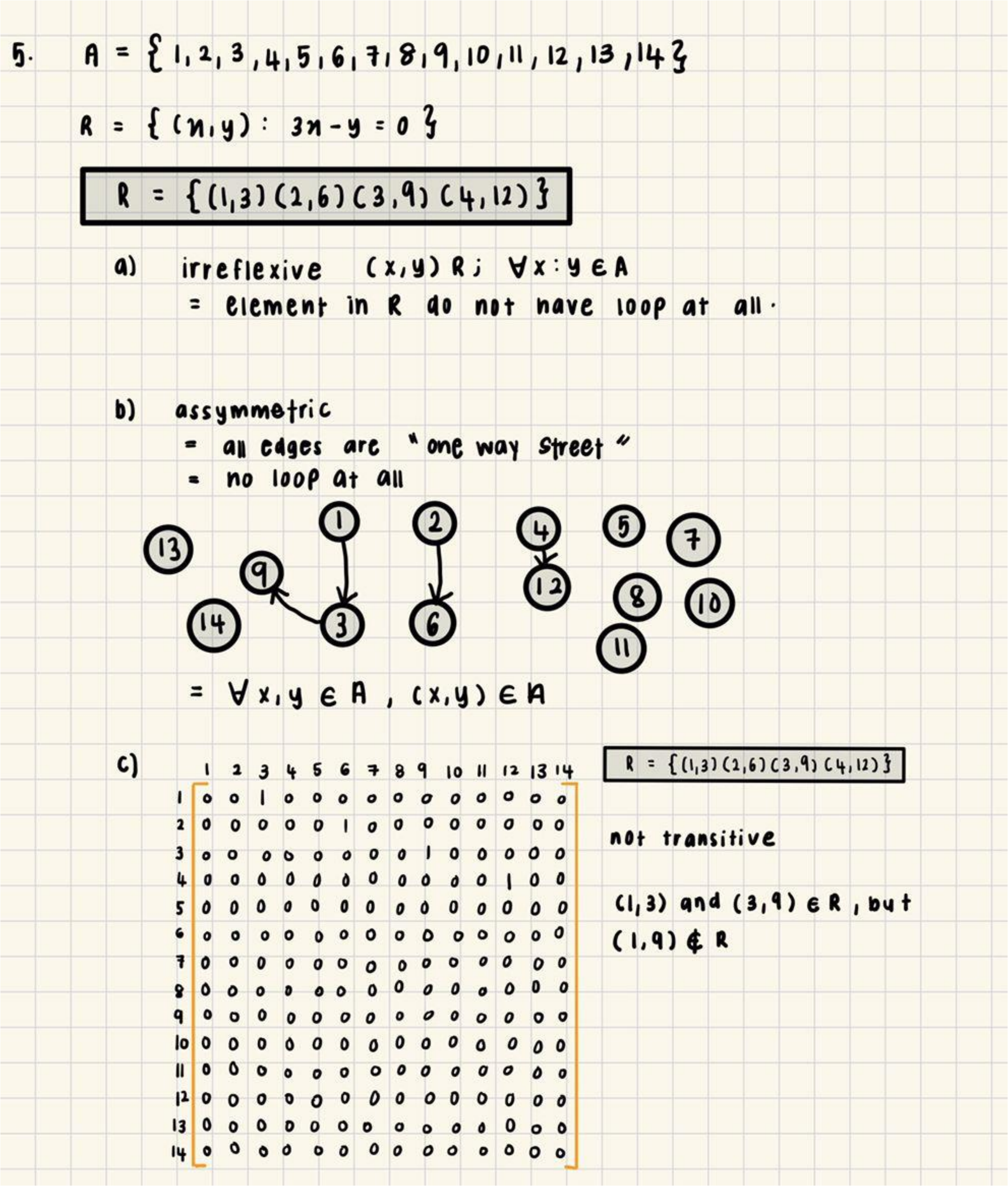
$$\begin{array}{c|ccccc} & 0 & 1 & 2 & 3 & 4 \\ \hline 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 1 & 1 & 1 & 0 \\ 3 & 1 & 0 & 1 & 1 & 1 \\ 4 & 1 & 0 & 0 & 1 & 1 \end{array}$$

5. Relation R in the set A = {1, 2, 3…13, 14} defined as R = {(x, y): 3x − y = 0}, Determine whether the relation is

   a. Reflexive
   b. Symmetric
   c. Transitive

   Support your answer with the reason.

(9 marks)



5. $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$

$R = \{(x, y) : 3x - y = 0\}$

$\boxed{R = \{(1,3)\,(2,6)\,(3,9)\,(4,12)\}}$

a) irreflexive $(x, y)\,R\,;\ \forall x : y \in A$

= element in R do not have loop at all.

b) assymmetric

= all edges are "one way street"

= no loop at all

= $\forall\, x, y \in A\ ,\ (x, y) \in A$

c)

$$\begin{array}{c|cccccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 \\
\hline
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
10 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
11 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
12 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
14 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

$\boxed{R = \{(1,3)\,(2,6)\,(3,9)\,(4,12)\}}$

not transitive

$(1, 3)$ and $(3, 9) \in R$ , but $(1, 9) \notin R$

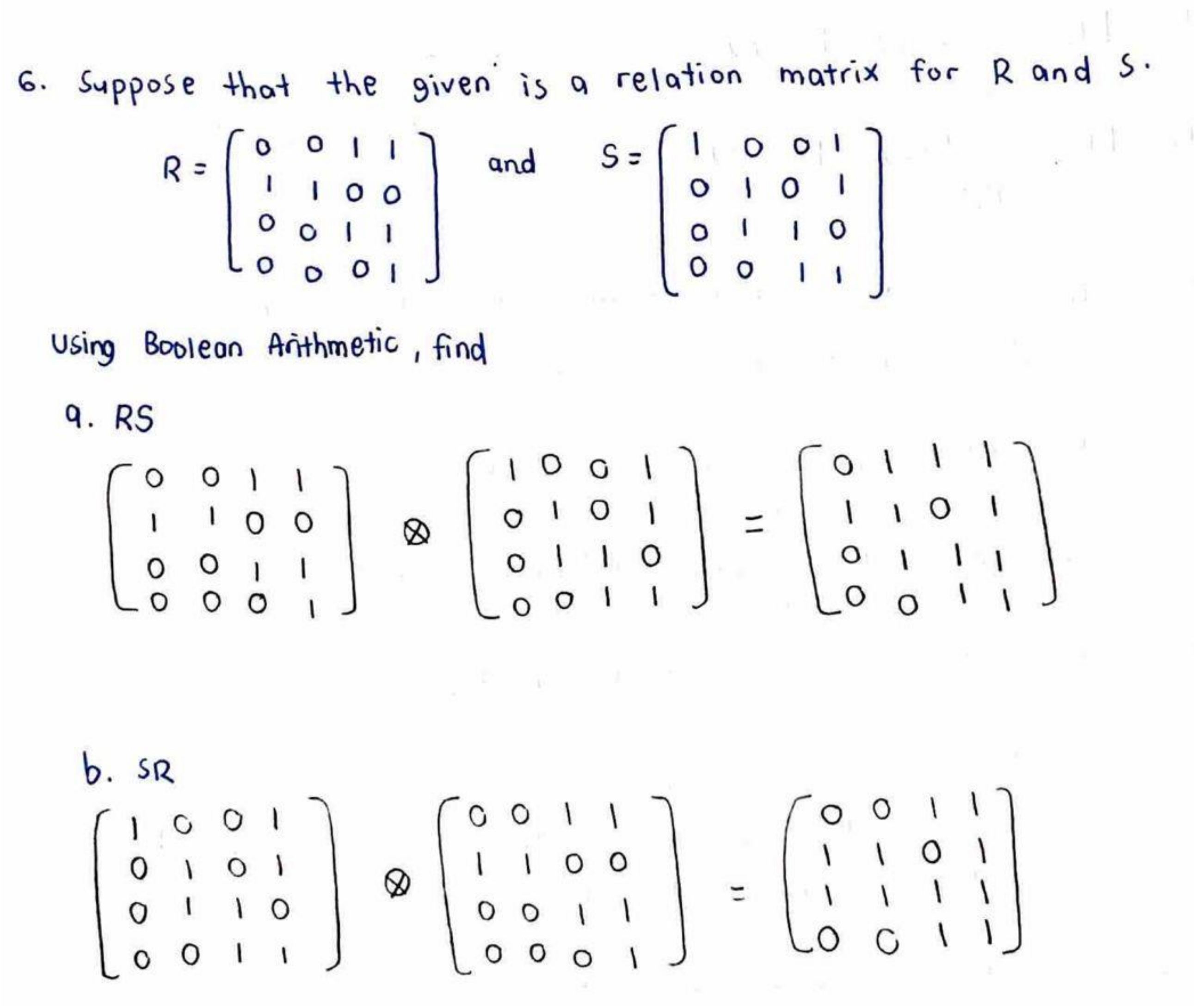
6. Suppose that the given is a relation matrix for R and S,

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Using Boolean Arithmetric, Find

a. RS
b. SR

(8 marks)



6. Suppose that the given is a relation matrix for R and S.

$$R = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ and } S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Using Boolean Arithmetic, find

a. RS

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

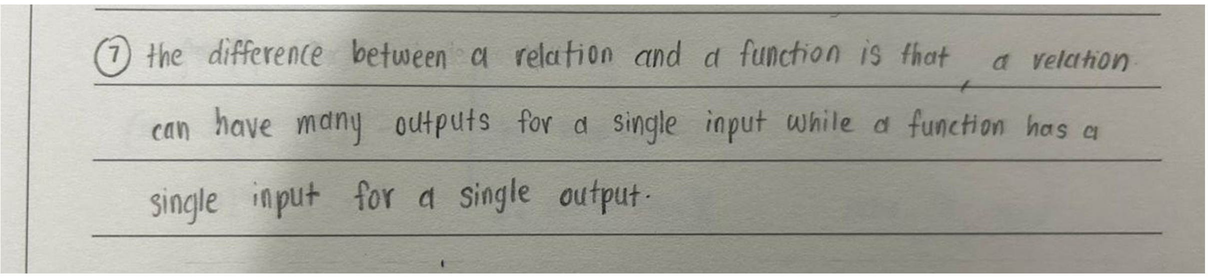
b. SR

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

**Q2. Function**

7. What is the different between Relation and Function?

(2 Marks)



⑦ the difference between a relation and a function is that, a relation can have many outputs for a single input while a function has a single input for a single output.

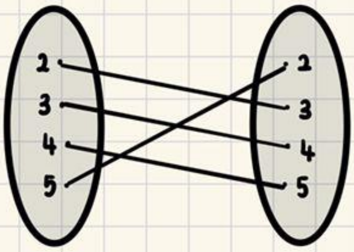
8. If A = {2, 3, 4, 5}, then write whether each of the following relations on set A is a function or not. Give reasons also.

(i) {(2, 3), (3, 4), (4, 5), (5, 2)}
(ii) {(2, 4), (3, 4), (5, 4), (4, 4)}
(iii) {(2, 3), (2, 4), (5, 4)}
(iv) {(2, 3), (3, 5), (4, 5)} (v) {(2, 2), (2, 3), (4, 4), (4, 5)}

(8 marks)

8. A = { 2, 3, 4, 5 )

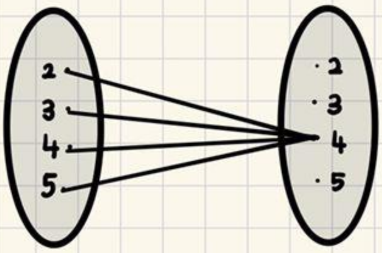
(i) { (2,3) (3,4) (4,5) (5,2) }



- domain R = { 2, 3, 4, 5 }
- range R = { 2, 3, 4, 5 }
- one-to-one

∴ the set is function #

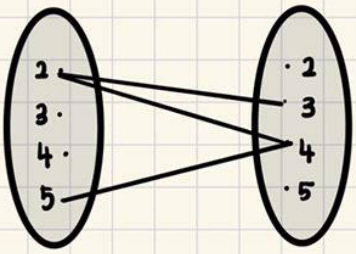
(ii) { (2,4) (3,4) (5,4) (4,4) }



- domain R = { 2, 3, 4, 5 }
- range R = { 4 }
- all has arrow from domain

∴ the set is function

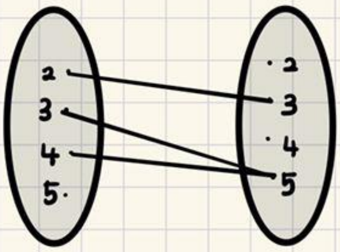
(iii) { (2,3) (2,4) (5,4) }



- domain R = { 2, 5 } ≠ set A
- range R = { 3, 4 }
- no arrow from domain 3, 4

∴ the set is not function

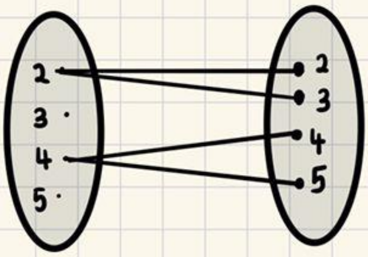
(iv) $\{ (2,3) (3,5) (4,5) \}$



- domain R $= \{ 2,3,4 \} \neq$ set A
- range R $= \{ 3,5 \}$
- no arrow from domain 5

∴ the set is not function.

(v) $\{ (2,2) (2,3) (4,4) (4,5) \}$



- domain R $= \{ 2,4 \} \neq$ set A
- range R $= \{ 2,3,4,5 \}$
- no arrow from domain 3,5

∴ the set is not function.

9. Given the relation of $R = \{(x, y)|y = x + 5\,, x \; is \; \mathbb{Z}^+ \; less \; than \; 6\}$. Depict this relationship using roster form. Write down the domain and the range.

(3 marks)

9. Given the relation of $R = \{(x,y) \mid y = x+5, x \text{ is } \mathbb{Z}^+ \text{ less than } 6\}$. Depict this relationship using roster form. write down the domain and the range.

$R = \{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$

Domain = 0, 1, 2, 3, 4, 5

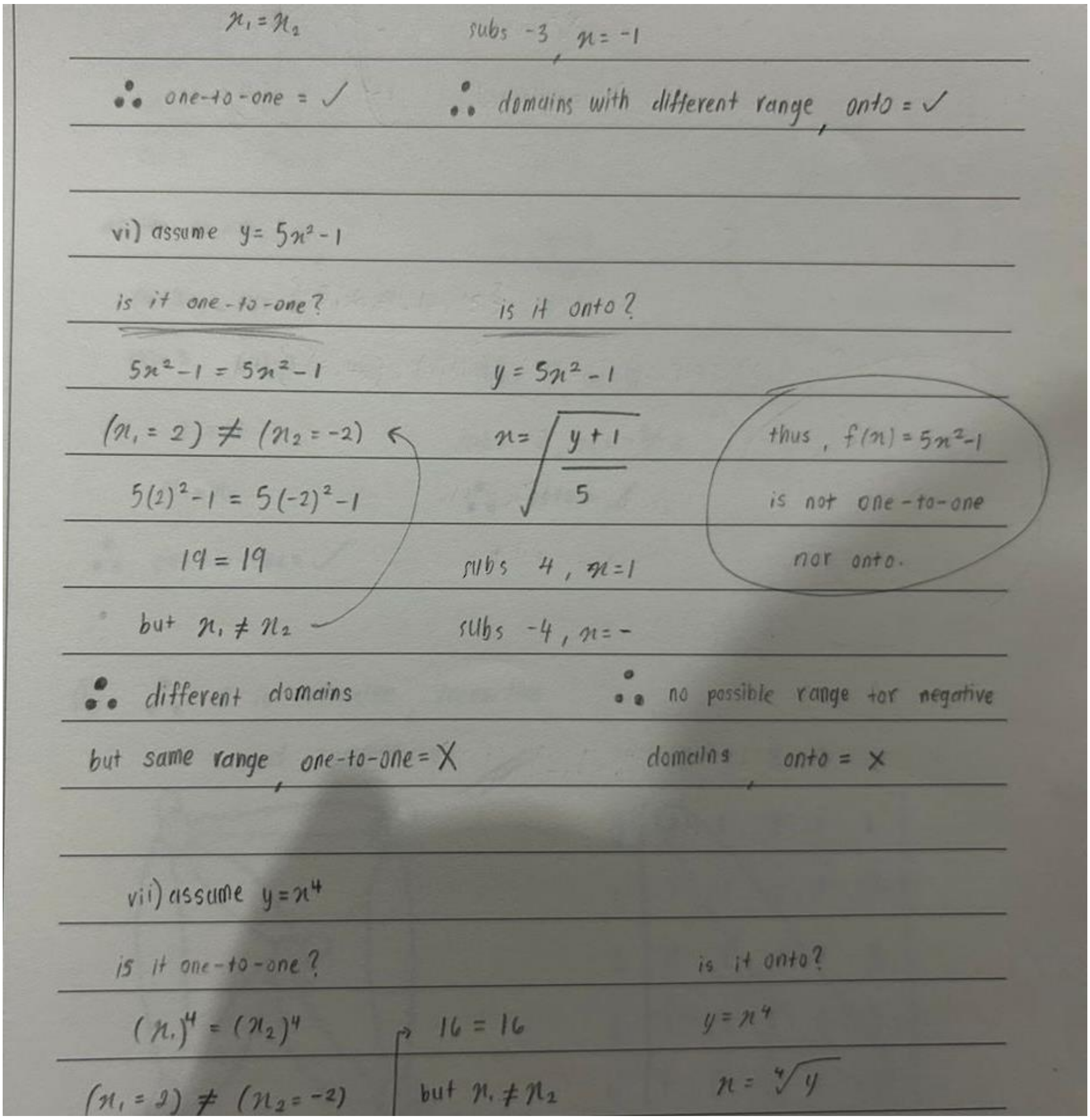
Range = 5, 6, 7, 8, 9, 10

| | | | | | | |
|---|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 |
| $y$ | 5 | 6 | 7 | 8 | 9 | 10 |

10. In the following cases, state whether the function is one-one, onto or bijective. Justify your answer.

- (v) $f = R \rightarrow R, f(x) = 1 - 2x$
- (vi) $f = R \rightarrow R, f(x) = 5x^2 - 1$
- (vii) $f = R \ \rightarrow R, f(x) = x^4$
- (viii) $f = R \rightarrow R, f(x) = \left(\frac{x-2}{x-3}\right)$

(8 marks)



$x_1 = x_2$

∴ one-to-one = ✓

subs -3, $x = -1$

∴ domains with different range, onto = ✓

vi) assume $y = 5x^2 - 1$

is it one-to-one?

$5x^2 - 1 = 5x^2 - 1$

$(x_1 = 2) \neq (x_2 = -2)$

$5(2)^2 - 1 = 5(-2)^2 - 1$

$19 = 19$

but $x_1 \neq x_2$

∴ different domains but same range, one-to-one = X

is it onto?

$y = 5x^2 - 1$

$x = \sqrt{\frac{y+1}{5}}$

subs 4, $x = 1$

subs -4, $x = -$

∴ no possible range for negative domains, onto = X

thus, $f(x) = 5x^2 - 1$ is not one-to-one nor onto.

vii) assume $y = x^4$

is it one-to-one?

$(x_1)^4 = (x_2)^4$

$(x_1 = 2) \neq (x_2 = -2)$

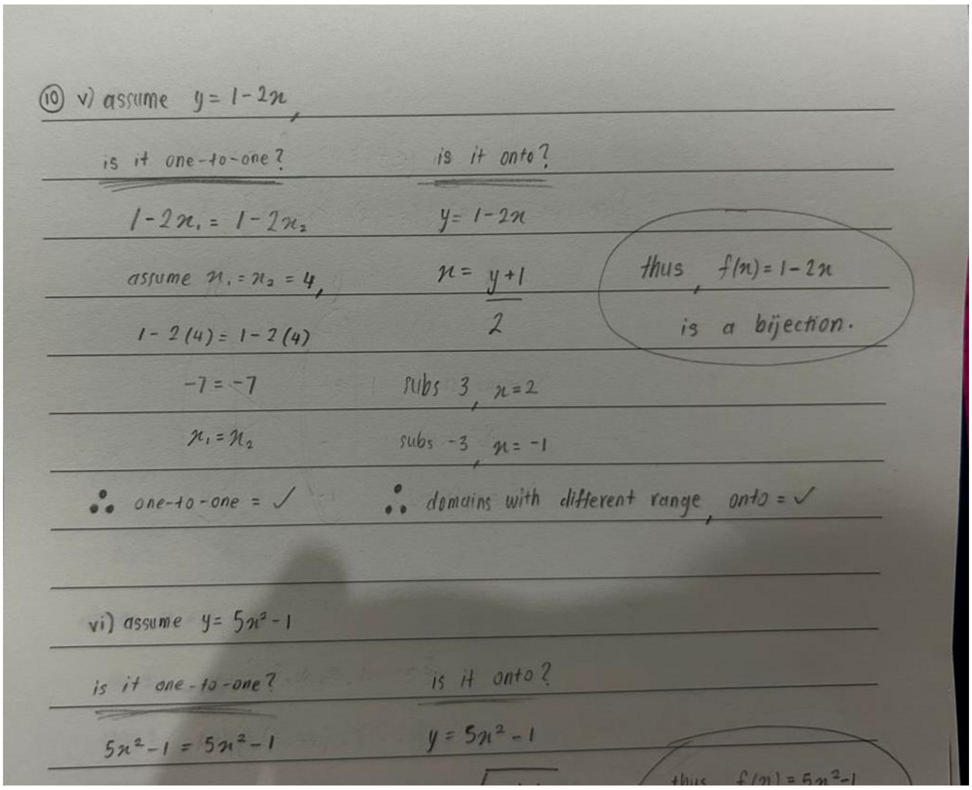
$16 = 16$

but $x_1 \neq x_2$

is it onto?

$y = x^4$

$x = \sqrt[4]{y}$

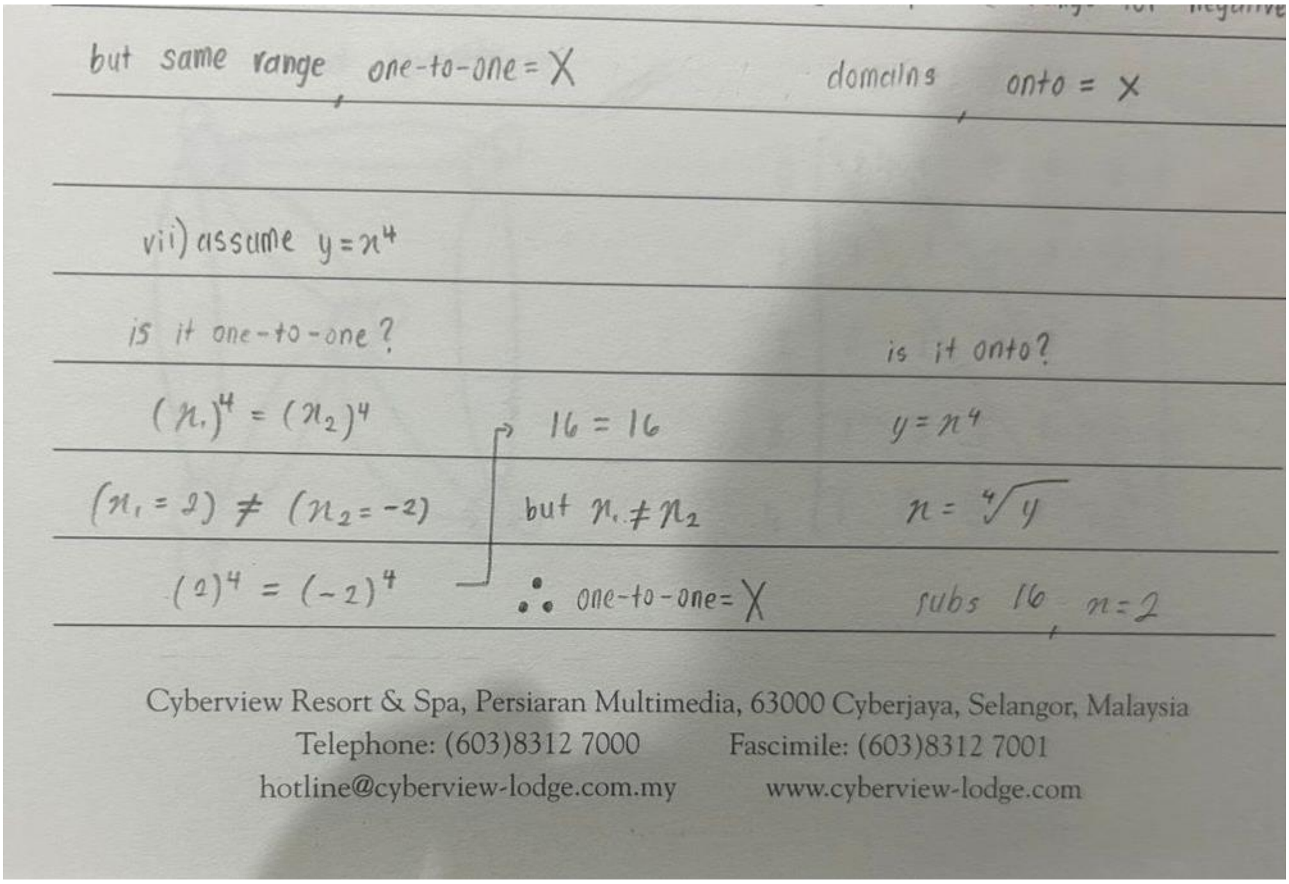


⑩ v) assume $y = 1-2x$,

| is it one-to-one? | is it onto? | |
|---|---|---|
| $1-2x_1 = 1-2x_2$ | $y = 1-2x$ | |
| assume $x_1 = x_2 = 4$, | $x = \frac{y+1}{2}$ | thus, $f(x) = 1-2x$ is a bijection. |
| $1-2(4) = 1-2(4)$ | | |
| $-7 = -7$ | subs 3, $x=2$ | |
| $x_1 = x_2$ | subs -3, $x=-1$ | |
| ∴ one-to-one = ✓ | ∴ domains with different range, onto = ✓ | |

vi) assume $y = 5x^2-1$

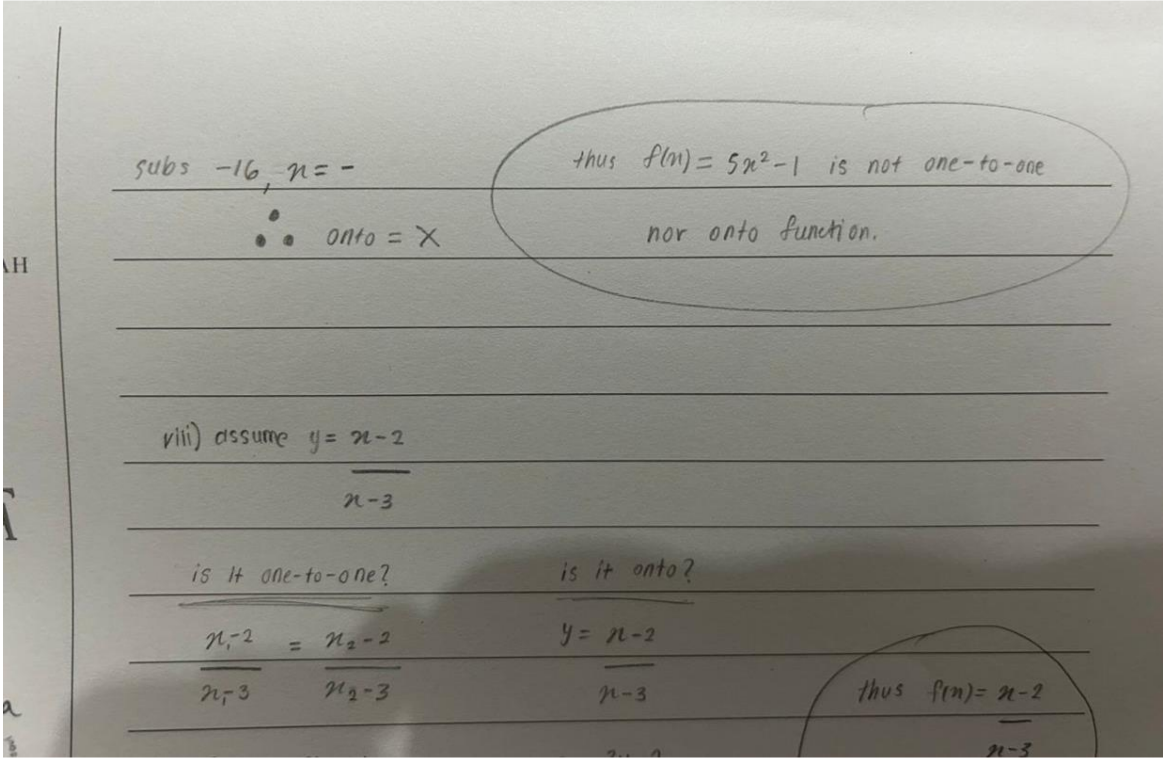
| is it one-to-one? | is it onto? | |
|---|---|---|
| $5x^2-1 = 5x^2-1$ | $y = 5x^2-1$ | thus $f(x) = 5x^2-1$ |



but same range, one-to-one = X      domains, onto = X

vii) assume $y = x^4$

| is it one-to-one? | | is it onto? |
|---|---|---|
| $(x_1)^4 = (x_2)^4$ | $\to 16 = 16$ | $y = x^4$ |
| $(x_1 = 2) \neq (x_2 = -2)$ | but $x_1 \neq x_2$ | $x = \sqrt[4]{y}$ |
| $(2)^4 = (-2)^4$ | ∴ one-to-one = X | subs 16, $x=2$ |



subs $-16$, $x = -$

$\therefore$ onto = ✗

thus $f(x) = 5x^2 - 1$ is not one-to-one nor onto function.

viii) assume $y = \frac{x-2}{x-3}$

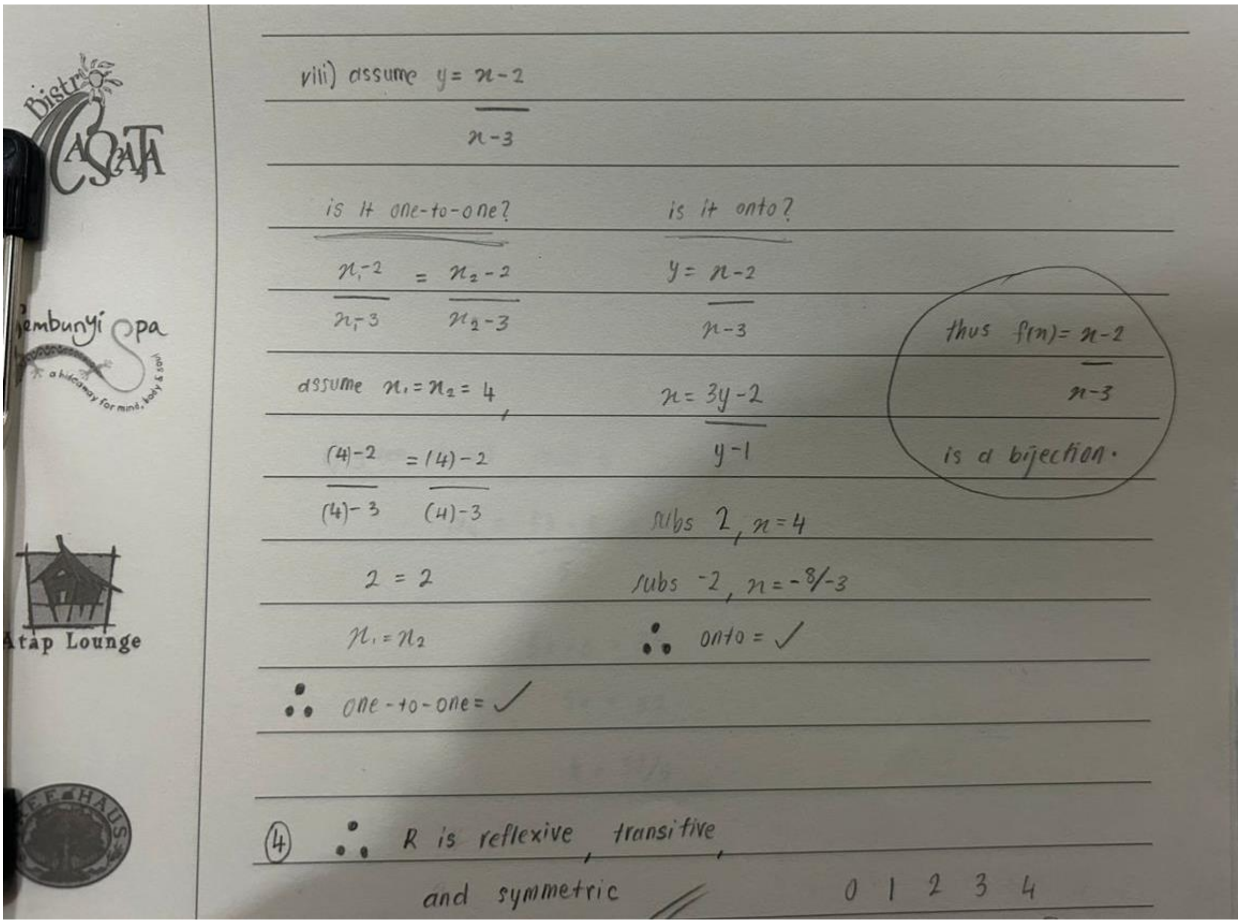
is it one-to-one?

$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

is it onto?

$y = \frac{x-2}{x-3}$

thus $f(x) = \frac{x-2}{x-3}$



viii) assume $y = \frac{x-2}{x-3}$

is it one-to-one?

$\frac{x_1-2}{x_1-3} = \frac{x_2-2}{x_2-3}$

assume $x_1 = x_2 = 4$,

$\frac{(4)-2}{(4)-3} = \frac{(4)-2}{(4)-3}$

$2 = 2$

$x_1 = x_2$

$\therefore$ one-to-one = ✓

is it onto?

$y = \frac{x-2}{x-3}$

$x = \frac{3y-2}{y-1}$

subs $2$, $x = 4$

subs $-2$, $x = -8/-3$

$\therefore$ onto = ✓

thus $f(x) = \frac{x-2}{x-3}$ is a bijection.

④ $\therefore$ R is reflexive, transitive, and symmetric

0 1 2 3 4

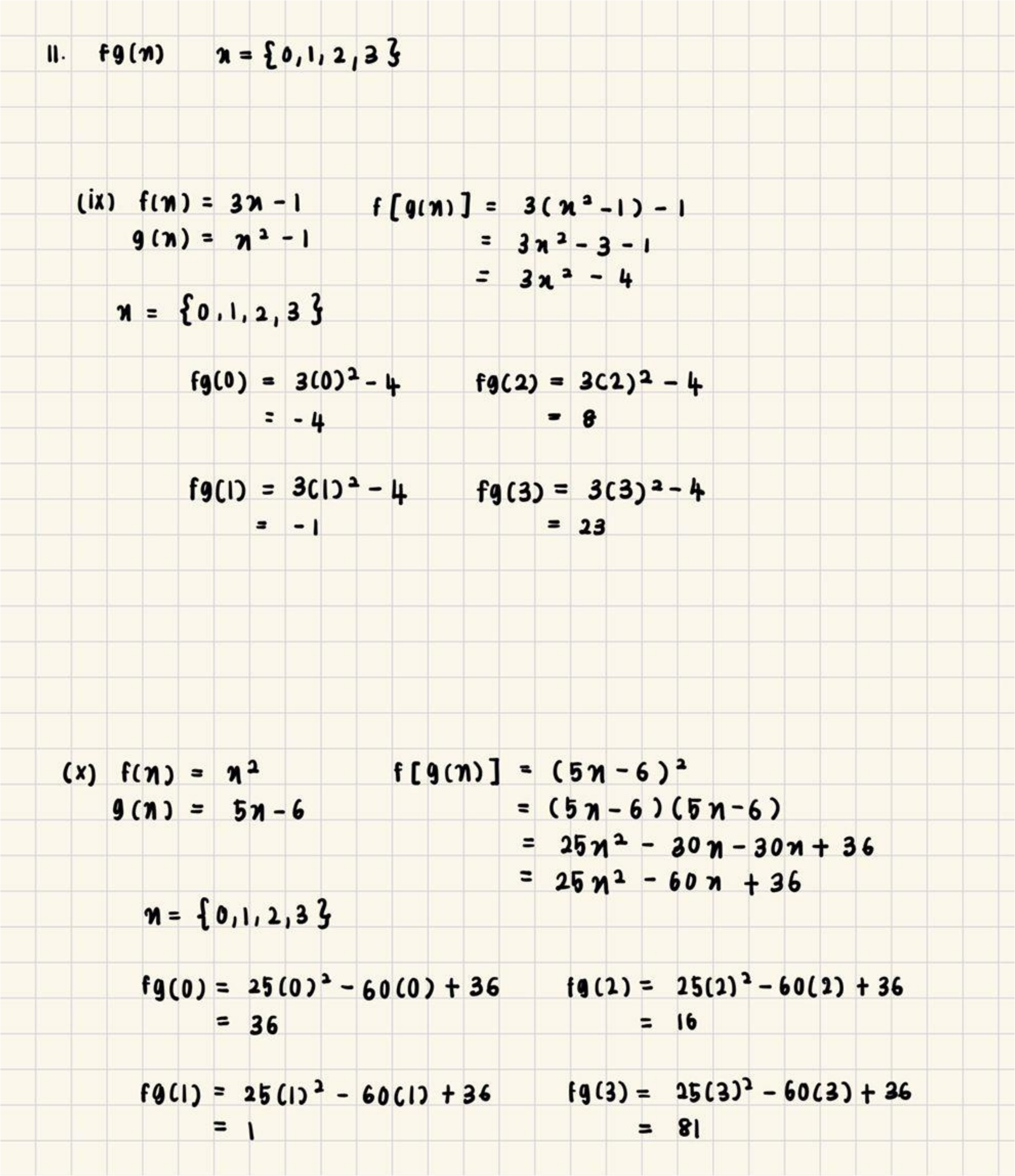
11. Given the following functions, find the function $f(g(x))$ and find the value of the function if $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1\,;\, g(x) = x^2 - 1$

(x) $f(x) = x^2\,;\, g(x) = 5x - 6$

(xi) $f(x) = x - 1\,;\, g(x) = x^3 + 1$

(9 marks)



11. $fg(x)$ $x = \{0, 1, 2, 3\}$

(ix) $f(x) = 3x - 1$

$g(x) = x^2 - 1$

$f[g(x)] = 3(x^2 - 1) - 1$

$= 3x^2 - 3 - 1$

$= 3x^2 - 4$

$x = \{0, 1, 2, 3\}$

$fg(0) = 3(0)^2 - 4$

$= -4$

$fg(1) = 3(1)^2 - 4$

$= -1$

$fg(2) = 3(2)^2 - 4$

$= 8$

$fg(3) = 3(3)^2 - 4$

$= 23$

(x) $f(x) = x^2$

$g(x) = 5x - 6$

$f[g(x)] = (5x - 6)^2$

$= (5x - 6)(5x - 6)$

$= 25x^2 - 30x - 30x + 36$

$= 25x^2 - 60x + 36$

$x = \{0, 1, 2, 3\}$

$fg(0) = 25(0)^2 - 60(0) + 36$

$= 36$

$fg(1) = 25(1)^2 - 60(1) + 36$

$= 1$

$fg(2) = 25(2)^2 - 60(2) + 36$

$= 16$

$fg(3) = 25(3)^2 - 60(3) + 36$

$= 81$

(xi) $f(x) = x - 1$

$g(x) = x^3 + 1$

$f[g(x)] = (x^3 + 1) - 1$

$= x^3 + 1 - 1$

$= x^3$

$x = \{0, 1, 2, 3\}$

$fg(0) = 0^3$

$= 0$

$fg(1) = 1^3$

$= 1$

$fg(2) = 2^3$

$= 8$

$fg(3) = 3^3$

$= 27$

## Q3. Recurrence Relation

12. Solve the recurrence relation given;

(xii) $a_n = 6a_{n-1} - 9a_{n-2}$ ; initial conditions $a_0 = 1\ and\ a_1 = 6$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ ;
initial conditions $a_0 = 2\,, a_1 = 5\ and\ a_2 = 15$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$
initial conditions $a_0 = 1\,, a_1 = -2\ and\ a_2 = -1$

(12 marks)

i) $a_n = 6a_{n-1} - 9a_{n-2}$ ; initial condition $a_0 = 1$ and $a_1 = 6$

$a_2 = 6a_1 - 9a_0 = 6(6) - 9(1)$
$= 27$

$a_3 = 6a_2 - 9a_1 = 6(27) - 9(6)$
$= 108$

$a_4 = 6a_3 - 9a_2 = 6(108) - 9(27)$
$= 405$

$a_5 = 6a_4 - 9a_3 = 6(405) - 9(108)$
$= 1458$

new recurrence relations :
1, 6, 27, 108, 405, 1458, ...

ii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ ; initial condition $a_0 = 2, a_1 = 5$ and $a_2 = 15$

$a_3 = 6a_2 - 11a_1 + 6a_0 = 6(15) - 11(5) + 6(2)$
$= 47$

$a_4 = 6a_3 - 11a_2 + 6a_1 = 6(47) - 11(15) + 6(5)$
$= 147$

$a_5 = 6a_4 - 11a_3 + 6a_2 = 6(147) - 11(47) + 6(15)$
$= 455$

$a_6 = 6a_5 - 11a_4 + 6a_3 = 6(455) - 11(147) + 6(47)$
$= 1395$

new recurrence relations : 2, 5, 15, 47, 147, 455, 1395, ...

iii) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$ ; initial condition $a_0 = 1$, $a_1 = -2$ and $a_2 = -1$

$a_3 = -3a_2 - 3a_1 + a_0 = -3(-1) - 3(-2) + 1$
$= 10$

$a_4 = -3a_3 - 3a_2 + a_1 = -3(10) - 3(-1) + (-2)$
$= -29$

$a_5 = -3a_4 - 3a_3 + a_2 = -3(-29) - 3(10) + (-1)$
$= 56$

$a_6 = -3a_5 - 3a_4 + a_3 = -3(56) - 3(-29) + (10)$
$= -71$

new recurrence relations : 1, -2, -1, 10, -29, 56, -71, ...

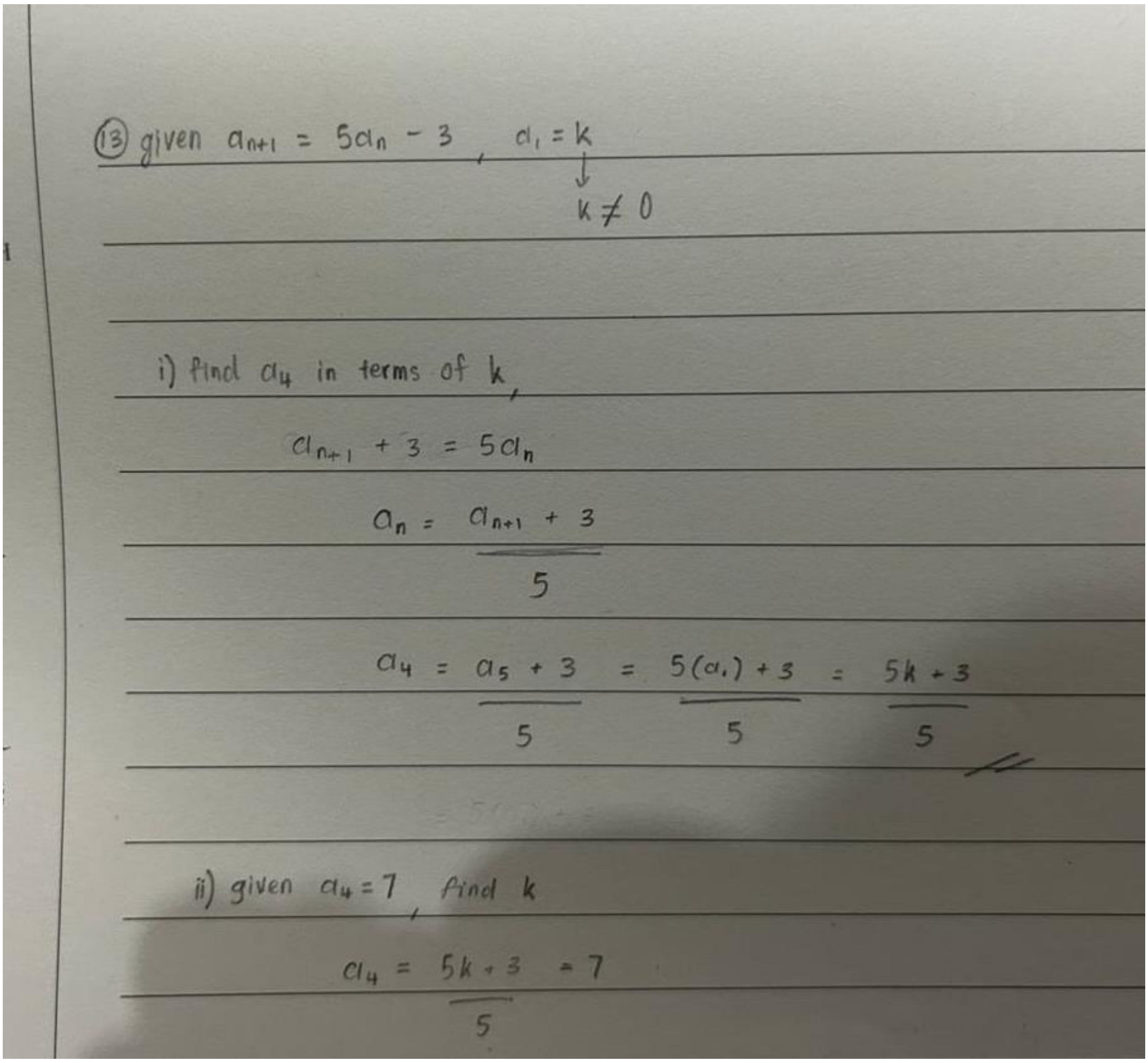
13. A sequence $a_1$, $a_2$ , $a_3$ , $a_4$ , … is given by

$$a_{n+1} = 5a_n - 3 \; ; \; a_1 = k$$

where k is a non-zero constant.

(i) Find the value of $a_4$ in terms of k .
(ii) Given that $a_4$= 7 , determine the value of k .

(8 marks)



⑬ given $a_{n+1} = 5a_n - 3$ , $a_1 = k$

$\downarrow$

$k \neq 0$

i) find $a_4$ in terms of k,

$$a_{n+1} + 3 = 5a_n$$

$$a_n = \frac{a_{n+1} + 3}{5}$$

$$a_4 = \frac{a_5 + 3}{5} = \frac{5(a_1) + 3}{5} = \frac{5k + 3}{5} //$$

ii) given $a_4 = 7$, find k

$$a_4 = \frac{5k + 3}{5} = 7$$

5

ii) given $a_4 = 7$, find k

$$a_4 = \frac{5k+3}{5} = 7$$

$$5k + 3 = 35$$

$$5k = 32$$

$$k = \frac{32}{5}$$ //