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Group Assignment 4

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**Description**

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1   2CNF(cnfFormula)
2       implicationGraph = empty directed graph
3
4       for each clause of the form (u || v) in cnfFormula
5           add edges (ū , v) and  (v̄ , u) to implicationGraph
6
7       SCCs = strongly connected components of implicationGraph
8
9       if any strongly connected component in SCCs contains both v and v̄ for any literal v
10          return False (formula is not satisfiable)
11      else
12          return True (formula is satisfiable)
```

**Runtime**

| Line(s) | Runtime | Explanation |
|---|---|---|
| 2 | O(1) | |
| 4-5 | O(n) | Each clause is used in exactly 2 constant-time operations. |
| 7 | O(n) | If the directed graph is represented as an adjacency list, Kosaraju's Algorithm can be used to find strongly connected components in linear-time [1]. |
| 9 | O(n) | This check requires looking at each literal and its negation exactly once. |
| 10-12 | O(1) | |

Total Runtime: O(n)

Reduction

The following steps reduce the inside/outside problem to 2CNF:

Let *cnf* be an empty boolean expression in conjunctive normal form where each clause contains exactly two literals.

Label each road from the inside/outside problem with a unique literal.

Let a positive literal represent that a road will be built inside the ring road and a negative literal represent that a road will be built outside the ring road.

Let U represent all of the roads. For each road, u ϵ U, let V represent the set of roads that would intersect u if both were built inside the ring road. For each road v ϵ V, add the following to *cnf*:

$$(u \,||\, v) \;\&\&\; (\bar{u} \,||\, \bar{v})$$

Correctness:

For every pair of roads that have the potential to intersect, the clauses added to the CNF formula dictate that one must be built inside the ring road and one must be built outside of the ring road if the formula is to be satisfied. This ensures that the formula is satisfied by an assignment iff the assignment does not contain any representations of road intersections as defined in the reduction.

## References

[1] J. Park, "Basic Graph Algorithms," in *Stanford University*, 2015. [Online]. Available: http://web.stanford.edu/class/cs97si/06-basic-graph-algorithms.pdf. Accessed: Dec. 1, 2016.