A geometric method to determine the electric field due to a uniformly charged line segment

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NOTES AND DISCUSSIONS

A geometric method to determine the electric field due to a uniformly charged line segment

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A geometrical method to calculate the electric field due to a uniformly charged rod is presented. The result is surprisingly simple and elegant. Using only lengths and angles, the direction of the electric field at any point due to this charge configuration can be graphically determined. The method is not new but seems to have been all but forgotten. A full understanding of this result can lead to a deeper appreciation of symmetry in a seemingly un-symmetric system.

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I. INTRODUCTION

The calculation of electric fields created by continuous charge distributions is a challenging part of an introductory physics course. It is a difficult task for many students because they are applying the concept of integration for the first time to an actual physical situation. To help students understand the process, nearly every calculus-based physics textbook starts with the example of calculating the electric field of a uniformly charged line segment.[1–8] This result can be simplified when the line segment is infinitely long, allowing comparison to a simple algebraic derivation using Gauss's law.

Applications of Gauss's law require a high degree of symmetry. Spherical charge distributions are the only finite systems for which the required symmetry is exact. Cylindrical or planar symmetry requires the system to be infinitely large—an infinitely long wire or infinite planar sheet—to obtain the field from Gauss's law. In the case of an infinite line with a uniform charge density, the electric field possesses cylindrical symmetry, which enables the electric flux through a Gaussian cylinder of radius r and length l to be expressed as $\Phi_E = 2\pi r l E = \lambda l/\epsilon_0$, implying $E(r) = \lambda/2\pi\epsilon_0 r = 2k\lambda/r$, where $k = 1/4\pi\epsilon_0$. For a finite line segment, however, symmetry determines the direction of the electric field only on the axis of the line and on the plane bisecting the line. At any other point, the direction of the electric field is not intuitively apparent. While it is straightforward to calculate the electric field components, the results often appear quite complicated.

This situation naturally leads to questions such as "are there simple ways to find the electric field?" or "are there hidden symmetries in the problem?"

II. ANALYSIS

The problem is to find the electric field everywhere due to a thin rod of length L with a uniform charge density λ. Without loss of generality, place the rod on the x-axis from $x = a$ to $x = b$ with $b = a + L$. The calculation of the electric field on the x-axis is fairly straightforward, because the direction is the same for every infinitesimal contribution dE from a dq on the charged rod:

$$E(x') = \int_a^b \frac{k\,dq}{r^2} = \int_a^b \frac{k\lambda dx}{(x'-x)^2} \tag{1}$$

The calculation of electric field at a point $P(y)$ on the y-axis is more involved because of the nonzero components in both the x and y directions.

There are two common approaches to finding the net electric field. Most often the approach taken is to integrate each component in terms of x (see Fig. 1):

$$E_x = -\int_a^b \frac{k\,dq}{r^2}\sin\theta = -\int_a^b \frac{k\lambda\,dx}{r^2}\frac{x}{r} = -\int_a^b \frac{k\lambda x\,dx}{(x^2+y^2)^{3/2}}$$
$$= k\lambda\left[\frac{1}{\sqrt{b^2+y^2}} - \frac{1}{\sqrt{a^2+y^2}}\right] \tag{2}$$

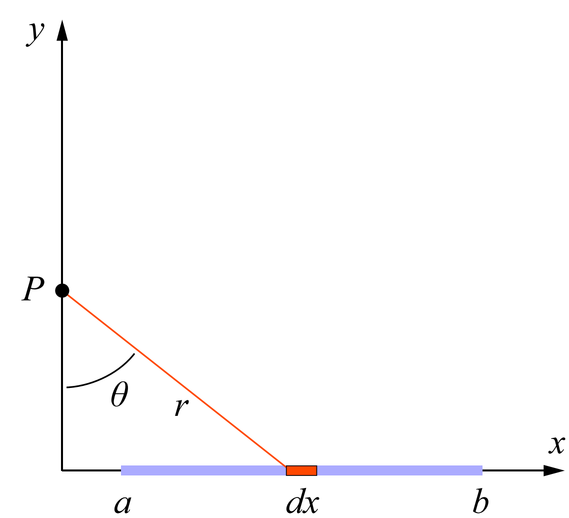

Fig. 1. The coordinate system and angle θ used in the calculation of the electric field at point P.

$$E_y = \int_a^b \frac{k\,dq}{r^2}\cos\theta = \int_a^b \frac{k\lambda\,dx}{r^2}\frac{y}{r} = \frac{k\lambda}{y}\left[\frac{b}{\sqrt{b^2+y^2}} - \frac{a}{\sqrt{a^2+y^2}}\right] \tag{3}$$

where the y-component involves a slightly more complicated integral

$$\int \frac{dx}{(x^2+y^2)^{3/2}} = \frac{1}{y^2}\frac{x}{\sqrt{x^2+y^2}}\,. \tag{4}$$

An alternative method, which avoids the integral above, is to use a change of variable from x to the angle θ. Using the relations $x = y\tan\theta$, $r = y/\cos\theta$, and $dx = yd\theta/\cos^2\theta$, the electric field components can be obtained as

$$E_x = -\int_a^b \frac{k\,dq}{r^2}\sin\theta = -\int_{\theta_a}^{\theta_b} \frac{k\lambda}{y}\sin\theta d\theta = \frac{k\lambda}{y}[\cos\theta_b - \cos\theta_a] \tag{5}$$

$$E_y = \int_a^b \frac{k\,dq}{r^2}\cos\theta = \int_{\theta_a}^{\theta_b} \frac{k\lambda}{y}\cos\theta d\theta = \frac{k\lambda}{y}[\sin\theta_b - \sin\theta_a]. \tag{6}$$

These results are consistent with Eqs. (2) and (3), since $\sin\theta_b = b/\sqrt{b^2+y^2}$ and $\sin\theta_a = a/\sqrt{a^2+y^2}$, but the integral is obviously much simpler in terms of the angle θ.

This example is normally done in textbooks using one of these two approaches. Limiting cases are then discussed, such as an infinitely long rod ($L \gg y$) or when the point of observation is far away compared to the size of rod ($y \gg L$). For intermediate cases, instructors and students alike are generally satisfied with knowing how to solve the problem while believing the answer is too complicated to contemplate or to make sense of.

However, it turns out that the answers above for the finite rod have a simple geometrical meaning. The infinitesimal contribution dE at a point P on the y-axis, due to the charge $\lambda\,dx$ on the x-axis, is given by

$$dE = \frac{k\,dq}{r^2} = \frac{k\lambda\,dx}{r^2} = \frac{k\lambda\,y\,d\theta/\cos^2\theta}{y^2/\cos^2\theta} = \frac{k\lambda\,y\,d\theta}{y^2}\,. \tag{7}$$

The last expression in Eq. (7) corresponds to the infinitesimal contribution from an arc segment of radius y and arc length $y\,d\theta$ with the same linear charge density λ. In other words, the electric field contribution from the charge on the x-axis can be mapped to the contribution by hypothetical charges on a circular segment of radius y, as shown in Fig. 2. For a circular arc, the symmetry axis is well defined. Thus, the total electric field due to a circular segment is along the direction bisecting the arc. If the lines connecting the ends are defined by the angles θ_a and θ_b, then the arc is defined by the angular spread of $\theta_b - \theta_a$, and the bisecting line will be pointing in the direction $\theta_a + (\theta_b - \theta_a)/2 = (\theta_b + \theta_a)/2$.

This geometric argument is consistent with the direction of the total electric field calculated in Eqs. (5) and (6) above and as shown in Fig. 3. The direction is defined by

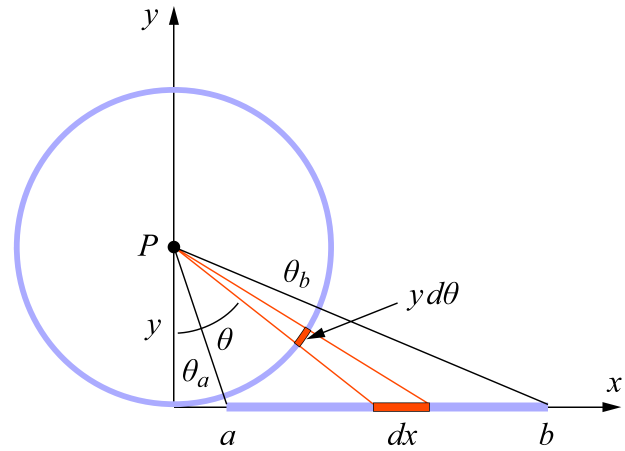

Fig. 2. Calculation of the electric field at point P. The infinitesimal contribution from dx is the same as that from an infinitesimal arc segment $y\,d\theta$.

$$\cot\varphi = \frac{E_y}{-E_x} = -\frac{\sin\theta_b - \sin\theta_a}{\cos\theta_b - \cos\theta_a} = \cot\frac{1}{2}(\theta_b + \theta_a). \tag{8}$$

The magnitude of the total electric field can be calculated for the simple arc to be

$$E = \frac{2k\lambda}{y}\sin\frac{1}{2}(\theta_b - \theta_a), \tag{9}$$

which can also be obtained directly from Eqs. (5) and (6).

The simplicity of the transformation between the line and the arc enables one to determine the direction and magnitude of the electric field due to any uniformly charged rod at any point in space using two purely geometrical quantities: the radius y (vertical distance from the point to the line) and the angle mid-way between the lines connecting the point to the ends of the rod.

One interesting example of this method is the calculation of the electric field on the y-axis due to a semi-infinite uniformly charged wire whose left end is at the origin, shown in Fig. 4. In this case, $\theta_b = \pi/2$ and $\theta_a = 0$ so the electric field always points in the direction $\pi/4$ or 45°, with a magnitude of $\sqrt{2}k\lambda/y$. The results look somewhat counterintuitive, as one might expect the field to point more toward the negative x-axis as y approaches zero. However, the mapped charge distribution will always lie along a quarter circle regardless of the radius y, so the field will always points in the same direction.

For a wire that is infinitely long in both directions, the transformation gives a half circle of radius y and $E = 2k\lambda/y$, the same result that is obtained from using Gauss's law.

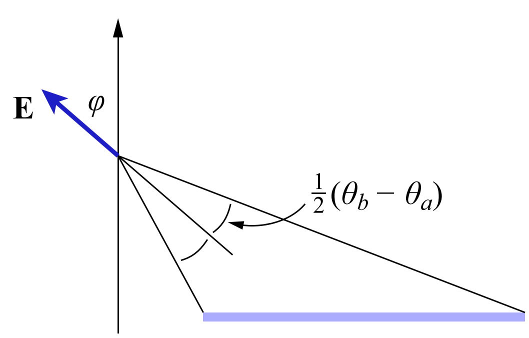

Fig. 3. The electric field points in the same direction as the line bisecting the angle from the point to the two ends of the rod.

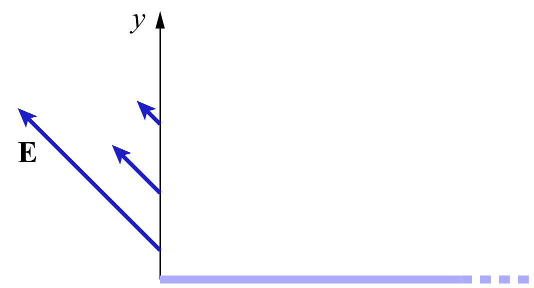

Fig. 4. Electric field on the y-axis due to a semi-infinite wire with its left end at the origin. Each of the field vectors points at a 45° angle to the wire.

The direction of the electric field can also be derived by first calculating the electric potential and then taking its gradient. It is known that the equipotential surface of a charged rod corresponds to an ellipsoid with the ends of the rod being the foci.[7–9] The gradient of the ellipsoidal surface indeed bisects the angle subtended between the point and the rod. However, the mathematical transformation and the calculation of the gradient are rather complicated and well beyond the level of introductory physics.

The mapping transformation can be used in other cases involving a $1/r^2$ dependence, such as the gravitational field due to a line segment of uniform mass density. However, it cannot be applied directly to the calculation of the magnetic field from a straight current-carrying wire, because of the cross product in $d\vec{B} = \mu_0 I\, d\vec{l} \times \hat{r}/4\pi r^2$. It would be of great interest to find analogous transformations in other systems or in higher dimensions.

III. LOOKING BACK IN HISTORY

It would be foolish to believe that this is the first time this transformation has been discovered. In fact, as pointed out by the referees of this paper, similar methods and results were obtained for the gravitational force of a rod by Edward Routh in his 1892 book *A Treatise on Analytical Statics with Numerous Examples*.[10] In 1879, Lord Kelvin (William Thomson) and Peter Guthrie Tait published a solution using pure geometrical arguments without calculus in their *Treatise on Natural Philosophy*.[11] Almost 50 years earlier, in 1828, George Green derived the ellipsoidal equipotential surface in "An Essay on the Application of Mathematical Analysis to the Theories of Electricity and Magnetism," which was self-published and was ignored by mathematicians for two decades.[12] The geometrical interpretation was discussed in the Appendix of *Mathematical Papers of the Late George Green*, edited by Ferrers in 1871.[13] Clearly, the electric field due to a line segment was known at that time. In addition, a similar result in terms of the ellipsoidal equipotential surface and its normal was published in French by Emile Durand about 60 years ago.[14]

Contemporary textbooks on electromagnetism have largely neglected the transformation between the line segment and a circular arc, not only at the introductory university physics level but also at the advanced undergraduate[7,8] and graduate levels.[9] In fact, this method does not appear to be known in some recent publications on this subject.[15,16]

It is not obvious why and how the geometric treatment, apparently well known in the late 19th century, has been all but lost. What is clear is that classic works by the pioneers should never be left in the dark and forgotten.[17]

IV. CONCLUSION

The contribution to the electric field from an infinitesimal charged line segment dx can be mapped to that of an infinitesimal arc segment of a fixed radius, leading to a mapping of a finite line segment to a finite circular arc. The symmetry axis of the arc is easily defined, thus pinpointing the direction and the magnitude of the total electric field.

This result can be traced back nearly two hundred years to George Green, yet is not found in today's textbooks. That such a simple and elegant transformation has not been preserved in current physics textbooks is surprising. Students in a calculus-based college physics course are certainly able to appreciate the beauty of this transformation.

The transformation of a seemingly non-symmetric charge distribution to a symmetric circular segment for the calculation of the electric field is interesting by itself. Discussion of this approach will enlighten students of science and engineering and encourage them to pursue simplicity and symmetry in complex problems.

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Comment on “Varying-G Cosmology with Type Ia Supernovae” [Am. J. Phys. 79, 57–62 (2011)]

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In a recent paper, Dungan and Prosper claim that the Type Ia supernovae data alone are not enough to distinguish between the standard ΛCDM model and other models with varying G. To substantiate this, they present two spatially flat variable G FRW models with $\Lambda = 0$ that fit well the Type Ia supernova data. In these models they assumed that the energy momentum tensor of the matter distribution is conserved. We show that this assumption is inconsistent with variable G cosmology when Λ is assumed to be constant, thus rendering the suggested models erroneous.

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I. INTRODUCTION

The idea of variability of the gravitational constant G is not a novel one. It was first suggested by Dirac[1] who claimed that $G \sim 1/t$, in light of his Large Number Hypothesis (LNH). Following Dirac's claim came the Brans-Dicke theory of gravity,[2] in which the gravitational constant G is replaced with a scalar field ϕ that couples to gravity via a constant ω. This theory was later generalized by Nordtvedt[3] to what are now called scalar-tensor theories, which use different variable coupling parameters $\omega(\phi)$. The variability of G has also been accounted for in observations, with various constraints on $|\dot{G}/G|$ obtained from lunar laser ranging,[4] the Viking Lander,[5] the spin rate of pulsars,[6] distant type Ia supernova,[7] helioseismological data,[8] and cosmological nucleosynthesis.[9] Working in the framework of general relativity, Lau[10] proposed a modification by introducing a variation of G and the cosmological constant Λ while preserving the form of Einstein's field equations. Since then, there have been numerous cosmological models with G and/or Λ as a function of time or scale factor (for a review of some of these models see Ref. 11). For example, it has been shown that a variable G can account for dark matter or some of its effects,[12] and recent dynamical dark-energy models utilizing a time dependent cosmological term[13] or a variable equation of state parameter $\omega(z)$[14] have also been considered. It is worth mentioning that many theoretical models with extra dimensions, such as string theory, contain a built in mechanism for possible time variation of the couplings.

II. VARYING-G MODELS

The Einstein's field equations with variable G and Λ are given by

$$R^{ij} - \frac{1}{2} R g^{ij} = \frac{8\pi G(t)}{c^4}\left[T^{ij} - \frac{\Lambda(t) c^4}{8\pi G(t)} g^{ij}\right]. \tag{1}$$

For the FLRW metric

$$ds^2 = -c^2 dt^2 + a(t)^2\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{2}$$

the field equations are

$$3\left(\frac{\dot{a}}{a}\right)^2 = 8\pi G(t)\rho - \frac{3kc^2}{a^2} + c^2\Lambda(t), \tag{3}$$

$$3\frac{\ddot{a}}{a} = -4\pi G(t)\left(\rho + \frac{3p}{c^2}\right) + c^2\Lambda(t), \tag{4}$$

which have the same form as those in standard GR with constant G and Λ. In a spatially flat ($k=0$) model with a matter distribution having an energy momentum tensor $T^{ij} = (\rho + p/c^2)u^i u^j + p g^{ij}$, where $u^i = (c, 0, 0, 0)$ is the four velocity vector in comoving coordinates and $p = wc^2\rho$, $0 \le w \le 1$ is the equation of state relating the pressure p and energy density ρ, the Bianchi identities lead to

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) + \frac{\dot{G}}{G}\rho + \frac{\dot{\Lambda}c^2}{8\pi G} = 0, \tag{5}$$

where the dots indicate differentiation with respect to the comoving time t. If one assumes the usual energy momentum conservation equation $T^{ab}_{\ ;b} = 0$ for the matter distribution, which leads to

$$\dot{\rho} + 3\frac{\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right) = 0, \tag{6}$$

then Eq. (5) implies a relation between the variation of G and Λ, given by

$$\dot{\Lambda} = -\frac{8\pi\dot{G}}{c^2}\rho. \tag{7}$$

Note that Eq. (7) is valid when the energy momentum tensors of the matter distribution and the cosmological term (vacuum energy) are conserved separately; i.e., there is no interchange of energy and momentum between the two components, which therefore requires that a variation in G is accompanied by a variation in the cosmological term Λ, as shown above. Cosmological models with a constant G and variable Λ (or a constant Λ and variable G) have been considered in the literature.[15,16] In this case, the vanishing of the divergence of the total energy momentum tensor leads to Eq. (5) with $\dot{G}=0$ (or $\dot{\Lambda}=0$), so that the respective energy momentum tensors cannot be conserved separately.

However, if besides the variable Λ and G parameters one also considers cosmological models in General Relativity with a time dependent speed of light $c(t)$ (see, for example, the perfect fluid cosmological models in Ref. 17), then the above Bianchi identity in Eq. (5) becomes

$$\dot{\rho}+3\frac{\dot{a}}{a}\left(\rho+\frac{p}{c^2}\right)+\frac{\dot{G}}{G}\rho+\frac{\dot{\Lambda}c^2}{8\pi G}-4\rho\frac{\dot{c}}{c}=0, \tag{8}$$

so that in this case the energy momentum tensor of the matter distribution can be separately conserved, even though Λ (or G) is taken as a constant. This situation is also possible in alternative theories of gravitation, such as Brans-Dicke (BD) theory,[2] where the time dependent G is represented by a scalar field $G=1/\psi$, and the field equations for the FRW metric in Eq. (2) with $\Lambda=0$ and $c=1$ are given by[18]

$$H^2\equiv\left(\frac{\dot{a}}{a}\right)^2=\frac{8\pi G\rho}{3}-\frac{k}{a^2}+\frac{\dot{G}}{G}H+\frac{\omega}{6}\left(\frac{\dot{G}}{G}\right)^2, \tag{9}$$

and

$$\dot{H}+2H^2+\frac{k}{a^2}=-\frac{4\pi TG}{3}-\frac{\omega}{6}\left(\frac{\dot{G}}{G}\right)^2-\frac{\Box G}{2G}. \tag{10}$$

Here, ω is a constant that determines the coupling between the scalar field and gravity, $T=T^{\mu}_{\mu}$, and $\Box\equiv\nabla^i\nabla_i$. The scalar field $\psi=1/G$ satisfies

$$\ddot{\psi}+3H\dot{\psi}=\frac{1}{2\omega+3}[8\pi(\rho-3p)]. \tag{11}$$

In this case it can be shown[19] that the Bianchi identities together with the above scalar field equation implies the conservation of the energy momentum tensor of the matter distribution $T^{i}_{j;i}=0$, so that one can have BD cosmological models with variable G and $\Lambda=0$ with the energy density of the matter distribution satisfying Eq. (6). Indeed Garcia-Berro *et al.*[20] have used such a model (see also Refs. 21 and 22 for other cosmological models in a generalized BD-theory) with $G(z)=G_0(1-0.01z+0.34z^2-0.17z^3)$, where $G_0=G(0)$ and redshift $z=(1-a)/a$, to fit the observational Hubble diagram of SNeIa.

Dungan and Prosper[23] assumed a spatially flat universe with $\Lambda=0$ and presented two variable-G cosmological models in General Relativity with $G(a)=G_0f(a)$, such that (i) $f(a)=e^{b(a-1)}$ and (ii) $f(a)=2/(1+e^{-b(a-1)})$, where b is a dimensionless parameter, a is the scale factor, and G_0 corresponds to the value of G at the present time when $a=1$. In these models, the strength of gravity increases with cosmic time, unlike most of the variable-G models found in the literature that contain a negative value of $\dot{G}/G$, in line with Dirac's LNH and the observational constraints cited above. After obtaining the expression for the distance modulus μ in terms of red shift z for the above two models, the authors showed that these models fit nicely the Type Ia supernova data compiled by Kowalski *et al.*[24] on 307 supernovae. This led them to the conclusion that the chosen models are consistent (on the basis of the supernovae data used) with the standard ΛCDM model having $\Omega_\Lambda\approx0.7$, and therefore they argued that the supernovae data alone are insufficient to distinguish between the standard and variable-G cosmological models.

In their paper, Dungan and Prosper write down the general Friedmann equation (3) in the form

$$\left(\frac{\dot{a}}{a}\right)^2=H_0^2\left[f(a)\Omega_M(a)+(1-\Omega_0)a^{-2}+\Omega_\Lambda\right], \tag{12}$$

where H_0 is the current value of the Hubble parameter, $\Omega_M(a)=\rho(a)/\rho_{c0}$ with $\rho_{c0}=3H_0^2/8\pi G_0$ being the current value of the critical density, $\Omega_\Lambda=\rho_\Lambda/\rho_{c0}$ with $\rho_\Lambda=\Lambda c^2/8\pi G_0$, and $\Omega_0=\Omega_M(1)+\Omega_\Lambda$ such that $-kc^2=H_0^2(1-\Omega_0)$. So in the field equations they assumed a constant value of Λ (which was later taken to be zero, along with the spatial curvature k) and also a constant vacuum density parameter Ω_Λ. They also made an assumption that the matter density parameter $\Omega(a)=\Omega_0a^{-3}$, which from the above definition implies that the density $\rho(a)=\rho_0a^{-3}$, i.e., the matter energy density satisfies the conservation law in Eq. (6). However, as shown above, unless one also allows a time dependent c in GR or uses an alternative theory of gravity such as Brans-Dicke theory, the separate conservation of the vacuum and matter components of the energy momentum tensor requires that the variation of G is accompanied by a corresponding variation in Λ, given by Eq. (7). Hence, the assumptions made by the authors that $\Lambda=\text{constant}=0$ and $G=G_0f(a)$ in the presence of conservation of the matter component of the energy momentum tensor are inconsistent.

In the conclusion to their paper, the authors make a stronger statement by saying that all varying-G FRW models with accelerated expansion are ruled out by current observational constraints on $\dot{G}/G$. They base their argument on the form of the Friedmann equation in Eq. (12), which, for a matter dominated universe, gives $H^2\sim G/a^3$ such that $\dot{G}/G\geq H_0\sim7\times10^{-11}\,\text{yr}^{-1}$. This value is about two orders of magnitude greater than the available observational constraints. However, in this case, as already pointed out above, the effect of a varying cosmological term Λ in the Friedmann equation cannot be ignored. The combined effects from the matter and vacuum terms in the Friedmann equation may even push the value of $\dot{G}/G$ below H_0, thus rendering the model compatible with observations. A case in point is given by the FRW cosmological model presented by Štefančić,[25] which contains a decreasing G and a growing cosmological term Λ, assuming separate conservation of the matter and vacuum components of the energy momentum tensor. This model exhibits accelerated expansion with the universe ending up in a "big rip" scenario represented by a de-Sitter spacetime with a constant asymptotic Λ and vanishing G, and it gives values of $\dot{G}/G$ that are consistent with observations.

So to conclude, although as suggested by the authors in their paper, a "mathematics first" approach to general relativity followed by applications is sometimes less desirable than a "physics" first approach, one still has to make sure that any conceptual approximations and assumptions used are mathematically correct and consistent with the underlying theory.

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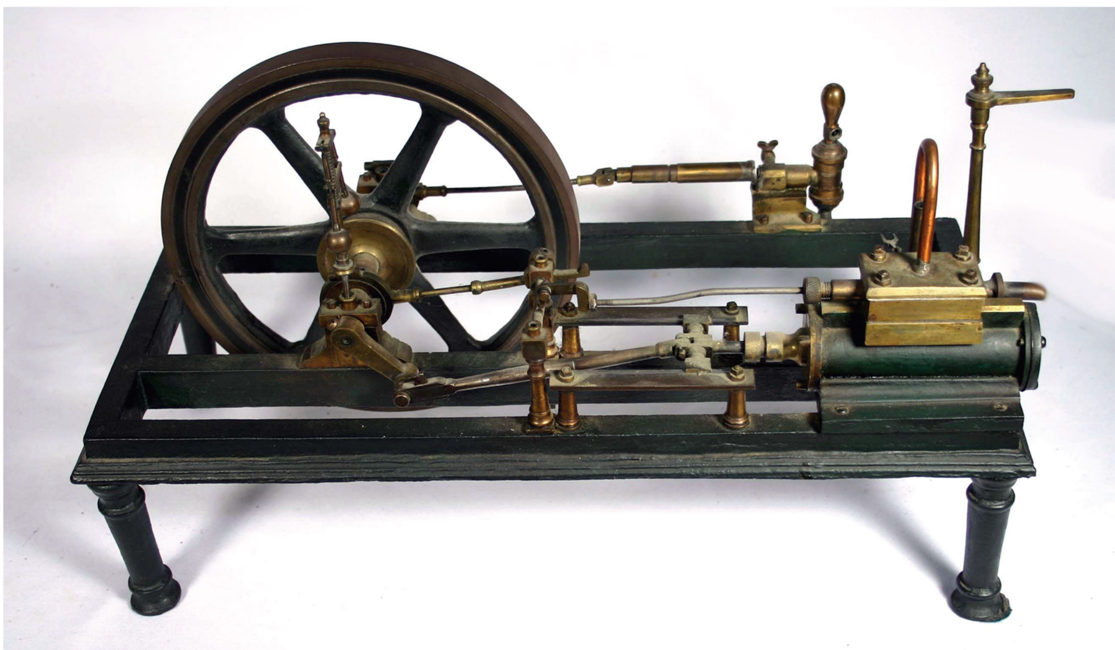

Steam Engine Model

This large working steam engine model (almost 50 cm long) is in the Greenslade Collection. On the left front side is a centrifugal governor, invented by James Watt in 1788, that is linked to the steam line leading to the steam chest atop the cylinder; this is an early example of a feedback mechanism. The sliding cross-head mechanism was standard practice for large, stationary steam engines used in the nineteenth century. (Notes and photograph by Thomas B. Greenslade, Jr., Kenyon College).