Modeling nonconvex dose-volume constraints for radiation therapy

Kelsey Maass, UW Department of Applied Mathematics
Aleksandr Aravkin, UW Department of Applied Mathematics
Minsun Kim, UW Department of Radiation Oncology

1. Summary

Fluence map optimization for intensity-modulated radiation therapy treatment planning can be formulated as a large-scale inverse problem with various constraints. Unfortunately, the clinically relevant dose-volume constraints are nonconvex, adding an extra layer of difficulty to the problem. We propose a new nonconvex relaxation for handling dose-volume constraints that is amenable to efficient algorithms based on partial minimization.

2. Problem Formulation

Fluence map optimization involves calculating beamlet intensities that deliver a prescribed dose of radiation to the tumor while keeping doses to healthy tissues low. The amount of radiation an organ can tolerate is often given in terms of max dose, mean dose, and dose-volume constraints (at most $p\%$ of the organ volume receives at least d Gy).

Original Problem

Treatment plans for a prostate tumor with a dose-volume constraint on the rectum can be computed by solving

$$\min_{x \geq 0} \frac{1}{2}\|A_t x - d_t\|_2^2 \quad \text{s.t.} \quad \|(A_r x - d_r)_+\|_0 \leq k, \tag{1}$$

where A_t, A_r, d_t, and d_r are the linear beamlet-to-voxel maps and dose vectors for the tumor and rectum, and k is the number of rectum voxels that may receive doses exceeding d_r.

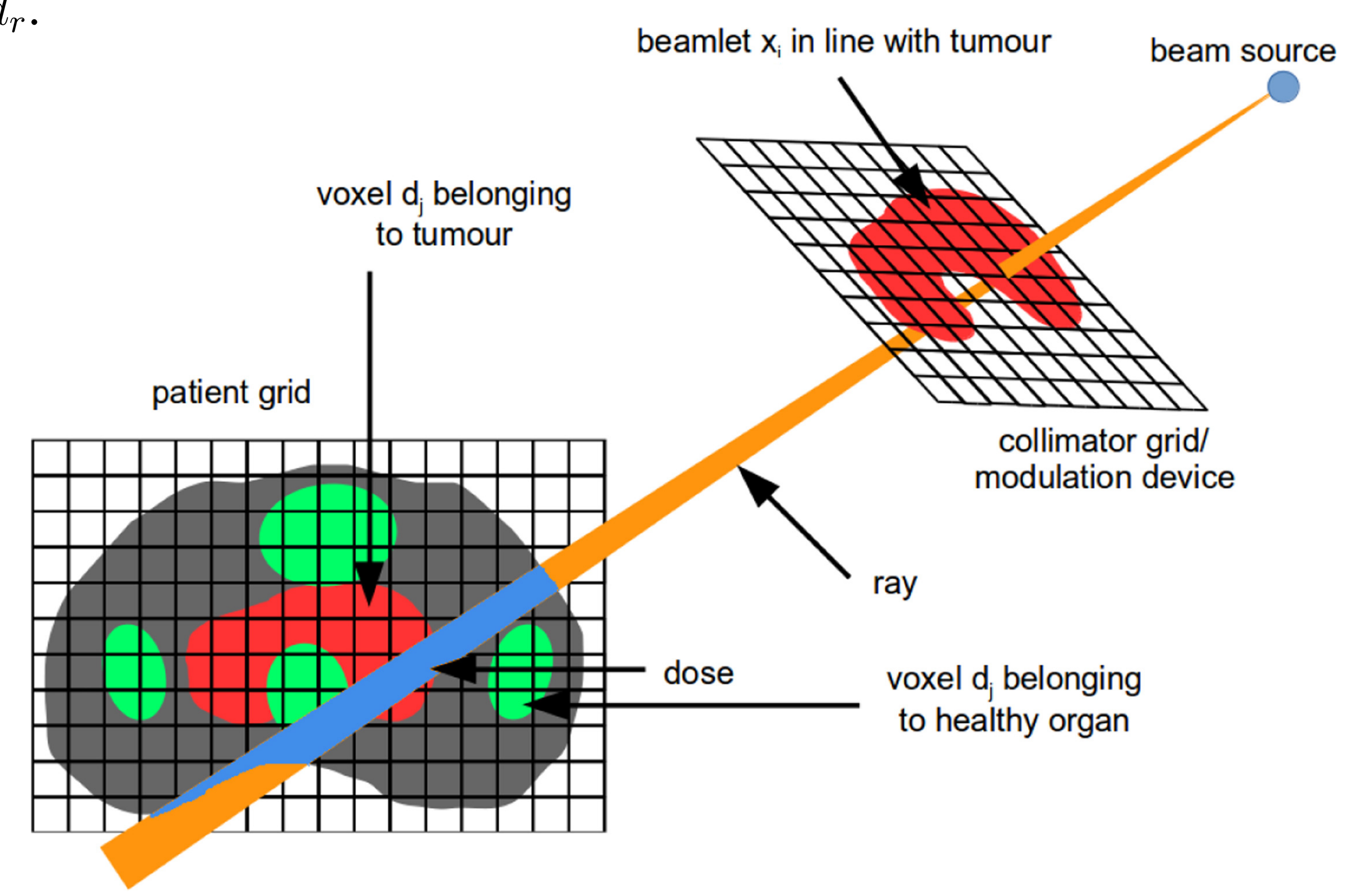

Figure 1: Radiotherapy problem decomposition illustrating the mapping from beamlet to voxel. Source: Breedveld, Sebastiaan, and Ben Heijmen. "Data for TROTS - The Radiotherapy Optimisation Test Set." Data in brief 12 (2017): 143-149.

Nonconvex Relaxation

We approximate solutions to (1) by imposing the dose-volume constraint on a new variable w so that minimizing over x becomes the non-negative least-squares problem

$$\min_{x \geq 0,\, w} \frac{1}{2n_t}\|A_t x - d_t\|_2^2 + \frac{\alpha}{2n_r}\|w - (A_r x - d_r)\|_2^2 \quad \text{s.t.} \quad \|w_+\|_0 \leq k, \tag{2}$$

where n_t and n_r are the number of voxels in the tumor and rectum, and the parameter α can be tuned to balance the tradeoff between meeting the tumor and rectum objectives.

3. Solution Method

Letting $A = \left[\frac{1}{\sqrt{n_t}}A_t; \sqrt{\frac{\alpha}{n_r}}A_r\right]$ and $d(w) = \left[\frac{1}{\sqrt{n_t}}d_t; \sqrt{\frac{\alpha}{n_r}}(d_r + w)\right]$, we solve (2) by alternating between partial minimization in x and projected gradient descent in w:

1. **Update x:** Solve the non-negative least-squares problem for x:

$$x_{i+1} = \arg\min_{x \geq 0} \frac{1}{2}\|Ax - d(w_i)\|_2^2$$

2. **Update w:** Do a step of projected gradient descent for w:

$$w_{i+1} = \text{proj}_C\left(w_i - \frac{\alpha}{Ln_r}(w_i - (A_r x_{i+1} - d_r))\right) \quad \text{where} \quad C = \{y : \|y_+\|_0 \leq k\}$$

4. Numerical Simulations

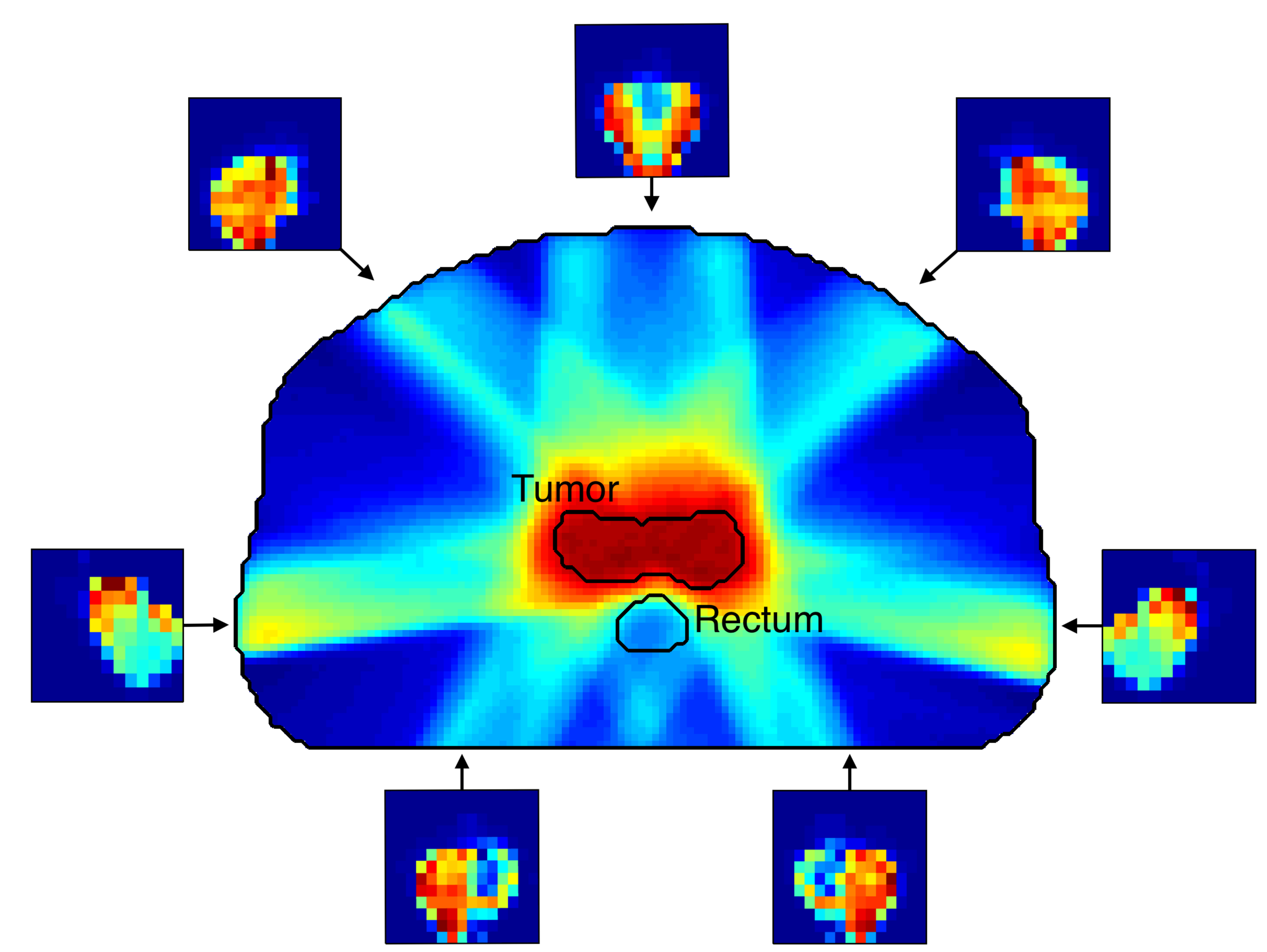

Figure 2: Dose distribution for a plan computed with a tumor prescription of 81 Gy and dose-volume constraint that at most 30% of the rectum volume receives at least 30 Gy.

We calculate treatment plans for the CORT Prostate dataset [1] with a tumor prescription of 81 Gy and various dose-volume constraints on the rectum. In our simulations, the tumor and rectum terms are weighted equally ($\alpha = 1$), and we add an L2 regularization term on x for stability. The beamlet vector x and variable w are initialized with a solution computed without healthy tissue constraints.

Starting with dose-volume constraints on the rectum that are satisfied by the initialization, we increase the difficulty of the constraint by progressively decreasing the dose and volume. In these simulations, we are able to approximately satisfy these increasingly difficult dose-volume constraints without significantly compromising the tumor dose distribution.

[1] Craft, David, Mark Bangert, Troy Long, Dávid Papp, and Jan Unkelbach. "Shared data for intensity modulated radiation therapy (IMRT) optimization research: the CORT dataset." GigaScience 3, no. 1 (2014): 37.

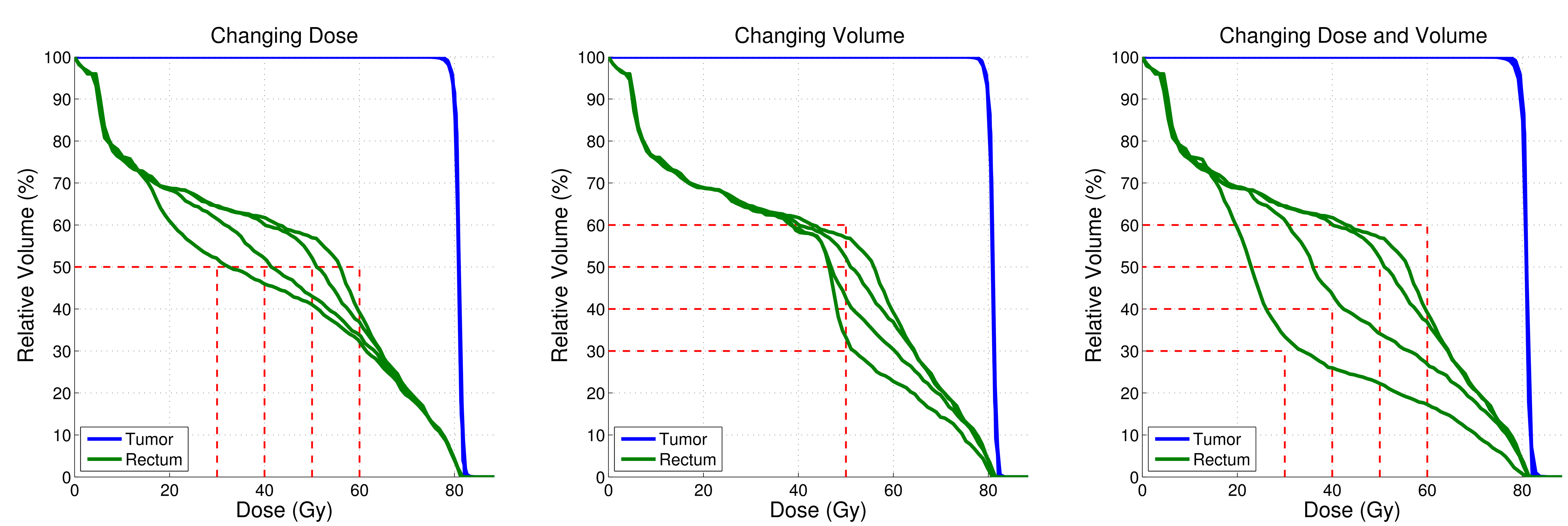

Figure 3: Dose-volume histograms for plans computed with a tumor prescription of 81 Gy and various dose-volume constraints on the rectum. Green lines lying below and to the left of the red dashed lines indicate that the dose-volume constraint is satisfied.

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