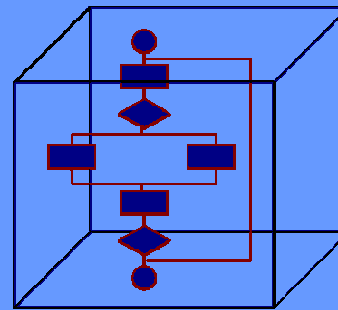


White-Box Testing



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Tutorial Outline

- What is White Box Testing?
- Flow Graph and Coverage Types
- Symbolic Execution:
 - Formal Definition
 - Examples

White-Box means Testing by Implementation

- Execution-based testing that uses the program's inner structure and logical properties
 - A.K.A Clear Box, Glass Box and Structural Testing
- There are different types of white-box testing
 - For example statement coverage where each statement is executed at least once
- *Flow Graph* helps us model and analyze different types of coverage

Flow Graph

$G = (V, E)$ where

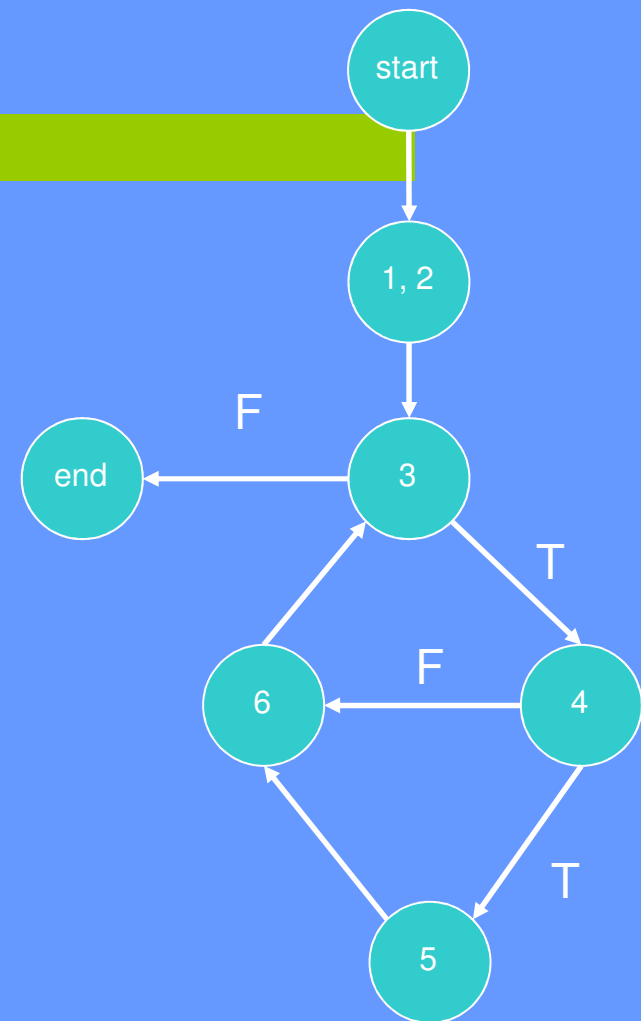
- V is the set of basic blocks
- E is the set of control branches

Example:

1. $a = \text{Read}(b)$
2. $c = 0$
3. **while** $(a > 1)$
4. **if** $(a^2 > c)$
5. $c = c + a$
6. $a = a - 2$

Input:
 $b = 2$

Output:
 $a = 0, c = 2$



White Box Testing

White Box Coverage Types

- *Statement* Coverage: Every statement is executed
- *Branch* Coverage: Every branch option is chosen
- *Path* Coverage: Every path is executed
- *Basic Path* Coverage:
 - We need to define basic path set first



Loops?

Basic Path Set

- An execution path is a set of nodes and directed edges in a flow graph that connects (in a directed fashion) the start node to a terminal node.
- Two execution paths are said to be independent if they do not include the same set of nodes and edges.
- A basic set of execution paths for a flow graph is an independent maximum set of paths in which all nodes and edges of the graph are included at least once.

Basic Path Coverage

- The number of Basic paths is $E - N + 2$ (Linear Complexity)

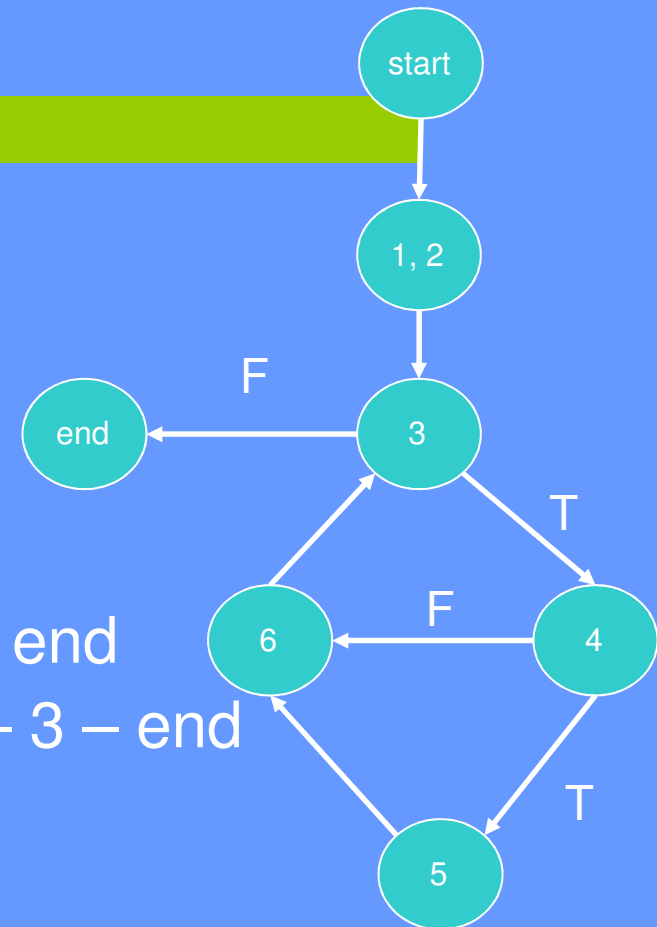
- Example**

p1 = start – 1,2 – 3 – end

p2 = start – 1,2 – 3 – 4 – 6 – 3 – end

p3 = start – 1,2 – 3 – 4 – 5 – 6 – 3 – end

$$E - N + 2 = 8 - 7 + 2 = 3$$



Path Function

- A function $f : D^n \rightarrow D^n$ represents the current values of the variables as function of their initial values
- Each variable X is represented by a projection function $f_X : D^n \rightarrow D$
- Function composition $(g \circ f)(\bar{v}) = g(f_{X_1}(\bar{v}), \dots, f_{X_n}(\bar{v}))$
 - For example

$$f(X, Y, Z) = (X + Y, X - Y, XZ)$$

$$f_X(X, Y, Z) = X + Y \quad f_Y(X, Y, Z) = X - Y \quad f_Z(X, Y, Z) = XZ$$

$$g(X, Y, Z) = (XY, X + Z, Z)$$

$$\begin{aligned} (g \circ f)(X, Y, Z) &= g(f_X(X, Y, Z), f_Y(X, Y, Z), f_Z(X, Y, Z)) = \\ &= g(X + Y, X - Y, XZ) = ((X + Y)(X - Y), (X + Y) + XZ, XZ) \end{aligned}$$

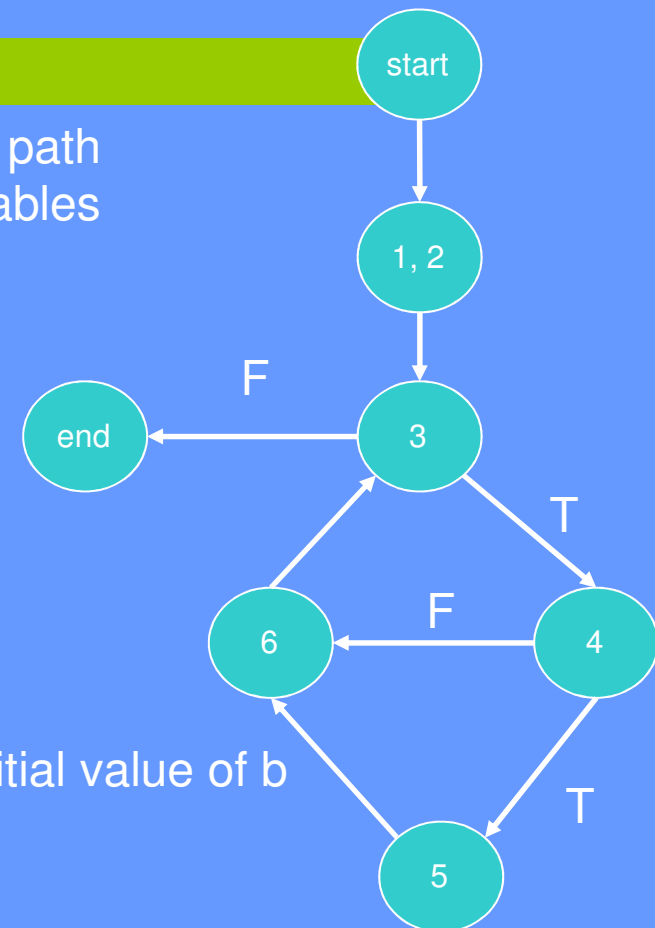
Path Condition

- A condition that ensures the execution of a path
- A constraint on the initial values of the variables

For Example: $p = \text{start} - 1,2 - 3 - \text{end}.$

1. $a = \text{Read}(b)$
2. $c = 0$
3. **while** ($a > 1$)
4. **if** ($a^2 > c$)
5. $c = c + a$
6. $a = a - 2$

The path condition is $B \leq 1$, where B is the initial value of b



White Box Testing

Symbolic Execution

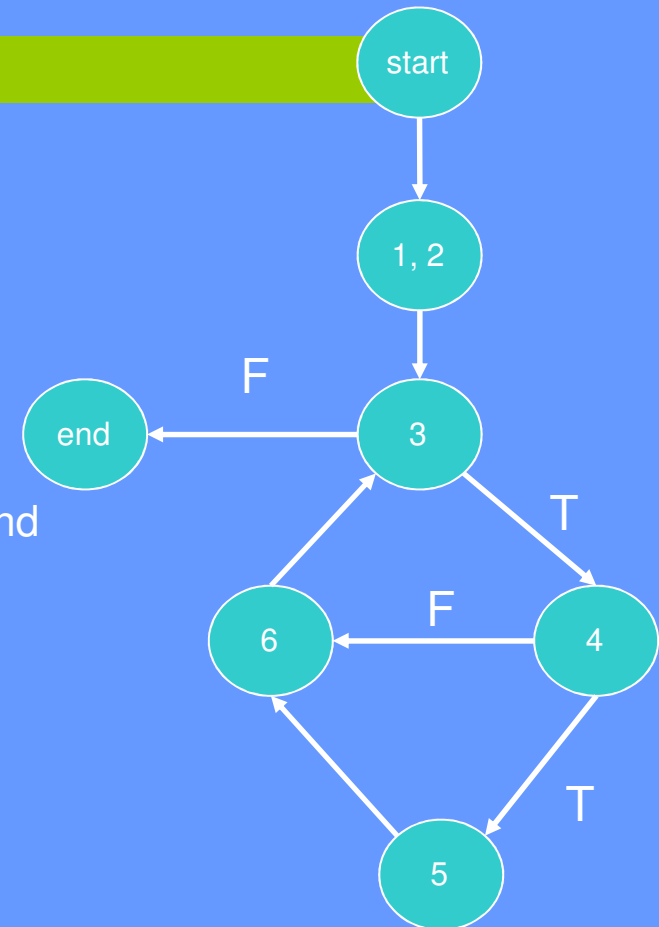
- A method for deriving test cases which satisfy a given path
 - Outputs path condition (input) and path function (expected result)
- Initially
 - Path function is the Identity function
 - Path condition is true
- Each step in the path induce a **symbolic composition** on the path function or a **logical constraint** on the path condition
 - Simple block $g(x)$: $f \leftarrow g \circ f$
 - Control branch: $C \leftarrow C \wedge \text{branch condition}$

Example: Symbolic Execution

1. $a = \text{Read}(b)$
2. $c = 0$
3. **while** ($a > 1$)
4. **if** ($a^2 > c$)
5. $c = c + a$
6. $a = a - 2$

Find test case for path:

$p = \text{start} - 1, 2 - 3 - 4 - 5 - 6 - 3 - 4 - 5 - 6 - 3 - \text{end}$



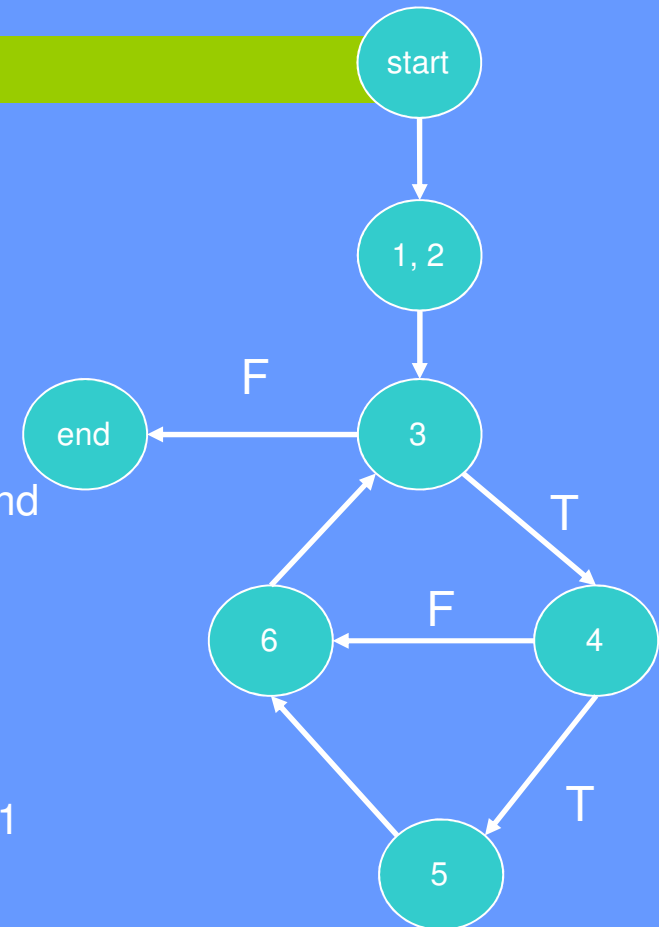
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Example: Symbolic Execution

1. $a = \text{Read}(b)$
2. $c = 0$
3. **while** ($a > 1$)
4. **if** ($a^2 > c$)
5. $c = c + a$
6. $a = a - 2$

$p = \text{start} - 1, 2 - 3 - 4 - 5 - 6 - 3 - 4 - 5 - 6 - 3 - \text{end}$

vertex	path function	path condition
start:	(A, B, C)	true
1,2	(A, B, C)	true
3	(B, B, 0)	true
4	(B, B, 0)	$(\text{true} \wedge B > 1) \leftrightarrow B > 1$
5	(B, B, 0)	$(B > 1 \wedge B^2 > 0) \leftrightarrow B > 1$



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Example: Symbolic Execution

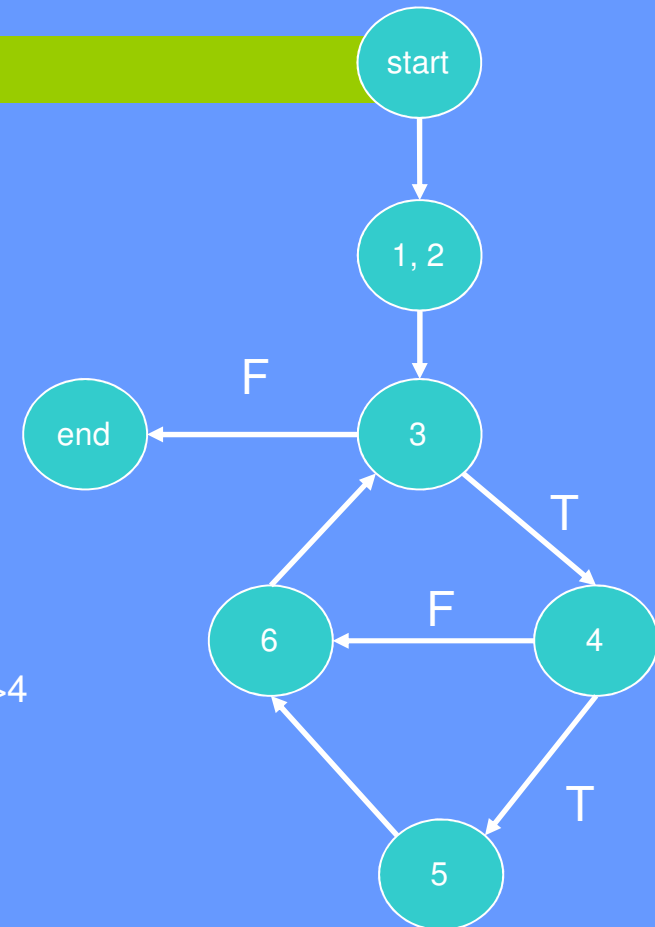
```

1.  a = Read(b)
2.  c = 0
3.  while (a > 1)
4.    if (a^2 > c)
5.      c = c + a
6.    a = a - 2

```

p = start - 1,2 - 3 - 4 - 5 - 6 - 3 - 4 - 5 - 6 - 3 - end

vertex	path function	path condition
6	(B, B, B)	$B > 1$
3	(B-2, B, B)	$B > 1$
4	(B-2, B, B)	$(B > 1 \wedge B-2 > 1) \leftrightarrow B > 3$
5	(B-2, B, B)	$(B > 3 \wedge (B-2)^2 > B) \leftrightarrow B > 4$
6	(B-2, B, 2B-2)	$B > 4$
3	(B-4, B, 2B-2)	$B > 4$
end	(B-4, B, 2B-2)	$(B > 4 \wedge B-4 \leq 1) \leftrightarrow B = 5$



White Box Testing

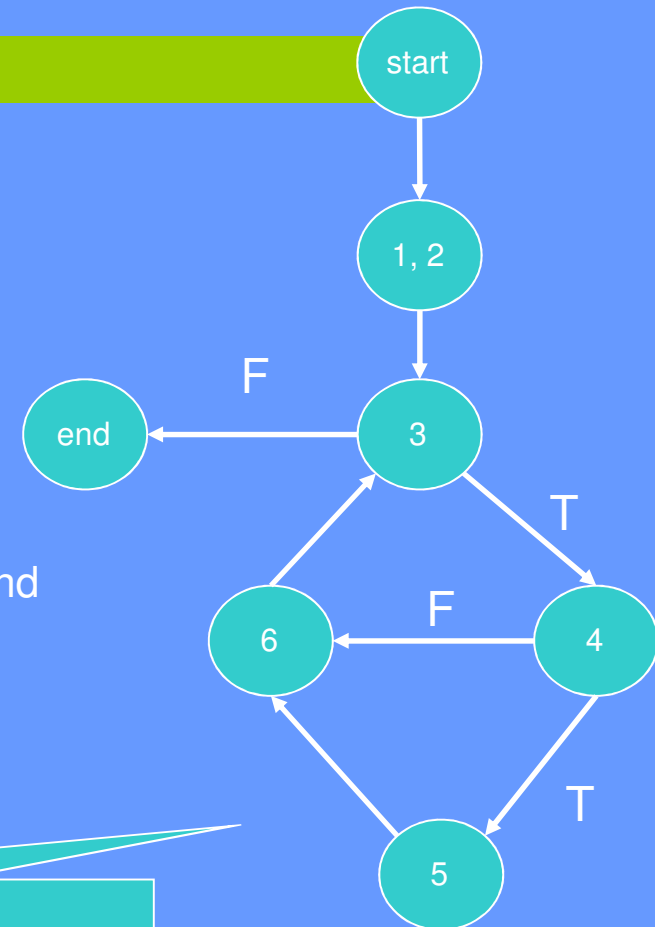
Example: Symbolic Execution

1. $a = \text{Read}(b)$
2. $c = 0$
3. **while** ($a > 1$)
4. **if** ($a^2 > c$)
5. $c = c + a$
6. $a = a - 2$

$p = \text{start} - 1, 2 - 3 - 4 - 5 - 6 - 3 - 4 - 5 - 6 - 3 - \text{end}$

end (B-4, B, 2B-2) B=5

Hence the test case is $B = 5$
and the expected result is $2B-2 = 8$



Is there a test case for
 $p = \text{start} - 1, 2 - 3 - 4 - 5 - 6 - 3 - 4 - 5 - 6 - 3 - 4 - 5 - 6 - 3 - \text{end} ?$

White Box Testing