Slides for Chapter 11

Capital Controls

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Introduction

• In Chapter 10, we saw that over the past 120 years sometimes capital flowed fairly freely across countries and sometimes there were significant deviations from free capital mobility.

• In the present chapter, we characterize the effects of capital controls on the current account, consumption, and welfare.

• We show that capital controls can be an effective tool to reduce current account deficits but that absent distortions or market power they are welfare decreasing.

• We also show that in the presence of borrowing externalities or market power capital control taxes can be desirable because they improve welfare.

11.1 Capital Controls and Interest Rate Differentials

Capital controls are restrictions imposed by governments on the flow of financial capital into or out of a country.

The imposition of capital controls gives rise to interest rate differentials that cannot be arbitraged away.

Suppose that a e country is borrowing from the rest of the world and start with no capital controls, ie free capital mobility.

Let i_t be the domestic interest rate on dollar loans (the onshore rate) and i_t^* the foreign interest rate on dollar loans (the offshore rate). As we have shown in Chapter 10, under free capital mobility the onshore interest rate must equal the offshore interest rate,

$$i_t = i_t^*.$$

Suppose now that the government imposes a tax τ_t per dollar borrowed internationally. The tax raises the cost of borrowing one dollar internationally to $i_t^* + \tau_t$. For agents to be indifferent between offshore and onshore borrowing, the domestic interest rate must equal the sum of the foreign interest rate and the capital control tax rate,

$$i_t = i_t^* + \tau_t.$$

The resulting onshore-offshore interest rate differential, $i_t - i_t^*$, equals the capital control tax rate, τ_t. The larger the capital control tax rate is, the larger the interest rate differential will be.

This is, capital control taxes will give rise to interest rate differentials. Next, we present a case study of capital control taxes imposed in Brazil that gave rise to interest rate differentials.

Capital Controls and Interest Rate Differentials in Brazil: 2009-2012

• In the wake of the global financial crisis of 2008 interest rates in the United States and other developed countries fell to near zero. In response to such low rates, global investors who were looking for higher yields started sending funds to emerging market economies where interest rates were higher.

• One country that was a recipient of large inflows was Brazil. The Brazilian authorities were concerned that these capital inflows would destabilize their economy and enacted capital control taxes on inflows. Specifically, between October 2009 and March 2012 Brazil imposed more than 10 major capital control taxes. The measures included taxes on portfolio equity inflows, taxes on fixed income inflows, and unremunerated reserve requirements. After March 2012 those restrictions were gradually removed.

• When capital inflow taxes are imposed on a specific asset or class of assets there is always the concern that market participants can find a way to circumvent them. One way to see if in this instance the capital control taxes were effective is to see whether they led to non-zero covered interest rate differentials.

Consider the covered interest rate differential between the Brazilian real and the U.S. dollar.

i_t 360-day interest rate in Brazil on domestic currency deposits (reais),
$\mathcal{E}_t$ the spot exchange rate (that is, the reais price of one U.S. dollar),
F_t the 360-day forward exchange rate of U.S. dollars, and
i_t^* the 360-day U.S. dollar Libor rate.

The covered interest rate differential between deposits inside Brazil and outside of Brazil is

$$(1 + i_t)\frac{\mathcal{E}_t}{F_t} - (1 + i_t^*).$$

The first term of this expression is called the cupom cambial, i_t^{cupom}

$$1 + i_t^{cupom} \equiv (1 + i_t)\frac{\mathcal{E}_t}{F_t}.$$

And there is a liquid market for cupom cambials, so that we have data on i_t^{cupom}.

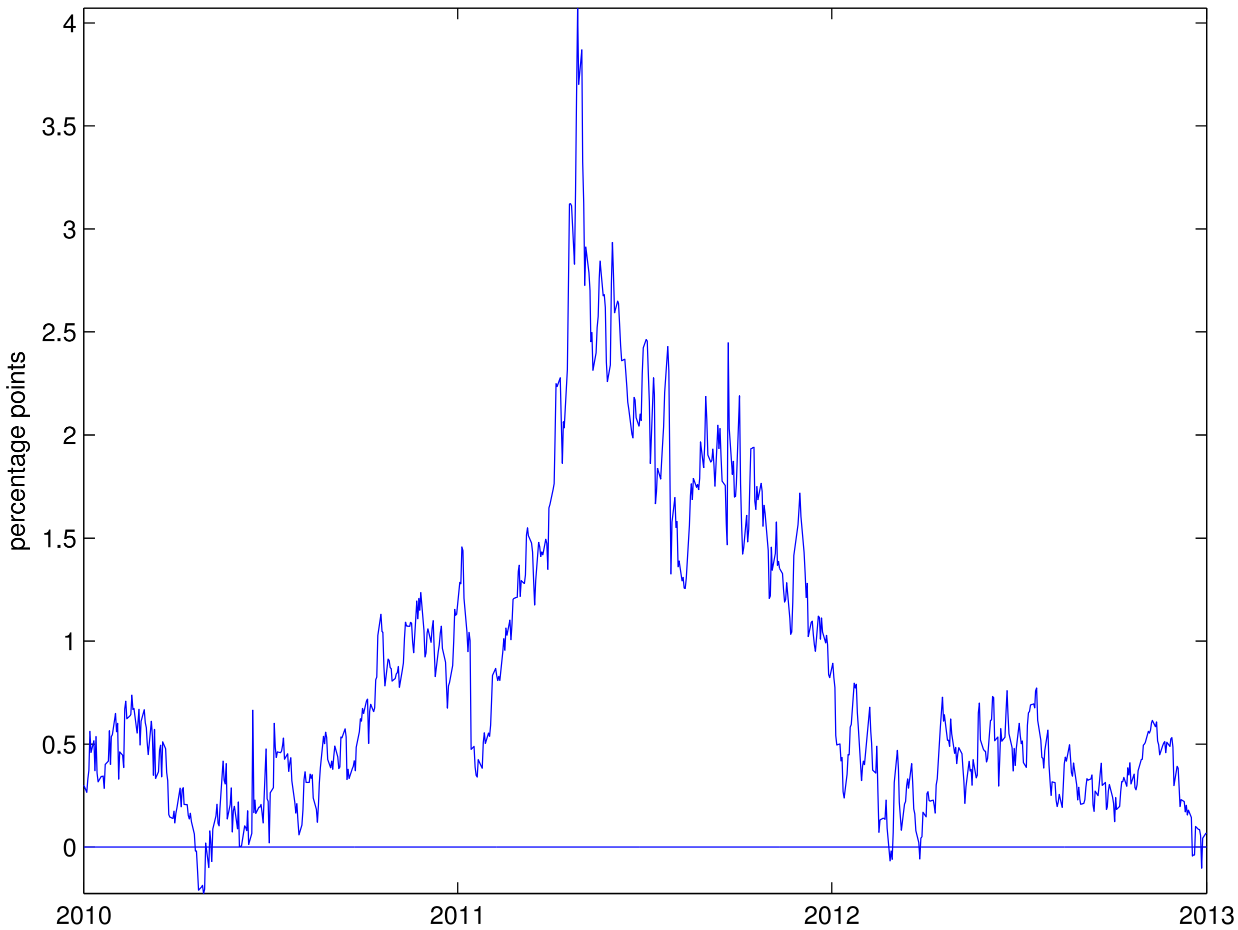
We can then express the covered interest rate differential as

$$\text{covered interest rate differential} \quad = \quad i_t^{cupom} - i_t^*.$$

Figure 11.1on the next slide plots daily data for the covered interest rate differential for the period January 1, 2010 to December 31, 2012.

If the capital inflow controls were successful, we should see that dollar interest rates inside Brazil, i_t^{cupom}, became higher than in London, i_t^*, that is, that the covered interest rate differential increased.

Figure 11.1: Brazilian Real-U.S. Dollar Covered Interest Rate Differentials: 2010-2012



The figure plots daily real-dollar covered interest rate differentials computed as the spread between the cupom cambial and the U.S. dollar Libor rate for the period January 1, 2010 to December 31, 2012. Data Source: Marcos Chamon and Márcio Garcia, 'Capital Controls in Brazil: Effective?', Journal of International Money and Finance 61, 2016, 163-187. We thank the authors for sharing the data.

Comments on the figure

• the covered interest rate differential was around 50 basis points until the fall of 2010 a level also observed prior to the imposition of capital controls. This means that the capital control measures enacted until then, which targeted mainly portfolio equity investment, were not effective in restricting arbitrage between the cupom cambial and the Libor rate.

• However, starting in the fall of 2010 as the government intensified capital controls, the differential starts rising and reaches a peak of 4 percentage points by April 2011, after an inflow tax of 6 percent on borrowing from abroad with maturities of less than 2 years was imposed. The size of the covered interest rate differentials suggest that the latter capital inflow taxes were indeed effective in the sense that they prevented interest rate equalization.

• By early 2012 arbitragers seem to have found ways to bypass the capital control tax as differentials return to normal levels of around 50 basis points.

• By June 2012 the capital control tax of 6 percent of borrowing from abroad with maturities of less than 2 years was removed.

11.2 Macroeconomic Effects of Capital Controls

Effects of Capital Controls on Consumption, Savings, and the Current Account

Consider a two-period small open endowment economy, like the one introduced in Chapter 3, with no initial assets ($B_0 = 0$).

Household preferences

$$U(C_1) + U(C_2)$$

Period-1 budget constraint: $C_1 + B_1 = Q_1$

Period-2 budget constraint: $C_2 = Q_2 + T_2 + (1 + i_1)B_1$

$T_2 =$ lump-sum transfer
$i_1 =$ interest rate on bonds held from period 1 to period 2 (taken as given by the household)

Optimality conditions associated with the household's problem:

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i_1 \tag{1}$$

$$C_2 = Q_2 + T_2 + (Q_1 - C_1)(1 + i_1) \tag{2}$$

Suppose that the government imposes a capital control tax on international borrowing. Let $\tau_1 > 0$ be the capital control tax rate. Then, if the economy is borrowing in period 1,

$$i_1 = i_1^* + \tau_1,$$

where i_1^* is the world interest rate.

The Euler equation (1) becomes

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i_1 = 1 + i_1^* + \tau_1. \tag{3}$$

This expression implies that a capital control tax on international borrowing ($\tau_1 > 0$) distorts the intertemporal allocation of consumption by increasing the interest rate faced by the household i_1. The tax creates incentives to save more and consume less in period 1.

The Government Budget Constraint

Revenue from the capital control tax: $\tau_1(-B_1)$.

Lump-sum transfers: T_2

Assume government returns the revenue from the capital control tax to households in a lump sum fashion. This implies the following period-2 government budget constraint:

$$T_2 = \tau_1(-B_1) \tag{4}$$

Equilibrium

Using the government budget constraint (4) to eliminate T_2 from the household's intertemporal budget constraint (2), yields the economy's resource constraint

$$C_2 = Q_2 + (Q_1 - C_1)(1 + i_1^*). \tag{5}$$

Note that the economy's intertemporal resource constraint is independent of the capital control tax rate, τ_1. This is because the government's tax revenue is returned to households, so no resources are lost as a consequence of the imposition of capital controls.

Equilibrium (continued)

An equilibrium in the economy with capital control taxes is an allocation (C_1, C_2) satisfying the Euler equation (3) and the economy's resource constraint (5), that is,

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i_1^* + \tau_1 \tag{3}$$

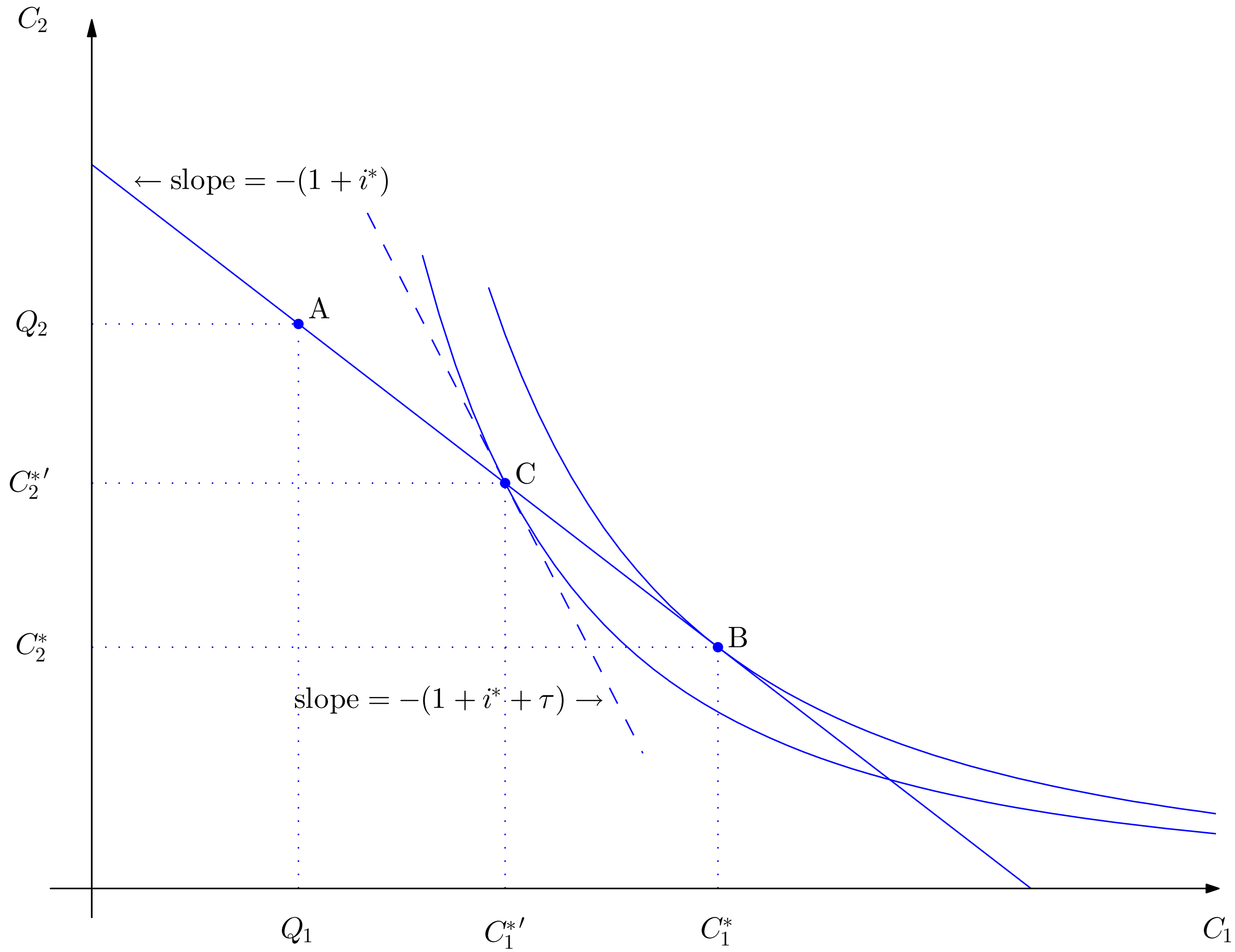
and

$$C_2 = Q_2 + (Q_1 - C_1)(1 + i_1^*), \tag{5}$$

given endowments Q_1, Q_2, the world interest rate, i_1^*, and the capital control tax, τ_1.

The figure on the next slide depicts the equilibrium effects of imposing a capital control tax $\tau_1 > 0$.

Figure 11.2: Equilibrium With and Without Capital Controls



Point A represents the endowment path, point B the equilibrium consumption path in the absence of capital controls, and point C the equilibrium consumption path with capital controls.

Comments on the figure:

The downward sloping straight line is the economy's intertemporal resource constraint, given by equation (5). The slope of this line is $-(1+i_1^*)$.

Point A represents the endowment path, (Q_1, Q_2).

Point B represents the optimal consumption path in the absence of capital controls, $\tau_1 = 0$. At point B, the indifference curve is tangent to the intertemporal resource constraint.

In the graph, when capital controls are zero, the economy runs a trade deficit equal to $C_1^* - Q_1 > 0$. Because $B_0 = 0$, $CA_1 = TB_1$.

Point C in the figure represents the equilibrium when the government imposes a capital control tax $\tau_1 > 0$. The economy's resource constraint is unchanged. However, households perceive an increase in the cost of borrowing from i_1^* to $i_1^* + \tau_1$. The negative of the slope of the indifference curve now is $1 + i_1^* + \tau_1 > 1 + i_1^*$. The imposition of the capital control tax τ_1 leads to a fall in C_1, and an improvement in TB_1, S_1, and CA_1.

Welfare is lower at point C than at point B.

Takeaway: The imposition of a capital control tax on international borrowing discourages current consumption and causes an increase in saving, a reduction in the trade deficit, a reduction in the current account deficit, a reduction in the country's net external debt, and a reduction in welfare.

Effects of Capital Controls on Investment

Consider the economy with investment of Chapter 5. To keep the present analysis self-contained, we go over its main elements.

Output in period 2: $Q_2 = \sqrt{I_1}$

$I_1 =$ the stock of capital available for production in period 2.

To build this stock of capital, firms invest in period 1. In period 1 firms borrow the amount I_1 at the interest rate i_1. In period 2, firms must pay back these loans, including interest. Thus, profits in period 2 are given by

$$\text{period-2 profits} = \sqrt{I_1} - (1 + i_1)I_1.$$

First-order condition for profit maximization:

$$\frac{1}{2\sqrt{I_1}} = 1 + i_1.$$

Solving for the optimal level of investment yields

$$I_1 = \left(\frac{1}{2(1+i_1)}\right)^2.$$

In equilibrium the domestic interest rate, i_1, will be equal to the world interest rate, i_1^* plus the capital control tax τ_1

$$i_1 = i_1^* + \tau_1.$$

Combining the above two expressions, we obtain

$$I_1 = \left(\frac{1}{2(1+i_1^*+\tau_1)}\right)^2,$$

which shows that capital controls lower investment.

Takeaway: Capital control taxes distort not only the consumption-saving choice of households but also the investment choice of firms and as such are, at least in the present model, welfare decreasing.

11.3 Quantitative Restrictions on Capital Flows

- Capital controls can also take the form of quantitative restrictions on international borrowing and lending.

- This form of capital controls, as we will show, is equivalent to those based on taxes on international capital flows.

• Suppose the government imposes the following limit on international borrowing

$$B_1 \geq -D; \qquad \text{where } D > 0.$$

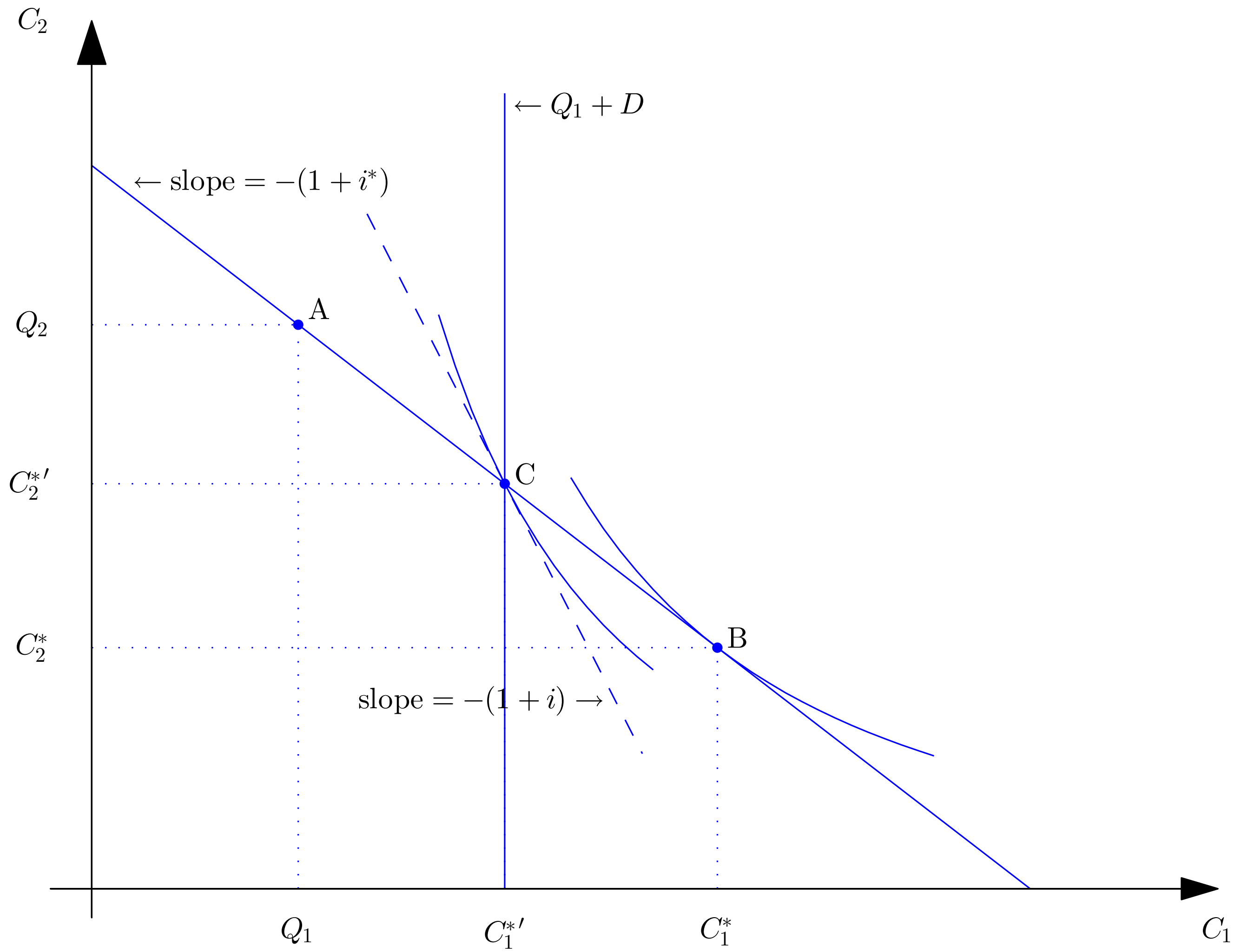
• This implies that: $C_1 \leq Q_1 + D$

The situation is depicted in Figure 11.3 on the next slide.

The endowment point is at point A and the optimal consumption path in the absence of quantitative capital control restrictions is at point B.

At point B, the economy borrows from the rest of the world in order to finance a level of consumption, C_1^*, that exceeds the period-1 endowment, Q_1, $Q_1 - C_1^* < 0$.

Figure 11.3 Equilibrium under Quantitative Capital Controls



The equilibrium under free capital mobility is at point B, where $C_1^* > Q_1$. Quantitative capital controls forbid borrowing more than D, pushing households to consume $Q_1 + D$ in period 1, point C. The domestic interest rate under quantitative capital controls (i_1) is given by the slope of the indifference curve at point C and is higher than the world interest rate i_1^*.

When quantitative capital controls are imposed, households choose point C, and the borrowing constraint is binding, $B_1 = -D$. In the constrained equilibrium, in period 1 the household consumes the endowment plus the maximum amount of borrowing allowed, D, so $C_1^{*'} = Q_1 + D$. In period 2, the household consumes its endowment, Q_2, net of debt obligations including interest, $(1 + i_1^*)D$, that is, $C_2 = Q_2 - (1 + i^*)D$. So we have that in response to the quantitative restrictions on capital inflows, current consumption falls from C_1^* to $C_1^{*'}$, the trade balance and the current account shrink from $Q_1 - C_1^*$ to $Q_1 - C_1^{*'} = -D$, and external debt falls from $C_1^* - Q_1$ to $C_1^{*'} - Q_1 = D$.

The quantitative capital control measure drive the domestic interest rate up to $i_1 > i_1^*$.

Graphically, $1+i_1$ is given by the negative of the slope of the indifference curve at point C .

Formally, under binding quantitative capital controls the domestic interest rate satisfies

$$\frac{U'(Q_1+D)}{U'(Q_2-(1+i_1^*)D)} = 1+i_1.$$

Because Q_1, Q_2, D, and i_1^* are exogenously given, this expression represents one equation in one unknown, i_1. The more stringent capital controls are (ie, the lower D is), the higher the domestic interest rate (i_1) will be. We therefore have that the interest rate differential, $i_1 - i_1^*$ is an increasing function of the severity of quantitative restrictions on capital inflows.

Comparing Figures 11.2 and 11.3, it is clear that quantity-based and tax-based capital controls give rise to the same equilibrium, in the sense that given a capital control tax τ_1 one can find a quantitative restriction D, such that in equilibrium, consumption, the trade balance, the current account, the stock of external debt, and the interest rate differential are the same under both capital-control policies.

11.4 Borrowing Externalities

and

Optimal Capital Controls

Overview of Section 11.4

• Sections 11.2 and 11.3 show that capital controls can be an effective instrument to curb external imbalances, but that if the economy is small and has well functioning markets, they are welfare decreasing.

• In this section, we introduce a debt-elastic interest rate, which is a financial friction whereby foreign lenders charge higher interest rates for larger external debt positions.

• The debt-elastic interest rate creates an externality: individual households, being atomistic participants in financial markets, fail to internalize that their individual borrowing decisions collectively determine the level of the interest rate. As a result, the economy overborrows.

• Under these circumstances, the government has an incentive to impose capital controls as a way to make households internalize the fact that their borrowing drives the interest rate up. The optimal level of capital controls is positive, eliminates overborrowing, and is welfare increasing.

11.4.1 An Economy with a Debt-Elastic Interest Rate

Suppose that the interest rate at which the small open economy can borrow in international capital markets is increasing in the country's cross-sectional average of borrowing, denoted $-\bar{B}_1$. If $\bar{B}_1 < 0$, the country borrows, if $\bar{B}_1 > 0$ the country saves.

The interest rate faced by the small open economy, denoted i_1^*, is assumed to be debt elastic. Specifically,

$$i_1^* = I(-\bar{B}_1)$$

where $I(\cdot)$ is a non-negative and non-decreasing function of external debt, $-\bar{B}_1$.

To see how a debt-elastic interest rate works, let's consider the following parametric example of $I(-\bar{B}_1)$:

$$I(-\bar{B}_1) = \begin{cases} \underline{\mathrm{i}} & \text{for } \bar{B}_1 \geq 0 \\ \underline{\mathrm{i}} + \delta(-\bar{B}_1) & \text{for } \bar{B}_1 < 0 \end{cases}, \tag{6}$$

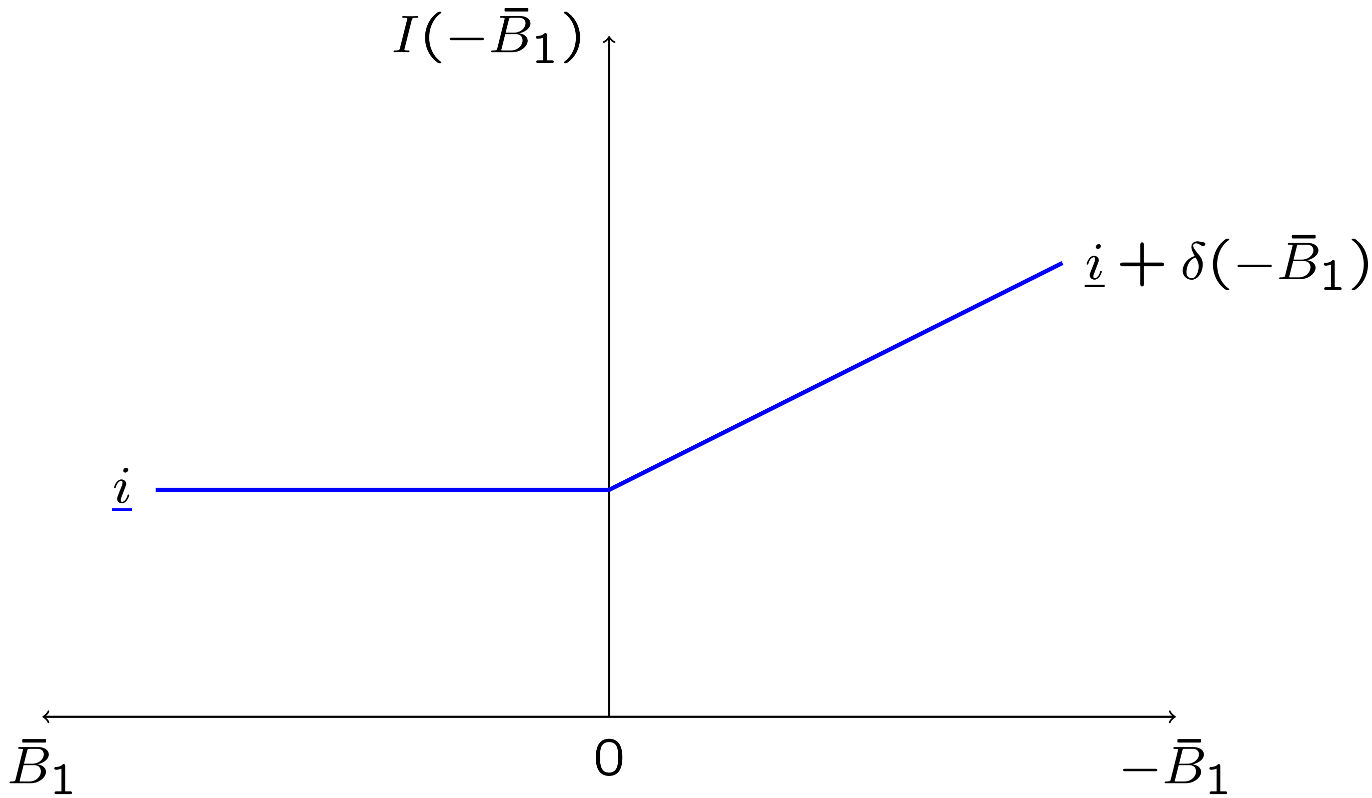
where $\underline{\mathrm{i}}$ and δ are positive parameters.

- the country lends at the constant interest rate $\underline{\mathrm{i}}$

- but borrows at an interest rate that increases linearly with the level of debt.

The figure on the next slide plots this debt-elastic interest rate schedule.

A Debt-Elastic Interest Rate



The figure displays an interest rate schedule, $I(-\bar{B}_1)$, which is weakly increasing in the level of external debt, $(-\bar{B}_1)$. Both $\underline{i}$ and δ are positive constants. For $\bar{B}_1 > 0$, the country is a net external lender, and the interest rate is constant and equal to $\underline{i}$. For $\bar{B}_1 < 0$, the country is a net external borrower, and the interest rate is an increasing function of the level of debt, $-\bar{B}_1$.

Let's now embed a debt-elastic interest rate schedule into a 2-period small open endowment economy of the type introduced in Chapter 3. Let's start with the household's problem.

Households:

Preferences: $U(C_1) + U(C_2)$

Endowments: Q_1 and Q_2.

No initial assets or debts, $B_0 = 0$.

Household can borrow or lend freely at the domestic interest rate i_1 via a bond, denoted B_1.

Budget constraint in period 1: $C_1 + B_1 = Q_1$

Budget constraint in period 2: $C_2 = Q_2 + (1 + i_1)B_1$.

Combine budget constraints in periods 1 and 2 to obtain the household's intertemporal budget constraint

$$C_2 = Q_2 + (1 + i_1)(Q_1 - C_1). \tag{7}$$

and use it to eliminate C_2 from the lifetime utility function

The household's maximization problem then becomes

$$\max_{\{C_1\}} [U(C_1) + U(Q_2 + (1 + i_1)(Q_1 - C_1))]$$

Taking derivative with respect to C_1 and equating it to zero, we obtain the optimality condition

$$U'(C_1) - U'(Q_2 + (1 + i)(Q_1 - C_1))(1 + i_1) = 0.$$

Using the fact that $Q_2 + (1 + i)(Q_1 - C_1) = C_2$ and rearranging, we obtain the usual Euler equation

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i_1. \tag{8}$$

The debt-elastic country interest rate

Suppose that the country has free capital mobility. Let i_1^* be the interest rate charged by foreign lenders to the country in international capital markets. Then, in equilibrium

$$i_1 = i_1^* \tag{9}$$

But what is i_1^* in the present model? We assume that i_1^* is an increasing function of the average external debt per household.

Debt of an individual household is given by $-B_1 = C_1 - Q_1$, where C_1 is consumption of an individual household in period 1 and Q_1 is the endowment of an individual household in period 1. Let $\bar{Q}_1$ and $\bar{C}_1$ denote cross-sectional averages of output and consumption in period 1. Then the average debt per household in period 1 is given by

$$-\bar{B}_1 = \bar{C}_1 - \bar{Q}_1. \tag{10}$$

We introduce a debt-elastic interest rate by assuming that i_1^* is given by

$$i_1^* = I(-\bar{B}_1),$$

where $I(\cdot)$ is a non-negative, non-decreasing function. Using (10) to replace $\bar{B}_1$ yields $i_1^* = I(\bar{C}_1)$, where $\bar{Q}_1$ is omitted as an argument of $I(\cdot)$. This is not a problem for the present analysis, because $\bar{Q}_1$ is an exogenous variable, which we will keep constant throughout.

Because all households are identical in preferences and endowments, in equilibrium they all consume the same amount of goods. This means that in equilibrium consumption per capita equals the individual level of consumption, $\bar{C}_1 = C_1$. So we can write the interest rate parity condition as

$$i_1 = i_1^* = I(C_1).$$

An equilibrium in the economy with a debt-elastic interest rate then are values for C_1, C_2, and i_1 satisfying

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i_1 \tag{11}$$

$$C_1 + \frac{C_2}{1+i_1} = Q_1 + \frac{Q_2}{1+i_1} \tag{12}$$

$$i_1 = I(C_1) \tag{13}$$

given the endowments Q_1 and Q_2.

Note that these are the same equilibrium conditions as those associated with the model of Chapter 3 with the only difference that in that model the interest rate parity condition, equation (13), takes the form $i_1 = i_1^*$, where i_1^* is an exogenous constant.

11.4.2 Competitive Equilibrium

without

Government Intervention

Use the interest rate parity condition (13) to eliminate the interest rate i_1 from the intertemporal budget constraint (12) and the Euler (11) to obtain

$$C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1) \tag{14}$$

and

$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1). \tag{15}$$

These two equations determine the equilibrium levels of consumption in periods 1 and 2.

The economy's resource constraint

Let's start by analyzing the effect of the debt-elastic interest rate on the economy's intertemporal resource constraint, equation (14).

• The key difference with the case of a constant interest rate is the slope. When the interest rate is constant, the slope is also constant and equal to minus $1 + i_1$. When the interest rate is debt elastic, the slope is

$$\text{slope of IRC} = \frac{\partial C_2}{\partial C_1} = -\left[1 + I(C_1) + I'(C_1)(C_1 - Q_1)\right]$$

• As in the case of a constant interest rate, the slope is negative: increasing C_1 requires sacrificing some C_2.

• If the country is a borrower ($C_1 > Q_1$), the slope is greater than $1+i_1$ in absolute value. Intuitively, if the country borrows an additional unit for consumption in period 1, in period 2 it must pay not only $1 + i_1$ but also the increase in the interest rate, $I'(C_1)$, caused by the increase in debt.

Figure 11.5 on slide 45 plots the resource constraint.

Equilibrium is shown in Figure 11.5 on the next slide.

The endowment is at point B, and the equilibrium is at point C. Because $C_1^e > Q_1$, the country is a borrower.

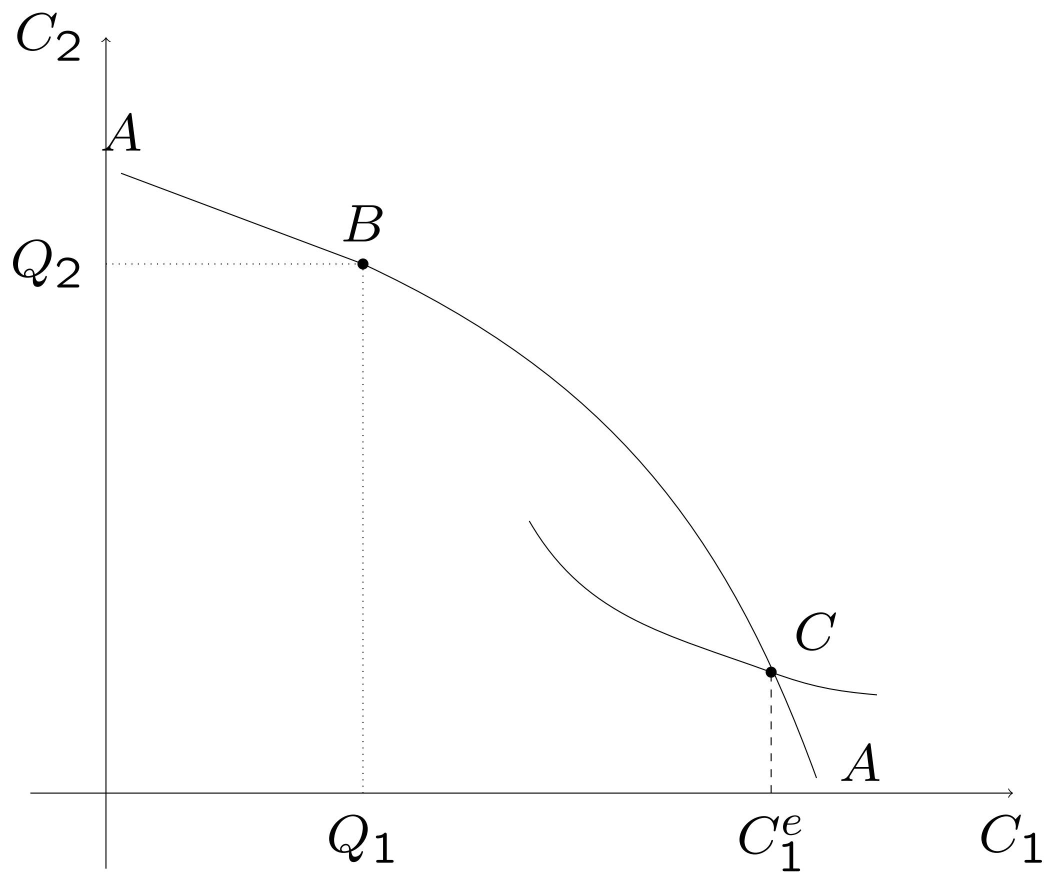
At point C, the indifference curve has a slope equal to minus $1 + i_1$, as dictated by the Euler equation (15), which is less (in absolute value) than the slope of the resource constraint, $1 + i_1 < 1 + i_1 + I'(C_1)(C_1 - Q_1)$.

Clearly, there are points on the intertemporal resource constraint that deliver higher levels of utility than point C. Therefore the equilibrium is inefficient.

The inefficiency originates in the fact that private households perceive the cost of C_1 to be $1+i_1$, whereas it is $1+i_1+I'(C_1)(C_1-Q_1) > 1 + i_1$

This misperception induces households to "overborrow", ie consume more in period 1 than is socially optimal.

Figure 11.5: Equilibrium in an Economy with Borrowing Externalities



The locus AA represents the economy's intertemporal resource constraint, $C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1)$. The endowment is at point B. The competitive equilibrium without government intervention is at point C. This equilibrium is inefficient because there are other allocations on the resource constraint that yield higher utility than point C.

11.4.3 The Efficient Allocation

Imagine a benevolent *social planner* who allocates C_1 and C_2 to maximize households' utility subject to the resource constraint (14)

$$\max_{\{C_1,C_2\}} U(C_1) + U(C_2),$$

subject to

$$C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1). \tag{14}$$

First-order condition:

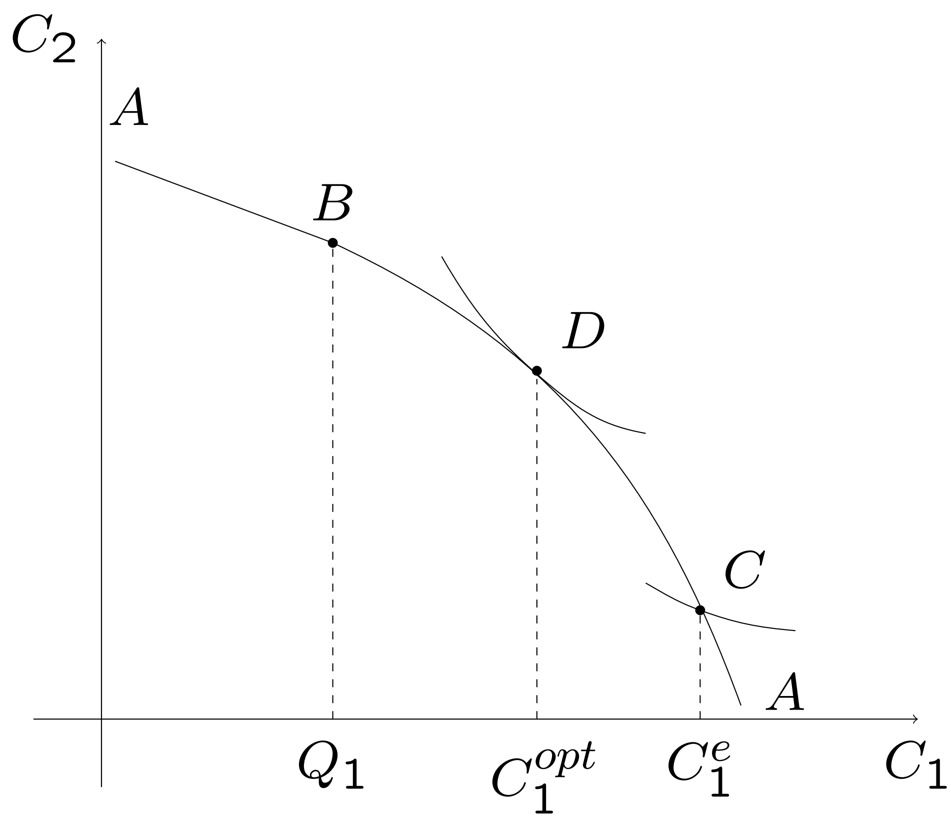
$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1) + I'(C_1)(C_1 - Q_1). \tag{16}$$

• At efficient allocation the slope of the indifference curve equals the slope of the economy's intertemporal resource constraint.

• In the competitive equilibrium, the slope of the indifference curve, $U'(C_1)/U'(C_2)$, is equated to the private cost of funds, $1 + I(C_1)$, whereas in the efficient allocation it is equated to the social cost of funds, $1 + I(C_1) + I'(C_1)(C_1 - Q_1)$.

• The excess external borrowing in the competitive equilibrium is known as *overborrowing*.

Figure 11.6: The Efficient Allocation in an Economy with Borrowing Externalities



The locus AA represents the economy's resource constraint. The endowment is at point B. The competitive equilibrium without government intervention is at point C. The efficient allocation is at point D, where an indifference curve is tangent to the resource constraint. At point C the economy overborrows, ie, borrows more than is socially optimal.

11.4.4 Optimal Capital Control Policy

Can the efficient allocation be achieved in a market economy as opposed to a centrally planned economy? The answer is yes. The government can eliminate overborrowing and achieve the efficient allocation by imposing a capital control tax $\tau_1 > 0$. Suppose $i_1 = i_1^* + \tau_1$, with

$$\tau_1 = I'(\bar{C}_1)(\bar{C}_1 - \bar{Q}_1)$$

In equilibrium $\bar{C}_1 = C_1$, $\bar{Q}_1 = Q_1$, and $i_1^* = I(C_1)$. Given this τ_1, we have

$$i_1 = I(C_1) + I'(C_1)(C_1 - Q_1).$$

Using this expression, the household's optimality condition (8) becomes

$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1) + I'(C_1)(C_1 - Q_1),$$

which is identical to the optimality condition of the social planner (16).

• Because the government rebates the revenue generated by the capital control tax to the households in a lump-sum fashion, the economy's resource constraint (14) is unchanged.

• Intuitively, the capital control tax increases the effective cost of borrowing perceived by households, which induces them to cut consumption in period 1. Thus, the role of the capital control tax is to make households internalize that the social cost of an extra unit of consumption is not just $1 + I(C_1)$, but $1 + I(C_1) + I'(C_1)(C_1 - Q_1)$.

• **Takeaways:**
– in the presence of a borrowing externality free capital mobility ceases to be optimal.
– in the economy without govt intervention, households consume more and borrow more in period 1 than is socially optimal.
– capital controls now are desirable as a way to eliminate overborrowing and to increase welfare.

Overview of Sections 11.5-11.9

The current account of a large economy affects the world interest rate. Therefore, a large economy has incentives to use capital controls to manipulate the world interest rate in its favor.

- Section 11.5 characterizes analytically eqm in a two-country model under free capital mobility.

- Section 11.6 does the same but graphically using an Egdeworth box.

- Section 11.7 characterizes analytically optimal capital control policy (ie optimal manipulation of the world interest rate)

- Section 11.8 does the same but graphically using an Edgeworth box.

- Section 11.9 considers the case of optimal capital controls with retaliation.

11.5 Capital Mobility in a Large Economy

• When a large economy like the United States, the eurozone, or China increases its demand for international funds, the world interest rate will in general experience upward pressure.

• Each individual household in the large economy takes the interest rate as exogenously given. But for the large country as a whole, the interest rate is an endogenous variable. This means that the government of a large economy might be able to apply policies to manipulate world interest rates in the country's favor. For example, if the country is running a current account deficit, the government could impose capital controls to curb the country's aggregate external borrowing and induce a fall in the world interest rate.

• In this section, we characterize equilibrium in a large economy under free capital mobility. This analysis will serve as a building block to characterize national policies aimed at manipulating the world interest rate, which we will take up in Section 11.7.

A two-country model

Consider a two-period world consisting of two countries, the home country, denoted h, and the foreign country, denoted f.

The home country receives a constant endowment over time.

By contrast, the foreign country receives a lower endowment in period 1 than in period 2.

The two economies are identical in all other respects. In particular, both have the same preferences for consumption and start period 1 with no assets or debts.

Preferences in both countries take the form

$$\ln C_1^j + \ln C_2^j, \tag{17}$$

where C_1^j and C_2^j denote consumption in periods 1 and 2, respectively, in country $j = h, f$.

Budget constraint of households in country j in period 1

$$C_1^j + B_1^j = Q_1^j, \tag{18}$$

Budget constraint of households in country j in period 2

$$C_2^j = Q_2^j + (1 + i^j)B_1^j, \tag{19}$$

where i^j denotes the interest rate in country j, for $j = h, f$. Optimization implies:

$$\frac{C_2^j}{C_1^j} = 1 + i^j \tag{20}$$

and

$$C_1^j = \frac{1}{2}\left(Q_1^j + \frac{Q_2^j}{1 + i^j}\right). \tag{21}$$

Consider the home country

Assume constant endowments over time: $Q_1^h = Q_2^h = Q$
Setting $j = h$ in (21), then yields

$$C_1^h = \frac{1}{2}\left(Q + \frac{Q}{1+i^h}\right). \tag{22}$$

Trade balance

$$TB_1^h = \frac{Q}{2}\frac{i^h}{1+i^h}.$$

Current account schedule (recall $B_0^h = 0$).

$$CA_1^h = \frac{Q}{2}\frac{i^h}{1+i^h}. \tag{23}$$

Net foreign asset position

$$B_1^h = \frac{Q}{2}\frac{i^h}{1+i^h}. \tag{24}$$

Consider the foreign country

Assume $Q_1^f = Q/2$ and $Q_2^f = Q$

Set $j = f$ in (21) to get

$$C_1^f = Q\frac{3 + i^f}{4(1 + i^f)}. \tag{25}$$

$\Rightarrow C_1^f > Q/2$, for interest rate below 100 percent (i.e., for any $i^f < 1$).
Trade balance

$$TB_1^f = \frac{Q}{4}\frac{i^f - 1}{(1 + i^f)},$$

Current account schedule (recall$B_0^f = 0$)

$$CA_1^f = \frac{Q}{4}\frac{i^f - 1}{(1 + i^f)}. \tag{26}$$

Net foreign asset position

$$B_1^f = \frac{Q}{4}\frac{i^f - 1}{(1 + i^f)}. \tag{27}$$

Equilibrium

Market clearing in world financial markets

$$CA_1^h + CA_1^f = 0. \tag{28}$$

Interest rate parity: under free capital mobility,

$$i^h = i^f. \tag{29}$$

Let this interest rate be denoted i^* and let's refer to it as the world interest rate. That is, let $i^h = i^f = i^*$.

Solve for the equilibrium value of the world interest rate, i^*

Replace i^h and i^f by i^* in the home and foreign current account schedules, equations (23) and (26). Then, use the resulting expressions to eliminate CA_1^h and CA_1^f from (28) to obtain

$$\frac{Q}{2}\frac{i^*}{1+i^*}+\frac{Q}{2}\frac{i^*-1}{2(1+i^*)}=0.$$

Solve for i^*, the equilibrium level of the world interest rate

$$i^*=\frac{1}{3},$$

$\Rightarrow$ under free capital mobility the world interest rate is 33 percent.

By (26), the foreign country runs a current account deficit and hence the domestic country a current account surplus. It follows from (27), that the foreign country borrows internationally in period 1. In turn, if the foreign country borrows, then domestic country must save in period 1.

Solving for the equilibrium values of C_t^h and B_1^h

Setting $i^h = i^* = 1/3$ in equations (22) and (24) yields

$$C_1^h = \frac{7}{8}Q < Q; \qquad \text{and} \qquad B_1^h = \frac{1}{8}Q > 0.$$

From (19) we have:

$$C_2^h = \frac{7}{6}Q > Q > C_1^h.$$

Interpretation: In spite of having a flat path of endowments, which, if consumed, would produce a perfectly smooth path of consumption, households in the home country choose to consume less than their endowment in period 1 and to save. This is because foreign demand for funds (discussed next) drives the world interest rate up, inducing the home country to postpone consumption. As a result, in period 2 the home country can enjoy a level of consumption higher than its endowment.

Solving for the equilibrium values of C_t^f and B_1^f

Proceeding in an analogous fashion, we obtain the following equilibrium values for the foreign country's levels of consumption and bond holdings:

$$C_1^f = \frac{5}{8}Q > \frac{1}{2}Q,$$

$$C_2^f = \frac{5}{6}Q < Q,$$

and

$$B_1^f = -\frac{1}{8}Q < 0.$$

Intuitively, facing an upward sloping path of endowments, the foreign country borrows in period 1 to smooth consumption over time. So it consumes above its endowment in period 1, below its endowment in period 2, and maintains a short bond position in period 1.

The Level of Welfare under Free Capital Mobility

Welfare under free capital mobility can be found by evaluating the utility function (17) at the respective equilibrium consumption levels. This yields

$$\ln C_1^h + \ln C_2^h = \ln\left(\frac{49}{48}Q^2\right) \tag{30}$$

for the home country, and

$$\ln C_1^f + \ln C_2^f = \ln\left(\frac{25}{48}Q^2\right),$$

for the foreign country.

Exercise 11.9 asks you to show that both countries are better off under free capital mobility than under financial autarky. An implication of this result is that it does not pay for either country to impose capital controls so high that all intertemporal trade stops.

But is there is a capital control policy that induces an equilibrium in which the level of welfare is higher (for the country imposing the controls) than in the eqm with free capital mobility?

11.6

Graphical Analysis of Equilibrium

under Free Capital Mobility

in a Large Economy

Thus far we analyzed the equilibrium under free capital mobility in a two-country world algebraically for a specific functional form of the utility function, $U(C) = \ln C$.

We will now repeat the analysis but instead of following an algebraic approach we will use two powerful graphical objects in general equilibrium analysis, the offer curve and the Edgeworth box, both created by the Irish economist Francis Ysidro Edgeworth (1845-1926).

The Offer Curve

Given endowments, Q_1 and Q_2, we wish to characterize the optimal consumption path (C_1, C_2) for different values of the interest rate.

Let i^0 denote the interest rate such that $C_1 = Q_1$ and $C_2 = Q_2$. Next find the optimal (C_1, C_2) for other values of i_1.

Starting from $i_1 = i^0$ consider a decline in the interest rate. This leads to $C_1 \uparrow$ and $C_2 \downarrow$.*

Starting from $i_1 = i^0$, consider higher values of i_1, by the substitution effect $C_1 \downarrow$ and $C_2 \uparrow$.

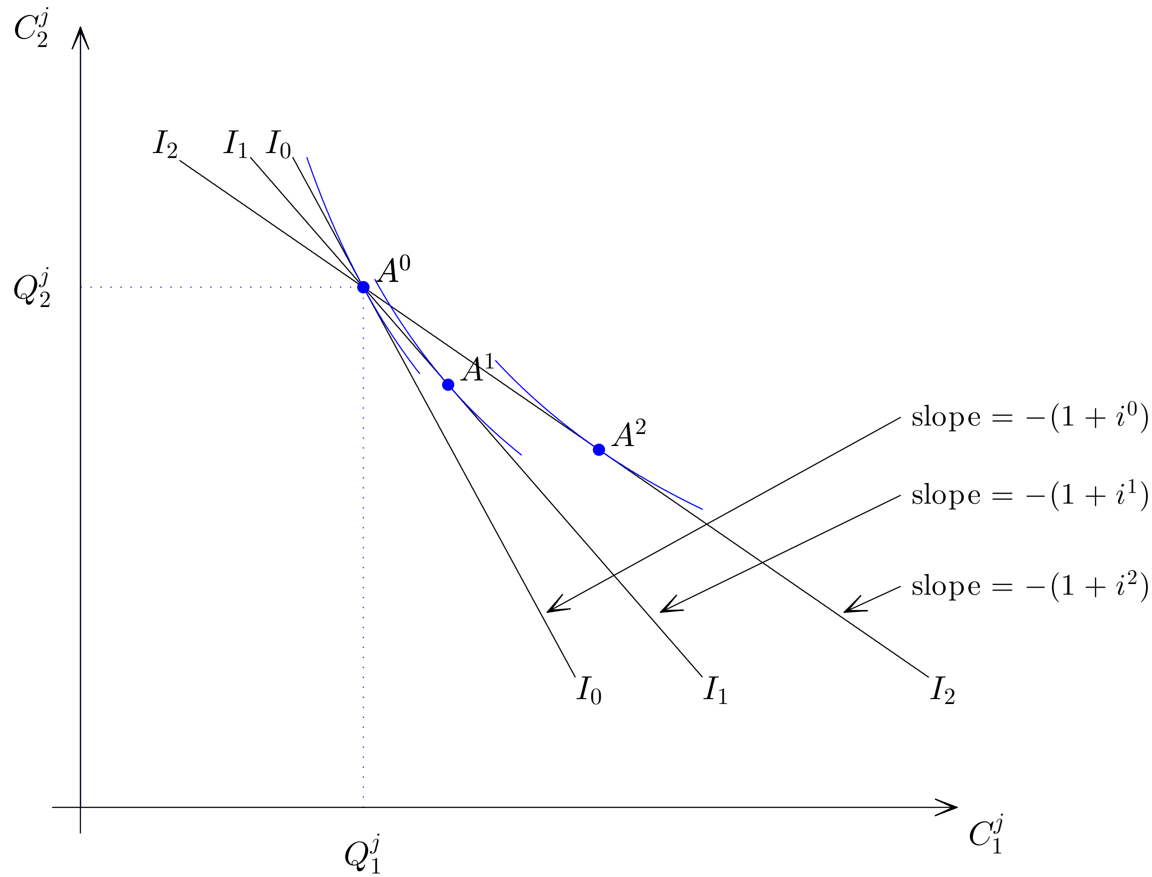
The figure on the next slide shows how (C_1, C_2) change with the interest rate.

It follows that the optimal consumption path as the interest rate declines describes a downward sloping locus in the space (C_1, C_2) that crosses the endowment point. This locus is the <u>offer curve</u>.

The figure on the slide after the next plots the offer curve.

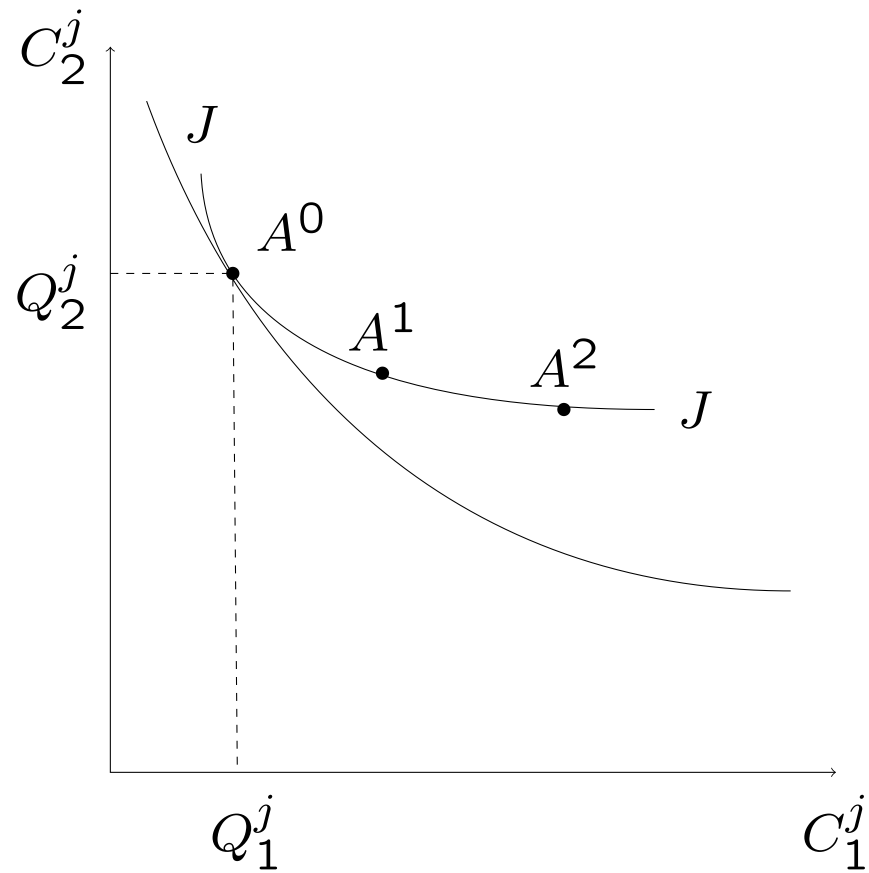
*If the income effect associated with a decline in interest rates dominates the substitution effect, it is possible that $C_2 \uparrow$. In what follows we assume that the substitution effect always dominates.

Figure 11.7: Optimal Intertemporal Consumption Choice at Different Interest Rates



The figure displays the optimal consumption choice for three different values of the interest rate, i^0, i^1, and i^2, satisfying $i^0 > i^1 > i^2$. Each interest rate is associated with a different intertemporal budget constraint. The higher the interest rate is, the steeper the intertemporal budget constraint will be. The intertemporal budget constraint $\overline{I^0 I^0}$ is induced by the highest of the three interest rates, and the intertemporal budget constraint $\overline{I^2 I^2}$ by the lowest. The associated optimal consumption path induced by the interest rate associated with budget constraint $\overline{I^0 I^0}$ is the endowment point, A^0. The intertemporal budget constraints $\overline{I^1 I^1}$ and $\overline{I^2 I^2}$ produce optimal consumption choices given by points A^1 and A^2, respectively. The offer curve (not shown) connects points A^0, A^1, A^2.

Figure 11.8: The Offer Curve



The offer curve is the locus JJ, which connects all optimal consumption allocations at different interest rates. The offer curve crosses the endowment point A^0. The figure also shows the indifference curve that crosses the endowment point. At the endowment point this indifference curve is tangent to the offer curve. All points on the offer curve other than the endowment point are preferred to the endowment point itself.

The Edgeworth Box

We now construct the Edgeworth Box, which is shown on the next slide.

• The length of the horizontal side of the box is the global endowment of goods in period 1, $Q_1^h + Q_1^f$.

• The height of the box is the global endowment in period 2, $Q_2^h + Q_2^f$.

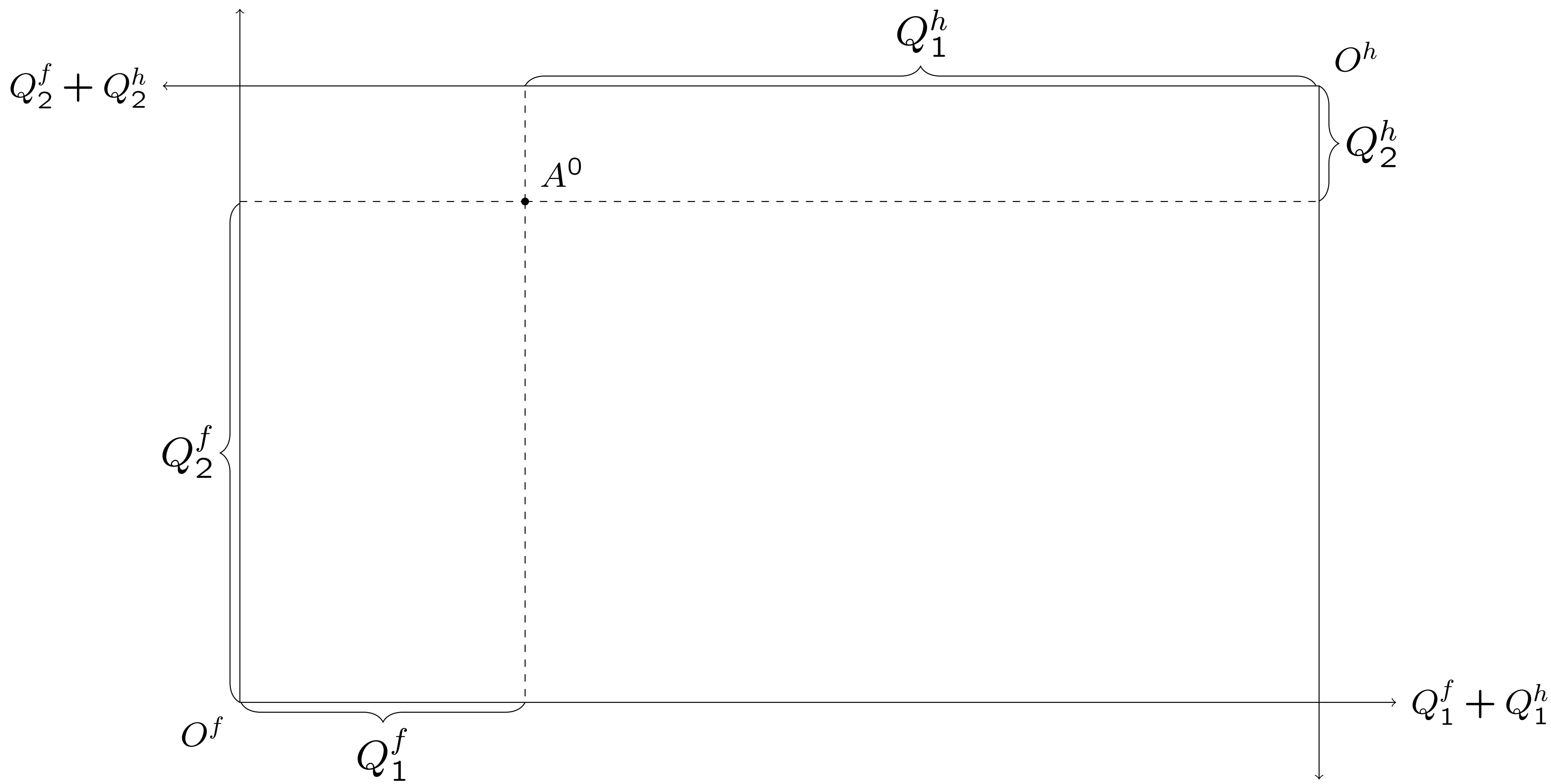
• The southwest corner of the box is the origin of the foreign country and is indicated by the symbol O^f. For the foreign country, consumption and the endowment in period 1 are measured on the horizontal axis from the origin O^f to the right, and consumption and the endowment in period 2 are measured on the vertical axis from O^f upward. Welfare of households in the foreign country rises as the allocation (C_1^f, C_2^f) moves northeast in the box.

• The northeast corner of the box is the origin of the home country and is indicated by the symbol O^h. For this country, consumption and the endowment in period 1 are measured on the horizontal axis from O^h to the left, and consumption and the endowment in period 2 are measured on the vertical axis from O^h downward. Welfare of households in the home country rises as the allocation (C_1^h, C_2^h) moves southwest in the box.

• The endowments of the two countries are given by point A^0.

• Any point in the box represents an allocation of consumption across time and countries that can be achieved with the existing global endowments.

The Edgeworth Box — before adding offer curves



The width of the box is equal to the world endowment of goods in period 1, $Q_1^f + Q_1^h$. The heights of the box is equal to the world endowment of goods in period 2, $Q_2^f + Q_2^h$. The origin of the foreign country is O^f and the origin of the home country is O^h. The endowment point is A^0.

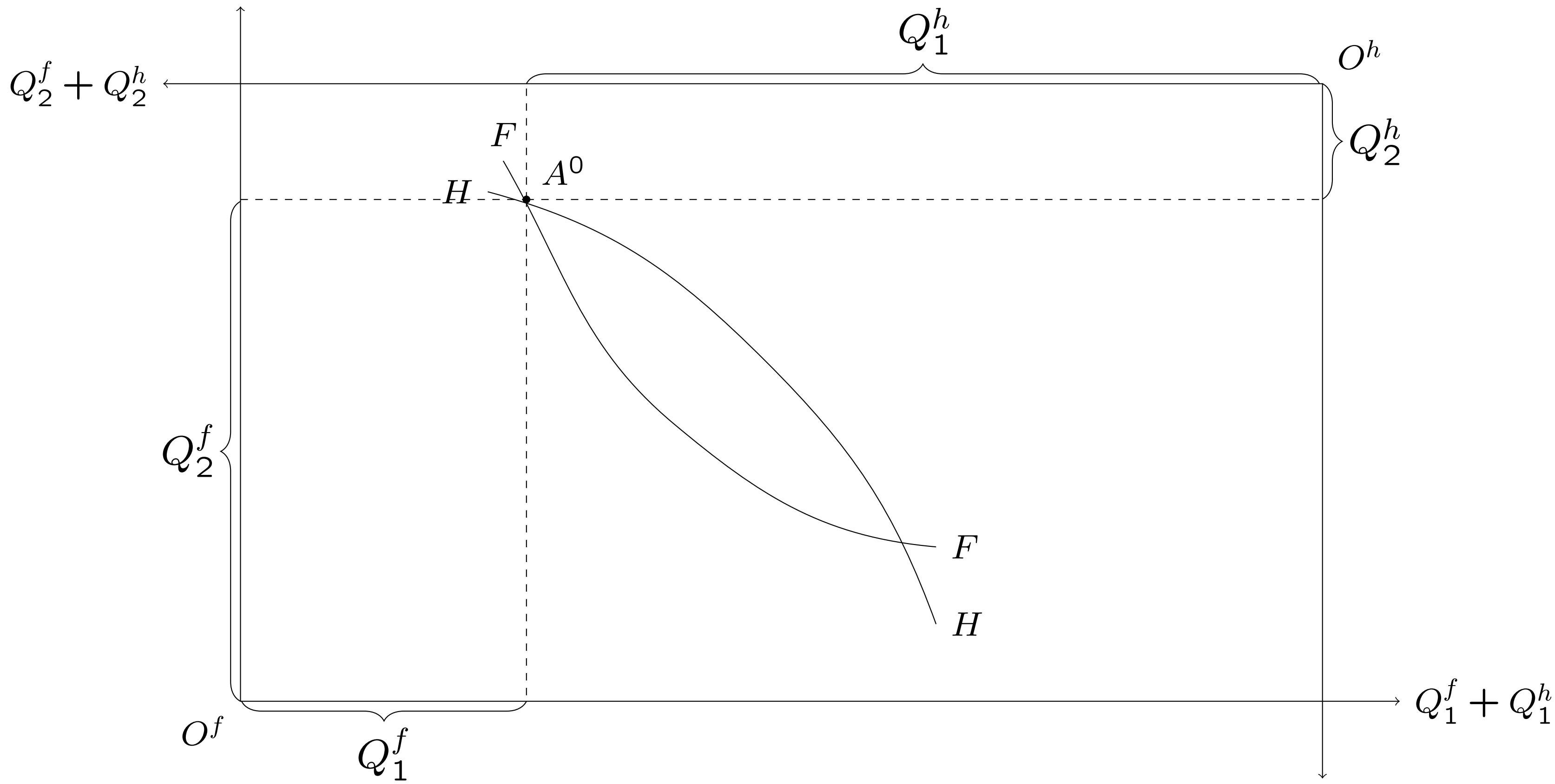
• Next, we add the offer curves to the Edgeworth box. This is shown in the figure on the next slide.

• The offer curve of the foreign country is the locus FF.

• The offer curve of the home country is the locus HH.

• Both offer curves must cross the endowment point A^0.

The Edgeworth Box — after adding the offer curves



The offer curve of the foreign country is the locus FF, and the offer curve of the home country is the locus HH. Both offer curves must cross the endowment point, A^0. For a given country, any point on the country's offer curve is preferred to the endowment point. The slope of a straight line from a point on the offer curve, of country f say, to A_0 is equal to $-(1 + i^f)$. For points to the right of A_0 the interest rate i^f is lower than the autarky interest rate and for points on the offer curve FF to the left of A_0 the interest rate supporting that allocation, i^f, is higher.

Equilibrium Under Free Capital Mobility

The figure on the next slide presents the equilibrium allocation.

• The equilibrium under free capital mobility is given by point B, where the two offer curves intersect for a second time.

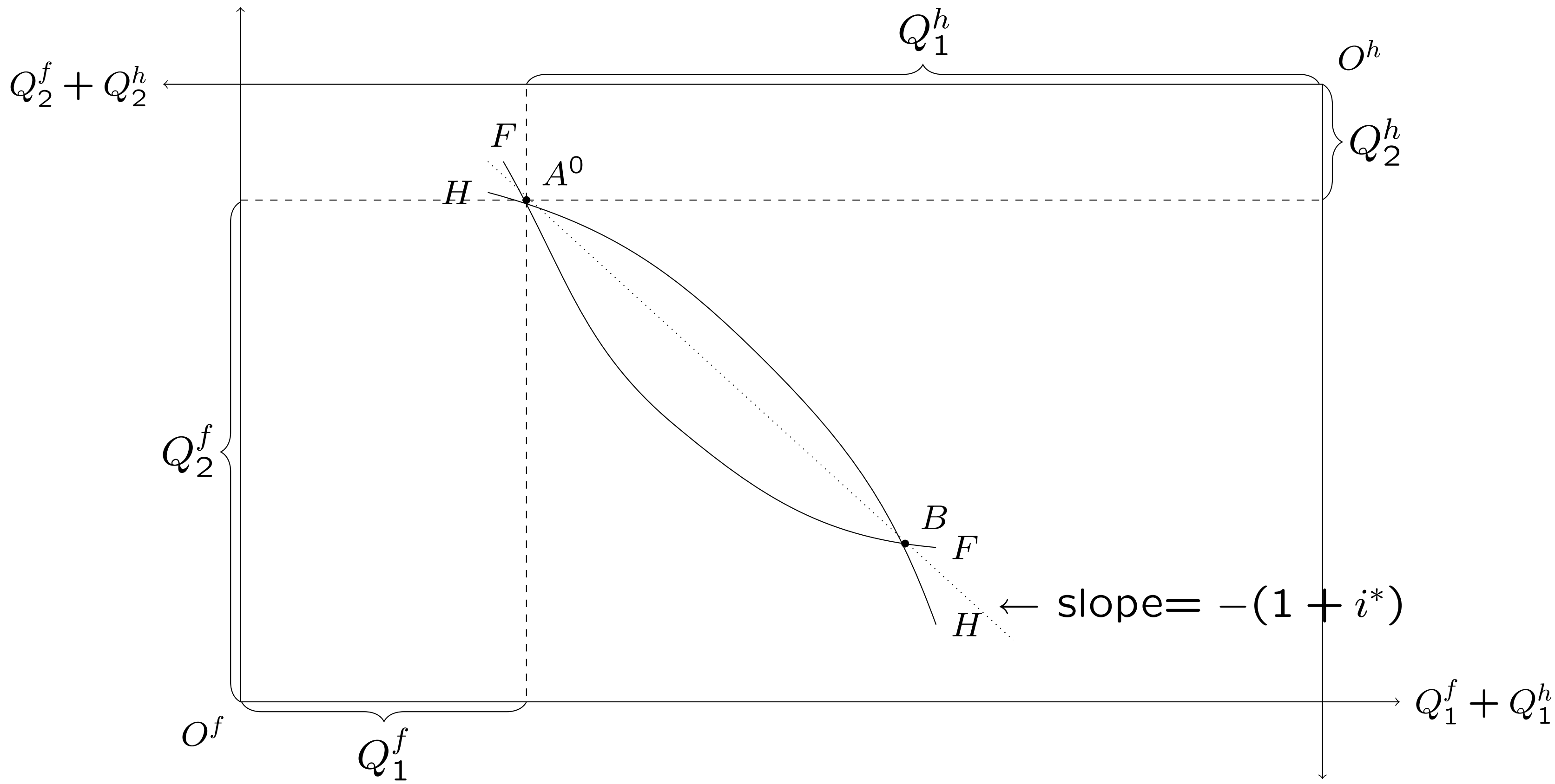
• In equilibrium, the foreign country, which has a relatively low endowment in period 1, borrows from the home country.

• The equilibrium world interest rate, i^*, is determined by the slope of the line that connects points A^0 and B. This line is the intertemporal budget constraint faced by the domestic and foreign households at the equilibrium world interest rate i^*. This interest rate is lower than the domestic interest rate in the foreign country under financial autarky, which is determined by the slope of the foreign household's indifference curve at the endowment point A^0. By the same logic, we have that the equilibrium interest rate under free capital mobility, i^*, is higher than the domestic interest rate in the home country under financial autarky.

• We have therefore established that allowing for free capital mobility eliminates interest rate differentials.

• Because in equilibrium both countries are on their respective offer curves, they are both better off than under autarky. Thus free capital mobility is welfare improving.

Figure 11.9: Equilibrium Under Free Capital Mobility in a Two-Country Model



The equilibrium under free capital mobility is point B. The slope of the line that connects points A^0 and point B is $-(1+i^*)$, where i^* is the equilibrium world interest rate under free capital mobility.

The Allocation Under Free Capital Mobility is Pareto Optimal

- At point B the indifference curves of the home and foreign households both have a slope equal to $-(1+i^*)$.

- Thus, at point B the indifference curves of the home and foreign households are tangent to each other.

- This implies that at no point inside the Edgeworth box can both countries be better off than at point B. In other words, any other attainable consumption allocation makes at least one country worse off relative to the allocation associated with the equilibrium under free capital mobility.

- When an equilibrium has this property, we say that it is Pareto optimal.

11.7 Optimal Capital Controls in a Large Economy

• A large country that is borrowing has monopsony power in international funds markets. Unlike the country as a whole, individual households do not have market power in financial markets as they are atomistically small. Thus, exploiting the country's market power can only be achieved via government intervention.

• For a large country that is a borrower, capital controls have two opposing macroeconomic effects: (1) they distort the intertemporal allocation, which is welfare decreasing. (2) they lower the world interest rate, which is welfare increasing. We will now present an economy in which this tradeoff is resolved in favor of (2).

• Let's go back to the algebraic example introduced in Section 11.5. Recall the foreign country was the borrower and the home country the lender.

• Assume that in response to capital controls imposed by the foreign country, the home country does not retaliate by imposing its own capital controls.

$\Rightarrow$ demand for international funds by home country still is

$$B_1^h = \frac{Q}{2}\frac{i^*}{1+i^*}. \tag{31}$$

• We turn to the setting of the optimal capital control tax, τ, in country f next.

In Section 11.1 we analyzed an economy with capital control taxes and here we will repeat some of these results to keep the presentation self-contained.

• The capital control tax τ creates a wedge between the world interest rate and the interest rate in the foreign country,

$$i^f = i^* + \tau. \tag{32}$$

• Government rebates capital control tax revenue via lump sum transfers, T

$$T = -\tau B_1^f. \tag{33}$$

• Budget constraints of the household:

$$\begin{aligned} C_1^f + B_1^f &= \frac{Q}{2} && (34) \\ C_2^f &= Q + T + (1 + i^f) B_1^f && (35) \end{aligned}$$

Combining (32), (33), and (35), in eqm we have that

$$C_2^f = Q + (1+i^*)B_1^f, \tag{36}$$

which implies that country f, in eqm, does not loose any resources due to the capital control tax.

The government of country f internalizes that:
(1) in eqm world capital market clears: $B_1^f + B_1^h = 0$
(2) demand by country h for funds is: $B_1^h = \frac{Q}{2}\frac{i^*}{1+i^*}$
(3) in eqm total period 2 resources are: $C_2^f = Q + (1+i^*)B_1^f$, and
(4) $C_1^f + B_1^f = \frac{Q}{2}$.

From here we can express C_1^f and C_2^f as functions of the world interest rate i^*

$$C_1^f = \frac{Q}{2}\frac{1+2i^*}{1+i^*} \tag{37}$$

and

$$C_2^f = \frac{Q}{2}(2-i^*). \tag{38}$$

Now use these two expressions to eliminate C_1^f and C_2^f from the utility function of the foreign household to obtain

$$\ln\left(\frac{Q}{2}\frac{1+2i^*}{1+i^*}\right)+\ln\left(\frac{Q}{2}(2-i^*)\right).$$

This is the indirect lifetime utility function because it is expressed in terms of a price, the world interest rate, i^*, instead of consumption, C_1^f and C_2^f.

The government of country f solves:

$$\max_{\{i^*\}} \left[\ln\left(\frac{Q}{2}\frac{1+2i^*}{1+i^*}\right) + \ln\left(\frac{Q}{2}(2-i^*)\right)\right]$$

Taking the derivative of the indirect utility function with respect to i^* and setting it to zero yields

$$i^{*2} + 2i^* - \frac{1}{2} = 0,$$

The two solutions are: $i^* = -1 \pm \sqrt{\frac{3}{2}}$. The only economically sensible (ie $i^* > -1$) solution is

$$i^* = -1 + \sqrt{\frac{3}{2}} = 0.22,$$

or 22 percent. Under optimal capital controls the world interest rate is **lower** than under free capital mobility (22 versus 33 percent).

Intuitively, because the foreign country is a borrower it benefits from a lower world interest rate. (By the same token, the home country, a lender, does not benefit.)

Country f welfare: $\ln(\frac{25.2122}{48}Q^2)$ under optimal capital controls (versus $\ln(\frac{25}{48}Q^2)$ under free capital mobility)

Country h welfare? Find consumption from:

$$C_1^h = Q_1^h + Q_1^f - C_1^f; \quad \text{and} \quad C_2^h = Q_2^h + Q_2^f - C_2^f$$

Country h welfare: $\ln(1.0103Q^2)$ under optimal capital controls (of country f) (versus $\ln(1.0208Q^2)$ under free capital mobility).

Optimal capital controls raise welfare in country f and lower it in country h.

The domestic interest rate in country f under optimal capital controls

Find i^f: From Euler equation:

$$1+i^{=}\frac{C_2^f}{C_1^f}=1.5$$

That is, the domestic interest rate increases from $i^f = 0.33$ under free capital mobility to $i^f = 0.5$. Why? Because the gov't must incentivate private households to borrow less.

Find τ:

$$\tau = i^f - i^* = 0.5 - 0.22 = 0.28$$

The tax rate on capital inflows is a hefty 28 percent.

Because the capital controls raise the domestic interest rate (i^f), the current account deficit of the foreign economy falls from

$B_1^f = -0.1250Q$ under free capital mobility to

$B_1^f = -0.0918Q$ under optimal capital controls.

11.8 Graphical Analysis of Optimal Capital Controls in a Large Economy

Let's now use the Edgeworth box to analyze optimal capital controls in the foreign country, see Figure 11.11 on the next slide.

• In setting capital controls, the objective of the foreign country is to attain a point on the home country's offer curve, HH, that maximizes the foreign country's utility. The foreign country is constrained to pick a point on the home country's offer curve because, by construction, only the allocations on the offer curve can be obtained as a market outcome, that is, by an appropriate choice of the world interest rate. Consequently, the allocation associated with the optimal capital control policy is one at which an indifference curve of the foreign country is tangent to the offer curve of the home country.

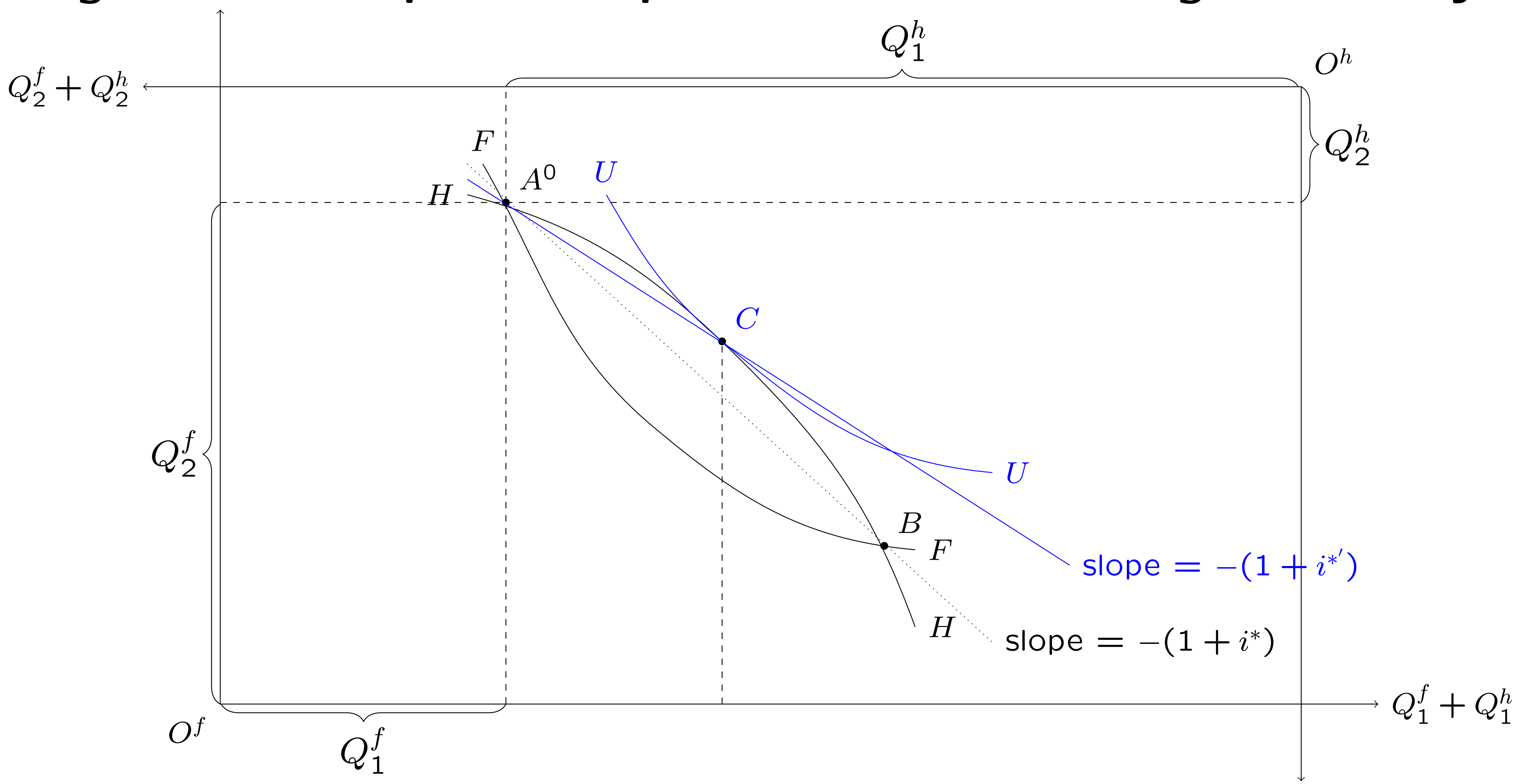
• This allocation is point C in Figure 11.11. The indifference curve attained by the foreign country under optimal capital controls is UU.

• The world interest rate under optimal capital controls ($i^{*'}$), is defined by the slope of the line that connects points A^0 and C. Clearly, this line is flatter than the one connecting points A^0 and B. This means that the imposition of optimal capital controls causes the world interest **rate to fall**.

• Also, the optimal capital control policy in the foreign country makes the foreign country better off at the expense of the home country, whose welfare goes down. Recall that the equilibrium under free capital mobility is Pareto optimal, so the improvement in the foreign country's welfare must be welfare decreasing for the home country.

• These results echo those obtained algebraically in Section 11.7.

Figure 11.11 Optimal Capital Controls in a Large Economy



The offer curve of the home country is HH and that of the foreign country is FF. The endowment is at point A^0. The equilibrium under free capital mobility is at point B, and the equilibrium under optimal capital controls in the foreign country is at point C. The interest rate under free capital mobility is i^* and under optimal capital controls $i^{*'} < i^*$. The indifference curve attained by the foreign country under optimal capital controls is UU. The fact that this indifference curve is not tangent to the intertemporal budget constraint that crosses point C implies that the equilibrium with capital controls is Pareto inefficient.

The Allocation Under Optimal Capital Controls is Inefficient

• The equilibrium under optimal capital controls in the foreign country fails to be efficient. Let's show why.

• At point C, by construction the indifference curve of the home country (not shown) must be tangent to the intertemporal budget constraint associated with the interest rate $i^{*'}$.

•The indifference curve of the foreign country (the locus UU), on the other hand, is tangent to the offer curve of country h (the locus HH).

• Since the budget constraint and the offer curve (the locus HH) intersect at C, the slopes of the home and foreign indifference curves at point C are not the same.

• This implies that the equilibrium allocation under capital controls is inefficient in the sense that the home country could be made better off without making the foreign country worse off.

11.9 Retaliation

• Thus far, we have assumed that, as the foreign country imposes controls on capital inflows, the home country does not retaliate by imposing its own restrictions on capital flows.

• Because the home country is a large economy, it will in general have an incentive to retaliate. In turn, the foreign country would have an incentive to readjust its capital control policy in response to the retaliation of the home country.

• The equilibrium that will emerge under this strategic interaction depends on what type of game the two countries play in setting capital controls. We will focus on one type of game known as *Nash equilibrium.*

• Essentially, in a Nash equilibrium each country sets its own capital control tax optimally taking as given the capital tax rate of the other country. An equilibrium is reached when the capital control tax that each country takes as given is indeed the tax rate that is optimal for the other country.

Equilibrium Requirements

As shown earlier, the intertemporal resource constraint with capital controls is the same as in the case without capital controls, that is, for $j = h, f$, we have

$$C_1^j + \frac{C_2^j}{1+i^*} = Q_1^j + \frac{Q_2^j}{1+i^*}. \tag{39}$$

And the Euler equations continue to be requirements of equilibrium:

$$\frac{C_2^j}{C_1^j} = 1 + i^j. \tag{40}$$

Consumption in the Foreign Country

The foreign country imposes controls on capital inflows:

$$i^f = i^* + \tau^f,$$

Setting $j = f$ in (39) and (40) and solving for C_1^f and C_2^f, yields

$$C_1^f = K^f(i^*, \tau^f) \equiv \frac{Q_1^f + \frac{Q_2^f}{1+i^*}}{1 + \frac{1+i^*+\tau^f}{1+i^*}}$$

and

$$C_2^f = L^f(i^*, \tau^f) \equiv (1 + i^* + \tau^f)\frac{Q_1^f + \frac{Q_2^f}{1+i^*}}{1 + \frac{1+i^*+\tau^f}{1+i^*}}$$

where i^* and τ^f are endogenous variables.

Consumption in the Home Country

The home country imposes controls on capital outflows.

$$i^h = i^* - \tau^h.$$

Setting $j = h$ in (39) and (40) and solving for C_1^h and C_2^h, yields

$$C_1^h = K^h(i^*, \tau^h) \equiv \frac{Q_1^h + \frac{Q_2^h}{1+i^*}}{1 + \frac{1+i^*-\tau^h}{1+i^*}}$$

and

$$C_2^h = L^h(i^*, \tau^h) \equiv (1 + i^* - \tau^h)\frac{Q_1^h + \frac{Q_2^h}{1+i^*}}{1 + \frac{1+i^*-\tau^h}{1+i^*}},$$

where i^* and τ^h are endogenous variables to be determined in equilibrium.

Market clearing in the goods market in period 1 requires that global consumption equal the global endowment,

$$K^f(i^*, \tau^f) + K^h(i^*, \tau^h) - Q_1^f - Q_1^h = 0.$$

This equation expresses the world interest rate as an implicit function of the tax rates in the home and foreign countries. We then write

$$i^* = I(\tau^f, \tau^h).$$

Using this relation to eliminate i^* from consumption in both periods in the home and foreign countries we can write

$$C_1^f = \tilde{K}^f(\tau^f, \tau^h) \equiv K^f(I(\tau^f, \tau^h), \tau^f),$$

$$C_2^f = \tilde{L}^f(\tau^f, \tau^h) \equiv L^f(I(\tau^f, \tau^h), \tau^f),$$

$$C_1^h = \tilde{K}^h(\tau^f, \tau^h) \equiv K^h(I(\tau^f, \tau^h), \tau^h),$$

$$C_2^h = \tilde{L}^h(\tau^f, \tau^h) \equiv L^h(I(\tau^f, \tau^h), \tau^h).$$

The government of the foreign country picks τ^f to maximize the utility of the foreign household taking as given the tax rate in the home country, τ^h.

$$\max_{\{\tau^f\}} \left[\ln \tilde{K}^f(\tau^f, \tau^h) + \ln \tilde{L}^f(\tau^f, \tau^h) \right]$$

The first-order condition is

$$\frac{\tilde{K}_1^f(\tau^f, \tau^h)}{\tilde{K}^f(\tau^f, \tau^h)} + \frac{\tilde{L}_1^f(\tau^f, \tau^h)}{\tilde{L}^f(\tau^f, \tau^h)} = 0,$$

where $\tilde{K}_1^f(\tau^f, \tau^h)$ and $\tilde{L}_1^f(\tau^f, \tau^h)$ denote, respectively, the partial derivatives of $\tilde{K}^f(\tau^f, \tau^h)$ and $\tilde{L}^f(\tau^f, \tau^h)$ with respect to the first argument, τ^f. This optimality condition implicitly defines the tax rate in the foreign country, τ^f, as a function of the tax rate in the home country, τ^h. We write the solution for τ^f as

$$\tau^f = R^f(\tau^h).$$

This relationship is called the *reaction* function of the foreign country. It represents the optimal tax response of the foreign country as a function of the tax rate in the home country.

Likewise, **the government of the home country** chooses τ^h to

$$\max_{\{\tau^h\}} \left[\ln \tilde{K}^h(\tau^f, \tau^h) + \ln \tilde{L}^h(\tau^f, \tau^h) \right],$$

taking as given τ^f.

The associated first-order condition is

$$\frac{\tilde{K}_1^h(\tau^f, \tau^h)}{\tilde{K}^h(\tau^f, \tau^h)} + \frac{\tilde{L}_1^h(\tau^f, \tau^h)}{\tilde{L}^h(\tau^f, \tau^h)} = 0.$$

Solving this expression for τ^h, we can write

$$\tau^h = R^h(\tau^f),$$

which is the *reaction function* of the home country.

Figure 11.12: on the next slide displays the reaction functions of the home and foreign countries in the space (τ^h, τ^f) for the algebraic example studied in Section 11.5.

- The Nash equilibrium is at point A, where the two reaction functions intersect.

- In the Nash equilibrium $\tau^f = 0.18$ and $\tau^h = 0.30$.

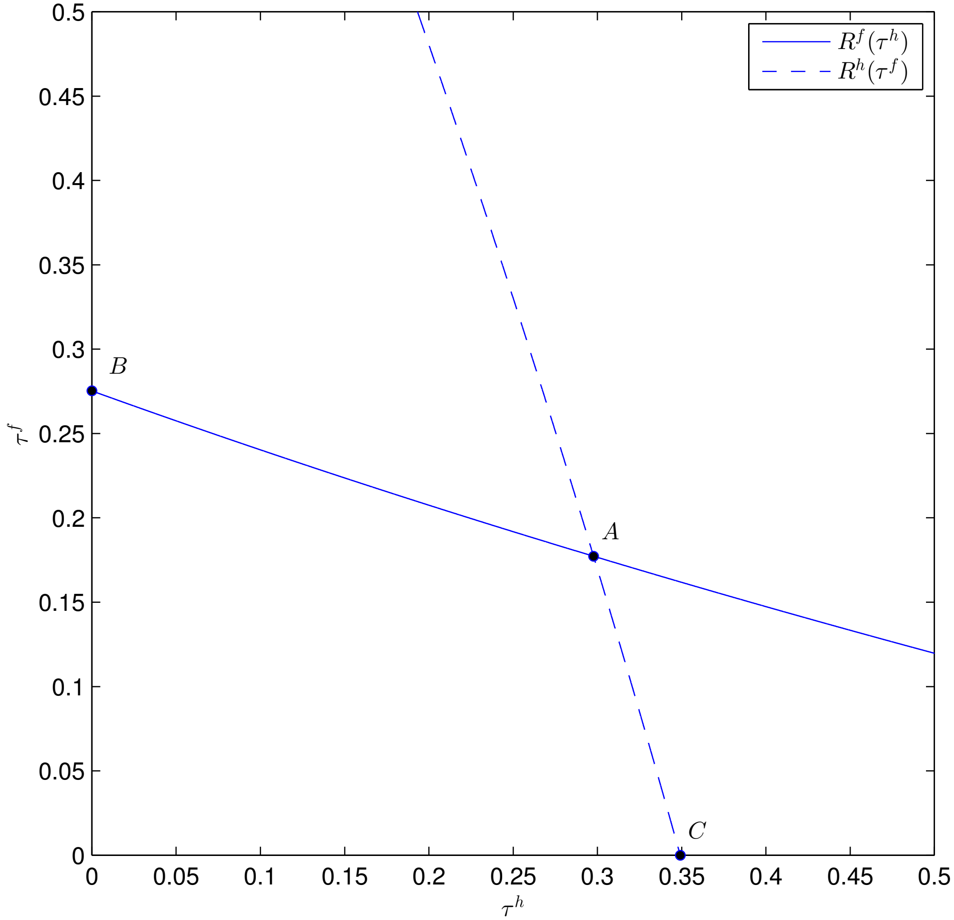
- The cross country interest rate differential, $i^f - i^h = \tau^f + \tau^h$, widens from 28 (=28+0) percent when the home country is passive to 48(=18+30) percent when the home country retaliates.

- Point B in the figure corresponds to the case in which the foreign country behaves strategically and the home is passive, which is the case we studied in Section . Comparing points A and B, we see that retaliation by the home country makes the foreign country lower its capital control tax rate (28 versus 18 percent).

- Point C in the figure corresponds to the case in which the home country behaves strategically and the foreign country is passive.

- Regardless of whether a country is borrowing or lending, retaliation by the other country lowers its own capital control taxes relative to the situation in which the other country is passive.

Figure 11.12: Capital Control Reaction Functions of the Home and Foreign Governments



The function $R^f(\tau^h)$ is the reaction function of the foreign country. It expresses the optimal capital control tax rate of the foreign country as a function of the tax rate of the home country. Similarly, $R^h(\tau^f)$ is the reaction function of the home country, representing the optimal tax rate in the home country as a function of the foreign country's tax rate. The intersection of the two reaction functions gives the Nash equilibrium capital control tax rates in the two countries. Replication file: `tauf_tauh_num.m` in `two_country.zip`.

Table 11.1: Comparison of Equilibria Under Alternative Capital Control Policies

					Welfare	
Policy	i^*	τ^f	τ^h	CA^f	f	h
Autarky	–	–	–	0	$\ln(0.5000Q^2)$	$\ln(1.0000Q^2)$
Free Capital Mobility	0.33	0	0	-0.125Q	$\ln(0.5208Q^2)$	$\ln(1.0208Q^2)$
Home Country Passive	0.22	0.28	0	-0.092Q	$\ln(0.5253Q^2)$	$\ln(1.0103Q^2)$
Foreign Country Passive	0.55	0	0.35	-0.073Q	$\ln(0.5082Q^2)$	$\ln(1.0318Q^2)$
Retaliation—Nash Eqm	0.45	0.18	0.30	-0.060Q	$\ln(0.5111Q^2)$	$\ln(1.0219Q^2)$

Comments:

- Not surprisingly, intertemporal trade, as measured by the absolute size of the current account, is the largest under free capital mobility and the smallest under optimal capital controls with Nash retaliation.
- Also not surprisingly, a country's welfare is the highest when it imposes optimal capital controls and the other country is passive.
- It is somewhat surprising, however, that the home country is better off under optimal capital controls with retaliation than under free capital mobility. Thus, it is optimal for the home country to impose capital controls regardless of whether this triggers a capital control war or not. This is not the case for the foreign country, which prefers free capital mobility to a capital control war.
- An interesting question is whether it pays for the foreign country to compensate the home country for abiding to free capital mobility.
- Finally, if one country imposes optimal capital controls unilaterally, it is in the interest of the other country to retaliate. To see this, note that in both countries welfare is higher under retaliation than in the absence thereof.

Summing Up

• Capital controls drive a wedge between the domestic interest rate and the world interest rate.

• If under free capital mobility, a small country borrows from the rest of the world, then the imposition of capital controls drives domestic interest rates up, depresses current consumption, and improves the current account.

• If under free capital mobility, a small country lends to the rest of the world, then the imposition of capital outflow controls lowers domestic interest rates, increases current consumption, and worsens the current account.

• In a small open economy without distortions, capital controls are always welfare decreasing.

Summing Up (continued)

• In the presence of borrowing externalities, capital controls can be welfare increasing, as they can be effective in eliminating overborrowing.

• In a two-country world, free capital mobility is in general preferred to financial autarky.

• In a two-country world, free capital mobility results in a Pareto optimal allocation, that is, any other feasible allocation makes at least one country worse off.

• For an economy that has market power in global financial markets it might be welfare improving to impose capital controls to move the world interest rate in its favor.

Summing Up (concluded)

• A large economy that runs a current account deficit benefits from imposing controls on capital inflows that drive the world interest rate down provided the rest of the world does not retaliate.

• A large economy that runs a current account surplus benefits from imposing controls on capital outflows that drive the world interest rate up provided the rest of the world does not retaliate.

• In a two-country world, the allocation under optimal capital controls fails to be Pareto efficient, that is, there is a feasible reallocation of resources that would make at least one country better off without making the other country worse off.

• In a two-country world, if one country imposes optimal capital controls unilaterally, it is in the interest of the other country to retaliate.

• In a two-country world, it may be welfare improving for one country to initiate a capital control war.