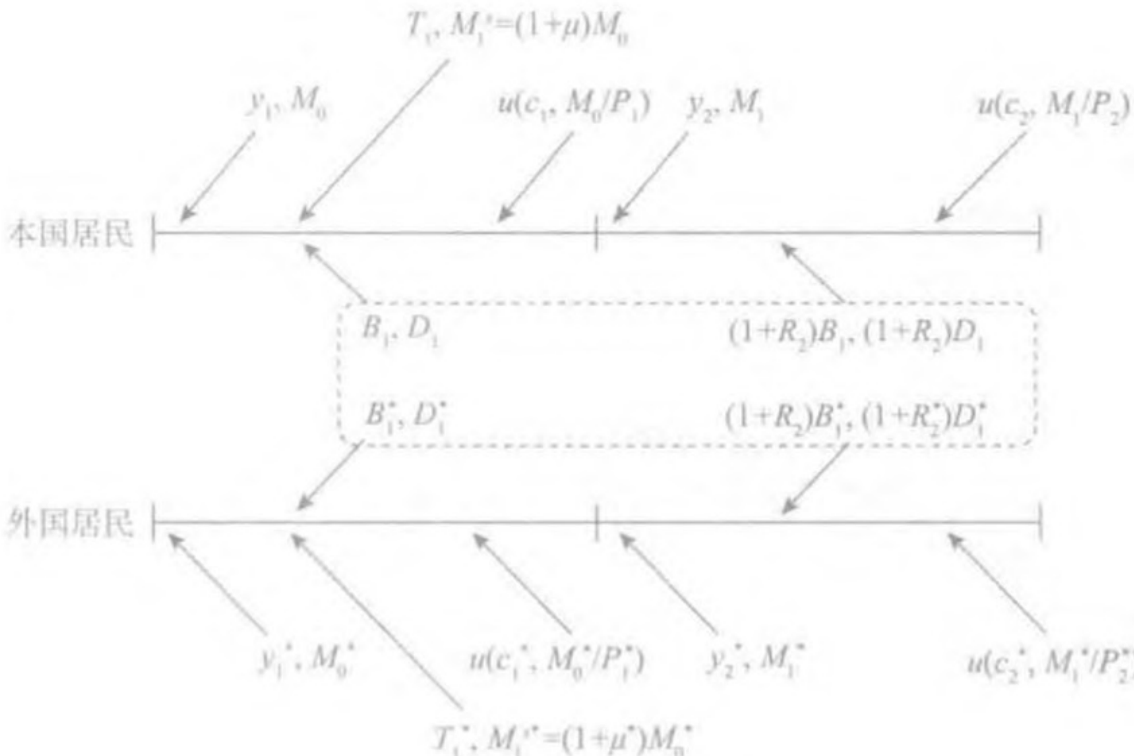
汇率决定的模型分析

#笔记



$$U=u(c_1)+v(M_0/P_1)+\beta[u(c_2)+v(M_1^d/P_2)]$$

$$P_1c_1+T_1+B_1+E_1D_1+M_1^d\leqslant M_0+P_1y_1$$

$$P_2c_2\leqslant M_1^d+P_2y_2+(1+R_2)B_1+E_2(1+R_2^*)D_1$$

$$c_1+t_1+b_1+d_1+(1+\pi_2)m_1^d\leqslant m_0+y_1$$

$$\frac{M_1^d}{P_1} = \frac{M_1^d}{P_2} \cdot \frac{P_2}{P_1} = (1+\pi_2)m_1^d$$

$$\frac{P_2 - P_1}{P_1} = \frac{P_2}{P_1} - 1 = \pi_2$$

$$\Rightarrow 1+\pi_2 = \frac{P_2}{P_1}$$

$$d_1 \equiv \frac{D_1}{P_1^*} = \frac{D_1 \cdot E_1}{P_1}$$

$$c_2 \leqslant m_1^d + y_2 + \frac{1+R_2}{1+\pi_2} b_1 + \frac{1+R_2^*}{1+\pi_2^*} d_1$$

$$\frac{E_2(1+R_2^*)D_1}{P_2} = \frac{(1+R_2^*)D_1}{P_2^*} = \frac{1+R_2^*}{P_2^*/P_1^*} \frac{D_1}{P_1^*} = \frac{1+R_2^*}{1+\pi_2^*} d_1$$

$$\begin{aligned}\mathcal{L} = {} & u(c_1) + v(m_0) + \beta[u(c_2) + v(m_1^d)] \\ & + \lambda_1 [m_0 + y_1 - c_1 - t_1 - b_1 - d_1 - (1+\pi_2)m_1^d] \\ & + \lambda_2 \left[m_1^d + y_2 + \frac{1+R_2}{1+\pi_2} b_1 + \frac{1+R_2^*}{1+\pi_2^*} d_1 - c_2 \right]\end{aligned}$$

$$\frac{\partial \mathcal{L}}{\partial c_1}=0 \Rightarrow u'(c_1)=\lambda_1$$

$$\frac{\partial \mathcal{L}}{\partial c_2}=0 \Rightarrow \beta u'(c_2)=\lambda_2$$

$$\frac{\partial \mathcal{L}}{\partial m_1^d}=0 \Rightarrow \beta v'(m_1^d)-\lambda_1(1+\pi_2)+\lambda_2=0$$

$$\frac{\partial \mathcal{L}}{\partial b_1}=0 \Rightarrow \lambda_1=\lambda_2\frac{1+R_2}{1+\pi_2}$$

$$\frac{\partial \mathcal{L}}{\partial d_1}=0 \Rightarrow \lambda_1=\lambda_2 \frac{1+R_2^*}{1+\pi_2^*}$$

消去其中的拉格朗日乘子可得

$$u'(c_1)=\frac{1}{1+\pi_2}\beta[u'(c_2)+v'(m_1^d)]$$

$$u'(c_1)=\beta u'(c_2)\left(\frac{1+R_2}{1+\pi_2}\right)$$

$$u'(c_1)=\beta u'(c_2)\left(\frac{1+R_2^*}{1+\pi_2^*}\right)$$

$$\frac{1+R_2}{1+\pi_2}=\frac{1+R_2^*}{1+\pi_2^*}$$

$$R_2-R_2^*=\pi_2-\pi_2^*$$

$$\frac{E_2 - E_1}{E_1} = R_2 - R_2^*$$