Dixit & Stiglitz (1977)

$$F(A,B) = (A^a + B^a)^{\frac{1}{a}}$$

① 家庭 $U = (\int_0^n q(w)^\rho dw)^{\frac{1}{\rho}}, \ 0<\rho<1$ (1)

$$L = U^\rho - \lambda(\int_0^n p(w)q(w)\,dw)$$

$$\frac{\partial L}{\partial q(w)} = \rho\, q(w)^{\rho-1} - \lambda p(w) = 0 \quad (2)$$

$$\Rightarrow q(w) = \left(\frac{\lambda p(w)}{\rho}\right)^{\frac{1}{\rho-1}} \quad (3)$$

取 $\begin{cases} q(w_1) = \left(\frac{\lambda p(w_1)}{\rho}\right)^{\frac{1}{\rho-1}} \\ q(w_2) = \left(\frac{\lambda p(w_2)}{\rho}\right)^{\frac{1}{\rho-1}} \end{cases}$

$$\Rightarrow \frac{q(w_1)}{q(w_2)} = \left(\frac{p(w_1)}{p(w_2)}\right)^{\frac{1}{\rho-1}} \quad (4)$$

记 $\delta = \frac{1}{1-\rho}$ 替代弹性 $\delta = \frac{-d\ln q(w_1)/q(w_2)}{-d\ln(q(w_2)/q(w_1))}$

$\rho = 1 - \frac{1}{\delta} = \frac{\delta-1}{\delta}$

$$q(w_1) = q(w_2)\left(\frac{p(w_1)}{p(w_2)}\right)^{-\delta}$$

同乘 $p(w_1)$ 并积分

$$\int_0^n p(w_1)\,q(w_1)\,dw = \int_0^n [q(w_2)\,p_{w_1}^{1-\delta}\,p(w_2)^\delta]\,dw_1$$

$$\Downarrow$$

$$I = q(w_2)\,p(w_2)^\delta \int_0^n p(w_1)^{1-\delta}\,dw_1$$

w_2的需求 $\Rightarrow q(w_2) = \frac{I\,p(w_2)^{-\delta}}{\int_0^n p(w)^{1-\delta}\,dw_1}$

定义价格指数：$P = (\int_0^n p(w)^{1-\delta}dw)^{\frac{1}{1-\delta}}$ (5)

需求：$q(w) = p(w)^{-\delta} P^{\delta-1} I = \left(\frac{p(w)}{P}\right)^{-\delta}\frac{I}{P}$ (5)

$\rho = \frac{\delta-1}{\delta}$

$$U = (\int_0^n q(w)^\rho dw)^{\frac{1}{\rho}}$$
$$= (\int_0^n p(w)^{1-\delta} I^\rho P^{(\delta-1)\rho} dw)^{\frac{1}{\rho}}$$
$$= I P^{\delta-1} (\int_0^n p(w)^{1-\delta} dw)^{\frac{\delta}{\delta-1}}$$
$$= I P^{\delta-1} P^{-\delta} = \frac{I}{P}$$

设对称：$I = \int_0^n pq\,dw = npq$

$$\Rightarrow q = \frac{I}{np}$$

$U = (\int_0^n q(w)^\rho dw)^{\frac{1}{\rho}} = (n(\frac{I}{np})^\rho)^{\frac{1}{\rho}} = (n^{\frac{1-\rho}{\rho}})\frac{I}{p} = n^{\frac{1}{\delta-1}}\frac{I}{p}$ 多样化偏好.

② 厂商：

垄断 劳动需求：$l(q) = f + cq$ (6)

$\pi = pq - wcq - wf$ 设定 p 最大化 (7)

$$\frac{\partial \pi}{\partial p} = q + (p - wc)\frac{\partial q}{\partial p} = 0.$$

$$p = wc + \frac{-q}{\frac{\partial q}{\partial p}} \quad (8)$$

$p = wc + \frac{p}{\delta} \Rightarrow \frac{wc}{(1-\frac{1}{\delta})} = p = \frac{wc}{\rho}$

(5) → $\frac{\partial q}{\partial p} = -\delta\, p^{-\delta-1} P^{\delta-1} I$ → $-q/\frac{\partial q}{\partial p} = \frac{-p^{-\delta}P^{\delta-1}I}{-\delta\, p^{-\delta-1}P^{\delta-1}I} = \frac{p}{\delta}$

③ 自由进入

$\pi = pq - wcq - wf = 0$

$=$

$\Rightarrow q = \frac{f}{c}(\delta-1)$